NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

MID - SEM EXAMINATION, 2022

SESSION: 2022 – 2023 (Autumn)

B.Tech. 3rd Semester

Subject Name: Discrete Structures **Subject code: CS2003** **Dept. Code: CS**

Full Marks: 30 Duration: 2 Hours

Answer ALL questions. Unnecessary and extra writing will attract negative marks.

1) Prove that $(P \to Q) \to (R \to S)$ and $(P \to R) \to (Q \to S)$ are not logically equivalent. **[2]**
2) Prove that $3+3\cdot5+3\cdot5^2+...+3\cdot5^n = 3(5^{n+1}-4)/4$ whenever n is a non-negative integer using mathematical induction theorem. **[3]**
3) Apply strong induction method to prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps. **[3]**
4) Consider these statements, and show that the first three are premises and the fourth is a valid conclusion.
   "All hummingbirds are richly colored."
   "No large birds live on honey."
   "Birds that do not live on honey are dull in color."
   "Hummingbirds are small." **[3]**
5) Use rules of inference to show that if $\forall x\ (P(x) \to (Q(x) \wedge S(x)))$ and $\forall x\ (P(x) \wedge R(x))$ are true, then $\forall x\ (R(x) \wedge S(x))$ is true. **[3]**
6) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using
   a) a proof by contraposition, and b) a proof by contradiction. **[3]**
7) Suppose that a and b are integers, $a \equiv 4(mod\ 13)$, and $b \equiv 9(mod\ 13)$. Find the integer c with $0 \leq c \leq 12$ such that
   i) $c \equiv 9a(mod\ 13)$.
   ii) $c \equiv 11b(mod\ 13)$.
   iii) $c \equiv a + b(mod\ 13)$.
   iv) $c \equiv 2a + 3b(mod\ 13)$.
   v) $c \equiv a^2 + b^2(mod\ 13)$.
   vi) $c \equiv a^3 - b^3(mod\ 13)$. **[3]**
8) Find the sum and the product of each of these pairs of numbers. Express your answers as a binary expansion.
   a) $(1000111)_2$, $(1110111)_2$,
   b) $(11101111)_2$, $(10111101)_2$,
   c) $(1010101010)_2$, $(111110000)_2$,
   d) $(1000000001)_2$, $(1111111111)_2$. **[3]**
9) Suppose that g is a function from A to B and f is a function from B to C. Show that if both f and g are one-to-one functions, then $f \cdot g$ is also one-to-one. **[2]**
10) Proof if a, b, and m are integers, $m \geq 2$ and $a \equiv b(mod\ m)$, then $gcd(a, m) = gcd(b, m)$. **[3]**
11) Use the construction in the proof of the Chinese remainder theorem to find all solutions to the system of congruences $x \equiv 1(mod\ 2)$, $x \equiv 2(mod\ 3)$, $x \equiv 3(mod\ 5)$, and $x \equiv 4(mod\ 11)$. **[2]**

************************** **ALL THE BEST** ******************************