NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

Mid Semester Examination, Autumn 2019

Subject: Discrete Structures **Subject Code: CS2003** **Full Marks: 30**

Each question carries 3 marks. Mere answer without justification will not fetch any mark.

1. Let $f_n = f_{n-1} + f_{n-2}$, $f_0 = 0$, $f_1 = 1$. Let A denote the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Let A^n denote $A \times A \times \ldots \times A$ (n times), where $\times$ is matrix multiplication. Prove that

$$A^{n-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} f_n \\ f_{n-1} \end{bmatrix}.$$

2. How many numbers should be chosen from a set $S = \{1, \ldots, n\}$ such that there exists a pair of numbers whose sum is $\frac{n}{2}$.

3. How many solutions are there to the inequality $x_1 + x_2 + x_3 \leq 11$, where x_1, x_2, and x_3 are nonnegative integers?

4. A bit string consisting of 0's and 1's is a palindrome if its reversal is same as the original string. For instance, 000111 is not a palindrome where as 1010101 and 101101 are palindromes. How many bit strings of length n are palindromes.

5. There is someone in the government who is not working properly. Everyone in the government is either a bureaucrat or a politician. Every bureaucrat is working properly. Prove, using Predicate logic and rules of inferences, that some politician is not working properly. Please use the following predicates.
   - $G(x)$: x is in the government.
   - $W(x)$: x is working properly.
   - $P(x)$: x is a politician.
   - $B(x)$: x is a bureaucrat.

6. Let p, q and r be propositions. Consider the following argument. The premise/hypothesis of the argument are $p \vee q$, $q \vee r$ and $r \vee p$. The conclusion of the argument is $p \wedge q \wedge r$. Is the argument valid?

7. Let f be an one to one function from set A to set B. Let X and Y be two subsets of A. Prove that $f(X \cap Y) = f(X) \cap f(Y)$.

8. Let A, B, C and D be sets which are subsets of an universal set U . Prove/disprove that $(A - B) - (C - D) = (A - C) - (B - D)$, where $-$ denotes the set minus operation.

9. Let a, b and c be integers. Consider the following propositions.
   - P: $a + b + c$ is a odd integer.
   - Q: a, b and c are all odd integers.
   - R: abc is a odd integer.

   Check whether P, Q and R are equivalent (P is equivalent to Q if $P \rightarrow Q$ and $Q \rightarrow P$).

10. Show that if A and B are countable sets, then $A \times B$ is also a countable set (Set A is countable if there is an injective mapping from set A to the set of positive integers).

11. Suppose that five ones and four zeros are arranged around a circle. Between any two equal bits you insert a 0 and between any two unequal bits you insert a 1 to produce nine new bits. Then you erase the nine original bits. Show that when you iterate this procedure, you can never get nine zeros.

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