Sample Question for CS2003

Each Question has 2.5 marks

The solutions of these questions are in the Kenneth Rosen Book, page numbers are mentioned in the end of each question.

1) What are the negations of the statements $\forall x\ (x^2 > x)$ and $\exists x\ (x^2 = 2)$? [47]
2) Express the statement "Everyone has exactly one best friend" as a logical expression involving predicates, quantifiers with a domain consisting of all people, and logical connectives. [62]
3) Show that if x and y are integers and both xy and x + y are even, then both x and y are even. [95]
4) Express gcd(252, 198) = 18 as a linear combination of 252 and 198. [270]
5) Find 7^{222} mod 11. [281]
6) Use mathematical induction to prove that $2^n < n!$ for every integer n with $n \geq 4$. (Note that this inequality is false for n = 1, 2, and 3.) [320]
7) How many bit strings of length four do not have two consecutive 1s? [395]
8) How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards? [411]
9) If X and Y are independent random variables on a sample space S, then E(XY) = E(X)E(Y). [486]
10) If $H_n = 2H_{n-1} + 1$ then show that $H_n = 2^n - 1$. [504]
11) What is the solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with a_0 = 2 and a_1 = 7? [516]
12) Use generating functions to find the number of ways to select r objects of n different kinds if we must select at least one object of each kind. [545]
13) Suppose that there are 1807 freshmen at your school. Of these, 453 are taking a course in computer science, 567 are taking a course in mathematics, and 299 are taking courses in both computer science and mathematics. Howmany are not taking a course either in computer science or in mathematics? [554]
14) What is the composite of the relations R and S, where R is the relation from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$ with R = $\{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$ and S is the relation from $\{1, 2, 3, 4\}$ to $\{0, 1, 2\}$ with $S = \{(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)\}$? [580]
15) Let R be the relation on the set of real numbers such that $x\ R\ y$ if and only if x and y are real numbers that differ by less than 1, that is $|x - y|\ <\ 1$. Show that R is not an equivalence relation. [610]
16) What are the sets in the partition of the integers arising from congruence modulo 4? [614]
17) Which elements of the poset $(\{2, 4, 5, 10, 12, 20, 25\}, |)$ are maximal, and which are minimal? [624]
18) Show that an undirected graph has an even number of vertices of odd degree. [653]
19) Proof: A connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree. [696]
20) Let G be a connected planar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then r = e - v + 2. [720]

******* This is a snapshot of the entire syllabus covered during this semester, focusing on the later part of mid-sem *****