NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA
Mid Semester Examination, 2018

Subject: Discrete Structures No. of pages: 2 Full Marks: **60**
Subject Code: CS2003

ANSWER ANY THREE QUESTIONS.
Mere answer without justification will not fetch any mark.

Q.No.	Particulars	Marks
1.	(a) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology (Tautology is a proposition whose truth value is always true). (b) Compute the statement that is equivalent to solving a 2*2 sudoko puzzle (Hint: let p(i,j,k) denote the proposition that the (i,j) cell of the puzzle contains the number k). (c) Let P(x), Q(x) and R(x) denote the statements that “x is a professor”, “x is ignorant”, and “x is vain”, respectively. The domain consists of all people. Express the statements below using quantifiers. 1. No professor is ignorant. 2. All ignorant people are vain. 3. No professor is vain. 4. Does 3 follow from 1 and 2. (d) Show that the premises “A student in this class has not read the book,” and “Everyone in this class passed the first exam” imply the conclusion “Someone who passed the first exam has not read the book.” Use C(x), B(x) and P(x) to denote the statements “x is in class”, “x has read the book”, and “x has passed the exam” respectively.	5×4=20
2.	(a) Prove that $1+1/2+1/3+1/4+...+1/n$ is $\Theta(\log n)$. (b) If A and B are countable sets, then $A \cup B$ is also countable. (c) The halting problem asks that given a program P and an input to the program I, whether P halts on execution of I as input. Prove that halting problem is unsolvable. (d) Find the inverses of the functions below if they exist. 1. $f(x)=\text{Sin}(x)$. 2. $f(x)=x^2$, $x>0$. 3. $f(x)=(x+4)/(3x-2)$ 4. $f(x)=\lvert x\rvert$, where $\lvert x\rvert = x$ if x is nonnegative and $-x$ if x is negative. 5. $f(x)=x^3$.	5×4=20

Q.No.	Particulars	Marks

3.	(a) Let A be an m*k matrix and B be an k*n matrix. Then, determine the time complexity of the following procedure, assuming additions and multiplications take O(1) time.
	Procedure matrix operation(A,B: matrices)
	for i:=1 to m
	    for j:=1 to n
	        $\mathbf{c_{ij} = 0}$
	        for r:=1 to k
	            $\mathbf{c_{ij} = c_{ij} + a_{ir}\, b_{rn}}$
	return C{C=[$\mathbf{c_{ij}}$] is the output m*n matrix }
	
	(b) Prove that if a=b*q+r, then gcd(a,b)=gcd(q,r). Given that gcd(1000,15)=5,write 5 as a linear combination of 1000 and 15.
	
	(c) If p is a prime and a is an positive integer not divisible by p, then $a^{p-1} \cong 1$ (mod p). This is Fermats little theorem. Prove this using induction and binomial theorem (Hint: expand (1+a)^p and look termwise).
	
	(d) A primitive root modulo p is a integer r in Z_p (Z_p = {0,1,...,p}) such that every nonzero element of Z_p is a power of r.
	Suppose that p is a prime, r is a primitive root modulo p, and a is an integer between 1 and p − 1 inclusive. If r^e mod p = a and $0 \leq e \leq p - 1$, we say that e is the discrete logarithm of a modulo p to the base r and we write $\log_r a = e$ (where the prime p is understood). Find the discrete logarithm of 3 and 5 modulo 11 base 2. (Note that 2 is a primitive root).
		5×4=20

4.	(a) Prove that if n is an composite integer, then it has a prime divisor that is at most $\sqrt{n}$. (b) The prime number theorem states that the number of primes not exceeding n is approximately equal to $n/\ln n$. Use this fact to prove that there are infinitely many primes. (c) Prove that $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n$ (d) A standard deck of 52 cards to guarantee that at least 3 cards of the same symbol (ace, king, queen, jack, 2, 3, … or 10) are chosen?	5×4=20