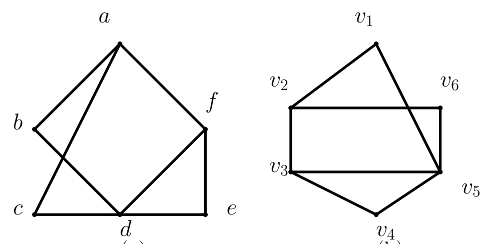
NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

End Semester Examination, Autumn 2019

Subject: Discrete Structures **Subject Code: CS2003** **Full Marks: 100**

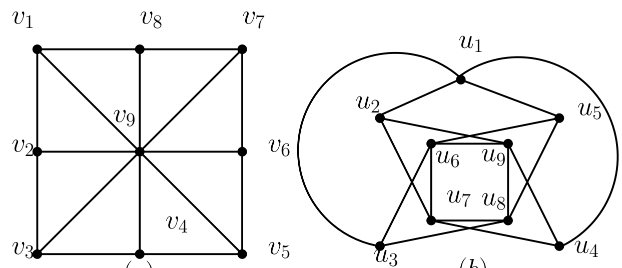
Answer all questions. Mere answer without justification will not fetch any mark.

1. (a) Determine whether the graphs given in the following figure are isomorphic or not. Please give the details of the Isomorphism if exists. 5*4=20



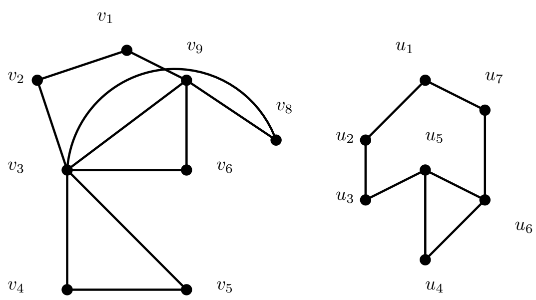
(a) (b)

(b) Determine the chromatic number of the graphs given in the following figure with detailed explanation.



(a) (b)

(c) Determine whether the graphs given in the following figure have Euler cycles, Euler paths or Hamiltonian cycles with detailed explanation.



(a) (b)

(d) For any connected bipartite planar graph on $\mathbf{n}$ vertices and $\mathbf{e}$ edges, prove that $\mathbf{e} \leq \mathbf{2n} - \mathbf{4}$ (Hint: use the fact that for connected planar graphs on $\mathbf{n}$ vertices, $\mathbf{f}$ faces and $\mathbf{e}$ edges, $\mathbf{n} - \mathbf{e} + \mathbf{f} = \mathbf{2}$).

2. (a) A DNA sequence consists of a string of four bases, namely, A, T, G and C. How many distinct DNA sequences of length $\mathbf{n}$ are possible with 10+5+5=20
(i) only A's and T's but no G's and C's.
(ii) exactly 2 G's.
(iii) at least $\mathbf{n/2}$ C's.
(iv) number of A's + number of C's=number of T's + number of G's.

(b) Let total number of ways a board of $3 \times n$ dominos can be filled with 3×1 tiles be denoted as f_n. Find a recurrence relation for f_n. Find the initial conditions and find a closed form formula.

(c) Prove the identity $\binom{\mathbf{n}}{\mathbf{r}}\binom{\mathbf{r}}{\mathbf{k}} = \binom{\mathbf{n}}{\mathbf{k}}\binom{\mathbf{n-k}}{\mathbf{r-k}}$, whenever n, r, and k are nonnegative integers with $r \leq k \leq n$, (i) using a combinatorial argument; (ii) using an argument based on the formula for the number of r-combinations of a set with n elements.

3. (a) Show that a regular bipartite graph G with the two independent partite sets L and R has $|L| = |R|$. 5*4=20

   (b) Prove that any graph where every vertex has degree at least 2 has a cycle.

   (c) Let Δ and δ be two partial order relations on a set A and σ be the intersection of Δ and δ. Is σ a partial order relation?

   (d) Let R be a binary relation on the set of all strings of $0's$ and $1's$ where aRb if a and b have the same number of 0's. Is R and equivalance relation/partial order relation?

4. (a) Answer these questions for the poset 10+5+5=20
   $(\{\{\mathbf{1}\}, \{\mathbf{2}\}, \{\mathbf{4}\}, \{\mathbf{1}, \mathbf{2}\}, \{\mathbf{1}, \mathbf{4}\}, \{\mathbf{2}, \mathbf{4}\}, \{\mathbf{3}, \mathbf{4}\}, \{\mathbf{1}, \mathbf{3}, \mathbf{4}\}, \{\mathbf{2}, \mathbf{3}, \mathbf{4}\}\}, \subseteq)$.
   (i) Find the maximal elements.
   (ii) Find the minimal elements.
   (iii) Is there a greatest element?
   (iv) Is there a least element?
   (v) Find all upper bounds of $\{\{\mathbf{2}\}, \{\mathbf{4}\}\}$.
   (vi) Find the least upper bound of $\{\{\mathbf{2}\}, \{\mathbf{4}\}\}$, if it exists.
   (vii) Find all lower bounds of $\{\{\mathbf{1}, \mathbf{3}, \mathbf{4}\}, \{\mathbf{2}, \mathbf{3}, \mathbf{4}\}\}$.
   (viii) Find the greatest lower bound of $\{\{\mathbf{1}, \mathbf{3}, \mathbf{4}\}, \{\mathbf{2}, \mathbf{3}, \mathbf{4}\}\}$, if it exists.
   (ix) Is this poset a lattice?
   (x) Let (S, R) be a poset which is a lattice and $A \subseteq S$. Is (A, R) always a lattice?

   (b) How many ways are there to assign m employess n jobs such that every job is performed by at least one employee and every employee receives exactly one job. How many such assignments exists if we have 5 employees and 4 jobs.

   (c) Prove the following statement, where $A_1, \ldots, A_n$ are finite sets.

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = \sum_{1 \le i \le n} |A_i| - \sum_{1 \le i < j \le n} |A_i \cap A_j| + \sum_{1 \le i < j < k \le n} |A_i \cap A_j \cap A_k| - \ldots$$
$$(-1)^{n+1} |A_1 \cap \ldots \cap A_n|.$$

5. (a) Show that the premise "A student in the class has not read the book" and "Everyone in the class passed the exam" imply the conclusion "Someone who has passed the exam has not read the book". Let $C(x)$, $B(x)$ and $P(x)$ denote that x is in class, x has read the book and x passed the exam, respectively. 4*5=20

   (b) Show that $\exists \mathbf{x} \mathbf{P}(\mathbf{x}) \wedge \exists \mathbf{x} \mathbf{Q}(\mathbf{x})$ and $\exists \mathbf{x}(\mathbf{P}(\mathbf{x}) \wedge \mathbf{Q}(\mathbf{x}))$ are not logically equivalent.

   (c) Use mathematical induction to prove that $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$.

   (d) How many one to one functions are there from a set of m elements to a set of n elements.

   (e) How many different cards must be chosen from a standard deck of 52 cards such that at least three cards of the same suit are chosen. How many cards must be selected such that at least three hearts are chosen. (A standard deck of cards consists of 52 cards with 13 cards each of the four suits, the cards are numbered $1, 2, \ldots, 10, J, Q, K$)

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