National Institute of Technology Rourkela
Department of Mathematics
(Backlog)End-Semester Examination:(2020-2021)

Sub. Code: MA 2203 Introduction to Probability & Statistics Dept. Code: MA
No of pages: 1 Full Marks: 50 Duration: 2 Hours

- Answer all questions.
- All parts of a question should be answered at one place.

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1. A bag contains 5 white and 2 black balls and balls are drawn one by one with replacement. What is the probability of drawing the second white ball before the second black ball? [5]

2. A continuous random variable X has the probability density function,

$$f(x) = a + bx, \quad 0 \leq x \leq 1.$$

If the mean of the distribution is $\frac{1}{2}$, find a and b. [5]

3. Suppose that two-dimensional continuous random variable (X, Y) has joint pdf

$$\begin{aligned} f(x,y) &= kx^2y, \quad 0 < x < 1, 0 < y < 1 \\ &= 0, \quad \text{otherwise.} \end{aligned}$$

Find the value of k. Further find the (a) $P(0 < X < 3/4, 1/3 < Y < 2)$ (b) $P(X > Y)$. [5]

4. Determine a 99% confidence interval for the mean of a normal population with standard deviation 2.5, using the sample 30.8, 30.0, 29.9, 30.1, 31.7, 34.0. [5]

5. Find the sample regression line of Y on X using the samples $(2, 12)$, $(5, 24)$, $(9, 33)$, $(14, 50)$. Further estimate the value of Y when the value of $X = 10$. [5]

6. Find the maximum likelihood estimator for μ and σ in normal distribution $N(\mu, \sigma^2)$. What is the MLEs of μ and σ using the samples 12, 15, 18, 20, 25, -24, -14, -12 from $N(\mu, \sigma^2)$. [5]

7. Two dice are thrown simultaneously. let X be the maximum of the numbers that the two dice show. Is this X a random variable? if yes find the cumulative distribution function of X. [5]

8. If X is a Poisson random variable such that $\frac{3}{2}P(X = 1) = P(X = 3)$. Find (i) $P(X \geq 1)$, (ii) $P(X \leq 3)$, (iii) $P(2 \leq X \leq 5)$. [5]

9. Discuss the similarities and/or dissimilarities between the hyper-geometric and the binomial distributions. Further derive the mean and variance of the hyper geometric distribution. [5]

10. The random variables X and Y are independent if and only if the correlation coefficient between them vanishes. Is the statement true? Justify your answer with example/counterexample. [5]

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