National Institute of Technology Rourkela
Department of Mathematics
Mid-Semester Examination-2018-2019

Sub. Code: MA 2203	Sub. Name: Intro. to Prob. & Stat.	Dept. Code: MA
No of pages: 1	Full Marks: 30	Duration: 2 Hours

- Answer all questions.
- All parts of a question should be answered at one place.

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1. A bag contains 5 white and 2 black balls and balls are drawn one by one without replacement. What is the probability of drawing the second white ball before the second black ball? [3]

2. Let $A_1, A_2, \ldots, A_n$ be n arbitrary events. Then prove that, [3]
$$P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i).$$

3. State the difference between mutually independent and pairwise independent events. Let A and B be two independent events. Prove that A^c and B^c are also independent. [3]

4. Define moment generating function of a random variable X. Find the moment generating function of the random variable X whose k^{th} order moments about origin are given by $(k+1)!2^k$. [3]

5. Let X be a binomial random variable with mean 4 and variance 3. Find the skewness of the random variable X. [3]

6. Throw a die once. Let S be the sample space of all possible outcomes. Suppose X is the random variable which denotes the face value, that is $X : S \to \mathbb{R}$, defined by $X(1) = 1$; $X(2) = 2$; $X(3) = 3$; $X(4) = 4$; $X(5) = 5$; $X(6) = 6$. Then find the cumulative distribution function $F(x)$ of X. [3]

7. Does the function [3]
$$f(x) = \begin{cases} \frac{(1+x)e^{-\frac{x}{\theta}}}{\theta(1+\theta)}, & \text{if } x > 0 \\ 0, & \text{otherwise,} \end{cases}$$
where $\theta > 0$ is a constant, define a PDF? If yes justify and find the CDF.

8. If X is a Poisson random variable such that, $\frac{3}{2}P(X = 1) = P(X = 3)$. Find (i) $P(X \geq 1)$ (ii) $P(X \leq 3)$ (iii) $P(2 \leq X \leq 5)$. [3]

9. A continuous random variable X has a probability density function, [3]
$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$
Find a and b such that, (i) $P(X \leq a) = P(X > a)$ and (ii) $P(X > b) = 0 \cdot 05$.

10. The marks obtained in Mathematics in a certain examination found to be normally distributed. If 15% of the students have secured 60 marks or more and 40% have got less than 30 marks, then find the mean mark and the standard deviation of the distribution. [3]

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