



**NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA**

**B. Tech. End-Semester Examination - 2019**

**(THIRD SEMESTER)**

**Subject Name: Introduction to Probability & Statistics**

**Subject Code : MA-2203**

**Full Marks : 50**

**Duration of Examination : 3 hours**

*All questions carry equal marks. Answer all the ten questions.*

*Use statistical tables wherever necessary.*

1. The probability density function of a continuous random variable  $X$  is  $f(x) = \alpha e^{-\beta x}$  if  $x > 0$  and  $f(x) = 0$  otherwise ( $\alpha, \beta > 0$ ). If the mean of  $X$  is 2, find the values of  $\alpha, \beta$  and hence find the variance of  $X$ .
2. A fair coin is tossed 100 times and  $X$  heads are obtained. Using normal approximation, find  $P(48 \leq X \leq 55)$ .
3. What are the mean thickness and standard deviation of transformer cores each consisting of 100 layers of metal sheets and 99 insulating paper layers if the metal sheets have mean thickness 0.7mm and standard deviation 0.07mm and the paper layers have mean thickness 0.1mm and standard deviation 0.03 mm?
4. Let  $(X, Y)$  have the joint pdf  $f(x, y) = k$  if  $x > 0, y > 0$  and  $x + y < 5$  and  $f(x, y) = 0$  otherwise. Find  $k$ ,  $P(X + Y < 2)$  and  $P(X < 2.5, Y < 2.5)$ .
5. Let  $X$  and  $Y$  be continuous random variables with joint pdf  $f(x, y) = x + y$  if  $0 < x < 1, 0 < y < 1$  and  $f(x, y) = 0$  otherwise. Find the conditional mean and conditional variance of  $X$  given  $Y = 0.25$ .
6. The heights of 10 randomly chosen students of NIT Rourkela are given by 61, 66, 64, 71, 70, 60, 59, 66, 65 and 62 inches respectively. Assuming a normal distribution of height, find 99% confidence intervals for the population mean height  $\mu$  and variance  $\sigma^2$ .
7. The prices of 10 different electronic items of two brands are given below:

Brand I (in ₹)	220	180	350	750	240	130	390	410	660	270
Brand II (in ₹)	200	160	330	730	220	110	370	390	640	250

Find the two lines of regression. Estimate the price of a particular Brand I item whose corresponding Brand II price is ₹ 450.

8. Using the method of moments, estimate the parameters  $\alpha$  and  $\beta$  of the uniform distribution  $f(x) = \frac{1}{\beta - \alpha}$ , if  $\alpha < x < \beta$  and  $f(x) = 0$  otherwise, using the random sample  $x_1, x_2, \dots, x_n$ .
9. In 100 tosses of a coin, what would be the minimum number of heads leading to the acceptance of the null hypothesis that the coin is fair at 5% level of significance?
10. The CGPA of 10 students after first and second semesters are given in the following table:

CGPA 1 <sup>st</sup> Semester	8.7	7.1	6.9	6.0	7.5	6.5	7.0	6.8	9.1	7.7
CGPA 2 <sup>nd</sup> Semester	9.0	6.6	7.1	6.8	8.1	7.5	6.9	7.3	8.8	7.6

Find the rank correlation coefficient between their CGPA of both semesters.

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