National Institute of Technology Rourkela
Department of Mathematics
End-Semester Examination: 2021-2022 (Autumn)

Code: MA2203	Name: Intro. to Prob. & Stat.	Dept. Code: MA
Pages: 2	Full Marks: 30	Duration: 2 Hours

- Answer all questions within two pages.
- All parts of a question should be answered at one place.

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1. From 6 positive and 8 negative numbers, 4 numbers are chosen at random (without replacement) and multiplied. What is the probability that the product is positive? [3]

2. Cards are dealt one by one from a well shuffled pack of 52 cards until an ace appears. Find the probability that exactly n cards are dealt before the ace appears. [3]

3. A, B and C play a game and the chances of their winning it in an attempt are 2/3, 1/2 and 1/4 respectively. A has the first chance, followed by B and then C. This cycle is repeated till one of them wins the game. What is the probability that B wins the game? [3]

4. The outcomes of an experiment are represented by the points in the square bounded by $x = 0$, $y = 0$, $x = a$, $y = a$ in the $xy-$plane. If the probability is distributed uniformly, determine the probability that $x^2 + y^2 > a^2$, $a > 0$. [3]

5. A box contains 10 LED bulbs, out of which 5 are 20 Watt each and another 5 are 17 Watt each. 4 bulbs are drawn from this box and put into another box. From this second box a bulb is drawn and is found to be of 20 Watt. What is the probability of drawing a 20 Watt bulb again in the next draw? (The first 20 Watt bulb drawn is not replaced.) [3]

6. Let X be a continuous random variable with probability density function [3]

$$f(x) = \begin{cases} \frac{x}{2}, & \text{if } 0 \le x \le 1, \\ \frac{1}{2}, & \text{if } 1 \le x \le 2, \\ -\frac{x}{2} + \frac{3}{2}, & \text{if } 2 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

Find the cumulative distribution function $F(x)$. Also, find the values of a and b such that $P(X < a) = 1/4$, and $P(a < X \le b) = 1/2$. [3]

7. Let X be a discrete random variable with the probability distribution $P(X = -3) = \frac{1}{6}$, $P(X = 6) = \frac{1}{2}$, $P(X = 9) = \frac{1}{3}$. Find the mean and variance of $Y = 2X + 1$. [3]

8. Suppose that Y is a random variable with MGF $H(t)$. Further, suppose that X is also a random variable with MGF $M(t)$ which is given by,

$$M(t) = \frac{1}{3}(2e^{3t} + 1)H(t).$$

Given that the mean and variance of the random variable Y are 10 and 12, respectively, then find the mean and variance of the random variable X. [3]

9. A distributor of bean seeds determine from extensive tests that 2% of a large batch of seeds will not germinate. He sales seeds in packets of 200 and guarantees 95% germination. Use Poison distribution to determine the probability that a particular packet will violate the guarantee. [3]

10. The breaking strength X(kg) of a certain type of plastic block is normally distributed with a mean of 1500 kg and a standard deviation of 50 kg. What is the maximum load such that we can expect no more than 5% of the blocks to break? [3]

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