National Institute of Technology Rourkela
Department of Mathematics
End-Semester Examination: 2021-2022 (Autumn)

Code: MA2203	Name: Intro. to Prob. & Stat.	Dept. Code: MA
Pages: 2	Full Marks: 50	Duration: 2 Hours

- Answer all questions within three (3) pages, sequentially.
- All parts of a question should be answered at one place.

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1. Out of $(2n+1)$ tickets consecutively numbered, three are drawn at random. Find the probability that the numbers on them are in arithmetic progression (A.P). [5]

2. Let X be a continuous random variable with probability density given by

$$f(x) = \begin{cases} \frac{x}{2}, & \text{if } 0 \leq x < 1, \\ \frac{1}{2}, & \text{if } 1 \leq x < 2, \\ -\frac{x}{2} + \frac{3}{2}, & \text{if } 2 \leq x < 3, \\ 0, & \text{elsewhere.} \end{cases}$$

If x_1, x_2 and x_3 are three independent observations from X, what is the probability that exactly one of these three numbers is larger than 1.5? [5]

3. If X is a Poisson random variable that satisfies $P(X=2) = 9P(X=4) + 90P(X=6)$, then find the skewness of X. [5]

4. A two dimensional random variable (X, Y) has the joint probability mass function given by

$$f(x,y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}, \ y = 0, 1, 2, \ldots, x; \ x = 0, 1, 2, \ldots,$$

where λ and p are constants with $\lambda > 0$, $0 < p < 1$. Find the marginal probability mass functions of X and Y. [5]

5. A two dimensional random variable (X, Y) has the joint density function given by

$$f(x,y) = \begin{cases} \frac{1}{8}(6 - x - y), & \text{if } 0 \leq x < 2, \ 2 \leq y < 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the probabilities, (i) $P(X < 1)$, and (ii) $P(Y < 3)$. [5]

6. Suppose that X is a discrete random variable with the probability mass function, $P(X=0) = \frac{2\theta}{3}$, $P(X=1) = \frac{\theta}{3}$, $P(X=2) = \frac{2(1-\theta)}{3}$, $P(X=3) = \frac{(1-\theta)}{3}$, where $0 \leq \theta \leq 1$ is a parameter. Find the maximum likelihood estimator of θ, based on a random sample 3, 0, 2, 1, 3, 2, 1, 0, 2, 1 which is taken from this distribution. [5]

7. Obtain 99% confidence intervals for the mean and variance of a normal distribution using the sample values 30.8, 30.0, 29.9, 30.1, 31.7, 34.0. [5]

8. Suppose you play a game that involves throwing three dice in succession of trials. Your winning are directly proportional to the number of fours recorded. Suppose, the number of trials is 100. Then using a chi-square test of goodness of fit, based on the following observed data conclude whether it is likely that the dice have been fairly weighted. (take $\alpha = 0.05$.). [5]

Number of Fours	0	1	2	3
Observed Count	47	35	15	3

9. The following dataset is available to us based on elastic band experiment. [5]

Mass (g) (X)	50	100	150	200	250	300	350	400
Lengths (mm) (Y)	37	48	60	71	80	90	102	109

(i) Obtain the correlation coefficient between Mass and Length, (ii) Obtain the regression line of Y on X.

10. Suppose we have ranks of 5 students in three subjects: Computer, Physics and Statistics. Using rank correlation coefficient determine which pair of subjects are highly correlated. [5]

Rank in Computer (R_1)	2	4	5	1	3
Rank in Physics (R_2)	5	1	2	3	4
Rank in Statistics (R_3)	2	3	5	4	1

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