National Institute of Technology Rourkela
Department of Mathematics
End-Semester Examination: 2022-2023 (Autumn)

Code: MA2203	Name: Intro. to Prob. & Stat.	Dept. Code: MA
Pages: 1	Full Marks: 30	Duration: 2 Hours

- Answer all questions.
- All parts of a question should be answered at one place.

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1. Prove the following statements. (i) If A and B are two events in an experiment, such that, $A \subset B$, then $P(B - A) = P(B)$–$P(A)$. (ii) If A, B and C are mutually independent events then $A \cup B$ and C are also independent. [3]

2. Let the matrix $M = \begin{pmatrix} x & y \\ z & 1 \end{pmatrix}$. If x, y, and z are chosen at random from the set $\{1, 2, 3\}$, and repetition is allowed, what is the probability that the given matrix M is a singular? [3]

3. An urn contains 5 white and 5 black balls, 4 balls are drawn from this urn and put into another urn. From this second urn a ball is drawn and is found to be white. What is the probability of drawing a white ball again at the next draw. (The first white ball drawn is not replaced.) [3]

4. Two dice are thrown and let X be the sum of the face values on them. Is X a random variable? If yes, justify and find the mean and the variance of X. [3]

5. A discrete random variable X has the mean 6 and variance 2. If it is assumed that the distribution is binomial then find the probability that $5 \leq X \leq 7$. [3]

6. Let X be a continuous random variable with probability density function [3]

$$f(x) = \begin{cases} k(1 - x^2), & \text{if } 0 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

   (i) Find the value of k. (ii) Find the mean and variance. (iii) Find $P(0.1 < X < 0.2)$. [3]

7. Suppose X is a random variable with moment generating function $M_X(t) = \frac{3e^{2t}}{(3-t)}$, $t < 3$. Find the skewness of the random variable X. [3]

8. If the chance that any of the 10 telephone lines is busy at an instant is 0.2. What is the most probable number of busy lines and what is the probability of this number. [3]

9. In a distribution exactly normal, 10.03% of the items are under 25 kilogram weight and 89.97% of the items are under 70 kilogram weight. What are the mean and standard deviation of the distribution? [3]

10. If $X \sim N(\mu, \sigma^2)$, find the moment generating function of X. Hence find the kurtosis of the distribution. [3]

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