Probability And Statistics

① Learn defⁿ of probability (different versions)

② For mutually exclusive events $A_1, A_2, A_3 \ldots A_n$

$$P\left(\bigcup_{n=1}^{n} A_i\right) = \sum_{i=1}^{n} A_i$$

③ Boole's Inequality

$$\rightarrow P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$

$$\rightarrow P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

Prove using PMI

④ Bonferronio's Inequality

$$\Sigma P(A_i) \geq P\left(\bigcup_{i=1}^{n} A_i\right) \geq \Sigma P(A_i) - \Sigma P(A_i \cap A_j)$$

⑤ Multiplication Theorem for n events

$$P(A_1 \cap A_2 \cap A_3 \ldots \cap A_n) = P(A_1) P\left(\frac{A_2}{A_1}\right) P\left(\frac{A_3}{A_1 \cap A_2}\right) P\left(\frac{A_4}{A_1 \cap A_2 \cap A_3}\right) \ldots P\left(\frac{A_n}{A_1 \cap A_2 \ldots \cap A_{n-1}}\right)$$

⑥ Independence of events

⑦ Baye's Theorem

$$P(E_i/A) = \frac{P(E_i) P\left(\frac{A}{E_i}\right)}{\Sigma P(E_i) P(A/E_i)}$$

⑧ $P(a < x \leq b) = P(b) - P(a)$

$\hookrightarrow$ CDF

⑨ $\frac{d(F(x))}{dx} = f(x)$

$\hookrightarrow$ PMF/pdf

⑩ Mean of R V X

$$\mu = E(X) = \begin{cases} \sum_{-\infty}^{\infty} x P(x = x_i) & \text{[discrete]} \\ \int_{-\infty}^{\infty} x f(x)\, dx & \text{[continuous]} \end{cases}$$

11) Variance of X $= \sigma^2 = \begin{cases} \sum (x_i - \mu)^2 P(X = x_i) & [\text{discrete}] \\ \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx. & [\text{cont}] \end{cases}$

12) Standard Deviation $= \sqrt{\sigma^2} = \sigma$

$$EX^2 - (EX)^2 = \text{Variance}$$

13) Mode, Median

$P(X < x) \geq 1/2$ & $P(X > x) \geq 1/2$ (discrete)

$\int_{-\infty}^{M} f(x)\,dx = \int_{M}^{\infty} f(x)\,dx = 1/2$ (cont)

The value of x for which the pmf/pdf attains its max value.

14) for RV $Y = ax + b$

$\mu^* = a\mu + b$

$\sigma_*^2 = a^2\sigma^2$

15) for $Z = \frac{X-\mu}{\sigma}$ $\Rightarrow$ mean $= 0$; variance $= 1$

16) Moment of order k about origin $\Rightarrow$

$EX^k = \begin{cases} \sum x_i^k P(X = x_i) & (\text{discrete}) \\ \int x^k f(x)\,dx & (\text{cont.}) \end{cases}$

17) Central Moments

$E(X-\mu)^k = \begin{cases} \sum (x_i - \mu)^k P(X = x_i) & (\text{dis}) \\ \int (x-\mu)^k f(x)\,dx & (\text{cont}) \end{cases}$

18) $E(k) = k$; k → const. Variance of k $= 0$; k → constant

$E(X-\mu)^k = \mu_k$; $EX^k = \mu_k'$

19) Skewness (Symmetry)

$$\alpha = \frac{\mu_3}{\mu^{3/2}} = \frac{E(X-\mu)^3}{\sigma^3}$$

20) Kurtosis (peakedness)

$$\alpha_4 = \frac{\mu_4}{\mu_2^2} = \frac{E(X-\mu)^4}{\sigma^4}$$

Relation between μ_k and μ_k'.

$$E(X-\mu)^k = {}^kC_0\,\mu_k' - {}^kC_1\,\mu_{k-1}' \times \mu + {}^kC_2\,\mu_{k-2}'\,\mu^2 \ldots\ldots + (-1)^k \mu^k$$

central moment.

about origin

22) MGF:- $M_x(t) = E\{e^{tx}\} = \begin{cases} \sum_{-\infty}^{\infty} e^{tx_i} p(X = x_i) \\ \int_{-\infty}^{\infty} e^{tx} f(x)\,dx \end{cases}$

Coefficient of $\frac{t^r}{r!}$ given $E(X^r)$

r^{th} moment abt origin

23) $$\frac{d^r}{dt^r} M_X(t) \Big|_{t=0} = E(X^r)$$

24) For the distribution,

$P(X=k) = q^{k-1} p \quad ; p+q=1$

Mean $= 1/p$; Var $= q/p^2$

25) Degenerate RV (discrete type)

~~P(X=k)~~

$P(X=k) = 1$

$P(X \neq k) = 0$

26) Two point Distribution.

$p(X=x_1) = p \quad ; P(X=x_2) = q \quad \& \ p+q=1$

27) Binomial distribution.

$p(X=x) = \cancel{\ldots}\ {}^nC_x\, p^x q^{n-x}$.

mean $= np$

variance $= npq$

$E X^2 = E\{X(X-1)\} + E X$

Skewness of binomial $\Rightarrow \frac{1-2p}{\sqrt{npq}}$

* Poisson Distribution. $\qquad E X^2 = E(X(X-1)$

28) $P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$ (Indefinite no-of trials $p \to 0$; np should converge to λ)

Mean = Var = λ

$M_X(t) = e^{(e^t-1)\lambda}$

* Hypergeometric Distribution

With Replacement $\longrightarrow$ Trials are independent

Without Replacement $\rightarrow$ Trials aren't "

$\rightarrow P(X=x) = \frac{{}^{M}C_x \; {}^{N-M}C_{n-x}}{{}^{N}C_n}$

Mean = $\frac{nM}{N}$ $\qquad$ Variance = $\left(\frac{N-n}{N-1}\right)\left(\frac{nM}{N}\right)\left(1-\frac{M}{N}\right)$

* Geometric Distribution

$p(x) = q^x p$

Mean = $1/p$; $\sigma^2 = q/p^2$

* CONTINUOUS

$\Rightarrow$ Uniform Distribution

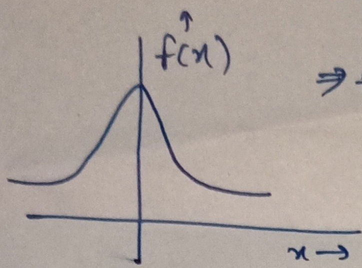
$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x < b \\ 0 & \text{elsewhere} \end{cases}$$

Mean = $\frac{a+b}{2}$; $\sigma^2 = \frac{(b-a)^2}{12}$

$M_X(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$

Normal Distribution

$$f(x, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



⇒ f(x) is symmetric about $x = \mu$ (general property)

If $\sigma \to 0$ then $f(x) \to 0$. (very fast)

If $X \to N(\mu, \sigma^2)$ then $Z \to (0,1)$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$$\text{cdf} = \int_{-\infty}^{z} \phi(t)\,dt = \Phi(z)$$

$$\Gamma k = \int_0^\infty e^{-t} t^{k-1}\,dt$$

Odd moments $= \mu_{2k+1} = 0$

$$\mu_{2k} = 1 \cdot 3 \cdot 5 \cdots (2k-1)\sigma^{2k}$$

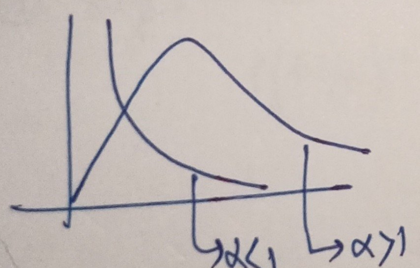
$$\Gamma(k+1) = k\Gamma k$$

$$\mu_{2k} = \sigma^2(2k-1)\mu_{2k-2}$$

kurtosis = 3

* Gamma Distribution

$$\Gamma\alpha = \int_0^\infty e^{-x} x^{\alpha-1}\,dx$$



$$f(x) = \begin{cases} \frac{1}{\Gamma\alpha\, \beta^\alpha} e^{-x/\beta} x^{\alpha-1} & \text{for } x > 0 \\ 0 & \text{Otherwise} \end{cases}$$

(8) $$M_X(t) = \left(\frac{1}{1-\beta t}\right)^\alpha \text{ for } t < \frac{1}{\beta}$$

(9) Mean $= \alpha\beta$

Var $= \alpha\beta^2$

If $\alpha = 1 \Rightarrow f(x) = \frac{1}{\beta} e^{-x/\beta}$ $x > 0$

If $\alpha = \frac{n}{2}$, $\beta = 2$,

$$f(x) = \frac{1}{\Gamma\frac{n}{2}\, 2^{n/2}} e^{-x/2} x^{(n/2 - 1)} \qquad \underline{x > 0}$$

$\Downarrow$

mean for this $= n$

Variance $= 2n$