1. Solve the following

a) $x(n) = x(n-1) + 5$ for $n > 1$ $\quad x(1) = 0$

$x(n) = x(n-1) + 5$

$x(n) = x(n-5) + 5$ - (1) $\quad n = 2$

$x(n-1) = x(n-2) + 5$ - (2) $\quad x(2) = 0 + 5$

$x(n-2) = x(n-2-1) + 5$ $\quad = 5$

$x(n-2) = x(n-3) + 5$ - (3)

sub (2) in (1)

$x(n) = (x(n-2) + 5) + 10$

$= x(n-2) + 10$ - (4)

sub (3) in (4)

$x(n) = x(n-3) + 5 + 10$

$= x(n-3) + 15$

$x(n) = n(n-1) + 5 \, x(n-2) + 10 \, . \, (n-(k-1)) + 5k$

$x(n) = x(n-k) + 5k$

$n - k = 1$ $\quad x(n) = x(n-n-1) + 5(n-1)$

$n - 1 = k$ $\quad = 0 + 5n - 5$

Time complexity is $o(n)$

b)

$x(n) = 3x(n-1)$ for $n > 1$ $x(1) = 4$

$x(n) = 3x(n-1)$ - (1)

$x(n-1) = 3x(n-1-1)$ $\quad n = 2$

$x(n-1) = 3x(n-2)$ - (2) $\quad x(2) = 3(x)(1) = 12$

$n = 3$

$x(n-2) = 3x(n-2-1)$ $\quad x(3) = 3x(2) = 36$

$x(n-2) = 3x(n-3)$ - (3)

sub ③ in ②

$x(n) = 3(3x(n-2))$

$x(n) = 9x(n-2)$ — ④

sub ④ in ③

$x(n) = 9(3x(n-3))$

$= 27x(n-3)$

~~sub~~ 1

$x(n) = 3(n-1) + 9(n-2) + 27(n-3) \ldots$

$3^k(n-1)$

$x(n) = 3^k x(n-k)$

$n-k=1$

$k=n-1$

$\cancel{x(n) = x(n-n-1)+5(n-1)}$

$\cancel{= x(1) + 5n - 5}$

$\cancel{= 0 + 5n - 5}$

$=$

$x(n) = 3^k x\ n-k$

$= 3^{n+1} x(1)$

$= O(3^n)$

2) Evaluate

i) $T(n) = T(n/2) + 1$ where $n = 2k$

$T(2^k) = T\left(\frac{2k}{2}\right) + 1$

$= T(2^{k-1}) + 1$

$T(2^k) = T(2^{k-2}) + 2$

$T(2^k) = T(2^{k-k}) + k$

$T(2^0) \neq k = T(1) = k$

$T(n) = \log n + 1$

ii) $T(n) = T(n/3) + T(2n/3) + cn$

where c is constant

n

$n/3 \quad 2n/3$

$n/9 \quad 2n/9 \quad 2n/9 \quad 4n/9$

$T(n) =$ sum of all

lengths $\log_3 n$

$T(n) > \log_{3/2} n$

depth $= \log_{3/2} n$

$T(n) = \log_{3/2} n$

iii) $x(n/2) + n \quad n > 1 \; (x(1) = 1)$

$n = 1 \; ; x(1) = 1$

$n = 2 \; ; x(2) = x(2/2) + n$

$= 1 + 2 = 3$

$n = 8 \; ; n(8/2) + 8$

$= n(4) + 8$

$= 7 + 8$

$= 15$

$n(2^k) = n(2^{k-1}) + 2^k$

$x(2^k) = 2^{k+1} - 1$

$2^k = n$

$x(n) = x(2^k) = 2^{(\log_2 n) + 1} - 1$

$= 2 \cdot 2^{\log_2 n} - 1$

$= 2n - 1$

Time complexity $= O(n)$

d) $x(n) = x(n/3) + 1$

solve $n = 3^k$

$x(3) = x(1) + 1 = 2$

$x(9) + x(3) + 1 = 3$

$x(27) = x(9) + 1 = 4$

$x(n) = 1 + \log_3 n$

Time complexity is $O(\log n)$

3) consider the following recursion

Algorithm

a) what does this algorithm compute?

1. Best case ($n=1$)

if $n=1$

only one element it return

2. Recursive case:

-> if $n>1$, create temp

-> call recursively (A [0 to n, 2]

-> comparing temp with last element

[A (n-1)]

if temp < A (n-1)

return temp

else

return A[n-1]

b) setup a recurrence relation for algorithm

Best case : $T(1) = c_1$ (constant (c_1)]

recursive case $= T(n) = T(n-1) + c_2$

$c_2 \rightarrow$ constans

$T(n) = c_2 n^2 + (c_1 - c_2))$

$T(n) = O(n^2)$

4) Analyze the order of growth

i) $f(n) = 2n^2 + 5$

$g(n) = 7n$ $\quad f(n) \geq c \cdot g(n)$

$n = 1$; $7 = 7$

$n = 2$; $13 > 14$

$n = 3$

$23 > 21$

$n = 4$

$37 > 28$

$n = 5$

$55 > 35$