PERFORMANCE MEASURES

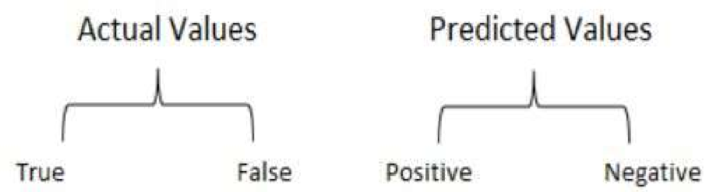
CONFUSION MATRIX

- A confusion matrix is a performance measurement technique for Machine learning classification.
- It is an N x N matrix used for evaluating the performance of a classification model, where N is the number of target classes.
- For a binary classification problem, it is a two-by-two table that contains four outcomes produced by a binary classifier.

		Actual Values	
		Positive (1)	Negative (0)
Predicted Values	Positive (1)	TP	FP
	Negative (0)	FN	TN

CONFUSION MATRIX

- **True Positive(TP):**
  - *Interpretation*: You predicted positive and it's true.
  - You had predicted that France would win the world cup, and it won.
- **True Negative(TN):**
  - *Interpretation:* You predicted negative and it's true.
  - You had predicted that England would not win, and it lost.
- **False Positive(FP): (Type 1 Error)**
  - *Interpretation*: You predicted positive and it's false.
  - You had predicted that England would win, but it lost.
- **False Negative(FN): (Type 2 Error)**
  - *Interpretation*: You predicted negative and it's false.
  - You had predicted that France would not win, but it won.
- Just Remember, We describe predicted values as Positive and Negative and actual values as True and False.



Example 1

ID	Actual Sick?	Predicted Sick?	Outcome
1	1	1	TP
2	0	0	TN
3	0	0	TN
4	1	1	TP
5	0	0	TN
6	0	0	TN
7	1	0	FP
8	0	1	FN
9	0	0	TN
10	1	0	FP

		Actual		
		Positive (1)	Negative (0)	Total
Predicted	Positive(1)	2(TP)	2(FP)	4
	Negative(0)	1(FN)	5(TN)	6
	Total	3	7	10

Example 2

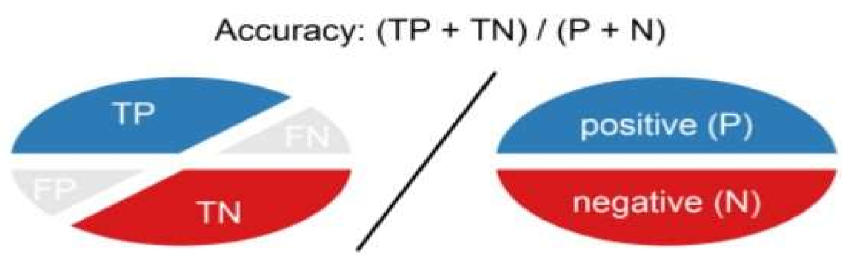
y	y pred	output for threshold 0.6
0	0.5	0
1	0.9	1
0	0.7	1
1	0.7	1
1	0.3	0
0	0.4	0
1	0.5	0

→ TN
→ TP
→ FP
→ TP
→ FN
→ TN
→ FN

Predicted	Actual: Positive (1)	Actual: Negative (0)
Positive(1)	2(TP)	1(FP)
Negative(0)	3(FN)	2(TN)

ACCURACY

- Accuracy (ACC) is calculated as the number of all correct predictions divided by the total number of the dataset.
- The best accuracy is 1.0, whereas the worst is 0.0.
- Overall, how often is the classifier correct?



Accuracy is calculated as the total number of two correct predictions (TP + TN) divided by the total number of a dataset (P + N).

ACCURACY(CONT)

- The accuracy would be calculated by the following formula

		Actual		
		Positive (1)	Negative (0)	Total
Predicted	Positive(1)	2(TP)	2(FP)	4
	Negative(0)	1(FN)	5(TN)	6
Total		3	7	10

- $$ACC = \frac{TP + TN}{TP + TN + FN + FP} = \frac{TP + TN}{P + N}$$

ACC= (2+5)/10 = 0.7

So the model is saying I can predict sick people 70% of the time.

ACCURACY(CONT)

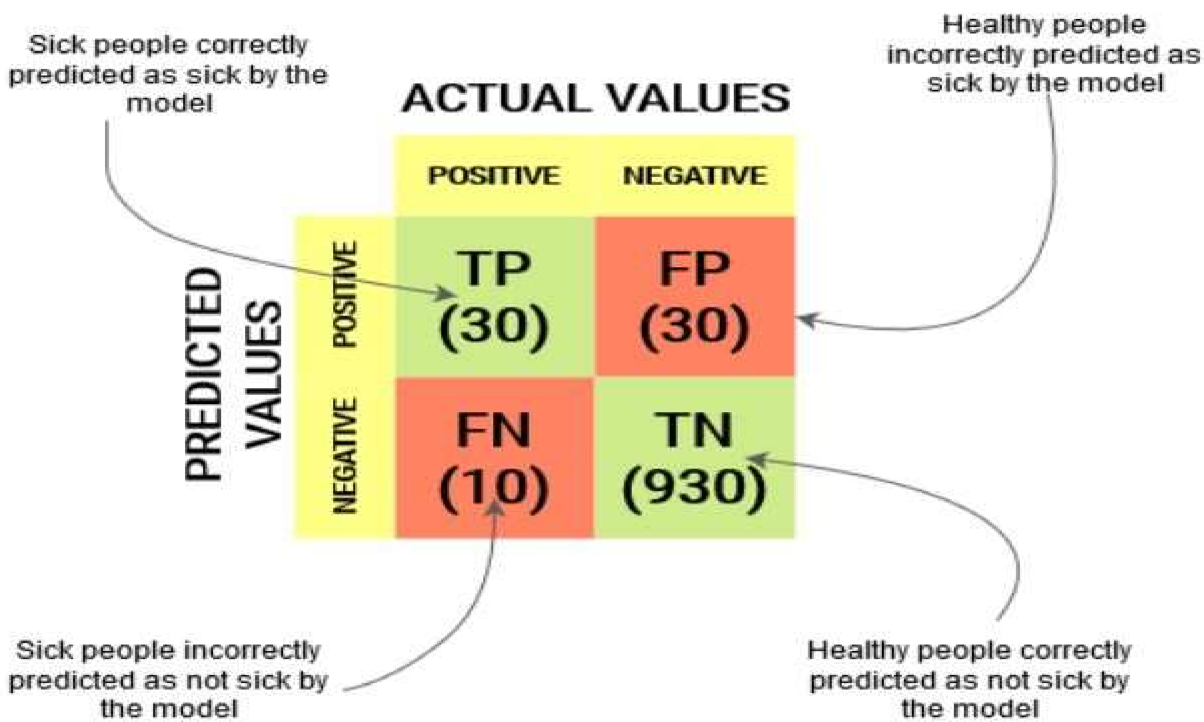
- Let's take another example.

		Actual		
		Positive (1)	Negative (0)	Total
Predicted	Positive(1)	30(TP)	30(FP)	60
	Negative(0)	10(FN)	930(TN)	940
Total		40	960	1000

ACC=(TP+TN)/P+N = (30+930)/1000 = 0.96

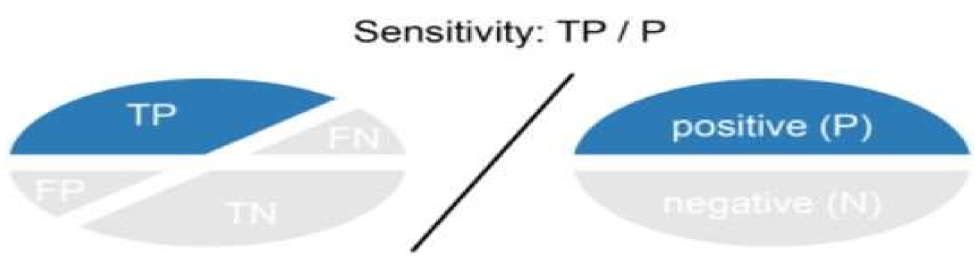
- Our model is saying "I can predict sick people 96% of the time".

CONFUSION MATRIX



RECALL

- Recall (REC) is calculated as the number of correct positive predictions divided by the total number of positives.
- It is also called sensitivity (SN) or true positive rate (TPR).
- The best recall is 1.0, where as the worst is 0.0.
- Recall tells us how many of the actual positive cases we were able to predict correctly with our model.



Sensitivity is calculated as the number of correct positive predictions (TP) divided by the total number of positives (P).

RECALL(CONT)

- The recall would be calculated by the following formula

		Actual		
		Positive (1)	Negative (0)	Total
Predicted	Positive(1)	30(TP)	30(FP)	60
	Negative(0)	10(FN)	930(TN)	940
	Total	40	960	1000

$$Recall = \frac{TP}{TP + FN}$$

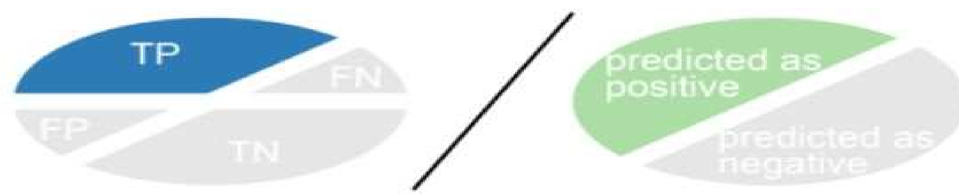
REC= 30/(30+10) = 0.75

- 75% of the positives were successfully predicted by our model. Awesome!

PRECISION

- Precision (PREC) is calculated as the number of correct positive predictions divided by the total number of positive predictions.
- It is also called positive predictive value (PPV).
- The best precision is 1.0, whereas the worst is 0.0.
- Precision tells us how many of the correctly predicted cases actually turned out to be positive.

Precision: TP / (TP + FP)



Precision is calculated as the number of correct positive predictions (TP) divided by the total number of positive predictions (TP + FP).

PRECISION(CONT.)

- The precision would be calculated by the following formula

Predicted	Actual: Positive (1)	Actual: Negative (0)	Total
Positive(1)	30(TP)	30(FP)	60
Negative(0)	10(FN)	930(TN)	940
Total	40	960	1000

$$Precision = \frac{TP}{TP + FP}$$

Precision= 30/(30+30)

- 50% percent of the correctly predicted cases turned out to be positive cases
- This would determine whether our model is reliable or not.

F-SCORE

- In practice, when we try to increase the precision of our model, the recall goes down, and vice-versa.
- The F1-score captures both the trends in a single value.
- **F1-score is a harmonic mean of Precision and Recall**, and so it gives a combined idea about these two metrics.
- It is maximum when Precision is equal to Recall.

- $F_\beta = \frac{(1+\beta^2)(\mathrm{PREC} \cdot \mathrm{REC})}{(\beta^2 \cdot \mathrm{PREC} + \mathrm{REC})}$

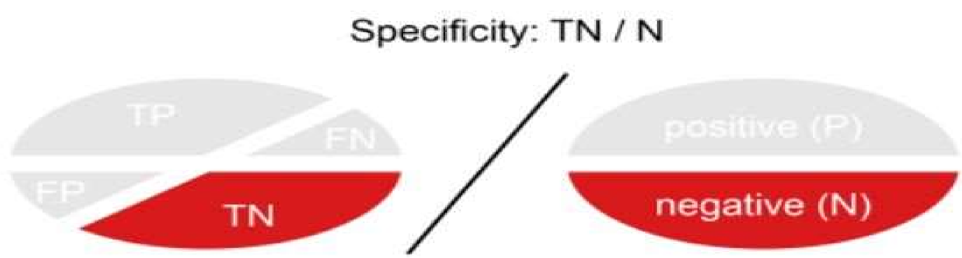
F-SCORE(CONT)

β is commonly 0.5, 1, or 2.

- $F_{0.5} = \dfrac{1.25 \cdot \mathrm{PREC} \cdot \mathrm{REC}}{0.25 \cdot \mathrm{PREC} + \mathrm{REC}}$
- $F_1 = \dfrac{2 \cdot \mathrm{PREC} \cdot \mathrm{REC}}{\mathrm{PREC} + \mathrm{REC}}$
- $F_2 = \dfrac{5 \cdot \mathrm{PREC} \cdot \mathrm{REC}}{4 \cdot \mathrm{PREC} + \mathrm{REC}}$

SPECIFICITY (TRUE NEGATIVE RATE)

- Specificity (SP) is calculated as the number of correct negative predictions divided by the total number of negatives.
- It is also called true negative rate (TNR).
- The best specificity is 1.0, whereas the worst is 0.0.



Specificity is calculated as the number of correct negative predictions (TN) divided by the total number of negatives (N).

SPECIFICITY (TRUE NEGATIVE RATE)

- The specificity would be calculated by the following formula

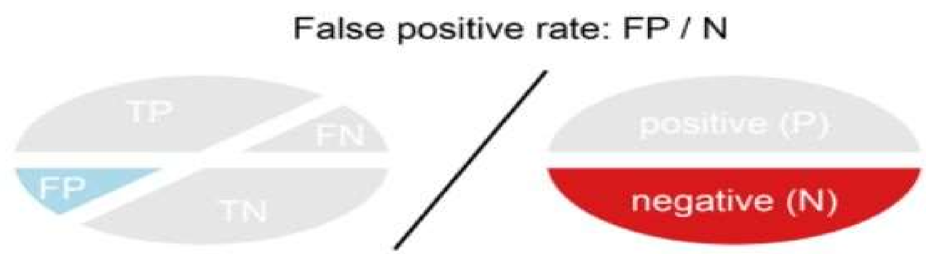
		Actual: Positive (1)	Actual: Negative (0)	Total
Predicted	Positive(1)	30(TP)	30(FP)	60
Predicted	Negative(0)	10(FN)	930(TN)	940
	Total	40	960	1000

- $$SP = \frac{TN}{TN + FP} = \frac{TN}{N}$$

SP=930/960 = 0.96

FALSE POSITIVE RATE

- False positive rate (FPR) is calculated as the number of incorrect positive predictions divided by the total number of negatives.
- The best false positive rate is 0.0 whereas the worst is 1.0.
- It can also be calculated as 1 – specificity.



False positive rate is calculated as the number of incorrect positive predictions (FP) divided by the total number of negatives (N).

FALSE POSITIVE RATE

- The specificity would be calculated by the following formula

		Actual: Positive (1)	Actual: Negative (0)	Total
Predicted	Positive(1)	30(TP)	30(FP)	60
Predicted	Negative(0)	10(FN)	930(TN)	940
Total		40	960	1000

- $\mathrm{FPR} = \frac{\mathrm{FP}}{\mathrm{TN} + \mathrm{FP}} = 1 - \mathrm{SP}$

FPR=30/960 = 0.031

PROBABILITY OF PREDICTION

- A machine learning classification model can be used to predict the actual class of the data point directly or predict its probability of belonging to different classes.
- We can determine our own threshold to interpret the result of the classifier.
- Setting different thresholds for classifying positive class for data points will inadvertently change the Sensitivity and Specificity of the model.
- One of these thresholds will probably give a better result than the others, depending on whether we are aiming to lower the number of False Negatives or False Positives.

ID	Actual	Prediction Probability	>0.6	>0.7	> 0.8	Metric
1	0	0.98	1	1	1	
2	1	0.67	1	0	0	
3	1	0.58	0	0	0	
4	0	0.78	1	1	0	
5	1	0.85	1	1	1	
6	0	0.86	1	1	1	
7	0	0.79	1	1	0	
8	0	0.89	1	1	1	
9	1	0.82	1	1	1	
10	0	0.86	1	1	1	
			0.75	0.5	0.5	TPR
			1	1	0.66	FPR
			0	0	0.33	TNR
			0.25	0.5	0.5	FNR

The AUC-ROC curve solves just that problem!

AUC-ROC CURVE

- The **Receiver Operator Characteristic (ROC)** curve is an evaluation metric for binary classification problems.
- It is a probability curve that plots the **TPR** against **FPR** at various threshold values and essentially **separates the 'signal' from the 'noise'**.
- The **Area Under the Curve (AUC)** is the measure of the ability of a classifier to distinguish between classes and is used as a summary of the ROC curve.
- In a ROC curve, a higher X-axis value indicates a higher number of False positives than True negatives.
- While a higher Y-axis value indicates a higher number of True positives than False negatives. So, the choice of the threshold depends on the ability to balance between False positives and False negatives.

Example Problem-

- **Suppose**, a classification model yields the classification probability as shown in Table 1.

Step 1 - Let us set the different Threshold as α = [0 , 0.2, 0.4, 0.8, 1]

Step 2 - Taking first threshold as α = [0], we will get-

Step 3 - TPR = $\frac{4}{4+0} = 1$

& FPR = $\frac{2}{2+0} = 1$

Therefore TPR, FPR at α = [0] is (1,1)

Output	Class. Prob. (Y)	At α = [0] Y_1
1	0.8	1
0	0.96	1
1	0.4	1
1	0.3	1
0	0.2	1
1	0.7	1

Table 2. Predicted Output at α = [0]

Output	Class. Prob. (Y)
1	0.8
0	0.96
1	0.4
1	0.3
0	0.2
1	0.7

Table 1. Actual Output vs Classification yielded Prob.

Cntd…

Step 4 - Repeating all the steps from **Step 1** to **Step 3** at $\boldsymbol{\alpha}$ = [0.2], we will get-

$$\text{TPR} = \frac{4}{4+0} = 1$$

$$\& \text{ FPR} = \frac{1}{1+1} = 0.5$$

Therefore TPR, FPR at $\boldsymbol{\alpha}$ = [0.2] is (1 , 0.5)

Output	Class. Prob. (Y)	At $\boldsymbol{\alpha}$ = [0.2] Y_1
1	0.8	1
0	0.96	1
1	0.4	1
1	0.3	1
0	0.2	0
1	0.7	1

Table 3. Predicted Output at $\boldsymbol{\alpha}$ = [0.2]

Step 5 - Again repeating all the steps from **Step 1** to **Step 3** at $\boldsymbol{\alpha}$ = [0.4], we will get-

$$\text{TPR} = \frac{2}{2+2} = 0.5$$

$$\& \text{ FPR} = \frac{1}{1+1} = 0.5$$

Therefore TPR, FPR at $\boldsymbol{\alpha}$ = [0.4] is (0.5 , 0.5)

Output	Class. Prob. (Y)	At $\boldsymbol{\alpha}$ = [0.4] Y_1
1	0.8	1
0	0.96	1
1	0.4	0
1	0.3	0
0	0.2	0
1	0.7	1

Table 4. Predicted Output at $\boldsymbol{\alpha}$ = [0.4]

Cntd...

Step 6 - Repeating all the steps from **Step 1** to **Step 3** at α = [0.8], we will get-

TPR = $\frac{0}{0+4}$ =0

& FPR = $\frac{1}{1+1} = 0.5$

Therefore TPR, FPR at α = [0.8] is (0 , 0.5)

Output	Class. Prob. (Y)	At α = [0.8] Y_1
1	0.8	0
0	0.96	1
1	0.4	0
1	0.3	0
0	0.2	0
1	0.7	0

Table 5. Predicted Output at α = [0.8]

Step 7 - Again repeating all the steps from **Step 1** to **Step 3** at α = [1], we will get-

TPR = $\frac{0}{0+4} = 0$

& FPR = $\frac{0}{0+2} = 0$

Therefore TPR, FPR at α = [1] is (0 , 0)

Output	Class. Prob. (Y)	At α = [1] Y_1
1	0.8	0
0	0.96	0
1	0.4	0
1	0.3	0
0	0.2	0
1	0.7	0

Table 6. Predicted Output at α = [1]

Cntd...

Step 8 – Finally, plotting the **ROC** curve at different threshold setting we will get-

Threshold	TPR	FPR
0	1	1
0.2	1	0.5
0.4	0.5	0.5
0.8	0	0.5
1	0	0

Table 7. TPR, FPR values at different threshold settings.

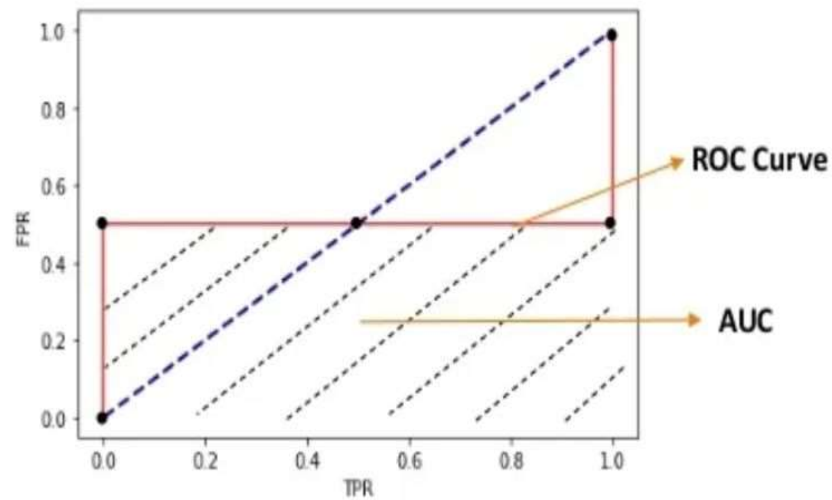


Fig. 2. ROC-AUC Curve.