National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Probability and Statistics	Course Code:	MT2005
Program:	BSE/BSCS/BDS	Semester:	Fall 2023
Duration:	60 Minutes	Total Marks:	40
Paper Date:	11-11-2023	Weight	15%
Section:	ALL SECTIONS	Page(s):	7
Exam Type:	MID-II	Moderator	Ms. Sarah Ahmad

Student : Name Roll No Section:- BDS3 C

Instruction/Notes:

Marks Obtained
27.5

1. It is great to have choices in life but here all the questions are compulsory. So attempt all the subsections properly (Utilize the given space for each section)*Write Roll no. on each page. You can use the last page to extend any part if needed. No extra sheets allowed to attach for marking. However, you can demand for one rough sheet but do not attach it.
2. Pencil Work wouldn't be marked. Necessary Statistical tables are attached. You are not allowed to bring any statistical table.
3. We know, sharing is caring but here exchange of calculators is not allowed. You can only use your own scientific calculator (programmable calculators are not allowed).
4. Don't get panic. If you found any ambiguity in the data then do not ask anything to the invigilator, just make assumption and continue solving your paper.
5. Believe in yourself & do not waste your time by looking in answer sheets of your fellows and copying them.
6. Now if you regret not being prepared for this exam then Crying is allowed but do it so quietly in order to avoid disturbance.
7. If you are thinking that it's a revenge. No, it is not. It is just an exam. We want you to be a most successful person in life. All the Best!

Don't Hurry. Don't Worry. Do your Best and Let it rest.

1.5

Question 1: **[CLO-5, Marks: 07]**

The amount of time in hours that a computer functions before breaking down is a continuous random variable with probability density function given by

Pdf → CDF
CDF → PDF.

$$F(x) = \begin{cases} 1 - e^{-x/100} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

(i) Obtain probability density function (pdf). **(3)**

1.5

$$\int_{a}^{b} f(x)\,dx$$

$$\int_{a}^{b} 1 - e^{-x/100}$$

x

$\int_0$

$$= \int_a^b (1)\,dx - \int_a^b e^{-x/100}\,dx$$

$$= \Big[x\Big]_a^b - \left[\frac{e^{-x/100}}{-1/100}\right]_a^b$$

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$$\int_{50}^{150} 1-e^{-x/100}$$

(ii) What is the probability that a computer will function between 50 and 150 hours before breaking down? **(4)**

$$\Rightarrow \int_{50}^{150} f(x)\,dx \qquad F(150)-F(50).$$

$$= \cancel{[(150)-(50)]} - \left[\frac{e^{-x}}{-1/100}\right].$$

$$= 61.65$$

Question 2: 9 **[CLO-4, Marks: 09]**

On the average, 1 in 800 computers crashes during a severe thunderstorm. A certain company had 4,000 working computers when the area was hit by a severe thunderstorm.

a) Compute the expected value and variance of the number of crashed computers. **(4)**

4

Here, $n = 4000$

$p = \frac{1}{800} = 0.00125$

$q = 0.99875$

Mean $= n.p = 5$ 1 1

Variance $= n.p.q = 4.993$ 1 1

b) Compute the probability that at least three computers crashed. **(5)**

5

Using Poisson Distribution Method:

$P(X \geq 3) = 1-(X \leq 2)$ 1

$$= 1-\left(\frac{5^0 \times e^{-5}}{0!}\right)-\left(\frac{5^1 \times e^{-5}}{1!}\right)-\left(\frac{5^2 \times e^{-5}}{2!}\right)$$ 1 1 1

$= 1-0.1246$

$= 0.8754$ 1

Question 3: 15 **[CLO-4, Marks: 15]**

A small-business website contains 100 pages. It was found that 60%, 30%, and 10% of the pages contain low, moderate, and high graphic content, respectively. A sample of two pages is selected without replacement. Let X and Y denote the number of pages with moderate and high graphics output respectively in the sample. Determine:

a) The joint probability function **(2)**

$$f(x,y) = \frac{\binom{30}{x}\binom{10}{y}\binom{60}{2-x-y}}{\binom{100}{2}}$$

2

b) The joint probability distribution of X and Y **(3)**

3 0.5 each prob

$f(x,y)$	0	1	2	$h(y)$
y 0	59/165	4/11	29/330	89/110
1	4/33	2/33	0	2/11
2	1/110	0	0	1/110
$g(x)$	161/330	14/33	29/330	1

c) $P(Y=1|X=1)$ (3)

$$P(Y=1/_{X=1}) = \frac{f(1,1)}{g(1)}$$

$$= \frac{2/33}{14/33} = 1/7$$

1

1 1

3

d) Verify whether E (XY) = E(X) E(Y) or not? (7)

7

$E(X) = \Sigma x . g(x)$ 1

$= 0.(161/330) + 1(14/33) + 2(\frac{24}{330})$

$= 3/5 = 0.6$ 1

$E(Y) = \Sigma y . h(y)$ 1

$= 0(89/110) + 1(2/11) + 2(1/110)$

$= 1/5 = 0.2$ 1

$E(X).E(Y) = 0.12 = 3/25$

$E(X.Y) = \Sigma\Sigma xy f(x,y)$ 1

$= (0.0)(59/165) + (0.1)(4/33)$
$+ (0.2)(1/110) + (1.0)(4/11) + (1.1)(2/33)$
$+ (2.0)(29/330)$
$= 2/33 = 0.06$ 2

Hence both are not equal. 1

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Question 4: 2 **[CLO-5, Marks: 09]**

a) Consumer test ratings for a new line of products have averaged 67.5 with a standard deviation of 23.3 follows normal distribution. Jeff Erickson has developed a new device which he wishes to market. His supervisor tells him that in order to put it into production, the device must receive a rating of at least 70. How likely is that Jeff's product is will reach the assembly line? **(5)**

1

$\mu = 67.5$

$\sigma = 23.3$

$P(X \geq 70)$

$Z = \dfrac{\overset{70}{69.5} - 67.5}{23.3}$

$= 0.085$

So, $P(X \geq 70) = P(Z \geq 0.085)$

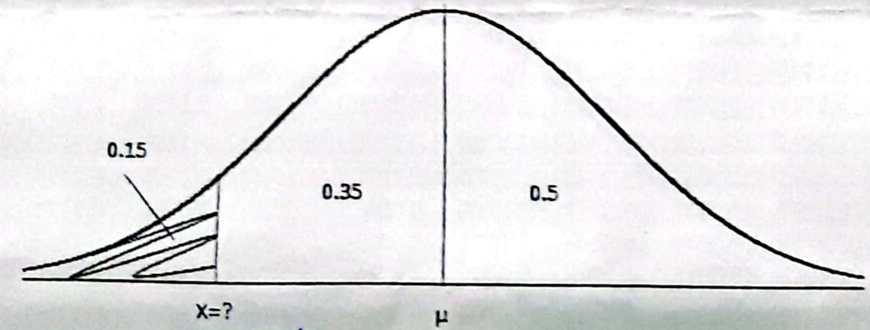
$P(Z \geq 0.086)$

$= 1 - P(Z \leq 0.086)$

$= 1 - 0.5359$

$= 0.4641$

b) Let X be normally distributed with mean = \$11.151 million and standard deviation = \$3.550 million. Consider the diagram given below and compute the required solution. **(4)**



mean = 11151

std = 3550

$Z = 0.5596$ for 0.15

As,

$Z = \dfrac{X - \mu}{\sigma} \Rightarrow X = Z\sigma + \mu$

$= -9164.42$

1

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