National University of Computer and Emerging Sciences, Lahore Campus

	Course Name:	Discrete Structure	Course Code:	CS-1005
	Degree Program:	BS (CS), BS(SR), BS(DS)	Semester:	Fall 2023
	Exam Duration:	60 Minutes	Total Marks:	30
	Paper Date:		Weight	15 %
	Sections:	All	No of Page(s):	4
	Exam Type:	Midterm-1		

Student Name: ______________________________ Roll No.________________ Section:_______

Instruction/Notes:
i. Attempt all questions neatly on question paper.
ii. Answer sheet is not required.

Question 1: (marks: 5+5)

i) Write the following English sentences in symbolic form-

a) Ali is poor but honest.

p ∧ q where- p : Ali is poor and q : Ali is honest

b) I will study only if you teach.

p → q where- p : I will study q : You teach

c) It is hot or else it is both cold and cloudy.

p ∨ (q ∧ r) where- p : It is hot q : It is cold r : It is cloudy

d) Birds fly if and only if sky is clear.

p ↔ q where- p : Birds fly q : Sky is clear

e) The presence of cycle in a multi-instance Resource Allocation Graph is a necessary but not sufficient condition for deadlock.

(q → p) ∧ ~(p → q) where- p : Presence of cycle in a multi instance RAG q : Presence of deadlock

ii) Write following statement in the form of if p then q
"Unless you report to the exam cell before 9 am, you will not be permitted to write quizzes."
Also, write inverse, converse and contrapositive.

Let P : You report to the exam cell before 9am.

Q : You will be permitted to write quizzes.
Therefore the statement is $\sim P \rightarrow \sim Q$ (if you don't report to the exam cell before 9 am then you will not be permitted to write quizzes.
Converse: $\sim Q \rightarrow \sim P$
If you are not permitted to write quizzes, then you have not reported to the exam cell before 9am.
Inverse: $P \rightarrow Q$
If you report to the exam cell before 9am, then you will be permitted to write quizzes.
Contrapositive: $Q \rightarrow P$

If you are permitted to write quizzes, then you reported to the exam cell before 9am.

Question 2: (marks: 5+5)

i) **Use rules of interference and quantifiers to determine if the following argument is valid, where the universe of discourse (domain) is all people. "Some students in the class did not read the textbook", "Every student in the class passed the test." Therefore, someone who passed the test did not read the textbook.**

First we list our propositions.
_ Let C(x) denote "x is in the class".
_ Let P(x) denote "x passed the test".
_ Let T(x) denote "x read the textbook".
In the arguments that follow 'a' is used to represent a specified element of the domain (a student). We want the conclusion $\exists x(P(x) \wedge \neg T(x))$.

Step	Reason
1- $\exists x(C(x) \wedge \sim T(x))$	P_1.
2- $\forall x(C(x) \rightarrow P(x))$	P_2.
3- $C(a) \wedge \sim T(a)$ for some 'a'	Ex. Instantiation on I
4- $C(a)$	Simplification from 3
5- $C(a) \rightarrow P(a)$ for an arbit. 'a'	UI from 2
6- $P(a)$	MP (4,5)
7- $\sim T(a)$	Simplification (3)
8- $P(a) \wedge \sim T(a)$ for some 'a'	Conjuction (6,7)
9- $\exists x(P(x) \wedge \sim T(x))$	Existential Generalization from 8.

ii) **Let $n \in \mathbb{Z}$. Prove that $(n+1)^2 - 1$ is even if and only if n is even.**

Solution. First we prove the implication $\Leftarrow$, namely, that if n is even then $(n+1)^2-1$ is even. Suppose n is even. Then, by definition, $n = 2k$ for some $k \in \mathbb{Z}$. Therefore

$$(n+1)^2 - 1 = (2k+1)^2 - 1 = 4k^2 + 4k = 2(2k^2 + 2k).$$

Since $2k^2 + 2k \in \mathbb{Z}$, we conclude that $(n+1)^2 - 1$ is even.

Now we prove the converse implication $\Rightarrow$, namely, that if $(n+1)^2 - 1$ is even then n is even. Suppose $(n+1)^2 - 1$ is even. Then $(n+1)^2 - 1 = 2k$ for some $k \in \mathbb{Z}$. Thus

$$(n+1)^2 - 1 = n^2 + 2n = 2k$$

so $n^2 = 2(k-n)$. Consequently, n^2 is even.

This implies that n is even as we know that

if n^2 is even, then n is even.

<u>Question2</u>: (marks: 5+5)

i) **Use Laws of equivalence to determine whether the compound proposition $\sim(p \rightarrow q) \vee (\sim p \vee (p \wedge q))$ is a tautology or not.**

We have-

$\sim(p \rightarrow q) \vee (\sim p \vee (p \wedge q))$

$= \sim(\sim p \vee q) \vee (\sim p \vee (p \wedge q))$ $\{ \because p \rightarrow q = \sim p \vee q \}$

$= (p \wedge \sim q) \vee (\sim p \vee (p \wedge q))$ { Using De Morgans law }

$= (p \wedge \sim q) \vee ((\sim p \vee p) \wedge (\sim p \vee q))$ { Using Distributive law }

$= (p \wedge \sim q) \vee (T \wedge (\sim p \vee q))$ { Using Complement law }

$= (p \wedge \sim q) \vee (\sim p \vee q)$ { Using Identity law }

$= ((p \wedge \sim q) \vee \sim p) \vee q$ { Using Associative law }

$= ((p \vee \sim p) \wedge (\sim q \vee \sim p)) \vee q$ { Using Distributive law }

$= (T \wedge (\sim q \vee \sim p)) \vee q$ { Using Complement law }

$= (\sim q \vee \sim p) \vee q$ { Using Identity law }

$= \sim p \vee (q \vee \sim q)$

$= \sim p \vee T$ { Using Complement law }

$= T$ { Using Identity law }

- Clearly, the result is T.
- So, given proposition is a tautology.

ii) Answer following questions

a) Find truth value of of $\forall x \forall y(xy > y)$, where the domain for all variables consists of all integers. Give counter example if it is false.

F , counter example is x = -1, y = 17

b) Use predicates, quantifiers, logical connectives, and mathematical operators to express, "The product of two negative real numbers is positive."

$\forall x \, \forall y \, ((x < 0) \land (y < 0) \rightarrow (xy > 0))$

c) Translate nested quantifications $\forall x \forall y((x \neq 0) \land (y \neq 0) \leftrightarrow (xy \neq 0))$ into an English statement that expresses a mathematical fact. The domain consists of all real numbers.

The product of two real numbers is nonzero if and only if both numbers are nonzero.

d) Write the negation of **"Some student has solved every exercise in this book"** in simple English. (Don't use the phrase "It is not the case that.")

Let S(x; y) mean that student x has solved exercise y . The statement is $\exists x \forall y \, S(x, y)$. The negation is $\forall x \exists y \sim S(x; \, y)$. In English, for every student in this class, there is some exercise that he or she has not solved.

Or

Every student in class has not solved some of the exercise.

e) find a domain for which the following statement is true and a domain for which the statement is false.

"There is someone older than 21 years."

If the domain is all residents of Pakistan, then this is true. If the domain is the set of pupils in a grade one class, it is false.