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National University of Computer and Emerging Sciences, Lahore Campus

	Course:	Linear Algebra	Course Code:	MT-1004
	Program:	BS (CS, DS, SE)	Semester:	Fall 2022
	Duration:	3 Hours	Total Marks:	100
	Date:	31-12-2022	Weight	50%
	Section:	All	Page(s):	3
	Exam:	Final	Roll No:	
Name:				

Instruction/Notes:

Use of programmable calculators is not allowed. Exchange of stationary is strictly prohibited. Show complete working in all questions. Attempt all question parts together. Question attempted in separate parts will not be marked.

Q#8 is a BONUS Question. (You will not lose marks if unable to attempt).

Question#1: [5+5 +5+5 marks, CLO#1, 5]

a) Use **Inversion Algorithm** to find the Inverse of matrix $A = \begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$.

b) Write down A^{-1} as a product of elementary matrices $A^{-1} = E_k E_{k-1} \ldots\ldots E_3 E_2 E_1$.

c) Verify that $A = E_1^{-1}\, E_2^{-1}\, E_3^{-1} \ldots\, E_{k-1}^{-1}\, E_k^{-1}$ for some k.

d) Use the decomposition done in part (c) to discuss the Geometric Effect on the Unit Square of the elementary matrices.

Do the following steps for part (d):

i) Show the effect of $E_1^{-1}\, E_2^{-1}\, E_3^{-1} \ldots\, E_{k-1}^{-1}\, E_k^{-1}$ on the unit square step by step.

ii) Show the action of each elementary matrix mathematically and graphically.

Question#2: [5+5 marks, CLO #2]

a) Find the area of the triangle in 3-space that has the given vertices.

$$P = (1, -1, \ 2), \ Q = (0, 3, \ 4), \quad R = (6, \ 1, \ 8).$$

b) Find the distance between the point $(2, 1, -3)$ and the plane $2x - y - 2z = 6$.

Question#3: [10 marks, CLO #3]

Find the geometric and algebraic multiplicity of each eigenvalue of the matrix A,

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

and determine whether A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A and find $P^{-1}AP$.

Question#4: [10 marks, CLO #03]

Suppose you are designing a simple video game where the player controls a shooter to hit a moving targets. If the shooter is an arrowhead whose vertices at any point is in the span of row vectors of A where

$$A = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & -7 & 0 & -6 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$$

(**Hint:** Take Transpose of given matrix A)

a) Find the basis for the row space of A consisting entirely of the row vectors from A.

b) Use Dimension theorem to find the rank and nullity of A and A^t.

Question#5: [20 marks, CLO #04]

Suppose $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ define the column vectors u_1, u_2 and u_3 as,

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \ \& \ u_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

a) Use Gram – Schmidt process to find the orthogonal set of vectors $\{v_1, v_2, v_3\}$ and then find orthonormal set of vectors $\{q_1, q_2, q_3\}$ by considering standard inner product between vectors.

b) Considering standard inner product between the vectors find QR- decomposition of the given matrix. Also verify that $A = QR$ where,

$Q = [\ q_1 \mid q_2 \mid q_3\ ]$ and R is given below

$$R = \begin{bmatrix} \langle u_1, q_1 \rangle & \langle u_2, q_1 \rangle & \langle u_3, q_1 \rangle \\ 0 & \langle u_2, q_2 \rangle & \langle u_3, q_2 \rangle \\ 0 & 0 & \langle u_3, q_3 \rangle \end{bmatrix}.$$

Question#6: [5+5 marks, CLO #04]

a) Find $\|u\|$ and $d\,(u, v)$ relative to the weighted inner product defined as $\langle u, v \rangle = 2u_1v_1 + 3u_2v_2$ on R^2 where, $u = (-3, 2)$ and $v = (1, 7)$.

b) Considering the weighted inner product defined in part (a) check that whether the vectors are orthogonal.

Question#7: [10 marks, CLO #05]

Let $T: R^2 \to R^3$ be the linear transformation defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -x_1 \\ -3x_1 + 2x_2 \\ -4x_1 + 3x_2 \end{bmatrix}$. Find the matrix for the transformation T i.e. $[T]_{B',B} = [\ [T(u_1)]_{B'} \mid [T(u_2)]_{B'}]$ relative to the basis $B = \{u_1, u_2\}$ and $B' = \{v_1, v_2, v_3\}$, where

$$u_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Question#8: [5+5 marks, CLO-3, 5] (Bonus Question)

$$\text{If } C = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \text{ and } D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \text{ then}$$

Do the following steps:

a. Find a matrix P (consisting of the Eigen vectors of the matrix C) using Eigenvalues of C and show that $P^{-1}CP = D$. Also, find the dimension of Eigen Spaces associated with each Eigen value.

b. Show that C and D represents same linear operator $T : R^2 \to R^2$ by showing $P^{-1}CP = D$, where $P = P_{B' \to B} = [[u_1']_B \ \ [u_2']_B]$ and $P^{-1} = P_{B \to B'}$, $B' = \{u_1', u_2'\}$, $B = \{e_1, e_2\}$, $u_1' = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $u_2' = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Here P & P^{-1} shows the transition matrices.