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National University of Computer and Emerging Sciences

Advanced Statistics (DS2003)

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Course Instructor(s)

Dr. Muhammad Ahmad Raza

Sessional-I Exam

Total Time: 1 Hours

Total Marks: 60 35+13 = 48

Total Questions: 03

Semester: SP-2024

Campus: Lahore

Dept: School of Computing

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CLO 1: CLO statement for question Q1&Q2

Learn about basic statistical methods

Q1: Question statement ....[15+15=30] 25

Suppose that orders at a restaurant are IID random variables with mean $\mu = \$8$ and standard deviation $\sigma = \$2$. Answer the followings:

a) Estimate the probability that the first 100 customers spend a total of between \$780 and \$820.

$\mu_x = 100(8) = 800$ | $N = 100$ $\mu = 8$ $\sigma = 2$

$\sigma^2 = 2^2(100) = 400$ | $\sigma^2 = 4$

$P(780 < x < 820) =$

$P(x < 820) - P(x < 780)$

$P\left(z < \frac{820-800}{\sqrt{400}}\right) - P\left(z < \frac{780-800}{\sqrt{400}}\right)$

$P(z < 1.00) - P(z < -1.00) = 0.8413 - 0.1587$

$= 0.6826 \approx 68.26\%$ ~~≈ 68.26%~~ 15

Probability

b) After how many orders can we be 90% sure that the total spent by all customers is more than $1000?

Is 0.90

Probability to which we sure is 90% = 0.90

$P(x > 1000) = 0.90$

$P(x < 1000) = 1 - 0.90$

$P(x < 1000) = 0.1$

$$\frac{1000 - \mu_x}{\sigma_x} = -1.28$$

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$$\frac{1000 - 8N}{2\sqrt{N}} = -1.28 \Rightarrow 1000 - 8N = -2.56\sqrt{N}$$

$N = 15594$

$1000 = -1.28(2\sqrt{N}) + 8N$

$$\frac{1000}{-2.56}\sqrt{n} = 8N \Rightarrow \frac{1000}{-2.56(8)} = N\sqrt{n}$$

Take sq on b-S

$$\left(\frac{1000}{-2.56}\right)^2 = 64N \cdot N \quad \left(\frac{1000}{-2.56(8)}\right)^2 = N^3$$

$$\left(\frac{1000}{-2.56(8)}\right)$$

$2384.19 = N^3$

~~$N = 13.36$~~

Q2: Question statement ....[10]

The PMF, mean and variance of a Poisson random variable are given as follows:

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$$p_X(k) = \frac{\lambda^k}{k!}e^{-\lambda}$$

$$E(X) = \lambda; \quad Var(X) = \lambda$$

Now consider a population following the Poisson distribution with mean of 2. Suppose that a sample of size 16 is obtained from this population. Use the central limit theorem to estimate the probability that the sample mean is greater than 2.5.

Ans mean = 2

$E(x) = \lambda$ $\boxed{2 = \lambda}$

$Var(x) = 2$

i.e

$\mu = 2$, $\sigma^2 = 2$

Using CLT for sample Mean

$\mu_{\bar{x}} = 2$, $\sigma^2_{\bar{x}} = \frac{2}{16} \rightarrow \frac{\sigma^2}{n}$

$P_{\bar{x}}(\bar{x} > 2.5) = \cancel{P(z} \quad 1 - P_{\bar{x}}(\bar{x} < 2.5)$

$= 1 - P_z\left(z < \frac{2.5-2}{\sqrt{\frac{2}{16}}}\right)$

$= 1 - F_z(1.4142) \approx 1 - F_z(1.41)$

$= 1 - 0.9207$

$= 0.0793 \approx \cancel{7.93} \; 7.93\%$

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CLO 2: CLO statement for question Q3

Understand the reasons for developing confidence intervals and performing hypothesis tests

Q3: Question statement ....[10+10=20]

a) An electrical firm manufactures light bulbs. The life of each bulb is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life (sample mean) of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

$\bar{X} = 780$, $\sigma = 40$, $N = 30$

$100(1-\alpha) = 96$ | $Z_{\frac{\alpha}{2}} = Z_{0.02} = -2.05$

$1-\alpha = 0.96$

$\alpha = 1-0.96 = 0.04 \Rightarrow \frac{\alpha}{2} = 0.02$

$\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$780 - (+2.05) \frac{40}{\sqrt{30}} < \mu < 780 + \cancel{t} (-2.05) \frac{40}{\sqrt{30}}$

~~794.97 < μ <~~

$765.02 < \mu < 794.97$

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b) How large a sample is needed if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

$\cancel{040}$

$780 - (2.05)\frac{40}{\sqrt{n}} < \cancel{40}\ 10$

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$\approx$

$780 - 2.05(40) \cancel{<} = 10\sqrt{n}$

~~0~~

$698\ \cancel{<} = 10\sqrt{n}$

$69.8 = \sqrt{n}$

$n = 48\cancel{72}.04$