Practice Sheet-2

QUESTION 1:

Six boxes of components are ready to be shipped by a certain supplier. The number of defective components in each box is as follows:

Box	1	2	3	4	5	6
No of defectives	0	2	0	1	2	0

Obtain the probability distribution of number of defective components.

QUESTION 2:

Obtain the probability distribution of a random variable X, where X denotes the number of aces in bridge hand.

QUESTION 3:

A store carries flash drives with either 1 GB, 2 GB, 4 GB, 8 GB, or 16 GB of memory. The accompanying table gives the distribution of Y 5 the amount of memory in a purchased drive:

Y	1	2	4	8	16
P(y)	0.05	0.10	0.35	0.40	0.10

Determine F(y) for each of the five possible values of Y.

QUESTION 4:

Let X be a continuous random variable with pdf

$$f(x) = 6x(1-x) \qquad 0 \le x \le 1$$

$$= 0 \qquad \text{otherwise}$$

a) Check that f(x) is a proper pdf.

b) Obtain an expression for distribution function of X.

c) Compute $P(\frac{1}{3} < X < \frac{2}{3})$

Question 5:

From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find

(a) The joint probability distribution of X and Y
(b) The marginal probability distribution of X and Y
(c) The Mathematical Expectation of X and Y

Question 6:

In a certain city district, the need for money to buy drugs is stated as the reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district, at most 3 resulted from the need for money to buy drugs.

QUESTION 7:

Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error, and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function

$$f(x) = \begin{cases} k(3 - x^2), & -1 \le x \le 1 \\ 0, & elsewhere \end{cases}$$

(a) Determine k and verify that f(x) is a valid density function.

(b) Find the probability that a random error in measurement is less than 1/2.

QUESTION 8:

Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses with mean 0.2.

(a) What is the probability that a disk has exactly one missing pulse?
(b) What is the probability that a disk has at least two missing pulses?

QUESTION 9:

10 containers, all are of the same size, have lost their labels. It is known that 5 contains Desktop computers and 5 contains Laptops. If 5 are selected at random, what is the probability that
(a) All contain desktops.
(b) Three or more contains desktops.

QUESTION 10:

Two hundred passengers have made reservations for an airplane flight. If the probability that the passenger who has a reservation will not show up is 0.01. What is the probability that exactly 3 will not show up?

QUESTION 11:

The length of life for an automatic dishwasher is approximately normally distributed, with a mean of 3.5 years and a standard deviation of 1.0 years. If this type of dishwasher is guaranteed for 1 year, what is the probability that the dishwasher will require replacement?

QUESTION 12:

Five individuals from an animal population thought to be near extinction in a certain region have been caught, tagged, and released to mix into the population. After they have had an opportunity to mix, a random sample of 10 of these animals is selected. Let X is the number of tagged animals in the second sample. Suppose there are actually 25 animals of this type in the region. Find the probability of

(a) Exactly two of the animals in the second sample are tagged.
(b) At most two of the animals in the recapture sample are tagged.
(c) Determine the mean and variance of X.

QUESTION 13:

Mopeds (small motorcycles with an engine capacity below 50 cm^3) are very popular in Europe because of their mobility, ease of operation, and low cost. A rolling bench test for determining maximum vehicle speed was described. A normal distribution with mean value 46.8 km/h and standard deviation 1.75 km/h is postulated. Consider randomly selecting a single such moped.

(a) What is the probability that maximum speed is at most 50 km/h?
(b) What is the probability that maximum speed is at least 48 km/h?

QUESTION 14:

Let X denote the number of traps (defects of a certain kind) in a particular type of metal oxide semiconductor transistor, and the average number of traps was found to be 2. What is the probability that there are

(a) Exactly three traps
(b) At most three traps

QUESTION 15:

An automobile manufacturer has developed a new type of bumper, which is supposed to absorb impacts with less damage than previous bumpers. The manufacturer has used this bumper in a sequence of 25 controlled crashes against a wall, each at 10 mph, using one of its compact car models. Let X is the number of crashes that result in no visible damage to the automobile. The parameter to be estimated is the proportion of all such crashes that result in no damage. If X is observed to be 15, then estimate the proportion of no visible damage.

QUESTION 16:

(a) Can you reject a claim that the average age of members of parliament is at least 50 with the standard deviation of 3.1 years? If a random sample of 36 members has a mean age of 48.7, assume all ages are normally distributed. Take $\alpha = 0.01$.
(b) Calculate 99% confidence bounds of average age of parliament members

QUESTION 17:

A random sample of 100 deaths in United States in the past years showed an average lifespan of 71.8 years. Assuming the population standard deviation of 8.9 years, does this seem to indicate that the mean lifespan is greater than 70 years?

QUESTION 18:

The study "Smoking Abstinence Impairs Time Estimation Accuracy in Cigarette Smokers" aimed to explore whether the ability to estimate time accurately is affected when smokers stop smoking for 24 hours. They tested this by asking 20 smokers to estimate the duration of a 45-second interval after the period of abstinence. On average, these smokers estimated it to be 59.30 seconds, with a standard deviation of 9.84 seconds. The study aimed to determine if this average estimation significantly differed from the known duration of 45 seconds.

QUESTION 19:

In an experiment to measure the stiffness of a spring, the length of the spring under different loads was measured as follows:

X= Loads (1b)	**Y= length (in)**
03	10
05	12
06	15
09	18
10	20
12	22
15	27
20	30
22	32
28	34

1) Develop an estimated regression equation to predict
   (a) The length, given the weight on the spring.
   (b) The weight, given the length of the spring.
2) Taking length as a dependent variable, calculate
   (a) Total variation
   (b) The unexplained variation
   (c) Explained variation
   (d) Coefficient of determination, also interpret the result.
   (e) Correlation Coefficient