


National University of Computer and Emerging Sciences, Lahore Campus				
	Course Name:	Differential Equations	Course Code:	MT-1006
	Degree Program:	BCS, BDS, BSR	Semester:	Spring 2023
	Exam Duration:	3 Hours	Total Marks:	70
	Paper Date:	09-06-2023	Weight	50%
	Section:	All	Page(s):	2
	Exam Type:	Final		
Student : Name _____ Roll No. _____ Section: BDS-2A Instruction/Notes: Attempt all questions. Use of programmable calculators is not allowed. Exchange of stationary is strictly prohibited. Best of luck!				

Question no. 1: (CLO-01) (5+5 marks)

- a) Use the ratio test to find whether the given series converges or diverges. Also, find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x+2)^n}{n 2^n}$ and check the convergence or divergence at endpoints.
- b) Show that the p -series,

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$$

(p is a real constant) converges if $p > 1$, and diverges if $p \leq 1$.

Question no. 2: (CLO-02) (10 marks)

The sugar-making process contains a step called "inversion" in which cane sugar is dissolved in water. The rate at which the quantity of unconverted sugar is changing is proportional to the amount present. If 550 kg are present initially and 400 kg are present after 10 hours, how much is left after 15 hours? At what rate is the unconverted sugar changing after 15 hours?

Question no. 3: (CLO-03) (10 marks)

Determine whether the equation $(2xy + x)dy + (2y^2 + 2y + 4x^2)dx = 0$ is exact. If not, make it exact and solve the differential equation.

Question no. 4: (CLO-04) (10 marks)

Determine the solution of the given differential equation by using undetermined coefficients approach.

$$y'' - 3y' = 8e^{3x} + 4\sin x.$$

Question no. 5: (CLO-04) (1+3+6 marks)

Identify the given differential equation and find the general solution of given differential equation by using appropriate technique.

$$x^2 y'' + xy' - y = \ln x.$$

Question no. 6: (CLO-05) (10 marks)

Find the Fourier series for the function

$$f(x) = \begin{cases} x, & -\pi < x < 0, \\ \pi - x, & 0 < x < \pi. \end{cases}$$

Question no. 7: (CLO-05) (5+2+3 marks)

a) Check that the given set of functions is orthogonal on the indicated interval.

$$\left\{ 1, \sin \frac{m\pi}{p} x, \cos \frac{n\pi}{p} x \right\}; n = 1, 2, 3, \dots, m = 1, 2, 3, \dots; [-p, p].$$

b) Classify the given partial differential equation as hyperbolic, parabolic or elliptic.

$$2 \frac{\partial^2 u}{\partial x \partial y} - 3 \frac{\partial^2 u}{\partial y^2} + 2 \frac{\partial u}{\partial x} = 0.$$

c) Verify that the indicated function $u(x, t) = \ln(x^2 + t^2)$ is a solution of given partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0.$$

(OR)

Question no. 7: (CLO-05) (10 marks)

Use separation of variables technique to find the product solutions for the given partial differential equation.

$$c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}.$$