Probability & Statistics

Assignment 3

(1) Roll 12 dice simultaneously, and let X denotes the number of 6's that appear. Calculate the probability of getting 7, 8 or 9, 6's using R. (Try using the function pbinom; If we set S = {get a 6 on one roll}, P(S) = 1/6 and the rolls constitute Bernoulli trials; thus X ~ binom(size=12, prob=1/6) and we are looking for $P(7 \leq X \leq 9)$.

```
#Q1
sum(dbinom(7:9,12,1/6))
a=dbinom(7,size=12,prob=1/6)
b=dbinom(8,size=12,prob=1/6)
c=dbinom(9,size=12,prob=1/6)

result=a+b+c
print(result)

sum(pbinom(7:9,12,1/6))
```

Output:

```
> #Q1
> sum(dbinom(7:9,12,1/6))
[1] 0.001291758
> a=dbinom(7,size=12,prob=1/6)
> b=dbinom(8,size=12,prob=1/6)
> c=dbinom(9,size=12,prob=1/6)
>
> result=a+b+c
> print(result)
[1] 0.001291758
>
> sum(pbinom(7:9,12,1/6))
[1] 2.99983
```

(2) Assume that the test scores of a college entrance exam fits a normal distribution. Furthermore, the mean test score is 72, and the standard deviation is 15.2. What is the percentage of students scoring 84 or more in the exam?

```
#Q2
pnorm(84,72,15.2,lower.tail = FALSE)
```

Output:

```
> #Q2
> pnorm(84,72,15.2,lower.tail = FALSE)
[1] 0.2149176
```

(3) On the average, five cars arrive at a particular car wash every hour. Let X count the number of cars that arrive from 10AM to 11AM, then X ~Poisson($\lambda = 5$). What is probability that no car arrives during this time. Next, suppose the car wash above is in operation from 8AM to 6PM, and we let Y be the number of customers that appear in this period. Since this period covers a total of 10 hours, we get that Y ~ Poisson($\lambda = 5\times10 = 50$). What is the probability that there are between 48 and 50 customers, inclusive?

```
#Q3
dpois(0,5)
result=dpois(48,50)+dpois(49,50)+dpois(50,50)
print(result)
```

Output:

```
> #Q3
> dpois(0,5)
[1] 0.006737947
> result=dpois(48,50)+dpois(49,50)+dpois(50,50)
> print(result)
[1] 0.1678485
```

(4) Suppose in a certain shipment of 250 Pentium processors there are 17 defective processors. A quality control consultant randomly collects 5 processors for inspection to determine whether or not they are defective. Let X denote the number of defectives in the sample. Find the probability of exactly 3 defectives in the sample, that is, find P(X = 3).

```
#Q4
dhyper(3,17,233,5)
```

Output:

```
> #Q4
> dhyper(3,17,233,5)
[1] 0.002351153
```

(5) A recent national study showed that approximately 44.7% of college students have used Wikipedia as a source in at least one of their term papers. Let X equal the number of students in a random sample of size n = 31 who have used Wikipedia as a source.

(a) How is X distributed?

(b) Sketch the probability mass function.

(c) Sketch the cumulative distribution function.

(d) Find mean, variance and standard deviation of X.

```
#Q5
pro=dbinom(1:31,31,447/1000)
e<-data.frame(x=1:31,pro)
print(e)
plot(e,type="l")
```

z=pbinom(1:31,31,0.447)

plot(1:31,z,type="l")

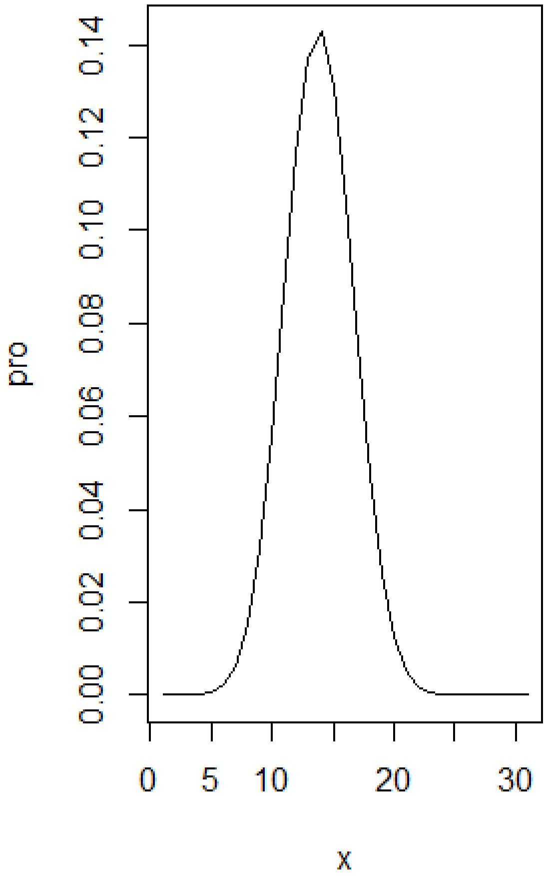
print(mean(pro))

print(var(pro))

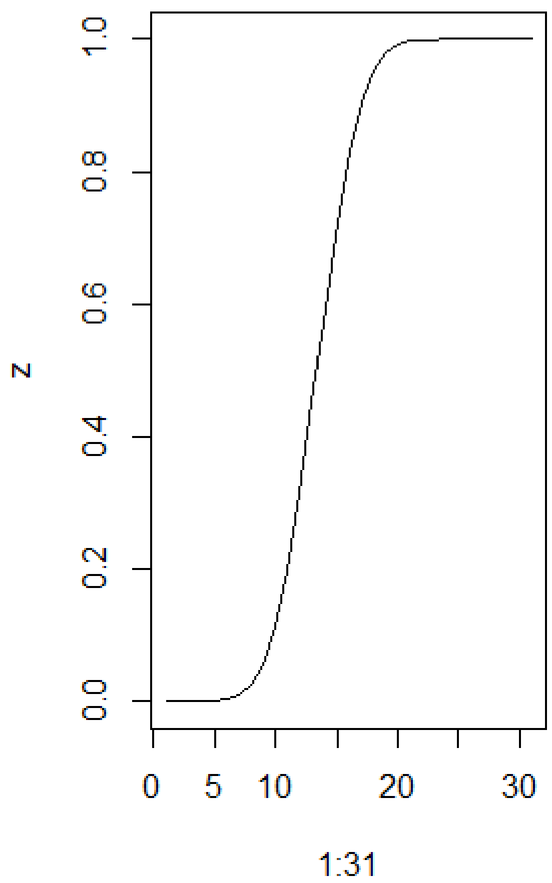
print(sd(pro))

Output:

```
> #Q5
> pro=dbinom(1:31,31,447/1000)
> e<-data.frame(x=1:31,pro)
> print(e)
    x          pro
1   1 2.651082e-07
2   2 3.214377e-06
3   3 2.511632e-05
4   4 1.421138e-04
5   5 6.203153e-04
6   6 2.172786e-03
7   7 6.272510e-03
8   8 1.521055e-02
9   9 3.142047e-02
10 10 5.587504e-02
11 11 8.622373e-02
12 12 1.161604e-01
13 13 1.372305e-01
14 14 1.426190e-01
15 15 1.306524e-01
16 16 1.056088e-01
17 17 7.532248e-02
18 18 4.735464e-02
19 19 2.618995e-02
20 20 1.270189e-02
21 21 5.378041e-03
22 22 1.975986e-03
23 23 6.250013e-04
24 24 1.684000e-04
25 25 3.811382e-05
26 26 7.109560e-06
27 27 1.064220e-06
28 28 1.228898e-07
29 29 1.027594e-08
30 30 5.537484e-10
31 31 1.443887e-11
> plot(e,type="l")
```




```
> z=pbinom(1:31,31,0.447)
> plot(1:31,z,type="l")
```




```
> print(mean(pro))
[1] 0.03225806
> print(var(pro))
[1] 0.00230823
> print(sd(pro))
[1] 0.04804404
```