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UCS654 Predictive Analytics using Statistics Assignment

Parameter Estimation

Q1. $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$X_1, X_2, X_3, \ldots X_m$ are sample of size m.

$L(X_1, X_2, X_3, \ldots, X_m) = f(x_1) \cdot f(x_2) \cdots f(x_m)$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right)\cdots\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_m-\mu)^2}{2\sigma^2}}\right)$$

taking ln on both sides.

$$\ln(L) = -\frac{m}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{m}\left|\frac{(x_i-\mu)^2}{2\sigma^2}\right| \quad \text{—} ①$$

$$\frac{d\ln(L)}{d\mu} = 0 + \sum_{i=1}^{m} -\left(\frac{2(x_i-\mu)}{2\sigma^2}\right) = 0$$

$$\sum_{i=1}^{m}(x_i - \mu) = 0$$

$$m\bar{x} - n\mu = 0$$

$$\boxed{\bar{x} = \mu}$$

$\boxed{\theta_1 = \bar{x}}$ is therefore sample mean.

$$\frac{d\ln(L)}{d\sigma^2} = -\frac{m}{2\sigma^2} + \sum_{i=1}^{m} -\frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$m = \sum_{i=1}^{m}\left(\frac{x_i-\mu}{2\sigma^2}\right) = 0$$

$$\sigma^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i-\mu)^2$$

$$\boxed{\theta_2 = \frac{1}{m}\sum_{i=1}^{m}(x_i-\mu)^2}$$

② Binomial Distribution $\quad {}^{n}C_{xi}\left(\theta^{xi}(1-\theta)^{n-xi}\right)$

$$L = \prod_{i=1}^{n} {}^{n}C_{xi}\left(\theta^{xi}(1-\theta)^{n-xi}\right)$$

taking log on both sides

$$\log L = \sum_{i=1}^{m} \left[\log({}^{n}C_{xi}) + \log \theta^{xi} + \log(1-\theta)^{n-xi}\right]$$

$$\log L = \sum_{i=1}^{n} \log({}^{n}C_{xi}) + \log\theta \sum_{i=1}^{n} xi + \log(1-\theta)\sum_{i=1}^{n}(n-xi)$$

$$\frac{d\log(L)}{d\theta} = 0$$

$$\frac{1}{\theta}\sum xi - \frac{1}{1-\theta}\sum(n-xi) = 0$$

$$\frac{1}{\theta}\sum xi - \frac{n^2}{1-\theta} + \frac{1}{1-\theta}\sum xi = 0$$

$$\frac{1}{\theta(1-\theta)}\sum xi = \frac{n^2}{1-\theta}$$

$$\Rightarrow \boxed{\theta = -\frac{\sum xi}{n^2}}$$