Matrix Assignment

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Problem Statement **- Show that the diagonals of a parallelogram divide it into four triangles of equal area.**

1. **AB=DC and AD=BC**
2. **O is the midpoint of AC and BD**

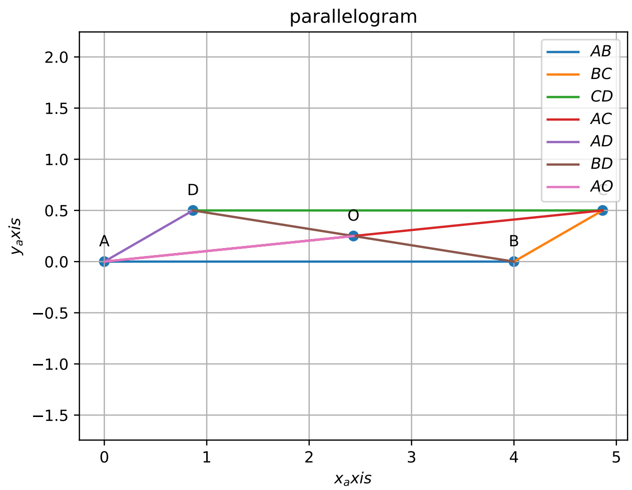

Figure 1: Parallelogram ABCD With centre O

Solution

Part 1

Construction

The input parameters are the lengths of AB and AD and angle between AB and ADs

Symbol	Value	Description
d	4	AB
r	3	AD
θ	$\pi/6$	$\angle$A
D	$r\begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}$	Point **D**
C	**B+D**	Point **C**

Part 2

The diagnols of a parallelogram bisect each other.

$$\mathbf{O} - \mathbf{A} = \mathbf{C} - \mathbf{O} \tag{1}$$

$$\mathbf{O} - \mathbf{D} = \mathbf{B} - \mathbf{O} \tag{2}$$

$$(\mathbf{C} - \mathbf{A}) = (\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{B}) \tag{3}$$

$$(\mathbf{B} - \mathbf{D}) = (\mathbf{B} - \mathbf{A}) - (\mathbf{C} - \mathbf{B}) \tag{4}$$

$$(\mathbf{O} - \mathbf{A}) = (\mathbf{C} - \mathbf{A})/2 \tag{5}$$

$$(\mathbf{O} - \mathbf{D}) = (\mathbf{B} - \mathbf{D})/2 \tag{6}$$

In parallelogram ABCD

1. **consider $\triangle$ AOB**

Area of triangle is given as

$$\frac{1}{2}|(\mathbf{O} - \mathbf{A} \times \mathbf{B} - \mathbf{O})| \tag{7}$$

$$\frac{1}{2}|(\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{B})/2 \times (\mathbf{B} - \mathbf{A}) - (\mathbf{C} - \mathbf{B})/2| \tag{8}$$

$$\textbf{Area of } \triangle\mathbf{AOB} = \frac{1}{4}|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \tag{9}$$

2. **consider $\triangle$ BOC**

Area of triangle is given as

$$\frac{1}{2}|(\mathbf{B} - \mathbf{O}) \times (\mathbf{C} - \mathbf{O})| \tag{10}$$

$$\frac{1}{2}|((\mathbf{B} - \mathbf{A}) - (\mathbf{C} - \mathbf{B}))/2 \times ((\mathbf{B} - \mathbf{A}) + (\mathbf{C} - \mathbf{B}))/2| \tag{11}$$

$$\frac{1}{4}|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \tag{12}$$

$$\textbf{Area of } \triangle\mathbf{BOC} = \frac{1}{4}|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{B})| \tag{13}$$

3. **consider $\triangle$ COD**

Area of triangle is given as

$$\frac{1}{2}|(\mathbf{C}-\mathbf{O})\times(\mathbf{D}-\mathbf{O})| \tag{14}$$

$$\frac{1}{4}|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{B})| \tag{15}$$

$$\textbf{Area of }\triangle\textbf{COD} = \frac{1}{4}|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{B})| \tag{16}$$

4. **consider △ DOA**

Area of triangle is given as

$$\frac{1}{2}|(\mathbf{A}-\mathbf{O})\times(\mathbf{D}-\mathbf{O})| \tag{17}$$

$$\frac{1}{2}|(-(\mathbf{B}-\mathbf{A})-(\mathbf{C}-\mathbf{B}))/2\times(-(\mathbf{B}-\mathbf{A})+(\mathbf{C}-\mathbf{B}))/2| \tag{18}$$

$$\frac{1}{4}|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{B})| \tag{19}$$

$$\textbf{Area of }\triangle\textbf{AOD} = \frac{1}{4}|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{B})| \tag{20}$$

Hence from the above results we can conclude that

ar(△AOB)=ar(△BOC)=ar(△COD)=ar(△DOA)
= 1/4*|B-A × C-B|

Hence We prooved that the diagonals of a parallelogram divide it into four triangles of equal area.