Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

AI5030: PROBABILITY AND STOCHASTIC PROCESSES

QUIZ 3

DATE: 11 SEPTEMBER 2024

Question	1(a)	1(b)	2(a)	2(b)	Total
Marks Scored					

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.
Assume that all random variables appearing in the questions below are defined with respect to $\mathscr{F}$.

1. Suppose that two batteries are chosen simultaneously and uniformly at random from the following group of 12 batteries : 3 new, 4 used (yet working), 5 defective. You may assume that all batteries within a particular group are identical.
   Let X be the number of new batteries chosen, and let Y be the number of used batteries chosen.

   (a) **(2 Marks)**
   Determine the joint PMF of X and Y.
   Solution: Note that $X + Y \leq 2$ with probability 1. Furthermore,

$$p_{X,Y}(x,y) = \begin{cases} \frac{\binom{5}{2}}{\binom{12}{2}}, & x=0, y=0, \\ \frac{\binom{4}{1}\cdot\binom{5}{1}}{\binom{12}{2}}, & x=0, y=1, \\ \frac{\binom{4}{2}}{\binom{12}{2}}, & x=0, y=2, \\ \frac{\binom{3}{1}\cdot\binom{5}{1}}{\binom{12}{2}}, & x=1, y=0, \\ \frac{\binom{3}{1}\cdot\binom{4}{1}}{\binom{12}{2}}, & x=1, y=1, \\ \frac{\binom{3}{2}}{\binom{12}{2}}, & x=2, y=0, \\ 0, & \text{otherwise.} \end{cases}$$

   (b) **(1 Mark)**
   Compute $\mathbb{P}(\{X = Y\})$.
   Solution: We have

$$\mathbb{P}(\{X = Y\}) = p_{X,Y}(0,0) + p_{X,Y}(1,1) = \frac{22}{66} = \frac{1}{3}.$$

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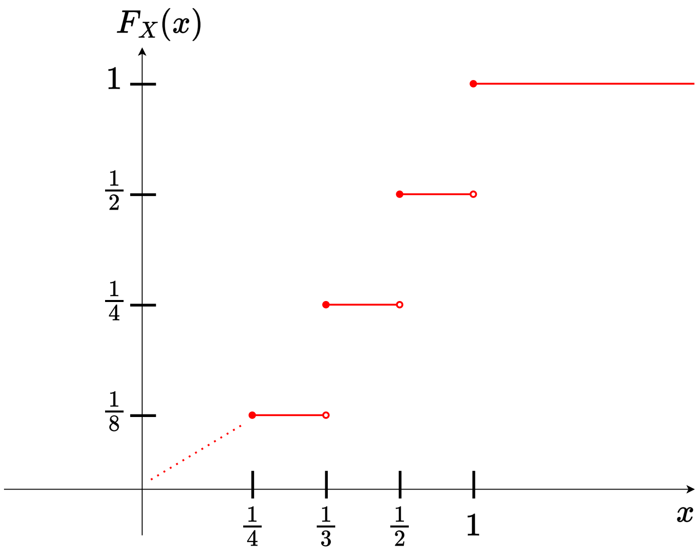
2. Suppose that X is a random variable whose CDF is given by

$$F_X(x) = \sum_{n=1}^{\infty} \frac{1}{2^n} \mathbf{1}_{\left[\frac{1}{n},\,+\infty\right)}(x), \qquad x \in \mathbb{R}.$$

(a) **(1 Mark)**
Sketch the above CDF (roughly).
Solution: A rough sketch of the CDF is presented below figure (not to scale).



(b) **(1 Mark)**
Let $\mathbb{P}_X$ denote the probability law of X. Determine $\mathbb{P}_X\left(\left[0,\, \frac{1}{2}\right)\right)$.
Solution: We have

$$\begin{aligned}
\mathbb{P}_X\left(\left[0,\, \frac{1}{2}\right)\right) &= \mathbb{P}_X\left(\left[0,\, \frac{1}{2}\right]\right) - \mathbb{P}_X\left(\left\{\frac{1}{2}\right\}\right) \\
&= F_X\left(\frac{1}{2}\right) - \mathbb{P}\left(\left\{X = \frac{1}{2}\right\}\right) \\
&= \frac{1}{2} - \frac{1}{4} \\
&= \frac{1}{4}.
\end{aligned}$$