



## Al5030: PROBABILITY AND STOCHASTIC PROCESSES

Quiz 4

**DATE: 16 OCTOBER 2024** 

Question	1	2(a)	2(b)	2(c)	Total
Marks Scored					

## **Instructions:**

- Fill in your name and roll number on each of the pages.
- You may use any result covered in class directly without proving it.
- Unless explicitly stated in the question, DO NOT use any result from the homework without proof.

Fix a probability space  $(\Omega, \mathscr{F}, \mathbb{P})$ .

Assume that all random variables appearing in the questions below are defined with respect to  $\mathscr{F}$ .

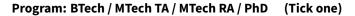
## 1. (2 Marks)

Let N be a discrete random variable taking values in  $\mathbb N$  and having PMF  $p_N$ . Construct an example for  $p_N$  under which  $\mathbb E[N]=+\infty$  and

$$\lim_{n \to \infty} n \, \mathbb{P}(\{N > n\}) > 0.$$

Show your working clearly.

Name: Roll Number: Department:





- 2. Suppose that X(0), X(1), and X(2) are independent, discrete random variables defined with respect to  $\mathscr{F}$ . For each  $i \in \{0,1,2\}$ , the random variable X(i) takes possible values 1 and 2 with probabilities  $p_i$  and  $1-p_i$  respectively.
  - (a) (1 Mark) Show, from first principles, that Y=X(X(0)) is a random variable with respect to  $\mathscr{F}.$
  - (b) (1 Mark) Derive the PMF of Y.
  - (c) **(1 Mark)** Compute  $\mathbb{E}[Y]$ .