Assignment 1

Due: 27/8/2024 11.59 pm

z=(abcde) are the last five digits of your roll number. For example if your roll no is ee21btech11027, then x=11027 and a=1, b=1, c=0,d=2 and e=7.

1. Is $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, in the span of the two vectors $\begin{bmatrix} 4 \\ d \\ 2 \end{bmatrix}$ and $\begin{bmatrix} e \\ -1 \\ 3 \end{bmatrix}$? If so, write it as a linear combination of these two vector. If not, show it is not.
2. Consider the matrix

$$\boldsymbol{A} = \begin{bmatrix} a & 0 & 2 & e & b & -1 \\ 2 & -1 & -2 & 5 & 4 & 0 \\ d & -1 & 0 & 8 & 5 & -1 \\ 4 & -1 & 2 & c & 8 & 0 \end{bmatrix}$$

(a) What is the dimension of the column space of $\boldsymbol{A}$?
(b) What is the dimension of the null space of $\boldsymbol{A}$?
(c) Find a basis for the column space of $\boldsymbol{A}$.
(d) Find a basis for the null space of $\boldsymbol{A}$.

3. Two vectors are said to be collinear when one can be written as a scalar multiple of the other. Consider two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ that are not collinear. Consider a vector w that does not belong to the linear span of $\boldsymbol{u}$ and $\boldsymbol{v}$. Prove that $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ are linearly independent.
4. A matrix is said to be upper triangular if $a_{ij} = 0$ for $i > j$. Consider a generic 3 -by- 3 upper triangular matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

(a) If $a_{11} = b, a_{22} = c$, and $a_{33} = e$ find the solution to $Ax = 0$.
(b) If either $a_{11} = 0$, or $a_{22} = 0$ or $a_{33} = 0$, then prove that the columns are linearly dependent. (Consider all three cases separately.)
(c) If $a_{22} = 0$, find a nonzero element in the nullspace of A.

5. Show that the vector space of polynomials of degree less or equal to d is of dimension d + 1. (For instance, the space of quadratic polynomials is of dimension 3.) Use this result to show that the vector space of all real functions cannot have finite dimension.

6. Prove that if V and W are two-dimensional subspaces of $\mathbb{R}^3$ then V and W must have a non-zero vector in common.

7. Suppose **A** is a m × n matrix. Give the conditions on the rank of **A** such that left and right inverses exist, respectively. Can the left and right inverses of a matrix exist simultaneously ?

8. If the product of two matrices is the zero matrix, **AB** = 0, show that the column space of **B** is contained in the null space of **A**.

9. (Fredlholm's alternative) For any A and b, one and only one of the following systems has a solution: (1) $\mathbf{Ax} = \mathbf{b}$ (2) $\{\mathbf{A}^T \mathbf{y} = 0, \mathbf{y}^T \mathbf{b} \neq 0\}$. Show that it is contradictory for (1) and (2) both to have solutions.

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