Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

AI5030: Probability and Stochastic Processes

Quiz 5

Date: 14 November 2024

Question	1(a)	1(b)	2	Total
Marks Scored				

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.
Assume that all random variables appearing in the questions below are defined with respect to $\mathscr{F}$.

1. Let X and Y be jointly continuous, with

$$f_{X,Y}(x,y) = \begin{cases} cy, & -1 \le x \le 1,\ 0 \le y \le |x|, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) **(1 Mark)**
   Determine the constant c.

   (b) **(2 Marks)**
   Compute $\mathrm{Cov}(X, Y)$.

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2. **(2 Marks)**

Let $X_1, X_2, \ldots \overset{\text{i.i.d.}}{\sim}$ Exponential(1), and let $N \sim$ Geometric$(1/2)$ be independent of $\{X_1, X_2, \ldots\}$. Let $S_N = \sum_{i=1}^{N} X_i$. Compute $\mathbb{E}[S_N \mathbf{1}_{\{N=4\}}]$. Justify your steps clearly.