Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

AI5030: Probability and Stochastic Processes

Quiz 1

Date: 14 August 2024

Question	1	2	Total
Marks Scored			

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

1. **(2 Marks)**

   Show that the collection of all finite-length strings of natural numbers is countably infinite.
   In your proof, if needed, you may use the fact that $\mathbb{N}^d$ is countably infinite for every $d \in \mathbb{N}$.

2. **(3 Marks)**

   Let Ω be a sample space (finite, countably infinite, or uncountable).
   Fix $n \in \mathbb{N}$. Let $A_1, \ldots, A_n$ be a *partition* of Ω, i.e., $A_1, \ldots, A_n$ satisfy the following properties:

   - $A_i \subseteq \Omega$ for each $i \in \{1, \ldots, n\}$.
   - $A_i \cap A_j = \emptyset$ for all $i \neq j$.
   - $\bigcup_{i=1}^{n} A_i = \Omega$.

   Let $\mathscr{C} = \{A_1, \ldots, A_n\}$, and let $\mathscr{F}$ denote the smallest σ-algebra constructed from the sets in $\mathscr{C}$. Show that $|\mathscr{F}| = 2^n$.