Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

AI5030: Probability and Stochastic Processes

Quiz 4

Date: 16 October 2024

Question	1	2(a)	2(b)	2(c)	Total
Marks Scored					

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.
Assume that all random variables appearing in the questions below are defined with respect to $\mathscr{F}$.

1. **(2 Marks)**

   Let N be a discrete random variable taking values in $\mathbb{N}$ and having PMF p_N.
   Construct an example for p_N under which $\mathbb{E}[N] = +\infty$ and

   $$\lim_{n\to\infty} n\,\mathbb{P}(\{N > n\}) > 0.$$

   Show your working clearly.

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2. Suppose that $X(0)$, $X(1)$, and $X(2)$ are independent, discrete random variables defined with respect to $\mathscr{F}$.
For each $i \in \{0, 1, 2\}$, the random variable $X(i)$ takes possible values 1 and 2 with probabilities p_i and $1 - p_i$ respectively.

   (a) **(1 Mark)**
   Show, from first principles, that $Y = X(X(0))$ is a random variable with respect to $\mathscr{F}$.

   (b) **(1 Mark)**
   Derive the PMF of Y.

   (c) **(1 Mark)**
   Compute $\mathbb{E}[Y]$.