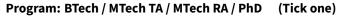
Name: Roll Number: Department:





Al5030: Probability and Stochastic Processes

Quiz 1

DATE: 14 AUGUST 2024

Question	1	2	Total
Marks Scored			

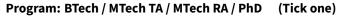
Instructions:

- Fill in your name and roll number on each of the pages.
- You may use any result covered in class directly without proving it.
- Unless explicitly stated in the question, DO NOT use any result from the homework without proof.

1. (2 Marks)

Show that the collection of all finite-length strings of natural numbers is countably infinite. In your proof, if needed, you may use the fact that \mathbb{N}^d is countably infinite for every $d \in \mathbb{N}$.

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2. (3 Marks)

Let Ω be a sample space (finite, countably infinite, or uncountable). Fix $n \in \mathbb{N}$. Let A_1, \ldots, A_n be a partition of Ω , i.e., A_1, \ldots, A_n satisfy the following properties:

- $A_i \subseteq \Omega$ for each $i \in \{1, \ldots, n\}$.
- $A_i \cap A_j = \emptyset$ for all $i \neq j$.
- $\bigcup_{i=1}^n A_i = \Omega$.

Let $\mathscr{C} = \{A_1, \dots, A_n\}$, and let \mathscr{F} denote the smallest σ -algebra constructed from the sets in \mathscr{C} . Show that $|\mathscr{F}| = 2^n$.