Name:
Roll Number:
Department:
Program: BTech / MTech TA / MTech RA / PhD (Tick one)

AI5030: Probability and Stochastic Processes

Quiz 3

Date: 11 September 2024

Question	1(a)	1(b)	2(a)	2(b)	Total
Marks Scored					

Instructions:

- **Fill in your name and roll number on each of the pages.**
- **You may use any result covered in class directly without proving it.**
- **Unless explicitly stated in the question, DO NOT use any result from the homework without proof.**

Fix a probability space $(\Omega, \mathscr{F}, \mathbb{P})$.
Assume that all random variables appearing in the questions below are defined with respect to $\mathscr{F}$.

1. Suppose that two batteries are chosen simultaneously and uniformly at random from the following group of 12 batteries : 3 new, 4 used (yet working), 5 defective. You may assume that all batteries within a particular group are identical. Let X be the number of new batteries chosen, and let Y be the number of used batteries chosen.

   (a) **(2 Marks)**
   Determine the joint PMF of X and Y.

   (b) **(1 Mark)**
   Compute $\mathbb{P}(\{X = Y\})$.

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2. Suppose that X is a random variable whose CDF is given by

$$F_X(x) = \sum_{n=1}^{\infty} \frac{1}{2^n} \mathbf{1}_{\left[\frac{1}{n}, +\infty\right)}(x), \qquad x \in \mathbb{R}.$$

(a) **(1 Mark)**
Sketch the above CDF (roughly).

(b) **(1 Mark)**
Let $\mathbb{P}_X$ denote the probability law of X. Determine $\mathbb{P}_X\left(\left[0, \frac{1}{2}\right)\right)$.