Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SURECE TENSION

SYNOPSIS:

Surface tension: $T = \frac{F}{l}$ or $F = Tl$

(i) Surface tension force on a wire of length l, placed on water F = 2Tl

(ii) Surface tension force on circular disc of radius r $F = 2\pi rT$

(iii) Surface tension force on a ring of radius R $F = 2(2\pi R \times T)$

Surface means a thin layer of approximately 10-15 molecular diameters

Surface energy: A molecule in the surface has greater potential energy than a molecule which inside the liquid. The extra energy that a surface film has is called the surface energy

Work done in increasing the area of the surface film, $W = T\Delta A$

(i) W.d in breaking a liquid drop $W = T\left[n \times 4\pi r^2 - 4\pi R^2\right]$ has $r = \frac{R}{n^{1/3}}$

(ii) Work done in blowing a soap bubble from zero to radius R $W = T \times 2\left(4\pi R^2\right) = 8\pi TR^2$

Pressure difference:

(i) In a liquid drop $P_i - P_0 = \frac{2T}{R}$

(ii) In a soap bubble $P_i - P_0 = \frac{4T}{R}$

(iii) In general for one free surface $P_i \sim P_0 = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Angle of contact:

(i) Angle of contact is the property of the material in contact

(ii) It decreases with increase in temperature

(iii) It decreases with the addition of soap and detergent

(iv) It increases with the addition of sugar and salt

(v) Angle of contact of water with glass is 8° and for mercury in glass is 140°

Capillary rise: If r is the radius of the capillary tube, then $h = \frac{2T\cos\theta}{r\rho g}$

In terms of radius of curvature $R = \frac{r}{\cos\theta}$, $h = \frac{2T}{R\rho g}$

In case of square tube of side a $h = \frac{4T\cos\theta}{a\rho g}$

Capillary tube insufficient length:

If h is the free rise of the liquid and l is the length of tube, being $l < h$ then $hR = lR'$

Here R' is the radius of curvature of the meniscus of the liquid at the top of the tube

The liquid will not splitout. As $l < h$, so $R' > R$

Apparent angle of contact: If θ and θ' are the true and apparent angle of contact then

$$\cos\theta' = \frac{l}{h}\cos\theta$$

Force required to pull the plates apart having some liquid between them:

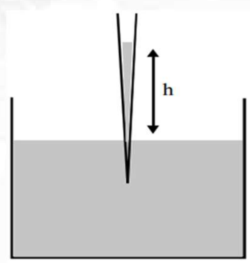
$$F = (P_0 - P_i)A = \frac{2TA}{d}$$

Here d is the separation between the plates and A is the area of each plate

SINGLE OPTION QUESTIONS

Topic: Surface energy, Surface tension, force of cohesion and Adhesion

01. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)



1. $\frac{2S}{b\rho g}\cos(\theta-\alpha)$ **2.** $\frac{2S}{b\rho g}\cos(\theta+\alpha)$ **3.** $\frac{2S}{b\rho g}\cos(\theta-\alpha/2)$ **4.** $\frac{2S}{b\rho g}\cos(\theta+\alpha/2)$

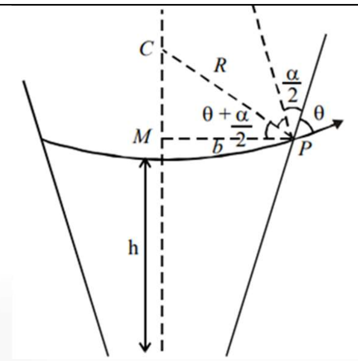
Key 4

Sol **Here R be the radius of the meniscus formed with a constant angle θ and P_0 be the atmosphere pressure**

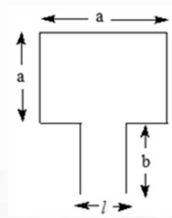
In $\Delta PCM \cos\left(\theta+\frac{\alpha}{2}\right) = \frac{b}{R} \Rightarrow R = \frac{b}{\left(\theta+\frac{\alpha}{2}\right)}$

Also $\left(\rho_0 - \frac{2S}{R}\right) + h\rho g = P_0 \Rightarrow h\rho g = \frac{2S}{R}$

$$h = \frac{2S}{R\rho g} = \frac{2S}{b\rho g}\cos(\theta+\alpha/2)$$



02. A number o little droplet of a liquid of surface tension T, density ρ, all of same radius, r, combine to form a single drop of radius R. If the total energy released in this process is converted into kinetic energy of the bigger drop, velocity of the bigger drop is



1. $\sqrt{\frac{3T(R-r)}{Rr\rho}}$ **2.** $\sqrt{\frac{6T(R-r)}{Rr\rho}}$ **3.** $\sqrt{\frac{T(R-r)}{Rr}}$ **4.** $\sqrt{\frac{2T(R-r)\rho}{R}}$

Key 2

Sol

$R^3 = Nr^3 \text{ so } N = \frac{R^3}{r^3}$

Energy released $= T \times 4N\left(Nr^2 - R^2\right) = 4\pi T\left(\frac{R^3}{r^3}r^2 - R^2\right) = 4\pi TR^2\left(\frac{R-r}{r}\right)$

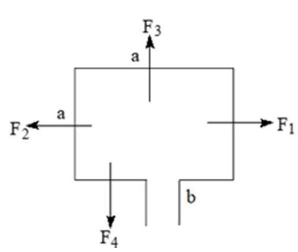
So, $\frac{1}{2}mv^2 = 4\pi TR^2\left(\frac{R-r}{r}\right) \Rightarrow V = \sqrt{\frac{6T(R-r)}{Rr\rho}}$

03. A light open rigid wire frame floats on the surface of water as shown in figure. What force will act on the frame, immediately after some soap solution is dropped inside it? α_1 and α_2 are the surface tension of water and soap respectively $(\alpha_1 > \alpha_2)$

1. zero **2.** $(\alpha_1 - \alpha_2)l$ **3.** $(\alpha_1 + \alpha_2)(4a + 2b - l)$ **4.** $(\alpha_1 - \alpha_2)(4a + 2b - l)$

Key 2

Sol F_1 and F_2 are balanced



Resultant force = F_3 and $F_4 = \alpha_1 I - \alpha_2 I \Rightarrow = (\alpha_1 - \alpha_2)l$

04. The surface tension of soap solution is $3.5\times10^{-2}\ \text{Nm}^{-1}$. The amount of work done required to increase the radius of soap bubble from 10 cm to 20cm is _____ $\times10^{-4}$ J.

1. 264 **2. 462** **3. 384** **4.130**

Key 1

Sol Initial radius, r_1 = 10 cm Final radius, $r_2 = 20$ cm

$$W = T.(\Delta A) \Rightarrow W = 2T\left(4\pi\left(r_2^2 - r_1^2\right)\right) \Rightarrow W = 8\times\frac{22}{7}\times3.5\times10^{-2}\left(\left(20\times10^{2}\right)^2 - \left(10\times10^{-2}\right)^2\right)$$

$$= 8\times\frac{22}{7}\times3.5\times10^{-2}\times300\times10^{-4} \Rightarrow W = 264\times10^{-4}\ \text{J}$$

05. A mercury drop of radius 10^{-3}m is broken into 125 equal size droplets. Surface tension of mercury is $0.45\ \text{Nm}^{-1}$. The gain in surface energy is:

1) 5×10^{-5} J 2) 2.26×10^{-5} J 3) 17.5×10^{-5} J 4) 28×10^{-5} J

Key 2

Sol Given, radius of mercury drop, r = 10^{-3} m

Initial surface energy E_1 = surface tension $\times$ Area

$$E_1 = 0.45\times4\pi r^2 = 0.45\times4\pi\left(10^{-3}\right)^2$$

Initial volume = $n\times\text{final volume}$

$$\frac{4}{3}\pi\left(10^{-3}\right)^3 = 125\times\frac{4\pi}{3}R_{new}^3 \quad \frac{4}{3}\pi\left(10^{-3}\right)^3 = 125\times\frac{4\pi}{3}R_{new}^3$$

So, final surface energy, $E_2 = 0.45\times125\times4\pi\left(\frac{10^{-3}}{5}\right)^2$

Increase in energy = $E_2 - E_1$ Increase in energy $= 0.45\times4\pi\times\left(10^{-3}\right)^2\left[\frac{125}{25}-1\right]$

$$= 4\times0.45\times4\pi\times10^{-6} = 2.26\times10^{-5}\ \text{J}$$

06. If 1000 droplets of water of surface tension 0.07 N/m. having same radius 1mm each, combine to from a single drop. In the process the released surface energy is:

$\left(\text{Take}\ \pi = \frac{22}{7}\right)$

1) 7.92×10^{-6} J 2) 7.92×10^{-4} J 3) 9.68×10^{-4} J 4) 8.8×10^{-5} J

Key 2

Sol We have $V_f = V_i \Rightarrow \frac{4}{3}\pi r_f^3 = 1000\times\frac{4}{3}\pi r_i^3 \Rightarrow r_f^3 = 1000 r_i^3 \quad \Rightarrow r_f = 10 r_i$

So, released energy = Initial surface energy – final surface energy

$$= 1000\times T\times4\pi r_i^2 - T\times4\pi r_f^2 \quad = 4\pi T\left(1000 r_i^2 - r_f^2\right) \quad = 4\pi\times0.07\left(1000 r_i^2 - 100 r_i^2\right)$$

$$= 4\pi\times0.07\times900 r_i^2 \quad = 4\pi\times63\times10^{-6} = 7.92\times10^{-4}\ \text{J}$$

07. Surface tension of a soap bubble is $2.0\times10^{-2}\ \text{Nm}^{-1}$. Work done to increase the radius of soap bubble from 3.5 cm to 7cm will be: $\left[\text{Take } \pi=\frac{22}{7}\right]$

1) 0.72×10^{-4} J 2) 5.76×10^{-4} J 3) 18.48×10^{-4} J 4) 9.24×10^{-4} J

Key **3**

Sol Given, surface tension of soap bubble, $T=2\times10^{-2}\ \text{Nm}^{-1}$

Surface area of soap bubble $=2\times4\pi R^2$

Work done = change in surface area of soap bubble $\times T$

$=T\times8\pi\times\left(R_2^2-R_1^2\right)$ $=2\times10^{-2}\times8\pi\left(7^2-(3.5)^2\right)\times10^{-4}$

$=2\times10^{-2}\times8\times\frac{22}{7}\times49\times\frac{3}{4}\times10^{-4}$ $=18.48\times10^{-4}$ J

08. Given below are two statements: One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Clothes containing oil or grease stains cannot be cleaned by water wash.

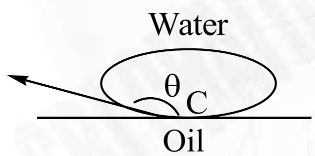
Reason (R): Because the angle of contact between the oil/grease and water is obtuse. In the light of the above statements, choose the correct answer from the option given below.

1)Both A and R are true and R is the correct explanation of A

2) Both A and R are true but R is not the correct explanation of A

3)A is true but R is false 4) A is true but R is true

Key **1**



Sol

As $\theta_C>90\Rightarrow$ So adhesive force will dominate between water and oil. And water molecule will stick to oil rather than removing it.

09. A water drop of radius 1 cm is broken into 729 equal droplets. If surface tension of water is 75 dyne/cm, then the gain in surface energy upto first decimal place will be $(\text{Given } \pi=3.14)$

1) 8.5×10^{-4}J 2) 8.2×10^{-4}J 3) 7.5×10^{-4}J 4) 5.3×10^{-4}J

Key **3**

Sol Given that radius of water drop, $R=1\text{cm}=10^{-2}$ m

Surface tension of water $T=75\ \text{dyne/cm}=\frac{75\times10^{-5}}{10^{-2}}$ $=75\times10^{-3}$ N / m

From volume conservation $\frac{4}{3}\pi R^3=729\times\frac{4}{3}\pi r^3$ $\Rightarrow R=9r\Rightarrow r=\frac{R}{9}=\frac{10^{-2}}{9}$ m

Initial surface energy, $E_1=T\times4\pi R^2$ Final surface energy, $E_2=T\times\left[4\pi r^2\times729\right]$

Change in surface energy $= E_2 - E_1 = T\left[4\pi r^2 \times 729 - 4\pi R^2\right]$

$$= 4\pi \times 75 \times 10^{-3}\left[\left(\frac{10^{-2}}{9}\right)^2 \times 729 - (9)^2 \times \left(\frac{10^{-2}}{9}\right)^2\right]$$

$$= 4\pi \times 75 \times 10^{-3}\left[(9-1)\times 10^{-4}\right] \quad = 4\pi \times 75 \times 10^{-3} \times 8 \times 10^{-4} = 7.5 \times 10^{-4}\text{ J}$$

10. A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension $7.5\times 10^{-4}\text{ Ncm}^{-1}$. The radius of drop in cm will be: (Take g = 10 m/s^2)

1) $\frac{15}{\sqrt{2\rho-\sigma}}$ 2) $\frac{15}{\sqrt{\rho-\sigma}}$ 3) $\frac{3}{3\sqrt{\rho-\sigma}}$ 4) $\frac{3}{20\sqrt{2\rho-\sigma}}$

Key **1**

Sol As liquid drop is in equilibrium. So $F_{net} = 0$

Boyant force + surface tension = mg $\sigma\frac{V}{2}g + 2\pi RT = \rho Vg$

$$\Rightarrow 2\pi RT = \frac{(2\rho-\sigma)}{2}\frac{4}{3}\pi R^3 g\left[\because V = \frac{4}{3}\pi R^3\right]s \Rightarrow R^3 = \frac{3T}{(2\rho-\sigma)g} \Rightarrow R = \sqrt{\frac{3\times 7.5\times 10^{-2}\text{ N}-\text{m}^{-1}}{(2\rho-\sigma)\times 10}}$$

$$\Rightarrow R = \frac{3}{20\sqrt{(2\rho-\sigma)}}\text{m} = \frac{15}{\sqrt{2\rho-\sigma}}\text{cm}$$

11. A water drop of diameter 2 cm in broken into 64 equal droplets. The surface tension of water is 0.075 N/m. In This process the gain in surface energy will be:

1) 2.8×10^{-4} J 2) 1.5×10^{-3} J 3) 1.9×10^{-4} J 4) 9.4×10^{-5} J

Key 1

Sol $E_i = T\times 4\pi r_i^2$ We have, $V_i = V_f \Rightarrow \frac{4}{3}\pi r_i^3 = 64\times\frac{4}{3}\pi r_f^3 \Rightarrow r_i^3 = 64 r_f^3 \quad \Rightarrow r_i = 4r_f$

So, $E_f = T\times 64\times 4\pi r_f^2 = 256T\times\pi\frac{r_i^2}{16} = 16\pi T r_i^2$ So, $\Delta E = E_f - E_i - 12\pi T r_i^2 = 12\pi\times 0.075\times 0.1^2$

$= 2.82\times 10^{-4}\text{ J} = 2.8\times 10^{-4}\text{ J}$

12. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be

1) $\frac{2T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ 2) $\frac{3T}{rJ}$ 3) $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ 4) $\frac{2T}{rJ}$

Key 3

Sol As volume remains unchanged $n\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow nr^3 = R^3$

Work done = T × Increment in surface area $W = T\left[4\pi nr^2 - 4\pi R^2\right]$....(i)

By conservation of energy

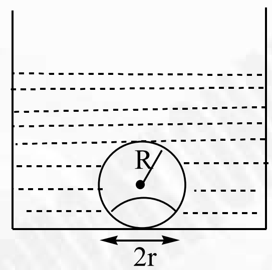
Loss in thermal energy = work done against surface tension JQ = W

Where J = mechanical equivalent of heat $Q = \frac{W}{J} = \frac{4\pi T\left(nr^2 - R^2\right)}{J}$ $\left[\text{Using }(i)\right]$ Heat

energy per unit volume $Q = \frac{4\pi T}{Jn.\frac{4}{3}\pi r^3}\left[nr^2 - R^2\right] = \frac{4\pi T}{J\times\frac{4}{3}\pi}\left[\frac{1}{r} - \frac{R^2}{nR^3}\right]$

$= \frac{3T}{J}\left[\frac{1}{r} - R^2\right] = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$

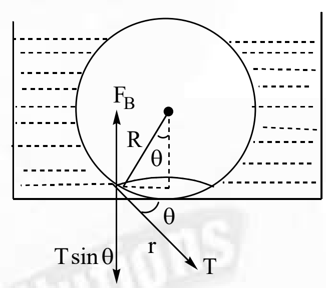
13. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r << R and the surface tension of water is T, value of r just before bubbles detach is: (density of water is ρ_w)



1) $R^2\sqrt{\frac{2\rho_w g}{3T}}$ 2) $R^2\sqrt{\frac{\rho_w g}{6T}}$ 3) $R^2\sqrt{\frac{\rho_w g}{T}}$ 4) $R^2\sqrt{\frac{3\rho_w g}{T}}$

Key 1

Sol When the bubble gets detached, Buoyant force = force due to surface tension from diagram



$\sin\theta = \frac{r}{R}$ $\rho_w Vg = \left(T\sin\theta\right)\times\left(2\pi r\right)$

$\Rightarrow \rho_w \times \frac{4}{3}\pi R^3 g = T.\frac{r}{R}\times 2\pi r$

$\Rightarrow \rho_w \frac{4}{3}R^3\ g = \frac{2T}{R}r^2 \Rightarrow r = R^2\sqrt{\frac{2\rho_w g}{3T}}$

14. A large number of liquid drops each of radius r coalesce to from a single drop of radius R. The energy released in the process is converted into kinetic energy of the big drop so formed. The speed of the big drop is (given, surface tension of liquid T, density ρ)

1) $\sqrt{\frac{T}{\rho}\left(\frac{1}{r} - \frac{1}{R}\right)}$ 2) $\sqrt{\frac{2T}{\rho}\left(\frac{1}{r} - \frac{1}{R}\right)}$ 3) $\sqrt{\frac{4T}{\rho}\left(\frac{1}{r} - \frac{1}{R}\right)}$ 4) $\sqrt{\frac{6T}{\rho}\left(\frac{1}{r} - \frac{1}{R}\right)}$

Key 4

Sol When drops combine to form a single drop of radius R.

$\Delta E = T\left[4\pi R^2 - n\times 4\pi r^2\right]$ $= 4\pi TR^3\left[\frac{1}{R} - \frac{1}{r}\right]$

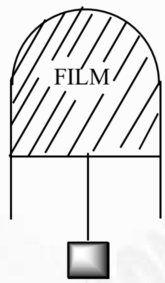
So energy released, $|E| = 4\pi TR^3\left[\frac{1}{r}-\frac{1}{R}\right]$

If this energy is converted into kinetic energy then

$\frac{1}{2}mv^2 = 4\pi R^3 T\left[\frac{1}{r}-\frac{1}{R}\right]$ $\quad \frac{1}{2}\times\left[\frac{4}{3}\pi R^3\rho\right]v^2 = 4\pi R^3 T\left[\frac{1}{r}-\frac{1}{R}\right]$

$\Rightarrow v = \frac{6T}{\rho}\left[\frac{1}{r}-\frac{1}{R}\right]$ $\quad \Rightarrow v = \sqrt{\frac{6T}{\rho}\left[\frac{1}{r}-\frac{1}{R}\right]}$

15. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see cm and its weight is negligible. The surface tension of the liquid film is



1) $0.0125\ Nm^{-1}$ 2) $0.1\ Nm^{-1}$ 3) $0.05\ Nm^{-1}$ 4) $0.025\ Nm^{-1}$

Key 4

Sol Let T is the force due to surface tension per unit length, then

$F = 2\ell T$ ℓ = length of the slider, At equilibrium, F = W $\therefore 2T\ell = mg$

$\Rightarrow T = \frac{mg}{2\ell} = \frac{1.5\times10^{-2}}{2\times30\times10^{-2}} = \frac{1.5}{60} = 0.025\ Nm^{-1}$

Applications of Surface tension (Excess pressure)

16. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube. [Take surface tension of water T = $7.3\times10^{-2}\ Nm^{-1}$, angle of contact = 0, g = 10 ms^{-2} and density of water = $=1.0\times10^3\ kg\,m^{-3}$]

1) 3.62 mm 2) 2.19 mm 3) 5.34 mm 4) 4.97 mm

Key 2

Sol Pressureat same horizontal level is equal

So, $P_1 = P_2$ $\quad P_3 = P_{atm} - \frac{2T}{r_1}$ $\quad$ & $P_4 = P_{atm} - \frac{2T}{r_2}$

$P_2 = P_4 + \rho g h_2 = P_{atm} - \frac{2T}{r_2} + \rho g h_2$ & $P_1 = P_3 + \rho g(h_1 + h_2) = P_{atm} - \frac{2T}{r_1} + \rho g(h_1 + h_2)$

$\therefore P_{atm} - \frac{2T}{r_1} + \rho g(h_1 + h_2) = P_{atm} - \frac{2T}{r_2} + \rho g h_2$ $\quad \therefore \rho g h_1 = 2T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= 2\times7.3\times10^{-2}\left[\frac{1}{2.5\times10^{-3}} - \frac{1}{4\times10^{-3}}\right]$ $\quad \Rightarrow h_1 = 2.19\times10^{-3} m = 2.19\,mm$

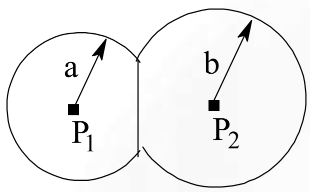
17. When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is

1) $\frac{ab}{b-a}$ 2) $\frac{ab}{a+b}$ 3) $\frac{b-a}{ab}$ 4) $\frac{a+b}{ab}$

Key 1

Sol Let R be the radius of curvature of common surface

$P_1 = P_0 + \frac{4T}{a}$ and $P_2 = P_0 + \frac{4T}{b}$



And $P_1 - P_2 = \frac{4T}{R}$ $\left(P_0 + \frac{4T}{a}\right) - \left(P_0 + \frac{4T}{b}\right) = \frac{4T}{R}$ $\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{R}$ $\therefore R = \frac{ab}{(b-a)}$

18. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is:

1) 4:1 2) 0.8:1 3) 8:1 4) 2:1

Key 3

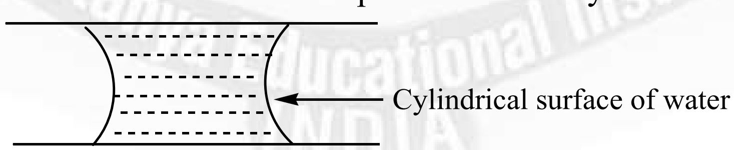
Sol According to question, pressure inside, 1st soap bubble.

$\Delta P_1 = P_1 - P_0 = 0.01 = \frac{4T}{R_1}$.......(i) and $\Delta P_2 = P_2 - P_0 = 0.02 = \frac{4T}{R_2}$.......(ii)

Dividing, equation (ii) by (i), $\frac{1}{2} = \frac{R_2}{R_1} \Rightarrow R_1 = 2R_2$

Volume $V = \frac{4}{3}\pi R^3 \Rightarrow \frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = \frac{8}{1}$

19. If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension of water is T then the pressure in water between the plates is lower by:



1) $\frac{2T}{R}$ 2) $\frac{4T}{R}$ 3) $\frac{T}{4R}$ 4) $\frac{T}{R}$

Key 1

Sol Inside a drop or an air bubble, towards concave side

$P = P_0 + \frac{2T}{R} \Rightarrow \Delta P = \frac{2T}{R}$

Here, the figure shows that the water between the plates formed a curved surface, similar to that formed by air bubble inside the water, So, pressure in water between the plates is lowered by $\frac{2T}{R}$.

20. An air bubble of radius 0.1 cm is in a liquid having surface tension 0.06 N/m and density 10^3 kg/m^3. The pressure inside the bubble is 1100 Nm^{-2} greater than the atmospheric pressure. At what depth is the bubble below the surface of the liquid? (g = 9.8 ms^{-2})

1) 0.1 m 2) 0.15 m 3) 0.20 m 4) 0.25 m

Key **1**

Sol As we know, $\rho_{Excess} = h\rho g + \frac{2s}{r}$ $\Rightarrow 1100 = h \times 10^3 \times 9.8 + \frac{2\times 6\times 10^{-2}}{10^{-3}}$

$\Rightarrow 1100 = 9800h + 120 \Rightarrow 9800h = 1100 - 120$ $\Rightarrow h = \frac{980}{9800} = 0.1m$

21. If two soap bubbles of different radii are connected by a tube

1) air flows from the smaller bubble to the bigger

2) air flows from bigger bubble to the smaller bubble till the sizes are interchanged

3) air flows from the bigger bubble to the smaller bubble till the sizes become equal

4) there is no flow of air.

Key **1**

Sol Let pressure outside be P_0 and r and R be the radius of smaller bubble and bigger bubble respectively.

$\therefore$ Pressure P_1 for smaller bubble $= P_0 + \frac{2T}{r}$ P_2 for bigger bubble $= P_0 + \frac{2T}{R}$ $(R > r)$

$\therefore P_1 > P_2$ Hence air moves from smaller bubble to bigger bubble

Angle of Contact Relation between force of cohesion and adhesion for different angles of contact

22. Assuming the Xylen tissues through which water rises from roots of the branches in a tree to be of uniform cross-section. The maximum radius of xylem tube in a 10m high coconut tree so that water can rise to the top is 10^{-6}m. If surface tension of water is 0.1 Nm^{-1}, then find the angle of contact $\left[\text{Take } g = 10\,ms^{-2}\right]$

1) 30° 2) 45° 3) 60° 4) 75°

Key: 3

Sol: $\rho h f \pi r^2 = 2\pi r s\cos\theta$ $\cos\theta = \frac{\rho g h r}{2s} = \frac{10^3 \times 10 \times 10 \times 10^{-6}}{2\times 0.1}$ $\cos\theta = \frac{1}{2}$ $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

Capillary rise

23. Two narrow bores of diameters 4.0mm and 6.0mm are joined together to form a U – tube open at both ends if the U – tube contains water, What is the difference in its level in the two limbs of the tube? Surface tension of water at the temperature of experiment is

$7.2\times10^{-2}Nm^{-1}$. Take the angle of contact to be zero, and density of water to be $1.0\times10^{3}kgm^{-3}(g = 9.8m/sec^{2})$

1. 3 min **2.** 1.83 min **3.** 4 mm **4.** 3.5 mm

Key 2

Sol $h=\frac{2T}{dg}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]\Rightarrow h=\frac{2\times7.2\times10^{-2}}{1\times10^{3}\times9.8}\left[\frac{1}{4}-\frac{1}{8}\right]\times\frac{1}{10^{-3}}\Rightarrow h=1.83mm$

24. A long capillary tube of radius 0.5mm open at both ends i filled with water and placed vertically. What will be the height of water column left in the capillary? (surface tension of water is $7.2\times10^{-2}Nm^{-2}$)

1. 4 cm **2.** 5.87cm **3.** 6.5 cm **4.** 7 cm

Key 3

Sol $\pi r^{2}hdg=2(2\pi r)T\cos\theta \quad h=\frac{4T}{rdg}\Rightarrow h=\frac{4\times7.2\times10^{-2}}{0.5\times10^{-3}\times1\times10^{3}\times9.8}=5.87\times10^{-2}met=5.87cm$

25. Water rises in a capillary tube through a height '*l*' if the tube is inclined to the liquid surface at 45° the required will rise in the tube upto its length equal to

1 . $\frac{1}{2}$ **2.** $\frac{\sqrt{3}1}{2}$ **3.** $\sqrt{2}1$ **4.** $\frac{21}{\sqrt{3}}$

Key 2

Sol $1'=\frac{1}{\sin45^{\circ}}=\sqrt{2}1$

26. If a capillary tube is tilted to 30° and 45° from the vertical then the ratio of lengths l_1,l_2 of liquid column in it will be

1. $1:\sqrt{2}$ **2.** $\sqrt{3}:\sqrt{2}$ **3.** $\sqrt{2}:\sqrt{3}$ **4.** $\sqrt{2}:1$

Key 2

Sol $\frac{l_1}{l_2}=\frac{\cos\theta_1}{\cos\theta_2}\Rightarrow\frac{l_1}{l_2}=\frac{\cos30^{\circ}}{\cos45^{\circ}}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}=\sqrt{\frac{3}{2}}$

27. A capillary tube is immersed vertically into water in a vessel kept in a stationary lift. The rise of water in it is 'h'. If the lift moves upwards with an acceleration 'a' then the rise of water in the capillary tube is

1. $\frac{hg}{a-g}$ **2.** $\frac{hg}{a}$ **3.** $\frac{hg}{a+g}$ **4. h**

Key 3

Sol $T=\frac{hrdg}{2\cos\theta}\Rightarrow h\,\alpha\frac{1}{g},h'\alpha\frac{1}{g+a}\Rightarrow\frac{h'}{h}=\frac{g}{g+a}$

28. A capillary tube is immersed vertically into water in a vessel kept in a stationary lift. The rise of water in it is 3cm, if the lift moves upwards with an acceleration '$\frac{g}{2}$' then the rise of water in the capillary tube is

1. 2cm **2.** 4 cm **3.** 1.5 cm **4.** 5 cm

Key **1**

Sol $h\alpha\frac{1}{g}$ $T=\frac{hrdg}{2\cos\theta}$ $\frac{h'}{h}=\frac{g}{g+a}\Rightarrow h'=\frac{hg}{g+a}\Rightarrow h'=\frac{3\times g}{g+g/2}$ **=2cm**

29. A drop of water of volume 3.6cm^{-3} pressed between two glass plates so as to spread to an area 2mm^2 if T is the surface to separate to the glass plates is $\left[T=7.2\times10^{-2}\text{Nm}^{-2}\right]$

1. 16×10^{-2}N **2.** 12×10^{-4}N **3.** 4×10^{-2}N **4.** 6×10^{-2}N

Key **1**

Sol $F=\frac{2+A^2}{v}\Rightarrow\frac{2\times7.2\times10^{-2}\left[2\times\left(10^{-3}\right)\right]^2}{3.6\times\left(10^{-2}\right)^3}$ = $F=16\times10^{-2}$N

30. A drop of water of volume 48 cc is pressed the two glass plates so as to spread to an area 2mm^2. If T is surface tension the normal force required to separate the glass plate is

1. 5×10^{-2}N **2.** 12×10^{-2}N **3.** 3×10^{-2}N **4.** 7×10^{-2}N

Key **2**

Sol $F=\frac{2TA^2}{V}=\frac{2\times7.2\times10^{-2}\times\left(2\times10^{-3}\right)^2}{4.8\times\left(10^{-2}\right)^3}=12\times10^{-2}\text{N}$

****

ONE ORE MORE THAN ONE QUESTIONS

Topic: Surface energy, Surface tension, force of cohesion and Adhesion

31. A capillarity tube of radius ‘r’ is lowered into water whose surface tension is ‘α’ and density ‘d’. The liquid rises to a height. Assume that the contact angle is zero. Choose the correct statement(s)

1. Magnitude of work done by force of surface tension is $\frac{4\pi\alpha^2}{dg}$

2. Magnitude of work done by force of surface tension is $\frac{2\pi\alpha^2}{dg}$

3. Potential energy acquired by the water is $\frac{2\pi\alpha^2}{dg}$ $\frac{2\pi\alpha^2}{dg}$

4. the amount of heat developed is $\frac{2\pi\alpha^2}{dg}$

Key **ACD**

Sol h = capillary rise $h = \frac{2\alpha}{dgr}$ Work done by surface tension $= Fh = (2\pi r\alpha)\left(\frac{2\alpha}{dgr}\right) = \frac{4\pi\alpha^2}{dg}$

$P.E = mg\left(\frac{h}{2}\right) = \left(d\pi r^2 hg\right)\left(\frac{\alpha}{dgr}\right) = \frac{2\pi\alpha^2}{dg}$ Remaining energy is liberated into heat

32. A vertical capillary is brought in contact with the water surface. [surface tension of water is T, density of water is ρ]

1. work done by surface tension in raising the liquid is $\frac{4\pi T^2}{\rho g}$

2. Increase in gravitational potential energy of the risen liquid is $\frac{2\pi T^2}{\rho g}$

3. The heat evolved during the rise of the liquid is $\frac{2\pi T^2}{\rho g}$

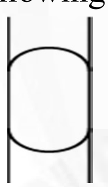
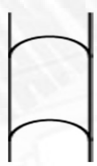
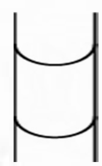
4. Heat is not evolved during the rise of the liquid

Key **ABC**

Sol $w = 2\pi rTh$ & $h = \frac{2T}{r\rho g}$ $P.E = mg\frac{h}{2}$ Heat $= W - P.E$

Applications of Surface tension (Excess pressure)

33. A vertical glass capillary tube, open at both ends, contains some water. Which of the following shapes may not be taken by the water is the tube?

1.  **2.**  **3.**  **4.** 

Key **ABC**

Sol Usually, the water contact angle is smaller than 90o with glass surface and so the surface will be hydrophilic (water loving). shape of water surface on top will be concave. if gravity is taken into account, water will take the shape as shown in option D.

34. Two separate bubbles (radii 0.002m and 0.001m) formed of the same soap solution (surface tension 0.07Nm^{-1}) comes together to form a double. The internal film common to both the bubbles has radius R. Then

1. R = 0.004m

2. R = -0.003m

3. Internal film is concave towards smaller bubble

4. Internal film is concave towards bigger bubble

Key 0.004

Sol Excess pressure is always present on concave side. So, interface is concave towards small bubble

$p_1 = p_0 + \frac{4T}{r_1}$; $p_2 = p_0 = \frac{4T}{r_2}$

Excess pressure = $p_1 - p_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

This excess pressure for the interface is also equal to $\frac{4T}{R}$

$4T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{4T}{R}$ $\quad R = \frac{r_1 r_2}{r_2 - r_1} = \frac{(0.002)(0.004)}{0.004 - 0.002} = 0.004\text{m}$

Angle of Contact Relation between force of cohesion and adhesion for different angles of contact

35. Which of the following is/are true about the angle of contact?
 1) The value of angle of contact for pure water and glass is zero
 2)Angle of contact increases with increase in temperature of liquid.
 3)Angle of contact doesn't depend on the inclination of the solid surface to the liquid surface.
 4)If the angle of contact of a liquid and a solid surface is less than 90°, then the liquid spreads on the surface of solid.

Key ABCD

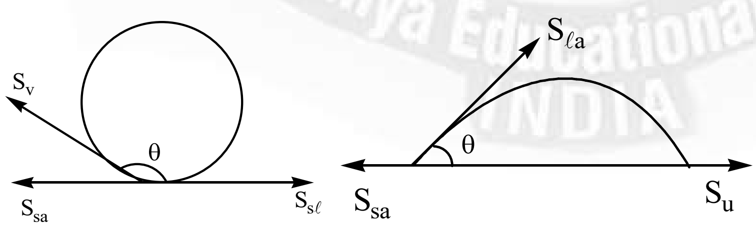
Sol **Option A**: For pure water, the angle of contact is 0°, whereas for ordinary water, if lies between 8° and 18°. Hence this is true.

Option B: with increase in temperatures surface tension of liquid decreases. Due to this, the liquid surface on the solid surface becomes more flat, so the angle of contact of lliqiud increases. So it is true.

Option C: Angle of contact doesn't depend on the inclination of the solid surface to the liquid surface, it actually depends on the inclination of the solid surface. Hence, this statement is not true

Option D: For the angle of contact less than 90°, the difference in the surface tensions of air-solid interface and liquid-solid interface is less than the air-liquid interface. Hence it spreads over the surface of solid. Statement is true.

36. The three interfacial tensions at all the three interfaces, liquid-air $(S_{\ell a})$, solid – air $(S_{s\ell})$ and solid – liquid $(S_{s\ell})$ respectively are shown the figures.



1) $S_{s\ell} < S_{\ell a}$ in case of water-plastic interface

2) $S_{s\ell} > S_{\ell a}$, in case of water-leaf interface.

3) At the line of contact net surface force between the three media exists.

4)Water proofing agents decrease the angle of contact and water wetting agents increase the angle of contact

37. From the figure (a)

$S_{sa} + S_{\ell a}\cos(180-\theta) = S_{s\ell}$

$S_{sa} - S_{\ell a}\cos\theta = S_{s\ell}$

(a) For water plastic interface, $\theta < 90°$

Hence, $\cos\theta$ is +ve

$\therefore S_{s\ell} < S_{\ell a}$

(b) For water – leaf interface, $\theta > 90°$

$\therefore \cos\theta$ is –ve and $S_{s\ell} > S_{\ell a}$

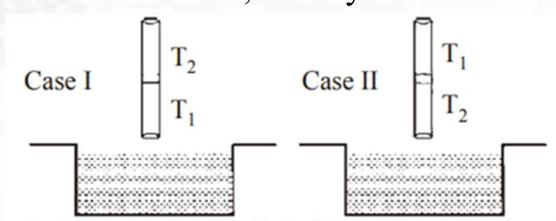
(c) At the line of contact, the surface forces between the three media must be in equilibrium.

(d) When the water proofing agents are added to liquid, they create large angle of contact between water & fibres and when the water wetting agents are added to liquids angle of contact is small so that these may penetrate well and become effective.

Capillary rise

38. Cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0^0 and 60^0 respectively. The Capillary tube is dipped vertically in water in two different configurations, case I and case 11 as shown in figure. Which of the following options is (are) correct?

[Surface tension of water 20.075 N/m , density of water = 1000kg/m^3 , take g= 10m/s^2]



1) For case 1, if the joint is kept at 8cm above the water surface, the height of the water column in the ruler will be 7.5cm

(Neglect the weight of the water in the meniscus)

2) For case 1, capillary joint is 5cm above the water surface, the height of the water column raised in the tube will be more than 8.75am

(Neglect the weight of the water in the meniscus)

3) The correction in the height of water coloumn raised in the tube, due to weight of water contained in the meniscus, will be different for both cases

4) For case 11, the Capillary joint is 5am above the water surface, the height of the water column raised in the tabs will be 3.5 am

(Neglect the weight of the water in the meniscus)

Key **ABD**

Sol Heights if only singh material tubes are used of sofficiant length

$n_1 = \frac{2TCOS\theta}{\rho rg} = \frac{2\times 0.075\times\cos 0^0}{1000\times 2\times 10^{-4}\times 10} = 7.5cm$

$n_2 = \frac{2T\cos\theta}{\rho rg} = \frac{2\times 0.075\times\cos 60^0}{1000\times 2\times 10^{-4}\times 10} = 3.75cm$

The correction in the height of water column raised in the tube due to weight of water contained in the meniscus will be different for both side

In case II if the capillary joint is 5 cm above the water surface then water in capillary will not reach the interface. Water will reach only till 23.75 cm

39. A Uniform Capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height hin the capillary tube above the water surface in to beaker. The surface tension of water is .The angle of contact between water and the wall of the capillary tube is θ. I gone the mass of water in the meniscus which of the following Statements in (are) true.

A) For a given material of the capillary tube , n decreases with The rease in r.

B) For a given material of the capillary tube, h is independent of ρ

C) If this experiment is Performed in a lift going up with constant acceleration, then h decreases,

D) h is proportional to contact angle θ **.**

Key **AC**

Sol As we know $h = \frac{2\sigma\cos\theta}{r\rho g_{eff}}$

As 'r' increase h decrease $h\alpha\frac{1}{r}$ [all other parameters remaining constant]

Also $h\alpha\sigma$ Further if lift is going up with an acceleration 'a' then $g_{eff} = g + a$. As g_{eff} increases h decreases Also $h\alpha\ \cos\theta \text{not} h\alpha\theta$

PARAGRAPH QUESTIONS

Topic: Surface energy, Surface tension, force of cohesion and Adhesion

Paragraph Question: When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wise to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper

40. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming r << R) is

1. $2\pi rT$ **2.** $2\pi rRT$ **3.** $\frac{2\pi r^2 T}{R}$ **4.** $\frac{2\pi R^2 T}{r}$

Key **3**

Sol $2\pi rT\frac{r}{R}=F$

41. If $r=5\times10^{-4}m, \bar{n}=10^{3}\,kgm^{-3}, g=10ms^{-2}$, T =0.11 Nm^{-1}. The radius of the drop when it detaches from the dropper is approximately

1. $1.4\times10^{-3}m$ **2.** $3.3\times10^{-3}m$ **3.** $2.0\times10^{-3}m$ **4.** $4.1\times10^{-3}m$

Key 1

Sol $2\pi r^{2}\frac{T}{R}=mg$

Paragraph 2: When liquid medicine of density ρ is to put in the eye, it is done with the help of a dropper. As the bulp on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

42. If the radius of the opening of the dropper is r, the vertical for a due to the surface tension on the drop of radius R (assuming $r << R$) is

a) $2\Pi rT$ **b)** $L\Pi RT$ **c)** $\frac{2\Pi r^{L}T}{R}$ **d)** $\frac{2\Pi R^{2}T}{r}$

Key (c)

Sol **Surface Tension fore =** $2\Pi rT\frac{r}{R}=\frac{L\Pi r^{L}T}{R}$

43. If $r=5\times16^{-4}m, \rho=10^{3}kg/m^{3}, g=10m/s^{2}, T=0.11N/m,$ the radius of the drop when if detaches from the dropper is approximatcly **.**

a) $1.4\times10^{-3}m$ **b)** $3.3\times10^{-3}m$ **c)** $2.0\times10^{-3}m$ **d)** $4.1\times10^{-3}m$

Key (A)

Sol $\frac{2\Pi r^{2}T}{R}=mg=\frac{4}{3}\Pi R^{3}\rho g$

44. After the drop detaches, its surface energy is

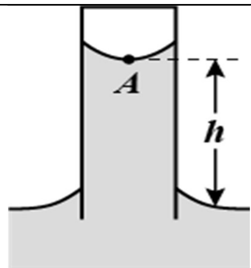
a) $1.4\times10^{-6}J$ **b)** $2.7\times10^{-6}J$ **c)** $5.4\times10^{-6}J$ **d)** $8.1\times10^{-6}J$ **s**

Key (B)

Sol **Surface energy = T** (ΠR^{2})**=2.7×**10^{-6} **J**

Paragraph 3: The figure shows a capillary tube of radius r dipped into water. The atmospheric pressure is P_{0} and the capillary rise of water is h, the surface tension for water glass is S.

45. The pressure inside water at the point A (lowest point of the mesiscus) is



A) P_0 B) $Pot\frac{2S}{r}$ C) $P_0-\frac{2S}{r}$ D) $P_0-\frac{4S}{r}$

Key 2

Sol - $P_0 > P \Rightarrow P_0 - P = \frac{25}{T}; P = P_0 - \frac{25}{r}$

46. **Initially h=10cm, if the capillary tube is now inclined at 45^0, the length of water rising in the tube will be**

a) 10cm **b) $10\sqrt{2}$ cm** **c)** $\frac{10}{\sqrt{2}}cm$ **d) 5**

Key - $T \times 2\Pi a \times 8 = W$ **= Weight of the mosquito**

Sol - $l = \frac{h}{\cos\alpha} = \frac{10\times\sqrt{2}}{1} = 10\sqrt{2}cm$

47. **A mosquito with 8 legs stands on water surface and each leg makes depression of radius 'a'. If the surface tension and angle of contact are 'T' and 3 zero respectively. Then the weight of mosquito's**

A) 8Ta **B) 16ΠTa** **C)** $\frac{Ta}{8}$ **D)** $\frac{Ta}{16\Pi}$

Key : B

Sol : $T \times 2\Pi a \times 8 = W$ **= Weight of the mosquito**

Applications of Surface tension (Excess pressure)

Paragraph: Molecular forces exist between the molecules of a liquid in a container. The molecular on the surface have unequal force leading to a tension on the surface. If this is not compensatedby a force, the equilibrium of the liquid will be a difficult task. This leads to an excess pressure on the surface. The nature of the meniscus can inform us of the direction of the excess pressure. The angle of contact of the liquid decided by the force between the molecules, air and container can make in the angle of contact

48 The direction of the excess pressure in the meniscus of liquid of angle of contact $2\pi/3$ is

1. upward **2.** downward **3.** horizontal **4.** Cannot be determined

Key 1

Sol Excess pressure is always on the concave side of the surface. For angle of contact of $2\pi/3$, the liquid should have a convex surface So, the excess pressure should be in the upward direction

49. If the express pressure in a soap bubble is ρ, the excess pressure in an air bubble is

1. $\frac{p}{2}$ **2.** p **3.** 2p **4.** 4p

Key 1

Sol In a soap bubble there will be two free surface. So, excess pressure will be double than that in an air bubble. So air bubble is p/2

50. In a meniscus of radius r, with excess pressure p in atmospheric pressure, p_e, the force experienced is

1. $(p-p_0)\pi r^2$ **2.** $(p-p_0)2\pi r$ **3.** $p\pi r^2$ **4.** $p_0 2\pi r$

Key 3

Sol Difference in pressure between the inner and outer surface is $(p_i - p_0)$

Force $(p_i - p_0)\pi r^2$ Since excess pressure is given $p = (p_i - p_0)$ $p = p\pi r^2$

Angle of Contact Relation between force of cohesion and adhesion for different angles of contact

Paragraph Questions

The angle between the tangent drawn to the liquid surface at the point of contact and the solid surface inside the liquid is called angle of contact. Angle of contact for water and glass is about 8°, for mercury and glass it's about 138° whereas for pure water and silver, it is 90°.

51. Concave meniscus forms for

1) water and glass 2) water and silver
3) mercury and glass 4) all of the above

52. Cohesive force = Adhesive force when

1) the water is filled in the glass tube
2) the water is filled in the silver tube
3) the mercury is filled in the glass tube
4) the mercury is filled in the silver tube

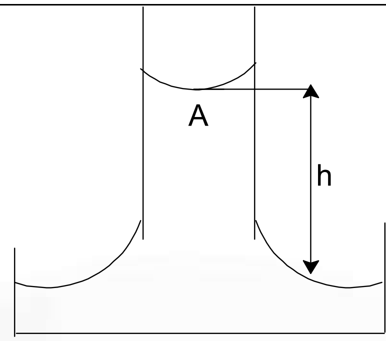
Key 9-1,10-2

51. For water and glass, Adhesive force is more than cohesive force, because angle of contact $\theta < 90°$. Hence concave menisus is formed.

52. When the pure water is filled in the silver tube, $\theta = 90°$, then cohesive force = Adhesive force

Capillary rise

Paragraph 2: figure shows a capillary tube of radius r dipped into water. The atmospheric pressure is P_0 and the capillary rise of water is h.s is the surface tension for water glass



53 The pressure inside water at the point A (lowest point of the meniscus) is

1. P_0 **2.** $P_0 + \frac{2s}{r}$ **3.** $P_0 - \frac{2s}{r}$ **4.** $P_0 - \frac{4s}{r}$

Key 3

Sol $P_A - P_B = \frac{2s}{r}$

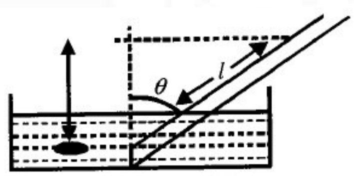
The pressure inside a concave meniscus is less than the pressure outside (atmospheric). Assuming the meniscus to the spherical (as for thin capillaries) excess pressure is 2s/r where r is the radius of the hemispherical surface

54. Initially h = 10cm. If the capillary tube is now inclined at 45° the length of water rising in the tube will be

1. 10cm **2.** $10\sqrt{2}$ cm **3.** $\frac{10}{\sqrt{2}}$cm **4.** None of these

Key 2

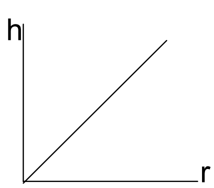
Sol When the capillary tube is tilted by an angle θ with the vertical, the capillary rise to distance l in the tube will be such that the meniscus will remain at the same vertical height above the level of water in the container.

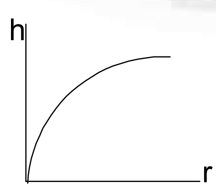


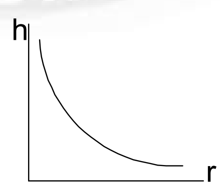
Hence, $h = l\cos\theta$

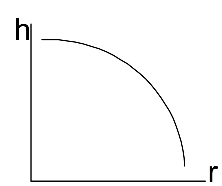
$l = \frac{h}{\cos\theta} = \frac{10}{\cos 45°} = 10\sqrt{2}\,\text{cm}$

55. Which of the following graphs may represented the tension between the capillary rise h and the radius r of the capillary**?**

1. 

2. 

3. 

4. 

Key 3

Sol The capillary rise h is given by $h = \frac{2S\cos\theta}{pgr}$

Where hr = constant for a given glass and liquid. This is shown correctly in (3)

NUMERICAL QUESTIONS

Topic: Surface energy, Surface tension, force of cohesion and Adhesion

56. A drop of liquid of radius R = 10^{-2}m having surface tension $S = \frac{0.1}{4\pi} Nm^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J If $K = 10^{\alpha}$ then the value of α is

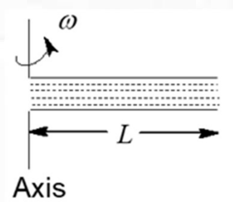
Key 6

Sol $\frac{4}{3}\pi R^2 = k \times \frac{4}{3}\pi r^3 \quad \therefore R = K^{1/3} r \quad \Delta U = S\left[k \times 4\pi r^2 - 4\pi R^2\right]$

$\Delta U = 4\pi s\left[k \times \frac{R^2}{k^{2/3}} - R^2\right] = 4\pi SR^2\left[k^{1/3} - 1\right] \quad 10^{-3} = 4\pi \times \frac{0.1}{4\pi} \times \left(10^{-2}\right)^2\left[10^{\alpha/3} - 1\right]$

$10^2 = 10^{\alpha/3} - 1$ neglecting $10^2 = 10^{\alpha/} \Rightarrow \frac{\alpha}{3} = 2 \quad \alpha =$

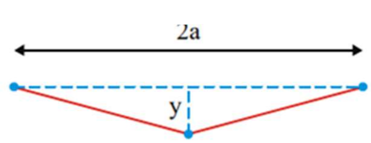
57. A thin uniform horizontal glass tube open at both ends (radius = R, length = L) is completely filled with water (density $= \rho$, surface tension = S) is being rotated with constant angular speed as shown in figure. Maximum angular speed with which it can be rotated such that water does not come out is $\omega_{max} = \sqrt{\frac{xS}{RL^2\rho}}$. Find the value of x



Key 8

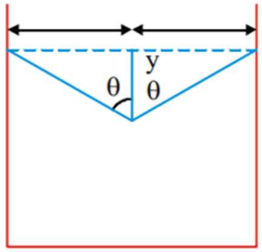
Sol $4\pi RS = \pi\rho R^2 L^2 \omega^2 / 2 \qquad \omega_{max} = \sqrt{\frac{8S}{RL^2\rho}}$

58. A thins film of a liquid is maintained between two very long, thin, parallel, horizontal wires separated by a distance 2a. A long wire of mass per unit length λ is gently placed over the liquid film at the middle parallel to the wires. As a result the liquid surface is depressed by a vertical distance y (y<<a) at equilibrium. The surface tension of the liquid is $\frac{\lambda ga}{ky}$ then k is ......



Key 2

Sol $W=\lambda lg;\ 2Tloc\theta = W = \lambda lg$



$$T=\frac{\lambda g}{2\cos\theta};\text{where}\cos\theta=\frac{y}{\sqrt{a^2+y^2}}=\frac{y}{a}$$

59. A long capillary tube of radius r dipped in water such that $\sqrt{\frac{3}{4}}$ of the height it can rise in the capillary is available above the tubel of water the angle of contact at the top level in the tube is $\frac{\pi}{k}$ what value of k.

Key 6

Sol - $h=\frac{2T}{rdg}$ _______ **(1)** **If** $\theta=o$ **(Normal conditions)**

Now $n^1=\frac{2TCOS\theta}{rdg}$ $n^1=\frac{\sqrt{3}}{2}n=\frac{2TCOS\theta}{rdg}$ ______**(2)**

$\frac{2}{1}\Rightarrow\cos\theta=\frac{\sqrt{3}}{2}$ $\theta=30^0=\frac{\Pi}{6}\Rightarrow k=6$

Applications of Surface tension (Excess pressure)

60. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 Nm^{-1} and density 1000 kg m^{-3} at a depth of 10 cm below the free surface. The amount by which the pressure inside the bubble is greater than the atmospheric pressure is ___ Pa (g = 10 ms^{-2})

Key 1150

Sol Radius of air bubble, $r=1\times10^3 m$ Surface tension, $T=0.075\ \frac{N}{m}$

Density of liquid, $\rho=1000\ kg/m^3$ at depth, h = 10 cm = 0.10 m

P_0 = atmospheric pressure Pressure inside the bubble,

$$P=P_0+h\rho g+\frac{2T}{r}\quad P-P_0=h\rho g+\frac{2T}{r}=0.1\times1000\times10+\frac{2\times.075}{10^{-3}}$$

$$=1000+(0.15)(1000)=1150\,Pa$$

61. The excess pressure inside a liquid drop is 500 Nm^{-2}. If the radius of the drop is 2mm, the surface tension of liquid is $x\times10^{-3}\,Nm^{-1}$. The value of x is ___.

Key 500

Sol $P=P_0+\frac{2T}{R}\Rightarrow P-P_0=\frac{2T}{R}$ $500=\frac{2\times T}{2\times10^{-3}}$ $T=500\times10^{-3}$ So, x = 500

62. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is ___ cm.

Key 2

Sol Excess pressure inside bigger bubble $=\frac{4T}{r_2}$

SO, Excess pressure inside the smaller soap bubble $\Delta P_1=\frac{4T}{r_1}+\frac{4T}{r_2}$

The excess pressure inside the equivalent soap bubble. $\Delta P_2=\frac{4T}{R_{eq}}$

$\Delta P_1=\Delta P_2 \Rightarrow \frac{4T}{R_{eq}}=\frac{4T}{r_1}+\frac{4T}{r_2} \Rightarrow R_{eq}=\frac{r_1 r_2}{r_1+r_2}$ Putting $r_1=3cm,\ r_2=6cm \Rightarrow R_{eq}=2cm$

Angle of Contact Relation between force of cohesion and adhesion for different angles of contact

63. The lower end of a glass capillary tube is dipped in water. Then water rises to a height of 9cm. The tube is than broken at a height of 5cm. The angle of contact will be $\cos^{-1}\left(\frac{x}{y}\right)$.

Find the value of $\frac{x+y}{2}$

Key 9

Sol When a capillary tube is broken at a height of 5cm. Meaning that water will rise to height of 5cm.

Hence, $h_2 = 5cm$

From the capillary phenomenon

$h=\frac{25\cos\theta}{r\rho g} \Rightarrow \frac{h}{\cos\theta}=\text{constant} \Rightarrow \frac{h_1}{\cos\theta_1}=\frac{h_2}{\cos\theta_2}$ for glass $\theta_1=0^\circ$ $\frac{9}{\cos\theta}=\frac{5}{\cos\theta_2}$

$\cos\theta_2=\frac{5}{9} \Rightarrow \theta_2=\cos^{-1}\left(\frac{5}{9}\right)=\cos^{-1}\left(\frac{x}{y}\right)$ $x=5\ \&\ y=9$ $\frac{x+y}{2}=\frac{5+9}{2}=7$

64. A capillary tube of radius r is lowered into a liquid of surface tension T and density ρ. The angle of contact between the solid and the free surface of the liquid is $\theta=0^\circ$. During the process, in which the liquid rises in the capillary, the work done by the surface tension is $\frac{n\pi T^2}{\rho g}$. Find the value of n

Key 4

Sol The liquid rise in capillary tube is $h_0=\frac{2T}{\rho g r}$

The workdone by the surface tension will be $w = Fh_0$

$$\Rightarrow w = (2\pi\ Tr)\left(\frac{2T}{\rho g r}\right) \quad \Rightarrow w = \frac{4\pi r^2}{\rho g} = \frac{n\pi T^2}{\rho g} \quad \Rightarrow n = 4$$

Capillary rise

65. A drop of water of volume 0.05cm^3 is pressed between two glass plates as a consequence of which it spreads and occupies an area of 25cm^2 if the surface tension of water 70 Dyu/cm. The normal force required to separated out two glass plates in 1.75×10^{-2} New tan then find the value of 'x' is

Key 3

Sol $$F = \frac{2+A^2}{v} \Rightarrow \frac{2\times7.2\times10^{-5}/10^{-2}\left[25\times10^{-2}\right]^2}{0.05\times\left(10^{-2}\right)^3}$$ **=1750 N** **=** 1.750×10^3N

66. A capillary tube is vertically dipped in a liquid. The height of the liquid in the tube is 4cm and the total setup is kept in a lift. If it is moving down with an acceleration g/3 then the height of liquid in the tube is (x+1) cm then find value of x is............

Key 6

Sol $$\frac{h'}{h} = \frac{g}{g-a} \Rightarrow h' = \frac{g\times h'}{g-g/3} \Rightarrow h' = \frac{4\times g}{\left(\frac{2g}{3}\right)} = 6\text{cm}$$

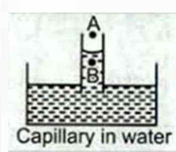
****

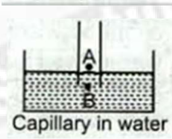
MATCHING QUESTIONS

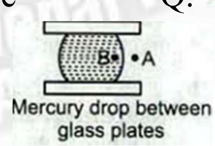
Topic: Surface energy, Surface tension, force of cohesion and Adhesion

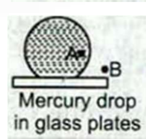
67. Capillary rise and shape of droplets on a plate due to surface tension are shown in column II match the following

Column I **Column II**

a. Adhesive force is greater than cohesive force P. 

b. Cohesive force is greater than adhesive force Q. 

c. Pressure at A = pressure at B R. 

d. Pressure at B > pressure at A S. 

Key **A-P** **B-QRS** **C-PRS** **D-Q**

Sol Conceptual

68. Match the following

Column I	**Column II**
A. Splitting of bigger drop into small droplets	**P.** Temperature increases
B. Formation of bigger drop from small dropplets	**Q.** Temperature decreases
C. Spraying of liquid	**R.** Surface energy increases
	S. Surface energy decreases

Key **A – QR,** **B –PS,** **C-QR**

Sol For A adn C in this case, the final surface area is greater than the initial surface area and hence surface energy increases the expanse of internal energy hence it causes cooling

For B reverse of the above reasoning

Applications of Surface tension (Excess pressure)

69. A column of alcohol is raised in a capillary tube of internal radius of r = 0.6mm. The lower meniscus of the column hangs from the bottom end of the tube. Match the height h of the alcohol column given in column II with the radius of curvature R of the lower meniscus given in column I. Consider that wetting is complete. Surface tension of alcohol T = 0.02N/m and density of alcohol $\rho = 790 kg/m^3$

Column I	**Column II**
A) if R = 3r	P. 8.3mm
B) if R = 2r	Q. 12.5 mm
C) if R = 4r	R. 5.5 mm
D) if R = -3r	S. 22.2 mm

Key **A – R** **B-P** **C-S** **D-Q**

Sol $P_0 - \frac{2T}{r} + h\rho g = P_0 - \frac{2T}{R} \Rightarrow h\rho g = wt\left(\frac{1}{r} - \frac{1}{R}\right); h = \frac{2T}{\rho g}\left(\frac{1}{r} - \frac{1}{R}\right)$

Angle of Contact Relation between force of cohesion and adhesion for different angles of contact

70. Match the following question.

Column – I		**Column – II**	
A.	Height of liquid in a capillary tube of radius r	**P.**	$\frac{hr}{\cos\theta} = \text{constant}$
B.	Rise of liquid in a tube of insufficient length (radius r)	**Q.**	$h + \frac{r}{3}$
C.	$0 < \theta < 90°$	**R.**	Capillary fall
D.	$90° < \theta < 180°$	**S.**	Capillary rise

1) $A \to Q,S; B \to P; C \to S; D \to R$ 2) $A \to Q; B \to PQ; C \to S; D \to R$

3) $A \to PQ; B \to Q; C \to R; D \to S$ 4) $A \to S; B \to QR; C \to P; D \to S$

Key 1

Sol A) surface tension $s = \frac{1}{2}\frac{rdg\left(h + \frac{r}{3}\right)}{\cos\theta}$ $\therefore$ effective height is $h + \frac{r}{3}$

B) $R = \frac{r}{\cos\theta}, h = \frac{25}{Rdg} \Rightarrow h \propto \frac{1}{R} \propto \frac{\cos\theta}{r} \Rightarrow \frac{hr}{\cos\theta} = \text{constant}$

C)& D) If angle of contact (θ) is $< 90°$ liquid rises and if $\theta > 90°$, liquid falls

71. Match the following question.

Column – I		**Column – II**	
A.	Capillarity	**P.**	Rising of liquid
B.	Cohesive force is greater than adhesive	**Q.**	Angle of contact becomes small
C.	Adhesive force is greater than cohesive	**R.**	Angle of contact becomes large
D.	Temperature change	**S.**	Falling liquid

1) $A \to PQRS; B \to P; C \to S; D \to R$ 2) $A \to Q; B \to PQ; C \to PQRS; D \to R$

3) $A \to PQ; B \to PQRS; C \to R; D \to S$ 4) $A \to PS; B \to RS; C \to PQ; D \to PQRS$

Key 4

Sol A) Rise or fall when liquid in a capillary tube is called capillarity

B) When cohesive force > Adhesive force, angle of contact $(\theta) > 90°$, then liquid falls in tube

C) When adhesive force > cohesive force angle of contact $(\theta) < 90°$, then liquid rises in the tube.

D) Change in temperature changes the surface tension.

Capillary rise

72. Match List-1 with List II

Angle of contact

a) 0^0 **e) livid will rise**

b)$=90^0$ **f) Rose water silver**

c)$<90^0$ **g) liquid will not wet**

d)$>90^0$ **h) pure water and glass**

Key a-h ; b-f ; c- e; d-g

Sol Conceptual

73. COLUMN-I

COLUMN-II

A) effective height of liquid in a capillary tube of radius r **P)** $\frac{hr}{\cos\theta}$ = Constanit

B) Rise of liquid in a tube of insufficient length (radius r) **Q)** $ht\frac{r}{3}$

C) $0<\theta<90^0$ **R)** Capillary fall

D) $90^0<\theta<180^0$

S) Capillary Rose

Key: A-QS; B-P; C-S; D-R

Sol: $l=\frac{h}{\cos\alpha}=\frac{10\times\sqrt{2}}{1}=10\sqrt{2}cm$