Q1 - 24 June - Shift 1

Let a line having direction ratios 1, -4, 2 intersect the lines $\frac{x-7}{3}=\frac{y-1}{-1}=\frac{z+2}{1}$ and $\frac{x}{2}=\frac{y-7}{3}=\frac{z}{1}$ at the point A and B. Then $(AB)^2$ is equal to _____ .

Space for your notes:

Q2 - 24 June - Shift 2

If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

(A) 16 (B) 6

(C) 12 (D) 15

Space for your notes:

Q3 - 24 June - Shift 2

Let the points on the plane P be equidistant from the points (–4, 2, 1) and (2, –2, 3). Then the acute angle between the plane P and the plane 2x + y + 3z = 1 is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

Space for your notes:

Q4 - 25 June - Shift 1

Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S : x + y + z = 5. If a line L passing through (1, –1, –1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to:

(A) 2 (B) 5

(C) 7 (D) 11

Space for your notes:

Q5 - 25 June - Shift 1

Let the lines

$L_1 : \vec{r} = \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in R$

$L_2 : \vec{r} = (\hat{i} + 3\hat{j} + \hat{k}) + \mu\left(\hat{i} + \hat{j} + 5\hat{k}\right); \mu \in R$

intersect at the point S. If a plane ax + by – z + d = 0 passes through S and is parallel to both the lines L_1 and L_2, then the value of a + b + d is equal to ________

Space for your notes:

Q6 - 25 June - Shift 2

Let P be the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=3$, and the point $(2,1,-2)$. Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points

(A) X and X + Y are on the same side of P

(B) Y and Y – X are on the opposite sides of P

(C) X and Y are on the opposite sides of P

(D) X + Y and X – Y are on the same side of P

Space for your notes:

Q7 - 25 June - Shift 2

Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to

Space for your notes:

Q8 - 26 June - Shift 1

If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}$, $z = 2$ and $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ perpendicular, then an angle between the lines l_2 and $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

(A) $\cos^{-1}\left(\frac{29}{4}\right)$ (B) $\sec^{-1}\left(\frac{29}{4}\right)$

(C) $\cos^{-1}\left(\frac{2}{29}\right)$ (D) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Space for your notes:

Q9 - 26 June - Shift 1

Let the plane $2x + 3y + z + 20 = 0$ be rotated through a right angle about its line of intersection with the plane $x - 3y + 5z = 8$. If the mirror image of the point $\left(2, -\frac{1}{2}, 2\right)$ in the rotated plane is $B(a, b, c)$, then :

(A) $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$ (B) $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$

(C) $\frac{a}{8} = \frac{b}{-5} = \frac{c}{4}$ (D) $\frac{a}{4} = \frac{b}{5} = \frac{c}{2}$

Space for your notes:

Q10 - 26 June - Shift 2

If the plane $2x + y - 5z = 0$ is rotated about its line of intersection with the plane $3x - y + 4z - 7 = 0$ by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point :

(A) $(2, -2, 0)$ (B) $(-2, 2, 0)$

(C) $(1, 0, 2)$ (D) $(-1, 0, -2)$

Space for your notes:

Q11 - 26 June - Shift 2

If the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ and $\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$ are co-planar, then distance of the plane containing these two lines from the point (•, 0, 0) is :

(A) $\frac{2}{9}$ (B) $\frac{2}{11}$

(C) $\frac{4}{11}$ (D) 2

Space for your notes:

Q12 - 27 June - Shift 1

If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :

(A) 6 (B) 4

(C) 3 (D) 2

Space for your notes:

Q13 - 27 June - Shift 1

Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to ________.

Space for your notes:

Q14 - 27 June - Shift 2

Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}$ be P. Then the distance of P from the plane $3x+4y+12z+23=0$

(A) 5
(B) $\frac{50}{13}$
(C) 4
(D) $\frac{63}{13}$

Space for your notes:

Q15 - 27 June - Shift 2

The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$ is :

(A) $\frac{18}{\sqrt{5}}$
(B) $\frac{22}{3\sqrt{5}}$
(C) $\frac{46}{3\sqrt{5}}$
(D) $6\sqrt{3}$

Space for your notes:

Q16 - 28 June - Shift 1

If two distinct point Q, R lie on the line of intersection of the planes $-x + 2y - z = 0$ and $3x - 5y + 2z = 0$ and $PQ = PR = \sqrt{18}$ where the point P is (1, –2, 3), then the area of the triangle PQR is equal to

(A) $\frac{2}{3}\sqrt{38}$ (B) $\frac{4}{3}\sqrt{38}$

(C) $\frac{8}{3}\sqrt{38}$ (D) $\sqrt{\frac{152}{3}}$

Space for your notes:

Q17 - 28 June - Shift 1

The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points (2, 1, 3) and (0, 1, 2), respectively, is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

Space for your notes:

Q18 - 28 June - Shift 1

Let the plane $P: \vec{r}\cdot\vec{a}=d$ contain the line of intersection of two planes $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=6$ and $\vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k})=7$. If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

(A) 90 (B) 93

(C) 95 (D) 97

Space for your notes:

Q19 - 28 June - Shift 2

Let the plane ax + by + cz = d pass through (2, 3, –5) and is perpendicular to the planes 2x + y – 5z = 10 and 3x + 5y – 7z = 12.

If a, b, c, d are integers d > 0 and gcd (|a|, |b|, |c|, d) = 1, then the value of a + 7b + c + 20d is equal to

(A) 18 (B) 20

(C) 24 (D) 22

Space for your notes:

Q20 - 28 June - Shift 2

Let the image of the point P(1, 2, 3) in the line $L: \frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ be Q. let $R(\alpha,\beta,\gamma)$ be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha+\beta+\gamma)$ is equal to

Space for your notes:

Q21 - 29 June - Shift 1

If the mirror image of the point (2, 4, 7) in the plane $3x - y + 4z = 2$ is (a, b, c), the $2a + b + 2c$ is equal to :

(A) 54 (B) 50

(C) -6 (D) -42

Space for your notes:

Q22 - 29 June - Shift 1

Let d be the distance between the foot of perpendiculars of the points P(1, 2 – 1) and Q(2, – 1, 3) on the plane $-x + y + z = 1$. Then d^2 is equal to ________.

Space for your notes:

Q23 - 29 June - Shift 1

Let $P_1 : \vec{r}.(2\hat{i} + \hat{j} - 3\hat{k}) = 4$ be a plane. Let P_2 be another plane which passes through the points (2, -3, 2) (2, − 2, − 3) and (1, −4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of α + β is equal to ______.

Space for your notes:

Q24 - 29 June - Shift 2

Let $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}$ lie on the plane px – qy + z = 5, for some p, $q \in \mathbb{R}$. The shortest distance of the plane from the origin is:

(A) $\sqrt{\frac{3}{109}}$ (B) $\sqrt{\frac{5}{142}}$

(C) $\sqrt{\frac{5}{71}}$ (D) $\sqrt{\frac{1}{142}}$

Space for your notes:

Q25 - 29 June - Shift 2

Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane x + 2y + 2z = 16. Let T be a plane passing through the point Q and contains the line $\vec{r}=-\hat{k}+\lambda\left(\hat{i}+\hat{j}+2\hat{k}\right), \lambda \in \mathbb{R}$. Then, which of the following points lies on T?

(A) (2, 1, 0) (B) (1, 2, 1)

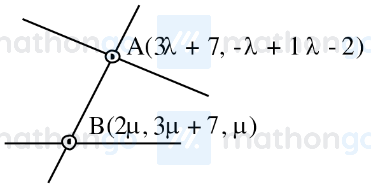
(C) (1, 2, 2) (D) (1, 3, 2)

Space for your notes:

Answer Key

Q1 (84)	**Q2** (A)	**Q3** (C)	**Q4** (B)
Q5 (5)	**Q6** (C)	**Q7** (51)	**Q8** (B)
Q9 (A)	**Q10** (C)	**Q11** (B)	**Q12** (A)
Q13 (137)	**Q14** (A)	**Q15** (A)	**Q16** (B)
Q17 (A)	**Q18** (B)	**Q19** (D)	**Q20** (125)
Q21 (C)	**Q22** (26)	**Q23** (28)	**Q24** (B)
Q25 (B)			

Q1 (84)



DR's of AB

$(3\lambda-2\mu+7,-\lambda-3\mu-6,\lambda-\mu-2)$

$$\frac{3\lambda-2\mu+7}{1}=\frac{-\lambda-3\mu-6}{-4}=\frac{\lambda-\mu-2}{2}$$

Taking first (2) $-12\lambda+8\mu-28=-\lambda-3\mu-6$

$\lambda-\mu+2=0$

Taking second & third

$-2\lambda-6\mu-12=-4\lambda+4\mu+8$

$\lambda-5\mu-10=0$

After solving above two equation $\lambda=-5,\mu=-3$

A = (-8, 6, 7)

B = (-6, -2, -3)

$(AB)^2 = 4 + 64 + 16 = 84$

Q2 (A)

SHORTEST distance $\dfrac{|(a_2-a_1)\cdot(b_1\times b_2)|}{|b_1\times b_2|}$

$a_1 = (1, 2, 3)$

$a_2 = (2, 4, 5)$

$\vec{b}_2 = 2\hat{i}+3\hat{j}+\lambda\hat{k}$

$\vec{b}_2 = \hat{i}+4\hat{j}+5\hat{k}$

$$\text{S.D.} = \frac{\left|\left((2-1)\hat{i}+(4-2)\hat{j}+(5-3)\hat{k}\right)\cdot(\vec{b}_1\times\vec{b}_2)\right|}{|b_1\times b_2|}$$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & \lambda\\ 1 & 4 & 5\end{vmatrix}$$

$= \hat{i}(15-4\lambda) + \hat{j}(\lambda-10) + \hat{k}(5)$

$= (15-4\lambda)\hat{i} + (\lambda-10)\hat{j} + 5\hat{k}$

$|\vec{b}_1\times\vec{b}_2| = \sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}$

Now

$$\text{S.D.} = \frac{\left|\left(\hat{i}+2\hat{j}+2\hat{k}\right)\cdot\left[(15-4\lambda)\hat{i}+(\lambda-10)\hat{j}+5\hat{k}\right]\right|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}$$

$$\frac{|15-4\lambda+2\lambda-20+10|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}} = \frac{1}{\sqrt{3}}$$

square both side

$3(5-2\lambda)^2 = 225 + 16\lambda^2 - 120\lambda + \lambda^2 + 100 - 20\lambda + 25$

$12\lambda^2 + 75 - 60\lambda = 17\lambda^2 - 140\lambda + 350$

$5\lambda^2 - 80\lambda + 275 = 0$

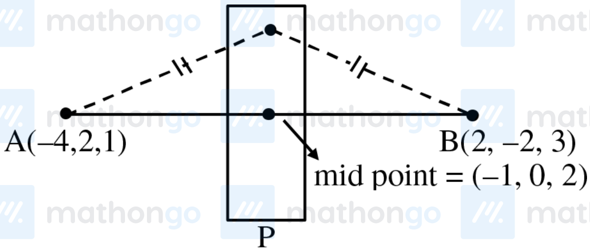
$\lambda^2 - 16\lambda + 55 = 0$

$(\lambda-5)(\lambda-11) = 0$

$\Rightarrow \lambda = 5, 11$

(A) is correct option.

Q3 (C)



Normal vector = $\overrightarrow{AB} = \left(\overrightarrow{OB} - \overrightarrow{OA}\right)$

$= \left(6\hat{i} - 4\hat{j} + 2\hat{k}\right)$

or $2\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$

$P \equiv 3(x + 1) - 2(y) + 1\ (z - 2) = 0$

$P \equiv 3x - 2y + z + 1 = 0$

$P' \equiv 2x + y + 3z - 1 = 0$

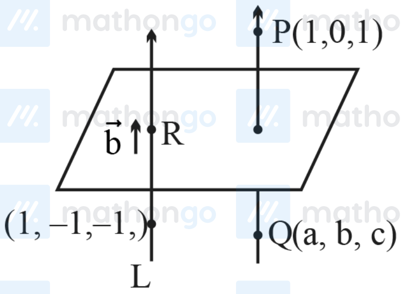
angle between P & P' = $\left|\dfrac{\hat{n}_1 \cdot \hat{n}_2}{|n_1||n_2|}\right| = \cos\theta$

$$\theta = \cos^{-1}\left(\frac{6 - 2 + 3}{\sqrt{14} \times \sqrt{14}}\right)$$

$$\theta = \cos^{-1}\left(\frac{7}{14}\right) = = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Option C is correct.

Q4 (B)



Let parallel vector of L = $\vec{b}$

mirror image of Q on given plane x+y+z=5

$$\frac{a-1}{1}=\frac{b-0}{1}=\frac{c-1}{1}=\frac{-2(2-5)}{3}$$

a = 3, b = 2, c = 3

$Q \equiv (3, 2, 3)$

$\because \quad \vec{b} \,||\, \overrightarrow{PQ}$

so, $\vec{b}=(1,1,1)$

Equation of line

$$L:\frac{x-1}{1}=\frac{y+1}{1}=\frac{z+1}{1}$$

Let point R, $(\lambda+1, \lambda-1, \lambda-1)$

lying on plane x + y + z = 5,

so, $3\lambda - 1 = 5$

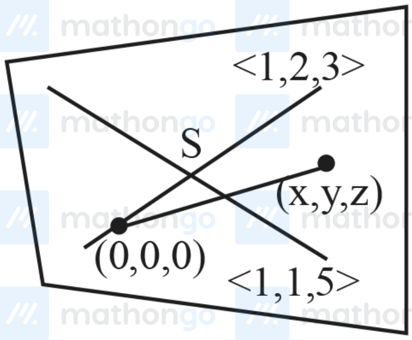
$\Rightarrow \quad \lambda = 2$

Point R is (3, 1, 1)

$QR^2 = 5$ **Ans.**

Q5 (5)

Both the lines lie in the same plane



$\therefore$ equation of the plane

$$\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 0$$

$\Rightarrow 7x - 2y - z = 0$

$\therefore \quad a + b + d = 5$

Q6 (C)

$P_1 + \lambda P_2 = 0$

$\Rightarrow (x + 3y - z - 5) + \lambda(2x - y + z - 3) = 0$

$(2, 1, -2)$ lies on this plane

$\therefore \lambda = 1 \Rightarrow$ plane is $3x + 2y - 8 = 0$

Q7 (51)

$$8x + 4\sqrt{2}y = 1,\ z = 0$$

$$\Rightarrow \frac{x - \frac{1}{8}}{1} = \frac{y - 0}{-\sqrt{2}} = \frac{z - 0}{0} = \lambda$$

$$-8x - 6\sqrt{3}z = 1, y = 0$$

$$\Rightarrow \frac{x + \frac{1}{8}}{3\sqrt{3}} = \frac{y - 0}{0} = \frac{z - 0}{-4}$$

$$\begin{vmatrix} \frac{1}{4} & 0 & 0 \\ 1 & -\sqrt{2} & 0 \\ 3\sqrt{3} & 0 & -4 \end{vmatrix} = \sqrt{2}$$

$$d = \frac{1}{\sqrt{51}}$$

$$\frac{1}{d^2} = 51$$

Q8 (B)

$$l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$$

$$l_2 : \frac{x-1}{1} = \frac{y+3/2}{\alpha/2} = \frac{z+5}{2}$$

$$l_3 : \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z-0}{4}$$

$$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$$

angle between l_2 & l_3

$$\cos\theta = \frac{|1\times(-3)+(-2)(\alpha/2)+2\times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$$

$$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$$

put $\alpha = 3$

$$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

Q9 (A)

Let equation of rotated plane be :

$(2x + 3y + z + 20) + \lambda(x - 3y + 5z - 8) = 0$

$(2 + \lambda)x + (3 - 3\lambda)y + (1 + 5\lambda)z + 20 - 8\lambda = 0$

Above plane is perpendicular to $2x + 3y + z + 20 = 0$

So, $(2 + \lambda).2 + (3 - 3\lambda).3 + (1 + 5\lambda).1 = 0 \Rightarrow \lambda = 7$

$\Rightarrow$ Equation of rotated plane : $x - 2y + 4z - 4 = 0$

Mirror image of $A\left(2, \frac{-1}{2}, 2\right)$ in rotated plane is

B(a, b, c)

Equation of AB : $\frac{x-2}{1} = \frac{y+1/2}{-2} = \frac{z-2}{4} = k$

Let coordinate of B be $\left(2+k, \frac{-1}{2} - 2k, 2 + 4k\right)$

midpoint of AB is $\left(2+\frac{k}{2}, \frac{-1}{2} - k, 2+2k\right)$ which

will lie on the plane $x - 2y + 4z - 4 = 0$

Hence $k = \frac{-2}{3}$

Therefore B is $\left(\frac{4}{3}, \frac{5}{6}, \frac{-2}{3}\right) \equiv \left(\frac{8}{6}, \frac{5}{6}, \frac{-4}{6}\right)$

So, $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$

Q10 (C)

$(2x + y - 5z) + \lambda(3x - y + 4z - 7) = 0$

Rotated by $\pi/2$

$(2 + 3\lambda)x + (1 - \lambda)y + (-5 + 4\lambda)z - 7\lambda = 0$

$2x + y - 5z = 0$

$2(2 + 3\lambda) + (1 - \lambda) - 5(-5 + 4\lambda) = 0$

$\Rightarrow 4 + 6\lambda + 1 - \lambda + 25 - 20\lambda = 0$

$30 = 15\lambda$

$\lambda = 2$

Required plane :- $8x - y + 3z - 14 = 0$

Check options

Q11 (B)

$\vec{r}=\left(\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(3\hat{j}-\hat{k}\right)$ ….. L1

$\vec{r}=\left(\alpha\hat{i}-\hat{j}\right)+\mu\left(2\hat{i}-3\hat{k}\right)$ ….. L2

• L1 and L2 are coplanar

$$\therefore\begin{vmatrix}0 & 3 & -1\\ 2 & 0 & -3\\ (1-\alpha) & 0 & 1\end{vmatrix}=0$$

–3 (2 + 3 (1 – •)) = 0

2 + 3 – 3• = 0

• 3• = 5

$\Rightarrow \alpha=\frac{5}{3}$

Now,

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 3 & -1\\ 2 & 0 & -3\end{vmatrix}=\hat{i}(-9)-\hat{j}(2)+k(-6)$$

= (9, 2, 6)

Equation of plane :

9 (x – 1) + 2(y + 1) + 6(z – 1) = 0

9x + 2y + 6z – 13 = 0

Perpendicular distance from (•, 0, 0)

$$=\left|\frac{\left(9\cdot\frac{5}{3}+0+0-13\right)}{\sqrt{81+36+4}}\right|=\frac{2}{\sqrt{121}}=\frac{2}{11}$$

Q12 (A)

$l + m - n = 0$

$3l^2 + m^2 + cl(l + m) = 0$

$n = l + m$

$3l^2 + m^2 + cl^2 + clm = 0$

$(3 + c)l^2 + clm + m^2 = 0$

$$(3+c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0 \;\ldots..(1)$$

$\because$ lies are parallel.

Roots of (1) must be equal

$\Rightarrow D = 0$

$c^2 - 4(3 + c) = 0$

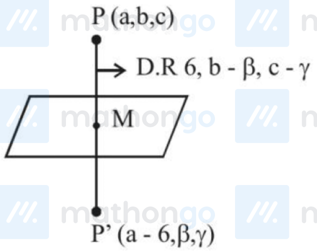
$c^2 - 4c - 12 = 0$

$(c - 6)(c + 2) = 0$

$c = 6$ or $c = -2$

+ve value of $c = 6$

Q13 (137)



$$M = \left(a-3, \frac{\beta+b}{2}, \frac{\gamma+c}{2}\right)$$

Since M lies on $3x+4y+12z+19=0$

$$\Rightarrow 6a-4b+12c-4\beta+12\gamma+20=0 \quad \ldots.(1)$$

Since PP' is parallel to normal of the plane then

$$\frac{6}{3}=\frac{b-\beta}{-4}=\frac{c-\gamma}{12}$$

$$\Rightarrow \beta=b+8, \quad \gamma=c-24$$

$$a+b+c=5 \Rightarrow a+\beta-8+\gamma+24=5$$

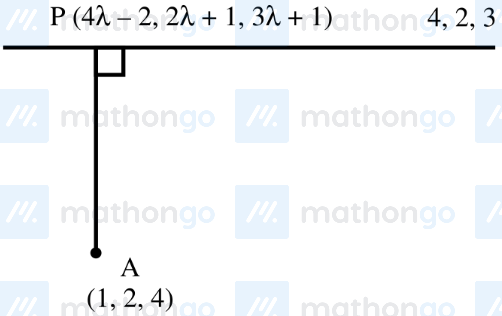
$$\Rightarrow a=-\beta-\gamma-11$$

Now putting these values in (1) we get

$$6(-\beta-\gamma-11)-4(\beta-8)+12(\gamma+24)-4\beta+12\gamma+20=0$$

$$\Rightarrow 7\beta-9\gamma=170-33 = 137$$

Q14 (A)



$$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = \lambda$$

$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1)$

$\overrightarrow{AP} = (4\lambda - 3)\ \hat{i} + (2\lambda - 1)\ \hat{j} + (3\lambda - 5)\ \hat{k}$

$\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AP} \cdot \vec{b} = 0$

$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0$

$29\lambda = 12 + 2 + 15 = 29$

$\lambda = 1$

$P = (2, 3, 2)$

$3x + 4y + 12z + 23 = 0$

$$d = \left|\frac{6+12+24+23}{\sqrt{9+16+144}}\right|$$

$$d = \left|\frac{65}{13}\right| = 5$$

Q15 (A)

$$\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$$

$$\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$$

$A=(3,2,1)$ $\quad B=(-3,6,5)$

$\overrightarrow{n_1}=2\hat{i}+3\hat{j}-\hat{k}$

$\overrightarrow{n_2}=2\hat{i}+\hat{j}-3\hat{k}$

$\overrightarrow{BA}=6\hat{i}-4\hat{j}-4\hat{k}$

$$\text{SHORTEST DISTANCE}=\frac{\left[\overrightarrow{BA}\ \overrightarrow{n_1}\ \overrightarrow{n_2}\right]}{\left|\overrightarrow{n_1}\times\overrightarrow{n_2}\right|}$$

$$\overrightarrow{n_1}\times\overrightarrow{n_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&-1\\2&1&3\end{vmatrix}$$

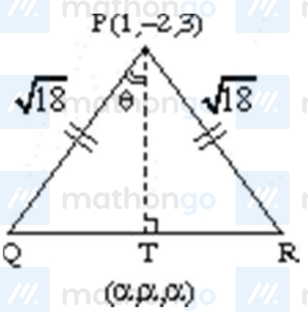
$=10\hat{i}-8\hat{j}-4\hat{k}$

$[\overrightarrow{BA}\ \overrightarrow{n_1}\ \overrightarrow{n_2}]=60+32+16=108$

$|\overrightarrow{n_1}\times\overrightarrow{n_2}|=\sqrt{100+64+16}=\sqrt{180}$

$$\text{S.D}=\frac{108}{\sqrt{180}}=\frac{108}{6\sqrt{5}}=\frac{18}{\sqrt{5}}$$

Q16 (B)



$-x+2y-z=0$

$3x-5y+2z=0$

$$\vec{n}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&2&-1\\3&-5&2\end{vmatrix}$$

$=\hat{i}(-1)-\hat{j}(1)+\hat{k}(-1)$

$\vec{n}=-\hat{i}-\hat{j}-\hat{k}$

Equation of LOI is $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

DR: of PT $\rightarrow \alpha-1, \alpha+2, \alpha-3$

DR: of QR $\rightarrow \quad 1, \quad 1, \ 1$

$$\Rightarrow (\alpha-1)\times 1+(\alpha+2)\times 1+(\alpha-3)\times 1=0$$

$$3\alpha=2$$

$$\boxed{\alpha=\frac{2}{3}}$$

$$PT^2=\frac{1}{9}+\frac{64}{9}+\frac{49}{9}$$

$$PT^2=\frac{114}{9}$$

$$\boxed{PT=\frac{\sqrt{114}}{3}}$$

$$\cos\theta=\frac{\sqrt{114}}{3}\times\frac{1}{3\sqrt{2}}=\frac{\sqrt{57}}{9}=\frac{\sqrt{19\times 3}}{3\times 3}$$

$$=\frac{\sqrt{19}}{3\sqrt{3}}$$

$$\cos 2\theta=\frac{2\times 19}{27}-1=\frac{11}{27}$$

$$\sin 2\theta=\sqrt{1-\left(\frac{11}{27}\right)^2}=\frac{\sqrt{38}\sqrt{16}}{27}$$

$$=\frac{4}{27}\sqrt{38}$$

$$\text{Area }=\frac{1}{2}\times\sqrt{18}\sqrt{18}\times\frac{4}{27}\sqrt{38}$$

$$=\frac{18}{2}\times\frac{4}{27}\sqrt{38}=\frac{36}{27}\sqrt{38} \qquad =\frac{4}{3}\sqrt{38}$$

Q17 (A)

Equation of plane passing through the intersection of planes 5x + 8y + 13z –29 = 0 and 8x – 7y + z – 20 = 0 is

$5x+8y+3z-29+\lambda(8x-7y+z-20)=0$ and

if it is passing through (2,1,3) then $\lambda=\frac{7}{2}$

P_1: Equation of plane through intersection of

$5x+8y+13z-29=0$ and $8x-7y+z-20=0$

and the point (2, 1, 3) is

$$5x+8y+3z-29+\frac{7}{2}(8x-7y+z-20)=0$$

$$\Rightarrow 2x-y+z=6$$

Similarly P_2 : Equation of plane through intersection of

5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0

and the point (0,1,2) is

$$\Rightarrow x+y+2z=5$$

Angle between planes $=\theta=\cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right)=\frac{\pi}{3}$

Q18 (B)

Equation of plane passing through line of intersection of planes $P_1 : \vec{r}\left(\left(\hat{i}+3\hat{j}-\hat{k}\right)=6\right.$ and $P_2 : \vec{r}\cdot\left(-6\hat{i}+5\hat{j}-\hat{k}\right)=7$ is

$P_1+\lambda P_2=0$

$\left(\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)-6\right)+\lambda\left(\vec{r}\cdot\left(-6\hat{i}+5\hat{j}-\hat{k}\right)-7\right)=0$

and it passes through point $\left(2,3,\frac{1}{2}\right)$

$\Rightarrow\left(2+9-\frac{1}{2}-6\right)+\lambda\left(-12+15-\frac{1}{2}-7\right)=0$

$\Rightarrow\lambda=1$

Equation of plane is $\vec{r}\cdot\left(-5\hat{i}+8\hat{j}-2\hat{k}\right)=13$

$|\vec{a}|^2=25+64+4=93$; d = 13

Value of $\frac{|13\vec{a}|^2}{d^2}=93$

Q19 (D)

DR'S normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

$\therefore$ eq^n of plane

$18x - y + 7z = d$

It passes through $(2, 3, -5)$

$36 - 3 - 35 = d \qquad \therefore d = -2$

$\therefore$ Eq^n of plane

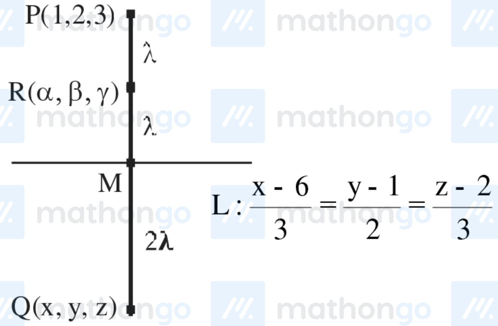
$18x - y + 7z = -2$

$-18x + y - 7z = 2$

$\therefore$ $a = -18$, $b = 1$, $c = -7$, $d = 2$

$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$

Q20 (125)



Let M be the mid-point of PQ

$\therefore M = (3\lambda+6,\ 2\lambda+1,\ 3\lambda+2)$

Now, $\overrightarrow{PM} = (3\lambda+5)\hat{i} + (2\lambda-1)\hat{j} + (3\lambda-1)\hat{k}$

$\because \overrightarrow{PM} \perp (3\hat{i}+2\hat{j}+3\hat{k})$

$\therefore 3(3\lambda+5)+2(2\lambda-1)+3(3\lambda-1)=0$

$\lambda = \frac{-5}{11}$

$\therefore M\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$

Since R is mid-point of PM

$22(\alpha+\beta+\gamma)=125$

Q21 (C)

$\frac{a-2}{3} = \frac{b-4}{-1} = \frac{c-7}{4} = \frac{-2(6-4+28-2)}{3^2+1^2+4^2}$

$\Rightarrow a = \frac{-84}{13}+2, b = \frac{28}{13}+4, C = \frac{-112}{13}+7$

$\Rightarrow 2a+b+2c=-6$

Q22 (26)

Points P(1, 2, –1) and Q (2, –1, 3) lie on same side of the plane.

Perpendicular distance of point P from plane is

$$\left|\frac{-1+2-1-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

Perpendicular distance of point Q from plane is

$$= \left|\frac{-2-1+3-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $\overrightarrow{PQ}$ is parallel to given plane. So, distance between P and Q = distance between their foot of perpendiculars.

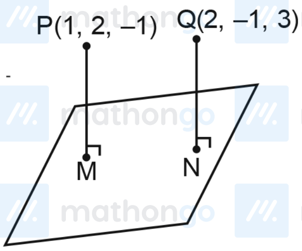
$$\Rightarrow \left|\overrightarrow{PQ}\right| = \sqrt{(1-2)^2+(2+1)^2+(-1-3)^2}$$

$= \sqrt{26}$

$\left|\overrightarrow{PQ}\right|^2 = 26 = d^2$

Alternate

$$-x+y+z-1=0$$



$M(x_1, y_1, z_1)$

$$\frac{x_1-1}{-1} = \frac{y_1-2}{1} = \frac{z_1+1}{1} = \frac{1}{3}$$

$$x_1 = \frac{2}{3}, y_1 = \frac{7}{3}, z_1 = \frac{-2}{3}$$

$$M\left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3}\right)$$

$N(x_2, y_2, z_2)$

$$\frac{x_2-2}{-1} = \frac{y_2+1}{1} = \frac{z_2-3}{1} = \frac{1}{3}$$

$$x_2 = \frac{5}{3}, y_2 = \frac{-2}{3},\ z_2 = \frac{10}{3}$$

$$N = \left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3}\right)$$

$d^2 = 1^2 + 3^2 + 4^2 = 26$

Q23 (28)

$P_1 : \vec{r}.(2\hat{i}+\hat{j}-3\hat{k}) = 4$

P_1: $2x + y - 3z = 4$

$$P_2 \begin{vmatrix} x-2 & y+3 & z-2 \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix} = 0$$

$\Rightarrow -5x + 5y + z + 23 = 0$

Let a, b, c be the d'rs of line of intersection

Then $a = \frac{16\lambda}{15}$; $b = \frac{13\lambda}{15}$; $c = \frac{15\lambda}{15}$

$\therefore \quad \alpha = 13 : \beta = 15$

Q24 (B)

(2, –1, –3) satisfy the given plane.

So $2p + q = 8$ .... (i)

Also given line is perpendicular to normal plane so

$3p + 2q - 1 = 0$ .... (ii)

$\Rightarrow p = 15, q = -22$

Eq. of plane $15x - 22y + z - 5 = 0$

its distance from origin $= \frac{6}{\sqrt{710}} = \sqrt{\frac{5}{142}}$

Q25 (B)

Image of P(1, 2, 1) in $x + 2y + 2z - 16 = 0$

is given by Q(4, 8, 7)

$$\text{Eq. of plane } T = \begin{vmatrix} x & y & z+1 \\ 4 & 8 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2x - z = 1$ so B(1, 2, 1) lies on it.