Sri Chaitanya IIT Academy.,India.
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A right Choice for the Real Aspirant
ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS-BT | Paper -2(2021-P2) | Date: 01-10-2023
Time: 02.00Pm to 05.00Pm | GTA-03 | Max. Marks: 180

KEY SHEET

PHYSICS

1	**AD**	2	**CD**	3	**BC**	4	**CD**	5	**AC**	6	**AC**
7	**2**	8	**0.33**	9	**22.5**	10	**90**	11	**11.86 - 11.87**	12	**1.89 - 1.90**
13	**D**	14	**B**	15	**A**	16	**A**	17	**9**	18	**1**
19	**6**										

CHEMISTRY

20	**ABCD**	21	**BCD**	22	**C**	23	**ABCD**	24	**AC**	25	**ACD**
26	**5.6**	27	**0.98**	28	**12**	29	**5**	30	**5**	31	**1**
32	**A**	33	**A**	34	**B**	35	**A**	36	**6**	37	**5**
38	**5**										

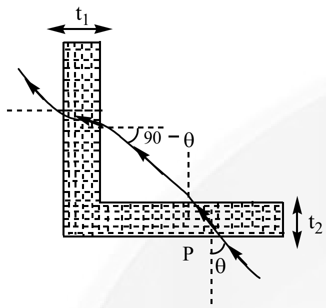
MATHEMATICS

39	**AD**	40	**CD**	41	**C**	42	**ABC**	43	**BC**	44	**ABC**
45	**144**	46	**8**	47	**734**	48	**48**	49	**10**	50	**46**
51	**D**	52	**B**	53	**D**	54	**D**	55	**3**	56	**5**
57	**7**										

SOLUTIONS
PHYSICS

1. $\Delta E = \dfrac{hc}{\lambda}$ $\Delta E = \dfrac{hc}{\lambda(in\,nm)}(in\,eV)$

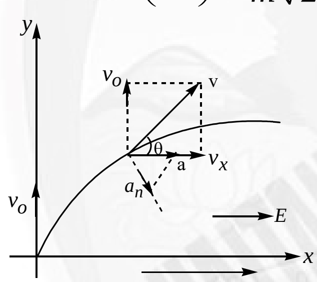
2.



3. $v^2 = v_0^2 + 2\left(\dfrac{qE_0}{m}\right)x_0$

$v = \sqrt{2}v_0$ $a_n = a\sin\theta$

$a_n = \dfrac{qE_0}{m}\left(\dfrac{v_0}{v}\right) = \dfrac{qE_0}{m\sqrt{2}}$



$R = \dfrac{v^2}{a_n} = \dfrac{\left(\sqrt{2}v_0\right)^2 m\sqrt{2}}{qE_0}$

$R = \left(\dfrac{mv_0^2}{qE_0}\right)\left(2\sqrt{2}\right)$ $R = 4\sqrt{2}x_0$

4. CONCEPTUAL

5. $g = \dfrac{GM}{R^2} = \dfrac{6.6742\times10^{-11}\times6\times10^{24}}{6.4^2\times10^{12}} = 9.77666$

$\dfrac{dg}{g} = \dfrac{dG}{G} + \dfrac{dM}{M} + \dfrac{2dR}{R}$

$= \dfrac{0.001}{6.6742} + 0.0001 + 2\times0.0002 = 0.00065$

$\therefore$ $g = (9.77767 \pm 0.0007)m/s^2$

6. The fan is running at 200 V, consuming

1000W, then $I = \dfrac{1000}{200} = 5A$

But as coil resistance is $1\,\Omega$, power dissipated by internal resistance as heat is $P_1 = I^2 R = 25W$.

If V is the net e.m.f across the coil, then

$$\frac{V^2}{R} = 25W \text{ or } V = 5V$$

Net e.m.f=source e.m.f-back e.m.f

or $V = VS - e \Rightarrow e = 195V$

7. From graph $\int \mu_3 dx = Area = \frac{1}{2}(1+3t) \Rightarrow 2t$

$$\Delta x = \mu_1 SS_2 + \mu_2 S_2P - [\mu_1 SS_1 + \mu_2 S_1P - \mu_2 t + 2t]$$

$$\Delta x = \mu_1 SS_2 + \mu_2 S_2P \quad \Delta x = 0$$

$$\Delta x = -\mu_1[SS_2 - SS_1] + \mu_2[S_2P - S_1P] + \mu_2 t - 2t$$

$$SS_2 - SS_1 = \sqrt{D^2 + d^2} - D \Rightarrow D\left[1 + \frac{d^2}{D^2}\right]^{\frac{1}{2}} - D \Rightarrow D\left[1 + \frac{1}{2}\frac{d^2}{D^2}\right] - D$$

$$SS_2 - SS_1 = \frac{1}{2}\frac{d^2}{D}\ \frac{1}{6} mm$$

$$2t - \mu_2 t = 2 \times \frac{1}{2}\frac{\left(10^{-3}\right)^2}{(1m)}\ 2t - \frac{3t}{2} = 10^{-6}\ \frac{t}{2} = 10^{-6}$$

$$t = 2 \times 10^{-6} m (t = 2\mu m)$$

8. $\Delta x = \mu_1[SS_2 - SS_1] + \mu_2[S_2P - S_1P] + \mu_2 t - 2t$

For central maxima

$$0 = 2\frac{1}{2}\frac{d^2}{D} + \frac{3}{2}(S_2P - S_1P) + \frac{3t}{2} - 2t$$

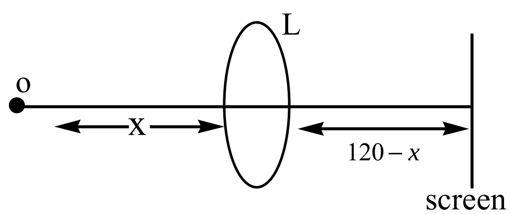
$$\frac{3}{2}(S_2P - S_1P) = 2t - \frac{3t}{2} - \frac{d^2}{D}$$

$$(S_2P - S_1P) = \frac{2}{3}\left[\frac{t}{2} - 10^{-6}\right] \quad \Rightarrow \frac{2}{3}\left[\frac{10^{-6}}{2} - 10^{-6}\right]$$

$$S_2P - S_1P \Rightarrow -\frac{1}{3}\mu m \qquad d\sin\theta \cong d\tan\theta$$

$$d\frac{y}{D} = -\frac{1}{3}\mu m \qquad y = -\frac{1}{3}\frac{10^{-6} \times 1}{10^{-3}} m \quad y = -\frac{1}{3} mm$$

9.



$m_1:m_2=1:9 \quad m_1.m_2=1$

$\therefore m_1=\frac{1}{3} \quad m_2=3 \quad m=\frac{v}{u}$

$\frac{1}{3}=\frac{120-x_1}{x_1} \quad 4x_1=360$

$x_1=90 \quad x_2=120-90=30$

$D=120 \quad d=x_1-x_2=60 \quad f=\frac{D^2-d^2}{4D}=22.5$

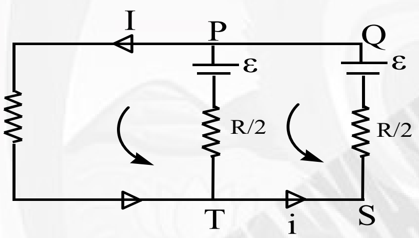
10. CONCEPTUAL

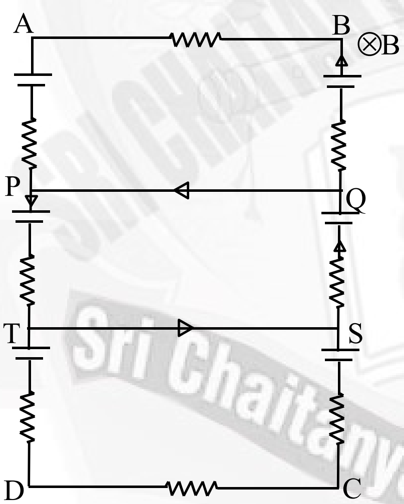
11. $V_A-V_C=V_A-V_B=\frac{\lambda}{2\pi\in_0}\ln\frac{r_B}{a}$

$=\frac{6\times10^{-7}\times2}{4\pi\in_0}\ln\frac{7.5}{2.5} \quad =12\times10^{-7}\times9\times10^9\ln3 \quad =11.86488\times10^3$

12. $\Delta K=e(V_A-V_B) \ =1.6\times10^{-19}\times11.86\times10^{-3} \quad =1.89\times10^{-15}$

13.





(7)(D) Current in side AB and CD=0

$\varepsilon=BV_0l$

Apply KVL

$$I\times R+(I-i)\times\frac{R}{2}-\varepsilon=0.....(1)$$

$$I\times\frac{R}{2}-\varepsilon+\varepsilon-(I-i)\times\frac{R}{2}=0.....(2) \quad \Rightarrow I=\frac{4\varepsilon}{5R}, i=\frac{2\varepsilon}{5R}$$

A) Current in $R = \dfrac{4Bv_0 I}{5R}$

B) Current in AB=p

C) Current in CD=0

D) Current in ST-$i = \dfrac{2\varepsilon}{5R} = \dfrac{2BV_0 l}{5R}$

(8) (D) Magnetic force on wire

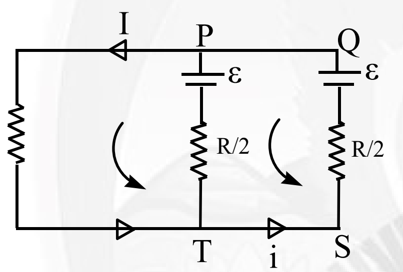
Force=force on QS+ force on PT $= BiI + B(I-i)l = \dfrac{4B^2 l^2 V_0}{5R}$

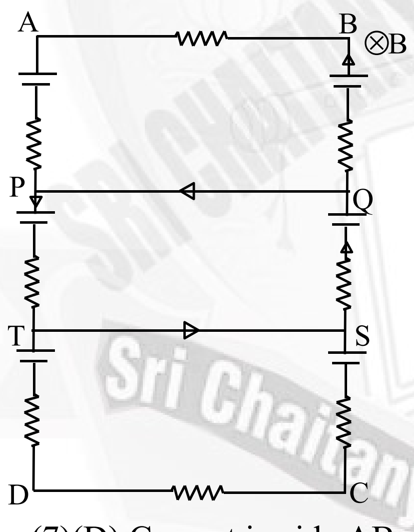
Let potential at S is zero

$V_s = 0 \; p \; V_C = -\dfrac{\varepsilon}{2} \quad V_Q = \varepsilon - i \times \dfrac{R}{2} = \dfrac{4\varepsilon}{5R} \Rightarrow V_B = V_Q + \dfrac{\varepsilon}{2}$

$V_{BC} = V_B - V_C = V_Q + \dfrac{\varepsilon}{2} + \dfrac{\varepsilon}{2} \qquad V_{BC} = \dfrac{9\varepsilon}{5} = \dfrac{9BV_0 I}{5}$

14.





(7)(D) Current in side AB and CD=0

$\varepsilon = BV_0 l$

Apply KVL

$I \times R + (I-i) \times \dfrac{R}{2} - \varepsilon = 0.....(1)$

$I \times \dfrac{R}{2} - \varepsilon + \varepsilon - (I-i) \times \dfrac{R}{2} = 0.....(2) \quad \Rightarrow I = \dfrac{4\varepsilon}{5R}, i = \dfrac{2\varepsilon}{5R}$

A) Current in $R = \dfrac{4Bv_0 I}{5R}$

B) Current in AB=0

C) Current in CD=0

D) Current in ST=$i=\frac{2\varepsilon}{5R}=\frac{2BV_0 l}{5R}$

(8)(D) Magnetic force on wire

Force=force on QS + force on PT $\quad =BiI+B(I-i)l=\frac{4B^2l^2V_0}{5R}$

Let potential at S is zero

$V_s=0\ p\ V_C=-\frac{\varepsilon}{2}$

$V_Q=\varepsilon-i\times\frac{R}{2}=\frac{4\varepsilon}{5R}\Rightarrow V_B=V_Q+\frac{\varepsilon}{2}$

$V_{BC}=V_B-V_C=V_Q+\frac{\varepsilon}{2}+\frac{\varepsilon}{2} \qquad V_{BC}=\frac{9\varepsilon}{5}=\frac{9BV_0I}{5}$

15. $\eta_{th}=\frac{\text{Net workdone}}{\text{Net heat added}}$

Since processes 1-2 and 3-4 are adiabatic processes, the heat transfer during the cycle takes place only during processes 2-3 and 4-1 respectively. Therefore, thermal efficiency can be written as,

$\eta_{th}=\frac{\text{Heat added-Heat rejected}}{\text{Heat added}}$

Consider 'm' kg of working fluid,

Heat added= $mC_v=(T_3-T_2)$

Heat Rejected= $mC_v=(T_4-T_1)$

$\eta_{th}=\frac{mC_v(T_3-T_2)-mC_v(T_4-T_1)}{mC_v(T_3-T_2)}=1-\frac{T_4-T_1}{T_3-T_2}$

For the reversible adiabatic processes 3-4 and 1-2, we can write,

$\frac{T_4}{T_3}=\left(\frac{v_3}{v_4}\right)^{\gamma-1}$ and $\frac{T_1}{T_2}=\left(\frac{v_2}{v_1}\right)^{\gamma-1}$

$v_2=v_3$ and $v_4=v_1$

$\frac{T_4}{T_3}=\frac{T_1}{T_2}=\frac{T_4-T_1}{T_3-T_2}=\left(\frac{V_2}{V_1}\right)^{\gamma-1}$

$\eta_{th}=1-\frac{T_1}{T_2}=1-\left(\frac{V_2}{V_1}\right)^{\gamma-1}$

The ratio $\frac{V_1}{V_2}$ is called as compression ratio, r.

$\eta_{th}=1-\left(\frac{1}{r}\right)^{\gamma-1}$

Net work done= $mC_v\{(T_3-T_2)-(T_4-T_1)\}$

Displacement volume $=(V_1-V_2)$

$=V_1\left(1-\frac{1}{r}\right)=\frac{mRT_1}{P_1}\left(\frac{r-1}{r}\right)$

$=\frac{mC_v(\gamma-1)T_1}{P_1}\left\{\frac{r-1}{r}\right\}$

Since, $R=C_v(\gamma-1)$

16. mep=$\dfrac{mC_v\left[(T_3\text{-}T_2)\text{-}(T_4\text{-}T_1)\right]}{\dfrac{mC_v(\gamma\text{-}1)T_1}{P_1}\left\{\left(\dfrac{r\text{-}1}{r}\right)\right\}}$

$=\left(\frac{1}{\gamma-1}\right)\left(\frac{P_1}{T_1}\right)\left(\frac{r}{r-1}\right)\{(T_3-T_2)-(T_4-T_1)\}$

17. F=$\int_0^R \frac{Q}{\pi R^2}2\pi rdr\ \omega rB_0 \Rightarrow \frac{2}{3}Q\omega B_0R=mg$

$\omega=9\times10^2 rad/s$

18. One particle t/2 before max height, other t/2 after max height
Relative velocity perpendicular to line joining them=gt
Relative separation=ut
Relative angular velocity=gt/ut=g/u

19. For any small change of pressure dp, there will be a change of volume dV and

$dp=-B\frac{dV}{V}$. In this change, work is done on the system and the energy stored in the material is

$dW=-pdV\left(\frac{V}{B}\right)pdp$

In the change mentioned in the question, the total work done is

$W=-\int_v^v pdV=\int_v^v \frac{V}{B}pdp$

The change in volume is negligible and volume can be treated as constant.

$W=\frac{V_0}{B}\int_{p_0}^{p} pdp=\frac{V_0}{B}\left(\frac{p^2\text{-}p_0^2}{2}\right)$

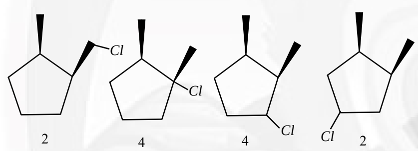
Extra energy stored per unit volume is

$\frac{W}{V_0}=\frac{1}{2B}\left(p^2\text{-}p_0^2\right)$

CHEMSIRTY

20. Conceptual
21. Conceptual
22. Conceptual
23. Hint: A) Nitro benzene will not undergo alkylation and acylation
    B) Alkene is more reactive than benzene
    D)Niroso electrophile will not react with bezene
24. ANS-AC
    Sol: $(i) HOCl < HClO_2 < HClO_3 < HClO_4 - acidic\ strength$
    $(ii) HCl < HBr < HI < HF : Boiling\ \ point$
25. ANS-ACD
    Sol: $Co_2(CO)_8 : EAN = \frac{(2\times 27)+16+2}{2} = 36$
26. Conceptual
27. Conceptual

28, 29.
Sol:



30. ANS-5
31. ANS-1

SOL- $Br_2+6NaOH \rightarrow 5NaBr+NaBrO_2+3H_2O$

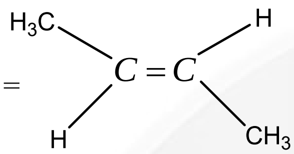
32. Sol: AB and CD are process $\frac{1}{V}=KT \quad \frac{1}{K}=VT$
    Or VT= constant
    Put $T=\frac{PV}{R} \quad V.\frac{PV}{R}=\text{conastant} \quad \Rightarrow PV^2=\text{conastant}$
    Compare with $PV^x=\text{conastant} \quad X=2$
    $C_m=C_{vm}+\frac{R}{(1-x)} \quad C_m=\frac{3R}{2}+\frac{R}{(1-2)} \quad C_m=\frac{R}{2}$
    For process BC, $\left(\frac{1}{V}\right)$ is constant it means V is constant
    $\Delta S=nC_{vm}\ln\frac{T_C}{T_B} \quad =1\times\frac{3R}{2}\ln\frac{300}{200} \quad =3\ln\frac{3}{2}\text{calorie/K}$
33. Ans: A
    $Q=Q_{AB}+Q_{BC}+Q_{CD}\left[T_A=T_D\right]$
    $=nC_m\left[T_B-T_A\right]+nC_{Vm}\left[T_C-T_B\right]+nC_m\left[T_D-T_C\right]$
    $=\frac{R}{2}\left[200-T_A\right]+\frac{3R}{2}\left[300-200\right]+\frac{R}{2}\left[T_D-300\right]$

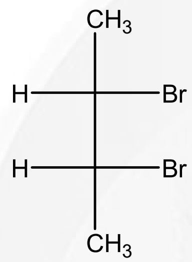
$$\frac{R}{2}\left[200-T_A+T_D-300\right]+\frac{3R}{2}\times 100$$

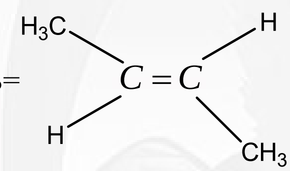
$$=\frac{R}{2}\times(-100)+\frac{3R}{2}\times 100 \quad =100R \qquad =200\,\text{calorie}$$

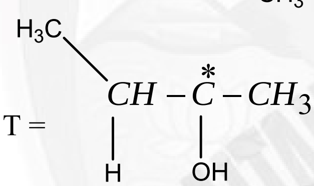
34. 35 Ans: A

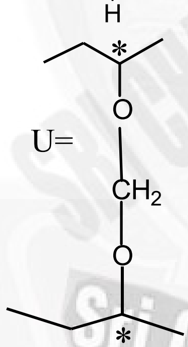
SOL-32-33: Ans: $P = CH_3-C\equiv C-CH_3$

Q = 

R = 

S= 

T = 

U= 

36. (i),(ii),(v),(vii),(ix),(x)

37. ANS-5

Sol: Azurite, Calamine, Siderite, Magnesite, Dolomite

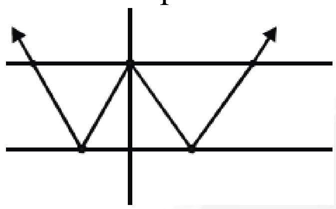
38. ANS-5

Sol: $KO_2, \left[Cu\left(NH_3\right)_4\right]^{2+}, \left[Ni\left(NH_3\right)_6\right]^{2+}, \left[Cr\left(NH_3\right)_6\right]^{3+}, O_2$

MATHEMATICS

39. $f(x)=f\left(\frac{x}{2}\right)=k=\frac{3\pi}{2}$

Number of points where function is not differentiable = 5



40. $f(x)-e^{x}+\frac{1}{f(x)}-e^{-x}=0$

$\left(f(x)-e^{x}\right)+\frac{e^{x}-f(x)}{e^{x}f(x)}=0$

$\left(f(x)-e^{x}\right)\left(e^{x}f(x)-1\right)=0 \quad \Rightarrow f(x)=e^{x}$ or e^{-x}

Or $\begin{cases} e^{x}, & x\ge 0 \\ e^{-x}, & x<0 \end{cases}$ or $\begin{cases} e^{x}, & x<0 \\ -e^{-x}, & x\ge 0 \end{cases}$

$\int_0^1 e^{x}\,dx=e-1 \qquad \int_0^1 e^{-x}\,dx=1-\frac{1}{e}$

$\therefore$ Area can be $e-1+1-\frac{1}{e}$ Or $e-1+e-1$ or $1-\frac{1}{e}+1-\frac{1}{e}$

41. Observe that

$$\frac{k+1}{k}\left(\frac{1}{\binom{n-1}{k}}-\frac{1}{\binom{n}{k}}\right)=\frac{k+1}{k}\frac{\binom{n}{k}-\binom{n-1}{k}}{\binom{n}{k}\binom{n-1}{k}}=\frac{k+1}{k}\frac{\binom{n-1}{k-1}}{\binom{n}{k}\binom{n-1}{k}}=\frac{1}{\binom{n}{k+1}}$$

Now apply this with $k=2008$ and sum across all n from 2009 to ∞. We get

$$\sum_{n=2009}^{\infty}\frac{1}{\binom{n}{2009}}=\frac{2009}{2008}\sum_{n=2009}^{\infty}\frac{1}{\binom{n-1}{2008}}-\frac{1}{\binom{n}{2008}}$$

All terms from the sum on the right-hand-side cancel, except for the initial $\frac{1}{\binom{2008}{2008}}$,

which is equal to 1,

So, we get $\sum_{n=2009}^{\infty}\frac{1}{\binom{n}{2009}}=\frac{2009}{2008}$

42. $y = \dfrac{x^2+2x+a}{x^2+4x+3a}$

After supplication we get

$a(a-1) \le 0$

If $a = 0,1$ the range of f is not equal to R so $0 < a < 1$

43. As, $AA^T = I_2 \qquad \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a = 0, b = \pm 1, d = 0, c = \pm 1$

Total 8 matrices are possible

They are

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Also, $|A - I_2| = |A - AA^T| = |A||I_2 - A^T|$

$= |A|\left|\left(I_2 - A^T\right)^T\right| = |A||I_2 - A| \qquad = |A||A - I_2|$

$\Rightarrow |A| = 1 \ \left(As, |A - I_2| \ne 0\right)$

except $A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Where $|A| = 1$ but

$|A - I_2| = 0$

44. Let $h(x) = f(x) - 5g(x)$

$h(x)$ is continuous, differentiable & $f'(x)$ is

Also, continuous, differentiable

$h(2) = 6 = h(0) = h(1) = h(3)$ & $(f - 5g)''$never vanishes

$\Rightarrow$ Exactly one root in $(-2,0),(0,1)$ & $(1,3)$

45. Let $z \in A$ *and* $\omega \in B$ *then* $z = e^{\frac{12k_1x}{18}}, \omega = e^{\frac{12k_2x}{48}}$ $k_1, k_2 \in I$

$\therefore z\omega = e^{\frac{12x}{6}\left(\frac{k_1}{3}+\frac{k_2}{8}\right)} = e^{12\left(\frac{8k_1+3k_2}{144}\right)\pi} = e^{\frac{12kx}{144}}$ *where* $k = 8k_1 + 3k_2$

$\therefore z\omega$ can have 144 different values

46. Let $C_k = e^{\frac{i(2k-1)\pi}{n}}, k = 1,2,\ldots\ldots,n$ *then* $|C_k - C_1| = |\cos\theta + i\sin\theta - 1|$

where $\theta = \dfrac{(2k-1)\pi}{n} = 2\sin\dfrac{\theta}{2}$

$$\therefore \left|C_2-C_1\right|+\left|C_3-C_1\right|+......+\left|C_n-C_1\right|=2\left[\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+......+\sin\frac{(n-1)\pi}{n}\right]$$

$$=2\cot\frac{\pi}{2n}=2\cot\frac{180^\circ}{2\times 144}=2\cot\left(\frac{5}{8}\right)^\circ$$

47 & 48

We want to have $a_n=k \text{ if } \frac{k(k-1)}{2}<n\le\frac{k(k+1)}{2}$

$\therefore n$ is an integer, this is equivalent to $\frac{k(k-1)}{2}+\frac{1}{8}<n\frac{k(k+1)}{2}+\frac{1}{8}$

$$\Rightarrow k^2-k+\frac{1}{4}<2n<k^2+k+\frac{1}{4}$$

$$\Rightarrow k-\frac{1}{2}<\sqrt{2n}<k+\frac{1}{2}\Rightarrow k<\sqrt{2n}+\frac{1}{2}<k+1$$

Hence, $a_n=\left[\sqrt{2n}+\frac{1}{2}\right]\Rightarrow \alpha=2=\beta$

47. Now, $a=2,b=3,c=5$

Let A = Number of numbers which are divisible by 2
B = Number of numbers which are divisible by 3
C = Number of numbers which are divisible by 5

Required number $=A+B+C-A\cap B-B\cap C-C\cap A+A\cap B\cap C$

$$=\left[\frac{1000}{2}\right]+\left[\frac{1000}{3}\right]+\left[\frac{1000}{5}\right]-\left[\frac{1000}{6}\right]-\left[\frac{1000}{15}\right]-\left[\frac{1000}{10}\right]+\left[\frac{1000}{30}\right]=734$$

48. $a=2,b=3,c=5,d=7$

Hence, the given number is $2^5\cdot 3^5\cdot 5^3\cdot 7^3$

$\because 4n+1$ is odd number therefore the factor 2 will not occur in divisor, 3 and 7 are of $4n+3$ form,

Odd powers of 3 and 7 will be of $4n+3$ form and even powers will be $4n+1$ form

5 is $4n+1$ form and any power of 5 will be of $4n+1$ form

$\therefore$ Number of divisors of $4n+1$ type

= Number of terms in the product $\left(1+3^2+3^4\right)\left(1+5+5^2+5^3\right)\left(1+7^2\right)$ + Number of terms in the product $\left(3+3^2+3^5\right)\left(1+5+5^2+5^3\right)\left(7+7^3\right)=48$

49. For $\lambda=10\Rightarrow t_1=2,t_2>2 \text{ and if } x+1/x=2\Rightarrow x=1 \text{ and } x+1/x=t_2$ So there will be two distinct values of x

50. Since domain of $f(t)=2t^3-9t^2+30, t=x+1/x, |t|\ge 2$ $f(-2)=-22$, $f(2)=10$, critical points at $t=0\ \&\ 3$; $f(3)=3$

51 & 52

$$PA+PB<2 \text{ and } PB+PC<2$$

$$PA+PB<2$$

Region inside ellipse with foci A and B

$2a = 2, a = 1 \quad 2ae = \frac{3}{2} - \frac{1}{2} = 1$

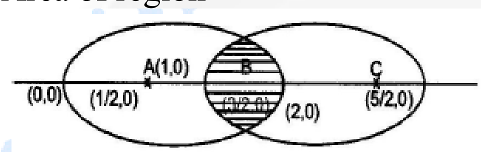
$b^2 = a^2 - a^2e^2 = 1 - \frac{1}{4} = \frac{3}{4}$

Equation of ellipse is $\frac{(x-1)^2}{1} = \frac{y^2}{3/4} = 1$

$PB + PC < 2 \Rightarrow P$ lies inside ellipse with foci B and C whose equation is $\frac{(x-2)^2}{1} + \frac{y^2}{3/4} = 1$

Locus of P is shown by shaded region which is symmetric about x-axis.

Area of region



$$= 4\int_1^{3/2} \frac{\sqrt{3}}{2}\sqrt{1-(x-2)^2}\,dx = \sqrt{3}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

53. $\ell n f(x) = \lim_{n\to\infty}\left[\log\prod_{r=1}^{n}\left(x+\frac{r}{n}\right) - \log\prod_{r=1}^{n}\left(\frac{r}{n}\right)\right]$

$$= \int_0^1 \log(x+t)\,dt - \int_0^1 \log t\,dt \quad = (x+1)\log(x+1) - (x+1) - x\log x + x + 1$$

$$= (x+1)\log(x+1) - x\log x$$

$$\frac{f'(x)}{f(x)} = \log(x+1) - \log x = \log\left(1+\frac{1}{x}\right) > 0$$

$$\Rightarrow \lim_{x\to\infty}\frac{xf'(x)}{f(x)} = \lim_{x\to\infty} x.\log\left(1+\frac{1}{x}\right) = 1$$

54. $\because f'(x) > 0 \Rightarrow f(x)\uparrow x\in(0,\infty) \Rightarrow f(2) > f(1)$

$\Rightarrow \ell n f(x) \in (0,\infty) \Rightarrow f(x)\in(1,\infty)$ i.e. Range of $f(x)$

55. $\sum_{i=1}^{10}(x_i - \bar{x})(y_i - \bar{y}) = 80$

$\sum_{i=1}^{10} x_i y_i - \bar{y}\sum_{i=1}^{10} x_i - \bar{x}\sum_{i=1}^{10} y_i + \sum_{i=1}^{10}\overline{xy} = 80$ which implies $\sum_{i=1}^{10} x_i y_i - 10\overline{yx} = 80$

$$\sigma^2 = \frac{\sum_{i=1}^{10}(x_i - y_i)^2}{10} - (\bar{y} - \bar{x}) = 9$$

56. $\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+\hat{3}}{\sqrt{14}}$ $\Rightarrow |\vec{a}|=1, |\vec{b}|=1$ and $\vec{a}.\vec{b}=0$

$Now, (2\vec{a}+\vec{b}).\{(\vec{a}\times\vec{b})-(\vec{a}-2\vec{b})\}$

$=(2\vec{a}+\vec{b}).\left[\vec{a}^2\vec{b}-(\vec{a}.\vec{b}).\vec{a}+2\vec{b}^2.\vec{a}-\vec{2}(\vec{b}.\vec{a}).\vec{a}\right]$ $=4.1+1=5$

57. Let vertices $A_1, A_2, ...A_7$ are 7^{th} roots of unity. Let $A_1(1), A_2(\alpha), A_3(\alpha^2), A_4(\alpha^3)$

$A_5=(\alpha^4)=\overline{\alpha^3}, A_6=(\alpha^5)=\overline{\alpha^2}, A_7=(\alpha^6)=\overline{\alpha}$

$(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4)(1-\alpha^5)(1-\alpha^6)=7$ $\Rightarrow \left(|1-\alpha||1-\alpha^2||1-\alpha^3|\right)^2=7$

$p_1=(A_1A_2)(A_1A_3)(A_1A_4)(A_1A_5)(A_1A_6)(A_1A_7)$

$=\left(|1-\alpha||1-\alpha^2||1-\alpha^3|\right)^2=(\sqrt{7})^2$

Similarly

$p_2=|1-\alpha||1-\alpha^2||1-\alpha^3||1-\alpha^2|$

$\sqrt{7}\left(|1-\alpha^3||1-a^2|\right)$ $p_3=\sqrt{7}\left(|1-\alpha^2|\right)$

$p_4=\sqrt{7}, p_5=|1-\alpha||1-\alpha^2|$ and $p_6=|1-\alpha|$

$\Rightarrow p_1.p_2.p_3.p_4.p_5.p_6=(\sqrt{7})^7$