Sri Chaitanya IIT Academy.,India.

A.P ✪ T.S ✪ KARNATAKA ✪ TAMILNADU ✪ MAHARASTRA ✪ DELHI ✪ RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS & STERLING_BT Paper -1(Adv-2021-P1-Model Date: 13-08-2023

Time: 09.00Am to 12.00Pm RPTA-02 Max. Marks: 180

KEY SHEET

PHYSICS

1	**B**	2	**B**	3	**A**	4	**A**	5	**45**	6	**49**
7	**2.5**	8	**4**	9	**0.25**	10	**0.9**	11	**BC**	12	**ABC**
13	**D**	14	**ABD**	15	**AD**	16	**ABD**	17	**2**	18	**6**
19	**3**										

CHEMISTRY

20	**C**	21	**C**	22	**C**	23	**A**	24	**59.01**	25	**137.91**
26	**3**	27	**1.33**	28	**6**	29	**27**	30	**ABC**	31	**ACD**
32	**AC**	33	**AD**	34	**ABC**	35	**AB**	36	**8**	37	**7**
38	**4**										

MATHEMATICS

39	**D**	40	**C**	41	**D**	42	**B**	43	**29.33**	44	**0**
45	**3**	46	**2**	47	**6.4**	48	**10**	49	**AC**	50	**D**
51	**CD**	52	**AB**	53	**BC**	54	**ACD**	55	**4**	56	**5**
57	**3**										

SOLUTIONS
PHYSICS

1. Parallel to mirror , $\upsilon_I = \upsilon_0 = 3\hat{i} + 4\hat{j}$

Perp to mirror , $\upsilon_I = -\upsilon_0 + 2\upsilon_M = -(5\hat{k}) + 2(8\hat{k}) = 11\hat{k}$

2. Object at x from concave mirror.

Image at $y = x\left(\frac{f}{x-f}\right)$

Then, $\left(\frac{x+y}{2}\right) = 22.5$

3. $\frac{1}{\upsilon} - \frac{\left(\frac{3}{2}\right)}{(+30)} = \frac{\left(1-\frac{3}{2}\right)}{(+20)}$

4. $f_2 = f_1 + d \Rightarrow d = 20 - 10 = 10\text{cm}$

5. $2\pi = A \Rightarrow \pi = 30^0$

$\frac{\sin i}{\sin \pi} = \sqrt{2} \Rightarrow i = 45^0$

6. $i = 90^0,\ r_1 = 45^0,\ r_2 = A - r_1 = 15^0,\ \frac{\sin e}{\sin 15} = \sqrt{2}$

$e = \sin^{-1}\left(\sqrt{2}\sin 15\right) = \sin^{-1}(0.32) = 19^o$

$\delta = i + e - A = 90 + 19 - 60 = 49^o$

7. Conceptual

8. $\frac{I_1}{I_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{5}{2.5}\right)^2 = 4$

9. $\omega = \frac{\mu_r - \mu_\pi}{\mu_y - 1}$

10. $\delta = (\mu_V - \mu_R)A = 0.9^o$

11. $\mu = \frac{\sin i}{\sin r} = \frac{1}{\text{slope}} = \sqrt{3} \Rightarrow$ x is denser medium

12. Conceptual

13. Conceptual

14. $\frac{n_2}{\vartheta_1} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{+R}$ and $\frac{n_3}{\vartheta} - \frac{n_2}{\vartheta_1} = \frac{(n_3 - n_2)}{-R}$

$\Rightarrow \frac{n_3}{\vartheta} - \frac{n_1}{u} = \frac{(2n_2 - n_1 - n_3)}{R}$

$u \to \infty$ and $\vartheta > 0 \Rightarrow$ converging lens

$u \to \infty$ and $\vartheta < 0 \Rightarrow$ diverging lens

15. $\frac{1}{\vartheta} - \frac{1.5}{\infty} = \frac{(1-1.5)}{(-6)} \Rightarrow \vartheta = +12\text{cm}$

$\frac{1.5}{\vartheta} - \frac{1}{\infty} = \frac{(1.5-1)}{(+6)} \Rightarrow \vartheta = +18\text{cm}$ and $\frac{(18-6)}{1.5} = \frac{h_2}{1} \Rightarrow h_2 = 8\text{cm}$

16. $\frac{\sin 90}{\sin \pi} = \mu \Rightarrow \pi = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\delta = i - \pi = 90^0 - \sin^{-1}\left(\frac{1}{\mu}\right) = \cos^{-1}\left(\frac{1}{\mu}\right)$

17. $\frac{\left(\frac{4}{3}\right)}{\vartheta} - \frac{\left(\frac{3}{2}\right)}{\left(\frac{-15}{2}\right)} = \frac{\left(\frac{4}{3}-\frac{3}{2}\right)}{(-5)} \Rightarrow \vartheta = -8\text{cm}$

$\frac{(10+8)}{\left(\frac{4}{3}\right)} = \frac{h_2}{1} \Rightarrow h_2 = 13.5\text{cm}$

18. Distance of object from free surface =x

Distance of first image from free Surface = μx

Distance of first image from free Mirror = $(\mu x + y)$

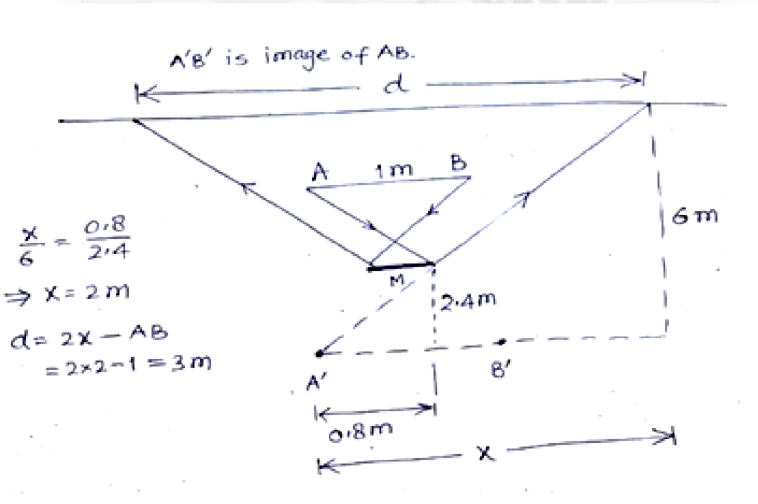
Distance of Secondary image from free mirror = $(\mu x + y)$

Distance of Secondary image from free Surface = $(\mu x + 2y)$

Distance of Third image from free Surface = $\frac{(\mu x + 2y)}{\mu}$ = $x + \frac{2y}{\mu}$

Speed of image = $\frac{d}{dt}\left(x + \frac{2y}{\mu}\right) = \frac{2}{\mu}\frac{dy}{dt} = 2x\frac{3}{4}x4 = 6\text{cm/s}$

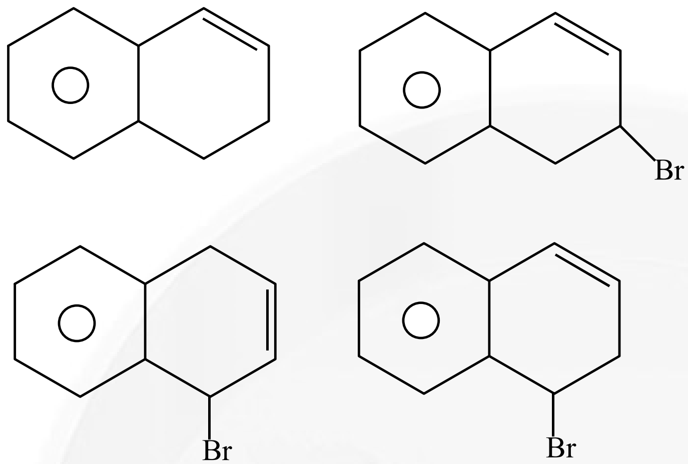
19.



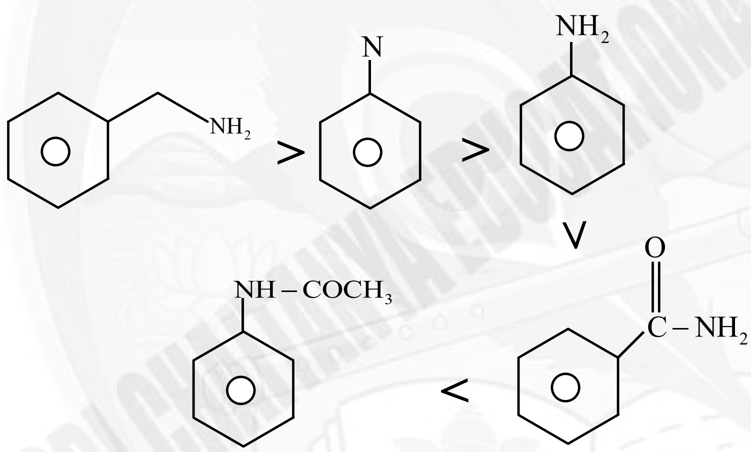
CHEMISTRY

20. In II , Si shows $P\pi - d\pi$ back bonding

21.

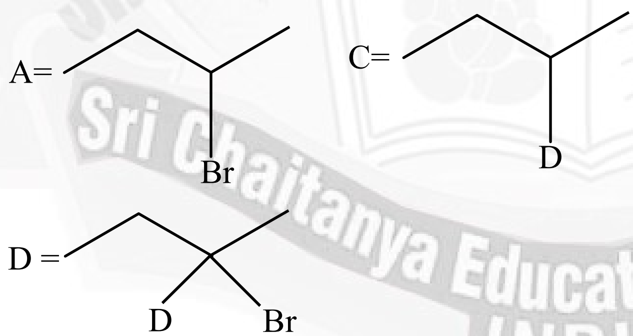


22. Maleic acid $Ka_1 >$ fumaric acid Ka_1

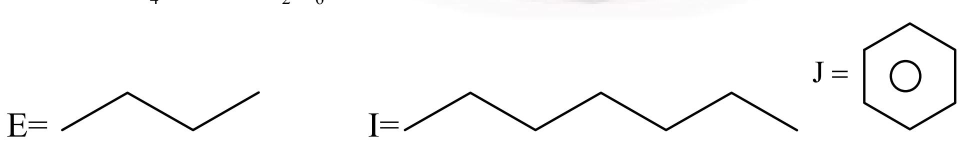


23.

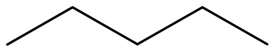
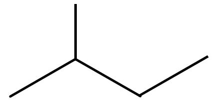
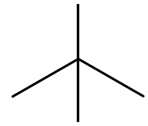
24 & 25.



26 & 27. $A = CH_4$ $\quad C = C_2H_6$



A, C, & E are gases at R.T $\frac{\cancel{24}}{\cancel{8}} = \frac{4}{3} = 1.33$

28 & 29. X=  6 Y=  Z= 

30. C Show nitro so oxime tautomerism

31. B is anti-aromatic and cannot be stable

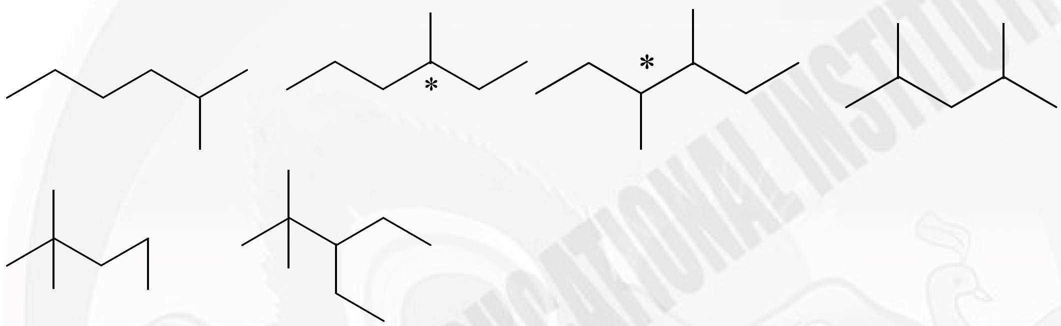
32. A and C contain both acidic and basic groups

33. Picric acid > chloro acetic acid > HCOOH > Ph-COOH > CH_3COOH

34. Compounds have equivalent resonating structures can have same bond lengths

35. Conjugate base of 3 is stabilized by $P\pi - d\pi$ resonance.

36. M. F is C_7H_4



37. **x=4, y=3**

38. $x = 2,\ y = 3,\ z = 4,\ P = 5$

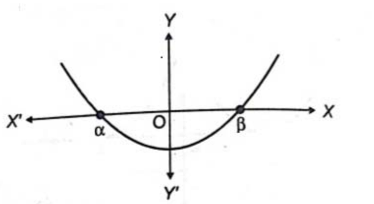
MATHEMATICS

39. $f(x)=(4-p)x^3+(p-2)x^2+(p^2-25)x+2$

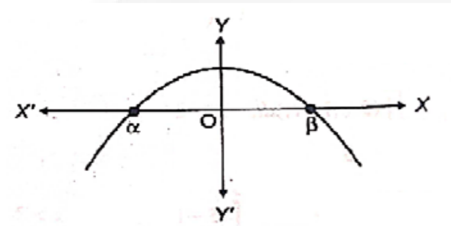
$f^1(x)=3(4-p)x^2+2(p-2)x+(p^2-25)$

One root positive and one root negative.

Let 4 – p > 0;



Here Minima occurs at α and maxima occurs at β which is allowed



Let 4 – p < 0

4 < p

f(0) > 0;

$p^2-25>0$

$\Rightarrow p\in(-\infty,-5)\cup(5,\infty)$

Thus, $p\in(5,\infty)$

40. $e^x=\frac{k}{x-3};x\neq3$

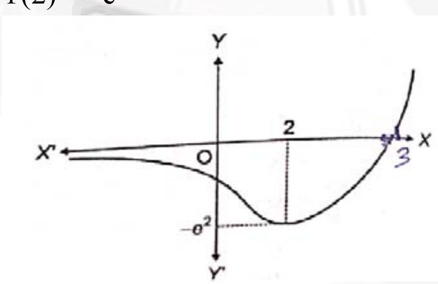
$e^x(x-3)=k$

Let $f(x)=e^x(x-3)$

$f'(x)=e^x(x-3)+e^x.1$

$=e^x(x-2)=0;\ x=2$

$f(2)=-e^2$



$-e^2<k<0$

$k\in\{-7,-6,-5,-4,-3,-2,-1\}$ Seven values

41. $f(x)=\left\{\frac{x-\sin x}{5}\right\}$

Consider $g(x)=\frac{x-\sin x}{5};g'(x)=\frac{1-\cos x}{5}\geq0$ for $x\in(0,20\pi)$

Thus, g(x) is an increasing function, also the range of

$g(x)=\frac{x-\sin x}{5}$ is $(0,4\pi)$.

At all integral values, f(x) will not be derivable. Hence there are $[4\pi]=12$ points, where f(x) is not derivable.

Thus, there are 12 values of c, for which $g'(c)=\frac{g(b)-g(a)}{b-a}$ has same values.

Thus, by Rolle's Theorem for $g'(x), g''(x)$ will vanish at (12 – 1) =11 points (minimum).

Hence n=11

Thus, $\left[\frac{n}{2}\right]=5$.

42. Ans. 0003

Thus is based on the concept of points of inflection. For any cubic curve, if we draw tangent at the point of inflection, let say P. If we take any general point Q on this tangent, other than P, then we can always draw two distinct tangents.

Here, given points (h, 2 – 5h) lies on the line $y=2-5x$ where $h\neq 1$ given

Hence, for point of inflection P; x = 1; y = -3 $(\because y=2-5x)$

Curve must satisfy point of inflection,

$$y=x^3-3x^2-ax+b$$

$$-3=1-3-a+b; a-b=1$$

Slope of tangent and curve, will be same at $P(1,-3)$

$$\frac{dy}{dx}=3x^2-6x-a$$

$$\left.\frac{dy}{dx}\right|_{at\,x=1}=3-6-a=-3-a$$

$$y=2-5x$$

$$\left.\frac{dy}{dx}\right|_{at\,x=1}=-5$$

$$-3-a=-5$$

$$\Rightarrow a=2$$

$$\therefore b=1 \qquad [\because a-b=1]$$

43 & 44 . $f(x)=\frac{x^3}{2}+1-x\int_0^x g(t)dt$

Let $\int_0^1 f(t)dt=K$

$g(x)=x-K$

$$f(x)=\frac{x^3}{2}+1-x\int_0^x (t-K)dt$$

$$=\frac{x^3}{2}+1-x\left[\frac{t^2}{2}-Kt\right]_0^x$$

$$=\frac{x^3}{2}+1-x\left(\frac{x^2}{2}-Kx\right)=1+Kx^2$$

As $\int_0^1 f(t)dt=K$

So, $\int_0^1 \left(1+Kt^2\right)dt = K;\quad \left[t+\frac{Kt^3}{3}\right]_0^1 = K$

$1+\frac{K}{3}=K;\quad 1=\frac{2K}{3};\quad K=\frac{3}{2}$

$$g(x)=x-\frac{3}{2}$$

$$f(x)=1+\frac{3}{2}x^2$$

$$\lim_{x\to 0}\frac{1}{x^3}\int_0^x \left(f(t)-1\right)dt=\lim_{x\to 0}\frac{\int_0^x \frac{3}{2}t^2dt}{x^3}$$

$$\lim_{x\to 0}\frac{\frac{3}{2}x^2}{3x^2}=\frac{1}{2}$$

$f(|x|)=f(x)$. Hence differentiable everywhere.

$f'(x)=3x.f'(x)\big|_{at\ x=2}=6;$ Slope of normal, $m=\frac{1}{6}$

$$y-7=-\frac{1}{6}(x-2)\qquad (\text{Put } y=0)$$

$$x=44.$$

45 & 46 . $f(x)=\cos 2x+2x\lambda^2+(2\lambda+1)(\lambda-1)x^2,\ \lambda\in R$

$\lambda=1;\quad f(x)=\cos 2x+2x.$

$f'(x)=2-2\sin x=2(1-2\sin x)$

$f'(x)\ge 0$ thus f(x) is increasing.

$f\left(3x^2-2x+1\right)<f\left(x^2-2x+9\right)$

$\Rightarrow 3x^2-2x+1<x^2-2x+9$

$\Rightarrow 2x^2<8;\quad x^2<4;\quad -2<x<2$

$\Rightarrow x=-1,\ 0,\ 1$

$f'(x)=-2\sin 2x+2\lambda^2+(2\lambda+1)(\lambda-1)2x$

At $\lambda=1;\ f'(x)=2(1-\sin 2x)\ge 0\ \forall x\in R$

At $\lambda=-\frac{1}{2};\ f'(x)=\frac{1}{2}-2\sin 2x$

Thus, only one value of λ (i.e., $\lambda=1$) is allowed.

47-48 : $f''(x)=k(x-1)$

$f'(x)=\frac{k(x-1)^2}{2}+C$

$f'(-1)=2k+C=0;\ C=-2k$

$f'(x)=\frac{k(x-1)^2}{2}-2k$

$f(x)=\frac{k(x-1)^3}{6}-2kx+B$

$$f(0) = -\frac{k}{6} + B = 5 \qquad ....(1)$$

$$f(-1) = -\frac{8k}{6} + 2k + B = 10 \qquad ....(2)$$

$$-\frac{7k}{6} + 2k = 5; \quad \frac{5k}{6} = 5; \quad k = 6, \quad B = 6$$

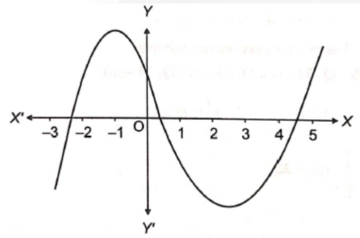
$$f'(x) = 3(x-1)^2 - 12 = 3\left((x-1)^2 - 4\right)$$

$$f'(x) = 3(x+1)(x-3) \text{ and } f(x) = (x-1)^3 - 12x + 6$$

$$\text{At } x = -1,\ f'(x) = 0; \quad f(-1) = 10$$

$$\text{At } x = 3,\ f'(x) = 0; \quad f(3) = 8 - 36 + 6 = -22$$

Distance between two horizontal tangents = 32



$$f(x) = (x-1)^3 - 12x + 6$$

$$f(4)f(5) < 0; \quad 4 < x_1 < 5; \quad [x_1] = 4.$$

49. Given, $f(x) = x^3 - 9x - 1$

$$f'(x) = 3x^2 - 9$$

$$m = f^1(x_0) = 3x_0^2 - 9$$

Let Q is (x_1, y_1); $\quad y_1 = x_1^3 - 9x_1 - 1$

Also,

$$3x_0^2 - 9 = \frac{y_1 - f(x_0)}{x_1 - x_0}$$

$$3x_0^2 - 9 = \frac{(x_1^3 - 9x_1 - 1) - (x_0^3 - 9x_0 - 1)}{(x_1 - x_0)}$$

$$= \frac{(x_1^3 - x_0^3) - 9(x_1 - x_0)}{(x_1 - x_0)}$$

$$= \frac{(x_1 - x_0)^3 + 3x_1x_0(x_1 - x_0) - 9(x_1 - x_0)}{(x_1 - x_0)}$$

$$= (x_1 - x_0)^2 + 3x_1x_0 - 9$$

$$3x_0^2 = x_1^2 + x_1x_0 + x_0^2$$

$$x_1^2 + x_1x_0 - 2x_0^2 = 0$$

$$x_1^2 + 2x_1x_0 - x_1x_0 - 2x_0^2 = 0$$

$$x_1(x_1 + 2x_0) - x_0(x_1 + 2x_0) = 0$$

$$(x_1 - x_0)(x_1 + 2x_0) = 0$$

$$x_1 = x_0 \text{ or } x_1 = -2x_0$$

Thus, $x_1 = -2x_0$

$$m_Q = 3x_1^2 - 9 = 3(-2x_0)^2 - 9 = 12x_0^2 - 9$$

$$m_p = 3x_0^2 - 9$$

$$m_Q - 4m_p = 12x_0^2 - 9 - 4(3x_0^2 - 9) = 27$$

$$m_{OP} = \frac{f(x_0) - 0}{x_0 - 0} = \frac{(x_0^3 - 9x_0 - 1)}{x_0} = -9 \text{ at } x_0 = 1$$

$$m_{OQ} = \frac{y_1 - 0}{x_1 - 0} = \frac{(x_1^3 - 9x_1 - 1)}{x_1}$$

$$= \frac{(-8 + 18 - 1)}{(-2)} \text{ at } x_1 = -2x_0 = -2$$

$$= -\frac{9}{2}$$

50. By LMVT, $f'(\alpha) = \dfrac{f\left(\frac{\pi}{2}\right) - f(0)}{\frac{\pi}{2} - 0} = \dfrac{1-0}{\frac{\pi}{2} - 0} = \dfrac{2}{\pi}$

$$\frac{2}{\pi} = \sqrt{1 - f^2(\alpha)}; \quad \frac{4}{\pi^2} = 1 - f^2(\alpha)$$

$$f^2(\alpha) = 1 - \frac{4}{\pi^2}; \quad f(\alpha) = \sqrt{\left(1 - \frac{4}{\pi^2}\right)}$$

It may be true for some $\alpha \in \left(0, \frac{\pi}{2}\right)$ but not for all α.

$$f'(\alpha) = \frac{2}{\pi};$$

$$\frac{f(\alpha_1) - 0f(0)}{\alpha_1 - 0} = \frac{2}{\pi}; \quad f(\alpha_1) = \frac{2}{\pi}\alpha_1$$

$$0 \le \alpha_1 \le \frac{\pi}{2}; \quad 0 \le \frac{2\alpha_1}{\pi} \le 1; \quad 0 \le f(\alpha_1) \le 1$$

Which is possible for not for all $\alpha_1 \in \left(0, \frac{\pi}{2}\right)$

$$f(\alpha).f^1(\alpha) = \frac{1}{\pi}$$

$$f(\alpha)\frac{\left(f\left(\frac{\pi}{2}\right) - f(0)\right)}{\frac{\pi}{2} - 0} = \frac{1}{\pi}$$

$$f(\alpha)\frac{(1-0)}{\frac{\pi}{2}} = \frac{1}{\pi}$$

$f(\alpha) = \frac{1}{2}$ which will be valid for at least one $\alpha \in \left(0, \frac{\pi}{2}\right)$

$f^1(\alpha) = \frac{8}{\pi^2}\alpha; \quad \frac{2}{\pi} = \frac{8}{\pi^2}\alpha; \ \alpha = \frac{\pi}{4}$ which is true for $\alpha \in \left(0, \frac{\pi}{2}\right)$

51. $f: R \to (-\infty, -1]$

$$f(x) = (ab + 2a - b - 2)x^5 - (a^3 - 2a + 1)x^3 + (a^2 - 2a - 3)x^2 + (a^2 + 2b)x - 5$$

$$= (a-1)(b+2x)x^5 - (a^3 - 2a + 1)x^3 + (a^2 - 2a - 3)x^2 + (a^2 + 2b)x - 5$$

If f(x) is a polynomial of odd degree then its range will be R.

Thus, a=1

$$f(x) = (1-2-3)x^2 + (1+2b)x - 5$$

$$= -4x^2 + (1+2b)x - 5$$

Max. Value of f (x) = -1

$$-\frac{D}{4a} = -1; -\frac{(1+2b)^2 - 4(-4)(-5)}{4(-4)} = -1$$

$$(1+2b)^2 - 80 = -16$$

$$(1+2b)^2 = 64$$

$$1+2b = \pm 8$$

2b = 7; 2b = -9

$$b = \frac{7}{2} \text{ or } -\frac{9}{2}$$

52. $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$

Diff. w.r.t $x \Rightarrow f'\left(\frac{x}{y}\right)\frac{1}{y} = \frac{f'(x)}{f(y)}$

Put $x \neq y \Rightarrow f'(1)\frac{1}{x} = \frac{f'(x)}{f(y)} \Rightarrow \frac{2}{x} = \frac{f'(x)}{f(y)} \Rightarrow 2\ell n\, x = \ell n(f(x)) \Rightarrow f(x) = x^2$

$g(x) = 1 + x^2$

$P(\alpha, \beta); \beta = 1 + \alpha^2$

$g(\alpha) = 2\alpha$

Tangent equation is

$y = (1+\alpha^2) = 2\alpha(x-\alpha)$

$x = 0 \Rightarrow y = 1 - \alpha^2;\ x = 1 \Rightarrow Y = 1 + 2\alpha - \alpha^2$

Trapezium area=$\left(\frac{1}{2}\right)(1)(2 + 2\alpha - 2\alpha^2)$

Area in max When $\alpha = \frac{1}{2} \Rightarrow \beta = \frac{5}{4}$

53. $f(x) = e^{(p+1)} - e^x \quad f'(x) = 0 \text{ if } f(x) \text{ is minimum}$

$e^{(p+1)x}(p+1) - e^x(1) = 0 \qquad e^{(p+1)x} = \frac{e^x}{(p+1)}$

$e^{px} = \frac{1}{(p+1)} \qquad (px) = \text{In}\left(\frac{1}{p+1}\right)$

$\frac{x}{p} = \frac{-1}{p}\text{In}(p+1) \qquad g(t) = \int_t^{t+1} f(x)e^{t-x}dx$

$= \int_t^{t+1}\left[e^{(p+1)x} - e^x\right]e^{t-x}dx \qquad = \int_t^{t+1}(e^{px} - 1)e^t dx$

$= (e^t)\left[\frac{e^{px}}{p} - x\right]\int_t^{t+1} \qquad g(t) = e^t\left[\frac{e^{p(t+1)} - e^{pt}}{p}\right] - e(t+1-t)$

$$=e^t\left[\frac{e^{px}-e^{pt}}{p}\right]-e^t \qquad =e^{(p+1)t}\frac{(e^p-1)}{p}-e^x$$

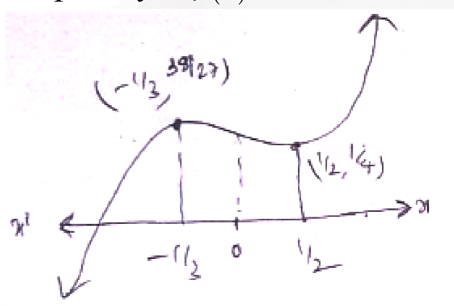
$$g'(t)=e^{(p+1)t}\frac{(p+1)}{p}(e^p-1)-e^t \qquad g'(t)=0$$

54. CHECK THE MONOTONIC NATURE OF $f(x)=\left(\frac{x+1}{x}\right)^{x+\frac{1}{2}}$

55. DOMAIN OF $f(x)=[-2,2]$

Maximum value of $f(x)=$Greatest value of $\{f(-2),f(0),f(2)\}$

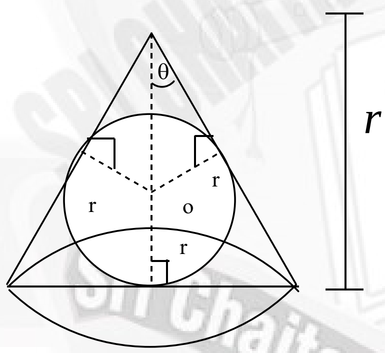
56. Graph of y= $f(x)$ is



$$g(x)=f(x),\ 0\le x<\frac{1}{2} \qquad \therefore g\left(\frac{1}{4}\right)=\frac{1}{2}$$

$$g(x)=\frac{1}{4},\ \frac{1}{2}\le x\le 1 \qquad g\left(\frac{3}{4}\right)=\frac{1}{4}$$

$$=3-x,\ 1<x\le 2 \qquad g\left(\frac{5}{4}\right)=\frac{7}{4}$$



57.

$$h=r+\frac{r}{\sin\theta}$$

Base radius of cone = $R=\left(r+\frac{r}{\sin\theta}\right)\tan\theta$

$$\therefore Volume\ V=\frac{1}{3}\pi\ R^2\ h \qquad =\frac{\pi r^3}{3}\frac{(1+\sin\theta)^3}{Sin\theta\,Cos^2\theta}$$

$$\frac{dV}{d\theta}=\frac{\pi r^3}{3}\frac{(1+\sin\theta)^3(3Sin\theta-1)}{Sin^2\theta\ Cos^3\theta}$$

For Min/max, $\Rightarrow \frac{dV}{d\theta}=0=1\quad Sin\theta=1/3.$