Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 26-08-2023
Time: 09.00Am to 12.00Pm RPTM-04 Max. Marks: 300

KEY SHEET

PHYSICS

1)	**1**	2)	**3**	3)	**2**	4)	**1**	5)	**3**
6)	**3**	7)	**2**	8)	**3**	9)	**3**	10)	**4**
11)	**2**	12)	**1**	13)	**3**	14)	**3**	15)	**1**
16)	**2**	17)	**1**	18)	**4**	19)	**1**	20)	**2**
21)	**4**	22)	**3**	23)	**4**	24)	**30**	25)	**5**
26)	**270**	27)	**300**	28)	**8**	29)	**16**	30)	**12**

CHEMISTRY

31)	**1**	32)	**1**	33)	**2**	34)	**1**	35)	**3**
36)	**1**	37)	**1**	38)	**3**	39)	**4**	40)	**1**
41)	**2**	42)	**2**	43)	**1**	44)	**2**	45)	**1**
46)	**3**	47)	**1**	48)	**3**	49)	**4**	50)	**3**
51)	**9**	52)	**6**	53)	**5**	54)	**6**	55)	**3**
56)	**5**	57)	**7**	58)	**4**	59)	**9**	60)	**3**

MATHEMATICS

61)	**2**	62)	**2**	63)	**1**	64)	**3**	65)	**2**
66)	**3**	67)	**4**	68)	**1**	69)	**1**	70)	**3**
71)	**1**	72)	**3**	73)	**2**	74)	**1**	75)	**2**
76)	**4**	77)	**1**	78)	**2**	79)	**2**	80)	**1**
81)	**1**	82)	**4**	83)	**2**	84)	**1**	85)	**7**
86)	**60**	87)	**4**	88)	**5**	89)	**5**	90)	**2**

SOLUTIONS

PHYSICS

1. For a pin hole of diameter a, the angular width of disc produced is given as $\sin\theta = \frac{m\lambda}{a}$. When a increases, θ decreases due to which the area of disc decreases so intensity will increases.

2. Least count of screw gauge can be given as $LC = \frac{0.5}{50} = 0.01\text{mm}$

Main scale, reading = 2.5 mm
Circular scale reading = 20
Measured diameter of the ball is given as

$D = 2.5\text{mm} + (20\times 0.01)\text{mm}$

$D = 2.5\text{mm} + 0.2\text{mm} = 2.7\text{mm}$

Density of the ball can be given as $\rho = \frac{m}{\frac{4\pi}{3}\left[\frac{D}{2}\right]^3}$

Fractional error in density is given as $\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta D}{D}$

Percentage error in density is given as

$\frac{\Delta\rho}{\rho}\times 100 = 2\% + 3\left(\frac{0.01}{2.7}\right)\times 100 = 3.1$

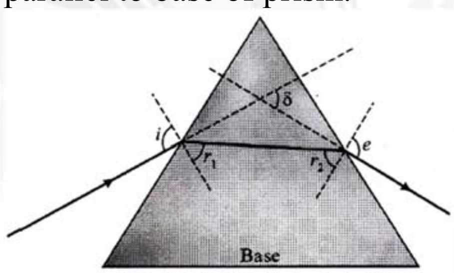
3. Main scale division(s)=0.05cm

Vernier scale division(v) $= \frac{49}{100} = .049$

Least count $= 0.5 - .049 = .001\text{cm}$

Diameter $= 5.10 + 24\times .001\text{cm} = 5.124\text{cm}$

4. Deviation is minimum in a prism when: $i = e, r_1 = r_2$ and refracted ray inside prism is parallel to base of prism.



5. Length of one division on vernier scale is given as $VSD = \frac{0.1\times 9}{10} = 0.09$

Given that MSD=0.1cm hence least count is given as $LC = 0.1 - 0.09 = 0.01\text{cm}$

In first case we can calculate zero which is calculated as

$z = 0.5 - 6\times 0.09 = 0.5 - 0.54 = -0.04\text{cm}$

In second case reading is taken as $R = 3.1 + 1\times LC$

$R = 3.1 + 1\times 0.01 = 3.11\text{cm}$

Thus diameter of sphere is given as $D = 3.11 - (-0.04) = 3.15\text{cm}$

6. $P\to 4; Q\to 2; R\to 1; S\to 3$

(P):Using molecular kinetic energy relation, we have $E=\frac{3}{2}kT$

Writing dimensions on the two sides, we have $\left[ML^2T^{-2}\right]=[k][K]\Rightarrow[k]=\left[ML^2T^{-2}K^{-1}\right]$

(Q): using Stokes rule, we have $F=6\pi\eta rv$

Writing dimensions on the two sides, we have $\left[MLT^{-2}\right]=[\eta][L]\left[LT^{-1}\right]\Rightarrow[\eta]=\left[ML^{-1}T^{-1}\right]$

(R):Using relation of energy of a photon, we have $E=hv$

Writing dimensions on the two sides, we have $\left[ML^2T^{-2}\right]=\frac{[h]}{[T]}\Rightarrow[h]=\left[ML^2T^{-1}\right]$

(S): Using relation of steady state of heat conduction, we have $\frac{dQ}{dt}=\frac{kA(\Delta T)}{\Delta x}$

$$\frac{\left[ML^2T^{-2}\right]}{[T]}=\frac{[k]\left[L^2\right][K]}{[L]}$$

$$[k]=\left[MLT^{-3}K^{-1}\right]$$

7. Total translational kinetic energy of all molecules of a gas is given as

$$E=\frac{3}{2}nRT=\frac{3}{2}PV$$

Thus statement-1 is correct. In a gas, molecule are in Brownian motion and travel randomly in all directions and at every collision direction of motion charges so velocity changes. Thus statenet-1 and 2 are true but statenet-2 is not explanation for statenet-1

8. Assertion is correct as mirror formula is valid for paraxial rays. Thus Assertion is correct but Reason is false because laws of reflection is valid for all type of reflecting surfaces.

9. acceleration $a=V\frac{dV}{dx}=(\text{velocity})(\text{slopeof given graph})$

10. As $\vec{a}_{B/A}=\vec{g}-\vec{g}=\vec{0},\vec{V}_{B/A}$ remains constant. Hence path of one projectile w.r.t another is a straight line.

$(B)H_B<H_A\Rightarrow\left(u_y\right)_B<\left(u_y\right)_A\Rightarrow T_B<T_A$

$(C)V_{min}=u\cos\theta\Rightarrow V_{min}\alpha\cos\theta$

From figure, $\theta_B<\theta_A\Rightarrow\cos\theta_B>\cos\theta_A$

$\therefore(V_{min})_B>(V_{min})_A$

11. Conceptual

12. Conceptual

13. From Wein's law $\lambda_m T=\text{constant}$

i.e.peak emission wavelength $\lambda_m\infty\frac{1}{T}$

as T increases λ_m decreases.

14. $\%A=\frac{1}{2}+\frac{3}{2}+1+1=4\%$

15. $g=\frac{k\ell}{T^2}\Rightarrow\frac{\Delta g}{g}=\frac{\Delta\ell}{\ell}+\frac{2\Delta T}{T}=0.2\%$

16. For a person if far away objects are not clear or blurry then problem is Myopia. When objects are looking distorted then it is due to Astigmatism.

17. The line of sight of the observer remains constant, making an angle of 45^0 with the normal.

$$\sin\theta = \frac{h}{\sqrt{h^2+(2h)^2}} = \frac{1}{\sqrt{5}}$$

18. Conceptual

19. If θ is angle of projection and α is angle of inclination, then for maximum range.
$2\theta-\alpha=90^0$ $2(30^0+\alpha)-\alpha=90^0 \Rightarrow \alpha=30^0$

20. Maximum separation is equal to maximum area under v_{rel} vs time graph
Maximum separation =1.25m

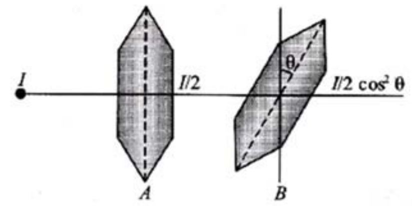
21. Substituting dimensions in given dimensional expression of magnetic field, we have
$[B]=[e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$
$\Rightarrow [M^1T^{-2}A^{-1}]=[AT]^\alpha [m]^\beta [ML^2T^{-1}]^\gamma [ML^3A^{-2}T^{-4}]^\delta$
$\Rightarrow M^1T^{-2}A^{-1}=m^{\beta+\gamma+\delta}L^{2r+3\delta}T^{\alpha-\gamma-4\delta}A^{\alpha-25}$
Comparing dimensions in LHS and RHS gives
$\beta+\gamma+\delta=1$
$2\gamma+3\delta=0$
$\alpha-\gamma+4\delta=-2$
$\alpha-2\delta=-1$
Solving above equations, we get
$\alpha=3, \beta=2, \gamma=-3, \delta=2$
$\Rightarrow \alpha+\beta+\gamma+\delta=4$

22. The distance d can be considered to depend on ρ,S and f with dimensions a,b and c. So we have
$d\,\alpha\,\rho^a S^b f^c$
Substituting the dimensional relation of all physical quantities in above expression, we have
$\Rightarrow [L]=[MT^{-3}]^a \left[\frac{M}{T^3}\right]^b \left[\frac{1}{T}\right]^c$
$\Rightarrow [LT^{-1}]=M^{a+b}L^{-3a}T^{-3b-c}$
Comparing dimensions of LHS and RHS, we get
$a+b=0, \Rightarrow b=-a$
$-3a=1 \Rightarrow a=-1/3$
$\Rightarrow b=1/3$
$\Rightarrow n=3$

23. Energy of system is given as $E(t)=A^2e^{-\alpha t}$
Taking natural log on both side gives
$\ln E=\ln A^2+\ln e^{-\alpha t}$
$\Rightarrow \ln E=2\ln A-\ln\alpha t$
Percentage error in measurement of E is given as
$$\frac{\Delta E}{E}\times100=2\frac{\Delta A}{A}\times100+\alpha\Delta t\times100$$
$$\Rightarrow \frac{\Delta E}{E}\times100=2\times1.25+\frac{2}{10}\times1.5\times5$$

$\Rightarrow \frac{\Delta E}{E} \times 100 = 2.25 + 1.5 = 4\%$

24. The situation described in question is shown in figure below



The intensity of light after polarizer A becomes I/2 then after passing through polarizer B it is given by Malus's law as

$I' = \frac{I}{2}\cos^2\theta$

$\Rightarrow \frac{3I}{8} = \frac{I}{2}\cos^2\theta$

$\Rightarrow \theta = 30^0$

25. $p_1V_1 = \mu_0 R(250)$

$p_2(2V_1) = (1.25\mu_0)R(2000)$

$\frac{p_2}{p_1} = 5$

26. Case-I $5C \times 50 + 5L = C_2 \times 30$

Case-II $80C(50-30) = C_2(80-50)$

By Eqs. (i) and (ii), we get

$1600C = 250 + 5L \qquad \therefore \frac{L}{C} = \frac{1350}{5} = 270^0 C$

27. Distance of first red violet fringe from central white fringe is given as

$y_1 = \frac{\lambda_r D}{d}$ and $y_2 = \frac{\lambda_v D}{d}$

$\Rightarrow \lambda_r - \lambda_v = (y_1 - y_2)\frac{d}{D} = (3.5-2)\,mm \times \frac{0.3mm}{1.5m} = 300nm$

28. $C_v = \frac{C_{v1} + C_{v2}}{2} = 2R \quad \frac{du}{3} + \frac{dw}{2} = du + dw$

$C = \frac{R}{\gamma - 1} + \frac{R}{1-n} \qquad \therefore n = \frac{11}{8}$

29. In YDSE setup intensity of light is proportional to the slit width. The ratio of maximum to minimum intensity is given as

$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \left[\frac{\sqrt{9I} + \sqrt{I}}{\sqrt{9I} + \sqrt{I}}\right]^2 \quad \Rightarrow \frac{I_{max}}{I_{min}} = \left[\frac{(3+1)\sqrt{I}}{(3-1)\sqrt{I}}\right]^2 = 4 = \frac{x}{4} \qquad \Rightarrow x = 16$

30. $v = -30, m = -\frac{v}{u} = -2 \qquad \therefore A' + B' = C'D' = 2 \times 1 = 2mm$

Now $\frac{B'C'}{BC} = \frac{A'D'}{AD} = \frac{v^2}{u^2} = 4$

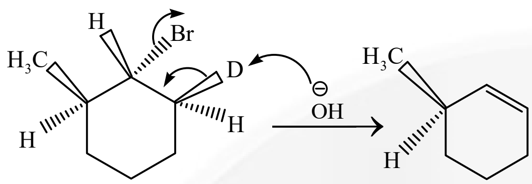
$\Rightarrow B'C' = A'D' = 4mm$

$\therefore$ Length = 2 + 2 + 4 + 4 = 12 mm

CHEMISTRY

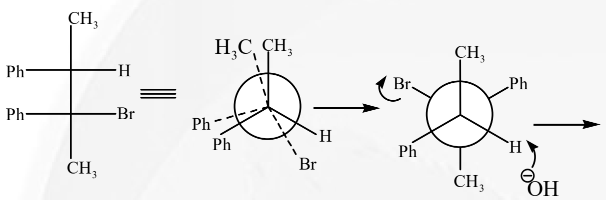
31. NCERT Pag.No:165

32.



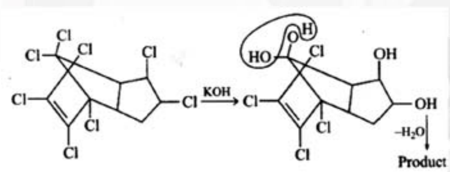
D and Br are anti to each other

33.



34. NCERT.Pg.No:171

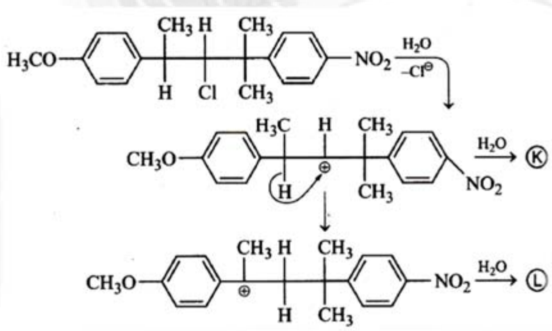
35.



No S_N2 reaction at bridgehead and double bonded C.

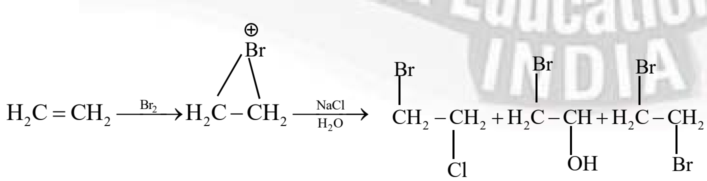
36. Conceptual

37.

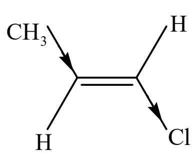


38. On the basis of hyper conjugation

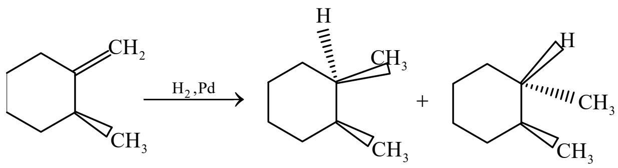
39.

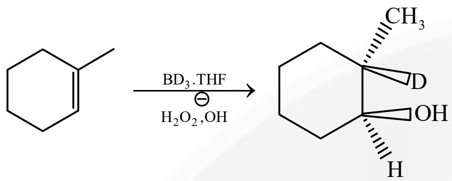


40.



Maximum dipole moment

41. 

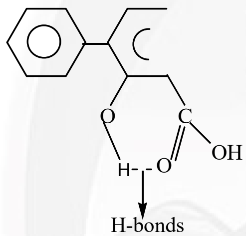
42. 

Addition is syn

43. Addition of –OH and –D occur

44. S is least basic as it is aromatic and 1p is delocalized P is most basic as it is aliphatic amine

45. P is most acidic

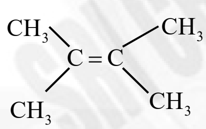


46. $[\theta]_T^\lambda = \frac{\theta_{obs}}{C \times \ell}$ $C =$ concentration in gm/mL

$\ell =$ length of tube in decimeter $[\theta]_T^\lambda = \frac{+13.4}{0.2 \times 2.5} = +26.8^0$

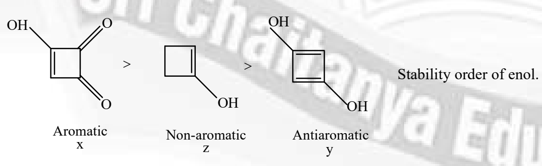
47. Conceptual

48.

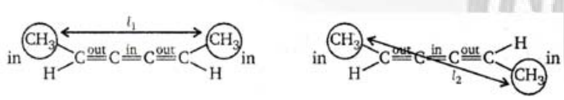


has highest C-C bond length (B.L); Because it has maximum hyperconjugation. More single bond character by hyperconjugation.

49.

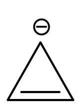


50.



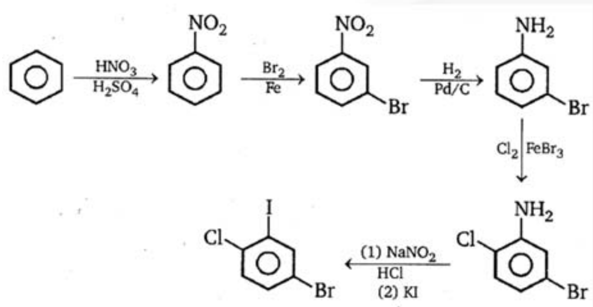
$\Rightarrow (\ell_2 > \ell_1)$

51.

 except this all are aromatic

52. 2 mole with each $-\overset{\overset{\displaystyle O}{\|}}{C}-Cl$, once mole with –OH and one mole with –SH
53. NCERT Pg.No:188

54. NCERT Pg.No:402
55.



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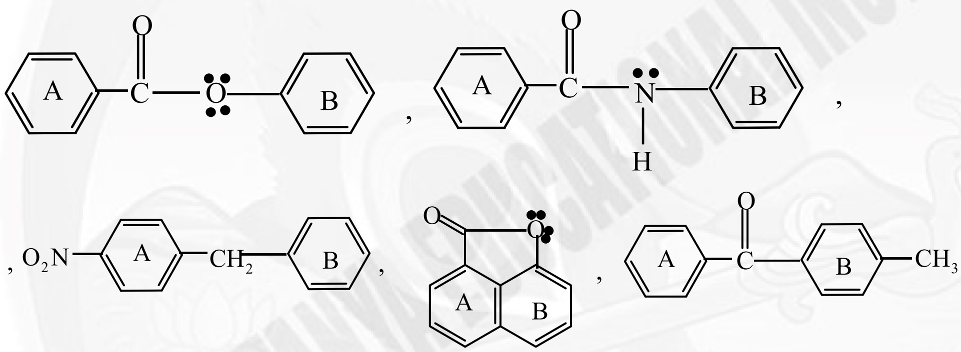
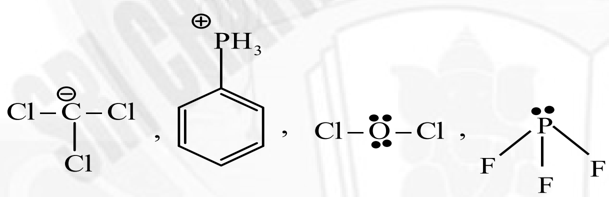
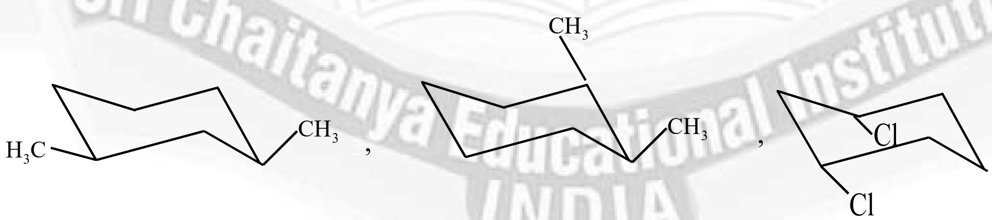


57. a,b,d,e,g,h,i
58.



59. Conceptual
60.



MATHEMATICS

61. Given, $f(x)=\pi\left(\frac{\sqrt{x+7}-4}{x-9}\right)$

Clearly, domain of $f(x)=[-7,\infty)-\{9\}$.

Now, $f(x)=\frac{\pi(x+7-16)}{(x-9)(\sqrt{x+7}+4)}$ (Rationalise) $=\frac{\pi}{\sqrt{x+7}+4}$

So, range of $f(x)$ is $\left(0,\frac{\pi}{4}\right]-\left\{\frac{\pi}{8}\right\}$.

Hence, range of $y=\sin(2f(x))$ is $(0,1]-\left\{\frac{1}{\sqrt{2}}\right\}$.

62. $\ell=\lim_{x\to0}\frac{(1+P(x))^{1/n}-1}{x}$ 9Using binomial expansion)

$=\lim_{x\to0}\frac{\left(1+\frac{1}{n}P(x)+.......\right)-1}{x}=\lim_{x\to0}\frac{1}{n}\left[\frac{a_1x+a_2x^2+a_3x^3+......}{x}\right]=\frac{a_1}{n}$

63. Conceptual

64. We have, $f(x)=x^3+2x^2+4x+\sin\left(\frac{\pi x}{2}\right)$

$\therefore f'(x)=3x^2+4x+4+\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)\Rightarrow f'(1)=11$

Also, $f(1)=8$

So, $g'(8)=\frac{1}{f'(1)}=\frac{1}{11}$

65. We have, $\int\frac{\sin^3 x}{(\cos^4 x+3\cos^2 x+1)\tan^{-1}(\sec x+\cos x)}dx$

$=\int\frac{\frac{\sin^3 x}{\cos^2 x}}{(\cos^4+3+\sec^2 x)\tan^{-1}(\sec x+\tan x)}dx$

$=\int\frac{1}{1+(\sec x+\cos x)^2}\times\frac{\sin x(1-\cos^2 x)}{\cos^2 x}\times\frac{1}{\tan^{-1}(sexx+\tan x)}dx$

$=\int\frac{1}{\tan^{-1}+(\sec x+\cos x)^2}\times\frac{1}{1+(\sec x+\cos x)^2}(\tan x\sec x-\sin x)dx$

$=\int\frac{1}{\tan^{-1}(\sec x+\cos x)}d\left|\tan^{-1}(\sec x+\cos x)\right|$

$=\log_e\left|\tan^{-1}(\sec x+\cos x)\right|+C$

66. $y=x^x\ln x; y=\frac{2^x-2}{\ln 2}$

At $x=1, m_1=\frac{dy}{dx}=x^x.\frac{1}{x}+x^x(\ln x+1)\ln x=1$

At $x=1, m_2=\frac{dy}{dx}=2^x\frac{\ln 2}{\ln 2}=2$

$\therefore \tan\theta=\left|\frac{2-1}{1+2}\right|=\frac{1}{3}$

67. Conceptual

68. We must have

$$\begin{vmatrix} x & 1 & 2 \\ f(x) & 3 & x^2 \\ 5x & 6 & 1 \end{vmatrix}=0$$

$\therefore x(3-6x^2)-1(f(x)-5x^3)+2(6f(x)-15x)=0$

$\therefore f(x)=\frac{x^3+27x}{11} \quad \Rightarrow f'(x)=\frac{3x^2+27}{11}>0\forall x\in R$

69. Let $I=\int\frac{(1+x)}{x(1+xe^x)^2}dx=\int\frac{(1+x)e^x}{(xe^x)(1+xe^x)^2}dx$,

Put $1+xe^x=t$

$\therefore (1+x)e^x dx=dt=\int\frac{dt}{(t-1).t^2}$, applying partial fraction,

We get $\frac{1}{(t-1)t^2}=\frac{A}{t-1}+\frac{B}{t}+\frac{C}{t^2}$

$\Rightarrow 1=A(t^2)+Bt(t-1)+C(t-1)$

For $t=1\Rightarrow A=1$

For $t=0\Rightarrow C=-1 \text{ and } B=-1$

$\therefore I=\int\left\{\frac{1}{t-1}-\frac{1}{t}-\frac{1}{t^2}\right\}dt=\log|t-1|-\log|t|+\frac{1}{t}+C$

$=\log|xe^x|-\log|1+xe^x|+\frac{1}{1+xe^x}+C \quad =\log\left|\frac{xe^x}{1+xe^x}\right|+\frac{1}{1+xe^x}+C$

70. $f(x)=2x^3-21x^2+78x+24$

$f'(x)=6(x^2-7x+13)$

$\Rightarrow f(x)$ is increasing function

Now, $f(f(f(x)-2x^3))\geq f(f(2x^3-f(x)))$

$\Rightarrow f(f(x)-2x^3)\geq f(2x^3-f(x))\Rightarrow f(x)-2x^3\geq 2x^3-f(x)$

$\Rightarrow f(x)\geq 2x^3\Rightarrow 7x^2-26x-8\leq 0 \quad \Rightarrow x\in\left[-\frac{2}{7},4\right]$

71. $I=\int\frac{(e^x+\cos x+1)-(e^x+\sin x+x)}{e^x+\sin x+x}dx$

$=\ln(^x+\sin x+x)-x+C$

$\therefore f(x)=e^x+\sin x+x \text{ and } g(x)=-x$

$\Rightarrow f(x)+g(x)=e^x+\sin x$

$\Rightarrow \frac{f(x)+g(x)}{e^x+\sin x}=1$

72. $f(x)$ is odd $\therefore f(-x)=-f(x)$

$f(0)=0$

$f(-1)=-f(1)=-2$

$f(-3)=-f(3)=-5$

$f(-5)=-f(5)=-1$

$Nr=f(f(f(-3)))+f(f(0))=f(f(-5))+f(0)=f(-1)+0=-2$

$Dr=3f(1)-2f(3)-f(5)=3(2)-2(5)-(1)=6-10-1=-5$ $\quad \frac{Nr}{Dr}=\frac{-2}{-5}=\frac{2}{5}$

73. Let $e^{x^3+x^2-1}\left(3x^4+2x^3+2x\right)dx$

$=\int x^2.e^{x^3+x^2-1}.\left(3x^2+2x\right)dx+\int e^{x^3+x^2-1}.(2x)dx$

$=x^2.e^{x^2+x^2-1}+C=h(x)+C \quad \therefore h(x)=x^2.e^{x^3+x^2-1} \quad \Rightarrow h(1).h(-1)=e^1.e^{-1}=1$

74. Using LMVT in [0,2]

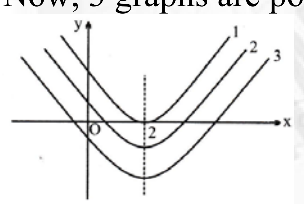
$f'(c)=\frac{f(2)-f(0)}{2-0}$ where $c\in(0,2)$ $\quad f'(c)=\frac{f(2)+3}{2}$

But $f'(x)\le 5$ $\quad \frac{f(2)+3}{2}\le 5\Rightarrow f(2)+3\le 10\Rightarrow f(2)\le 7$

75. $f(x)=x^2+bx+c, f(2+t)=f(2-t)$ $\quad \Rightarrow$ f is symmetric about a line x=2

$\therefore \frac{-b}{2}=2\Rightarrow b=-4 \quad \therefore f(x)=x^2-4x+c$

Now, 3 graphs are possible.



In (1) and (2) 'c' is positive and in (3) 'c' is negative.

$f(0)=c$

Let c is positive.

Now, $f(1)=c-3$

$f(2)=c-4 \quad f(4)=c$

Say $c=3$

then $f(1)=0; f(2)=-1; f(3)=3\Rightarrow f(2)<f(1)<f(3)$

Again c is negative. Let $c=-3$

$f(1)=-6; f(2)=-7; f(4)=-3 \quad \therefore f(2)<f(1)<f(4)\Rightarrow(B)$

Also,if c=0 the statement '2' is true.

76. Statement II Given , $f(x)=\frac{x^2}{x^3+200}$

$f'(x)=\frac{\left(x^3+200\right)2x-3x^2x^2}{\left(x^3+200\right)^2}=\frac{-x^4+400x}{\left(x^3+200\right)^2}$

$x\to 0^+ f(x)=0^+ \Rightarrow x=400^{1/3} f(x)=\frac{400^{2/3}}{600}$

$x \to \infty\ f(x) \to 0$

So, Statement II is true. But Statement I is false as $x \in N$.

Hence,(4) is the correct answer.

77. $\sin 51x(\sin x)^{49} = \left[\sin x.\cos 50x(\sin x)^{49} + \cos x \sin 50x(\sin x)^{49}\right] = \dfrac{\frac{d}{dx}\left((\sin x)^{50}\sin 50x\right)}{50}$

78. $\displaystyle\int \frac{1-\frac{1}{x^2}}{\left(x^2+\frac{1}{x^2}+3\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx;$

$\displaystyle\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx$

79. $\because \displaystyle\int \frac{x^{2009}}{1+x^{2010}}dx = \int \frac{x^{2010}}{x+x^{2011}}dx = \int \frac{1+x^{2010}}{x+x^{2011}}dx - \int \frac{1}{x+x^{2011}}dx$

$\Rightarrow \displaystyle\int \frac{x^{2009}}{1+x^{2010}}dx + \int \frac{1}{x+x^{2011}}dx = \int \frac{1+x^{2010}}{x\left(1+x^{2010}\right)}dx$

$\Rightarrow h(x) = \displaystyle\int \frac{dx}{x} = \ln x + C \qquad \because h(1)=0 \quad \because C=0 \quad \therefore h(x)=\ln x \quad \Rightarrow h(e)=\ln e = 1$

80. Let $f(x)=0$ has two roots say $x=r_1$ nad $x=r_2$, where $r_1, r_2 \in [a,b]$. $\Rightarrow f(r_1)=f(r_2)$

Hence, there must exist some $c \in (r_1, r_2)$, where $f'(c)=0$

But $f'(x)=x^6-x^5+x^4-x^3+x^2-x+1$

for $x \ge 1, f'(x)=\left(x^6-x^5\right)+\left(x^4-x^3\right)+\left(x^2-x\right)+1>0$

for $x \le 1, f'(x)=(1-x)+\left(x^2-x^3\right)+\left(x^4-x^5\right)+x^6>0$

Hence, $f'(x)>0$ for all x. $\therefore$ Rolle's theorem fails.

$\Rightarrow f(x)=0$ cannot have two or more roots.

81. $k^2-3k+2=0, k^2-1=0, k^2-6k+5=0, k^2-2k+1=0$

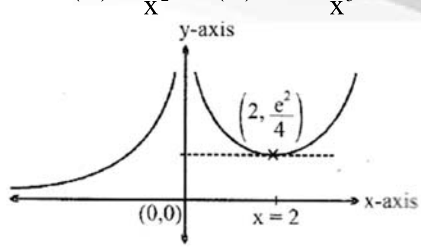
and $k^2-k=0$ must be satisfied simultaneously.

So, $k=1$

Hence, number of real values of k is one (i.e.,k=1)

82. Conceptual

83. Let $f(x)=\dfrac{e^x}{x^2}, f'(x)=\dfrac{(x-2)e^x}{x^3}$



So, from above graph, $c > \dfrac{e^2}{4}$

So, $c_{min}=2$

84. Conceptual

85. $f(x^2y) = x^2f(y) + yf(x^2), \forall x, y > 0$

Partial differentiation w.r.t.x keeping y constant

$f'(x^2y).2xy = 2xf(y) + yf'(x^2).2x$

$yf'(x^2y) = f(y) + yf'(x^2)$

Put $x = 1, yf'(y) = f(y) + yf'(1) = f(y) + y$

Differentiation w.r.t.y

$yf''(y) + f'(y) = f'(y) + 1 \Rightarrow f''(y) = \frac{1}{y}$

Put $y = \frac{1}{7} \Rightarrow f''\left(\frac{1}{7}\right) = 7$

86. We have $f(x) = (x+1)^3$

Now, $\int f(x)dx = \int (x+1)^3 dx = \frac{(x+1)^4}{4} + C \Rightarrow g(x) = \frac{(x+1)^4}{4}$

Hence, $g(x) - g(1) = \frac{4^4}{4} - \frac{2^4}{4} = 64 - 4 = 60$

87. $f'(x) = \frac{1}{1+\cos x}$;

Integrating, $f(x) = \int \frac{dx}{2\cos^2\frac{x}{2}} = \frac{1}{2}\int \sec^2\frac{x}{2}dx = \frac{1}{2}.2.\tan\frac{x}{2} + C = \tan\frac{x}{2} + C$

$f(0) = 3 \Rightarrow C = 3;\ f(x) = \tan\frac{x}{2} + 3; f\left(\frac{\pi}{2}\right) = 4$

88. $\int \frac{2x+3}{(x^2+3x)(x^2+3x+2)+1}dx$

Put $x^2 + 3x = t \quad \Rightarrow (2x+3)dx = dt$

$\Rightarrow \int \frac{dt}{t(t+2)+1}; \int \frac{dt}{(t+1)^2} = C - \frac{1}{t+1} = C - \frac{1}{x^2+3x+1} \quad \Rightarrow a = 1, b = 3, c = 1 \Rightarrow a+b+c = 5$

89. Conceptual

90. Let $x = t^2 \Rightarrow dx = 2t\,dt$

$$I = 2\int \frac{(t^2-1)tdt}{t(t+t^2+1)\sqrt{t(t^2+1)}} = 2\int \frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}+1\right)\sqrt{t+\frac{1}{t}}}dt$$

Put $t + \frac{1}{t} = y^2; \quad \left(1-\frac{1}{t^2}\right)dt = 2ydy$

$$I = 4\int \frac{ydy}{(y^2+1)y} = 4\tan^{-1}\sqrt{t+\frac{1}{t}} + C \quad = 4\tan^{-1}\sqrt{\sqrt{x}+\frac{1}{\sqrt{x}}} + C$$

$g(x) = \sqrt{\sqrt{x}+\frac{1}{\sqrt{x}}} \qquad g(1) = \sqrt{2}$