EXERCISE-I

1 . PROPERTIES OF VECTORS

Straight Objective-Including previous years questions

1. Let A,B,C be there points whose position vectors are respectively are $\bar{a} = \bar{i} + 4\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + \alpha\bar{j} + 4\bar{k}, \alpha \in R$, $\bar{c} = 3\bar{i} - 2\bar{j} + 5\bar{k}$

If α is the smallest positive integer for which $\bar{a}, \bar{b}, \bar{c}$ are non-collinear then the length of the Medianin ΔABC, through A is **(29 June 2022 S-II)**

(a) $\frac{\sqrt{82}}{2}$ (b) $\frac{\sqrt{62}}{2}$ (c) $\frac{\sqrt{69}}{2}$ (d) $\frac{\sqrt{66}}{2}$

SOL:- $\overline{AB} = \bar{b} - \bar{a} = 2\bar{i} + \alpha\bar{j} + 4\bar{k} - \bar{i} - 4\bar{j} - 3\bar{k}$

$= \bar{i} + (\alpha - 4)\bar{j} + \bar{k}$

$\overline{AC} = \bar{c} - \bar{a} = 3\bar{i} - 2\bar{j} + 5\bar{k} - \bar{i} - 4\bar{j} - 3\bar{k}$

$= 2\bar{i} - 6\bar{j} + 2\bar{k}$

$\overline{AB} \| \overline{AC}$ if $\frac{-1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2} \Rightarrow \alpha = 1$

$\bar{a}, \bar{b}, \bar{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

Midpoint of $\overline{BC} = M\left(\frac{5}{2}, 0, \frac{9}{2}\right)$

$$AM = \sqrt{\left(\frac{5}{2} - 2\right)^2 + (0-4)^2 + \left(\frac{9}{2} - 3\right)^2} = \sqrt{\frac{82}{4}} = \frac{\sqrt{82}}{2}$$

(Key: a)

2. The unit vector parallel to the resultant vector of $2\bar{i} + 4\bar{j} - 5\bar{k}$ and $\bar{i} + 2\bar{j} + 3\bar{k}$ is

a) $\frac{1}{7}(3\bar{i} + 6\bar{j} - 2\bar{k})$ (b) $\frac{1}{\sqrt{3}}(\bar{i} + \bar{j} + \bar{k})$ (c) $\frac{1}{\sqrt{6}}(\bar{i} + \bar{j} + 2\bar{k})$ (d) $\frac{1}{\sqrt{69}}(-\bar{i} - \bar{j} + 8\bar{k})$

Key: (a)

SOL:- Resultant vectors

$\bar{r} = 2\bar{i} + 4\bar{j} - 5\bar{k} + \bar{i} + 2\bar{j} + 3\bar{k}$ $= 3\bar{i} + 6\bar{j} - 2\bar{k}$

Unit vector parallel to $\bar{r} = \frac{\bar{r}}{|\bar{r}|} = \frac{3\bar{i} + 6\bar{j} - 2\bar{k}}{\sqrt{3^2 + 6^2 + (-2)^2}}$ $= \frac{3\bar{i} + 6\bar{j} - 2\bar{k}}{7}$

3. The vector $\bar{c}$, directed along the internal bisector of the angle between the vectors $\bar{a} = 7\bar{i} - 4\bar{j} - 4\bar{k}$ and $\bar{b} = -2\bar{i} - \bar{j} + 2\bar{k}$ with $|\bar{c}| = 5\sqrt{6}$

a) $\frac{5}{3}(\bar{i} - 7\bar{j} + 2\bar{k})$ (b) $\frac{5}{3}(5\bar{i} + 5\bar{j} + 2\bar{k})$ (c) $\frac{5}{3}(\bar{i} + 7\bar{j} + 2\bar{k})$ (d) $\frac{5}{3}(-5\bar{i} + 5\bar{j} + 2\bar{k})$

Key (a)

SOL:- Let $\bar{a} = 7\bar{i} - 4\bar{j} - 4\bar{k}$ and $\bar{b} = -2\bar{i} - \bar{j} + 2\bar{k}$

Now, required vector $\bar{c} = \lambda\left(\frac{\bar{a}}{|\bar{a}|}+\frac{\bar{b}}{|\bar{b}|}\right) = \lambda\left(\frac{7\bar{i}-4\bar{j}-4\bar{k}}{9}+\frac{-2\bar{i}-\bar{j}+2\bar{k}}{3}\right) \quad = \frac{\lambda}{9}\left(\bar{i}-7\bar{j}+2\bar{k}\right)$

$|\bar{c}|^2 = \frac{\lambda^2}{81}\times 54 = 150 \quad \Rightarrow \lambda = \pm 15 \qquad \therefore \bar{c} = \pm\frac{5}{3}\left(\bar{i}-7\bar{j}+2\bar{k}\right)$

4. The perimeter of a triangle with sides $3\bar{i}+4\bar{j}+5\bar{k}, 4\bar{i}-3\bar{j}-5\bar{k}$ and $7\bar{i}+\bar{j}$ is

$a)\sqrt{450}$ (b) $\sqrt{150}$ (c) $\sqrt{50}$ (d) $\sqrt{200}$

Key: (a)

SOL:- $l_1 = \sqrt{25+25} = \sqrt{50}$

$l_2 = \sqrt{25+25} = \sqrt{50}$

$l_3 = \sqrt{49+1} = \sqrt{50}$

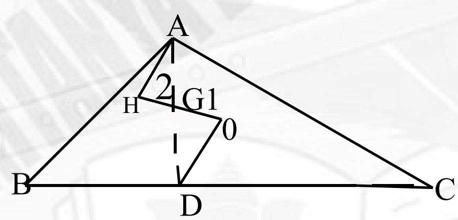
Hence $l_1 + l +_1 l_3 = 3\sqrt{50} = \sqrt{450}$

5. In Δ ABC, let O and H denote circum center and orthocenter respectively. Then $\overline{OA}+\overline{OB}+\overline{OC} =$

$a)\ \overline{OH}$ (b)$2\overline{OH}$ (c)$\frac{2}{3}\overline{OH}$ (d) $3\overline{OH}$

Key: (a)

Since D is midpoint of $\overline{BC}$



$\overline{OD} = \frac{\overline{OB}+\overline{OC}}{2} \Rightarrow \overline{OB}+\overline{OC} = 2\overline{OD}$

$\Delta^{l's}$ AHG, GOD are similar and G divides $\overline{HO}$ in the ratio 2:1

$\frac{AH}{OD} = \frac{AG}{GD} = \frac{HG}{GO} = \frac{2}{1} \Rightarrow AH = 2OD \Rightarrow \overline{AH} = 2\overline{OD}$

Now $\overline{OA}+\overline{OB}+\overline{OC} = \overline{OA}+2\overline{OD}$

$= \overline{OA}+\overline{AH} = \overline{OH}$

<u>Collinear vectors or Parallel vectors ,Coplanar and non-coplanar vectors</u>

6. If vectors $\overline{a_1} = x\bar{i}-\bar{j}+\bar{k}$ and $\overline{a_2} = \bar{i}+y\bar{j}+z\bar{k}$ are collinear, then a possible unit vector Parallel to the vector $x\bar{i}+y\bar{j}+z\bar{k}$ is**(2021, 26th Feb, S-II)**

a)$\frac{1}{\sqrt{2}}(-\bar{j}+\bar{k})$ b)$\frac{1}{\sqrt{2}}(\bar{i}-\bar{j})$ c)$\frac{1}{\sqrt{3}}(\bar{i}+\bar{j}-\bar{k})$ d)$\frac{1}{\sqrt{3}}(\bar{i}-\bar{j}+\bar{k})$

Key: (d)

SOL:- Given, $\overline{a_1}, \overline{a_2}$ are collinear then

$\frac{x}{1} = \frac{-1}{y} = \frac{1}{z} = \lambda (say)$

$x = \lambda$, $y = \frac{-1}{\lambda}$, $z = \frac{1}{\lambda}$

The unit vector parallel to $x\bar{i} + y\bar{j} + z\bar{k}$ will be

$$= \frac{\pm\left(\lambda\bar{i} - \frac{1}{\lambda}\bar{j} + \frac{1}{\lambda}\bar{k}\right)}{\sqrt{\lambda^2 + \frac{1}{\lambda^2} + \frac{1}{\lambda^2}}} = \frac{\pm\left(\lambda\bar{i} - \frac{1}{\lambda}\bar{j} + \frac{1}{\lambda}\bar{k}\right)\lambda}{\sqrt{\lambda^4 + 2}} = \frac{\pm\left(\lambda^2\bar{i} - \bar{j} + \bar{k}\right)}{\sqrt{\lambda^4 + 2}}$$

Put $\lambda = 1$ we have $= \frac{\pm\left(\bar{i} - \bar{j} + \bar{k}\right)}{\sqrt{3}}$

7.Let $\bar{a}$ and $\bar{b}$ be non-collinear vectors. If the vectors $(\lambda - 1)\bar{a} + 2\bar{b}$ and $3\bar{a} + \lambda\bar{b}$ are collinear vectors then value of λ is

a) 2 or 3 (b) -2 or 3 (c) -2 or -3 (d) 2 or -3

Key: (b)

SOL:- Since the vectors $(\lambda - 1)\bar{a} + 2\bar{b}$ and $3\bar{a} + \lambda\bar{b}$ one collinear vectors there exits scalar x such that $3\bar{a} + \lambda\bar{b} = x\left[(\lambda - 1)\bar{a} + 2\bar{b}\right]$ $= x(\lambda - 1)\bar{a} + 2x\bar{b}$

$\Rightarrow x(\lambda - 1) = 3$ and $2x = \lambda$

On solving $\frac{\lambda}{2}(\lambda - 1) = 3 \Rightarrow \lambda = -2$ or 3.

8. The points with position vectors $60\bar{i} + 3\bar{j}, 40\bar{i} - 8\bar{j}, a\bar{i} - 52\bar{j}$ collinear then value of a is equal to

a) 38 b) -40 c) 40 d) 58

Key: (b)

Sol: Given points be A,B,C then $\overline{AB} = \lambda\left(\overline{BC}\right)$

(or) $40\bar{i} - 8\bar{j} - 60\bar{i} - 3\bar{j} = \lambda\left((a - 40)\bar{i} - 44\bar{j}\right)$

$-20\bar{i} - 11\bar{j} = \lambda\left[(a - 40)\bar{i} - 44\bar{j}\right]$

Compare $-11 = -44\lambda \Rightarrow \lambda = \frac{1}{4}$

$-20 = \lambda(a - 40) \Rightarrow -20 = \frac{1}{4}(a - 40) \Rightarrow -80 = a - 40 \quad \therefore a = -40$

9. If $\bar{a}$ and $\bar{b}$ are non-zero and non-collinear vectors then the value of α for which the vectors $\bar{v}_1 = (\alpha - 2)\bar{a} + \bar{b}$ and $\bar{v}_2 = (2 + 3\alpha)\bar{a} - 3\bar{b}$ are collinear is

a) $\frac{3}{2}$ b) $\frac{2}{3}$ c) $\frac{-2}{3}$ d) $\frac{-3}{2}$

Key: (b)

SOL:- $\bar{v}_1 = \lambda \bar{v}_2 \Rightarrow (\alpha-2)-\lambda(2+3\alpha)\bar{a}+(1+3\lambda)\bar{b}=0$ $\Rightarrow \lambda = \frac{-1}{3}$ $\therefore \alpha = \frac{2}{3}$

10. If the lines $\bar{r}=\left(-\bar{i}+\bar{j}-\bar{k}\right)+\lambda\left(2\bar{i}+\bar{j}+3\bar{k}\right)$ and $\bar{r}=-2\bar{i}+\alpha\bar{j}+\bar{k}+\mu\left(2\bar{i}+3\bar{j}+4\bar{k}\right)(\lambda,\mu\in R)$ are coplanar then the value of α is

a) $\frac{-9}{2}$ b) $\frac{11}{2}$ c) $\frac{-11}{2}$ d) $\frac{15}{2}$

Key: (d)

SOL:- we must have $\begin{vmatrix} -1+2 & 1-\alpha & -1-1 \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0 \Rightarrow \alpha = \frac{15}{2}$

11. If $\bar{a},\bar{b}$ and $\bar{c}$ are non-coplanar vectors such that $(x+y-3)\bar{a}+(2x-y+2)\bar{b}+(2x+y+\lambda)\bar{c}=\bar{o}$ holds true for some x and y then λ is

a) $\frac{7}{3}$ b) 2 c) $\frac{-10}{3}$ d) $\frac{5}{3}$

Key: (c)

SOL: since $\bar{a},\bar{b}$ and $\bar{c}$ are non-coplanar vector

$x+y-3=0, 2x-y+2=0$ *and* $2x+y+\lambda=0$

$\begin{vmatrix} 1 & 1 & -3 \\ 2 & -1 & 2 \\ 2 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = \frac{-10}{3}$

12. If $\bar{a}=\bar{i}+\bar{j}+\bar{k}, \bar{b}=4\bar{i}+3\bar{j}+4\bar{k}, \bar{c}=4\bar{i}+\alpha\bar{j}+\beta\bar{k}$ be linearly dependent vectors and $|\bar{c}|=\sqrt{3}$ then

a) $\alpha=1,\beta=-1$ b) $\alpha=1,\beta=\pm1$ c) $\alpha=-1,\beta=\pm1$ d) $\alpha=\pm1,\beta=1$

Key : (d)

SOL:- since $\bar{a},\bar{b},\bar{c}$ are linearly dependent vectors

$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0 \Rightarrow 1(3\beta-4\alpha)-1(4\beta-4)+1(4\alpha-3)=0$

$\Rightarrow 3\beta-4\alpha-4\beta+4+4\alpha-3=0 \Rightarrow \beta=1$

$\because |\bar{c}|=\sqrt{3} \Rightarrow \sqrt{1+\alpha^2+\beta^2}=\sqrt{3} \Rightarrow 1+\alpha^2+\beta^2=3 \Rightarrow \alpha^2+\beta^2=2 \Rightarrow \alpha^2=2-1=1 \therefore \alpha=\pm1$

13. If the four points, whose position vectors are $3\bar{i}-4\bar{j}+2\bar{k}, \bar{i}+2\bar{j}-\bar{k}, -2\bar{i}-\bar{j}+3\bar{k}$ and $5\bar{i}-2\alpha\bar{j}+4\bar{k}$ are coplanar, then α is equal to **[Jee2023,25 jan S2]**

a) $\frac{73}{17}$ b) $\frac{-107}{17}$ c) $\frac{-73}{17}$ d) $\frac{107}{17}$ Key: (a)

Sol: Let $A=(3,-4,2)B=(1,2,-1)C=(-2,-1,3)D=(5,-2\alpha,4)$

A,B,C,D are coplanar

$$\begin{vmatrix} 1-3 & 2+4 & -1-2 \\ -2-3 & -1+4 & 3-2 \\ 5-3 & -2\alpha+4 & 4-2 \end{vmatrix}=0 \Rightarrow \alpha=\frac{73}{17}$$

14. Let S be the set of all (λ,μ) for which the vectors $\lambda\bar{i}-\bar{j}+\bar{k},\bar{i}+2\bar{j}+\mu\bar{k}$, and $3\bar{i}-4\bar{i}+5\bar{k}$, where $(\lambda-\mu)=5$ are coplanar, then $\sum_{(\lambda,\mu)\in s} 80(\lambda^2+\mu^2)$ is equal to

(JEE 2023 15th Apr S-I)

a)2370 b)2130 c)2210 d)2290 Key: (d)

Sol: Given vector are coplanar

$$\Rightarrow \begin{vmatrix} \lambda & -1 & 1 \\ 1 & 2 & \mu \\ 3 & -4 & 5 \end{vmatrix}=0$$

$\lambda(10+4\mu)+1(5-3\mu)+1(-10)=0$

$10\lambda+4\lambda\mu+5-3\mu-10=0$ $10\lambda+4\lambda\mu-3\mu-5=0$

Now $\lambda=5+\mu$ in the above

$10(5+\mu)+4\mu(5+\mu)-3\mu-5=0$

$5+10\mu+20\mu+4\mu^2-3\mu-5=0$ $4\mu^2+27\mu+45=0$ $(4\mu+15)(\mu+3)=0$

$\therefore \mu=-3,\frac{-15}{4}$ $\therefore \lambda=\mu+5$ so we get $\lambda=2,\frac{5}{4}$

Now $\sum_{(\lambda,\mu)\in s} 80(\lambda^2+\mu^2)=80\left[\left(2^2+(-3)^2\right]+\left(\left(\frac{5}{4}\right)^2+\left(\frac{-15}{4}\right)^2\right)\right.$ $=2290.$

15. The vectors $\bar{a}$ and $\bar{b}$ are non-collinear for what value of x, the vectors $\bar{c}=(x-2)\bar{a}+\bar{b}$ and $\bar{d}=(2x+1)\bar{a}-\bar{b}$ are collinear

a) $\frac{2}{3}$ b)1 c) $\frac{1}{3}$ d) $\frac{-1}{3}$

Key: (c)

SOL:- Here $\bar{c}$ and $\bar{d}$ are collinear so $\bar{d}=\lambda\bar{c}$ where $\lambda\in R$ (or)

$(2x+1)\bar{a}-\bar{b}=\lambda[(x-2)\bar{a}+\bar{b}]$ On comparing we have

$\lambda=-1$ and $2x+1=\lambda(x-2)\Rightarrow 2x+1=-1(x-2)=-x+2$ $\Rightarrow 3x=2-1=1$ $\therefore x=\frac{1}{3}$

16. Let the vector $\bar{a}=(1+t)\bar{i}+(1-t)\bar{j}+\bar{k}$, $\bar{b}=(1-t)\bar{i}+(1+t)\bar{j}+2\bar{k}$ and $\bar{c}=t\bar{i}-t\bar{j}+\bar{k}$, $t\in R$ for $\alpha,\beta,\gamma\in,R,\alpha\bar{a}+\beta\bar{b}+\gamma\bar{c}=\bar{o}\Rightarrow\alpha=\beta=\gamma=0$ then, the set of all values of t is

[2022,28th july,S-I]

a) A non-empty finite set b) Equal to N

c) Equal to $R-\{0\}$ d) Equal to R Key: (c)

here $\bar{a},\bar{b},\bar{c}$ are linearly in dependent vectors. So

$$\begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1+t & 2 & 1 \\ 1-t & 2 & 2 \\ t & 0 & 1 \end{vmatrix} \neq 0$$

$C_2 \to C_2 + C_1$

$$\Rightarrow 2\begin{vmatrix} 1+t & 1 & 1 \\ 1-t & 1 & 2 \\ t & 0 & 1 \end{vmatrix} \neq 0 \Rightarrow 2\left[(1+t)-(1-t-2t)-t\right] \neq 0 \qquad \Rightarrow 6t \neq 0 \Rightarrow t \neq 0$$

17. let $\alpha = (\lambda-2)\bar{a}+\bar{b}$ and $\beta = (4\lambda-2)\bar{a}+3\bar{b}$ be two given vectors where vectors $\bar{a}$ and $\bar{b}$ are non-collinear. The value of λ for which vectors α and β are collinear is

a)4 b)-3 c)3 d)-4 **[2019,10th Jan S-I]** Key: (d)

Sol: Since α,β are collinear $\Rightarrow \alpha = k\beta$ where $k \in R-\{0\}$

$\Rightarrow (\lambda-2)\bar{a}+\bar{b} = k[(4\lambda-2)\bar{a}+3\bar{b}]$

On comparing

3k = 1 $\Rightarrow$ k = $\frac{1}{3}$ and $\lambda-2 = k(4\lambda-2) \Rightarrow \lambda-2 = \frac{1}{3}(4\lambda-2) \Rightarrow -4 = \lambda$

18. Let $\bar{a},\bar{b}$ and $\bar{c}$ be three non-zero vectors which are pair wise non-collinear. If $\bar{a}+3\bar{b}$ is collinear with $\bar{c}$ and $\bar{b}+2\bar{c}$ is collinear with $\bar{a}$ then $\bar{a}+3\bar{b}+6\bar{c}$ is **[AIEEE 2011]**

a) $\bar{a}+\bar{c}$ b) $\bar{a}$ c) $\bar{c}$ d) $\bar{o}$ Key: (d)

Sol: As, $\bar{a}+3\bar{b}$ is collinear with $\bar{c}$ $\therefore \bar{a}+3\bar{b} = \lambda\bar{c}$ -----(1)

Also, $\bar{b}+2\bar{c}$ is collinear with $\bar{a}$ $\therefore \bar{b}+2\bar{c} = \mu\bar{a}$ ------(2)

from (1) we have $\bar{a}+3\bar{b}+6\bar{c} = \lambda\bar{c}+6\bar{c} = (\lambda+6)\bar{c}$

from (2) we have $\bar{a}+3\bar{b}+6\bar{c} = (1+3\mu)\bar{a}$

$\therefore (\lambda+6)\bar{c} = (1+3\mu)\bar{a}$

$\therefore \bar{a}$ is not collinear with $\bar{c}$

$\therefore \bar{a}+3b+\alpha\bar{c} = \bar{a}$

$\Rightarrow \lambda+6=0$ and $1+3\mu=0$

$\therefore \lambda = -6, \mu = \frac{-1}{3}$

$\therefore \bar{a}+3b+6\bar{c} = \bar{O}$

19. The position vectors of three points are $2\bar{a}-\bar{b}+3\bar{c}$, $\bar{a}-2\bar{b}+\lambda\bar{c}$ and $\mu\bar{a}-5\bar{b}$ Where $\bar{a},\bar{b},\bar{c}$ are non-coplanar vector, the points are collinear when

a) $\lambda=-2,\mu=\frac{9}{4}$ b) $\lambda=\frac{-9}{4},\mu=2$ c) $\lambda=\frac{9}{4},\mu=-2$ d) $\lambda=2,\mu=-2$ Key : (c)

Sol: $\left(\bar{a}-2\bar{b}+\lambda\bar{c}\right)-\left(2\bar{a}-\bar{b}+3\bar{c}\right) = t\left(\left(\mu\bar{a}-5\bar{b}\right)-\left(\bar{a}-2\bar{b}+\lambda\bar{c}\right)\right)$

Comparing both sides, we get

$-1=t(\mu-1)$ ------(i)

$-1=t(-5+2)$ -----(ii)

$\lambda-3=-\lambda t$ -------(iii)

From (ii) t = 1/3, from (iii) $\lambda-3=\frac{-\lambda}{3}\Rightarrow\lambda=\frac{9}{4}$

From (i) $-1=\frac{1}{3}(\mu-1)\Rightarrow\mu=-2$ $\quad\therefore\lambda=\frac{9}{4},\mu=-2$

20. Let $f(\vec{t})=[t]\bar{i}+(t-[t])\bar{j}+[t+1]\bar{k},[.]$ is G.I.F. If $f\left(\frac{\vec{5}}{4}\right)$ and $\bar{i}+\lambda\bar{j}+\mu\bar{k}$ are parallel vectors then $(\lambda,\mu)=$ Key:b

a)(1,1) b)($\frac{1}{4}$,2) c) $\left(\frac{1}{2},2\right)$ d) $\left(\frac{1}{4},4\right)$

SOL:- $f\left(\frac{\vec{5}}{4}\right) = \left[\frac{5}{4}\right]\bar{i}+\left(\frac{5}{4}-\left[\frac{5}{4}\right]\right)\bar{j}+\left[\frac{5}{4}+1\right]\bar{k}=\bar{i}+\frac{1}{4}\bar{j}+2\bar{k}$

Now $\bar{i}+\frac{1}{4}\bar{j}+2\bar{k}$ and $\bar{i}+\lambda\bar{j}+\mu\bar{k}$ are parallel

$\Rightarrow\frac{1}{1}=\frac{\lambda}{\frac{1}{4}}=\frac{\mu}{2}\Rightarrow\lambda=\frac{1}{4},\mu=2$ $\quad\therefore(\lambda,\mu)=\left(\frac{1}{4},2\right)$

21. If $\bar{a},\bar{b}$ and $\bar{c}$ are non-coplanar vectors and if $\bar{d}$ is such that $\bar{d}=\frac{1}{x}\left(\bar{a}+\bar{b}+\bar{c}\right)$ and $\bar{a}=\frac{1}{y}\left(\bar{b}+\bar{c}+\bar{d}\right)$ where x and y non-zero real numbers, then $\frac{1}{xy}\left(\bar{a}+\bar{b}+\bar{c}+\bar{d}\right)=$

a) $-\bar{a}$ b) $\bar{o}$ c) $2\bar{a}$ d) $3\bar{c}$

key:b

SOL:- $x\bar{d}=\bar{a}+\bar{b}+\bar{c}\Rightarrow x\bar{d}+\bar{d}=\bar{a}+\bar{b}+\bar{c}+\bar{d}=\bar{a}+y\bar{a}$ $=(x+1)\bar{d}=\bar{a}(1+y)$

$\Rightarrow(x+1)\bar{d}-\bar{a}(1+y)=\bar{o}\;\therefore x+1=0,y+1=0$

$\therefore x=-1,y=-1$ $\quad\therefore\bar{a}+\bar{b}+\bar{c}+\bar{d}=\bar{o}$

22. Three non-zero, non-collinear vectors $\bar{a},\bar{b},\bar{c}$ are sum that $\bar{a}+3\bar{b}$ is collinear with $\bar{c}$ while $3\bar{b}+2\bar{c}$ is collinear with $\bar{a}$ then $\bar{a}+3\bar{b}+2\bar{c}=$ [E 2014]

Key : d

a) $2\bar{a}$ b) $3\bar{b}$ c) $4\bar{c}$ d) $\bar{o}$

SOL:- $\bar{a}+3\bar{b}=\lambda\;\bar{c}$ ----(1)

$3\bar{b}+2\bar{c}=\mu\ \bar{a}$----(2)

$\bar{a}+3\bar{b}+2\bar{c}=\lambda\bar{c}+2\bar{c}=(\lambda+2)\bar{c}$

$\bar{a}+\mu\bar{a}=\bar{a}+3\bar{b}+2\bar{c}$

$(\lambda+2)\bar{c}=(1+\mu)\bar{a}$

$\therefore \lambda+2=0, \mu+1=0 \Rightarrow \lambda=-2, \mu=-1$

$\Rightarrow \bar{a}+3\bar{b}+2\bar{c}=\bar{o}$

23. Let a,b and c be distinct positive numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co planar ,then c is equal to :(JEE 2021 25 jul S2)

(a) $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ (b) $\frac{a+b}{2}$ (c) $\frac{1}{a}+\frac{1}{b}$ (d) $\sqrt{ab}$

Sol: Given vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar ,then

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}=0 \qquad \Rightarrow a(o-c)-a(b-c)+c(c-0)=0 \Rightarrow c^2=ab \therefore c=\sqrt{ab}.$$

Section Formulae,Angular Bisectors:

24. The position vectors of the point A and B with reflect to a fixed point O (origin) are $2\bar{i}+2\bar{j}+\bar{k}$ and $2\bar{i}+4\bar{j}+4\bar{k}$ .the length of internal bisector of $\underline{|BOA}$ of ΔAOB , is

a) $\frac{\sqrt{136}}{9}$ b) $\frac{\sqrt{136}}{3}$ c) $\frac{20}{3}$ d) $\frac{\sqrt{217}}{9}$ Key: (a)

SOL:- $|\overline{OA}|=3, |\overline{OB}|=6$

Now, position vector of $L=\overrightarrow{OL}$

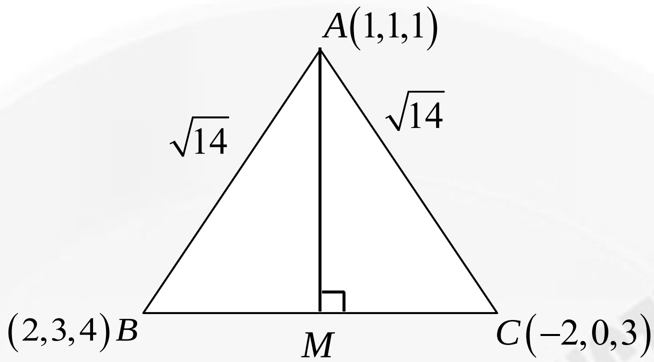
$$=\frac{|\overline{OA}|\left(2\bar{i}+4\bar{j}+4\bar{k}\right)+|\overline{OB}|\left(2\bar{i}+2\bar{j}+\bar{k}\right)}{|\overline{OA}|+|\overline{OB}|} = \frac{3\left(2\bar{i}+4\bar{j}+4\bar{k}\right)+6\left(2\bar{i}+2\bar{j}+\bar{k}\right)}{3+6}$$

$$=\frac{18\bar{i}+24\bar{j}+18\bar{k}}{3+6}=\frac{1}{3}\left(6\bar{i}+8\bar{j}+8\bar{k}\right) \qquad \text{So, } |\overline{OL}|=\frac{1}{3}\sqrt{36+64+86}=\frac{\sqrt{136}}{3}$$

25. If the position vectors of the vertices A,B,C of a triangle ABC are (1,1,1),(2,3,4),(-2,0,3) respectively, then the magnified of the vector representing the internal bisector $\underline{|BAC}$, is

a) $\frac{\sqrt{27}}{2}$ b) $\frac{\sqrt{30}}{2}$ c) $\frac{\sqrt{35}}{2}$ d) $\frac{\sqrt{33}}{2}$ b nKey: (b)

SOL:- $|\overline{AM}| = \sqrt{1+\frac{1}{4}+\frac{25}{4}} = \frac{\sqrt{30}}{2}$



26. If three points A,B and C are collinear, whose position vector are $\bar{i}-2\bar{j}-8\bar{k}$, $5\bar{i}-2\bar{k}$ and $11\bar{i}+3\bar{j}+6\bar{k}$ respectively, then the ratio in which B divides A C is

a)1:2 b)2:3 c)2:1 d)1:1 Key: (b)

Sol: Let B divides AC in the ratio $\lambda:1$ then

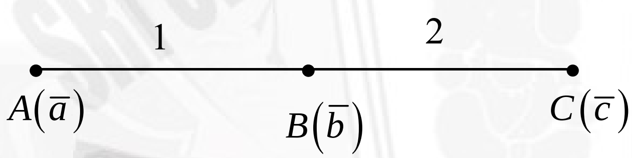
$$5\bar{i}-2\bar{k} = \frac{\lambda\left(11\bar{i}+3\bar{j}+7\bar{k}\right)+1\left(\bar{i}+2\bar{j}+8\bar{k}\right)}{\lambda+1}$$

$\Rightarrow 3\lambda-2=0 \Rightarrow \lambda=\frac{2}{3}$ i.e, ratio = 2:3

27. $\bar{a}$ and $\bar{b}$ are position vectors of A,B respectively and C is a point on AB produced such that AC=3AB. Then position vector of C is

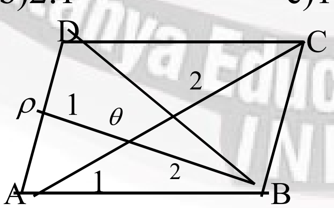
a) $3\bar{b}-2\bar{d}$ b) $2\bar{a}-3\bar{b}$ c) $2\bar{a}+3\bar{b}$ d) $3\bar{a}-2\bar{b}$ Key: (a)

SOL:- $\bar{b} = \frac{1\times\bar{c}+2\times\bar{a}}{1+2} = \frac{\bar{c}+2\bar{a}}{3}$ $\Rightarrow \bar{c} = 3\bar{b} - 2\bar{a}$.



28. ABCD is a parallelogram and P is the midpoint of the side AD. The line BP meets the diagonal AC in Q. Then the ratio $AQ:QC$ is

a)1:2 b)2:1 c)1:3 d)3:1 key : a



WKT In a parallelogram opposite sides are equal in length

$\therefore BC = AD$ ----(1)

Since P is mid point of the side AD we have AD= 2AP----(2)

$\therefore$ (1) $\Rightarrow$ BC = 2AP

Here ΔAQP similar to ΔCQB

$\Rightarrow \frac{AQ}{CQ}=\frac{AP}{BC}=\frac{AP}{2AP}=\frac{1}{2}$ $\qquad\qquad \therefore AQ:QC=1:2$

29. In ΔABC, P,Q,R are points on BC, CA and AB respectively, dividing them in the ratio 1:4, 3:2 and 3:7. The point S divides AB in the ratio 1:3. Then $\frac{|\overline{AP}+\overline{BQ}+\overline{CR}|}{|\overline{CS}|}=$

Key: b

a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{5}{2}$ d) $\frac{7}{10}$

SOL:- $\overline{OP}=\frac{\overline{OC}+4\overline{OB}}{5}, \overline{OQ}=\frac{3\overline{OA}+2\overline{OC}}{5}, \overline{OR}=\frac{3\overline{OB}+7\overline{OA}}{10}$

$\overline{OS}=\frac{\overline{OB}+3\overline{OA}}{4}$

$\overline{AP}+\overline{BQ}+\overline{CR}=\overline{OP}-\overline{OA}+\overline{OQ}-\overline{OB}+\overline{OR}-\overline{OC}$

$=\frac{3\overline{OA}+\overline{OB}-4\overline{OC}}{10}=\frac{4\overline{OS}-4\overline{OC}}{10}=\frac{4}{10}\overline{CS}=\frac{2}{5}\overline{CS}$

30. If the vectors $\overline{AB}=3\overline{i}+4\overline{k}$ *and* $\overline{AC}=5\overline{i}-2\overline{j}+4\overline{k}$ are the sides of a ΔABC, then the length of the median through A is **(Jee main 2013,2003)**

a) $\sqrt{18}$ b) $\sqrt{72}$ c) $\sqrt{33}$ d) $\sqrt{95}$

Key: (c)

Sol: Length of the median though A is

$=\frac{\left|\overline{AB}+\overline{AC}\right|}{2}=\frac{\left|3\overline{i}+4\overline{k}+5\overline{i}-2\overline{j}+4\overline{k}\right|}{2}$

$=\frac{\left|8\overline{i}-2\overline{j}+8\overline{k}\right|}{2}=\left|4\overline{i}-\overline{j}+4\overline{k}\right| \qquad =\sqrt{16+1+16}=\sqrt{33}$

31. The vector $\cos\alpha.\cos\beta\overline{i}+\cos\alpha.\sin\beta\overline{j}+\sin\alpha\overline{k}$ is a/an

(a) null vector b)unit vector

c)constant vector d)vector of magnitude 3 Key: (b)

Sol: $|\overline{a}|=\sqrt{\cos^2\alpha\cos^2\beta+\cos^2\alpha\sin^2\beta+\sin^2\alpha}$

$=\sqrt{\cos^2\alpha(\cos^2\beta+\sin^2\beta)+\sin^2\alpha}$

$=\sqrt{\cos^2\alpha+\sin^2\alpha}=1$

GEOMETRICAL APPLICATIONS:

32. A vector $\overline{a}$ has components 2p and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counter clock wise sense. If, w.r.t the new system, $\overline{a}$ has components $P+1$ and 1 then **(1986)**

a) $P=0$ b) $P=1$ (or) $P=\frac{-1}{3}$ c) $P=-1$ or $P=\frac{1}{3}$ d) $P=1$ or $P=-1$ Key : (b)

SOL:- Here $\bar{a}=2P\bar{i}+\bar{j}$, when a system is rotated the new component of $\bar{a}$ are $(P+1)$ and 1

i.e $\bar{b}=(P+1)\bar{i}+\bar{j} \Rightarrow |\bar{a}|^2=|\bar{b}|^2$

or $4P^2+1=(P+1)^2+1$

$\Rightarrow 4P^2=P^2+2P+1 \Rightarrow 3P^2-2P-1=0$

$\Rightarrow (3P+1)(P-1)=0$

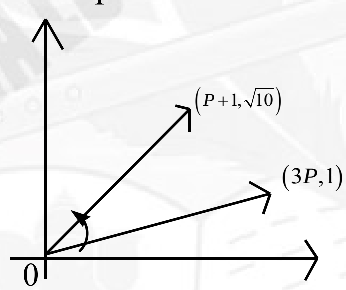
$\Rightarrow P=1,\frac{-1}{3}$

33. A vector $\bar{a}$ has components 3ρ and 1 with reflect to rectangular Cartesian system. This System is rotated through a certain angle about the origin in the counter clock wise sense. If, with respect to new system, $\bar{a}$ has components $P+1$ and $\sqrt{10}$ then Q value P is equal to **[2021,18 march S-I]**

a) 1 b) $\frac{-5}{4}$ c) $\frac{4}{5}$ d) -1 Key : (d)

Sol: After counter clock wise (or) anti clock-wise rotation, the length of the vector $\bar{a}$ remains constant.

i.e, $|\bar{a}|$ at old position = $|\bar{a}|$ at new position



$\Rightarrow (3P)^2+(1)^2=(P+1)^2+\left(\sqrt{10}\right)^2$

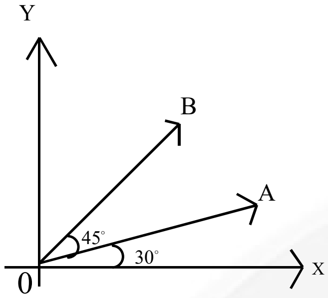
$\Rightarrow 8P^2-2P-10=0$

$\Rightarrow (P+1)(4P-5)=0 \Rightarrow P=\frac{5}{4},-1$

34. Let a vector $\alpha\bar{i}+\beta\bar{j}$ be obtained by rotating the vector $\sqrt{3}\bar{i}+\bar{j}$ by an angle 45^0 about the origin in counter clock wise direction in first quadrant then the area of triangle having vertices $(\alpha,\beta),(0,\beta)$ and(0,0) is equal to **[2021, march S-I]**

a) $\frac{1}{2}$ b) 1 c) $\frac{1}{\sqrt{2}}$ d) $2\sqrt{2}$ Key : (a)

Sol: let $\overline{OA}$ be $\sqrt{3}\bar{i}+\bar{j}$ and $\overline{OB}$ be $\alpha\bar{i}+\beta\bar{j}$



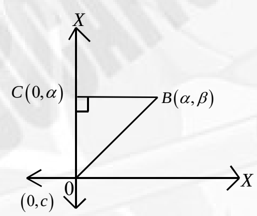
As, we can notice in $\overline{OA}$, $\frac{1}{\sqrt{3}} = \text{Tan}\,30^0$ So it makes an angle of 30^0 with the $x-$axis .

Now, When $\overline{OA}$ is rotated further by 45^0 anti clock wise the resultant vector $\overline{OB}$ Makes an angle of 75^0 with the X-axis.

So, $\overline{OB} = \left|\overline{OA}\right| \left(\cos 75^0\, \bar{i} + \sin 75^0\, \bar{j}\right)$

Let ΔOBC be the required triangle whose area we have to determine area of

$\Delta OBC = \frac{1}{2} \times base \times height$ $\qquad = \frac{1}{2} \times \beta \times \alpha$



$= \frac{1}{2} \times \left(2\sin 75^0\right)\left(2\cos 75^0\right)$

$= \frac{1}{2} \times \left(2\sin 75^0\right)\left(2\cos 75^0\right)$

$= 2\sin 75^0 \cos 75^0 = \sin 150^0 = \sin 30^0 = \frac{1}{2}$ Hence area is $\frac{1}{2}$ sq units.

35. If theposition vectors of the vertices A,B and C of a ΔABC are $7\bar{j} + 10\bar{k}, -\bar{i} + 6\bar{j} + 6\bar{k}$ and $-4\bar{i} + 9\bar{j} + 6\bar{k}$ respectively. The triangle is

a)Equilateral b) Isosceles

c) Scalen d) Right angled and Isosceles Key: (d)

SOL:- Given, position vectors A,B and C are

$7\bar{j} + 10\bar{k}$, $-\bar{i} + 6\bar{j} + 6\bar{k}$ and $-4\bar{i} + 9\bar{j} + 6\bar{k}$ respectively

$|\overline{AB}| = |-\bar{i} - \bar{j} - 4\bar{k}| = \sqrt{18}$ $\qquad |\overline{BC}| = |-3\bar{i} - 3\bar{j} - | = \sqrt{18}$

$|\overline{AC}| = |-4\bar{i} + 2\bar{j} - 4\bar{k}| = \sqrt{36}$

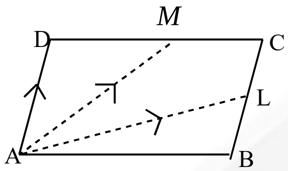
Clearly, AB = BC and $(AC)^2 = (AB)^2 + (BC)^2$

Hence, triangle is risht angled isosceles.

36. L and M are the midpoints of the sides BC and CD of a parallelogram ABCD then $\overline{AL} + \overline{AM}$ is equal to

a) $\frac{1}{2}\overline{AC}$ b) $\frac{2}{3}\overline{AC}$ c) $\frac{3}{2}\overline{AC}$ c) $\frac{4}{3}\overline{AC}$ Key: (c)

SOL:-

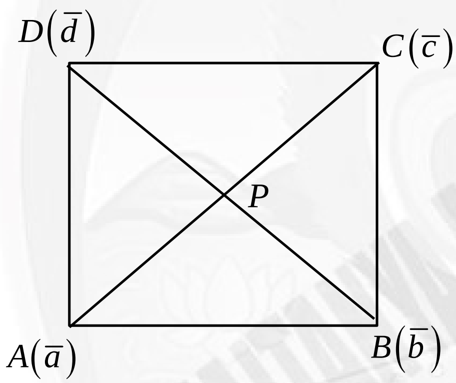


$\overline{AL} = \frac{1}{2}\left(2\bar{b} + \bar{d}\right)$ $\overline{AM} = \frac{1}{2}\left(\bar{b} + 2\bar{d}\right)$

$\therefore \overline{AL} + \overline{AM} = \frac{1}{2}\left(3\bar{b} + 3\bar{d}\right)$ $= \frac{3}{2}\left(\bar{b} + \bar{d}\right) = \frac{3}{2}\overline{AC}$

37. Let ABCD be a parallelogram whose diagonal intersect at P and Let O be the origin. Then $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD}$ equals

a) $\overline{OP}$ b) $2\overline{OP}$ c) $3\overline{OP}$ d) $4\overline{OP}$ Key: (d)



Sol: Diagonals of parallelogram intersect at P $\Rightarrow \frac{\bar{a} + \bar{c}}{2} = \bar{P} = \frac{\bar{b} + \bar{d}}{2}$

$\Rightarrow \bar{a} + \bar{b} + \bar{c} + \bar{d} = 2\bar{P} + 2\bar{P} = 4\bar{P}$

Taking position vectors $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} = 4\overline{OP}$

38. Let P, Q, R and S be the points on the plane with position vectors $-2\bar{i} - \bar{j}$, $4\bar{i}$, $3\bar{i} + 3\bar{j}$ and $-3\bar{i} + 2\bar{j}$ Respectively. the quadrilateral $PQRS$ must be a

[IIT-JEE 2010]

a)parallelogram, which is neither a Rhombus nor a rectangle
b)Square
c) Rectangle, but not a square
d)Rhombus, but not a square. Key: (a)

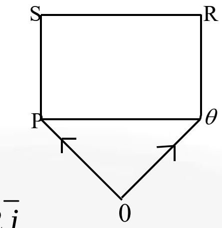
Sol: Let O be the origin

Then $\overline{OP} = -2\bar{i} - \bar{j}$

$\overline{OQ} = 4\bar{i}, \overline{OR} = 3\bar{i} + 3\bar{j}, \overline{OS} = -3\bar{i} + 2\bar{j}$

Here we have

$\overline{PQ} = 6\bar{i} + \bar{j}, \overline{QR} = -\bar{i} + 3\bar{j}, \overline{RS} = -6\bar{i} - \bar{j}$



And $\overline{PS} = -\bar{i} + 3\bar{j}, \overline{PR} = 5\bar{i} + 4\bar{j}, \overline{QS} = -7\bar{i} + 2\bar{j}$

$\overline{PS}.\overline{QS} = -35 + 8 = -27 \neq 0$

Diagonally are not perpendicular

And $|\overline{PQ}| = |\overline{RS}|, |\overline{QR}| = |\overline{PS}|$

Hence $PQRS$ is a parallelogram which is neither a Rhombus nor a rectangle.

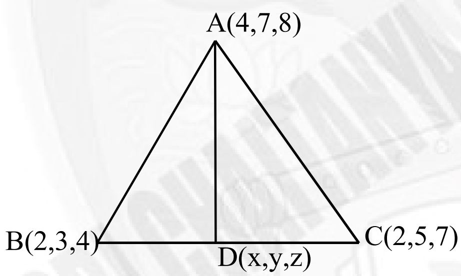
39. If the position vectors of the vertices A,B and C of a ΔABC are respectively $4\bar{i} + 7\bar{j} + 8\bar{k}, 2\bar{i} + 3\bar{j} + 4\bar{k}$ and $2\bar{i} + 5\bar{j} + 7\bar{k}$ then the position vector of the point, where the bisector of $\underline{|A}$ meets BC is

[Online April 15th, 2018]

a) $\frac{1}{2}\left(4\hat{i} + 8\hat{j} + 11\hat{k}\right)$ b) $\frac{1}{3}\left(6\hat{i} + 13\hat{j} + 18\hat{k}\right)$ c) $\frac{1}{4}\left(8\hat{i} + 14\hat{j} + 9\hat{k}\right)$ d) $\frac{1}{3}\left(6\hat{i} + 11\hat{j} + 15\hat{k}\right)$

Key: (b)

Sol:



Suppose angular bisector of A meets BC at D (x,y,z)

Using angular bisector theorem

$$\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{BD}{DC} = \frac{6}{3} = \frac{2}{1}$$

$BD : DC = 2:1$

SO $D(x,y,z) = \left(\frac{(2)(2)+(1)(2)}{2+1}, \frac{(2)(5)+(1)(3)}{2+1}, \frac{(2)(7)+(1)(4)}{2+1}\right)$

$$D(x,y,z) = \left(\frac{6}{3}, \frac{13}{3}, \frac{18}{3}\right)$$

There fore, position vector of point $P = \frac{1}{3}\left(6\hat{i} + 13\hat{j} + 18\hat{k}\right)$

40. Let ABC be a triangle whose circumcentre is at P. If the position vectors A,B,C and P are $\bar{a}, \bar{b}, \bar{c}$ and $\frac{\bar{a} + \bar{b} + \bar{c}}{4}$ respectively, then the position vector of the orthocenter of this triangle is

[online April 10th, 2016]

a) $-\left(\dfrac{\bar{a}+\bar{b}+\bar{c}}{2}\right)$ b) $\left(\bar{a}+\bar{b}+\bar{c}\right)$ c) $\dfrac{\bar{a}+\bar{b}+\bar{c}}{2}$ d) $\bar{0}$ **Key: (c)**

Sol: Position vector of centriod $\bar{G}=\dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

Position vector of circumcentre $\bar{C}=\dfrac{\bar{a}+\bar{b}+\bar{c}}{4}$

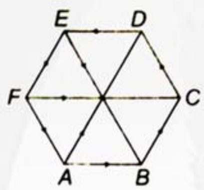
$\bar{G}=\dfrac{2\bar{C}+\bar{r}}{3} \Rightarrow 3\bar{G}=2\bar{C}+\bar{r} \Rightarrow \bar{r}=3\bar{G}-2\bar{C}$ $\quad \bar{a}+\bar{b}+\bar{c}-2\dfrac{\bar{a}+\bar{b}+\bar{c}}{4}$ $\quad =\dfrac{\bar{a}+\bar{b}+\bar{c}}{2}$

Regular polygon in vectors :

41. If ABCDEF is a regular hexagon, then $\overline{AD}+\overline{EB}+\overline{FC}$ is equal to

a) $\bar{O}$ (b) $2\overline{AB}$ (b) $3\overline{AD}$ (d) $4\overline{AB}$

SOL:- we have



$\overline{AD}+\overline{EB}+\overline{FC}$

$(\overline{AB}+\overline{BC}+\overline{CD})+(\overline{ED}+\overline{DC}+\overline{CB})+\overline{FC}$

$\overline{AB}+(\overline{BC}+\overline{CB})+(\overline{CD}+\overline{DC})+\overline{ED}+\overline{FC}$ $\quad =\overline{AB}+\bar{0}+\bar{0}+\overline{AB}+2\overline{AB}$

$=4\overline{AB}\left(\overline{ED}=\overline{AB},\overline{EC}=2\overline{AB}\right)$

Key: (d)

42. If ABCDE is a pentagon then

$\overline{AB}+\overline{AE}+\overline{BC}+\overline{DC}+\overline{ED}+\overline{AC}$ is equal to

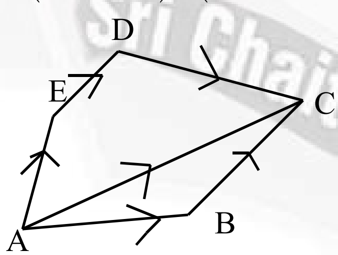
a) $6\overline{AC}$ (b) $5\,\overline{AC}$ (c) $4\,\overline{AC}$ (d) $3\,\overline{AC}$

Key: (d)

Sol: We have

$\overline{AB}+\overline{AE}+\overline{BC}+\overline{DC}+\overline{ED}+\overline{AC}$

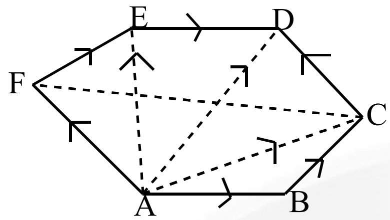
$=\left(\overline{AB}+\overline{BC}\right)+\left(\overline{AE}+\overline{ED}+\overline{DC}\right)+\overline{AC}$ $\quad \overline{AC}+\overline{AC}+\overline{AC}$ $\quad =3\overline{AC}$



43. ABCDEF be a regular hexagon whose centre is O. Then $\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}=$

a) $2\overline{AO}$ b) $3\,\overline{AO}$ c) $5\,\overline{AO}$ d) $\overline{AO}$

key: d



SOL:-

$$\begin{bmatrix} \because \overline{AF} = \overline{CD} \\ \because \overline{AB} = \overline{FD} \end{bmatrix}$$

$\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF} \qquad = \left(\overline{AE}+\overline{ED}\right)+\overline{AD}+\left(\overline{AC}+\overline{CD}\right)$

$= \overline{AD}+\overline{AD}+\overline{AD}=3\overline{AD}=6\overline{AO}$

44. If $A_1A_2.......A_n$ is a regular polygon. Then the vector $\overline{A_1A_2}+\overline{A_2A_3}+.........+\overline{A_nA_1}$ is equal to

a) $\overline{O}$ b) $n\left(\overline{A_1A_2}\right)$ c) $n\left(\overline{OA_1}\right)$ (O is the centre) d) $(n-1)\left(\overline{A_1A_2}\right)$

key: a

SOL:-Let Obe the centre

$\overline{A_iA_{i+1}}=\overline{OA_{i+1}}-\overline{OA_i}$

$\sum_{i=1}^{n-1}\overline{A_iA_{i+1}}=\sum_{i=1}^{n-1}\overline{OA_{i+1}}-\overline{OA_i}=\bar{o}$

45. ABCDEF be a regular hexagon in the xy plane and $\overline{AB}=4\bar{i}\, then\, \overline{CD}$ =

a) $6\bar{i}+2\sqrt{3}\bar{j}$ b) $2\left(-\bar{i}+\sqrt{3}\bar{j}\right)$ c) $2\left(\bar{i}+\sqrt{3}\bar{j}\right)$ d) $2\left(\bar{i}-\sqrt{3}\bar{j}\right)$

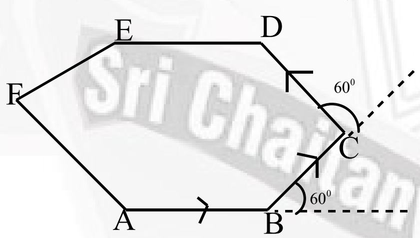
key:b

SOL:- we rotate the side AB by 60^0 in the antic clock wise direction we get

$$4e^{\frac{i\pi}{3}}=4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$

Thus

$$=4\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=2+i2\sqrt{3}$$



Now we need to rotate this also by 60^0 anti clock wise we get

$\left(2+i2\sqrt{3}\right)\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=1-3+i\left(\sqrt{3}+\sqrt{3}\right)$

$$=-2+i\left(2\sqrt{3}\right)$$

They we have $2\left(-\bar{i}+\sqrt{3}\bar{j}\right)$ is vector form.

(or)

From diagram $\overline{CD}=\overline{AF}$ when AB = AF = 4, $\underline{|FAB}=120^0$

$$F=\left(4\cos 120^0, 4\sin 120^0\right)=\left(-2, 2\sqrt{3}\right)$$

Now

$\Rightarrow \quad \overline{AF}=-2\bar{i}+2\sqrt{3}\,\bar{j} \qquad \therefore \overline{CD}=-2\bar{i}+2\sqrt{3}\,\bar{j}$

46. The point of inter section of the lines

$l_1=\bar{r}(t)=\left(\bar{i}-6\bar{j}+2\bar{k}\right)+t\left(\bar{i}+2\bar{j}+\bar{k}\right)$

$l_2=\bar{R}(u)=\left(4\bar{j}+\bar{k}\right)+u\left(2\bar{i}+\bar{j}+2\bar{k}\right)$ is $[E\ 2016]$

Key :c

a)(4,4,5) b)(6,4,7) c)(8,8,9) d)(10,12,11)

SOL:- Equating the coefficients we have

1+t = 2u $\Rightarrow$ 2u-t-1 = 0 ----(1)

-6+2t = 4+u $\Rightarrow$ u-2t+10 = 0 $\Rightarrow$ 2u-4t+20 = 0 ---(2)

Now (1) - (2) we have

3t = 21 $\Rightarrow$ t = 7

$\therefore$ Point of inter section is (8,8,9)

47. For three vectors $\bar{p},\bar{q}$ and $\bar{r}$, if $\bar{r}=3\bar{p}+4\bar{q}$ and $2\bar{r}=\bar{p}+3\bar{q}$ then

Key :b

a) $|\bar{r}|<2|\bar{q}|$ and $\bar{r},\bar{q}$ have the same direction

b) $|\bar{r}|>2|\bar{q}|$ and $\bar{r},\bar{q}$ have the opposite direction

c) $|\bar{r}|<2|\bar{q}|$ and $\bar{r},\bar{q}$ have the opposite direction

d) $|\bar{r}|>2|\bar{q}|$ and $\bar{r},\bar{q}$ have the same direction

SOL:- $\bar{r}=3\bar{p}+4\bar{q}$ and $2\bar{r}=\bar{p}-3\bar{q}$

$$\Rightarrow 6\bar{r}=3\bar{p}-9\bar{q}$$

On subtraction

$$5\bar{r}=-13\bar{q} \text{ and} \Rightarrow \bar{r}=\frac{-13}{5}\bar{q}$$

$$\Rightarrow |\bar{r}|=\frac{13}{5}|\bar{q}|$$

$|\bar{r}|>2|\bar{q}|$ and $\bar{r},\bar{q}$ have opposite direction.

48.Let $P_1,P_2,....,P_{15}$ be 15 points on a circle.The number of distinct triangles formed by points P_i,P_j *and* P_k such that $\hat{i}+\hat{j}+\hat{k}\neq 15$ is (2021,01sept,S-II)

a) 12 b)419 c)443 d)455

Sol: $\hat{i}+\hat{j}+\hat{k}=15$, *Where* $\hat{i}=1, \hat{j}+\hat{k}=14$

$\Rightarrow\left(\hat{j}=2,\hat{k}=12\right),\left(\hat{j}=3,\hat{k}=11\right)$

$\left(\hat{j}=4,\hat{k}=10\right),\left(\hat{j}=5,\hat{k}=9\right),\left(\hat{j}=6,\hat{k}=8\right)$.....5 *ways*

$Where\ \hat{i}=3, \hat{j}+\hat{k}=12$

$\Rightarrow\left(\hat{j}=4,\hat{k}=8\right),\left(\hat{j}=5,\hat{k}=7\right)....2\,ways$

$Where\ \hat{i}=4, \hat{j}+\hat{k}=11$ Key:c

$\Rightarrow\left(\hat{j}=5,\hat{k}=6\right)....1\,way$

$\therefore Total=12\,ways$

Then, number of possible triangles using vertices $P_i, P_j\ and\ P_k$ such that $\hat{i}+\hat{j}+\hat{k}\neq 15$ is $^{15}C_3-12=443$.

2. SCALAR PRODUCT

49. If the position vectors of A,B,C are respectively

$$2\bar{i}-\bar{j}+\bar{k},\ \bar{i}-3\bar{j}-5\bar{k}\ and\ 3\bar{i}-4\bar{j}-4\bar{k} \text{ then } \underline{|ACB}=\_\_\_\_\_$$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{3}$

Key: 2

SOL: Given $\overline{OA}=2\bar{i}-\bar{j}+\bar{k}, \overline{OB}=\bar{i}-3\bar{j}-5\bar{k}\ and\ \overline{OC}=3\bar{i}-4\bar{j}-4\bar{k}$

$$Cos\underline{|ACB}=\cos\left(\overline{CA}.\overline{CB}\right)=\frac{\overline{CA}.\overline{CB}}{\left|\overline{CA}\right|\left|\overline{CB}\right|}$$

Where $\overline{CA}=\overline{OA}-\overline{OC}=-\bar{i}+3\bar{j}+5\bar{k}$

$\overline{CB}=\overline{OB}-\overline{OC}=-2\bar{i}+\bar{j}-\bar{k}$

Here $\overline{CA}.\overline{CB}=2+3-5=0$

$\Rightarrow\overline{CA}\perp\overline{CB}$

$\Rightarrow\underline{|ACB}=90^0=\frac{\pi}{2}$

50. If $\bar{a}\ and\ \bar{b}$ are two vectors of lengths 2,1 respectively and $\left|\bar{a}-\bar{b}\right|=\sqrt{3}$ then $\left(\bar{a},\bar{b}\right)=\_\_\_\_\_\_\_\_$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

KEY:3

SOL: $\left|\bar{a}-\bar{b}\right|=\sqrt{3}\Rightarrow\left|\bar{a}-\bar{b}\right|^2=3$

$\Rightarrow\left|\bar{a}\right|^2+\left|\bar{b}\right|^2-2\left(\bar{a}.\bar{b}\right)=3$

$\Rightarrow\left|\bar{a}\right|^2+\left|\bar{b}\right|^2-2\left|\bar{a}\right|\left|\bar{b}\right|\cos\left(\bar{a},\bar{b}\right)=3$

$$\Rightarrow 4+1-4\cos(\bar{a},\bar{b})=3$$

$$\Rightarrow \cos(\bar{a},\bar{b})=\frac{1}{2}\Rightarrow(\bar{a},\bar{b})=\frac{\pi}{3}$$

51. If $\bar{a},\bar{b},\bar{c}$ are mutually perpendicular vectors equal magnitude, then

$$(\bar{a}+\bar{b}+\bar{c},\bar{a})= \_\_\_\_\_\_\_\_$$

1) $\cos^{-1}\frac{1}{\sqrt{2}}$ 2) $\cos^{-1}\frac{1}{\sqrt{3}}$ 3) $\cos^{-1}\frac{1}{3}$ 4) $\cos^{-1}\frac{1}{2}$

KEY: 2

SOL: $\bar{a},\bar{b},\bar{c}$ are mutually perpendicular vector of equal magnitude

$$\Rightarrow |\bar{a}|=|\bar{b}|=|\bar{c}|\ \&\ \bar{a}.\bar{b}=\bar{b}.\bar{c}=\bar{c}.\bar{a}=0\ \&\ |\bar{a}+\bar{b}+\bar{c}|=\sqrt{3}|\bar{a}|$$

$$\therefore \cos(\bar{a}+\bar{b}+\bar{c},\bar{a})=\frac{(\bar{a}+\bar{b}+\bar{c}).\bar{a}}{|\bar{a}+\bar{b}+\bar{c}||\bar{a}|}=\frac{|\bar{a}|^2}{\sqrt{3}|\bar{a}||\bar{a}|}=\frac{1}{\sqrt{3}}$$

$$\therefore (\bar{a}+\bar{b}+\bar{c},\bar{a})=\cos^{-1}\frac{1}{\sqrt{3}}$$

52. If $\bar{a},\bar{b}$ *and* $\bar{c}$ are vectors such that $\bar{a}+\bar{b}+\bar{c}=\bar{o}$ *and* $|\bar{a}|=7,|\bar{b}|=5$ *and* $|\bar{c}|=3$ then the angle between $\bar{b}$ and $\bar{c}$ is ____________

1) 60^0 2) 30^0 3) 45^0 4) 90^0

KEY: 1

SOL: $\bar{a}+\bar{b}+\bar{c}=\bar{o}\Rightarrow\bar{b}+\bar{c}=-\bar{a}$

$$\Rightarrow (\bar{b}+\bar{c})^2=|\bar{a}|^2$$

$$\Rightarrow |\bar{b}|^2+|\bar{c}|^2+2|\bar{b}||\bar{c}|\cos(\bar{b},\bar{c})=|\bar{a}|^2$$

$$\Rightarrow 25+9+2(5)(3)\cos(\bar{b},\bar{c})=49$$

$$\Rightarrow \cos(\bar{b},\bar{c})=\frac{1}{2}\Rightarrow(\bar{b},\bar{c})=60^0$$

53. If $\bar{a},\bar{b},\bar{c}$ are three vectors such that each is inclined at an angle $\frac{\pi}{3}$ with the other two and $|\bar{a}|=1,|\bar{b}|=2,|\bar{c}|=3$ then the scalar product of the vectors

$$2\bar{a}+3\bar{b}-5\bar{c}\ \textit{and}\ 4\bar{a}-6\bar{b}+10\bar{c}\ \text{is}\_\_\_\_\_$$

1) 188 2)-334 3)-522 4)-514

KEY: 2

SOL: Given $|\bar{a}|=1,|\bar{b}|=2,|\bar{c}|=3$ *and* $(\bar{a},\bar{b})=(\bar{b},\bar{c})=(\bar{c},\bar{a})=\frac{\pi}{3}$

$\therefore \bar{a}.\bar{b} = 1, \bar{b}.\bar{c} = 3, \bar{c}.\bar{a} = \frac{3}{2}$

$= 8|\bar{a}|^2 - 18|\bar{b}|^2 - 50|\bar{c}|^2 + 60\bar{b}.\bar{c}$

$= 8 - 72 - 450 + 180 = -334$

54. If $|\bar{a}| + |\bar{b}| = |\bar{c}|$ and $\bar{a} + \bar{b} = \bar{c}$ then the angle between $\bar{a}$ and $\bar{b}$ is________

1)0 2)$\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

KEY:-1

SOL: $\left(\bar{a} + \bar{b}\right)^2 = |\bar{c}|^2$

$\Rightarrow |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}.\bar{b} = |\bar{c}|^2$

$\Rightarrow |\bar{a}|^2 + |\bar{b}|^2 + 2|\bar{a}||\bar{b}|\cos\theta = \left(|\bar{a}| + |\bar{b}|\right)^2 = |\bar{a}|^2 + |\bar{b}|^2 + 2|\bar{a}||\bar{b}|$

$\Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$

55. If $\bar{a} = 2\bar{m} + \bar{n}, \bar{b} = \bar{m} - 2\bar{n}$, angle between the unit vectors $\bar{m}$ and $\bar{n}$ is 60^0, a,b are the sides of a parallelogram, then the lengths of the diagonals are___________

1) $\sqrt{7}, \sqrt{5}$ 2) $\sqrt{13}, \sqrt{5}$ 3) $\sqrt{7}, \sqrt{13}$ 4) $\sqrt{11}, \sqrt{13}$

KEY:- 3

SOL:- Given $|\bar{m}| = |\bar{n}| = 1, (\bar{m}, \bar{n}) = 60^0$

Lengths of diagonals are $|\bar{a} + \bar{b}|, |\bar{a} - \bar{b}|$

$\Rightarrow |3\bar{m} - \bar{n}|, |\bar{m} + 3\bar{n}|$

$\Rightarrow \sqrt{9|\bar{m}|^2 + |\bar{n}|^2 - 6\bar{m}.\bar{n}}, \sqrt{|\bar{m}|^2 + 9|\bar{n}|^2 + 6\bar{m}.\bar{n}}$

Here $\bar{m}.\bar{n} = \cos\theta = \cos 60^0 = \frac{1}{2}$

Lengths of diagonals are $\sqrt{9 + 1 - 6\left(\frac{1}{2}\right)}, \sqrt{1 + 9 + 6\left(\frac{1}{2}\right)} \Rightarrow \sqrt{7}, \sqrt{13}$

56. Angle between $\bar{a}$ & $\bar{b}$ is 120^0. If $|\bar{b}| = 2|\bar{a}|$, and the vectors $\bar{a} + x\bar{b}, \bar{a} - \bar{b}$ are right angles, then x is equal to__________

1) $\frac{1}{3}$ 2) $\frac{1}{5}$ 3) $\frac{2}{5}$ 4) $\frac{2}{3}$

KEY:- 3

SOL:-$\left(\bar{a}+x\bar{b}\right).\left(\bar{a}-\bar{b}\right)=0$

$\Rightarrow |\bar{a}|^2 - x|\bar{b}|^2 + (x-1)\bar{a}.\bar{b} = 0$

$\Rightarrow |\bar{a}|^2 - x(2|\bar{a}|)^2 + (x-1)|\bar{a}||\bar{b}|\cos 120^0 = 0$

$\Rightarrow |\bar{a}|^2 - 4x|\bar{a}|^2 + (x-1)|\bar{a}|.2|\bar{a}|.\left(-\frac{1}{2}\right) = 0$

$\Rightarrow |\bar{a}|^2(1-4x-x+1) = 0$

$\Rightarrow 5x = 2 \Rightarrow x = \frac{2}{5}$

57. If the angle θ between the vectors $\bar{a} = 2x^2\bar{i} + 4x\bar{j} + \bar{k}$ *and* $\bar{b} = 7\bar{i} - 2\bar{j} + x\bar{k}$ is such that $90^0 < \theta < 180^0$ then x lies in the interval

1) $\left(0, \frac{1}{2}\right)$ 2) $\left(\frac{1}{2}, 1\right)$ 3) $\left(1, \frac{3}{2}\right)$ 4) $\left(\frac{1}{2}, \frac{3}{2}\right)$

KEY:-1

SOL:- $90^0 < \theta < 180^0 \Rightarrow \cos\theta < 0$

$\Rightarrow \bar{a}.\bar{b} < 0$

$\Rightarrow 14x^2 - 8x + x < 0$ $\Rightarrow 14x^2 - 7x < 0$

$\Rightarrow 7x(2x-1) < 0 \Rightarrow 0 < x < \frac{1}{2}$

$\therefore x \in \left(0, \frac{1}{2}\right)$

58. If the vectors $\bar{i} - 2x\bar{j} - 3y\bar{k}$ *and* $\bar{i} + 3x\bar{j} - 2y\bar{k}$ are orthogonal to each other, then the locus of the point (x, y) is_____

1)Circle 2) Ellipse 3) Hyperbola 4)Pairs of lines

KEY:- 3

SOL:- $\bar{a}, \bar{b}$ are orthogonal $\Rightarrow \bar{a}.\bar{b} = 0$ $\Rightarrow 1 - 6x^2 + 6y^2 = 0$

$\Rightarrow \frac{x^2}{1/6} - \frac{y^2}{1/6} = 1 \Rightarrow$Hyperbola

59. Let $\bar{a}=2\bar{i}+\lambda\bar{j}+3\bar{k},\bar{b}=4\bar{i}+(3-\lambda_2)\bar{j}+6\bar{k}$ and $\bar{c}=3\bar{i}+6\bar{j}+(\lambda_3-1)\bar{k}$ be 3 vectors such that $\bar{b}=2\bar{a}$ and $\bar{a}$ is perpendicular to $\bar{c}$. Then a possible value of $(\lambda_1,\lambda_2,\lambda_3)$ is______

(2019 jan-S1)

1)(1,3,1) 2) (1,5,1) 3) $(-\frac{1}{2},4,0)$ 4) $(\frac{1}{2},4,-2)$

KEY:-3

SOL:- $\bar{b}=2\bar{a}\Rightarrow 3-\lambda_2=2\lambda_1$

$$\Rightarrow 2\lambda_1+\lambda_2=3............(1)$$

$$\Rightarrow \lambda_2=3-2\lambda_1$$

$\bar{a}\perp\bar{c}\Rightarrow\bar{a}.\bar{c}=0\Rightarrow 6+6\lambda_1+3(\lambda_3-1)=0$

$$\Rightarrow 2\lambda_1+\lambda_3=-1..........(2)$$

$$\Rightarrow \lambda_3=-1-2\lambda_1$$

$$\therefore(\lambda_1,\lambda_2,\lambda_3)=(\lambda_1,3-2\lambda_1,-1-2\lambda_1)$$

$$If\ \lambda=\frac{-1}{2}\Rightarrow(\lambda_1,\lambda_2,\lambda_3)=\left(\frac{-1}{2},4,0\right)$$

60. A vectors $\bar{a}=\alpha\bar{i}+2\bar{j}+\beta\bar{k}\,(\alpha,\beta\in R)$ lies in the plane of the vectors, $\bar{b}=\bar{i}+\bar{j}$ *and* $\bar{c}=\bar{i}-\bar{j}+4\bar{k}$. If $\bar{a}$ bisects the angle between $\bar{b}$ and $\bar{c}$ then **(Jan2020-S1)**

1) $\bar{a}.\bar{i}+3=0$ 2) $\bar{a}.\bar{k}+2=0$ 3) $\bar{a}.\bar{i}+1=0$ 4) $\bar{a}.\bar{k}+4=0$

KEY:-4

SOL:- Vectors bisects the angle between $\bar{b}$ *and* $\bar{c}$ is

$$\bar{a}=\lambda\left(\hat{b}+\hat{c}\right)=\lambda\left(\frac{i+j}{\sqrt{2}}\pm\frac{\bar{i}-\bar{j}+4\bar{k}}{\sqrt{18}}\right)$$

$$=\frac{\lambda}{3\sqrt{2}}\left(3\bar{i}+3\bar{j}\pm\left(\bar{i}-\bar{j}+4\bar{k}\right)\right)$$

$$=\frac{\lambda}{3\sqrt{2}}\left(4\bar{i}+2\bar{j}+4\bar{k}\right)(or)\frac{\lambda}{3\sqrt{2}}\left(2\bar{i}+4\bar{j}-4\bar{k}\right)$$

But $\bar{a}=\alpha\bar{i}+2\bar{j}+\beta\bar{k}$

$$\Rightarrow\frac{4\lambda}{3\sqrt{2}}=2\Rightarrow\lambda=\frac{3\sqrt{2}}{2}$$

$$\alpha=\frac{4\lambda}{3\sqrt{2}}=\frac{6\sqrt{2}}{3\sqrt{2}}=2,\beta=\frac{4\lambda}{3\sqrt{2}}=2\Rightarrow\bar{a}.\bar{k}=\beta=2$$

(Or) $\frac{4\lambda}{3\sqrt{2}} = 4 \Rightarrow \lambda = 3\sqrt{2}$

$\alpha = \frac{2\lambda}{3\sqrt{2}} = 2,\ \beta = \frac{-4\lambda}{3\sqrt{2}} = -4$

$\therefore \bar{a}.\bar{k} = \beta = -4 \Rightarrow \bar{a}.\bar{k} + 4 = 0$

61. Let $A(3,0,-1)\ B(2,10,6)$ *and* $C(1,2,1)$ be the vertices of a triangle and M be the midpoint of AC, If G divides BM in the ratio 2:1 then $Cos\underline{|GOA}$ (O being origin) in equal to _________ **(April 2019_S1)**

1) $\frac{1}{\sqrt{15}}$ 2) $\frac{1}{2\sqrt{15}}$ 3) $\frac{1}{\sqrt{30}}$ 4) $\frac{1}{6\sqrt{10}}$

KEY:- 1

SOL:- G=(2,4,2)

$$\cos\underline{|GOA} = \frac{\overline{OG}.\overline{OA}}{|\overline{OG}||\overline{OA}|} = \frac{(2\bar{i}+4\bar{j}+2\bar{k}).(3\bar{i}-\bar{k})}{\sqrt{4+16+4}\ \sqrt{9+1}} = \frac{4}{\sqrt{24}\sqrt{10}} = \frac{1}{\sqrt{15}}$$

62. Let $a,b,c \in R$ such that $a^2+b^2+c^2=1$. If $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right)$ where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\bar{i}+b\bar{j}+c\bar{k}$ *and* $b\bar{i}+c\bar{j}+a\bar{k}$ is_______(**Sep2020-S2**)

1) $\frac{\pi}{2}$ 2) 0 3) $\frac{2\pi}{3}$ 4) $\frac{\pi}{9}$

KEY:- 1

SOL:- Let $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right) = K$

$$\Rightarrow a = \frac{K}{\cos\theta}, b = \frac{K}{\cos\left(\theta+\frac{2\pi}{3}\right)}, c = \frac{K}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

Here

$$ab+bc+ca = K^2\left[\frac{1}{\cos\theta.\cos\left(\theta+\frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\theta+\frac{2\pi}{3}\right).\cos\left(\theta+\frac{4\pi}{3}\right)} + \frac{1}{\cos\left(\theta+\frac{4\pi}{3}\right).\cos\theta}\right]$$

$$=K^2\left[\frac{\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\theta}{\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=K^2(0)=0$$

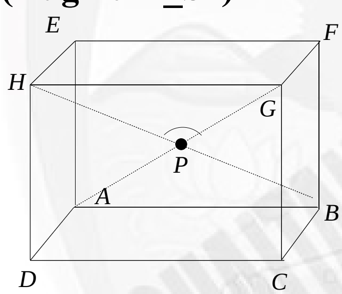
$$\left(\because \cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)=0\right)$$

$\therefore$ Angle b/w $a\bar{i}+b\bar{j}+c\bar{k}$ and $b\bar{i}+c\bar{j}+a\bar{k}$ is

$$\cos^{-1}\left(\frac{ab+bc+ca}{a^2+b^2+c^2}\right)=\cos^{-1}(0)=\frac{\pi}{2}$$

63. A hall has a square floor of dimension $10m\times 10m$ and vertical walls, If the $\lfloor\underline{GPH}$ between the diagonal AG and BH is $Cos^{-1}\frac{1}{5}$ then the height of the hall (in M) is___

(Aug 2021_S2)



1) 5 2) $2\sqrt{10}$ 3) $5\sqrt{3}$ 4) $5\sqrt{2}$

KEY:- 4

SOL:- Let $A=(0,0,0)$ $B=(10,0,0)$ $G=(10,10,h)$, $H=(0,10,h)$

$\overline{AG}=10\bar{i}+10\bar{j}+h\bar{k}$, $\overline{BH}=10\bar{i}+10\bar{j}+h\bar{k}$ $\therefore \overline{AG}.\overline{BH}=\left|\overline{AG}\right|\left|\overline{BH}\right|\cos\theta$

$$\Rightarrow -100+100+h^2=\sqrt{h^2+200}.\sqrt{h^2+200}\left(\frac{1}{5}\right)$$

$$5h^2=h^2+200\Rightarrow h^2=50\Rightarrow h=\sqrt{50}=5\sqrt{2}$$

64. Let S be the set of all $a\in R$ for which the angle between the vectors $\bar{u}=a(\log b)\bar{i}-6\bar{j}+3\bar{k}$ and $\bar{v}=(\log b)\bar{i}+2\bar{j}+2a(\log b)\bar{k}$ $(b>1)$ is acute then S is equal to___________**(July 2020_S2)**

1) $\left(-\infty,\frac{-4}{3}\right)$ 2) ϕ 3) $\left(\frac{-4}{3},0\right)$ 4) $\left(\frac{12}{7},\infty\right)$

KEY:- 2

SOL:- $\bar{u}.\bar{v}>0\Rightarrow a(\log b)^2-12+6a(\log b)>0$

$$\Rightarrow a(\log b)^2 + 6a(\log b) - 12 > 0$$
$$b > 1 \Rightarrow \log b > 0$$
$$Let\ \log b = \alpha$$
$$\therefore a\alpha^2 + 6a\alpha - 12 > 0$$
$$a\left(\alpha^2 + 6\alpha\right) > 12 \Rightarrow a \in \phi$$

65. Let $\bar{u}$ be vector coplanar with the vectors $\bar{a} = 2\bar{i} + 3\bar{j} - \bar{k}\ and\ \bar{b} = \bar{j} + \bar{k}$. If $\bar{u}$ is perpendicular to $\bar{a}$ and $\bar{u}.\bar{b} = 24$ then $|\bar{u}|^2$ is equal to__________(mains2018)

1)336 2)315 3)256 4)84

KEY:- 1

SOL:- Let $\bar{u} = \lambda\bar{a} + \mu\bar{b}$

$\bar{u}.\bar{a} = \lambda(\bar{a}.\bar{a}) + \mu(\bar{b}.\bar{a})$

$O = \lambda(14) + \mu(2) \Rightarrow \mu + 7\lambda = 0.....(1)$

$\bar{a}.\bar{b} = \lambda(\bar{a}.\bar{b}) + \mu(\bar{b}.\bar{b})$

$24 = \lambda(2) + \mu(2) \Rightarrow \lambda + \mu = 12$ ....(2)

Solving $(1) \& (2) \Rightarrow \lambda = -2, \mu = 14$

$\bar{u}.\bar{u} = \lambda(\bar{u}.\bar{a}) + \mu(\bar{a}.\bar{b})$ $\Rightarrow |\bar{u}|^2 = \lambda(0) + 14(24)$ $\Rightarrow |\bar{u}|^2 = 336$

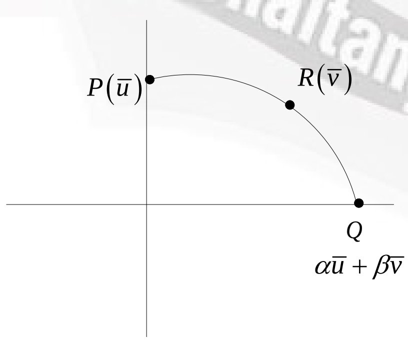
66. An arc $\widehat{PQ}$ of a circle subtends a right angle of its centre O. The midpoint of the arc $\widehat{PQ}$ is R. If $\overline{OP} = \bar{u}, \overline{OR} = \bar{v}$ and $\overline{OQ} = \alpha\bar{u} + \beta\bar{v}$ then α, β^2 are the roots of the equation (April2023_S1)

1) $x^2 - x - 2 = 0$ 2) $3x^2 + 2x - 1 = 0$

3) $x^2 + x - 2 = 0$ 4) $3x^2 - 2x - 1 = 0$

KEY:-1

SOL:-



$|\bar{u}| = |\bar{v}| = |\alpha\bar{u} + \beta\bar{v}|$ $\bar{u}(\alpha\bar{u} + \beta\bar{v}) = 0$

$\Rightarrow \alpha|\bar{u}|^2 + \beta|\bar{u}||\bar{v}|\cos 45^0 = 0$ $\Rightarrow \alpha|\bar{u}|^2 + \beta|\bar{u}||\bar{u}|\frac{1}{\sqrt{2}} = 0$

$\Rightarrow \alpha + \frac{\beta}{\sqrt{2}} = 0 \Rightarrow \alpha = \frac{-\beta}{\sqrt{2}}......(1)$ $r = |\alpha\bar{u} + \beta\bar{v}| = |\bar{u}|$

$\Rightarrow \alpha^2|\bar{u}|^2 + \beta^2|\bar{v}|^2 + 2\alpha\beta|\bar{u}||\bar{v}|\cos 45^0 = |\bar{u}|^2 \Rightarrow \alpha^2 + \beta^2 + \sqrt{2}\alpha\beta = 1\left(\because |\bar{u}| = |\bar{v}|\right).....(2)$

Solving $(1)\&(2) \Rightarrow \alpha = -1, \beta^2 = 2$ $\left(\alpha, \beta^2\right) = (-1, 2)$

Quadratic equation having α, β^2 as roots is $x^2 - (-1+2)x + (-1)(2) = 0$

$\Rightarrow x^2 - x - 2 = 0$

b) <u>Geometrical Interpretation of Scalar Product</u>:

67. If the vector OP in xy plane whose magnitude is $\sqrt{3}$ makes an angle 60^0 with y axis, the length of the component of the vector in direction of x axis is_____

1)1 2) $\sqrt{3}$ 3) $\frac{1}{2}$ 4) $\frac{3}{2}$

KEY:- 4

SOL:- Require length of component of vector in the direction of x axis $= \sqrt{3}\cos\left(90^0 - 60^0\right)$

$= \sqrt{3}\cos 30^0 = \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = \frac{3}{2}$

68. The projection of the vector $\bar{a} = 4\bar{i} - 3\bar{j} + 2\bar{k}$ on the vector making equal angles (a cute) with coordinate axes and having magnitude $\sqrt{3}$ is_________________

1) 3 2) $\sqrt{3}$ 3) $2\sqrt{3}$ 4)1

KEY:- 2

SOL:- Req. projection of $\bar{a}$ on $\bar{b} = \frac{\bar{a}.\bar{b}}{|\bar{b}|}$

Where $\bar{a} = 4\bar{i} - 3\bar{j} + 2\bar{k}, \bar{b} = \sqrt{3}\frac{\left(\bar{i} + \bar{j} + \bar{k}\right)}{\sqrt{3}} = \bar{i} + \bar{j} + \bar{k}$

$= \frac{4-3+2}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

69. $\bar{a}, \bar{b}, \bar{c}$ are 3 vectors such that $|\bar{a}|=1, |\bar{b}|=2, |\bar{c}|=3$ *and* $\bar{b}, \bar{c}$ are perpendicular to each other . If the projection of $\bar{b}$ along $\bar{a}$ is same as that of $\bar{c}$ along $\bar{a}$ then $|\bar{a}-\bar{b}+\bar{c}|=$ ________

1) $\sqrt{2}$ 2) $\sqrt{7}$ 3) $\sqrt{14}$ 4) 14

KEY:- 3

SOL:- Projection of $\bar{b}$ along $\bar{a}$ =projection of $\bar{c}$ along $\bar{a}$

$$\Rightarrow \frac{\bar{b}.\bar{a}}{|\bar{a}|} = \frac{\bar{c}.\bar{a}}{|\bar{a}|} \Rightarrow \bar{a}.\bar{b} = \bar{c}.\bar{a}$$

Given $\bar{b}, \bar{c}$ are perpendicular $\Rightarrow \bar{b}.\bar{c} = 0$

$$|\bar{a}-\bar{b}+\bar{c}|^2 = |a|^2 + |b|^2 + |c|^2 - 2\bar{a}.\bar{b} + 2\bar{a}.\bar{c} - 2\bar{b}.\bar{c}$$

$$= 1+4+9-2\bar{a}.\bar{b}+2\bar{a}.\bar{b}-2(0)=14$$

$$|\bar{a}-\bar{b}+\bar{c}| = \sqrt{14}$$

70. If $\bar{b} = 4\bar{i} + 3\bar{j}$ *and* $\bar{c}$ are two vectors perpendicular to each other in the xy plane the vector in the same plane having components 1,2 along $\bar{b}$ *and* $\bar{c}$ respectively is __

1) $\frac{-2\bar{i}+11\bar{j}}{5}$ 2) $\frac{2\bar{i}+11\bar{j}}{5}$ 3) $\frac{-2\bar{i}-11\bar{j}}{5}$ 4) $\frac{2\bar{i}-11\bar{j}}{5}$

KEY:- 1

SOL:- Let $\bar{d} = x\bar{i} + y\bar{j}$ *and* $\bar{c} = -3\bar{i} + 4\bar{j}$

Given $\frac{\bar{d}.\bar{b}}{|\bar{b}|} = 1 \Rightarrow \frac{4x+3y}{5} = 1 \Rightarrow 4x+3y=5$

$$\frac{\bar{d}.\bar{c}}{|\bar{c}|} = 2 \Rightarrow \frac{-3x+4y}{5} = 2 \Rightarrow -3x+4y=10$$

Solving the equations $\Rightarrow x = \frac{-2}{5}, y = \frac{11}{5}$

$$\therefore \bar{d} = \frac{-2}{5}\bar{i} + \frac{11}{5}\bar{j}$$

71. If $\bar{a} = 3\bar{i} + \bar{j} + 2\bar{k}$ *and* $\bar{b} = \bar{i} + 2\bar{j} + 3\bar{k}$ then a unit vector in the direction of the resultant of orthogonal projection of $\bar{b}$ *on* $\bar{a}$ and the projection of $\bar{b}$ on a line perpendicular to $\bar{a}$ is___________

1) $\frac{\bar{i}+2\bar{j}+3\bar{k}}{\sqrt{14}}$ 2) $\frac{2\bar{i}+\bar{j}+3\bar{k}}{\sqrt{14}}$ 3) $\frac{3\bar{i}+\bar{j}+2\bar{k}}{\sqrt{14}}$ 4) $\frac{\bar{i}+3\bar{j}+2\bar{k}}{\sqrt{14}}$

KEY:-1

SOL:- Req vector $= \hat{b} = \frac{\bar{b}}{|\bar{b}|} = \frac{\bar{i} + 2\bar{j} + 3\bar{k}}{\sqrt{14}}$

72. Let $\bar{a} = \bar{i} - \bar{j} + 3\bar{k}$ *and* $\bar{b} = 3\bar{i} - 5\bar{j} + 6\bar{k}$ then the magnitude of the projection of $2\bar{a} - \bar{b}$ on $\bar{a} + \bar{b}$ is________

1) $\frac{22}{\sqrt{133}}$ 2) $\frac{11\sqrt{2}}{\sqrt{10}}$ 3) $\frac{22}{\sqrt{10}}$ 4) $\frac{22}{\sqrt{5}}$

KEY:- 1

SOL:- Here $2\bar{a} - \bar{b} = 2\bar{i} - 2\bar{j} + 6\bar{k} - 3\bar{i} + 5\bar{j} - 6\bar{k} = -\bar{i} + 3\bar{j}$

$$\bar{a} + \bar{b} = 4\bar{i} - 6\bar{j} + 9\bar{k}$$

Proj. of $2\bar{a} - \bar{b}$ on $\bar{a} + \bar{b} = \left|\frac{(2\bar{a} - \bar{b}).(\bar{a} + \bar{b})}{|\bar{a} + \bar{b}|}\right|$

$$= \left|\frac{(-\bar{i} + 3\bar{j}).(4\bar{i} - 6\bar{j} + 9\bar{k})}{|4\bar{i} - 6\bar{j} + 9\bar{k}|}\right| = \left|\frac{-4 - 18}{\sqrt{16 + 36 + 81}}\right| = \frac{22}{\sqrt{133}}$$

73. Let $\bar{a} = 5\bar{i} - \bar{j} - 3\bar{k}$ *and* $\bar{b} = \bar{i} + 3\bar{j} + 5\bar{k}$ be two vectors then which of following statements is true. (FEB 2023_S2)

1) Projection of $\bar{a}$ *on* $\bar{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\bar{b}$

2) Projection of $\bar{a}$ *on* $\bar{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\bar{b}$

3) Projection of $\bar{a}$ *on* $\bar{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of projection vector is opposite to the direction of $\bar{b}$

4) Projection of $\bar{a}$ *on* $\bar{b}$ is $\frac{-13}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\bar{b}$

KEY:- 4

SOL:- Projection of $\bar{a}$ *on* $\bar{b} = \frac{\bar{a}.\bar{b}}{|\bar{b}|} = \frac{5 - 3 - 15}{\sqrt{1 + 9 + 25}} = \frac{-13}{\sqrt{35}}$

74. Let $\overline{a} = \overline{i} + \overline{j} + 2\overline{k}, \overline{b} = 2\overline{i} - 3\overline{j} + \overline{k}$ *and* $\overline{c} = \overline{i} - \overline{j} + \overline{k}$ be 3 given vector, Let $\overline{v}$ be a vector in the plane of $\overline{a}$ *and* $\overline{b}$ whose projection on $\overline{c}$ is $\frac{2}{\sqrt{3}}$ If $\overline{v} : J = 7$ *then* $\overline{v}\left(\overline{i} + \overline{k}\right) =$ _____ **(Jun 2022-S2)**

1) 6 2) 4 3)8 4) 19

KEY:-4

SOL:- $\overline{v} = \lambda\overline{a} + \mu\overline{b}$

$= (\lambda + 2\mu)\overline{i} + (\lambda - 3\mu)\overline{j} + (2\lambda + \mu)\overline{k}$

Given $\overline{v}.\overline{j} = 7; \frac{\overline{v}.\overline{c}}{|\overline{c}|} = \frac{2}{\sqrt{3}}$

$\Rightarrow \lambda - 3\mu = 7, \frac{\lambda + 2\mu - \lambda + 3\mu + 2\lambda + \mu}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\Rightarrow \lambda - 3\mu = 7 - (1), 2\lambda + 6\mu = 2 \Rightarrow \lambda + 3\mu = 1 - (2)$

Solving $(1) \& (2) \Rightarrow \mu = -1, \lambda = 4$

$\therefore \overline{v} = 2\overline{i} + 7\overline{j} + 7\overline{k}$

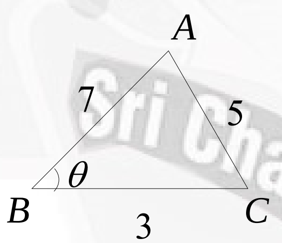
$\overline{v}.\left(\overline{i} + \overline{k}\right) = 2 + 7 = 9$

75. In ΔABC, if $\left|\overline{BC}\right| = 3, \left|\overline{CA}\right| = 5$ *and* $\left|\overline{BA}\right| = 7$ then the projection of the vector $\overline{BA}$ on $\overline{BC}$ is equal to___________(July 2021-S2)

1) $\frac{19}{2}$ 2) $\frac{13}{2}$ 3) $\frac{11}{2}$ 4) $\frac{15}{2}$

KEY:- 3

SOL:-



$\text{Cos}\underline{|ABC} = \frac{7^2 + 3^2 - 5^2}{2.7.3} = \frac{11}{14}$

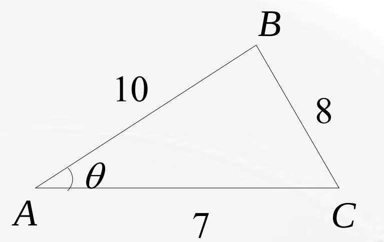
Projection of $\overline{BA}$ on $\overline{BC} = \left|\overline{BA}\right|.\cos\underline{|ABC} = 7 \times \frac{11}{14} = \frac{11}{2}$

76. In ΔABC, if $\left|\overline{BC}\right| = 8, \left|\overline{CA}\right| = 7, \left|\overline{AB}\right| = 10$ then the projection of the vector $\overline{AB}$ *and* $\overline{AC}$ is equal to _______________(March2021-S2)

1) $\frac{25}{4}$ 2) $\frac{85}{14}$ 3) $\frac{127}{20}$ 4) $\frac{115}{16}$

KEY:- 2

SOL:-



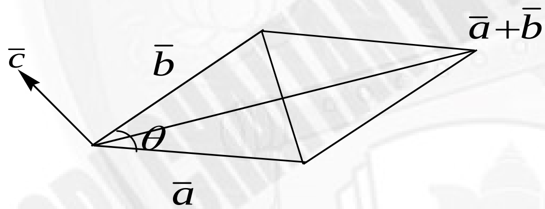
$\text{Cos}\underline{|BAC} = \frac{10^2 + 7^2 - 8^2}{2.10.7} = \frac{85}{140}$

$\therefore$ projection of $\overline{AB}$ on $\overline{AC} = \left|\overline{AB}\right|\cos\theta = 10 \times \frac{85}{140} = \frac{85}{14}$

77. Let $\overline{a} = \overline{i} + \overline{j} + \sqrt{2}\overline{k}, \overline{b} = b_1\overline{i} + b_2\overline{j} + \sqrt{2}\overline{k}$ and $\overline{c} = 5\overline{i} + \overline{j} + \sqrt{2}\overline{k}$ be 3 vectors such that the projection vector of $\overline{b}$ on $\overline{a}$ is $\overline{a}$. If $\overline{a} + \overline{b}$ is perpendicular to $\overline{c}$ then $\left|\overline{b}\right|$ is equal to___________(Jan 2019-S2)

1) 6 2)4 3) $\sqrt{22}$ 4) $\sqrt{32}$

Key: (1) SOL:-



Projection of $\overline{b}$ on $\overline{a} = \frac{\overline{b}.\overline{a}}{|\overline{a}|}$ $= \frac{b_1 + b_2 + 2}{2}$

Given Projection of $\overline{b}$ on $\overline{a} = |\overline{a}|$

$\Rightarrow \frac{b_1 + b_2 + 2}{2} = 2 \quad \Rightarrow b_1 + b_2 = 2 \;.....(1)$

$\overline{a} + \overline{b}$ is $\perp$ to $\overline{c} \Rightarrow \left(\overline{a} + \overline{b}\right).\overline{c} = 0$

$\Rightarrow 5(1 + b_1) + 1(1 + b_2) + \sqrt{2}\left(2\sqrt{2}\right) = 0$

$\Rightarrow 5b_1 + b_2 = -10 \;.....(2)$

Solving $(1)\,\&\,(2) \Rightarrow b_1 = -3, b_2 = 5$

$\therefore |\overline{b}| = \sqrt{b_1^2 + b_2^2 + 2} = 6$

78. If the vectors $\bar{a}=\lambda\bar{i}+\mu\bar{j}+4\bar{k}, \bar{b}=2\bar{i}+4\bar{j}-2\bar{k}$ *and* $\bar{c}=2\bar{i}+3\bar{j}+\bar{k}$ are coplanar and the projection of $\bar{a}$ on the vector $\bar{b}$ is $\sqrt{54}$ units, then sum of all possible values of $\lambda+\mu$ is_________(Jan 2023-S1)

1)0 2) 6 3) 24 4)18

KEY:- 3

SOL:- $\begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix}=0 \Rightarrow 5\lambda-\mu=28.....(1)$

$\frac{\bar{a}.\bar{b}}{|\bar{b}|}=\sqrt{54} \Rightarrow \lambda-2\mu=22.....(2)$

Solving $(1)\&(2) \Rightarrow \lambda+\mu=24$

C) <u>Properties of dot product of vectors</u>:

79. If $\bar{a}$ is collinear with $\bar{b}=3\bar{i}+6\bar{j}+6\bar{k}$ and $\bar{a}.\bar{b}=27$ then $\bar{a}=$______

1) $3(\bar{i}+\bar{j}+\bar{k})$ 2) $\bar{i}+3\bar{j}+3\bar{k}$ 3) $\bar{i}+2\bar{j}+2\bar{k}$ 4) $2\bar{i}+2\bar{j}+2\bar{k}$

KEY:- 3

SOL:- Let $\bar{a}=\lambda\bar{b} \Rightarrow \bar{a}.\bar{b}=\lambda(\bar{b}.\bar{b}) \quad \Rightarrow \lambda=\frac{\bar{a}.\bar{b}}{|\bar{b}|^2}=\frac{27}{9+36+36}=\frac{1}{3}$

$\therefore \bar{a}=\frac{1}{3}(3\bar{i}+6\bar{j}+6\bar{k})=\bar{i}+2\bar{j}+2\bar{k}$

80. Let $\bar{a}$ and $\bar{b}$ be two unit vectors and θ be the angle between them then $|\bar{a}+\bar{b}|^2-|\bar{a}-\bar{b}|^2=$___________

1) $\cos\theta$ 2) $2\cos\theta$ 3) $3\cos\theta$ 4) $4\cos\theta$

KEY:- 4

SOL:- $\bar{a},\bar{b}$ are unit vectors $\&(\bar{a},\bar{b})=\theta$

$\therefore(\bar{a}+\bar{b})^2=4\cos^2\frac{\theta}{2}\&(\bar{a}-\bar{b})^2=4\sin^2\frac{\theta}{2}$

$$(\bar{a}+\bar{b})^2-(\bar{a}-\bar{b})^2=4\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)=4\cos\theta$$

81. If $|\bar{a}|=2,|\bar{b}|=3$ *and* $|\bar{a}-\bar{b}|=1$ then $|\bar{a}+\bar{b}|=$____________

1)5 2)4 3)2 4)1

KEY:-1

SOL:- $|\bar{a}+\bar{b}|^2+|\bar{a}-\bar{b}|^2=2\left(|\bar{a}|^2+|\bar{b}|^2\right)$

$\left|\bar{a}+\bar{b}\right|^2+1=2(4+9)\Rightarrow\left|\bar{a}+\bar{b}\right|=5$

82. If $|\bar{a}|=1,|\bar{b}|=2,|\bar{a}-\bar{b}|^2+|\bar{a}+2\bar{b}|^2=20$ then $(\bar{a},\bar{b})=$ _____

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{2\pi}{3}$

KEY:- 4

SOL:- $|\bar{a}-\bar{b}|^2+|\bar{a}+2\bar{b}|^2=20$

$\Rightarrow 2|\bar{a}|^2+5|\bar{b}|^2+2(\bar{a}.\bar{b})=20$ $\bar{a}.\bar{b}=-1$

$\Rightarrow \cos\theta=\frac{-1}{2}\Rightarrow\theta=\frac{2\pi}{3}$

83. Let $|\bar{a}|=3$ and $|\bar{b}|=4$, the value of μ for which the vectors $\bar{a}+\mu\bar{b}$ and $\bar{a}-\mu\bar{b}$ are perpendicular is___________

1) $\frac{3}{4}$ 2) $\frac{2}{3}$ 3) $\pm\frac{3}{4}$ 4) $\frac{-2}{3}$

KEY:-3

SOL:- $(\bar{a}+\mu\bar{b}).(\bar{a}-\mu\bar{b})=0$

$\Rightarrow|\bar{a}|^2=\mu^2|\bar{b}|^2\Rightarrow 9=16\mu^2\Rightarrow\mu=\pm\frac{3}{4}$

84. If $\bar{a}+\bar{b}$ is perpendicular to $\bar{b}$ and $\bar{a}+2\bar{b}$ is perpendicular to $\bar{a}$ then

1) $|\bar{a}|=|\bar{b}|$ 2) $|\bar{a}|=\sqrt{2}|\bar{b}|$` 3) $|\bar{b}|=\sqrt{2}|\bar{a}|$ 4) $|\bar{a}|=\sqrt{3}|\bar{b}|$

KEY:- 2

SOL:- $(\bar{a}+\bar{b}).\bar{b}=0$ and $(\bar{a}+2\bar{b}).\bar{a}=0$

$\Rightarrow\bar{a}.\bar{b}+|\bar{b}|^2=0$ and $|\bar{a}|^2+2\bar{b}.\bar{a}=0$

$\Rightarrow\bar{a}.\bar{b}=-|\bar{b}|^2$ and $\bar{a}.\bar{b}=\frac{-|a|^2}{2}$ $\therefore -|\bar{b}|^2=\frac{-|\bar{a}|^2}{2}\Rightarrow|\bar{a}|^2=2|\bar{b}|^2$

$\Rightarrow|\bar{a}|=\sqrt{2}|\bar{b}|$

85. If $\bar{a},\bar{b},\bar{c}$ are unit vectors then $|\bar{a}-\bar{b}|^2+|\bar{b}-\bar{c}|^2+|\bar{c}-\bar{a}|^2$ does not exceed

1) 4 2) 9 3) 8 4) 6

KEY:-2

SOL:- $|\bar{a}+\bar{b}+\bar{c}|^2=0\Rightarrow 3+2(\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a})\geq 0$ $\Rightarrow 2(\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a})\geq -3$

$\therefore \left|\bar{a}-\bar{b}\right|^2+\left|\bar{b}.\bar{c}\right|^2+\left|\bar{c}-\bar{a}\right|^2=2\left(\left|\bar{a}\right|^2+\left|\bar{b}\right|^2+\left|\bar{c}\right|^2\right)-2\left(\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}\right)\ \le 2(3)-(-3)=9$

86. If the sum of two unit vector is a unit vector, then the magnitude of their difference is___________

1) 3 2) $\sqrt{3}$ 3) $\sqrt{13}$ 4) $\sqrt{7}$ KEY:- 2

SOL:- $\left|\bar{a}+\bar{b}\right|=1, \left|\bar{a}\right|=\left|\bar{b}\right|=1$

$\left|\bar{a}+\bar{b}\right|^2+\left|\bar{a}-\bar{b}\right|^2=2\left(\left|\bar{a}\right|^2+\left|\bar{b}\right|^2\right)\Rightarrow 1+\left|\bar{a}-\bar{b}\right|^2=2(1+1)\Rightarrow\left|\bar{a}-\bar{b}\right|=\sqrt{3}$

87. If θ is acute angle and the vector $(\sin\theta)\bar{i}+(\cos\theta)\bar{j}$ is perpendicular to the vector $\bar{i}-\sqrt{3}\bar{j}$ then θ=_________

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{5}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

KEY:- 4

SOL:- $\left((\sin\theta)\bar{i}+(\cos\theta)\bar{j}\right).\left(\bar{i}-\sqrt{3}\bar{j}\right)=0\Rightarrow\sin\theta-\sqrt{3}\cos\theta=0\Rightarrow \mathrm{Tan}\,\theta=\sqrt{3}$

$\Rightarrow\theta=\frac{\pi}{3}$

88. If $\bar{a}=-\bar{i}+\bar{j}+\bar{k}$ *and* $\bar{b}=2\bar{i}+\bar{k}$, then the vector $\bar{c}$ satisfying the conditions that (i) It is coplanar with $\bar{a}$ and $\bar{b}$

(ii) It is perpendicular to $\bar{b}$

(iii) $\bar{a}.\bar{c}=7$ is_______________

1) $\frac{-3}{2}\bar{i}+\frac{5}{2}\bar{j}+3\bar{k}$ 2) $-3\bar{i}+5\bar{j}+6\bar{k}$ 3) $-6\bar{i}+\bar{k}$ 4) $-\bar{i}+2\bar{j}+2\bar{k}$

KEY:- 1

SOL:- Verify the options satisfies $\bar{c}$ such that $\bar{c}.\bar{b}=0, \bar{a}.\bar{c}=7$

$\therefore\bar{c}=\frac{-3}{2}\bar{i}+\frac{5}{2}\bar{j}+3\bar{k}$

89. If $\bar{a},\bar{c}$ are unit parallel vectors $\left|\bar{b}\right|=6$ then $\bar{b}-3\bar{c}=\lambda\bar{a}$ if λ=______

1) -9,3 2) -3,6 3) 6,3 4) -3,4

KEY:- 1

SOL:- $\bar{b}=\lambda\bar{a}+3\bar{c}$ $\left|\bar{b}\right|^2=\lambda^2\left|\bar{a}\right|^2+9\left|\bar{c}\right|^2+6\lambda\,\bar{a}.\bar{c}$

$36=\lambda^2+9+6\lambda$ $\lambda^2+6\lambda-27=0$

$(\lambda+9)(\lambda-3)=0$ $\lambda=-9\ or\ 3$

90. If $\bar{a},\bar{b}$ *and* $\bar{c}$ are 3 unit vectors inclined to each other at an angle θ, then the maximum value of θ is_____________

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{2}$ 3) $\frac{2\pi}{3}$ 4) $\frac{5\pi}{6}$

KEY:-3

SOL:- $\left|\bar{a}+\bar{b}+\bar{c}\right|^2 \geq 0 \Rightarrow \left|\bar{a}\right|^2+\left|\bar{b}\right|^2+\left|\bar{c}\right|^2+2\left(\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}\right)\geq 0$

$\Rightarrow 3+2\left(3\cos\theta\right)\geq 0$ $\Rightarrow \cos\theta \geq \frac{-1}{2} \Rightarrow \theta \leq \frac{2\pi}{3}$

91. Let $\bar{a}$ *and* $\bar{b}$ be two vectors such that $\left|2\bar{a}+3\bar{b}\right|=\left|3\bar{a}+\bar{b}\right|$ and the angle between $\bar{a}$ *and* $\bar{b}$ is 60^0. If $\frac{1}{8}\bar{a}$ is a unit vector, then $\left|\bar{b}\right|$ is equal to ______**(Aug2021-S2)**

1) 4 2) 6 3) 5 4) 8

KEY:- 3

SOL:- $\left|2\bar{a}+3\bar{b}\right|=\left|3\bar{a}+\bar{b}\right|$ $\left|2\bar{a}+3\bar{b}\right|^2=\left|3\bar{a}+\bar{b}\right|^2$

$\Rightarrow 4\left|\bar{a}\right|^2+9\left|\bar{b}\right|^2+12\bar{a}.\bar{b}=9\left|\bar{a}\right|^2+\left|\bar{b}\right|^2+6\bar{a}.\bar{b}$

$\Rightarrow 5\left|\bar{a}\right|^2-6\bar{a}.\bar{b}-8\left|\bar{b}\right|^2=0......(1)$

$\frac{-a}{8}$ is a unit vector $\Rightarrow \left|\bar{a}\right|=8$

$(1)\Rightarrow 5(64)-6.8\left|\bar{b}\right|\cos 60^0-8\left|\bar{b}\right|^2=0$

$\Rightarrow \left|\bar{b}\right|^2+3\left|\bar{b}\right|-40=0$ $\Rightarrow \left(\left|\bar{b}\right|+8\right)\left(\left|\bar{b}\right|-5\right)=0 \Rightarrow \left|\bar{b}\right|=5$

92. Let $\bar{\alpha}=4\bar{i}+3\bar{j}+5\bar{k}$ *and* $\bar{\beta}=\bar{i}+2\bar{j}-4\bar{k}$ Let $\bar{\beta}_1$ be parallel to $\bar{\alpha}$ and $\bar{\beta}_2$ be perpendicular to $\bar{\alpha}$. If $\bar{\beta}=\bar{\beta}_1+\bar{\beta}_2$ then the value of $5\bar{\beta}_2\left(\bar{i}+\bar{j}+\bar{k}\right)$ is______**(Jan 2023-S2)**

1) 6 2)11 3)7 4)9

KEY:-3

SOL:- $\bar{\beta}=\bar{\beta}_1+\bar{\beta}_2,\ \bar{\beta}_1=\lambda\bar{\alpha}$ $\Rightarrow \bar{\beta}_2=\bar{\beta}-\bar{\beta}_1$

$=\bar{i}+2\bar{j}-4\bar{k}-\lambda\left(4\bar{i}+3\bar{j}+5\bar{k}\right)$

$=(1-4\lambda)\bar{i}+(2-3\lambda)\bar{j}-(4+5\lambda)\bar{k}$

$\bar{\beta}_2\perp\bar{\alpha}\Rightarrow\bar{\beta}_2.\bar{\alpha}=0$ $\Rightarrow \lambda=\frac{-1}{5}$ $\therefore \bar{\beta}_2=\frac{9}{4}\bar{i}+\frac{13}{5}\bar{j}-3\bar{k}$

$5\bar{\beta}_2.\left(\bar{i}+\bar{j}+\bar{k}\right)=9+13-15=7$

93. Let $\bar{a}=2\bar{i}+\bar{j}+\bar{k}$ *and* $\bar{b}$ *and* $\bar{c}$ be two non zero vectors such that $\left|\bar{a}+\bar{b}+\bar{c}\right|=\left|\bar{a}+\bar{b}-\bar{c}\right|$ and $\bar{b}.\bar{c}=0$ consider the following statement

A) $\left|\bar{a}+\lambda\bar{c}\right|\geq\left|\bar{a}\right|\forall\lambda\in R$

B) $\bar{a}$ and $\bar{c}$ are always parallel (jan 2023 S-1)

1) Only (B) is correct
2) Neither (A) nor (B) is correct
3) Only (A) is correct
4) Both (A) and (B) are correct

KEY:- 3

SOL:- $\left|\bar{a}+\bar{b}+\bar{c}\right|^2=\left|\bar{a}+\bar{b}-\bar{c}\right|^2\Rightarrow\bar{a}.\bar{c}=0\;(B)$ is in correct

$\left|\bar{a}+\lambda\bar{c}\right|^2\geq\left|\bar{a}\right|^2\Rightarrow\lambda^2c^2\geq0\forall\lambda\in R$ (A) is correct

d) <u>Cauchy Schwartz inequality</u>

94. If $a+2b+3c=4$ then the least value of $a^2+b^2+c^2$ is_________

1) $\frac{2}{7}$
2) $\frac{3}{7}$
3) $\frac{5}{7}$
4) $\frac{8}{7}$

KEY:- 4

SOL:- Let $\bar{u}=a\bar{i}+b\bar{j}+c\bar{k},\bar{v}=\bar{i}+2\bar{j}+3\bar{k}$

$\therefore\bar{u}.\bar{v}=a+2b+3c=4$

$\left|\bar{u}\right|=\sqrt{a^2+b^2+c^2},\left|\bar{v}\right|=\sqrt{1+4+9}=\sqrt{14}$

$\therefore\left|\bar{u}.\bar{v}\right|\leq\left\|\bar{u}\right\|.\left\|\bar{v}\right\|\quad\Rightarrow4\leq\sqrt{a^2+b^2+c^2}.\sqrt{14}$

$\Rightarrow\sqrt{a^2+b^2+c^2}\geq\frac{4}{\sqrt{14}}\qquad\Rightarrow a^2+b^2+c^2\geq\frac{16}{14}=\frac{8}{7}$

<u>CROSS PRODUCT OF TWO VECTORS:</u>

DEFINITION OF CROSS PRODUCT OF VECTORS

95. If $\bar{a}=3\hat{i}-5\hat{j},\bar{b}=6\hat{i}+3\hat{j}$ are two vectors and $\bar{c}$ is a vector such that $\bar{c}=\bar{a}\times\bar{b}$, then $\left|\bar{a}\right|:\left|\bar{b}\right|:\left|\bar{c}\right|=$

1) $\sqrt{34}:\sqrt{45}:\sqrt{39}$
2) $\sqrt{34}:\sqrt{45}:39$
3) $34:39:45$
4) $39:35:34$

Key: 2

SOL:

$$\bar{c}=\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-5&0\\6&3&0\end{vmatrix}=(9+30)\hat{k}=39\hat{k}$$

$\therefore |\bar{a}| = \sqrt{34}\,; |\bar{b}| = \sqrt{45}\ \ |\bar{c}| = 39$

96. If α, β are the roots of the equation $x^2 + 2x + 5 = 0$ and $\bar{a} = (\alpha+\beta)\hat{i} + \alpha\beta\hat{j}, \bar{b} = \alpha\beta\hat{i} + (\alpha+\beta)\hat{j} + (\alpha^2+\beta^2)\hat{k}$ then $\bar{a} \times \bar{b} =$

1) $\hat{i} + 12\hat{j} + 12\hat{k}$
2) $-30\hat{i} + 12\hat{j} - 5\hat{k}$
3) $-30\hat{i} - 12\hat{j} - 21\hat{k}$
4) $\hat{i} - 12\hat{j} - 29\hat{k}$

Key:3

SOL: $\alpha + \beta = -2, \alpha\beta = 5, \alpha^2 + \beta^2 = -6$

$\bar{a} = -2\hat{i} + 5\hat{j},\ \bar{b} = 5\hat{i} - 2\hat{j} - 6\hat{k}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 5 & 0 \\ 5 & -2 & -6 \end{vmatrix} = -30\hat{i} - 12\hat{j} - 21\hat{k}$$

97. . If $|\bar{a}| = 2, |\bar{b}| = 4, (\bar{a}, \bar{b}) = \frac{\pi}{6}$, then $|\bar{a} \times \bar{b}|$ is

1)4
2) $\sqrt{3}$
3) $15\sqrt{3}$
4) 6

Key: 1

SOL: $|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}|\sin(\bar{a}, \bar{b})$

$= (2)(4)\sin\frac{\pi}{6} = 4$

98. If $\bar{u} = \bar{a} - \bar{b}, \bar{v} = \bar{a} + \bar{b}$ *and* $|\bar{a}| = |\bar{b}| = 2, (\bar{a}, \bar{b}) = \frac{\pi}{3}$, then $|\bar{u} \times \bar{v}| =$

1) $\sqrt{3}$
2) $2\sqrt{3}$
3) $3\sqrt{3}$
4) $4\sqrt{3}$

Key: 4

SOL: $\bar{u} \times \bar{v} = (\bar{a} - \bar{b}) \times (\bar{a} + \bar{b}) = 2(\bar{a} \times \bar{b})$

$|\bar{u} \times \bar{v}| = 2|\bar{a} \times \bar{b}| = 2|\bar{a}||\bar{b}|\sin(\bar{a}, \bar{b})$

$= 2(2)(2)\sin\frac{\pi}{3}$ $= (8)\frac{\sqrt{3}}{2} = 4\sqrt{3}$

99. If $\bar{a}$ and $\bar{b}$ are two vectors, such that $\bar{a}.\bar{b} < 0$ and $|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$, then the angle between vectors $\bar{a}$ and $\bar{b}$ is

1) π 2) $\frac{7\pi}{4}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{4}$

Key: 4

SOL: $\left|\bar{a}.\bar{b}\right| = \left|\bar{a}\times\bar{b}\right|$

$$\left|\bar{a}\right|\left|\bar{b}\right|\left|\cos\left(\bar{a},\bar{b}\right)\right| = \left|\bar{a}\right|\left|\bar{b}\right|\sin\left(\bar{a},\bar{b}\right)$$

$$\Rightarrow -\cos\left(\bar{a},\bar{b}\right) = \sin\left(\bar{a},\bar{b}\right) \quad \left(90^0 < \left(\bar{a},\bar{b}\right)\right) < 180^0$$

$$\Rightarrow \tan\left(\bar{a},\bar{b}\right) = -1 \quad \left(\bar{a},\bar{b}\right) = \frac{3\pi}{4}$$

100. If $\left|\bar{a}\right| = \left|\bar{b}\right| = 1$ and $\left|\bar{a}\times\bar{b}\right| = \bar{a}.\bar{b}$ then $\left|\bar{a}+\bar{b}\right|^2 =$

1) $\sqrt{2}$ 2) $2+\sqrt{2}$ 3) 2 4) 1

Key: 2

SOL: $\left|\bar{a}\times\bar{b}\right| = \bar{a}.\bar{b}$

$$\Rightarrow \left|\bar{a}\right|\left|\bar{b}\right|\sin\left(\bar{a},\bar{b}\right) = \left|\bar{a}\right|\left|\bar{b}\right|\cos\left(\bar{a},\bar{b}\right)$$

$$\Rightarrow \tan\left(\bar{a},\bar{b}\right) = 1 \quad \Rightarrow \left(\bar{a},\bar{b}\right) = \frac{\pi}{4}$$

$$\left|\bar{a}+\bar{b}\right|^2 = \bar{a}^2 + \bar{b}^2 + 2\bar{a}.\bar{b} = \left|\bar{a}\right|^2 + \left|\bar{b}\right|^2 + 2\left|\bar{a}\right|\left|\bar{b}\right|\cos\left(\bar{a},\bar{b}\right)$$

$$= 1 + 1 + 2(1)(1)\frac{1}{\sqrt{2}} = 2 + \sqrt{2}$$

CONDITION FOR CROSS PRODUCT OF TWO VECTORS IS A NULL VECTOR:

101. If $\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right)\times\left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right) = \bar{O}$, then values of λ, μ are

1) 3,27 2) $3, \frac{27}{2}$ 3) $\frac{27}{2}, 3$ 4) $3, \frac{9}{2}$

Key: 2

SOL: $2\hat{i} + 8\hat{j} + 27\hat{k}, \hat{i} + \lambda\hat{j} + \mu\hat{k}$ are collinear $\Rightarrow \frac{1}{2} = \frac{\lambda}{6} = \frac{\mu}{27}$

$$\Rightarrow \lambda = 3, \mu = \frac{27}{2}$$

102. If $\bar{a}, \bar{b}, \bar{c}$ are three vectors such that $\left|\bar{a}+\bar{b}+\bar{c}\right| = 1\bar{c} = \lambda\left(\bar{a}\times\bar{b}\right)$ and $\left|\bar{a}\right| = \frac{1}{\sqrt{2}}, \left|\bar{b}\right| = \frac{1}{\sqrt{3}}, \left|\bar{c}\right| = \frac{1}{\sqrt{6}}$ then the angle between $\bar{a}$ and $\bar{b}$ is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Key: 4

SOL: $\bar{c}=\lambda\left(\bar{a}\times\bar{b}\right)$ $\Rightarrow \bar{c}$ is parallel to $\bar{a}\times\bar{b}$

$\Rightarrow \bar{c}$ is perpendicular to both $\bar{a}$ and $\bar{b}$ $\Rightarrow \bar{c}.\bar{a}=0=\bar{b}.\bar{c}$

Now $\left(\bar{a}+\bar{b}+\bar{c}\right)=1$ $\Rightarrow\left(\bar{a}+\bar{b}+\bar{c}\right)^2=1$

$\Rightarrow|\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2\bar{a}.\bar{b}+2\bar{b}.\bar{c}+2\bar{c}.\bar{a}=1$

$\Rightarrow\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+2.\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}\cos\left(\bar{a},\bar{b}\right)=1$

$\Rightarrow\frac{\sqrt{2}}{\sqrt{3}}\cos\left(\bar{a},\bar{b}\right)=1-1$

$Cos\left(\bar{a},\bar{b}\right)=0$ $\Rightarrow\left(\bar{a},\bar{b}\right)=\frac{\pi}{2}$

103. Let $\bar{a},\bar{b},\bar{c}$ be three vectors satisfying $\bar{a}\times\bar{b}=2\bar{a}\times\bar{c},|\bar{a}|=|\bar{c}|=1,\bar{b}=4$ and $\left|\bar{b}\times\bar{c}\right|=\sqrt{15}$. If $\bar{b}-2\bar{c}=\lambda\bar{a}$ then λ is

1) 1 2) -1 3) 2 4) ±4

Key: 4

SOL: $\left|\bar{b}\times\bar{c}\right|=\sqrt{15}\Rightarrow \sin ce\left(\bar{b},\bar{c}\right)=\frac{\sqrt{15}}{4}\Rightarrow\cos\left(\bar{b},\bar{c}\right)=\frac{1}{4}$

$\bar{a}\times\bar{b}=2\bar{a}\times\bar{c}$ $\Rightarrow\bar{a}\times\left(\bar{b}-2\bar{c}\right)=\bar{o}$

$\Rightarrow\bar{a}$ is parallel to $\bar{b}-2\bar{c}$

$\Rightarrow\bar{b}-2\bar{c}=\lambda\bar{a}$ $\left(\bar{b}-2\bar{c}\right)^2=\lambda^2\bar{a}^2$

$\bar{b}^2-4\bar{b}.\bar{c}+4\bar{c}^2=\lambda^2\bar{a}^2$

$16-4.4\frac{1}{4}+4(1)=\lambda^2(1)$

$\lambda^2=16$ $\lambda=\pm4$

104. If $\bar{r}\times\bar{a}=\bar{b}\times\bar{a},\bar{r}\times\bar{b}=\bar{a}\times\bar{b},\bar{a}\neq\bar{o},\bar{b}\neq\bar{o}$ $\bar{a}\neq\lambda\bar{b}$ and $\bar{a}$ is not perpendicular to $\bar{b}$, then $\bar{r}=$

1) $\bar{a}-\bar{b}$ 2) $\bar{a}+\bar{b}$ 3) $\left(\bar{a}\times\bar{b}\right)+\bar{a}$ 4) $\bar{a}\times\bar{b}+\bar{b}$

Key: 2

SOL: $\bar{r}\times\bar{a}=\bar{b}\times\bar{a}\Rightarrow\left(\bar{r}-\bar{b}\right)\times\bar{a}=\bar{o}$

$\Rightarrow \bar{r}-\bar{b}, \bar{a}$ are parallel $\Rightarrow \bar{r}-\bar{b}=t\bar{a}$

$\Rightarrow \bar{r}=t\bar{a}+\bar{b}.....(1)$ *and* $\bar{r}\times\bar{b}=\bar{a}\times\bar{b}\Rightarrow(\bar{r}-\bar{a})\times\bar{b}=\bar{o}$

$\Rightarrow \bar{r}-\bar{a}, \bar{b}$ are parallel

$\Rightarrow \bar{r}-\bar{a}=s\bar{b}$ $\Rightarrow \bar{r}=\bar{a}+s\bar{b}.....(2)$

From $(1)\&(2)$ $t=s=1$ $\therefore \bar{r}=\bar{a}+\bar{b}$

105. Let $\bar{a}=2\hat{i}+\hat{k}, \bar{b}=\hat{i}+\hat{j}+\hat{k}$ *and* $\bar{c}=4\hat{i}-3\hat{j}+7\hat{k}$ be three vectors. The vectors which satisfies $\bar{r}\times\bar{b}=\bar{c}\times\bar{b}$ *and* $\bar{r}.\bar{a}=0$ is

1) $\hat{i}+8\hat{j}+2\hat{k}$ 2) $-\hat{i}-8\hat{j}+2\hat{k}$ 3) $-\hat{i}-8\hat{j}-2\hat{k}$ 4) $\hat{i}+8\hat{j}-2\hat{k}$

Key: 2

SOL: $\bar{r}\times\bar{b}=\bar{c}\times\bar{b}$

$\Rightarrow(\bar{r}-\bar{c})\times\bar{b}=\bar{o}$ $\Rightarrow \bar{r}-\bar{c}=t\bar{b}$

$\Rightarrow \bar{r}=\bar{c}+t\bar{b}$ *and* $\bar{r}.\bar{a}=o\Rightarrow(\bar{c}+t\bar{b}).\bar{a}=0$

$\Rightarrow \bar{c}.\bar{a}+t\,\bar{a}.\bar{b}=0$

$\Rightarrow t=\frac{-\bar{c}.\bar{a}}{\bar{a}.\bar{b}}=\frac{-15}{3}=-5$

$\therefore \bar{r}=(4\hat{i}-3\hat{j}+7\hat{k})-5(\hat{i}+\hat{j}+\hat{k})$

$=-\hat{i}-8\hat{j}+2\hat{k}$

106. If $\bar{\alpha}=3\hat{i}+\hat{j}$ and $\bar{\beta}=2\hat{i}-\hat{j}+3\hat{k}$. If $\bar{\beta}=\bar{\beta}_1-\bar{\beta}_2$ where $\bar{\beta}_1$ is parallel to $\bar{\alpha}$ and $\bar{\beta}_2$ is perpendicular to $\bar{\alpha}$ then $\bar{\beta}_1\times\bar{\beta}_2$ is equal to

1) $3\hat{i}-9\hat{j}-5\hat{k}$ 2) $\frac{1}{2}(-3\hat{i}-9\hat{j}+5\hat{k})$

3) $-3\hat{i}+9\hat{j}+5\hat{k}$ 4) $\frac{1}{2}(3\hat{i}-9\hat{j}+5\hat{k})$

Key: 2

SOL: $\bar{\beta}_1$ is parallel to $\bar{\alpha}\Rightarrow\bar{\beta}_1=\lambda\bar{\alpha}=\lambda(3\hat{i}+\hat{j})$

$\bar{\beta}_2$ is perpendicular to $\bar{\alpha}\Rightarrow\bar{\beta}_2.\bar{\alpha}=0$

$\bar{\beta}=\bar{\beta}_1-\bar{\beta}_2\Rightarrow\bar{\alpha}.\bar{\beta}=\bar{\alpha}.\bar{\beta}_1-\bar{\alpha}.\bar{\beta}_2$

$\Rightarrow 5=\lambda(10)$ $\lambda=\frac{1}{2}$

$\bar{\beta}_1=\frac{1}{2}(3\hat{i}+\bar{j})$ *and* $\bar{\beta}_2=\bar{\beta}_1-\bar{\beta}$

$$=\frac{1}{2}\left(-\hat{i}+3\hat{j}-5\hat{k}\right)$$

$$\bar{\beta}_1\times\bar{\beta}_2=\frac{1}{2}\left(-3\hat{i}+9\hat{j}+5\hat{k}\right)$$

107. If the vector $\bar{b}=3\hat{j}+4\hat{k}$ is written as the sum of a vector $\bar{b_2}$, perpendicular to $\bar{b_1}$, parallel to $\bar{a}=\hat{i}+\hat{j}$ and a vector $\bar{a}$, then $\bar{b_1}\times\bar{b_2}$ is equal to

1) $3\hat{i}-3\hat{j}+9\hat{k}$ 2) $-3\hat{i}+3\hat{j}-9\hat{k}$ 3) $-6\hat{i}+6\hat{j}-\frac{9}{2}\hat{k}$ 4) $6\hat{i}-6\hat{j}+\frac{9}{2}\hat{k}$

Key: 4

SOL: $\bar{b}=\bar{b_1}+\bar{b_2}$

And $\bar{b_1}=\lambda\bar{a}=\lambda\left(\hat{i}+\bar{j}\right)$

$\bar{b_2}.\bar{a}=0$

Now $\bar{a}.\bar{b}=\bar{a}.\left(\bar{b_1}+\bar{b_2}\right)$

$3=\bar{a}.\bar{b_1}+\bar{a}.\bar{b_2}$ $3=2\lambda+0$

$\lambda=\frac{3}{2}$ $\bar{b_1}=\frac{3}{2}\left(\hat{i}+\hat{j}\right)$

$\bar{b_2}=\bar{b}-\bar{b_1}$ $=\frac{-3}{2}\hat{i}+\frac{3}{2}\hat{j}+4\bar{k}$ $\bar{b_1}\times\bar{b_2}=6\hat{i}-6\hat{j}+\frac{9}{2}\hat{k}$

108. If $\bar{r}\times\bar{b}=\bar{c}\times\bar{b},\bar{r}.\bar{a}=0,\bar{a}=2\hat{i}+3\hat{j}-\hat{k},\ \bar{b}=3\hat{i}-\hat{j}+\hat{k},\bar{c}=\hat{i}+\hat{j}+3\hat{k}$ then $\bar{r}=$

1) $\frac{1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$ 2) $2\hat{i}+\hat{j}+\hat{k}$ 3) $2\left(-\hat{i}+\hat{j}+\hat{k}\right)$ 4) $\frac{1}{2}\left(\hat{i}-\hat{j}+\hat{k}\right)$

Key: 3

SOL: $\left(\bar{r}-\bar{c}\right)\times\bar{b}=\bar{o}$ $\bar{r}=\bar{c}+t\bar{b}$

And $\bar{r}.\bar{a}=0\Rightarrow\left(\bar{c}+t\bar{b}\right).\bar{a}=0$

$t=\frac{-\bar{c}.\bar{a}}{\bar{a}.\bar{b}}=\frac{-2}{2}=-1$ $\bar{r}=\bar{c}-\bar{b}$

$=-2\hat{i}+2\hat{j}+2\hat{k}$ $=2\left(-\hat{i}+\hat{j}+\hat{k}\right)$

109. Let $\bar{a}=\hat{i}+\hat{j},\bar{b}=2\hat{i}-\hat{k}$. Then the point of intersection of the lines $\bar{r}\times\bar{b}=\bar{a}\times\bar{b}$ is

1) $3\hat{i}+\hat{j}-\hat{k}$ 2) $3\hat{i}-\hat{j}-\hat{k}$ 3) $3\hat{i}-3\hat{j}-\hat{k}$ 4) $3\hat{i}+3\hat{j}+\hat{k}$ Key: 1

SOL: $\bar{r}\times\bar{a}=\bar{b}\times\bar{a};\bar{r}\times\bar{b}=\bar{a}\times\bar{b}$ $\Rightarrow\bar{r}=\bar{a}+\bar{b}=3\hat{i}+\hat{j}-\hat{k}$

110. If $\bar{a}=\hat{i}+2\hat{k},\bar{b}=\hat{i}+\hat{j}+\hat{k},\bar{c}=7\hat{i}-3\hat{j}+4\hat{k},\bar{r}\times\bar{b}+\bar{b}\times\bar{c}=\bar{o}$ and $\bar{r}.\bar{a}=0$, then $\bar{r}.\bar{c}$ is equal to **(29th jan 2023_II)**

1) 34 2) 12 3)36 4) 30

Key: 1

SOL: $\bar{r}\times\bar{b}-\bar{c}\times\bar{b}=\bar{o}$ $(\bar{r}-\bar{c})\times\bar{b}=\bar{o}$

$\Rightarrow\bar{r}-\bar{c}=\lambda\bar{b}$ $\bar{r}=\bar{c}+\lambda\bar{b}$

And $\bar{r}.\bar{a}=0\Rightarrow\bar{c}.\bar{a}+\lambda\bar{a}.\bar{b}=0$

$\Rightarrow 15+\lambda(3)=0$ $\lambda=-5$

Now $\bar{r}=\bar{c}-5\bar{b}=2\hat{i}-8\hat{j}-\hat{k}$ $\bar{r}.\bar{c}=14+24-4=34$

111. Let $\bar{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\bar{b}=7\hat{i}+\hat{j}-6\hat{k}$. If $\bar{r}\times\bar{a}=\bar{r}\times\bar{b},\bar{r}.\left(\hat{i}+2\hat{j}+\hat{k}\right)=-3$, then $\bar{r}.\left(2\hat{i}-3\hat{j}+\hat{k}\right)$ is equal to (17th march 2021-(S-I))

a) 12 b) 8 c) 13 d) 10

Key: a

Sol: $\bar{r}\times\bar{a}=\bar{r}\times\bar{b}\Rightarrow\bar{r}\times\left(\bar{a}-\bar{b}\right)=\bar{0}$ $\Rightarrow\bar{r}=t\left(\bar{a}-\bar{b}\right)$

$\bar{r}=t\left(-5\hat{i}-4\bar{j}+10\hat{k}\right)$

Given $\bar{r}.\left(\hat{i}+2\bar{j}+\hat{k}\right)=-3$

$t(-5-8+10)=-3$

$t=1$

$\therefore\bar{r}=-5\hat{i}-4\hat{j}+10\hat{k}$

$\bar{r}.\left(2\hat{i}-3\hat{j}+\hat{k}\right)=-10+12+10=12$

112. Let $\bar{a}=\hat{i}+2\hat{j}-3\hat{k}$ *and* $\bar{b}=2\hat{i}-3\hat{j}+5\hat{k}$. If $\bar{r}\times\bar{a}=\bar{b}\times\bar{r}$. $\bar{r}.\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right)=3$ and $\bar{r}.\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right)=-1, \alpha\in R$, then the value of $\alpha+|r|^2$ is equal to (16th march 2021(s-2))

a) 9 b) 15 c) 13 d) 11

Key: b

Sol: $\bar{r}\times\bar{a}=\bar{b}\times\bar{r}\Rightarrow\bar{r}\times\bar{a}+\bar{r}\times\bar{b}=\bar{o}$ $\Rightarrow\bar{r}\times\left(\bar{a}+\bar{b}\right)=\bar{o}$

$\Rightarrow\bar{r}=t\left(\bar{a}+\bar{b}\right)$ $\Rightarrow\bar{r}=t\left(3i-\hat{j}+2\hat{k}\right)$

Given $\bar{r}.\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right)=3\Rightarrow 3t\alpha-2t+2t=3$ $\Rightarrow t\alpha=1$

And $\overline{r}.\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right)=-1 \Rightarrow 6t-5t-2t\alpha=-1$

$\Rightarrow t-2(1)=-1$ $\quad \Rightarrow t=1$ $\quad$ *and* $\alpha=1$

$\overline{r}=3\hat{i}-\hat{j}+2\hat{k},\ \alpha+|\overline{r}|^2=1+9+1+4=15$

113. Let $\overline{a}=\hat{i}-2\hat{j}+\hat{k}$ *and* $\overline{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\overline{c}$ is a vector such that $\overline{b}\times\overline{c}=\overline{b}\times\overline{a}$ *and* $\overline{c}.\overline{a}=0$ then $\overline{c}.\overline{b}$ is equal to (8th jan 2020(S-2))

a) $\frac{1}{2}$ $\quad$ b) $\frac{-3}{2}$ $\quad$ c) $\frac{-1}{2}$ $\quad$ d) -1

Key: c

SOL: $\overline{b}\times\overline{c}=\overline{b}\times\overline{a}$ $\quad \Rightarrow \overline{b}\times\overline{c}-\overline{b}\times\overline{a}=\overline{o}$

$\Rightarrow \overline{b}\times(\overline{c}-\overline{a})=\overline{o}$ $\quad \Rightarrow \overline{c}-\overline{a}=t\overline{b}$

$\overline{c}=\overline{a}+t\overline{b}$

Given $\overline{c}.\overline{a}=0 \Rightarrow \overline{a}.\overline{a}+t\overline{a}.\overline{b}=0$

$\Rightarrow 6+t4=0$ $\quad t=\frac{-6}{4}=\frac{-3}{2}$

$\therefore \overline{c}=\overline{a}-\frac{3\overline{b}}{2}=\frac{-1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

$\overline{c}.\overline{b}=\frac{-1}{2}(1-1+1)=\frac{-1}{2}$

114. If $\overline{a}$ *and* $\overline{b}$ are vectors such that $|\overline{a}+\overline{b}|=\sqrt{29}$ and $\overline{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)\times\overline{b}$, then a possible value of $\left(\overline{a}+\overline{b}\right).\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$ is (JEE ADV 2012 P-2)

a) 0 $\quad$ b) 3 $\quad$ c) 4 $\quad$ d) 8

Key: c

Sol: Given $\overline{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)\times\overline{b}$

$\Rightarrow\left(\overline{a}+\overline{b}\right)\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=0$ $\quad \Rightarrow \overline{a}+\overline{b}=\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$

Given that $|\overline{a}+\overline{b}|=\sqrt{29}$

$|\lambda|\sqrt{4+9+16}=\sqrt{29}$

$|\lambda|=\pm 1$

$\overline{a}+\overline{b}=\pm\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$

$\left(\overline{a}+\overline{b}\right).\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)=\pm(-14+6+12)=\pm 4$

VECTOR PERPENDICULAR TO BOTH GIVEN VECTORS:

115. The unit vector orthogonal to $\bar{a}=2\hat{i}+2\hat{j}+\hat{k}, \bar{b}=3\hat{i}+4\bar{j}-12\hat{k}$ and forming a right hended system with $\bar{a}$ *and* $\bar{b}$ is

1) $\dfrac{-28\hat{i}+27\hat{j}+2\hat{k}}{\sqrt{1517}}$ 2) $-28\hat{i}+27\hat{j}+2\hat{k}$ 3) $\dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ 4) $-\hat{i}+\hat{j}+\hat{k}$

Key: 1

Sol: $$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 1\\ 3 & 4 & -12\end{vmatrix}=-28\hat{i}+27\hat{j}+2\hat{k}$$

Required unit vector $=\dfrac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}=\dfrac{-28\hat{i}+27\hat{j}+2\hat{k}}{\sqrt{1517}}$

116. $\bar{c}$ is a unit vector orthogonal to $\bar{a},\bar{b}$ and a,b,c are R.H.S. $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}=2\hat{j}+2\hat{k}$ then $\bar{c}=$

1) $\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$ 2) $\dfrac{\hat{j}+\hat{k}}{\sqrt{2}}$ 3) $\dfrac{\hat{i}-\hat{k}}{\sqrt{2}}$ 4) $\dfrac{\hat{k}-\hat{j}}{\sqrt{2}}$

Key: 4

Sol: $\bar{c}=\dfrac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}=\dfrac{\hat{k}-\hat{j}}{\sqrt{2}}$

117. A vector of magnitude $\sqrt{b}$ which is perpendicular to both $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$ is

1) $\pm\left(2\hat{i}-\hat{j}-\hat{k}\right)$ 2) $\pm\left(\hat{i}+2\hat{j}+\hat{k}\right)$ 3) $\pm\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ 4) $\hat{i}+\hat{j}+\hat{k}$

Key: 1

Sol: Let $\bar{a}=\hat{i}+\hat{j}+\hat{k},\ \bar{b}=2\hat{i}+\hat{j}+3\hat{k}$

$\bar{a}\times\bar{b}=2\hat{i}-\hat{j}-\hat{k}$ $|\bar{a}\times\bar{b}|=\sqrt{6}$ Required vector $=\pm\sqrt{6}\dfrac{\left(\bar{a}\times\bar{b}\right)}{|\bar{a}\times\bar{b}|}=\pm\left(2\hat{i}-\hat{j}-\hat{k}\right)$

118. Let $\bar{a}=3\hat{i}+2\hat{j}+2\hat{k}$ *and* $\bar{b}=\hat{i}+2\hat{j}-2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\bar{a}+\bar{b}$ *and* $\bar{a}-\bar{b}$ has the magnitude 12 then one such vector is (12th april 2019, S-1)

1) $4\left(2\hat{i}-2\hat{j}-\hat{k}\right)$ 2) $4\left(-2\hat{i}-2\hat{j}+\hat{k}\right)$

3) $4\left(2\hat{i}+2\hat{j}+\hat{k}\right)$ 4) $4\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

Key: 1

Sol: $\bar{a}+\bar{b}=4\hat{i}+4\hat{j},\ \bar{a}-\bar{b}=2\hat{i}+4\hat{k}$

$\left(\bar{a}+\bar{b}\right)\times\left(\bar{a}-\bar{b}\right)=16\hat{i}-16\hat{j}-8\hat{k}$

Required vector $=\pm 12\dfrac{\left(\bar{a}+\bar{b}\right)\times\left(\bar{a}-\bar{b}\right)}{\left|\left(\bar{a}+\bar{b}\right)\times\left(\bar{a}-\bar{b}\right)\right|}$ $=\pm 4\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

119. Any vector which is perpendicular to each of the vectors $2\hat{i}+\hat{j}-\hat{k}$ *and* $3\hat{i}-\hat{j}+\hat{k}$ is normal to

1) x-axis 2)y-axis 3) z-axis 4) all the above

Key: 1

Sol: Let $\bar{a}=2\hat{i}+\hat{j}-\hat{k},\ \bar{b}=3\hat{i}-\hat{j}+\hat{k}$

Any vector perpendicular to $\bar{a},\bar{b}=\bar{a}\times\bar{b}\ =-5\hat{j}-5\hat{k}$

Which is normal to x-axis.

120. $A(1,2,5), B(5,7,9)$ *and* $C(3,2,-1)$ are given three points A unit vector normal to the plane of the triangle ABC

1) $\dfrac{15\hat{i}+16\hat{j}-5\hat{k}}{\sqrt{506}}$ 2) $\dfrac{-15\hat{i}+16\hat{j}-5\hat{k}}{\sqrt{506}}$

3) $\dfrac{-15\hat{i}+16\hat{j}+5\hat{k}}{\sqrt{506}}$ 4) $\dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

Key: 2

Sol: $\overline{OA}=\hat{i}+2\hat{j}+5\hat{k},\ \overline{OB}=5\hat{i}+7\hat{j}+9\hat{k},\ \overline{OC}=3\hat{i}+2\hat{j}-\hat{k}$

$\overline{AB}=4\hat{i}+5\hat{j}+4\hat{k},\ \overline{AC}=2\hat{i}-6\hat{k}$

$\overline{AB}\times\overline{AC}=-30\hat{i}+32\hat{j}-10\hat{k}$

Required unit vector $=\pm\dfrac{\overline{AB}\times\overline{AC}}{\left|\overline{AB}\times\overline{AC}\right|}$ $=\pm\dfrac{\left(-15\hat{i}+16\hat{j}-5\hat{k}\right)}{\sqrt{506}}$

121. A unit vector making an obtuse angle with x-axis and perpendicular to the plane containing the points $\hat{i}+2\hat{j}+3\hat{k}, 2\hat{i}+3\hat{j}+4\hat{k}$ and $\hat{i}+5\hat{j}+7\hat{k}$ also makes an obtuse angle with

1) y-axis 2) z-axis

3) both y and z-axis 4) both x and y- axes

Key:2

Sol: Let $\overline{OA}=\hat{i}+2\hat{j}+3\hat{k},\ \overline{OB}=2\hat{i}+3\hat{j}+4\hat{k}$

$\overline{OC}=\hat{i}+5\hat{j}+7\hat{k}$

$\overline{AB}\times\overline{AC}=\hat{i}-4\hat{j}+3\hat{k}$

Unit vector perpendicular to the plane $==\pm\dfrac{\overline{AB}\times\overline{AC}}{\left|\overline{AB}\times\overline{AC}\right|}=\pm\dfrac{\left(\hat{i}-4\hat{j}+3\hat{k}\right)}{\sqrt{26}}$

Since it makes an obtuse angle with x-axis Required vectors= $\dfrac{-\hat{i}+4\hat{j}-3\hat{k}}{\sqrt{26}}$

Which also makes an obtuse angle with –z-axis

122. Let O be the origin and the position vectors of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the points A,B and C are $-2\hat{i}+\hat{j}-3\hat{k}, 2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively then the projection of the vectors $\overline{OP}$ on a vector perpendicular to the vectors $\overline{AB}$ and $\overline{AC}$ IS

1) 3 2) $\frac{8}{3}$ 3) $\frac{10}{3}$ 4) $\frac{7}{3}$

Key: 1

Sol: $\overline{AB}=4\hat{i}+3\hat{j}+\hat{k}$

$\overline{AC}=-2\hat{i}+\hat{j}+2\hat{k}$

Now $\overline{AB}\times\overline{AC}=5\hat{i}-10\hat{j}+10\hat{k}$

$\overline{OP}=-\hat{i}-2\hat{j}+3\hat{k}$

The projection of $\overline{OP}$ on $\overline{AB}\times\overline{AC}=\dfrac{\overline{OP}.\left(\overline{AB}\times\overline{AC}\right)}{\left|\overline{AB}\times\overline{AC}\right|}$ $=\dfrac{-5+20+30}{\sqrt{25+100+100}}=\dfrac{45}{15}=3$

123. Let $A(0,0,0), B(1,1,1) C(3,2,1)$ *and* $D(2,3,1)$ be four points. The angle between the planes through the points A,B,C and through A,B,D is

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

Key: 4

Sol: Let n_1 *and* n_2 be the vectors normal to the planes $\overleftrightarrow{ABC}$ respectively.

$n_1=\overline{AB}\times\overline{AC}=-\hat{i}+2\hat{j}-\hat{k}$

$n_2=\overline{AB}\times\overline{AD}=-2\hat{i}+\hat{j}+\hat{k}$

Let θ be the angle between the planes, then $(n_1,n_2)=\theta$

$\therefore Cos\theta=\dfrac{n_1-n_2}{|n_1||n2|}=\dfrac{2+2-1}{\sqrt{6}\sqrt{6}}=\dfrac{1}{2}\Rightarrow\theta=\dfrac{\pi}{3}$

124. If $\overline{a}=\hat{i}+2\hat{j}+3\hat{k},\ \overline{b}=-\hat{i}+2\hat{j}+\hat{k}, \overline{c}=3\hat{i}+\hat{j}$ and $\overline{d}$ is normal to both $\overline{a}$ and $\overline{b}$, then $\left(\overline{c},\overline{d}\right)$ =

1) $\cos^{-1}\left(\frac{4}{\sqrt{30}}\right)$ 2) $sin^{-1}\left(\frac{4}{\sqrt{30}}\right)$ 3) $\cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$ 4) $sin^{-1}\left(\frac{2}{\sqrt{30}}\right)$

Key: 1

Sol: $\overline{d}$ is parallel $\overline{a}\times\overline{b}$

$\overline{d}=\lambda\left(\overline{a}\times\overline{b}\right)=\lambda\left(-4\hat{i}-4\hat{j}+4\hat{k}\right) \qquad =-4\lambda\left(\hat{i}+\hat{j}-\hat{k}\right)$

$Cos\left(\overline{c},\overline{d}\right)=\frac{\overline{c}.\overline{d}}{|\overline{c}||\overline{d}|}=\frac{4}{\sqrt{30}} \qquad \left(\overline{c},\overline{d}\right)=\cos^{-1}\left(\frac{4}{\sqrt{30}}\right)$

125. The vector $\overline{c}$ is perpendicular to both $\overline{a}=(1,-2,1),\overline{b}=(2,1,-1)$ and $\overline{c}$ also satisfies $\left|\overline{c}\times\left(\hat{i}-\hat{j}+\hat{k}\right)\right|=2\sqrt{6}$ then $\overline{c}=$

1) $\pm\frac{\hat{i}+3\hat{j}+5\hat{k}}{2}$ 2) $\pm\left(-4\hat{i}+5\hat{j}+\hat{k}\right)$

3) $\pm\left(\hat{i}+\hat{j}+\hat{k}\right)$ 4) $\pm2\left(\hat{i}+\hat{j}+\hat{k}\right)$

Key: 1

Sol: $\overline{c}$ is perpendicular to both $\overline{a},\overline{b}$

$\overline{c}$ is parallel to $\overline{a}\times\overline{b} \qquad \overline{c}=\lambda\left(\overline{a}\times\overline{b}\right)$

$\overline{c}=\lambda\left(\hat{i}+3\hat{j}+5\hat{k}\right)$ Given $\left|\overline{c}\times\left(\hat{i}-\hat{j}+\hat{k}\right)\right|=2\sqrt{6}$

$|\lambda|\left|\left(\hat{i}+3\hat{j}+5\hat{k}\right)\times\left(\hat{i}-\hat{j}+\hat{k}\right)\right|=2\sqrt{6}$

$|\lambda|\left|8\hat{i}+4\hat{j}-4\hat{k}\right|=2\sqrt{6} \qquad |\lambda|\left|4\sqrt{6}\right|=2\sqrt{6} \qquad |\lambda|=\frac{1}{2}$

$\lambda=\pm\frac{1}{2} \qquad \overline{c}=\pm\frac{\left(\hat{i}+3\hat{j}+5\hat{k}\right)}{2}$

MAGNITUDE OF CROSS PRODUCT OF VECTORS

126. Let $\overline{a},\overline{b}$ be two non collinear unit vectors. If $\overline{\alpha}=\overline{a}-\left(\overline{a}.\overline{b}\right)\overline{b},\ \overline{\beta}=\overline{a}\times\overline{b}$, then

1) $|\overline{\alpha}|=|\overline{\beta}|$ 2) $|\overline{\alpha}|^2=|\overline{\beta}|$ 3) $|\overline{\beta}|^2=|\overline{\alpha}|$ 4) $|\overline{\alpha}|=2|\overline{\beta}|$

Key: 1

Sol: $|\overline{\alpha}|^2=\left|\overline{a}-\left(\overline{a}.\overline{b}\right)\overline{b}\right|^2 \qquad =\overline{a}^2+\left(\overline{a}.\overline{b}\right)^2\overline{b}^2-2\overline{a}.\left\{\left(\overline{a}.\overline{b}\right)\overline{b}\right\}$

$=|\bar{a}|^2+(\bar{a}.\bar{b})^2|\bar{b}|^2-2(\bar{a}.\bar{b})^2$ $\quad =1-(\bar{a}.\bar{b})^2$ $\quad =1-|\bar{a}|^2|\bar{b}|^2\cos^2(\bar{a},\bar{b})$

$=1-\cos^2(\bar{a},\bar{b})$ $\quad =\sin^2(\bar{a},\bar{b})$ $\quad |\bar{\alpha}|=\sin(\bar{a},\bar{b})$

$|\bar{\beta}|=|\bar{a}\times\bar{b}|=|\bar{a}||\bar{b}|\sin(\bar{a},\bar{b})$ $\quad =\sin(\bar{a},\bar{b})$

127. Let $|\bar{a}|=2,|\bar{b}|=3$ and the angle between the vectors $\bar{a}$ and $\bar{b}$ be $\frac{\pi}{4}$. Then

$|(\bar{a}+2\bar{b})\times(2\bar{a}-3\bar{b})|^2$ is equal to (13th april 2023 (II)

1) 482 2) 441 3) 841 4) 882

Key: 4

Sol: $|\bar{a}|=2,|\bar{b}|=3$

$|(\bar{a}+2\bar{b})\times(2\bar{a}-3\bar{b})|^2$

$=|-3(\bar{a}\times\bar{b})+4(\bar{b}\times\bar{a})|^2$ $\quad =|-7(\bar{a}\times\bar{b})|^2$

$(\bar{b},\bar{a})=49|\bar{a}|^2|\bar{b}|^2\sin^2(\bar{a},\bar{b})$ $\quad =49(4)(9).\frac{1}{2}=882$

128. Let a vector $\bar{a}$ has magnitude 9. Let a vector $\bar{b}$ be such that for every $(x,y)\in R\times R-\{(0,0)\}$, the vector $x\bar{a}+y\bar{b}$ is perpendicular to the vector $6y\bar{a}-18x\bar{b}$.

Then, the value of $|\bar{a}\times\bar{b}|$ is equal to (28th july 2022 (I))

1) $9\sqrt{3}$ 2) $27\sqrt{3}$ 3) 9 4) 81

Key : 2

SOL: Given $|\bar{a}|=9$

and $x\bar{a}+y\bar{b}$ is perpendicular to $6y\bar{a}-18x\bar{b}$

$\Rightarrow(x\bar{a}+y\bar{b}).(6y\bar{a}-18x\bar{b})=0$ $\quad \Rightarrow 6xy\left(|\bar{a}|^2-3|\bar{b}|^2\right)+6(\bar{a}.\bar{b})(y^2-3x^2)=0$

Since $x,y\in R\times R$

$|\bar{a}|^2-3|\bar{b}|^2=0,\ \bar{a}.\bar{b}=0$ $\quad |\bar{a}|^2=3|\bar{b}|^2,(\bar{a},\bar{b})=\frac{\pi}{2}$ $\quad \Rightarrow|\bar{b}|=3\sqrt{3}$

Now $\Rightarrow|\bar{a}\times\bar{b}|=|\bar{a}||\bar{b}|\sin(\bar{a},\bar{b})$ $\quad =9(3\sqrt{3})=27\sqrt{3}$

129. Let $\bar{a}$ and $\bar{b}$ be unit vectors. If $\bar{c}$ is a vector such that the angle between $\bar{a}$ and $\bar{c}$ is $\frac{\pi}{12}$ and

$\bar{b}=\bar{c}+2(\bar{c}\times\bar{a})$ then $|6\bar{c}|^2$ is equal to (24th june 2022,(I))

1) $6\left(3-\sqrt{3}\right)$ 2) $3+\sqrt{3}$ 3) $6\left(3+\sqrt{3}\right)$ 4) $6\left(\sqrt{3}+1\right)$

Key: 3

Sol: $\bar{b}=\bar{c}+2\left(\bar{c}\times\bar{a}\right)$

$\Rightarrow \bar{b}.\bar{c}=\bar{c}^2 \Rightarrow \bar{b}.\bar{c}=|\bar{c}|^2$

And $\bar{b}-\bar{c}=2\left(\bar{c}\times\bar{a}\right)$

$\left(\bar{b}-\bar{c}\right)^2=4\left(\bar{c}\times\bar{a}\right)^2$

$\bar{b}^2-2\bar{b}.\bar{c}+\bar{c}^2=4|\bar{c}|^2|\bar{a}|^2\sin^2\left(\bar{a},\bar{c}\right)$

$1--2|\bar{c}|^2+|\bar{c}|^2=4|\bar{c}|^2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$

$1-|\bar{c}|^2=|\bar{c}|^2\left(2-\sqrt{3}\right)$ $\quad |\bar{c}|^2\left(3-\sqrt{3}\right)=1$

$|\bar{c}|^2=\frac{1}{3-\sqrt{3}}=\frac{3+\sqrt{3}}{6}$ $\quad |6\bar{c}|^2=36|\bar{c}|^2=6\left(3+\sqrt{3}\right)$

130. Let $\bar{a}$ and $\bar{b}$ be two unit vectors such that $\left|\bar{a}+\bar{b}\right|=\sqrt{3}$. If $\bar{c}=\bar{a}+2\bar{b}+3\bar{a}\times\bar{b}$, then $2|\bar{c}|$ is

1) $\sqrt{55}$ 2) $\sqrt{37}$ 3) $\sqrt{51}$ 4) $\sqrt{43}$

Key: 1

SOL: $\left|\bar{a}+\bar{b}\right|=\sqrt{3}, \quad Let\left(\bar{a},\bar{b}\right)=\theta$

$\Rightarrow\left|\bar{a}+\bar{b}\right|^2=3 \qquad \Rightarrow|a|^2+\left|\bar{b}\right|^2+2\bar{a}.\bar{b}=3$

$\Rightarrow 1+1+2(1)(1)\cos\theta=3$

$\cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$

$|\bar{c}|^2=\left(\bar{a}+2\bar{b}+3\left(\bar{a}\times\bar{b}\right)\right)^2$

$=|\bar{a}|^2+4\left|\bar{b}\right|^2+9\left|\bar{a}\times\bar{b}\right|^2+4\bar{a}.\bar{b}$

$=1+4(1)+9(1)(1)\sin^2\frac{\pi}{3}+4(1)(1)\cos\frac{\pi}{3}$

$=5+\frac{27}{4}+2=\frac{55}{4} \quad |\bar{c}|=\frac{\sqrt{55}}{2}$

b) PROPERTIES OF CROSS PRODUCT OF VECTORS:

131. If the vectors $\bar{a},\bar{b}\bar{c}$ from the sides $\overline{BC},\overline{CA}$ *and* $\overline{AB}$ *of* ΔABC, then

1) $\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}=0$ 2) $\bar{a}\times\bar{b}=\bar{b}\times\bar{c}=\bar{c}\times\bar{a}$

3) $\bar{a}.\bar{b}=\bar{b}.\bar{c}=\bar{c}.\bar{a}$ 4) $\bar{a}\times\bar{b}=\bar{b}\times\bar{c}=\bar{c}\times\bar{a}=\bar{o}$

Key: 2

Sol: In $\Delta ABC, \overline{BC}+\overline{CA}+\overline{AB}=\bar{O}$ $\Rightarrow \bar{a}+\bar{b}+\bar{c}=\bar{o}$

Now $\bar{a}\times\left(\bar{a}+\bar{b}+\bar{c}\right)=\bar{a}\times\bar{o}$

$\Rightarrow \bar{a}\times\bar{b}+\bar{a}\times\bar{c}=\bar{o}$

$\Rightarrow \bar{a}\times\bar{b}=\bar{c}\times\bar{a}$

And $\bar{b}\times\left(\bar{a}+\bar{b}+\bar{c}\right)=\bar{b}\times\bar{o}$

$\bar{b}\times\bar{a}+\bar{b}\times\bar{c}=\bar{o}$

$\bar{a}\times\bar{b}=\bar{b}\times\bar{c}$

$\therefore \bar{a}\times\bar{b}=\bar{b}\times\bar{c}=\bar{c}\times\bar{a}$

132. If $\bar{a}+2\bar{b}+3\bar{c}=\bar{o}$, then $\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}=$

1) $4\left(\bar{b}\times\bar{c}\right)$ 2) $5\left(\bar{b}\times\bar{c}\right)$ 3) $6\left(\bar{b}\times\bar{c}\right)$ 4) $7\left(\bar{b}\times\bar{c}\right)$

Key: 3

Sol: $l\bar{a}+m\bar{b}+n\bar{c}=\bar{o}$, then $\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}=\left(\dfrac{l+m+n}{l}\right)\bar{b}\times\bar{c}$

$=\left(\dfrac{6}{1}\right)\left(\bar{b}\times\bar{c}\right)$ $=6\left(\bar{b}\times\bar{c}\right)$

133. If $\bar{a}=2\hat{i}-\hat{j}+2\hat{k}$, then the values of $\left|\bar{a}\times\hat{i}\right|^2+\left|\bar{a}\times\hat{j}\right|^2+\left|\bar{a}\times\hat{k}\right|^2=$

1) 2 2) 4 3) 6 4) 18 Key: 4

Sol: $\left|\bar{a}\times\hat{i}\right|^2+\left|\bar{a}\times\hat{j}\right|^2+\left|\bar{a}\times\hat{k}\right|^2=2\left|\bar{a}\right|^2$ $=2\left(4+1+4\right)=18$

134. Let $\bar{a}=\hat{i}+4\hat{j}+2\hat{k}, \bar{b}=3\hat{i}-2\hat{j}+7\hat{k}$ *and* $\bar{c}=2\hat{i}-\hat{j}+4\hat{k}$. If a vector $\bar{d}$ satisfies $\bar{d}\times\bar{b}=\bar{c}\times\bar{b}$ and $\bar{d}.\bar{a}=24$, then $\left|\bar{d}\right|^2$ is equal to (13th April 2023-I)

1) 413 2) 423 3) 323 4) 313

Key: 1

Sol: $\bar{d}\times\bar{b}=\bar{c}\times\bar{b}\Rightarrow\left(\bar{d}-\bar{c}\right)\times\bar{b}=\bar{o}$ $\Rightarrow \bar{d}-\bar{c}=\lambda\bar{b}$

$\Rightarrow \bar{d}=\bar{c}+\lambda\bar{b}$ $\bar{d}.\bar{a}=24\Rightarrow\left(\bar{c}+\lambda\bar{b}\right).\bar{a}=24$

$\Rightarrow \bar{c}.\bar{a}+\lambda\bar{a}.\bar{b}=24 \Rightarrow 6+9\lambda=24\Rightarrow\lambda=2$

$\bar{d}=\bar{c}+2\bar{b}=8\hat{i}-5\hat{j}+18\hat{k}$ $\left|\bar{d}\right|^2=413$

135. Let $\lambda \in Z, \bar{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ *and* $\bar{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ Let $\bar{c}$ be a vector such that $(\bar{a} + \bar{b} + \bar{c}) \times \bar{c} = \bar{o}$, $\bar{a}.\bar{c} = -17$ *and* $\bar{b}.\bar{c} = -20$. Then $\left|\bar{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})\right|^2$ is equal to

1)62 2) 46 3) 53 4) 49

Key: 2

Sol: $(\bar{a} + \bar{b} + \bar{c}) \times \bar{c} = \bar{o} \Rightarrow (\bar{a} + \bar{b}) \times \bar{c} = \bar{o}$

$$\Rightarrow \bar{c} = t(\bar{a} + \bar{b})$$

Given $\bar{a}.\bar{c} = -17, \bar{b}.\bar{c} = -20$

$\bar{a}.\left[t(\bar{a} + \bar{b})\right] = -17; \bar{b}.\left[t(\bar{a} + \bar{b})\right] = -20$

$t\left[\bar{a}^2 + \bar{a}.\bar{b}\right] = -17; t\left[\bar{a}.\bar{b} + \bar{b}^2\right] = -20$

$t\left[\lambda^2 + 1 + 1 + 3\lambda - 1 - 2\right] = -17; t[3\lambda - 3 + 9 + 1 + 4] = -20$

$t\left[\lambda^2 + 3\lambda - 1\right] = -17;\ t[3\lambda + 11] = -20$

Solve $\lambda = 3$ *and* $t = -1$

$\bar{c} = -1(\bar{a} + \bar{b}) = -(6\hat{i} + \hat{k})$

$\bar{c} \times (3\hat{i} + \hat{j} + \hat{k}) = \hat{i} + 3\hat{j} - 6\hat{k}$

$\left|\bar{c} \times (3\hat{i} + \hat{j} + \hat{k})\right|^2 = 1 + 9 + 36 = 46$

136. Let $\bar{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \bar{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\bar{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\bar{d}$ is a vector perpendicular to both $\bar{b}$ *and* $\bar{c}$ and $\bar{a}.\bar{d} = 18$. Then $\left|\bar{a} \times \bar{d}\right|^2$ is equal to

(6th April, 2023-I)

1)640 2) 760 3) 680 4) 720

Key: 4

Sol: $\bar{d}$ is parallel to $\bar{b} \times \bar{c}$

$\Rightarrow \bar{d} = \lambda(\bar{b} \times \bar{c})$

$\bar{b} \times \bar{c} = 2\hat{i} - \hat{j} + \hat{k}$

$\bar{d} = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

$\bar{a}.\bar{d} = 18 \Rightarrow \lambda(4 - 3 + 8) = 18$

$$\Rightarrow \lambda = 2 \therefore \bar{d} = 4\bar{i} - 2\bar{j} + 4\bar{k}$$

$\overline{a}\times\overline{d}=20\hat{i}+8\hat{j}-16\hat{k}$

$\left|\overline{a}\times\overline{d}\right|^2=720$

137. Let $\overline{a}=2\hat{i}-7\hat{j}+5\hat{k}, \overline{b}=\hat{i}+\hat{k}$ *and* $\overline{c}=\hat{i}+2\hat{j}-3\hat{k}$ be three given vectors. If $\overline{r}$ is a vectors such that $\overline{r}\times\overline{a}=\overline{c}\times\overline{a}$ *and* $\overline{r}.\overline{b}=0$, then $\left|\overline{r}\right|$ is equal to

(1st Feb 2023-II)

1) $\frac{11}{7}\sqrt{2}$ 2) $\frac{11}{5}\sqrt{2}$ 3) $\frac{11}{7}$ 4) $\frac{\sqrt{914}}{7}$

Key: 1

SOL: $\overline{r}\times\overline{a}=\overline{c}\times\overline{a}\Rightarrow(\overline{r}-\overline{c})\times\overline{a}=\overline{o}$

$\Rightarrow\overline{r}-\overline{c}=\lambda\overline{a}$ $\Rightarrow\overline{r}=\overline{c}+\lambda\overline{a}$

And $\overline{r}.\overline{b}=0\Rightarrow(\overline{c}+\lambda\overline{a}).\overline{b}=0$

$\Rightarrow\overline{c}.\overline{b}+\lambda\overline{a}.\overline{b}=0$ $\Rightarrow-2+7\lambda=0$ $\lambda=\frac{2}{7}$

$\overline{r}=\overline{c}+\frac{2\overline{a}}{7}=\frac{7\overline{c}+2\overline{a}}{7}$ $=\frac{11\hat{i}-11\hat{k}}{7}$ $\left|\overline{r}\right|=\frac{11\sqrt{2}}{7}$

138. Let $\overline{a}=\hat{i}+2\hat{j}+3\hat{k},\overline{b}=\hat{i}-\hat{j}+2\hat{k}$ *and* $\overline{c}=5\hat{i}-3\hat{j}+3\hat{k}$ be three vectors. If $\overline{r}$ is a vector such that $\overline{r}\times\overline{b}=\overline{c}\times\overline{b}$ and $\overline{r}.\overline{a}=0$,then $25\left|\overline{r}\right|^2$ is equal to

(31st Jan 2023-II)

1) 449 2) 336 3) 339 4)560

Key: 3

SOL: $\overline{r}\times\overline{b}=\overline{c}\times\overline{b}\Rightarrow(\overline{r}-\overline{c})\times\overline{b}=\overline{o}$

$\Rightarrow\overline{r}-\overline{c}=\lambda\overline{b}$ $\Rightarrow\overline{r}=\overline{c}+\lambda\overline{b}$

Given $\overline{r}.\overline{a}=0\Rightarrow(\overline{c}+\lambda\overline{b}).\overline{a}=0$

$\Rightarrow\overline{c}.\overline{a}+\lambda\overline{a}.\overline{b}=0$ $\Rightarrow 8+5\lambda=0$

$\lambda=\frac{-8}{5}\,\overline{r}=\overline{c}\frac{-8}{5}\,\overline{b}=\frac{5\overline{c}-8\overline{b}}{5}=\frac{17\hat{i}-7\hat{j}-\hat{k}}{5}$

$\left|\overline{r}\right|=\frac{\sqrt{339}}{5}$

$25\left|\overline{r}\right|^2=339$

139. Let $\bar{a}=\alpha\hat{i}+\hat{j}-\hat{k}$ and $\bar{b}=2\hat{i}+\hat{j}-\alpha\hat{k},\alpha>0$ .If he projection of $\bar{a}\times\bar{b}$ on the vector $-\hat{i}+2\hat{j}-2\hat{k}$ is 30, then α is equal to (26th july 2022-I)

1) $\frac{15}{2}$ 2) 8 3) $\frac{13}{2}$ 4) 7

Key: 4

SOL: $\bar{a}\times\bar{b}=\hat{i}(1-\alpha)+\hat{j}\left(\alpha^2-2\right)+\hat{k}(\alpha-2)$

Projection of $\bar{a}\times\bar{b}$ $on -\hat{i}+2\hat{j}-2\hat{k}=30$

$\Rightarrow\frac{\left|\left(\bar{a}\times\bar{b}\right).\left(-\hat{i}+2\hat{j}-2\hat{k}\right)\right|}{\sqrt{1+4+4}}=30$

$\Rightarrow\left|-(1-\alpha)+2\left(\alpha^2-2\right)-2(\alpha-2)\right|=90$

$\Rightarrow\left|2\alpha^2-\alpha-1\right|=90$

$\Rightarrow 2\alpha^2-\alpha-91=0$ or $2\alpha^2-\alpha+89=0$

$2\alpha^2-\alpha-91=0\Rightarrow\alpha=7,\alpha=\frac{-13}{2}$

Since $\alpha>0,\alpha=7$

140. Let $\bar{a}=\alpha\hat{i}+3\hat{j}-\hat{k},\bar{b}=3\hat{i}-\beta\hat{j}+4\hat{k}$ and $\bar{c}=\hat{i}+2\hat{j}-2\hat{k}$ where $\alpha,\beta\in R$ be three vectors. If the projection of $\bar{a}$ on $\bar{c}$ is $\frac{10}{3}$ and $\bar{b}\times\bar{c}=-6\hat{i}+10\hat{j}+7\hat{k}$, then the value of $\alpha+\beta$ is equal to (29th June 2022-I)

1) 3 2) 4 3) 5 4) 6

Key: 1

Sol: Projection of $\bar{a}$ on $\bar{c}=\frac{10}{3}$

$\Rightarrow\frac{\bar{a}.\bar{c}}{|\bar{c}|}=\frac{10}{3}$

$\Rightarrow\frac{\alpha+8}{3}=\frac{10}{3}$

$\alpha=2$

$\bar{b}\times\bar{c}=-6\hat{i}+10\hat{j}+7\hat{k}$

$\Rightarrow(2\beta-8)\hat{i}+10\hat{j}+(6+\beta)\hat{k}=-6\hat{i}+10\hat{j}+7\hat{k}$

$2\beta-8=-6$

$2\beta=2$

$\beta = 1$

$\therefore \alpha + \beta = 2 + 1 = 3$

141. Let $\overline{a}$ be a vector which is perpendicular to the vector $3\hat{i} - \frac{1}{2}\hat{j} + 2\hat{k}$. If $\overline{a} \times \left(2\hat{i} + \hat{k}\right) = 2\hat{i} - 13\hat{j} - 4\hat{k}$ then the projection of the vector $\overline{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

(28th june 2012-II)

1) $\frac{1}{3}$ 2) 1 3) $\frac{5}{3}$ 4) $\frac{7}{3}$

Key: 3

Sol: Let $\overline{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\overline{a}.\left(3\hat{i} - \frac{1}{2}\hat{j} + 2\hat{k}\right) = 0$

$3x - \frac{y}{2} + 2z = 0.....(1)$

And $\overline{a} \times \left(2\hat{i} + \hat{k}\right) = 2\hat{i} - 13\hat{j} - 4\hat{k}$

$\Rightarrow y\hat{i} - (x - 2z)\hat{j} - 2y\hat{k} = 2\hat{i} - 13\hat{j} - 4\hat{k}$

$y = 2, \;\; x - 2z = 13$

$...(2) \quad ....(3)$

Solve $x = 3, \; z = -5$

$\overline{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ Projection of $\overline{a}$ on $2\hat{i} + 2\hat{j} + \hat{k} = \frac{\overline{a}.\left(2\hat{i} + 2\hat{j} + \hat{k}\right)}{3}$

$= \frac{6 + 4 - 5}{3} = \frac{5}{3}$

142. Let $\overline{a}$ and $\overline{b}$ be two non-zero vectors perpendicular to each other and $|\overline{a}| = |\overline{b}|$. If $|\overline{a} \times \overline{b}| = |\overline{a}|$, then the angle between the vectors $\overline{a} + \overline{b} + \left(\overline{a} \times \overline{b}\right)$ and $\overline{a}$ is equal to

(25th july 2021-I)

1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 3) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ 4) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Key: 2

Sol: Given $\left(\overline{a}, \overline{b}\right) = \frac{\pi}{2}, |\overline{a}| = |\overline{b}|$

And $\left|\bar{a}\times\bar{b}\right| = |\bar{a}|$

$|\bar{a}||\bar{b}|\sin\left(\bar{a},\bar{b}\right) = |\bar{a}|$

$\left|\bar{b}\right| = 1 = |\bar{a}|$

Now $\bar{a},\bar{b}$ are perpendicular unit vectors

Let $\bar{a} = \bar{i}, \bar{b} = \hat{j} \quad \Rightarrow \bar{a}\times\bar{b} = \hat{k}$

Now $\bar{a}+\bar{b}+\bar{a}\times\bar{b} = \hat{i}+\hat{j}+\hat{k}$ *and* $\bar{a} = \hat{i}$

$Cos\theta = \dfrac{\left(\hat{i}+\hat{j}+\hat{k}\right).\hat{i}}{\sqrt{3}(1)} = \dfrac{1}{\sqrt{3}} \qquad \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

143. Let $\bar{a},\bar{b}$ *and* $\bar{c}$ be three unit vectors such that $\bar{a}+\bar{b}+\bar{c} = \bar{o}$. If $\lambda = \bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}$ and $\bar{d} = \bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}$, then the ordered pair $\left(\lambda,\bar{d}\right)$ is equal to

(7th Jan 2020-I)

1) $\left(\dfrac{3}{2}, 3\bar{b}\times\bar{c}\right)$ 2) $\left(\dfrac{-3}{2}, 3\bar{c}\times\bar{b}\right)$ 3) $\left(\dfrac{-3}{2}, 3\bar{a}\times\bar{c}\right)$ 4) $\left(\dfrac{-3}{2}, 3\bar{a}\times\bar{b}\right)$

Key: 4

SOL: $|\bar{a}| = \left|\bar{b}\right| = |\bar{c}| = 1$

$\left(\bar{a}+\bar{b}+\bar{c}\right)^2 = 0$

$|\bar{a}|^2+\left|\bar{b}\right|^2+|\bar{c}|^2+2\bar{a}.\bar{b}+2\bar{b}.\bar{c}+2\bar{c}.\bar{a} = 0$

$3+2\left(\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}\right) = 0$

$\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a} = \dfrac{-3}{2}$

$\bar{a}+\bar{b}+\bar{c} = \bar{o} \Rightarrow \bar{a}\times\bar{b} = \bar{b}\times\bar{c} = \bar{c}\times\bar{a}$

$\bar{d} = 3\left(\bar{a}\times\bar{b}\right)$

144. Let $\bar{a} = 3\hat{i}+2\hat{j}+x\hat{k}$ *and* $\bar{b} = \hat{i}-\hat{j}+\hat{k}$, for some real x. Then $\left|\bar{a}\times\bar{b}\right| = r$ is possible if

(8th April 2019-I)

1) $0 < r \le \sqrt{\dfrac{3}{2}}$ 2) $\sqrt{\dfrac{3}{2}} < r \le 3\sqrt{\dfrac{3}{2}}$ 3) $3\sqrt{\dfrac{3}{2}} < r < 5\sqrt{\dfrac{3}{2}}$ 4) $r \ge 5\sqrt{\dfrac{3}{2}}$

Key: 4

Sol: $\bar{a}\times\bar{b} = (x+2)\hat{i}+(x-3)\hat{j}-5\hat{k}$

$\left|\bar{a}\times\bar{b}\right| = \sqrt{(x+2)^2+(x-3)^2+25} \qquad = \sqrt{2x^2-2x+38}$

$$=\sqrt{2\left(x-\frac{1}{2}\right)^2+\frac{75}{2}}$$

So $\left|\bar{a}\times\bar{b}\right|\geq\sqrt{\frac{75}{2}}$ $\qquad r\geq 5\sqrt{\frac{3}{2}}$

C) Lagrange's identity in vectors.

145. If $|\bar{a}|=2, |\bar{b}|=5$ and $|\bar{a}\times\bar{b}|=8,$ then $|\bar{a}.\bar{b}|$ is equal to (25th july 2021-II)

1) 3 2) 4 3) 5 4) 6

Key: 4

Sol: $\left(\bar{a}\times\bar{b}\right)^2+\left(\bar{a}.\bar{b}\right)^2=|\bar{a}|^2|\bar{b}|^2$

$64+\left(\bar{a}.\bar{b}\right)^2=(4)(25)$ $\qquad \left(\bar{a}.\bar{b}\right)^2=36$ $\qquad |\bar{a}.\bar{b}|=6$

146. If $\hat{i},\hat{j},\hat{k}$ are unit orthonormal vectors and $\bar{a}$ is a vector of magnitude 2 units satisfying $\bar{a}\times\hat{i}=\hat{j}$ then $\bar{a}.\hat{i}=$

1) $\pm\sqrt{3}$ 2) $\pm\sqrt{2}$ 3) 0 4) 1

Key: 1

Sol: $\left(\bar{a}\times\hat{i}\right)^2+\left(\bar{a}.\hat{i}\right)^2=|\bar{a}|^2|\hat{i}|^2$

$|\hat{j}|^2+\left(\bar{a}.\hat{i}\right)^2=4(1)$ $\left(\bar{a}.\hat{i}\right)^2=4-1=3$ $\qquad \bar{a}.\hat{i}=\pm\sqrt{3}$

147. Let $\bar{a}=\hat{i}-\hat{j}+2\hat{k}$ and Let $\bar{b}$ be a vector such that $\bar{a}\times\bar{b}=2\hat{i}-\hat{k}$ and $\bar{a}.\bar{b}=3$. Then the projection of $\bar{b}$ on the vector $\bar{a}-\bar{b}$ is (25th july 2022-I)

1) $\frac{2}{\sqrt{21}}$ 2) $2\sqrt{\frac{3}{7}}$ 3) $\frac{2}{3}\sqrt{\frac{7}{3}}$ 4) $\frac{2}{3}$

Key: 1

Sol: $\bar{a}=\hat{i}-\hat{j}+2\hat{k}\Rightarrow|\bar{a}|=\sqrt{6}$ $\qquad \bar{a}\times\bar{b}=2\hat{i}-\hat{k}\Rightarrow|\bar{a}\times\bar{b}|=\sqrt{5}$

$\bar{a}.\bar{b}=3$ $\qquad \left(\bar{a}\times\bar{b}\right)^2+\left(\bar{a}-\bar{b}\right)^2=|\bar{a}|^2|\bar{b}|^2$

$5+9=6|\bar{b}|^2$ $\qquad |\bar{b}|^2=\frac{7}{3}$

$|\bar{a}-\bar{b}|^2=|\bar{a}|^2-2\bar{a}.\bar{b}+|\bar{b}|^2$ $\qquad =6-6+\frac{7}{3}=\frac{7}{3}$

$\therefore$ The projection of $\bar{b}$ on the vector

$$\overline{a}-\overline{b}=\frac{\overline{b}.\left(\overline{a}-\overline{b}\right)}{\left|\overline{a}-\overline{b}\right|}=\frac{\overline{a}.\overline{b}-\left|\overline{b}\right|^2}{\left|\overline{a}-\overline{b}\right|}=\frac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}}=\frac{2}{3}\times\sqrt{\frac{3}{7}}$$

$$=\frac{2}{\sqrt{21}}$$

148. If $\overline{p}=\overline{a}-\overline{b},\overline{q}=\overline{a}+\overline{b},\left|\overline{a}\right|=\left|\overline{b}\right|=t$, then $\left|\overline{p}\times\overline{q}\right|$

1) $\sqrt[2]{t^2-\left(\overline{a}.\overline{b}\right)^2}$ 2) $\sqrt[3]{t^2-\left(\overline{a}.\overline{b}\right)^2}$ 3) $\sqrt{t^4-\left(\overline{a}.b\right)^2}$ 4) $2\sqrt{t^4-\left(\overline{a}.\overline{b}\right)^2}$

Key: 4

Sol: $\overline{p}\times\overline{q}=\left(\overline{a}-\overline{b}\right)\times\left(\overline{a}+\overline{b}\right)=2\left(\overline{a}\times\overline{b}\right)$

$$\left|\overline{p}\times\overline{q}\right|^2=4\left(\overline{a}\times\overline{b}\right)^2$$

$$=4\left(\left|\overline{a}\right|^2\left|\overline{b}\right|^2-\left(\overline{a}.\overline{b}\right)^2\right)$$

$$=4\left(t^4-\left(\overline{a}.\overline{b}\right)^2\right)$$

$$\left|\overline{p}\times\overline{q}\right|^2=2\sqrt{t^4-\left(\overline{a}.\overline{b}\right)^2}$$

149. If $\left|\overline{a}\right|=2,\left|\overline{b}\right|=7$ and $\overline{a}\times\overline{b}=3\hat{i}+2\hat{j}+6\hat{k}$, then $\overline{a}.\overline{b}=$

1) $\sqrt{147}$ 2) $\sqrt{145}$ 3) $\sqrt{143}$ 4) $\sqrt{142}$

Key: 2

Sol: $\left|\overline{a}\right|=2,\left|\overline{b}\right|=7,\left|\overline{a}\times\overline{b}\right|=\sqrt{9+4+36}=7$

$$\left(\overline{a}\times\overline{b}\right)^2\pm\left(\overline{a}-\overline{b}^2\right)=\left|\overline{a}\right|^2\left|\overline{b}\right|^2$$

$$49+\left(\overline{a}.\overline{b}\right)^2=(4)\times(49)$$

$$\left(\overline{a}.\overline{b}\right)^2=196-49=145$$

$$\overline{a}.\overline{b}=\sqrt{145}$$

150. $\left|\overline{a}\right|=\left|\overline{b}\right|=2,\overline{p}=\overline{a}+\overline{b},\overline{q}=\overline{a}-\overline{b}$, it $\left|\overline{p}\times\overline{q}\right|=2\left[k-\left(\overline{a}.\overline{b}\right)^2\right]^{1/2}$, then k=

1) 16 2) 8 3) 4 4) 1

Key: 1

Sol: $\left|\overline{p}\times\overline{q}\right|=\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}-\overline{b}\right)$

$=2\left(\overline{a}\times\overline{b}\right)$ $\left|\overline{p}\times\overline{q}\right|^2=4\left(\overline{a}\times\overline{b}\right)^2$ $=4\left\{\left|\overline{a}\right|^2\left|\overline{b}\right|^2-\left(\overline{a}.\overline{b}\right)^2\right\}$

$$= 4\left\{(4).(4) - \left(\bar{a}.\bar{b}\right)^2\right\}$$

$$\left|\bar{p}\times\bar{q}\right| = 2\sqrt{16-\left(\bar{a}.\bar{b}\right)^2}\ , \quad K = 16$$

SCALAR TRIPLE PRODUCT

Exercise-1

CONCEPT 1: Definition of Scalar triple product

151. Let $\bar{a} = \bar{i} - \bar{k},\ \bar{b} = \alpha\bar{i} + \bar{j} + (1-\alpha)\bar{k}$ and $\bar{c} = \beta\bar{i} + \alpha\bar{j} + (1+\alpha-\beta)\bar{k}$. Then $[\bar{a}\ \bar{b}\ \bar{c}]$ depends on [AIEEE 2005]

1)Only α 2) Only β 3) Both α and β 4) Neither α nor β

Key: 4

Sol: $\begin{vmatrix} 1 & 0 & -1 \\ \alpha & 1 & 1-\alpha \\ \beta & \alpha & 1+\alpha-\beta \end{vmatrix} = 1$

152. Let λ be the point of local maxima of $f(x)$ $f(x) = \left(\bar{a}\times\bar{b}\right).\bar{c}$, where $\bar{a} = x\bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = -2\bar{i} + x\bar{j} - \bar{k}$ and $\bar{c} = 7\bar{i} - 2\bar{j} + x\bar{k}$. Then the value of $\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}$ at $x = \lambda$, is

[Model: 2020, 4th Sep Shift 1]

1)4 2) 22 3) -22 4) -4

Key: 3

Sol: $f(x) = \left(\bar{a}\times\bar{b}\right).\bar{c} = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x^3 - 27x + 26$

$f'(x) = 0 \Rightarrow x = \pm 3$

$f''(x) = 6x \text{ and } f''(-3) < 0 \Rightarrow \lambda = -3$

Consider $\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a} = 3x - 13$. At $x = -3,\ \left(\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}\right)_{x=-3} = -22$

153. Let a vector $\bar{a}$ be coplanar with vectors $\bar{b} = 2\bar{i} + \bar{j} + \bar{k}$ and $\bar{c} = \bar{i} - \bar{j} + \bar{k}$. If $\bar{a}$ is perpendicular to $\bar{d} = 3\bar{i} + 2\bar{j} + 6\bar{k}$ and $|\bar{a}| = \sqrt{10}$. Then a possible value of $\left[\bar{a}\ \bar{c}\ \bar{b}\right] + \left[\bar{a}\ \bar{b}\ \bar{d}\right] - \left[\bar{a}\ \bar{d}\ \bar{c}\right]$ is

[Model: 2021, 22nd July Shift 2]

1) -2 2) -40 3) -42 4) 38

Key: 3

Sol: Let $\bar{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$. $|\bar{a}| = \sqrt{10} \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 10 \rightarrow (1)$

$\bar{a}.\left(\bar{b}\times\bar{c}\right) = 0 \text{ and } \bar{a}.\bar{d} = 0 \Rightarrow \alpha = 0, \beta = -21k\ \&\ \gamma = 7k, \text{ for some scalar } k$

$(1) \Rightarrow k = \pm\frac{1}{7}.\ \therefore \bar{a} = \pm\left(-3\hat{j} + \hat{k}\right)$

Consider $\left[\bar{a}\ \bar{c}\ \bar{b}\right] + \left[\bar{a}\ \bar{b}\ \bar{d}\right] - \left[\bar{a}\ \bar{d}\ \bar{c}\right] = \left[\bar{a}\ \bar{b}+\bar{c}\ \bar{d}\right] = \pm 42$

154. Let $\bar{a}=\bar{i}+\bar{j}+\bar{k}$, $\bar{b}$ and $\bar{c}=\bar{j}-\bar{k}$ be three vectors such that $\bar{a}\times\bar{b}=\bar{c}$ and $\bar{a}.\bar{b}=1$. If the length of the projection vector of the vector $\bar{b}$ on the vector $\bar{a}\times\bar{c}$, is l, then the value of $6l^2$, is

[Model: 2021, 27th July Shift 1]

1) 1 2) 2 3) 3 4) 4

Key: 4Sol: $(\bar{a}\times\bar{b}).\bar{c}=2 \Rightarrow l=\frac{\bar{b}.(\bar{a}\times\bar{c})}{|\bar{a}\times\bar{c}|}=-\frac{2}{\sqrt{6}} \Rightarrow 6l^2=4$

155. Let $\bar{v}=\alpha\hat{i}+2\hat{j}-3\hat{k}, \bar{w}=2\alpha\hat{i}+\hat{j}-\hat{k}$, and $\bar{u}$ be a vector such that $|\bar{u}|=\alpha>0$. If the minimum value of the scalar triple product $[\bar{u}\ \bar{v}\ \bar{w}]$ is $-\alpha\sqrt{3401}$, and $|\bar{u}.\hat{i}|^2=\frac{m}{n}$, where m and n are coprime natural numbers, then $m+n$ is equal to

(2023, 1st Feb. Shift 1)

1) 3501 2) 3401 3) 3400 4) 3500

Key: 1

Sol: $Min.\{\bar{u}.(\bar{v}\times\bar{w})\}=Min.\{|\bar{u}||\bar{v}\times\bar{w}|\cos\theta\}=-\alpha\sqrt{3401}$. Take

$\cos\theta=-1$ & $|\bar{u}|=\alpha(>0) \Rightarrow |\bar{v}\times\bar{w}|=\sqrt{3401} \Rightarrow \alpha=10$

$\therefore \bar{u}=\lambda(\bar{v}\times\bar{w})=\lambda(\hat{i}-5\alpha\hat{j}-3\alpha\hat{k}) \Rightarrow \alpha^2=\lambda^2(1+25\alpha^2+9\alpha^2) \Rightarrow \lambda^2=\frac{100}{3401}$

Consider $|\bar{u}.\hat{i}|^2=\lambda^2=\frac{100}{3401}=\frac{m}{n} \Rightarrow m+n=100+3401=3501$.

156. Let $\bar{a}=\bar{i}-\alpha\bar{j}+\beta\bar{k}$, $\bar{b}=3\bar{i}+\beta\bar{j}-\alpha\bar{k}$ and $\bar{c}=-\alpha\bar{i}-2\bar{j}+\bar{k}$, $\alpha,\beta\in\mathbb{Z}$. If $\bar{a}.\bar{b}=-1$ and $\bar{b}.\bar{c}=10$, then $(\bar{a}\times\bar{b}).\bar{c}$ is equal to

[2021, 27th July Shift 2]

1) 9 2) 6 3) 3 4) 1

Key: 1

Sol: $\bar{a}.\bar{b}=3-\alpha\beta-\alpha\beta=-1 \Rightarrow 2\alpha\beta=4 \Rightarrow \alpha\beta=2$

And $\bar{b}.\bar{c}=10 \Rightarrow -3\alpha-2\beta-\alpha=10 \Rightarrow -4\alpha-2\beta=10 \Rightarrow 2\alpha+\beta=-5$

$\therefore \alpha=-2$ & $\beta=-1$

Consider $(\bar{a}\times\bar{b}).\bar{c}=\begin{vmatrix} 1 & -\alpha & \beta \\ 3 & \beta & -\alpha \\ -\alpha & -2 & 1 \end{vmatrix}=1(\beta-2\alpha)=\alpha(3-\alpha^2)+\beta(-6+\alpha\beta)$

$=1(3)-2(-1)-1(-6+2)=3+2+4=9$

157. If $\bar{a},\bar{b}$ be non-zero and non-collinear vectors, then $[\bar{a}\ \bar{b}\ \bar{i}]\bar{i}+[\bar{a}\ \bar{b}\ \bar{j}]\bar{j}+[\bar{a}\ \bar{b}\ \bar{k}]\bar{k}$ is equal to

1) $\bar{a}+\bar{b}$ 2) $\bar{a}\times\bar{b}$ 3) $\bar{a}-\bar{b}$ 4) $\bar{b}\times\bar{a}$

Key: 2

Sol: $((\bar{a}\times\bar{b}).\bar{i})\bar{i}+((\bar{a}\times\bar{b}).\bar{j})\bar{j}+((\bar{a}\times\bar{b}).\bar{k})\bar{k}=(\bar{a}\times\bar{b})$

158. If $\bar{a},\bar{b},\bar{c}$ be the three vectors such that $\bar{a}\times\bar{b}=\bar{c}$ and $\bar{b}\times\bar{c}=\bar{a}$, then

a) $\bar{a},\bar{b},\bar{c}$ are orthogonal in pairs

b) $|\bar{a}|=|\bar{b}|=|\bar{c}|=1$

c) $|\bar{a}|=|\bar{b}|=|\bar{c}|\neq 1$

d) $|\bar{a}|\neq|\bar{b}|\neq|\bar{c}|$

Key: 1

Sol: $\bar{a}\times\bar{b}=\bar{c}$ *and* $\bar{b}\times\bar{c}=\bar{a}\Rightarrow \bar{a}.\bar{c}=0\ \&\ \bar{b}.\bar{c}=0\ \&\ \bar{a}.\bar{b}=0$

and $\left[\bar{a}\bar{b}\bar{c}\right]=|\bar{c}|^2=|\bar{a}|^2$

and $\left[\bar{a}\bar{b}\bar{c}\right]=|\bar{a}\times\bar{b}|^2=|\bar{a}|^2|\bar{b}|^2$ & $\left[\bar{a}\bar{b}\bar{c}\right]=|\bar{b}\times\bar{c}|^2=|\bar{b}|^2|\bar{c}|^2$

CONCEPT 2: ScalarTripleProduct Of 3 Mutually Perpendicular Vectors

159. If $\bar{a}$ and $\bar{b}$ are two vectors such that $\bar{a}.\bar{b}=0$ *and* $|\bar{a}\times\bar{b}|=3$, then the value of $\left[\bar{a}\ \bar{b}\ \bar{a}\times\bar{b}\right]$, is

1) 9 2) 3 3) 0 4) 1

Key: 1

Sol: $\bar{a}.\bar{b}=0\ \&\ |\bar{a}\times\bar{b}|=3$. Consider $\left[\bar{a}\ \bar{b}\ \bar{a}\times\bar{b}\right]=\left(\bar{a}\times\bar{b}\right).\left(\bar{a}\times\bar{b}\right)=|\bar{a}\times\bar{b}|^2=3^2=9$

CONCEPT 3: Conditions for Scalar Triple Product to be zero

160. Let $\bar{a}=\hat{i}-\hat{j};\bar{b}=\hat{j}-\bar{k};\bar{c}=\hat{k}-\hat{i}$. If $\bar{d}$ is a unit vector such that $\bar{d}.\bar{a}=0=\left[\bar{d}\ \bar{b}\ \bar{c}\right]$, then $\bar{d}$ is

1) $\pm\frac{1}{\sqrt{6}}\left(\bar{i}+\bar{j}-2\bar{k}\right)$ 2) $\pm\frac{1}{\sqrt{6}}\left(\bar{i}-\bar{j}+2\bar{k}\right)$

3) $\pm\frac{1}{\sqrt{6}}\left(\bar{i}-\bar{j}-2\bar{k}\right)$ 4) $\pm\frac{1}{\sqrt{6}}\left(\bar{i}+\bar{j}+2\bar{k}\right)$

Key: 1

Sol: Assume that $\bar{d}=\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k}$

$\bar{d}.\bar{a}=\alpha-\beta=0\rightarrow(1)$

$$\left[\bar{d}\ \bar{b}\ \bar{c}\right]=0\Rightarrow\begin{vmatrix}\alpha & \beta & \gamma\\ 0 & 1 & -1\\ -1 & 0 & 1\end{vmatrix}=0\Rightarrow\alpha(1)-\beta(-1)+\gamma(1)=0\Rightarrow\alpha+\beta+\gamma=0\rightarrow(2)$$

$|\bar{d}|=1\Rightarrow\alpha^2+\beta^2+\gamma^2=1\rightarrow(3)$

$(1)\Rightarrow\alpha=\beta=k(say);(2)\Rightarrow\gamma=-k-k=-2k$;

$(3)\Rightarrow k^2+k^2+4k^2=1\Rightarrow 6k^2=1\Rightarrow k=\pm\frac{1}{\sqrt{6}}$

$\therefore\ \bar{d}=k\bar{i}+k\bar{j}-2k\bar{k}=\pm\frac{1}{\sqrt{6}}\left(\bar{i}+\bar{j}-2\bar{k}\right)$

CONCEPT 4: Right handed system and Left handed system

161. If $\bar{a}=\alpha\bar{i}+3\bar{j}-5\bar{k};\bar{b}=\bar{i}+\bar{k}$ *and* $\bar{c}=3\alpha\bar{i}-3\bar{j}+\bar{k}$ and given that the vectors $\bar{a},\bar{b},\bar{c}$ form a right-handed system, then range of α is

1) $(-1,1)$ 2) $(-1,2)$ 3) $(-1,\infty)$ 4) $(1,4)$

Key: 3

Sol: $\left[\bar{a}\,\bar{b}\,\bar{c}\right]>0 \Rightarrow \begin{vmatrix} \alpha & 3 & -5 \\ 1 & 0 & 1 \\ 3\alpha & 3 & 1 \end{vmatrix}>0 \Rightarrow \alpha(3)-3(1-3\alpha)-5(-3)>0 \Rightarrow 12\alpha+12>0 \qquad \Rightarrow \alpha>-1$

162. If vectors $\bar{a}=2\hat{i}+3\hat{j}+4\hat{k}, \bar{b}=\hat{i}+\hat{j}+5\hat{k}$ and $\bar{c}$ form a left-handed system, then $\bar{c}$ is

1) $-11\hat{i}+6\hat{j}+\hat{k}$ 2) $11\hat{i}-6\hat{j}-\hat{k}$ 3) $11\hat{i}+6\hat{j}-\hat{k}$ 4) $11\hat{i}-6\hat{j}+\hat{k}$

Key: 1

Sol: $\left[\bar{a}\,\bar{b}\,\bar{c}\right]<0 \Rightarrow \bar{c}.\left(\bar{a}\times\bar{b}\right)<0$, where $\bar{a}\times\bar{b}=\begin{vmatrix} \bar{i} & \bar{j} & k \\ 2 & 0 & 4 \\ 1 & 1 & 5 \end{vmatrix}=\bar{i}\,(11)-\bar{j}\,(6)+\bar{k}\,(-1)$[Verify]

CONCEPT 5: Coplanarity of vectors

163. Let the vectors $(2+\alpha+\beta)\bar{i}+(\alpha+2\beta+\gamma)\bar{j}-(\beta+\gamma)\bar{k}$, $(1+\beta)\bar{i}+2\beta\bar{j}-\beta\bar{k}$ and $(2+\beta)\bar{i}+2\beta\bar{j}+(1-\beta)\bar{k}$, where $\alpha,\beta,\gamma\in\mathbb{R}$, be coplanar. Then which of the following is true? [Model: 2021, 25th July Shift 1]

1) $2\beta=\alpha+\gamma$ 2) $\alpha-\beta=2\gamma$ 3) $\alpha-2\beta=\gamma$ 4) $\alpha+\beta=3\gamma$

Key: 1

Sol: $\begin{vmatrix} 2+\alpha+\beta & \alpha+2\beta+\gamma & -\beta-\gamma \\ 1+\beta & 2\beta & -\beta \\ 2+\beta & 2\beta & 1-\beta \end{vmatrix}=0$

Apply $R_1\to R_1-R_2$

$\begin{vmatrix} 1+\beta & \alpha+\gamma & -\gamma \\ 1+\beta & 2\beta & -\beta \\ 2+\beta & 2\beta & 1-\beta \end{vmatrix}=0$

Apply $R_3\to R_3-R_2$

$\begin{vmatrix} 1+\alpha & \alpha+\gamma & -\gamma \\ 1+\beta & 2\beta & -\beta \\ 1 & 0 & 1 \end{vmatrix}=0 \Rightarrow 1\left((\alpha+\gamma)(-\beta)+2\beta\gamma\right)+1\left((1+\alpha)2\beta-(1+\beta)(\alpha+\gamma)\right)=0$

$\Rightarrow -\alpha\beta+\beta\gamma+2\beta+2\alpha\beta-\alpha-\gamma-\alpha\beta-\beta\gamma=0 \Rightarrow 2\beta-\alpha-\gamma=0$

164. If the four points, whose position vectors are $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ *and* $5\hat{i}-2\alpha\hat{j}+4\hat{k}$, are coplanar, then α is equal to (2023, 25th Jan. Shift 2)

1) $\frac{107}{17}$ 2) $\frac{73}{17}$ 3) $-\frac{107}{17}$ 4) $-\frac{73}{17}$

Key: 2

Sol: $\left[\overline{AB}\ \overline{AC}\ \overline{AD}\right]=\begin{vmatrix}-2 & 6 & -3\\ -5 & 3 & 1\\ 2 & 4-2\alpha & 2\end{vmatrix}=0$

$\Rightarrow -2(6-4+2\alpha)-6(-12)-3(-20+10\alpha-6)=0 \Rightarrow \alpha=\frac{146}{34}=\frac{73}{17}$.

165. Let O be the origin. Let $\overline{OP}=x\bar{i}+y\bar{j}-\bar{k}$ and $\overline{OQ}=-\bar{i}+2\bar{j}+3x\bar{k},\ x,y\in\mathbb{R},\ x>0,$ be such that $|PQ|=\sqrt{20}$ and the vector $\overline{OP}\perp\overline{OQ}$. If $\overline{OR}=3\bar{i}+z\bar{j}-7\bar{k},\ z\in\mathbb{R}$, is coplanar with $\overline{OP}$ and $\overline{OQ}$, then the value of $x^2+y^2+z^2$ is equal to [2021, 17th Mar Shift 2]

1) 9 2) 7 3) 2 4) 1

Key : 1

Sol: $|\overline{PQ}|=\sqrt{20}\Rightarrow x=\pm1$. $x>0\Rightarrow x=1$

$\therefore y=2$, because $\overline{OP}\perp\overline{OQ}\Rightarrow y=2x$

Now $\left[\overline{OP}\ \overline{OQ}\ \overline{OR}\right]=0\Rightarrow\begin{vmatrix}x & y & -1\\ -1 & 2 & 3x\\ 3 & z & -7\end{vmatrix}=0\Rightarrow x^2+y^2+z^2=9$

166. If $(1,5,35),(7,5,5),(1,\lambda,7)$ and $(2\lambda,1,2)$ are coplanar, then the sum of all possible values of λ, is [2021, 26th Feb Shift 1]

1) $-\frac{39}{5}$ b) $-\frac{44}{5}$ c) $\frac{39}{5}$ d) $\frac{44}{5}$

Key: 4

Sol: $\left[\overline{PQ}\ \overline{PR}\ \overline{PS}\right]=0\Rightarrow\begin{vmatrix}6 & 0 & -30\\ 0 & \lambda-5 & -28\\ 2\lambda-1 & -4 & -33\end{vmatrix}=0$

$\Rightarrow$ Sum of all possible values of λ ,is $\frac{44}{5}$.

167. Let three vectors $\bar{a},\bar{b}$ and $\bar{c}$ be such that $\bar{c}$ is coplanar with $\bar{a}$ and $\bar{b}$, $\bar{a}.\bar{c}=7$ and $\bar{b}$ is perpendicular to $\bar{c}$, where $\bar{a}=-\bar{i}+\bar{j}+\bar{k}$ and $\bar{b}=2\bar{i}+\bar{k}$, then the value of $2|\bar{a}+\bar{b}+\bar{c}|^2$, is [2021, 24th Feb. Shift 1]

1) 75 2) 85 3) 76 4) 86

Key: 1

Sol: $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=0$, let $\bar{c}=\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k}$ such that $\bar{a}.\bar{c}=7$ and $\bar{b}.\bar{c}=0$

$\Rightarrow\begin{vmatrix}\alpha & \beta & \gamma\\ -1 & 1 & 1\\ 2 & 0 & 1\end{vmatrix}=0\Rightarrow\alpha(1)-\beta(-3)+\gamma(-2)=0\Rightarrow\alpha+3\beta-2\gamma=0\rightarrow(1)$

$\& \bar{a}.\bar{c} = 7 \Rightarrow -\alpha + \beta + \gamma = 7 \rightarrow (2)$

$\& \bar{b}.\bar{c} = 0 \Rightarrow 2\alpha + \gamma = 0 \rightarrow (3)$

By solving, we get $\alpha = \frac{-3}{2}, \beta = \frac{5}{2} \& \gamma = 3 \quad \therefore 2\left|\bar{a} + \bar{b} + \bar{c}\right|^2 = 75$

168. If the vectors $\bar{p} = (a+1)\bar{i} + a\bar{j} + a\bar{k}$; $\bar{q} = a\bar{i} + (a+1)\bar{j} + a\bar{k}$ and $\bar{r} = a\bar{i} + a\bar{j} + (a+1)\bar{k}$, $(a \in \mathbb{R})$ are coplanar and $3(\bar{p}.\bar{q})^2 - \lambda|\bar{r} \times \bar{q}|^2 = 0$, then the value λ, is

[2020, 9th Jan. Shift 1]

1) 0 2) 1 3) 2 4) 4

Key: 2

Sol: $[\bar{p}\,\bar{q}\,\bar{r}] = 0 \Rightarrow a = \frac{-1}{3}$. Consider $\bar{r} \times \bar{q} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a & a & a+1 \\ a & a+1 & a \end{vmatrix}$

$\Rightarrow \bar{r} \times \bar{q} = -(1+2a)\bar{i} + a\bar{j} + a\bar{k}$ & $\bar{p}.\bar{q} = a(3a+2)$

Consider $\lambda = \frac{3(\bar{p}.\bar{q})^2}{|\bar{r} \times \bar{q}|^2} = 1$

169. Let $\alpha \in \mathbb{R}$ and the three vectors $\bar{a} = \alpha\bar{i} + \bar{j} + 3\bar{k}, \bar{b} = 2\bar{i} + \bar{j} - \alpha\bar{k}$ and $\bar{c} = \alpha\bar{i} - 2\bar{j} + 3\bar{k}$. Then the set $S = \{\alpha : \bar{a}, \bar{b} \text{ and } \bar{c} \text{ are coplanar}\}$

[2019, 12th Apr. Shift 2]

a) is singleton set

b) is empty set

c) contains exactly two positive numbers

d) contains exactly two numbers only, one of which is positive.

Key: 2

Sol: $\left[\bar{a}\,\bar{b}\,\bar{c}\right] = 0 \Rightarrow \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0 \Rightarrow \alpha(3-2\alpha) - 1(6+\alpha^2) + 3(-4-\alpha) = 0$

$\Rightarrow 3\alpha - 2\alpha^2 - 6 - \alpha^2 - 12 - 3\alpha = 0 \Rightarrow -3\alpha^2 - 18 = 0 \Rightarrow \alpha^2 = -6$

which is not possible. $\therefore \alpha \in \phi$

170. Let $\bar{a} = \bar{i} + 2\bar{j} + 4\bar{k}$, $\bar{b} = \bar{i} + \lambda\bar{j} + 4\bar{k}$ and $\bar{c} = 2\bar{i} + 4\bar{j} + (\lambda^2 - 1)\bar{k}$ be coplanar vectors. Then the non-zero vector $\bar{a} \times \bar{c}$ is [2019, 11th Jan. Shift 1]

1) $-10\bar{i} + 5\bar{j}$ 2) $-10\bar{i} - 5\bar{j}$ 3) $-14\bar{i} - 5\bar{j}$ 4) $-14\bar{i} + 5\bar{j}$

Key: 1

Sol: $\left[\bar{a}\,\bar{b}\,\bar{c}\right] = 0 \Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0 \quad \Rightarrow 1(\lambda^3 - \lambda - 16) - 2(\lambda^2 - 1 - 8) + 4(4 - 2\lambda) = 0$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0 \Rightarrow \lambda(\lambda^2 - 9) - 2(\lambda^2 - 9) = 0 \Rightarrow (\lambda - 2)(\lambda - 3)(\lambda + 3) = 0$

$\Rightarrow \lambda = 2\ (or)\ \lambda = 3\ (or)\ \lambda = -3$

If $\lambda = 2$, then $\bar{a} \times \bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = \bar{i}(-10) - \bar{j}(-5) + \bar{k}(0) = -10\bar{i} + 5\bar{j}$

171. The sum of the distinct real values of μ, for which the vectors $\mu\hat{i} + \hat{j} + \hat{k}, \hat{i} + \mu\hat{j} + \hat{k}, \hat{i} + \hat{j} + \mu\hat{k}$ are coplanar, is [2019, 12th Jan. Shift 1]

1) 2 2) 0 3) 1 4) -1

Key:4

Sol: $\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0 \Rightarrow \mu(\mu^2 - 1) - 1(\mu - 1) + 1(1 - \mu) = 0 \Rightarrow (\mu - 1).(\mu(\mu + 1) - 2) = 0$

$\Rightarrow (\mu - 1)(\mu^2 + \mu - 2) = 0 \Rightarrow \mu = 1\ (or)\ \mu = -2$

172. If the vectors $p\hat{i} + \hat{j} + \hat{k}, \hat{i} + q\hat{j} + \hat{k}\ and\ \hat{i} + \hat{j} + r\hat{k}$, where $p \neq q \neq r \neq 1$, are coplanar, then the value of $pqr - (p + q + r)$, is [AIEEE 2011]

1) -2 2) 2 3) 0 4) -1

Key:1

Sol: $\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0 \Rightarrow p(qr - 1) - 1(r - 1) + 1(1 - q) = 0 \Rightarrow pqr - p - q - r + 2 = 0$

173. If $\bar{u}, \bar{v}\ and\ \bar{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\bar{u}\ p\bar{v}\ p\bar{w}] - [p\bar{v}\ \bar{w}\ q\bar{u}] - [2\bar{w}\ q\bar{v}\ q\bar{u}] = 0$ holds for [AIEEE 2009]

a) Exactly two values of (p, q).

b) More than two, but not all values of (p, q).

c) All values of (p, q).

d) Exactly one value of (p, q).

Key:4

Sol: $(3p^2 - pq + 2q^2)[\bar{u}\ \bar{v}\ \bar{w}] = 0 \Rightarrow 3p^2 - pq + 2q^2 = 0$

$\therefore (p, q) = (0, 0)$ is the only solution

174. The vector $\bar{a} = \alpha\bar{i} + 2\bar{j} + \beta\bar{k}$ lies in the plane of the vectors $\bar{b} = \hat{i} + \hat{j}\ \&\ \bar{c} = \hat{j} + \hat{k}$ and bisects the angle between $\bar{b}$ and $\bar{c}$. Then which one of the following gives possible values of $\alpha\ and\ \beta$? [AIEEE 2008]

1) $\alpha = 1, \beta = 2$ 2) $\alpha = 1, \beta = 1$ 3) $\alpha = 2, \beta = 1$ 4) $\alpha = 2, \beta = 2$

Key:2

Sol: The equation of angular bisector of $\bar{b}$ *and* $\bar{c}$, is $\bar{r}=\lambda\left(\frac{\bar{b}}{|\bar{b}|}+\frac{\bar{c}}{|\bar{c}|}\right)=\frac{\lambda}{\sqrt{2}}\left(\bar{i}+2\bar{j}+\bar{k}\right)$

Now $\bar{a}$ can be $\bar{a}=\bar{b}+\mu\bar{c}\Rightarrow\frac{\lambda}{\sqrt{2}}\left(\bar{i}+2\bar{j}+\bar{k}\right)=\left(\bar{i}+\bar{j}\right)+\mu\left(\bar{j}+\bar{k}\right)$

$\Rightarrow\frac{\lambda}{\sqrt{2}}=1;\ \sqrt{2}\lambda=\mu+1;\frac{\lambda}{\sqrt{2}}=\mu\Rightarrow\lambda=\sqrt{2};\mu=1\quad\therefore\bar{r}=\bar{i}+2\bar{j}+\bar{k}$

$\therefore\alpha=1,\ \beta=1$

175. Let $\bar{a}=\hat{i}+\hat{j}+\hat{k},\ \bar{b}=\hat{i}-\hat{j}$ and $\bar{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vector $\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$, then x is equal to [AIEEE 2007]

1) 0 2) -4 3) 1 4) -2

Key:1

Sol: $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=0\Rightarrow\begin{vmatrix}1&1&1\\1&-1&0\\x&x-2&-1\end{vmatrix}=0\Rightarrow1(-1-x+2)+1(-1-x)=0$

$\Rightarrow-1-x+2-1-x=0\Rightarrow x=0$

176. Let $\bar{a},\bar{b}$ *and* $\bar{c}$ be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k},\ \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is [AIEEE 2005]

a) The Arithmetic mean of a *and* b.

b) The Geometric mean of a *and* b.

c) The Harmonic mean of a *and* b.

d) 0

Key:2

Sol: $\begin{vmatrix}a&a&c\\1&0&1\\c&c&b\end{vmatrix}=0\Rightarrow a(-c)-a(b-c)+c(c)=0\Rightarrow c^2=ab$

177. If $\bar{a},\bar{b}$ *and* $\bar{c}$ are non-coplanar vectors and $\lambda\in\mathbb{R}$, then $\left[\lambda\left(\bar{a}+\bar{b}\right)\ \lambda^2\bar{b}\ \lambda\bar{c}\right]=\left[\bar{a}\ \bar{b}+\bar{c}\ \bar{b}\right]$, for [AIEEE 2005]

a) Exactly two values of λ.

b) Exactly three values of λ.

c) No value of λ.

d) Exactly one value of λ.

Key:3

Sol: $\left[\lambda\bar{a}\ \lambda^2\bar{b}\ \lambda\bar{c}\right]+\left[\lambda\bar{b}\ \lambda^2\bar{b}\ \lambda\bar{c}\right]=\left[\bar{a}\,\bar{c}\,\bar{b}\right]\Rightarrow\lambda^4\left[\bar{a}\,\bar{b}\,\bar{c}\right]=-\left[\bar{a}\,\bar{b}\,\bar{c}\right]\Rightarrow\lambda^4=-1,$

which is not possible.

178. If $\bar{a}, \bar{b}$ *and* $\bar{c}$ are non-coplanar vectors and $\lambda \in \mathbb{R}$, then the vectors $\bar{a}+2\bar{b}+3\bar{c},\ \lambda\bar{b}+4\bar{c}$ *and* $(2\lambda-1)\bar{c}$ are non-coplanar, for [AIEEE 2004]

a) All values of λ .
b) All except one value of λ .
c) All except two values of λ .
d) No values of λ .

Key:3

Sol: Given that $\left[\bar{a}\,\bar{b}\,\bar{c}\right] \neq 0$; Consider $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda-1 \end{vmatrix} = 1\left(2\lambda^2-\lambda\right) \neq 0$

$\Rightarrow \lambda(2\lambda-1) \neq 0 \Rightarrow \lambda \neq 0\ \&\ \lambda \neq \frac{1}{2}$ $\quad \therefore \lambda \in R - \left\{0, \frac{1}{2}\right\}$

CONCEPT 6: Nature of Specific vectors

179. $\left[\bar{a}\times\bar{b}\ \ \bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}\right] = \lambda\left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$, then λ is equal to [JEE Mains 2014]

1) 0 2) 1 3) 2 4) 3

Key:2

Sol: $\left[\bar{a}\times\bar{b}\ \ \bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}\right] = \left(\left(\bar{a}\times\bar{b}\right)\times\left(\bar{b}\times\bar{c}\right)\right).\left(\bar{c}\times\bar{a}\right) = \left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$

180. If $\bar{a}, \bar{b}, \bar{c}$ are any three non-coplanar vectors, then the equation $\left[\bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}\ \ \bar{a}\times\bar{b}\right]x^2 + \left[\bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\ \ \bar{c}+\bar{a}\right]x + 1 + \left[\bar{b}-\bar{c}\ \ \bar{c}-\bar{a}\ \ \bar{a}-\bar{b}\right] = 0$ has roots which are

1) Real and distinct 2) Irrational 3) Equal 4) Imaginary

Key:3 Sol: $x^2+2x+1+\begin{vmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 0 \Rightarrow x^2+2x+1+\left(-1(-1)-1(1)\right) = 0$

$\Rightarrow x^2+2x+1=0 \Rightarrow (x+1)^2 = 0 \Rightarrow x=-1$

181. Let $\bar{a} = \hat{i}-\hat{k},\ \bar{b} = x\hat{i}+\hat{j}+(1-x)\hat{k},\ \bar{c} = y\hat{i}+x\hat{j}-(1+x+y)\hat{k}$. Then $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$ depends on

1) Only x 2) Only y 3) neither x nor y 4) both x and y

Key:1

Sol: $\begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & -1-x-y \end{vmatrix} = 1\left(-1-x-y-x+x^2\right) - 1\left(x^2-y\right) = \cancel{x^2}-2x-\cancel{y}-1-\cancel{x^2}+\cancel{y}$

$= -2x-1$

CONCEPT 7: Vectors based on any three non-coplanar vectors

182. If $\bar{u}, \bar{v}$ *and* $\bar{w}$ are three non-coplanar vectors, then $\left(\bar{u}+\bar{v}-\bar{w}\right).\left(\left(\bar{u}-\bar{v}\right)\times\left(\bar{v}-\bar{w}\right)\right)$ is equal to

1) $\bar{u}.\left(\bar{v}\times\bar{w}\right)$ 2) $\bar{u}.\left(\bar{w}\times\bar{v}\right)$ 3) 0 4) None of these

Key:1

Sol: $\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} [\bar{u}\,\bar{v}\,\bar{w}] = \bar{u}.(\bar{v}\times\bar{w})$

183. If $\bar{\alpha} = p(\bar{b}\times\bar{c}) + q(\bar{c}\times\bar{a}) + r(\bar{a}\times\bar{b})$ and $\bar{\alpha}.(\bar{a}+\bar{b}+\bar{c}) = 1,$ then $[\bar{a}\,\bar{b}\,\bar{c}]$ is

1) $p+q+r$ 2) $\dfrac{1}{p+q+r}$ 3) $2(p+q+r)$ 4) $\dfrac{2}{p+q+r}$

Key:2

Sol: $\bar{\alpha}.\bar{a} = p[\bar{a}\,\bar{b}\,\bar{c}]$; $\bar{\alpha}.\bar{b} = q[\bar{a}\,\bar{b}\,\bar{c}]$; $\bar{\alpha}.\bar{c} = r[\bar{a}\,\bar{b}\,\bar{c}]$

$$\Rightarrow \bar{\alpha}.(\bar{a}+\bar{b}+\bar{c}) = (p+q+r)[\bar{a}\,\bar{b}\,\bar{c}] \Rightarrow [\bar{a}\,\bar{b}\,\bar{c}] = \frac{1}{p+q+r}$$

184. If $\bar{b},\bar{c}$ be any two non-collinear unit vectors and $\bar{a}$ is any vector, then

$(\bar{a}.\bar{b})\bar{b} + (\bar{a}.\bar{c})\bar{c} + \dfrac{\bar{a}.(\bar{b}\times\bar{c})}{|\bar{b}\times\bar{c}|^2}(\bar{b}\times\bar{c})$ is

1) $\bar{a}$ 2) $\bar{b}$ 3) $\bar{c}$ 4) $\bar{a}+\bar{b}+\bar{c}$

Key:1

Sol: Take $\bar{b} = \bar{i}, \bar{c} = \bar{j} \Rightarrow \dfrac{\bar{b}\times\bar{c}}{|\bar{b}\times\bar{c}|} = \bar{k} \Rightarrow (\bar{a}.\bar{i})\bar{i} + (\bar{a}.\bar{j})\bar{j} + (\bar{a}.\bar{k})\bar{k} = \bar{a}$

185. If $x(\bar{a}\times\bar{b}) + y(\bar{b}\times\bar{c}) + z(\bar{c}\times\bar{a}) = \bar{r}$ and $[\bar{a}\,\bar{b}\,\bar{c}] = \dfrac{1}{8}$, then the value of $x+y+z$ is

1) $\bar{r}.(\bar{a}+\bar{b}+\bar{c})$ 2) $4(\bar{r}.(\bar{a}+\bar{b}+\bar{c}))$ 3) $8(\bar{r}.(\bar{a}+\bar{b}+\bar{c}))$ 4) 0

Key:3

Sol: $x[\bar{a}\,\bar{b}\,\bar{c}] = \bar{r}.\bar{c}$; $y[\bar{a}\,\bar{b}\,\bar{c}] = \bar{r}.\bar{a}$; $z[\bar{a}\,\bar{b}\,\bar{c}] = \bar{r}.\bar{b}$

$$\Rightarrow x = y = z = \frac{\bar{r}.(\bar{a}+\bar{b}+\bar{c})}{[\bar{a}\,\bar{b}\,\bar{c}]} = 8(\bar{r}.(\bar{a}+\bar{b}+\bar{c}))$$

<u>CONCEPT 8:</u> $[\bar{a}\,\bar{b}\,\bar{c}][\bar{l}\,\bar{m}\,\bar{n}] = \begin{vmatrix} \bar{a}.\bar{l} & \bar{a}.\bar{m} & \bar{a}.\bar{n} \\ \bar{b}.\bar{l} & \bar{b}.\bar{m} & \bar{b}.\bar{n} \\ \bar{c}.\bar{l} & \bar{c}.\bar{m} & \bar{c}.\bar{n} \end{vmatrix}$

186. If $\bar{a} = \hat{i}+4\hat{j}+3\hat{k}$, $\bar{b} = 3\hat{i}+4\hat{j}-2\hat{k}$, $\bar{c} = \hat{i}+\hat{j}+\hat{k}$ and $[3\bar{a}+\bar{b}\;\; 3\bar{b}+\bar{c}\;\; 3\bar{c}+\bar{a}] = \lambda \begin{vmatrix} \bar{a}.\hat{i} & \bar{a}.\hat{j} & \bar{a}.\hat{k} \\ \bar{b}.\hat{i} & \bar{b}.\hat{j} & \bar{b}.\hat{k} \\ \bar{c}.\hat{i} & c.\hat{j} & \bar{c}.\hat{k} \end{vmatrix}$, then

find the value of $\dfrac{\lambda}{7}$ is

1) 1 2) 2 3) 3 4) 4

Key:4

Sol: $\left[3\bar{a}+\bar{b}\ \ 3\bar{b}+\bar{c}\ \ 3\bar{c}+\bar{a}\right]=\lambda\left[\bar{a}\,\bar{b}\,\bar{c}\right]\left[\bar{i}\ \bar{j}\ \bar{k}\right] \Rightarrow \begin{vmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 3 \end{vmatrix}\left[\bar{a}\,\bar{b}\,\bar{c}\right]=\lambda\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

$\Rightarrow \lambda = 3(9) = 1(-1) = 28$ $\qquad \therefore \dfrac{\lambda}{7}=\dfrac{28}{7}=4$

CONCEPT 9: Volume Based Questions

187. If the volume of a parallelopiped whose coterminous edges are given by the vectors $\bar{a}=\hat{i}+\hat{j}+n\hat{k}, \bar{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\bar{c}=\hat{i}+n\hat{j}+3\hat{k}, (n\geq 0)$, is 158 cubic units, then

(2020, 5 Sep. Shift 1)

1) $n=9$ 2) $\bar{b}.\bar{c}=10$ 3) $\bar{a}.\bar{c}=17$ 4) $n=7$

Key:2

Sol: $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=158 \Rightarrow \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix}=158 \Rightarrow 1\left(12+n^2\right)-1(6+n)+n(2n-4)=158$

$\Rightarrow n^2+12-6-n+2n^2-4n-158=0 \Rightarrow 3n^2-5n-152=0$. Here $n\geq 0 \Rightarrow n=8$

$\bar{b}.\bar{c}=2+4n-3n=n+2=10$ and $\bar{a}.\bar{c}=1+n+3n=4n+1=33$

188. Let the volume of a parallelopiped whose coterminous edges are given by $\bar{u}=\hat{i}+\hat{j}+\lambda\hat{k}, \bar{v}=\hat{i}+\hat{j}+3\hat{k}$ and $\bar{w}=2\hat{i}+\hat{j}+\hat{k}$ be 1 cubic unit. If θ be the angle between the edges $\bar{u}$ and $\bar{w}$, then $\cos\theta$ can be

1) $\dfrac{5}{3\sqrt{3}}$ 2) $\dfrac{7}{6\sqrt{3}}$ 3) $\dfrac{7}{6\sqrt{6}}$ 4) $\dfrac{5}{7}$

Key:2

Sol: $[\bar{u}\,\bar{v}\,\bar{w}]=1 \Rightarrow \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix}=1 \Rightarrow \left|1(-2)-1(-5)+\lambda(-1)\right|=1 \Rightarrow |3-\lambda|=1 \Rightarrow \lambda=2\ (or)\ \lambda=4$

$\cos\theta=\dfrac{\bar{u}.\bar{w}}{|\bar{u}||\bar{w}|}=\dfrac{2+1+\lambda}{\sqrt{1+1+\lambda^2}\sqrt{4+1+1}}=\dfrac{3+\lambda}{\sqrt{\lambda^2+2}\sqrt{6}}=\dfrac{5}{\sqrt{6}\sqrt{6}}=\dfrac{5}{6}$ (OR) $\cos\theta=\dfrac{7}{\sqrt{18}\sqrt{6}}=\dfrac{7}{6\sqrt{3}}$

189. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}, \hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to

(2019, 12th Apr. Shift1)

1) $\dfrac{1}{\sqrt{3}}$ 2) $-\dfrac{1}{\sqrt{3}}$ 3) $\sqrt{3}$ 4) $-\sqrt{3}$

Key:1

Sol: $\begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix}=1(1)-\lambda\left(-\lambda^2\right)+1(-\lambda)=1+\lambda^3-\lambda=f(\lambda)$

$f'(\lambda)=3\lambda^2-1=0 \Rightarrow \lambda=\pm\frac{1}{\sqrt{3}}$

$f''(\lambda)=6\lambda<0,\ when\ \lambda=\frac{-1}{\sqrt{3}}$ *and* $f''(\lambda)=6\lambda>0,\ when\ \lambda=\frac{1}{\sqrt{3}}$

$\therefore f$ has minimum when $\lambda=\frac{1}{\sqrt{3}}$

190. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is $(2,a,4)$, $a\in\mathbb{N}$. If the volume of the tetrahedron OABC is $144\,unit^3$, then which of the following is NOT on P?

(2023, 31st Jan. Shift 2)

1) $(2,2,4)$ 2) $(0,4,4)$ 3) $(3,0,4)$ 4) $(0,6,3)$

Key:3

Sol: $\overline{OA} \perp \overline{AP} \Rightarrow 2(x-2)+a(y-a)+4(z-4)=0 \Rightarrow 2x+ay+4z=a^2+20$

$A=\left(\frac{20+a^2}{2},0,0\right);\ B=\left(0,\frac{20+a^2}{a},0\right);\ C=\left(0,0,\frac{20+a^2}{4}\right)$

Given that $\frac{1}{6}\left[\overline{OA}\ \overline{OB}\ \overline{OC}\right]=144 \Rightarrow a=2$

$\therefore$ Equation of plane is $2x+2y+4z=24 \Rightarrow x+y+2z-12=0$

191. The unit vector which is orthogonal to the vector to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is

1) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ 2) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$ 3) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ 4) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Key:3

Sol: Let the unit vector be $\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k}$.

$\Rightarrow \alpha^2+\beta^2+\gamma^2=1$ and it is perpendicular to $3\bar{i}+2\bar{j}+6\bar{k} \Rightarrow 3\alpha+2\beta+6\gamma=0$

And it is coplanar with $2\bar{i}+\bar{j}+\bar{k}$ and $\bar{i}-\bar{j}+\bar{k} \Rightarrow \begin{vmatrix} \alpha & \beta & \gamma \\ 2 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}=0 \Rightarrow 4\alpha-2\beta-6\gamma=0;$ By solving, $\alpha=0,\ \beta=\mp\frac{3}{\sqrt{10}},\ \gamma=\pm\frac{1}{\sqrt{10}}$

$\therefore$ Required vector is $\pm\frac{1}{\sqrt{10}}\left(-3\bar{j}+\bar{k}\right)$

192. The position vectors of the vertices A, B and C of a tetrahedron ABCD are $\hat{i}+\hat{j}+\hat{k},\ \hat{i}$ *and* $3\hat{i}$ respectively. The altitude drawn from vertex D to the opposite face ABC, meets the median line through A of the triangle ABC, at a point E. If the length of the side AD is 4

and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$, find the position vector of the point E for all its possible positions.

1) $3\hat{i}-\hat{j}-\hat{k}$ 2) $-\hat{i}-3\hat{j}-3\hat{k}$ 3) $3\hat{i}+\hat{j}-\hat{k}$ 4) $\hat{i}+3\hat{j}+3\hat{k}$

Key:1

Sol: Let $F=\frac{B+C}{2}$ and $AE:EF=\lambda:1$

$$\overline{OE}=\frac{\lambda(2\bar{i})+(\bar{i}+\bar{j}+\bar{k})}{\lambda+1}$$

$$Volume=\frac{1}{6}\left[\overline{AB}\,\overline{AC}\,\overline{AD}\right]=\frac{2\sqrt{2}}{3}\Rightarrow\frac{1}{6}\left(A\bar{B}\times A\bar{C}\right).A\bar{D}=\frac{2\sqrt{2}}{3}$$

If $\left(A\bar{B}\times AC, A\bar{D}\right)=\theta$, $\frac{1}{6}2\sqrt{2}(4)\cos\theta=\frac{2\sqrt{2}}{3}\Rightarrow\theta=\frac{\pi}{3}$

$$\sin 60^0=\frac{AE}{4}\Rightarrow AE=2\sqrt{3}\Rightarrow\left|O\bar{E}-O\bar{A}\right|=2\sqrt{3}$$

$$\Rightarrow\left|\frac{\lambda}{\lambda+1}\bar{i}-\frac{\lambda}{\lambda+1}\bar{j}-\frac{\lambda}{\lambda+1}\bar{k}\right|=2\sqrt{3}\Rightarrow\frac{3\lambda^2}{(\lambda+1)^2}=12\Rightarrow(3\lambda+2)(\lambda+2)=0$$

$$\therefore\ \lambda=-2\Rightarrow\overline{OE}=3\bar{i}-\bar{j}-\bar{k}\ \&\ \lambda=-\frac{2}{3}\Rightarrow\overline{OE}=-\bar{i}+3\bar{j}+3\bar{k}$$

193. A tetrahedron of volume V=5 has three of its vertices at the points $A(2,1,-1), B(3,0,1)\ and\ C(2,-1,3)$. The fourth vertex D lies on the y-axis. Then D is

1) $(0,8,0)$ 2) $(0,-7,0)$ 3) $(0,7,0)$ 4) $(0,8,0)\ or\ (0,-7,0)$

Key:4

Sol: Let $D=(0,\lambda,0)$. Given that V=$\frac{1}{6}\begin{vmatrix}2 & 1-\lambda & -1\\ 3 & -\lambda & 1\\ 2 & -1-\lambda & 3\end{vmatrix}=5$

$\Rightarrow|4\lambda-2|=30\Rightarrow\lambda=8,-7$ $\therefore D=(0,8,0)\ OR\ (0,-7,0)$

VECTOR TRIPLE PRODUCT:

Exercise-1 (Single Answer Type Questions)

CONCEPT 1: $\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}.\bar{c})\bar{b}-(\bar{a}.\bar{b})\bar{c}$ **and** $(\bar{a}\times\bar{b})\times\bar{c}=(\bar{c}.\bar{a})\bar{b}-(\bar{c}.\bar{b})\bar{a}$

194. If $\bar{a},\bar{b},\bar{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\bar{c}$ such that $\bar{a}=\alpha\bar{b}-\hat{n},(\alpha\neq 0)$ and $\bar{b}.\bar{c}=12$, then $\left|\bar{c}\times(\bar{a}\times\bar{b})\right|$ is equal to

(2023, 30th Jan. Shift 1)

1) 9 2) 6 3) 12 4) 15

Key:3

Sol: $|\hat{n}|=1\ \&\ \hat{n}.\bar{c}=0\ \&\ \bar{a}=\alpha\bar{b}-\hat{n},(\alpha\neq 0)\ \&\ \bar{b}.\bar{c}=12$

Consider $\bar{a}.\bar{c}=\alpha(\bar{b}.\bar{c})-\hat{n}.\bar{c}=12\alpha$

Consider $\left|\overline{c}\times\left(\overline{a}\times\overline{b}\right)\right|=\left|\left(\overline{c}.\overline{b}\right)\overline{a}-\left(\overline{c}.\overline{a}\right)\overline{b}\right|$

$=\left|12\overline{a}-12\alpha\overline{b}\right|=12\left|\overline{a}-\alpha\overline{b}\right|=12$

195. Let $\lambda\in\mathbb{R}, \overline{a}=\lambda\hat{i}+2\hat{j}-3\hat{k}, \overline{b}=\hat{i}-\lambda\hat{j}+2\hat{k}$. If $\left(\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}\times\overline{b}\right)\right)\times\left(\overline{a}-\overline{b}\right)$

$=8\hat{i}-40\hat{j}-24\hat{k}$, then $\left|\lambda\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}-\overline{b}\right)\right|^2$ is equal to (2023, 30th Jan. Shift 2)

1) 132 2) 136 3) 140 4) 144

Key:3

Sol: $\left(\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}\times\overline{b}\right)\right)\times\left(\overline{a}-\overline{b}\right)=\left(\left(\overline{a}-\overline{b}\right).\left(\overline{a}+\overline{b}\right)\right)\left(\overline{a}\times\overline{b}\right)$

$-\left(\left(\overline{a}-\overline{b}\right).\left(\overline{a}\times\overline{b}\right)\right)\left(\overline{a}+\overline{b}\right)=\left(\left(\overline{a}-\overline{b}\right).\left(\overline{a}+\overline{b}\right)\right)\left(\overline{a}\times\overline{b}\right) \quad =8\overline{i}-40\overline{j}-24\overline{k}$

Here $\overline{a}-\overline{b}=\lambda\overline{i}+2\overline{j}-3\overline{k}-\overline{i}+\lambda\overline{j}-2\overline{k}=(\lambda-1)\overline{i}+(2+\lambda)\overline{j}-5\overline{k}$

$\overline{a}+\overline{b}=(\lambda+1)\overline{i}+(2-\lambda)\overline{j}-\overline{k};\left(\overline{a}-\overline{b}\right)\left(\overline{a}+\overline{b}\right)=\lambda^2-1+4-\lambda^2+5=8$

$\Rightarrow 8\left(\overline{a}\times\overline{b}\right)=8\overline{i}-40\overline{j}-24\overline{k}\Rightarrow \overline{a}\times\overline{b}=\overline{i}-5\overline{j}-3\overline{k}$

$$\therefore\begin{vmatrix}\overline{i} & \overline{j} & \overline{k}\\ \lambda & 2 & -3\\ 1 & -\lambda & 2\end{vmatrix}=\overline{i}-5\overline{j}-3\overline{k}$$

$\Rightarrow \overline{i}(4-3\lambda)-\overline{j}(2\lambda+3)+\overline{k}\left(-\lambda^2-2\right)=\overline{i}-5\overline{j}-3\overline{k}\Rightarrow\lambda=1$

$\therefore\overline{a}=\overline{i}+2\overline{j}-3\overline{k};\overline{b}=\overline{i}-\overline{j}+2\overline{k}$

$\Rightarrow\overline{a}+\overline{b}=2\overline{i}+\overline{j}-\overline{k}\ \&\ \overline{a}-\overline{b}=3\overline{j}-5\overline{k}$

$$\therefore\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}-\overline{b}\right)=\begin{vmatrix}\overline{i} & \overline{j} & \overline{k}\\ 2 & 1 & -1\\ 0 & 3 & -5\end{vmatrix}=\overline{i}(-2)-\overline{j}(-10)+\overline{k}(6)$$

$\therefore\ \left|\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}-\overline{b}\right)\right|^2=4+100+36=140$

$\therefore\left|\lambda\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}-\overline{b}\right)\right|^2=140$

196. $\overline{a}=3\hat{i}+\hat{j}$ and $\overline{b}=\hat{i}+2\hat{j}+\hat{k}$. Let $\overline{c}$ be a vector satisfying $\overline{a}\times\left(\overline{b}\times\overline{c}\right)=\overline{b}+\lambda\overline{c}$. If $\overline{b}$ and $\overline{c}$ are non-parallel, then the absolute value of λ is

(2022, 29 July Shift 1)

1) -5 2) 5 3) 10 4) 15

Key:2

Sol: $\overline{a}\times\left(\overline{b}\times\overline{c}\right)=\overline{b}+\lambda\overline{c}\Rightarrow(\overline{a}.\overline{c})\overline{b}-\left(\overline{a}.\overline{b}\right)\overline{c}=\overline{b}+\lambda\overline{c}\Rightarrow\overline{a}.\overline{c}=1\ \&\ \overline{a}.\overline{b}=-\lambda$

$\Rightarrow\lambda=-\left(3\overline{i}+\overline{j}\right).\left(\overline{i}+2\overline{j}+\overline{k}\right)=-[3+2]=-5\Rightarrow|\lambda|=5$

197. Let $\overline{a}=2\hat{i}-\hat{j}+5\hat{k}$ and $\overline{b}=\alpha\hat{i}+\beta\hat{j}+2\hat{k}$. If $\left(\left(\overline{a}\times\overline{b}\right)\times\hat{i}\right).\hat{k}=\frac{23}{2}$, then $\left|\overline{b}\times 2\hat{j}\right|$ is

(2022, 27 July Shift 1)

1) 5 2) $\sqrt{17}$ 3) 4 4) $\sqrt{21}$

Key:1

Sol: $\left(\left(\overline{a}\times\overline{b}\right)\times\overline{i}\right).\overline{k}=\frac{23}{2}\Rightarrow\left(\left(\overline{a}.\overline{i}\right)\overline{b}-\left(\overline{b}.\overline{i}\right)\overline{a}\right).\overline{k}=\frac{23}{2}$

$\Rightarrow\left(\left(2\overline{i}-\overline{j}+5\overline{k}\right).\overline{i}\right)\overline{b}-\left(\overline{i}.\left(\alpha\overline{i}+\beta\overline{j}+2\overline{k}\right)\overline{a}\right).\overline{k}=\frac{23}{2}$

$\Rightarrow\left(2\overline{b}-\alpha\overline{a}\right).\overline{k}=\frac{23}{2}\Rightarrow\left(2\left(\alpha\overline{i}+\beta\overline{j}+2\overline{k}\right)-\alpha\left(2\overline{i}-\overline{j}+5\overline{k}\right)\right).\overline{k}=\frac{23}{2}$

$\Rightarrow 4-5\alpha=\frac{23}{2}\Rightarrow 5\alpha=4-\frac{23}{2}=\frac{-15}{2}\Rightarrow\alpha=\frac{-3}{2}$

$$\therefore\overline{b}\times 2\overline{j}=\begin{vmatrix}\overline{i} & \overline{j} & \overline{k}\\ \frac{-3}{2} & \beta & 2\\ 0 & 2 & 0\end{vmatrix}=-4\overline{i}-3\overline{k}\Rightarrow\left|\overline{b}\times 2\overline{j}\right|=\sqrt{16+9}=5$$

198. Let $\overline{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\overline{b}=-\hat{i}+2\hat{j}+3\hat{k}$. Then the vector product $\left(\overline{a}+\overline{b}\right)\times\left(\left[\overline{a}\times\left\{\overline{a}-\overline{b}\right\}\times\overline{b}\right]\times\overline{b}\right)$ is equal to

(2021, 27 July Shift 1)

1) $5\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$ 2) $7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$

3) $7\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$ 4) $5\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$

Key:2

Sol: Consider $\left(\overline{a}+\overline{b}\right)\times\left(\left(\overline{a}\times\left(\overline{a}-\overline{b}\right)\times\overline{b}\right)\times\overline{b}\right)$

$=\left(\overline{a}+\overline{b}\right)\times\left(\left(\left(\overline{a}.\overline{b}\right)\overline{a}-\left(\overline{a}.\overline{a}\right)\overline{b}\right)\times\overline{b}\right)\Rightarrow\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}.\overline{b}\right)\left(\overline{a}\times\overline{b}\right)$

$=\left(\overline{a}.\overline{b}\right)\left[\left(\overline{a}+\overline{b}\right)\times\left(\overline{a}\times\overline{b}\right)\right]=\overline{a}.\overline{b}\left[\left(\left(\left(\overline{a}+\overline{b}\right).\overline{b}\right)\overline{a}-\left(\overline{a}+\overline{b}\right).\overline{a}\right)\overline{b}\right].....(1)$

$\Rightarrow\overline{a}.\overline{b}=-1+2+6=7\Rightarrow\overline{a}+\overline{b}=3\overline{j}+5\overline{k}$

$\left(\overline{a}+\overline{b}\right).\overline{b}=\left(3\overline{j}+5\overline{k}\right).\left(-\overline{i}+2\overline{j}+3\overline{k}\right)=6+15=21$

$\left(\overline{a}+\overline{b}\right).\overline{a}=\left(3\overline{j}+5\overline{k}\right).\left(\overline{i}+\overline{j}+2\overline{k}\right)=3+10=13$

$(1)\Rightarrow 7\left(21\overline{a}-13\overline{b}\right)=7\left(21\overline{i}+21\overline{j}+42\overline{k}+13\overline{i}-26\overline{j}-39\overline{k}\right)$

$=7\left(34\overline{i}-5\overline{j}+3\overline{k}\right)$

199. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be three vectors such that $\bar{a} = \bar{b} \times (\bar{b} \times \bar{c})$. If magnitudes of the vectors $\bar{a}, \bar{b}$ and $\bar{c}$ are $\sqrt{2}, 1$ and 2, respectively and the angle between $\bar{b}$ and $\bar{c}$ is $\theta\left(0<\theta<\frac{\pi}{2}\right)$, then the value of $(1+\tan\theta)$ is equal to

(2021, 27 July Shift 2)

1) $\sqrt{3}+1$ 2) $\frac{\sqrt{3}+1}{\sqrt{3}}$ 3) 1 4) 2

Key:4

Sol: $|\bar{a}| = \sqrt{2}, |\bar{b}| = 1, |\bar{c}| = 2$ & $\bar{a} = \bar{b} \times (\bar{b} \times \bar{c})$

$= (\bar{b}.\bar{c})\bar{b} - (\bar{b}.\bar{b})\bar{c} = 2\cos\theta\bar{b} - \bar{c}$

$\Rightarrow |\bar{a}|^2 = 4\cos^2\theta + 4 - 4\cos\theta\bar{b}.\bar{c} \Rightarrow 2 = 4\cos^2\theta + 4 - 8\cos^2\theta$

$\Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{4}$ $\therefore 1 + Tan\theta = 1+1 = 2$

200. Let three vectors $\bar{a}, \bar{b}$ and $\bar{c}$ be such that $\bar{a} \times \bar{b} = \bar{c}$, $\bar{b} \times \bar{c} = \bar{a}$ and $|\bar{a}| = 2$. Then, which of the following is NOT true? (2021, 22 July Shift 2)

1) $\bar{a} \times ((\bar{b}+\bar{c}) \times (\bar{b}-\bar{c})) = \bar{O}$ 2) Projection of $\bar{a}$ on $(\bar{b} \times \bar{c})$ is 2

3) $[\bar{a}\ \bar{b}\ \bar{c}] + [\bar{c}\ \bar{a}\ \bar{b}] = 8$ 4) $|3\bar{a}+\bar{b}-2\bar{c}|^2 = 51$

Key:4

Sol: $\bar{a} \times \bar{b} = \bar{c}, \bar{b} \times \bar{c} = \bar{a}$ & $|\bar{a}| = 2$. From Options,

$(a)\ \bar{a} \times [(\bar{b}+\bar{c}) \times (\bar{b}-\bar{c})] = \bar{o}$

$\Rightarrow \bar{a} \times (-\bar{b} \times \bar{c} - \bar{b} \times \bar{c}) = \bar{o} \Rightarrow \bar{a} \times (-2\bar{a}) = \bar{o}$

$(b)\ \frac{\bar{a}.(\bar{b} \times \bar{c})}{|\bar{b} \times \bar{c}|} = \frac{\bar{a}.\bar{a}}{|\bar{a}|} = 2$

$(c)\ [\bar{a}\,\bar{b}\,\bar{c}] + [\bar{a}\,\bar{b}\,\bar{c}] = 8 \Rightarrow [\bar{a}\,\bar{b}\,\bar{c}] = 4 \Rightarrow \bar{a}.(\bar{b} \times \bar{c}) = 4 \Rightarrow |\bar{a}|^2 = 4$

$(d)\ |3\bar{a}+\bar{b}-2\bar{c}|^2 = 9|\bar{a}|^2 + |\bar{b}|^2 + 4|\bar{c}|^2 = 36+1+16 = 53 \neq 51$

201. Let $\bar{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{b} = \hat{i} + \hat{j}$. If $\bar{c}$ is a vector such that $\bar{a}.\bar{c} = |\bar{c}|$, $|\bar{c} - \bar{a}| = 2\sqrt{2}$ and the angle between $\bar{a} \times \bar{b}$ and $\bar{c}$ is $\frac{\pi}{6}$, then the value of $|(\bar{a} \times \bar{b}) \times \bar{c}|$ is

(2021, 20 July Shift 1)

1) $\frac{2}{3}$ 2) 4 3) 3 4) $\frac{3}{2}$

Key:4

Sol: $|\bar{c}-\bar{a}|=2\sqrt{2} \Rightarrow |\bar{c}|^2+|\bar{a}|^2-2\bar{c}.\bar{a}=8 \Rightarrow |\bar{c}|^2-2|\bar{c}|+1=0 \Rightarrow |\bar{c}|=1$

Given that $(\bar{a}\times\bar{b},\bar{c})=30^\circ$

$$\Rightarrow\left|\left(\bar{a}\times\bar{b}\right)\times\bar{c}\right|=|\bar{a}||\bar{b}||\bar{c}|\sin\left(\bar{a}\times\bar{b},\bar{c}\right)\sin\left(\bar{a},\bar{b}\right)$$

$$=\frac{1}{2}(3)\left(\sqrt{2}\right)(1)\frac{|\bar{a}\times\bar{b}|}{|\bar{a}||\bar{b}|}=\frac{3}{\sqrt{2}}\frac{1}{3\sqrt{2}}\times 3=\frac{3}{2}$$

202. If $\bar{a}$ and $\bar{b}$ are perpendicular, then $\bar{a}\times\left(\bar{a}\times\left(\bar{a}\times\left(\bar{a}\times\bar{b}\right)\right)\right)$ is equal to (2021, 26 Feb. Shift 1)

1) 0 2) $\frac{1}{2}|\bar{a}|^4\bar{b}$ 3) $\bar{a}\times\bar{b}$ 4) $|\bar{a}|^4\bar{b}$

Key:4

Sol: $\bar{a}.\bar{b}=0$. Consider $\bar{a}\times\left(\bar{a}\times\left(\bar{a}\times\left(\bar{a}\times\bar{b}\right)\right)\right)=\bar{a}\times\left(\bar{a}\times\left(\left(\bar{a}.\bar{b}\right)\bar{a}-\left(\bar{a}.\bar{a}\right)\bar{b}\right)\right)$

$$=-|\bar{a}|^2\left(\bar{a}\times\left(\bar{a}\times\bar{b}\right)\right)=-|\bar{a}|^2\left(\left(\bar{a}.\bar{b}\right)\bar{a}-\left(\bar{a}.\bar{a}\right)\bar{b}\right)=|\bar{a}|^4\bar{b}$$

203. Let $\bar{a}=\hat{i}-\hat{j}, \bar{b}=\hat{i}+\hat{j}+\hat{k}$ and $\bar{c}$ be a vector such that $\bar{a}\times\bar{c}+\bar{b}=\bar{O}$ and $\bar{a}.\bar{c}=4$, then $|\bar{c}|^2$ is equal to (2019, 9 Jan. Shift 1)

1) 8 2) $\frac{19}{2}$ 3) 9 4) $\frac{17}{2}$

Key:2

Sol: Consider $\left(\bar{a}\times\bar{c}\right)+\bar{b}=\bar{o} \Rightarrow \bar{a}\times\left(\bar{a}\times\bar{c}\right)+\bar{a}\times\bar{b}=\bar{o}$

$$\Rightarrow\left(\bar{a}.\bar{c}\right)\bar{a}-\left(\bar{a}.\bar{a}\right)\bar{c}+\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & -1 & 0\\ 1 & 1 & 1\end{vmatrix}=\bar{o}$$

$$\Rightarrow 4\left(\bar{i}-\bar{j}\right)-2\bar{c}+\left(-\bar{i}-\bar{j}+2\bar{k}\right)=\bar{o} \Rightarrow \bar{c}=\frac{3\bar{i}-5\bar{j}+2\bar{k}}{2}$$

$$\therefore |\bar{c}|^2=\frac{\left(\sqrt{9+25+4}\right)^2}{4}=\frac{38}{4}=\frac{19}{2}$$

204. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be three unit vectors, out of which vectors $\bar{b}$ and $\bar{c}$ are not parallel. If α and β are the angles which vector $\bar{a}$ makes with vectors $\bar{b}$ and $\bar{c}$ respectively and $\bar{a}\times\left(\bar{b}\times\bar{c}\right)=\frac{1}{2}\bar{b}$, then $|\alpha-\beta|$ is equal to

(2019, 12 Jan. Shift 2)

1) 30° 2) 45° 3) 90° 4) 60°

Key:1

Sol: $|\bar{a}|=|\bar{b}|=|\bar{c}|=1$ and $\left(\bar{a},\bar{b}\right)=\alpha \;\&\left(\bar{a},\bar{c}\right)=\beta \;\&\left(\bar{a}\times\left(\bar{b}\times\bar{c}\right)\right)=\frac{1}{2}\bar{b}$

$$\Rightarrow(\bar{a}.\bar{c})\bar{b}-(\bar{a}.\bar{b})\bar{c}=\frac{1}{2}\bar{b}\Rightarrow\bar{a}.\bar{c}=\frac{1}{2}\ \&\ \bar{a}.\bar{b}=0\Rightarrow\cos\beta=\frac{1}{2}\ \&\ \cos\alpha=0$$

$$\Rightarrow\alpha=\frac{\pi}{2}\ \&\ \beta=\frac{\pi}{3} \qquad \therefore|\alpha-\beta|=\frac{\pi}{6}$$

205. Let $\bar{a},\bar{b}$ and $\bar{c}$ be three unit vectors such that $\bar{a}\times(\bar{b}\times\bar{c})=\frac{\sqrt{3}}{2}(\bar{b}+\bar{c})$. If $\bar{b}$ is not parallel to $\bar{c}$, then the angle between $\bar{a}$ and $\bar{b}$ (JEE Main 2016)

1) $\frac{3\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{2\pi}{3}$ 4) $\frac{5\pi}{6}$

Key:4

Sol: $|\bar{a}|=|\bar{b}|=|\bar{c}|=1$. Consider $\bar{a}\times(\bar{b}\times\bar{c})=\frac{\sqrt{3}}{2}(\bar{b}+\bar{c})\Rightarrow(\bar{a}.\bar{c})\bar{b}-(\bar{a}.\bar{b})\bar{c}=\frac{\sqrt{3}}{2}\bar{b}+\frac{\sqrt{3}}{2}\bar{c}$

$$\Rightarrow\bar{a}.\bar{c}=\frac{\sqrt{3}}{2}\ \&\ \bar{a}.\bar{b}=\frac{-\sqrt{3}}{2}\Rightarrow(\bar{a},\bar{b})=\frac{5\pi}{6}$$

206. Let $\bar{a},\bar{b}$ and $\bar{c}$ be three non-zero vectors such that no two of them are collinear and $(\bar{a}\times\bar{b})\times\bar{c}=\frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$. If θ is the angle between vectors $\bar{b}$ and $\bar{c}$, then a value of $\sin\theta$ is (JEE Main 2015)

1) $\frac{2\sqrt{2}}{3}$ 2) $-\frac{\sqrt{2}}{3}$ 3) $\frac{2}{3}$ 4) $-\frac{2\sqrt{3}}{3}$

Key:1

Sol: $(\bar{a}\times\bar{b})\times\bar{c}=\frac{1}{3}|\bar{b}||\bar{c}|\bar{a}\ \&\ (\bar{b},\bar{c})=\theta\Rightarrow(\bar{c}.\bar{a})\bar{b}-(\bar{c}.\bar{b})\bar{a}=\frac{1}{3}|\bar{b}||\bar{c}|\bar{a}\Rightarrow\bar{c}.\bar{a}=0\ \&\ \bar{c}.\bar{b}=\frac{1}{3}|\bar{b}||\bar{c}|$

$$\Rightarrow(\bar{a},\bar{c})=\frac{\pi}{2}\ \&\ \cos\theta=\frac{1}{3}\Rightarrow\sin\theta=\pm\sqrt{1-\frac{1}{9}}=\pm\sqrt{\frac{8}{9}}=\pm\frac{2\sqrt{2}}{3}$$

207. Let $\bar{a}=\hat{j}-\hat{k}$ and $\bar{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\bar{b}$ satisfying $\bar{a}\times\bar{b}+\bar{c}=\bar{O}$ and $\bar{a}.\bar{b}=3$, is (AIEEE 2010)

1) $-\hat{i}+\hat{j}-2\hat{k}$ 2) $2\hat{i}-\hat{j}+2\hat{k}$ 3) $\hat{i}-\bar{j}-2\hat{k}$ 4) $\hat{i}+\hat{j}-2\hat{k}$

Key:1

Sol: Let $\bar{b}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ when $\bar{a}=\bar{j}-\bar{k},\bar{c}=\bar{i}-\bar{j}-\bar{k}$

$\bar{a}.\bar{b}=3\Rightarrow\beta-\gamma=3$ and $\bar{c}=-(\bar{a}\times\bar{b})\Rightarrow\alpha=-1\ \&\ \beta+\gamma=-1$

$\therefore\alpha=-1,\beta=1,\gamma=-2\Rightarrow\bar{b}=-\bar{i}+\bar{j}-2\bar{k}$

208. If $\bar{a}.\bar{b}=\beta$ and $\bar{a}\times\bar{b}=\bar{c}$, then $\bar{b}$ is

1) $\frac{\beta\bar{a}+(\bar{c}\times\bar{a})}{|\bar{a}|^2}$ 2) $\frac{\beta\bar{a}-(\bar{c}\times\bar{a})}{|\bar{a}|^2}$ 3) $\frac{\beta\bar{a}-(\bar{b}\times\bar{c})}{|\bar{b}|^2}$ 4) $\frac{\beta\bar{c}-(\bar{a}\times\bar{b})}{|\bar{b}|^2}$

Key:1

Sol: $\overline{a}.\overline{b}=\beta \Rightarrow \overline{a}\times\overline{b}=\overline{c}$. Consider $\left(\overline{a}\times\overline{b}\right)\times\overline{a}=\overline{c}\times\overline{a}\Rightarrow(\overline{a}.\overline{a})\overline{b}-\left(\overline{a}.\overline{b}\right)\overline{a}=\overline{c}\times\overline{a}$

$$\Rightarrow \overline{b}=\frac{\beta\overline{a}+\left(\overline{c}\times\overline{a}\right)}{|\overline{a}|^2}$$

CONCEPT 2: Relation between $\overline{a}\times\left(\overline{b}\times\overline{c}\right)$ and $\left(\overline{a}\times\overline{b}\right)\times\overline{c}$

209. If $\overline{a}\times\left(\overline{b}\times\overline{c}\right)=\left(\overline{a}\times\overline{b}\right)\times\overline{c}$, where $\overline{a},\ \overline{b}\ and\ \overline{c}$ are any three vectors such that $\overline{a}.\overline{b}\neq 0,\ \overline{b}.\overline{c}\neq 0$, then $\overline{a}\ and\ \overline{c}$ are (AIEEE 2006)

1) Inclined at an angle of $\frac{\pi}{6}$ between them.

2) Perpendicular.

3) Parallel.

4) Inclined at an angle of $\frac{\pi}{3}$ between them.

Key:3

Sol: $\overline{a}\times\left(\overline{b}\times\overline{c}\right)=\left(\overline{a}\times\overline{b}\right)\times\overline{c}$, where $\overline{a}.\overline{b}\neq 0\ \&\ \overline{b}.\overline{c}\neq 0 \Rightarrow (\overline{a}.\overline{c})\overline{b}-\left(\overline{a}.\overline{b}\right)\overline{c}=(\overline{c}.\overline{a})\overline{b}-\left(\overline{c}.\overline{b}\right)\overline{a}$

$\Rightarrow \overline{c}=\frac{\overline{b}.\overline{c}}{\overline{a}.\overline{b}}=\overline{a}$. $\therefore \overline{a},\overline{c}$ are parallel vectors

CONCEPT 3: Nature of the vectors $\overline{a}\times\left(\overline{b}\times\overline{c}\right),\ \overline{b}\times(\overline{c}\times\overline{a}),\overline{c}\times\left(\overline{a}\times\overline{b}\right)$

210. If $\overline{a}.\overline{b}=1,\ \overline{b}.\overline{c}=2\ and\ \overline{c}.\overline{a}=3$, then the value of $\left[\overline{a}\times\left(\overline{b}\times\overline{c}\right)\ \ \overline{b}\times(\overline{c}\times\overline{a})\ \ \overline{c}\times\left(\overline{b}\times\overline{a}\right)\right]$ is (2022, 26 June Shift 1)

1) 0 2) $-6\overline{a}.\left(\overline{b}\times\overline{c}\right)$ 3) $12\overline{c}.\left(\overline{a}\times\overline{b}\right)$ 4) $-12\overline{b}.\left(\overline{c}\times\overline{a}\right)$

Key:1

Sol: Given that $\overline{a}.\overline{b}=1,\overline{b}.\overline{c}=2\ \&\overline{c}.\overline{a}=3$

We have $\overline{a}\times\left(\overline{b}\times\overline{c}\right)=(\overline{a}.\overline{c})\overline{b}-\left(\overline{a}.\overline{b}\right)\overline{c}=3\overline{b}-\overline{c}$

$\overline{b}\times(\overline{c}\times\overline{a})=\left(\overline{b}.\overline{a}\right)\overline{c}-\left(\overline{b}.\overline{c}\right)\overline{a}=\overline{c}-2\overline{a}$

$\overline{c}\times\left(\overline{b}\times\overline{a}\right)=(\overline{c}.\overline{a})\overline{b}-\left(\overline{c}.\overline{b}\right)\overline{a}=3\overline{b}-2\overline{a}$

Consider $\left[\overline{a}\times\left(\overline{b}\times\overline{c}\right)\overline{b}\times(\overline{c}\times\overline{a})\ \overline{c}\times\left(\overline{b}\times\overline{a}\right)\right]=\begin{vmatrix}0 & 3 & -1\\ -2 & 0 & 1\\ -2 & 3 & 0\end{vmatrix}\left[\overline{a}\overline{b}\overline{c}\right]=-3(2)-1(-6)=0$

211. Let $\overline{a},\overline{b}\ and\ \overline{c}$ be three unit vectors which are mutually perpendicular to each other. If a vector $\overline{r}$ satisfies $\overline{a}\times\left(\left(\overline{r}-\overline{b}\right)\times\overline{a}\right)+\overline{b}\times\left((\overline{r}-\overline{c})\times\overline{b}\right)+\overline{c}\times\left((\overline{r}-\overline{a})\times\overline{c}\right)=0$, then $\overline{r}$ is equal to

(2021, 31 Aug. Shift 2)

1) $\frac{1}{3}\left(\overline{a}+\overline{b}+\overline{c}\right)$ 2) $\frac{2}{3}\left(\overline{a}+\overline{b}+\overline{c}\right)$ 3) $\frac{1}{2}\left(\overline{a}+\overline{b}+\overline{c}\right)$ 4) $\frac{1}{3}\left(2\overline{a}+\overline{b}+\overline{c}\right)$

Key:3

Sol: $|\overline{a}|=\left|\overline{b}\right|=|\overline{c}|\ \&\ \overline{a}.\overline{b}=\overline{b}.\overline{c}=\overline{c}.\overline{a}=0$

Consider $\sum \overline{a}\times\left(\left(\overline{r}-\overline{b}\right)\times\overline{a}\right)=\sum\left(\overline{a}.\overline{a}\right)\left(\overline{r}-\overline{b}\right)-\left(\overline{a}.\left(\overline{r}-\overline{b}\right)\right)\overline{a}$

$$=3\overline{r}-\left(\overline{a}+\overline{b}+\overline{c}\right)-\overline{r}=\overline{o}\Rightarrow 2\overline{r}=\overline{a}+\overline{b}+\overline{c}\Rightarrow\overline{r}=\frac{\overline{a}+\overline{b}+\overline{c}}{2}$$

<u>CONCEPT 4:</u> $\left[\overline{a}\times\overline{b}\ \ \overline{b}\times\overline{c}\ \ \overline{c}\times\overline{a}\right]=\left[\overline{a}\ \overline{b}\ \overline{c}\right]^2$

212. If $\overline{a},\overline{b},\overline{c}$ are three vectors such that $\left[\overline{a}\,\overline{b}\,\overline{c}\right]=1$, then the value of $\left[\overline{a}+\overline{b}\ \ \overline{b}+\overline{c}\ \ \overline{c}+\overline{a}\right]+\left[\overline{a}\times\overline{b}\ \ \overline{b}\times\overline{c}\ \ \overline{c}\times\overline{a}\right]+\left[\overline{a}\times\left(\overline{b}\times\overline{c}\right)\ \overline{b}\times\left(\overline{c}\times\overline{a}\right)\ \overline{c}\times\left(\overline{a}\times\overline{b}\right)\right]$, is

1) 3 2) 2 3) 1 4) 0

Key:1

Sol: $\left[\overline{a}\overline{b}\overline{c}\right]=1$. Now $2(1)+1^2+0=3$

<u>CONCEPT 5:</u> $\left(\overline{a}\times\overline{b}\right).\left(\overline{c}\times\overline{d}\right)=\begin{vmatrix}\overline{a}.\overline{b} & \overline{a}.\overline{c}\\ \overline{b}.\overline{c} & \overline{b}.\overline{d}\end{vmatrix}$

213. Let $\overline{a},\overline{b}\ and\ \overline{c}$ be three non-zero vectors such that $\overline{b}.\overline{c}=0$ and $\overline{a}\times\left(\overline{b}\times\overline{c}\right)=\frac{\overline{b}-\overline{c}}{2}$. If $\overline{d}$ be a vector such that $\overline{b}.\overline{d}=\overline{a}.\overline{b}$, then $\left(\overline{a}\times\overline{b}\right).\left(\overline{c}\times\overline{d}\right)$ is equal to

(2023, 25th Jan. Shift 1)

1) $\frac{3}{4}$ 2) $\frac{1}{4}$ 3) $\frac{1}{2}$ 4) $-\frac{1}{4}$

Key:2

Sol: $\overline{b}.\overline{c}=0\ \&\ \overline{a}\times\left(\overline{b}\times\overline{c}\right)=\frac{\overline{b}-\overline{c}}{2}\Rightarrow\overline{a}.\overline{c}=\frac{1}{2}\ \&\ \overline{a}.\overline{b}=\frac{1}{2}\Rightarrow\overline{b}.\overline{d}=\overline{a}.\overline{b}=\frac{1}{2}$

Consider $\left(\overline{a}\times\overline{b}\right).\left(\overline{c}\times\overline{d}\right)=\begin{vmatrix}\overline{a}.\overline{c} & \overline{a}.\overline{d}\\ \overline{b}.\overline{c} & \overline{b}.\overline{d}\end{vmatrix}=\left(\overline{a}.\overline{c}\right)\left(\overline{b}.\overline{d}\right)-\left(\overline{b}.\overline{c}\right)\left(\overline{a}.\overline{d}\right)$

$$=\overline{a}.\left(\left(\overline{b}.\overline{d}\right)\overline{c}-\left(\overline{b}.\overline{c}\right)\overline{d}\right)=\left(\overline{a}.\overline{c}\right)\left(\overline{b}.\overline{d}\right)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

214. If $\overline{a},\overline{b},\overline{c}$ are coplanar vectors and $\overline{a}$ is not parallel to $\overline{b}$, then $\left\{\left(\overline{c}\times\overline{b}\right).\left(\overline{a}\times\overline{b}\right)\right\}\overline{a}+\left\{\left(\overline{a}\times\overline{c}\right).\left(\overline{a}\times\overline{b}\right)\right\}\overline{b}$ is equal to

1) $\left\{\left(\overline{a}\times\overline{b}\right).\left(\overline{a}\times\overline{b}\right)\right\}\overline{c}$ 2) $\left(\overline{a}\times\overline{b}\right).\overline{c}$ 3) $\left|\overline{c}\right|\left(\overline{a}\times\overline{b}\right)$ 4) None of these

Key:1

Sol: $\left[\overline{a}\overline{b}\overline{c}\right]=0\ \&\ \overline{a}\ and\ \overline{b}$ are not parallel vectors. Consider

$\left\{\left(\overline{c}\times\overline{b}\right).\left(\overline{a}\times\overline{b}\right)\right\}\overline{a}+\left\{\left(\overline{a}\times\overline{c}\right).\left(\overline{a}\times\overline{b}\right)\right\}\overline{b}=\begin{vmatrix}\overline{c}.\overline{a} & \overline{c}.\overline{b}\\ \overline{b}.\overline{a} & \overline{b}.\overline{b}\end{vmatrix}\overline{a}+\begin{vmatrix}\overline{a}.\overline{a} & \overline{a}.\overline{b}\\ \overline{c}.\overline{a} & \overline{c}.\overline{b}\end{vmatrix}\overline{b}$

$$\Rightarrow\left(\left(\overline{c}.\overline{a}\right)\left|\overline{b}\right|^2-\left(\overline{b}.\overline{a}\right)\left(\overline{c}.\overline{b}\right)\right)\overline{a}+\left(\left|\overline{a}\right|^2\left(\overline{b}.\overline{c}\right)-\left(\overline{c}.\overline{a}\right)\left(\overline{a}.\overline{b}\right)\right)\overline{b}$$

$$=\left(\overline{c}.\overline{a}\right)\overline{a}\left|\overline{b}\right|^2-\left(\overline{b}.\overline{a}\right)\left(\overline{b}.\overline{c}\right)\overline{a}+\left|\overline{a}\right|^2\left(\overline{b}.\overline{c}\right)\overline{b}-\left(\overline{c}.\overline{a}\right)\left(\overline{a}.\overline{b}\right)\overline{b}$$

$$=\left(\bar{c}.\bar{a}\right)\left(\left(\bar{b}.\bar{b}\right)\bar{a}-\left(\bar{b}.\bar{a}\right)\bar{b}\right)+\left(\bar{b}.\bar{c}\right)\left(\left(\bar{a}.\bar{a}\right)\bar{b}-\left(\bar{a}.\bar{b}\right)\bar{a}\right)$$

$$=\left(\bar{c}.\bar{a}\right)\left(\bar{b}\times\left(\bar{a}\times\bar{b}\right)\right)+\left(\bar{b}.\bar{c}\right)\left(\bar{a}\times\left(\bar{b}\times\bar{a}\right)\right)$$

$$=\left(\bar{a}\times\bar{b}\right)\times\left(-\left(\bar{c}.\bar{a}\right)\bar{b}+\left(\bar{c}.\bar{b}\right)\bar{a}\right)=\left(\bar{a}\times\bar{b}\right)\times\left(-\left(\bar{c}.\bar{b}\right)\bar{a}+\left(\bar{c}.\bar{a}\right)\bar{b}\right)$$

$$=\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\left(\bar{a}\times\bar{b}\right)\right)=\left(\left(\bar{a}\times\bar{b}\right).\left(\bar{a}\times\bar{b}\right)\right)\bar{c}-0=\left|\bar{a}\times\bar{b}\right|^2\bar{c}$$

215. If $\bar{\alpha}\,\|\left(\bar{\beta}\times\bar{\gamma}\right)$, then $\left(\bar{\alpha}\times\bar{\beta}\right).\left(\bar{\alpha}\times\bar{\gamma}\right)$ equal to

1) $\left|\bar{\beta}\right|^2\left(\bar{\gamma}.\bar{\alpha}\right)$ 2) $\left|\bar{\alpha}\right|^2\left(\bar{\beta}.\bar{\gamma}\right)$ 3) $\left(\bar{\gamma}.\bar{\alpha}\right)\left(\bar{\beta}.\bar{\alpha}\right)$ 4) $\left|\bar{\gamma}\right|^2\left(\bar{\alpha}.\bar{\beta}\right)$

Key:2

Sol: $\bar{\alpha}=\lambda\left(\bar{\beta}\times\bar{\gamma}\right),\lambda\in\mathbb{R}$. $\left(\bar{\alpha}\times\bar{\beta}\right).\left(\bar{\alpha}\times\bar{\gamma}\right)=\begin{vmatrix}\left|\bar{\alpha}\right|^2 & \bar{\alpha}.\bar{\gamma}\\ \bar{\alpha}.\bar{\beta} & \bar{\beta}.\bar{\gamma}\end{vmatrix}=\left|\bar{\alpha}\right|^2\left(\bar{\beta}.\bar{\gamma}\right)$

$\left(\because\ \bar{\alpha}.\bar{\gamma}=\bar{\beta}.\bar{\alpha}=0\right)$

CONCEPT 6: $\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\left[\bar{a}\ \bar{b}\ \bar{d}\right]\bar{c}-\left[\bar{a}\ \bar{b}\ \bar{c}\right]\bar{d}=\left[\bar{a}\ \bar{c}\ \bar{d}\right]\bar{b}-\left[\bar{b}\ \bar{c}\ \bar{d}\right]\bar{a}$

216. If $\bar{a},\bar{b},\bar{c}$ are non-coplanar vectors such that $\left[\bar{b}\ \bar{c}\ \bar{d}\right]=24$ and

$\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)+\left(\bar{a}\times\bar{c}\right)\times\left(\bar{d}\times\bar{b}\right)+\left(\bar{a}\times\bar{d}\right)\times\left(\bar{b}\times\bar{c}\right)+k\bar{a}=\bar{O}$, then $\frac{k}{8}=$

1) 6 2) 7 3) 8 4) 9

Key:1

Sol: $\left[\bar{b}\bar{c}\bar{d}\right]=24\ \&\ \left[\bar{a}\bar{b}\bar{c}\right]\neq 0$. Consider

$$\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)+\left(\bar{a}\times\bar{c}\right)\times\left(\bar{d}\times\bar{b}\right)+\left(\bar{a}\times\bar{d}\right)\times\left(\bar{b}\times\bar{c}\right)$$

$$=\left[\bar{a}\bar{c}\bar{d}\right]\bar{b}-\left[\bar{b}\bar{c}\bar{d}\right]\bar{a}+\left[\bar{a}\bar{d}\bar{b}\right]\bar{c}-\left[\bar{b}\bar{c}\bar{d}\right]\bar{a}+\left[\bar{a}\bar{b}\bar{c}\right]\bar{d}-\left[\bar{b}\bar{c}\bar{d}\right]\bar{a}$$

$$=\left[\bar{b}\bar{c}\bar{d}\right]\bar{a}-3\left[\bar{b}\bar{c}\bar{d}\right]\bar{a}=-2\left(24\right)\bar{a}=-48\bar{a}\ \therefore k=48\Rightarrow\frac{k}{8}=6$$

217. Let $\bar{b}$ and $\bar{c}$ be unit vectors. For any arbitrary vector $\bar{a}$, $\left(\left(\left(\bar{a}\times\bar{b}\right)+\left(\bar{a}\times\bar{c}\right)\right)\times\left(\bar{b}\times\bar{c}\right)\right).\left(\bar{b}-\bar{c}\right)$ is equal to

1) $\left|\bar{a}\right|$ 2) $\frac{\left|\bar{a}\right|}{2}$ 3) $\frac{\left|\bar{a}\right|}{3}$ 4) 0

Key:4

Sol: $\left|\bar{b}\right|=\left|\bar{c}\right|=1$. Consider $\left(\left(\bar{a}\times\bar{b}\right)\times\left(\bar{b}\times\bar{c}\right)\right)+\left(\left(\bar{a}\times\bar{c}\right)\times\left(\bar{b}\times\bar{c}\right)\right).\left(\bar{b}-\bar{c}\right)$

$$=\left(\left[\bar{a}\,\bar{b}\,\bar{c}\right]\bar{b}+\left[\bar{a}\,\bar{b}\,\bar{c}\right]\bar{c}\right).\left(\bar{b}-\bar{c}\right)=\left[\bar{a}\bar{b}\bar{c}\right]\left(\left|\bar{b}\right|^2-\left|\bar{c}\right|^2\right)=0$$

218. If $\bar{a},\bar{b},\bar{c},\bar{d},\bar{e}$ *and* $\bar{f}$ are non-coplanar vectors and

$\left[\bar{a}\times\bar{b}\ \ \bar{c}\times\bar{d}\ \ \bar{e}\times\bar{f}\right]\times\left(\left[\bar{a}\,\bar{c}\,\bar{d}\right]\left[\bar{b}\,\bar{e}\,\bar{f}\right]-\left[\bar{b}\,\bar{c}\,\bar{d}\right]\left[\bar{a}\,\bar{e}\,\bar{f}\right]\right)>0$, then the value of

$$\log_k\left(\frac{\left[\bar{a}\times\bar{b}\ \ \bar{c}\times\bar{d}\ \ \bar{e}\times\bar{f}\right]}{\left[\bar{a}\,\bar{c}\,\bar{d}\right]\left[\bar{b}\,\bar{e}\,\bar{f}\right]-\left[\bar{b}\,\bar{c}\,\bar{d}\right]\left[\bar{a}\,\bar{e}\,\bar{f}\right]}\right)^{2023}$$, where $k\in\left\{x\in\mathbb{Z}^+ / \frac{2023-x}{2022}<1\right\}$ is equal to

1) 2023 2) 1 3) 0 4) None of these

Key:3

Sol: $\left[\bar{a}\times\bar{b}\ \ \bar{c}\times\bar{d}\ \ \bar{e}\times\bar{f}\right]=\left(\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)\right).\left(\bar{e}\times\bar{f}\right)=\left[\bar{a}\,\bar{c}\,\bar{d}\right]\left[\bar{b}\,\bar{e}\,\bar{f}\right]-\left[\bar{b}\,\bar{c}\,\bar{d}\right]\left[\bar{a}\,\bar{e}\,\bar{f}\right]\ OR$

$\left[\bar{a}\times\bar{b}\ \ \bar{c}\times\bar{d}\ \ \bar{e}\times\bar{f}\right]=\left(\bar{a}\times\bar{b}\right).\left(\left(\bar{c}\times\bar{d}\right)\times\left(\bar{e}\times\bar{f}\right)\right)=\left[\bar{a}\,\bar{b}\,\bar{d}\right]\left[\bar{c}\,\bar{e}\,\bar{f}\right]-\left[\bar{a}\,\bar{b}\,\bar{c}\right]\left[\bar{d}\,\bar{e}\,\bar{f}\right]$

CONCEPT 7: $\bar{a},\bar{b},\bar{c},\bar{d}$ **are coplanar vectors** $\Rightarrow\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\bar{O}$

219. Let $\bar{a},\bar{b},\bar{c}$ *and* $\bar{d}$ be such that $\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\bar{O}$. Let P_1 *and* P_2 be planes determined by the pair of vectors $\bar{a},\bar{b}$ *and* $\bar{c},\bar{d}$ respectively, then the angle between P_1 *and* P_2, is

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Key:1

Sol: $\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\bar{o}$. Let θ be angle between $\bar{a}\times\bar{b}$ & $\bar{c}\times\bar{d}$

Then $\sin\theta=\frac{\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)}{\left|\bar{a}\times\bar{b}\right|\left|\bar{c}\times\bar{d}\right|}\Rightarrow\theta=0^\circ$

220. If A, B, C, D and E be any five coplanar points with position vectors $\bar{a}=\overline{AB},\bar{b}=\overline{BC},\bar{c}=\overline{CD}$ *and* $\bar{d}=\overline{DE}$, then $\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=$

1) 0 2) $\bar{a}$ 3) $\bar{a}\times\bar{c}$ 4) $\bar{O}$

Key:4

Sol: A,B,C,D & E are coplanar points such that

$\bar{a}=\overline{AB},\bar{b}=\overline{BC},\bar{c}=\overline{CD}$ & $\bar{d}=\overline{DE}$

Now $\bar{a}\times\bar{b}$ *and* $\bar{c}\times\bar{d}$ are parallel $\Rightarrow\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\bar{o}$

GEOMETRICAL APPLICATION OF VECTORS

221. The Cartesian equation of the line passing through the point (2,-1,4) and parallel to the vector $\bar{i}+\bar{j}+2\bar{k}$ is

a) $\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-4}{-2}$ b) $\frac{x+2}{1}=\frac{y-1}{1}=\frac{z-4}{-2}$

c) $\frac{x-2}{2}=\frac{y+1}{1}=\frac{z-4}{-2}$ d) $\frac{x+2}{1}=\frac{y-1}{1}=\frac{z+4}{-2}$

key: a

SOL:- The Cartesian equation to the straight line passing through the point (a_1,a_2,a_3) and parallel to the vector $b_1\bar{i}+b_2\bar{j}+b_3\bar{k}$ is $\frac{x-a_1}{b}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}$

Here (a_1,a_2,a_3)=(2,-1,4) and $b_1\bar{i}+b_2\bar{j}+b_3\bar{k}=\bar{i}+\bar{j}-2\bar{k}$

$\therefore$ cartesian equation is $\frac{x-2}{2}=\frac{y+1}{1}=\frac{z-4}{-2}$

222.he vector equation of the line passing through the point $2\bar{i}+\overline{3j}-4\bar{k}$ and parallel to the vector $6\bar{i}+\overline{3j}-4\bar{k}$ is

a) $\bar{\gamma}=\left(2\bar{i}+\overline{3j}-4\bar{k}\right)+t\left(6\bar{i}+\overline{3j}-\bar{k}\right)$ b) $\bar{\gamma}=\left(2\bar{i}+\overline{3j}-4\bar{k}\right)+t\left(6\bar{i}+\overline{3j}-4\bar{k}\right)$

c) $\bar{\gamma}=\left(2\bar{i}+\overline{3j}-4\bar{k}\right)+t\left(6\bar{i}+\overline{3j}+4\bar{k}\right)$ d) $\bar{\gamma}=\left(2\bar{i}+\overline{3j}-4\bar{k}\right)+t\left(6\bar{i}-\overline{3j}+4\bar{k}\right)$

key:b

SOL:- The vector equation of the line passing through the point $\bar{a}$ and parallel to the vector $\bar{b}$ is $\bar{\gamma}=\bar{a}+t\left(\bar{b}\right),t\in R$

$\therefore$ here $\bar{a}=2\bar{i}+\overline{3j}-4\bar{k}$ and $\bar{b}=6\bar{i}+\overline{3j}-4\bar{k}$

$\therefore$ $\bar{\gamma}=\left(2\bar{i}+\overline{3j}-4\bar{k}\right)+t\left(6\bar{i}+\overline{3j}-4\bar{k}\right)$

223. The vector equation of the line passing through the point (1,-2,3) and parallel to the vector (-1,2,-1) is

a) $\bar{\gamma}=\left(\bar{i}-2\bar{j}+3\bar{k}\right)+t\left(\bar{i}+2\bar{j}-\bar{k}\right)$ b) $\bar{\gamma}=\left(\bar{i}-2\bar{j}+3\bar{k}\right)+t\left(\bar{i}+2\bar{j}+\bar{k}\right)$

c) $\bar{\gamma}=\left(\bar{i}+2\bar{j}-3\bar{k}\right)+t\left(-\bar{i}+2\bar{j}-\bar{k}\right)$ d) $\bar{\gamma}=\left(\bar{i}-2\bar{j}+3\bar{k}\right)+t\left(-\bar{i}+2\bar{j}-\bar{k}\right)$

key : d

SOL:- here $\bar{a}=\bar{i}-2\bar{j}+3\bar{k}$ and $\bar{b}=-\bar{i}+2\bar{j}-\bar{k}$

The vector equation of the line passing through the point $\bar{a}$ and parallel $\bar{b}$ is $\bar{\gamma}=\bar{a}+t\left(\bar{b}\right),t\in R$

$\therefore$ $\bar{\gamma}=\left(\bar{i}-2\bar{j}+3\bar{k}\right)+t\left(-\bar{i}+2\bar{j}-\bar{k}\right),t\in R$

224. If $A\left(3\bar{i}+2\bar{j}-\bar{k}\right),B\left(2\bar{i}-2\bar{j}+5\bar{k}\right),C\left(\bar{i}+3\bar{j}-\bar{k}\right)$ then the vector equation of the line passing through the centroid of Δ ABC and parallel to $\overline{BC}$ is

a) $\bar{\gamma}=\left(\bar{i}+5\bar{j}-4\bar{k}\right)+t\left(2\bar{i}+\bar{j}+\bar{k}\right)$ b) $\bar{\gamma}=\left(2\bar{i}+\bar{j}+\bar{k}\right)+t\left(-\bar{i}+5\bar{j}-6\bar{k}\right)$

c) $\bar{\gamma}=\left(3\bar{i}+2\bar{j}-\bar{k}\right)+t\left(-\bar{i}+5\bar{j}-6\bar{k}\right)$ d)None

key : b

SOL:-Centroid $=\frac{\overline{OA}+\overline{OB}+\overline{OC}}{3}=2\bar{i}+\bar{j}+\bar{k}$

$\overline{BC}=\overline{OC}=\overline{OB}=-\bar{i}+5\bar{j}-6\bar{k}$

Vector equation of the line is

$\bar{\gamma}=\left(2\bar{i}+\bar{j}+\bar{k}\right)+t\left(-\bar{i}+5\bar{j}-6\bar{k}\right)$

225. If $A\left(\bar{i}+2\bar{j}+3\bar{k}\right), B\left(-\bar{i}-\bar{j}+8\bar{k}\right), C\left(-4\bar{i}+4\bar{j}+6\bar{k}\right)$ are the vertices of a triangle then the equation of the line passing through the circumcentre and parallel to $\overleftrightarrow{AB}$ is

a) $\bar{\gamma}=\left(\frac{-4}{3}\bar{i}+\frac{5}{3}\bar{j}+\frac{17}{3}\bar{k}\right)+t\left(2\bar{i}+3\bar{j}-5\bar{k}\right)$ b) $\bar{\gamma}=\left(\frac{4}{3}\bar{i}+\frac{5}{3}\bar{j}+\frac{17}{3}\bar{k}\right)+t\left(2\bar{i}+3\bar{j}-5\bar{k}\right)$

c) $\bar{\gamma}=\left(\frac{-4}{3}\bar{i}+\frac{5}{3}\bar{j}+\frac{-17}{3}\bar{k}\right)+t\left(2\bar{i}+3\bar{j}-5\bar{k}\right)$ d) $\bar{\gamma}=\left(\frac{4}{3}\bar{i}-\frac{5}{3}\bar{j}+\frac{17}{3}\bar{k}\right)+t\left(2\bar{i}+3\bar{j}-5\bar{k}\right)$

key :a

SOL:- Here AB = BC =CA= $\sqrt{38} \Rightarrow \Delta ABC$ *is equilateral*, $\overline{AB}=\overline{OB}-\overline{OA}=2\bar{i}+3\bar{j}-5\bar{k}$

$\therefore$ Circumcentre = centroid $=\left(\frac{-4}{3}\bar{i}+\frac{5}{3}\bar{j}+\frac{17}{3}\bar{k}\right)+t\left(2\bar{i}+3\bar{j}-5\bar{k}\right)$

226. If $2\bar{i}-\bar{j}+\bar{k}$, $i-3\bar{j}-5\bar{k}$, $3\bar{i}-4\bar{j}-4\bar{k}$ are the vectices of a triangle then the vector equation of the Median passing through $2\bar{i}-\bar{j}+\bar{k}$ is

a) $\bar{\gamma}=\left(2\bar{i}-\bar{j}+\bar{k}\right)+t\left(2\bar{i}-\frac{7}{2}\bar{j}-\frac{9}{2}\bar{k}\right)$ b) $\bar{\gamma}=\left(2\bar{i}-\bar{j}+\bar{k}\right)+t\left(5\bar{i}+11\bar{k}\right)$

c) $\bar{\gamma}=\left(\bar{i}-3\bar{j}-5\bar{k}\right)+t\left(2\bar{i}-\frac{7}{2}\bar{j}-\frac{9}{2}\bar{k}\right)$ d)None

Key :b

SOL:-mid point of $i-3\bar{j}-5\bar{k}$, $3\bar{i}-4\bar{j}-4\bar{k}$ is $2\bar{i}-\frac{7}{2}\bar{j}-\frac{9}{2}\bar{k}$

Equation of the required median is

$\bar{\gamma}=\left(2\bar{i}-\bar{j}+\bar{k}\right)+2t\left(\left(2\bar{i}-\bar{j}+\bar{k}\right)-\left(2\bar{i}-\frac{7}{2}\bar{j}-\frac{9}{2}\bar{k}\right)\right)$

$=\left(2\bar{i}-\bar{j}+\bar{k}\right)+t\left(5\bar{j}+11\bar{k}\right)$

227. The vector equation of the line passing through the points (-2,3,5) ,(1,2,3) is

a) $\bar{\gamma}=(1-t)\left(-2\bar{i}+3\bar{j}+5\bar{k}\right)+t\left(\bar{i}+2\bar{j}+3\bar{k}\right)$ b) $\bar{\gamma}=(1-t)\left(2\bar{i}+\bar{j}+3\bar{k}\right)+t\left(-4\bar{i}+3\bar{j}-\bar{k}\right)$

c) $\bar{\gamma}=(1-t)\left(2\bar{i}-3\bar{j}+4\bar{k}\right)+t\left(4\bar{i}+2\bar{j}-3\bar{k}\right)$ d)None

key : a

Sol:- vector equation of the line passing through two points is

$\bar{\gamma}=(1-t)\bar{a}+t\bar{b}, t\in R$

$=(1-t)\left(-2\bar{i}+3\bar{j}+5\bar{k}\right)+t\left(\bar{i}+2\bar{j}+3\bar{k}\right)$

228. The Cartesian equation of the passing the points $2\bar{i}+\bar{j}+3\bar{k}$, $-4\bar{i}+3\bar{j}-3\bar{k}$ is

a) $\frac{x-2}{3}=\frac{y-1}{-1}=\frac{z-3}{2}$ b) $\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-3}{2}$

c) $\frac{x+2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ d) $\frac{x+2}{3}=\frac{y+1}{-1}=\frac{z+3}{2}$

key : a

SOL:- Vector parallel to the line = $\left(2\bar{i}+\bar{j}+3\bar{k}\right)-\left(-4\bar{i}+3\bar{j}-\bar{k}\right)$ $=\left(6\bar{i}-2\bar{j}+4\bar{k}\right)$

D.r's of the line are (6,-2,4) = (3,-1,2)

$\therefore$ Equation of the line is $\frac{x-2}{3}=\frac{y-1}{-1}=\frac{z-3}{2}$

229. The equation to the attitude of the triangle formed by (1,1,1),(1,2,3),(2,-1,1) through (1,1,1) is

a) $\bar{\gamma}=\left(\bar{i}+\bar{j}+\bar{k}\right)+t\left(\bar{i}-3\bar{j}-2\bar{k}\right)$ b) $\bar{\gamma}=\left(\bar{i}+\bar{j}+\bar{k}\right)+t\left(3\bar{i}+\bar{j}+4\bar{k}\right)$

c) $\bar{\gamma}=\left(\bar{i}+\bar{j}+\bar{k}\right)+t\left(\bar{i}-\bar{j}+2\bar{k}\right)$ d)None

key : c

Sol: Let A(1,1,1) B(1,2,3) C(2,-1,1) . Then AB =AC = $\sqrt{5}$

Midpoint of BC is D = $\left(\frac{3}{2},\frac{1}{2},2\right)$ and AD $\perp$ BC

$\overrightarrow{AD}=\frac{1}{2}\bar{i}-\frac{1}{2}\bar{j}+\bar{k}=\frac{1}{2}\left(\bar{i}-\bar{j}+2\bar{k}\right)$

Equation $\overrightarrow{AD}$ is $\bar{\gamma}=\left(\bar{i}+\bar{j}+\bar{k}\right)+t\left(\bar{i}-\bar{j}+2\bar{k}\right)$

230. The lines $\bar{\gamma}=(6-6s)\bar{a}+(4s-4)\bar{b}+(4-8s)\bar{c}$ and $\bar{\gamma}=(2t-1)\bar{a}+(4t-2)\bar{b}-(2t+3)\bar{c}$ intersect at

a)4$\bar{c}$ b)-4$\bar{c}$ c)3$\bar{c}$ d)-2$\bar{c}$

Key:b

SOL:-$(6-6s)\bar{a}+(4s-4)\bar{b}+(4-8s)\bar{c}=(2t-1)\bar{a}+(4t-2)\bar{b}-(2t+3)\bar{c}$

$\Rightarrow 6-6s=2t-1, 4s-4=4t-2, 4-8s=-(2t+3)$

$\Rightarrow s=1, t=\frac{1}{2}$ $\therefore$ Point of intersection = -4$\bar{c}$

231.The vector equation of the plane passing through the point $\bar{i}-2\bar{j}-3\bar{k}$ and parallel to the vectors $2\bar{i}-\bar{j}+3\bar{k}, 2\bar{i}+3\bar{j}-6\bar{k}$ is

Key : a

a) $\bar{\gamma}=\left(\bar{i}-2\bar{j}-3\bar{k}\right)+s\left(2\bar{i}-\bar{j}+3\bar{k}\right)+t\left(2\bar{i}+3\bar{j}-6\bar{k}\right)$

b) $\bar{\gamma}=(1-s-t)\left(\bar{i}-2\bar{j}-3\bar{k}\right)+s\left(2\bar{i}-\bar{j}+3\bar{k}\right)+t\left(2\bar{i}+3\bar{j}-6\bar{k}\right)$

c) $\bar{\gamma}=\left(\bar{i}-2\bar{j}-3\bar{k}\right)+s\left(4\bar{j}-9\bar{k}\right)$

d) $\bar{\gamma}=\left(4\bar{j}-9\bar{k}\right)+s\left(\bar{i}-2\bar{j}-3\bar{k}\right)$

Sol :- The vector equation of a plane passing through a point $\bar{a}$ and parallel to the non-collinear vectors $\bar{b}$ and $\bar{c}$ is $\bar{\gamma} = \bar{a} + s\bar{b} + t\bar{c}, s,t \in R$

Here $\bar{a} = \bar{i} - 2\bar{j} - 3\bar{k}, \bar{b} = 2\bar{i} - \bar{j} + 3\bar{k}, \bar{c} = 2\bar{i} + 3\bar{j} - 6\bar{k}$

232.The vector equation of the plane passing through the points $\bar{i} + 2\bar{j} + 5\bar{k}$, $-5\bar{j} + \bar{k}$, $-3\bar{i} + 5\bar{j}$ is

a) $\bar{\gamma} = (1-s-t)\left(\bar{i} + 2\bar{j} + 5\bar{k}\right) + s\left(-5\bar{j} + \bar{k}\right) + t\left(-3\bar{i} + 5\bar{j}\right)$, s,t are scalars

b) $\bar{\gamma} = (1-s-t)\left(\bar{i} + 2\bar{j} + 5\bar{k}\right) + s\left(3\bar{i} + 2\bar{j} + \bar{k}\right) + t\left(2\bar{i} + \bar{j} + 3\bar{k}\right)$, s,t are scalars

c) $\bar{\gamma} = (1-s-t)\left(2\bar{i} + \bar{j} + \bar{k}\right) + s\left(\bar{i} - \bar{j} - \bar{k}\right) + t\left(-\bar{i} + \bar{j} + 2\bar{k}\right)$, s,t are scalars

d) $\bar{\gamma} = (1-s-t)\left(\bar{i} - 2\bar{j} + 5\bar{k}\right) + s\left(-5\bar{j} - \bar{k}\right) + t\left(-3\bar{i} + 5\bar{j}\right)$, s,t are scalars

key : a

SOL:- The vector equation of a plane passing through three points $\bar{a}, \bar{b}, \bar{c}$ is $\bar{\gamma} = (1-s-t)\bar{a} + s\bar{b} + t\bar{c}$, s,t are scalars.

233. The Cartesian equation of the plane whose equation is $\bar{\gamma} = (1+\lambda-\mu)\bar{i} + (2-\lambda)\bar{j} + (3-2\lambda+2\mu)\bar{k}$ where λ, μ are scalars is

a)2x+y=5 b)2x-y=5 c)2x-z=5 d)2x-z=5

key :d

SOL:- Given

$$\bar{\gamma} = (1+\lambda-\mu)\bar{i} + (2-\lambda)\bar{j} + (3-2\lambda+2\mu)\bar{k}$$

$$= \left(\bar{i} + 2\bar{j} + 3\bar{k}\right) + \lambda\left(\bar{i} - \bar{j} - 2\bar{k}\right) + \mu\left(-\bar{i} + 2\bar{k}\right)$$

Cartesian equation of plane is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{vmatrix} = 0$$

$$(x-1)(-2) - (y-2)(0) + (z-3)(-1) = 0 \Rightarrow 2x + z = 5.$$

Applications of Vectors in Geometry:

234. The vector equation of the plane which is perpendicular to $2\bar{i} - 3\bar{j} + \bar{k}$ and at a distance of 5 units from the origin is

1) $\bar{r}.(2\bar{i} - 3\bar{j} + \bar{k}) = 5\sqrt{14}$ 2) $\bar{r}.(2\bar{i} - 3\bar{j} + \bar{k}) = 5$

3) $\bar{r}.\frac{(2\bar{i} - 3\bar{j} + \bar{k})}{\sqrt{14}} = 0$ 4) $\frac{\bar{r}.(2\bar{i} - 3\bar{j} + \bar{k})}{\sqrt{14}} = 0$

Key :1

Sol: Vector equation of the plane perpendicular to $\bar{n}$ and at a distance p from origin is $\bar{r}.\hat{n} = p$

$$\overline{r}.\frac{2\overline{i}-3\overline{j}+\overline{k}}{\sqrt{14}}=5\Rightarrow\overline{r}.(2\overline{i}-3\overline{j}+\overline{k})=5\sqrt{14}$$

235. The perpendicular distance from origin to the plane $3x-2y-2z=2$ is

1) $\frac{1}{\sqrt{17}}$ 2) $\frac{2}{\sqrt{17}}$ 3) $\frac{3}{\sqrt{17}}$ 4) $\frac{4}{\sqrt{17}}$

Key: 2

Sol: Perpendicular distance from origin to the plane $ax+by+cz+d=0$ is $\frac{|d|}{\sqrt{a^2+b^2+c^2}}=\frac{2}{\sqrt{17}}$

236. The Cartesian equation of the plane perpendicular to vector $3\overline{i}-2\overline{j}-2\overline{k}$ and passing through the point $2\overline{i}+3\overline{j}-\overline{k}$ is

1) $3x+2y+2z=2$ 2) $3x-2y+2z=2$

3) $3x+2y-2z=2$ 4) $3x-2y-2z=2$

Key: 4

Sol:

$$(\overline{r}-\overline{a}).\overline{b}=0\Rightarrow(\overline{r}-(2\overline{i}+3\overline{j}-\overline{k}).(3\overline{i}-2\overline{j}-2\overline{k})=0$$

$$\Rightarrow(x\overline{i}+y\overline{j}+z\overline{k}).(3\overline{i}-2\overline{j}-2\overline{k})=2\Rightarrow 3x-2y-2z=2$$

237. The vector equation of the plane passing through the point $(3,-2,1)$ and perpendicular to the vector $(4,7,-4)$ is

1) $\overline{r}.(4\overline{i}+7\overline{j}-4\overline{k})=-6$ 2) $\overline{r}.(3\overline{i}-2\overline{j}+\overline{k})=2$

3) $\overline{r}.(4\overline{i}+7\overline{j}-\overline{k})=2$ 4) $\overline{r}.(3\overline{i}-2\overline{j}+\overline{k})=1$

Key: 1

Sol: Required equation of plane is $(\overline{r}-\overline{a}).\overline{b}=0\Rightarrow\overline{r}.\overline{b}=\overline{a}.\overline{b}\Rightarrow\overline{r}.(4\overline{i}+7\overline{j}-4\overline{k})=-6$

238. Angle between the planes $\overline{r}.(2\overline{i}-\overline{j}+\overline{k})=3$ and $\overline{r}.(\overline{i}+\overline{j}+2\overline{k})=4$ is

1) 30^0 2) 60^0 3) 90^0 4) 45^0

Key: 2

Sol: $\cos\theta=\frac{\overline{a}.\overline{b}}{|\overline{a}||\overline{b}|}$ where $\overline{a}=2\overline{i}-\overline{j}+\overline{k}$ and $\overline{b}=\overline{i}+\overline{j}+2\overline{k}$ $\Rightarrow\cos\theta=\frac{1}{2}\Rightarrow\theta=60^0$

239. The locus of the point equidistant from two given points $\overline{a}$ and $\overline{b}$ is given by

1) $\left[\overline{r}-\frac{1}{2}(\overline{a}+\overline{b})\right].(\overline{a}-\overline{b})=0$ 2) $\left[\overline{r}-\frac{1}{2}(\overline{a}-\overline{b})\right].(\overline{a}+\overline{b})=0$

3) $\left[\overline{r}-\frac{1}{2}(\overline{a}+\overline{b})\right].(\overline{a}+\overline{b})=0$ 4) $\left[\overline{r}-\frac{1}{2}(\overline{a}-\overline{b})\right].(\overline{a}-\overline{b})=0$

Key: 1

Sol:

Let $\overline{OA}=\overline{a},\overline{OB}=\overline{b},\overline{OP}=\overline{r}$

Let M be the midpoint of AB=$\frac{1}{2}(\overline{a}+\overline{b})$

$\therefore \overline{MP}.\overline{BA}=0 \Rightarrow \left[\overline{r}-\frac{1}{2}(\overline{a}+\overline{b})\right].(\overline{a}-\overline{b})=0$

240. A particle acted upon by constant forces $4\overline{i}+\overline{j}-3\overline{k}$ and $3\overline{i}+\overline{j}-\overline{k}$ which displace it from a point $\overline{i}+2\overline{j}+3\overline{k}$ to the point $5\overline{i}+4\overline{j}+\overline{k}$, the workdone in standard units by the forces is given by

1)40 2)30 3)25 4)15

Key: 1

Sol: $w=(\overline{F}_1+\overline{F}_2).\overline{AB}=(7\overline{i}+2\overline{j}-4\overline{k}).(4\overline{i}+2\overline{j}-2\overline{k})=40$

241. The force $\overline{F}=3\overline{i}+\overline{j}-\overline{k}$ acts on a particle and it moves from the point $A(2\overline{i}-\overline{j})\,to\,B(2\overline{i}+\overline{j})$ the workdone by the force $\overline{F}$ =

1)1 2)2 3)3 4)4

Key: 2

Sol: $w=\overline{F}.\overline{AB}=(3\overline{i}+\overline{j}-\overline{k}).2\overline{j}=2$

242. The workdone by force $\overline{F}=a\overline{i}+\overline{j}+\overline{k}$ in moving a particle from (1,1,1) to (2,2,2) along a straightline is 5 units then a=

1)1 2)2 3)3 4)4

Key: 3

Sol: $\overline{F}.\overline{AB}=5 \Rightarrow (a\overline{i}+\overline{j}+\overline{k}).(\overline{i}+\overline{j}+\overline{k})=5 \Rightarrow a+1+1=5 \Rightarrow a=3$

Geometrical application of cross product:

I. Area of triangles:

243. The vector area of the triangle whose adjacent sides are $2\hat{i}+3\hat{j}$ *and* $-2\hat{i}+4\hat{j}$ is.

1) $5\hat{k}$ 2) $7\hat{k}$ 3) $9\hat{k}$ 4) $11\hat{k}$

Key: 2

Sol: Let $\overline{a}=2\hat{i}+3\hat{j},\overline{b}=-2\hat{i}+4\hat{j}$

The vector area of triangle $=\frac{1}{2}\overline{a}\times\overline{b}=7\hat{k}$

244. If G is centriod of ΔPQR where $\overline{GP}=2\hat{i}+\hat{j}+3\hat{k}$, $\overline{GQ}=\hat{i}-\hat{j}+2\hat{k}$ then the area of triangle PQR is

1) $\sqrt{35}$ 2) $\frac{3\sqrt{35}}{2}$ 3) $\frac{\sqrt{35}}{2}$ 4) $\frac{5\sqrt{35}}{2}$

Key: 2

Sol: Area of $\Delta PQR=3(Area\ of\ \Delta GPQ)$

$=\frac{3}{2}\left|\overline{GP}\times\overline{GQ}\right|$ $=\frac{3}{2}\left|5\hat{i}-\hat{j}-3\hat{k}\right|=\frac{3}{2}\sqrt{35}$

245. If $\bar{a}$ and $\bar{b}$ are such that $|\bar{a}|=3, |\bar{b}|=2, (\bar{a},\bar{b})=\frac{\pi}{3}$, then the area of the triangle with adjacent sides $\bar{a}+2\bar{b}$ and $2\bar{a}+\bar{b}$ in sq. units is

1) $3\sqrt{3}$ 2) $9\sqrt{3}$ 3) $\frac{9\sqrt{3}}{2}$ 4) $\frac{9}{2}$

Key: 3

Sol: Required area of triangle $=\frac{1}{2}\left|\left(\bar{a}+2\bar{b}\right)\times\left(2\bar{a}+\bar{b}\right)\right|$ $=\frac{1}{2}\left|3\bar{a}\times\bar{b}\right|$

$=\frac{3}{2}|\bar{a}||\bar{b}|\sin\left(\bar{a},\bar{b}\right)$ $=\frac{3}{2}(3)(2)\sin\frac{\pi}{3}=\frac{9\sqrt{3}}{2}$

246. If $\bar{a},\bar{b},\bar{c}$ are the vertices of a triangle ABC than $\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|=$

1) Area of the triangles ABC
2) Two times Area of the triangle ABC
3) Three times area of the ΔABC
4) Four times area of the ΔABC

Key: 2

Sol: Area of ΔABC $=\frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|$

$=\frac{1}{2}\left|\left(\bar{b}-\bar{a}\right)\times\left(\bar{c}-\bar{a}\right)\right|$ $=\frac{1}{2}\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|$

247. The area of the triangle formed by the points $A(1,2,3), B(2,3,1), C(3,1,2)$ is

1) $\frac{3\sqrt{3}}{2}$ 2) $3\sqrt{3}$ 3) $\frac{\sqrt{3}}{2}$ 4) $\sqrt{3}$

Key: 1

Sol: $\overline{AB}=\overline{OB}-\overline{OA}=\hat{i}+\hat{j}-2\hat{k}$

$\overline{AC}=\overline{OC}-\overline{OA}=2\hat{i}-\hat{j}-\hat{k}$

Area of $\Delta ABC=\frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|$ $=\frac{1}{2}\left|-3\hat{i}-3\hat{j}-3\hat{k}\right|=\frac{3\sqrt{3}}{2}$

248. The vector area of parallelogram whose adjacent sides are $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}-\hat{j}+2\hat{k}$ is

1) $3\left(\hat{i}+\hat{k}\right)$ 2) $3\left(\hat{i}-\hat{k}\right)$ 3) $2\hat{i}+\hat{j}-2\hat{k}$ 4) $-2\hat{i}-\hat{j}-2\hat{k}$

Key: 2

Sol: $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}=2\hat{i}-\hat{j}+2\hat{k}$ are adjacent sides of a parallelogram .

Required vector area$=\bar{a}\times\bar{b}$ $=3\hat{i}-3\hat{k}$

249. The area of a parallelogram whose adjacent sides are $3\hat{i}+2\hat{j}+\hat{k}$ *and* $3\hat{i}+\hat{k}$ is

1) $\sqrt{10}$ 2) $10\sqrt{2}$ 3) $2\sqrt{10}$ 4) 20

Key: 3

Sol: Area of parallelogram $=\left|\left(3\hat{i}+2\hat{j}+\hat{k}\right)\times\left(3\hat{i}+\hat{k}\right)\right|$

$=\left|2\hat{i}-6\hat{k}\right|=\sqrt{4+36}=2\sqrt{10}$

250. The vector area of a parallelogram whose diagonals are $\hat{i}+\hat{j}-\hat{k},\ 2\hat{i}-\hat{j}+2\hat{k}$ is

1) $\frac{1}{2}\left(\hat{i}+4\hat{j}-3\hat{k}\right)$
2) $\frac{1}{2}\left(\hat{i}-4\hat{j}+3\hat{k}\right)$
3) $\frac{1}{2}\left(\hat{i}+4\hat{j}+3\hat{k}\right)$
4) $\frac{1}{2}\left(\hat{i}-4\hat{j}-3\hat{k}\right)$

Key: 4

Sol: Vector area of parallelogram $=\frac{1}{2}\left\{\left(\hat{i}+\hat{j}-\hat{k}\right)\times\left(2\hat{i}-\hat{j}+2\hat{k}\right)\right\}$

$=\frac{1}{2}\left(\hat{i}-4\hat{j}-3\hat{k}\right)$

251. The area of the parallelogram whose diagonal are $\hat{i}-3\hat{j}+2\hat{k};\ -\hat{i}+2\hat{j}$ is (in sq units)

1) $4\sqrt{29}$
2) $\frac{1}{2}\sqrt{21}$
3) $10\sqrt{3}$
4) $\frac{1}{2}\sqrt{270}$

Key: 2

Sol: Required area $=\frac{1}{2}\left|\left(\hat{i}-3\hat{j}+2\hat{k}\right)\times\left(-\hat{i}+2\hat{j}\right)\right|$

$=\frac{1}{2}\left|-4\hat{i}-2\hat{j}-\hat{k}\right|$ $=\frac{1}{2}\sqrt{16+4+1}=\frac{1}{2}\sqrt{21}$

252. If ABCD is a quadrilateral such that $\overline{AB}=\hat{i}+2\hat{j},\overline{AD}=\hat{j}+2\hat{k}$ and $\overline{AC}=-2\left(\hat{i}+2\hat{j}\right)+3\left(\hat{j}+2\hat{k}\right)$, then area of the quadrilateral ABCD is

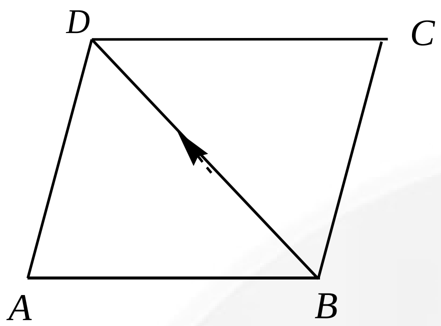
1) $\frac{5\sqrt{21}}{2}$
2) $\frac{3\sqrt{21}}{2}$
3) $\frac{\sqrt{21}}{2}$
4) $\frac{7}{2}$

Key: 3

Sol: $\overline{BD}=\overline{BA}+\overline{AD}$

$=-\hat{i}-2\hat{j}+\hat{j}+2\hat{k}$

$=-\hat{i}-\hat{j}+2\hat{k}$



Area of quadrilateral $=\frac{1}{2}\left|\overline{AC}\times\overline{BD}\right| \quad =\frac{1}{2}\left|4\hat{i}-2\hat{j}-\hat{k}\right| \quad =\frac{1}{2}\sqrt{21}$

253. ABCD is a quadrilateral with $\overline{AB}=\overline{a},\overline{AD}=\overline{b}$ and $\overline{AC}=2\overline{a}+3\overline{b}$. If its area is α times the area of parallelogram with AB, AD as adjacent sides , then $\alpha=$

1) $\frac{5}{2}$ 2) $\frac{3}{2}$ 3) $\frac{9}{2}$ 4) $\frac{7}{2}$

Key: 1

Sol: $\overline{BD}=\overline{BA}+\overline{AD}=-\overline{a}+\overline{b}=\overline{b}-\overline{a}$

Area of quadrilateral ABCD $=\frac{1}{2}\left|\overline{AC}\times\overline{BD}\right|$

$$=\frac{1}{2}\left|\left(2\overline{a}+3\overline{b}\right)\times\left(\overline{b}-\overline{a}\right)\right|$$

$$=\frac{1}{2}\left|5\left(\overline{a}\times\overline{b}\right)\right|=\frac{5}{2}\left|\overline{a}\times\overline{b}\right|$$

Area of parallelogram $ABCD=\left|\overline{AB}\times\overline{AD}\right|=\left|\overline{a}\times\overline{b}\right|$ $\qquad \alpha=\frac{5}{2}$

254. Let $\overline{a}=\alpha\hat{i}+2\hat{j}-\hat{k}$ and $\overline{b}=-2\hat{i}+\alpha\hat{j}+\hat{k}$ where $\alpha\in R$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\overline{a}$ *and* $\overline{b}$ is $\sqrt{15\left(\alpha^2+4\right)}$, then the value of $2\left|\overline{a}\right|^2+\left(\overline{a}.\overline{b}\right)\left|\overline{b}\right|^2$ is equal to (28 th june 2022-I)

1) 10 2)7 3) 9 4) 14

Key: 4

Sol: Area of parallelogram $=\sqrt{15\left(\alpha^2+4\right)}$

$$\left|\overline{a}\times\overline{b}\right|=\sqrt{15\left(\alpha^2+4\right)}$$

$$\left|(2+\alpha)\hat{i}-(\alpha-2)\hat{j}+\left(\alpha^2+4\right)\hat{k}\right|=\sqrt{15\left(\alpha^2+4\right)}$$

$$(2+\alpha)^2+(\alpha-2)^2+\left(\alpha^2+4\right)^2=15\left(\alpha^2+4\right)$$

$\alpha^4 - 5\alpha^2 - 36 = 0$ $\qquad \alpha^2 = 9$

$\alpha = \pm 3$

$|\bar{a}|^2 = \alpha^2 + 4 + 1 = 14$

$|\bar{b}|^2 = 4 + \alpha^2 + 1 = 14$

$\bar{a}.\bar{b} = -2\alpha + 2\alpha - 1 = -1$

Now $2|\bar{a}|^2 + (\bar{a}.\bar{b})|\bar{b}|^2 = 2(14) - 1(14) = 14$

255. Let $\bar{a}$ and $\bar{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\bar{a}$ and $\bar{b}$ be acute, $|\bar{a}| = 1,$ *and* $|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$. If $\bar{c} = 2\sqrt{2}(\bar{a} \times \bar{b}) - 2\bar{b}$, then an angle between $\bar{b}$ *and* $\bar{c}$ is

1) $\frac{\pi}{4}$ 2) $\frac{-\pi}{4}$ 3) $\frac{5\pi}{6}$ 4) $\frac{3\pi}{4}$

Key: 4

Sol: $(\bar{a}, \bar{b})$ is acute and $|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}| \Rightarrow (\bar{a}, \bar{b}) = \frac{\pi}{4}$

Area of parallelogram $= 2\sqrt{2}$

$\frac{1}{2}|\bar{a} \times \bar{b}| = 2\sqrt{2}$

$|\bar{a}||\bar{b}|\sin\frac{\pi}{4} = 4\sqrt{2} \Rightarrow |\bar{b}| = 8$

$\bar{c} = 2\sqrt{2}(\bar{a} \times \bar{b}) - 2\bar{b}$

$|\bar{c}|^2 = 8(\bar{a} \times \bar{b})^2 + 4\bar{b}^2$

$= 8|\bar{a}|^2|\bar{b}|^2\sin^2\frac{\pi}{4} + 4|\bar{b}|^2$

$= 8(1)(64)\frac{1}{2} + 4(64)$

$= 8 \times 64$

$|\bar{c}| = 16\sqrt{2}$

$\bar{b}.\bar{c} = \bar{b}.\left[2\sqrt{2}(\bar{a} \times \bar{b}) - 2\bar{b}\right]$

$= -2|\bar{b}|^2 = -128$

$|\bar{b}||\bar{c}|\cos(\bar{b}, \bar{c}) = -128$

$(8)\left(16\sqrt{2}\right)\cos\left(\bar{b},\bar{c}\right)=-128$

$\cos\left(\bar{b},\bar{c}\right)=\dfrac{-1}{\sqrt{2}}$

$\left(\bar{b},\bar{c}\right)=\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$

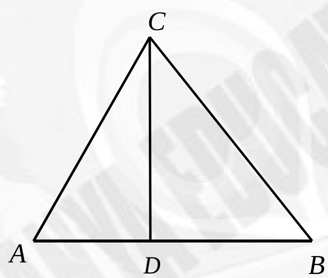
LENGTH OF $\perp^{r}$ TER FROM POINT TO THE SIDE:

256. If $\overline{OA}=(1,2,-5),\overline{OB}=(-2,2,1),\overline{OC}=(4,3,-1)$ then perpendicular distance fro m C to the line AB is

1) $\sqrt{19}$ 2) $\sqrt{21}$ 3) $\sqrt{23}$ 4) $\sqrt{25}$

Key: 2

Sol: Area of $\Delta ABC=\dfrac{1}{2}(base)(height)$



$=\dfrac{1}{2}\left|\overline{AB}\times\overline{AC}\right|$

$\Rightarrow height\ CD=\dfrac{\left|\overline{AB}\times\overline{AC}\right|}{\left|\overline{AB}\right|}$

$\overline{AB}=(-3,0,6)$

$\overline{AC}=(3,1,4)$

$\overline{AB}\times\overline{AC}=-6\hat{i}+30\hat{j}-3\hat{k}$

$=-3\left(2\hat{i}-10\hat{j}+\hat{k}\right)$

Length of perpendicular from C to $AB=\dfrac{\left|\overline{AB}\times\overline{AC}\right|}{\left|\overline{AB}\right|}$

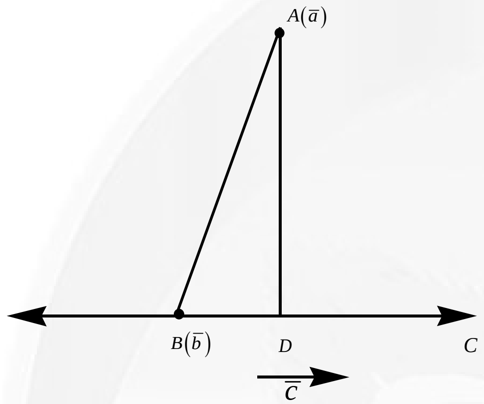
$=\dfrac{3\sqrt{4+100+1}}{\sqrt{9+36}}=\dfrac{\sqrt{105}}{\sqrt{5}}=\sqrt{21}$

257. The perpendicular distance of any point $\bar{a}$ on to the line $\bar{r}=\bar{b}+t\bar{c}$ is

1) $\frac{\left|\left(\bar{a}-\bar{b}\right)\times\bar{c}\right|}{\left|\bar{b}\right|}$ 2) $\frac{\left|\left(\bar{a}-\bar{b}\right)\times\bar{c}\right|}{\left|\bar{c}\right|}$ 3) $\frac{\left|\bar{a}-\bar{b}\right|}{\left|\bar{c}\right|}$ 4) $\frac{\left|\left(\bar{a}-\bar{b}\right)\times\bar{c}\right|}{\left|\bar{a}\right|}$

Key: 2

Sol: Area $=\frac{1}{2}(base)(heigth)\frac{1}{2}\left|\overline{BA}\times\overline{C}\right|$



$\Rightarrow \overline{AD}=\frac{\left|\overline{BA}\times\bar{c}\right|}{\left|\bar{c}\right|} \quad =\frac{\left|\left(\bar{a}-\bar{b}\right)\times\bar{c}\right|}{\left|\bar{c}\right|}$

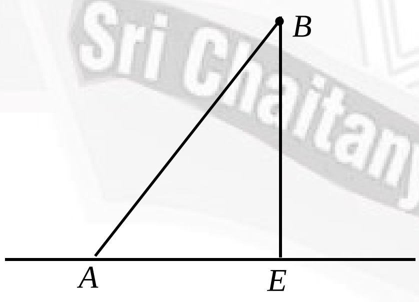
258. The distance of the point B with position vector $\hat{i}+2\hat{j}+3\hat{k}$ from the line through A with position vector $4\hat{i}+2\hat{j}+2\hat{k}$ from the line through A with position vector $4\hat{i}+2\hat{j}+2\hat{k}$ and parallel to the vector $2\hat{i}+3\hat{j}+6\hat{k}$ is

1) $\sqrt{10}$ 2) $\sqrt{5}$ 3) $\sqrt{6}$ 4) 2

Key: 1

Sol: $\overline{OA}=4\hat{i}+2\hat{j}+2\hat{k}, \overline{OB}=\hat{i}+2\hat{j}+3\hat{k}$. Let $\bar{c}=2\hat{i}+3\hat{j}+6\hat{k}$

Length of $\perp^r$ ter from B to the given line $=\frac{\left|\overline{AB}\times\bar{c}\right|}{\left|\bar{c}\right|}$



$=\frac{\left|\left(-3\hat{i}+\hat{k}\right)\times\left(2\hat{i}+3\hat{j}+6\hat{k}\right)\right|}{\sqrt{49}} \quad =\frac{\left|-3\hat{i}+20\hat{j}-9\hat{k}\right|}{7}$

$$=\frac{\sqrt{9+400+81}}{7}=\frac{\sqrt{490}}{7}=\sqrt{10}$$

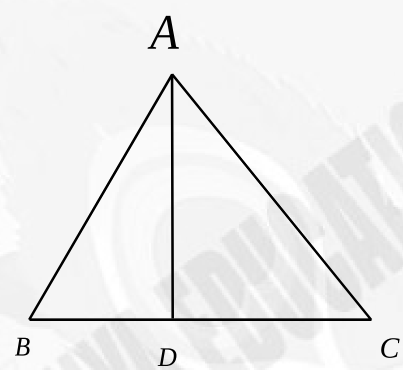
259. If $\overline{AB}=\overline{b}$ *and* $\overline{AC}=\overline{c}$, then the length of the perpendicular from A to the line BC is

1) $\frac{|\overline{b}\times\overline{c}|}{|\overline{b}+\overline{c}|}$ 2) $\frac{|\overline{b}\times\overline{c}|}{|\overline{b}-\overline{c}|}$ 3) $\frac{1}{2}\frac{|\overline{b}\times\overline{c}|}{|\overline{b}-\overline{c}|}$ 4) $\frac{2|\overline{b}\times\overline{c}|}{|\overline{b}-\overline{c}|}$

Key: 2

Sol: $\overline{AB}=\overline{b},\overline{AC}=\overline{c}\Rightarrow\overline{BC}=\overline{c}-\overline{b}$

The length of perpendicular from $A to BC=\frac{|\overline{BC}\times\overline{BA}|}{|\overline{BC}|}$



$$=\frac{|(\overline{c}-\overline{b})\times(-\overline{b})|}{|\overline{c}-\overline{b}|}=\frac{|-\overline{c}\times\overline{b}|}{|\overline{b}-\overline{c}|}=\frac{|\overline{b}\times\overline{c}|}{|\overline{b}-\overline{c}|}$$

Application of vector algebra in physics (cross product)

Torque or vector moment:

260. The torque about the point $2\hat{i}+\hat{j}-\hat{k}$ of a force represented by $4\hat{i}+\hat{k}$ acting through the point $\hat{i}-\hat{j}+2\hat{k}$ is.

1) $2\hat{i}+13\hat{j}+8\hat{k}$ 2) $2\hat{i}+13\hat{j}-8\hat{k}$

3) $2\hat{i}-13\hat{j}+8\hat{k}$ 4) $-2\hat{i}+13\hat{j}+8\hat{k}$

Key: 4

Sol: $\overline{F}=4\hat{i}+\hat{k},\overline{OA}=\hat{i}-\hat{j}+2\hat{k},\overline{OP}=2\hat{i}+\hat{j}-\hat{k}$

Now $\overline{r}=\overline{PA}=\overline{OA}-\overline{OP}=-\hat{i}-2\hat{j}+3\hat{k}$

Torque $=\overline{v}\times\overline{F}$

$=\left(-\hat{i}-2\hat{j}+3\hat{k}\right)\times\left(4\hat{i}+\hat{k}\right)$

$=-2\hat{i}+13\hat{j}+8\hat{k}$

261. A force $\overline{F} = 2\hat{i} - \lambda\hat{j} + 5\hat{k}$ is applied at the point $A(1,2,5)$. If its moment about the point $(-1,-2,3)$ *is* $16\hat{i} - 6\hat{j} + 2\lambda\hat{k}$, then $\lambda =$

1) -2 2) -1 3) 0 4) 2

Key: 1

Sol: $\overline{F} = 2\hat{i} - \lambda\hat{j} + 5\hat{k}, \overline{OA} = \hat{i} + 2\hat{j} + 5\hat{k}, \overline{OP} = -\hat{i} - 2\hat{j} + 3\hat{k}$

Now $\overline{r} = \overline{PA} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

Torque $= \overline{r} \times \overline{F} = (20 + 2\lambda)\hat{i} - 6\hat{j} + (-2\lambda - 8)\hat{k}$

$\Rightarrow 16\hat{i} - 6\hat{j} + 2\lambda\hat{k} = (20 + 2\lambda)\hat{i} - 6\hat{j} + (-2\lambda - 8)\hat{k}$

$\therefore 20 + 2\lambda = 16 \Rightarrow 2\lambda = -4 \Rightarrow \lambda = -2$

262. Forces $2\hat{i} + 7\hat{j}, 2\hat{i} - 5\hat{j} + 6\hat{k}, -\hat{i} + 2\hat{j} - \hat{k}$ act at a point P whose position vector is $4\hat{i} - 3\hat{j} - 2\hat{k}$. The vector moment of resultant of three forces acting at P about the point Q whose position vector is $6\hat{i} + \hat{j} - 3\hat{k}$ is

1) $24\hat{i} + 13\hat{j} + 4\hat{k}$ 2) $-24\hat{i} - 13\hat{j} + 4\hat{k}$

3) $-24\hat{i} + 13\hat{j} + 4\hat{k}$ 4) $-24\hat{i} - 13\hat{j} - 4\hat{k}$

Key: 3

Sol: Resultant force $\overline{F} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\overline{OP} = 4\hat{i} - 3\hat{j} - 2\hat{k}, \overline{OQ} = 6\hat{i} + \hat{j} - 3\hat{k}$

$\overline{r} = \overline{QP} = \overline{OP} - \overline{OQ} \quad = -2\hat{i} - 4\hat{j} + \hat{k}$

Torque $= \overline{r} \times \overline{F}$

$= (-2\hat{i} - 4\hat{j} + \hat{k}) \times (3\hat{i} + 4\hat{j} + 5\hat{k}) \quad = -24\hat{i} + 13\hat{j} + 4\hat{k}$

<u>CONCEPT 1: Skew lines and shortest distance between them</u>

263. The shortest distance between the lines $\overline{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} + 2\hat{j} + \hat{k})$ and $\overline{r} = -\hat{i} - \hat{j} + \hat{k} + \mu(7\hat{i} - 6\hat{j} + \hat{k})$ is

1) $\frac{16}{5\sqrt{5}}$ 2) $\frac{26}{5\sqrt{5}}$ 3) $\frac{46}{5\sqrt{5}}$ 4) $\frac{36}{5\sqrt{5}}$

Key:2

Sol: $\frac{[\overline{a} - \overline{c}\ \overline{b}\ \overline{d}]}{|\overline{b} \times \overline{d}|} = \frac{26}{5\sqrt{5}}$

<u>CONCEPT 2: Condition for the lines to intersect</u>

264. The point of intersection of the lines $\overline{r} \times \overline{a} = \overline{b} \times \overline{a}$ and $\overline{r} \times \overline{b} = \overline{a} \times \overline{b}$

1) $\overline{a} - \overline{b}$ 2) $\overline{a} + \overline{b}$ 3) $2\overline{a} + 3\overline{b}$ 4) $3\overline{a} - 2\overline{b}$

Key:2

Sol: $\overline{r} \times \overline{a} = \overline{b} \times \overline{a}$ and $\overline{r} \times \overline{b} = \overline{a} \times \overline{b}$

$\Rightarrow \left(\bar{r}-\bar{b}\right)\times\bar{a}=\bar{O}$ & $\left(\bar{r}-\bar{a}\right)\times\bar{b}=\bar{O}$

$\Rightarrow \bar{r}=\bar{a}+t\bar{b}=\bar{b}+s\bar{a}$

$\therefore t=s=1\Rightarrow \bar{r}=\bar{a}+\bar{b}$

265. The line passing through the point $P(\bar{a})$, parallel to the line of intersection of the planes $\bar{r}.\bar{n_1}=1,\ \bar{r}.\bar{n_2}=1$, is

1) $\bar{r}=\bar{a}+t\left(\bar{n_1}+\bar{n_2}\right)$ 2) $\bar{r}=\bar{a}+t\left(\bar{n_1}-\bar{n_2}\right)$

3) $\bar{r}=\bar{a}+t\left(\bar{n_1}\times\bar{n_2}\right)$ 4) $\bar{r}=t\left(\bar{n_1}+\bar{n_2}\right)$

Key:3

Sol: Line of intersection is parallel to $\bar{n_1}\times\bar{n_2}$. Required line is

$\bar{r}=\bar{a}+\lambda\left(\bar{n_1}\times\bar{n_2}\right),\ \lambda\in\mathbb{R}$

CONCEPT 3: Vector Equation of planes

266. The vector equation of the plane passing through $\hat{i}+\hat{j}+\hat{k}$ and parallel to the vectors $2\hat{i}+3\hat{j}-\hat{k},\ \hat{i}+2\hat{j}+3\hat{k}$ is

1) $\left[\bar{r}-\left(\hat{i}+\hat{j}+\hat{k}\right)\ \ 2\hat{i}+3\hat{j}-\hat{k}\ \ \hat{i}+2\hat{j}+3\hat{k}\right]=0$ 2) $\left[\bar{r}\ \ 2\hat{i}+3\hat{j}-\hat{k}\ \ \hat{i}+2\hat{j}+5\hat{k}\right]=0$

3) $\left[\bar{r}-\left(\hat{i}+\hat{j}+\hat{k}\right)\ \ \hat{i}+2\hat{j}-2\hat{k}\ \ \hat{j}+2\hat{k}\right]=0$ 4) $\left[\bar{r}\ \ \hat{i}+2\hat{j}-2\hat{k}\ \ \hat{j}+2\hat{k}\right]=0$

Key:1

Sol: Use the formula: $\left[\bar{r}-\bar{a}\ \bar{b}\ \bar{c}\right]=0$

267. The cartesian equation of the plane passing through the points $4\hat{i}+\hat{j}-2\hat{k},\ 5\hat{i}+2\hat{j}+\hat{k}$ and parallel to the vector $3\hat{i}-\hat{j}+4\hat{k}$, is

1) $2x-6y+5z-1=0$ 2) $2x-6y-5z-1=0$

3) $5x-y-z+3=0$ 4) $7x+5y-4z-41=0$

Key:4

Sol: Use the formula: $\left[\bar{r}-\bar{a}\ \ \bar{b}-\bar{a}\ \ \bar{c}\right]=0$

268. The equation of the plane passing through the points $P(2,3,-1), Q(4,5,2)$ *and* $R(3,6,5)$, is

1) $3x-9y-4z-24=0$ 2) $3x+9y+4z+25=0$

3) $3x-9y+4z+25=0$ 4) $3x-9y-4z+25=0$

Key:3

Sol: Use the formula: $\left[\bar{r}-\bar{a}\ \ \bar{b}-\bar{a}\ \ \bar{c}-\bar{a}\right]=0$

269. The equation of the plane passing through the points $A(3,-2,-1)$, and parallel to the vectors $\hat{i}-2\hat{j}+4\hat{k}$ *and* $3\hat{i}+2\hat{j}-5\hat{k}$, is

1) $2x-3y-8z-26=0$ 2) $2x+17y+8z+36=0$

3) $2x-17y-8z-36=0$ 4) $3x-2y-18z+9=0$

Key:2

Sol: Use the formula: $\left[\bar{r}-\bar{a}\ \bar{b}\ \bar{c}\right]=0$

270. The equation of the plane containing the line $\bar{r}=\bar{a}+s\bar{b}$ and parallel to the line $\bar{r}=\bar{c}+t\bar{d}$, is

1) $\left[\bar{r}-\bar{a}\ \bar{b}\ \bar{d}\right]=0$ 2) $\left[\bar{r}-\bar{c}\ \bar{b}\ \bar{d}\right]=0$ 3) $\left[\bar{r}-\bar{d}\ \bar{a}\ \bar{b}\right]=0$ 4) $\left[\bar{r}-\bar{b}\ \bar{c}\ \bar{d}\right]=0$

Key:1

Sol: Use the formula: $\left[\bar{r}-\bar{a}\ \bar{b}\ \bar{c}\right]=0$

271. The equation of the plane containing the lines $\bar{r}=\bar{a}+t\bar{b}$ and $\bar{r}=\bar{b}+s\bar{a}$, is

1) $\left[\bar{r}\,\bar{a}\,\bar{b}\right]=0$ 2) $\bar{r}.\bar{a}=\bar{r}.\bar{b}$ 3) $\bar{r}.\bar{a}=\bar{a}.\bar{b}$ 4) $\bar{r}.\bar{b}=\bar{a}.\bar{b}$

Key:1

Sol: $\left[\bar{r}-\bar{a}\ \bar{a}\ \bar{b}\right]=0 \Rightarrow \left[\bar{r}\ \bar{a}\ \bar{b}\right]=0$

272. The distance between the line $\bar{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda\left(\hat{i}-\hat{j}+4\hat{k}\right)$ and the plane $\bar{r}.\left(\hat{i}+5\hat{j}+\hat{k}\right)=5$, is

1) $2\sqrt{3}$ 2) $\frac{10}{3\sqrt{3}}$ 3) $\frac{\sqrt{3}}{2}$ 4) $\frac{10}{3\sqrt{2}}$

Key:2

Sol: Observe that the straight line is parallel to the given plane. Perpendicular distance required, is the distance from the point $(2,-2,3)$ to the plane $x+5y+z-5=0$. i.e., $\frac{|2-10+3-5|}{\sqrt{1+25+1}}=\frac{10}{3\sqrt{3}}$.

273. The vector equation of the plane passing through the point with position vector $\bar{a}$ and perpendicular to $\bar{b}$, is

1) $(\bar{r}-\bar{a}).\bar{b}=0$ 2) $\bar{r}.\left(\bar{a}\times\bar{b}\right)=0$ 3) $\bar{r}=\bar{a}\times\bar{b}$ 4) $\bar{r}=\bar{b}\times\bar{a}$

Key:1

Sol: Required plane is $\bar{r}.\bar{b}=k,$ where $k\in\mathbb{R}^{+}.\Rightarrow \bar{a}.\bar{b}=k$

$\therefore$ The plane is $\bar{r}.\bar{b}=\bar{a}.\bar{b}\Rightarrow(\bar{r}-\bar{a}).\bar{b}=0$

CONCEPT 4: Length of the perpendicular from origin to the plane containing three points

274. The perpendicular distance from the origin to the plane passing through the points $2\hat{i}-2\hat{j}+\hat{k},\ 3\hat{i}+2\hat{j}-\hat{k},\ 3\hat{i}-\hat{j}-2\hat{k}$ is

1) $\frac{12}{\sqrt{30}}$ 2) $\frac{25}{\sqrt{110}}$ 3) $\frac{10}{\sqrt{60}}$ 4) $\frac{15}{\sqrt{187}}$

Key:2

Sol: Perpendicular distance = $\left|\frac{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}{\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|}\right|=\frac{25}{\sqrt{110}}$

275. The perpendicular distance from origin to the plane passing through the points $2\hat{i}+\hat{j}+3\hat{k},\ \hat{i}+3\hat{j}+2\hat{k},\ 3\hat{i}+2\hat{j}+\hat{k}$, is

1) $2\sqrt{3}$ 2) $3\sqrt{2}$ 3) $3\sqrt{3}$ 4) $2\sqrt{2}$

Key:1

Sol: Perpendicular distance = $\left|\frac{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}{\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|}\right|=2\sqrt{3}$

CONCEPT 6: Problems based on planes

276. The distance of the point $P(\bar{a})$ from the plane $\bar{r}.\bar{n}=q$, measured parallel to the line $\bar{r}=\bar{b}+t\bar{c}$

1) $\left|\dfrac{q-\bar{b}.\bar{n}}{|\bar{c}.\bar{n}|}\right||\bar{c}|^2$ 2) $\dfrac{\bar{q}-\bar{a}.\bar{n}}{|\bar{c}|}$ 3) $\bar{q}+\dfrac{|\bar{b}-\bar{c}|}{|\bar{n}|}$ 4) $\dfrac{|q-\bar{a}.\bar{n}|}{|\bar{c}.\bar{n}|}|\bar{c}|$

Key:4

Sol: Let M$\left(\bar{d}\right)$ be a point on the plane $\bar{r}.\bar{n}=q$ such that the line joining A$\left(\bar{a}\right)$ and M$\left(\bar{d}\right)$ is parallel to the vector $\bar{c}$. $\overline{AM} \,||\, \bar{c} \Rightarrow \bar{d}-\bar{a}=\lambda\bar{c}, \lambda\in\mathbb{R} \Rightarrow \bar{d}=\bar{a}+\lambda\bar{c},$ which lies in $\bar{r}.\bar{n}=q$.

$\Rightarrow (\bar{a}+\lambda\bar{c}).\bar{n}=q \Rightarrow \lambda=\dfrac{q-\bar{a}.\bar{n}}{\bar{c}.\bar{n}} \Rightarrow |\bar{d}-\bar{a}|=\left|\dfrac{q-\bar{a}.\bar{n}}{\bar{c}.\bar{n}}\right||\bar{c}|$

CONCEPT 8: Projection of a vector

277. Let $\bar{a}=\hat{i}+2\hat{j}+\hat{k}$, $\bar{b}=\hat{i}-\hat{j}+\hat{k}$, $\bar{c}=\hat{i}+\hat{j}-\hat{k}$. A vector coplanar to $\bar{a}$ *and* $\bar{b}$ has a projection along $\bar{c}$ of magnitude $\dfrac{1}{\sqrt{3}}$, then the vector is (2006, Adv.)

1) $4\hat{i}-\hat{j}+4\hat{k}$ 2) $4\hat{i}+\hat{j}-4\hat{k}$ 3) $2\hat{i}+\hat{j}+\hat{k}$ 4) None of these

Key:1

Sol: If $\bar{d}$ is required vector, then $\bar{d}.\left(\bar{a}\times\bar{b}\right)=0$ and

$\left|\dfrac{\bar{d}.\bar{c}}{|\bar{c}|}\right|=\dfrac{1}{\sqrt{3}} \Rightarrow |\bar{d}.\bar{c}|=1 \Rightarrow \alpha=\gamma \;\&\; |\alpha+\beta+\gamma|=1$ $\therefore \alpha=\gamma \;\&\; \beta=\pm1$

Verify the options to get satisfied the above conditions