Sri Chaitanya IIT Academy.,India.

✪ A.P ✪ T.S ✪ KARNATAKA ✪ TAMILNADU ✪ MAHARASTRA ✪ DELHI ✪ RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_(NUCLEUS,STERLING) & LIIT _BT JEE-MAIN Date: 20-01-2023
Time: 02.00Pm to 05.00Pm GTM-10 Max. Marks: 300

KEY SHEET

PHYSICS

1)	2	2)	3	3)	4	4)	2	5)	1
6)	4	7)	4	8)	3	9)	1	10)	3
11)	2	12)	3	13)	2	14)	3	15)	4
16)	2	17)	2	18)	4	19)	2	20)	2
21)	5	22)	2	23)	8	24)	3	25)	9
26)	2	27)	5	28)	2	29)	5	30)	5

CHEMISTRY

31)	4	32)	3	33)	3	34)	1	35)	1
36)	3	37)	4	38)	3	39)	3	40)	2
41)	2	42)	3	43)	3	44)	4	45)	3
46)	1	47)	3	48)	3	49)	2	50)	2
51)	3	52)	16	53)	2	54)	4	55)	8
56)	500	57)	1	58)	7	59)	3	60)	600

MATHEMATICS

61)	2	62)	1	63)	2	64)	3	65)	1
66)	1	67)	1	68)	2	69)	1	70)	3
71)	2	72)	4	73)	1	74)	1	75)	3
76)	1	77)	4	78)	3	79)	3	80)	3
81)	1	82)	1	83)	1	84)	0	85)	78
86)	2	87)	0	88)	4	89)	2	90)	8

SOLUTIONS

PHYSICS

1. $v\cos(90^0-\theta)=u\cos\theta$ or $v\sin\theta=u\cos\theta$; $v=u\cot\theta$

$$\frac{v_T^2}{R}=a_c;\frac{u^2\cot^2\theta}{g\sin\theta}=R$$

2. a) when $t<\frac{mg\sin\theta}{k}$, f will be upwards & f= $mg\sin\theta-kt$

b) At $t=\frac{mg\sin\theta}{k}, f=0$

C) At $t>\frac{mg\sin\theta}{k}$ till the body starts to moves f will be down wards and f= $kt-mg\sin\theta$

d) Once it start moving f = constant

3. Conceptual

4. a) $(mv_o\sin\theta)5R=mv_1R....(1)$

b) $\frac{1}{2}mv_0^2-\frac{GMm}{5R}=\frac{1}{2}mv_1^2-\frac{GMm}{R}....(2)$

Solving $\theta=\sin^{-1}\left[\frac{1}{5}\sqrt{1+\frac{8GM}{5v_0^2R}}\right]$

5. $\sqrt{2gy}L^2=\sqrt{2g(4y)}\times\pi R^2$

$\Rightarrow R=\frac{L}{\sqrt{2\pi}}$

6. $2T\left[\frac{1}{r_1}-\frac{1}{r_2}\right]=\rho gh$

$\Rightarrow h=11.36\,mm$

7. Resolving power is directly proportional to diameter of objective

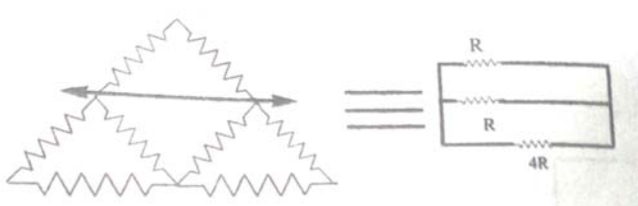
8. Electromagnetic waves interact with matter via their electric and magnetic field which in oscillation of charges present in all matter. The detailed interaction and so the mechanism of absorption, scattering, etc. depend of the wavelength of the electromagnetic wave, and the nature of the atoms and molecules in the medium.

9. Conceptual

10. Potential drop across C_1 is maximum

Hence energy stored in C_1 is maximum as Energy α (potential drop)

11. $r=\frac{4R}{9}$



By symmetric method

The internal resistance must be equal to external resistance for maximum power transfer

The R_{eq} for circuit= $\frac{4R}{9}$

Thus, $R_{eq} = \frac{4R}{9}$.Thus $r = \frac{4R}{9}$

12. Current density $\bar{J} = j_0 r\hat{k}$

Current within a distance d $I = \int_{r=0}^{d} j.ds = \int_0^d J_0 r.2\pi r dr = 2\pi J_0 \frac{d^3}{3}$

From ampere's Law $\int_C \bar{B}.\bar{dl} = \mu_0 2\pi J_0 \frac{d^3}{3}$

Here loop is a circle of radius, r, B is magnetic field at r

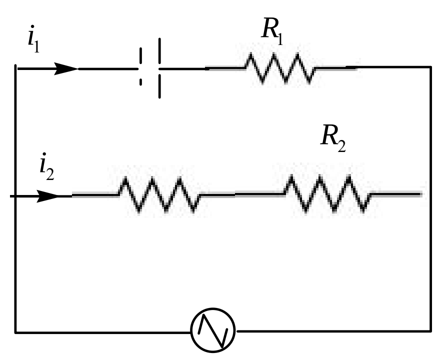
$$2\pi r.B = \mu_0 2\pi J \frac{d^3}{3}$$

$$\Rightarrow B_{r(=d)} = \frac{\mu_0 J_0 d^2}{3}$$

13. Use Fleming's left hand rule, we find that a force is acting in the radially outward direction through the circumference of the conducting loop.

14.
$$i_{1rms} = \frac{E_{rms}}{\sqrt{x_c^2 + R_1^2}} = \frac{130}{13} = 10A$$

$$i_{2rms} = \frac{E_{rms}}{\sqrt{x_L^2 + R_2^2}} = 13A$$



Power dissipated

$i_{1rms}^2 R_1 + i_{2rms}^2 R_2 = 10^2 \times 5 + 13^2 + 6$ = 1514 W = power delivered by battery

15. $\delta = (45^0 - 30^0) + (180^0 - 60^0) + (45^0 - 30^0)$

$= 150^0$ clockwise

16. fringe width=$\frac{\lambda D}{d}$

$12\lambda_1 = k\lambda_2$. Hence $k = \frac{12 \times 600}{400} = 18$

17. $\frac{N}{N_0}=\frac{1}{2^{t/t_{1/2}}}$

18. Ferro-magnetic substances become paramagnetic above Curie temp.

19. CONCEPTUAL

20. Conceptual

21. Given $Q=x^{2/5}y^{-1}t^{-1/2}z^3$

$$\frac{\Delta Q}{Q}\times 100=\frac{2}{5}\frac{\Delta x}{x}\times 100+\frac{\Delta y}{y}\times 100+\frac{1}{2}\frac{\Delta t}{t}\times 100+3\frac{\Delta z}{z}\times 100$$

$$=\frac{2}{5}\times 2.5+2+3\times 0.5+\frac{1}{2}\times 1=5\%$$

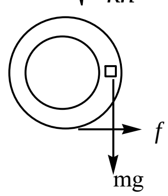
22. lose in gravitational P.E= gain in spring P.e

$$mgh=\frac{1}{2}k(h\cot\alpha-h)^2$$

$$or(\cot\alpha-1)=\sqrt{\frac{2mg}{kh}}$$

$$\cot\alpha=1+\sqrt{\frac{2mg}{kh}}$$

23.



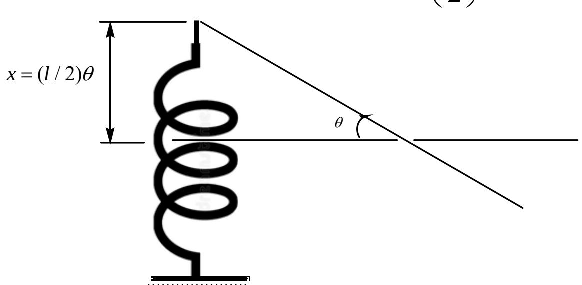
Sol:

$$(mg-f)r=(3mr^2+mr^2)\alpha\ \ldots\ldots(1)$$

f =4ma --- (2)

Solving $\alpha=\frac{g}{8r}$

24. restoring torque is given by $kx\left(\frac{l}{2}\right)=I\alpha$



$$kx\left(\frac{l}{2}\theta\right)\left(\frac{l}{2}\right)=\frac{ml^2}{2}(\alpha)=\frac{3k}{m}\theta\Rightarrow\omega=\sqrt{\frac{3k}{m}}$$

25. $\frac{E_r}{E_i}=\left(\frac{A_r}{A_i}\right)^2=\left(\frac{v_2-v_1}{v_1+v_2}\right)^2=1/9$

Therefore, $\frac{E_r}{E_i}=8/9$

26. $f = \frac{330-v}{330-22}\times 176;\ f_2 = \frac{330+v}{330}\times 165$

$f_1 - f_2 = 0 \Rightarrow v = 22m/s$

27. $a)\frac{K\times 1\times 27}{9\times L} = PAV......(1)$

$b)PA\frac{L}{2} = nR\times 300.......(2)$

$Hence\ \frac{K\times 3}{L} = \frac{2nR\times 300V}{L} \Rightarrow V = \frac{K}{100R}$

28. $\int_2^v dv = -\int_{(0,0)}^{(1,2)} \bar{E}.d\bar{r}\ \ d\bar{r} = dx\hat{i} + dy\hat{j}$

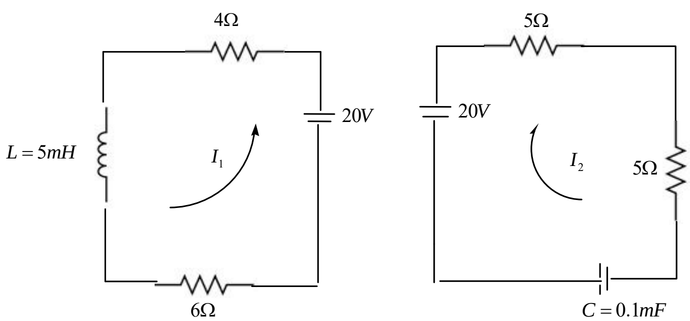
$\bar{E}.d\bar{r} = (2xy+y)dx + (x^2+x)dy = d(x^2y+xy)$

$\int_2^v dv = -\int_{(0,0)}^{(1,2)} (x^2y+xy)$

$V-2 = -\left[\left(1^2\times 2+1\times 2\right)-0\right]$

$V-2 = -4, V = -2volts$

29. $I_1 = \frac{20}{10}\left(1-e^{-\frac{10t}{5\times 10^{-3}}}\right) = \frac{3}{2} = 1.5A$



$I_1 = \frac{20}{10}e^{-\frac{t}{1\times 10^{-3}}} = 1.0A$

From superposition $I = I_1 + I_2 = 2.5A$

30. for first line of balmar series

$\frac{1}{\lambda} = R\left(\frac{1}{4}-\frac{1}{9}\right) \Rightarrow R = \frac{36}{5\lambda}$

Wave length of the first line λ_L of the Lyman series is given by

$\frac{1}{\lambda_L} = R\left(1-\frac{1}{4}\right) = \frac{36}{5\lambda}\times\frac{3}{4} = \frac{27}{5\lambda} \Rightarrow \lambda_L = \frac{5\lambda}{27}$

CHEMISTRY

31. Greater the polarity of solvent more will be its interaction with substance which will effect R_f. TLC is an example of adsorption chromatography.

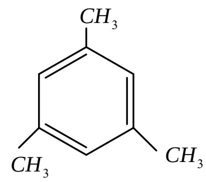
32. Isomeric ethers have relatively low boiling point than alcohols. Cresol is soluble in aqueous $NaOH$.

33. $BF_3, CO_3^{2-}, NO_3^- \rightarrow Planar triangular$

$SO_4^{2-}, CrO_4^{2-}, CF_4 \rightarrow tetrahedral$

$NH_3 \rightarrow Pyramidal$

$SF_4 \rightarrow sea-saw$

34. $CH_3-CH=CHBr \xrightarrow[ii)red.hot.Fe]{i)NaNH_2}$  For electrophilic substitution on the ring all positions are similar

35. Due to low hydration enthalpies of both ions in CsI it is less soluble in water

36. Deacon's process------ $CuCl_2$, Ostwald's process------ $Pt-Rh$ wire guage

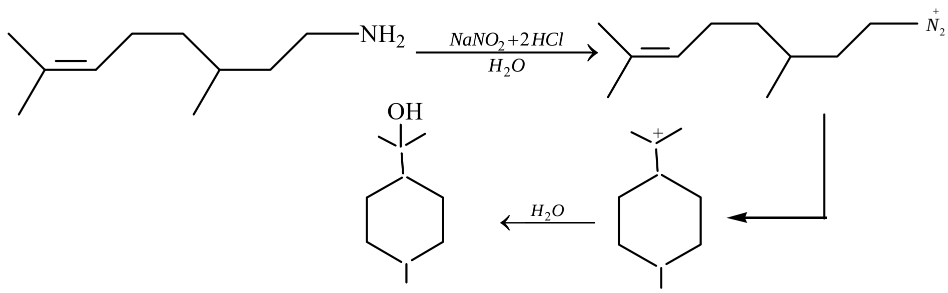
Contact process----------- V_2O_5, Haber's process----------- Fe

37. Aldehydes will give silver mirror with Tollens reagent

38. Ring expansion and more stable tertiary carbocation formation is involved

39. $Mg(OH)_2$ is less soluble in water than $Ca(OH)_2$

40.



41. PbS – black ppt

$Pb(NO_3)_2$ – soluble in water

PbI_2 -yellow ppt

42. Diborane is non-planar molecule

43. More number of resonance structures formed, greater will be its stability

44.

$$-\!\!\left(CH_2 - \underset{\displaystyle OCOCH_3}{\underset{|}{CH}} \right)_{\!n} \xrightarrow{H_3O^+} -\!\!\left(CH_2 - \underset{\displaystyle OH}{\underset{|}{CH}} \right)_{\!n}$$

45. Benzaldehyde is produced

46. XeF_5^- has square pyramidal shape with different $Xe-F$ bond lengths due to lone pair-bond pair repulsions.

47. electron withdrawing groups will increase reactivity towards SN_2 mechanism

48. $Cu + 8HNO_3(dilute) \rightarrow 3Cu(NO_3) + 2NO + 4H_2O$

49. In amylose $\alpha-D-$glucose units were joined via C_1-C_4 glycosidic link

50. calamine--- $ZnCO_3$ Cryolite---- Na_3AlF_6

Siderite --- $FeCO_3$ Magnetite--- Fe_3O_4

51. it gives two geometrical isomers out of which one is optically active

52.

$$0.42 = \frac{-0.06}{1}\log\left(\frac{[Ag^+]\ saturated}{(0.1)}\right)$$

$[Ag^+] = 1\times10^{-8}$

53. $\Delta T_f = (1.86)\left(\frac{1}{2}\right) = 0.93K$

$\Delta T_b = (0.52)(2) = 1.04K$

$\Delta T_b + \Delta T_f = 1.97 \simeq 2$

54. 15 moles of $NaOH$ will neutrilize 5 moles of $KHC_2O_4.H_2C_2O_4.2H_2O$

55. $3P \rightarrow 6\, electrons$

$4S \rightarrow 2\ electrons$

56. $K_p = \frac{80}{20} = 4 - (2)(T)\left[\ell n(4)\right] = 1400 - 5.6T$

$-2.8T = 1400 - 5.6T$

$2.8T = 1400$

$T = \frac{1400}{2.8}K = 500K$

57. $K = \frac{1}{0.2}\ell n\left(\frac{2}{1.6}\right)$

58. $Cubic \rightarrow 3$

$Tetragonal \rightarrow 2$

$Hexagonal \rightarrow 1$

$R\text{hom}bohedral \rightarrow 1$

59. weight of 1 lit of solution=1100g

Weight of solvent in 1 lit solution

=1100-360

=740g

Molality $= 2 \times \frac{1000}{740}$

$= 2.7 mole.Kg^{-1}$

60. $A_2 + B_2 \rightarrow 2\ AB \quad \Delta H = -300KJ$

$\Delta H_r = \left[\left(B.E\ A_2\right) + \left(B.E\ \ B_2\right)\right] - \left[2(B.E\ \ AB)\right]$

MATHEMATICS

61. $(p \wedge \sim q) \wedge (\sim p \wedge q) \equiv (\sim q \wedge p) \wedge (\sim p \wedge q)$

$\equiv \sim q \wedge (p \wedge \sim p) \wedge q \equiv \sim q \wedge F \wedge q \equiv (\sim q \wedge q) \wedge F \equiv F \wedge F \equiv F$

Statement-1 is true $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$

$\sim q \Rightarrow \sim p \Rightarrow \sim(\sim q) \vee \sim p \Leftrightarrow q \vee \sim p \Leftrightarrow \sim p \vee q$

Statement-2 is true

Thus, both the statements are true and

statement-2 is not the correct explanation for statement-1

62. The given equation is dx – x (ydx+ xdy)= x^5y^4(ydx + xdy)

$\Rightarrow \frac{dx}{x} = (1 + x^4y^4)d(xy) \Rightarrow \ln x = xy + \frac{1}{5}x^5y^5 + \ln c \Rightarrow x = ce^{xy+\frac{1}{5}x^5y^5}$.

63. The roots of first equation are –1 and a^2 –1. Now the roots of second equation are 1, $a^2 + 4a$.

According to given condition $a^2 - 1 < 1$ and $a^2 - 1 < a^2 + 4a$

$a \in (-\sqrt{2}, \sqrt{2})$ and $a > -\frac{1}{4} \Rightarrow a \in \left(-\frac{1}{4}, \sqrt{2}\right)$

64. Since $\sqrt{x^2 - 3x + 2} \geq 0, \quad 0 \leq \tan^{-1}\sqrt{x^2 - 3x + 2} < \frac{\pi}{2}$

And $\sqrt{4x - x^2 - 3} \geq 0 \Rightarrow 0 < \cos^{-1}\sqrt{4x - x^2 - 3} \leq \frac{\pi}{2}$.

Adding, we have $0 < \text{L.H.S.} < \pi$

$\Rightarrow$ The given equation has no solution.

65. $S = (n-1)\cos\frac{2\pi}{n} + (n-2)\cos\frac{4\pi}{n} + (n-3)\cos\frac{6\pi}{n} + ... + \cos\frac{2(n-1)\pi}{n}$

$S = 1\cos\frac{2\pi}{n} + 2\cos\frac{4\pi}{n} + ... + (n-1)\cos\frac{2(n-1)\pi}{n}$

$2S = n\left(\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + ... + \cos\frac{2(n-1)\pi}{n}\right)$

$$2S = n\frac{\sin(n-1)\frac{\pi}{n}}{\sin\frac{\pi}{n}}\cos\left(\frac{\frac{2\pi}{n} + \frac{2(n-1)\pi}{n}}{2}\right) = -n$$

66. Let θ be the required angle, then

$$\sin\theta = \cos(90^\circ - \theta) = \left|\frac{(\vec{a}\times\vec{b}).\vec{b}\times\vec{c}}{|\vec{a}\times\vec{b}||\vec{b}\times\vec{c}|}\right| = \left|\frac{[(\vec{a}\times\vec{b})\times\vec{b}].\vec{c}}{\sin^2\alpha}\right| = \left|\frac{[(\vec{a}.\vec{b})\vec{b} - (\vec{b}.\vec{b})\vec{a}].\vec{c}}{\sin^2\theta}\right|$$

$$= \left|\frac{\cos\alpha\,\vec{b}.\vec{c} - \vec{a}.\vec{c}}{\sin^2\alpha}\right| = \left|\frac{\cos^2\alpha - \cos\alpha}{\sin^2\alpha}\right| = \left|-\frac{\cos\alpha(1-\cos\alpha)}{\sin\alpha\sin\alpha}\right| = \left|-\frac{\cos\alpha\, 2\sin^2\frac{\alpha}{2}}{\sin\alpha\, 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right|$$

$$= \left|-\cot\alpha\tan\frac{\alpha}{2}\right| = \left|\cot\alpha\tan\frac{\alpha}{2}\right| \therefore \theta = \sin^{-1}\left(\tan\frac{\alpha}{2}|\cot\alpha|\right)$$

67. $\frac{dy}{dx} = -\frac{x_1^2}{y_1^2}$

Tangent equation is $x_1^2x + y_1^2y = x_1^3 + y_1^3$ $\Rightarrow x_1^2x + y_1^2y = a^3$

Since, it passes through (x_2, y_2)

$$\therefore x_1^2x_2 + y_1^2y_2 = a^3 \quad (1)$$

and $$x_1^3 + y_1^3 = a^3 \quad (2)$$

$$x_2^3 + y_2^3 = a^3 \quad (3)$$

By solving (1), (2), (3) we get result

68. There will exist two common tangents when both the circles are intersecting.

Solving the equation $4 - 4\lambda x + 9 = 0 \Rightarrow x = \frac{13}{4\lambda} \Rightarrow y^2 + \left(\frac{13}{4\lambda}\right)^2 = 4$ or $y = \pm\sqrt{4-\left(\frac{13}{4\lambda}\right)^2}$.

It should have two real and distinct values so $4 - \left(\frac{13}{4\lambda}\right)^2 > 0$

69. $$f'(x) = (x-2)^{2/3}2 + (2x+1)\frac{2}{3}(x-2)^{-1/3} = \frac{6(x-2)+2(2x+1)}{3(x-2)^{1/3}} = \frac{10x-10}{3(x-2)^{1/3}}$$

$x = 1$ is a point of local maximum and $x = 2$ is a point of local minimum

$\therefore$ No.of extremum points is 2

70. We have to find the number of integral solutions if $x_1 + x_2 + x_3 + x_4 + x_5 = 6$ and that equals ${}^{5+6-1}C_{5-1} = {}^{10}C_4$

Thus Statement-1 is false.

Number of different ways of arranging 6A's and 4B's in a row

$= \frac{\lfloor 10}{\lfloor 6 \times \lfloor 4} = {}^{10}C_4 =$ Number of different way the child can buy the six ice-creams.

$\therefore$Statement-2 is true

So, Statement-1 is false, Statement-2 is true.

71.
$$\int \frac{x^2-1}{x^2\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}}dx = \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}dx \quad \text{put } x+\frac{1}{x} = t$$

$$= \int \frac{dt}{t\sqrt{t^2-2}} \quad t = \sqrt{2}\sec\theta \quad = \int \frac{\sqrt{2}\sec\theta\tan\theta d\theta}{\sqrt{2}\sec\theta\sqrt{2}\tan\theta} = \frac{1}{\sqrt{2}}\theta + C$$

72. 5x+3y-2=0, 3x-y-4=0

(x, y)=(1, -1)

x-y+1=0, 2x-y-2=0

(x, y)=(3, 4)

Required line passing through (1, -1) and (3, 4)

73. $ln\, f(x) = ln(x+1) + ln(x+2) + ... + ln(x+100)$ $\therefore$Differentiating

$$\frac{f'(x)}{f(x)} = \frac{1}{x+1} + \frac{1}{x+2} + ... + \frac{1}{100}$$

Again differentiating $$\frac{f(x)f''(x) - f'(x)^2}{f(x)^2} = \frac{-1}{(x+1)^2} - \frac{1}{(x+2)^2}......\frac{-1}{(x+100)^2} < 0$$

$$\therefore f(x)f''(x) - \left(f'(x)\right)^2 < 0 \ \therefore g(x) < 0 \quad \therefore g(x) = 0 \text{ has no solution}$$

74. For f to be defined we must have $\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)<-1 \Leftrightarrow 1+\frac{1}{\sqrt[4]{x}}>\left(2^{-1}\right)^{-1}=2$ which is possible if and only if $\frac{1}{\sqrt[4]{x}}>1$ i.e. 0<x<1. Hence the domain of the given function is $\{x: 0<x<1\}$

75. The chord of contact $y y_o=2(x+x_o)$ of the point $P(x_o, y_o)$ w.r.t the parabola is tangent to the hyperbola $x^2-y^2=1$ *iff* $2x_0^2+y_0^2=4$. Locus of P is the ellipse $2x^2+y^2=4$

76.

$$C=AB=\begin{bmatrix} x & 1 & 2 \\ 3x & -1 & \frac{1}{4x^2+1}\end{bmatrix}\begin{bmatrix} \frac{1}{x^2} & \frac{1}{x} \\ 2x & 2 \\ 3 & x\end{bmatrix}$$

$$\Delta(x)=\sum_{1\le i,j\le 2} c_{ij}=c_{11}+c_{12}+c_{22}=\frac{1}{x}+2x+6+1+2+2x+3-2+\frac{x}{4x^2+1}=\frac{1}{x}+4x+10+\frac{1}{4x+\frac{1}{x}}$$

Let $4x+\frac{1}{x}=t \Rightarrow t\ge 4$

$$\Delta(x)_{\min}=4+10+\frac{1}{4}=\frac{57}{4}$$

77. Given $xRy \Leftrightarrow \sin^2 x+\cos^2 y=1$

Now $\sin^2 x+\cos^2 x=1$, So R is Reflexive. i.e, xRx

Let $xRy \Rightarrow \sin^2 x+\cos^2 y=1$

$\Rightarrow 1-\cos^2 x+1-\sin^2 y=1$

$\Rightarrow \sin^2 y+\cos^2 x=1$

So $xRy \Rightarrow yRx$ $\quad\therefore$ R is symmetrix

Now Let xRy and yRz holds

i.e, $\sin^2 x+\cos^2 y=1$ and $\sin^2 y+\cos^2 z=1$

So $\sin^2 x+\cos^2 z=1$ (from above '2' equations)

So $xRy \;\&\; yRz \Rightarrow xRz$

$\therefore$ R is Transitive

Hence R is an equivalence relation

78. Using L.M.V.T for $x\in[-7,-1]$, we have

$$\frac{f(-1)-f(-7)}{-1+7}\le 2 \Rightarrow \frac{f(-1)+3}{6}\le 2\left(\because\ f(-3)=-3\right)\Rightarrow f(-1)\le 9$$

Also using L.M.V.T for $x\in[-7,0]$, we have

$$\frac{f(0)-f(-7)}{0+7}\le 2 \Rightarrow \frac{f(0)+3}{7}\le 2 \Rightarrow f(0)\le 11 \therefore\ f(0)+f(-1)\le 20$$

79. n is even so n = 2m

$$E=2.\frac{m!\ \ m!}{(2m)!}.\left[C_0^2-2C_1^2+3C_2^2......+(-1)^{2m}(2m+1)C_{2m}^2\right]\to(1)$$

$C_{2m} = C_0, \quad C_{2m-1} = C_1$

Write the terms in reverse order

$$E = \frac{2m!m!}{(2m)!}\left[(2m+1)C_0^2 - 2mC_1^2 + (2m-1)C_2^2 + ...C_{2m}^2\right] \to (2)$$

$$2E = \frac{2m!m!}{(2m)!}(2m+2)\left(C_0^2 - C_1^2 + C_2^2 + ...C_{2m}^2\right)$$

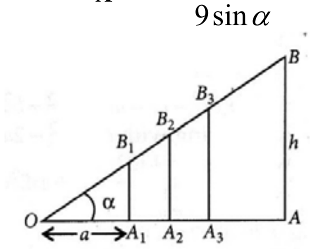
$$= \frac{2m!m!(m+1)}{(2m)!}(-1)^m 2mC_m\left[C_0^2 - C_1^2 + C_2^2 .... + C_n^2\right] = (-1)^{n/2}.nC_{n/2}$$

$$= 2(-1)^m(m+1) \qquad = 2(-1)^{n/2}\left(\frac{n}{2}+1\right) \qquad = (-1)^{n/2}(n+2)$$

80. Let the distance between each of the pole be x

$$\frac{h}{a+9x} = \tan\alpha$$

$$x = \frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$$



81. $$e^{i\alpha} + e^{i\beta} = (\cos\alpha + i\sin\alpha) + (\cos\beta + i\sin\beta) = e^{i\gamma}$$

Let $a = e^{i\alpha}, b = e^{i\beta}, c = e^{i\gamma}$ then $a + b = c$. Also $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$.

$\Rightarrow ab = c^2 \Rightarrow e^{i(\alpha+\beta)} = e^{i(2\gamma)} \Rightarrow \sin(\alpha+\beta) = \sin 2\gamma$.

82. S.D. between the lines $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and $\frac{x-\alpha'}{l'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$ is given by

$$\text{S.D.} = \begin{vmatrix} \alpha-\alpha' & \beta-\beta' & \gamma-\gamma' \\ l & m & n \\ l^- & m' & n' \end{vmatrix} \div \sqrt{\Sigma(mn' - m'n)^2}$$

$$\therefore \Sigma(mn' - m'n)^2 = \sqrt{\left[(-1)^2 + (2)^2 + (1)^2\right]} = \sqrt{6} \qquad \therefore S.D. = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \div \sqrt{6} = \frac{1}{\sqrt{6}}$$

83. Required area

$$= \int_0^1 \left(2x - 2x^2 - x\log x\right)dx \qquad = \left[x^2 - \frac{2x^3}{3}\right]_0^1 - \left\{\frac{x^2}{2}\log x - \frac{x^2}{4}\right\}_0^1$$

$$= \frac{1}{3} + \frac{1}{4} = \frac{7}{12} = 0.583 \qquad \left[\because \log_{x\to 0^+} x^2 \log x = 0\right]$$

84. a = P(A getting 6), b = P(B getting 7)

$$\lambda = (1-a)(1-b)$$

$$P(A)=\frac{a}{1-\lambda}=\frac{\frac{5}{36}}{1-\frac{31}{36}.\frac{5}{6}}=\frac{30}{61}=0.4918$$

85. We have $\Sigma x=170,\ \Sigma x^2=2830$

Increase in $\Sigma x=10$

$\Rightarrow \Sigma x^1=170+10=180$

Increase in $\Sigma x^2=900-400=500$

$\Rightarrow \Sigma x^2=2830+500=3330$

$$\therefore\ \sigma^2=\frac{1}{15}(3330)-\left(\frac{1}{15}\times 180\right)^2=222-(12)^2=78$$

$$\left(\therefore\ \sigma^2=\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2\right)$$

86. $g(2-x)=g(2+x)$

$\&g(2+x)\sin x$ is an odd function $\quad\therefore\ \mathrm{I}_1=0$

Now $g(2-(2-x))=g(2+2-x)\Rightarrow g(x)=g(4-x)\Rightarrow g^1(x)=-g^1(4-x)$

So $I_2=\int_0^4\frac{1}{1+e^{g^1(x)}}dx\to(1)\quad=\int_0^4\frac{dx}{1+e^{g^1(4-x)}}\quad=\int_0^4\frac{dx}{1+e^{-g^1(x)}}$

$=\int_0^4\frac{e^{g^1(x)}}{1+e^{g^1(x)}}dx\to(2)\ (1)+(2):\quad 2\mathrm{I}_2=\int_0^4 1dx\Rightarrow I_2=2$

87. $\lambda^3\hat{i}+\hat{k},\hat{i}-\lambda^3\hat{j}$ and $\hat{i}+(2\lambda-\sin\lambda)\hat{j}-\lambda\hat{k}$

$$\begin{vmatrix}\lambda^3 & 0 & 1\\ 1 & -\lambda^3 & 0\\ 1 & 2\lambda-\sin\lambda & -\lambda\end{vmatrix}=0$$

$$\therefore 2\lambda-\sin\lambda+\lambda^3-\lambda\left(-\lambda^6\right)=0\Rightarrow\lambda^7+\lambda^3+2\lambda-\sin\lambda=0\Rightarrow\lambda=0$$

88. Max values of $\sin x+\cos x$ and $1+\sin 2x$ are $\sqrt{2}$ and 2 respectively.

Also $\left(\sqrt{2}\right)^2=2$

$\therefore$ the equation can hold only when $\sin x+\cos x=\sqrt{2}$ and $1+\sin 2x=2$

Now $\sin x+\cos x=\sqrt{2}$

$\Rightarrow\cos\left(x-\frac{\pi}{4}\right)=1\qquad\Rightarrow\qquad x=2n\pi+\frac{\pi}{4}$

$1+\sin 2x=2\Rightarrow\sin 2x=1\Rightarrow x=\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}$

The value in $[-\pi,\pi]$ satisfying both the equations is $\frac{\pi}{4}$. [when $n=0$]

89.

$$\lim_{x\to0+}f(x)=\lim_{x\to0+}\frac{4+e^{\frac{1}{x}}}{1+e^{\frac{4}{x}}}+2\frac{\sin x}{x}=\lim_{x\to0+}\frac{4e^{\frac{-1}{x}}+1}{e^{\frac{-1}{x}}+e^{\frac{3}{x}}}+2\frac{\sin x}{x}=0+2=2$$

$$\lim_{x\to 0-} f(x)=\lim_{x\to 0-}\frac{4+e^{\frac{1}{x}}}{1+e^{\frac{4}{x}}}-2\frac{\sin x}{x}=4-2=2$$

90. We have $\sin x\sqrt{8\cos^2 x}=1 \qquad \Rightarrow \sin x\,|\cos x| = \frac{1}{2\sqrt{2}}$

Case – 1 when $\cos x > 0$

In this case $\sin x \cos x = \frac{1}{2\sqrt{2}} \qquad \Rightarrow \sin 2x=\frac{1}{\sqrt{2}} \Rightarrow 2x=\frac{\pi}{4},\frac{3\pi}{4},\frac{9\pi}{4},\frac{13\pi}{4}$

$\Rightarrow x=\frac{\pi}{8},\frac{3\pi}{8},\frac{9\pi}{8},\frac{13\pi}{8}$

As x lies between 0 and 2π and $\cos x>0$, $x=\frac{\pi}{8},\frac{3\pi}{8}$

Case – 2 when $\cos x < 0$

In this case $\frac{1}{2\sqrt{2}} \Rightarrow \sin x\cos x = -\frac{1}{2\sqrt{2}}$ or $\sin 2x=-\frac{1}{\sqrt{2}}$

$\Rightarrow x=\frac{5\pi}{8},\frac{7\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8} \qquad \Rightarrow x=\frac{5\pi}{8},\frac{7\pi}{8}$ as $\cos x < 0$

Thus the value of x satisfying the given equation which lie between 0 and 2π are $\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8}$ These are in A.P. with common difference $\frac{\pi}{4}$.