Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT	JEE-MAIN	Date: 11-01-2023
Time: 09.00Am to 12.00Pm	GTM-05	Max. Marks: 300

KEY SHEET

PHYSICS

1)	4	2)	3	3)	2	4)	2	5)	1
6)	2	7)	2	8)	1	9)	2	10)	3
11)	4	12)	1	13)	1	14)	4	15)	1
16)	2	17)	4	18)	2	19)	1	20)	1
21)	10	22)	100	23)	2	24)	7	25)	2
26)	6	27)	1	28)	5	29)	7	30)	45

CHEMISTRY

31)	1	32)	3	33)	3	34)	2	35)	3
36)	3	37)	2	38)	1	39)	4	40)	1
41)	2	42)	4	43)	2	44)	4	45)	3
46)	2	47)	1	48)	4	49)	2	50)	2
51)	57	52)	1	53)	3	54)	3	55)	6
56)	8	57)	3	58)	3	59)	3	60)	4

MATHEMATICS

61)	3	62)	1	63)	1	64)	2	65)	3
66)	2	67)	4	68)	2	69)	1	70)	2
71)	4	72)	1	73)	3	74)	1	75)	1
76)	3	77)	1	78)	2	79)	2	80)	4
81)	216	82)	8	83)	2	84)	1	85)	7
86)	2	87)	4	88)	64	89)	5	90)	4

SOLUTIONS

PHYSICS

1. $t_{mean} = t_{true} = \dfrac{53+52+55+54+51}{5} = 53\sec$

Mean error $\dfrac{0+1+2+1+2}{5} = \dfrac{6}{5} = 1.2$

Least count is 1 sec, means round off 1.2 to 1 sec.

$\therefore t = 53 \pm 1\sec$

2. The rolling sphere has rotational as well as translational kinetic energy.

$$KE = \frac{1}{2}mu^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mu^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2$$

$$= \frac{1}{2}mu^2 + \frac{1}{5}mu^2 = \frac{7}{10}mu^2 \therefore (u = r\omega)$$

From energy conservation i.e., $mgh = \dfrac{7}{10}mu^2$ or $h = \dfrac{7u^2}{10}$

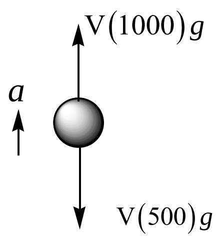
3. $V = V_{CE} + I_C R_C$

$\Rightarrow 15 = 7 + I_C \times 2 \times 10^3 \Rightarrow i_C = 4mA$

$\because \beta = \dfrac{i_C}{i_B} \Rightarrow i_B = \dfrac{4}{100} = 0.04mA$

4. Changing polarity is termed as AC.

5. Velocity of ball when it reaches to surface of liquid



$a = \dfrac{1000gV - 500gV}{500V}$ where V is the volume of the ball

$a = 10m/\sec^2 \ (\uparrow)$

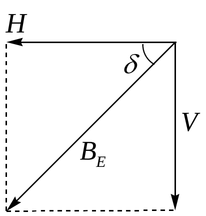
Apply $v = u + at \rightarrow 0 = \sqrt{2gh} - 10t$

$0 = \sqrt{2gh} - 10t \ \sqrt{2gh} = 10 \times (2)$

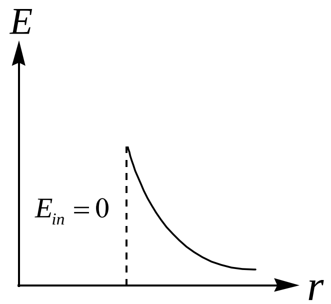
$2 \times 10 \times h = 400 \Rightarrow h = 20m$

6. $V = B_E \sin\delta$

$H = B_E \cos\delta$



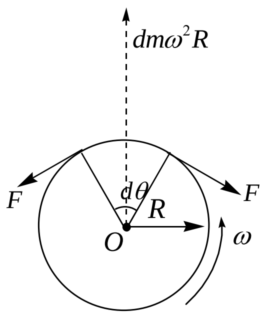
7. For a metal sphere $E_{in} = 0$ and $\vec{E}_{out} = \frac{Kq}{r^2}\hat{r}$



8. Numerical aperture $= \frac{0.61\lambda}{d} \Rightarrow d = \frac{0.61 \times 0.5\mu m}{1.25} = 0.24\mu m$

9. $Fd\theta = dm\omega^2 R$

$Fd\theta = (\rho ARd\theta)\omega^2 R$



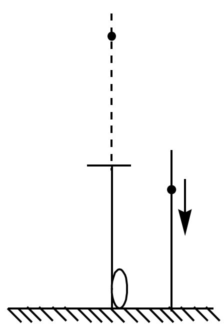
$\therefore$ stress, $F/A = \rho\omega^2 R^2$

Now, $\frac{F}{A} = Y\frac{\Delta l}{l} = Y\frac{\Delta R}{R}$

$[\because l = 2\pi R, \Delta l = 2\pi\Delta R]$

$\rho\omega^2 R^2 = Y\frac{\Delta R}{R} \therefore \Delta R = \frac{\rho\omega^2 R^3}{Y}$

10.



$$F_T = \lambda V^2 = \frac{m}{L}\left(\sqrt{2 \times g\frac{1}{2}}\right)^2$$

$$F_T = mg \qquad F_{Net} = mg + \frac{mg}{2} = \frac{3mg}{2}$$

11. $(Te)_i = (Te)_\infty - \frac{GMm}{R} + \frac{1}{2}m(3v_e)^2 = \frac{1}{2}m(v_\infty)^2$

$$-\frac{1}{2}mv_e^2+9\left(\frac{1}{2}mv_e^2\right)=\frac{1}{2}m\left(V_\infty\right)^2\ V_\infty=2\sqrt{2}V_e$$

12. $\eta=\dfrac{\text{workdone}}{\text{heat suplied}}=\dfrac{+200+600-300}{200+600}=\dfrac{5}{8}$

13. $U(x)=\left(x^2-3x\right)J$

For a conservative field, Force $F=-\dfrac{du}{dx}$

$\therefore F=-\dfrac{d}{dx}\left(x^2-3x\right)=-\left(2x-3\right)=-2x+3$

At equilibrium position, $F=0\ \Rightarrow -2x+3=0\Rightarrow x=\dfrac{3}{2}m=1.5m$

14. Eddy current effect is not used in electric heater

15. Let $\phi_1=4eV$, then $\phi_2=2eV$

$(E-\phi)$ represent kinetic energy of most energetic electron.

$E-\phi_2=2\left(E-\phi_1\right)\ \Rightarrow E=6eV$

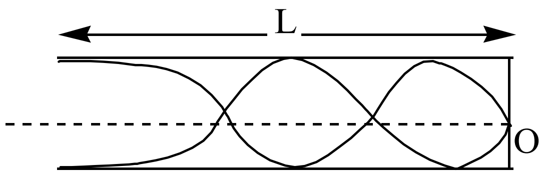
16. Let $a>b>c$, so

$P_1=\dfrac{V^2bc}{\rho a},P_2=\dfrac{V^2ac}{\rho b},P_3=\dfrac{V^2ab}{\rho c}$

Volume of cuboid $=abc=\dfrac{m}{d}\Rightarrow 4\sqrt{2}c^3\dfrac{m}{d}\Rightarrow c=\sqrt[3]{\dfrac{m}{4\sqrt{2}d}}$

17. $\dfrac{5\lambda}{4}=L\Rightarrow\lambda=\dfrac{4L}{5}\Rightarrow k=\dfrac{5\pi}{2L}$

$\dfrac{2\pi}{k}=\dfrac{4L}{5}$ $\qquad k=\dfrac{5\pi}{2L}$



Equation of wave from open end:

At $t=0$ $P=\dfrac{P_0}{2}\Rightarrow\dfrac{1}{2}=\sin\dfrac{5\pi x}{2L}\Rightarrow x=\dfrac{\pi}{6}\times\dfrac{2L}{5\pi}=\dfrac{L}{15}$

18. Range $=x=\sqrt{2h_TR}$

$=\sqrt{2\times150\times6400\times10^3}$

Area $=\pi x^2$

Population density $=\dfrac{50\times10^5}{\pi\times2\times150\times6400\times10^3}$

$=\dfrac{1}{6\pi\times64}m^{-2}$ $\qquad =\dfrac{10^6}{6\pi\times64}(km)^{-2}=828.6km^{-2}$

19. $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$ $\vec{F} = \perp \vec{v}$

Work done $= \vec{F}.\vec{S}$

Work done $= 0$

20. C_p is always greater than C_v in gases.

Work done at constant pressure is more than at constant volume.

21. Mass of rope $= 10 \times 0.5 = 5kg$

Given force $= 25n$

Acceleration $= F/m = 25/5 = 5m/s^2$

Length of remaining rope $= 4m$

Hence mass of remaining rope $= 4/2 = 2kg$

Hence tension on the rope at a point 6m away $= ma = 2 \times 5 = 10$

22. Under resonance $iA = \frac{V_\Theta}{R} = 5A$ Voltmeter reading $V_\Theta = 500V$

23. $\lambda = \frac{h}{\sqrt{2meV}}$ and $\lambda_0 = \frac{hc}{eV} = \frac{2mc\lambda^2}{h} \Rightarrow n = 2$

24. For maximum intensity on the screen $d\sin\theta = n\lambda$

$\Rightarrow \sin\theta = \frac{n\lambda}{d}$ $\sin\theta = \frac{n(2000)}{7000} = \frac{n}{3.5}$

Since $\sin\theta >\neq 1$ $n = 0,1,2,3$ only

Thus only seven maximas can be obtained on both sides of the screen

25. The two bodies will collide at the highest point if both cover the same vertical height in the same time. So, $\frac{v_1^2 \sin^2 30°}{2g} = \frac{v_2^2}{2g}$

Or $\frac{v_2}{v_1} = \sin 30° = \frac{1}{2} = 0.5$

26. Energy $= FAT^{x/3}$

$M^1L^2T^{-2} = \left[M^1L^1T^{-2}\right]\left[L^1T^{-2}\right]\left[T\right]^{x/3}$

By equating power of time, $-2 = -4 + x/3$ $\Rightarrow x = 6.00$

27. $A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$, $\frac{dN_B}{dt} = \lambda_1 N_A - \lambda_2 N_B = 0$, $\frac{N_A}{N_B} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{\lambda_1 N_A}{\lambda_2 N_B} = 1$

28. Energy of diatomic gas due to its thermal motion is

$\left(\therefore U = \frac{5}{2}nRT = \frac{5}{2}PV\right) = \frac{5}{2}PV = \frac{5}{2}P\left(\frac{m}{\rho}\right) = 5 \times 10^4 J$

29. $\frac{T^\gamma}{P^{\gamma-1}}$ = constant (or) $P \propto T^{\frac{\gamma}{(\gamma-1)}}$ $\therefore C = \frac{\gamma}{\gamma - 1} = \frac{7/5}{2/5} = \frac{7}{2}$.

30. A and B are parallel so $\frac{I}{2}\cos^2\theta \times \cos^2\theta = \frac{I}{8} \Rightarrow \theta = 45°$

CHEMISTRY

31. $C_xH_{2y}O_y + 2xO_2 \rightarrow xCO_2 + yH_2O + xO_2$

Number of moles after cooling =2x

Volume after cooling=2.24 litres

Number moles of CO_2=0.05

$\therefore$ EF=CH_2O

$P^0 = 17.5mm$

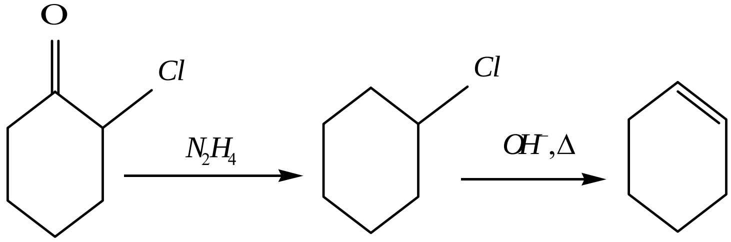
$P^0 - P = 0.104mm$

M=151.4

MF=$C_5H_{10}O_5$

32. For the given question only C & D compounds are possible. In that H of C is less acidic.

33.



34. Both are true statements.

35. Basic strength order 3>4>2>1>5.

36. $XeF_6 + H_2O \rightarrow XeOF_4 + HF$

$XeF_4 : sp^3d$

37. $Cu^{+2} + S^{-2} \rightarrow CuS \xrightarrow{HNO_3} Cu^{+2} + S$

NH_3 deep blue solution

38. $NaCl + H_2SO_4 \rightarrow HCl + NaHSO_4$

$MnO_2 + HCl \rightarrow MnCl_2 + Cl_2 + H_2O$

39. A is false but R is true

40. $[Cr(H_2O)_6]^{3+}$ - $d^2sp^3,3$, $[Co(CN)_4]^{2-}$ - $dsp^2,1$, $[Ni(NH_3)_6]^{2+}$ - $sp^3d^2,2$, $[MnF_6]^{4-}$ - $sp^3d^2,5$

41. Assertion and reason both are correct statements and reason explains the assertion.

42. A is correct but R is incorrect

43. For formation of NH_3 and PCl_5 change in entropy in negative

44. $NH_2^- > OH^- > NH_3$ is correct order of basic strength.

45. $AB_{2(g)} + A_{(s)} \rightleftharpoons 2AB_{(g)}$

	AB_2	AB
Initial	0.7	0
	0.7-x	2x
Final	y	(0.4 - y)

$K_p = \dfrac{(2x)^2}{0.7-x} = \dfrac{1}{4} \times \dfrac{1}{0.45}$

$P_{tot} = 0.95$

$= 0.7 + x \therefore x = 0.25$

$$\frac{(0.4-y)^2}{y}=\frac{5}{9}\ \ y=0.13$$

At second equilibrium $V_{AB_2}=\frac{0.13}{0.4}\times 100=32.5\%$

46. $m = Zit$

$$q = Area\ of\ \ figure = [100\times 10\times 10^{-3}]+2[\frac{1}{2}\times 10\times 10^{-3}]=2$$

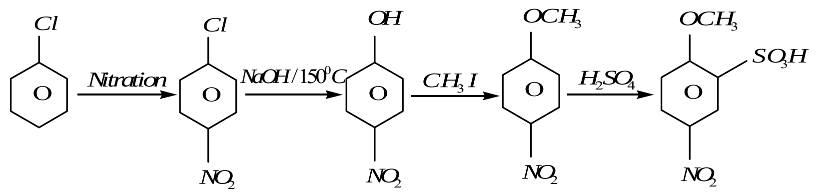
$$\therefore m = Z\times 2\ \ Z=\frac{m}{2}$$

47. $Z=\frac{d\times a^3.No}{M}$ $\qquad a^3 = volume = 10A^0\times 10A^0\times \sin 60^0\times 15A^0$

$$=\frac{2\sqrt{3}\times 750\sqrt{3}\times 10^{-24}\times 6\times 10^{23}}{450}\ \ Z=6$$

48. A is Prop –1–en–2–ol. On tautomerization, it changes to ketone

49.



50.

$$CH_3COCH_3 \xrightarrow{PhMgBr} CH_3-\underset{OH}{\overset{ph}{C}}-CH_3 \xrightarrow{I_2/KOH} \text{-ve}$$

A)

$$CH_3CH_2CHO \xrightarrow{MeMgBr} CH_3-CH_2-\underset{OH}{CH}-CH_3 \xrightarrow{I_2/KOH} +ve$$

B)

$$HCOOEt \xrightarrow{PhMgBr} PhCHO \xrightarrow{PhMgBr} Ph-\underset{OH}{CH}-Ph \xrightarrow{I_2/KOH} -ve$$

C)

$$CH_3COOEt \xrightarrow{PhMgBr} CH_3-\underset{OH}{\overset{Ph}{C}}-Ph \xrightarrow{I_2/KOH} -ve$$

D.

51. $\frac{21.4\times 10^{-3}}{M\times 2}=conc.of\ \ salt\ solution$ (molecular weight=1070)

$$P^H = 7-\frac{1}{2}[\log c-\log K_b]$$

$$5 = 7-\frac{1}{2}\left[\log\frac{21.4\times 10^{-3}}{2M}+9\right]$$

52. Only (iv) is wrong.

53. $\Delta T_f = \frac{1000\times K_f\times w}{m\times w}$

For the solution in benzene using the data given

$$1.28 = \frac{1000 \times 5.12 \times w}{m_w \times 100} \qquad ....1$$

For the solution in water in which solute dissociates

$$1.40 = \frac{1000 \times 1.86 \times w}{m_{exp} \times 100} \qquad ....2$$

Dividing eq. (ii) by (i)

$$i = \frac{m_N}{m_{exp}} = \frac{1.40}{1.28} \times \frac{5.12}{1.86} = 3.01 = 3.0$$

Now, suppose that formula of solute is

$$A_x B_y \rightleftharpoons xA^+ + yB^-$$

$1 \qquad 0 \qquad 0$

$(1-\alpha) \qquad x\alpha \qquad y\alpha$

$i = 1 - \alpha + x\alpha + y\alpha$

$i = 3 \;\; and \;\; \alpha = 1 \qquad (Given\, that \;\; \alpha = 1)$

No of ions given (x+y) = 3

54. 1 micelle $\rightarrow 2.4 \times 10^{13}$ *molecules*

$1.2 \times 10^{-3} M \rightarrow 1mm^3 = 10^{-3} cm^3 \Rightarrow 1.2 \times 10^{-3} \times 10^{-6}\, moles$

$= 1.2 \times 10^{-9}\, moles$

$1.2 \times 10^{-9}\, moles \Rightarrow 1.2 \times 6 \times 10^{14}\, molecules$

$1.2 \times 6 \times 10^{14} = 2.4 \times 10^{13} x$

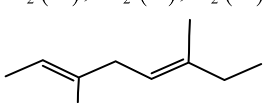
$$\Rightarrow x = \frac{1.2 \times 6 \times 10}{2.4} = 30$$

55. With no 2s – 2p mixing the order of energy of MO's would be

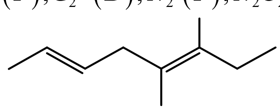
$\sigma_{1s} < \sigma_{1s}^* < \sigma_{2s}\sigma_{2s}^* < \sigma_{2pz} < \left|\pi_{2P_x} = \pi_{2P_y}\right| < \left|\pi_{2p_y}^* = \pi_{2p_x}^*\right| < \sigma_{2p_2}$

$H_2(D), He_2^+(P), Li_2(D), Be_2(D), C_2(P), C_2^{2-}(D), N_2^+(P), N_2O_2(P), O_2^-(P), S_2(P), F_2(D).$

56.



and



both can show G.I.

$\therefore 8\, alkenes\, of \;\; x$

57.

$$K = \frac{2.303}{t_2 - t_1}\log\frac{R_1}{R_2} = \frac{2.303}{60}\log\frac{1.24 \times 10^{-2}}{0.2 \times 10^{-2}} = \frac{2.303}{60}\log 6.2$$

$= 0.0304 = 3 \times 10^{-2}$

[ref : NCERT solved example 4.5]

58. H_2O_2 in basic medium reduces Fe^{+3} to Fe^{+2}.

59. Mg_3N_2 *with* D_2O *gives* ND_3 *having* $M.wt\; 20$

Al_4C_3 *and* Be_2C *gives* CD_4 *having* $M.wt\; 20$

60. Group 1 bicarbonates exists in solid state except lithium.

MATHEMATICS

61. $f(x)$ and $f^{-1}(x)$ can only intersect on the line $y=x$

$\therefore y=x$ must be tangent

Solving $3x^2-7x+c=x \Rightarrow 3x^2-8x+c=0$

The above equation has real and equal roots

$\Rightarrow 64-12c=0 \quad c=\dfrac{16}{3}$

62. Let $g(x)=f(x+T/2)-f(x)$

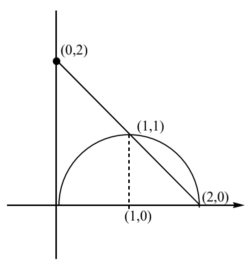
then $g(k)=f(k+T/2)-f(k)$ …… (1)

and $g(k+T/2)=f(k+T)-f(k+T/2)$

$=f(k)-f(k+T/2)=-g(k)$

Hence by intermediate value property there exist an $x_0\in[k,k+T/2]$ for which g(x) = 0

63.



64. By LMVT, $\exists a\in(0,4) \ni \dfrac{f(4)-f(0)}{4-0}=f^1(a)\Rightarrow f(4)-f(0)=4f^1(a)$

$\because \dfrac{f(4)+f(0)}{2}$ lies between $f(0)$ and $f(4)$, by Intermediate value theorem

$\exists b\in(0,4)\ni \dfrac{f(4)+f(0)}{2}=f(b)$ hence, $\left(f(4)^2\right)-\left(f(0)\right)^2=8 \quad f^1(a)f(b)$

65. We know that if $d_i=\dfrac{x_i-A}{h}$ then $\sigma_x=|h|\sigma_d$.

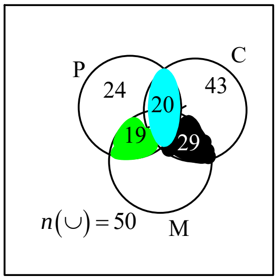
In this case $-2x_i-3=\dfrac{x_i-3/2}{-1/2}$.

So. $h=\dfrac{1}{2}$

This. $\sigma_d=\dfrac{1}{|h|}\sigma_x=2\times3.5=7$

66. $(2008)^8=2^{24}\times251^8$ has $25\times9=225$ positive divisors, including $(2008)^4=\sqrt{(2008)^8}$. There is a one to one correspondence between the positive divisors less than $(2008)^4$ and those larger than $(2008)^4$. It follows that there are $\dfrac{1}{2}(225-1)=112$ positive divisors less than $(2008)^4$.

67.



Since, $n(M \cup P \cup C)=50$

$n(M)=37, n(P)=24, n(C)=43$

$n(M \cap P) \leq 19,\ n(M \cap C) \leq 29$

And $n(P \cap C) \leq 20$

Since, we know that

$n(P \cup C \cup M)=n(P)+n(C)+n(M)$

$-n(P \cap C)-n(C \cap M)-n(M \cap P)+n(P \cap C \cap M)$

$\Rightarrow\ 50 \geq 37+24+43-n(M \cap P)-n(P \cap C)\ -n(M \cap C)+n(M \cap P \cap C)$

$\Rightarrow\ n(M \cap P \cap C) \leq N(M \cap P)+n(M \cap C)+M(P \cap C)-54$

$19+29+20-54=14$

68. Equation of the chord AB having (a, b)

$as\, M.P\, S_1=S_{11} \Rightarrow ax+by-\left(a^2+b^2\right)=0$

$Chord\ length=2\sqrt{r^2-a^2-b^2}$

$$c=\left(\frac{-ar^2}{a^2+b^2}, \frac{br^2}{a^2+b^2}\right)$$

$$h=\frac{r^2-a^2-b^2}{\sqrt{a^2+b^2}}$$

$$Area=\frac{1}{2}bh$$

69. $16\left(\sin^5 x+\cos^5 x\right)-11(\sin x+\cos x)=0$

$\Rightarrow(\sin x+\cos x)\left\{16\left(\sin^4 x-\sin^3 x\cos x+\sin^2 x\cos^2 x-\sin x\cos^3 x+\cos^4 x\right)-11\right\}=0$

$\Rightarrow(\sin x+\cos x)\left\{16\left(1-\sin^2 x\cos^2 x-\sin x\cos x\right)-11\right\}=0$

$\Rightarrow(\sin x+\cos x)(4\sin x\cos x-1)(4\sin x\cos x+5)=0$

$As\ 4\sin x\cos x+5 \neq 0, We\ have$

The required values are

$\pi/12, 5\pi/12, 9\pi/12, 13\pi/12, 17\pi/12, 21\pi/12$

They are 6 solutions on $[0,2\pi]$

70. $\frac{z+2i}{z-2i}=\lambda i \Rightarrow \frac{2z}{4i}=\frac{\lambda i+1}{\lambda i-1}, \frac{z}{2i}=\frac{\lambda i+1}{\lambda i-1} \Rightarrow |z|=2$

71. Tangent on ellipse having slope 2 will be

$y=2x\pm\sqrt{4a^2+b^2}$ $\therefore$ It is normal to circle

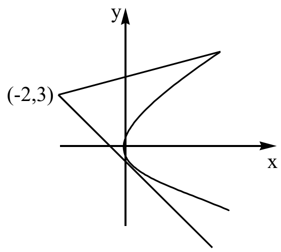
$\therefore$ (–2, 0) is on it. $\therefore$ (–2, 0) is on it.

$\Rightarrow \quad 0=-4\pm\sqrt{4a^2+b^2} \quad \Rightarrow \quad 4a^2+b^2=16$

$\because$ A.M. $\Rightarrow$ G.M. $\Rightarrow \frac{4a^2+b^2}{2}\geq 2ab \quad \Rightarrow 8\geq 2ab \quad \Rightarrow ab\leq 4$

$\therefore$ Maximum value of $ab=4$ **Ans.**

72. Equation of tangent $y=mx+\frac{2}{m}$, Passes through $(-2,3)$



$\Rightarrow 2m^2+3m-2=0 \qquad \Rightarrow (2m-1)(m+2)=0$, $m=\left(\frac{y-3}{x+2}\right)_{max.}=\frac{1}{2}$

73. $S_1:((\sim p)\vee q)\vee((\sim p)\vee r)\equiv\sim p\vee(q\vee r)$

$S_2: p\rightarrow(q\vee r)\equiv\sim p\vee(q\vee r)$ By Conditional Law

$S_1\equiv S_2$

74. $I=\int_{\frac{-3\pi}{4}}^{\frac{5\pi}{4}}\frac{\sin x+\cos x}{1+e^{-\left(x-\frac{\pi}{4}\right)}}dx$; $\quad I=\int_{\frac{-3\pi}{4}}^{\frac{5\pi}{4}}\frac{(\cos x+\sin x)}{1+e^{\left(x-\frac{\pi}{4}\right)}}e^{\left(x-\frac{\pi}{4}\right)}dx$

$\therefore \int_{\frac{-3\pi}{4}}^{\frac{5\pi}{4}}(\cos x+\sin x)dx=(\sin x-\cos x)\Big]_{\frac{-3\pi}{4}}^{\frac{5\pi}{4}}=\left(\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-\left(\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$

$2I=0 \Rightarrow \quad I=0$. **Ans.**

75. $\frac{dy}{dx}-f(x)y=0$

$\frac{dy}{y}=f(x)dx$

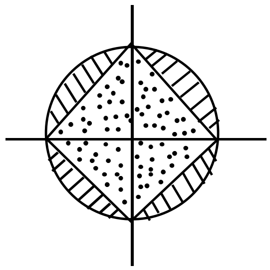
$In\, y=\int f(x)dx$

$y_1(x)=e^{\int f(x)dx}$ Then for given equation I.F = $e^{\int f(x)dx}$

Hence Solution $y.y_1(x)=\int r(x)y_1(x)dx$, $y=\frac{1}{y_1x}\int r(x)y_1(x)dx$

76. $-1 \le \left||x|-|y|\right| \le 1$

$\left||x|-|y|\right| \le 1 \wedge \left||x|-|y|\right| \ge 1$



Required area $= \pi(1)^2 = \pi$

77. $\left(1+x+x^2+x^3+x^4\right)^{1001}(1-x)^{1002}$

$=(1-x)\left(1-x^5\right)^{1001}$

so all the powers of x will be of the $5m$ or $5m+1(m \in 1)$

So coff. of x^{2009} will be 0

78. Coordinates of any point Q on the given line are

$(2r+1,-3r-1,8r-10)$ for some $r \in R$

So, the direction ratios of PQ are $2r,-3r-1,-8r-10$

Now PQ is perpendicular to the given line

if $2(2r)-3(-3r-1)+8(8r-10)=0$

$\Rightarrow 77r-77=0 \Rightarrow r=1$

and the coordinates of Q, the foot of the perpendicular from P on the line are (3, –4, –2).

Let R (a, b, c) be the reflection of P in the given lines when Q is the mid-point of PR

$\Rightarrow \frac{a+1}{2}=3, \frac{b}{2}=-4, \frac{c}{2}=-2$

$\Rightarrow a=5, b=-8, c=-4$

and the coordinates of the required point are $(5,-8,-4)$.

79. Since $\bar{u},\bar{m},\bar{r}$ are mutually perpendicular vectors of same magnitude, we can resolve $\bar{e}$ along the directions $\bar{u},\bar{m},\bar{r}$ let $|\bar{u}|=|\bar{m}|=|\bar{r}|=K$

Let $\bar{e}=a\bar{u}+b\bar{m}+c\bar{r}$

$\bar{u} \times \{(\bar{e}-\bar{m}) \times \bar{u}\}=(\bar{u}.\bar{u})(\bar{e}-\bar{m})-(\bar{u}.(\bar{e}-\bar{m}))\bar{u}$

$=K^2(\bar{e}-\bar{m})-(\bar{u}.\bar{e})\bar{u}\,(\because \bar{u}.\bar{m}=\bar{m}.\bar{r}=\bar{m}.\bar{u}=0)$

$=K^2(\bar{e}-\bar{m})-(\bar{u}.\bar{e})\bar{u}$

Similar $\bar{m}\times\{(\bar{e}-\bar{r})\times\bar{m}\}=(\bar{m}.\bar{m})(\bar{e}-\bar{r})-\bar{m}.(\bar{e}-\bar{r})\bar{m}$

$=K^2(\bar{e}-\bar{r})-(\bar{m}.\bar{e})\bar{m}$

$\bar{r}\times\{(\bar{e}-\bar{u})\times\bar{r}\}=(\bar{r}.\bar{r})(\bar{e}-\bar{u})-(\bar{r}.(\bar{e}-\bar{u})).\bar{r}$

$=K^2(\bar{e}-\bar{u})-(\bar{r}.\bar{e})r$

Substitute above, in the given question $\bar{e}=\frac{1}{2}(\bar{u}+\bar{m}+\bar{r})$

80. $Pqr\left(a^3+b^3+c^3\right)-abc\left(p^3+q^3+r^3\right)$

$\Rightarrow pqr\left(a^3+b^3+c^3-3abc\right)-abc\left(p^3+q^3+r^3-3pqr\right)$

$\Rightarrow pqr\left(a^3+b^3+c^3-3abc\right)-abc(p-q+r)\left(p^2+q^2+r^2-pq-qr-rp\right)$

$=pqr\left(a^3+b^3+c^3-3abc\right)$

81. Toys in group 1 1 2 $\rightarrow\frac{4!}{1!\,1!\,2!\,2!}\times 3!=36$

Marbles O O Ø Ø $={}^4C_2=6$

$\Rightarrow$ Total ways $=36\times 6=216$

82. H_1 : Three numbers drawn are 1, 2 and 3 in any order.

H_2 : Three numbers drawn are 1, 2 and 2.

$P(H_1)=6\left(\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\right)=\frac{2}{9}$ think! $P(H_2)=\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}=\frac{1}{27}$

$\therefore\quad P(H_2/H_1\cup H_2)=\frac{P(H_2)}{P(H_1)+P(H_2)}=\frac{1}{27}\cdot\frac{27}{7}=\frac{1}{7}\equiv\frac{a}{b}$

$\therefore\quad (a+b)=8.$

83. The equation of any plane containing the given line is

$(x+y+2z-3)+\lambda(2x+3y+4z-4)=0$

$\Rightarrow(1+2\lambda)x+(1+3\lambda)y+(2+4\lambda)z-(3+4\lambda)=0\quad ....(1)$

If the plane is parallel to z-axis whose direction cosines are 0, 0, 1; then the normal to the plane will be perpendicular to z-axis

$\therefore\quad (1+2\lambda)(0)+(1+3\lambda)(0)+(2+4\lambda)(1)=0\Rightarrow\quad \lambda=-\frac{1}{2}$

Put in eq. (1), the required plane is

$(x+y+2z-3)-\frac{1}{2}(2x+3y+4z-4)=0\Rightarrow y+2=0...(2)$

$\therefore$ S.D. = distance of any point say (0, 0, 0) on z-axis from plane (2) $=\frac{2}{\sqrt{(1)^2}}=2$

84. $\frac{|\text{adj } B|}{|C|}=\frac{|\text{adj}(\text{adj } A)|}{|5A|}=\frac{|A|^{(3-1)^2}}{5^3|A|}=\frac{|A|^3}{125}$

Now $|A|=5$ $\therefore$ $\frac{|\text{adj } B|}{|C|}=1$ Ans.

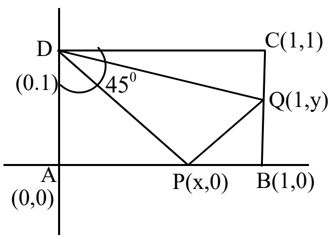
85. The equation of the tangent at $\left(5\cos\theta, 2\sin\theta\right)$ is $\frac{x}{5}\cos\theta+\frac{y}{2}\sin\theta=1$

If it is a tangent to the circle then $\frac{1}{\sqrt{\frac{\cos^2\theta}{25}+\frac{\sin^2\theta}{4}}}=4\Rightarrow\cos\theta=\frac{10}{4\sqrt{7}},\sin\theta=\frac{\sqrt{3}}{2\sqrt{7}}$

Let A and B be the points where the tangent meets the coordinate axis then

$A\left(\frac{5}{\cos\theta},0\right),\ B\left(0,\frac{2}{\sin\theta}\right),\quad L=\sqrt{\frac{25}{\cos^2\theta}+\frac{4}{\sin^2\theta}}=\frac{14}{\sqrt{3}}$

86.



$\tan\theta_1=x$ and $\tan\theta_2=1-y$

Since, $\theta_1-\theta_2=45^0\Rightarrow\frac{\tan\theta_1+\tan\theta_2}{1-\tan\theta_1\tan\theta_2}=1$

$\Rightarrow\frac{x+(1-y)}{1-x(1-y)}=1\Rightarrow y=\frac{2x}{1+x}....(i)$

Now, Perimeter $=1-x+y+\sqrt{(1-x)^2+y^2}$

By using (i), we get

Perimeter=2

87. Normal to xy $=c^2$ is $y-\frac{c}{t}=t^2\left(x-ct\right)$

Solving with $xy=-c^2$ we get

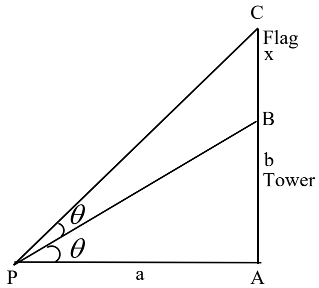
$x\left(\frac{c}{t}+t^2\left(x-ct\right)\right)+c^2=0\quad t^2x^2+\left(\frac{c}{t}-ct^3\right)x+c^2=0$

For equal roots $\left(\frac{1}{t}-t^3\right)^2-4t^2=0\Rightarrow 4\ \textit{values are possible}$

88.

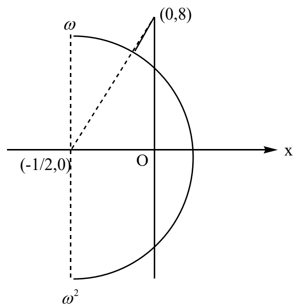
$\frac{b}{a}=\tan\theta,\ \frac{b+x}{a}=\tan 2\theta$ or $\frac{b+x}{a}=\frac{2\tan\theta}{1-\tan^2\theta}=\frac{2\cdot\frac{b}{a}}{1-\frac{b^2}{a^2}}$

$$\frac{b+x}{a}=\frac{2ab}{a^2-b^2}\quad\therefore\quad x=\frac{2a^2b}{a^2-b^2}-b$$



or $x = \dfrac{a^2b + b^3}{a^2 - b^2} = \dfrac{b\left(a^2 + b^2\right)}{a^2 - b^2}$

89.



$$\arg\left(\frac{z^2-\omega^2}{z^2-\omega}\right)=\frac{\pi}{2}$$

z^2 lies on a circle whose center is $\left(\dfrac{-1}{2},0\right)$ and radius is equal to $\dfrac{\sqrt{3}}{2}$ units.

$\left|z-2(1+i)\right|\left|z+2(1+i)\right|=\left|z^2-4(2i)\right|=\left|z^2-8i\right|$

Minimum value of $\left|z^2-8i\right|$ is equal to

$$\sqrt{\frac{1}{4}+64}-\frac{\sqrt{3}}{2}=\frac{\sqrt{257}-\sqrt{3}}{2}\equiv\frac{\sqrt{a}-\sqrt{b}}{2}\ \therefore\frac{a+b}{\sqrt{2}}=\frac{257+3}{52}=5$$

90. $R: A \in B$ under given condition a < b is given by

$$R=\begin{Bmatrix}(1,3) & (1,5) & (2,3)\\(2,5) & (3,5) & (4,5)\end{Bmatrix},\ R^{-1}=\begin{Bmatrix}(3,1) & (5,1) & (3,2)\\(5,2) & (5,3) & (5,4)\end{Bmatrix}$$

RoR^{-1}: for composing RoR^{-1} we will pick up an element of R^{-1} first and then of R

$(3,1)\in R^{-1},\ (1,3)\in R,\ \Rightarrow (3,3)\in R\ o\ R^{-1}$

$\therefore R\ o\ R^{-1}=\{(3,3),(3,5),(5,3),(5,5)\}$ only

Elements are not repeated in a set