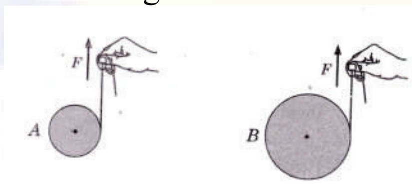
PHYSICS:

31. Two solid uniform disks of equal mass but different radii r_A and r_B shown in the figure are mounted on fixed horizontal axles passing through their centers. On each of the disks light inextensible long threads are wound. Initially both the disks are motionless. If free ends of both the threads are pulled with equal forces for equal amount of time, lengths ℓ_A and ℓ_B of the threads are unwound from the disks A and B respectively. Which of the following statements is correct?



(1) $\ell_A > \ell_B$ (2) $\ell_A < \ell_B$
(3) $\ell_A = \ell_B$ (4) More information needed.

32. A uniform square plate is placed on a rough horizontal floor. When it is given angular velocity ω_0 about a vertical axis passing through one of its corner as shown in figure-I, it takes time t_0 to a complete stop.

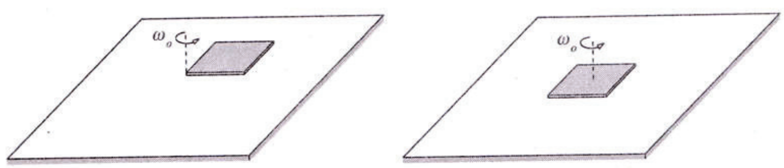
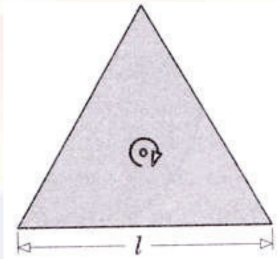


Figure-I Figure-II

Now the same square plate is given the same angular velocity to rotate about another vertical axis passing through its center as shown in figure-II. How long will it take to a complete stop?

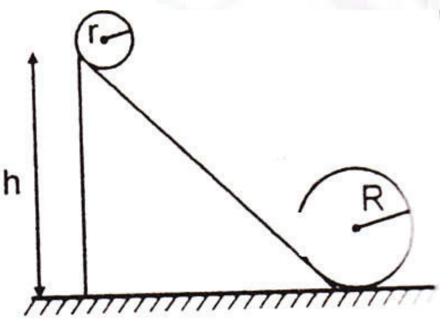
(1) $\frac{t_0}{2}$ (2) $\frac{2t_0}{3}$ (3) $\frac{3t_0}{2}$ (4) $2t_0$

33. A uniform equilateral triangular lamina of side ℓ has mass m. Its moment of inertia about the axis through the centroid and perpendicular to the plane of the lamina is



(1) $\frac{m\ell^2}{3}$ (2) $\frac{m\ell^2}{6}$ (3) $\frac{m\ell^2}{12}$ (4) $m\ell^2$

34. A solid sphere rolls without slipping along the track shown in figure. The sphere starts from rest from a height h above the bottom of a loop of radius R which is much larger than the radius of the sphere r. The minimum value of h for the sphere to complete the loop is

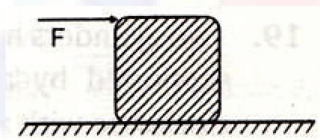


(1) $2.1R$ (2) $2.3R$ (3) $2.7R$ (4) $2.5R$

35. A wheel of radius R rolls on the ground with a uniform velocity v. The relative acceleration of topmost point of the wheel with respect to the bottommost point is

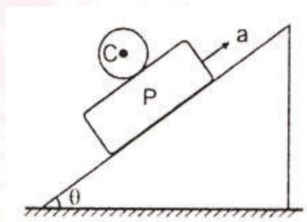
(1) $\frac{v^2}{R}$ (2) $\frac{2v^2}{R}$ (3) $\frac{v^2}{2R}$ (4) $\frac{4v^2}{R}$

36. A force F is applied on the top of a cube as shown in figure. The coefficient of friction between the cube and the ground is μ. If F is gradually increased, the cube will topple before sliding if



(1) $\mu > 1$ (2) $\mu < \frac{1}{2}$ (3) $\mu > \frac{1}{2}$ (4) $\mu < 1$

37. The acceleration a of the plank P required to keep the centre C of a cylinder in a fixed position during the motion is (no slipping takes place between cylinder and plank)

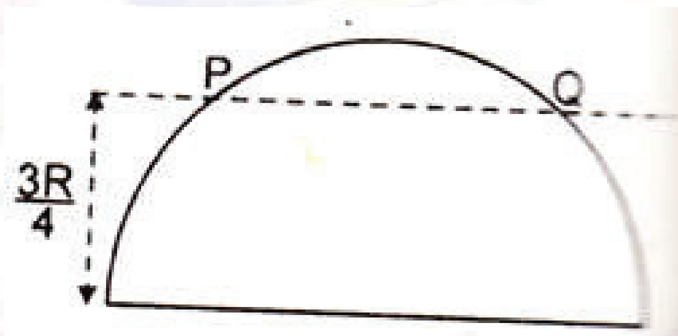


(1) $\frac{g}{2}\sin\theta$ (2) $2g\sin\theta$ (3) $g\sin\theta$ (4) $\sqrt{2}g\sin\theta$

38. A spherical body of radius R is allowed to roll down on an incline without slipping and it reaches with a speed v_0 at the bottom. The incline is then made smooth by waxing and the body is allowed to slide without rolling and now the speed attained is $\frac{5}{4}v_0$. The radius of gyration of the body about an axis passing through its centre is

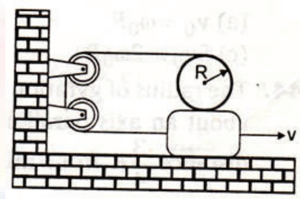
(1) $\frac{4}{3}R$ (2) $\frac{3}{4}R$ (3) $\frac{5}{2}R$ (4) $\frac{2}{5}R$

39. The radius of gyration of a solid uniform hemisphere of mass M and radius R about an axis parallel to the diameter of plane cross section at a distance $\frac{3}{4}R$ from this plane is given by



(1) $\frac{3R}{\sqrt{10}}$ (2) $\frac{5R}{4}$ (3) $\frac{5R}{8}$ (4) $\sqrt{\frac{2}{5}}R$

40. In the figure shown, the plank is being pulled to the right with a constant speed v. If the cylinder does not slip then:



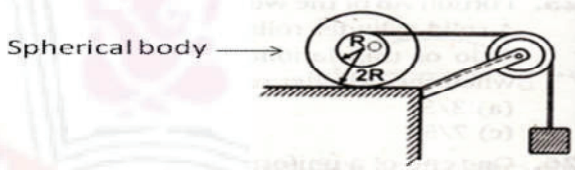
(1) the speed of the centre of mass of the cylinder is $2v$.

(2) the speed of the centre of mass of the cylinder is zero.

(3) the angular velocity of the cylinder is v/R.

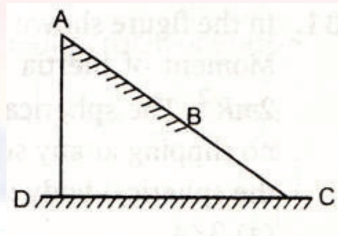
(4) the angular velocity of the cylinder is zero.

41. In the figure shown mass of both, the spherical body and block is m. Moment of inertia of the spherical body about centre of mass (which is at centre O of the body) is $2mR^2$. The spherical body rolls on the horizontal surface. There is no slipping at any surfaces in contact. The ratio of kinetic energy of the spherical body to that of block is



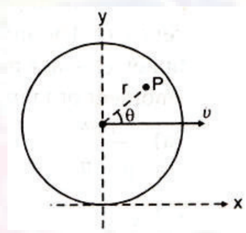
(1) $\frac{3}{4}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2}$

42. Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. If AB = BC, then ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C is



(1) $\frac{3}{5}$ (2) 5 (3) $\frac{7}{5}$ (4) $\frac{8}{3}$

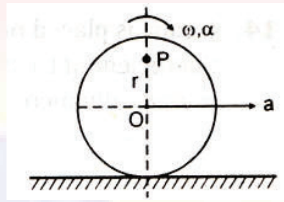
43. A disc of radius R rolls without slipping at speed v along positive x-axis. Velocity of point P at the instant shown in figure is



(1) $\vec{V}_P = \left(v + \frac{vr\sin\theta}{R}\right)\hat{i} + \frac{vr\cos\theta}{R}\hat{j}$ (2) $\vec{V}_P = \left(v + \frac{vr\sin\theta}{R}\right)\hat{i} - \frac{vr\cos\theta}{R}\hat{j}$

(3) $\vec{V}_P = \frac{vr\sin\theta}{R}\hat{i} + \frac{vr\cos\theta}{R}\hat{j}$ (4) $\vec{V}_P = \frac{vr\sin\theta}{R}\hat{i} - \frac{vr\cos\theta}{R}\hat{j}$

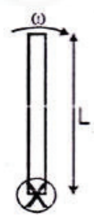
44. A disc of radius $'r'$ rolls on a horizontal ground with linear acceleration a and angular acceleration α as shown in figure. The magnitude of acceleration of point P shown in figure at an instant when its angular velocity is ω, will be



(1) $\sqrt{(a+r\alpha)^2+(r\omega^2)^2}$ (2) $\frac{ar}{R}$

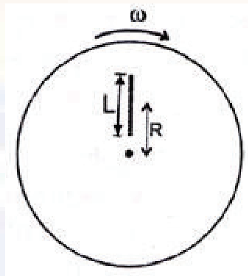
(3) $\sqrt{r^2\alpha^2+r^2\omega^4}$ (4) $r\alpha$

45. A uniform rod hinged at its one end is allowed to rotate in vertical plane. Rod is given an angular velocity ω in its vertical position as shown in figure. The value of ω for which the force exerted by the hinge on rod is zero in this position is



(1) $\sqrt{\frac{g}{L}}$ (2) $\sqrt{\frac{2g}{L}}$ (3) $\sqrt{\frac{g}{2L}}$ (4) $\sqrt{\frac{3g}{L}}$

46. A uniform rod of mass M and length L lies radially on a disc rotating with angular speed ω in a horizontal plane about its axis. The rod does not slip on the disc and the centre of the rod is at a distance R from the centre of the disc. Then the kinetic energy of the rod is



(1) $\frac{1}{2}m\omega^2\left(R^2+\frac{L^2}{12}\right)$ (2) $\frac{1}{2}m\omega^2R^2$

(3) $\frac{1}{24}m\omega^2L^2$ (4) $m\omega^2L^2$

47. A solid sphere, a hollow sphere and a disc, all having same mass and radius are placed at the top of an inclined plane and released. The friction coefficients between the objects and the inclined plane are same and not sufficient to allow pure rolling. Least time will be taken in reaching the bottom by

(1) the solid sphere (2) the hollow sphere

(3) the disc (4) all will take same time.

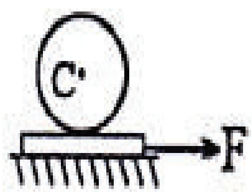
48. A uniform disc of radius R lies in the x-y plane with its centre at origin. Its moment of inertia about z-axis is equal to its moment of inertia about line $y = x + c$. The value of c will be

(1) $-\frac{R}{2}$ (2) $\pm\frac{R}{\sqrt{2}}$ (3) $\frac{+R}{4}$ (4) $-R$

49. A ring of radius R rolls without sliding with a constant velocity. The radius of curvature of the path followed by any particle of the ring at the highest point of its path will be:

(1) R (2) $2R$ (3) $4R$ (4) $5R$

50. A cylinder is placed on a rough plank which in turn is placed on a smooth surface. The plank is pulled with a constant force F. The friction force can be given by which of the following diagram:



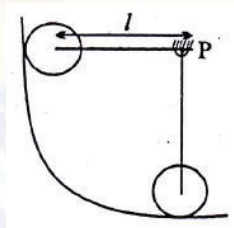
(1) 

(2) 

(3) 

(4) cannot be interpreted

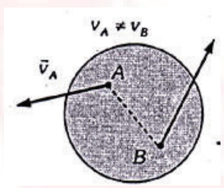
51. A sphere of mass M radius R is attached by a light rod of length ℓ to a point P. The sphere rolls without slipping on a circular track as shown. It is released from the horizontal position. The angular momentum of the system about P when the rod becomes vertical is:

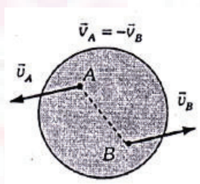


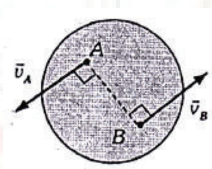
(1) $M\sqrt{\frac{10}{7}g\ell}[\ell+R]$ (2) $M\sqrt{\frac{10}{7}g\ell}\left[\ell+\frac{2}{5}R\right]$

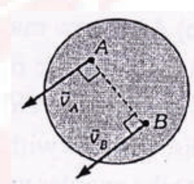
(3) $M\sqrt{\frac{10}{7}g\ell}\left[\ell+\frac{7}{5}R\right]$ (4) $M\sqrt{\frac{10}{7}g\ell}\left[\ell-\frac{2R}{5}\right]$

52. A laminar rigid body is confined to move in its own plane. At some instant velocities of any two points on the body are shown in following figures. Which of the following physical situation is not possible?

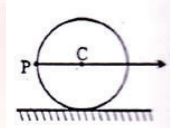
(1) 

(2) 

(3) 

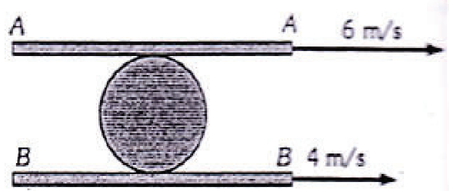
(4) 

53. A disc of radius R is rolling purely on a flat horizontal surface with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is



(1) zero (2) 45^0 (3) 135^0 (4) $\tan^{-1}\left(\frac{1}{2}\right)$

54. A circular roller of radius $0.5m$ is in contact at the top and bottom points of its circumference with two conveyor belts AA and BB, as shown in the figure. If the belts run at $6m/s$ and $4m/s$ both towards right, then the angular and linear speed of the roller are respectively [Consider no slipping at both contacts]

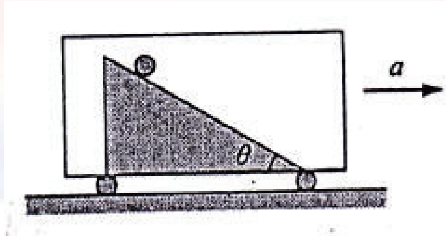


(1) 2 rad/s, 1 m/s (2) 10 rad/s, 9 m/s (3) 2 rad/s, 5 m/s (4) 10 rad/s, 5 m/s

55. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(1) I (2) $I\sin^2\theta$ (3) $I\cos^2\theta$ (4) $I\cos^2\left(\frac{\theta}{2}\right)$

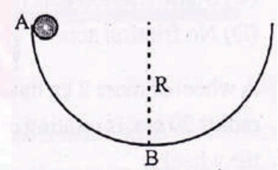
56. Figure shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angle of incline θ is related to the acceleration a of the car as $a = g\tan\theta$. If the sphere is set in pure rolling on the incline.



(1) it will continue pure rolling (2) it will slip down the plane.

(3) its linear velocity will increase. (4) its linear velocity will slowly decreases.

57. A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius $R(>>r)$ as shown in figure. If the sphere starts from rest at the top point of the hemisphere, find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.

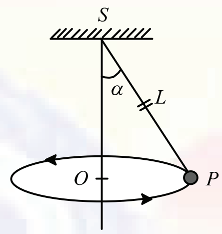


(1) $\frac{10}{7}mg$ (2) $\frac{17}{7}mg$ (3) $\frac{5}{7}mg$ (4) $\frac{7}{5}mg$

58. A particle of mass $2\,kg$ located at the position $(\hat{i}+\hat{j})m$ has a velocity $2(+\hat{i}-\hat{j}+\hat{k})m/s$. Its angular momentum about z-axis in $kg\text{-}m^2/s$ is

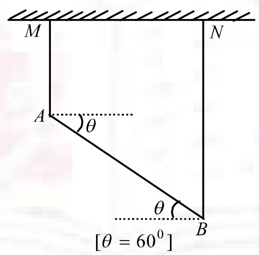
(1) zero (2) +8 (3) 12 (4) -8

59. In a conical pendulum, angular speed of particle P about 'O' is ω. Its angular speed about 'S' is



(1) $\omega\cos\alpha$ (2) $\omega\sin\alpha$ (3) $\frac{\omega}{\sin\alpha}$ (4) zero

60. AB is a rod of uniform mass M and length L suspended with the help of two light, inextensible strings (which are vertical). Now string NB is cut. What is tension in string MA just after string NB is cut.



(1) $\frac{4Mg}{7}$ (2) Mg (3) $\frac{Mg}{2}$ (4) zero

Sec: Jr.SUPER60 **Date: 20-08-16**
Time: 07:30 AM to 10:30 AM **WTM-16** **Max. Marks: 360**

KEY SHEET

MATHEMATICS:

1	2	**2**	2	**3**	1	**4**	3	**5**	3	**6**	4
7	4	**8**	4	**9**	1	**10**	3	**11**	4	**12**	3
13	1	**14**	2	**15**	3	**16**	2	**17**	3	**18**	4
19	1	**20**	3	**21**	4	**22**	4	**23**	4	**24**	4
25	2	**26**	2	**27**	3	**28**	2	**29**	3	**30**	2

PHYSICS:

31	3	**32**	1	**33**	3	**34**	3	**35**	2	**36**	3
37	2	**38**	2	**39**	4	**40**	3	**41**	3	**42**	2
43	2	**44**	1	**45**	2	**46**	1	**47**	4	**48**	2
49	3	**50**	2	**51**	4	**52**	2	**53**	2	**54**	3
55	1	**56**	1	**57**	2	**58**	4	**59**	1	**60**	1

CHEMISTRY:

61	4	**62**	4	**63**	4	**64**	1	**65**	1	**66**	3
67	2	**68**	3	**69**	4	**70**	4	**71**	1	**72**	1
73	4	**74**	3	**75**	3	**76**	1	**77**	3	**78**	2
79	2	**80**	3	**81**	2	**82**	4	**83**	4	**84**	3
85	4	**86**	1	**87**	3	**88**	3	**89**	1	**90**	3

PHYSICS:

31. (3)

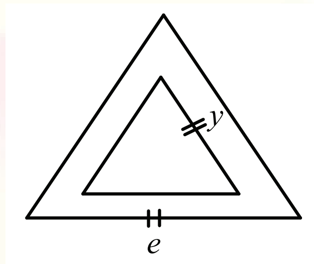
$r\theta$ is same for two cases, where θ is angle turned in a given time interval.

32. (1)

Fixed torque due to friction is 1^{st} case in double of fixed torque due to friction in 2^{nd} case.

$\therefore\ \alpha_2 = 2\alpha_1$

33. (3)



$$dm = 2\frac{m}{\ell^2}ydy$$

$$dt = \frac{dm}{6}y^2 = \frac{m}{3\ell^2}y^3dy$$

$$\therefore\ t = \int dt = \frac{m}{3\ell^2}\int_0^e y^2dy = \frac{m\ell^2}{12}.$$

34. (3)

At the topmost point of the loop minimum value of linear speed of centre of sphere should be:

$v = \sqrt{gR}$ or translational kinetic energy

$$K_T = \frac{1}{2}mv^2 = \frac{1}{2}mgR.$$

In case of pure rolling of a solid sphere the ratio of rotational to translational kinetic energy is $\frac{K_R}{K_T} = \frac{2}{5}$.

$\therefore$ Total kinetic energy at topmost point should be:

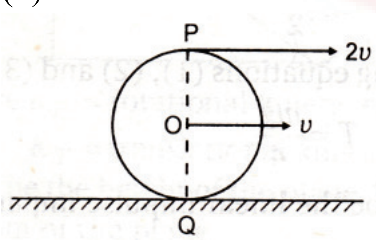
$$K = \frac{5+2}{5}\cdot K_T = \frac{7}{5}\left(\frac{1}{2}mgR\right) = \frac{7}{10}mgR.$$

Now from conservation of mechanical energy:

$$\frac{7}{10}mgR = mg(h-2R)$$

$$\therefore\ h = 2.7R$$

35. (2)

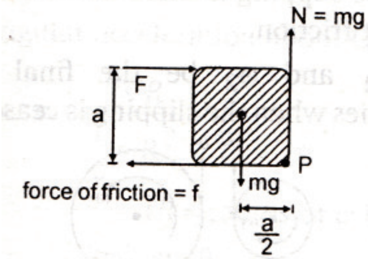


$$v_r = v_{PQ} = 2v$$

$\therefore\ a_r = \frac{v_r^2}{2R} = \frac{4v^2}{2R} = \frac{2v^2}{R}$

36. (3)

Let m be the mass of cube and $'a'$ the side of cube. The cube will slide if:



$F > \mu mg$ ….. (1)

and it will topple if torque of F about P is greater than torque of mg about P i.e.,

$F \cdot a > \left(\frac{a}{2}\right)mg$ or $F > \frac{1}{2}mg$ ….. (2)

From eqs (1) and (2) we see that cube will topple before sliding if $\mu > \frac{1}{2}$

37. (2)

Linear acceleration of cylinder is zero i.e., $mg\sin\theta$ = frictional force (f) upwards (m = mass of cylinder)

$\therefore$ Angular acceleration about C is $\alpha = \frac{\tau}{I}$

or $\alpha = \frac{f \cdot R}{\frac{1}{2}mR^2} = \frac{2f}{mR}$ (R = radius of cylinder)

or $\alpha = \frac{2mg\sin\theta}{mR} = \frac{2g\sin\theta}{R}$

For no slipping between cylinder and plank

$a = R\alpha = 2g\sin\theta$.

38. (2)

Mechanical energy is conserved in both the cases

Hence, $\frac{1}{2}m\left(\frac{5}{4}v_0\right)^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2$

or $\frac{25}{16}v_0^2 = v_0^2 + \frac{I}{m}\left(\frac{v_0}{R}\right)^2$

or $\frac{25}{16} = 1 + \frac{mK^2}{mR^2}$ ($I = mK^2$, K = radius of gyration)

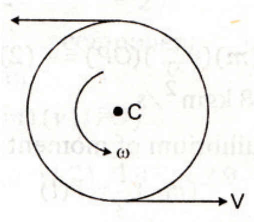
or $K^2 = \frac{9}{16}R^2$ or $K = \frac{3}{4}R$

39. (4)

Conceptual

40. (3)

$\omega = \frac{v}{R}$ and $v_C = 0$



41. (3)

Let v be the linear velocity of centre of mass of the spherical body and ω its angular velocity about centre of mass. Then

$\omega = \frac{v}{2R}$

K.E. of spherical body

$K_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ or $K_1 = \frac{1}{2}mv^2 + \frac{1}{2}(2mR)^2\left(\frac{v^2}{4R^2}\right) = \frac{3}{4}mv^2$ .... (1)

Speed of the block will be

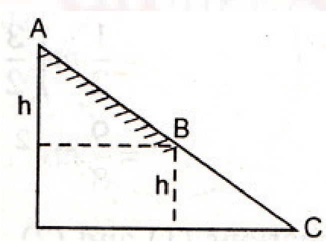
$v' = (\omega)(3R) = 3R\omega = (3R)\left(\frac{v}{2R}\right) = \frac{3}{2}v$

$\therefore$ K.E. of block $K_2 = \frac{1}{2}mv'^2 = \frac{1}{2}m\left(\frac{3}{2}v\right)^2 = \frac{9}{8}mv^2$ .... (2)

From eqs. (1) and (2) $\frac{K_1}{K_2} = \frac{2}{3}$

42. (2)

For a cylinder $\frac{K_T}{K_R} = 2$ in case of pure rolling upto B:



$K_T = \frac{2}{3}mgh$ and $K_R = \frac{1}{3}mgh$

After B: rotational kinetic energy is constant however translational kinetic energy increases:

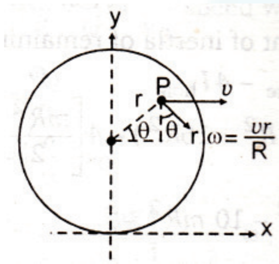
At C: $K_T = \frac{2}{3}mgh + mgh = \frac{5}{3}mgh$

While $K_R = \frac{1}{3}mgh$

$\therefore \frac{K_T}{K_R} = 5$

43. (2)

$\omega = \frac{v}{R}$ for pure rolling.



Velocity of point P is resultant of two velocity vectors shown as arrows in figure.

Hence $\vec{V}_P = v_x\hat{i} + v_y\hat{j}$ or $\vec{V}_P = \left(v + \frac{vr\sin\theta}{R}\right)\hat{i} - \left(\frac{vr\cos\theta}{R}\right)\hat{j}$.

44. (1)

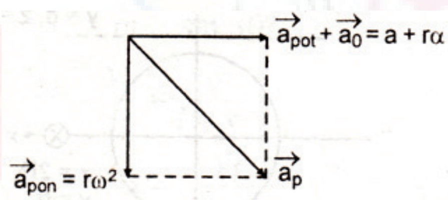
$\vec{a}_P = \vec{a}_{P_0} + \vec{a}_0$

Here $\vec{a}_{P0}$ = acceleration of P with respect to O.

$= \vec{a}_{p_0t} + \vec{a}_{p_0n}$

$\therefore \vec{a}_p = \vec{a}_{p_0t} + \vec{a}_{p_0n} + \vec{a}_0$

Here $\vec{a}_{p_0t}$ = tangential component of $\vec{a}_{p_0}$ and $\vec{a}_{p_0n}$ = normal component of $\vec{a}_{p_0}$



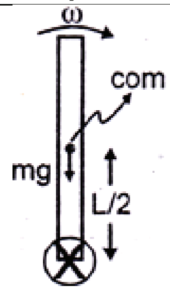
$|\vec{a}_0 + \vec{a}_{p_0t}| = a + r\alpha$

$|\vec{a}_{p_0n}| = r\omega^2$

$\therefore |\vec{a}_p| = \sqrt{(a + r\alpha)^2 + (r\omega^2)^2}$.

45. (2)

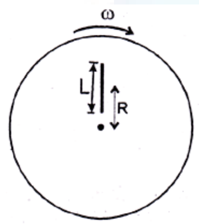
For the circular motion of COM:



$$mg = m\left(\frac{L}{2}\right)\omega^2 \Rightarrow \omega = \sqrt{\frac{2g}{L}}$$

46. (1)

Moment of inertia of the rod w.r.t the axis through centre of the disc is: (by parallel axis theorem).



$I = \frac{mL^2}{12} + mR^2$ and K.E. of rod w.r.t disc $= \frac{1}{2}I\omega^2$

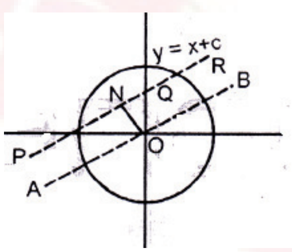
$$= \frac{1}{2}m\omega^2\left[R^2 + \frac{L^2}{12}\right]$$

47. (4)

Since linear acceleration is same for all $(a = g\sin\theta - \mu g\cos\theta)$ as they have same mass M and same μ all will reach the bottom simultaneously.

48. (2)

$I_{PQR} = I_{AOB} + M(ON)^2$



$$I_{PQR} = \frac{1}{4}MR^2 + M\left(\frac{c}{\sqrt{2}}\right)^2$$

But $I_{PQR} = \frac{1}{2}MR^2$

$\therefore\ c = \pm\frac{R}{\sqrt{2}}$.

49. (3)

Radius of curvature = $\frac{(\text{velocity})^2}{\text{Normal acceleration}}$

$= \frac{(2v)^2}{v^2/R} = 4R$

50. (2)

Conceptual

51. (4)

$mgL = \frac{7}{10}MV^2$

Angular moment (required) = $mv\left(\ell - \frac{2R}{5}\right)$.

52. (2)

Conceputal

53. (2)

Conceptual

54. (3)

$v_0 + R\omega = 6$

$v_0 - R\omega = 4$

$\therefore\ v_0 = 5\,ms^{-1}$ and $\omega = 2\,rad/s$

55. (1)

Conceptual

56. (1)

Net force on sphere along downward incline is zero

57. (2)

$\frac{7}{10}mv^2 = mgR$

$N = mg + \frac{10mg}{7} = \frac{17mg}{7}$

58. (4)

$\vec{L} = \vec{r} \times m\vec{v}$.

59. (1)

Conceptual

60. (1)

$Mg - T = Ma_{c.m.}, \alpha = \frac{3T}{ML}, \alpha\frac{L}{2}\cos\theta = g - \frac{T}{M}$

Final Key

S.NO	SUB	Q.NO	GIVEN KEY	FINALIZED KEY	EXPLANATION
3	PHY	40	3	2 or3	Option 2 is Also Possible
4	PHY	59	1	2	Key Change