Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS & STERLING_BT **Paper -2(Adv-2022-P2-Model)** **Date: 24-09-2023**

Time: 02.00Pm to 05.00Pm **CTA-05 & CTA-07** **Max. Marks: 180**

KEY SHEET

MATHEMATICS

1	**6**	2	**2**	3	**2**	4	**5**	5	**2**	6	**7**
7	**10**	8	**9**	9	**AD**	10	**BCD**	11	**BCD**	12	**ACD**
13	**BCD**	14	**AB**	15	**A**	16	**A**	17	**D**	18	**A**

PHYSICS

19	**5**	20	**5**	21	**1**	22	**6**	23	**4**	24	**8**
25	**1**	26	**6**	27	**AD**	28	**BCD**	29	**ABCD**	30	**BC**
31	**ABCD**	32	**BC**	33	**B**	34	**B**	35	**C**	36	**A**

CHEMISTRY

37	**0**	38	**3**	39	**6**	40	**3**	41	**4**	42	**7**
43	**5**	44	**5**	45	**ABD**	46	**ABC**	47	**AB**	48	**BC**
49	**AC**	50	**AC**	51	**B**	52	**D**	53	**D**	54	**C**

SOLUTIONS

MATHEMATICS

1. $$\prod_{k=1}^{\infty}\left(\frac{1+2\cos\frac{2x}{3^k}}{3}\right),\ \prod_{k=1}^{\infty}\frac{1}{3}\left[1+2-4\sin^2\frac{x}{3^k}\right]=\prod_{k=1}^{\infty}\frac{1}{3\sin\frac{x}{3^k}}\left\{3\sin\frac{x}{3^k-4}\sin^3\frac{x}{3^k}\right\}$$

$$\prod_{k=1}^{\infty}\left[\frac{\sin\left(\frac{x}{3^{k-1}}\right)}{3\sin\frac{x}{3^k}}\right],\quad \lim_{k\to\infty}\frac{1}{3^k}\left\{\frac{\sin x}{\sin\frac{x}{3}}\times\frac{\sin\frac{x}{3}}{\sin\frac{x}{9}}\ldots\ldots\frac{\sin\frac{x}{3^{k-1}}}{\sin\frac{x}{3^k}}\right\}$$

$$\lim_{k\to\infty}\frac{\sin x}{\dfrac{\sin\left(\frac{x}{3^k}\right)}{\left(\frac{x}{3^k}\right)}}\Rightarrow f(x)=\frac{\sin x}{x}$$

$$xf(x)=\sin x$$

$$[\sin x]+|\sin x|+(x-1)|(x-1)(x-2)|$$

Non–Differentiable at 6 points.

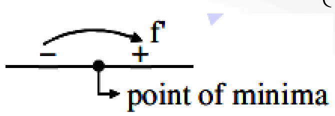
2. $$f(x)=\int_0^{x\tan^{-1}x}\frac{e^{t-\cos t}}{1+t^{2023}}dt$$

$$f'(x)=\frac{e^{x\tan^{-1}x-\cos\left(x\tan^{-1}x\right)}}{1+\left(x\tan^{-1}x\right)^{2023}}.\left(\frac{x}{1+x^2}+\tan^{-1}x\right)$$

For $x<0.\tan^{-1}x\in\left(-\frac{\pi}{2},0\right)$

For $x\ge 0,\tan^{-1}x\in\left[0,\frac{\pi}{2}\right)$ $\quad\Rightarrow\quad x\tan^{-1}x\ge 0\ \forall\ x\in R$

And $$\frac{x}{1+x^2}+\tan^{-1}x=\begin{cases}0 & For\ \ x>0\\ 0 & For\ \ x<0\\ 0 & For\ \ x=0\end{cases}$$



Hence minimum value is $f(0)=\int_0^0\frac{e^{t-\cos t}}{1+t^{2023}}dt=0$

3. $\bar{r}.\bar{p}=1,\bar{r}.\bar{q}=1$ are two planes

Perpendicular distance from origin to the line of intersection of planes is $\left|\frac{\bar{p}-\bar{q}}{\left|\bar{p}\times\bar{q}\right|}\right|$

$\bar{p}=a\hat{i}+b\hat{j}+c\hat{k}$

$\bar{q}=a'\hat{i}+b'\hat{j}+c'\hat{k}$

$\left|\bar{p}-\bar{q}\right|=\sqrt{(a-a')^2+(b-b')^2+(c-c')^2}=\sqrt{8}=2\sqrt{2}$

$\left|\bar{p}\times\bar{q}\right|=\sqrt{(ab'-a'b)^2+(bc'-b'c)^2+(ca'-c'a)^2}=\sqrt{2}$

distance $=\left|\frac{2\sqrt{2}}{\sqrt{2}}\right|=2$

4. $(1+x)^{\frac{1}{x}}-e+\frac{ex}{2}=e^{\frac{1}{x}\ln(1+x)}-e+\frac{ex}{2}$

$$=e\left[1+\left(-\frac{x}{2}+\frac{x^2}{3}-.....\right)+\frac{\left(-\frac{x}{2}+\frac{x^2}{3}-......\right)}{2!}\right]-e+\frac{ex}{2}=e\left(\frac{1}{3}+\frac{1}{8}\right)x^2+..........$$

$\therefore$ The limit $=e\left(\frac{1}{3}+\frac{1}{8}\right)=\frac{11e}{24}$ $\qquad \frac{A+B}{7}=\frac{11+24}{7}=5$

5. $f\left((x-y)^2\right)=\left(f(x)\right)^2-2x\,f(y)+y^2$ -------------(1)

Put y = 0 in (1)

$f\left(x^2\right)=\left(f(x)\right)^2-2x\,f(0)$ -------------(2)

Put x = 0 in (1)

$f\left(y^2\right)=\left(f(0)+y^2\right)$ ---------(2)

Taking y = 0 in (3) $\Rightarrow f(0)=0$ or 1

Taking x = y in (1) we get

$f(0)=\left(f(x)\right)^2-2x\,f(x)+x^2 \qquad =\left(f(x)-x\right)^2$

If $f(0)=0\Rightarrow f(x)=x$

If $f(0)=1\Rightarrow f(x)=x+1,x-1$

But $f(x)=x-1$ will not satisfy the hypothesis

Functions are $f(x)=x$ or $x+1$

6. $\left(\sin^2(x)+\cos^2(x)\right)^4=\sin^8(x)+\cos^8(x)+4\sin^2(x)\cos^2(x)$

$\left(\sin^4(x)+\cos^4(x)\right)+6\sin^4(x)\cos^4(x)$

$$=\sin^8(x)+\cos^8(x)+\frac{3}{8}\sin^4(2x)+\sin^2(2x)\left(1-\frac{1}{2}\sin^2(2x)\right).$$

So, we have that

$$\sin^8(x)+\cos^8(x)+\frac{3}{8}\sin^4(2x)=1-\sin^2(2x)\left(1-\frac{1}{2}\sin^2(2x)\right)$$

$$=1-\left(\frac{1-\cos(4x)}{2}\right)\left(\frac{3+\cos(4x)}{4}\right)$$

$$=\frac{5}{8}+\frac{1}{4}\cos(4x)+\frac{1}{8}\cos^2(4x).$$

Now $f'(x)$ is

$$-\sin(4x)-\cos(4x)\sin(4x)=-\sin(4x)-\frac{1}{2}\sin(8x).$$

Hence, $f^{(2020)}(x)$ is

$$4^{2019}\cos(4x)+\frac{1}{2}8^{2019}\cos(8x)).$$

Taking $x=15^0$, we have $\cos(4x)=\frac{1}{2}$ and $\cos(8x)=-\frac{1}{2}$.So, we have that

$$f^{(2020)}(15^0)=\frac{1}{2}4^{2019}-\frac{1}{4}8^{2019}=2^{4037}(1-2^{2018}).$$

So, the largest power of two dividing the expression is $\boxed{4037}$

7. Plane : $8x+y+2z=0$

Given line AB : $\frac{x-2}{5}=\frac{y-4}{10}=\frac{z+3}{-4}=\lambda$

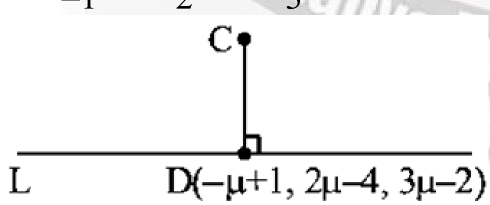
Any point on line $(5\lambda+2,10\lambda+4,-4\lambda-3)$

Point of intersection of line and plane

$8(5\lambda+2)+10\lambda+4-8\lambda-6=0$

$\lambda=-\frac{1}{3}$ $\quad C\left(\frac{1}{3},\frac{2}{3},-\frac{5}{3}\right)$

$L:\frac{x-1}{-1}=\frac{y+4}{2}=\frac{z+2}{3}=\mu$



$$\overrightarrow{CD}=\left(-\mu+\frac{2}{3}\right)\hat{i}+\left(2\mu-\frac{14}{3}\right)\hat{j}+\left(3\mu-\frac{1}{3}\right)\hat{k}$$

$$\left(-\mu+\frac{2}{3}\right)(-1)+\left(2\mu-\frac{14}{3}\right)2+\left(3\mu-\frac{1}{3}\right)3=0$$

$\mu = \dfrac{11}{14}$ $\qquad \overrightarrow{CD} = \dfrac{-5}{42}, \dfrac{-130}{42}, \dfrac{85}{42}$

Direction ratios $\rightarrow (-1, -26, 17)$

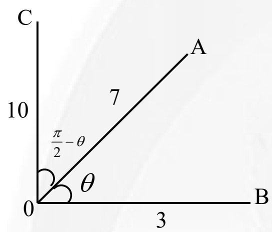
$|a+b+c| = 10$

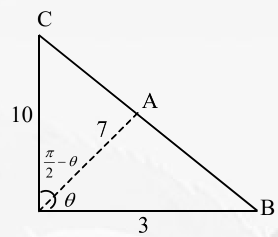
8. $\sqrt{58-42x} + \sqrt{149-140\sqrt{1-x^2}}, -1 \le x \le 1$

Let $x = \cos\theta = \sqrt{58-42\cos\theta} + \sqrt{149-140\sin\theta}$

$= \sqrt{3^2+7^2-2.3.7\cos\theta} + \sqrt{10^2+7^2-2.10.7\cos\left(\dfrac{\pi}{2}-\theta\right)}$

$= (AB+AC)_{\min} \quad = BC$ @ A,B,C are collinear





9. $S = (0,1) \cup (1,2) \cup (3,4)$

$T = \{0,1,2,3,4\}$

Number of functions :

Each element of S have 5 choice

Let n be the number of element in set S.

Number of function $= 5^n$

Here $n \rightarrow \infty$ $\qquad \Rightarrow$ Option (A) is correct.

Option (B) is incorrect

Number of strictly increasing functions $= {}^5C_3 = 10$

(C) For continuous function

Each interval will have 5 choices. $\qquad \Rightarrow$ Number of continuous functions

$= 5 \times 5 \times 5 = 125 \quad \Rightarrow$ Option (C) is incorrect.

(D) Every continuous function is piecewise constant functions $\Rightarrow$ Differentiable.

Option (D) is correct.

10. $f(x) = \dfrac{x^3}{3} - x^2 + \dfrac{5x}{9} + \dfrac{17}{36}$ $\qquad f'(x) = x^2 - 2x + \dfrac{5}{9}$

$f'(x) = 0$ at $x = \dfrac{1}{3}$ in $[0,1]$

A_R = Area of Red region

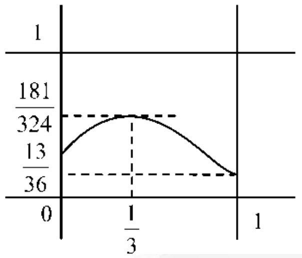
A_G = Area of Green region

$A_R = \int_0^1 f(x)\,dx = \dfrac{1}{2}$

Total area = 1 $\qquad \Rightarrow A_G = \dfrac{1}{2}$

$\int_0^1 f(x)\,dx = \frac{1}{2}$ $\quad A_G = A_R$

$f(0) = \frac{17}{36}$ $\quad f(1) = \frac{13}{36} \approx 0.36$ $\quad f\left(\frac{1}{3}\right) = \frac{181}{324} \approx 0.558$



(A) Correct when $h = \frac{3}{4}$ but $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ $\Rightarrow (A)$ is incorrect

(B) is correct

(C) When $h = \frac{181}{324}, A_R = \frac{1}{2}, A_G < \frac{1}{2}$ $\quad h = \frac{13}{36}, A_R < \frac{1}{2}, A_G = \frac{1}{2}$

$\Rightarrow A_R = A_G$ for some $h \in \left(\frac{13}{36}, \frac{181}{324}\right)$ $\quad \Rightarrow (C)$ is correct

(D) Option (D) is remaining coloured part of option (C), hence option (D) is also correct

11. Consider $O(\bar{o})$ $A(\bar{a})$ $B(\bar{b})$ $C(\bar{c})$

$OA = OB = OC \quad \Rightarrow \quad |\bar{a}| = |\bar{b}| = |\bar{c}|$

$D \to \frac{\bar{b}+\bar{c}}{2}, E \to \frac{\bar{a}+\bar{c}}{2}, F \to \frac{\bar{a}+\bar{b}}{2} \quad P \to \frac{2\bar{a}+\bar{b}+3\bar{c}}{6}$

$Q \to \frac{3\bar{a}+2\bar{b}+\bar{c}}{6} \quad R \to \frac{\bar{a}+3\bar{b}+2\bar{c}}{6}$

12. $x^9 + \frac{9}{8}x^6 + \frac{27}{64}x^3 - x + \frac{219}{512} = 0$

$\left(x^3 + \frac{3}{8}\right)^3 = x - \frac{3}{8} \Rightarrow x^3 + \frac{3}{8} = \left(x - \frac{3}{8}\right)^{1/3}$ $\quad f(x) = f^{-1}(x)$

Solutions will be along f (x) = x

$x^3 + \frac{3}{8} = x \quad x = \frac{-1+\sqrt{13}}{4}, \frac{-1-\sqrt{13}}{4}, \frac{1}{2}$

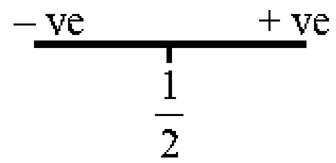
13. $f(x) = \int 4.e^{x^2}.x^{\frac{1}{2}}dx - \int \underbrace{e^{x^2}}_{I}.\underbrace{x^{\frac{-3}{2}}}_{II}dx$

$= \int 4.e^{x^2}.x^{\frac{1}{2}}dx - \left(e^{x^2}.\frac{x^{\frac{-1}{2}}}{\left(\frac{-1}{2}\right)} - \int e^{x^2}.2x\left(\frac{x^{\frac{-1}{2}}}{\frac{-1}{2}}\right)dx\right)$

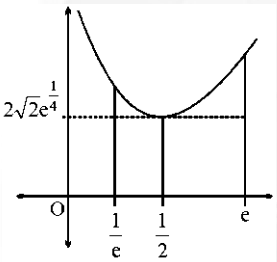
$$f(x)=\frac{2e^{x^2}}{\sqrt{x}}+C$$

Putting x = 1, f (1) = 2e + $C \Rightarrow C=0$

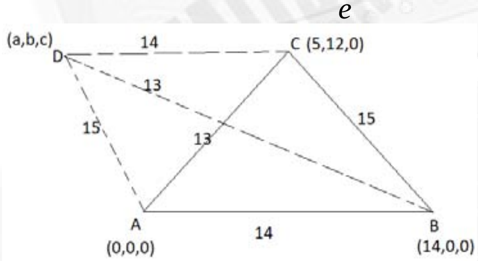
$\{\because f(1)=2e\}$ $\quad\therefore f(x)=\frac{2e^{x^2}}{\sqrt{x}}$



$$f'(x)=\frac{e^{x^2}\left(4x^2-1\right)}{x^{\frac{3}{2}}}=\frac{e^{x^2}(2x-1)(2x+1)}{x^{\frac{3}{2}}}$$



$f(x)|_{\min}=2\sqrt{2}e^{\frac{1}{4}}$ $\quad A=\int_{\frac{1}{e}}^{e} f(x)\,dx$ $\qquad A>\left(e-\frac{1}{e}\right).2\sqrt{2}e^{\frac{1}{4}}$

14. 

$a^2+b^2+c^2=15^2$ $\qquad (14-a)^2+b^2+c^2=13^2$

$(5-a)^2+(12-b)^2+c^2=14^2$

Solving the three equations, we get $\quad a=9, b=\frac{9}{2}, c=\frac{3\sqrt{55}}{2}$

Volume of tetrahedron $=\left(\frac{1}{6}\right)(14)(12)\left(\frac{3\sqrt{55}}{2}\right)=42\sqrt{55}$

15. $$\int_0^x \frac{x\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)}{(2x-\pi)}dx=\int_0^{\pi}\frac{\frac{1}{2}(2x-\pi+\pi)\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)}{(2x-\pi)}dx$$

$$=\frac{1}{2}\int_0^{\pi}\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)dx+\frac{1}{2}\pi\int_0^{\pi}\frac{\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)}{(2x-\pi)}dx \qquad I_1=\int_0^{\pi}\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)dx$$

$$I_2=\int_0^{\pi}\frac{\sin x\sin\left(\frac{\pi}{2}\cos x\right)}{(2x-\pi)}dx=\int_0^{\pi}\frac{\sin 2(\pi-x)\sin\left(\frac{\pi}{2}\cos(\pi-x)\right)}{\left[2(\pi-x)-\pi\right]}dx \qquad =-I_2\Rightarrow I_2=0$$

$$I_1=\int_0^{\pi}\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)dx=\int_0^{\pi}2\cos x.\sin\left(\frac{\pi}{2}\cos x\right)\sin x\,dx$$

$$\frac{\pi}{2}\cos x=t\Rightarrow\cos x=\frac{2}{\pi}t\Rightarrow-\sin x\,dx=\frac{2}{\pi}dt \quad x=0,t=\frac{\pi}{2} \quad x=\pi,t=-\frac{\pi}{2}$$

$$I_1=\int_{\frac{\pi}{2}}^{\frac{-\pi}{2}}2\left(\frac{2}{\pi}t\right)\sin(t)\left(\frac{-2}{\pi}\right)dt=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{8}{\pi^2}+\sin t\,dt=2\times\frac{8}{\pi^2}\int_0^{\frac{\pi}{2}}t\sin t\,dt \qquad =\frac{16}{\pi^2}(1)$$

$$I=\frac{1}{2}I_1+\frac{1}{2}\pi I_2=\frac{1}{2}\left(\frac{16}{\pi^2}\right)+\frac{1}{2}\pi(0)=\frac{8}{\pi^2}$$

16. $\left(3y^2-5x^2\right)y.dx+2x\left(x^2-y^2\right)dy=0 \quad \Rightarrow\frac{dy}{dx}=\frac{y\left(5x^2-3y^2\right)}{2x\left(x^2-y^2\right)}$

Put $y=mx \quad \Rightarrow \quad m+x.\frac{dm}{dx}=\frac{m\left(5-3m^2\right)}{2\left(1-m^2\right)}$

$$x.\frac{dm}{dx}=\frac{\left(5-3m^2\right)m-2m\left(1-m^2\right)}{2\left(1-m^2\right)} \qquad \Rightarrow\frac{dx}{x}=\frac{2\left(m^2-1\right)}{m\left(m^2-3\right)}dm$$

$$\Rightarrow\frac{dx}{x}=\left(\frac{2}{m}-\frac{\frac{4}{3}}{m}+\frac{\frac{4m}{3}}{m^2-3}\right)dm \qquad \Rightarrow\int\frac{dx}{x}=\int\frac{\left(\frac{2}{3}\right)}{m}+\int\left(\frac{2m}{m^2-3}\right)dm$$

$$\Rightarrow\ln|x|=\frac{2}{3}\ln|m|+\frac{2}{3}\ln\left|m^2-3\right|+C \qquad \text{Or, }\ln|x|=\frac{2}{3}\ln\left|\frac{y}{x}\right|+\frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2-3\right|+C$$

Put $(x=1,y=1)$:we get $c=-\frac{2}{3}\ln(2) \quad \Rightarrow\ln|x|=\frac{2}{3}\ln\left|\frac{y}{x}\right|+\frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2-3\right|-\frac{2}{3}\ln(2)$

$$\Rightarrow\left(\frac{y}{x}\right)\left[\left(\frac{y}{x}\right)^2-3\right]=2.\left(x^{\frac{3}{2}}\right)$$

Put $x=2$ to get y (2)$\Rightarrow y\left(y^2-12\right)=4\times 2\times 2\times 2\sqrt{2}$ $\Rightarrow y^3-12y=32\sqrt{2}$

$\Rightarrow \left|y^3(2)-12y(2)\right|=32\sqrt{2}$

17. $2\alpha+4\beta+3\gamma=5$ ……….(1)

$2\alpha+9\beta+8\gamma=0$ ……….(2)

$10\alpha+3\beta+4\gamma=0$ ……….(3)

$8\alpha+8\beta+8\gamma=0$ ……….(4)

$-6\alpha+\beta=0$

$\beta=6\alpha$ ……….(5)

From equation (4)

$8\alpha+48\alpha+8\gamma=0$

$\gamma=-7\alpha$ ……….(6)

From equation(1)

$8\alpha+24\alpha-21\alpha=5$ $5\alpha=5$ $\alpha=1$

$\beta=+6,$ $\gamma=-7 \therefore 6\alpha+9\beta+7\gamma=6+54-49=11$

18. $\dfrac{dy}{dx}+\left(\dfrac{-3x^5\tan^{-1}x^3}{\left(1+x^6\right)^{\frac{3}{2}}}\right)y=2e^{\left\{\frac{x-\tan x}{\sqrt{1+x^6}}\right\}}$ $I.F.=e^{\int\frac{-3x^5\tan^{-1}x^3}{\left(1+x^6\right)^{\frac{3}{2}}}dx}=e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}$

Solution of differential equation

$y.e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}=\int 2xe^{\left(\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right)}.e^{\left(\frac{\tan^{-1}\left(x^3\right)-x^3}{\sqrt{1+x^6}}\right)}dx$ $=\int 2x\,dx+c$

$y.e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}=x^2+c$

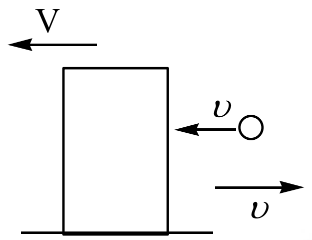
Also it passes through origin c = 0

$y(1).e^{\frac{\frac{\pi}{4}-1}{\sqrt{2}}}=1$

$y(1).e^{\frac{\pi-4}{4\sqrt{2}}}=1$

$y(1)=\dfrac{1}{e^{\frac{\pi-4}{4\sqrt{2}}}}=e^{\frac{4-\pi}{4\sqrt{2}}}$

PHYSICS

19. 

$-\frac{d\upsilon}{dt} = \cancel{2}V\frac{\upsilon}{\cancel{2}\ell}$ $\quad \int_{10}^{\upsilon}\frac{d\upsilon}{\upsilon} = \int_{\ell}^{\ell/2}\frac{d\ell}{\ell}$ $\quad \upsilon = 5\ m/\sec$

20. $\frac{3}{2}Mg\sin\alpha - f_r = \frac{3}{2}Ma$ .... (1)

$\underline{f_r = Ma}$

$a = \frac{3g\sin\alpha}{5}$

$g\sin(\alpha-\beta) = a\cos\beta$ $\quad \tan\beta = \frac{2}{5}$

n = 5

21. $N\cos 60^0 - F = ma_x$ $\quad mg - 2N\sin 60^0 = ma_y$

$FR = \left(\beta m R^2\right)\left(a_x/R\right)$ $\quad a_x = \sqrt{3}\,a_y$

$a_y = \frac{g}{7+6\beta}$ $\quad a = 1\ m/s^2$

22. $\frac{3}{2}mg\ell\,\sin\theta = \frac{1}{2}mv^2 + 2\times\frac{3}{4}\frac{m}{2}\frac{v^2}{4}$

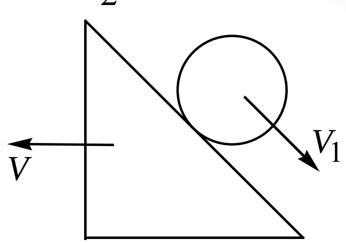
$\frac{3}{2}mg\ell\sin\theta = \frac{1}{2}mv^2 + \frac{3mv^2}{16}$ $\quad = \frac{11mv^2}{16}$

$3g\,\ell\sin\theta = \frac{11}{8}v^2$ $\quad 3g\sin\theta\cancel{v} = \frac{11^2}{4}\cancel{v}\frac{dv}{dt}$

$\frac{dv}{dt} = \frac{12g\sin\theta}{11} = \frac{72}{11}m/\sec^2$

23. $V = \frac{V_1\cos\theta}{2}$

$a = \frac{a_1\cos\theta}{2}$



$$mg\ell = \frac{1}{2}mV^2 + \frac{1}{2}mV^2 + \frac{1}{2}mV_1^2\sin^2\theta + \frac{1}{2}\frac{mR^2}{2}\left(\frac{V_1}{R}\right)^2$$

$$mg\ell = \frac{mV_1^2}{8} + \frac{mV_1^2}{4} + \frac{mV_1^2}{4} = \frac{5V_1^2}{8}m$$

$$g\cancel{V_1} = \frac{5}{4}\cancel{V_1}\frac{dV_1}{dt}$$

$$a_1 = \frac{4g}{5} \qquad a = \frac{\sqrt{2}g}{5}m/s^2$$

24. $I = 2\left(\frac{I}{18} + \frac{m}{2}\times\frac{\ell^2}{9}\right)$ $\qquad \frac{8I}{9} = \frac{m\ell^2}{9}$ $\qquad I = \frac{m\ell^2}{8}$

25. $\omega = \Omega\hat{z} - \omega'\hat{x}_3 = \Omega\hat{z} - \left(\frac{R}{r}\right)\Omega\,\hat{x}_3$

But $\hat{z} = \sin\theta\hat{x}_2 + \cos\theta\hat{x}_3$, so we can write ω in terms of the principal axes as

$$\omega = \Omega\sin\theta\hat{x}_2 - \Omega\left(\frac{R}{r} - \cos\theta\right)\hat{x}_3$$

The principal moments are $I_3 = \left(\frac{1}{2}\right)mr^2$, and $I_2 = \left(\frac{1}{4}\right)mr^2$

The angular momentum is $L = I_2\omega_2\hat{x}_2 + I_3\omega_3\hat{x}_3$, so its horizontal component has length $L_\perp = I_2\omega_2\cos\theta - I_3\omega_3\sin\theta$, with leftward taken to be positive. Therefore, the magnitude of $\frac{dL}{dt}$ is $\left|\frac{dL}{dt}\right| = \Omega L_\perp$ $\qquad = \Omega\left(I_2\omega_2\cos\theta - I_3\omega_3\sin\theta\right)$

$$= \Omega\left[\left(\frac{1}{r}mr^2\right)(\Omega\sin\theta)\cos\theta - \left(\frac{1}{2}mr^2\right)\left(-\Omega\left(\frac{R}{r} - \cos\theta\right)\right)\sin\theta\right]$$

$$= \frac{1}{4}mr\Omega^2\sin\theta\left(2R - r\cos\theta\right)$$

With a positive quantity corresponding to $\frac{dL}{dt}$ pointing out of the page (at the instant shown).

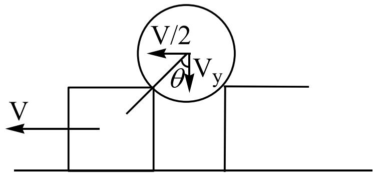
The torque (relative to the CM) comes from the force at the contact point. There are two components to this force. The vertical component is mg, and the horizontal component is $m\left(R - r\cos\theta\right)\Omega^2$ leftward, because the CM moves in a circle of radius $\left(R - r\cos\theta\right)$. The torque therefore has magnitude.

$$|\tau| = mg\left(rco\theta\right) - m\left(R - r\cos\theta\right)\Omega^2\left(r\sin\theta\right)$$

Without of the page taken to be positive. Equating this $|\tau|$ with the $\left|\frac{dL}{dt}\right|$ in eq. gives

$$\Omega^2 = \frac{g}{\frac{3}{2}R\tan\theta - \frac{5}{4}r\sin\theta}$$

26. With every collision ball deflect by θ, so total collisions = 6

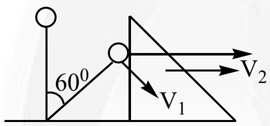
27. 

$$V\sin\theta = \frac{V}{2}\sin\theta + V_y\cos\theta \qquad V_y = \frac{V}{2}\tan\theta$$

$$mgR\left(1-\frac{1}{\sqrt{2}}\right) = \frac{1}{2}mV^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 = \frac{3}{4}mV^2$$

$$V^2 = \frac{gR}{3}\left(4-2\sqrt{2}\right) \qquad N = \frac{mg}{\sqrt{2}} - \frac{mV^2}{2R}$$

$$= \frac{mg}{\sqrt{2}} - \frac{mg}{3}\left(2-\sqrt{2}\right) = \frac{mg}{6}\left(3\sqrt{2}-4+2\sqrt{2}\right) \; = \frac{mg}{6}\left(5\sqrt{2}-4\right)$$

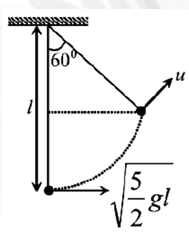
28. 

$$\frac{mg\ell}{2} = \frac{1}{2}mV_1^2 + \frac{M}{8}V_1^2 \qquad \frac{V_1^2}{\ell}\frac{\sqrt{3}}{2} = \frac{g\sqrt{3}}{4} \qquad V_1^2 = \frac{g\ell}{2}$$

$$\frac{mg\ell}{2} = \frac{mg\ell}{4} + \frac{M}{16}g\ell \qquad \frac{1}{4}m = \frac{M}{16}$$

29. $$T - mg\cos 60^0 = \frac{mv^2}{\ell} \qquad \ldots\ldots\text{ (i)}$$

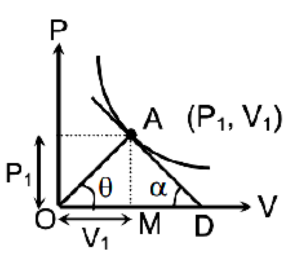
$$\frac{1}{2}m\left(\sqrt{\frac{5}{2}g\ell}\right)^2 - mg\frac{\ell}{2} = \frac{1}{2}mv^2 \qquad \ldots\ldots\text{ (ii)}$$



$v = \sqrt{\frac{3}{2}g\ell}$ and T = 2 mg

Net force is in horizontal direction when $\theta = 60^0$

30. $$\tan\alpha = -\left[slope\ at\left(P_1, V_1\right)\right] = \frac{P_1}{V_1} \qquad \tan\theta = \frac{P_1}{V_1}$$



$\therefore \alpha = \theta \qquad OM = MD = V_1 \qquad \therefore OD = 2V_1$

Area of triangle $AOD = \frac{1}{2}(AM \times OD) = \frac{1}{2}(P_1 \times 2V_1) = P_1V_1 = nRT$

31. At the time of maximum elongation angular speed of B and C are equal, let speed of B is 2v and C is v, by conserving angular momentum of the system about the centre.

$v = \frac{2v_0}{5}, \qquad v_B = \frac{4v_0}{5}, \qquad v_C = \frac{2v_0}{5}$

Conserving energy of the system $\frac{1}{2}mv_0^2 = \frac{1}{2}kx_{max}^2 + \frac{1}{2}m\left(\frac{4v_0}{5}\right)^2 + \frac{1}{2}m\left(\frac{2v_0}{5}\right)^2$

$\therefore x_{max} = \sqrt{\frac{m}{5k}}v_0 \quad \therefore$ (a), (b), (c) and (d)

32. Path difference = 0

$\frac{d^2}{D} = \frac{yd}{2D} - \left(\frac{\mu_2}{\mu_3} - 1\right)t \qquad$ y = 2 mm

When slab is removed then path difference = $\frac{d^2}{D} - \frac{y_1 d}{2D} = 0, y_1 = 2d = 4\ mm$

33. $mg\ell + 2mg\ell = \left(m\ell^2 + 4M\ell^2\right)\alpha$

$\alpha = \frac{(m+2M)g}{\ell(m+4M)} \qquad T = (m+M)g - \frac{(m+2M)^2 g}{m+4M} \qquad = \frac{mMg}{m+4M}$

34. By balancing torques on the first rod, we see that the normal force on the right end is twice the force exerted on the left end; that is, the first rod is acting as a lever with mechanical advantage 2. By similar logic, the second rod acts as a lever with mechanical advantage 1/4. Thus, the combination of the two has mechanical advantage 2(1/4) = 1/2, so $F' = F/2$.

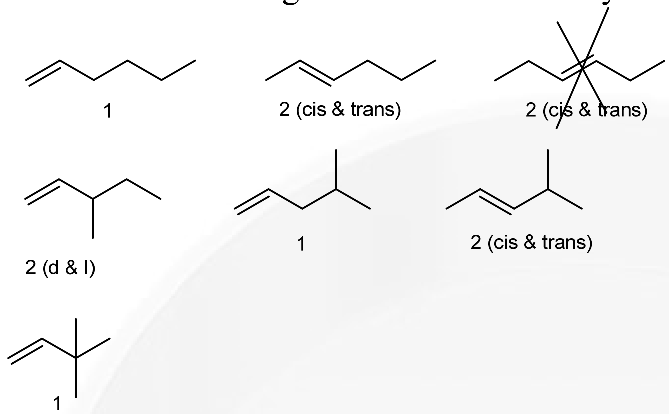
35. The force of the spring on the platform is $F_s = kx$, where x is the displacement of the object from equilibrium. The maximum value of x can be found by conservation of energy, $\frac{1}{2}m\upsilon^2 = \frac{1}{2}kx^2$

Which gives a maximum spring force $F_s = \upsilon\sqrt{km}$ acting on the platform. The maximum possible friction force on the platform is $(M+m)\mu g$. Setting these two equal, the maximum velocity is $\upsilon = \frac{(M+m)\mu g}{\sqrt{km}} = 0.25\ m/s$

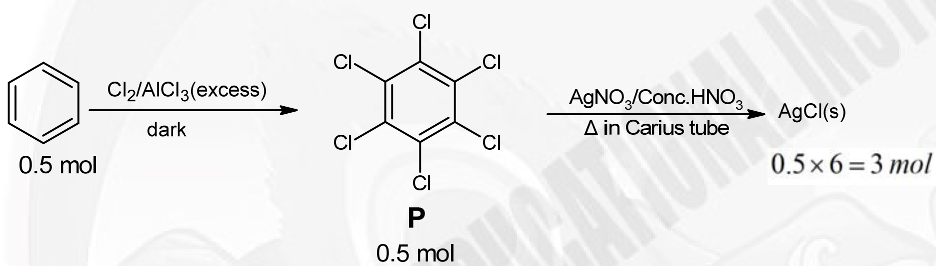
36. Work in the reference frame with x–axis along the plane and y–axis perpendicular to the plane. Then in the y–direction, the ball simply bounces regularly up and down, so the collisions are uniformly spaced in time. However, gravity provides an acceleration in the x–direction, so the distance increase.

CHEMISTRY

37. The formula of **X** must be C_6H_{12}.
The isomers which give two distinct aldehydes on ozonolysis are



38.



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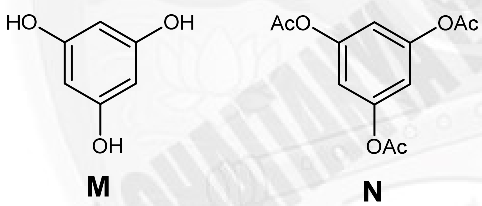
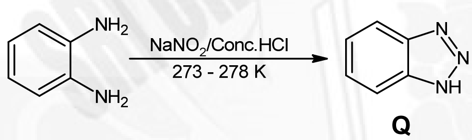
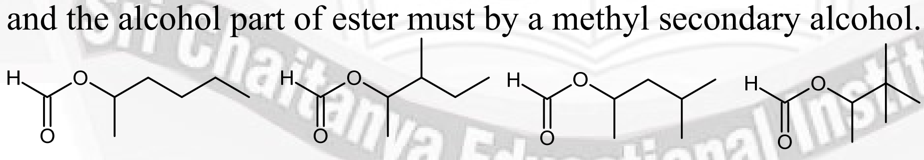


40.



41. The esters must be formate esters so that they can give propan-2-ol with excess MeMgI, and the alcohol part of ester must by a methyl secondary alcohol.



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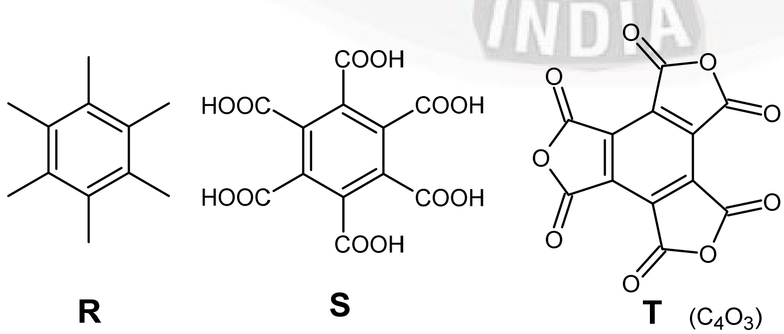

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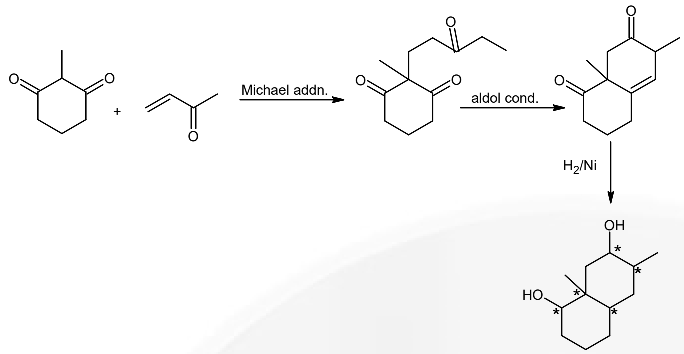
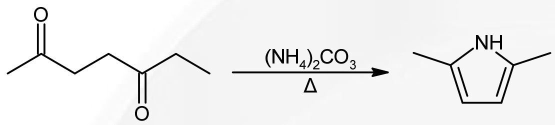




44.

45. I, II, and VI must be metals as they have low $\Delta_r H_1$.

III and IV must be non-metals as they have large negative $\Delta_{eg} H$.

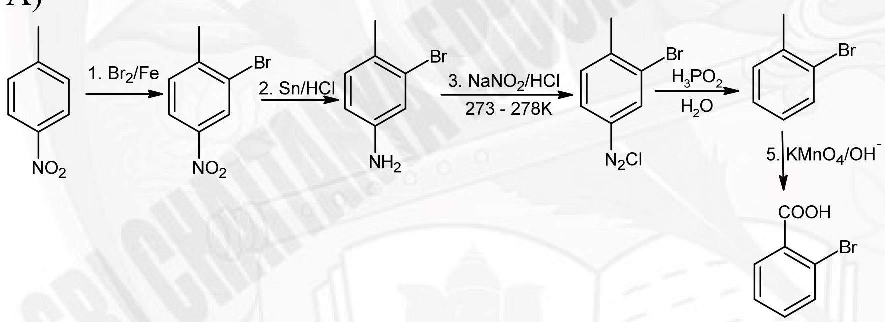
V must be a noble gas as it has high IEs as well as +ve $\Delta_{eg} H$.

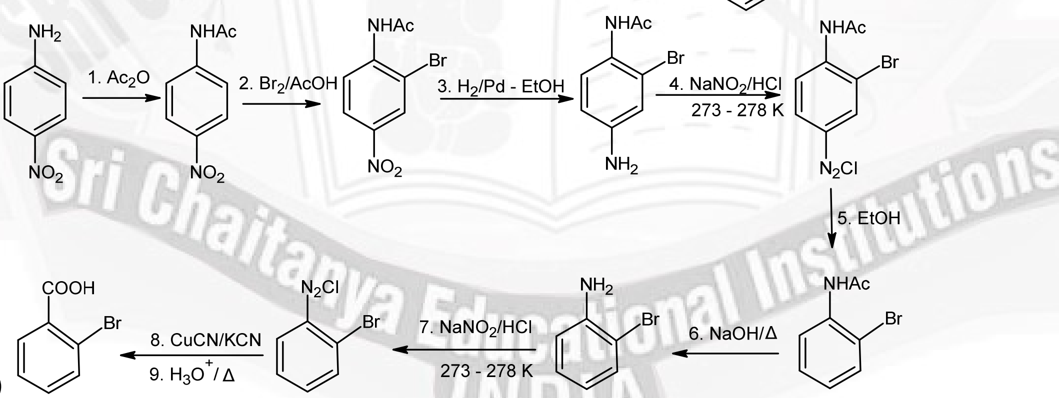
VI may be a 2nd group metal as $\Delta_r H_1$ relatively small and $\Delta_r H_2$ is just about double.

46. $CaSO_4$ is soluble in water while camphor is soluble in ether.

Camphor is sublimable.

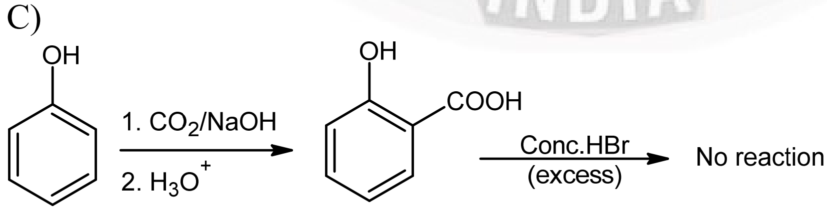
47. A)



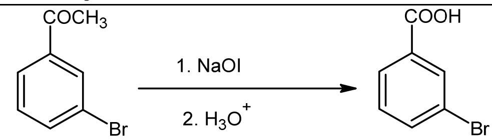


B)

C)

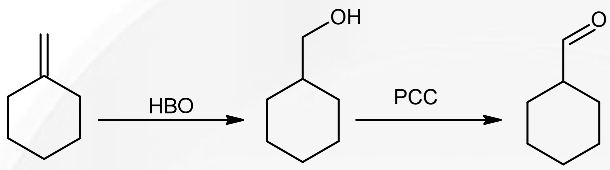


D)

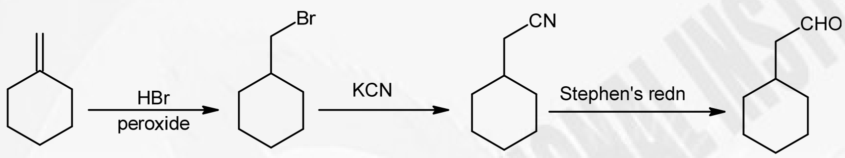


48. X – PhCH(Br)COOH & Y – PhCH(NH_2)COOH
C) Both X and Y give PhCH(OH)COOH
D) X is not a naturally occurring amino acid. $PhCH_2CH(NH_2)COOH$ is naturally occurring.

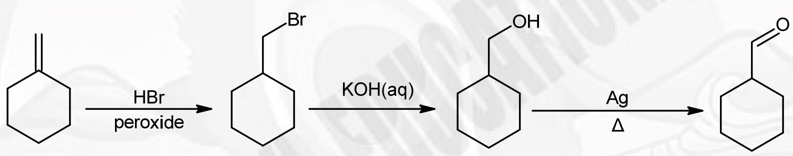
49. A)



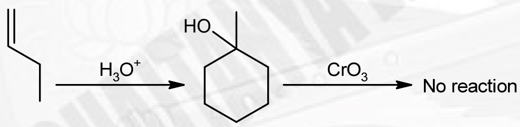
B)



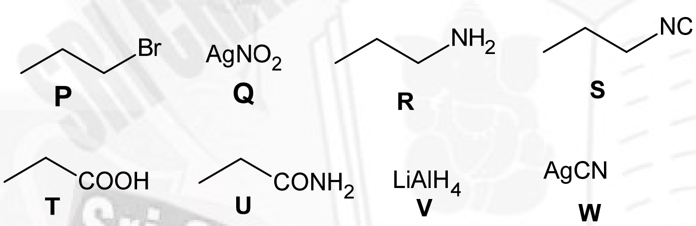
C)



D)



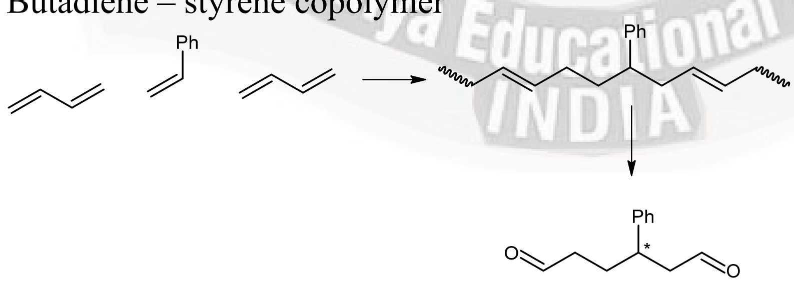
50.



51. MO configuration of N2: $(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\pi_{2p})^4(\sigma_{2p})^2$

52. D) S-II: Hydrogen bonding dominates over inductive effect in aqueous solution.

53. Butadiene – styrene copolymer



54. Sugar unit must be ribose ⇒ A & B can't be the answers.
RNA does not contain thymine ⇒ D can't be the answer.