Sri Chaitanya IIT Academy.,India.

A.P ✪ T.S ✪ KARNATAKA ✪ TAMILNADU ✪ MAHARASTRA ✪ DELHI ✪ RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec:Sr.Super60_NUCLEUS & STERLING_BT Paper -2(Adv-2021-P2-Model) Date: 27-08-2023

Time: 02.00Pm to 05.00Pm CTA-03 Max. Marks: 180

KEY SHEET

PHYSICS

1	**BC**	2	**ABCD**	3	**ABCD**	4	**AB**	5	**AD**	6	**AD**
7	**4**	8	**18**	9	**1.33**	10	**189**	11	**2.4**	12	**32**
13	**B**	14	**C**	15	**D**	16	**A**	17	**2**	18	**2**
19	**2**										

CHEMISTRY

20	**ABC**	21	**AC**	22	**A**	23	**ABC**	24	**ACD**	25	**ABC**
26	**58.50**	27	**35.20**	28	**1.50**	29	**0.80**	30	**52.80**	31	**2**
32	**B**	33	**C**	34	**D**	35	**D**	36	**4**	37	**3**
38	**5**										

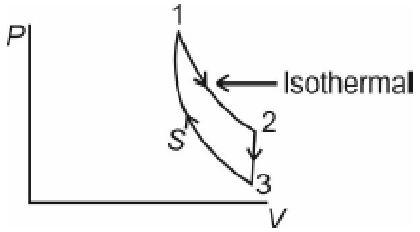
MATHEMATICS

39	**BD**	40	**ACD**	41	**BD**	42	**BC**	43	**AC**	44	**A**
45	**6.00**	46	**1.33 - 1.34**	47	**3.00**	48	**1.00**	49	**0.10**	50	**0.40**
51	**A**	52	**D**	53	**A**	54	**B**	55	**3**	56	**8**
57	**0**										

SOLUTIONS

PHYSICS

1. The P-V graph of the process is shown.
Equation of the process $3 \to 1$



$V = -\frac{3}{10}T + 140 \qquad V = -\frac{3}{10}\left(\frac{PV}{nR}\right) + 140$

2. $\Delta Q_{AB} = \Delta U_{AB}$
Write equation of straight line BC and the ideal gas equation
$\Delta U_{AB} = C_V\left[T_B - T_A\right]$
Equation of straight line BC is $P = -\frac{P_0}{V_0}V + 4P_0$

3. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$
$P_1 = 1\times10^5\, Nm^{-2} \quad T_1 = 300\,K$
$V_1 = 2.4|\times10^{-2}m^3 \quad P_2 = P_1 = \frac{Kx}{A}, V_2 = V_1 + Ax$
$dU = n\mu dT = n\frac{R}{\gamma-1}dT = \left(\frac{P_1V_1}{RT_1}\right)\frac{R}{\gamma-1}dT$

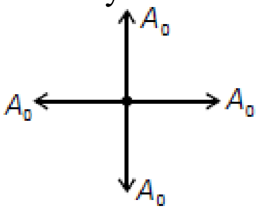
4. $f = \frac{D^2-d^2}{4D} \quad = \frac{(100)^2-(80)^2}{400}$
$f = \frac{10000-6400}{400} = 9\ cm$
$m_1 = \frac{D+d}{D-d} = \frac{100+80}{100-80} = \frac{180}{20} = 9 \qquad m_1.m_2 = 1$

5. For minima
$\Delta x_1 + \Delta x_2 = \frac{\lambda}{2} \quad \Delta x_1 = \sqrt{D^2+d^2} - D$
$= D\left(1+\frac{d^2}{D^2}\right)^{\frac{1}{2}} - D \qquad = \frac{d^2}{2D}$
$\frac{d^2}{2D} + \frac{d^2}{2D} = \frac{\lambda}{2} \qquad \frac{2d^2}{D} = \lambda \qquad d = \sqrt{\frac{\lambda D}{2}}$
For maxima
$\frac{d^2}{2D} + \frac{d^2}{2D} = \lambda$
$d = \sqrt{\lambda D}$

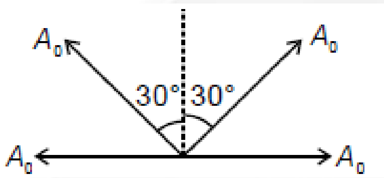
6. When $d = \frac{\lambda}{4} =$ path difference

Phase difference $= \frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$

Intensity



When $d = \frac{\lambda}{6}$ $\Delta\phi = \frac{2\pi}{\lambda}\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$



$A_{net} = 2A_0 \cos 30°$

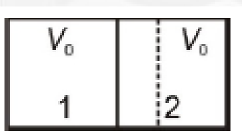
$A_{net} = \frac{\sqrt{3}}{2} A \times 2 = \sqrt{3} A_0$

$l_{net} = 3l_0$

7 & 8. Average of change in velocity is same in magnitude but opposite in direction.

Find the equation of trajectory and use simple kinematic relations $y^2 = \frac{ud}{v_0} x$

9 & 10.



Let the cross sectional area be *a*.

Then the work done from a displacement of x to

$xadx$ is $(P_2 - P_1) dV$

$P_1(V_0 + ax) = P_2(V_0 - ax) = P_0V_0$

$P_1 = \frac{P_0V_0}{V_0 + ax}, P_2 = \frac{P_0V_0}{V_0 - ax}$

$P_2 - P_1 = P_0V_0\left(\frac{V_0 + ax - V_0 + ax}{V_0^2 - a^2x^2}\right) = \frac{2axP_0V_0}{V_0^2 - a^2x^2}$

$W = \int (P_2 - P_1) dV = \int (P_2 - P_1) adx$

$= \int \frac{2a^2 P_0 V_0 x dx}{V_0^2 - a^2x^2}$

$V_0^2 - a^2x^2 = b \Rightarrow -a^2 2xdx = db$

$\frac{a^2 P_0 V_0}{-a^2} \int \frac{db}{b} = -P_0V_0\left[\ln\left(V_0^2 - a^2x^2\right)\right]$

$P_0V_0 \ln\left(\frac{V_0^2}{V_0^2 - a^2x^2}\right)$

$ax = \frac{al}{4}$ and $\frac{al}{2} = V_0$

$ax=\frac{V_0}{2}\therefore W=P_0V_0\ln\left(\frac{4}{3}\right)$

Since $dU=0, d\theta=dW\Rightarrow Q=W=nRT\ln\left(\frac{4}{3}\right)$

If walls are insulated but piston is conducting the temperature of gas in both the chambers remains the same. Let the rise in temperature be dT.

$\therefore \Delta U=nC_V dT$

$dU=-dW$ but $dW=$ Negative since work is done on the gas

$2nC_V dT=dU=-nRT\int\frac{db}{b}$

$2\frac{C_V}{R}\int_{T_1}^{T_2}\frac{dT}{T}=dU=-\int\frac{db}{b}\Rightarrow 2\frac{C_V}{R}\ln\left(\frac{T_2}{T_1}\right)=-\ln\frac{3}{4}$

$\left(\frac{T_2}{T_1}\right)^{\frac{2C_V}{R}}=\frac{4}{3}, C_V=2R$

$\frac{T_2}{T_1}=\sqrt{\frac{4}{3}}\Rightarrow T_2=T_1\frac{2\sqrt{3}}{3}=1.154\ T_1$

$T_2=(1.154)(400)=461.6$

$\Delta U=2nC_V(T_2-T_1)=2nR(62)=124nR$

11. As the collision is elastic

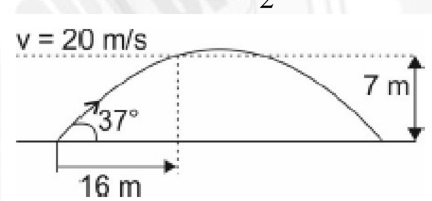
$\therefore$ Range of the projectile, $R=12+3=15m$

Now using $y=x\tan\theta\left(1-\frac{x}{R}\right)$

$x=12m\ or\ \theta=45°$ $\quad R=15m$

we get y = 2.4 m.

12. Using $y=u\sin\theta t-\frac{1}{2}gt^2$ for vertical direction motion



$y=7m\quad u\sin\theta=12m/s\qquad g=10m/s^2$

we get t = 1 & or 1.4 s

But ball will collide for the first time at $t=1.0s\ x=u\cos\theta t=16m$

Now after collision the ball will flow the path which is mirror image of the motion that would have taken place after 2nd collision if the wall were absent.

$\therefore$ Desired range, $R=2\times16=32m$

13. $r=\left(\frac{1-a}{1+a}\right)$

$\frac{\Delta r}{r}=\frac{\Delta(1-a)}{(1-a)}+\frac{\Delta(1+a)}{(1+a)}=\frac{\Delta a}{(1-a)}+\frac{\Delta a}{(1+a)}\qquad=\frac{\Delta a(1+a+1-a)}{(1-a)(1+a)}$

$$\therefore \Delta r = \frac{2\Delta a}{(1-a)(1+a)(1+a)} = \frac{2\Delta a}{(1+a)^2}$$

14. $N = N_{0^{e^{-\lambda t}}}$ $\qquad \ln N = \ln N_0 - \lambda t$

$$\frac{dN}{N} = -d\lambda t$$

Converting to error,

$$\frac{\Delta N}{N} = \Delta\lambda t \qquad \therefore \Delta\lambda = \frac{40}{2000 \times L} = 0.02$$ (N is number of nuclei left undecayed)

15 & 16. $V_r = \frac{400}{3} \times 30 = 4 km/hr$

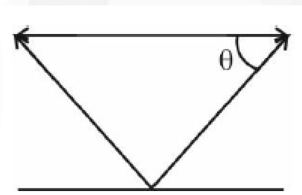
$$V_{mr} = \frac{80 \times 30}{1000} = 2.4 km/hr$$

To travel minimum distance

$V_{mr} \perp V_{mg}$

$\theta = 37^0$

$$d_0 = 80\sec(53^0) = \frac{400}{3} m$$



17. $\sqrt{(3\lambda)^2 + x^2} - x = \frac{5\lambda}{2}$ $\quad 9\lambda^2 + x^2 = \left(\frac{5\lambda}{2} + x\right)^2$

$9\lambda^2 = \frac{25}{4}\lambda^2 + 5\lambda x \qquad \frac{11}{4}\lambda^2 = 5\lambda x \qquad x = \frac{11}{20}\lambda$

18. $L.C = \frac{1}{50 \times 2} = 0.01mm$

$\therefore \text{Diameter} = 3 + 35 \times 0.01 + 0.03$

$= 3.38\text{mm} \qquad = \left(3 + \frac{38}{100}\right) mm$

19. $D = 1000 \times 1m$

$V_{rms} = 1000\, m/s$

$V_{rms} = \sqrt{\frac{3RT}{M}} \qquad \Rightarrow 10^6 = \frac{3 \times 25 \times T}{3 \times 4 \times 10^{-3}}$

$\Rightarrow T = \left(\frac{1000}{25}\right) \times 4 = 160$

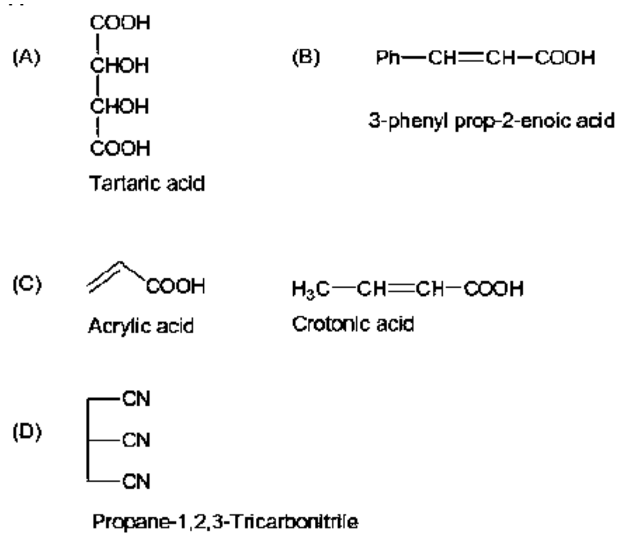
$\Rightarrow x = 02$

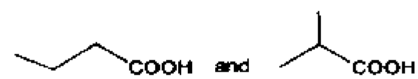
CHEMISTRY

20. Neither the lone pair of oxygen nor the pi-bonds are in conjugation in (D).

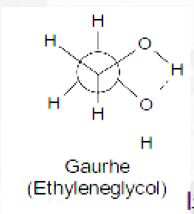
21.

22.



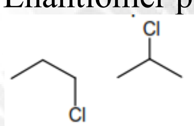
23. A) 

are chain isomers

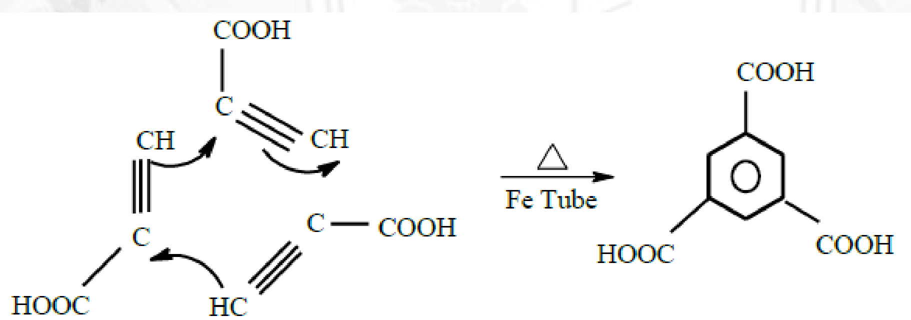


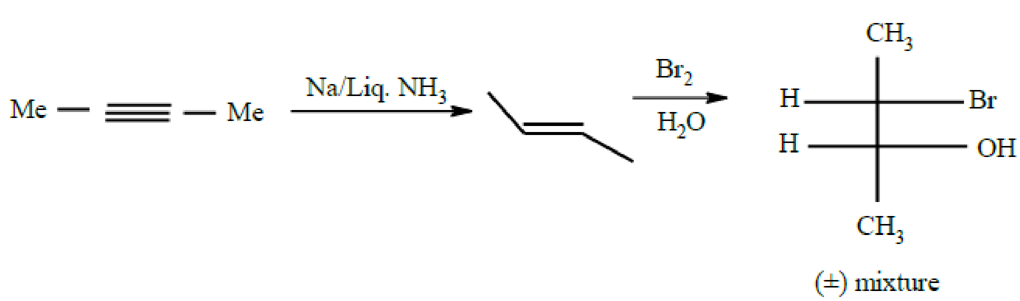
B) intramolecular hydrogen bonding

C) Enantiomer pair

D) 

24. (A)



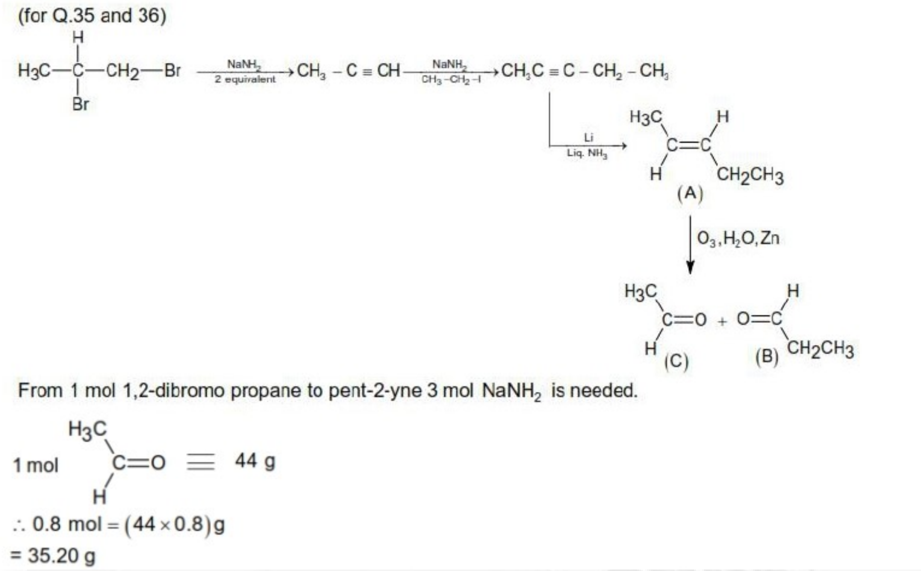


(B)

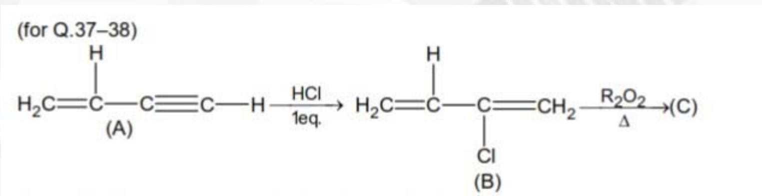
25. Stereospecific syn addition takes place in case of (A), (B) and (C). (D) is the reagent for hydration (HBO) not hydrogenation.

26.

27.

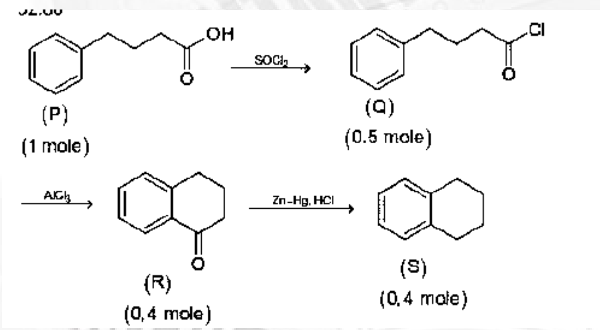


28 & 29.



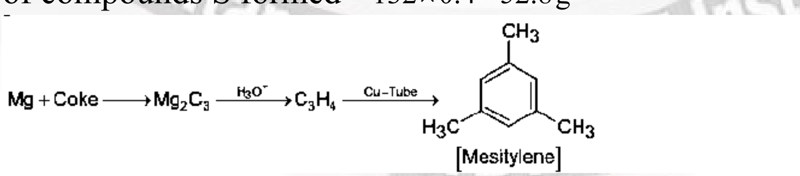
C is artificial number (polymerization, free radical)

30.

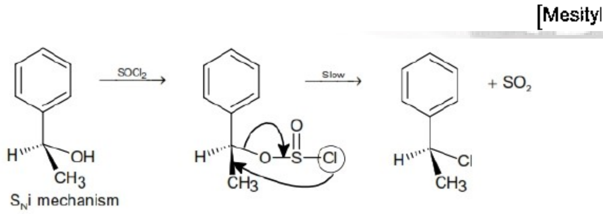


Molecular weight of S = 132.

$\therefore$ WL of compounds S formed $=132\times0.4=52.8\,g$



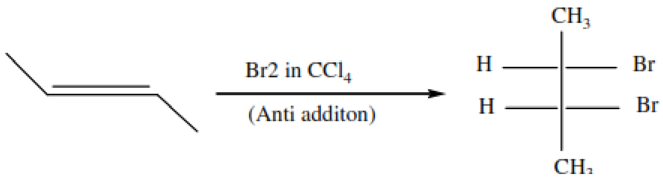
31.



32.

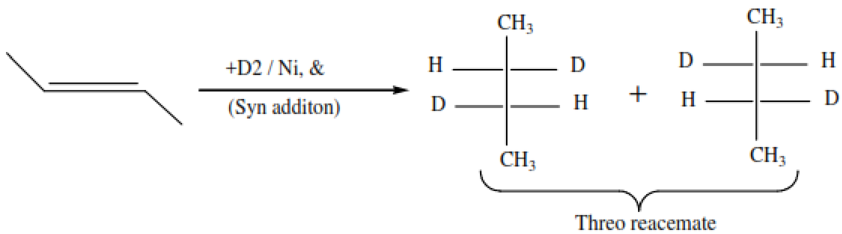
33.

34 & 35



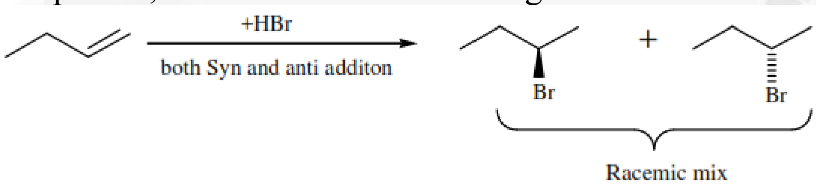
I)

It is stereo specific and stereo selective but not regioselective.



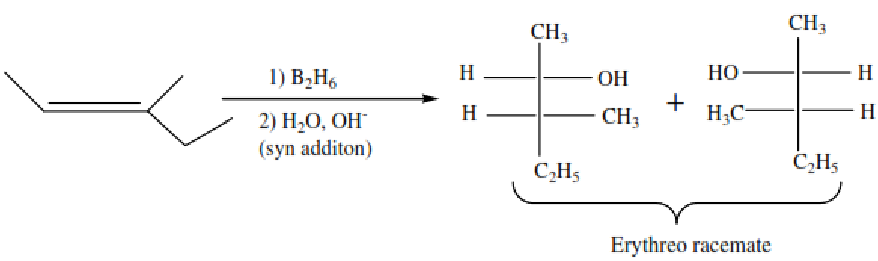
II)

Stereo specific, stereo selective but not regeoselective



III)

It is regeoselective, neither stereo selective not stereo specific.



IV)

It is regeoselective, stereo specific, stereo selective.

36. The product are $CH_3CH_2CH_2NO_2, CH_3\underset{\displaystyle NO_2}{\underset{|}{C}}HCH_3, CH_3CH_2NO_2$ and CH_3NO_2 because C-C

And C – H bond cleavage takes place in this reaction.

37. Use concept of conformational analysis of given organic compound.

38. a, b, c, g and h give carbocation more Stable than isopropyl cation.

MATHEMATICS

39. $f(x)=2^{2\left(x^2+2x+1^2-1^2\right)}=2^{2\left\{(x-1)^2-1\right\}}=2^{2(x-1)^2-2} \quad \Rightarrow a=2^{-2}=\frac{1}{4}$

$g(x)=1+\frac{1}{\cos x+2}\Rightarrow 1+\frac{1}{2}\le g(x)\le 1+\frac{1}{1} \quad \Rightarrow \frac{3}{2}\le g(x)\le 2$

40. $f(x)=\begin{cases} 2\sin\frac{1}{x} & ; \ x^2>1 \\ x & ; \ x^2<1 \\ \frac{2\sin 1+1}{2} & ; \ x^2=1 \end{cases}$ Now verify

41. $\lim_{x\to 2}\left\{\frac{(a-1)-\frac{2a^2\ln(\cos(x-2))}{(x-2)^2}}{(x-2)^2-2\ln\cos(x-2)}\right\}=\frac{1}{4}$

$\left\{\frac{(a-1)+a^2}{1+1}\right\}^2=\frac{1}{4}$

$a^2+a-1=\pm 1\Rightarrow a^2+a-1=-1\Rightarrow a=0,-1$

$a^2+a-1=1\Rightarrow a^2+a-2=0\Rightarrow (a+2)(a-1)=0\Rightarrow a=-2,1$

Reject a = 0, -1

42. a) Denominator only integer real values

b) Fundamental period = $\frac{2\pi\times 2\pi^2}{2}=2\pi^3$

c) clear from graph

43. (A) is true

$\lim_{h\to 0}\frac{f(c+h)-f(c)+f(c)-f(c-h)}{h}$

$=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}+\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h}$

$=f'(c)+f'(c)=2f'(c)$

(f is differentiable)

(B) is false. Existence of limit is no guarantee for differentiability

(C) is true

(D) is false

44. If n > 1, sin x >$\sin^n$x. If 0 < n < 1, sin x <$\sin^n$x.

$\therefore$ if $n>1,\ f(x)=\frac{2\left(\sin x-\sin^n x\right)+\left(\sin x-\sin^n x\right)}{2\left(\sin x-\sin^n x\right)-\left(\sin x-\sin^n x\right)}=3.$

if $0<n<1,\ f(x)=\frac{2\left(\sin x-\sin^n x\right)-\left(\sin x-\sin^n x\right)}{2\left(\sin x-\sin^n x\right)+\left(\sin x-\sin^n x\right)}=\frac{1}{3}.$

$\therefore$ if $n>1,\ g(x)=3,\ x\in(0,\pi)$

$\therefore$ g(x) is continuous and differentiable at $x=\frac{\pi}{2}.$

If $0<n<1, g(x)=0,\ x\in\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right) \qquad 3,\ x=\frac{\pi}{2}.$

Then $g\left(\frac{\pi}{2}+0\right)=0, g\left(\frac{\pi}{2}-0\right)=0, g\left(\frac{\pi}{2}\right)=3.$ So, g(x) is not continuous at $x=\frac{\pi}{2}.$ Hence, g(x) is also not differentiable at $x=\frac{\pi}{2}.$

45. $a+b+c=6$

$52. g=f^{-1}$ *and* $f\left(g\left(x\right)\right)=x$

$$\frac{d}{dx}\left(g\left(x\right)\times\left(g\left(x\right)\right)\right)=\frac{d}{dx}\left(g\left(x\right).x\right) \qquad =x.g'\left(x\right)g\left(x\right)$$

$at x=4,$ the derivative

$$=4.g'\left(x\right)+g\left(x\right) \quad =4.\frac{1}{3}+0$$

Hence $f^{-1}\left(4\right)=0=g\left(x\right)$ and $g'\left(x\right)=\frac{1}{f^{1}\left(0\right)}$

46. $a+b+c=6$

$52. g=f^{-1}$ *and* $f\left(g\left(x\right)\right)=x$

$$\frac{d}{dx}\left(g\left(x\right)\times\left(g\left(x\right)\right)\right)=\frac{d}{dx}\left(g\left(x\right).x\right) \qquad =x.g'\left(x\right)g\left(x\right)$$

$at x=4,$ the derivative

$$=4.g'\left(x\right)+g\left(x\right) \quad =4.\frac{1}{3}+0$$

Hence $f^{-1}\left(4\right)=0=g\left(x\right)$ and $g'\left(x\right)=\frac{1}{f^{1}\left(0\right)}$

47. $\lambda=1\Rightarrow f\left(x\right)=\cos 2x+2x$

$f'\left(x\right)\geq 0\Rightarrow f$ is increasing

$\Rightarrow 3x^2-2x+1\leq x^2-2x+9$

$\Rightarrow -2<x<2 \qquad \Rightarrow x=-1,0,1$

48. f is increasing $\forall x\in R\Rightarrow f'\left(x\right)=-2\sin 2x+2\lambda^2+\left(2\lambda+1\right)\left(\lambda-1\right)/2x>0$

at $\lambda=\frac{-1}{2}, f'\left(x\right)<0 \qquad \therefore$ this is valid only at $\lambda=1$

49. $y^2=t\Rightarrow f\left(x\right)=\frac{1}{2}\int\frac{t^3-t^2+t-1}{\left(t+1\right)\left(t^4-t^3+t^2-t+1\right)}dt$

$$=\frac{1}{2}\int\left(\frac{A}{t+1}+\frac{Bt^3+Ct^2+Dt+E}{t^4-t^3+t^2-t+1}\right)dt$$

Clearly $A=-\frac{4}{5},B=\frac{4}{5},C=-\frac{3}{5},D=\frac{2}{5},E=-\frac{1}{5}$

$$\therefore f\left(x\right)=\frac{1}{2}\left[-\frac{4}{5}\ln\left|t+1\right|\right]+\frac{1}{5}\ln\left|t^4-t^3+t^2-t+1\right|+C\Rightarrow B=-\frac{2}{5}$$

50. $y^2=t\Rightarrow f\left(x\right)=\frac{1}{2}\int\frac{t^3-t^2+t-1}{\left(t+1\right)\left(t^4-t^3+t^2-t+1\right)}dt$

$$=\frac{1}{2}\int\left(\frac{A}{t+1}+\frac{Bt^3+Ct^2+Dt+E}{t^4-t^3+t^2-t+1}\right)dt$$

Clearly $A=-\frac{4}{5},B=\frac{4}{5},C=-\frac{3}{5},D=\frac{2}{5},E=-\frac{1}{5}$

$$\therefore f(x)=\frac{1}{2}\left[-\frac{4}{5}\ln|t+1|\right]+\frac{1}{5}\ln\left|t^4-t^3+t^2-t+1\right|+C\Rightarrow B=-\frac{2}{5}$$

51. $f'(\sin x)<0$ *and* $f''(\sin x)>0\forall x\in[0,\pi/2]$

$g(x)=f(\sin x)+f(\cos x)$

$g'(x)=f'(\sin x)\cos x+f'(\cos x)(-\sin x)$

$g''(x)=f''(\sin x).\cos^2 x-\sin x+(\sin x)$

$+ \quad + \quad + \quad -$

$g''(x)>0\Rightarrow g''(x)$ is increasing

52. $g'\left(\frac{\pi}{4}\right)=0\Rightarrow g'(x)>0,$ *for* $x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

and $g'(x)<0$ *for* $x\in\left(0,\frac{\pi}{4}\right)$

g is increasing in $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

g is decreasing in $\left(0,\frac{\pi}{4}\right)$

53. $f(x)=e^{2x}$

54. $g(x)=x^2$

55. $(f(f(f(x))))=3\Rightarrow f(f(x))=2$

$f(x)=-3;$ $\quad f(x)=\frac{1}{2};$ $\quad f(x)=3;$

Zero solutions $\quad$ 2 solution $\quad$ One solution

Total solutions = 3

56. $I=\int\frac{\tan^4 x-1}{1-\tan^2 x}dx+\int\frac{1}{1-\tan^2 x}dx$

$=-\int(1+\tan^2 x)\ln+\int\frac{\cos^2 x}{\cos 2x}dx \qquad =-\tan x+\frac{1}{2}\int\frac{1+\cos 2x}{\cos 2x}dx$

$=-\tan x+\frac{1}{2}\int\sec 2x dx+\frac{1}{2}\int dx \qquad =-\tan x+\frac{1}{4}\ln|\sec 2x+\tan 2x|+\frac{1}{2}x+D$

$A=-1,B=\frac{1}{4},C=\frac{1}{2}\Rightarrow GE=8$

57. $f(x)=\int\left(\frac{1}{\sqrt{1+x^2}}-\frac{1}{1+x^2}\right)dx \qquad =\ln\left(x+\sqrt{1+x^2}\right)-\tan^{-1}x+C$

$f(0)=0\Rightarrow C=0$

$f(1)=\ln\left(\sqrt{2}+1\right)-\frac{\pi}{4}\in(0,1)\Rightarrow G.E=0$