Sri Chaitanya IIT Academy.,India.
A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI
A right Choice for the Real Aspirant
ICON Central Office - Madhapur - Hyderabad

Sec:Sr.Super60_ STERLING_BT Paper -2(Adv-2020-P2-Model) Date: 01-10-2023
Time: 02.00Pm to 05.00Pm CTA-08 Max. Marks: 180

KEY SHEET

PHYSICS

1	**6**	2	**1**	3	**5**	4	**6**	5	**1**	6	**5**
7	**BC**	8	**AB**	9	**ACD**	10	**ACD**	11	**AC**	12	**AC**
13	**7.2**	14	**1.04**	15	**4.25**	16	**0.83**	17	**20.32**	18	**0.56**

CHEMISTRY

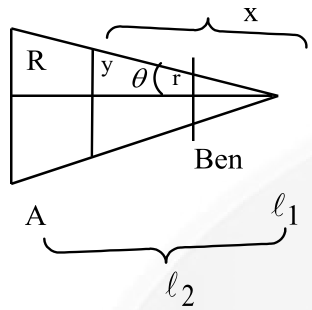
19	**3**	20	**6**	21	**4**	22	**5**	23	**7**	24	**3**
25	**ABCD**	26	**ABCD**	27	**CD**	28	**ABCD**	29	**AD**	30	**ABCD**
31	**6**	32	**11**	33	**4**	34	**2**	35	**8**	36	**14**

MATHEMATICS

37	**3**	38	**2**	39	**3**	40	**4**	41	**2**	42	**7**
43	**ABD**	44	**ABD**	45	**ABD**	46	**ABC**	47	**ABC**	48	**C**
49	**7**	50	**0.2**	51	**0.1**	52	**0.08**	53	**25**	54	**6**

SOLUTIONS
PHYSICS

1.



$$\frac{r}{\ell_1}=\frac{R}{\ell_2}=\frac{y}{x}$$

$$y=r+\frac{(R-r)}{(\ell_2-\ell_1)}(x-\ell_1)$$

$P.O.C: \pi R^2 V_1=\pi r^2 v_2 \qquad \pi y^2 v \qquad v_2=\sqrt{2gh}$

Bernoulli's : $P_x+\frac{1}{2}\rho v^2=\rho_0+\frac{1}{2}\rho(2gh)$

$F=\int(\rho_x-\rho_0)\sin\theta(2\pi\, y\, dz)$

$dz=dy\sec\theta; \qquad \tan\theta=\frac{R-r}{\ell_2-\ell_1} \qquad \therefore F=\rho gh\frac{\pi\left(R^2-r^2\right)}{R^2}$

2. $f=\mu mg \qquad T_{AB}=\mu mg r$

$\alpha=\frac{\mu mg r}{mr^2/2}=\frac{2\mu g}{r} \qquad \omega=\alpha t=\frac{2\mu g}{r}.t$

T on larger disc = $\mu\, mg\, L=\frac{mR^2}{2}.\alpha_0$

$\alpha_0=\frac{2\mu mg L}{MR^2} \qquad \omega_L=\omega_0-\alpha_0 t$

No slipping, $\omega_L \quad L=\omega r$

$$t=\frac{MR^2\omega_0 L}{2\mu g\left(MR^2+mL^2\right)}$$

3. $m(1)(22-20)=5\times0.2\times(40-22)$

$m(1)(23-20)=5\times s\times(40-23)$

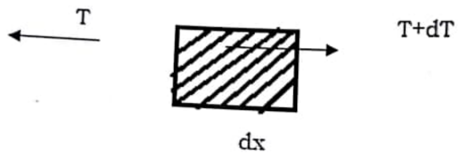
Dividing $\frac{17s}{0.2(18)}=\frac{3}{2} \qquad \Rightarrow s=\frac{3}{2}\times\frac{0.2\times18}{17}=\frac{27}{85}$ cal/gm/k

4. On 'dx' element shown,

$-dT=dmx\omega^2$

$$T=\frac{m\omega^2}{2L}(L^2-x^2)$$

The required force is difference of value of T at $x=0$ and $x=\frac{L}{2}$



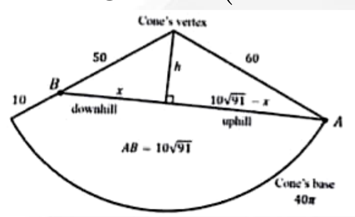
5. $\int_0^{\ell/2}(\lambda dx)\, x\,\omega^2$

6. $d = D\theta$

7. $450^\circ = 1\,rot^n + \frac{1}{4}^{th} rot^n \quad = 1mm + 0.25mm$

Reading $= 18 + (34\times 0.01) + (1.25)$

8.



By the Pythagorean Theorem:

$$(10\sqrt{91}-x)^2 + h^2 = 60^2$$

$$x^2 + h^2 = 50^2$$

$$9100 - 2(10\sqrt{91})x = 60^2 - 50^2$$

$$(20\sqrt{91})x = 8000$$

$$x = \frac{400}{\sqrt{91}}$$

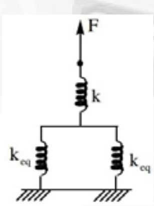
9. a) densities are different. So levels should be different

b) $\Gamma_{body} = \Gamma_{liq}$ means body just submerges.

c) Level should be same

10. $\frac{C-O}{100} = \frac{F-32}{180}$

11.



$\therefore \frac{2K_{eq}.K}{2K_{eq}+K} = K/eq \qquad \Rightarrow 2K = 2K_{eq} + K \Rightarrow K_{eq} = K/2$

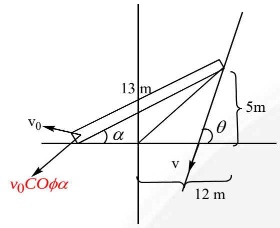
12. Weight of ice ball ice ball $= V_{in}\,\Gamma_\ell\, g$

13. a=F/9 $\qquad 5\frac{F}{9} \le (0.1)40$

14. $\sin\theta_c = \frac{\mu_2}{\mu_1} \qquad 1-(0)^2\,\theta_c = \frac{\mu_2^2}{\mu_1^2}, 1-(\hat{n}\cdot\hat{p})^2 = \left(\frac{\mu_2}{\mu_1}\right)^2, \mu_2 = \frac{3\sqrt{3}}{5}$

15. $J\dfrac{\ell}{2}=\left(\dfrac{m\ell^2}{3}+m\ell^2\right)\omega_1,\ J\dfrac{\ell}{2}=\left(\dfrac{m\ell^2}{3}+\left(\dfrac{m\ell^2}{12}+m\ell^2\right)\right)\omega_2\quad \therefore \dfrac{4\omega_1}{\omega_2}=\dfrac{17}{4}=4.25$

16. $e=\dfrac{\sqrt{2gh_2}}{2gh_1}=\sqrt{\dfrac{11}{16}}=\sqrt{0.6875=\sqrt{0.69}}=0.83$



17.

$=v_0.\dfrac{12}{13}\qquad x^2=20y\qquad \dfrac{dy}{dx}=\dfrac{x}{10}=\tan\theta,\theta=45^\circ$

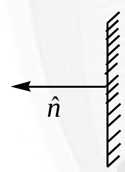
$v_0\dfrac{12}{13}=V\cos\theta=\dfrac{V}{\sqrt{2}},\ V=\dfrac{12\sqrt{2}}{13},\qquad \omega_{rel}=\dfrac{V\sin\theta+V_0\sin\alpha}{13}$

18. $m(-3\hat{i}+4\hat{j}),e=\dfrac{9}{16}\qquad (i)\vec{P_i}=m\vec{v_i}=m\left(4\hat{i}-\hat{j}\right)$

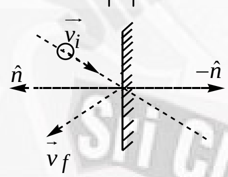
$\overrightarrow{P_f}=m\overrightarrow{v_f}=m\left(\hat{i}-3\hat{j}\right)\quad \therefore$ Impulse $1=\vec{P}_f-\vec{P}_i=m\left(-3\hat{i}+4\hat{j}\right)$

(ii) Impulse is imported on ball in the direction of common normal between wall & ball

$\therefore$ unit vector along normal to wall



$\hat{n}=\hat{i}=\dfrac{\vec{j}}{\left|\vec{j}\right|}=\dfrac{m\left(-3\hat{i}+4\hat{j}\right)}{m.5}=\left(\dfrac{-3\hat{i}+4\hat{j}}{5}\right)$



$\therefore$ Component in $\vec{v_i}$ in the direction of $(-\hat{n})$ is $=\dfrac{\vec{v_i}.(-\hat{n})}{|\hat{n}|=1}$

$=\dfrac{1}{5}(12+4)=\dfrac{16}{5}\Rightarrow$ R.V.A. just before collision

Component of $\vec{v}_f$ in the direction of $\hat{n}$ is $=\dfrac{\vec{v}_f.\hat{n}}{|\hat{n}|=1}=\dfrac{1}{5}(-3+12)=\dfrac{9}{5}$

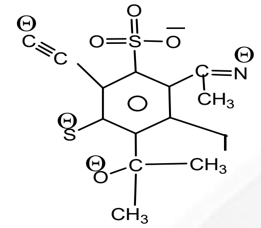
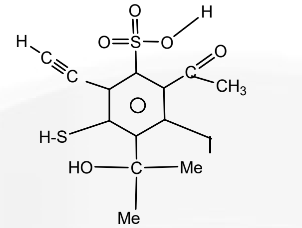
R.V.S just after collision

$\therefore e=\dfrac{R.V.S}{R.V.A}=\dfrac{9/5}{16/5}\qquad e=\dfrac{9}{16}$

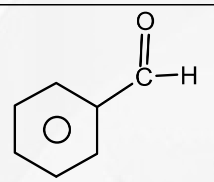
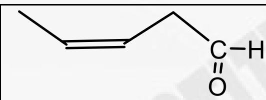
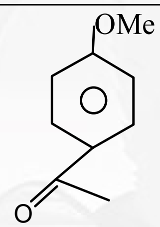
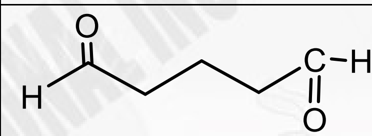
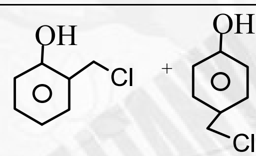
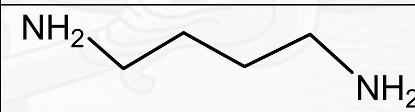
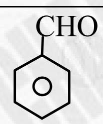
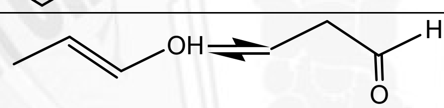
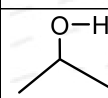
CHEMISTRY

19. ADE

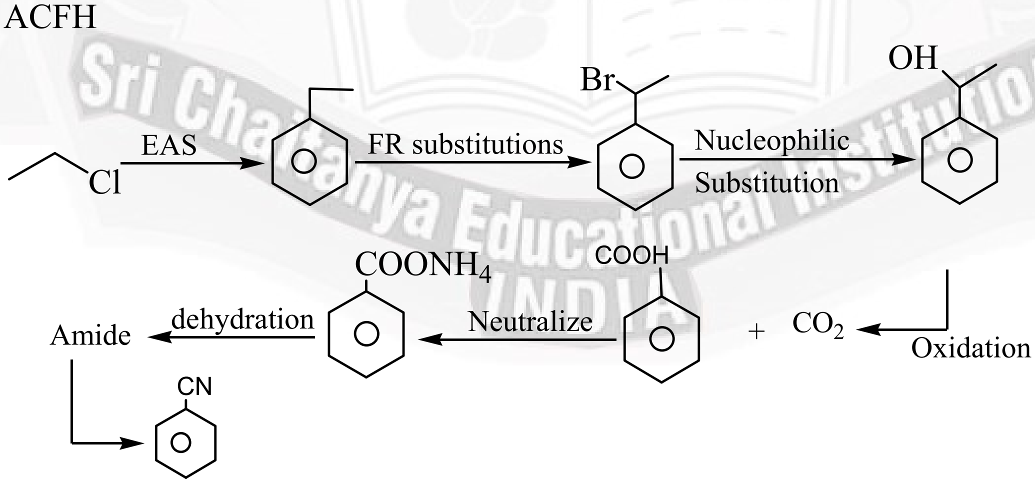
$x = 7$; $y = z$ hence $x + y = 9$

Structure of A	Structure of B
	

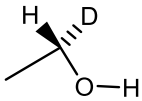
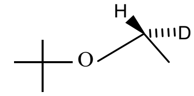
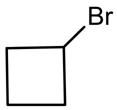
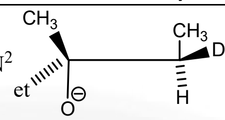
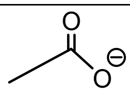
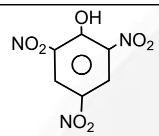
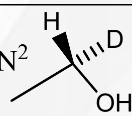
20. ABDEFG

A		F	
B		G	
C		H	
D		I	Hofmann Bromide reaction $R - NH_2 + CO_3^{2-}$
E		J	

21. ACFH

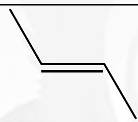
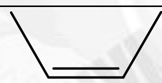
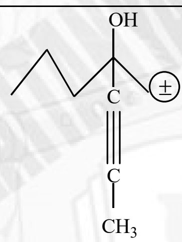
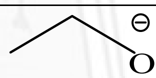
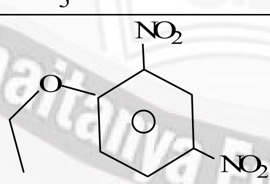
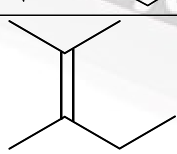
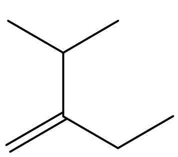
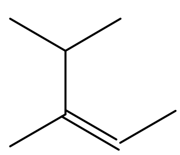


22. AEFGIJ

A	 SN2	F	Williamson's synthesis SN2 
B		G	SN2 
C		H	Bromamide reaction; Retension in configuration
D		I	Finkelstein SN2 —— Inversion
E	SN2 		

23. $\Rightarrow$ A is $CH_3-C\equiv \overset{\ominus}{C}$ Anionic salt

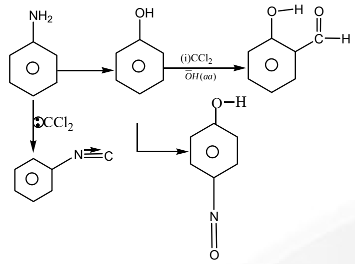
B is $CH_3-C\equiv C-CH_3$

C is	
D is	
E is	
F is	
G is	$CH_3-C\equiv C-H$
H is	
I can be	 ;  ;  trans and cis

24. B; C; F

Refer NCERT table

25.

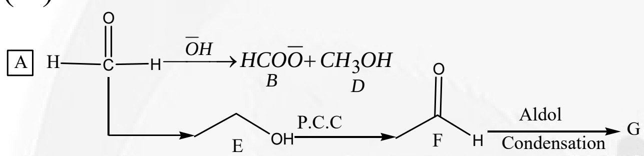


Refer NCERT for Colored compounds.

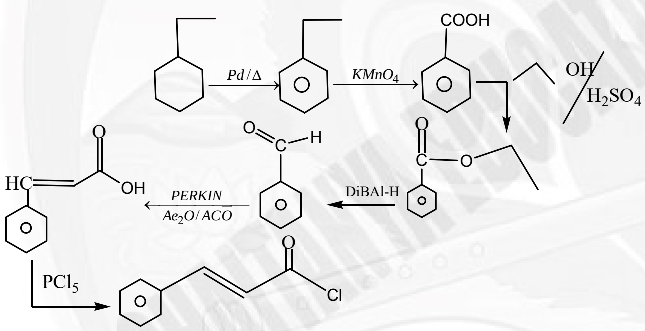
26. (i) A & F reduce F.S. (ii) E & F give CHI_3 + Carboxylate

(iii) Due to presence of EWG in Conjugation with alkene $\Rightarrow$ undergoes 1,4- conjugate Addition.

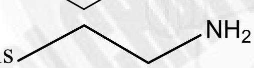
(iv) Refer NCERT for Named reactions.



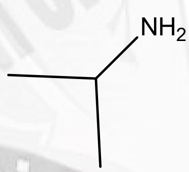
27.



28. X is



Y is

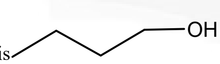


Z is

Refer NCERT for named reactions

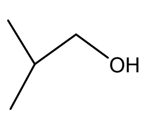
29.

P is

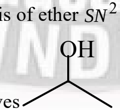


A) williamson's synthesis of ether SN^2

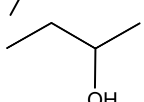
Q is



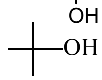
B) 2 mole CH_3MgCl gives



R is

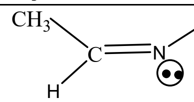


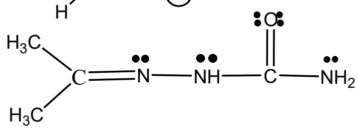
S is

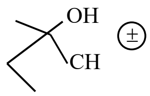


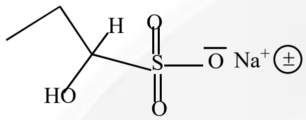
D) SN^2[NCERT-215 page]

30.

A)  (Z;E)

B)  5 Lone pairs

C) 

D) 

31. Aldehydes & α -hydroxy ketones reduce Ag^+ to Ag

$H_3C-C\equiv C-H \xrightarrow{TS} H_3C-C\equiv C^{\Theta}Ag^+$ white PPt (not silver mirror $Ag_{(s)}$)

32.

SiO_4^{4-} _____1

$C\ell O_4^-$ _____1.75

PO_4^{3-} _____1.25

$\left.\begin{array}{l} O_2^- \text{\_\_\_\_\_\_\_} 1.5 \\ N_2^+ \text{\_\_\_\_\_\_\_} 2.5 \end{array}\right\} MOT$

SiO_3^{2-} _____1.33

$C\ell O_3^-$ _____1.66

Total_________11

33. $\left[\frac{56}{11.2}\times 2\right]2=\left(n_{KMnO_4}\right)5$ $n=4$

34. $\Rightarrow 1;4$ are saline hydrides

35.

$x=4$ __ B_2H_6

BF_3

CCl_2

$AlCl_3$

$y=4$ __ SiF_4

$SiCl_4$

BCl_3

BF_3 ___ Partial Hydrolysis

36.

$R_2Culi \Rightarrow 1+11$

$Rmgx \Rightarrow 2$

Total $\Rightarrow 14$

MATHEMATICS

37. $\int e^x\left(f(x)+f'(x)\right)dx$

38. $\lim\limits_{n\to\infty}\sum\limits_{K=0}^{n}\dfrac{{}^nC_K}{n^K(K+3)}=\lim\limits_{n\to\infty}\sum\limits_{K=0}^{n}\dfrac{1}{K+3}\,{}^nC_K.\dfrac{1}{n^K}$

$=\lim\limits_{n\to\infty}\sum\limits_{K=0}^{n}{}^nC_K.\dfrac{1}{n^K}\int_0^1 x^{K+2}dx\left(\because \dfrac{1}{K+3}=\int_0^i x^{K+2}dx\right)$

$=\int_0^1\left(x^2\lim\limits_{n\to\infty}\sum\limits_{K=0}^{n}{}^nC_K.\left(\dfrac{x}{n}\right)^K\right)dx \qquad =\int_0^1 x^2\left\{\lim\limits_{n\to\infty}\left(1+\dfrac{x}{n}\right)^n\right\}dx$

$=\int_0^1 x^2.e^x dx \qquad \left[\because \lim\limits_{n\to\infty}\left(1+\dfrac{x}{n}\right)^n=e^x\right]$

$=(x^2.e^x)_0^1-\int_0^1 2x.e^x dx=e-2\left\{(xe^x)_0^1-\int_0^1 e^x dx\right\}$

$=e-2\{e-e+1\}=e-2$

39. If A denotes the coefficient matrix, then $|A|=\lambda^2(\lambda+3)$

For $\lambda=0$ and, $\lambda=-3$, the system is inconsistent.

40. Using $a^2+b^2+c^2=0$, we can write Δ as

$$\Delta=\begin{vmatrix}-a^2 & ab & ac\\ ab & -b^2 & bc\\ ac & bc & -c^2\end{vmatrix}=abc\begin{vmatrix}-a & a & a\\ b & -b & b\\ c & c & -c\end{vmatrix}$$

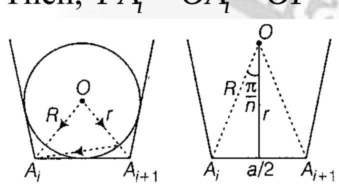
[taking a, b, c common from C_1, C_2, C_3 respectively]

$$=a^2b^2c^2\begin{vmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}=a^2b^2c^2\begin{vmatrix}0 & 2 & 1\\ 0 & 0 & 1\\ 2 & 0 & -1\end{vmatrix}$$

[using $C_1\to C_1+C_2$ and $C_2\to C_2+C_3$] $=4a^2b^2c^2$ Thus, $k=4$

41. Let the centre of the incircle be the reference point.

Then, $PA_i=OA_i-OP$



$PA_i.PA_i=(OA_i-OP).(OA_i-OP)$

$(PA_i)=\left(|OA_i|\right)^2+\left(|OP|\right)^2-OA_i.OP$

$\sum\limits_{i=1}^{n}(PA_i)^2=\sum\limits_{i=1}^{n}\left(|OA_i|\right)^2+\left(|OP|\right)^2-2OA_i.OP$

$$= nR^2 + nr^2 - 2OP.\sum_{i=1}^{n} OA_i \qquad (i)$$

$$= n(R^2 + r^2) - 2OP.(0).$$

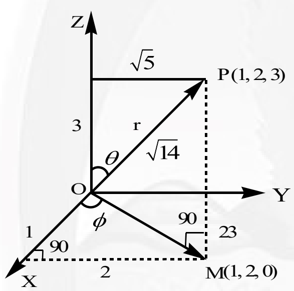
Now, $R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}, r = \frac{a}{2}\cot\frac{\pi}{n}$ ...(ii)

$$\therefore \quad R^2 + r^2 = n.\frac{a^2}{4}\left(\operatorname{cosec}^2\frac{\pi}{n} + \cot^2\frac{\pi}{n}\right)$$

$$= n.\frac{a^2}{4}\left(\frac{1+\cos^2\pi/n}{\sin^2\pi/n}\right) \qquad ...(iii)$$

$\therefore$ From Eqs. (i) and (iii), we get $\Rightarrow \sum_{i=1}^{n}(PA_i)^2 = n\frac{a^2}{4}\left(\frac{1+\cos^2\pi/n}{\sin^2\pi/n}\right)$

42.



$$\Rightarrow 1 = r\sin\theta.\cos\phi, 2 = r\sin\theta\sin\phi, 3 = r\cos\theta \qquad ...(i)$$

$$\Rightarrow 1^2 + 2^2 + 3^2 = r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$$

$$= r^2\sin^2\theta\left(\cos^2\phi + \sin^2\phi\right) + r^2\cos^2\theta = r^2\sin^2\theta + r^2\cos^2\theta = r^2$$

$\Rightarrow \quad r = \pm\sqrt{14} \quad \therefore$ From Eq. (i), we have

$$\sin\theta\cos\phi = +\frac{1}{\sqrt{14}},$$

$$\sin\theta\sin\phi = \frac{2}{\sqrt{14}}, \cos\theta = \frac{3}{\sqrt{14}}$$

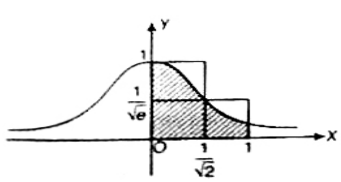
(neglecting –ve sign as acute angles)

$$\therefore \quad \frac{\sin\theta\sin\phi}{\sin\theta\cos\phi} = \frac{2}{1} \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{5}}{3}$$

$$\Rightarrow \quad \tan\phi = 2 \text{ and } \tan\theta = \frac{\sqrt{5}}{3}$$

43. Conceptual

44.



Since, $x^2 \le x$ when $x \in [0,1]$

$\Rightarrow \quad -x^2 \ge -x$ or $e^{-x^2} \ge e^{-x}$

$\therefore \quad \int_0^1 e^{-x^2}dx \ge \int_0^1 e^{-x}dx \Rightarrow \quad S \ge -\left(e^{-x}\right)_0^1 = 1-\frac{1}{e} \qquad ...(i)$

Also, $\int_0^1 e^{-x^2}dx \le$ Area of two rectangles

$\le \left(1\times\frac{1}{\sqrt{2}}\right)+\left(1-\frac{1}{\sqrt{2}}\right)\times\frac{1}{\sqrt{e}}$

$\le \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right) \qquad ...(ii)$

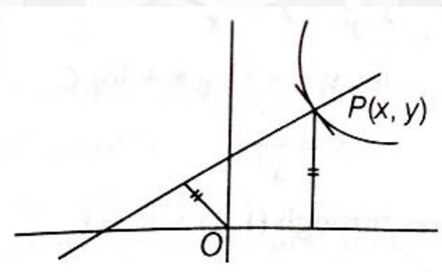
$\therefore \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right) \ge S \ge 1-\frac{1}{e}$ [from Eqs.(i) and (ii)]

45. Equation of normal

$Y-y=-\frac{1}{m}(X-x) \quad \Rightarrow \quad -my+mY=X-x$

$x+mY=X+my$

$X-mY+my-x=0$



Perpendicular from $(0,0)=\left|\frac{x+my}{\sqrt{1+m^2}}\right|=y \Rightarrow x^2+2xym=y^2$

$\frac{dy}{dx}=\frac{y^2-x^2}{2xy} \Rightarrow$ homogeneous $\qquad$ Also $x.2y.\frac{dy}{dx}-x^2=y^2$

Put $y^2=t; \quad 2y\frac{dy}{dx}=\frac{dt}{dx}; x.\frac{dt}{dx}+x^2=t$

$\frac{dt}{dx}-\frac{1}{x}t=-x$ Which is linear differential equation.

Hence, (a). (b) and (d) are the correct answers.

46. Replacing x by 2, $\Rightarrow \quad 2f(2)+2f\left(\frac{1}{2}\right)-2f(1)=4$

$\Rightarrow \quad f(2)+f\left(\frac{1}{2}\right)=2+f(1)$

Replacing x by 1, $\qquad f(1)=-1$

Replacing x by $\frac{1}{2}, 2f\left(\frac{1}{2}\right)+\frac{1}{2}f(2)+2=\frac{5}{2}$

$\therefore \quad 2f(2)+\frac{1}{2}f\left(\frac{1}{2}\right)=\frac{1}{2}$

From Eqs. (i) and (iii), we get

$f(2)=1, f\left(\frac{1}{2}\right)=0$

47. : $|a-b|^2+|b-c|^2+|c-a|^2$

$=2(|a|^2+|b|^2+|c|^2-a.b-b.c-c.a)$

$\Rightarrow \quad a.b+b.c+c.a=-\frac{9}{2}$

Now, $|a+b+c|^2+|a|^2+|b|^2 2(a.b+b.c+c.a) \quad =3+3+3-2\left(\frac{9}{2}\right)=0$

$\therefore \quad a+b+c=0 \quad ...(i)$

Also, $\quad |a+b+c|^2 \geq 0$

$\Rightarrow \quad a.b+b.c+c.a \geq -\frac{9}{2} \quad ...(ii)$

Thus, least value is -9/2

48. Let angle between a and b be θ_1, c and d be θ_2 and $a\times b$ and $b\times d$ be θ.

Since, $(a\times b).(c\times d)=1 \Rightarrow \sin\theta_1.\sin\theta_2.\cos\theta=1$

$\Rightarrow \quad \theta_1=90^\circ, \theta_2=90^\circ, \theta=0^\circ$

$\Rightarrow a\perp b, c\perp d, (a\times b)\|(cd)$

So, $a\times b=k(c\times d)$ and $a\times b=k(c\times d)$

$\Rightarrow \quad (a\times b).c=k(c\times d).c$ and $(a\times b).d=k(c\times d).d$

$\Rightarrow a,b,c$ and a,b,d are coplanar vectors, so options (a) and (b) are incorrect.

Let $b\|d \quad \Rightarrow b=\pm d$

As $\quad (a\times b).(c\times d)=1 \Rightarrow (a\times b).(c\times b)=\pm 1$

$\Rightarrow \quad [a\times b \ cb]=\pm 1 \Rightarrow [cba\times b]=\pm 1$

$\Rightarrow c.[b\times(a\times b)]=\pm 1 \Rightarrow \quad c.[a-(b.a)b]=\pm 1$

$\Rightarrow \quad c.a=\pm 1 \quad [\because a.b=0]$

Which is a contradiction, so option (c) is correct.

Let option (d) is correct.

$\Rightarrow \quad d=\pm a$ and $c=\pm b$

As $\quad (a\times b).(c\times d)=1$

$\Rightarrow \quad (a\times b).(b\times a)=\pm 1$

49. Let $I\int_0^{\sqrt{3}} \frac{1}{1+x^2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Now, $\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\begin{cases}2\tan^{-1}x, & \text{if } -1\le x\le 1\\ \pi-2\tan^{-1} & \text{if } x>1\end{cases}$

$$\therefore I=\int_0^1\frac{1}{1+x^2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx+\int_1^{\sqrt{3}}\frac{1}{1+x^2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

$$=\int_0^1\frac{2\tan^{-1}x}{1+x^2}dx+\int_1^{\sqrt{3}}\frac{\pi-2\tan^{-1}x}{1+x^2}dx$$

$$=2\int_0^1\frac{\tan^{-1}x}{1+x^2}dx+\pi\int_1^{\sqrt{3}}\frac{1}{1+x^2}dx-2\int_1^{\sqrt{3}}\frac{\tan^{-1}x}{1+x^2}dx$$

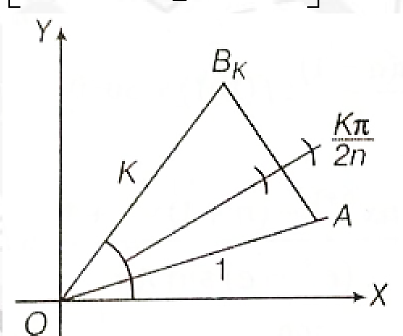
$$=2\int_0^{\pi/4}t\,dt+\pi(\tan^{-1}x)_1^{\sqrt{3}}-2\int_{\pi/4}^{\pi/3}t\,dt\quad(\text{put }\tan^{-1}x=t)$$

$$=2\left(\frac{t^2}{2}\right)_0^{\pi/4}+\pi\{\tan^{-1}\sqrt{3}-\tan^{-1}1\}-\left(\frac{t^2}{2}\right)_{\pi/4}^{\pi/3}$$

$$=\frac{\pi^2}{16}+\pi\left\{\frac{\pi}{3}-\frac{\pi}{4}\right\}-\left\{\frac{\pi^2}{9}-\frac{\pi^2}{16}\right\}=\frac{7}{72}\pi^2$$

50. Here, $OB_K=K$ and $\angle AOB_K=\frac{K\pi}{2n}$

$$\therefore\quad S_K=\frac{1}{2}.(1)(K)\sin\left(\frac{Kn}{2n}\right)\left[\text{using},\Delta=\frac{1}{2}ab\sin\theta\right]$$



$$\text{Then, } L=\lim_{x\to\infty}\frac{1}{n^2}\sum_{K=1}^{n}\frac{K}{2}\sin\left(\frac{K\pi}{2n}\right)$$

$$=\frac{1}{2}.\lim_{n\to\infty}\frac{K}{n}.\sin\left(\frac{\pi}{2}.\frac{K}{n}\right)=\frac{1}{2}.\int_0^1 x.\sin\left(\frac{\pi}{2}x\right)dx$$

$$=\frac{1}{2}\left[\left(\underbrace{\frac{-2}{\pi}.x.\cos\frac{\pi x}{2}}_{0}\right)_0^1+\frac{2}{\pi}\int_0^1\cos\frac{\pi x}{2}.dx\right]$$

$$=\frac{1}{2}\left[\frac{2}{\pi}.\frac{2}{\pi}.\left(\sin\frac{\pi x}{2}\right)_0^1\right]=\frac{2}{\pi^2}$$

$$\therefore \quad \frac{\pi^2}{10}L=\frac{\cancel{\pi^2}}{10}.\frac{2}{\cancel{\pi^2}}$$

$$=\frac{1}{5}=0.2$$

51. Given $f\left(x+y^3\right)=f\left(x\right)+\left[f\left(y\right)\right]^3 \qquad ...(i)$

And $f'\left(0\right)>0 \qquad ...(ii)$

On replacing x, y by 0,

$f\left(0\right)=f\left(0\right)+f\left(0\right)^3 \Rightarrow f\left(0\right)=0 \qquad ...(iii)$

Also, $f'\left(0\right)=\lim\limits_{h\to 0}\frac{f\left(0+h\right)-f\left(0\right)}{h}=\lim\limits_{h\to 0}\frac{f\left(h\right)}{h} \qquad ..(iv)$

Let $I=f'(0)=\lim\limits_{h\to 0}\frac{f(0+(h^{1/3})^3)-f\left(0\right)}{(h^{1/3})^3}$

$$=\lim_{h\to 0}\frac{f((h^{1/3})^3)}{(h^{1/3})^3}=\lim_{h\to 0}\left(\frac{f(h^{1/3})}{(h^{1/3})}\right)^3=I^3$$

$\Rightarrow \quad I=I^3$ or $I=0, 1, -1$ as $f'\left(0\right)>0$

$\therefore \quad f'(0)=1 \qquad ...(v)$

Thus, $f'\left(x\right)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{f(x+(h^{1/3})^3)-f(x)}{(h^{1/3})^3}$$

$f'(x)=\lim\limits_{h\to 0}\frac{f\left(x\right)+(f(h^{1/3}))^3-f\left(x\right)}{(h^{1/3})^3}$ [using Eq.(i)]

$\Rightarrow \quad f'(x)=\lim\limits_{h\to 0}\left(\frac{f(h^{1/3})}{(h^{1/3})}\right)=(f'(0))^3$

$\Rightarrow \quad f'(x)=1 \qquad$ [as $f'(0)=1$, using Eq. (v)]

On integrating both sides, we get $f(x)=x+c$

As $f(0)=0$

$\Rightarrow \quad f\left(x\right)=x$

Thus, $f\left(100\right)=100$

52. Applying $R_2 \to R_2 - R_3$, we get

$$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ -\sin^2 x & 0 & 0 \\ 1 & \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$$

$$= -(-\sin^2 x)\begin{vmatrix} \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$$

$$= \sin^2 x\left[\cos x \operatorname{cosec}^2 x - \cos^2 x(\sec^2 x + \cot x \operatorname{cosec} x)\right]$$

$$= \cos x - \sin^2 x - \cos^3 x = \cos x \sin^2 x - \frac{1}{2}(1-\cos^2 x)$$

Thus, $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(x)\,dx = \left[\frac{1}{3}\sin^3 x - \frac{1}{2}x + \frac{1}{4}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$$= \frac{1}{3} - \frac{\pi}{4} - \frac{1}{3}.\frac{1}{2\sqrt{2}} + \frac{\pi}{8} - \frac{1}{4}$$

$$= \frac{1}{12} - \frac{\pi}{8} - \frac{1}{6\sqrt{2}}$$

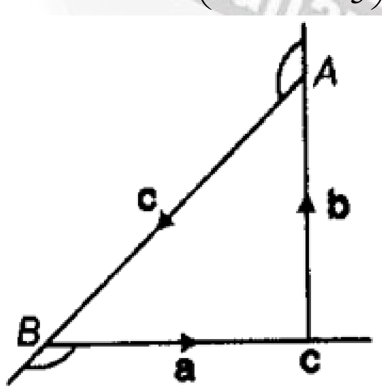
53. We observe, $|a|^2 + |b|^2 = 3^2 + 4^2 = 5^2 = |c|^2 \quad a.b = 0$

$$b.c = |b|\,|c|.\cos\left(\pi - \cos^{-1}\frac{4}{5}\right)$$

$$= 4 \times 5\left\{-\cos\left(\cos^{-1}\frac{4}{5}\right)\right\}$$

$$= 4 \times 5 \times \left(-\frac{4}{5}\right) = -16$$

$$c.a = |c|\,|a|.\cos\left(\pi - \cos^{-1}\frac{3}{5}\right)$$



$$= 5.3\left\{-\cos\left(\cos^{-1}\frac{3}{5}\right)\right\} = 5.3\left(-\frac{3}{5}\right) = -9$$

$\therefore\ a.b + b.c + c.a = 0 - 16 - 9 = -25$

$\therefore \left|\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}\right| = 25$

54. Equation of the plane containing the lines

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

and $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$

is $a(x-2) + b(y-3) + c(z-4) = 0$

where, $3a + 4b + 5c = 0$

$2a + 3b + 4c = 0$

and $a(1-2) + b(2-3) + c(2-3) = 0$

i.e. $a + b + c = 0$

From Eqs. (ii) and (iii), $\dfrac{a}{1} = \dfrac{b}{-2} = \dfrac{c}{11}$, which satisfy Eq. (iv).

$\therefore$ Planes must be parallel, so $A = 1$ and then

$$\frac{|d|}{\sqrt{6}} = \sqrt{6} \Rightarrow \quad |d| = 6$$