# Sri Chaitanya IIT Academy.,India.

A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 19-08-2023

Time: 09.00Am to 12.00Pm RPTM-03 Max. Marks: 300

## KEY SHEET

## PHYSICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1) | 3 | 2) | 1 | 3) | 1 | 4) | 2 | 5) | 3 |
| 6) | 1 | 7) | 4 | 8) | 2 | 9) | 2 | 10) | 2 |
| 11) | 4 | 12) | 1 | 13) | 4 | 14) | 3 | 15) | 1 |
| 16) | 2 | 17) | 3 | 18) | 3 | 19) | 3 | 20) | 3 |
| 21) | 42 | 22) | 25 | 23) | 60 | 24) | 15 | 25) | 50 |
| 26) | 6 | 27) | 6 | 28) | 8 | 29) | 5 | 30) | 4 |

## CHEMISTRY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 31) | 4 | 32) | 3 | 33) | 1 | 34) | 4 | 35) | 3 |
| 36) | 3 | 37) | 3 | 38) | 2 | 39) | 2 | 40) | 3 |
| 41) | 3 | 42) | 3 | 43) | 3 | 44) | 2 | 45) | 1 |
| 46) | 4 | 47) | 4 | 48) | 3 | 49) | 1 | 50) | 1 |
| 51) | 3 | 52) | 4 | 53) | 2 | 54) | 4 | 55) | 64 |
| 56) | 8 | 57) | 8 | 58) | 5 | 59) | 5 | 60) | 6 |

## MATHEMATICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 61) | 2 | 62) | 3 | 63) | 2 | 64) | 1 | 65) | 1 |
| 66) | 3 | 67) | 4 | 68) | 3 | 69) | 1 | 70) | 2 |
| 71) | 2 | 72) | 4 | 73) | 3 | 74) | 4 | 75) | 3 |
| 76) | 4 | 77) | 1 | 78) | 1 | 79) | 3 | 80) | 1 |
| 81) | 1399 | 82) | 27 | 83) | 769 | 84) | 7 | 85) | 7 |
| 86) | 3 | 87) | 9 | 88) | 16 | 89) | 8 | 90) | 2 |

# SOLUTIONS

# PHYSICS

**1.** $W_1 = mg - Vd_a g$

$W_2 = mg - V'd'_a g = mg - V(1+50\gamma_b)\dfrac{d_a g}{(1+50\gamma a)}$

$= mg - Vd_{ag}\left[\dfrac{1+50\gamma_b}{1+50\gamma_a}\right]$

Given $\gamma_b < \gamma_a$

$\therefore \quad 1+50\ \gamma_b < 1+50\gamma_a \quad or, \ \dfrac{1+50\gamma_b}{1+50\gamma_a} < 1$

$\therefore \quad W_2 > W_1 \quad or \quad W_1 < W_2$

**2.** $M_1 C_{ice} \times (10) + M_1 L = M_2 C_\omega (50)$

$\Rightarrow \quad M_1 \times C_{ice}(=0.5) \times 10 + M_1 L = M_2 \times 1 \times 50$

$\Rightarrow \quad L = \dfrac{50M_2}{M_1} - 5$

**3.** From (i) Stefan – Boltzmann law, $P = \sigma A T^4$ and (ii) Wein's displacement law $= \lambda_m \times T =$ constant

$$\frac{P_A}{P_B} = \frac{A_A}{A_B}\frac{T_A^4}{T_B^4} = \frac{A_A}{A_B} \times \frac{\lambda_B^4}{\lambda_A^4}$$

$$\therefore \quad \frac{\lambda_A}{\lambda_B} = \left[\frac{A_A}{A_B} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{R_A^2}{R_B^2} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{400 \times 400}{10^4}\right]^{\frac{1}{4}}$$

$\therefore \quad \dfrac{\lambda_A}{\lambda_B} = 2$

**4.** From Newton's Law of cooling,

$$\frac{T_1 - T_2}{t} = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

Here, $T_1 = 50^0 C, T_2 = 40^0 C$

An $T_0 = 20^0 C, t = 600S = 5$ minutes

$\Rightarrow \quad \dfrac{50-40}{5Min} = K\left(\dfrac{50+40}{2} - 20\right)$ …………… (i)

Let T be the temperature of sphere after next 5 minutes. Then

$\dfrac{40-T}{5} = K\left(\dfrac{40+T}{2} - 20\right)$ ………… (ii)

Dividing eqn. (ii) by (i), we get

$$\frac{40-T}{10}=\frac{40+T-40}{50+40-40}=\frac{T}{50}$$

$$\Rightarrow \quad 40-T=\frac{T}{5}\Rightarrow 200-5T=T \therefore T=\frac{200}{6}=33.3^0 C$$

**5.** Efficiency, $\eta=\frac{Work\ done}{Heat\ absorbed}=\frac{W}{\Sigma Q}$

$$=\frac{Q_1+Q_2+Q_3+Q_4}{Q_1+Q_3}=0.5$$

Here, $Q_1=1915J, Q_2=-40J$ and $Q_3=125J$

$$\therefore \quad \frac{1915-40+125+Q_A}{1915+125}=0.5$$

$$\Rightarrow \quad 1915-40+125+Q_4=1020\Rightarrow Q_4=1020-2000$$

$$\Rightarrow \quad Q_4=-Q=-980\ J \Rightarrow Q=980\ J$$

**6.** Focal length of concave lens, $f_2=-\frac{3}{2}f_1$

$f_1$ = focal length of convex lnes.

$$\frac{1}{f}=\frac{1}{f_1}-\frac{1}{f_2}\Rightarrow \frac{1}{30}=\frac{1}{f_1}=\frac{2}{3f_1}$$

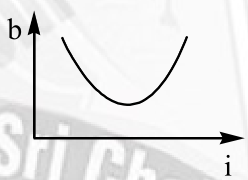
$$\therefore \quad f_1=10\ cm \text{ and } f_2=-\frac{3}{2}\times 10=-15\ cm$$

**7.** For virtual object (u = positive),

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \therefore \quad \frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

If $u<f$, then $v=-ve$ i.e., image may be real.

**8.** For minimum deviation,




And $\quad r_1=r_2$

$$\because \quad A=r_1+r_2 \quad \therefore \quad r_1=r_2=\frac{A}{2}$$

**9.** As light enters from air to glass it suffers a phase change on $\pi$ and therefore at centre there will be destructive interference.

**Statement 1** is true as light enters from air to glass it suffers a phase change on $\pi$ ($\therefore$ rarer to denser propagation)

**Statement 2** is true, the centre of interference pattern is dark, showing that the phase difference between two interfering waves is $\pi$.

**10.** **i)** If for the normal relaxed eye of an average person, the power at the far point be $P_f$.

The required power

$$P_f = \frac{1}{f} = \frac{1}{0.1} + \frac{1}{0.2} = 60D$$

By the corrective lens the object distance at the far point is $\infty$.

The power required is, $P_f' = \frac{1}{f'} = \frac{1}{\infty} + \frac{1}{0.02} = 50D$

Now for eye + lens system, we have the sum of the eye and that of the glasses $P_g$

$$P'_f = P_f + P_g \Rightarrow 50D = 60D + P_g$$

Which gives, $P_g = -10D$

ii) For the normal eye his power of accommodation is 4D. Let the power of the normal eye for near vision be $P_n$.

Then, $4 = P_n - P_f \;\; or \;\; P_n = 64D$

Let his near point be $x_n$, then

$$\frac{1}{x_n} + \frac{1}{0.02} = 64 \;\; or \;\; \frac{1}{x_n} + 50 = 64$$

$$\frac{1}{x_n} = 14 \Rightarrow x_n = \frac{1}{14} m = 0.07\ m$$

**11.** I is the intensity of incident beam ab. The interfering waves are bc and ef, reflected from surface of I and II plate, respectively. Reflection coefficient of intensity,

$r = 25\% = 0.25$

Transmission coefficient of intensity,

$t = 75\% = 0.25$

The intensity of beam $bc, I_1 = 0.25I = \frac{1}{4}I$

The intensity of beam $bd = 0.75I$

The intensity of beam $de = 0.25 \times 0.75I$

The intensity of beam ef,

$$I_2 = 0.75 \times 0.25 \times 0.75I = \frac{9}{64}I$$

Ratio of maximum and minimum intensities.

$$\frac{\sqrt{I_{max}}}{\sqrt{I_{min}}} = \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = 7$$

**12.** $\frac{I_1}{I_2} = \frac{a^2}{b^2} = \beta \;\therefore\; \frac{a}{b} = \sqrt{\beta}$

Fringe visibility is given by

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a+b)^2 - (a-b)^2}{(a+b)^2 + (a-b)^2} = \frac{4ab}{2\left(a^2 + b^2\right)} = \frac{2a/b}{\left(\frac{a^2}{b^2} + 1\right)} = \frac{2\sqrt{\beta}}{\beta + 1}$$

**13.** The extra path travelled by rays reaching $S_2$ is

$$\Delta x = \mu d \sin\theta = \left(10^{-3} \times \frac{1}{2}\right)\frac{4}{3} = 5 \times 10^{-4} \times \frac{4}{3}$$

$$\lambda = 0.4\ mm$$

$$\frac{\Delta x}{\lambda} = \lambda + \frac{2}{3}\lambda \Rightarrow \Delta\phi = \frac{2\pi}{\lambda}\frac{2}{3}\lambda = \frac{4\pi}{3}$$

$$I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$= 4I_0 \cos^2\left(\frac{2\pi}{3}\right) = I_0$$

**14.** $2\mu_0 t = n \times 7500 = (n+1) \times 5000$

$\therefore \quad n = 2 \Rightarrow t = \frac{3}{5}\mu m$

**15.** Refer to the following figure . a ray of light travelling in air $(\mu_1 = 1)$ falls normally on a thin layer $(\mu_2 = 1.8)$ of thickness t. It is partly reflected at point P as wave 1 and partly reflected as wave 2 on meeting the surface of the glass plate $(\mu_3 = 1.5)$ is reflected at point Q and travels along QP.

$\Delta_2 =$ Refractive index of layer $x2(PQ)$

$= \mu_2 \times 2t = 2\mu_2 t$

Optical path difference between waves 1 and 2 at point p is

$$\Delta = \Delta_2 - \Delta_1 = 2\mu_2 t - \frac{\lambda}{2}$$

Now for constructive interference $\Delta = n\lambda, n = 0,1,2.....$

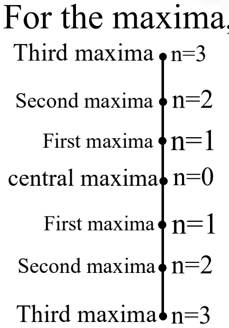
Or $2\mu_2 t - \frac{\lambda}{2} = n\lambda$ or $2\mu_2 t = \left(n + \frac{1}{2}\right)\lambda$

Or $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$

The minimum value of t corresponds to $n = 0$. Hence $t_{\min} = \dfrac{\lambda}{4\lambda_2} = \dfrac{648nm}{4 \times 1.8} = 90nm$

**16.** Here, $\lambda$ and d both are comparable. Hence path difference i.e. $\Delta x = d\sin\theta$

For the maxima, $\Delta x = \pm n\lambda$



$\Rightarrow \quad d\sin\theta = \pm n\lambda \Rightarrow \sin\theta = \pm\frac{2n}{7}$

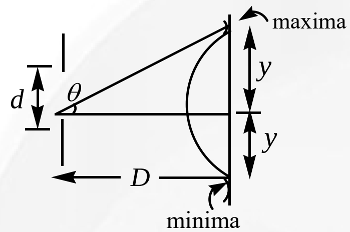
[where only possible values of $n = 0,1,2,3$]

Hence, total maxima = 7

**17.** Given, $2y = 2\times 10^{-3} m$

$d\sin\theta = \lambda$ for first minima

$\sin\theta \approx \tan\theta = \frac{y}{D}$




So, $d\left(\frac{y}{D} = \lambda\right)$

$d = \frac{\lambda D}{y} = \frac{5.89\times 10^{-7}\times 0.5}{1\times 10^{-3}}$

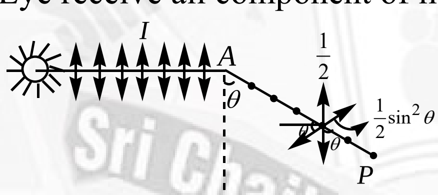
$= 2.945\times 10^{-4} m$

**18.** For 1st minimum $b\sin\theta = \lambda \Rightarrow b\sin 30 = \lambda$ ................(1)

For 1st secondary maximum $b\sin\theta = \frac{3\lambda}{2}$ ...............(2)

$\frac{(1)}{(2)} \Rightarrow \frac{\sin 30}{\sin\theta} = \frac{\lambda}{\frac{3\lambda}{2}} \Rightarrow \sin\theta = \frac{3}{4} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{4}\right)$

**19.** Eye receive all component of light which is along the line. AP




And perpendicular component is $\frac{1}{2}\sin^2\theta$

Net intensity received by light is $\frac{1}{2} + \frac{1}{2}\sin^2\theta$

**20.** Here, $i_p = 60^0, \upsilon = ?$

$\mu = \frac{c}{\upsilon} = \tan i_p = \tan 60^0 = \sqrt{3}$

$\therefore \quad \upsilon = \frac{c}{\sqrt{3}} = \frac{3\times 10^8}{\sqrt{3}} = \sqrt{3}\times 10^8 \ ms^{-1}$

**21.** Heat absorbed by water per min $= 2\times 4200\times 40 = 336000 J$

Now, $8\times10^3 J$ of heat is produced by 1 gm.

So, 336000 J of heat is produced by 1 gm $=\frac{1}{8\times10^3}\times336000gm$

So, rate of combustion = 42 gm/min.

**22.** From first law of thermodynamics

$Q=\Delta U+\Delta W=\Delta U+\frac{Q}{5}\Rightarrow \quad \Delta U=\frac{4Q}{5}\Rightarrow\frac{5R}{2}\Delta T=\frac{4}{5}Q\left[\because \quad \Delta U=\frac{f}{2}nR\Delta T\right]$

Or, $\frac{Q}{\Delta T}=\frac{2}{4}=\frac{25R}{8}$

Therefore molar heat capacity of the gas during the process $C=\frac{Q}{\Delta T}=\frac{25}{8}R\therefore\ x=25$

**23.** ${}_g^m\mu=\frac{{}_g\mu}{{}_m\mu}=\frac{1.5}{4/3}=1.125$

Using $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ [Lens-maker's formula]

$\frac{1}{15}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ ................. (i)

And $\frac{1}{f'}=(1.125-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ ......... (ii)

Diving eq. (i) by (ii)

$$\frac{f'}{15}=\frac{1.5-1}{1.125-1}=\frac{0.5}{0.125}=4$$

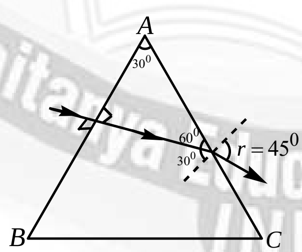
$\therefore \quad f'=60\ cm$

**24.** At face AB of prism ray falls normally using Snell's law for the refraction at AC,

$\mu\sin i=(1)\sin r \quad \sqrt{2}\sin30^0=\sin r\Rightarrow\ r=45^0$

Angle of deviation at face $AC$

$=45^0-30^0=15^0$




**25.** Given: Length of compound microscope, $L=10\ cm$

Focal length of objective $f_0=1\ cm$ and of eye-piece,

$f_e=5\ cm$

$u_0=f_e=5\ cm$

Final image formed at infinity $(\infty), v_e=\infty$

$v_0=10-5=5$

Using lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}\Rightarrow \frac{1}{5}-\frac{1}{u_0}=\frac{1}{1}\Rightarrow u_0=-\frac{5}{4}cm$

Or, $\frac{5}{4}=\frac{N}{40}$ $\therefore$ $N=\frac{200}{4}=50\ cm$.

**26.** In reflected light, $I_1=0.2\ I_0, I_2=0.8\times0.2\times0.8I_0$, using $\frac{I_{\max}}{I_{\min}}=\frac{(a_1-a_2)^2}{(a_1-a_2)^2}, x=81$

similarly in transmitted light, $I_1=0.8\times0.8\ I_0, I_2=0.8\times0.2\times0.2\times0.8I_0$, using

$\frac{I_{\max}}{I_{\min}}=\frac{(a_1+a_2)^2}{(a_1-a_2)^2},\ y=\frac{9}{4}$

**27.** Slit of fringe pattern $=(\mu-1)\frac{tD}{d}$

$\therefore$ $\frac{30D\left(4800\times10^{-10}\right)}{d}=(0.6)t\frac{D}{d}$

$30\times4800\times10^{-10}=0.6$

$t=\frac{30\times4800\times10^{-10}}{0.6}=\frac{1.44\times10^{-5}}{0.6}=2.4\times10^{-5}$

**28.** $N\frac{4\lambda_1 D}{d}=\frac{3\lambda_2 D}{d}$

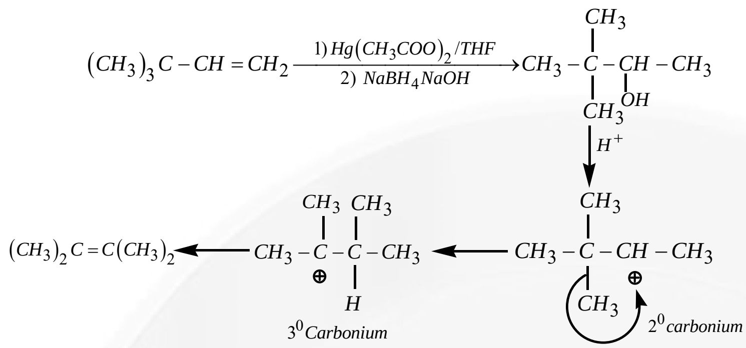
$\frac{\lambda_1}{\lambda_2}=\frac{3}{4}=\frac{6}{8}$

**29.** $I_A/I_B=A_A^2/A_B^2=\left(A^2+4A^2\right)/\left(A^2+4A^2-2*A*2A\right)=5$

**30.** $I=\frac{I_0}{2}\cos^2\theta=\frac{I_0}{2}\cos^2 45=\frac{I_0}{2}\left(\frac{1}{2}\right)=\frac{I_0}{4}$

# CHEMISTRY

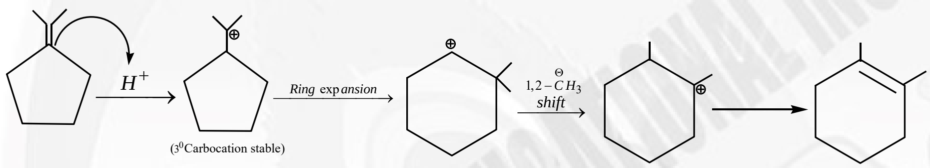
**31.** Properties of geometrical isomers.




**32.**

**33.** $CH_3-C\equiv C-CH_3 \xrightarrow{Na/Liq.NH_3} CH_3CH=CHCH_3$ $(Trans)$

**34.**

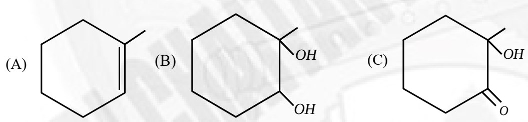



**35.**

$(A)\ Ph-\underset{\substack{|\\ Br}}{CH}-CH_2-Br$ $(B)\ Ph-C\equiv C-H$

$(C)\ Ph-C\equiv C-CH_3$

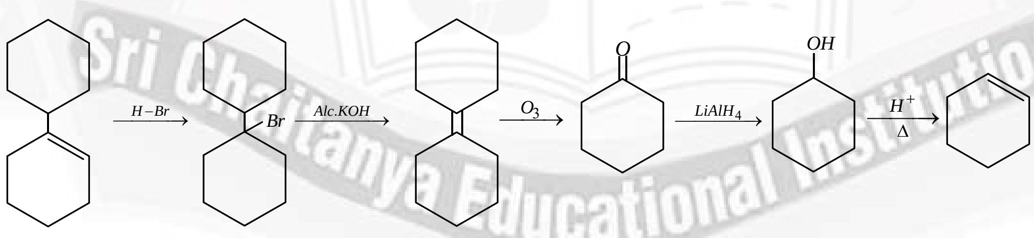
**36.**




**37.**

Anti markovnikoff rule

**38.**




**39.** Most stable alkene less heat of hydrogenation. Heat of hydrogenation $d > a > c > b$

$a \longrightarrow 4\ \alpha\ H$

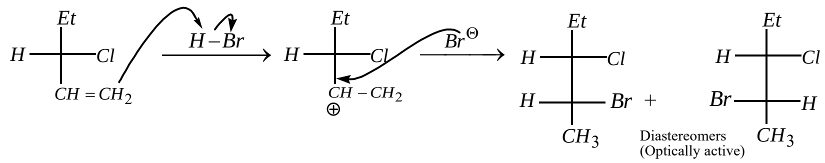
$b \longrightarrow 10\ \alpha\ H$

$c \longrightarrow 6\ \alpha\ H$

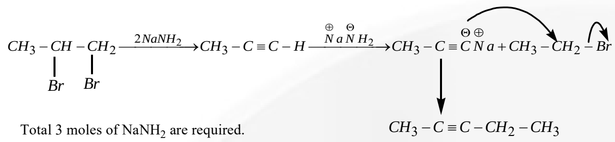
$d \longrightarrow 3\ \alpha\ H$

**40.** Markovnikoff's product.

**41.**

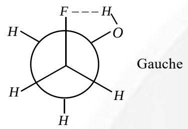


**42.**

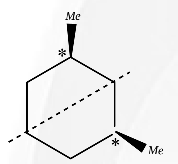


Total 3 moles of $NaNH_2$ are required.

**43.**

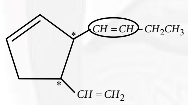


**44.**



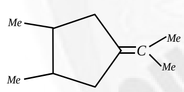
Meso $\rightarrow$ Compounds having chiral centres and have plane or centre of symmetry are known as meso compound.

**45.**



, total isomers $= 2^3 = 8$

**46.**



can show G.I. Along the 5-membered ring.

**47.** CONCEPTUAL

**48.** CONCEPTUAL

**49.** CONCEPTUAL

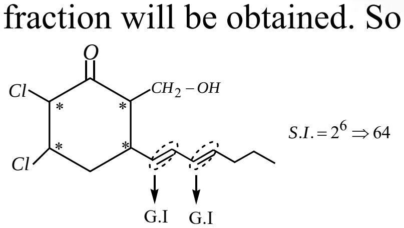
**50.** CONCEPTUAL

**51.** CONCEPTUAL

**52.** a) $C-C-C-C-OH$ b) $C-C-\underset{\large OH}{\underset{|}{C}}-C$ c) $C-\overset{\large C}{\overset{|}{C}}-C-OH$ d) $C-\underset{\large OH}{\underset{|}{\overset{\large C}{\overset{|}{C}}}}-C$

**53.** Benzoic acid and benzene Sulphonic acid are more acidic compounds than $H_2CO_3$ so these are capable to react with $NaHCO_3$ to give a salt and $CO_2$ gas.

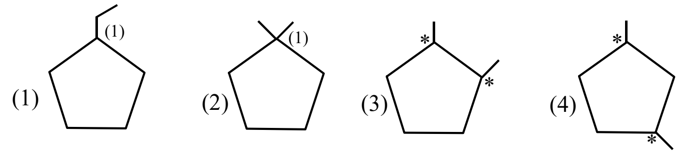
**54.** Value of N is 6, that includes two enantiomeric pairs. On fractional distillation four fraction will be obtained. So answer is 4



**55.**

Compound unsymmetrical.

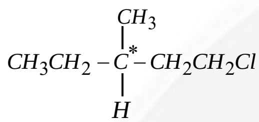
**56.** $[C_7H_{14}]$



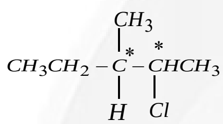
S.I. = 3 (symmetrical) S.I. = 3

Total = 8

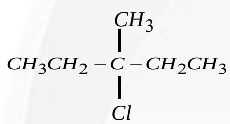
**57.**



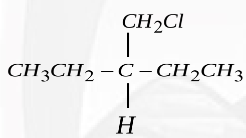
Enantiomeric pair = 2



Two enantiomeric pairs = 4



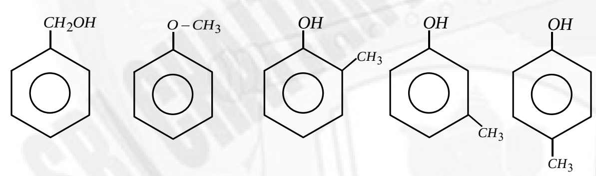
= 1



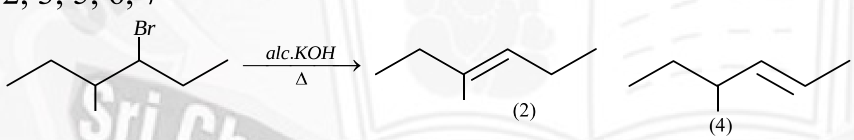
= 1

Total $= 2+4+1+1=8$

**58.**



**59.** 2, 3, 5, 6, 7



**60.**

# MATHEMATICS

**61.** For defined $\log_{(x+1)}(x-2)$

$x-2>0 \Rightarrow x>2$
$x+1>0 \Rightarrow x>-1$
$x+1 \neq 1 \Rightarrow x \neq 0$ and $x>0$

And Denominator

$x^2-2x-3 \neq 0; (x-3)(x+1) \neq 0$

$x \neq -1, 3$

So, domain is $(2,\infty)-\{3\}$

**62.** Let, $f(x_1)=f(x_2) \Rightarrow 2x_1=2x_2 \Rightarrow x_1=x_2$.

Thus, f is one-one

Let $y \in R$ (co-domain) and $f(x)=y \Rightarrow 2x=y$

$\Rightarrow \quad x=\frac{1}{2}y \in \mathbf{R}$, the codomain. That is, for every

$y \in \mathbf{R}$ there exists a real number $\frac{y}{2} \in R$, such that $f\left(\frac{y}{2}\right)=y$. Hence $f$ is onto.

Clearly, $f(2)=f(-2)$ but $2 \neq -2$. Thus, $f$ is not one-one.

Now, $-2 \in R$ (codomain), then $x^2=-2$ has no real solution $\Rightarrow$ $f$ is not onto.

**63.** $\lim\limits_{x\to 2} \dfrac{2^x+2^{3-x}-6}{\sqrt{2^{-x}}-2^{1-x}}$

$$= \lim_{x\to 2} \frac{\left(2^x\right)^2-6\times 2^x+2^3}{\sqrt{2^x}-2} \text{ [Multiplying } N^r \text{ and } D^r \text{ by } 2^x]$$

$$= \lim_{x\to 2} \frac{\left(2^x-4\right)\left(2^x-2\right)\left(\sqrt{2^x}+2\right)}{\left(\sqrt{2^x}-2\right)\left(\sqrt{2^x}+2\right)}$$

$$= \lim_{x\to 2} \frac{\left(2^x-4\right)\left(2^x-2\right)\left(\sqrt{2^x}+2\right)}{\left(2^x-4\right)}$$

$$= \lim_{x\to 2} \left(2^x-2\right)\left(\sqrt{2^x}+2\right)=\left(2^2-2\right)(2+2)=8$$

**64.** $\because \lim\limits_{x\to 0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\frac{x+2}{x^2}}$ is of the form $1^{\infty}$

So, let $y=e^{\lim\limits_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)\times(x+2)}$

Now, $\lim\limits_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$$=\lim_{x\to 0}\frac{\sin x\sqrt{\cos 2x}-\cos x\times\frac{1}{2\sqrt{\cos 2x}}\times(-2\sin 2x)}{2x} \quad \text{(by L' Hospital rule)}$$

$$=\lim_{x\to 0}\frac{\sin x\cos 2x+\sin 2x.\cos x}{2x}=\frac{1}{2}+1=\frac{3}{2}$$

So, $y=e^{\lim\limits_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x}\right)(x+2)}=e^{\frac{3}{2}(0+2)}=e^{\frac{3}{2}\times 2}=e^3$

$\because \lim\limits_{x\to 0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}=e^a \Rightarrow e^3=e^a \Rightarrow a=3$

**65.** **Conceptual**

**66.** Since, $\lim\limits_{x\to 0}\dfrac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{1-\cos(2x)}=17$

$$\Rightarrow \lim_{x\to 0}\frac{ae^{ax}+b\sin(bx)-\frac{c}{2}\left(e^{-cx}-cxe^{-cx}\right)}{2\sin 2x}=17 \quad \text{(by using L.Hospital Rule)}$$

$$\Rightarrow a-\frac{c}{2}=0$$

$$\lim_{x\to 0}\frac{a^2e^{ax}+b^2\cos bx+\frac{c^2}{2}e^{-cx}+\frac{c^2}{2}\left(e^{-cx}-xce^{-x}\right)}{4\cos 2x}=17$$

$$\Rightarrow \frac{a^2+b^2+\frac{c^2}{2}+\frac{c^2}{2}}{4}=17 \Rightarrow 5a^2+b^2=68$$

**67.** $\lim\limits_{x\to\frac{\pi}{2}}\tan^2 x\left[\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right]$

Rationalize the functions apply the limit in the denominator.

$$=\lim_{x\to\frac{\pi}{2}}\frac{\tan^2 x\left[\sin^2 x-3\sin x+2\right]}{\sqrt{9}+\sqrt{9}} \qquad =\lim_{x\to\frac{\pi}{2}}\frac{\tan^2 x(\sin x-1)(\sin x-2)}{6}$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}}\frac{\sin^2 x(1-\sin x)}{(1-\sin x)(1+\sin x)}=\frac{1}{12}$$

$$\lim_{x\to 2}\frac{x^2 f(2)-4f(x)}{x-2}\left[\frac{0}{0}\ form\right]$$

Apply L' Hospital Rule

$$=\lim_{x\to 2}\left(\frac{2xf(2)-4f'(x)}{1}\right)=\frac{4(4)-4}{1}=12$$

**68.** Given function is

$$f(x)=\begin{cases}\dfrac{\ln(1+5x)-\ln(1+\alpha x)}{x}: & x\neq 0\\ 10: & x=0\end{cases}$$

$$\lim_{x\to 0}\frac{\ln(1+5x)-\ln(1+\alpha x)}{x}=10$$

Apply expansion of $\ln(1+x)$.

$$\lim_{x\to 0}\frac{(5x+.....)-(ax+......)}{x}=10$$

$$\lim_{x\to 0}(5-\alpha)=10 \quad 5-\alpha=10\Rightarrow \alpha=-5$$

**69.** $$R.H.L=\lim_{x\to 2^+}f(x)=\lim_{x\to 2^+}e^{\frac{\tan(x-2)}{x-2}}=e^1\left[\because \lim_{x\to 2^+}[x]=2\right]$$

$$L.H.L=\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}\frac{-\lambda(x-2)(x-3)}{\mu(x-2)(x-3)}=-\frac{\lambda}{\mu}$$

Given that $f(x)$ is continuous at $x=2$

$$\therefore \quad \mu=e=-\frac{\lambda}{\mu}\Rightarrow \mu=e,\lambda=-e^2$$

**70.** $$f(x)=\begin{cases}1+\left[\cos\frac{\pi x}{2}\right], & 1<x\leq 2\\ 1-\{x\}, & 0\leq x<1\\ |\sin\pi x|, & -1\leq x<0\end{cases}=\begin{cases}1-1, & 1<x\leq 2\\ 1-x, & 0\leq x<1\\ -\sin\pi x, & -1\leq x<0\end{cases}$$

$f(x)$ is continuous at $x=1$ but not differentiable.

**71.** $$f(x)=\begin{cases}-x & x<1\\ a+\cos^{-1}(x+b) & 1\leq x\leq 2\end{cases}$$

$f(x)$ is continuous

$$\Rightarrow \quad \lim_{x\to 1^-}f(x)=\lim_{x\to 1^+}f(x)=f(x)$$

$\Rightarrow \quad -1 = a + \cos^{-1}(1+b)$

$\cos^{-1}(1+b) = -1 - a$ ................ (i)

$f(x)$ is differentiate

$\Rightarrow \quad LHD = RHD \Rightarrow -1 = \dfrac{-1}{\sqrt{1-(1+b)^2}}$

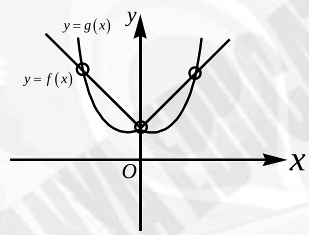
$\Rightarrow \quad 1-(1+b)^2 = 1 \Rightarrow b = -1$ .......... (ii)

From $(i) \Rightarrow \cos^{-1}(0) = -1 - a$ $\quad \therefore \quad -1 - a = \dfrac{\pi}{2}$

$a = -1 - \dfrac{\pi}{2} \Rightarrow a = \dfrac{-\pi-2}{2}$ ........... (iii)

$\therefore \quad \dfrac{a}{b} = \dfrac{\pi+2}{2}$

**72.** $f(x) = \begin{cases} x+1, & x \ge 0 \\ -x+1, & x < 0 \end{cases}$ $\qquad g(x) = x^2 + 1$




From graph, it is clear that there are 3 points at which $h(x)$ is not differentiable.

**73.** $(a^2 - 2a - 15)e^{ax} + (b^2 - 2b - 15)e^{bx} = 0$

Or $(a^2 - 2a - 15) = 0$ and $b^2 - 2b - 15 = 0$

Or $(a-5)(a+3) = 0$ and $(b-5)(b+3) = 0$

i.e., $a = 5$ or $-3$ and $b = 5$ or $-3$

$\therefore \quad a \ne b$

Hence $a = 5$ and $b = -3$ or $a = -3$ and $b = 5$

Or $ab = -15$

**74.** $f(2^+) = 2 + 2\sin(0) = 2$

$f(2^-) = 3 + 2\sin 1$

Hence $f(x)$ is discontinuous at $x = 2$

Also $f(0^+) = 2(0) - 0 - 0\sin(0-0) = 0$

And $f\left(0^{-}\right)=2(0)-(-1)-0\sin\left(0-(-1)\right)=1$

Hence $f(x)$ is discontinuous at $x=0$

**75.** Given function is, $f(x)=4\log_e(x-1)-2x^2+4x+5, x>1$

Differentiate w.r.t 'x'

$f'(x)=\dfrac{4}{x-1}-4(x-1)$ ................$(i)$

Now, check option wise

Take $1<x<2 \Rightarrow f'(x)>0$

Take $x>2 \Rightarrow f'(x)<0$

So, option (a) is correct.

Take $f(x)=-1$.

We have

$\log_e(x-1)^2=(x-3)(x+1)$.

Therefore, it has two solutions

So option (b) is correct.

Take $f(e)>0, f(e+1)<0$

$f(e).f(e+1)<0$

So, option (d) is correct.

Now, put $x=e$ in eq. (i) and again diff. eq. (i).

$f'(e)-f''(2)=\dfrac{4}{e-1}-4(e-1)+8>0$

Therefore, option (c) is incorrect.

**76.** Since, normal of line is parallel to line $x+90y+2=0$ is $m=-\dfrac{1}{90}$

$$\Rightarrow \quad -\left(\frac{dx}{dy}\right)_{(x_1,y_1)}=-\frac{1}{90}\Rightarrow\left(\frac{dy}{dx}\right)_{(x_1y_1)}=90$$

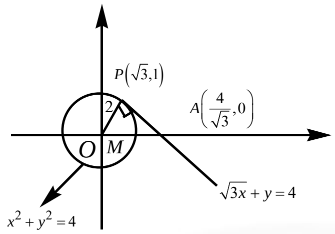
Now, $\dfrac{dy}{dx}=270x^4-540x^3-210x^2+360x+210=90$

After solving we get,

$x=1,2,\dfrac{-2}{3},\dfrac{-1}{3}$

There are total 4 normals

**77.** Equation of tangent to circle at point $\left(\sqrt{3},1\right)$ is $\sqrt{3}x+y=4$




Coordinates of the point $A=\left(\frac{4}{\sqrt{3}},0\right)$

Area $=\frac{1}{2}\times OA\times PM=\frac{1}{2}\times\frac{4}{\sqrt{3}}\times 1=\frac{2}{\sqrt{3}}$ sq. units

**78.** Since, given $\ell_1+\ell_2=20\Rightarrow\frac{d\ell_2}{d\ell_1}=-1$

Now, $A_1=\left(\frac{\ell_1}{4}\right)^2$ and $A_2=\pi\left(\frac{\ell_2}{2\pi}\right)^2$

Let $S=2A_1+3A_2=\frac{\ell_1^2}{8}+\frac{3\ell_2^2}{4\pi}$

For max or min

$\frac{ds}{d\ell_1}=0\Rightarrow\frac{2\ell 1}{8}+\frac{6\ell_2}{4\pi}.\frac{d\ell_2}{d\ell_1}=0$

$\Rightarrow\quad\frac{\ell_1}{4}=\frac{6\ell_2}{4\pi}\Rightarrow\frac{\pi\ell_1}{\ell_2}=6$

**79.** Let $f(x)=ax^3+bx^2+cx+d$

$\Rightarrow\quad f'(x)=3ax^2+2bx+c\Rightarrow f''(x)=6ax+2b$

$\Rightarrow\quad f''(-1)=0\Rightarrow -6a+2b=0\Rightarrow b=3a$

$\Rightarrow\quad f'(1)=0\Rightarrow 3a+6a+c=0\Rightarrow c=-9a$

$\Rightarrow\quad f(1)=-10\Rightarrow -5a+d=-10$ .......... (i)

$\Rightarrow\quad f(-1)=6\Rightarrow 11a+d=6$ ......... (ii)

Subtract, (ii) from (i),

We get $a=1,d=-5,b=3,c=-9$

Then $f(x)=x^3+3x^2-9x-5$

So, $f(2)=8+12-18-5=-3$

**80.** $y(x)=ax^3+bx^2+cx+5$ is passing through

$(-2,0)$ then $8a-4b+2c=5$ .......... (i)

$y'(x)=3ax^2+2bx+c$ touches $x-axis$ at $(-2,0)$

$12a-4b+c=0$ ............ (ii)

Acc. To question

$y(-2)=0$

$y'(-2)=0$

$y'(0)=3$

Again, for $x=0, y'(x)=3 \Rightarrow c=3$ .......... (iii)

Solving eqs (i), (ii) & (iii) $a=-\frac{1}{2}, b=-\frac{3}{4}$

$\Rightarrow \quad y'(x)=3ax^2+2bx+c$

Or $y'(x)=-\frac{3}{2}x^2-\frac{3}{2}x+3$

$y(x)$ has local maxima at $x=1 \Rightarrow y(1)=\frac{27}{4}$

**81.** Let $f(n)=\left[\frac{1}{3}+\frac{3n}{100}\right]n$

Where $[n]$ is greatest integer function $=\left[0.33+\frac{3n}{100}\right]n$

For $n=1,2,......,22,$ we get $f(n)=0$

And for $n=23,24,.....,55,$ we get $f(n)=1\times n$

For $n=56, f(n)=2\times n$

So, $\sum_{n=1}^{56} f(n)=1(23)+1(24)+....+1(55)+2(56)$

$=(23+24+......+55)+112=\frac{33}{2}[46+32]+112$

$=\frac{33}{2}(78)+112=1399.$

**82.** $\lim_{x\to\infty} \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$

$$\lim_{x\to\infty} x^3\times\left\{\frac{x^3\left\{\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right\}}{x^6\left\{\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right\}}\right\}$$

$=\frac{\left(2\sqrt{3}\right)^6+0}{2^6+0}=3^3=(27)$

**83.** $f(x)=\sin\left(\cos^{-1}\left(\frac{1-(2^x)^2}{1+(2^x)^2}\right)\right)$

$=\sin\left(2\tan^{-1}2^x\right)\left[\because\ 2\tan^{-1}\alpha=\cos^{-1}\left(\frac{1-\alpha^2}{1+\alpha^2}\right)\right]$

$f'(x)=\cos\left(2\tan^{-1}2^x\right).2.\frac{1}{1+(2^x)^2}\times 2^x.\log_e 2$

$f'(1)=\cos\left(2\tan^{-1}2\right)\frac{2}{1+4}\times 2\times\log_e 2$

$\Rightarrow\quad f'(1)=\cos\cos^{-1}\left(\frac{1-2^2}{1+2^2}\right).\frac{4}{5}\log_e 2\qquad =-\frac{12}{25}\log_e 2$

Now, compare the above with $-\frac{b}{a}\log_e 2$,

$\Rightarrow\quad a=25, b=12\ \therefore\quad \left|a^2+b^2\right|=\left|625+144\right|=769$

**84.** $x=t^2; y=t^3$

$\frac{dx}{dt}=2t;\frac{dy}{dt}=3t^2$

$\frac{dy}{dx}=\frac{3t}{2}$

Equation of tangent at P is

$y-t^3=\frac{3t}{2}\left(x-t^2\right)$

$2k-2t^3=3th-3t^3$

$\therefore\quad t^3-3th+2k=0\quad (1)$

Product of roots, $t_1t_2t_3=-2k$

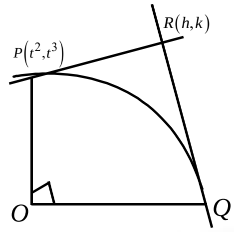
Putting $t_1t_2=-1, t_3=2k$.

Now, $t_3$ must satisfy equation (1). Therefore,

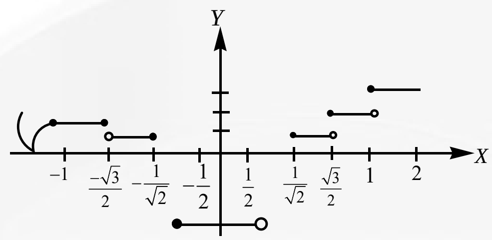
$(2k)^3-3(2k)h+2k=0$

I.e., $4y^2-3x+1=0$ or $4y^2=3x-1$

Or $a+b=7$




**85.** Let's draw the graph of the given function



Here, there are 7 points $\pm\frac{1}{2},\pm\frac{1}{\sqrt{2}},\pm\frac{\sqrt{3}}{2}$ and 1 at which function is discontinuous.

**86.** The dimensions of the box after cutting equal squares of side $x$ on the corner will be $21-2x, 16-2x,$ and height $x$.

$V = x(21-2x)(16-2x)$

$= x\left(336-75x+4x^2\right) = 4x^3+336x-74x^2$

$\therefore\ \frac{dV}{dx} = 12x^2+336-148x$

$\frac{dV}{dx}=0$ gives $x=3$ for which $\frac{d^2V}{dx^2}$ is $-ve$ and, hence, maximum.

**87.** Let 'r' be the radius of spherical balloon and S is Surface area. $S=4\pi r^2$

Differentiate both sides w.r.t. 't'.

$\frac{dS}{dt} = 8\pi r \times \frac{dr}{dt} = k$ (constant)

Take integral both sides,

$4\pi r^2 = kt + C$ (C is constant of integration) ............ (i)

Put the values of $'t'(\&'r')$ in equation (i).

At $t=0, r=3 \Rightarrow 36\pi = C$; At $t=5, r=7 \Rightarrow k=32\pi$

Put the values of C and k in eq. (i)

$4\pi r^2 = 32\pi t + 36\pi \Rightarrow r^2 = 8t+9$; Put $t=9; r^2=81 \Rightarrow r=9$

**88.** $y^4 = x$ ....... (i)

And $xy = k$ ...... (ii)

On solving equations (i) and (ii), we get

Point of intersection is $\left(k^{\frac{4}{5}}, k^{\frac{1}{5}}\right)$

Diff. (i) w.r. to $x$. Now, $m_1 = \frac{dy}{dx} = \frac{1}{4y^3} = \frac{1}{4k^{3/5}}$

Diff. (ii) w.r to $x$ $\quad m_2 = \frac{dy}{dx} = -\frac{y}{x} = -\frac{1}{k^{3/5}}$

Since curve intersect at right – angle:

$\therefore \quad m_1.m_2 = -1 \Rightarrow \frac{1}{4k^{6/5}} = 1 \Rightarrow 4k^{6/5} = 1$.

So, $(4k)^{12} = 16$.

**89.** $f'(x) = x^2 + 2b + ax = 0$ ..........$(i)$

$g'(x) = x^2 + a + 2bx = 0$ ………….. (ii)

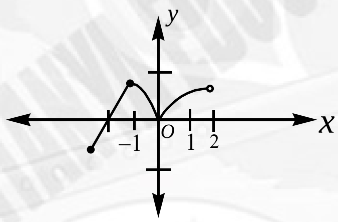
$(i)-(ii)$, we get

$(2b-a) - x(2b-a) = 0 \;\therefore x = 1$ si the common root

Put $x = 1$ in $f'(x) = 0$ or $g'(x) = 0$

$\Rightarrow \quad 1 + 2b + a = 0 \quad or \quad 9 + 2b + a = 8$

**90.** Given function : $f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$




The graph of $y = f(x)$ is as shown in the figure. From graph, clearly, there is one local maximum at $x = -1$ and one local minima at $x = 0$

$\therefore$ Total number of local maxima or minima $= 2$.