Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_STERLING&NUCLEUS_BT **JEE-MAIN** **Date: 05-08-2023**

Time: 09.00Am to 12.00Pm **RPTM-01** **Max. Marks: 300**

KEY SHEET

PHYSICS

1)	2	2)	1	3)	2	4)	3	5)	2
6)	1	7)	3	8)	2	9)	2	10)	3
11)	3	12)	3	13)	1	14)	1	15)	2
16)	3	17)	4	18)	4	19)	2	20)	1
21)	5	22)	10	23)	0	24)	389	25)	750
26)	80	27)	160	28)	30	29)	90	30)	2

CHEMISTRY

31)	2	32)	2	33)	1	34)	3	35)	2
36)	1	37)	2	38)	4	39)	4	40)	4
41)	4	42)	2	43)	4	44)	3	45)	4
46)	2	47)	3	48)	3	49)	2	50)	2
51)	6	52)	4	53)	3	54)	4	55)	7
56)	4	57)	4	58)	5	59)	8	60)	4

MATHEMATICS

61)	3	62)	2	63)	2	64)	3	65)	3
66)	4	67)	2	68)	3	69)	3	70)	3
71)	1	72)	3	73)	3	74)	1	75)	3
76)	4	77)	3	78)	2	79)	4	80)	1
81)	4	82)	2	83)	8	84)	11	85)	8
86)	7	87)	5	88)	0	89)	1	90)	4

SOLUTIONS

PHYSICS

1. If the sheet is heated then both d_1 and d_2 will increase since the thermal expansion of isotropic solid is similar to true photographic enlargement.

2. Isothermal process, T = constant $\Rightarrow dU = 0$

   Adiabatic process, $dQ = 0 \Rightarrow dU + dW = 0$

   Isochoric process, $dV = 0,\ dW = 0 \Rightarrow dQ = dU$

   Isobaric process, P = constant

3. Wien's law $\lambda_m \propto \frac{1}{T}$ *or* $V_m \propto T$

   v_m increases with temperature. So the graph will be straight line.

4. According to Wien's law $\lambda_0 T_0 = \lambda T$

   According to Stefan's law $\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^4$

   As $P = \frac{256}{81} P_0 \Rightarrow \lambda = \frac{3}{4}\lambda_0$

   $\therefore$ Wavelength shift $\Delta\lambda = \lambda - \lambda_0 = -\frac{\lambda_0}{4}$

5. Let V be volume of either liquid
   Mass of water $= V \times 1\text{g}$
   Mass of alcohol $= V \times 0{,}8 = 0.8V\ \text{g}$
   Rate of cooling of the water calorimeter
   $= \frac{1}{100}\left[V \times (50^\circ - 40^\circ) + V \times 1 \times (50^\circ - 40^\circ)\right]$
   $= (1/5)\text{Vcal/s}$
   Rate of cooling of alcohol calorimeter
   $= \frac{1}{74}\left[\text{V} \times (50^\circ - 40^\circ) + 0.5\text{V} \times s(50^\circ - 40^\circ)\right]$
   $= (1/74)(10\,\text{V} + 8\,\text{V}s)\text{cal/s}$
   As, rate of cooling of both is same
   $5V = (1/74)(10\,\text{V} + 8Vs)$
   $s = 0.6\text{cal/g}^\circ\text{C}$

6. According to Newton's law of cooling,
   $\left[\frac{\theta_1 - \theta_2}{t}\right] = K\left[\left(\frac{\theta_1 + \theta_2}{2}\right) - \theta_0\right]$

   So that $\left[\frac{60-40}{7}\right] = K\left[\left(\frac{60+40}{2}\right) - 10\right]$

   $\Rightarrow K = \frac{1}{14}$ (i)

Now if after cooling from $40^{\circ}\mathrm{C}$ to $7\,\mathrm{min}$ the temperature of the body becomes θ, according to Newton's law of cooling, $\left[\frac{40-\theta}{7}\right]=K\left[\left(\frac{40+\theta}{2}\right)-10\right]$

Which in the light of Eq. (i), i.e., $K=(1/14)$, gives $\left[\frac{40-\theta}{7}\right]=\frac{1}{14}\left[\left(\frac{20+\theta}{2}\right)\right]$

$160-4\theta=20+\theta;\theta=28^{\circ}\mathrm{C}$

7. Statements 1 and 2 are true but Statement 2 is not correct explanation for Statement 2.
8. temperature of gas decreases, volume of gas also decreases PV = nRT ;
$P\rightarrow$constant
9. If m is the total mass of the gas, then its kinetic energy $=1/2mv^2$.When the vessel is suddenly stopped, total kinetic energy will increase the temperature of the gas (because process will be adiabatic), i.e.,

$\frac{1}{2}mv^2=\mu C_v\Delta T \quad =\frac{m}{M}C_v\Delta T \quad \Rightarrow \frac{m}{M}\frac{R}{\gamma-1}\Delta T=\frac{1}{2}mv^2 \quad \left(\text{As } C_v=\frac{R}{\gamma-1}\right)$

$\Rightarrow \Delta T=\frac{Mv^2(\gamma-1)}{2R}$

10. For isothermal process

$P_1V=P_2'\frac{V}{2}\Rightarrow P_2'=2P_1 \quad$ (i)

For adiabatic process

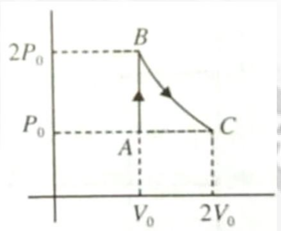
$P_1V^{\gamma}=P_2\left(\frac{V}{2}\right)^{\gamma} \quad \Rightarrow P_2=2^{\gamma}P_1 \quad$ (ii)

Since $\gamma>1, P_2>P_2'$

11. In free expansion pressure outside is zero. So no work done by ideal gas but in real gas work gas may be done against internal force exist between molecules.
12. $Q_{AB}=\Delta U_{AB}+W_{AB}$

$W_{AB}=0$

$\Delta U_{AB}=\frac{f}{2}nR\Delta T$



$\frac{f}{2}(\Delta PV)\Delta U_{AB}=\frac{5}{2}(\Delta PV)$

13. In an adiabatic process, there is no exchange of heat.

i.e., $\Delta Q=0$

$\Delta Q=\Delta U+\Delta W=0 \quad \Rightarrow \Delta U=-\Delta W$

14. $PdV=nC_vdT\Rightarrow\frac{nRT}{V}dV=nC_vdT \quad \frac{dV}{V}=\frac{3}{2}\frac{dT}{T} \quad \therefore V^2=CT^3$, where C is a constant..
15. $K=-\frac{\Delta p}{\Delta v/V}$ & $Vp^n=cont\Rightarrow \log V+n\log\ p=\log\ cont$

$=\frac{\Delta v}{V}=-n\frac{\Delta p}{p}$ $\quad\quad \therefore K=\frac{p}{n}$

16. $nc\Delta T=Q$ $\quad\quad W=\frac{Q}{4}$

So $\Delta U=nC_V\Delta T=\frac{3Q}{4}\Rightarrow\frac{C}{C_V}=\frac{4}{3}$ $\quad\quad C=\frac{4}{3}C_V$

17. A-PQ B-QRS C-PQRS D-PQRS

Root mean square speed of molecules $=\sqrt{\frac{3\ RT}{M}}=1.732\sqrt{\frac{RT}{M}}$

Most probable speed of molecues $=\sqrt{\frac{2\ RT}{M}}=1.44\sqrt{\frac{RT}{M}}$

Average velocity of a molecule is zero

Speed of any individual molecule may be anything.

18. At any temperature , other than zero Kelvin a black body emits all wavelengths. This is known as total radiation

19. Wien's law is valid for low wavelength region. Hence the part QA of curve represents Wien's law.

20. Since volume if constant and pressure varies linearly with temperature so we obtain the 1st graph.

21. Let the mass of the bullet = m.

Heat required to take the bullet from $27°C$ to $327°C$

$$= m\times(125\ J/kg-K)(300\,K)$$

$$= m\times(3.75\times10^4\ J/kg)$$

Heat required to melt the bullet

$$= m\times(2.5\times10^4\ J/kg)$$

If the initial speed be v, the kinetic energy is $\frac{1}{2}mv^2$ and hence the heat developed is

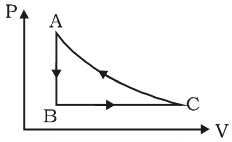
$$\frac{1}{2}\left(\frac{1}{2}mv^2\right)=\frac{1}{4}mv^2.\ \text{Thus,}$$

$$\frac{1}{4}mv^2=m(3.75+2.5)\times10^4\ J/kg$$

or, $\quad\quad$ v = 500 m/s

22. The change in internal energy during the cyclic process is zero. Hence, the heat supplied to the gas is equal to the work done by it. Hence,

$\Delta Q = W_{AB} + W_{BC} + W_{CA}$ .....
(i)



The work done during the process AB is zero

$W_{BC} = P_B(V_C - V_B)$

$= nR(T_C - T_B)$

$= (3\ \text{mol})(25/3\ \text{J/mol} - \text{K})(500\ \text{K})$

$= 12500$ J

As $W_{CA} = -2500$ J (given)

$\therefore$ $\Delta Q = 0 + 12500 - 2500$ [from ..... (i)]

$\Delta Q = 10$ kJ

23. In this case rod rests on a horizontal base which is the free expansion on heating. Hence no strain is developed in the rod i.e.
Strain = 0

24 The efficiency of cycle is $\eta = 1 - \frac{T_2}{T_1}$

for adiabatic process $TV^{\gamma-1}$ = constant, For a diatomic gas, $\gamma = 7/5$.
We know that

$$P = \frac{1}{3}\rho \bar{v}^2$$

$$\therefore \quad v_{rms} = \left(\frac{3P}{\rho}\right)^{1/2}$$

Given that $\rho = 1.98\ \text{kg/m}^3$ and

$$P = 1.0 \times 10^5\ \text{N/m}^2$$

$$\therefore \quad v_{rms} = \sqrt{\frac{3 \times 1.0 \times 10^5}{1.98}}$$

$\therefore$ $v_{rms} = 389$ m/s

25. For 1st case,

Efficiency $= \eta = \left(1 - \frac{T_1}{T_2}\right) \times 100$

$\left(1 - \frac{T_1}{500}\right) \times 100 = 40$

$T_1 = 300\text{K}$

For 2nd case, $\eta = \left(1 - \frac{300}{T_2}\right) \times 100 = 60$ $T_2 = 750\text{K}$

26. $U \propto \sqrt{V}$

As $U \propto T$

$\therefore$ $T \propto V^{1/2}$

Or $TV^{-1/2} =$ constant

or $PV^{1/2} =$ constant

Comparing with $PV^x =$ constant, we have $x = \dfrac{1}{2}$ $\therefore$ Molar specific heat

$C = \dfrac{R}{\gamma - 1} + \dfrac{R}{1-x}$ $= \dfrac{R}{7/5-1} + \dfrac{R}{1-1/2}$ $(\gamma = 7/5)$ $= \dfrac{5}{2}R + 2R = \dfrac{9R}{2}$ $Q = nC\Delta T$

$\Delta U = nC_V \Delta T$ $\therefore W = Q - \Delta U = n(C - C_V)\Delta T$

$\dfrac{W}{\Delta U} = \dfrac{C - C_V}{C_V}$ $\therefore W = \left(\dfrac{C - C_V}{C_V}\right)\Delta U = \left(\dfrac{9/2 - 5/2}{5/2}\right)(100) = 80\,\text{J}$

27. In 1 sec, molecules make 500 hit in a cubical vessel of side 1m.Therefore $v_{rms} = 1000\,m/s$ Because between two successive collisions a molecule will travel 2m.

Using $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ $\therefore$ $T = \dfrac{Mv_{rms}^2}{3R} = \dfrac{(4\times10^{-3})(10^3)^2}{3\times 8.31} = 160K$

28. From $\Delta l = l\alpha\Delta\theta$ we have, $0.05 = 25\alpha_A(100)$

$\alpha_A = 0.00002$ per $^\circ$C

$0.04 = 40\alpha_B(100)$

$\alpha_B = 0.00001$ per $^\circ$C

In third case let l is the length of rod A. Then length of rod B will be $(50 - l)$cm

$\Delta l = \Delta l_1 + \Delta l_2$ or $0.03 = l(0.00002)(50) + (50 - l)(0.00001)(50)$

Solving we get $l = 10\,\text{cm}$ and $50 - l = 40\,\text{cm}$

29. Let heat capacity of flask be C and latent heat of fusion of ice L.

Then, $C(70-40) + 200\times1\times(70-40) = 50L + 50\times1\times(40-0)$

or $3C - 5L = -400$ ....(i)

Further, $C(40-10) + 250\times1\times(40-10) = 80L + 80\times1\times(10-0)$

or $3C - 8L = -670$ ....(ii)

Solving Eq. (i) and (ii), we have, $L = 90\,\text{cal/g}$

30. $\eta = \dfrac{\text{Wnet}}{\text{Total heat supplied}}$

$= \dfrac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_4} = \dfrac{2000}{10000}\times 100$

$= 20\%$ so x = 2

CHEMISTRY

31. It has 1 chiral carban
32. Conceptual
33. It has 3- stereogenic centres-terminal groups are same

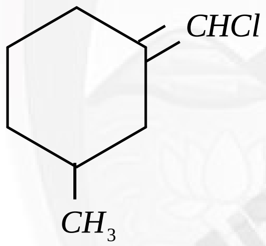
    No of stereoisomers $=2^{n-1}+2^{\left(\frac{n-1}{2}\right)}$

    Both cis trans isomers have 2 enantiomers each
34. Both CH_3 groups at equatorial position.
35. n=2 terminal groups are same

    No of stereoisomers= $=2^{n-1}+2^{\frac{n}{2}-1}$

36. (2s,3s)

$$\begin{array}{c} CH_3 \\ {}^{1}| \\ H-C-OH \\ {}^{2}| \\ H-C-CH_3 \\ {}^{3}| \\ \mathrm{OH} \end{array}$$

37. Due to intramolecular hydrogen bond
38.  exhibits (E & Z) isomerism.
39. Conceptual
40. Conceptual
41. Conceptual
42.

$$\begin{array}{ccccccc} & & & CH_3 & & \mathrm{O} & \\ & & & | & & \| & \\ CH_3- & CH_2- & & CH- & & C- & OC_2H_5 \\ 4 & 3 & & 2 & & 1 & \end{array}$$

    Ethyl-2-methyk-butanoate
43. No of sp^3 hybrid orbitals=8

    No of sp^2 hybrid orbitals=6

    No. of sp hybrid orbitals=6
44. All trans isomers not have zero dipolement
45. I)

$$\begin{array}{c} \mathrm{O} \\ \| \\ CH_3-C-COOH \end{array}$$

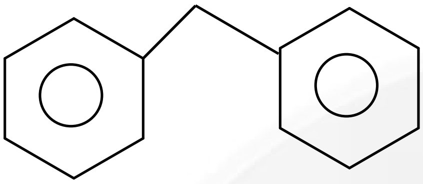
    II)

$$\begin{array}{c} \mathrm{H} \\ | \\ CH_3-C-COOH \\ | \\ \mathrm{OH} \end{array}$$

III) $CH_3-CH_2-COOH \,\&\, CH_3-\overset{\overset{O}{||}}{C}-O-CH_3$

46. Conceptual

47. Conceptual

48. 

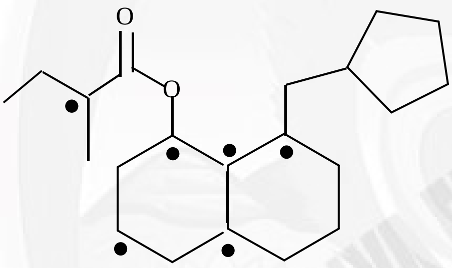
49. I) $CH_3-\overset{\overset{O}{||}}{C}-NH_2$

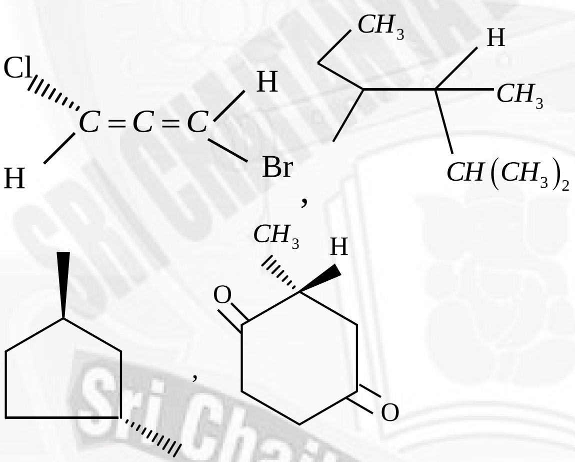
II)Conceptual

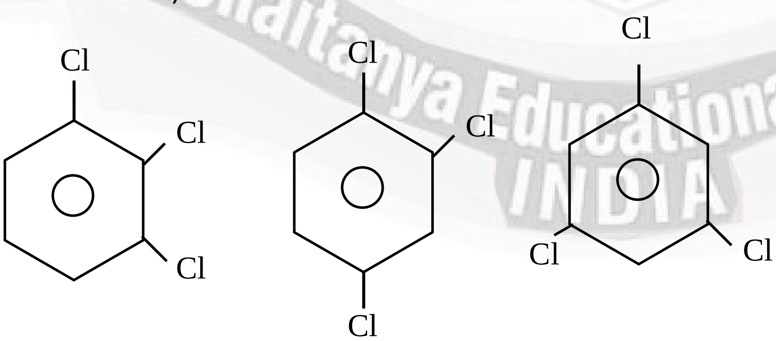
III)Conceptual

50. Conceptual

51.



52. 

53. 

54. Conceptual

55. $$\begin{array}{c} \quad\;\; H \\ \quad\;\; | \\ CH_3-C-C_3H_7 \\ \quad\;\; | \\ \quad\;\; C_2H_5 \end{array}$$

56. $$\begin{array}{c} \quad\;\; CH_3 \\ \quad\;\; | \\ CH_3-C-CH_3 \\ \quad\;\; | \\ \quad\;\; CH_3 \end{array}$$

57. 3° amine, amide, alcohol,phenyl

58. $$H-C\equiv C-\overset{H}{\overset{|}{C}}=C=\overset{H}{\overset{|}{C}}-C\equiv C-H$$

59. $CH_3-CH_2-CH_2-CH_2-CH_2-OH$

$$CH_3-CH_2-CH_2-\overset{OH}{\overset{|}{CH}}-CH_3$$

$$CH_3-CH_2-\overset{OH}{\overset{|}{CH}}-CH_2-CH_3$$

$$CH_3-CH_2-\overset{CH_3}{\overset{|}{CH}}-CH_2-OH$$

$$CH_3-\overset{CH_3}{\overset{|}{CH}}-CH_2-CH_2-OH$$

$$CH_3-\underset{CH_3}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_2-OH$$

$$CH_3-CH_2-\underset{OH}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-CH_3$$

$$CH_3-\overset{OH}{\overset{|}{CH}}-\overset{CH_3}{\overset{|}{CH}}-CH_3$$

60. Conceptual

MATHEMATICS

61.

	Case I	Case II	Case III
$f(x)=1$	T	F	F
$f(y)\neq 1$	F	T	F
$f(z)\neq 2$	F	F	T

Case I $f(y)=1, f(z)=2$, then $f(x)=1$ (not true)
Case II $f(z)=2, f(y)\neq 1, f(x)\neq 1$ (not possible)
Case III $f(y)=1, f(z)=3, f(x)=2$ $\therefore f(y)=1$

62. We have,
$f(x)=x^2+x-1$
$f'(x)=2x+1$
which is one-one, when $x>-\frac{1}{2}$.
In $[0,3]$, $f(x)$ is one-one and its range is $[f(0), f(3)]=[1,13]$ which is codomain.
Hence, its invertible and its inverse is $\frac{-1+\sqrt{4x-3}}{2}$ $\therefore$ Statement I is correct.
and Statement II is also correct but not correct explanation of Statement I.

63. $\dfrac{\frac{1}{|\sin x|}+\frac{1}{|\cos x|}}{2}\geq\sqrt{\frac{1}{|\sin x\cdot\cos x|}}\Rightarrow\frac{1}{|\sin x|}+\frac{1}{|\cos x|}\geq 2\sqrt{2}$

64. $\underset{n\to\infty}{\mathrm{Lt}}\left(\frac{x+2x+-+nx-([x]+[2x]+-+[nx])}{n^2}\right)=\frac{x}{2}-\frac{x}{2}=0$

65. Taking log on b.s And differentiating 2 times we get $-\left(\frac{1}{2^{2n}}\right)\cos ec^2\left(\frac{x}{2^n}\right)+\cos ec^2 x$

66. $\text{Limit}=\lim_{h\to 0}\frac{2-\sqrt[3]{8+h}}{2h\cdot\sqrt[3]{8+h}}=\lim_{h\to 0}\frac{8-(8+h)}{2h\cdot\sqrt[3]{8+h}\left\{8^{2/3}+8^{1/3}\cdot(8+h)^{1/3}+(8+h)^{2/3}\right\}}=-\frac{1}{48}$

67. 3 does not belongs to the range of $f\Rightarrow 2$ also does not belongs to the range of f for otherwise $f(x)$ for some $x\in R$. If $f(x+2p)=f(x)$, then

$$f(x+2p)=f(x+p+p)=\frac{\frac{f(x)-5}{f(x)-3}-5}{\frac{f(x)-5}{f(x)-3}-3}=\frac{-4f(x)+10}{-2f(x)+4}=\frac{2f(x)-5}{f(x)-2}$$

So that $(f(x)-2)^2=-1$ which is absured.
Hence $f(x+2p)\neq f(x)$, Similarly $f(x+3p)\neq f(x)$

$$f(x+4p)=\frac{f(x+3p)-5}{f(x+3p)-3}=\frac{\left(\frac{3f(x)-5}{f(x)-1}-5\right)}{\left(\frac{3f(x)-5}{f(x)-1}-3\right)}=\frac{-2f(x)}{-2}=f(x)$$

68. $f(x)=g(x)\forall x\Rightarrow f(-3)=K\Rightarrow K=-6$

69. Let $f(x)=ax^3-bx-(\tan x)\mathrm{sgn}(x)$ & $f(-x)=f(x)\Rightarrow 2\left(ax^2-b\right)x=0\forall x\in R$

$\therefore a=0\ \&\ b=0 \Rightarrow [k]^2-5[k]+4=0\ \&\ 6\{k\}^2-5\{k\}+1=0$

$\Rightarrow k=1,4\ \&\ \{k\}=\frac{1}{3},\frac{1}{2} \qquad \therefore k=1+\frac{1}{3},1+\frac{1}{2},4+\frac{1}{3},4+\frac{1}{2}$

70. $g(-x)=-g(x)\ \&\ f(-x)=f(x) \Rightarrow x^2f(-x)-2f\left(\frac{-1}{x}\right)=g(-x) \Rightarrow g(x)=0$

$\therefore x^2f(x)-2f\left(\frac{1}{x}\right)=0 \Rightarrow \frac{1}{x^2}f\left(\frac{1}{x}\right)-2f(x)=0 \Rightarrow f(x)=0\,\forall x\in R-\{0\}$

71. Limit $=\lim\limits_{x\to 0}\frac{\log_e(1+x)+x^2-x}{x^2}=\lim\limits_{x\to 0}\frac{\left(x-\frac{1}{2}x^2+\frac{1}{3}x^3-\ldots\right)+x^2-x}{x^2}=\frac{1}{2}$

72. Limit $=\lim\limits_{x\to 1}\frac{f(1)g'(x)-f'(x)g(1)}{g'(x)-f'(x)}=\lim\limits_{x\to 1}\frac{2\{g'(x)-f'(x)\}}{g'(x)-f'(x)}=\lim\limits_{x\to 1}2=2$

73. limit $=\lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x^2}{\lambda x+\mu}}\right)^{2x}=\lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x^2}{\lambda x+\mu}}\right)^{\frac{x^2}{\lambda x+\mu}\cdot\frac{2(\lambda x+\mu)}{x}}$

$=\left\{\lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x^2}{\lambda x+\mu}}\right)^{\frac{x^2}{x\to\infty}}\right\} \quad =e^{\lim\limits_{x\to\infty}\left(2\lambda+\frac{2\mu}{x}\right)=e^{2\lambda}} \quad \therefore e^{2\lambda}=e^2 \quad \therefore \lambda=1$

74. $\lim\limits_{n\to\infty}f\left(\frac{n}{\sqrt{9n^2+1}}\right)=f\left\{\lim\limits_{x\to\infty}\frac{n}{\sqrt{9n^2+1}}\right\}=f\left(\frac{1}{3}\right)=1$

75. $f(1+0)=\lim\limits_{h\to 0}\left\{\left[(1+h)^2\right]-[1+h]^2\right\}=\lim\limits_{h\to 0}\{1-1\}=0$

$f(1-0)=\lim\limits_{h\to 0}\left\{\left[(1-h)^2\right]-[1-h]^2\right\}=\lim\limits_{h\to 0}\{0-0\}=0$

Also $f(1)=0$, So, $f(x)$ is continuous at $x=1$

76. $f(x)=\frac{x\left(1-e^{\frac{1}{x}}\right)}{1+e^{\frac{1}{x}}}\ x\neq 0,\ f(0)=0$

For $f(x)$ left hand limit = right hand limit = 0

77. When x is not an integer, both the functions [x] and $\cos\left(\frac{2x-1}{2}\right)\pi$ are continuous.

Therefore, $f(x)$ is continuous on all non-integral points.

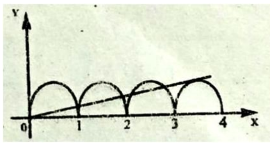
For $x=n\in I$, $\quad \lim\limits_{x\to n-}f(x)=\lim\limits_{x\to n-}[x]\cos\left(\frac{2x-1}{2}\right)\pi \quad =(n-1)\cos\left(\frac{2n-1}{2}\right)\pi=0$

$\lim\limits_{x\to n^+}f(x)=\lim\limits_{x\to n+}[x]\cos\left(\frac{2x-1}{2}\right)\pi \quad =n\cos\left(\frac{2n-1}{2}\right)\pi=0$

Also, $f(n)=n\cos\frac{(2n-1)\pi}{2}=0$

Therefore, f is continuous at all integral points as well. Thus, f is continuous everywhere.

78. $f(x) = \max\left\{\frac{x}{n}, |\sin \pi x|\right\}$

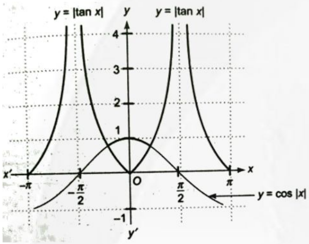


Thus for the maximum points of non differentiability, graphs of

$y = \frac{x}{n}$ and $y = |\sin \pi x$ must intersect at maximum number of points which occurs when $n > 3.5$ Hence, the least value of n is 4

79. $\sin(|x|) - |x| \quad = |x| - \frac{|x|^3}{3!} + \frac{|x|^5}{5!} ...... - |x|,$ Differentiable.

80.



The function is not differentiable and continuous at two points between $x = -\pi/2$ and $x = \pi/2$. Also, the function is not continuous at $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$. Hence, at four points, the function is not differentiable.

82. $g'\left(3^-\right) = \lim\limits_{h \to 0} \frac{g(3-h) - g(3)}{-h} = \lim\limits_{h \to 0} \frac{a\sqrt{4-h} - (3b+2)}{-h}$ (1)

For existence of limit, $\lim\limits_{h \to 0} = 0$

$\therefore 2a - 3b = 2$

Now, $g'\left(3^+\right) = \lim\limits_{h \to 0} \frac{b(3+h) + 2 - (3b+2)}{h} = b$ (2)

Substituting $3b + 2 = 2a$ in equation (1), we get (3)

$g'\left(3^-\right) = \lim\limits_{h \to 0} \frac{a\sqrt{4-h} - 2a}{-h} = \lim\limits_{h \to 0}\left(\frac{(4-h) - 4}{(-h)(\sqrt{4-h} + 2)}\right) = \frac{a}{4}$

Hence, $g'\left(3^-\right) = g'\left(3^+\right)$ $\quad \frac{a}{4} = b$ or $a = 4b$

From equations (2) and (4), $\quad 8b - 3b = 2$ or $b = \frac{2}{5}$ and $a = \frac{8}{5}$ or $a + b = 2$

83. $f\left(x\right) = \lim\limits_{h \to \infty} \frac{f\left(x+h\right) - f\left(x\right)}{h} = \lim\limits_{h \to 0} \frac{f(x) + f(h) + 2xh(x+h) - \frac{1}{3} - \left(f(x) + f(0) - \frac{1}{3}\right)}{h}$

$= \lim\limits_{h \to 0} \frac{f(h) - f(0)}{h} + 2x^2 = f'(0) + 2x^2 \qquad \lim\limits_{h \to 0} \frac{3f(h) - 1}{6h} = \lim\limits_{h \to 0} \frac{f(h) - \frac{1}{3}}{2h} = \lim\limits_{h \to 0} \frac{f(h) - f(0)}{2h}$

$= \frac{f'(0)}{2} = \frac{2}{3} \qquad \therefore f'(0) = \frac{4}{3} \qquad \therefore f(x) = \frac{4}{3} + 2x^2$

$f(x) = \lambda + \frac{4}{3}x + \frac{2x^3}{3}$ or $f(0) = \lambda = \frac{1}{3}$ $\therefore f(x) = \frac{2x^3}{3} + \frac{4}{3}x + \frac{1}{3}$ or $f(2) = \frac{25}{3}$

84. Given function is a constant function.

85. We have $f(x)=[x]+[x+1/3]+[x+2/3]=[3x]$,
which is discontinuous when $3x=k$ or $x=k/3, k\in I$.
Hence, points of discontinuity are $1/3, 2/3, 3/3, 4/3, 5/3, 6/3,\ 7/3, 8/3$.

86. Let $g(x)=(\ln x)(\ln x)....\infty$

$$g(x)=\begin{cases}0, & 1<x<e\\ 1, & x=e\\ \infty, & x>e\end{cases}$$

$$\therefore f(x)=\begin{cases}x, & 1<x<e\\ x/2, & x=e\\ 0, & e<x<3\end{cases}$$

Hence, $f(x)$ is non-differentiable at $x=e$.

87. Put $x=\dfrac{1}{y}$ so that as $x\to\infty \Rightarrow y\to 0$

$$\therefore \lim_{x\to\infty}\left(\frac{y^2+y+1}{y(y+1)}-\frac{a}{y}-b\right)=4 \Rightarrow \lim_{y\to 0}\left(\frac{y^2+y+1-a(y+1)-by(y+1)}{y(y+1)}\right)=4$$

$$\Rightarrow \lim_{y\to 0}\left(\frac{(1-b)y^2+(1-a-b)y+(1-a)}{y(y+1)}\right)=4 \therefore 1-a=0 \text{ or } a=1$$

$$\therefore \lim_{x\to 0}\frac{[(1-b)y+1-a-b]}{y+1} \qquad \therefore -b=4 \text{ or } b=-4$$

88. Let $y=\sqrt{(\tan x-\sin x)+\sqrt{(\tan x-\sin x)+\sqrt{\tan x-\sin x+....\infty}}}$

Let $z=\sqrt{x^3+\sqrt{x^3+\sqrt{x^3+.......\infty}}}$ By simplify we get limit value is 1/2

89. Replace x by $\dfrac{2023}{x}$ in given equation

$$f\left(\frac{2023}{x}\right)+2f(x)=3\left(\frac{2023}{x}\right)$$

$$2f\left(\frac{2003}{x}\right)+4f(x)=6\left(\frac{2023}{x}\right)$$

$$2f\left(\frac{2023}{x}\right)+f(x)=3x$$

$$= \quad 3f(x)=3\left\{\frac{(2023)}{x}-x\right\}$$

$$\therefore f(2)=2023-2=2021.$$

90. $$\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)-\frac{x^3}{2}}{x^n}$$

By using expansions we get $\lim_{x\to 0}\dfrac{\left(\frac{x^3}{2}+\frac{x^4}{2}+\frac{x^5}{12}....\right)-\frac{x^3}{2}}{x^n}=non\ zero$

n = 4