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Central Office, Bangalore

DIFFERENTIAL EQUATIONS

EXERCISE - III

ONE OR MORE CORRECT TYPE QUESTIONS

Order and Degree of Differential Equations :

1. If differential equation is formed to the family of all the central conics centred at origin, then

(A) order = 2 (B) order = 3 (C) degree = 1 (D) degree = order = 3

PRACTICE QUESTIONS

2. The differential equation representing the family of curves $y^2 = 2c\left(x+\sqrt{c}\right)$, where c is a positive parameter, is of **[Adv. 1999]**

A) Order 1 B) Order 2 C) Degree 3 D) Degree 4

Formation of D.E :

3. Consider the family f all circles whose centers lie on the straight line $y = x$. If this family of circle is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and $y'\left(\text{here } y' = \dfrac{dy}{dx}, y'' = \dfrac{d^2y}{dx^2}\right)$, then which of the following statement is (are) true? **[Adv.2015]**

A) $P = y + x$ B) $P = y - x$

C) $P + Q = 1 - x + y + y' + \left(y'\right)^2$ D) $P - Q = x + y - y' - \left(y'\right)^2$

PRACTICE QUESTIONS

4. If solution of $x^2\dfrac{d^2y}{dx^2} - 9x\dfrac{dy}{dx} + 21y = 0$ is of the from $y = c_1x^m + c_2x^n$ (c_1, c_2 are arbitrary constants, $m, n \in N$) then values of m,n can be

A) m = 3, n = 0 B) m = 3, n = 7 C) m = 7, n = 3 D) m = 0, n = 7

Inspection D.E :

5. Solution of the differential equation : $\dfrac{x+y\dfrac{dy}{dx}}{y-x\dfrac{dy}{dx}}=\dfrac{x\sin^2(x^2+y^2)}{y^3}$

(A) $-\cot(x^2+y^2)=\left(\dfrac{x}{y}\right)^2+c$ (B) $\dfrac{y^2}{x^2+y^2c}=-\tan(x^2+y^2)$

(C) $-\cot(x^2+y^2)=\left(\dfrac{y}{x}\right)^2+c$ (D) $\dfrac{y^2+x^2c}{x^2}=-\tan^2(x^2+y^2)$

6. Solution of the differential equation :

$(3\tan x + 4\cot y - 7)\sin^2y\,dx - (4\tan x + 7\cot y - 5)\cos^2x\,dy = 0$ is

(A) $\dfrac{3}{2}\cot^2 x-7\cot x+\dfrac{7}{2}\tan^2 y-5\tan y+4\cot x.\tan y=c$

(B) $\dfrac{3}{2}\tan^2 x-7\tan x+\dfrac{7}{2}\cot^2 y-5\cot y+4\tan x.\cot y=c$

(C) $3\tan^2y - 14\cot x.\tan^2y + 7\cot^2x - 10\tan y\cot^2x + 8\cot x.\tan y + 2c\cot^2x\tan^2y = 0.$

(D) $3\cot^2y - 14\cot x.\cot^2y + 7\cot^2x + 10\cot y\tan^2x + 8\tan x.\cot y = 0.$

PRACTICE QUESTIONS

7. If the solution of $y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2=x, y(0)=y(1)=1$ is given by $y^2=f(x)$ then

a) f(x) is monotonically increasing $\forall x\in(1,\infty)$

b) f(x) =0 has only one root

c) f(x) is neither even nor odd

d) f(x) has 3 real roots

Variable separable form :

8. The solution of $p^2+(2y\cot x)p=y^2$ *where* $p=\dfrac{dy}{dx}$ *is*

A) $y(1+\cos x)=c$ B) $y(1-\cos x)=c$

C) $x=2\sin^{-1}\sqrt{c/2y}$ D) $x=2\sin^{-1}\left(\sqrt{2y}\right)+C$

PRACTICE QUESTIONS

9. The function $f(\theta)=\dfrac{d}{d\theta}\int_0^\theta\dfrac{dx}{1-\cos\theta\cos x}$ *satisfies* the differential equation

A) $\dfrac{df}{d\theta}+2f(\theta)\cot\theta=0$ B) $\dfrac{df}{d\theta}-2f(\theta)\cot\theta=0$

C) $f(\theta)=1+\cot^2\theta$ D) $f(\theta)=1+\csc^2\theta$

Lagrange's Linear D.E :

10. For any real numbers α and β, let $y_{\alpha,\beta}(x), x \in \mathbb{R}$, be the solution of the differential equation. $\frac{dy}{dx} + \alpha y = xe^{\beta x}, y(1) = 1$. Let $S = \{y_{\alpha,\beta}(x) : \alpha, \beta \in \mathbb{R}\}$. Then which of the following function belong(s) to the set S? **[Adv. 2021]**

A) $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$ B) $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

C) $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{2}\right)e^{-x}$ D) $f(x) = \frac{e^x}{2}\left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{2}\right)e^{-x}$

11. Let $f : [0, \infty) \to \mathbb{R}$ be a continuous function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$ for all $x \in [0, \infty)$. Then , Which of the following statement (s) is (are) TRUE? **[Adv. 2018]**

A) The curve $y = f(x)$ passes through the point (1, 2)

B) The curve $y = f(x)$ passes through the point (2, -1)

C) The area of the region $\left\{(x, y) \in [0,1] \times \mathbb{R} : f(x) \le y \le \sqrt{1 - x^2}\right\}$ is $\frac{\pi - 2}{4}$

D) The area of the region $\left\{(x, y)\} \in [0,1] \times \mathbb{R} : f(x) \le y \le \sqrt{1 - x^2}\right\}$ is $\frac{\pi - 1}{4}$

12. Let $f : (0, \infty) \to \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \ne 1$. Then **[Adv. 2016]**

A) $\lim\limits_{x \to 0+} f'\left(\frac{1}{x}\right) = 1$ B) $\lim\limits_{x \to 0+} xf\left(\frac{1}{x}\right) = 2$

C) $\lim\limits_{x \to 0+} x^2 f'(x) = 0$ D) $|f(x)| \le 2$ for all $x \in (0, 2)$

PRACTICE QUESTIONS

13. If $y(x)$ satisfies the differential equation $y' - y\tan x = 2x\sec x$ and $y(0) = 0$, then

A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$ C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$ D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

Geometrical Application of D.E :

14. A normal is drawn at a point $P(x, y)$ of a curve. It meet the x-axis at Q. If PQ is of constant length k. Then

a) The differential equation describing such curves is $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

b) The curve is passing through (0, k)

c) The curve is passing through (k, 0)

d) The equation of the curve represents circle with centre as origin

15. The curve whose sub tangent is 'n' times the abscissa of the point of contact and passes through the point (2, 3), then

a) for n=1 equation of the curve is $2y = 3x$

b) for n=1 equation of the curve is $2y^2 = 9x$

c) for n=2 equation of the curve is $2y = 3x$

d) for n=2 equation of the curve is $2y^2 = 9x$

16. Let C be a curve such that the normal at any point P on it meets x-axis and y-axis at A and Y respectively. If BP:PA=1:2 (internally) and the curve passes through the point (0,4) then which of the following alternative(s) is/are correct?

(A) The curves passes through $\left(\sqrt{10}, -6\right)$

(B) The equation of tangent at $\left(4, 4\sqrt{3}\right)$ is $2x + \sqrt{3}y = 20$

(C) The differential equation for the curve is $yy' + 2x = 0$

(D) The curve represent a hyperbola

PRACTICE QUESTIONS

17. The coordinates of a point P(*x,y*) are functions of time t and satisfy the relations $\frac{dx}{dt} + \frac{dy}{dt} = t$ *and* $\frac{dx}{dt} - 2\frac{dy}{dt} = t^2$ at any instant of time t .The locus of point P(x.y) is a curve given by (assume x(0)=y(0)=0)

A) $(x+y)^3 = (x-2y)^2$

B) $x = \frac{t^2}{3} + \frac{t^3}{9}, y = \frac{t^2}{6} - \frac{t^3}{9}$

C) $9(x+y)^3 = 8(x-2y)^2$

D) $8(x+y)^3 = 9(x-2y)^2$

18. A curve passes through (2,0) and the slope of tangent at P(x,y) equals $\frac{(x+1)^2+y-3}{x+1}$ *then*

A) curve is $y=x^2-2x$

B) curve is $y=x^3-8$

C) area bounded by the curve and X-axis in fourth quadrant is $\frac{2}{3}$ square units

D) area bounded by the curve and X-axis in fourth quadrant is $\frac{4}{3}$ square units

19. The normal at a general point (a, b) on curve makes an angle θ with x-axis which satisfies $b(a^2\tan\theta-\cot\theta)=a(b^2+1)$. The equation of curve can be

A) $y=e^{\frac{x^2}{2}}+c$ B) $\log ky^2=x^2$ C) $y=ke^{\frac{x^2}{2}}$ D) $x^2-y^2=k$

Orthogonal Trajectory :

20. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2=\frac{a}{x}$ are

A) $9a(y+c)^2=4x^3$

B) $y+c=\frac{-2}{3\sqrt{a}}x^{3/2}$

C) $y+c=\frac{2}{3\sqrt{a}}x^{3/2}$

D) none of these

KEY SHEET

1.	BC	**2.**	AC	**3.**	BC	**4.**	ABCD	**5.**	AB
6.	BC	**7.**	ABC	**8.**	ABC	**9.**	AC	**10.**	AC
11.	BC	**12.**	ABC	**13.**	AD	**14.**	AB	**15.**	AD
16.	AD	**17.**	BD	**18.**	AD	**19.**	BCD	**20.**	ABC

HINTS & SOLUTIONS

1. Equation of such conics are

$ax^2+by^2+cxy=1 \Rightarrow \text{order}=3=\text{parameters}$

$\deg\text{ree}=1$ (no parameters is being repeated)

2. $y^2=2c\left(x+\sqrt{c}\right) \Rightarrow 2yy_1=2c \Rightarrow c=yy_1$

Eliminating c, we get $y^2=2yy_1\left(x+\sqrt{yy_1}\right)$ or $\left(y-2xy_1\right)^2=4yy_1^{\;3}$

It involves only 1st order derivative, its order is 1 but its degree is 3 as $y_1^{\;3}$ is there

3. Let the equation of circle be $x^2+y^2+2gx+2gy+c=0$

$\Rightarrow 2x+2yy'+2g+2gy'=0 \Rightarrow x+yy'+g+gy'=0$ ….(i)

On differentiating again, we get $1+yy''+(y')^2+gy''=0 \Rightarrow g=\left[\dfrac{1+(y')^2+yy''}{y''}\right]$

On substituting the value of g in equation (i) , we get

$$x+yy'-\frac{1+(y')^2+yy''}{y''}-\left(\frac{1+(y')^2+yy''}{y''}\right)y'=0$$

$\Rightarrow xy''+yy'y''-1-(y')^2-yy''-y'(y')^3-yy'-yy'y''=0$

$\Rightarrow (x-y)y^n-y'(1+y'+(y')^2)=1 \Rightarrow (y-x)y^n+[1+y'+(y')^2]y^3+1=0$

$\therefore\ Py''+Qy'+1=0$

$\therefore\ P=y-x, Q=1+y'+(y')^2$

Hence, $P+Q=1-x+y+y'+(y')^2$

4. $y=c_1x^m+c_2x^n$ or $y_1=mc_1x^{m-1}+nc_2x^{n-1}, y_2=m(m-1)c_1x^{m-2}+n(n-1)c_2x^{n-2}$ substituting gives

$m^2-10m+21=0, n^2-10n+21=0 \ \Rightarrow m=3$ or 7 and $n=3$ or 7

5. $\dfrac{xdx+ydy}{ydx-xdy}=\dfrac{x\sin^2\left(x^2+y^2\right)}{y^3} \Rightarrow \dfrac{1}{2}\cos ec^2\left(x^2+y^2\right)d\left(x^2+y^2\right)=\dfrac{x}{y}d\left(\dfrac{x}{y}\right)$

$$\Rightarrow \frac{1}{2}\cot\left(x^2+y^2\right)=\left(\frac{x}{y}\right)^2+C \Rightarrow =\cot\left(x^2+y^2\right)=\frac{x^2}{y^2}+K$$

$$\Rightarrow \frac{y^2}{x^2+ky^2} = -\tan\left(x^2+y^2\right)$$

6. Dividing throughout by $\cos^2 x \sin^2 y$, the given differential equation becomes

$(3 \tan x + 4 \cot y - 7)\sec^2 x\, dx - (4 \tan x + 7 \cot y - 5) \operatorname{cosec}^2 y\, dy = 0$

$\Rightarrow 3 \tan x \sec^2 x\, dx - 7 \sec^2 x\, dx - 7 \cot y \operatorname{cosec}^2 y\, dy + 5 \operatorname{cosec}^2 y dy + 4 \cot y \sec^2 x\, dx - 4 \tan x \operatorname{cosec}^2 y\, dy = 0$

$\Rightarrow 3 \tan x\, d(\tan x) - 7\, d(\tan x) + 7 \cot y\, d(\cot y) - 5\, d(\cot y) + 4\, d(\tan x \cot y) = 0$

On integrating, we obtain $\frac{3}{2}\tan^2 x - 7\tan x + \frac{7}{2}\cot^2 y - 5\cot y + 4\tan x \cot y = C.$

7. Given

$$\frac{d}{dx}\left(y\frac{dy}{dx}\right) = x \Rightarrow y\frac{dy}{dx} = \frac{x^2}{2} + c$$

$$\Rightarrow \frac{d}{dx}\left(\frac{y^2}{2}\right) = \frac{x^2}{2} + c$$

$$\Rightarrow y^2 = \frac{x^3}{3} + \alpha x + \beta \text{ given } y(0) = 1, y(1) = 1 \Rightarrow \beta = 1, \alpha = \frac{-1}{3}$$

$$\therefore y^2 = f(x) = \frac{x^3 - x + 3}{3}, f'(x) = 3x^2 - 1 > 0 \text{ for } x > 1$$

$$\therefore f(x) \uparrow$$

$f\left(\frac{1}{\sqrt{3}}\right) f\left(-\frac{1}{\sqrt{3}}\right) > 0\, f(x) = 0$ has only one real root .

8. $p^2 + 2py\cot x - y^2 = 0$

$$p = \frac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2} \Rightarrow \frac{dy}{dx} = \left(-\cot x \pm \operatorname{cosec} x\right) y \Rightarrow \frac{dy}{dx} + y\frac{(\cos x \pm 1)}{\sin x} = 0$$

On integration $\log y + 2\log\sin\frac{x}{2} = \log c \Rightarrow y(1-\cos x) = 2c \;\&\; y\left(1+\cos x\right) = c$

$$\Rightarrow y\left(2\sin^2 x/2\right) = C \Rightarrow \sin^2 x/2 = \frac{C}{2y} \Rightarrow x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$$

9. $f(\theta) = \frac{d}{d\theta}\int_0^\theta \frac{dx}{1-\cos x\cos\theta} = \frac{1}{1-\cos\theta} = \csc^2\theta \Rightarrow \frac{df}{d\theta} = -2\csc^2\theta\cot\theta$

$$\frac{df}{d\theta} + 2f(\theta)\cot\theta = 0 \;\&\; f(\theta) = \operatorname{cosec}^2\theta$$

10. Integrating factor = $e^{\int \alpha dx} = e^{\alpha x}$

Solution : $ye^{\alpha x} = \int xe^{(\alpha+\beta)x}dx$

Case I :

If $\alpha+\beta=0$ then $ye^{\alpha x}=\frac{x^2}{2}+C$

Put $x = 1$ and $y = 1 \Rightarrow C=e^{\alpha}-\frac{1}{2}$

So, $ye^{\alpha x}=\frac{x^2}{2}+e^{\alpha}-\frac{1}{2} \Rightarrow y=\frac{x^2}{2}.e^{-ax}+\left(e^{a}-\frac{1}{2}\right)e^{-\alpha x}$

For $\alpha=1$, $y=\frac{x^2}{2}e^{-x}+\left(e-\frac{1}{2}\right)e^{-x}$

Option (a) is correct.

Case II :

If $\alpha+\beta\neq 0$

$$ye^{\alpha x}=\frac{x.e^{(\alpha+\beta)x}}{\alpha+\beta}-\frac{1}{\alpha+\beta}\int e^{(\alpha+\beta)x}dx \Rightarrow ye^{\alpha x}=\frac{x.e^{(\alpha+\beta)x}}{\alpha+\beta}-\frac{x.e^{(\alpha+\beta)x}}{(\alpha+\beta)^2}+C$$

Put $x=1$ and $y=1$, we get $c=e^{\alpha}-\frac{e^{\alpha+\beta}}{\alpha+\beta}+\frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}$

$$c=e^{\alpha}-\frac{e^{\beta x}}{(\alpha+\beta)^2}+\left((\alpha+\beta)x-1\right)+e^{-\alpha x}\left(e^{x}-\frac{e^{\alpha+\beta}}{\alpha+\beta}+\frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}\right)$$

For $\alpha=\beta=1$, $y=\frac{e^{x}}{4}(2x-1)+e^{-x}\left(e-\frac{e^2}{2}+\frac{e^2}{4}\right)$

$$y=\frac{e^{x}}{4}\left(x-\frac{1}{2}\right)+e^{-x}\left(e-\frac{e^2}{4}\right)$$

So, option (c) is correct.

11. $f(x)=1-2x+\int_0^x e^{x-1}f(t)dt \Rightarrow f(x)=1-2x+e^{x}\int_0^x e^{-t}f(t)dt$

$\Rightarrow f'(x)=-2+e^{x}\int_0^x e^{-t}f(t)dt+e^{x}\left[e^{-x}f(x)\right]$

$\Rightarrow f'(x)=-2+\left[f(x)-1+2x\right]+f(x) \Rightarrow f'(x)-2f(x)=2x-3$

It's a linear differential equation

IF = $e^{\int -2dx=e^{-2x}}$

Solution $f(x)\times e^{-2x}=\int e^{-2x}(2x-3)dx$

$$f(x)\times e^{-2x}=\frac{e^{-2x}}{-2}(2x-3)-\int\frac{e^{-2x}}{-2}\times 2\,dx$$

$\Rightarrow f(x)=-x+\frac{3}{2}+\frac{1}{-2}+ce^{2x} \Rightarrow f(x)=-x+1+ce^{2x}$

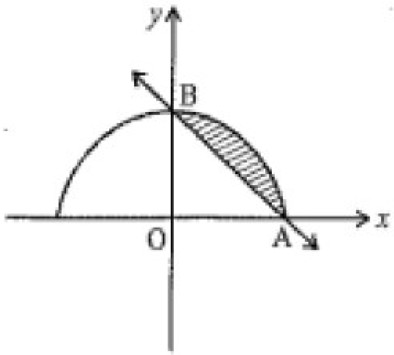
From definition of function, $f(0)=1$

$\therefore 1=1+c \Rightarrow c=0$, $\therefore f(x)=1-x$

Clearly curve $y=1-x$, does not pass through (1, 2) but it passes through (2, -1)

$\therefore$ (a)is false and (b) is true.

Also the area of the region $1-x \le y \le \sqrt{1-x^2}$, is shown in the figure by



Area of quadrant – Area ΔOAB

$-\frac{1}{4}\times\pi\times 1^2-\frac{1}{2}\times 1\times 1=\frac{\pi-2}{4}$

$\therefore$ (c) is true and (d) is false.

12. Here, $f(x)=2-\frac{f(x)}{x}$ or $\frac{dy}{dx}+\frac{y}{x}=2$ [i.e.linear differential equation in y]

Integrating Factor, IF = $e^{\int\frac{1}{x}dx}=e^{\log x}=x$

$\therefore$ Required solution is $y.(IF)=\int Q(IF)dx+C \Rightarrow y(x)=\int f2(x)+C$

$\Rightarrow yx=x^2+C$

$\therefore y=x+\frac{C}{x}$ $[\because C=0, as\ f(1)=1]$

A) $\lim\limits_{x\to 0^-} f'\left(\frac{1}{x}\right)= \lim\limits_{x\to 0^-}\left(1-Cx^2\right)=1$

option (a) is correct

B) $\lim\limits_{x\to 0^-} xf'\left(\frac{1}{x}\right)= \lim\limits_{x\to 0^-}\left(1+Cx^2\right)=1$

option (b) is correct

C) $\lim\limits_{x\to 0^+} x^2 f(x_{\_}\left(\frac{1}{x}\right)= \lim\limits_{x\to 0^+}\left(x^2-C\right)=-C=0$

Option (c) is correct

D) $f(x)=x+\frac{C}{x}, C=0$

For $C>0,\ \lim\limits_{x\to 0^+} f(x)=\infty$

$\therefore$ Function is not bounded in (0, 2)

$\therefore$ Option (d) is incorrect

13. $\frac{dy}{dx}-y\tan x=2x\sec x,$

$IF=e^{-\int\tan x dx}=\cos x$

$\therefore y.\cos x=\int 2xdx=x^2\sec x \Rightarrow y'=2x\sec x+x^2\sec x\tan x$

$$Now,\ y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{16}\times\sqrt{2}=\frac{\pi^2}{8\sqrt{2}}$$

$$y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}\times 2=\frac{2\pi^2}{9}$$

$$y'\left(\frac{\pi}{4}\right)=\frac{2\pi}{4}\times\sqrt{2}+\frac{\pi^2}{8\sqrt{2}}\times 1=\frac{\pi^2}{8\sqrt{2}}+\frac{\pi}{\sqrt{2}}$$

$$y'\left(\frac{\pi}{3}\right)=\frac{2\pi}{3}\times 2+\frac{2\pi^2}{9}\times\sqrt{3}=\frac{2\pi^2}{3\sqrt{3}}+\frac{4\pi}{3}$$

14. Equation of the normal at a point $P(x,y)$ is given by

$$Y-y=-\frac{1}{dy/dx}(X-x) \qquad \text{---(1)}$$

Let the point Q at the x-axis be $(x_1,0)$. From (1) we get

$$y\frac{dy}{dx}=x_1-x \qquad \text{--(2)}$$

Now given that $PQ^2=k^2$

We have $(x-x_1)^2+y^2=k^2$ or $x-x_1=\pm\sqrt{k^2-y^2}$,

Hence using (2) we obtain $y\frac{dy}{dx}=\pm\sqrt{k^2-y^2}$ ---(3)

(3) is the required differential equation for such curves

Now solving (3) we get $\int\frac{-ydy}{\sqrt{k^2-y^2}}=\int -dx$

or $x^2+y^2=k^2$ which passes through (0,k)

15. If (x,y) is any point on the curve, the sub tangent at (x,y) $=y\frac{dx}{dy}$

$\therefore\ y\frac{dx}{dy}=nx$ (given) or $n\frac{dy}{y}=\frac{dx}{x}$

Integrating $n\log y=\log x+\log c$ or $\log y^n=\log cx$

or $y^n=cx......(i)$ which is the required equation of the family of curves.

Putting $x=2$, $y=3$ in (i), we have $3^n=2c$ or $c=\frac{3^n}{2}$

Putting this value of c in (i)

$$y^n=\frac{3^n}{2}x \text{ or } 2y^n=3^n x \qquad \text{(ii)}$$

Which is the particular curve passing through the point (2,3)

Putting n=1 in (ii), we have $2y = 3x$

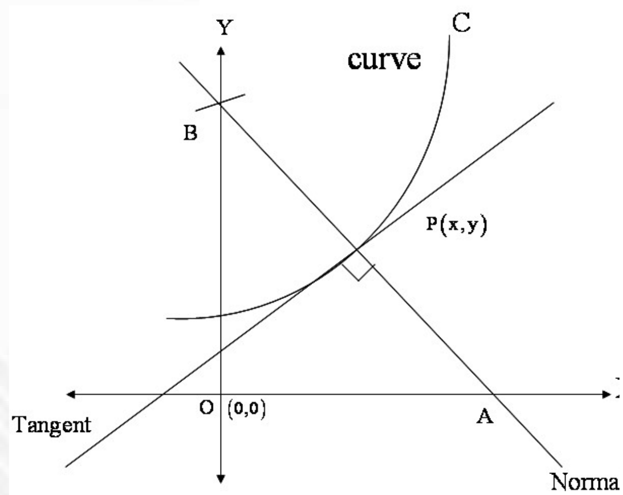
Which is a straight line

Putting n = 2 in (ii) we have $2y^2 = 9x$

Which is a parabola.

16.

The equation of normal of $P(x,y)$ is

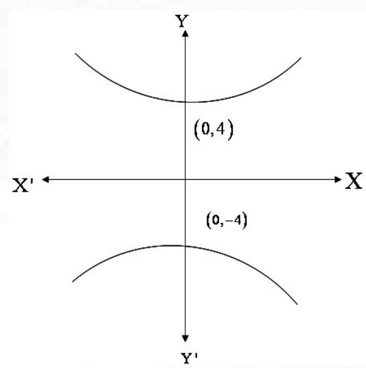


$$(Y-y) = \frac{-1}{\frac{dy}{dx}}(X-x)$$

$$\therefore A\left(x + y\frac{dy}{dx}, 0\right) \text{ and } B\left(0, y + \frac{x}{\frac{dy}{dx}}\right)$$

Now $\frac{1\left(x + y\frac{dy}{dx}\right) + 2(0)}{1+2} = x \Rightarrow x + y\frac{dy}{dx} = 3x$

$$y\frac{dy}{dx} = 2x \text{ …..(1)}$$



$$\Rightarrow \int y dy = \int 2x\ dx \Rightarrow \frac{y^2}{2} = x^2 + C$$

Also (0,4) satisfy it, so C =8

$\therefore y^2 = 2x^2 + 16$ (equation of curve)

Which represent a hyperbola

Also $\left.\frac{dy}{dx}\right]_{(4,4\sqrt{3})} = \frac{2(4)}{4\sqrt{3}} = \frac{2}{\sqrt{3}}$

17. From the given equations we get $x = \frac{t^2}{3} + \frac{t^3}{9}$ *and* $y = \frac{t^2}{6} - \frac{t^3}{9}$.Eliminating t gives

$$8(x+y)^3 = 9(x-2y)^2$$

18. $\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1} = x + 1 + \frac{y-3}{x+3}$

Putting x+1=X and y-3=Y

$$\frac{dY}{dX} = X + \frac{4}{X} \Rightarrow \frac{dY}{dX} - \frac{Y}{X} = X$$

Solution is $\frac{Y}{X} = X + c \Rightarrow y - 3 = (x+1)^2 + (2+1)$

It passes Through $(2,0) \Rightarrow c = -4$

Equation of curve is $y = x^2 - 2x$

Area bounded=$\int_{o}^{2}(2x - x^2)\,dx = \frac{4}{3}s.u$

19. $y\left(x^2\left(-\frac{dx}{dy}\right) + \frac{dy}{dx}\right) = x\left(y^2 + 1\right) \Rightarrow yy'^2 - xy^2y' - xy' - yx^2 = 0$

$\therefore \frac{dy}{dx} = xy \;\; or \;\; \frac{dy}{dx} = \frac{x}{y}$

$\therefore y = ke^{\frac{x^2}{2}} \; (or) \; \log y^2 = x^2 - \log k$

Also $\therefore y = ke^{\frac{x^2}{2}} \Rightarrow \log ky^2 = x^2$

20. Replacing $\frac{dy}{dx} by - dx/dy$, we get

$$\left(\frac{dy}{dx}\right)^2 = \frac{x}{a} \Rightarrow dy = \pm\left(\frac{x^{1/2}}{a^{1/2}}\right)dx$$

Integrating we get $y + c = \pm\frac{2}{3a^{1/2}}x^{3/2} \Rightarrow 9a(y+c)^2 = 4x^3$