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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS & STERLING_BT   **Paper -2**(Adv-2021-P2-Model)   **Date: 08-10-2023**

Time: 02.00Pm to 05.00Pm   **CTA-06 & CTA-09**   **Max. Marks: 180**

08-10-2023_*Sr.Super60*_NUCLEUS & STERLING_BT_Jee-Adv(2021-P2)_CTA-06 & CTA-09_Syllabus

PHYSICS : Thermal physics, Geometrical optics, Wave optics, General Physics, Kinematics, NLM, Friction, Circular motion, WPE, COM & Collisions, Rigid body Dynamics 1, Rigid body Dynamics 2, Fluid statics & Dynamics, Properties of matter, Experiments

CHEMISTRY : Nomenclature, Isomerism, GOC, Alkanes, Alkene & Alkyne, Benzene, Alkyl Halides & Aryl Halides, Alcohols, Phenols , Ethers & Amines, Alcohols, Phenols, Ethers, Amines, Aldehydes & Ketones, Carboxylic acid & Derivatives, Aldehydes & Ketones, Carboxylic acids & derivatives, Biomolecules, Polymers, Chemistry in Everyday Life & POC, Biomolecules, Polymers, Chemistry in Everyday Life, POC, 1.Classification of Elements and Periodicity in Properties, 2.Chemical bonding and Molecular Structure, 3.Hydrogen, 1.s-Block Elements, 2.Group-13, 3.Group 14, 1.Group-15, 2.Group-16, 3.Group-17

MATHEMATICS : Functions & LCD, Application of Differentiation (AOD), TOTAL DIFFERENTIAL CALCULUS, Indefinite Integration, Definite Integration, Areas & Differential Equations, Total Integral Calculus, Vectors & 3D-Geometry, Matrices & Determinants, Stragith Lines, Pair of Straight Lines & Complete Circles

Name of the Student: _____   **H.T. NO:** ☐☐☐☐☐☐☐☐☐☐

JEE-ADVANCE-2021-P2-Model

Time: 3:00Hr's   ## IMPORTANT INSTRUCTIONS   **Max Marks: 180**

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 6)	Questions with Multiple Correct Choice with Partial mark	+4	-2	6	24
Sec – II(Q.N : 7 – 12)	Paragraph Questions with Numerical Value Answer Type	+2	0	6	12
Sec – III(Q.N : 13 – 16)	Paragraph Questions with Single Answer Type	+3	-1	4	12
Sec – IV(Q.N : 17 – 19)	Questions with Non-negative Integer Value Type	+4	0	3	12
Total				**19**	**60**

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 20 – 25)	Questions with Multiple Correct Choice with Partial mark	+4	-2	6	24
Sec – II(Q.N : 26 – 31)	Paragraph Questions with Numerical Value Answer Type	+2	0	6	12
Sec – III(Q.N : 32 – 35)	Paragraph Questions with Single Answer Type	+3	-1	4	12
Sec – IV(Q.N : 36– 38)	Questions with Non-negative Integer Value Type	+4	0	3	12
Total				**19**	**60**

MATHEMATICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 39 – 44)	Questions with Multiple Correct Choice with Partial mark	+4	-2	6	24
Sec – II(Q.N : 45 – 50)	Paragraph Questions with Numerical Value Answer Type	+2	0	6	12
Sec – III(Q.N : 51 – 54)	Paragraph Questions with Single Answer Type	+3	-1	4	12
Sec – IV(Q.N : 55 – 57)	Questions with Non-negative Integer Value Type	+4	0	3	12
Total				**19**	**60**

## PHYSICS   Max. Marks: 60

SECTION-1(Maximum Marks: 24)
One or More Type

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks: +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks: +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks: +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks: 0 If unanswered;
  Negative Marks: −2 In all other cases.

1. Young's modulus of a uniform cross section rod, linearly varies from $Y_0 \, (end \, A) \, to \, 2Y_0 \, (end \, B)$. Two equal forces are acting at ends of rod (area of cross-section =A, length $=\ell$)

    A) Stress at end A = stress at midpoint of rod

    B) Stress at end A $\neq$ stress at midpoint of rod

    C) Total extension in rod is $\dfrac{F\ell}{AY_0}\ell n2$

    D) Total extension in rod is $\dfrac{2F\ell}{AY_0}\ell n2$

2. A uniform rod of mass m and length 2ℓ lies on smooth horizontal surface. A particle of same mass m is connected to string of length ℓ, whose other end is connected to rod.

    Initially string is taut and both rod and string lies in same horizontal plane with 90° angle between them. If particle is given initial velocity v_0 perpendicular to string, then just after giving velocity v_0 to particle: -
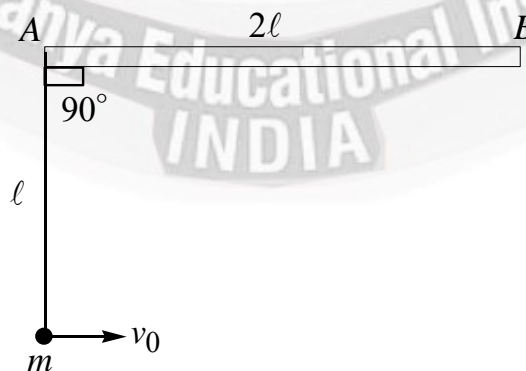
A) Linear acceleration of centre of mass of rod will be $\dfrac{v_0^2}{4\ell}$

B) Angular acceleration of rod will be $\dfrac{6v_0^2}{5\ell}$

C) Tension in string will be $\dfrac{mv_0^2}{5\ell}$

D) Angular acceleration of rod will be $\dfrac{3v_0^2}{5\ell^2}$

3. At time $t = 0$, two point objects A and B respectively are at pole and centre of curvature of a fixed concave mirror of focal length f; the velocity vectors of A and B are always $\vec{V}_A = u\hat{i}$ and $\vec{V}_B = -u\hat{i}$ respectively, where $\hat{i}$ is unit vector along principal axis directed from pole towards focus and u is a positive constant
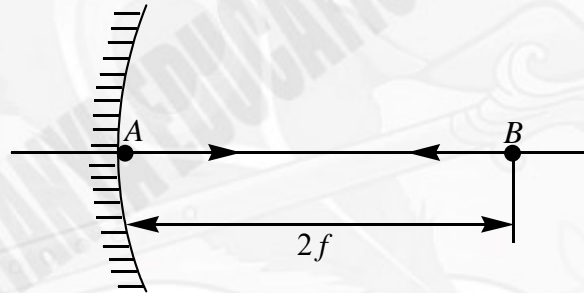
A) The distance between images of A and B will be 4f at time $t = \dfrac{f}{2u}$

B) Magnitude of relative velocity of image of A and image of B at $t = 0$ is 2u

C) Starting from $t = 0$ and before the particles come in contact, distance between image of A and image of B increases

D) Starting from $t = 0$ and before the particles come in contact, distance between image of A and image of B first increases and then decreases

4. Cylindrical tank, having a cross-sectional area S, stands fixed on a horizontal surface (Figure). In its wall is a small hole, cross-sectional area of which is S_1, is located at a distance h_1 from the surface of water in the tank and the h_2 from the bottom. Cross-sectional area of the jet emanating from the holes at the place of its falling on the horizontal surface is S_2. $h_1 + h_2 = constant.(S \gg S_1)$
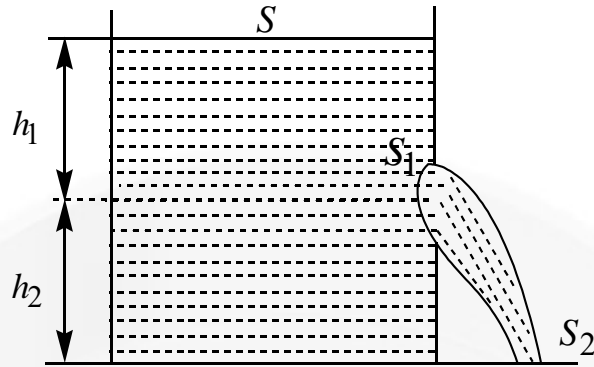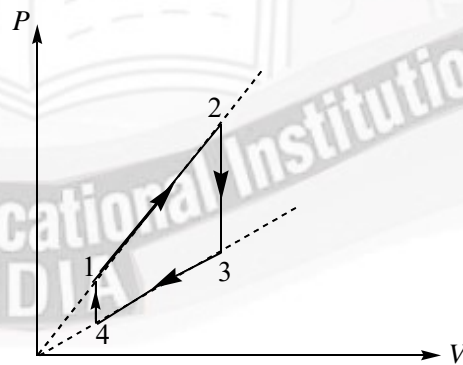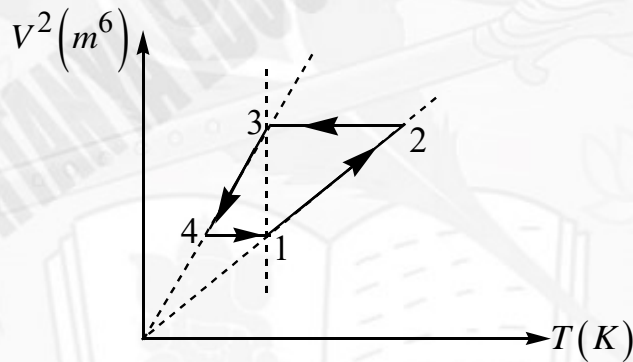
A) $S_2 = S_1 \sqrt{\dfrac{h_2}{h_1 + h_2}}$

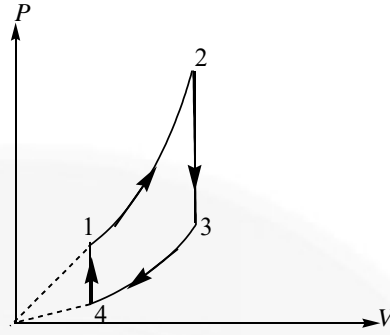**B)** S_2 will increase if h_1 is increased

C) S_2 will increase if h_2 is increased

D) $S_2 = S_1 \sqrt{\dfrac{h_1}{h_1 + h_2}}$

5. One mole of an ideal monatomic gas goes through a cyclic process, shown in the graph. Mark the correct statements: -


A) PV diagram for the cyclic process is

B) PV diagram for the cyclic process is

C) Molar specific heat capacity of the gas for the process is same for process 1-2 and 3-4.

D) Modulus of change in internal energy is same for process 1-2 and 2-3.

6. Two small identical discs A and B each of mass $m = 2kg$ placed on a frictionless horizontal floor are connected by light inextensible string of length $l = 2m$. Now the midpoint C of cord is pulled perpendicular to the line joining centres of the discs by a constant force $F = 8N$. In the subsequent motion, when they are about to collide:
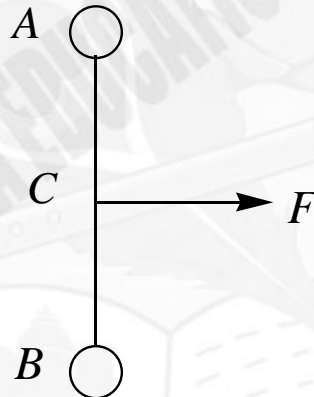
A) Tension in the string is 8N
B) Velocity of approach of discs A and B is 4m/s
C) Acceleration of midpoint C of the cord is rightward
D) Magnitude of acceleration of midpoint C of the cord is $2m/s^2$

SECTION-2(Maximum Marks: 12)
Paragraph with Numerical

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +2   If ONLY the correct numerical value is entered at the designated place;
Zero Marks: 0   In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

In a Vernier caliper, there are 20 divisions on Vernier scale which are equal to 25 divisions on main scale. One main scale division is equal to 1mm, and there is no zero error in the Vernier caliper. m^{th} Vernier scale division coincides with n^{th} main scale division, when reading is 12.25 mm

7. Largest value of m is:

8. Smallest value of n is:

Question Stem for Question Nos. 9 and 10

Question Stem

In a modified YDSE, plane of screen is not parallel to plane of slits but making an angle $37°$ between them as shown in figure. Parallel beam of monochromatic light perpendicular to the plane of slits is illuminating the slits. Screen size is very large, and is extended both above and below. Slit separation is $d = 5.5\lambda$, where λ is wavelength of light.
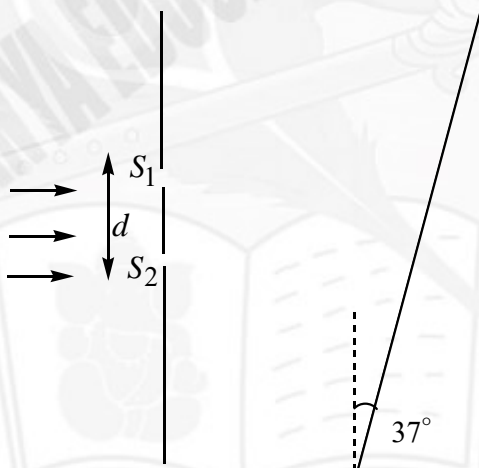
9. Highest order maxima below the central maxima is

10. Highest order maxima above the central maxima is

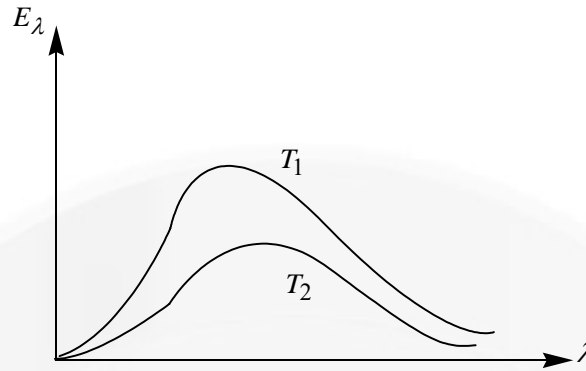## Question Stem for Question Nos. 11 and 12

Question Stem

E_λ (spectral emissive power) vs λ (radiation wavelength) graph at two different absolute temperatures $T_1 \ and \ T_2$ of a black body is shown. Corresponding area under curve for complete domain of wavelength are $A_1 \ and \ A_2$. If $\dfrac{A_1}{A_2} = 4$, then

11.   Ratio of temperature $\dfrac{T_1}{T_2}$ is

12.   Ratio of emissive power $\dfrac{E_1}{E_2}$ is

SECTION-3(Maximum Marks: 12)
Paragraph with Single Answer Type

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +3 If ONLY the correct option is chosen;
Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks: −1   In all other cases.

Paragraph-I

Motion of a spinning top is quite intriguing. When a spinning top is placed on the floor and its tip held in one position, it starts to precess about a vertical axis as shown.
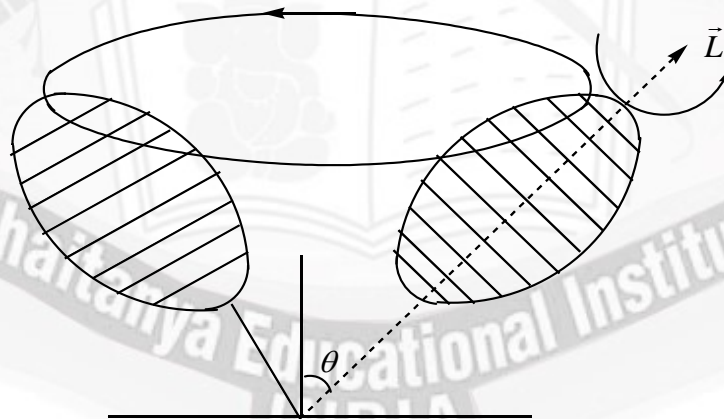

Let us take the mass of top as m, its moment of inertia about spinning axis as I, distance of its centre of mass from pivot point is ℓ, and its spinning rate is ω_s. The rate of precession, that is angular speed at which the top starts to rotate about vertical is Ω. Generally Ω is much smaller than ω, so in our present discussion we will assume that Ω does not

contribute to angular momentum $\vec{L}$ and it arises only due to ω_s alone. So $\vec{L} = I\vec{\omega}_s$. As the top precess, horizontal component of its angular momentum changes. This change is brought by the torque due to weight of top, about the pivot. If top precess with a steady rate, then

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$$

Rate of change of horizontal component of $\vec{L}$ can be calculate easily, as described below
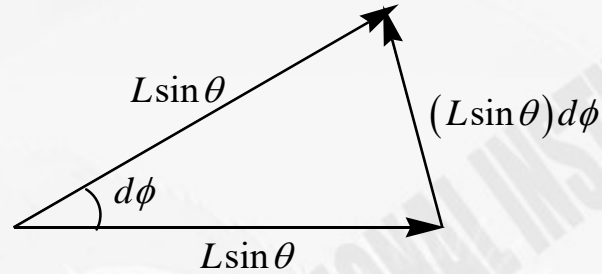
$$\frac{dL_H}{dt} = L\sin\theta\frac{d\phi}{dt}$$

13. If the top described in the problem is precessing uniformly, then its rate of precession is given by

A) $\dfrac{mg\ell}{I\omega_s}$ B) $\dfrac{mg\ell\tan\theta}{I\omega_s}$ C) $\dfrac{mg\ell\sin\theta}{I\omega_s}$ D) $\dfrac{mg\ell\cos\theta}{I\omega_s}$

14. Sometimes when a ceiling fan is fitted loosely on the ceiling, it performs a conical pendulum like motion with frequency Ω, while running. If a fan is running at 100 rpm, find the value of Ω assuming it to be much smaller as compared to spinning frequency. Take mass of fan 5kg, its moment of inertia about spinning axis is $2kg - m^2$ and distance of centre of mass of system from point of hanging is 0.5m $\left(g = 10m/s^2\right)$

A) 3.14 rpm B) 6 rpm C) 9.2 rpm D) 11.4 rpm

Paragraph-II

Sign convention is taken as +ve direction in the direction of light ray and the graph is drawn between $\dfrac{1}{u}$ and v for a spherical mirror.
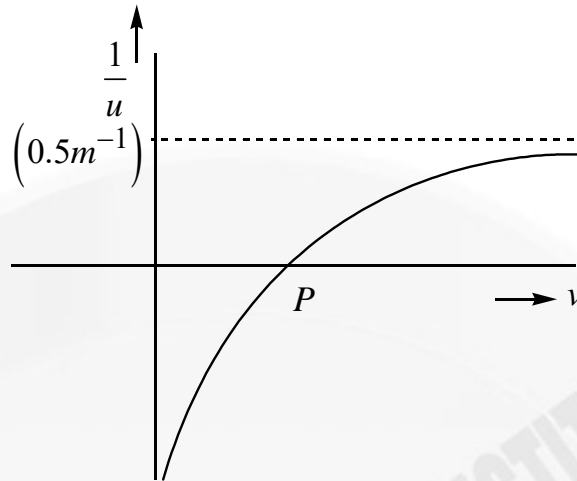
15. What is the focal length of the mirror?

A) -50cm **B)** -200 cm **C)** +200 cm **D)** +50 cm

16. Magnitude of slope of curve at point P

A) $1m^{-2}$ **B)** $0.25m^{-2}$ **C)** $2m^{-2}$ **D)** $4m^{-2}$

SECTION-4(Maximum Marks: 12)
Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +4 If ONLY the correct integer is entered;
Zero Marks: 0   In all other cases.

17. Two small balls of masses m_1 and m_2 (m_1 slightly above m_2) are released from height h. All collisions will be elastic. The maximum height upto which m_1 can rebound is nh after all possible collision. Find the value of n. (Consider all possible values of m_1 & m_2)
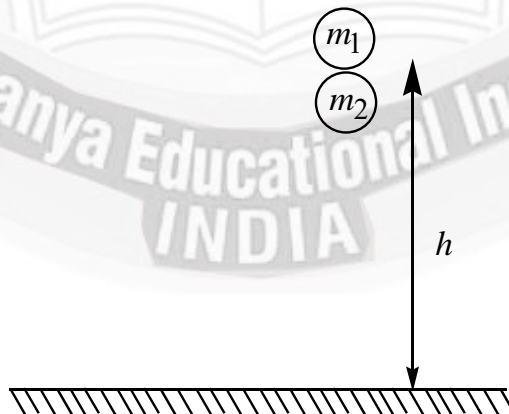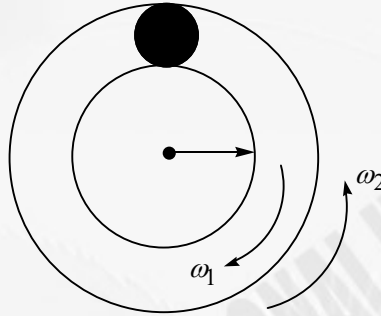
18. A solid spherical ball is stuck up between two coaxial rotating cylinders as shown whose centers are fixed. There is no slipping anywhere. Find the kinetic energy of the ball (in J). Radius of inner cylinder = 1m and its angular velocity $\omega_1 = 10\, rad/s$; radius of outer cylinder =1.5 m and it's angular velocity $\omega_2 = 20\, rad/s$. Take mass of ball $= \dfrac{1}{130}\, kg$.

19. At the middle of the mercury barometer tube there is a little column of air with the length ℓ_0 and there is vacuum at the top as shown. Under the normal atmospheric pressure and the temperature of 300 kelvin, $\ell_0 = 10cm$. Neglect thermal expansion of the tube and mercury. The length of the air column if the temperature rises to 330 kelvin will be α cm, then find $\alpha/11$:
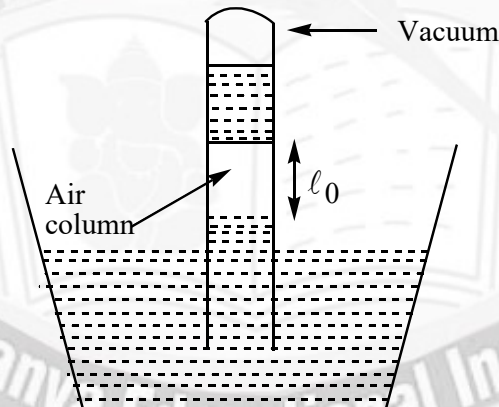
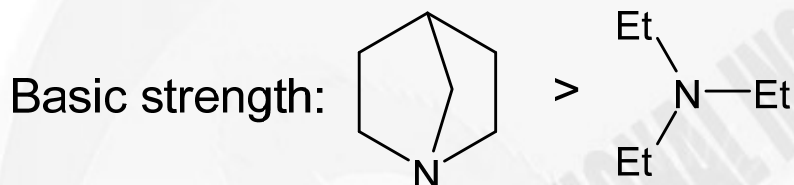**CHEMISTRY**   **Max. Marks: 60**
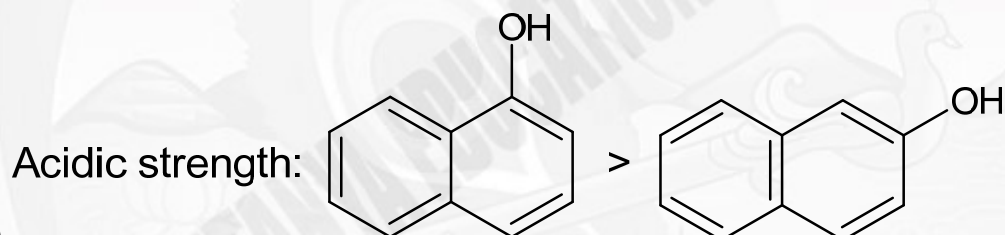
SECTION-1(Maximum Marks: 24)
One or More Type

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks: +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks: +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks: +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
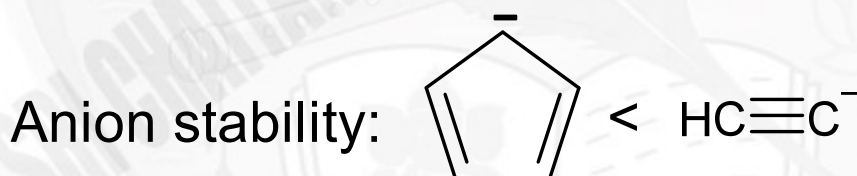 Zero Marks: 0 If unanswered;
  Negative Marks: −2 In all other cases.

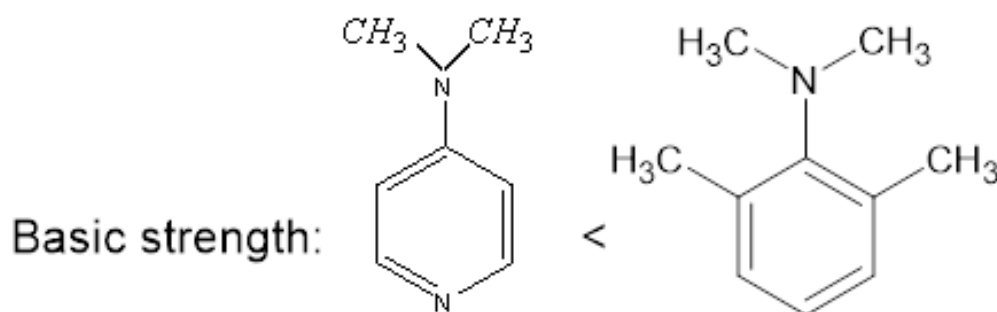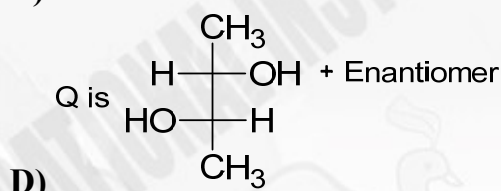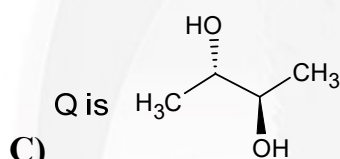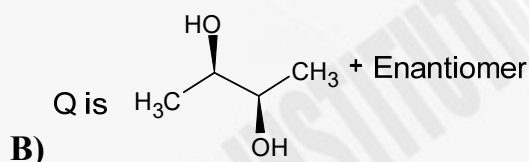20.   Select the CORRECT option against mentioned property

A)   Basic strength:

B)   Acidic strength:

C)   Anion stability:

D)   Basic strength:

21.

$$P \xleftarrow{\begin{array}{l}1)NaNH_2(1eq)\\2)CH_3I(1eq)\\3)NaNH_2(1eq)\\4)CH_3I(1eq)\\5)Na,NH_3(liq.)\\6)OsO_4,NaHSO_3,H_2O\end{array}} HC \equiv CH \xrightarrow{\begin{array}{l}1)NaNH_2(1eq)\\2)CH_3I(1eq)\\3)NaNH_2(1eq)\\4)CH_3I(1eq)\\5)H_2,Lindlar\ catalyst\\6)m-CPBA\\7)H_3O^+\end{array}} Q$$

A) P is

$$\begin{array}{c}CH_3\\ H-\!\!-OH\\ H-\!\!-OH\\ CH_3\end{array}$$

B) Q is H_3C ... CH_3 + Enantiomer (with HO and OH)

C) Q is H_3C ... CH_3 (with HO and OH)

D) Q is

$$\begin{array}{c}CH_3\\ H-\!\!-OH\\ HO-\!\!-H\\ CH_3\end{array} + Enantiomer$$
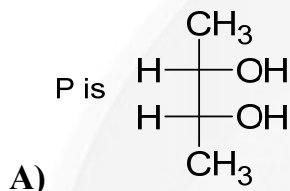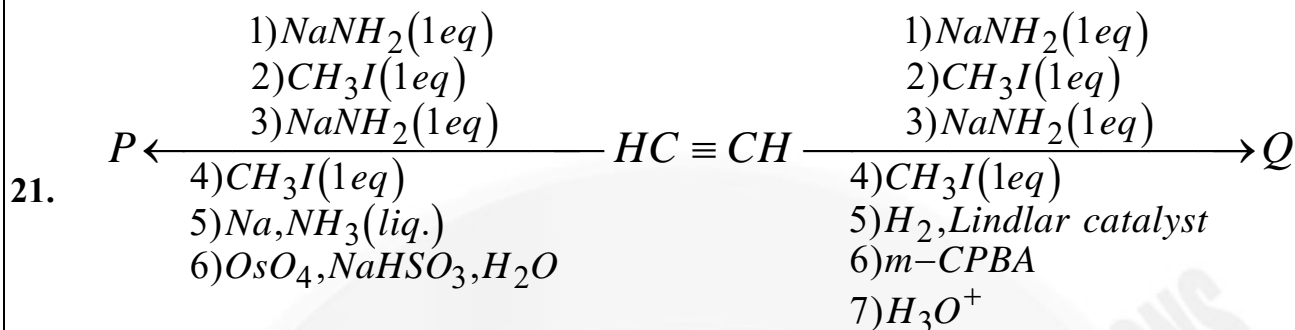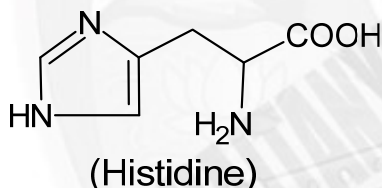
22. Which of the following is true regarding Histidine

(Histidine)

A) At pH = 1, net charge of histidine is '+2'

B) At pH = 4, net charge of histidine is '+1'

C) At pH = 7.6, net charge of histidine is '0'

D) At pH = 11, net charge of histidine is '-2'

23. Consider the following reactions:

$$SiO_2 + 2NaOH \rightarrow X + Y$$

$$SiO_2 + 4HF \rightarrow R + 2Y$$

$$R + 2HF \rightarrow Q$$

$$3R + 4H_2O \rightarrow Q + M$$

Then identify the correct statements?

A) M is also obtained as one of the product by the hydrolysis of $SiCl_4$

B) In the anionic part of Q, the central atom is in the sp^3d^2 hybridized state

C) M on strong heating gives SiO_2 as one of the product

D) Similar to $SiCl_4$, R also readily undergoes complete hydrolysis under normal conditions

24. In which of the following arrangements the order is/are correct according to property indicated?

A) $Al^{3+} < Mg^{2+} < Na^+ < F^-$: Increasing ionic size

B) $B < C < O < N$: Increasing first ionization enthalpy

C) $Cl < F < Br < I$: Increasing electron affinity

D) $Li < Na < K < Rb$: Increasing metallic radius

25. Select the incorrect order?

A) $Na_3N > Mg_3N_2 > AlN$: Lattice energy (magnitude)

B) $Na^+_{(g)} < Mg^{2+}_{(g)} < Al^{3+}_{(g)}$: Ionic radius

C) $Li^+(aq) > Na^+(aq) > K^+(aq)$: Hydrated ionic radius

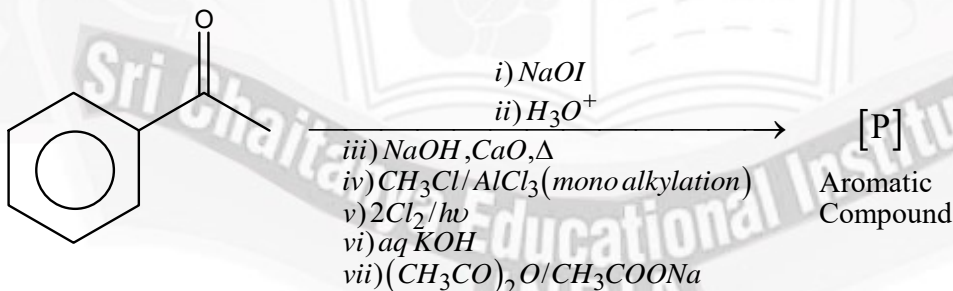**D)** $F^- > Cl^- > I^-$: Order of polarisability

SECTION-2(Maximum Marks: 12)
Paragraph with Numerical

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks: +2   If ONLY the correct numerical value is entered at the designated place;
  Zero Marks: 0   In all other cases.

Question Stem for Question Nos. 26 and 27

Question Stem

$$
\xrightarrow[\substack{iii)\,NaOH,CaO,\Delta \\ iv)\,CH_3Cl/AlCl_3(mono\,alkylation) \\ v)\,2Cl_2/h\upsilon \\ vi)\,aq\,KOH \\ vii)\,(CH_3CO)_2O/CH_3COONa}]{\substack{i)\,NaOI \\ ii)\,H_3O^+}}
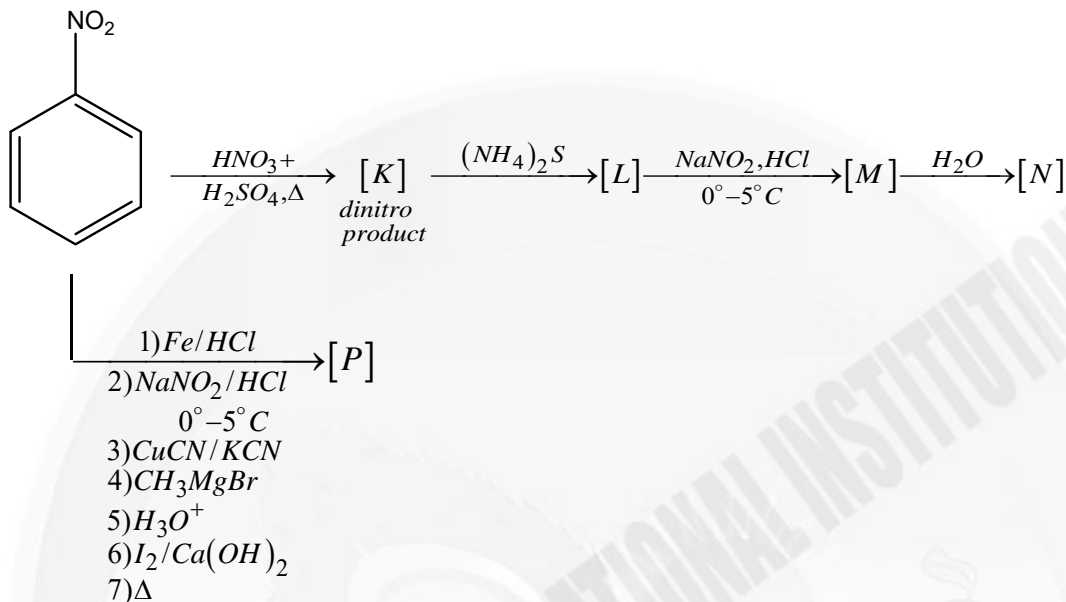$$

[P]
Aromatic Compound

Compound P upon reaction with Br_2 / Na_2CO_3, followed by heating at 473 K with moist KOH furnishes Q as the major product.

26. Degree of unsaturation in the product 'P' is _____

27. Find the number of pure atomic orbitals in the product 'Q' is_____?

Question Stem for Question Nos. 28 and 29

Question Stem

$$\xrightarrow[H_2SO_4,\Delta]{HNO_3+} \underset{\substack{dinitro\\product}}{[K]} \xrightarrow{(NH_4)_2S} [L] \xrightarrow[0°-5°C]{NaNO_2,HCl} [M] \xrightarrow{H_2O} [N]$$

$$\xrightarrow[\substack{2)NaNO_2/HCl\\0°-5°C\\3)CuCN/KCN\\4)CH_3MgBr\\5)H_3O^+\\6)I_2/Ca(OH)_2\\7)\Delta}]{1)Fe/HCl} [P]$$

28. Degree of unsaturation in the product P is?

29. How many maximum atoms are present in one plane for product 'N' in any possible conformation

Question Stem for Question Nos. 30 and 31

Question Stem

Experiment I:

Aluminium trifluoride is insoluble in anhydrous HF but dissolves on addition of NaF due to the formation of a soluble compound 'X'.

Experiment II:

When gaseous BF_3 is bubbled through the soluble compound 'X', a white crystalline compound (Y) is precipitated out.

Experiment III:

In Experiment II along with 'Y', 'Z' is also obtained as another product

30. Co-ordination number of the central atom of the anionic part of 'X' is____

31. Co-ordination number of the central atom of the anionic part of 'Z' is_____

SECTION-3(Maximum Marks: 12)
Paragraph with Single Answer Type

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +3 If ONLY the correct option is chosen;

Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks: −1 In all other cases.

Paragraph-I:

The two major contributors of conformers of 1,2-dichloroethane are anti and gauche. At 32^0C in gas phase, the measured dipole moment of 1,2-dichloroethane is 1.12 D. The dipole moment of a mixture of X and Y is given by the relationship

$$\mu^2 = N_x\mu_x^2 + N_y\mu_y^2$$

Here, N= mole fraction of each kind of molecule. From bond moment measurement, it has been estimated that gauche conformer of 1,2-dichloroethane should have a dipole moment of about 3.2 D.

32. What percent of conformers of 1,2-dichloroethane is anti, at $32\,^oC$?

A) 12 %   B) 36 %   C) 60 %   D) 88%

33. What happens if temperature is increased to $52\,^oC$?

A) Percentage gauche conformers decrease and that of anti-conformers increases
B) Percentage of both gauche and anti-conformers increases
C) Percentage of anti-conformers decreases and percentage of gauche increases
D) Percentage of both anti and gauche conformers decreases

Paragraph-II:

An element(A) reacts quickly with water liberating a coloulress, odourless gas and a solution of 'B'.

Passing CO_2 gas through 'B' yields a white precipitate 'Y' which dissolves on passing excess CO_2 gas.

Precipitate 'Y' effervesced with dilute HCl and gave a brick red colouration to a Bunsen burner flame.

Heating 'Y' gave a caustic white compound 'D'(as one of the product) which when further heated with carbon at 1000^0C, gave a solid 'E'(as one of the product) of immense commercial importance.

34. Weight (in grams) of nitrogen containing product(X) obtained if 6.4 grams of 'E' is heated with excess N_2:

(Ignore the weight of unreacted N_2 present after the reaction)

A) 16   B) 8   C) 32   D) 4

35. Identify the incorrect statement regarding 'X' which is obtained in the previous question number 34?

A) Anion of 'X' is isoelectronic and isostructural with CO_2

B) Anion of 'X' is a pseudo halide ion

C) Anion of 'X' contains 3σ and 2π bonds

D) 'X' on hydrolysis with steam (H_2O) gives NH_3 as one of the product
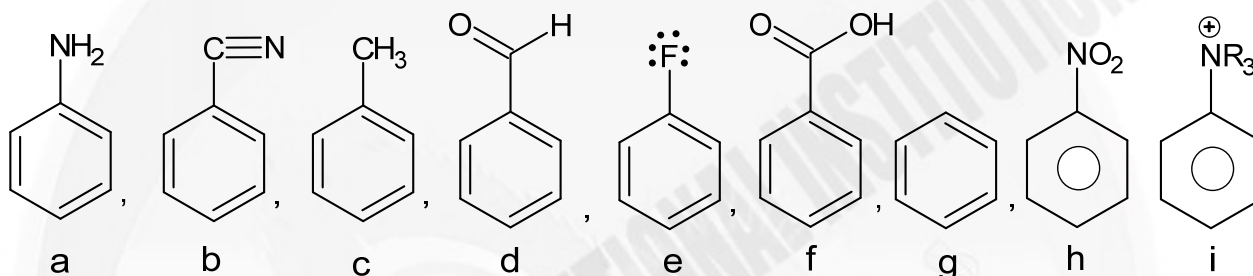
SECTION-4(Maximum Marks: 12)
Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
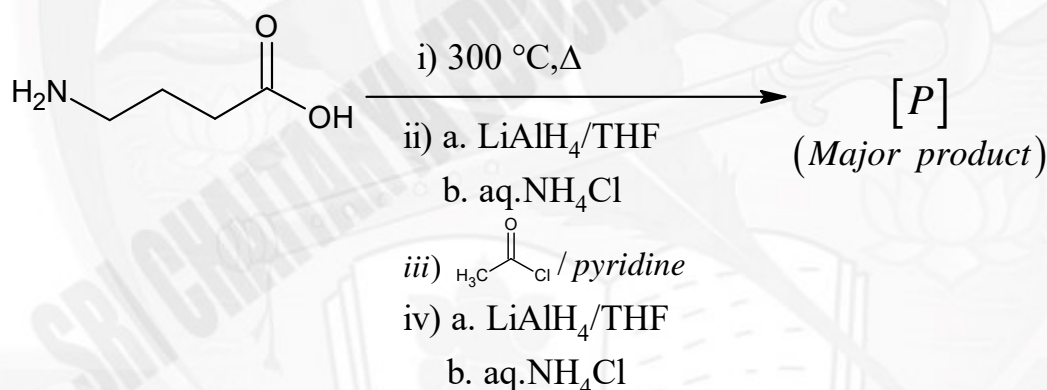- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +4 If ONLY the correct integer is entered;

Zero Marks: 0   In all other cases.

36. Examine the structural formulas shown below and find out how many compounds cannot give Friedel Crafts reaction.

NH_2 (a), $C\equiv N$ (b), CH_3 (c), CHO (d), $\ddot{F}$ (e), COOH (f), (g), NO_2 (h), $\overset{\oplus}{N}R_3$ (i)

37.

H_2N—(CH$_2$)$_3$—COOH

i) 300 °C, Δ

ii) a. LiAlH$_4$/THF
    b. aq.NH$_4$Cl

iii) $H_3C-\overset{O}{\underset{Cl}{C}}$ / pyridine

iv) a. LiAlH$_4$/THF
    b. aq.NH$_4$Cl

→ $[P]$ (Major product)

If x is the total number of secondary carbon atoms present in the product P and y is the total number of oxygen atoms present in product P. Then find the value of x+y _____

38. Find the number of species having X – O – X linkage: (X = Si, S)

Anion of beryl, Anion of Marshall's acid, Anion of Caro's acid,

Disulphate ion, Tetrathionate ion, Thiosulphate ion

MATHEMATICS   Max. Marks: 60

SECTION-1(Maximum Marks: 24)
One or More Type

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks: +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks: +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks: +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks: 0 If unanswered;
  Negative Marks: −2 In all other cases.

39. Let $f(x) = \lim\limits_{m \to \infty} \left\{ \sqrt[n]{e^{\sin^2 x} \sqrt[n]{e^{\sin^2 x} \sqrt[n]{e^{\sin^2 x} \sqrt[n]{e^{\sin^2 x} \dots \sqrt[n]{e^{\sin^2 x}}}}}} \right\}$ where $n = \sec^2 x$ and given

m is the number of radicals in $\sqrt[n]{e^{\sin^2 x} \sqrt[n]{e^{\sin^2 x} \sqrt[n]{e^{\sin^2 x} \sqrt[n]{e^{\sin^2 x} \dots \sqrt[n]{e^{\sin^2 x}}}}}}$,

then select INCORRECT alternative(s) (consider meaning of radical $\sqrt[y]{z}$ means

$z^{1/y}, \forall y \in R^+$)

A) The number 1 is in the range of function $f(x)$ and it is the minimum value of the

function

B) The function is unbounded

C) The number e is in the range of the function, and it is the maximum value of the

function

D) There is only one integer in the range of $f(x)$

40. Given $A = \left\{ x^2 : x \in R \right\}, B = \left\{ \log_2^{3+x^2} : x \in R \right\}$ and $C = \left\{ -x^2 + 3x - 2 : x \in R \right\}$

Given three functions $f(x), g(x), h(x)$ with domain D, where D is $A \cup B \cup C$. The

given functions will follow the inequality $f(x) \le g(x) \le h(x) \ \forall x \in D - \{\alpha\}$, where α is

fixed positive integer.

And $\lim\limits_{x \to \alpha} f(x) = \lim\limits_{x \to \alpha} h(x) = l \ and \ f(\alpha) = h(\alpha) = l$

then which of the following is/are not INCORRECT?

A) The value of $\lim\limits_{x \to \alpha} g(x)$ is l.

B) The value of $g(\alpha)$ is l

C) The value of $\lim\limits_{x \to \alpha} \left[g(x) \right]$ must be l, where $[.]$ is greatest integer function

D) $f(\alpha) = g(\alpha) = h(\alpha) = l$

41. Given two lines

$$L_1 : \quad \frac{x - 2023}{a_1} = \frac{y - 2022}{b_1} = \frac{z + 2021}{c_1},$$

$$L_2 : \quad \frac{x - 2023}{a_2} = \frac{y - 2022}{b_2} = \frac{z - 2021}{c_2}$$

And L_1, L_2 are parallel lines $\left(a_i, b_i, c_i \neq 0, \forall i = 1, 2 \right)$ and a_1, b_1, c_1 are direction cosines of L_1, and a_2, b_2, c_2 are direction cosines of L_2. And $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \lambda, \text{ where } \lambda \in R - \{0\}$

which of the following options is/are CORRECT?

A) The value of $\lim\limits_{x \to \lambda} \dfrac{x^2 - 4}{x - 2}$ can be 2023

B) The value of $\lim\limits_{x \to \lambda} \dfrac{x^2 - 4}{x - 2}$ can be 2022

C) The value of $\lim\limits_{x \to \lambda} \dfrac{x^2 - 4}{x - 2}$ can be 1

D) The value of $\lim\limits_{x \to \lambda} \dfrac{x^2 - 4}{x - 2}$ can be 3

42. A ray is sent from the point $(1, 1, 2)$ along the line $\dfrac{x - 1}{1} = \dfrac{y - 1}{2} = \dfrac{z - 2}{3}$ and it reflected from the plane $x + y + z = 10$, then which of the following options is/are CORRECT?

A) Reflected ray lies along the line is $\dfrac{x - 2}{3} = \dfrac{y - 3}{2} = \dfrac{z - 5}{1}$

B) Reflected ray lies along the line is $\dfrac{x - 11}{3} = \dfrac{y - 9}{2} = \dfrac{z - 8}{1}$

C) Point$(-1, 1, 4)$ lies on reflected ray.

D) Point$(8, 7, 7)$ lies on reflected ray.

43. Suppose that $f : R \to R$ is a continuous function and $f(x)f(f(x)) = 1$, for all real x.

Let $f(10) = 5$. Then for all such functions, which of the following statements is/are CORRECT?

A) $f(4) = \dfrac{1}{4}$

B) $f(6) = \dfrac{1}{6}$

C) $f(x) = \dfrac{x}{4}$ has at least 3 solutions

D) $f(x)$ can have at most of 9 integers in its range

44. Let a 2×2 matrix A satisfy $A^2 - 1013A + 2022I = O$, where I and O are 2×2 identity and null matrices respectively. Here tr(A) denotes the sum of principal diagonal elements of a matrix A and $|A|$ stands for the determinant of A. Then, which of the following option(s) can be CORRECT?

A) $\text{tr}(A)$ can be 1013   **B)** $\text{tr}(2A)$ can be 8

C) $|A|$ can be 2022   **D)** $|A|$ can be $(1011)^2$

SECTION-2(Maximum Marks: 12)
Paragraph with Numerical

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +2   If ONLY the correct numerical value is entered at the designated place;
Zero Marks: 0   In all other cases.

Question Stem for Question Nos. 45 and 46

Question Stem

For a differentiable function in $[a,b]$,

The global maximum values is given by $\max\{f(\alpha), f(\beta), f(\gamma), f(a), f(b)\}$ where α, β, γ are local extremum points of $f(x)$.

The global minimum values is given by $\min\{f(\alpha), f(\beta), f(\gamma), f(a), f(b)\}$ where α, β, γ are local extremum points of $f(x)$.

45. The global minimum value of

$$\left|\cot x - 1\right| + \left|\cot x - 2\right| + \left|\cot x - 31\right| + \left|\cot x - 32\right| + \left|\cot x - 24\right| + \left|\cot x - 5\right| + \left|\cot x - 6\right|$$

$$+ \left|\cot x - 17\right| + \left|\cot x - 8\right| + \left|\cot x - 9\right| + \left|\cot x - 10\right| + \left|\cot x - 11\right| + \left|\cot x - 12\right|$$

occurs at $x = \sec^{-1}\left(\dfrac{\alpha}{\beta}\right)$, then the value of $\left[\dfrac{\alpha^2}{\beta^2}\right]$ is, (where [.] denotes the greatest

integer function)

46. If $f(x) = \left\|x - 1\right| + \left|x - 3\right| - \left|2x - 1\right\| + \left\|x - 1\right| + \left|x - 3\right| + \left|2x - 1\right\|$, then minimum value of $f(x)$

is

Question Stem for Question Nos. 47 and 48

Question Stem

The number of solutions of the equation $f(x) = g(x)$ can be found by plotting the curves

of $y = f(x)$ and $y = g(x)$ and checking their points of intersection.

47. The number of solution(s) of $\displaystyle\sum_{r=1}^{2023} r^{2023x} = 2024^{2023x}$ is/are

48. Let g, f, h functions are defined as follows

$$g : R - \{0\} \to R, g(x) = x - \dfrac{1}{x},$$

$$f : \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 8, 9\} \to R, f(x) = \tan\left(\tan^{-1} x\right), \text{ and}$$

$$h : R \to R, h(x) = \begin{cases} x & x \in [1,8] \\ 2x + 1 & x \in (8, \infty) \end{cases}.$$

Let α, β be the number of real solutions of $x, x \in R$ which satisfy the equations

$f(x) = g(x)$ and $f(x) = h(x)$ respectively, then the value of $\alpha + \beta$ is...

Question Stem for Question Nos. 49 and 50

Question Stem

Let $f : R \to R$ is a function defined by $f(x) = \begin{cases} 1, & \text{if } x = 1 \\ e^{\left(x^{10} - 1\right)} + (x - 1)^2 \sin\left(\dfrac{1}{x - 1}\right), & \text{if } x \neq 1 \end{cases}$

49. The value of $f'(1)$ is

50. If $\displaystyle\lim_{x \to \infty}\left(x\left(\sum_{k=1}^{100} f\left(1 + \dfrac{k}{x}\right) - 100\right)\right) = \lambda,$ then the value of $\dfrac{\lambda}{100}$ is

SECTION-3(Maximum Marks: 12)
Paragraph with Single Answer Type

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +3 If ONLY the correct option is chosen;
Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks: −1 In all other cases.

Paragraph-I:

If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non collinear, unit vectors. Such that a non-zero vector $\vec{r}$ satisfies the relation

$$\vec{a} + 2\vec{b} - 3\vec{c} = \vec{r}.\left[\left(\vec{b} - 2\vec{a}\right) \times \vec{c} + \left(2\vec{c} + 2\vec{b}\right) \times \vec{a}\right]\left(\vec{a} + 2\vec{b}\right) + \vec{r}.\left[\left(\vec{c} - 2\vec{b}\right) \times \vec{a} + \left(2\vec{a} + 2\vec{c}\right) \times \vec{b}\right]\left(\vec{b} + 2\vec{c}\right)$$

$$+\vec{r}.\left[\left(\vec{a} - 2\vec{c}\right) \times \vec{b} + \left(2\vec{b} + 2\vec{a}\right) \times \vec{c}\right]\left(\vec{c} + 2\vec{a}\right)$$

51. If projection of $\vec{r}$ on $\left(\vec{b} \times \vec{c}\right)$ is $\dfrac{2}{9}$ units, then the product of all possible angles between $\vec{b}$ and $\vec{c}$ is…

A) $\dfrac{5\pi^2}{36}$ 　　B) $\dfrac{5\pi^2}{18}$ 　　C) $\dfrac{3\pi^2}{16}$ 　　D) $\dfrac{7\pi^2}{36}$

52. If $\vec{a}.\vec{b} = 12\left[\vec{a}\ \vec{b}\ \vec{c}\right], \left(\vec{a}.\vec{c}\right) = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right],$, where $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \dfrac{1}{25}$, then the length of projection of $\vec{r}$ on $\vec{a}$ is

A) $\dfrac{41}{27}$ 　　B) $\dfrac{43}{18}$ 　　C) $\dfrac{35}{9}$ 　　D) $\dfrac{43}{9}$

Paragraph-II:

A 2×2 involutary matrix is formed with the elements in $\{-3, -2, -1, 0, 1, 2, 3\}$ (repetition allowed). Based on this, answer the following: (Here tr(A) defines the sum of principle diagonal elements and $|A|$ denotes the determinant of the matrix A.)

53. The number of such matrices is

A) 32 　　B) 34 　　C) 38 　　D) 36

54. For all such matrices, the number of distinct values of $\dfrac{\left(tr(A)\right)^2}{1 + |A|}$, whenever the expression is defined, is

A) 1 　　B) 2 　　C) 0 　　D) 28

SECTION-4(Maximum Marks: 12)
Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +4 If ONLY the correct integer is entered;
Zero Marks: 0   In all other cases.

55. Out of four statements below, some statement(s) are correct and some are incorrect. If the sequence of their truth value is (say) TFTT i.e., S_1 is True, S_2 is False, S_3 is True, S_4 is True, then enter 1011 in response.(For e.g. if the sequence of their truth value is FFTT then enter 0011=11,FTFF then enter 0100=100 etc, 1=0001 would mean FFFT,101=0101 would mean FTFT etc)

$S_1:$ $\lim\limits_{x\to 0^+}\dfrac{e^{x\ln x}-e^{[\cos x]}}{x\ln x}=0,$ where $[\,.\,]$ represents greatest integer function.

$S_2:$ Given $\lim\limits_{x\to a}\big(f(x)+g(x)\big)$ and $\lim\limits_{x\to a}\big(f(x)-g(x)\big)$ exist, then it is not necessary that $\lim\limits_{x\to a}f(x)$ and $\lim\limits_{x\to a}g(x)$ both exist separately.

$S_3:$ The value of $\lim\limits_{x\to 0}\dfrac{(1-\cos ax)}{x^2}$ is $\dfrac{a^2}{4}$

$S_4:$ Given function $f:R-\{0\}\to R, f(x)=\dfrac{(\sin ax)(1-\cos 2ax)\left(e^{2x}-1\right)}{x^4}$, $a\in R$ then the value of $\lim\limits_{a\to 0}\dfrac{(\sin ax)(1-\cos 2ax)\left(e^{2x}-1\right)}{x^4}$ is $4a^3$

Write the correct sequence of the above four statements as per the given order...?

56. Let S_1 be the locus of point P such that $\angle APB=\dfrac{\pi}{2}$ where $A(3,-4)$ and $B(-3,4)$. Then the number of distinct possible value(s) of absolute value of $3x-2y$ where $x\in I, y\in I$ and $(x,y)\in S_1$ is/are ___

57. Given lines $L_1:x+2y=1; L_2:2x-y=2$. Let S_1 be the set of all the lines passing through point of intersection of L_1 and L_2. Let L be the line which belongs to S_1, which is at a distance of $\sqrt{5}$ from the point $(2,-3)$. Then the sum of length(s) of y intercept of all possible distinct line(s) of L is P. then the value of 2P is ___ (If only one L is possible then write answer is length of y intercept of L)