Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS & STERLING_BT Paper -1(Adv-2020-P1-Model) Date: 17-09-2023

Time: 09.00Am to 12.00Pm RPTA-07 Max. Marks: 198

KEY SHEET

PHYSICS

1	**A**	2	**D**	3	**B**	4	**C**	5	**A**	6	**C**
7	**ABC**	8	**AB**	9	**ABC**	10	**BC**	11	**ABC**	12	**BC**
13	**1**	14	**0.50**	15	**2.20**	16	**0.81**	17	**0.33**	18	**4.23**

CHEMISTRY

19	**D**	20	**C**	21	**B**	22	**C**	23	**D**	24	**D**
25	**ABCD**	26	**BCD**	27	**AC**	28	**ABCD**	29	**ABCD**	30	**ABCD**
31	**6**	32	**4900**	33	**135**	34	**4**	35	**23**	36	**6**

MATHEMATICS

37	**B**	38	**B**	39	**B**	40	**A**	41	**B**	42	**A**
43	**CD**	44	**BD**	45	**BC**	46	**CD**	47	**ABD**	48	**AD**
49	**3**	50	**1**	51	**7**	52	**2**	53	**0**	54	**2**

SOLUTIONS

PHYSICS

01. $I = I_{base} + I_{y'}$

$$= \frac{M\left(\frac{L}{2}\right)^2}{6} + \left(\frac{ML^2}{24} + M\left(\frac{L}{2}\right)^2\right) \qquad = \frac{ML^2}{3}$$

02. When the rod becomes vertical, apply conservation of energy

$\frac{MgL}{2} = \frac{1}{2}\frac{ML^2}{3}w^2 \Rightarrow w = \sqrt{3g/L}$

Centre of mass of the lower two-third part moves in a circle of radius 2L/3

We apply Newton's second law on this part

$T - \frac{2Mg}{3} = \left(\frac{2M}{3}\right)\left(\frac{2L}{3}\right)w^2$ (or) $T = 2Mg$. Hinge reaction = 2.5 Mg $\therefore \ \Delta T = \frac{Mg}{2}$

03. coriolis force $= 2M\left(-u\hat{i} \times w\hat{k}\right)$

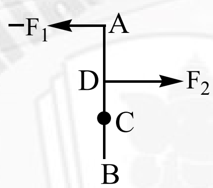
$= 2Muw\hat{j} \qquad \therefore \qquad x = -2MuwL$

04. At the top $\frac{mv^2}{R} = mg$

$mg\left(h - 2R\right) \qquad = \frac{3}{4}mv^2 \qquad \therefore \qquad h = \frac{11R}{4}$

05. Let x be the distance of centre point C of rod from D.

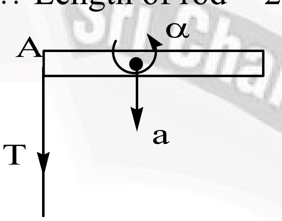
$F_2 - F_1 = ma$ or $F_1 = 3N$



Further, $\tau_C = 0 \qquad \therefore F_2 x = F_1(0.2 + x)$ (or) $x = 0.3m$

$\therefore$ Length of rod = 2 (x + 0.2) = 1.0m

06.



$a = \frac{T}{2m}; \ T\ell = \frac{2m(2\ell)^2}{12}\alpha$

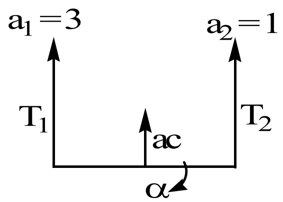
Acceleration of point

$A = a + \ell\alpha = \frac{T}{2m} + \frac{3T}{2m} = \frac{2T}{m}$

w.r.t A particle M moves in circle

$\therefore$ from frame of A $T + m \times \frac{2T}{m} = \frac{mv_0^2}{\ell} \qquad \therefore T = \frac{mv_0^2}{3\ell}; \ \alpha = \frac{v_0^2}{6\ell}; \ \alpha = \frac{v_0^2}{2\ell^2}$

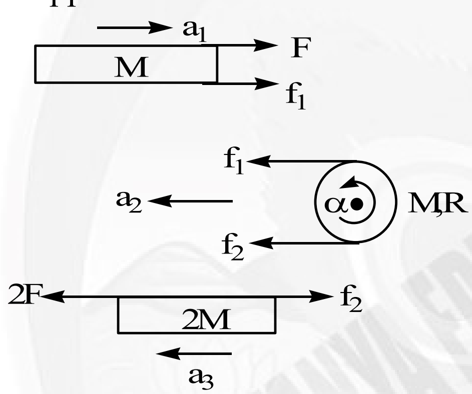
07. $a_C + L\alpha/2 = a_1$; $a_C - L\alpha/2 = a_2$



$\alpha = \dfrac{a_1 - a_2}{L} = 2\,rad/s^2$

08. Anticlockwise torque due to N reduces angular momentum

09. Point of contact P is at rest. Taking torque of all the forces about point P we see that:
Torque is clockwise if F_1 is applied i.e., spool will rotate clockwise.
Torque of F_2 is zero i.e., spool will not rotate if F_2 is applied.
Torque of $F_{3=}$ and $F4=$ is anticlockwise i.e., spool will rotate anticlockwise if only F_3 and F_4 is applied.



10.

Equations of motion are, $F + f_1 = Ma_1$, $f_1 + f_2 = Ma_2$

$2F - f_2 = 2Ma_3$ $\alpha = \dfrac{(f_1 - f_2)R}{\frac{1}{2}mR^2}$

For no slipping $a_2 + R\alpha = -a_1$; $a_2 - R\alpha = a_3$

Solving above equations we get $a_1 = \dfrac{21F}{26M}$ and $a_2 = \dfrac{F}{26M}$ $f_1 = -\dfrac{5F}{26}$

11. $mg\ell \times \dfrac{d\omega}{dt} = I \times \omega \times \omega^1$

$\omega^1 = 2\pi n = \pi$, So $\omega^1 = \dfrac{g^1\ell}{\pi R^2 n} = 4\times10^2$ rad/s

12. There are two angular motions of the cone, one in the horizontal plane with angular velocity $\omega_1 = \dfrac{v}{R\cot\alpha}$ where $R\cot\alpha =$ radius of the circle in which c moves. The direction of this vector is upward. The other angular velocity ω_2 is about its own axis. Since it rolls without slipping, $v = \omega_2 R \Rightarrow \omega_2 = \dfrac{v}{R}$. The direction of this vector is horizontal and towards O.

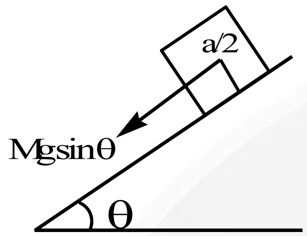
$\omega_{results} = \sqrt{\omega_1^2 + \omega_2^2} = \sqrt{\dfrac{v^2}{R} + \dfrac{v^2}{R}\tan^2\alpha};\ \dfrac{v}{R}\sqrt{1+\tan^2\alpha} = \dfrac{v}{R}\sec\alpha$

Substituting the values of $V\,\&\,R$ $100\sqrt{2}$

Let θ be the angle made by the resultant with the vertical. Then

$\tan\theta = \frac{\omega_2}{\omega_1} \quad \theta = 45^0$

13. $\tau_0 = \vec{r} \times \vec{F} = R \times mg \qquad = \frac{u^2 \sin 2\theta}{g} \times mg = mu^2 \sin 2\theta$

14. 

$\tau = \frac{a}{2} \times Mg(\sin\theta + \cos\theta)$ due to shift of N

15. μ_{min} for pure rolling $= \tan\theta\left(\frac{\beta}{1+\beta}\right) = 0.4$

Given $\mu = 0.2 < 0.4$

$$W_1 = \frac{12\mu^2 mg}{1-\mu} = 0.6$$

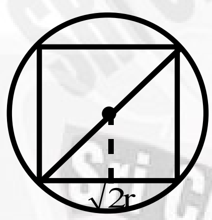
$$W_2 = -8\mu mg = -1.6$$

16. Torque about point of contact

$\tau = I_{P\alpha;}\alpha;\ mg\frac{\ell}{2} = \frac{4}{3}m\ell^2; F = ma$. $\alpha = \frac{3g}{8\ell}$ $\therefore\ N = \frac{13mg}{16} = 0.81\ mg$

17. $mg \times \frac{\ell}{2}\sin 60^0 = Mg \times \frac{\ell}{2}\sin 30^0$

$m \times \sqrt{3} = M \times 1; \frac{M}{m} = \sqrt{3}\ \therefore\ \frac{m}{M} = \sqrt{0.33}$

18. 

I = moment of inertia of system about centre of mass of system

$$= 4\left[\frac{1}{12}m(\sqrt{2}r)^2 + m\left(\frac{r}{\sqrt{2}}\right)^2\right] + mr^2 = \frac{11mr^2}{3} = 5mK^2$$

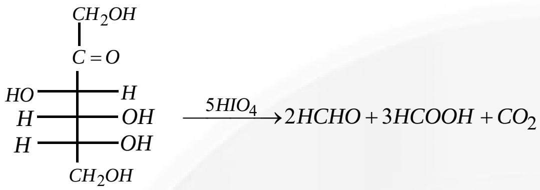
$$\mu = \frac{Tan\theta}{1+\frac{R^2}{K^2}} = \frac{11}{26} = 0.42$$

CHEMISTRY

19. FACT

20. at $P^H = 5.6$, net charge on threomine is zero, glutamic acid converte in to amion, & histidine exists as cation.

21.



22. In fibers and ealstomers interactive forces are not equal.

23. All the molecules give same osazame.

24. Without CN^-, X^-, S^{2-}, Lessaigne's test is not possible.

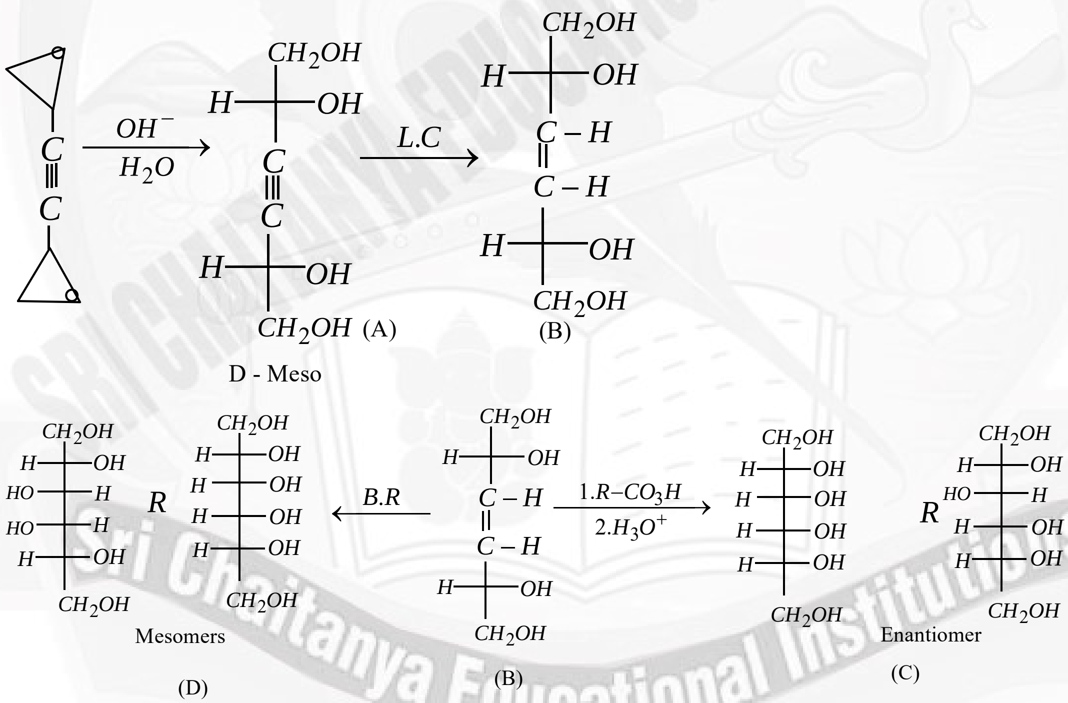
25. FACT

26. FACT

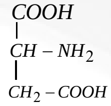
27. FACT

28. Kjeldahl method is not applicable to nitro, azo compounds.

29.

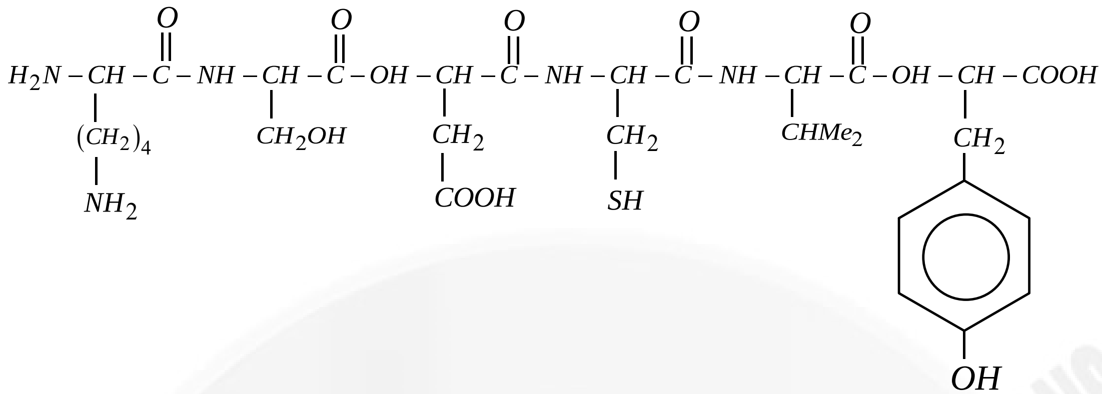


30. F is aspartic acid



$$P^I = \frac{P^{k_1} + P^{k_2}}{2} = \frac{1.88 + 3.65}{2} = 2.765$$

31.



At $P^H = 8$ only carboxylic acid functions will be deprotonated.

At $P^H = 12.5$ carboxylic acid functions, phenolichydroxy, thiol function will be deprotonated.

32. $M_1 = 2800; M_2 = 5600$

$$\bar{M}_w = \frac{2\times 2800^2 + 3\times 5600^2}{2\times 2800 + 3\times 5600} = 4900$$

33. $\%N = \frac{1.4}{10}\times 100 \quad = 14$

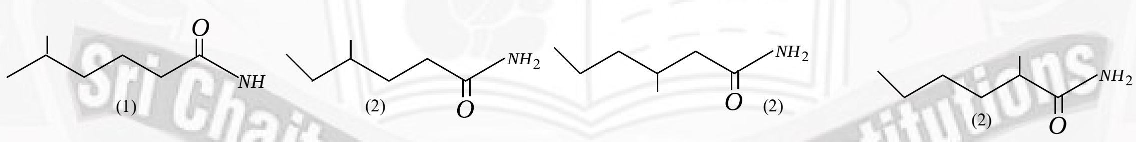
$\%C = 63 \qquad \%H = 11 \qquad \%O = 12$

$C \to \frac{63}{12} = 5.25 \to \frac{5.25}{0.75} = 7 \qquad H \to \frac{11}{1} = 11 \to \frac{11}{0.75} \approx 15$

$N \to \frac{14}{14} = 1 \to \frac{1}{0.75} \approx 1 \qquad O \to \frac{12}{16} = 0.73 \to \frac{0.75}{0.75} = 1$

$M.F \to C_7H_{15}ON$

Monomethyl primaryamides.



34. CONCEPTUAL

35. Total no. of $AAr = 12$

Total no. of different $AAr = 8$

Total no. of water molecules produced = 12

Total no. of glycine units = 3

36. FACT

MATHEMATICS

37. $f(n)=n+\sum_{r=1}^{n}\frac{16r+(9-4r)n-3n^2}{4rn+3n^2}=n+\sum_{r=1}^{n}\frac{(16r+9n)-(4n+3n^2)}{4rn+3n^2}=\sum_{r=1}^{n}\frac{(16r+9n)}{4rn+3n^2}$

$\Rightarrow \lim_{n\to\infty} f(n)=\lim_{n\to\infty}\Sigma\frac{16r+9n}{4rn+3n^2}=\lim_{n\to\infty}\frac{\left(16\left(\frac{r}{n}\right)+9\right)\frac{1}{n}}{4\left(\frac{r}{n}\right)+3}=\int_0^1\frac{16x+9}{4x+3}dx=\int_0^1 4dx-\int_0^1\frac{3dx}{4x+3}$

$=4-\frac{3}{4}\left(\ell n|4x+3|\right)_0^1=4-\frac{3}{4}\ell n\frac{7}{3}$

38. Let $k=\int_2^4 xh(x)dx$

Using by parts, $k=\left(xf^{-1}(x)\right)_2^4-\int_2^4 f^{-1}(x)dx=4f^{-1}(4)-2f^{-1}(2)-\int_2^4 f^{-1}(x)dx$

$\because \quad f^{-1}(4)=1, f^{-1}(2)=0$

We know, $\int_a^b f(x)dx+\int_{f(a)}^{f(b)} f^{-1}(x)dx=bf(b)-af(a)$

$\int_0^1 f(x)dx+\int_2^4 f^{-1}(x)dx=1.4-0.2=4$

Now, $\int_0^1 f(x)dx=\int_0^1\left(x^3+x+\sin 2\pi x+2\right)dx$

$=\left(\frac{x^4}{4}+\frac{x^2}{2}-\frac{\cos 2\pi x}{2\pi}+2x\right)_0^1=\left(\frac{1}{4}+\frac{1}{2}-\frac{1}{2\pi}+2\right)-\left(-\frac{1}{2\pi}\right)=\frac{11}{4}$

So, $\frac{11}{4}+\int_2^4 f^{-1}(x)dx=4\Rightarrow \int_2^4 f^{-1}(x)dx=\frac{5}{4}$

Hence, $k=4-\frac{5}{4}=\frac{11}{4}$

39. We have $|f(x)-f(y)|\le|x-y|^{100}$

$\Rightarrow \quad \left|\frac{f(x)-f(y)}{x-y}\right|\le|x-y|^{99}$

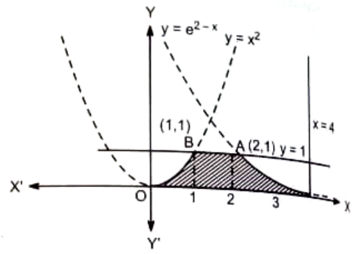
Now, $\left|\lim_{y\to x}\frac{f(x)-f(y)}{x-y}\right|\le\lim_{y\to x}|x-y|^{99}$

$\Rightarrow \quad |f'(x)|\le 0$

i.e., $\quad f'(x)=0$

$\Rightarrow \quad f(x)=c$ (constant)

Given, $f(1000)=1 \quad \Rightarrow \quad c=1 \quad \therefore \quad f(x)=1$



Now, $y=\min.\left\{f(x),x^2,e^{2-x}\right\} \quad =\min.\left\{1,x^2,e^{2-x}\right\}$

$\therefore$ The required area $=\int_0^1 x^2dx+(2-x)\times 1+\int_2^4 e^{2-x}dx$

$=\left[\frac{x^3}{3}\right]_0^1+1+e^2\left[\frac{e^{-x}}{-1}\right]_2^4 \quad =\frac{1}{3}+1+e^2\left(-e^{-4}+e^{-2}\right)$

$=\frac{4}{3}+1-e^{-2} \quad =\left(\frac{7}{3}-\frac{1}{e^2}\right)$ sq. units

40. The given differential equation can be written as $y'(x)=y(x)+\int_0^1 y(x)dx$

Differentiating both sides w.r.t. x, then $y''(x)=y'(x)$

Or $\frac{y''(x)}{y'(x)}=1$ Or $\int\frac{y''(x)}{y'(x)}dx=\int dx$ Or $\ln\, y'(x)=x+\ln c$

Or $\ln\left(\frac{y'(x)}{c}\right)=x$, Or $y'(x)=c\,e^x$

And $y(x)=ce^x+d \Rightarrow \quad y(0)=c+d=1 \quad (\because\ y(0)=1)$

$\therefore \quad y(x)=ce^x+1-c$

From (i), (ii) and (iii), we get $c\,e^x=c\,e^x+1-c+\int_0^1\left(c\,e^x+1-c\right)dx$

$\Rightarrow \quad c-1=\left[ce^x+(1-c)x\right]_0^1 \quad \Rightarrow \quad c-1=ce+1-c-c$

$\therefore \quad c=\frac{2}{3-e}$

From (iii),

$y(1)=ce+1-c=\frac{e+1}{3-e}$ [From (iv)]

$=\frac{2718+1}{3-2718}=\frac{3.718}{0.282}=13.18$

$[y(1)-7]=[6.18]=6$

41. We have

$f(x)=\sin x+\cos x=\sqrt{2}\sin(x+\pi/4)$

And $g(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 2, & x=0\end{cases} \quad =\begin{cases}1, & x>0\\ -1, & x<0\\ 2, & x=0\end{cases}$

$\therefore \qquad gof(x)=g(f(x))=\begin{cases}1, & f(x)>0\\ -1, & f(x)<0\\ 2, & f(x)=0\end{cases}$

$=\begin{cases}1, & x\in\left(-\frac{\pi}{4},\frac{3\pi}{4}\right)\cup\left(\frac{7\pi}{4},2\pi\right)\\ -1, & x\in\left(\frac{3\pi}{4},\frac{7\pi}{4}\right)\\ 2, & x=-\frac{\pi}{4},\frac{3\pi}{4},\frac{7\pi}{4}\end{cases}$

$\therefore \qquad \int_{-\pi/4}^{2\pi} gof(x)\,dx=\int_{-\pi/4}^{3\pi/4}(-1)\,dx+\int_{7\pi/4}^{2\pi}(1)\,dx$

$=1.\left(\frac{3\pi}{4}-\left(-\frac{\pi}{4}\right)\right)-\left(\frac{7\pi}{4}-\frac{3\pi}{4}\right)+\left(2\pi-\frac{7\pi}{4}\right) \qquad =\pi-\pi+\frac{\pi}{4}=\frac{\pi}{4}$

42. $2\int_1^2\frac{dx}{(x-1)^2+1}=2\int_0^1\frac{dt}{1+t^2}=\frac{\pi}{2}$

43. $\frac{x}{y}=x^y \Rightarrow \ln x-\ln y=y\ln x \ \Rightarrow \qquad (1-y)dx=\left(\ln x^x+\frac{x}{y}\right)dy$

$\therefore \qquad I=\int dy;\ J=\int dx$

44. Let $I=\int\sqrt{\left(\frac{1-\cos x}{\cos\alpha-\cos x}\right)}dx$

$=\int\sqrt{\left(\frac{(1-\cos x)(1+\cos x)}{(\cos\alpha-\cos x)(1+\cos x)}\right)}dx \qquad =\int\frac{\sin x}{\sqrt{(\cos\alpha-\cos x)(1+\cos x)}}dx$

Put $1+\cos x=t^2 \qquad =-2\int\frac{dt}{\sqrt{(1+\cos\alpha)-t^2}}$

$=2\cos^{-1}\left(\frac{\cos x/2}{\cos\alpha/2}\right)+c$ or $-2\sin^{-1}\left(\frac{\cos x/2}{\cos\alpha/2}\right)+c$

45. A.T.Q

$\int_0^\alpha\left(x^2-x^5\right)dx=\int_0^1\left(x^2-x^5\right)dx \qquad 2\int_0^\alpha\left(x^2-x^5\right)dx=\int_0^1\left(x^2-x^5\right)dx$

$2\left[\frac{x^3}{3}-\frac{x^6}{6}\right]_0^\alpha=\left[\frac{x^3}{3}-\frac{x^6}{6}\right]_0^\alpha \quad 2\left[\frac{\alpha^3}{3}-\frac{\alpha^6}{6}\right]=\left[\frac{1}{3}-\frac{1}{6}\right]$

$$2\frac{\left[2\alpha^3-\alpha^6\right]}{6}=\frac{1}{6} \Rightarrow \quad 2\alpha^6-4\alpha^3+1=0 \quad \frac{2\alpha^6+1}{4\alpha^2}=\alpha$$

Let $f(\alpha)=2\alpha^6-4\alpha^3+1$

$$f(3/4)=2\times\frac{9}{16}-4\times\frac{3}{4}+1=\frac{9}{8}+1-3<0 \qquad f(0)>0.$$

46. LET $I=\int\sec^2\theta(\sec\theta+\tan\theta)^2\,d\theta$

PUT $\tan\theta=t \quad \therefore \quad \sec^2\theta d\theta=dt$, then

$$I=\int\left(t+\sqrt{\left(1+t^2\right)}\right)^2 dt$$

Now let $t+\sqrt{1+t^2}=z$

$$\Rightarrow \quad dt=\frac{1}{2}\left(1+\frac{1}{z^2}\right)dz \qquad \therefore \qquad I=\frac{1}{2}\int z^2\left(1+\frac{1}{z^2}\right)dz$$

$$=\frac{z}{6}\left(z^2+3\right)+c \quad =\frac{\left(t+\sqrt{\left(1+t^2\right)}\right)}{6}\left[\left(t+\sqrt{\left(1+t^2\right)}\right)^2+3\right]+c$$

47. $f(x)=\dfrac{e^x}{x^2}$

48. WE HAVE $\dfrac{dy}{dx}=\dfrac{\sin y+x}{\sin 2y-x\cos y} \Rightarrow \quad \cos y\dfrac{dy}{dx}=\dfrac{\sin y+x}{2\sin y-x}$

Put $\sin y=v \Rightarrow \cos y\dfrac{dy}{dx}=\dfrac{dv}{dx}$, then equation (i) reduces to $\dfrac{dv}{dx}=\dfrac{v+x}{2v-x}$

On integrating, we get $2\sin^2 y=2x\sin y+x^2+c \quad \therefore \quad \sin^2 y\ge\dfrac{2}{3}$

Or $\sin y\in\left[-1,-\sqrt{\dfrac{2}{3}}\right]\cup\left[\sqrt{\dfrac{2}{3}},1\right]$

49. $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\dfrac{x}{y}\right)dy=0$

$$\Rightarrow \quad dx+e^{\frac{x}{y}}dy+y\left(-\frac{x}{y^2}\right)e^{\frac{x}{y}}dy+e^{\frac{x}{y}}dx=0$$

$$\Rightarrow \quad dx+e^{\frac{x}{y}}dy+y.d\left(e^{\frac{x}{y}}\right)=0 \Rightarrow \quad dx+d\left(y.e^{x/y}\right)=0$$

$\Rightarrow \qquad x + y.e^{\frac{x}{y}} = c$

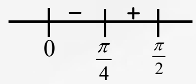
Passes through $(1,1)$, so $c = 1 + e$

Putting $x = 0; y = c = 1 + e$

i.e; $k = 1 + e \qquad \therefore \qquad [k] = [1 + e] = 3$.

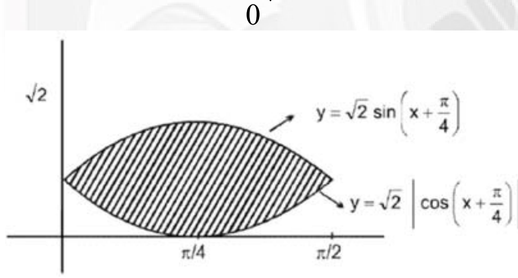
50. Given $y = \sin x + \cos x \qquad x \in [0, \pi/2]$

$$\frac{dy}{dx} = \cos x - \sin x$$



$$y = |\cos x - \sin x| = \begin{bmatrix} \cos x - \sin x & x \in \left[0, \frac{\pi}{4}\right] \\ \sin x - \cos x & x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \end{bmatrix}$$

Required area $= \int_0^{\frac{\pi}{4}} \left|(\sin x + \cos x) - (\cos x - \sin x)\right| dx + \int_{\pi/4}^{\pi/2} |2\cos x| dx$



$$= \int_0^{\pi/4} |2\sin x| dx + \int_{\pi/4}^{\pi/2} |2\cos x| dx \qquad = 2(-\cos x)_0^{\pi/4} + 2(\sin x)_{\pi/4}^{\pi/2}$$

$$= 2\left[-\frac{1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}}\right] \qquad = 2\left(2 - \frac{2}{\sqrt{2}}\right)$$

$$= 2\left(2 - \sqrt{2}\right) = 4 - 2\sqrt{2} = 2\sqrt{2}\left(\sqrt{2} - 1\right).$$

51. $I_n = \int \frac{\sin 7x}{\sin x} dx$

$$I_n - I_{n-2} = \int \frac{\sin x - \sin(n-2)x}{\sin x} dx \quad = 2\int \cos(n-1) dx$$

$$I_n - I_{n-2} = 2\frac{\sin(n-1)x}{n-1} + c$$

$$I_n = \frac{2\sin(n-1)x}{n-1} + I_{n-2}$$

$$I_7 = \int \frac{\sin 7x}{\sin x} dx \qquad = \frac{2\sin 6x}{6} + I_5$$

$$=\frac{2\sin 6x}{6}+\frac{2\sin 4x}{4}+I_3 \qquad =\frac{2\sin 6x}{6}+\frac{2}{4}\sin 4x+\frac{2\sin 2x}{2}+I_1$$

$$=\frac{\sin 6x}{3}+\frac{2}{4}\sin 4x+\sin 2x+x+c$$

52. $g(x)=-f(x)\sin x+C$

$$g\left(\frac{\pi}{2}\right)=0\Rightarrow +2+C=0 \qquad \Rightarrow \quad C=-2$$

$$\Rightarrow \quad g(x)=-f(x)\sin x-2 \qquad \Rightarrow \quad \lim_{x\to 0} g(x)=-2$$

53. $X=\lim\limits_{n\to\infty}\frac{1}{e^{n+2}}\int_0^n \frac{e^{-x}.e^{-x}dx}{1+e^{-x}\left(\frac{2}{e}-\frac{1}{e^3}\right)}$

$$\Rightarrow \quad X=\lim_{n\to\infty}\frac{1}{e^{n+2}}\int_{1/e^n}^{1}\frac{tdt}{1+t\left(\frac{2}{e}-\frac{1}{e^3}\right)} \quad \text{Put } t=e^{-x}$$

$$=\lim_{n\to\infty}\frac{1}{e^{n+2}}\left(t-\frac{\ln(at+1)}{a}\Bigg|_{\frac{1}{e^n}}^{1}\right) \quad \text{where } a=\frac{2}{e}-\frac{1}{e^3}$$

$$=\lim_{n\to\infty}\frac{1}{a}e^{n+2}\left(\left(1-\frac{\ln(a+1)}{a}\right)-\left(\frac{1}{e^n}-\frac{\ln\left(\frac{a}{e^n}+1\right)}{a}\right)\right)$$

54. $\int\frac{2\cos x+1}{(\cos x+2)^2}dx \qquad =\int\frac{2\cot x\operatorname{cosec} x+\operatorname{cosec}^2}{(\cot x+2\operatorname{cosec} x)^2}dx$