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A right Choice for the Real Aspirant

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AREAS UNDER THE CURVE

SYNOPSIS

➢ **Area bounded by curve and axes :**

1. The area of the region bounded by the curve $y = f(x)$, X-axis and the lines $x = a,\ x = b$ is $\left|\int_a^b f(x)dx\right|$ (If curve does not cross $x - axis$ between $x = a\ and\ x = b$)
2. The area of the region bounded by the curve $x = f(y)$, Y-axis and the lines $y = c,\ y = d$ is $\left|\int_c^d f(y)dy\right|$ (If curve does not cross $y - axis$ between $y = c\ and\ y = d$)
3. If $f(x) > 0\ \forall x \in [a, c]\ and\ f(x) < 0,\ \forall x \in [c, b]$, then the area bounded by the curve $y = f(x)$, X-axis the lines $x = a,\ x = b$ is $\int_a^c f(x)dx - \int_c^b f(x)dx$.

➢ **Area bounded by curve and line :**

4. Let $y = f(x)$ and $y = g(x)$ are two curves. Then the area between the two curves and the lines $x = a,\ x = b$ is $\int_a^b |f(x) - g(x)|dx$.
5. Let $y = f(x)$ and $y = g(x)$ are two curves intersect at $x = c(a < c < b)$ then the area between the given curves and $x = a,\ x = b$ is $\left|\int_a^c \left(f(x) - g(x)\right)dx\right| + \left|\int_c^b \left(f(x) - g(x)\right)dx\right|$

➢ **Area bounded between standard Geometrical figures :**

1. The area of the region bounded by $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16|ab|}{3}$ sq.units.
2. The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab sq.units.
3. The area of the circle $x^2 + y^2 = a^2$ is πa^2 sq.units

➢ **Area bounded by curves involving standard Algebraic function :**

1. The area of the region bounded by $y = ax^2 + bx + c\ and\ y = mx + k$ is $\frac{\Delta^{\frac{3}{2}}}{6a^2}$ sq.units.

   Where Δ is the discriminant of $ax^2 + (b - m)x + (c - k)$.

2. The area of the region bounded by $y = ax^2 + bx + c$ and $y = px^2 + qx + r$ is $\frac{\Delta^{\frac{3}{2}}}{6a^2}$ sq.units where Δ is discriminant of $ay^2 + (b-m)y + (c-k)$.

3. The area of the region bounded by $y = ax^2 + bx + c$ and $y = px^2 + qx + r$ is $\frac{\Delta^{\frac{3}{2}}}{6a^2}$ sq.units where Δ is discriminant of $(a-p)x^2 + (b-q)x + (c-r)$.

➢ **Area bounded by curves involving standard Trigonometric functions :**

1. The area of the region bounded by one arch of $Sin\ ax$ or $Cos\ ax$ and X-axis is $\frac{2}{|a|}sq.units$.

2. The area of the region bounded by the curve one arch of $y = Sin\ ax$ or $Cos\ ax$ and X-axis is $\frac{2n}{|a|}sq.units$

EXERCISE - I

SINGLE CORRECT ANSWER TYPE QUESTIONS

Total Area of Curve :

1. Area bounded by the curve $y = x\sin x$ and x-axis between $x=0$ and $x=2\pi$ is

A) 2π sq. unit B) 3π sq. unit C) 4π sq. unit D) 5π sq. unit

PRACTICE QUESTIONS

2. The area between the curve $y=\cos^2 x$, x-axis and ordinates $x=0$ and $x=\pi$ in the interval $(0,\pi)$ is

A) π sq. units B) $\frac{\pi}{4}$ sq. units C) $\frac{\pi}{2}$ sq. units D) 2π sq. units

Area bounded by curve and axes :

3. The area (in square units) of the region bounded by the curves $2x = y^2 - 1$ and $x=0$ is ____ sq.units.

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) 2

4. The area between the curve $y = x(x-1)(x-2)$ and x-axis is ….. sq. units

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 0

PRACTICE QUESTIONS

5. The area of the region bounded by $x^2 = 8y$, $x=4$ and $X-$axis is ______ sq.units

A) $\frac{2}{3}$ B) $\frac{4}{3}$ C) $\frac{8}{3}$ D) $\frac{10}{3}$

Area bounded by curve and line :

6. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$,then λ is equal to **[Main 2019]**

A) 48 B) 24 C) $4\sqrt{3}$ D) $2\sqrt{6}$

7. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is **[Main 2021]**

A) 4 B) 6 C) 9 D) 10

8. The area bounded by the curves $y = |x^2 - 1|$ and y = 1 is **[Main 2022]**

A) $\frac{2}{3}\left(\sqrt{2}+1\right)$ B) $\frac{4}{3}\left(\sqrt{2}-1\right)$ C) $2\left(\sqrt{2}-1\right)$ D) $\frac{8}{3}\left(\sqrt{2}-1\right)$

9. The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ and the x-axis is **[Main 2022]**

A) $\frac{9}{4}$ B) $\frac{45}{16}$ C) $\frac{27}{8}$ D) $\frac{63}{16}$

10. The area (in sq. units)bounded by the parabola $y = x^2 - 1$,the tangent at the point (2,3) to it and the y-axis is **[Main 2019]**

A) $\frac{32}{3}$ B) $\frac{8}{3}$ C) $\frac{56}{3}$ D) $\frac{14}{3}$

11. The area enclosed by the curves $y^2+4x=4$ and $y-2x=2$ is :

A) $\frac{25}{3}$ B) $\frac{22}{3}$ C) 9 D) $\frac{23}{3}$

12. The area bounded by the curve $y=x(x-1)^2$. Y-axis and the line $y=2$ is ___ sq.units

A) $\frac{10}{3}$ B) $\frac{5}{3}$ C) $\frac{20}{3}$ D) $\frac{40}{3}$

13. The area bounded by $y^2=4x$ with the lines $x=2\,and\;x=5$ is ____ sq.units

A) $\frac{8}{3}\left(5\sqrt{5}-2\sqrt{2}\right)$ B) $\frac{4}{3}\left(5\sqrt{5}-2\sqrt{2}\right)$ C) $5\sqrt{5}-2\sqrt{2}$ D) $\frac{16}{3}$

14. The area bounded by the curve $y=f(x)$ the coordinate axis and the line $x=t$ is given by te^t then $f(x)=$

A) $e^x(x+1)$ B) $e^x(x-1)$ C) $x\left(1+e^x\right)$ D) $x\left(1-e^x\right)$

PRACTICE QUESTIONS

15. The area (in sq. units) of the region bounded by the curve $x^2=4y$ and the straight line $x=4y-2$ is **(Jee Mains_2019)**

A) $\frac{7}{8}$ B) $\frac{5}{4}$ C) $\frac{9}{8}$ D) $\frac{3}{4}$

16. The area (in sq. units) in the first quadrant bounded by the parabola, $y=x^2+1$,the tangent to it at the point (2,5) and the coordinate axes is **(Jee Mains_2019)**

A) $\frac{187}{24}$ B) $\frac{8}{3}$ C) $\frac{14}{3}$ D) $\frac{37}{24}$

17. The area (in sq. units)of the region bounded by the parabola $y=x^2+2$ and the lines, $y=x+1$, $x=0$ and $x=3$ **(Jee Mains_2019)**

A) $\frac{15}{2}$ B) $\frac{21}{2}$ C) $\frac{15}{4}$ D) $\frac{17}{4}$

18. The area (in sq. units) of the region enclosed between the parabola $y^2=2x$ and the line $x+y=4$ is …. Sq.units **(Jee Mains_2022)**

A) 20 B) 18 C) 36 D) 6

19. The area enclosed by the curves $y^2+4x=4$ and $y-2x=2$ is : **(Jee Mains_2023)**

A) $\frac{25}{3}$ B) $\frac{22}{3}$ C) 9 D) $\frac{23}{3}$

20. The area of the region bounded by $y-x=2$ and $x^2=y$ is equal to **[Main 2021]**

A) $\frac{2}{3}$ B) $\frac{4}{3}$ C) $\frac{9}{2}$ D) $\frac{16}{3}$

21. The area bounded by the curve $y=|x^2-9|$ and the line y = 3 is :

(Main June 26, 2022)

A) $4(2\sqrt{3}+\sqrt{6}-4)$ B) $4(4\sqrt{3}+\sqrt{6}-4)$

C) $8(4\sqrt{3}+3\sqrt{6}-9)$ D) $8(4\sqrt{3}+\sqrt{6}-9)$

22. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is ______ sq.units

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{1}{6}$

23. The area bounded by the curves $y = x^3$, $y = x^2$ and the ordinates $x = 1, x = 2$ is ___ sq.units

A) $\frac{17}{12}$ B) $\frac{12}{13}$ C) $\frac{2}{7}$ D) $\frac{7}{2}$

24. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point $(-1,-1)$ is

A) $\frac{27}{4}$ B) $\frac{19}{4}$ C) $\frac{23}{4}$ D) $\frac{31}{4}$

Area bounded between standard geometrical figures :

25. Area (in sq. units)of the region outside $\frac{x}{2}+\frac{y}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is **(Jee Mains_2020)**

A) $3(4-\pi)$ B) $6(4-\pi)$ C) $6(\pi-2)$ D) $3(\pi-2)$

26. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$,is 1 square unit. Then k is **(Jee Mains_2019)**

A) $\sqrt{3}$ B) $\frac{1}{\sqrt{3}}$ C) $\frac{\sqrt{3}}{1}$ D) $\frac{2}{\sqrt{3}}$

27. If two circles each of unit radius intersect orthogonally, the common area of the circles is … sq. units

A) $\frac{\pi}{4}$ B) $\frac{\pi}{4}+1$ C) $\frac{\pi}{2}+1$ D) $\frac{\pi}{2}-1$

PRACTICE QUESTIONS

28. The region represented by $|x-y| \le 2$ and $|x+y| \le 2$ is bounded by a **(Jee Mains_2019)**

A)Square of side length $2\sqrt{2}$ B)Square of area 16 sq. units
C) Rhombus of side length 2 units D) Rhombus of area $8\sqrt{2}$ sq. units

29. The area enclosed between the curves $y = ax^2$ and $x = ay^2 (a > 0)$ is 1 sq.unit, then the value of a is **(2004 S)**

A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{2}$ C) 1 D) $\frac{1}{3}$

30. The area of the region $\left\{(x,y)/x^2+y^2 \le 1 \le x+y\right\}$ is … sq. units.

A) $\frac{\pi}{4}+\frac{1}{2}$ B) $\pi+1$ C) $\frac{\pi}{4}-\frac{1}{2}$ D) $\frac{\pi}{4}+\frac{3}{4}$

Area bounded by two or more curves :

31. The area of the region, enclosed by the circle $x^2+y^2=2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$,is **[Main 2020]**

A) $\frac{1}{6}(24\pi-1)$ B) $\frac{1}{6}(12\pi-1)$ C) $\frac{1}{3}(12\pi-1)$ D) $\frac{1}{3}(6\pi-1)$

32. The area enclosed by the curves $y = log_e(x+e^2),\ x = \log_e\left(\frac{2}{y}\right)$ *and* $x = \log_e 2$, above the line $y = 1$ is : **[Main 2022]**

A) $2+e-\log_e 2$ B) $1+e-\log_e 2$

C) $e-\log_e 2$ D) $1+\log_e 2$

33. The area of the smaller region enclosed by the curves $y^2 = 8x+4$ *and* $x^2+y^2+4\sqrt{3}x-4=0$ is equal to **[Main 2022]**

A) $\frac{1}{3}\left(2-12\sqrt{3}+8\pi\right)$ B) $\frac{1}{3}\left(2-12\sqrt{3}+6\pi\right)$

C) $\frac{1}{3}\left(4-12\sqrt{3}+8\pi\right)$ D) $\frac{1}{3}\left(4-12\sqrt{3}+6\pi\right)$

34. The area of the region enclosed between the parabola $y^2 = 2x-1$ *and* $y^2 = 4x-3$ is

A) $\frac{1}{3}$ B) $\frac{1}{6}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$ **[Main 2022]**

35. Let A_1 be the area of the region bounded by the curves $y=\sin x,\ y=\cos x$ *and* $y-axis$ in the first quadrant. Also, let A_2, be the area of the region bounded by the curves $y = six,\ y=\cos x,\ x-axis$ and $x=\frac{\pi}{2}$ in the first quadrant. Then, **[Main 2021]**

A) $A_1 = A_2$ *and* $A_1 + A_2 = \sqrt{2}$ B) $A_1 : A_2 = 1:\sqrt{2}$ *and* $A_1 + A_2 = 1$

C) $A_1 : A_2 = 1:2$ *and* $A_1 + A_2 = 1$ D) $2A_1 = A_2$ *and* $A_1 + A_2 = 1+\sqrt{2}$

36. Let the functions $f:\mathbb{R}\to\mathbb{R}$ *and* $g:\mathbb{R}\to\mathbb{R}$ be defined by $f(x) = e^{x-1} - e^{-|x-1|}$ and $g(x) = \frac{1}{2}(e^{x-1}+e^{1-x})$. Then the area of the region in the first quadrant bounded by the curves $y=f(x),\ y=g(x)$ and $x=0$ is **[Adv. 2020]**

A) $(2-\sqrt{3})+\frac{1}{2}(e-e^{-1})$ B) $(2+\sqrt{3})+\frac{1}{2}(e-e^{-1})$

C) $(2-\sqrt{3})+\frac{1}{2}(e+e^{-1})$ D) $(2+\sqrt{3})+\frac{1}{2}(e+e^{-1})$

37. The area bounded by the curves $y=\sqrt{\frac{1+\sin x}{\cos x}}$ *and* $y=\sqrt{\frac{1-\sin x}{\cos x}}$ between the lines $x=0,\ x=\frac{\pi}{4}$ is **[Adv.2014]**

A) $\int_0^{\sqrt{2}-1}\frac{tdt}{(1+t^2)\sqrt{1-t^2}}$ B) $\int_0^{\sqrt{2}-1}\frac{4t\ dt}{(1+t^2)\sqrt{1-t^2}}$

C) $\int_0^{\sqrt{2}+1}\frac{4t\ dt}{(1+t^2)\sqrt{1-t^2}}$ D) $\int_0^{\sqrt{2}+1}\frac{t\ dt}{(1+t^2)\sqrt{1-t^2}}$

38. The area of the region enclosed by the curves $y=x^2$ *and* $y=x^3$ is ___ sq.units

A) $\frac{1}{12}$ B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) 1

39. The area of the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$ is ____sq.units

A) $\pi - \frac{2}{3}$ B) $\pi + \frac{2}{3}$ C) $\frac{\pi}{3}$ D) $\frac{2\pi}{3}$

40. Area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ is

A) $\frac{3\pi - 8}{3}$ sq. units B) $\frac{\pi - 8}{3}$ sq. units C) $\frac{2\pi - 8}{3}$ sq. units D) $\frac{\pi + 2}{3}$ sq. units

PRACTICE QUESTIONS

41. The area (in sq.units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to : **(Main 2021)**

A) $\frac{3}{5}$ B) $\frac{1}{3}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$

42. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to **(Jee Mains_2020)**

A) $\frac{7}{2}$ B) $\frac{4}{3}$ C) $\frac{8}{3}$ D) $\frac{16}{3}$

43. The odd natural number a, such that the area of the region bounded by y = 1, y = 3, x = 0, $x = y^a$ is $\frac{364}{3}$, is equal to : **(Main July 26, 2022)**

A) 3 B) 5 C) 7 D) 9

44. Given: $F(x) = \begin{cases} x & , \quad 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , \quad x = \frac{1}{2} \\ 1 - x & , \quad \frac{1}{2} < x < 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2, x \in R$. Then the area of (in sq. units) of the region bounded by the curves $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$ **(Jee Mains_2020)**

A) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ B) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ C) $\frac{1}{2} - \frac{\sqrt{3}}{4}$ D) $\frac{1}{2} + \frac{\sqrt{3}}{4}$

45. The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x$, $x = 1$, $y = 2\sqrt{2}$, is equal to : **(Main June 29, 2022)**

A) $\frac{16\sqrt{2}}{6}$ B) $\frac{11\sqrt{2}}{6}$ C) $\frac{13\sqrt{2}}{6}$ D) $\frac{5\sqrt{2}}{6}$

46. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to **[Main-2022]**

A) $\frac{32}{3}$ B) $\frac{40}{3}$ C) 16 D) 9

47. The area (in sq.units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is

A) $12\pi - 3\sqrt{3}$ B) $24\pi + 3\sqrt{3}$ C) $12\pi + 3\sqrt{3}$ D) $24\pi - 3\sqrt{3}$

48. If $x=a(>0)$ divides the area bounded by $X-axis$, part of the curve $y=1+\frac{8}{x^2}$ and the ordinates $x=2, x=4$ into equal parts then $a=$

A) 2 B) $\sqrt{2}$ C) $\frac{1}{\sqrt{2}}$ D) $2\sqrt{2}$

49. $y=\frac{x^2}{1+x^2}$ divides the area enclosed by $y=\frac{1}{1+x^2}$, x-axis and y-axis in the first quadrant in the ratio

A) $\frac{\pi-2}{2}$ B) $\frac{4-\pi}{4}$ C) $\frac{2\pi-4}{\pi}$ D) $\frac{\pi-1}{\pi+1}$

Area bounded by two or more cure inequalities :

50. If the area (in sq. units) of the regions $\{(x,y): y^2\le 4x, x+y\le 1, x\ge, y\ge 0\}$ is $a\sqrt{2}+b$ then $a-b$ is equal to **[Main-2019]**

A) $-\frac{2}{3}$ B) 6 C) $\frac{10}{3}$ D) $\frac{8}{3}$

51. The area (in sq. units) of the region $\{(x,y)\in R^2 \,|\, 4x^2\le y\le 8x+12\}$ is **[Main-2020]**

A) $\frac{128}{3}$ B) $\frac{125}{3}$ C) $\frac{127}{3}$ D) $\frac{124}{3}$

52. The area (in sq. units)of the region $\left\{(x,y): 0\le y\le x^2+1, 0\le y\le x+1, \frac{1}{2}\le x\le 2\right\}$ is **[Main-2020]**

A) $\frac{79}{16}$ B) $\frac{23}{6}$ C) $\frac{79}{24}$ D) $\frac{23}{16}$

53. The area of the region $A=\{(x,y): 0\le y\le x|x|+1 \, and \, -1\le x\le 1\}$ in sq. units is **[Main-2019]**

A) 2 B) $\frac{4}{3}$ C) $\frac{2}{3}$ D) $\frac{1}{3}$

54. Let $A=\left\{(x,y)\in R^2: y\ge 0, 2x\le y\le \sqrt{4}-(x-1)^2\right\}$ and $B=\left\{(x,y)\in R\times R: 0\ge y\le y\le \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}$. Then the ratio of the area of A to the area of B is **[Main 2023]**

1) $\frac{\pi-1}{\pi+1}$ 2) $\frac{\pi}{\pi-1}$ 3) $\frac{\pi}{\pi+1}$ 4) $\frac{\pi+1}{\pi-1}$

55. The area of the region $\left\{(x,y); |x-1|\le y\le \sqrt{5-x^2}\right\}$ is equal to **[Main-2022]**

A) $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right)-\frac{1}{2}$b B) $\frac{5\pi}{4}-\frac{3}{2}$ C) $\frac{3\pi}{4}+\frac{3}{2}$ D) $\frac{5\pi}{4}-\frac{1}{2}$

56. Let the straight line x = b divide the area enclosed by $y=(1-x)^2$, $y=0$ and $x=0$ into two parts $R_1(0\le x\le b)$ and $R_2(b\le x\le 1)$ such that $R_1-R_2=\frac{1}{4}$. Then b equals **[Adv.2011]**

A) $\frac{3}{4}$ B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$

57. The area of the region $A=\left\{(x,y):|\cos x-\sin x|\le y\le \sin x,\ 0\le x\le \frac{\pi}{2}\right\}$ **[Main 2023]**

A) $1-\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{5}}$ B) $\sqrt{5}+2\sqrt{2}-4.5$ C) $\frac{3}{\sqrt{5}}-\frac{3}{\sqrt{2}}+1$ D) $\sqrt{5}-2\sqrt{2}+1$

58. The area bounded by the curves $|y+x|\le 1, |y-x|\le 1$ and $2x^2+2y^2\ge 1$ is … sq. units

A) $\left(2+\frac{\pi}{2}\right)$ B) $\left(2-\frac{\pi}{2}\right)$ C) $\left(4-\frac{\pi}{2}\right)$ D) $\left(4+\frac{\pi}{2}\right)$

59. Let Δ be the area of the region $\left\{(x,\ y)\in\mathbb{R}^2 : x^2+y^2\le 21,\ y^2\le 4x,\ x\ge 1\right\}$.

$\frac{1}{2}\left(\Delta-21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to

A) $2\sqrt{3}-\frac{1}{3}$ B) $\sqrt{3}-\frac{2}{3}$ C) $2\sqrt{3}-\frac{2}{3}$ D) $\sqrt{3}-\frac{4}{3}$

60. Let q be the maximum integral value of p in [0, 10] for which the roots of the equation $x^2-px+\frac{5}{4}p=0$ are rational. Then the area of the region $\left\{(x,y):0\le y\le (x-q)^2,\ 0\le x\le q\right\}$ is

A) 243 B) 25 C) $\frac{125}{3}$ D) 164

PRACTICE QUESTIONS

61. The area (in sq. units) of the region $\left\{(x,y)\in R^2 : x^2\le y\le 3-2x\right\}$ is **(Jee Mains_2020)**

A) $\frac{31}{3}$ B) $\frac{29}{3}$ C) $\frac{34}{3}$ D) $\frac{32}{3}$

62. Consider a region $R=\left\{(x,y)\in R^2 : x^2\le y\le 2x\right\}$ .if a line $y=\alpha$ divides the area of region R into two equal parts, then which of the following is true? **(Jee Mains_2020)**

A) $3\alpha^2-8\alpha+8=0$ B) $\alpha^3-6\alpha^{3/2}-16=0$ C) $3\alpha^3-8\alpha^{3/2}+8=0$ D) $\alpha^3-6\alpha^2+16=0$

63. The area (in sq. units) of the region $A=\left\{(x,y)\in R\times R \mid 0\le x\le 3, 0\le y\le 4, y\le x^2+3x\right\}$ is

A) $\frac{26}{3}$ B) $\frac{59}{6}$ C) 8 D) $\frac{53}{6}$ **(Jee Mains_2019)**

64. Let $S(a)=\left\{(x,y): y^2\le x, 0\le x\le \alpha\right\}$ and $A(\alpha)$ is area of the region $S(\alpha)$ .If for $0<\lambda<4$, $A(\lambda):A(4)=2:5$ then λ equals **(Jee Mains_2019)**

A) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ B) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ C) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ D) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

65. The area (in sq. units) of the region $\left\{(x,y): x^2\le y\le x+2\right\}$ is **(Jee Mains_2019)**

A) $\frac{31}{6}$ B) $\frac{10}{3}$ C) $\frac{9}{2}$ D) $\frac{13}{6}$

66. The area (in sq. units) of the region $A=\left\{(x,y):\frac{y^2}{2}\le x\le y+4\right\}$ is **(Jee Mains_2019)**

A) 18 B) 16 C) $\frac{53}{3}$ D) 30

67. The area (in sq. units) of the region $A=\{(x,y):(x-1)[x]\le y\le 2\sqrt{x},0\le x\le 2\}$, where [t] denotes the greatest integer function is **(Jee Mains_2020)**

A) $\frac{8}{3}\sqrt{2}-1$ B) $\frac{4}{3}\sqrt{2}+1$ C) $\frac{8}{3}\sqrt{2}-\frac{1}{2}$ D) $\frac{4}{3}\sqrt{2}-\frac{1}{2}$

68. The area (in sq. units) of the region $A=\{(x,y):|x|+|y|\le 1,2y^2\ge|x|\}$ is **(Jee Mains_2020)**

A) $\frac{1}{6}$ B) $\frac{7}{6}$ C) $\frac{8}{3}$ D) $\frac{1}{3}$

69. Let q be the maximum integral value of p in [0, 10] for which the roots of the equation $x^2-px+\frac{5}{4}p=0$ are rational. Then the area of the region **[Main 2023]**

$\{(x,y):0\le y\le (x-q)^2,\ 0\le x\le q\}$ is

A) 243 B) 25 C) $\frac{125}{3}$ D) 164

70. The area (in sq. units) of the region, given by the set $\{|(x,y)\in R\times R|x\ge 0,\ 2x^2\le y\le 4-2x\}$ is **[Main 2021]**

A) $\frac{8}{3}$ B) $\frac{13}{3}$ C) $\frac{7}{3}$ D) $\frac{17}{3}$

71. If the area of the bounded region $R=\left\{(x,\ y):\max\{0,\log_e x\}\le y\le 2^x,\frac{1}{2}\le x\le 2\right\}$ is, $\alpha(\log_e 2)^{-1}+\beta(\log_e 2)+\gamma$, then the value of $(\alpha+\beta-2\gamma)^2$ is equal to **[Main 2021]**

A) 2 B) 8 C) 4 D) 1

72. The area of the region : $R=\{(x,y):5x^2\le y\le 2x^2+9\}$ is

A) $9\sqrt{3}$ *square units* B) $6\sqrt{3}$ *square units*
C) $11\sqrt{3}$ *square units* D) $12\sqrt{3}$ *square units*

73. The area of the region $\left\{(x,y):0\le x\le\frac{9}{4},\ 0\le y\le 1,\ x\ge 3y,\ x+y\ge 2\right\}$ is **[Main 2021]**

A) $\frac{11}{32}$ B) $\frac{35}{96}$ C) $\frac{37}{96}$ D) $\frac{13}{32}$

74. The area of the region $\left\{(x,y):0\le x\le\frac{9}{4},0\le y\le 1,\ x\le 3y,\ x+y\ge 2\right\}$ **(Adv. 2022)**

A) $\frac{11}{32}$ B) $\frac{35}{96}$ C) $\frac{37}{96}$ D) $\frac{13}{32}$

75. The area (in sq.units) of the region $\{(x,y):\ y\le x^2+1,\ 0\le y\le x+1,\ \frac{1}{2}\le x\le 2\}$

A) $\frac{23}{16}$ B) $\frac{79}{24}$ C) $\frac{79}{16}$ D) $\frac{23}{6}$ **[Main 2019]**

76. The area of the region $\left\{(x, y: xy \le 8, 1 \le y \le x^2\right\}$ is **(Adv.2018)**

A) $8\log_e 2 - \frac{14}{3}$ B) $16\log_e 2 - \frac{14}{3}$ C) $8\log_e 2 - \frac{7}{3}$ D) $16\log_e 2 - 6$

77. Area of the region $\left\{(x, y) \in \mathbb{R}^2 : y \ge \sqrt{|x+3|}, 5y \le x+9 \le 15\right\}$ is equal to **(Adv.2016)**

A) $\frac{1}{6}$ B) $\frac{4}{3}$ C) $\frac{3}{2}$ D) $\frac{5}{3}$

78. The area of the region enclosed by $y \le 4x^2$, $x^2 \le 9y$ and $y \le 4$, is equal to **(Main 2022)**

A) $\frac{40}{3}$ B) $\frac{56}{3}$ C) $\frac{112}{3}$ D) $\frac{80}{3}$

79. The area of the region $S = \{(x, y): y^2 \le 8x,\ y \ge \sqrt{2}x, x \ge 1\}$ is **(Main 2022)**

A) $\frac{13\sqrt{2}}{6}$ B) $\frac{11\sqrt{2}}{6}$ C) $\frac{5\sqrt{2}}{6}$ D) $\frac{19\sqrt{2}}{6}$

80. Area of the region $\left\{(x,\ y): x^2 + (y-2)^2 \le 4,\ x^2 \ge 2y\right\}$ is

A) $2\pi - \frac{16}{3}$ B) $\pi - \frac{8}{3}$ C) $\pi + \frac{8}{3}$ D) $2\pi + \frac{16}{3}$

81. The area of the region $\left\{(x,\ y): x^2 \le \left|x^2 - 4\right|, y \ge 1\right\}$ is

A) $\frac{3}{4}\left(4\sqrt{2}-1\right)$ B) $\frac{4}{3}\left(4\sqrt{2}-1\right)$ C) $\frac{4}{3}\left(4\sqrt{2}+1\right)$ D) $\frac{3}{4}\left(4\sqrt{2}+1\right)$

Least area bounded by two or more curves :

82. The area bounded by $y = f(x) = x^4 - 2x^3 + x^2 + 3$, x-axis and ordinates corresponding to minimum of the function $f(x)$ is …. Sq. units

A) $\frac{10}{3}$ B) $\frac{91}{30}$ C) $\frac{2}{3}$ D) $\frac{1}{2}$

Miscelleneous based on area bounded by two or more curves :

83. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx} + \frac{x+a}{y-2} = 0,\ y(1) = 0$ is 4π. Let P and Q be the points of intersection of the curve C and the y-axis. If normal at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is **[Main 2023]**

A) $2\sqrt{3}$ B) $\frac{2\sqrt{3}}{3}$ C) 2 D) $\frac{4\sqrt{3}}{3}$

84. Let $g(x) = \cos x^2,\ f(x) = \sqrt{x}\ and\ \alpha,\ \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$ Then the area (in sq.units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha\ x = \beta\ and\ y = 0$, is : **(Adv. 2018)**

A) $\frac{1}{2}(\sqrt{3}+1)$ B) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$ C) $\frac{1}{2}(\sqrt{2}-1)$ D) $\frac{1}{2}(\sqrt{3}-1)$

85. For $a>0$, let the curves $C_1: y^2 = ax$ and $C_2: x^2 = ay$ intersect at origin O and a point P. Let the line $x=b\,(0<b<a)$ intersect the chord OP and the x-axis at the points Q and R, respectively. If the line $x=b$ bisects the area bounded by the curves C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation **(Jee Mains_2020)**

A) $x^6+6x^3-4=0$ B) $x^6-12x^3-4=0$ C) $x^6-6x^3+4=0$ D) $x^6-12x^3+4=0$

86. Let $f:[-1,2]\to[0,\infty)$ be a continuous function such that $f(x)=f(1-x)$ for all $x\in[-1,2]$ Let $R_1=\int_{-1}^{2} xf(x)dx$, and R_2 be the area of the region bounded by $y=f(x), x=-1, x=2$, and the x-axis. Then **(Main 2021)**

A) $R_1=2R_2$ B) $R_1=3R_2$ C) $2R_1=R_2$ D) $3R_1=R_2$

87. The area cut off from a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex then $k=$

A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$

88. P is a variable point in the square formed by the lines $x=\pm 1$ and $y=\pm 1$, P moves such that its distance from the origin is less than its distance from any side of the square. The area traced by the point P is …. Sq. units.

A) $\frac{4}{3}\left(4\sqrt{2}+1\right)$ B) $\frac{4}{3}\left(4\sqrt{2}-1\right)$ C) $\frac{4}{3}\left(4\sqrt{2}-3\right)$ D) $\frac{4}{3}\left(4\sqrt{2}-5\right)$

89. Integral part of the area of figure bounded by the tangents at the ends of latusrecta of ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and directrices of hyperbola $\frac{x^2}{9}-\frac{y^2}{72}=1$ is

A) 7 B) 9 C) 10 D) 8

90. Let $A=\left(\frac{1}{2},0\right), B=\left(\frac{3}{2},0\right), C=\left(\frac{5}{2},0\right)$ be 3-points on xy-plane P is another point on the same plane satisfying $\max\{PA+PB+PC\}<2$. If the area of the region of P is $\sqrt{3}\left(\frac{\pi}{a}-\frac{\sqrt{3}}{b}\right)$ then $a+b=$ _____

A) 7 B) 5 C) 9 D) 11

91. The area enclosed by the curve $|x-60|+|y|=\left|\frac{x}{4}\right|$

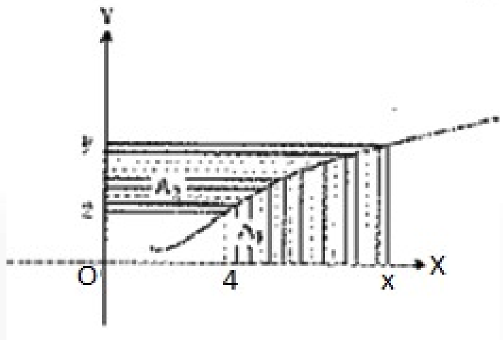
A) 240 sq.units B) 360 sq.units C) 480 sq.units D) 600 sq.units

PRACTICE QUESTIONS

92. The area bounded by the curve $4y^2=x^2(4-x)(x-2)$ is equal to **[Main 2021]**

A) $\frac{\pi}{8}$ B) $\frac{\pi}{16}$ C) $\frac{3\pi}{8}$ D) $\frac{3\pi}{2}$

93. Consider a curve y = y(x) in the first quadrant as shown in the figure. Let the Area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line $2x-12y=15$ is passing through the point. **[Main 2021]**



A) (6, 21) B) (8, 9) C) (10, -4) D) (12, -15)

94. Left $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y=f(x)$, x-axis and the ordinates $x=\frac{\pi}{4}$ and $x=\beta>\frac{\pi}{4}$ is $\beta\sin\beta+\frac{\pi}{4}\cos\frac{\beta}{4}\cos\beta+\sqrt{2}\beta$ sq. units, then $f^1\left(\frac{\pi}{2}\right)$ is

A) $\left(\frac{\pi}{2}-\sqrt{2}-1\right)$ B) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ C) $-\frac{\pi}{2}$ D) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

95. If $P=\lim\limits_{a\to\infty}\frac{\int_0^a \sin^6 x\cos^4 x\,dx}{a}$ *and* $f(x)=\frac{x^3}{2}+1-x\int_0^x g(t)dt,\ g(x)=x-\int_0^1 f(t)dt$ such that the area of the region bounded by $y=f(x)$ with x-axis between the ordinates $x=0\,and\ x=4$ is Q then the value of $\frac{64PQ}{3}$ is

A) 6 B) 2 C) 3 D) 9

96. If $p(x), q(x), r(x)\ and\ s(x)$ in x such that

$\int_t^x p(t)q(t)dt\int_t^x r(t)s(t)dt-\int_t^x p(t)s(t)dt\int_t^x q(t)r(t)dt$ is divisible by $(x-1)^\lambda$, $\lambda\in N$

$\lambda_{\max}=a\ and\ \int_b^e|\sin x|dx=8\ and\ \int_0^{b+c}|\cos x|dx=9$ such that the area of the region bounded by f(x) = x sin x with x-axis and between the ordinates $x=b\ and\ x=c-14b$ is A, then the value of $\frac{a\sqrt{2}}{A}$ = ……

A) $\frac{4}{\pi}$ B) $\frac{2}{\pi}$ C) $\frac{8}{\pi}$ D) $\frac{1}{\pi}$

Area bounded by curves involving standard algebraic function :

97. The area bounded by $y=e^x$, $y=e^{-x}\ and\ x=1$ is ______ sq.units

A) $e+\frac{1}{e}+2$ B) $e+\frac{1}{e}-2$ C) $e-\frac{1}{e}+2$ D) $e-\frac{1}{e}-2$

98. The area bounded by $y=2-|2-x|,\ y=\frac{3}{|x|}$ is

A) $\left(\frac{5-4\ln 2}{3}\right)sq.unit$ B) $\left(\frac{2-\ln 3}{2}\right)sq.unit$

C) $\left(\frac{4-3\ln 3}{2}\right)sq.unit$ D) $\left(\frac{4+3\ln 3}{2}\right)sq.unit$

99. Area bounded by the curves $y=\log_e^x$ and $y=(\log_e x)^2$ is

A) e-2 sq. units B) 3-e sq. units C) e sq. units D) e-1 sq. units

100. Area of the region bounded by the curve $y=25^x+16$ and curve $y=b.\ 5^x+4$ whose angent at the point $x=1$, makes and angle $\tan^{-1}\left(40\log^5\right)$ with the x-axis is …. sq. units

A) $2\log_5\left(\frac{e^4}{27}\right)$ B) $4\log_5\left(\frac{e^4}{27}\right)$ C) $3\log_5\left(\frac{e^4}{27}\right)$ D) None of these

PRACTICE QUESTIONS

101. The area (in sq. units) of the region bounded by the curves $y=2^x$ and $y=|x+1|$,in the first quadrant is: **(Main_2019)**

A) $\frac{3}{2}-\frac{1}{\log_e 2}$ B) $\frac{1}{2}$ C) $\log_e 2+\frac{3}{2}$ D) $\frac{3}{2}$

102. The area of the region given by max $(|x|,|y|)\le 2$ and $e^{|x|}\left(|y|+\frac{1}{2}\right)$ is …. Sq. units

A) $14+ln2$ B) $14-2ln2$ C) $14+2ln2$ D) $14-ln2$

Area bounded by a function and its Trignometic function :

103. The positive value of parameter "a" for which the area of the figure bounded by $y=\sin ax,\ y=0, x=\frac{\pi}{a}$ and $x=\frac{\pi}{3a}$ is 3 is …. Sq. units

A) 1 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$

104. The area bounded by $y=\sec^{-1}x, y=\cos ec^{-1}x$ and line $x-1=0$ is

A) $\log\left(3+2\sqrt{2}\right)-\frac{\pi}{2}$ sq. units B) $\frac{\pi}{2}-\log\left(3+2\sqrt{2}\right)$ sq. units

C) $\pi-\log_e 3$ sq. units D) $\pi+\log_e 3$ sq. units

105. If $f(x)=\max\left\{\sin x.\cos x,\frac{1}{2}\right\}$ then the area of the region bounded by the curves $y=f(x)$, x-axis, y-axis and the line $x=\frac{5\pi}{3}$ is

A) $\frac{5\pi}{12}+\sqrt{3}\,sq.units$ B) $\frac{5\pi}{12}+\frac{\sqrt{3}}{2}sq.units$

C) $\frac{5\pi}{12}+\sqrt{3}+\sqrt{2}\,sq.units$ D) $\frac{5\pi}{12}+\frac{\sqrt{3}}{2}+\sqrt{2}\,sq.units$

PRACTICE QUESTIONS

106. The area bounded by y-axis and the curve $x = e^y.\sin \pi y, y = 0, y = 1$ is …. Sq. units

A) $\frac{e+1}{\pi^2+1}$ B) $\frac{e-1}{\pi^2+1}$ C) $\frac{(e+1)\pi}{\pi^2+1}$ D) $\frac{(e-1)\pi}{\pi^2+1}$

107. A square ABCD is inscribed in a circle of radius 4. A point P moves inside the circle such that $d(P, AB) \le \min\{d(P,BC), d(P,DA)\}$ where $d(P, AB)$ is the distance of a point P from line AB. The area of region covered by the moving point Pin square units is

A) 4π B) 8π C) $8\pi - 16$ D) $4\pi - 4$

108. The area, enclosed by the curves y = sin x + cos x and y = |cosx – sinx| and the lines x = 0, $x = \frac{\pi}{2}$, is : **(Main Sep 1, 2021)**

A) $2\sqrt{2}(\sqrt{2}-1)$ B) $2(\sqrt{2}+1)$

C) $4(\sqrt{2}-1)$ D) $2\sqrt{2}(\sqrt{2}+1)$

Area bounded by a function and its Inverse:

109. Let $f(x) = x^3 - x^2 + 2x - 8$ and $g(x)$ is inverse of $f(x)$. Then area bounded by $y = g(x)$, $x - axis$ between $x = -12$, and $x = 16$ is

A) $\frac{325}{6} sq.units$ B) $\frac{325}{4} sq.units$ C) $\frac{325}{7} sq.units$ D) $\frac{325}{9} sq.units$

110. Let $g(x)$ be the inverse $f(x) = x^3 + 3x + 1$. If the area bounded by $y = g(x)$, x-axis $x = -3$ *and* $x = 5$ is Δ then the value of $[\Delta]$ equals $([.]GIF)$.

A) 4 B) 3 C) 2 D) 1

Area bounded by a function and Y = X :

111. The area of the region enclosed by the curves $y = x,\ x = e,\ y = \frac{1}{x}$ and the positive x-axis is ______

A) $\frac{3}{2}$ square units B) $\frac{5}{2}$ square units C) $\frac{1}{2}$ square units D) square units

112. If A_n is the area bounded by the curve $y = x$ and $y = x^n$, $n \in N - \{1\}$ in the first quadrant if $(A_2)(A_3)(A_4).....(A_n) = \frac{1}{(an^2+bn).2^{n+c}}$ then $a + b + c =$ …….

A) 0 B) $\frac{1}{2}$ C) 2 D) $\frac{1}{4}$