GRAVITATION

SYNOPSIS

I) Gravitational Force and laws:

A) Kepler's laws of planetary motion:

i) Law (Law of orbits):

a) Every planetrevolves around the sun in an elliptical orbit, with the sun at on eof the foci.

b) During the orbital motion of a planet around the sun in an elliptical orbit, at a particular instant the planet is very close to the sun (the point is called perihelion) and at another particular instant it is far away from the sun (the point is called apiheleon)

ii) Law (Law of areas):

a) The line joining the sun and the planet sweeps out equal areas in equal intervals of time or areal velocity of radius vector is constant.

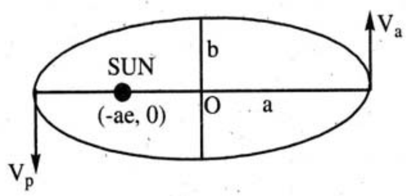
Areal velocity, $V_A = \frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2}r^2 d\theta\right) = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \frac{vr}{2} =$ constant

Where 'L' is the angular momentum of the planet of mass 'm' in the given orbit.

b) Kepler's II law is a consequence of the law of conservation of Angular momentum. According to II law, a planet moves faster when it is near to sun and moves slower when it is far away from the sun.

c) $V_{max} r_{min} = V_{min} r_{max}$

d) From geometry of the ellipse of eccentricity e, semi major axis a , semi minor axis b,



The aphelion distance $r_a = a(1+e)$

The perihelion distance $r_p = a(1-e)$

e) $V_a(1+e) = V_P(1-e)$

f) Relation between a and b is $b = a\sqrt{1-e^2}$

iii) Law(law of periods) :

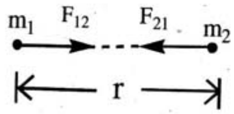
a) The square of the time period of revolution of a planet around the sun is proportional to the cube of the mean distance of the planet from the sun or semi-major axis.

$$T^2 \propto R^3 \Rightarrow \left[\frac{T_1}{T_2}\right]^2 = \left[\frac{R_1}{R_2}\right]^3$$

b) According to III law, as the distance of the planet increases from the sun, then the duration of the year of the planet increases.

B) Newton's Law Of Gravitation:

a) The gravitational force of attraction between two particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



$$\overrightarrow{F_{12}} = \frac{Gm_1m_2}{r^2}\hat{r}_{12} = \frac{Gm_1m_2}{r^3}\vec{r}_{12}; F = \frac{Gm_1m_2}{r^2}, G = 6.67\times10^{-11}\,Nm^2Kg^{-2}$$

b) The value of 'G' is constant for any two bodies, in any medium and at any place in the universe.

c) The gravitational force acts as action-reaction pair on two different bodies. It is central force and conservative in nature.

d) It is independent of medium between the masses and always attractive in nature.

e) Law of gravitation is a consequence of "action at a distance concept".

f) The resultant of gravitational force $\vec{F}$ on a particle due to number of point masses is equal to the in general for continuous mass distribution, use integration for basic summation of these forces.

g) In double star system, if two stars are revolving about their center of mass the gravitational force of attraction provides the necessary centripetal force.

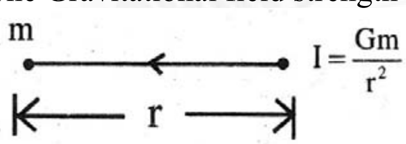
$$\frac{Gm_1m_2}{R^2} = m_1 r_{cm}\omega^2$$

II) Gravitational field strength:

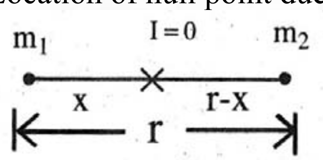
It is the gravitational force experienced by a unit mass kept at a point in a gravitational field.

$I = \dfrac{F}{m}Nkg^{-1}$ (It is a vector)

a) The Gravitational field strength at a point due to a point mass



b) Location of null point due to two point masses

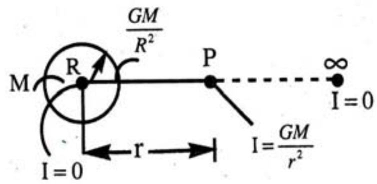


$x = \dfrac{r}{\sqrt{\dfrac{m_2}{m_1}}+1}$ from the centre of weaker mass $(m_1 < m_2)$

c) The resultant gravitational field strength at a point due to system of particles is given by the vectorical sum of individual fields. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_1 + ...$

d) The Gravitational field strength at a point is equal to the acceleration due to gravity at that point.

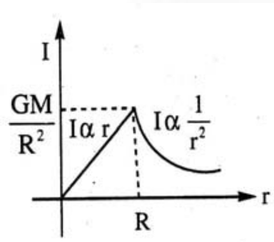
e) The Gravitational field strength at a point due to a uniform sphere of mass M



i) If $r > R, I = \dfrac{GM}{r^2} \therefore I \propto \dfrac{1}{r^2}$

ii) If $r = R; I = \dfrac{GM}{R^2}$

iii) If $r < R(r \neq 0), I = \dfrac{GM}{R^3}(r), I \propto r$



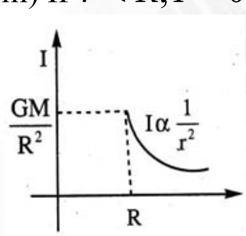
iv) E versus r graph

f) The gravitational field strength at a point due to a spherical shell of mass M

i) If $r > R, I = \dfrac{GM}{r^2} \therefore I \propto \dfrac{1}{r^2}$

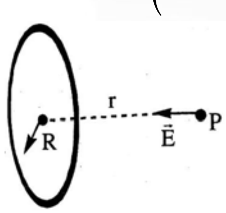
ii) If $r = R; I = \dfrac{GM}{R^2}$

iii) If $r < R, I = 0$



g) Gravitational field strength due to a uniform circular ring at a point on its axis:

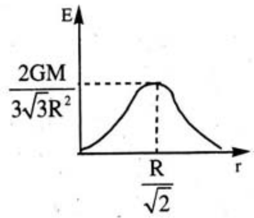
i) $E(r) = \dfrac{GMr}{\left(R^2 + r^2\right)^{\frac{3}{2}}}$



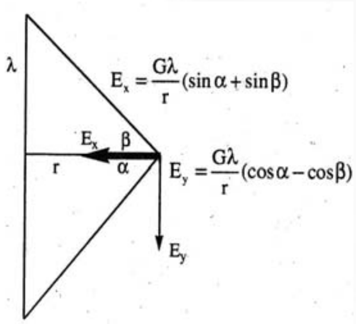
ii)This is directed towards the centre of the ring. It is zero at the centre of the ring and maximum at $r = \dfrac{R}{\sqrt{2}}$ (can be obtained by putting $\dfrac{dE}{dr} = 0$).

The maximum value is $E_{\max} = \dfrac{2GM}{3\sqrt{3}R^2}$

iii) E-r graph is as shown in figure.



h) The gravitational fields strength due to a thread of a mass density $\lambda\ kg/m$ at a distance r from the thread is



The gravitational field strength due to a very long thread $\left(\alpha = \beta = 90^0\right)$ is, $E_x = \dfrac{2G\lambda}{r}$

i) Gauss's law in gravitation: $\int \vec{E}.d\vec{s} = -4\pi G\left(m_{in}\right)$

i) Relation Between g & G:

a) $g = \dfrac{GM}{R^2} = \dfrac{4}{3}\pi Rg\rho$

b) $W \propto g \propto \dfrac{M}{R^2} \propto R\rho$; W-weight of an object

c) Acceleartion due to gravity on a planet depends on the mass, radius and density of the planet. It is independent of mass of the object.

ii) Variation of 'g' with altitude:

If R is the radius of the earth, M is the mass of the earth and h is the altitude, then

a) $g_h = \dfrac{GM}{r^2} = \dfrac{GM}{(R+h)^2} = g\left[\dfrac{R}{R+h}\right]^2$

b) If $h << R$, then $g_h = g\left(1 - \dfrac{2h}{R}\right)$

c) Fractional decrease of 'g' for smaller altitude is $\dfrac{\Delta g}{g} = \dfrac{2h}{R}$

iii) Varification of 'g' with depth

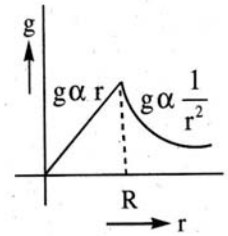
a) $g_d = g\left(1 - \dfrac{d}{R}\right)\left(\because g = \dfrac{4}{3}\pi RG\rho\right)$ (ρ is the mean density of the earth)

b) Fractional decreases of 'g' and weight of an object according to depth 'd' is $\frac{\Delta W}{W} = \frac{\Delta g}{g} = \frac{d}{R}$

c) At the centre of the earth, $g_d = 0 \Rightarrow W = 0$

d) If $g_h = g_d$ and $(h << R, d << R)$, then $2h = d$.

e) The variation of g with distance from the centre of the earth is



iv) Variation of 'g' with latitude (effect of rotation of earth):

a) If 'g' is the acceleration due to gravity when the earth does not rotate and g_φ is the acceleration due to gravity at an latitude ϕ due to rotation of earth, then

$g_\phi = g - R\omega^2 \cos^2 \phi$

b) As ϕ increases, g also increases

c) At equator $\phi = 0^0 \Rightarrow g_e = g - R\omega^2$ (minimum)the value of g at equator depends on angular velocity of earth

d) At poles $\phi = 90^0 \Rightarrow g_p = g$ (maximum)

e) The value of 'g' at the poles is independent of the rotation of the earth.

f) If a body feels weightless at equator, $mg_e = 0 \Rightarrow g_e = 0 \Rightarrow \omega = \sqrt{\frac{g}{R}} = \frac{1}{800} rad/s$

The few time period of the earth rotation is $T = 2\pi\sqrt{\frac{R}{g}}$

g) The value of 'g' at the equator become zero when the angular velocity of rotation increases to 17 times of the present value.

v) Variation of g according to shape of the earth and local conditions

As $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

a) g is minimum at equator and maximum at poles, because earth is not perfect sphere. It is in ellipsoidal in shape. $R_e > R_p$

b) The value of g is slightly more at the location of mineral deposits.

c) The value of g is slightly less on the top of mountains and also inside the mines.

d) If g_1, g_2 and g_3 are the acceleration due to gravities on the surface of the earth, on the top of a mountain and inside a mine, then $g_2 > g_1 > g_3$.

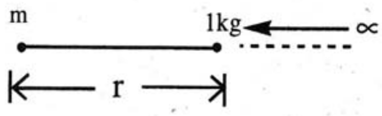
IV) Gravitational Potential:

a) It is the amount of work done in bringing a unit mass from infinity to a point in the gravitational field.

b) It is always –Ve and its maximum value is zero at infinity

c) The Gravitational potential at a point due to a point mass

$$V = \frac{-Gm}{r}$$



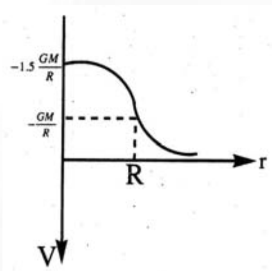
d) The gravitational potential at appoint due to a uniform sphere of mass M

i) $r > R, V = \frac{-GM}{r}$

ii) $r = R, V = \frac{-GM}{R}$

iii) $r < R, V = \frac{-GM}{2R^3}\left[3R^2 - r^2\right]$

iv)



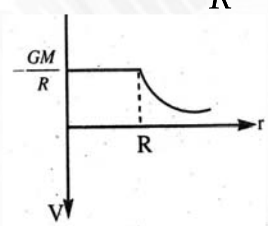
e) The gravitational potential at point due to a spherical shell of mass M and radius R

i) $r > R, V = -\frac{GM}{r}$

ii) $r = R, V = -\frac{GM}{R}$

iii) $r < R, V = -\frac{GM}{R}$

iv)



f) The gravitational potential at a distance r from the centre on the axis of a ring of mass M and radius R is given by, $V(r) = -\frac{GM}{\sqrt{R^2 + r^2}} \quad 0 \le r \le \infty$

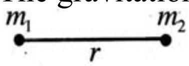
g) Relation between Gravitational Field and potential: The gravitational field intensity is equal to negative gravitational potential gradient. The negative of the slope of V-r curve gives E

$$\vec{E} = -gradient\ V; \vec{E} = -\frac{dv}{dr}; \vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

h) If $\vec{E}$ is given V can be calculated by the formula $V = \int_{\infty}^{r} dv = -\int_{\infty}^{r} \overline{E}.\overline{dr}$

Gravitational Potential energy:

a) The gravitational PE of the system of two particles



; $U = \frac{-Gm_1 m_2}{r}$

b) The gravitational potential energy of a body at a point in the gravitational field of earth

i) The potential energy on the surface of earth is $U = \frac{-GMm}{R} = -mgr$

ii) The potential energy at distance "R+h" from the center of earth is

$$U = \frac{-GMm}{R+h} = \frac{-GMm}{r} (\because r = R+h)$$

iii) The potential energy at "∞" is $U = 0$

iv) Potential energy at the center of earth is $U = -\frac{3GMm}{2R}$

c) Work done in lifting a body from the surface of the earth to a height h against the gravitational force is.

$$W = \left(\frac{-GMm}{R+h} + \frac{GMm}{R}\right) = \frac{GMm}{(R+h)R} = \frac{mgh}{\left(1+\frac{h}{R}\right)} \text{ if } h <<< R, W = mgh$$

V) Orbital Velocity (V_0):

a) It is the minimum velocity required by a satellite to revolve in a particular orbit around a planet.

b) Direction of V_0 is always tangential to the orbit

c) If satellite revolving at height h from the surface of the planet is $V_0 = \sqrt{\frac{GM}{R+h}}$

d) If the orbit is very near the surface of the planet (for a surface satellite)

$$V_0 = \sqrt{\left(\frac{GM}{R}\right)} = \sqrt{gR}$$

e) $V_0 \propto \sqrt{\frac{M}{R}} \propto R\sqrt{\rho}$ $\qquad \rho \rightarrow$ mean density of the planet

f) V_0 is independent of the mass of the orbiting body

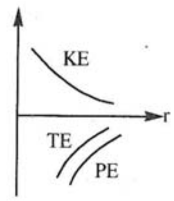
g) V_0 depends on mass of the planet and radius of the orbit

h) For an earth's surface satellite $V_0 = 7.98\ kms^{-1}$

i) Orbital angular velocity satellite is $\omega_0 = \sqrt{\frac{GM}{(R+h)^3}}$

j) Time period of satellite is $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

k) Energies of an orbiting satellite



i) $PE = \frac{-GMm}{r} = -\frac{GMm}{R+h}$

ii) $KE = \frac{GMm}{2r} = \frac{GMm}{2\left(R+h\right)}$

iii) $TE = \frac{-Gmm}{2r} = \frac{-GMm}{2\left(R+h\right)}$

iv) $PE : KE : TE = -2 : 1 : -1$

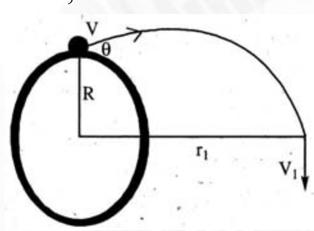
l) Angular momentum of a satellite

$$L = mv_0 r = m\sqrt{\frac{GM}{r}}r = \sqrt{GMm^2 r} = \sqrt{GMm^2\left(R+h\right)}\left(\because r = R+h\right)$$

For a surface satellite $\left(h << R\right)$ $L = \sqrt{GMm^2 R} = \sqrt{gR^3 m^2}$

m) For a satellite with increase in height of the orbit from the surface of the planet, then its
i) PE increases
ii) KE decreases
iii) Orbital velocity decreases
iv) Total energy increases
v) Period of revolution increases

n) If a body was projected with a velocity V from the earth surface at an angle θ with the horizontal, let V_1 be the velocity of the body at the maximum distance r_1 from the centre of the earth, then $mVR\cos\theta = mV_1 r_1$

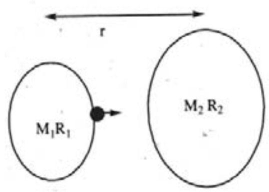


From law of conservation of energy, $-\frac{GMm}{R} + \frac{1}{2}mV^2 = -\frac{GMm}{r_1} + \frac{1}{2}mV_1^2$

o) If a satellite revoles in the direction opposite to the earth rotation, i.e., from east to west in the equatorial plane, then

$$\omega' = \omega_s + \omega_e \Rightarrow T' = \frac{T_s T_e}{T_s + T_e}$$

p) If two stars of masses M_1, M_2 and radii R_1, R_2 centres are separated by a distance r, then the minimum speed of projection of a body from the surface of one star towards another should be sufficient to cross the null point.



If the null point is at a distance x from $M_1, x = \dfrac{r}{\sqrt{\dfrac{M_2}{M_1}}+1}$

Apply law of conservation of energy at the surface of first star and null point.

$$-\frac{GM_1m}{R_1}-\frac{GM_2m}{(r-R_1)}+\frac{1}{2}mV^2=\frac{GM_1m}{x}-\frac{GM_2m}{(r-x)}$$

Escape Velocity (V_e):

a) It is the minimum velocity of an object to escape from the gravitational field.

b) From the surface of planet $V_e=\sqrt{\dfrac{2GM}{R}}=\sqrt{2gR}, V_e \propto \sqrt{\dfrac{M}{R}} \propto R\sqrt{\rho}$

c) $(V_e)_{moon}=2.38\ kms^{-1}, (Ve)_{earth}=11.2\,kms^{-1}$ & $(V_e)_{sun}=42\,kms^{-1}$

d) It depends on the mass and radius of the planet

e) It is independent of the mass of the body and the direction of projection

f) There is no atmosphere on the surface of the moon, because r.m.s velocity of the air molecules is greater than the escape velocity on the moon.

g) $V_e=\sqrt{2}V_0$

h) If the speed of the orbital satellite is made $\sqrt{2}$ times or increased by 41.4% or its KE is doubled, then it will escape to infinity.

i) For a satellite,

i) If $V<V_0$ then it falls to the surface of earth in parabolic path $(TE<0)$

ii) If $V=V_0$ then in revolves around the earth in circular orbit $(TE<0)$

iii) If $V_0<V<V_e$, then it revolves in an elliptical orbit $(TE<0)$

iv) If $V=V_e$, then it escapes from the gravitational field by parabolic path. $(TE=0)$

v) If $V>V_e$, then it escapes to infinity and enters into interstellar space with some velocity in hyperbolic path.

$V_\infty=\sqrt{V_P^2-V_e^2}, v_e \rightarrow$ escape velocity $(TE>0)$

$V_\infty \rightarrow$ Velocity at interstellar space or at infinity

Self energy:

It is the amount of work done by an external agent in assembling the body from infinitesimal small elements that are initially at infinite distance apart.

i) Self energy of a uniform sphere of mass M and radius R is $U=-\dfrac{3GM^2}{5R}$

ii) Self energy of a thin spherical shell of mass M and radius R is $U = -\frac{GM^2}{2R}$

Geo-stationary or communication or parking satellite:

a) Its time period of revolution around the earth is 24 hr
b) Its relative velocity w.r.t the earth is zero
c) Its orbit is called parking orbit and its height from the surface of the earth is 36,000 km or from the center of the earth is 42, 400 km
d) It revolves from west to east in the equatorial plane.

Polar satellite:

A satellite whose orbital plane is perpendicular to equatorial plane is called polar satellite. Since its time period is around 100 minutes, it crosses any altitude many times a day.

The angle between the equatorial plane and the orbital plane of polar satellite is 90^0

Weightlessness:

If $g = 0$. The object appears weightlessness this is possible when the present angular velocity of earth has to increase to 17 times.

If $g < R\omega^2$, object files off from the surface of the planet.

If $g > R\omega^2$, object will remain struck with the surface of planet.

Exercise – I

(Straight Objective – Including Previous Year Questions)

1. Gravitational force and laws

a) Universal law of gravitation

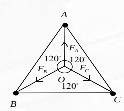
01. Suppose a tunnel could be dug through the earth from one side to the other along a diameter and particle of mass M is dropped into it. If all frictional forces are neglected, the particle will

1) Enter from one side and come out from the other with a velocity greater than that at the centre
2) Stop at the centre of the earth as earth attracts all bodies towards its centre
3) Undergo pendulum-like motion and never stop
4) Take a spiral path in the tunnel till it comes out from the other end

Key: 3

Sol:

02. A spherical hollow cavity is made in a lead sphere of radius R, such that its surface touches the outside surface of the lead sphere and passes through its centre. The mass of the sphere before hollowing was M. With what gravitational force will the hollowed out lead sphere attract a small sphere of mass ‘m’, which lies at a distance ‘d’ from the centre of the lead sphere on the straight line connecting the centres of the spheres and that of the hollow, if $d = 2R$:

1) $\dfrac{7GMm}{18R^2}$ 2) $\dfrac{7GMm}{36R^2}$ 3)  4) $\dfrac{7GMm}{72R^2}$

Key: 2

Sol:

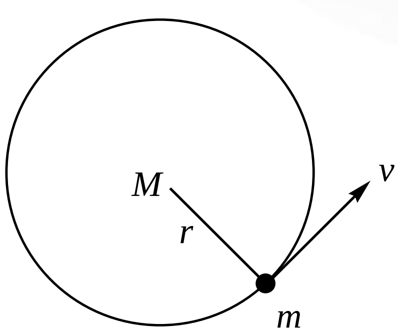
01. Consider a spherical gaseous cloud of mass density $\rho(r)$ in a free space where r is the radial distance from its centre. The gaseous could is made of particles of equal mass m moving in circular orbits about their common centre with the same kinetic energy K . The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time. The particle number density $n(r) = \dfrac{\rho(r)}{m}$ is : (G = universal gravitational constant)

(27 – May – 2019)

1) $\dfrac{K}{6\pi r^2 m^2 G}$ 2) $\dfrac{K}{\pi r^2 m^2 G}$ 3) $\dfrac{3K}{\pi r^2 m^2 G}$ 4) $\dfrac{K}{2\pi r^2 m^2 G}$

Key: 4

Sol:



$$\frac{GMm}{r^2}=\frac{mv^2}{r}=\frac{2}{r}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \frac{GMm}{r^2}=\frac{2K}{r}\Rightarrow M=\frac{2Kr}{Gm}$$

$$\Rightarrow dM=\frac{2K}{Gm}dr\Rightarrow 4\pi r^2 dr\rho=\frac{2K}{Gm}dr$$

$$\therefore \rho=\frac{K}{2\pi Gmr^2}$$

03. Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centers equal to 12R. if they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is **(09 – Jan – 2020 (M)) (D)**

1) 2.5R 2) 4.5R 3) 7.5R 4) 1.5R

Key: 7.5R

Sol: Just before collision, total distance travelled by both = 12R – R – 2R = 9R

If x is the distance travelled by block of mass M and 9R – x the distance travelled by block of mass 5M.

As center of mass of system not change, then

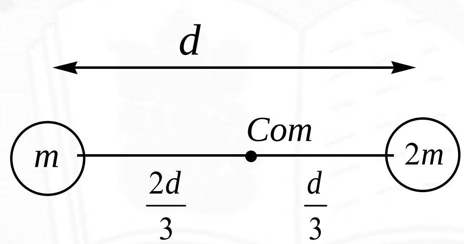
$$Mx = 5M(9R - x)\Rightarrow x = 7.5R$$

04. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is: **(24 – FEB – 2021(M))**

1) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$ 2) $2\pi\sqrt{\frac{d^3}{3Gm}}$ 3) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ 4) $2\pi\sqrt{\frac{3Gm}{d^3}}$

Key: **2**

Sol:



$$F=\frac{G(2m)m}{d^2}=(2m)\omega^2\frac{d}{3}$$

$$\frac{Gm}{d^2}=\omega^2\frac{d}{3}$$

$$\Rightarrow \omega^2=\frac{3Gm}{d^3}$$

$$\Rightarrow \omega=\sqrt{\frac{3Gm}{d^3}}$$

$$T=\frac{2\pi}{\omega}$$

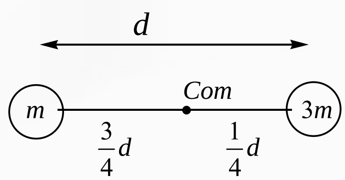
$$= 2\pi\sqrt{\frac{d^3}{3Gm}}$$

05. Two stars of masses m and $3m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is: **(24 – FEB – 2021(M)) (D)**

1) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$ 2) $2\pi\sqrt{\frac{d^3}{3Gm}}$ 3) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ 4) $2\pi\sqrt{\frac{d^3}{4Gm}}$

Key: **4**

Sol:



$$F = \frac{G(3m)m}{d^2} = (3m)\omega^2\frac{d}{4}$$

$$\frac{Gm}{d^2} = \omega^2\frac{d}{4}$$

$$\omega^2 = \frac{4Gm}{d^3}$$

$$\omega = \sqrt{\frac{4Gm}{d^3}}$$

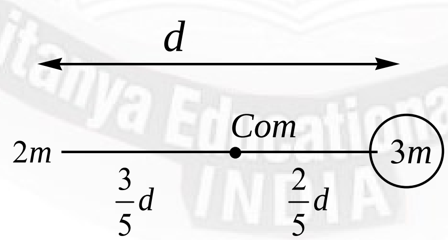
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{4Gm}}$$

06. Two stars of masses $2m$ and $3m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is: **(24 – FEB – 2021(M)) (D)**

1) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$ 2) $2\pi\sqrt{\frac{d^3}{5Gm}}$ 3) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ 4) $2\pi\sqrt{\frac{3Gm}{d^3}}$

Key: **2**

Sol:



$$F = \frac{G(3m)m}{d^2} = (3m)\omega^2\frac{2}{5}d$$

$$\frac{2Gm}{d^3} = \omega^2\frac{2}{5}$$

$$\omega^2 = \frac{5Gm}{d^3}$$

$$\omega = \sqrt{\frac{5Gm}{d^3}} \qquad T = \frac{2\pi}{\omega} \qquad T = 2\pi\sqrt{\frac{d^3}{5Gm}}$$

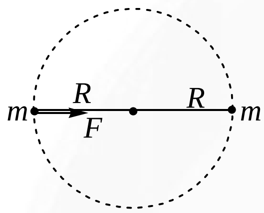
07. Two identical particles of mass $1kg$ each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is:

(27 – July – 2021(E))

1) $\sqrt{\frac{G}{2R^3}}$ 2) $\frac{1}{2}\sqrt{\frac{G}{R^3}}$ 3) $\frac{1}{2R}\sqrt{\frac{1}{G}}$ 4) $\sqrt{\frac{2G}{R^3}}$

Key: 2

Sol:



$$F = \frac{Gm^2}{(2R)^2} = mR\omega^2$$

$$\omega = \frac{1}{2}\sqrt{\frac{G}{R^3}}$$

08. Two identical particles of mass $4kg$ each go round a circle of radius 'r', under the action of their mutual gravitational attraction. The angular speed of each particle is:

(27 – July – 2021(E)) (D)

1) $\sqrt{\frac{G}{3r^3}}$ 2) $\sqrt{\frac{4G}{r^3}}$ 3) $\sqrt{\frac{G}{r^3}}$ 4) $\sqrt{\frac{2G}{r^3}}$

Key: 3

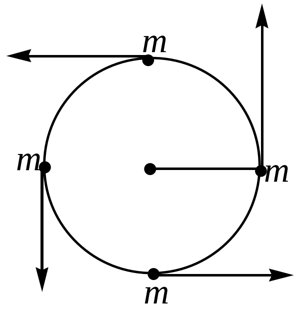
Sol:

$$\frac{Gmm}{(2r)^2} = mr\omega^2$$

$$\omega = \sqrt{\frac{Gm}{4r^3}}$$

$$= \sqrt{\frac{G \times 4}{4r^3}} = \sqrt{\frac{G}{r^3}}$$

09. Four similar particles of mass 'm' are orbiting in a circle of radius 'r' in the same angular direction because velocity of a particle is given by

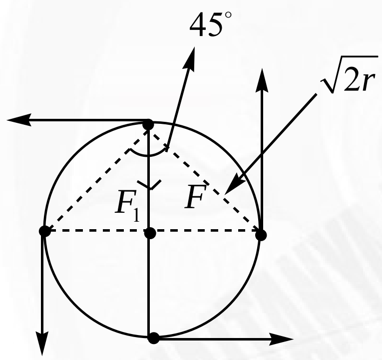


1) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ 2) $\sqrt{\frac{Gm}{r}}$

3) $\sqrt{\frac{Gm}{r}\left(1+2\sqrt{2}\right)}$ 4) $\left[\frac{1}{2}\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{2}\right)\right]^{\frac{1}{2}}$

Key: 1

Sol:



Centripetal force = net gravitational force

$\Rightarrow mV_0^2 = 2F\cos 45^\circ + F_1$

$\Rightarrow mV_0^2 = 2F\cos 45^\circ + F_1 = \frac{2Gm^2}{\left(\sqrt{2}\right)^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{4r^2}$

$V_0 = \left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$

10. Two objects of equal masses placed at certain distance from each other attracts each other with a force of F. If one third mass of one object is transferred to the other object, then new force will be **(28 – JUNE – 2022(E))**

1) $\frac{2}{9}F$ 2) $\frac{16}{9}F$ 3) $\frac{8}{9}F$ 4) F

Key: 3

Sol: $F = \frac{Gm_1m_2}{d^2}$

$F = \frac{Gm^2}{d^2}$

$$m_1 = m - \frac{m}{3} = \frac{2m}{3}$$

$$m_2 = m + \frac{m}{3} = \frac{4m}{3}$$

$$F' = G\frac{2m}{3}\frac{4m}{3d^2} = \frac{8Gm^2}{9d^2}$$

$$F' = \frac{8}{9}d$$

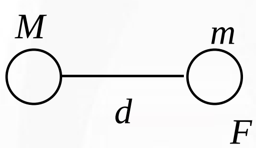
11. Gravitational force between a point mass m and M separated by a distance is F. Now if a point mass $3m$ is placed next to m in contact with it. The force on M is

(28 – JUNE – 2022(E)) (D)

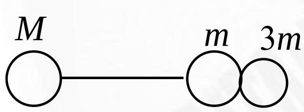
1) F 2) 2F 3) 3F 4) 4F

Key: 4

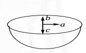
Sol: $F = \frac{Gm_1m_2}{d^2}$



$$F = \frac{GmM}{d^2}$$



$$F' = G\frac{4mM}{d^2} \qquad m_2 = m + 3m$$



$$F' = 4F$$

12. If the mass of one particle is increased by 50% and the mass of another particle is decreased by 50% the force between them is **(28 – JUNE – 2022(E)) (D)**

1) Decreases by 25% 2) Decreases by 75% 3) Increases by 25% 4) Does not change

Key: 1

Sol: $F = \frac{Gm_1m_2}{d^2}$

$$m_1 = m_1 + \frac{m_1}{2} = \frac{3m_1}{2}$$

$$m_2 = m_2 - \frac{m_2}{2} = \frac{m_2}{2}$$

$$F' = \frac{G}{d^2}\frac{3m_1 \times m_2}{2 \times 2}$$

$$F' = \frac{3}{4}\frac{Gm_1m_2}{d^2}$$

$$F' = \frac{3}{4}F$$

$$\left(\frac{F'}{F} - 1\right) \times 100 = \left(\frac{3}{4} - 1\right) \times 100$$

$= -25\%$

decreases 25%

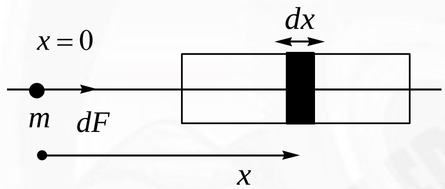
b) Principle of super position-Gravitational force

13. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force is exerts on a point mass 'm' at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by: **(12 – JAN – 2019 (M))**

1) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$ 2) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

3) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$ 4) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$

Key: 2



Sol:

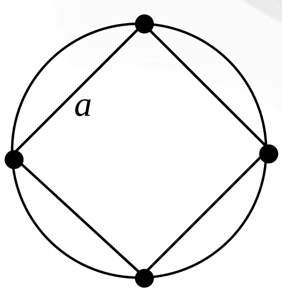
$$dm = \left(A + Bx^2\right)dx$$

$$dF = \frac{GM\,dm}{x^2} \qquad = F = \int_a^{a+L} \frac{GM}{x^2}\left(A + Bx^2\right)dx$$

$$= GM\left[-\frac{A}{x} + Bx\right]_a^{a+L}$$

$$= GM\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

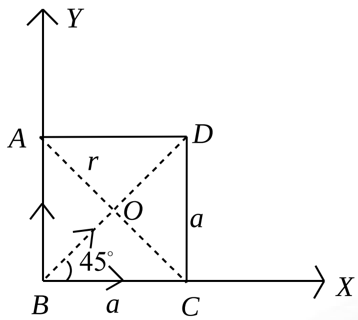
14. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others gravitational field in a circular orbit circumscribing the square? **(08 – Apr – 2019 (M))**



1) $1.35\sqrt{\frac{GM}{a}}$ 2) $1.16\sqrt{\frac{GM}{a}}$ 3) $1.21\sqrt{\frac{GM}{a}}$ 4) $1.41\sqrt{\frac{GM}{a}}$

Key: 2

Sol:



$AC = a\sqrt{2}, r = \frac{AC}{2} = \frac{a}{\sqrt{2}}$

Resultant force on the body

$$B = \frac{GM^2}{a^2}\hat{i} + \frac{GM^2}{a^2}\hat{j} + \frac{GM^2}{\left(a\sqrt{2}\right)^2}\left(\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}\right)$$

$$\Rightarrow |F| = \frac{GM^2}{a^2}\left(\sqrt{2}\right) + \frac{GM^2}{2a^2}$$

$\frac{Mv^2}{r} =$ Resultant force towards centre

$$\frac{Mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$\Rightarrow v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow v = 1.16\sqrt{\frac{GM}{a}}$$

15. Three particles each of mass 'm' are situated at the vertices of an equilateral triangle of side length 'a'. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original separation 'a'. Find the initial velocity that should be given to each particle?

1) $v = \sqrt{\frac{Gm}{a}}$ 2) $v = -\sqrt{\frac{Gm}{a}}$ 3) $v = -\sqrt{\frac{Gm^2}{a}}$ 4) $v = \sqrt{\frac{Gm^2}{a}}$

Key: 1

Sol:

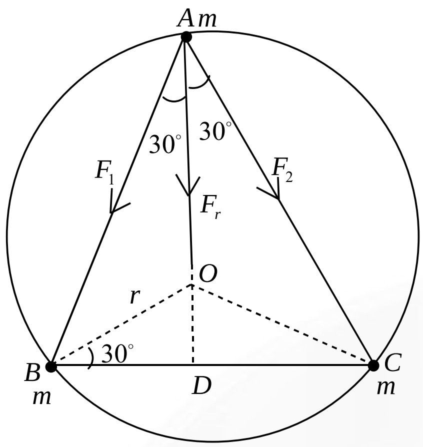


Figure shows three particles located at vertices A, B and C of an equilateral triangle of sides AB = BC= CA = a. These particles move in a circle with O as the centre and radius
r = OA = OB = OC

$$r = \frac{BD}{\cos 30^\circ} = \frac{\frac{a}{2}}{\frac{\sqrt{3}}{2}} = \frac{a}{\sqrt{3}}$$

Let us find the net gravitational force acting on one particle, say at A, due to particles at B and C.

Particle at A is attracted towards B with a force, $F_1 = \frac{Gmm}{a^2}$ and towards C with a force,

$$F_2 = \frac{Gmm}{a^2}$$

$$F_1 = F_2 = F(say) = \frac{Gm^2}{a^2}$$

The angle between these equal forces is $\theta = \angle BAC = 60^\circ$.
The resultant force on the particle at A is

$$F_r = \left(F^2 + F^2 + 2F^2 \cos 60^\circ\right)^{\frac{1}{2}} \Rightarrow F_r = \sqrt{3}\, F$$

$F_r = \sqrt{3}\frac{Gm^2}{a^2}$ directed along AO.

Thus the net force on particle A is radial. Similarly, the net force on particle at B and C is F_r , each directed towards centre O. This force provides the necessary centripetal force. If v is the required initial velocity of each particle ,

Then $\frac{mv^2}{r} = \sqrt{3}\frac{Gm^2}{a^2}$ or $v^2 = \sqrt{3}\frac{Gm}{\sqrt{3}\,a} = \frac{Gm}{a}$

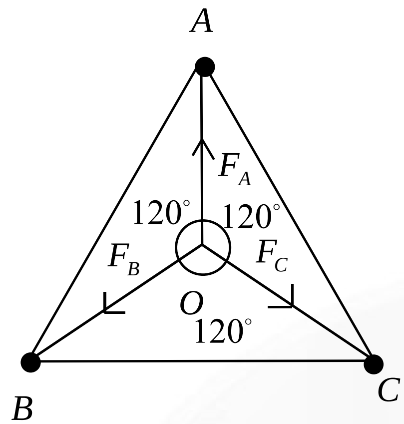
$$v = \sqrt{\frac{Gm}{a}}$$

16. The point masses each of mass m are placed at the vertices of an equilateral triangle of side a. What is the gravitational field?

1) 0 2) 1 3) 2 4) 3

Key: 1

Sol:



To find gravitational field at point O due to the three masses, we find the net force exerted on a unit mass placed at point O, due to masses m at vertices A, B and C.

Since the three masses are equal and their distance from O are also equal, they will exert forces F_A, F_B and F_C of equal magnitudes. Their directions are shown in figure. It follows from the symmetry of forces that the resultant force at Point O is zero. (Three equal forces in magnitude are acting at an $\angle 120^\circ$ with each other so resultant is zero)

Hence the gravitational field at O (resultant force per unit mass at O) is zero.

17. A test particle is moving in circular orbit in the gravitational field produced by mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T: **(08 – Apr – 2019 (E))**

1) $\frac{T}{R}$ is a constant
2) $\frac{T^2}{R^3}$ is a constant
3) $\frac{T}{R^2}$ is a constant
4) TR is a constant

Key: 1

Sol: $F = \frac{GMm}{r} = \int a \frac{\rho(dV)m}{r^2} = mG\int_0^R \frac{k}{r^2}\frac{4\pi r^2 dr}{r^2} = 4\pi kG\left(\frac{1}{r}\right)_0^R = -\frac{4\pi kGm}{R}$

Using Newton's second law, we have

$\frac{mv_0^2}{R} = \frac{4\pi kGm}{R}$ or $v_0 = C(constant)$

Time period , $T = \frac{2\pi R}{v_0} = \frac{2\pi R}{C}$ or $\frac{T}{R} = constant$

18. A cavity of radius $\frac{R}{2}$ is made inside a solid sphere of radius R . The centre of the cavity is located at a distance $\frac{R}{2}$ from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance $\frac{R}{2}$ from the centre of the sphere on the line joining both the centres of sphere and cavity is_____ (Opposite to the centre of gravity)

(Here $g = \frac{GM}{R^2}$, where M is the mass of the sphere)

1) $\dfrac{mg}{2}$ 2) $\dfrac{3mg}{8}$ 3) $\dfrac{mg}{16}$ 4) $\dfrac{mg}{8}$

Key: 2

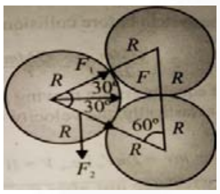
Sol: Conceptual

19. If three uniform spheres, each having mass M and radius R, are kept in such a way that each touches the other two, the magnitude of the gravitational force on any sphere due to the other two is **(08 – Jan – 2020 (M)) (D)**

1) $\dfrac{GM^2}{4r^2}$ 2) $\dfrac{2GM^2}{r^2}$ 3) $\dfrac{2GM^2}{4r^2}$ 4) $\dfrac{\sqrt{3}GM^2}{4r^2}$

Key: 4

Sol:



$\vec{F} = \vec{F}_1 + \vec{F}_2$

As $\left|\vec{F}_1\right| = \left|\vec{F}_2\right|$

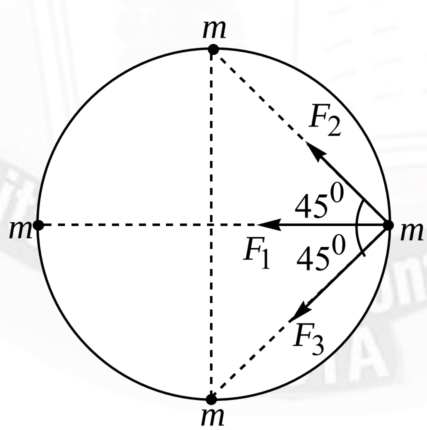
$\therefore \left|\vec{F}\right| = 2F_1 \cos 30^\circ \Rightarrow \left|\vec{F}\right| = 2\dfrac{GM^2}{(2r)^2}\dfrac{\sqrt{3}}{2} \Rightarrow \left|\vec{F}\right| = \dfrac{\sqrt{3}}{4}\dfrac{GM^2}{r^2}$

20. Four identical particles of equal masses $1kg$ made to move along the circumference of a circle of radius $1m$ under the action of their own mutual gravitational attraction. The speed of each particle will be: **(24 – FEB – 2021(M))**

1) $\sqrt{\dfrac{G}{2}\left(1+2\sqrt{2}\right)}$ 2) $\sqrt{G\left(1+2\sqrt{2}\right)}$ 3) $\sqrt{\dfrac{G}{2}\left(2\sqrt{2}-1\right)}$ 4) $\sqrt{\dfrac{\left(1+2\sqrt{2}\right)G}{2}}$

Key: 4

Sol:



$m = 1kg$

$R = 1m$

$$F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$$

$$F_2 = \frac{Gmm}{\left(\sqrt{2}R\right)^2} = \frac{Gm^2}{2R^2}$$

$$F_3 = \frac{Gmm}{\left(\sqrt{2}R\right)^2} = \frac{Gm^2}{2R^2}$$

$$\Rightarrow F_{net} = F_1 + F_2\cos 45^0 + F_3\cos 45^0$$

$$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}$$

$$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

$$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = \frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right)$$

$$F_{net} = \frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right) = \frac{mv^2}{R}$$

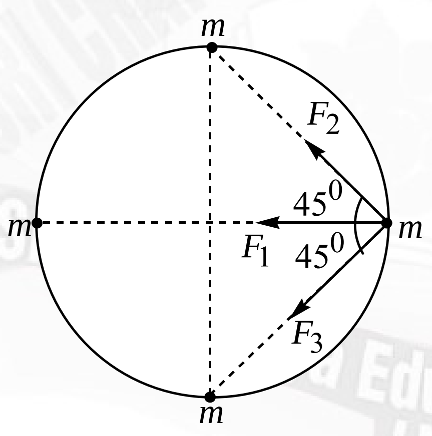
$$\Rightarrow v = \frac{\sqrt{G\left(1+2\sqrt{2}\right)}}{2}$$

21. Four identical particles of equal masses $2\,kg$ made to move along the circumference of a circle of radius $1\,m$ under the action of their own mutual gravitational attraction. The speed of each particle will be: **(24 – FEB – 2021(M)) (D)**

1) $\sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$ 2) $\sqrt{G\left(1+2\sqrt{2}\right)}$ 3) $\sqrt{\frac{G}{2}\left(2\sqrt{2}-1\right)}$ 4) $\sqrt{2G\left(1+2\sqrt{2}\right)}$

Key: 1

Sol:



$m = 2\,kg$

$R = 1\,m$

$$F_1 = \frac{Gm^2}{4R^2}$$

$$F_2 = F_3 = \frac{Gm^2}{2R^2}$$

$$\Rightarrow F_{net} = F_1 + F_2\cos 45^0 + F_3\cos 45^0$$

$$=\frac{Gm^2}{4R^2}+\frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}+\frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}$$

$$F_{net}=\frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right)$$

$$F_{net}=\frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right)=\frac{mv^2}{R}$$

$$\Rightarrow v=\sqrt{\frac{Gm\left(1+2\sqrt{2}\right)}{4R}}$$

m = 2 kg, R = 1 m

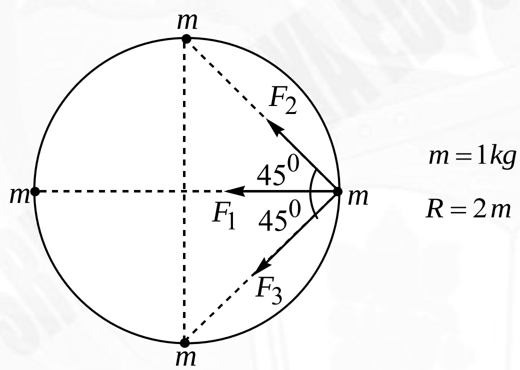
$$v=\sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$$

22. Four identical particles of equal masses $1kg$ made to move along the circumference of a circle of radius $2m$ under the action of their own mutual gravitational attraction. The speed of each particle will be: **(24 – FEB – 2021(M)) (D)**

1) $\sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$ 2) $\sqrt{G\left(1+2\sqrt{2}\right)}$ 3) $\sqrt{\frac{G}{8}\left(1+2\sqrt{2}\right)}$ 4) $\sqrt{\frac{G}{4}\left(1+2\sqrt{2}\right)}$

Key: 3

Sol:



$$F_1=\frac{Gm^2}{4R^2}$$

$$F_2=F_3=\frac{Gm^2}{2R^2}$$

$$\Rightarrow F_{net}=F_1+F_2\cos 45^0+F_3\cos 45^0$$

$$=\frac{Gm^2}{4R^2}+\frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}+\frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}$$

$$F_{net}=\frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right)$$

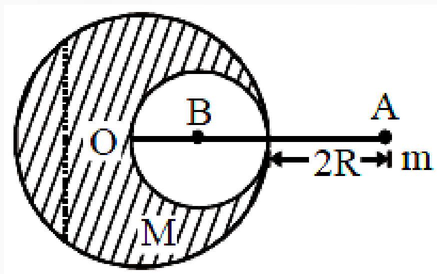
$$F_{net}=\frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right)=\frac{mv^2}{R}$$

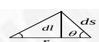
$$\Rightarrow v=\sqrt{\frac{Gm\left(1+2\sqrt{2}\right)}{4R}}$$

m = 1 kg, R = 2 m

$$v=\sqrt{\frac{G}{8}\left(1+2\sqrt{2}\right)}$$

23. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is: **(25 – FEB – 2021(M))**

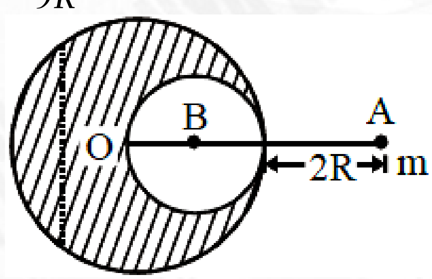


1) $25:36$ 2)  3)  4) $41:50$

Key: 3

Sol: Let initial mass of sphere is m'. Hence mass of removed portion will be $m'/8$

$$F_1 = m.E. = \frac{m.Gm'}{9R^2}$$



$$F_2 = m\left[\frac{G.m'}{(3R)^2}-\frac{G.m'/8}{(5R/2)^2}\right]$$

$$=\frac{Gm'}{9R^2}-\frac{Gm'\times 4}{8\times 25}$$

$$=\left(\frac{1}{9}-\frac{1}{50}\right)\frac{Gm'}{R^2}$$

$$F_2=\frac{41}{50\times 9}\cdot\frac{Gm'}{R^2}$$

$$\frac{F_1}{F_2}=\frac{1}{9}\times\frac{50\times 9}{41}=\frac{50}{41}$$

24. A solid sphere of radius R gravitationally attracts a particle placed at 2R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is: **(25 – FEB – 2021(M)) (D)**

1) $41:50$ 2) $36:25$ 3) $50:41$ 4) $9:7$

Key: **4**

Sol: Gravitational field intensity at A before making cavity

$$g_1 = \frac{GM}{(2R)^2} = \frac{GM}{4R^2} \quad -------(1)$$

after making cavity

$$g_2 = \frac{GM}{(2R)^2} - \frac{GM/8}{(R+R/2)^2} = \frac{GM}{4R^2} - \frac{GM}{8 \times \frac{9R^2}{4}}$$

$$= \frac{GM}{(2R)^2} - \frac{GM}{18R^2} \quad \Rightarrow \frac{9GM - 2GM}{36R^2} = \frac{7GM}{36R^2} \quad -------(2)$$

$$F_1 : F_2 = mg_1 : mg_2 = g_1 : g_2$$

$$\Rightarrow F_1 : F_2 = \frac{GM}{4R^2}; \ \frac{7GM}{36R^2}$$

$$\frac{F_1}{F_2} = \frac{\frac{GM}{4R^2}}{\frac{7GM}{36R^2}} = \frac{9}{7}$$

25. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{4}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is: **(25 – FEB – 2021(M)) (D)**

1) $36:25$ 2) $36:35$ 3) $41:50$ 4) $7:9$

Key: **2**

Sol: Gravitational intensity $E_g = \frac{GM}{9R^2} \left(before\ cavity\right)$

after cavity

$$Eg' = \frac{GM}{9R^2} - \frac{GM/64}{\left(\frac{9R}{4}\right)^2} = \frac{GM}{9R^2} - \frac{\frac{GM}{64}}{\frac{81R^2}{16}} = \frac{GM}{9R^2} - \frac{GM}{324R^2}$$

$$= \frac{36GM - GM}{324R^2} = \frac{35GM}{324R^2}$$

$$F_1 = mg, \ F_2 = mg'$$

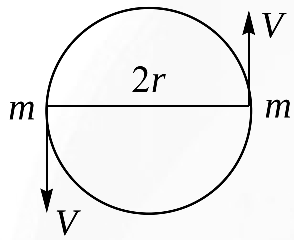
$$\frac{F_1}{F_2}=\frac{Eg}{Eg'}=\frac{\frac{GM}{9R^2}}{\frac{35GM}{324R^2}}=\frac{324}{9\times 35}=\frac{36}{35}$$

26. Two particles of equal mass $'m'$ more in a circle of radius $'r'$ under the action of their mutual gravitational attraction. The speed of each particle will be **(29 – Jan – 2023(M))**

1) $\sqrt{\frac{GM}{2r}}$ 2) $\sqrt{\frac{4GM}{r}}$ 3) $\sqrt{\frac{GM}{r}}$ 4) $\sqrt{\frac{GM}{4r}}$

Key: 4

Sol:



$$\frac{Gm^2}{4r^2}=\frac{mV^2}{r}$$

$$V=\sqrt{\frac{GM}{4r}}$$

27. Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T on what power of $'r'$ will the square of time period depend of the gravitational force of attraction between the planet and the star is proportional to $r^{5/2}$ **(29 – Jan – 2023(M)) (D)**

1) $r^{5/2}$ 2) $r^{3/2}$ 3) $r^{7/2}$ 4) $r^{9/2}$

Key: 3

Sol: $\frac{mV^2}{r}=\frac{k}{r^{5/2}}\Rightarrow V^2=\frac{k}{mr^{3/2}}$

$$T=\frac{2\pi r}{V}=2\pi r\sqrt{\frac{mr^{3/2}}{k}}$$

$$\therefore \boxed{T^2\propto r^{7/2}}$$

28. Two particles of masses $1kg$ & $2kg$ are placed at a distance of $50cm$. The initial acceleration of the first particle due to gravitational force is **(29 – Jan – 2023(M)) (D)**

1) $5.3\times10^{-10}\,m/s^2$ 2) $1.25\times10^{-8}\,m/s^2$

3) $4.28\times10^{-10}\,m/s^2$ 4) $8.2\times10^{-8}\,m/s^2$

Key: 1

Sol: $F=\frac{Gm_1m_2}{r^2}=\frac{6.67\times10^{-11}\times1\times2}{(0.5)^2}=5.3\times10^{-10}\,N$

Acceleration of 1 kg particle is $a_1 = \frac{F}{m_1} = \frac{5.3\times10^{-10}}{1} = 5.3\times10^{-10}\ ms^{-2}$ towards 2 kg mass

c) Kepler's laws

29. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is: **(09 – JAN – 2019 (M))**

1) $\frac{4L}{m}$ 2) $\frac{L}{m}$ 3) $\frac{L}{2m}$ 4) $\frac{2L}{m}$

Key: 3

Sol: $\frac{dA}{dt} = \frac{L}{2m}$

30. Angular momentum of planet of mass $10^6\ kg$ is moving around the sun in a circular orbit is **(09 – JAN – 2019 (M)) (D)**

1) $2\times10^8\ m^2/s$ 2) $4\times10^{14}\ m^2/s$ 3) $4\times10^8\ m^2/s$ 4) $2\times10^6\ m^2/s$

Key: 1

Sol:

31. Choose the correct statement from among the following

1) A planet moves in an elliptical orbit with the centre of mass located at the intersection of major and minor axes of the ellipse
2) The position vector for a planet, no matter from where it is measured, sweeps out equal area in equal time intervals
3) The time period of a planet is directly proportional to the cube root of the semi-major axis
4) The ratio of the square of the time period to the cube of the semi-major axis is approximately the same for all planets.

Key: 4

Sol:

32. Find the correct statement from among the following

1) The angular moments of all planets about the sun do not necessarily remain constant
2) Kepler's second law is valid for any central force, that is, any force directed towards the sun
3) Kepler's first law requires that a force of gravitation can yield planetary orbits which are circular only with the sun at its centre
4) The gravitational field concept was developed and used by Newton

Key: 2

Sol:

33. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $\left(M_P + M_Q\right)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then

1) $V_Q > V_R > V_P$ 2) $V_R > V_Q > V_P$ 3) $\frac{\upsilon_R}{\upsilon_P} = 3$ 4) $\frac{\upsilon_P}{\upsilon_Q} = \frac{1}{2}$

Key: 2

Sol:

34. A planet revolving in elliptical orbit has:

(A) a constant velocity of revolution.
(B) has the least velocity when it is nearest to the sun.
(C) its areal velocity is directly proportional to its velocity.
(D) areal velocity is inversely proportional to its velocity.
(E) to follow a trajectory such that the areal velocity is constant.
Choose the correct answer from the options given below: **(26 – FEB – 2021(M))**
1) A only 2) D only 3) C only 4) E only

Key: 4

Sol: As per Keppler's 2nd law, areal velocity is constant.

35. If the distance between the sun and the earth is doubled then the duration of year will be **(26 – FEB – 2021(M)) (D)**
1) $2\,year$ 2) $2\sqrt{2}year$ 3) $4year$ 4) $8year$

Key: 2

Sol: $T^2 \propto r^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{r}{2r}\right)^3$$

$T_2 = 2\sqrt{2}\,T_1$

36. The law states that the line joins any planet to the sun sweeps equal areas in equal interval of time are: **(26 – FEB – 2021(M)) (D)**
1) Law of gravitation 2) Law of periods 3) Law of areas 4) law of orbits

Key: 3

Sol: According to law of areas

37. Maximum and minimum distances of a comet from the Sun are $1.6\times10^{12}met$ and $8\times10^{10}met$ if speed of the comet at nearest point is $6\times10^4\ m/s$, speed at farthest point is m/s: **(16 – March – 2021 (M))**
1) 1.5×10^2 2) 6×10^2 3) 3×10^3 4) 4.5×10^3

Key: 3

Sol: $m_eV_1r_1 = mV_2r_2 \Rightarrow V_1 = \dfrac{48\times10^4}{1.6\times10^{12}} = 3000\dfrac{m}{s} = 3\times10^3\ m/s$

38. Maximum and minimum distance of a comet from sun are 0.8×10^{12} *meter* & 4×10^{10} *meter*, if speed of comet at nearest point is $3\times10^4\dfrac{m}{s}$, speed at farthest point is **(16 – March – 2021 (M)) (D)**
1) $1000\,m/s$ 2) $3000\,m/s$ 3) $1500\,m/s$ 4) $6000\,m/s$

Key: 3

Sol: $mV_1r_1 = mV_2r_2$

$$V_1 = \frac{V_2r_2}{r_1} = \frac{3\times10^4\times4\times10^{10}}{0.8\times10^{12}} = 1500\,m/s$$

39. A geostationary satellite is orbiting around an arbitrary point P at a height of 11R above surface of P, R is radius of P, the time period of another satellite in hours at a height of 2R from surface of P is _____ **(17 – March – 2021 (E))**

1) $6\sqrt{2}$ 2) $\frac{6}{\sqrt{2}}$ 3) 3 4) 5

Key: 3

Sol: $T \propto R^{3/2}$

$$\frac{24}{T}\frac{\propto (11R+R)^{3/2}}{\propto (2R+R)^{3/2}} \Rightarrow T = 3\,hour$$

40. A geostationary satellite around an arbitrary point P at a height 7R above surface of P, R is radius of P, the time period of another satellite in hours at a height of 3R from surface of P is **(17 – March – 2021 (E)) (D)**

1) $6\sqrt{2}$ 2) $\frac{6}{\sqrt{2}}$ 3) 3 4) 5

Key: 1

Sol: $T \propto R^{3/2}$

$$\frac{24}{T} = \left[\frac{7R+R}{3R+R}\right]^{3/2}$$

$$\frac{24}{T} = 2\sqrt{2} \Rightarrow T = 6\sqrt{2}$$

41. Time period of a satellite in a circular orbit of radius R is T, the period of another satellite in a circular orbit of radius 9R is **(18 – March – 2021 (M))**

1) 9T 2) 27T 3) 12T 4) 3T

Key: 2

Sol: $$\left(\frac{T'}{T}\right)^2 = \left(\frac{9R}{R}\right)^3$$

$$\Rightarrow \left(T'\right)^2 = T^2\left(9^3\right) \Rightarrow T' = 27T$$

42. Time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 4R is **(18 – March – 2021 (M)) (D)**

1) 8T 2) 64T 3) 32T 4) 128T

Key: 1

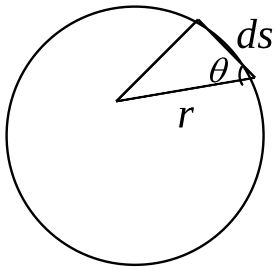
Sol: $$\left(\frac{T'}{T}\right)^2 = \left(\frac{4R}{R}\right)^3 \Rightarrow \frac{T'}{T} = 4^{3/2} \Rightarrow \frac{T'}{T} = 8 \Rightarrow T' = 8T$$

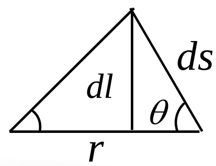
43. The angular momentum of a planet of mass M moving around sun in an elliptical orbit is L, the magnitude of a real velocity of planet is **(18 – March – 2021 (E))**

1) $\frac{4L}{M}$ 2) $\frac{L}{M}$ 3) $\frac{2L}{M}$ 4) $\frac{L}{2M}$

Key: 4

Sol: For small displacement





$$A=\frac{1}{2}r\,dl=\frac{1}{2}base\times height$$

$$=\frac{1}{2}r\left(ds\sin\theta\right)\qquad \because \sin\theta=\frac{dl}{ds}$$

$$\frac{dA}{dt}=\frac{1}{2}\left(r\sin\theta\right)\frac{ds}{dt}=\frac{\left(r\sin\theta\right)V}{2}\frac{m}{m}=\frac{mVr\sin\theta}{2m}=\frac{L}{2m}$$

44. A planet of mass m is moving in an elliptical orbit around sun of mass M, if the maximum and minimum distances of the planet from sun be l_1 and l_2, the angular momentum of planet about sun will be **(18 – March – 2021 (E)) (D)**

1) $\dfrac{GMm^2}{\sqrt{l_1+l_2}}$ 2) $m\sqrt{\dfrac{l_1+l_2}{GMl_1l_2}}$ 3) $m\sqrt{\dfrac{2GMl_1l_2}{l_1+l_2}}$ 4) 0

Key: 3

Sol: For a planet moving around the sun in an orbit angular momentum (L) is constant

$$L=mV_1l_1=mV_2l_2 \qquad \Rightarrow V_1=\frac{V_2l_2}{l_1}$$

By conserving energy between the 2 points farthest and nearest
total energy = total energy

$$-\frac{GMm}{l_1}+\frac{1}{2}mV_1^2=-\frac{GMm}{l_2}+\frac{1}{2}mV_1^2$$

$$GM\left(\frac{1}{l_1}-\frac{1}{l_2}\right)=\frac{1}{2}\left[V_2^2\left(\frac{l_2^2}{l_1^2}\right)-V_2^2\right]$$

$$GM\frac{(l_2-l_1)}{l_1l_2}=\frac{V_2^2}{2}\left[\frac{(l_2-l_1)(l_2+l_1)}{l_1^2}\right]$$

$$V_2^2=\frac{2GMl_1}{(l_1+l_2)l_2}\Rightarrow V_2=\sqrt{\frac{2GMl_1}{(l_1+l_2)l_2}}$$

$$L=mV_2l_2$$

Substitute for V_2

$$=m\sqrt{\frac{2GMl_1}{(l_1+l_2)l_2}}l_2$$

$$L=m\sqrt{\frac{2GMl_1l_2}{l_1+l_2}}$$

45. A satellite is launched into a circular orbit of radius 'R' around earth, while a second

satellite is launched into a circular orbit of radius 1.02R. The percentage difference in the time periods of the two satellite is: **(20 – July – 2021(E))**

1) 1.5 2) 2.0 3) 0.7 4) 3.0

Key: 4

Sol: $T^2 \propto R^3$

$T = kR^{3/2}$

$\frac{dT}{T} = \frac{3}{2}\frac{dR}{R}$

$= \frac{3}{2} \times 0.02 = 0.03$

$\% change = 3\%$

46. A satellite launched into a circular orbit of radius 'R' around earth, while a second satellite is launched into a circular orbit of radius 1.003R. The percentage difference in the time periods of the two satellite is **(20 – July – 2021(E)) (D)**

1) 0.97 2) 0.45 3) 2.34 4) 5.78

Key: 2

Sol: $T^2 \propto R^3$

$2\frac{dT}{T} = 3\frac{dR}{R}$

$2\frac{dT}{T} \times 100\% = \frac{3(1.003R - R)}{R} \times 100\%$

$\frac{dT}{T} \times 100\% = 1.5 \times 0.3\% = 0.45\%$

47. Consider a binary star system of star 'A' and star 'B' with masses m_A and m_B revolving in a circular orbit of radii 'r_A' and 'r_B' respectively. If T_A and T_B are the time period of star 'A' and star 'B' respectively then: **(20 – July – 2021(E))**

1) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$ 2) $T_A = T_B$

3) $T_A > T_B \left(if\ \ m_A > m_B\right)$ 4) $T_A > T_B \left(if\ \ r_A > r_B\right)$

Key: 2

Sol: $T_A = T_B$ $\left(\sin ce\ \omega_A = \omega_B\right)$

48. Consider a binary star system of star 'P' and star 'Q' with masses m_P and m_Q revolving in a circular orbit of radii 'r_P' and 'r_Q' respectively. If T_P and T_Q are the time period of star 'P' and star 'Q' respectively then **(20 – July – 2021(E)) (D)**

1) $\frac{T_P}{T_Q} = \left(\frac{r_P}{r_Q}\right)^{2/3}$ 2) $T_P = T_Q$

3) $T_P > T_Q \left(If\ \ m_P > m_Q\right)$ 4) $T_P > T_Q \left(If\ \ r_A > r_B\right)$

Key: 2

Sol: $T_P = T_Q \left(\because \omega_T = \omega_Q\right)$

49. The minimum and maximum distances of a planet revolving around the sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be: **(25 – July – 2021(M))**

1) $\dfrac{v_0 x_1^2}{x_2^2}$ 2) $\dfrac{V_0 x_2^2}{x_1^2}$ 3) $\dfrac{v_0 x_1}{x_2}$ 4) $\dfrac{v_0 x_2}{x_1}$

Key: **4**

Sol: Angular momentum conservation equation $v_0 x_2 = v_1 x_1$

$$v_1 = \frac{v_0 x_2}{x_1}$$

50. The minimum and maximum distances of a planet revolving around the sun are $5\times10^7\, km$ and $4\times10^8\, km$. If the minimum speed of the planet on its trajectory is $'v_0'$then its maximum speed will be: **(25 – July – 2021(M)) (D)**

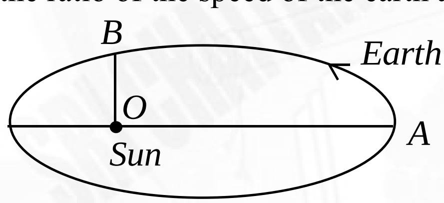
1) $8v_0$ 2) $5v_0$ 3) $1.2v_0$ 4) $0.12v_0$

Key: **1**

Sol: $mv_0 \times 4\times10^8 = mv\times5\times10^7 \left(\because mvr = constent\right)$

$V = 8v_0$

51. The earth moves around the sun in an elliptical orbit as shown in figure. The ratio $\dfrac{OA}{OB} = x$, the ratio of the speed of the earth at B to that at A is nearly B



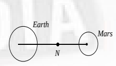
1) $\sqrt{x}$ 2) x 3) $x\sqrt{x}$ 4) x^2

Key: 2

Sol: Conservation of angular momentums at A and B

$$mV_A \times O_A = mV_B \times O_B \Rightarrow \frac{V_B}{V_A} = \frac{OA}{OB} = x$$

52. The distance between sun and earth is R. the duration of year if the distance between sun and earth becomes 3R will be **(24 – JUNE – 2022(E))**

1) $\sqrt{3}\, years$ 2) $3\, years$ 3) 

4) $3\sqrt{3}\, years$

Key: **4**

Sol: $R_1 = R$ $T_1 = T$

$R_2 = 3R$ $T_2 = ?$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$\frac{T_1}{T_2}=\left(\frac{R}{3R}\right)^{3/2}$$

$$T_2=3^{3/2}T=\sqrt{3^3}T$$

$$T_2=3\sqrt{3}T$$

53. The period of revolution of an earth's satellite close to the surface of earth is 60 minutes. The period of another earth's satellite in an orbit at a distance of 3 times earth radius from its surface will be in minutes **(24 – JUNE – 2022(E)) (D)**

1) 90 2) $90\sqrt{8}$ 3) 270 4) 480

Key: 4

Sol: $R_1=R$ $T_1=60\min$

$R_2=4R$ $T_2=?$

$T^2\alpha R^3 \quad \Rightarrow T\,\alpha\, R^{3/2}$

$$\frac{T_2}{T_1}=\left(\frac{R_2}{R_1}\right)^{3/2}$$

$$\frac{T_2}{60}=\left(\frac{4R}{R}\right)^{3/2}$$

$$T_2=60\times(4)^{3/2}$$

$$=60\times\left(2^2\right)^{3/2}$$

$$T_2=480\min$$

54. The time period of a satellite of earth is 5 hours if the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become. **(24 – JUNE – 2022(E)) (D)**

1) 10 hours 2) 80 hours 3) 30 hours 4) 40 hours

Key: 4

Sol: $T_1=5H$ $T_2=?$

$R_1=R$ $R_2=4R$

$T^2\,\alpha\,R^3$

$T\,\alpha\,R^{3/2}$

$$\frac{T_2}{T_1}=\left(\frac{R_2}{R_1}\right)^{3/2}=\left[\frac{4R}{R}\right]^{3/2}$$

$$T_2=8T_1=8\times 5$$

$$T_2=40\,hours$$

55. Two planet A and B of equal mass are having their of revolutions T_A and T_B such that $T_A=2T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits? **(28 – JUNE – 2022(M))**

1) $2r_A^2 = r_B^2$ 2) $r_A^2 = 2r_B^2$ 3) $r_A^3 = 4r_B^3$ 4) $T_A^2 - T_B^2 = \frac{\pi}{GM}\left(r_B^3 - 4r_A^3\right)$

Key: 3

Sol: $T_A = 2T_B$

$T^2 \propto r^3$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$\left(\frac{2T_B}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$4 = \frac{r_A^3}{r_B^3}$

$r_A^3 = 4r_B^3$

56. The time period of a satellite of earth is 8h. If the separation between the earth and the satellite is increased to 9 times the previous value the new time period will become

(28 – JUNE – 2022(M)) (D)

1) 216h 2) 80h 3) 40h 4) 20h

Key: 1

Sol: $T_1 = 8h$ $T_2 = ?$

$R_1 = R$ $R_2 = 9R$

$T^2 \propto R^3$

$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} = \left(\frac{9R}{R}\right)^{3/2}$

$\frac{T_2}{8} = 3^2$

$T_2 = 27 \times 8$

$T_2 = 216\ hr$

57. The time period of a geostationary satellite at a height $36000\,km$ is $24h$. A spy satellite orbits earth at a height $6400\,km$. What will be the time period of sky satellite?

Radius of the earth = $6400\,km$ **(28 – JUNE – 2022(M)) (D)**

1) 5h 2) 4h 3) 3h 4) 12h

Key: 2

Sol: $h_1 = 36000\,km$

$T_1 = 24\,hr$

$T_2 = ?$

$h_2 = 6400\ km$

$T^2 \propto R^3$

$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{6400+6400}{6400+36000}\right)^3$

$$T_2^2 = (24)^2 \times \left(\frac{16}{53}\right)^3$$

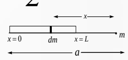
$T_2 = 4h$

58. The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then new time period of the satellite will be **(29 – JUNE – 2022(E))**

1) 40 hours 2) 36 hours 3) 30 hours 4) 25 hours

Key: 2

Sol: $T_1 = 7h$ $T_2 = ?$



$R_1 = R$

$T^2 \propto R^3$

$$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2}$$

$$T_2 = \left(\frac{3R}{R}\right)^{3/2} \times 7$$

$= 3^{3/2} \times 7$

$= 3\sqrt{3} \times 7$

$= 21 \times 1.732$

≈ 36.37

$T_2 = 36h$

59. The ratio of mean distances of 3 planets from the sun are $0.5:1:1.5$ then square of time periods are in the ratio **(29 – JUNE – 2022(E)) (D)**

1) $1:4:9$ 2) $1:9:4$ 3) $1:8:27$ 4) $2:1:3$

Key: 3

Sol: $T^2 \propto R^3$

$$r_1 : r_2 : r_3 = \frac{1}{2} : 1 : \frac{3}{2}$$

$$T_1^2 : T_2^2 : T_3^2 = \frac{1}{8} : 1 : \frac{27}{8}$$

$T_1^2 : T_2^2 : T_3^2 = 1:8:27$

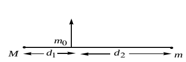
60. A body is projected from earth's surface to become its satellite, its time period of revolution will hot depend upon **(29 – JUNE – 2022(E)) (D)**

1) mass of earth 2) it's own mass 3) gravitational constant 4) radius of orbit

Key: 2

Sol: $$T = \frac{2\pi (r)^{3/2}}{\sqrt{GM}}$$

T is independent of mass of body (m) or satellite

61. If the distance of the earth from sun is . Then the distance of an imaginary planet from sun, if its period of revolution is 2.83 year is **(24 – Jan – 2023(E))**

1) $6 \times 10^7\ km$ 2) $6 \times 10^6\ km$ 3) $3 \times 10^6\ km$ 4) $3 \times 10^7\ km$

Key: 3

Sol: $T^2 \propto R^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \left(\frac{1}{2.83}\right)^2 = \left(\frac{1.5\times10^6}{R_2}\right)^3$$

$$R_2 = \left[2.83\times\left(1.5\times10^6\right)^3\right]^{1/3} = 8^{1/3}\times1.5\times10^6 = 3\times10^6\ km$$

62. The mean distance of a planet from the sun is approximately $\frac{1}{4}$ times that of earth from sun. The number of years required for planet to make one revolution about the sun.
(24 – Jan – 2023(E)) (D)

1) 0.5 years 2) 0.25 years 3) 0.125 years 4) 1.5 years

Key: 3

Sol: $R_P = \frac{1}{4}R_E \qquad T_E = 1\ year$

$$T^2 \propto R^3 \qquad \left(\frac{T_P}{T_E}\right)^2 = \left(\frac{R_P}{R_E}\right)^3 \Rightarrow T_P = T_E\left(\frac{R_P}{R_E}\right)^{3/2}$$

$$T_P = \left(\frac{R_E}{4R_E}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2} = 0.125\ years$$

63. A satellite moving in a circular path of radius R around earth has time period T. If its radius slightly increases by 4%. Then percentage change in time period is
(24 – Jan – 2023(E)) (D)

1) 1% 2) 6% 3) 3% 4) 9%

Key: 2

Sol: $T \propto R^{3/2}$

$$\frac{\Delta T}{T} = \frac{3}{2}\times\frac{\Delta R}{R} = \frac{3}{2}\times4 = 6\%$$

64. The time period of a satellite of earth is $24\,hrs$. If the separation between the earth and satellite is decreased to one fourth of the previous value, then its new time period will become.
(30 – Jan – 2023(M))

1) 4 hours 2) 6 hours 3) 12 hours 4) 3 hours

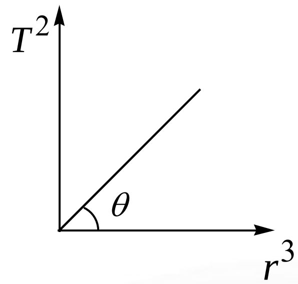
Key: 4

Sol: $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \left(\frac{R}{R/4}\right)^3 \Rightarrow \frac{T_1^2}{T_2^2} = 64$$

$$T_2 = \frac{24}{8} = 3\ hours$$

65. If a graph is plotted between T^2 and r^3 for a planet then its slope will be



(30 – Jan – 2023(M)) (D)

1) $\frac{4\pi^2}{GM}$ 2) $\frac{GM}{4\pi^2}$ 3) $4\pi GM$ 4) Zero

Key: 1

Sol: $\frac{GMm}{r^2} = mr\omega^2 \Rightarrow \frac{GM}{r^2} = \frac{4\pi^2}{T^2}\left(\because \omega = \frac{2\pi}{T}\right)$

$T^2 = \frac{4\pi^2}{GM} \cdot r^3$

66. The period of a satellite on orbit of radius R is T. Its period of revolution in an orbit of radius of 4R will be **(30 – Jan – 2023(M)) (D)**

1) $2T$ 2) $2\sqrt{2}T$ 3) $4T$ 4) $8T$

Key: 4

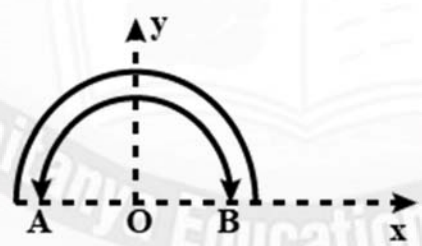
Sol: $T^2 \propto R^3$

$\frac{T^2}{T_1^2} = \frac{R^3}{(4R)^3}$

$T_1^2 = 64T^2 \Rightarrow T_1 = 8T$

e) Gravitational force of a uniform spherical shell on a particle

67. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass ‘m’ and length l is:



1) $\frac{Gm}{l^2} along + x\,axis$ 2) $\frac{Gm}{\pi l^2} along + y\,axis$

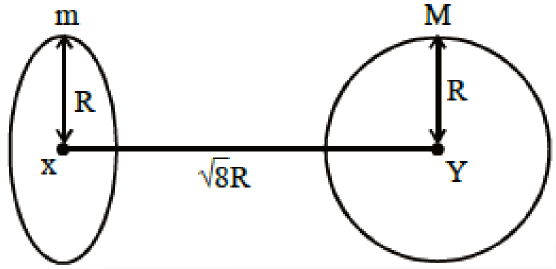
3) $\frac{2\pi Gm}{l^2} along + x\,axis$ 4) $\frac{2\pi Gm}{l^2} along + y\,axis$

Key: 4

Sol:

68. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$

is the distance between the centres of a ring (of mass ‘m’) and a sphere (mass ‘M’) where both have equal radius ‘R’. **(26 – FEB – 2021(M))**



1) $\frac{\sqrt{8}}{9}\cdot\frac{GmM}{R}$ 2) $\frac{2\sqrt{2}}{3}\cdot\frac{GMm}{R^2}$ 3) $\frac{1}{3\sqrt{8}}\cdot\frac{GMm}{R^2}$ 4) $\frac{\sqrt{8}}{27}\cdot\frac{GmM}{R^2}$

Key: 4

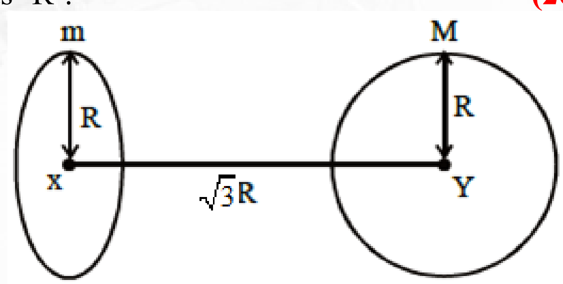
Sol: Gravitational field of ring

$$=\frac{Gmx}{\left(R^2+x^2\right)^{3/2}}$$

Force between sphere & ring

$$=\frac{GmM\left(\sqrt{8}R\right)}{\left(R^2+8R^2\right)^{3/2}}=\frac{GmM}{R^2}\times\frac{\sqrt{8}}{27}$$

69. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{3}R$ is the distance between the centres of a ring (of mass ‘m’) and a sphere (mass ‘M’) where both have equal radius ‘R’. **(26 – FEB – 2021(M)) (D)**



1) $\frac{3GMm}{1000R^2}$ 2) $\frac{\sqrt{3}GMm}{8R^2}$ 3) $\frac{\sqrt{3}GMm}{16R^2}$ 4) $\frac{3GMm}{8R^2}$

Key: 2

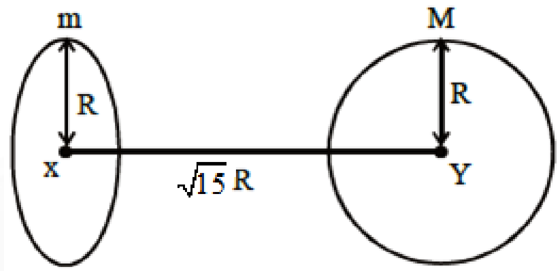
Sol: Gravitational field of the ring

$$=\frac{Gmx}{\left(R^2+x^2\right)^{3/2}} \qquad r=\sqrt{3}R$$

Force between ring & sphere

$$=-\frac{GmM\left(\sqrt{3}R\right)}{\left(R^2+3R^2\right)^{3/2}}=\frac{GMm}{\left(4R^2\right)^{3/2}}\sqrt{3}R=\frac{GMm}{8R^3}\sqrt{3}R=\frac{GMm\sqrt{3}}{8R^2}$$

70. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{15}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'. **(26 – FEB – 2021(M)) (D)**



1) $\dfrac{GMm\sqrt{15}}{64R^2}$ 2) $\dfrac{GMm\sqrt{13}}{64R^2}$ 3) $\dfrac{GMm\sqrt{13}}{16R^2}$ 4) $\dfrac{GMm}{64R^2}$

Key: **1**

Sol: Gravitation field of the ring

$$=\frac{Gm\sqrt{15}R}{\left(R^2+\left(\sqrt{15}R\right)^2\right)^{3/2}}$$

Force between sphere & ring

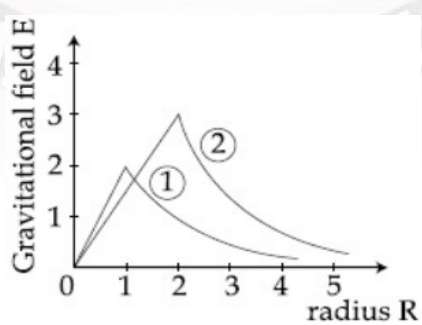
$$=\frac{GmM\sqrt{15}R}{\left(16R^2\right)^{3/2}} \quad =\frac{GMm\sqrt{15}}{64R^2}$$

2. Gravitational field intensity

a) Gravitation field strength

01. Consider two solid spheres of radii $R_1 = 1\text{m}, R_2 = 2\text{m}$ and masses m_1 and m_2 respectively. The gravitational field due to sphere 1 and 2 are shown. The value of is

(08 – Jan – 2020 (M))

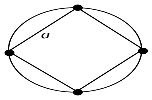


1) $\dfrac{2}{3}$ 2) $\dfrac{1}{6}$ 3) $\dfrac{1}{2}$ 4) $\dfrac{1}{3}$

Key: 2

Sol: Gravitation field at the surface

$$E = \frac{GM}{r^2}$$

$\therefore E_1 = \frac{Gm_1}{r_1^2}$ and 

From the diagram given in question,

$$\frac{E_1}{E_2} = \frac{2}{3} \ (r_1 = 1m, r_2 = 2m \text{ given})$$

$$\therefore \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2 \left(\frac{m_1}{m_2}\right) \quad \frac{m_1}{m_2} = \frac{1}{6}$$

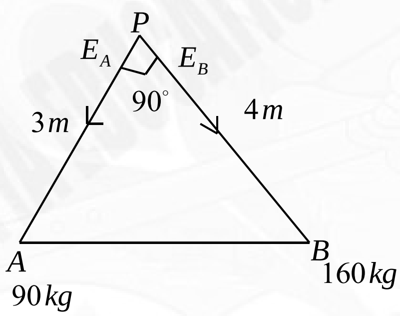
b) Gravitational field due to a point mass

02. Two masses $90\,kg$ and $160\,kg$ are at a distance $5\,m$ part. Compute the magnitude of intensity of the gravitational field at a point distance $3\,m$ from the $90\,kg$ and $4\,m$ from the $160\,kg$ mass.

1) $10G$ 2) $10\sqrt{2}\,G$ 3) $20G$ 4) $20\sqrt{2}\,G$

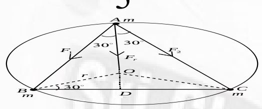
Key: **2**

Sol:



Gravitational intensity at P due to mass at A

$$E_A = G\frac{90}{3^2} = 10G$$



$$E = \sqrt{E_A^2 + E_B^2} = \sqrt{(10G)^2 + (10G)^2} = 10\sqrt{2}\,G$$

c) Principle of superposition-Gravitational field

03. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness $2a$ and mass $2M$. The gravitational field at distance '$3a$' form the centre will be (9th April 2019, Shift-1)

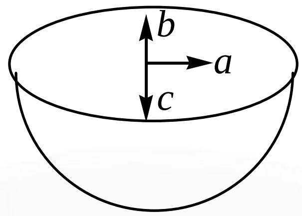
1) $\frac{2GM}{9a^2}$ 2) $\frac{GM}{9a^2}$ 3) $\frac{GM}{3a^2}$ 4) $\frac{2GM}{3a^2}$

Key: 3

Sol: $E_g = \frac{GM}{(3a)^2} + \frac{G(2M)}{(3a)^2} = \frac{GM}{3a^2}$

d) Field due to common shapes

04. The gravitation intensity at the centre of the drumhead defined by a hemispherical shell has the direction indicated by the arrow (see figure)

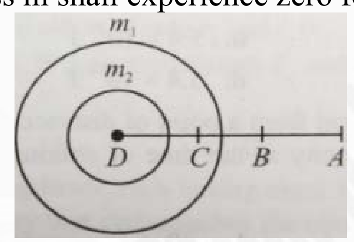


1) a   2) b   3) c   4) zero

Key: 3

Sol: Conceptual

05. The following figure shows two shells of masses and m_2. The shells are concentric. At which point, a particle of mass m shall experience zero force? **(08 – Jan – 2020 (M)) (D)**



1) A   2) B   3) C   4) D

Key: 4.

Sol: The gravitational field intensity at a point inside the spherical shell is zero.

06. The mass density of a planet of radius R varies with the distance (r) from its centre as $\rho(r)=\rho_0\left[1-\frac{r^2}{R^2}\right]$ then, the gravitational field is maximum at **(3 – Sep – 2020 (E))**

1) $r=\sqrt{\frac{3}{4}}R$ 2) $r=R$ 3) $r=\sqrt{\frac{5}{9}}R$ 4) $r=\sqrt{\frac{1}{9}}R$

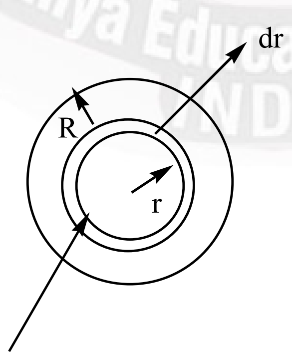
Key: 3

Sol: Gravitational intensity at a distance x from centre of planet is $E=\frac{GM}{x^2}$

Where M = mass enclosed with in a spherical volume of radius x

So $E=\frac{G}{x^2}\int_o^x\left(4\pi r^2 dr\right)[\rho(r)]...1$

Here $M=\int_o^x dm=\int_o^x 4\pi r^2 dr\left(\rho(r)\right)$



$dm=\left(4\pi r_2 dr\right)\rho$

So, from equation 1, we get

$$E=\frac{G}{x^2}\int_o^x 4\pi r^2\ \rho_0\left[1-\frac{r^2}{R^2}\right]dr$$

$$=\frac{G}{x^2}\int_o^x 4\pi\rho_0\left[r^2-\frac{r^4}{R^2}\right]d \quad =\frac{4\pi G\rho_0}{x^2}\left[\frac{x^3}{3}-\frac{x^5}{5R^2}\right]=4\pi G\rho_0\left[\frac{x}{3}-\frac{x^3}{5R^2}\right]$$

For gravitational field to be maximum $\frac{dE}{dx}=0$

$$\Rightarrow\frac{d}{dx}\left[4\pi G\rho_0\left(\frac{x}{3}-\frac{x^3}{5R^2}\right)\right]=0$$

$$4\pi G\rho_0\left[\frac{1}{3}-\frac{3x^2}{5R^2}\right]=0$$

$$\therefore x=\sqrt{\frac{5}{9}}R$$

So, E is maximum at a distance of $\sqrt{\frac{5}{9}}R$ from centre of planet.

07. Inside a uniform spherical shell

A) The gravitational field is zero

B) The gravitational potential is zero

C) The gravitational field is same every where

D) The gravitational potential is same every where

E) All of the above

Choose the most appropriate answer from the option given below **(26 – Aug – 2021 (M))**

1) A, B & D only 2) E only 3) A, B & C only 4) B, C & D only

Key:

Sol: Conceptual

3. Acceleration due to gravity and its variation

a) Acceleration due to gravity on the surface of earth

01. Two bodies each of mass M, are kept fixed with a separation 2L. A particle of mass ‘m’ is projected from the mid-point of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are)

1) The minimum initial velocity of the mass ‘m’ to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

2) The minimum initial velocity of the mass ‘m’ to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

3) The minimum initial velocity of the mass ‘m’ to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

4) The energy of the mass ‘m’ remains constant

Key: 2

Sol:

02. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is $2s$. The period of oscillation of the same pendulum on the planet would be:- **(11 – JAN – 2019 (E))**

1) $\frac{2}{\sqrt{3}}s$ 2) $2\sqrt{3}s$ 3) $\frac{\sqrt{3}}{2}s$ 4) $\frac{3}{2}s$

Key: **2**

Sol: $\because g = \frac{GM}{R^2}$

$$\frac{g_p}{g_e} = \frac{M_e}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also $T \propto \frac{1}{\sqrt{g}}$ $\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3} \Rightarrow T_p = 2\sqrt{3}\,s$

03. What is the acceleration due to the gravity of the Moon at the center of the Earth?

1) $3.31\times10^{-5}\,m/s^2$ 2) $3.21\times10^{-5}\,m/s^2$ 3) $3.32\times10^{-5}\,m/s^2$ 4) $3.4\times10^{-5}\,m/s^2$

Key: **3**

Sol: Conceptual

04. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9:4. The mass of the planet is $\frac{1}{9}th$ of that of the earth. If 'R' is the radius of the earth, what is the radius of the planet? (Take the planets to have the same mass density) **(12 – Apr – 2019 (E))**

1) $\frac{R}{3}$ 2) $\frac{R}{4}$ 3) $\frac{R}{9}$ 4) $\frac{R}{2}$

Key: 4

Sol: $\frac{W_e}{W_p} = \frac{mg_e}{mg_p} = \frac{9}{4} or \frac{g_e}{g_p} = \frac{9}{4}$

$$or \frac{\frac{GM}{R^2}}{G(M/9)/R_p^2} = \frac{9}{4}$$

$$R_p = \frac{R}{2}$$

05. A mass of $5kg$ is first weighted on a balance at the top of a tower $20m$ high. The mass is then suspended from a fine wire $20m$ long and reweighed. Find the difference in weight. Assume that the radius of the earth is $6400\,km$, the mass of the earth $6\times10^{24}\,kg$ and $G = 6.67\times10^{-11}\,Nm^2\,kg^{-2}$.

1) 2) 3) 4)

Key:

Sol: Original force = $\frac{6.67\times10^{-11}\times6\times10^{24}}{\left(6400.02\times10^3\right)^2}N = 48.8522N$

Force at surface = $\dfrac{6.67\times10^{-11}\times6\times10^{24}\times5}{\left(6400\times10^{3}\right)^{2}}N=48.8525N$

Therefore change in weight = 0.0003 N

If $g=10\,N\,kg^{-1}$, this is equivalent to the weight of a $0.03\,g$ mass at earth's surface.

06. The acceleration due to gravity at the Moon's surface is $1.67\,ms^{-2}$. If the radius of the Moon is $1.74\times10^{6}\,m$, calculate the mass of the Moon. Use the known value of G.

1) $7.58\times10^{21}\,kg$ 2) $7.85\times10^{21}\,kg$ 3) $7.58\times10^{22}\,kg$ 4) $7.4\times10^{21}\,kg$

Key: 3

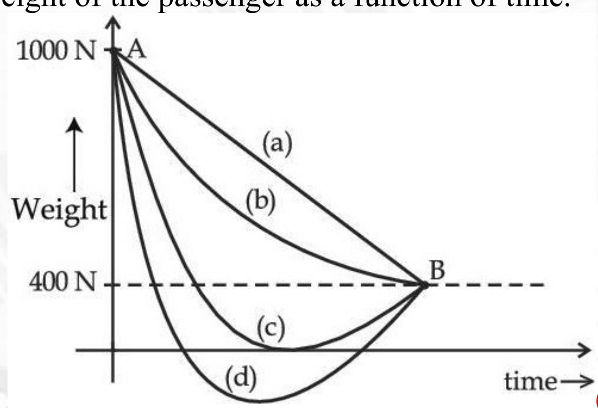
Sol: $g=\dfrac{GM}{R^{2}}\,or\,M=\dfrac{gR^{2}}{G}$

This relation is true not only for earth but for any heavenly body which is assumed to be spherical.

Here $g=1.67\,ms^{-2},R=1.74\times10^{6}\,m\,\&\,G=6.67\times10^{-11}\,Nm^{2}\,kg^{-2}$

Mass of Moon M = $\dfrac{1.67\times\left(1.74\times10^{6}\right)^{2}}{6.67\times10^{-11}}kg=7.58\times10^{22}\,kg$

07. A person whose mass is $100\,kg$ travels from earth to mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the earth and mars as $10\,m/s^{2}$ and $4\,m/s^{2}$ respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.



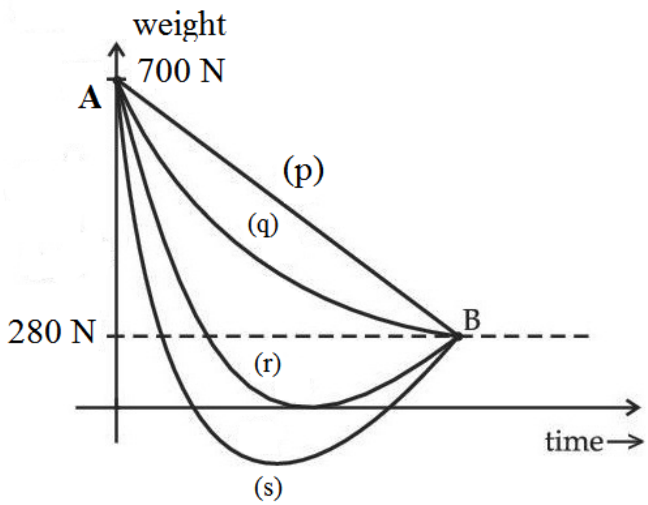
(20 – July – 2021(M))

1) (c) 2) (a) 3) (d) 4) (b)

Key: 1

Sol: At neutral point g = 0 so graph (c) is correct

08. A person whose mass is $70\,kg$ travels from earth to mars in a space ship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the earth and mars as $10\,m/s^{2}$ and $4\,m/s^{2}$ respectively. Identify from the below figures the curve that fits best for the weight of the passenger as a function of time.

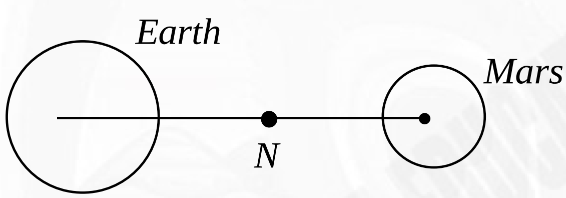


(20 – July – 2021(M)) (D)

1) (p) 2) (q) 3) (r) 4) (s)

Key: 3

Sol: There will be a neutral point – N where weight will be zero because at this point net gravitational field due to earth and mars is zero



09. The density of a newly discovered planet is twice that of the earth's. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth's. If the radius of the earth is R and the radius of the planet is R^1 then what is the value of $\frac{R}{R^1}$

Key: 2

Sol: $g = \frac{GM}{R^2} = G\frac{4}{3}\pi R^3 \rho$ $\qquad g \; \alpha \; R\rho$

Given = $\frac{g}{g^1} = 1, \frac{R}{R^1} = 2$

10. If the radius of the earth were t shrink by one percent, its mass remaining the same, acceleration due to gravity on the earth's surface would

1) Decrease 2) Remains same 3) Increase 4) be zero

Key:

Sol: Conceptual

11. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface. The mean density of the earth is

1) $\frac{4\pi G}{3gR}$ 2) $\frac{3\pi R}{4gG}$ 3) $\frac{3g}{4\pi RG}$ 4) $\frac{\pi R}{12G}$

Key: 3

Sol: $g = \frac{Gm}{R^2}, M = \frac{4}{3}\omega R^2 \rho$

So $\rho = \frac{3g}{4\pi RG}$

b) Variation due to shape of the earth

12. A box weighs 196N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (take $g = 10 ms^{-2}$ at the north pole and the radius of the earth = 6400 km) **(07 – Jan – 2020 (E))**

1) 195.66N 2) 194.32N 3) 194.66N 4) 195.32N

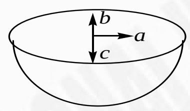
Key: 4

Sol: Weight at pole, w = mg = 196N $\Rightarrow$ m = 19.6 kg

weight at equator, w' = mg' = $m\left(g-\omega^2 R\right)$

$$w' = 19.6\left(10-\left(\frac{2\pi}{24\times3600}\right)^2 6400\times10^3\right)N \Rightarrow 195.33N$$

13. A box weighs 'W' N on a spring balance at the equator. Its weight recorded on the same balance when shifted to north pole is (g – acceleration due to gravity at north pole, R – Radius of the earth). **(07 – Jan – 2020 (E)) (D)**

1) $2W\left(1+\frac{mR\omega^2}{W}\right)$ 2) $W\left(1+\frac{mR\omega^2}{W}\right)$ 3) 

4) $\frac{W}{2}\left(2-\frac{R\omega^2}{g}\right)$

Key: 2

Sol: Weight at equator, $W = mg - mR\omega^2 \Rightarrow mg = W + mR\omega^2 N$

Weight at pole, $W' = mg = W + mR\omega^2$

14. The height at which the weight of a body will be the same as that at the same depth 'h' from the surface of Earth is [Radius of Earth is R and effect of the rotation of the Earth is neglected] **(02 – Sep – 2020 (E))**

1) $\frac{\sqrt{5}}{2}(R)-R$ 2) $\frac{R}{2}$ 3) $\frac{\sqrt{5}R-R}{2}$ 4) $\frac{\sqrt{3}R-R}{2}$

Key: 3

Sol: Given weight of body at height h=weight of body at depth 'h'

$$mg_h = mg_d$$

$$g_h = g_d$$

$$g\left[\frac{R^2}{(R+h)^2}\right] = g\left[1-\frac{d}{R}\right](\because d = h)$$

$$\frac{R^2}{(R+h)^2} = 1-\frac{h}{R}$$

$$\frac{(R)^2}{(R+h)^2} = \frac{(R-h)}{R}$$

$$R^3 = (R-h)(R+h)^2$$

$$R^3 = R^3 + Rh^2 + 2R^2h - hR^2 - h^3 - 2Rh^2$$

$$O = Rh^2 + 2R^2h - hR^2 - h^3 - 2Rh^2$$

$$O = -Rh^2 + R^2h - h^3$$

$$O = -Rh + R^2 - h^2$$

$$h^2 + Rh - R^2 = O$$

$$h=\frac{-R\pm\sqrt{R^2-4(1)(-R^2)}}{2(1)}$$

$$h=\frac{-R\pm\sqrt{5}R}{2}$$

As ‘h’ is a positive number

$$h=\frac{-R+\sqrt{5}R}{2} \text{ or } \frac{\sqrt{5}R-R}{2}$$

15. If the change in the value of $'g'$ at a height $'h'$ above the surface of earth is the same as at a depth $'x'$ below it, and when both x and h are much smaller than the radius of the earth.

(02 – Sep - 2020 (E)) (D)

1) $x=h$ 2) $x=2h$ 3) $x=\frac{h}{2}$ 4) $x=h^2$

Key: **2**

Sol: For smaller height, the value of $'g'$ at a height $'h'$ above the surface of the earth

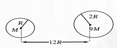
$$g_h=g\left[1-\frac{2h}{R}\right] \quad \because (d=x)$$

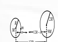
The value of $'g'$ at depth $g_d=g\left[1-\frac{d}{R}\right]=g\left[1-\frac{x}{R}\right]$

But $g_h=g_d$

$$g\left[1-\frac{2h}{R}\right]=g\left[1-\frac{x}{R}\right]$$

$$\frac{2h}{R}=\frac{x}{R}$$



16. The acceleration due to gravity on the earth’s surface at the poles is $'g'$ and angular velocity of the earth about the axis passing through the pole is  an object is weighed at the equator and at a height h above the pole by using a spring balance. If the weight are found to be same, then h is $(h << R)$, where R is the radius of the earth **(5 – Sep - 2020 (E))**

1) $\frac{R^2w^2}{2g}$ 2) $\frac{R^2w^2}{g}$ 3) $\frac{R^2w^2}{4g}$ 4) $\frac{R^2w^2}{8g}$

Key: **1**

Sol: Weight same at poles and at $'h'$ $(so\, g_1=g_2)$

$$g_1=g-Rw^2 \ldots 1$$

$$g_2=g\left[1-\frac{2h}{R}\right] \ldots 2$$

$$\because g_1=g_2$$

$$g-Rw^2=g\left[1-\frac{2h}{R}\right]$$

$$g-Rw^2=g-\frac{2gh}{R}$$

$$Rw^2=\frac{2gh}{R}$$

$$h=\frac{R^2w^2}{2g}$$

17. A body weighs 49N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator? **(24 – FEB – 2021(E))**

(Use $g = \frac{GM}{R^2} = 9.8\,ms^{-2}$ and radius of earth, $R = 6400\,km$)

1) 49 N 2) 48.83 N 3) 49.83 N 4) 49.17 N

Key: 2

Sol: Weight of pole = mg = 49 N

At equator due to rotation $= g_e = g - R\omega^2$

so $W = mg_e = m\left(g - R\omega^2\right)$

$\therefore W_P > W_e$ $\quad W_P = 49\,N$

so, $W_e = 48.83\,N$ $\quad W_e < 49\,N$

18. A body weighs 98 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?

(Use $g = \frac{GM}{R^2} = 9.8\,ms^{-2}$ and radius of earth, $R = 6400\,km$) **(24 – FEB – 2021(E)) (D)**

1) 100 N 2) 97.66 N 3) 75 N 4) 86.4 N

Key: 2

Sol: at poles, apparent weight is same as the true weight

$98N = mg = m\left(9.8\,m/s^2\right) \Rightarrow \; so\; m = 10\,kg$

at equator: $mg' = mg - mRW^2 \quad \Rightarrow 98 - 10\times\left(7.27\times10^{-5}\right)^2\; 6400\,km$

$\Rightarrow 99.66\,N$

19. A body weighs 284 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?

(Use $g = \frac{GM}{R^2} = 9.8\,ms^{-2}$ and radius of earth, $R = 6400\,km$) **(24 – FEB – 2021(E)) (D)**

1) 282.98 N 2) 284 N 3) 280 N 4) 274 N

Key: 1

Sol: At poles, apparent weight is same as the true weight, $284N = mg = m\left(9.8\,m/s^2\right)$

$so\; m = 30\,kg$

at equator $mg' = mg - mRw^2$

$\Rightarrow 284 - 30\times\left(7.27\times10^{-5}\right)^2 \times 6400\times10^3$

$\Rightarrow 282.98\,N$

20. Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be: **(25 – July – 2021(E))**

1) $2W$ 2) W 3) $2^{\frac{1}{3}}W$ 4) $\sqrt{2}W$

Key: 3

Sol: Density is same $M=\frac{4}{3}\pi R^3\rho,\ \ 2m=\frac{4}{3}\pi R^{33}\rho$

$$R^{'}=2^{1/3}R$$

$$\omega=\frac{GMm}{R^2}$$

$$\omega_2=\frac{G_2Mm}{R^{'2}}$$

$$\omega_2=2^{1/3}\omega$$

c) Variation with height from surface of earth

21. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is radius of earth. The value of $\frac{T_2}{T_1}$ is

1) 1 2) $\sqrt{2}$ 3) 4 4) 2

Key: **4**

Sol: $g_2=g\left(\frac{R}{R+h}\right)^2$

When $h=R$

$$\frac{T_1}{T_2}=\sqrt{\frac{g_2}{g_1}}$$

22. The value of acceleration due to gravity at earth's surface is $9.8\,ms^{-2}$. The altitude above its surface at which the acceleration due to gravity decreases to $4.9\,ms^{-2}$, is close to (Radius of earth = $6.4\times10^6 m$) (9th April 2019, Shift-1)

1) $2.6\times10^6 m$ 2) $6.4\times10^6 m$ 3) $9.0\times10^6 m$ 4) $1.6\times10^6 m$

Key: 1

Sol: $g_{eff}=\frac{g}{\left(1+\frac{h}{R}\right)^2}$

$$\Rightarrow g_{eff}=\frac{g}{\left(1+\frac{h}{R}\right)^2}\Rightarrow\sqrt{2}=1+\frac{h}{R}\Rightarrow\frac{h}{R}=\sqrt{2}-1$$

$$\Rightarrow h=\left(\sqrt{2}-1\right)\times6400\times10^3 m=2.6\times10^6 m$$

23. What is the fractional decrease in the value of free – fall acceleration g for a particle when it is lifted from the surface to an elevation h? (h<<R). **(07 – Jan – 2020 (E)) (D)**

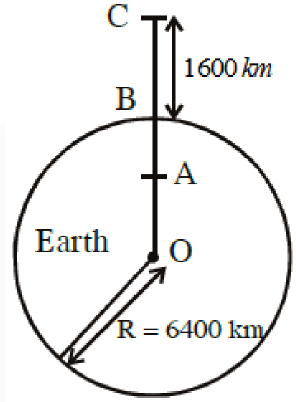
1) $-3\left(\frac{h}{R}\right)$ 2) $-\frac{h}{R}$ 3) $-2\left(\frac{h}{R}\right)$ 4) $-\frac{h}{2R}$

Key: 3

Sol: $g=\frac{GM}{R^2}\Rightarrow\frac{dg}{dR}=-\frac{2GM}{R^3}$

$$\frac{dg}{g}=-\frac{2GM}{R^2}\frac{1}{R}\Rightarrow\frac{dg}{g}=-2\left(\frac{h}{R}\right)$$

24. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of $OA:AB$ will be $X:Y$. The value of X is ……. **(26 – FEB – 2021(E)) (D)**



1) $\frac{25}{26}$ 2) $\frac{36}{25}$ 3) $\frac{5}{6}$ 4) $\frac{16}{9}$

Key: 4

Sol: $g_A=\frac{GMr}{R^3}$

$$g_C=\frac{GMr}{\left(R+\frac{R}{4}\right)^2}$$

$$g_A=g_C$$

$$\frac{r}{R}=\frac{1}{\frac{25}{16}}$$

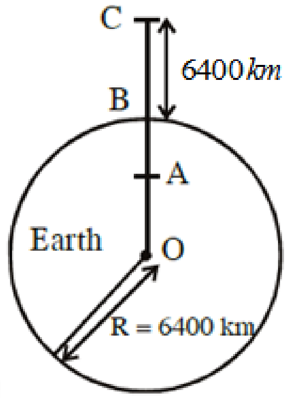
$$r=\frac{16R}{25}=OA$$

$$AB=R-r=R-\frac{16R}{25}$$

$$=\frac{25R-16R}{25}\qquad=\frac{9R}{25}$$

$$OA:AB=\frac{16R}{25}:\frac{9R}{25}=\frac{16}{9}$$

25. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of $OA:AB$ will be $X:Y$. The value of X is ……. **(26 – FEB – 2021(E)) (D)**



1) $\frac{3}{1}$ 2) $\frac{1}{3}$ 3) $\frac{5}{6}$ 4) $\frac{16}{9}$

Key: 2

Sol: $g_A = \frac{GMr}{R^3}$

$g_C = \frac{GMr}{(R+R)^2}$

$g_A = g_C$

$\frac{r}{R} = \frac{1}{4}$

$r = \frac{R}{4}$

$OA : AB = \frac{R}{4} : \frac{3}{4}R = \frac{1}{3}$

26. The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{3}$ of it's weight on the surface of earth is

Radius of earth $R = 6400\,km$ and $\sqrt{3} = 1.732$ **(24 – JUNE – 2022(M))**

1) $3840\,km$ 2) $4685\,km$ 3) $2133\,km$ 4) $4267\,km$

Key: 2

Sol: $g_h = \frac{g}{3}$ $h = ?$

formula

$g_h = g\left[\frac{R}{R+h}\right]^2$

$\frac{g}{3} = g\left[\frac{R}{R+h}\right]^2$

$\sqrt{3} = 1 + \frac{h}{R}$

$$\frac{h}{R} = \sqrt{3} - 1 = 1.732 - 1$$
$$h = 0.732R$$
$$h = 0.732 \times 6400$$
$$h = 4685\,km$$

27. How much above the surface of earth does the acceleration due to gravity reduced by 36% of it's value on the surface of earth. **(24 – JUNE – 2022(M)) (D)**
1) $1600\,km$ 2) $6100\,km$ 3) $1060\,km$ 4) $600\,km$

Key: 1

Sol: $g_h = 36\%\ reduce\ in\ g$

$$g_h = 64\%\ of\ g$$
$$g_h = \frac{64}{100}g \qquad h = ?$$
$$g_h = g\left[\frac{R}{R+h}\right]^2$$
$$\frac{64}{100}g = g\left[\frac{R}{R+h}\right]^2$$
$$\frac{5}{4} = R + \frac{h}{R}$$
$$\frac{h}{R} = \frac{5}{4} - 1 = \frac{1}{4}$$
$$h = \frac{R}{4} = \frac{6400}{4} = 1600\ km$$

28. A body of $200\,kg\,wt$ is lying on the surface of the earth. Find its weight at a place R above the surface of the earth. [Radius of the earth is R] **(24 – JUNE – 2022(M)) (D)**
1) $25\,kg\,wt$ 2) $100\,kg\,wt$ 3) $50\,kg\,wt$ 4) Remains same

Key: 3

Sol: $W = mg = 200\,kg\,wt$

$$W = 200\,g\,N$$
$$h = R \qquad R = R \qquad W' = ?$$
$$g_h = g\left[\frac{R}{R+h}\right]^2$$
$$g_h = g\left[\frac{R}{R+h}\right]^2 = g\left[\frac{1}{2}\right]^2$$
$$g_h = \frac{g}{4} \quad \Rightarrow W' = Mg_h$$
$$W' = 200 \times g$$
$$W' = 50\,kg\,wt$$

29. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at a point P will be [g = acceleration due to gravity at the surface of earth] **(25 – JUNE – 2022(M))**

1) $\frac{g}{2}$ 2) $\frac{g}{4}$ 3) $\frac{g}{8}$ 4) $\frac{g}{9}$

Key: 4

Sol: $h = 2R$ $g' = ?$

$r = R + h = R + 2R = 3R$

$g' = \frac{GM}{r^2}$

$g' = \frac{GM}{(3R)^2} = \frac{GM}{9R^2} = \frac{g}{9}$

$\boxed{\therefore g' = \frac{g}{9}}$ $\left[\because g = \frac{GM}{R^2}\right]$

30. If the radius of the earth is made three times keeping the mass constant, then the weight of a body on the earth's surface will be compared to its previous value is

(25 – JUNE – 2022(M)) (D)

1) $\frac{1}{3}rd$ 2) $\frac{1}{9}th$ 3) 3 times 4) 9 times

Key: 2

Sol: $g = \frac{GM}{R^2}$

$w = mg$

$R = R$ $R' = 3R$

$w' = mg' = m\frac{GM}{R^{12}}$

$w' = \frac{GMm}{9R^2} = \frac{w}{9}$ $w' = \frac{w}{9}$

31. The height vertically above the earth's surface at which the acceleration due to gravity becomes 1% of its valve at the surface is (R is the radius of the earth)

(25 – JUNE – 2022(M)) (D)

1) 8R 2) 9R 3) 10R 4) 20R

Key: 2

Sol: $g_h = 1\%\ of\ g$

$g_h = \frac{g}{100}$

$g_h = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$

$$\frac{g}{100}=\frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$\left(1+\frac{h}{R}\right)^2=100$$

$$1+\frac{h}{R}=10$$

$$\frac{h}{R}=9$$

$$h=9R$$

32. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of $3200\,km$ above the earth's surface is (given Radius of earth $R_e=6400\,km$)

(24 – Jan – 2023(M))

1) 9.8 N   2) 4.9 N   3) 19.6 N   4) 8 N

Key: 4

Sol: $$g'=\frac{g}{\left[1+\frac{h}{R}\right]^2}\Rightarrow mg'=\frac{mg}{\left[1+\frac{h}{R}\right]^2}$$

$$mg'=\frac{18}{\left[1+\frac{3200}{6400}\right]^2}=\frac{18}{\left[1+\frac{1}{2}\right]^2}=8\,N$$

33. The height at which acceleration due to gravity becomes $\frac{g}{9}$ in terms of radius of the earth (R)

(24 – Jan – 2023(M)) (D)

1) 3R   2) 2R   3) R   4) $\frac{R}{2}$

Key: 2

Sol: $$\frac{g}{9}=g\left[\frac{R}{R+h}\right]^2\Rightarrow\frac{R}{R+h}=\frac{1}{3}\Rightarrow 3R=R+h\Rightarrow h=2R$$

34. How much above the surface of earth does acceleration due to gravity reduces by 36% of its value on the surface of earth (R $\rightarrow$ Radius of earth) **(24 – Jan – 2023(M)) (D)**

1) $\frac{R}{2}$ 2) R 3) $\frac{R}{4}$ 4) $\frac{3R}{4}$

Key: 3

Sol: The value of g is 100 – 36 = 64

$$g'=\frac{64}{100}g$$

$$g'=g\left[\frac{R}{R+h}\right]^2=\frac{64}{100}g=g\left[\frac{R}{R+h}\right]^2$$

$$\frac{8}{10}=\frac{R}{R+h}\Rightarrow 8R+8h=10R$$

$$8h=2R\Rightarrow h=\frac{R}{4}$$

35. A body of weight 'w' is projected vertically upwards from earth's surface to reach a height 9 times the radius of earth. The weight of the body at that height will be

(31 – Jan – 2023(E))

1) $\frac{w}{91}$ 2) $\frac{w}{100}$ 3) $\frac{w}{9}$ 4) $\frac{w}{3}$

Key: 2

Sol: $g' = g\left[\frac{R}{R+h}\right]^2$ $\quad mg' = mg\left[\frac{R}{R+h}\right]^2$

$$mg' = w\left[\frac{R}{R+9R}\right]^2 \Rightarrow w' = \frac{w}{100}$$

36. The height at which the value of 'g' is half that on the surface of earth of radius R is

(31 – Jan – 2023(E)) (D)

1) R 2) $2R$ 3) $0.414R$ 4) $0.75R$

Key: 3

Sol: $g' = g\left[\frac{R}{R+h}\right]^2$ $\quad g_1 = \frac{g}{2}$

$$\frac{g}{2} = g\left[\frac{R}{R+h}\right]^2 \Rightarrow \frac{R}{R+h} = \frac{1}{\sqrt{2}} \Rightarrow \sqrt{2}R = R+h$$

$$h = \left(\sqrt{2}-1\right)R$$

$$h = 0.414R$$

37. A particle hanging from a massless spring stretches it by 2cm at earth's surface. How much will the same particle stretch the spring at a height of 2624 km from the surface of earth $(R = 6400\,km)$

(31 – Jan – 2023(E)) (D)

1) $1cm$ 2) $2cm$ 3) $3cm$ 4) $4cm$

Key: 1

Sol: $mg = kx$ $\quad g' = g\left[\frac{R}{R+h}\right]^2$ $\quad h = 2624\,km$

$x \propto g$ $\quad \frac{x_1}{x_2} = \frac{g}{g'}$

$x_1 = 2\,cm$ on solving $x_2 = 1\,cm$

d) Variation with depth

38. What will be the value of g at the bottom of sea $7\,km$ deep? Diameter of earth is $12800\,km$ and g on the surface of earth is $9.8\,ms^{-2}$.

1) $9.89 ms^{-2}$ 2) $9.789 ms^{-2}$ 3) $9.879 ms^{-2}$ 4) $9.9 ms^{-2}$

Key: 2

Sol: Depth of sea $d = 7 km, g = 9.8 ms^{-2}$

Diameter of earth, $D = 12800 km$

Let $g_d =$ value of g the bottom of sea then

$$g_d = g\left(1-\frac{d}{R}\right) = 9.8\left(1-\frac{7}{6400}\right) ms^{-2} = \frac{9.8\times 6393}{6400} ms^{-2} = 9.789 ms^{-2}$$

39. What will be the value of g at the bottom of sea $7 km$ deep? Diameter of earth is $12800 km$ and g on the surface of earth is $9.8 ms^{-2}$.

1) $9.89 ms^{-2}$ 2) $9.789 ms^{-2}$ 3) $9.879 ms^{-2}$ 4) $9.9 ms^{-2}$

Key: 2

Sol: Depth of sea $d = 7 km, g = 9.8 ms^{-2}$

Diameter of earth, $D = 12800 km$

Let $g_d =$ value of g the bottom of sea then

$$g_d = g\left(1-\frac{d}{R}\right) = 9.8\left(1-\frac{7}{6400}\right) ms^{-2} = \frac{9.8\times 6393}{6400} ms^{-2} = 9.789 ms^{-2}$$

40. The value of the acceleration due to gravity is g at a height $h = \frac{R}{2}$ (where, $R =$ radius of the earth) from the surface of the earth. It is again equal to g, at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals **(5 – Sep - 2020 (M))**

1) $\frac{7}{9}$ 2) $\frac{1}{3}$ 3) $\frac{4}{9}$ 4) $\frac{5}{9}$

Key: 4

Sol: Given that acceleration due to gravity at height h from the surface of earth=acceleration due to gravity at depth d below the surface of earth g

Given $h = \frac{R}{2}$

$g_1 = g_{high} = g_{depth}$

$$\frac{GM}{(R+h)^2} = g\left[1-\frac{d}{R}\right];\ \frac{GM}{\left(R+\frac{R}{2}\right)^2} = g\left[1-\frac{d}{R}\right];\ \frac{GM}{\left(\frac{2R+R}{2}\right)^2} = g\left[1-\frac{d}{R}\right]$$

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = g\left[1-\frac{d}{R}\right]\quad \because \left[g = \frac{GM}{R^2}\right];\ \frac{4GM}{9R^2} = \frac{GM}{R^2}\left[1-\frac{d}{R}\right];\ \frac{4}{9} = \left[1-\frac{d}{R}\right]$$

$$\frac{d}{R} = 1-\frac{4}{9} = \frac{5}{9}$$

41. Find the depth at which 'g' becomes 4% less than value at surface

(5 – Sep - 2020 (M)) (D)

1) 0.2 R 2) 0.04 R 3) 0.3 R 4) 0.03 R

Key: 2

Sol: Given $g_d = 96\%(g)$

$$g_d = g\left(1-\frac{d}{R}\right);\ 96\%(g) = g\left(1-\frac{d}{R}\right);\ \frac{96}{100} = \left(1-\frac{d}{R}\right)$$

$$\frac{d}{R} = 1-\frac{96}{100} = \frac{100-96}{100}$$

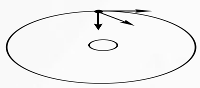
$$\frac{d}{R} = \frac{4}{100} = \frac{1}{25} = 0.04$$

$$d = 0.04R$$

42. A man has weight W on the earth's surface how much deep inside the earth (radius=R) he must go so that his weight becomes 3/4th that on the earth's surface **(5 – Sep - 2020 (E)) (D)**

1) $\frac{R}{4}$ 2) $\frac{R}{2}$ 3) $\frac{3R}{4}$ 4) $\frac{R}{8}$

Key: 1

Sol: Given 

$$g_d = \frac{3}{4}g\ ,\ \left[1-\frac{d}{R}\right] = \frac{3}{4}$$

$$\frac{d}{R} = 1-\frac{3}{4} = \frac{1}{4};\ d = \frac{R}{4}$$

43. Consider a planet in some solar system which has a mass three times of the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be: **(25 – July – 2021(E)) (D)**

1) W 2) $3W$ 3) $\sqrt{3}W$ 4) $3^{1/3}W$

Key: 4

Sol: $d_E = d_P$

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{\frac{4}{3}\pi R_P^3} \Rightarrow R_P = 3^{1/3}R$$

$$g = \frac{GM}{R^2}$$

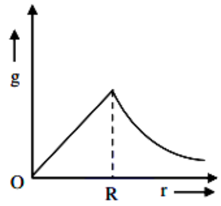
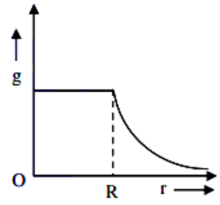
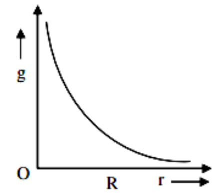
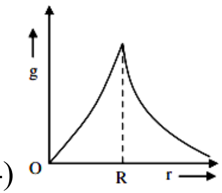
$$g_P = \frac{G(3M)}{R_P^2}$$

$$\frac{g}{g_P} = \frac{R_P^2}{3R^2}$$

$$\frac{g}{g_P} = \frac{3^{2/3}\cdot R^2}{3R^2} \Rightarrow g_P = 3^{1/3}g \qquad \Rightarrow mg_P = 3^{1/3}mg$$

$$\Rightarrow \omega_P = 3^{1/3}\omega\ (\because \omega = mg)$$

44. The variation of acceleration due to gravity (g) with distance (r) from the centre of the earth is correctly represented by [Given R = Radius of earth] **(26 – JUNE – 2022(M))**

1)  2)  3)  4) 

Key: **1**

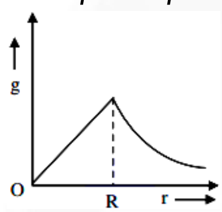
Sol: For $r < R$

$$g = \frac{GMr}{R^3} = kr$$

$$g \propto r$$

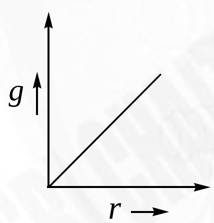
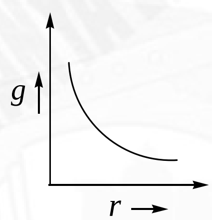
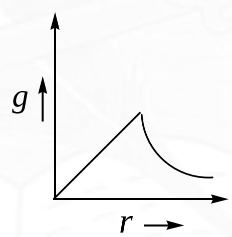
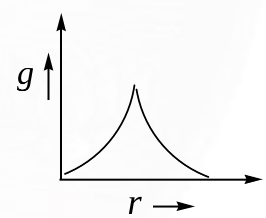
For $r > R$

$$g = \frac{GM}{r^2} = \frac{k'}{r^2}$$



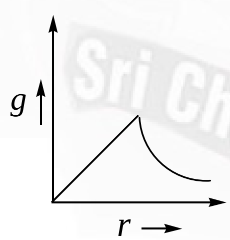
$$g \propto \frac{1}{r^2}$$

45. The variation of acceleration due to gravity as one moves away from earth's centre is given by **(26 – JUNE – 2022(M)) (D)**

1)  2)  3)  4) 

Key: **3**

Sol:



46. At a certain depth 'd' below surface of earth value of acceleration due to gravity becomes four times that of its value at a height 3R above earth's surface (where R is radius of earth R = 6400 km). Then depth 'd' is equal to **(31 – Jan – 2023(M))**

1) 5260 km 2) 640 km 3) 2560 km 4) 4800 km

Key: **4**

Sol: $\frac{GM}{R^2}\left[1-\frac{d}{R}\right] = 4\times\frac{GM}{(4R)^2}$ $\qquad 1-\frac{d}{R} = \frac{1}{4} \Rightarrow d = \frac{3R}{4} = 4800\ km$

47. The height at which the value of acceleration due to gravity becomes 50% of that at the surface of the earth in km is $(R = 6400\,km)$ **(31 – Jan – 2023(M) (D))**

1) $2630\,km$ 2) $2640\,km$ 3) $2650\,km$ 4) $2660\,km$

Key: 3

Sol: $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$ $g' = \frac{g}{2}$

$$\frac{1}{2} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$h = 2650\,km$

48. The depth at which the value of 'g' becomes 25% of that at the surface of earth is $(in\ km)$ **(31 – Jan – 2023(M) (D))**

1) $4800\,km$ 2) $2400\,km$ 3) $3600\,km$ 4) $1200\,km$

Key: 1

Sol: $g' = g\left[1-\frac{d}{R}\right]$ $g' = \frac{g}{4}$

$$\frac{g}{4} = g\left[1-\frac{d}{R}\right]$$

$$1-\frac{d}{R} = \frac{1}{4}$$

$$d = \frac{3R}{4} = \frac{3}{4}\times 6400 = 4800\,km$$

4. Gravitational potential and potential energy

a) Gravitational potential energy

01. A body is projected vertically upwards from the surface of earth with a velocity, sufficient to carry it to infinity. Find n if the time taken by it to reach a height of thrice the radius of earth (Take radius of earth = R and acceleration due to gravity as g) is $\left(\frac{n}{3}\right)\left(\sqrt{\frac{2R}{g}}\right)$

1) 3 2) 4 3) 5 4) 7

Key: 4

Sol: $[T.F_1]_{earth} = [T.F_2]_m = 3R$

$$-\frac{GM}{R} + \frac{1}{2}mv^2 = 0 + \frac{1}{2}on v_0^2$$

Find $V_0 = ?$

02. If the acceleration due to gravity at the surface of earth is g, the work done in slowly lifting a body of mass m from the earth's surface to a height R equal to the radius of the earth is

1) $\frac{1}{2}mgR$ 2) $2mgR$ 3) mgR 4) $\frac{1}{4}mgR$

Key: 1

Sol: $w = (T.E)_R - (T.E)_{2R}$

$$w = -\frac{gMm}{R} - \left[-\frac{gMm}{2R}\right]$$

03. A body is released at a distance far away from the surface of the earth. Calculate its speed when it is near the surface of earth. (Given $g = 9.8ms^{-2}$, radius of earth $R = 6.37 \times 10^6 m$).

(08 – Jan – 2020 (E)) (D)

1) $11.2 \times 10^6\, ms^{-1}$ 2) $22.4 \times 10^3\, ms^{-1}$ 3) $11.2 \times 10^3\, ms^{-1}$ 4) $11.2 \times 10^{-3}\, ms^{-1}$

Key: 3

Sol: Conservation of energy implies

$$0 + 0 = \frac{1}{2}mv^2 - \frac{GMm}{R} \Rightarrow v = \sqrt{\frac{2GM}{R}} \Rightarrow v = \sqrt{\frac{2(gR^2)}{R}} \Rightarrow v = 11.2 \times 10^3 ms^{-1}$$

04. A satellite of mass 1000kg is rotating around the earth in a circular orbit of radius 3R. What extra energy should be given to this satellite if it is to be lifted into an orbit of radius 4R?

(08 – Jan – 2020 (E)) (D)

1) $2.614 \times 10^9 J$ 2) $0.614 \times 10^8 J$ 3) $4.614 \times 10^6 J$ 4) $5 \times 10^6 J$

Key: 1

Sol: Energy required = $(T.E)_f - (T.E)_i$

$$= \left(-\frac{GMm}{2(4R)}\right) - \left(-\frac{GMm}{2(3R)}\right) = -\frac{GMm}{8R} + \frac{GMm}{6R} \Rightarrow \frac{GMm}{24R} \Rightarrow \frac{gR^2m}{24R}$$

$\Rightarrow 2.614 \times 10^9 J$

05. Energy required to move a body of mass m from an orbit of radius 2R to 3R is

(08 – Jan – 2020 (E)) (D)

1) $\frac{GMm}{12R^2}$ 2) $\frac{GMm}{3R^2}$ 3) $\frac{GMm}{8R}$ 4) $\frac{GMm}{6R}$

Key: 4

Sol: Gravitational potential energy of body will be $E = -\frac{GMm}{r}$

At $r = 2R \Rightarrow E_1 = -\frac{GMm}{2R}$

At $r = 3R \Rightarrow E_2 = -\frac{GMm}{3R}$

Energy required to move a body of mass m from an orbit of radius 2R to 3R is

$$\Delta E = \frac{GMm}{R}\left[\frac{1}{2} - \frac{1}{3}\right] \Rightarrow \Delta E = \frac{GMm}{6R}$$

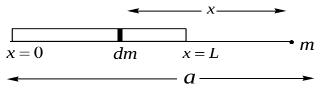
06. If M is mass of the earth and R is the radius, then the velocity with which a body should be projected from the surface of the earth such that it just reaches maximum height equal to 'R' above the surface is

(3 – Sep – 2020 (M)) (D)

1) $\sqrt{\frac{2GM}{R}}$ 2) $\sqrt{\frac{GM}{2R}}$ 3) $\sqrt{\frac{GM}{R}}$ 4) $\frac{1}{2}\sqrt{\frac{GM}{R}}$

Key: 3

Sol: From conservation of energy $(PE+KE)$ on the surface of the earth $= P.E$ at height R



$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R}\left[\frac{1}{2}\right]$$

$$v^2 = \frac{GM}{R}$$

$$v = \sqrt{\frac{GM}{R}}$$

07. Energy required is moving as body of mass m from a distance 2R to 3R from centre of earth of mass M is **(6 – Sep – 2020 (E)) (D)**

1) $\frac{GMm}{12R^2}$ 2) $\frac{GMm}{3R^2}$ 3) $\frac{GMm}{8R}$ 4) $\frac{GMm}{6R}$

Key: 4

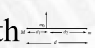
Sol: Change in potential energy in displacing a body from r_1 to r_2 is given by

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = GMm\left[\frac{1}{2R} - \frac{1}{3R}\right] = \frac{GMm}{6R}\begin{pmatrix}\because r_1 = 2R \\ r_2 = 3R\end{pmatrix}$$

08. If g is acceleration due to gravity on the surface of earth, find the gain in potential energy of an object of mass m raised from the surface of earth to height equal to radius of earth **(16 – March – 2021 (E)) (D)**

1) mgR 2) $\frac{m}{2}gR$ 3) $\frac{3m}{2}gR$ 4) $\frac{5}{2}mgR$

Key: 2

Sol: Let mass of earth 

Radius of earth $= R$

$$g = \frac{GM}{R^2} \Rightarrow gR^2 = GM \quad ----(1)$$

Potential energy of object on surface of earth $u_1 = -G\frac{Mm}{R}$

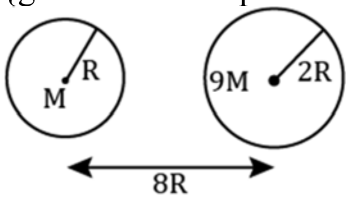
Potential energy of object at a height equal to radius of earth $u_2 = -G\frac{Mm}{2R}$

Gain in potential energy $= u_2 - u_1 = -\frac{GMm}{2R} - \left(\frac{-GMm}{R}\right) = \frac{GMm}{2R} \quad -----(2)$

substitute (1) is (2) gain in potential energy $= \frac{gR^2m}{2R} = \frac{mgR}{2}$

09. Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9M respectively have a centre to centre separation 8R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'v' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is ____

{given: The two planets are fixed in their position] **(27 – July – 2021(M))**

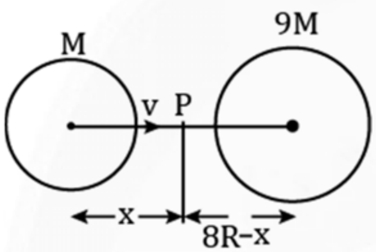


Key: 4

Sol: Acceleration due to gravity will be zero at P therefore,

$$\frac{GM}{x^2} = \frac{G9M}{(8R - x)^2} \Rightarrow 8R - x = 3x$$

$$\Rightarrow x = 2R$$



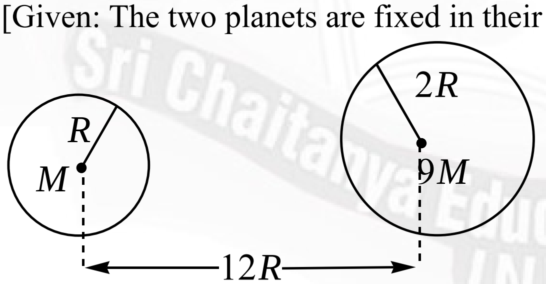
Apply conservation of energy and consider velocity at P is zero.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{G9Mm}{7R} = 0 - \frac{GMm}{2R} - \frac{G9Mm}{6R}$$

$$\therefore V = \sqrt{\frac{4}{7}\frac{GM}{R}}$$

10. Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9M respectively have a centre to centre separation 12R as shown in the figure. A satellite of mass $'m'$ is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed $'v'$ required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{33}\frac{GM}{R}}$ then the value of 'a' is ____

[Given: The two planets are fixed in their position]

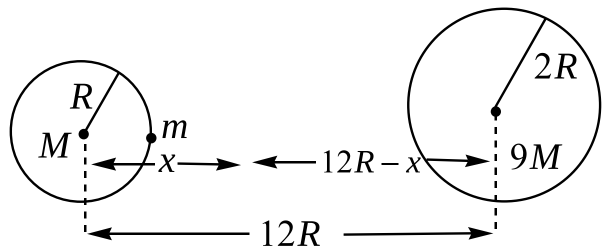


(27 – July – 2021(M)) (D)

1) 32 2) 33 3) 31 4) 27

Key: 1

Sol: Let gravitational force on $'m'$ is zero at a distance 'x' from centre of planet of mass 'M'.



$$\frac{GmM}{x^2}=\frac{Gm9M}{(12R-x)^2}\Rightarrow\left(\frac{12R-x}{x}\right)^2=9$$

$$\frac{12R-x}{x}=3\Rightarrow x=3R$$

By conservation of energy

$$-\frac{GmM}{R}-\frac{Gm(9M)}{11R}+\frac{1}{2}mV^2=-\frac{GmM}{3R}-\frac{Gm(9M)}{9R}$$

$$\frac{1}{2}mV^2=\frac{16}{33}\frac{GmM}{R}\Rightarrow V=\sqrt{\frac{32}{33}\frac{GM}{R}}$$

11. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height 'h' is

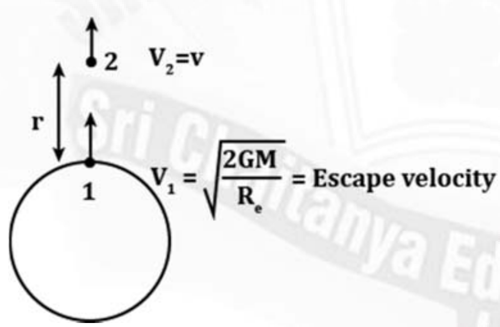
(22 – July – 2021(M))

1) $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$ 2) $\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

3) $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$ 4) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

Key: 4

Sol:



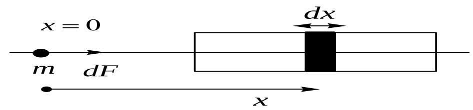
Applying energy conservation from (1) to (2)

$$\frac{1}{2}m\cdot\left(\frac{2GM}{R_e}\right)-\frac{GMm}{R_e}=\frac{1}{2}mv^2-\frac{GMm}{R+r}$$

$$\Rightarrow\frac{1}{2}mv^2=\frac{GMm}{R+r}$$

$$\Rightarrow v=\sqrt{\frac{2GM}{R+r}}=\frac{dr}{dt}$$

$$\Rightarrow \sqrt{2GM}\int_0^t dt = \int_{R_e}^{R_e th}\left(\sqrt{R+r}\right)dr$$



$$t = \frac{2}{3}\sqrt{\frac{R_e^{\,3}}{2GM}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

$$\frac{GM}{R_e^{\,2}} = g$$

$$t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

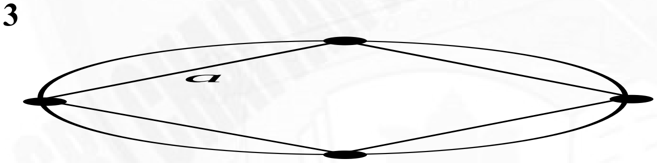
12. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height 3h is

(22 – July – 2021(M)) (D)

1) $\sqrt{\frac{R^3}{2GM}}\left[\left(1+\frac{3h}{R}\right)^{3/2} - 1\right]$
2) $\frac{1}{3}\sqrt{\frac{R^3}{GM}}\left[\left(1+\frac{3h}{R}\right)^{3/2} - 1\right]$
3) $\frac{2}{3}\sqrt{\frac{R^3}{2GM}}\left[\left(1+\frac{3h}{R}\right)^{3/2} - 1\right]$
4) $\frac{2}{3}\sqrt{\frac{R^3}{GM}}\left[\left(1+\frac{3h}{R}\right)^{3/2} - 1\right]$

Key: 3

Sol:



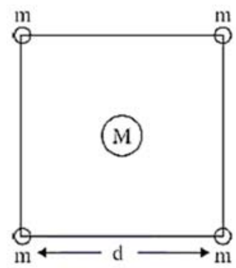
$$\frac{1}{2}mV^2 = \frac{GmM}{R+3h} \Rightarrow V = \sqrt{\frac{2GM}{R+3h}} \Rightarrow V = \sqrt{\frac{2GM}{r}} \quad (\because r = R+3h)$$

$$V = \frac{dr}{dt} = \sqrt{\frac{2GM}{r}} \Rightarrow \int dt = \int_R^{R+3h}\frac{r^{1/2}}{\sqrt{2GM}}dr$$

$$t = \frac{2}{3}\frac{1}{\sqrt{2GM}}\left[(R+3h)^{3/2} - R^{3/2}\right]$$

$$t = \frac{2}{3}\sqrt{\frac{R^3}{2GM}}\left[\left(1+\frac{3h}{R}\right)^{3/2} - 1\right]$$

13. Four sphere each of mass form a square of side d. A fifth sphere of mass M is situated at the centre of square. The total gravitational potential energy of the system is

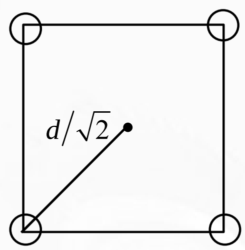


(27 – JUNE – 2022(E))

1) $-\frac{Gm}{d}\left[\left(4+\sqrt{2}\right)m+4\sqrt{2}M\right]$ 2) $-\frac{Gm}{d}\left[\left(4+\sqrt{2}\right)M+4\sqrt{2}m\right]$

3) $-\frac{GM}{d}\left[3m^2+4\sqrt{2}M\right]$ 4) $-\frac{GM}{d}\left[6m^2+4\sqrt{2}M\right]$

Key: 1
Sol:



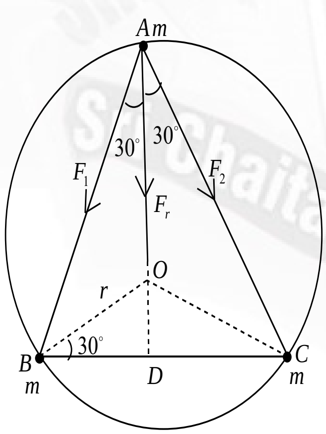
$$PE=-\left[\frac{4Gm^2}{d}+\frac{2Gm^2}{\sqrt{2}d}+\frac{4GmM}{d/\sqrt{2}}\right]$$

$$PE=-\frac{Gm}{d}\left[4\sqrt{2}M+m\left(4+\sqrt{2}\right)\right]$$

14. The energy of 4 particles each of mass $1kg$ placed at the four vertices of a square of side length $1m$ is **(27 – JUNE – 2022(E)) (D)**

1) $+4.0G$ 2) $-7.5G$ 3) $-5.4G$ 4) $+6.3G$

Key: 3
Sol:



$AC=BD=\sqrt{2}\,a$

here $a=1m$

$\therefore AC=BD=\sqrt{2}$

Total potential energy

$$U=-\left[\frac{Gm_1m_2}{r_{12}}+\frac{Gm_2m_3}{r_{23}}+\frac{Gm_3m_4}{r_{34}}+\frac{Gm_4m_1}{r_{41}}+\frac{Gm_1m_3}{r_{13}}+\frac{Gm_2m_4}{r_{24}}\right]$$

$$U=-G\left[\frac{4\times1}{1}+\frac{2\times1}{\sqrt{2}}\right]$$

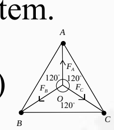
$$U=-4G-\frac{2G}{\sqrt{2}}$$

$$=-2G\left[2+\frac{1}{\sqrt{2}}\right]$$

$$=-2G(2+0.707)$$

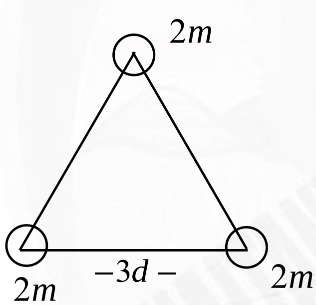
$$U=-5.4G$$

15. Three particles each of mass 2m are placed at the corners of an equilateral triangle of side $3d$ calculate the potential energy of the system. **(27 – JUNE – 2022(E)) (D)**

1) $\frac{3Gm^2}{2d}$ 2) $-\frac{4Gm^2}{d}$ 3)  4) $-\frac{4Gm^2}{3d}$

Key: 2

Sol:



$$U=U_1+U_2+U_3$$

$$U=-\frac{Gm_1m_2}{r_{12}}-\frac{Gm_2m_3}{r_{23}}-\frac{Gm_3m_1}{r_{31}}$$

$$U=-\frac{(3G\ 2m\times2m)}{3d}$$

$$U=-\frac{3G\ 4m^2}{3d}$$

$$U=-\frac{4Gm^2}{d}$$

16. A body of mass m is taken from earth surface to a height $'h'$ equal to twice the radius of radius of earth (R). The increase in potential energy will be **(25 – Jan – 2023(E))**

1) $3mgR$ 2) $\frac{mgR}{3}$ 3) $\frac{2mgR}{3}$ 4) $\frac{mgR}{2}$

Key: 3

Sol: $\Delta U=\frac{mgh}{1+\frac{h}{R}}$

$$h = 2R \Rightarrow \frac{mg \times 2R}{1 + \frac{2R}{R}} = \frac{2}{3} mgR$$

17. A body of mass m is taken from earth surface to a height $'h'$ equal to four times radius of earth (R). The increase in potential energy will be **(25 – Jan – 2023(E)) (D)**

1) $4mgR$ 2) $\frac{4mgR}{5}$ 3) $\frac{5mgR}{4}$ 4) $\frac{mgR}{4}$

Key: 2

Sol: $\Delta U = \frac{mgh}{1 + \frac{h}{R}}$

$$h = 4R \Rightarrow \frac{mg \times 4R}{1 + \frac{4R}{R}} = \frac{4}{5} mgR$$

18. If g is acceleration due to gravity on the earth's surface the gain in potential energy of the body at a height equal to 3 times the radius of earth (R) will be **(25 – Jan – 2023(E)) (D)**

1) mgR 2) $\frac{mgR}{2}$ 3) $\frac{mgR}{3}$ 4) $\frac{3mgR}{4}$

Key: 4

Sol: $\Delta U = \frac{mgh}{1 + \frac{h}{R}}$

$$h = 3R \Rightarrow \frac{mg \times 3R}{1 + \frac{3R}{R}} = \frac{3}{4} mgR$$

19. An object is allowed to fall from a height 'R' above the earth, where is radius of earth. Its velocity when it strikes the earth's surface ignoring air resistance, will be

(30 – Jan – 2023(E))

1) $2\sqrt{gR}$ 2) $\sqrt{gR}$ 3) $\sqrt{\frac{gR}{2}}$ 4) $\sqrt{2gR}$

Key: 2

Sol: loss in PE = Gain in KE

$$-\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{1}{2} mV^2 \Rightarrow V^2 = \frac{GM}{R} = g^R$$

$$V = \sqrt{gR}$$

20. If mass of earth is M, radius is R, and gravitational constant is G, then the work done to take $1\,kg$ mass from earth surface to infinity will be **(30 – Jan – 2023(E)) (D)**

1) $\sqrt{\frac{GM}{2R}}$ 2) $\frac{GM}{R}$ 3) $\sqrt{\frac{2GM}{R}}$ 4) $\frac{GM}{2R}$

Key: 2

Sol: P.E of 1kg mass on earth's surface $= \dfrac{-GM}{R}$

P.E at infinity = 0

Work done $= \Delta PE = \dfrac{GM}{R}$

21. The work done to increase the radius of the orbit of a satellite of mass $'m'$ revolving around a planet of mass M from orbit of radius R into another orbit of radius 3R is

(30 – Jan – 2023(E)) (D)

1) $\dfrac{2GMm}{3R}$ 2) $\dfrac{GMm}{3R}$ 3) $\dfrac{GMm}{6R}$ 4) $\dfrac{GMm}{24R}$

Key: 2

Sol: work done = change in T.E

$$work\ done = \frac{GMm}{2R} - \frac{GMm}{6R} = \frac{GMm}{3R}$$

b) Gravitational potential

22. The gravitational potential of two homogeneous spherical shells A and B of same surface density at their respective centres are in the ration 3: 4. If the two shells coalesce into single one such that surface charge density remains same then the ratio of potential at an internal point of the new shell to shell A is equal to ______

1) 3 : 2 2) 4 : 3 3) 5 : 3 4) 5 : 6

Key: 3

Sol: $GP = -\dfrac{GM}{r}$

$GP\ \alpha\ \dfrac{1}{r}$

$R = \sqrt{r_1^2 + r_2^2}$

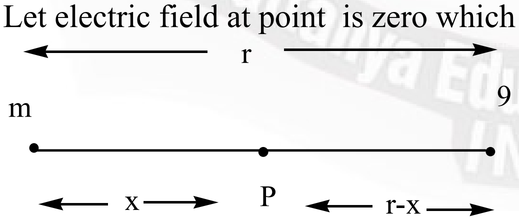
23. Two bodies of masses m and $9m$ are placed at a distance r the gravitational potential at a point on the line joining them, where gravitational field is zero, is (G is universal gravitational constant)

(3 – Sep – 2020 (E)) (D)

1) $\dfrac{-14GM}{r}$ 2) $\dfrac{-16GM}{r}$ 3) $\dfrac{-12GM}{r}$ 4) $\dfrac{-8GM}{r}$

Key: 2

Sol: Let electric field at point is zero which is situated at a distance x from mass m



$\dfrac{Gm}{x^2} = \dfrac{G(9m)}{(r-x)^2}$

$or\ \dfrac{(r-x)^2}{x^2} = 9$

$\dfrac{r-x}{x} = 3;\ x = \dfrac{r}{4}$

Potential at point $P, V = \frac{-GM}{x} - \frac{G(9m)}{r-x}$

$$V = \frac{-GM}{(r/4)} - \frac{G(9m)}{\left(\frac{3r}{4}\right)}$$

$$V = \frac{-Gm}{r}(4+12) = \frac{-16GM}{r}$$

24. On the x-axis and at a distance x from the orgin, the gravitational field due to a mass distribution is given by $\frac{Ax}{\left(x^2+a^2\right)^{3/2}}$ in the x direction. The magnitude of gravitational potential on the x axis at a distance x, taking its value to be zero at infinity is

(4 – Sep – 2020 (M))

1) $\frac{A}{\left(x^2+a^2\right)^{1/2}}$ 2) $A\left(x^2+a^2\right)^{3/2}$ 3) $\frac{A}{\left(x^2+a^2\right)^{3/2}}$ 4) $A\left(x^2+a^2\right)^{1/2}$

Key: 1

Sol: Here gravitational field $E_G = \frac{Ax}{\left(x^2+a^2\right)^{3/2}}$ and also gravitational potential at infinity $V_\infty = 0$

Now, $E_G = -\frac{dV}{dx}$

$$dV = -E_G dx$$

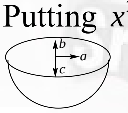
$$V_\infty = 0 \int_{V_\infty}^{V_x} dV = -\int_\infty^x E_G dx$$

$$V_x - V_\infty = -\int_\infty^x \frac{Ax}{\left(x^2+a^2\right)^{3/2}} dx$$

$$V_x - 0 = -A\int_\infty^x \frac{x}{\left(x^2+a^2\right)^{3/2}} dx$$

$$V_x = -A\int_\infty^x \frac{x}{\left(x^2+a^2\right)^{3/2}} dx \ldots 1$$

Putting $x^2+a^2 = t$ then $2xdx + 0 = dt$



At $x = \infty, (\infty)^2 + a^2 = t$

$t = \infty$ (lower limt)

Again, at $x = x$, $x^2 + a^2 = t$

$t = x^2 + a^2$ (upper limit)

From equation 1, we get

$$V_x = -A\int_\infty^x x^2+a^2 \frac{(dt/2)}{(t)^{3/2}}$$

$$= -\frac{A}{2}\int_\infty x^2 + a^2 (t)^{-3/2} dt$$

$$= -\frac{A}{2}\left[\frac{(t)^{-1/2}}{-\frac{1}{2}}\right]_{\infty}^{x^2+a^2}$$

$$= A\left[\frac{1}{(t)^{1/2}}\right]_{\infty}^{x^2+a^2} = A\left[\frac{1}{\left(x^2+a^2\right)^{1/2}} - \frac{1}{(\infty)^{1/2}}\right] = A\left[\frac{1}{\left(x^2+a^2\right)^{1/2}} - 0\right] = \left[\frac{A}{\left(x^2+a^2\right)^{1/2}}\right]$$

25. The gravitational field due to a mass distribution is $E = \frac{K}{x^3}$ in the $x-$direction, where K is a constant taking the gravitational potential to be zero at infinity, its value at a distance x is **(4 – Sep – 2020 (M)) (D)**

1) $\frac{K}{x}$ 2) $\frac{K}{2x}$ 3) $\frac{K}{x^2}$ 4) $\frac{K}{2x^2}$

Key: 4

Sol: Gravitation potential $V = \int E dx = \int_x^{\infty} \frac{K}{x^3} dx = K\int_x^{\infty} x^{-3} dx = K\left[\frac{x^{-3+1}}{-3+1}\right]_x^{\infty}$

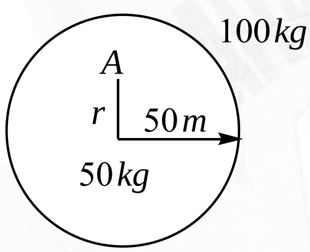
$$= K\left[\frac{x^{-2}}{-2}\right]_x^{\infty} = K\left[-\frac{1}{2x^2}\right]_x^{\infty} = \frac{K}{2x^2}$$

26. A mass of $50\,kg$ is placed at the centre of a uniform spherical shell of mass $100\,kg$ and radius $50\,m$. If the gravitational potential at a point $25\,m$ from the centre is $V\,kg/m$, the value of V is **(27 – Aug – 2021 (M))**

1) $-60G$ 2) $2G$ 3) $-20G$ 4) $-4G$

Key: 4

Sol:



$$V_A = \left[\frac{-GM_1}{r} - \frac{-GM_2}{R}\right] = \left[\frac{-50G}{25} - \frac{100G}{50}\right] = -4G$$

5. Motion of planets and satellites

b) Escape speed

01. Two stars of masses $3\times10^{31}\,kg$ each, and at distance $2\times10^{11}\,m$ rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is: (Take Gravitational constant $G = 6.67\times10^{-11}\,Nm^2\,kg^{-2}$) **(10 – JAN – 2019 (E))**

1) $1.4\times10^5\,m/s$ 2) $24\times10^4\,m/s$ 3) $3.8\times10^4\,m/s$ 4) $2.8\times10^5\,m/s$

Key: 4

Sol: By energy convervation between 0 & ∞.

$$-\frac{GMm}{r}+\frac{-GMm}{r}+\frac{1}{2}mV^2=0+0$$

(M is mass of star m is mass of meteroite)

$$\Rightarrow v=\sqrt{\frac{4GM}{r}}=2.8\times10^5\ m/s$$

02. A satellite is revolving in a circular orbit at a height h from the earth surface, such that $h << R$ where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escapte from the gravitational field of earth is: **(11 – JAN – 2019 (M))**

1) $\sqrt{gR}\left(\sqrt{2}-1\right)$ 2) $\sqrt{2gR}$ 3) $\sqrt{gR}$ 4) $\sqrt{\frac{gR}{2}}$

Key:

Sol: Conceptual

03. A satellite is revolving in a circular orbit of a height "h" from earths surface, such that $h << R$ when R is the radius of earth. Assuming that the effect of earths atmosphere can be neglected, the minimum increade in the speed required so that the satellite could escape from the gravitational field of earth is $\left(g=9.8m/s^2, R=6400km\right)$ **(11 – JAN – 2019 (M)) (D)**

1) $3.2\,km/s$ 2) $8\,km/s$ 3) $11.2\,km/s$ 4) $4\,km/s$

Key: 1

Sol: For satellite very close to earth, orbital speed is $8\,km/s$. Escape speed on earth is $11.2\,km/s$.

So $\Delta v=11.2-8=3.2\,km$

04. Planet A has mass M and radius R. Planet B has half the mass and half the radius of planet A. If the escape velocities from the planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B}=\frac{n}{4}$. Then the value of n is **(09 – Jan – 2020 (E))**

1) 4 2) 3 3) 2 4) 1

Key: 1

Sol: Escapevelocity of the plane A is $v_A=\sqrt{\frac{2GM_A}{R_A}}$

Where M_A and R_A be the mass and radius of the planet A

According to given problem

$$M_B=\frac{M_A}{2},\ R_B=\frac{R_A}{2}$$

$$\therefore v_B=\sqrt{\frac{2G\frac{M_A}{2}}{\frac{R_A}{2}}}\ \therefore \frac{v_A}{v_B}=\sqrt{\frac{\frac{2GM_A}{R_A}}{\frac{2G\left(M_A/2\right)}{\left(R_A/2\right)}}}\Rightarrow\frac{v_A}{v_B}=\frac{n}{4}=1$$

05. A projectile is fired vertically upwards form the surface of the earth with a velocity kv_e where v_e is the escape velocity and k<1. If R is the radius of the earth. The maximum height to which it will rise measured from the center of earth will be (neglect air resistance).

(09 – Jan – 2020 (E)) (D)

1) $\frac{1-k^2}{R}$ 2) $R(1-k^2)$ 3) $\frac{R}{(1-k^2)}$ 4) $\frac{R}{(1+k^2)}$

Key: 3

Sol: Using conservation of energy:

$$\frac{1}{2}m(kv_e)^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

Use $v_e = \sqrt{2gR}$ and $GM = gR^2$

And to solve we get $h + R = \frac{R}{(1-k^2)}$

06. Escape velocities on two planets are in the ratio 1:2. Ratio of their densities is 2:1, then ratio of the radii of the two planets is **(4 – Sep – 2020 (E)) (D)**

1) $\sqrt{2}:1$ 2) $1:\sqrt{2}$ 3) $1:2\sqrt{2}$ 4) $2\sqrt{2}:1$

Key: 3

Sol: $V_e = \sqrt{2gR} = \sqrt{2\left(\frac{GM}{R^2}\right)^R} = \sqrt{\frac{2GM}{R}}$

$$m = \rho V = \int \frac{4}{3}\pi R^3$$

$$V_e = \sqrt{\frac{2G\int\left(\frac{4}{3}\right)\pi R^3}{R}} = \sqrt{\frac{2G\rho 4\pi R^2}{3}}$$

$$V_e \alpha \sqrt{\rho R^2}\ ;\ \frac{V_1}{V_2} = \sqrt{\frac{\rho_1}{\rho_2}\times\frac{R_1^2}{R_2^2}}$$

$$= \frac{1}{2}\sqrt{\frac{2}{1}\times\left(\frac{R_1}{R_2}\right)^2}\ ;\ = \frac{1}{4} = \frac{2}{1}\times\left(\frac{R_1}{R_2}\right)^2 ;\ \left(\frac{R_1}{R_2}\right)^2 = \frac{1}{8}$$

$$\frac{R_1}{R_2} = \frac{1}{2\sqrt{2}}$$

07. The escape velocity for a body projected vertically upwards from the surface of earth is $11\,km/s$. If the body is projected at an angle of 60^0 with horizontal. The escape velocity will be **(25 – FEB – 2021(M)) (D)**

1) $11\sqrt{2}\,km/s$ 2) $22\,km/s$ 3) $11\,km/s$ 4) $\frac{11}{\sqrt{2}}km/s$

Key:

Sol: $V_e = \sqrt{2gk}$

Clearly escape velocity does't depend on the angle at which the body is projected

08. The escape velocity of a body depend upon mass as **(25 – FEB – 2021(M)) (D)**

1) m^0 2) m^1 3) m^3 4) m^2

Key:

Sol: $V_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

$V_e \propto m^0$

M, R are the mass and radius of planet respectively

09. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 5R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value x of will be ____.

(25 – FEB – 2021(E)) (D)

1) $\sqrt{\frac{20GM}{11R}}$ 2) $\sqrt{\frac{5GM}{3R}}$ 3) $\sqrt{\frac{10GM}{3R}}$ 4) $\sqrt{\frac{10GM}{11R}}$

Key: 2

Sol: $h = 5R + R = 6R$

According to L.C.E

$$-\frac{GMm}{6R} = \frac{-GMm}{R} + \frac{1}{2}mV^2$$

$$\frac{1}{2}mV^2 = \frac{GMm}{R} - \frac{6Gm}{6R}$$

$$\frac{1}{2}mV^2 = \frac{5GMm}{6R}$$

$$V = \sqrt{\frac{5GM}{3R}}$$

10. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 4R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value x of will be ____.

(25 – FEB – 2021(E)) (D)

1) $\sqrt{\frac{8}{5}\frac{GM}{R}}$ 2) $\sqrt{\frac{10}{11}\frac{GM}{R}}$ 3) $\sqrt{\frac{5}{8}\frac{GM}{R}}$ 4) $\sqrt{\frac{11}{10}\frac{GM}{R}}$

1) 2) 3) 4)

Key: 1

Sol: $h = 4R + R = 5R$

according to L.C.E

$$-\frac{GMm}{5R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{5R}$$

$$V^2 = \frac{8GM}{5R}$$

$$V=\sqrt{\frac{8GM}{5R}}$$

11. Radius in km to which $R_E = 6400\,km$ to be compresses so that escape velocity is raised to 5 times is ___in km **(17 – March – 2021 (M)) (D)**

1) 256 2) 128 3) 64 4) 32

Key: 1

Sol: $$\frac{V_e=\sqrt{\frac{2GM}{R}}}{5V_e=\sqrt{\frac{2GM}{R'}}} \Rightarrow 5=\sqrt{\frac{R}{R'}}$$

square on both sides

$$\Rightarrow R' = \frac{R}{25}=\frac{6400}{25}=256$$

12. A spaceship goes into a circular orbit close to the earth's surface. What additional velocity must be imparted to the ship so that it is able to escape. The gravitational pull of the earth? $(R=6400\,km, g=9.8\,m/s)$

1) $1.68\,km/s$ 2) $2.61\,km/s$ 3) $3.27\,km/s$ 4) $4.16\,km/s$

Key: 3

Sol: The orbital velocity $V=\sqrt{gR}$

The velocity required to escape $V_e=\sqrt{2gR}$

Additional velocity required is $V_e - V = \left(\sqrt{2}-1\right)\sqrt{gR}$

$$V_e - V = 0.414\times\sqrt{9.8\times 6400\times 10^3} = 3.27\,km/s$$

13. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' a part. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses (31st August 2021, Shift-1)

1) $V=\frac{1}{2}\sqrt{\frac{4G(M_1+M_2)}{r}}$ 2) $V=\sqrt{\frac{4G(M_1+M_2)}{r}}$

3) $V=\frac{1}{2}\sqrt{\frac{2G(M_1+M_2)}{r}}$ 4) $V=\frac{\sqrt{2G}(M_1+M_2)}{r}$

Key: 2

Sol: Total energy at middle point = $K.E + P.E \text{ of } M_1 \& m + P.E \text{ of } M_2 \& m$

To get escape velocity total energy should be zero

$$\frac{1}{2}mV^2 - \frac{GM_1 m}{\frac{r}{2}} - \frac{GM_2 m}{\frac{r}{2}} = 0$$

$$\Rightarrow \frac{1}{2}mV^2 = \frac{2Gm}{r}(M_1+M_2)$$

$$V=\sqrt{\frac{4G(M_1+M_2)}{r}}$$

14. The escape velocity of a body on a planet A is $12\,kms^{-1}$. The escape velocity of the body on planet B, whose density is four times and radius is half of the planet A is

(29 – JUNE – 2022(M))

1) $12\,kms^{-1}$ 2) $24\,kms^{-1}$ 3) $36\,kms^{-1}$ 4) $6\,kms^{-1}$

Key: 1

Sol: $V_A = 12\,km/s$ $\rho_A = \rho$ $R_A = R$

$V_B = ?$ $\rho_B = 4\rho$ $R_B = \dfrac{R}{2}$

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\rho\frac{4}{3}\pi R^3}$$

$$V_e \propto R\sqrt{\rho}$$

$$\frac{V_B}{V_A} = \frac{R_B}{R_A}\sqrt{\frac{\rho_B}{\rho_A}}$$

$$= \frac{R}{2R}\sqrt{\frac{4\rho}{\rho}}$$

$$\frac{V_B}{V_A} = \frac{1}{1}$$

$$V_B = 1 \times V_A$$

$$= 1 \times 12$$

$$V_B = 12\ km/s$$

15. The escape velocity for a body of mass $1kg$ from earth surface is $11.2\ km/s$. The escape velocity for a body of mass $1000\,kg$ would be **(29 – JUNE – 2022(M)) (D)**

1) $112 \times 10^2\ km/s$ 2) $112\ km/s$ 3) $11.2\ km/s$ 4) $11.2 \times 10^{-2}\ km/s$

Key: 2

Sol: $V_e = 11.2\,km/s$

$m_1 = 1kg$

$m_2 = 1000\,kg$

$V_e = 1000\,kg$

$$V_e = \sqrt{\frac{2GM}{R}}$$

V_e does not depend on mass of body

16. A planet has twice the radius but the mean density is $\dfrac{1}{4}^{th}$ as compared to earth. What is the ratio of escape velocity from earth to that from the planet? **(29 – JUNE – 2022(M)) (D)**

1) $3:1$ 2) $1:2$ 3) $1:1$ 4) $2:1$

Key: 3

Sol: $R_2 = 2R$

$$\rho_2 = \frac{\rho}{4}$$

$\frac{V_e}{V_\rho}=?$ $\qquad V_e=\sqrt{\frac{2GM}{R}}$

$V_e \propto \sqrt{\rho}\,R$ $\qquad V_e=\sqrt{\frac{2G4\pi R^3\rho}{3R}}$

$$\frac{V_e}{V_\rho}=\sqrt{\frac{\rho_e}{\rho_p}}\frac{R_\rho}{R_p}$$

$$\frac{V_e}{V_\rho}=\sqrt{\frac{4\rho}{\rho}}\frac{R}{2R}$$

$$\frac{V_e}{V_\rho}=2\times\frac{1}{2}=1$$

$$V_e:V_\rho=1:1$$

17. If earth has a mass nine times and radius twice to that of planet P. Then $\frac{V_e}{3}\sqrt{x}\,ms^{-1}$ will be the minimum velocity required by a rocket to pull out of gravitational force of P, where V_e is escape velocity on earth. The value of x is **(25 – Jan – 2023(M))**

1) 2 2) 3 3) 18 4) 1

Key: 1

Sol: $V_P=\sqrt{\frac{2GM_P}{R_P}}=\sqrt{\frac{2G\left(\frac{M_e}{9}\right)}{\left(\frac{R_e}{2}\right)}}=\sqrt{\frac{2GM_e}{R_e\times 3}}\sqrt{2}=\frac{V_e\sqrt{2}}{3}$

18. The mass of earth is 9 times that of mass, radius of earth is twice that of mass. If escape velocity of earth is $12\,km/s$. The escape velocity of mass is ___ $km/\sec$ **(25 – Jan – 2023(M)) (D)**

1) $4\sqrt{2}\ km/s$ 2) $2\sqrt{2}\ km/s$ 3) $6\sqrt{2}\ km/s$ 4) $8\sqrt{2}\ km/s$

Key: 1

Sol: $\frac{V_m}{V_e}=\sqrt{\frac{\mu_m}{\mu_e}\times\frac{R_e}{R_m}}$

$$\frac{V_m}{12}=\sqrt{\frac{1}{9}\times\frac{2}{1}}\Rightarrow V_m=\frac{12\sqrt{2}}{3}=4\sqrt{2}$$

19. The escape velocity from earth is $11\,km/\sec$. The escape velocity from a planet having 9 times the radius and one third of density as earth is ____ $km/\sec$ **(25 – Jan – 2023(M)) (D)**

1) 11 2) $22\sqrt{3}$ 3) $33\sqrt{3}$ 4) $44\sqrt{3}$

Key: 3

Sol: $V\propto R\sqrt{\rho}$

$$\frac{V_E}{V_P}=\frac{R_E}{R_P}\times\sqrt{\frac{\rho_E}{\rho_P}}$$

$$\frac{11}{9}=\frac{1}{9}\times\sqrt{\frac{3}{1}}\Rightarrow V_P=\frac{99}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=33\sqrt{3}$$

c) Orbital speed of a satellite in circular orbit

20. If the angular velocity of earth spin is increased such that the bodies at the equator start floating, the duration of day would be approximately

$\left[g=10\,m/s^2; R=6400\,km; \pi=3.14\right]$ **(18 – March – 2021 (E))**

1) 60 mints 2) No change 3) 1200 mints 4) 84 mints

Key: 4

Sol: $mg=mRw^2$

$$\sqrt{\frac{g}{R}}=w=\frac{2\pi}{T}$$

$$T=2\pi\sqrt{\frac{R}{g}}=2\pi\sqrt{\frac{6400000}{10}}=83.7\,\text{minutes}$$

21. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital of $9.0\times10^3\,km$. Find the mass of Mars. $\left\{Given\ \frac{4\pi^2}{G}=6\times10^{11}\,N^{-1}m^{-2}kg^2\right\}$

(27 – July – 2021(E))

1) $5.96\times10^{19}\,kg$ 2) $3.25\times10^{21}\,kg$ 3) $7.02\times10^{25}\,kg$ 4) $6.00\times10^{23}\,kg$

Key: 4

Sol: $T^2=\frac{4\pi^2}{GM}\cdot r^3$

$$M=\frac{4\pi^2}{G}\cdot\frac{r^3}{T^2}$$

by putting values

$M=6\times10^{23}$

22. The planet mars has two moons, if one of them has a period 8 hours and an orbital radius of $8\times10^3\,km$. Find the mass of mars $\left(\frac{4\pi^2}{G}=6\times10^{11}\ N^{-1}m^{-2}kg^2\right)$

(27 – July – 2021(E)) (D)

1) $8\times10^{25}\,kg$ 2) $14\times10^{24}\,kg$ 3) $5\times10^{29}\,kg$ 4) $7\times10^{17}\,kg$

Key: 1

Sol: $V_0=\sqrt{\frac{GM}{r}}=r\left(\frac{2\pi}{T}\right)\Rightarrow T^2=\frac{4\pi^2r^3}{GM}$

$$(8\times 60\times 60)^2=\frac{6\times 10^{11}\left(8\times 10^{6}\right)^{-3}}{M}$$

$M=8\times 10^{25}\,kg$

23. An orbiting satellite will escape if

A) Its speed is increased by 41%

B) Its speed in the orbit is made $\sqrt{1.5}$ times of its initial value

C) Its K.E is doubled

D) It stops moving in the orbit

1) A, B & C only 2) A & C only 3) B & D only 4) D only

Key: **2**

Sol: Conceptual

24. Two satellites revolves around a planet in co –planer circular orbits in anticlockwise direction their period of revolution are $1hr$ and $8hr$ respectively. The radius of the orbit of nearer satellite is $2\times 10^3\,km$. The angular speed of the lather satellite as observed from the nearer satellite at the instant when both satellites are closest is $\frac{\pi}{x}rad/h$ where x is ___________ **(01 – Sep – 2021(M))**

Key: **3**

Sol: $T_1=1hr\Rightarrow W_1=2\pi\,rad/s$

$T_2=8hr\Rightarrow W_1=\frac{\pi}{4}rad/s$

$R_1=2\times 10^3\,km$

From the Kepler's law of orbits

As $T^2\,\alpha\,R^3\Rightarrow\frac{R_2}{R_1}=\left(\frac{8}{1}\right)^{\frac{2}{3}}=4$

$R_2=8\times 10^3\,km$

$V_1=W_1R_1=4\pi\times 10^3\,km/h$

$V_2=W_2R_2=2\pi\times 10^3\,km/h$

$$W=\frac{V_{rel}}{R_{rel}}=\frac{V_1-V_2}{R_2-R_1}=\frac{2\pi\times 10^3}{6\times 10^3}=\frac{\pi}{3}rad/h$$

d) Angular speed of a satellite in circular orbit

25. Consider two satellites S_1 and S_2 with period of revolution $1hr$ and $8hr$. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is: **(24 – FEB – 2021(M))**

1) 8: 1 2) 1: 4 3) 2: 1 4) 1: 8

Key: **1**

Sol: $\frac{T_1}{T_2}=\frac{1}{8}$

$$\frac{2\pi/\omega_1}{2\pi/\omega_2} = \frac{1}{8}$$

$$\frac{\omega_1}{\omega_2} = \frac{8}{1}$$

26. Consider two satellites S_1 and S_2 with period of revolution $1hr$ and $9hr$. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is: **(24 – FEB – 2021(M)) (D)**

1) 8: 1 2) 9: 1 3) 1: 9 4) 1: 8

Key: 2

Sol: $\frac{T_1}{T_2} = \frac{1}{9}$

$$\frac{T_1}{T_2} = \frac{2\pi/\omega_1}{2\pi/\omega_2}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{9}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{9}{1}$$

27. Consider two satellites S_1 and S_2 with period of revolution $2hr$ and $8hr$. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is: **(24 – FEB – 2021(M)) (D)**

1) 8: 1 2) 4: 1 3) 1: 4 4) 1: 9

Key: 2

Sol: $\frac{T_1}{T_2} = \frac{2}{8} = \frac{1}{4}$

$$\Rightarrow \frac{T_1}{T_2} = \frac{2\pi/\omega_1}{2\pi/\omega_2} = \frac{\omega_2}{\omega_1} = \frac{1}{4}$$

$$\frac{\omega_1}{\omega_2} = \frac{4}{1}$$

e) Orbital speed of a satellite in elliptical orbit

28. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\frac{v}{2}$ the instantaneous velocity 'v' of A. The collision is completely inelastic. Then, the combined body: **(09 – Jan – 2020 (M))**

1) Continues to move in a circular orbit.
2) Escapes from the planet's Gravitational field
3) Falls vertically downwards towards the planet
4) Starts moving in an elliptical orbit around the planet.

Key: 4

Sol: By conservation of linear momentum and taking velocity in line for maximum momentum transfer in single direction.

$$\frac{m}{2}\times\frac{v}{2}+mv=\left[m+\frac{m}{2}\right]v_f$$

$\Rightarrow v_f=\frac{5v}{6}$; where V is orbital velocity.

Escape velocity will be v<V and at velocity less than escape velocity but greater than orbital velocity (V), the path will be elliptical.

At orbital velocity (V), path will be circular.

At velocity less than orbital velocity path will remain part of ellipse and it will either orbit in elliptical path whose length of semi major axis will be less than radius of circular orbit or start falling down and collide with the planet but it will not fall vertically down as path will remain part of ellipse.

Hence the resultant mass will start moving in an elliptical orbit around the planet.

29. A comet is in highly elliptical orbit around the Sun. the period of the comet's orbit is 90days. Some statements are given regarding the collision between the comet and the earth. Mark the correct statement [Mass of the Sun = 2×10^{30} kg, mean distance between the earth and the Sun 1.5×10^{11} m] **(09 – Jan – 2020 (M)) (D)**

1) Collision is there
2) Collision is not possible
3) Collision may or may not be there
4) Enough information is not given.

Key: 2

Sol: Using Kepler's third law, the semi – major axis of the comet's orbit is given by

$$T^2=\frac{4\pi^2}{GM_s}\times a^3\Rightarrow a=\left(\frac{T^2\times GM_s}{4\pi^2}\right)^{\frac{1}{3}}$$

$$\Rightarrow a=\left(\frac{(90\times24\times3600)^2\times6.67\times10^{-11}\times2\times10^{30}}{4\pi^2}\right)^{\frac{1}{3}}\Rightarrow a=5.89\times10^{10}\text{m}$$

Since the radius of comet is less the radius of the earth, it won't collide.

30. A satellite of mass m revolves around the earth of radius R at a height x from its surface.If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **(09 – Jan – 2020 (E)) (D)**

1) $\left(\frac{gR^2}{R+x}\right)^{1/2}$
2) $\frac{gR}{R-x}$
3) $\frac{gR^2}{R+x}$
4) gx

Key: 1

Sol: The gravitational force exerted on satellite at a height x is

$F_G=\frac{GM_em}{(R+x)^2}$ where M_e = mass of earth

Since, gravitational force provides the necessary centripetal force, so

$\frac{GM_em}{(R+x)^2}=\frac{mv_o^2}{(R+x)}$ where v_o is orbital speed of satellite.

$$\frac{GM_em}{(R+x)}=mv_o^2\Rightarrow\frac{gR^2m}{(R+x)}=mv_o^2\left(\because g=\frac{GM_e}{R^2}\right)$$

$$\Rightarrow V_0 = \left(\frac{gR}{(R+x)}\right)^{1/2}$$

31. An artificial satellite revolves round earth at a height $=1000\,km$, Radius of earth $= 6400\,km$, mass of earth $= 6\times10^{24}\,kg$, $G = 6.67\times10^{-11}\ N\ m^2\ kg^{-2}$, find orbital speed?

(18 – March – 2021 (E)) (D)

1) $7300\frac{m}{S}$ 2) $7364\frac{m}{S}$ 3) $8300\frac{m}{S}$ 4) $8364\frac{m}{S}$

Key: 2

Sol: Here $h = 1000\,km = 1000\times10^3\,km = 1000\times10^3 = 10^6\,meter$

$r = R + h = 7400\,km = 7400\times10^3\,meter$

$$V_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.6\times10^{-11}\times6\times10^{24}}{74\times10^5}} = 7364\frac{m}{S}$$

32. The ratio of the orbital speeds of two satellites of the earth if the satellites are at heights $6400\,km$ and $70{,}400\,km$ [Radius of the earth = $6400\,km$] **(25 – JUNE – 2022(E)) (D)**

1) $\sqrt{2}:1$ 2) $\sqrt{3}:1$ 3) $2:1$ 4) $3:1$

Key: 2

Sol: $V_0 = \sqrt{\frac{GM}{R-h}}$

$R_1 = 6400\,km$

$h_1 = 6400\,km$

$h_2 = 3200\,km$

$$\frac{V_1}{V_2} = \sqrt{\frac{R+h_2}{R+h_1}} = \sqrt{\frac{6400+32000}{6400+6400}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{3}{1}}$$

$V_1 : V_2 = \sqrt{3}:1$

33. An artificial satellite is revolving in a circular orbit at height of $1200\,km$ above the surface of the earth. If the radius of the earth is $6400\,km$ and mass is $6\times10^{24}\,kg$. The orbital velocity is $\left[G = 6.67\times10^{-11}\,Nm^2/kg^2\right]$ **(25 – JUNE – 2022(E)) (D)**

1) $7.26\ kms^{-1}$ 2) $4.26\ kms^{-1}$ 3) $9.36\ kms^{-1}$ 4) $2.26\ kms^{-1}$

Key: 1

Sol: $h = 1200\,km = 12\times10^5\,m$

$R = 6400\,km = 64\times10^5\,m$

$M = 6\times10^{24}\,ks$

$G = 6.6\times10^{-11}\,Nm^2/kg^2$

$$V_0 = \sqrt{\frac{GM}{R+h}}$$

$$V_0 = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{64\times10^5+12\times10^5}}$$

$$V_0 = 7.26\,km/s$$

f) Relation between escape and orbital speed

34. A satellite is moving with a constant speed 'v' in a circular orbit above the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection the kinetic energy of the object is

(07 – Jan – 2020 (M)) (D)

1) $\frac{1}{2}mv^2$ 2) mv^2 3) $\frac{3}{2}mv^2$ 4) $2mv^2$

Key: 2.

Sol: $v_e = \sqrt{2}v_o$

$$K.E = \frac{1}{2}mv_e^2 \Rightarrow K.E = \frac{1}{2}m\left(\sqrt{2}v_o\right)^2 \Rightarrow K.E = mv_o^2 \qquad (\because v_o = v)$$

35. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself, then, the ratio of the speed of this body in the orbit to the escape velocity from the planet is **(4 – Sep – 2020 (E))**

1) 1 2) $\frac{1}{\sqrt{2}}$ 3) $\sqrt{2}$ 4) 2

Key: 2

Sol: Orbital velocity of a body revolving around a planet $V_o = \sqrt{\frac{GM}{R}}$

M=mass of the planet
R=orbital radius of revolving body
And G=universal gravitational constant

Escape velocity of any body from the surface of a planet $V_e = \sqrt{\frac{2GM}{R}}$

Ratio of V_o and V_e

$$\frac{V_o}{V_e} = \frac{\left(\sqrt{\frac{GM}{R}}\right)}{\left(\sqrt{\frac{2GM}{R}}\right)} = \frac{1}{\sqrt{2}}$$

g) Energy of a satellite in circular orbit

36. The energy required to take a satellite to a height 'h' above earth surface (radius of earth $=6.4\times10^3\,km$) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is: **(09 – JAN – 2019 (E))**

1) $1.28\times10^4\,km$ 2) $6.4\times10^3\,km$ 3) $3.2\times10^3\,km$ 4) $1.6\times10^3\,km$

Key: 3

Sol: $U_{surface}+E_1=U_h$

KE of satellite is zero at earth surface & at height h

$$-\frac{GM_em}{R_e}+E_1=-\frac{GM_em}{(R_e+h)}$$

$$E_1=GM_em\left(\frac{1}{R_e}-\frac{1}{R_e+h}\right)$$

$$E_1=\frac{GM_em}{(R_e+h)}\times\frac{h}{R_e}$$

Gravitational attraction $F_G=ma_C=\frac{mv^2}{(R_e+h)}$

$$E_2\Rightarrow\frac{mv^2}{(R_e+h)}=\frac{GM_em}{(R_e+h)^2}$$

$$mv^2=\frac{GM_em}{(R_e+h)}$$

$$E_2=\frac{mv^2}{2}=\frac{GM_em}{2(R_e+h)}$$

$$E_1=E_2$$

$$\frac{h}{R_e}=\frac{1}{2}\Rightarrow h=\frac{R_e}{2}=3200\,km$$

37. The energy required to take a satellite to a height $'h'$ above earth surface (radius of earth $=6.4\times10^3\,km$) is E_1 and $k\cdot E_2$ required for the satellite to be in a circular orbit at this height is E_2. The value of h for which $E_2=2E_1$ is **(09 – JAN – 2019 (E)) (D)**

1) $3200\,km$ 2) $1000\,km$ 3) $6400\,km$ 4) $12800\,km$

Key: 2

Sol:

38. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escape from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is: **(10 – JAN – 2019 (M))**

1) $\frac{3}{2}mv^2$ 2) mv^2 3) $2mv^2$ 4) $\frac{1}{2}mv^2$

Key: 2

Sol: At height r from center of earth. Orbital velocity

$$V=\sqrt{\frac{GM}{r}}$$

$\therefore$ By energy conservation

$$KE\ of\ 'm' + \left(-\frac{GMm}{r}\right) = 0 + 0$$

(At infinity, PE = KE = 0)

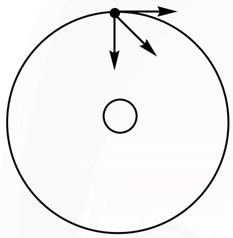
$$\Rightarrow KE\ of\ 'm' = \frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2$$

39. A satellite of mass m is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be: **(12 – JAN – 2019 (M))**

1) In a circular orbit of a different radius 2) In the same circular orbit of radius R
3) In an elliptical orbit 4) Such that it escapes to infinity

Key: 3

Sol:



$$mv\hat{i} + mv\hat{j}$$

$$= 2m\vec{v}^{1}$$

$$\vec{v} = \frac{1}{\sqrt{2}} \times \sqrt{\frac{GM}{R}}$$

40. Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B, is: **(12 – JAN – 2019 (E))**

1) 2 2) $\sqrt{\frac{1}{2}}$ 3) 1 4) $\frac{1}{2}$

Key: 3

Sol: Orbital velocity $V = \sqrt{\frac{GMe}{r}}$

$$T_A = \frac{1}{2} m_A V_A^2$$

$$T_B = \frac{1}{2} m_B V_B^2$$

$$\Rightarrow \frac{T_A}{T_B} = \frac{m \times \frac{Gm}{R}}{2m \times \frac{Gm}{2R}}$$

$$\Rightarrow \frac{T_A}{T_B} = 1$$

41. A satellite of mass m is launched vertically upwards with an initial speed 'u' from the surface of the earth. After it reaches height R (R – Radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G – gravitational constant M – mass of the earth) **(07 – Jan – 2020 (M))**

1) $\frac{3m}{8}\left(u+\sqrt{\frac{5GM}{6R}}\right)^2$
2) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$
3) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$
4) $\frac{m}{20}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$

Key: 3

Sol:

$$T.E_{surface} = T.E_R$$

$$\frac{1}{2}mu^2+\left(-\frac{GMm}{R}\right)=\frac{1}{2}mv^2+\left(-\frac{GMm}{2R}\right)$$

$$v^2=u^2+\left(-\frac{GM}{R}\right)\Rightarrow v=\sqrt{u^2+\left(-\frac{GM}{R}\right)}$$

The rocket splits at height R. since, separation of rocket is impulsive therefore conservation of momentum in both radial and tangential direction can be applied.

$$\frac{m}{10}v_T=\frac{9m}{10}\sqrt{\frac{GM}{2R}}\Rightarrow v_T^2=81\frac{GM}{2R}$$

$$\frac{m}{2x}v_r=m\sqrt{u^2-\frac{GM}{R}}$$

Ejects a rocket of mass $\frac{m}{10}$

$$\text{Kinetic energy of rocket}=\frac{1}{2}\times\frac{m}{10}\left(v_T^2+v_R^2\right)$$

$$=\frac{m}{20}\left(81\frac{GM}{2R}+100u^2-100\frac{GM}{R}\right)$$

$$=5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$$

42. A satellite of mass m is launched vertically upwards with an initial speed $\frac{u}{2}$ from the surface of the earth. After it reaches height R (R – Radius of the earth), it ejects a rocket of mass $\frac{m}{20}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G – gravitational constant M – mass of the earth)

(07 – Jan – 2020 (M)) (D)

1) $\frac{5m}{2}\left(u^2+\frac{439}{200}\frac{GM}{R}\right)$
2) $m\left(u^2+\frac{439}{200}\frac{GM}{R}\right)$

3) $\frac{5m}{2}\left(u^2 - \frac{439}{200}\frac{GM}{R}\right)$ 4) $5m\left(u^2 - \frac{439}{200}\frac{GM}{R}\right)$

Key: 3

Sol: $T.E_{surface} = T.E_R$

$$\frac{1}{2}m\left(\frac{u}{2}\right)^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2 + \left(-\frac{GMm}{2R}\right)$$

$$v^2 = \left(\frac{u}{2}\right)^2 + \left(-\frac{GM}{R}\right) \Rightarrow v = \sqrt{\left(\frac{u}{2}\right)^2 + \left(-\frac{GM}{R}\right)}$$

The rocket splits at height R. since, separation of rocket is impulsive therefore conservation of momentum in both radial and tangential direction can be applied.

$$\frac{m}{20}v_T = \frac{19m}{20}\sqrt{\frac{GM}{2R}} \Rightarrow v_T^2 = 361\frac{GM}{2R}$$

$$\frac{m}{20}v_r = m\sqrt{\left(\frac{u}{2}\right)^2 - \frac{GM}{R}}$$

Ejects a rocket of mass $= \frac{m}{20}$

Kinetic energy of rocket $= \frac{1}{2} \times \frac{m}{20}\left(v_T^2 + v_R^2\right)$

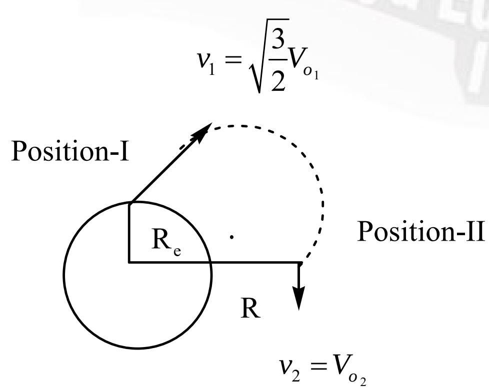
$$= \frac{m}{40}\left(361\frac{GM}{2R} + 400\left[\left(\frac{u}{2}\right)^2 - \frac{GM}{R}\right]\right)$$

$$= \frac{5m}{2}\left(u^2 - \frac{439}{200}\frac{GM}{R}\right)$$

43. A satellite is moving in a low nearly circular orbit around the earth, its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion. So that it becomes $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is **(3 – Sep - 2020 (M))**

1) $4\ R_e$ 2) $2.5\ R_e$ 3) $3\ R_e$ 4) $2\ R_e$

Key: **4**



Sol:

Using law of conservation of energy for position (I) & position (II)

$$(PE+KE)_I = (PE+KE)_{II}$$

$$\frac{-GMm}{R_e}+\frac{1}{2}mv_1^2=\frac{-GMm}{R}+\frac{1}{2}mv_2^2$$

$$\frac{-GMm}{R_e}+\frac{1}{2}m\left[\frac{3}{2}v_{o1}^{\ 2}\right]=\frac{-GMm}{R}+\frac{1}{2}mv_{o2}^{\ 2}$$

As orbital speed is

$$v_o=\sqrt{\frac{GM}{R}} \quad \text{Where } R=\text{orbital radius}$$

$$\therefore \frac{-GMm}{R_e}+\frac{1}{2}m\left[\frac{3}{2}\frac{GM}{R_e}\right]=\frac{-GMm}{R}+\frac{1}{2}m\left[\frac{GM}{R}\right]$$

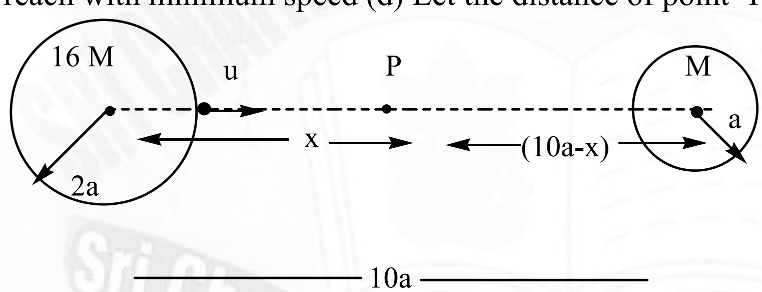
$$-\frac{1}{4}\left[\frac{GMm}{R_e}\right]=-\frac{1}{2}\left[\frac{GMm}{R}\right] \qquad R=2R_e$$

44. Two planet's have masses M and 16 M and their radii are 'a' and 2a respectively, the separation between the centres of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is **(6 – Sep – 2020 (E))**

1) $2\sqrt{\frac{Gm}{a}}$ 2) $4\sqrt{\frac{Gm}{a}}$ 3) $\sqrt{\frac{GM^2}{ma}}$ 4) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

Key: 4

Sol: For the body to be able to reach at the surface of smaller planet it just has to once come the gravitational field of bigger planet and at that point (let say 'P' in given figure) net force will be zero. After crossing this 'P' point body will move towards smaller planet it self and will reach with minimum speed (d) Let the distance of point 'P' from the bigger planet be 'x'



At point 'P', $F_{net}=0$

$$F_{16M}=F_M\ ;\ \frac{G(16M)(m)}{x^2}=\frac{G(M)(m)}{(10a-x)^2}\Rightarrow x=8a----1$$

Since, there is no net external force on system, so we can apply conservation of mechanical energy from the surface of bigger planet to point P

$$K_i+U_i=K_f+U_f$$

$$\frac{1}{2}mu^2+\left(\frac{-G(16M)(m)}{2a}\frac{GMm}{8a}\right)=0+\left[\frac{-G(16M)(m)}{8a}\frac{GMm}{2a}\right]$$

$$\frac{1}{2}mu^2-\frac{65}{8}\frac{GMm}{a}=\frac{-5}{2}\frac{GMm}{a}$$

$$\frac{1}{2}mu^2 = \frac{45}{8}\frac{GMm}{a} \Rightarrow u = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

45. A satellite of mass $1000\,kg$ is rotating around the earth in a circular orbit of radius $3R$. What extra energy should be given to this satellite if it is to be lifted into an orbit of radius 4R?

1) $2.6\times10^9\,J$ 2) $1.8\times10^9\,J$ 3) $1.1\times10^9\,J$ 4) $3.2\times10^9\,J$

Key: 1

Sol: Energy required = $TE_f - TE_i = \left[\frac{-GMm}{2(4R)}\right] - \left[\frac{-GMm}{2(3R)}\right] = \frac{GMm}{24R}$

$$= \frac{\left[9.8\times\left(6400\times10^3\right)^2\times1000\right]}{24\left(6400\times10^3\right)} = 2.6\times10^9\,J$$

i) Time period of satellite

46. An asteroid whose mass is 2.0×10^{-4} times the mass of earth, revolves in a circular orbit around the Sun at a distance that is twice earth's distance from the Sun, Calculate the period of revolution of the asteroid in years______

1) 2.4 years 2) 2.8 years 3)2.9 years 4) 2.7 years

Key: 2

Sol: Use the law of periods: $T^2 = \frac{4\pi^2}{GM}r^3$ where M is the mass of the sun $\left(1.99\times10^{30}\,kg\right)$ and r is the radius of the orbit. The radius of the orbit is twice the radius of earth's orbit:

$$r = 2r_e = 2\left(150\times10^9\,m\right) = 300\times10^9\,m$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$T = \sqrt{\frac{4\pi^2\left(300\times10^9\,m\right)^3}{\left(6.67\times10^{-11}\,m^3/s^2.kg\right)\left(1.99\times10^{30}\,kg\right)}} = 8.96\times10^7\,s$$

Divide by (365 d/y) (24 h/d) (60 min/h) (60 sec/min) to obtain T = 2.8 y

47. Suppose earth's orbital motion around the Sun is suddenly stopped. What time will the earth take to fall into the Sun? (approximately)

1) more than 2 months 2) less than 2 months

3) 2 months 4) greater than 3 months

Key: 1

Sol: When the earth's motion is suddenly stopped, it would fall into the sun and (suppose) it comes back. If the effect of temperature of Sun is ignored, we can say that the earth would continue to move along a strongly extended flat ellipse whose extreme points are located at the earth's orbit and at the centre of the sun.

The semi major axis of such ellipse is $\frac{R}{2}$.

Now $\frac{T'^2}{T^2} = \left[\frac{R}{2}\right]^3 \left[\frac{1}{R^3}\right]$

Where T is the time period of normal orbit of earth.

$or\, T'^2 = \frac{T^2}{8} or\, T' = \frac{T}{2\sqrt{2}}$

Now time required to fall into the sun,

$t = \frac{T'}{2} = \frac{T}{4\sqrt{2}} = \frac{365}{4\sqrt{2}} \approx 65\,day$

So, the earth would take slightly more than 2 months to fall into the sun.

48. The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator just barely provides the centripetal force needed for the rotation. Calculate the rotation period assuming a density of $3.0\,g/cm^3$, typical of many planet, satellites and steroids. No astronomical object has ever found to be spinning with a period shorter than that determined by this analysis.

1) 1.8 h 2) 1.7 h 3) 1.9 h 4) 1.6 h

Key: 3

Sol: $T = \sqrt{\frac{3\pi}{G\rho}}$

$\rho = 3.0 \times 10^3\,kg/m^3$

$T = \sqrt{\frac{3\pi}{\left(6.67 \times 10^{-11}\,m^3/s^2 kg\right)\left(3.0 \times 10^3\,kg/m^3\right)}} = 6.86 \times 10^3\,s = 1.9h$

49. A spaceship orbits around a planet at a height of $20\,km$ from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?

(Mass of the planet $= 8 \times 10^{22}\,kg$, Radius of planet $= 2 \times 10^6\,m$, gravitational constant $G = 6.67 \times 10^{-11}\,Nm^2/kg^2$) **(10 – Apr - 2019 (E))**

1) 9 2) 17 3) 13 4) 11

Key: 4

Sol: $T = \frac{2\pi r}{v}$

$v = \sqrt{\frac{GM}{r}}$

$\therefore T = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$

Substituting the values we get

$T = 2\pi\sqrt{\frac{(202)^3 \times 10^{12}}{6.67 \times 10^{-11} \times 8 \times 10^{22}}}\,\text{sec}$

$T = 7812.2\,s$

$T \simeq 2.17\,hr$

$\Rightarrow n = \frac{24}{2.17} \simeq 11\,revolutions$

50. The planet Neptune travels around the Sun with a period of 165 year. Show that the radius of its orbit is approximately thirty times that of Earth's orbit, both being considering as circular.

1) 2) 3) 4)

Key:

Sol: $T_1 = T_{earth} = 1\,year, T_2 = T_{Neptune} = 165\,year$

Let R_1 and R_2 be the radii of the circular orbits of earth and Neptune respectively.

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

$$R_2^3 = \frac{R_1^3 T_2^2}{T_1^2}\,or\,R_2^3 = \frac{R_1^3 \times 165^2}{1^2}$$

$$R_2^3 = 165^2 R_1^3\,or\,R_2 \approx 30R_1$$

51. Object at rest on the earth's surface move in circular paths with a period of 24 hours. Are they in orbit in the sense that an earth satellite is in orbit? What would the length of the day have to be put such objects in true orbit?

1) 2) 3) 4)

Key:

Sol: We know that the apparent weight of a body in a satellite is zero. However, the weight of a body at the equator, considering the motion of earth, i.e, $W = m\left(g - R\omega^2\right) \neq 0$. So bodies at rest on the surface of the earth (through moving in circular path) are not in orbit in the sense that an earth satellite is in the orbit.

Now if the body on the surface of earth is to be in 'true orbit'

$$W = m\left(g - R\omega^2\right) = 0 \text{ i.e } \omega = \sqrt{\frac{g}{R}}$$

So that , $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6400\times10^3}{10}} \approx 84.6\,\text{min}ute \approx 1.4hr$

i.e, if the earth rotates on its axis so fast that the length of day becomes 1.4 hour, a body on its surface will be in 'true satellite orbit'.

52. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance r from its centre in that region, a small star is in a circular orbit of radius R, then the period of revolution T depends on R as **(02 – Sep - 2020 (M))**

1) $T\alpha R$ 2) $T^2\alpha R^3$ 3) $T^2\alpha R$ 4) $T^2\alpha\frac{1}{R^3}$

Key: 3

Sol: density $\rho = \frac{M}{V}$

$$M = \rho V$$

$$\int_O^R \rho dv$$

$$= \int_O^R \frac{K}{r} 4\pi r^2 dv$$

$$= 4\pi K \int_O^R r dv$$

$= 4\pi K \frac{R^2}{2}$

$M = 2K\pi R^2 ...1$

Gravitational pull of this mass provides necessary centripetal force for rotation of star

So, $\frac{GMm}{R^2} = \frac{mv^2}{R}$

$v^2 = \frac{GM}{R}$

$v^2 = \frac{G\left[2K\pi R^2\right]}{R}$

$v = \sqrt{2GK\pi R}$

Time period of rotation

$T = \frac{2\pi R}{V} = \frac{2\pi R}{\sqrt{2GK\pi R}}$

$T = \frac{2\pi\sqrt{R}}{\sqrt{2GK\pi}}$

$T \alpha \sqrt{R}$

$T^2 \alpha R$

53. Imagine a light planet revolving around a massive star in circular orbit of radius 'R' with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$ then T^2 is proportional to **(2 – Sep - 2020 (M)) (D)**

1) R^3 2) $R^{7/2}$ 3) $R^{5/2}$ 4) $R^{3/2}$

Key: 2

Sol: Given $f \alpha R^{-5/2}$

$f \alpha \frac{1}{R^{5/2}}$

Gravitational force=centripetal force

$\frac{GMm}{(R)^{5/2}} = mRW^2 \qquad \left[W = \frac{2\pi}{T}\right]$

$\frac{GM}{(R)^{5/2}} = R\left[\frac{2\pi}{T}\right]^2$

$\frac{GM}{(R)^{5/2}} = R\frac{4\pi^2}{T^2}$

$T^2 = \frac{R4\pi^2[R]^{5/2}}{GM}$

$T^2 = \frac{4\pi^2}{GM}R^{7/2}$

$T^2 \alpha R^{7/2}$

54. Two satellites A and B of masses $200\,kg$ and $400\,kg$ are revolving round the earth at height of $600\,km$ and $1600\,km$ respectively. If T_A and T_B are the time periods of A & B respectively then the value of $T_B - T_A$ $\left(Given\ R_e = 6400\,kg\right.$, Man of earth $= 6\times10^{24}\,kg$)

(25 – FEB – 2021(M))

1) $1.33\times10^3\,s$ 2) $3.33\times10^2\,s$ 3) $4.24\times10^3\,s$ 4) $4.24\times10^2\,s$

Key: 1

Sol: $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$T_A = 2\pi\sqrt{\frac{\left(6400+600\right)\times 10^3}{GM}}$

$T_A = 2\pi \times 10^9\sqrt{\frac{7^3}{GM}}$

$T_B = 2\pi \times 10^9\sqrt{\frac{8^3}{GM}}$

$T_B - T_A = \frac{2\pi 10^9}{\sqrt{GM}}\left[8\sqrt{8}-7\sqrt{7}\right]$

$= 314 \times 4.107$

$= 1289.64$

$= 1.289 \times 10^3\, s$

55. Two satellites A and B of masses $200\,kg$ and $400\,kg$ are revolving round the earth at height of $1600\,km$ and $2600\,km$ respectively. If T_A and T_B are the time periods of A & B respectively then the value of $T_B - T_A$ $\left(Given\ R_e = 6400\,kg\right.$, Man of earth $\left.= 6\times 10^{24}\,kg\right)$

(25 – FEB – 2021(M)) (D)

1) $1.37\times 10^3\, s$ 2) $3.33\times 10^3\, s$ 3) $4.27\times 10^3\, s$ 4) $4.24\times 10^3\, s$

Key: 1

Sol: $T = 2\pi\sqrt{\frac{r^3}{GMe}}$

$T_B - T_A = 2\pi\sqrt{\frac{r_B{}^3}{GMe}} - 2\pi\sqrt{\frac{r_A{}^3}{GMe}}$

$= \frac{2\pi}{\sqrt{GMe}}\left[\sqrt{r_B{}^3} - \sqrt{r_A{}^3}\right]$

$= 2\pi \times \frac{109}{20.005\times 10^6}\left[9\sqrt{9}-8\sqrt{8}\right]$

$= 1.37\times 10^3\, s$

56. Two satellites A and B of masses $200\,kg$ and $400\,kg$ are revolving round the earth at height of $600\,km$ and $2600\,km$ respectively. If T_A and T_B are the time periods of A & B respectively then the value of $T_B - T_A$ $\left(Given\ R_e = 6400\,kg\right.$, Man of earth $\left.= 6\times 10^{24}\,kg\right)$

(25 – FEB – 2021(M)) (D)

1) $1.33\times 10^3\, s$ 2) $2.66\times 10^3\, s$ 3) $4.28\times 10^3\, s$ 4) $4.23\times 10^3\, s$

Key: 2

Sol: $T = 2\pi\sqrt{\frac{r^3}{GMe}}$

$$T_B - T_A = 2\pi\sqrt{\frac{r_B^{\,3}}{GMe}} - 2\pi\sqrt{\frac{r_A^{\,3}}{GMe}}$$

$$= \frac{2\pi}{\sqrt{GMe}}\left[\sqrt{r_B^{\,3}} - r_A^{\,3}\right]$$

$$= \frac{2\pi \times 10^9}{\sqrt{6.67\times10^{-11}\times 6\times 10^{24}}}\left[9\sqrt{9} - 7\sqrt{7}\right]$$

$$= 313.92\left[27 - 18.52\right] = 313.92\left[8.48\right] \quad = 2662.04 \; = 2.66\times10^3\,s$$

k) Weightlessness in a satellite

57. An astronault inside an earth satellite experiences weightlessness because

a) No external force is acting on him

b) He is falling freely

c) No reaction is exerted by the floor of the satellite

d) He is far away from the earth's surface

1) a & b are correct 2) b & c are correct 3) c & a are correct 4) a & c are correct

Key: **2**

Sol: Conceptual

l) Angular momentum of a satellite

58. A planet of mass m moves along an ellipse around the sun so that its maximum and minimum distances from the sun are equal to r_1 and r_2 respectively. Find the angular momentum of this planet relative to the centre of the sun.

1) $m\sqrt{\left[\frac{2GMr_1r_2}{(r_1+r_2)}\right]}$ 2) $2m\sqrt{\left[\frac{2GMr_1r_2}{(r_1+r_2)}\right]}$ 3) $m\sqrt{\left[\frac{3GMr_1r_2}{(r_1+r_2)}\right]}$ 4) $m\sqrt{\left[\frac{GMr_1r_2}{2(r_1+r_2)}\right]}$

Key: 1

Sol: $L_1 = L_2$

$m_1v_1r_1 = m_2v_2r_2$

$$\frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{g(1-e)}{g(1+e)}$$

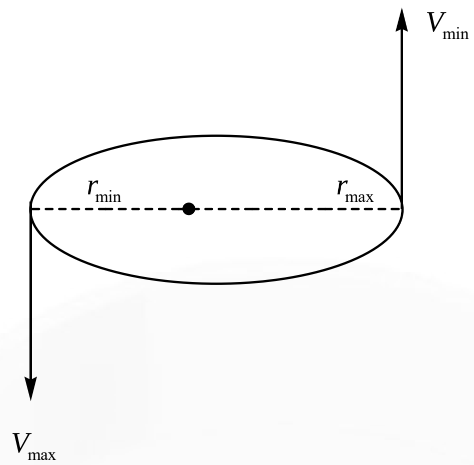
$L = mvr$

59. A satellite is in an elliptical orbit around a planet P it is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet, the ratio of distances between the satellite and the planet at closest and farthest points is **(6 – Sep – 2020 (M))**

1) 1:6 2) 1:3 3) 1:2 4) 3:4

Key: **1**

Sol: Applying, angular momentum $L_{closest} = L_{farthest}$



$mV_{max}r_{min} = mV_{min}r_{max}$ **or** $\frac{r_{min}}{r_{max}} = \frac{V_{min}}{V_{max}}$

Given that $V_{max} = 6V_{min}$ **or** $\frac{r_{min}}{r_{max}} = \frac{V_{min}}{6V_{min}} = \frac{1}{6}$

EXERCISE – II

(MULTI CORRECT OPTION QUESTIONS)

1. Gravitational force and laws

a) Universal law of gravitation

01. A double star is a system of 2 stars rotating about their centre of mass only under their centre of mass only under their mutual gravitational attraction. Let the stars have masses n and 2n and let their separate ion be ℓ. Their angular velocity of rotation about their centre of man will be proportional to **(24 – Feb – 2021(M)) (D)**

1) $\ell^{-\frac{3}{2}}$ 2) ℓ 3) $n^{-\frac{1}{2}}$ 4) $n^{\frac{1}{2}}$

Key: 1, 4

Sol: $m\left(\frac{2\ell}{3}\right)w^2 = G\frac{m \times 2m}{\ell^2}$

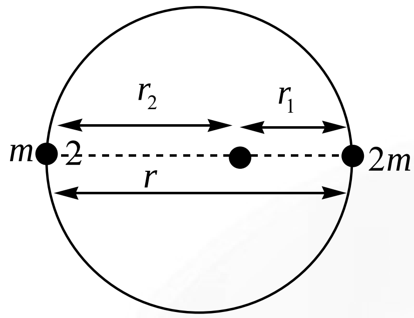
$w = \sqrt{G \times \frac{3m}{\ell^3}} \Rightarrow w \propto m^{\frac{1}{2}}, w \propto \ell^{\frac{-3}{2}}$

02. A Double stair is a system of two stars of masses m and $2m$,rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to **(27 – May – 2019) (D)**

1) $r^{\frac{3}{2}}$ 2) r 3) $m^{\frac{1}{2}}$ 4) $m^{-\frac{1}{2}}$

Key: 1, 4

Sol:



$$r_2 = \frac{2mr}{m+2m} = \frac{2r}{3}$$

$$T_2^2 = \frac{4\pi^2 r_2^2}{Gm} \Rightarrow T_2^2 = \frac{32\pi^2 r^3}{27Gm}$$

$$T_2 \ \alpha \ r^{\frac{3}{2}} \ and \ T_2 \ \alpha \ m^{\frac{1}{2}}$$

b) Principle of super position-Gravitational force

03. The minimum and maximum distance of a satellite from the centre of earth R and 2R respectively. Where R is the radius of the earth are M is the mass of earth its minimum speed is **(25 – Feb – 2021(M)) (D)**

1) $\sqrt{\frac{GM}{6R}}$ 2) $\sqrt{\frac{2}{3}\frac{GM}{R}}$ 3) $\sqrt{\frac{GM}{3R}}$ 4) $\sqrt{\frac{2GM}{2R}}$

Key: **2, 4**

Sol: $mV_1R_1 = mV_2R_2$

$$V_1R = V_2 2R$$

$$\Rightarrow \frac{V_1}{V_2} = 2 \qquad ...(1)$$

Total energy conserved

$$U_A + K_B = U_B + K_B$$

$$\frac{-Gmm}{R} + \frac{1}{2}mV_1^2 = \frac{-GMm}{2R} + \frac{1}{2}mV_2^2$$

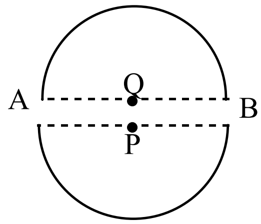
$$V_1^2 - V_2^2 = \frac{-GM}{2R} + \frac{GM}{R} = \frac{GM}{2R}$$

$$V_2^2\left(\frac{V_1^2}{V_2^2} - 1\right) = \frac{GM}{2R} \Rightarrow V_2^2 = \sqrt{\frac{GM}{6R}}$$

2. Gravitational field intensity

d) Field due to common shapes

08. A spherical shell of radius 'R' is cut along a chord AB and slightly displaced as shown in fig. Two points P and Q are such that with P in lower part and Q in upper part of the shell. Then **(26 – Feb – 2021(M)) (D)**



1) The gravitational field at P and Q are zero
2) The sum of gravitational fields at P and Q is zero
3) The gravitational potentials at P and Q are equal
4) The gravitational potentials at P and Q are unequal

Key: 2, 4

Sol: Inside shell $\overline{E}_g = 0$ if $\overline{E}_1$ and $\overline{E}_2$ are the gravitational field strength at P and Q, then $E_1 + E_2 = 0$

3. Acceleration due to gravity and its variation

a) Acceleration due to gravity on the surface of earth

06. Assuming the earth to be a sphere of uniform density acceleration due to gravity **(24 – Feb – 2021(M)) (D)**

1) At a point outside the earth is inversely proportional to the square of its distance from the centre
2) At a point outside the earth is inversely proportional to its distance from the centre
3) At a point inside is zero
4) At a point inside is proportional to its distance from the centre

Key: 1, 4

Sol: Out side earth $g = \frac{GM}{r^2}$, Inside earth $g = \frac{GM}{R^3} r$

c) Variation with height from surface of earth

05. Which of the folliwng statements are true about acceleration due to gravity? **(24 – Feb – 2021(M)) (D)**

1) g decrease in moving away from the centre if $r > R$
2) g decreases in moving away from the centre if $V < R$
3) g is zero at the centre of earth
4) g decrease if earth stops rotating on its axis

Key: 1, 3

Sol: $g_h = g\left(1 - \frac{2h}{R}\right)$ $\Rightarrow$ g decrease at altitudes

$g_d = g\left(1 - \frac{d}{R}\right)$ $\Rightarrow d = 0, gd = 0$

5. Motion of planets and satellites

b) Escape speed

02. The correct option regarding orbital and escape velocity is/are **(24 – Feb – 2021(M)) (D)**

1) Orbital velocity $V \propto \frac{1}{\sqrt{r}}$ for different orbit

2) Orbital velocity $V \propto \frac{1}{\sqrt{r}}$ holds good for different points on the same elliptical orbit

3) Escape velocity depend on acceleration due to gravity

4) Orbital velocity does not depend on the mass orbiting object

Key: 1, 3, 4

Sol: Orbital velocity $=\sqrt{\frac{GM}{r}}$

Escape velocity $=\sqrt{2gR}$

07. The spherical planets have the same mass but densities in the ratio 1:8. For those planets the **(25 – Feb – 2021(E)) (D)**

1) acceleration due to gravity will be in the ratio $1:4$

2) acceleration due to gravity will be in the ratio $4:1$

3) Escape velocities from their substances will be in the ratio $1:\sqrt{2}$

4) Escape velocities from their substances will be in the ratio $\sqrt{2}:1$

Key: 1, 3

Sol: $M=\frac{4}{3}\pi R \Rightarrow R=\left(\frac{3M}{4\pi r}\right)^{\frac{1}{3}}$

i) $g \propto \rho^{\frac{2}{3}}$ $\qquad \frac{g_1}{g_2}=\frac{1}{4}$

ii) $V_e=\sqrt{\frac{2GM}{R}} \Rightarrow V_e \propto (\rho)^{\frac{1}{6}}$

$\frac{V_1}{V_2}=\frac{1}{\sqrt{2}}$

c) Orbital speed of a satellite in circular orbit

04. For a satellite with increase in height of the orbit from the surface of the planet. Then its **(25 – Feb – 2021(M)) (D)**

1) P.E increase 2) K.E increase 3) orbital velocity decrease

4) Period of revolution decrease

Key: 1, 3

Sol: $P.E=-\frac{GMems}{(\mathrm{Re}+h)}, K.E=\frac{GMems}{2(\mathrm{Re}+h)}, V_0=\sqrt{\frac{GM}{R+h}}$

09. Maximum and minimum distance of a comet from sun are $0.2\times10^{12}\,m$ and $1\times10^{10}\,m$, if the speed of comet at nearest point is ____speed at farthest point is ____ **(16 – March – 2021 (M)) (D)**

1) $5000\,m/s; 10{,}000\,m/s$ 2) $1000\,m/s; 20{,}000\,m/s$

3) $8000\,m/s; 20,000\,m/s$ 4) $2000\,m/s; 40,000\,m/s$

Key: 2, 3

Sol: $mV_1r_1 = mV_2r_2 \Rightarrow \frac{V_1}{V_2} = \frac{r_2}{r_1}$

$$\frac{V_1}{V_2} = \frac{10^{10}}{0.2\times10^{12}} = \frac{1}{20}$$

10. A geostationary satellite is orbiting around an arbitrary point P at a height 17R above surface of P, R is radius of P, time period of another satellite in hours at a height of ___ is

(17 – March – 2021 (E)) (D)

1) $h_1 = R \;\; T_1 = \frac{8}{9}$ 2) $h_2 = 3R \;\; T_2 = \sqrt{\frac{8}{3}}$ 3) $h_3 = 4R \;\; T_3 = \sqrt{\frac{18}{3}}$ 4) $h_4 = 5R \;\; T_4 = \sqrt{\frac{24}{7}}$

Key: 1, 2

Sol: $T \propto R^{3/2}$

$$\frac{24}{T_1} \propto \left[\frac{17R+R}{R+R}\right]^{3/2} \Rightarrow T_1 = \frac{8}{9}\,hrs \quad h_1 = R$$

$$\frac{24}{T_2} \propto \left[\frac{17R+R}{2R+R}\right]^{3/2} \Rightarrow T_2 = \sqrt{\frac{8}{3}}\,hrs \quad h_2 = 3R$$

11. Time period of a satellite in a circular orbit of radius R is T, the period of another satellite in a circular orbit of radius ____is____ **(18 – March – 2021 (M)) (D)**

1) 16R, 64T 2) 64R, 512T 3) 25R, 625T 4) 36R, 316T

Key: 1, 2

Sol: $\left(\frac{T}{T_1}\right)^2 = \left(\frac{R}{16R}\right)^3 \Rightarrow T_1 = 16^{3/2}T \Rightarrow T_1 = 64T$

$$\left(\frac{T}{T_2}\right)^2 = \left(\frac{R}{64R}\right)^3 \Rightarrow T_2 = (64)^{3/2}T \Rightarrow T_2 = 512T$$

12. If 2 satellites of different masses are revolving in the same orbit they have the same

(18 – March – 2021 (E)) (D)

1) Angular momentum 2) Energy 3) Time period 4) Speed

Key: 3, 4

Sol: If 2 satellites of different masses are revolving in the same orbit. Then they have the same time period and speed because

$$V_e = \sqrt{\frac{GM}{r}}$$

$$T = \frac{2\pi}{w} = \frac{2\pi}{\frac{V}{r}} = \frac{2\pi r}{V} = 2\pi\sqrt{\frac{r^3}{GM}}$$

angular momentum $= mVr$

energy $= \frac{GMm}{2r}$

both depend on the mass of the satellites

$\therefore$ 3 and 4 are correct

Potential energy of a system of particles

13. A small mass m is moved slowly from surface of earth to a height h above surface, work done by an external agent in doing this is **(16 – March – 2021 (E)) (D)**

1) mgh for all values of h
2) mgh for $h << R$
3) $\frac{mgR}{2}$ for $h = R$
4) $-\frac{mgR}{2}$ for $h = R$

Key: 2, 3

Sol: Work done $= PE_2 - PE_1 = -\frac{GMm}{R+h} - \frac{(-GMm)}{R}$

$= -GMm\left[\frac{1}{R+h} - \frac{1}{R}\right]$

$work = gR^2 m\left[\frac{h}{R(R+h)}\right] = \frac{mgRh}{R+h}$

$for\ h << R \qquad w = mgh$

$for\ h = R \qquad w = \frac{mgR}{2}$

14. Which of the following are not correct? **(17 – March – 2021 (M)) (D)**

1) The escape velocity for the moon is $6\,km/s$

2) Escape velocity from the surface of moon is V, the orbital velocity for a satellite to orbit very close to surface of moon is $\frac{V}{2}$

3) When an earth satellite is moved from one stable orbit to a further stable orbit, the gravitational potential energy increases

4) The orbital velocity of a satellite revolving in a circular orbit close to the planet is independent of density of planet

Key: 1, 4

Sol: 1) For moon escape velocity $= 2.4\,km/s$

4) $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{M} \times \frac{4}{3}\pi R^3 \rho} \quad \Rightarrow V_e \propto \sqrt{\rho}$

$here\ \ \rho = density\ \ of\ \ \ planet$

15. An astronaut inside an earth satellite, experience weight lessness because, **(18 – March – 2021 (E)) (D)**

1) No external force acting on him
2) He is falling freely
3) No reaction force is exerted by the floor of the satellite
4) He is far away from earths surface

Key: 2, 3

Sol: Centripetal force = Centripetal reaction force act away from center of earth

No net force on person.

EXERCISE – III

(NUMERICAL/INTEGER TYPE – INCLUDING PREVIOUS YEAR QUESTIONS)

1. Gravitational force and laws

a) Universal law of gravitation

20. Two stars of masses m and m at a distance d rotate about their common centre of mass in free space. The time period of revolution is $2\pi\sqrt{\frac{d^3}{xGm}}$ where $'x'$ is

(24 – FEB – 2021(M)) (D)

Key: 2

Sol: $F = \frac{Gm^2}{d^2} = m\omega^2 \frac{d}{2}$

$\omega = \sqrt{\frac{2Gm}{d^3}}$, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{2Gm}}$

04. A particle of mass m is subjected to an attractive central force of magnitude $\frac{k}{r^2}$, k being a constant. If at the instant when the particle is at an extreme position in its closed orbit, at a distance a from the centre of force, its speed is $\frac{k}{2ma}$, if the distance of other extreme position is b . Find $\frac{a}{b}$? **(27 – May – 2019) (D)**

Key: 3

Sol: $F = -\frac{K}{r^2}$ (negative sign is for attractive force)

Potential energy $U = -\int F dr = \int \frac{K}{r^2} dr = -\frac{K}{r}$

Conversation of energy gives (let at other extreme position $r = b$)

$K_1 + U_1 = K_2 + U_2$

$\frac{1}{2}mv_1^2 - \frac{K}{a} = \frac{1}{2}mv_2^2 - \frac{K}{b}$ ----1

Where $v_1 = \sqrt{\frac{K}{2ma}}$

Conservation of angular momentum gives

$mv_1 a = mv_2 b$

$v_2 = \frac{a}{b}\sqrt{\frac{K}{2ma}}$

Therefore equation 1 we get

$\Rightarrow \frac{1}{2}m\frac{K}{2ma} - \frac{K}{a} = \frac{1}{2}m\left(\frac{a}{b}\right)^2 \frac{K}{2ma} - \frac{K}{b}$

$-\frac{3K}{4a} = \frac{aK}{4b^2} - \frac{K}{b} \Rightarrow b^2 - \frac{4a}{3}b + \frac{a^3}{3} = 0$

Hence $b = \frac{a}{3} \Rightarrow \frac{a}{b} = 3$

c) Kepler's laws

22. Two planets orbit the sun in circular orbits, with their radius of orbit as $R_1 = R$ and $R_2 = 2R$, ratio of their periods $\left(\frac{T_1}{T_2}\right)$ around the sun will be $x:\gamma$, where x is

(26 – FEB – 2021(M)) (D)

Key: 1

Sol: $T^2 \propto r^3$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r}{2r}\right)^3$$

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{1}{8}$$

$$\frac{T_1}{T_2} = \frac{1}{\sqrt{8}}$$

Where x is 1

2. Gravitational field intensity

a) Gravitation field strength

07. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be xF. Then the value of 'x' is________.

(08 – Jan – 2020 (M)) (D)

Key: 1

Sol: According to Newton's law of gravitation = the force between two spheres is given by $F = \frac{GMm}{r^2}$

From the relation, we can say the gravitational force does not depend on the medium between two spheres hence, it remains same i.e. F

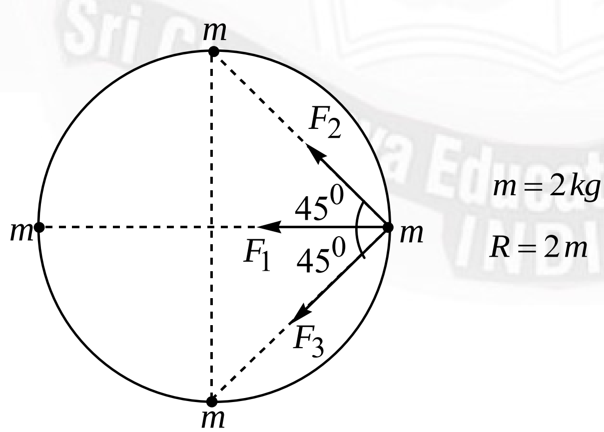
Comparing we get x = 1.

b) Gravitational field due to a point mass

21. Four identical particles of equal masses $2kg$ made to move along the circumference of a circle of radius $2m$ under the action of their own mutual gravitational attraction. The speed of each particle will be $\sqrt{\frac{G}{N}\left(1+2\sqrt{2}\right)}$ where $N = ?$: **(24 – FEB – 2021(M)) (D)**

Key: **4**

Sol:



$$F_1 = \frac{Gm^2}{4R^2}$$

$$F_2 = F_3 = \frac{Gm^2}{2R^2}$$

$$\Rightarrow F_{net} = F_1 + F_2\cos 45^0 + F_3\cos 45^0$$

$$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}$$

$$F_{net} = \frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right)$$

$$F_{net} = \frac{Gm^2}{4R^2}\left(1+2\sqrt{2}\right) = \frac{mv^2}{R}$$

$$\Rightarrow v = \sqrt{\frac{Gm\left(1+2\sqrt{2}\right)}{4R}}$$

m = 1 kg, R = 2 m

$$v = \sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$$

31. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is $\frac{x}{24}$ where x is **(25 – Feb – 2021(M))**

Key: $x = 25$

Sol: gravitational Intensity $Eg_1^2 = \frac{GM}{4R^2}$ (before cavity)

after cavity

$$Eg_2^1 = \frac{GM}{4R^2} - \frac{\frac{GM}{64}}{\frac{25R^2}{16}} \Rightarrow \frac{GM}{4R^2} - \frac{GM}{100R^2}$$

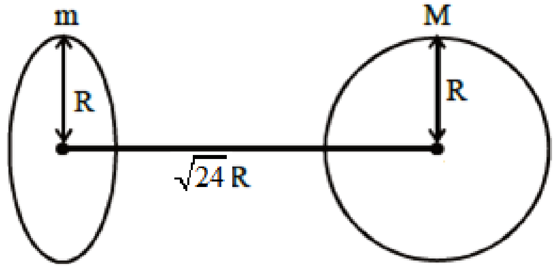
$$\Rightarrow \frac{25GM - GM}{100R^2} = \frac{24GM}{100R^2}$$

$$F_1 : F_2 = mg_1 : mg_2 = Eg_1 : Eg_2 = \frac{\frac{GM}{4R^2}}{\frac{24GM}{100R^2}} = \frac{25}{24}$$

d) Field due to common shapes

32. The gravitational force of attraction between the ring and sphere is as shown in the figure is $\frac{GMm}{125R^2}2\sqrt{x}$, where the plane of the ring is perpendicular to the line joining the centres. If

$\sqrt{24}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R', where the value of 'x' is **(26 – Feb – 2021(M)) (D)**



Key: **6**

Sol: Gravitational field of the ring $=\dfrac{Gmx}{\left(R^2+x^2\right)^{3/2}}$ $\left(x=\sqrt{24}R\right)$

Force between ring and sphere

$$=\frac{GMm\left(\sqrt{24}R\right)}{\left(R^2+24R^2\right)^{3/2}}$$

$$=\frac{GMm\left(\sqrt{24}R\right)}{(5R)^3}$$

$$=\frac{GMm\sqrt{24}}{125R^2}$$

3. Acceleration due to gravity and its variation

a) Acceleration due to gravity on the surface of earth

28. A body weighs 49 N on a spring balance at the north pole its weight recorded on the same weighty machine it is shifted to the equator is $\dfrac{97.66}{N}N$ where $N=?$

(24 – Feb – 2021(E)) (D)

Key: **2**

Sol: At poles 49N

$$mass=\frac{49}{g}=\frac{49}{9.8}=5\,kg$$

at equator $mg'=mg-mRw^2 \Rightarrow 49-\left(5\times7.27\times10^{-3}\right)^2\times64\times10^5$

$\Rightarrow 48.83$

$$\Rightarrow\frac{97.66}{2}=48.83\,N$$

$N=2$

b) Variation due to shape of the earth

06. A box weighs $49 \times N$ on a spring balance at the north pole. Its wight observed on the same balance if it is shifted to the equator is close to 195 N ($g = 10ms^{-2}$ at the north pole and the radius of the earth = 6400 km). The value of 'x' is__________.

(07 – Jan – 2020 (E)) (D)

Key: 4

Sol: Weight at pole, w = mg = 49xN $\Rightarrow$ $m = \dfrac{49 \times x}{g}$ kg

weight at equator, w' = mg' = $m\left(g - \omega^2 R\right)$

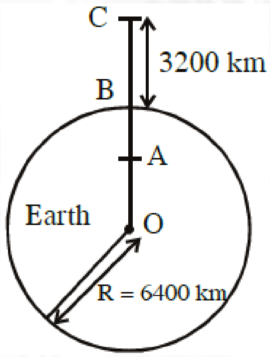
$$w' = \frac{49 \times x}{g}\left(10 - \left(\frac{2\pi}{24 \times 3600}\right)^2 6400 \times 10^3\right) N \Rightarrow 195N$$

solving we get x = 4

c) Variation with height from surface of earth

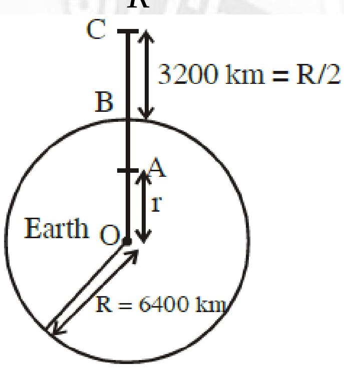
29. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of $OA : AB$ will be $X : Y$. The value of X is …….

(26 – Feb – 2021(E))



Key: **4**

Sol: $g_A = \dfrac{GM\left(r\right)}{R^3}$



$$g_C = \frac{GM}{\left(R + \frac{R}{2}\right)^2}$$

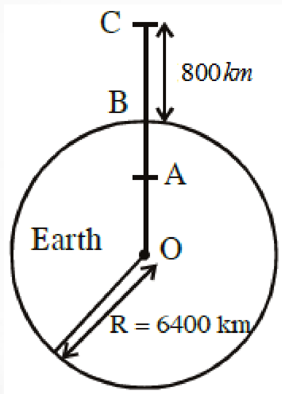
$g_A = g_C$

$\frac{r}{R^3} = \frac{1}{\frac{9}{4}R^2} \quad \Rightarrow r = \frac{4R}{9}$

So $OA = \frac{4R}{9}$; $AB = R - r = \frac{5R}{9}$

$OA : AB = \frac{4R}{9}; AB = R - r = \frac{5R}{9}$

30. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of $OA : AB$ will be $X : Y$. The value of X is ……. **(26 – Feb – 2021(E)) (D)**



Key: 64

Sol: $g_A = \frac{GM(r)}{R^3}$

$g_C = \frac{GM}{\left(R + \frac{R}{8}\right)^2} = \frac{GM}{\frac{81R^2}{64}}$

$g_A = g_C$

$\frac{r}{R} = \frac{64}{81}$

$r = \frac{64R}{81} = OA$

$AB = R - r = OR - \frac{64R}{81}$

$= \frac{17R}{81}$

$\frac{OA}{AB} = \frac{\frac{64R}{81}}{\frac{17R}{81}} = \frac{64}{17}$

4. Gravitational potential and potential energy

a) Gravitational potential energy

08. An asteroid is moving directly towards the center of the earth. When at a distance of 10R (R is the radius of the earth) from the earth's center, it has a speed of 12 kms^{-1}. Neglecting the effect of earth's atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity form the earth is 11.2 kms^{-1})? Give your answer to the nearest integer in kms^{-1} is _________. **(08 – Jan – 2020 (E))**

Key: 16.00

Sol: Using law of conservation of energy

Total energy at height 10R = Total energy at earth surface

$$\frac{GM_e m}{10R}+\frac{1}{2}mv_o^2=-\frac{GM_e m}{R}+\frac{1}{2}mv^2$$

($\because$ Gravitational potential energy = $-\frac{GMm}{R}$)

$$\Rightarrow \frac{GM_e}{R}\left(1-\frac{1}{10}\right)+\frac{v_o^2}{2}=\frac{v^2}{2}\Rightarrow v^2=v_o^2+\frac{9}{5}gR$$

$$v=\sqrt{v_o^2+\frac{9}{5}gR}\Rightarrow v=16kms^{-1}$$

5. Motion of planets and satellites

b) Escape speed

10. A particle is projected vertically upwards from the surface of the earth (radius R_e) witha kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of the earth is xR. The value of x is? **(09 – Jan – 2020 (E))**

Key: 2

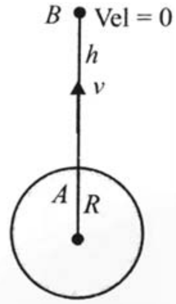
Sol: Applying energy conservation,
Total energy at A = Total energy at B

$$\frac{1}{2}mv^2=\left(-\frac{GMm}{R}\right)=\left(-\frac{GMm}{R+h}\right) \qquad \text{---------1}$$

Escape velocity $v_e=\sqrt{\frac{2GM}{R}}$

$\therefore$ kinetic energy required for escape velocity is

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$



The kinetic energy gives half of this value. Therefore,

$$\frac{1}{2}mv_e^2 = \frac{GMm}{2R} \qquad \text{--------2}$$

From equations 1 and 2

$$\frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\Rightarrow -\frac{1}{2R} = -\frac{1}{R+h} \Rightarrow R+h = 2R$$

25. Two planets A & B, there masses in the ratio $1:1$, radius $4:1$, then their escape velocity ratio is $X:Y$ value of Y is ___ **(25 – Feb – 2021(M)) (D)**

Key: 2

Sol: $\frac{V_A}{V_B} = \sqrt{\frac{M_A}{M_B}}\sqrt{\frac{R_B}{R_A}}$

$= \sqrt{\frac{1}{1}}\sqrt{\frac{1}{4}} = \frac{1}{2}$

Y is 2

26. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value x of will be ____.

(25 – Feb – 2021(E))

Key: 10

Sol: $\frac{-GMm}{11R} = \frac{-GMm}{R} + \frac{1}{2}mv^2$

$v = \sqrt{\frac{20GM}{11R}}$

27. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 8R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value x of will be ____.

(25 – Feb – 2021(E)) (D)

Key: 8

Sol: According to L.C.E

$$-\frac{GMm}{9R}=-\frac{GMm}{R}+\frac{1}{2}mV^2$$

$$\frac{1}{2}V^2=-\frac{GM}{9R}+\frac{GM}{R}$$

$$V^2=\frac{8GM}{9R}$$

$$V=\sqrt{\frac{8GM}{9R}}=\sqrt{\frac{8}{9}}\times V_C$$

g) Energy of a satellite in circular orbit

05. A satellite of mass 'm' is launched vertically upwards with an initial speed 'u' from the surface of earth, it ejects a rocket of mass $\frac{m}{2x}$ so that subsequently the satellite moves in a circular orbit. If the kinetic energy of the rocket is $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$ (G – universal gravitational constant and M – mass of the earth) then the value of 'x' is________

(07 – Jan – 2020 (M)) (D)

Key: 5

Sol: $T.E_{surface}=T.E_R$

$$\frac{1}{2}mu^2+\left(-\frac{GMm}{R}\right)=\frac{1}{2}mv^2+\left(-\frac{GMm}{2R}\right)$$

$$v^2=u^2+\left(-\frac{GM}{R}\right)\Rightarrow v=\sqrt{u^2+\left(-\frac{GM}{R}\right)}$$

Ejects a rocket of mass $\frac{m}{2x}$

The rocket splits at height R. since, separation of rocket is impulsive therefore conservation of momentum in both radial and tangential direction can be applied.

$$\frac{m}{2x}v_T=\frac{(2x-1)}{2x}\sqrt{\frac{GM}{2R}}\Rightarrow v_T=(2x-1)\sqrt{\frac{GM}{2R}}$$

$$\frac{m}{2x}v_r=m\sqrt{u^2-\frac{GM}{R}}\Rightarrow v_r=2x\sqrt{u^2-\frac{GM}{R}}$$

Kinetic energy of rocket $=\frac{1}{2}\times\frac{m}{2x}\left(v_T^2+v_R^2\right)$

Kinetic energy of rocket $=\frac{m}{4x}\left((2x-1)^2\frac{GM}{2R}+4x^2\left(u^2-\frac{GM}{R}\right)\right)$

Equating with K.E. of rocket= $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$

And solving we get x = 5.

i) Time period of satellite

23. Consider two satellites S_1 and S_2 with period of revolution $2\,hr$ and $3\,hr$. Respectively ratios of angular velocity of satellite S_1 to angular velocity of satellite S_2 is 3: n, where n = ?

Key: 2

Sol: $\frac{T_1}{T_2}=\frac{2}{3}$

$\Rightarrow \frac{T_1}{T_2}=\frac{2\pi/\omega_1}{2\pi/\omega_2}$

$\Rightarrow \frac{\omega_2}{\omega_1}=\frac{2}{3}$

$\Rightarrow \frac{\omega_1}{\omega_2}=\frac{3}{2}$

24. Two satellites A and B of masses $300\,kg$ and $400\,kg$ are revolving round the earth at height of $600\,km$ and $2600\,km$ respectively. If T_A and T_B are the time periods of A & B respectively then the value of T_B-T_A is $1.3\times10^{x}\,s$, where x is

(25 – Feb – 2021(M)) (D)

Key: 3

Sol: $T=2\pi\sqrt{\frac{r^3}{GMe}}$

$T_B-T_A=2\pi\sqrt{\frac{r_B{}^3}{GMe}}-2\pi\sqrt{\frac{r_A{}^3}{GMe}}$

$=\frac{2\pi}{\sqrt{GMe}}\left[\sqrt{r_B{}^3}-\sqrt{r_A{}^3}\right]$

$=\frac{2\pi\times10^9}{\sqrt{6.67\times10^{-12}\times6\times10^{24}}}\left[8\sqrt{8}-7\sqrt{7}\right]$

$=314\times4.107$

$=1.3\times10^3\,s$

l) Angular momentum of a satellite

09. The ratio of the earth's orbital angular momentum (about the Sun) to its mass is $4.4\times10^{15}\,\mathrm{m^2s^{-1}}$. The area enclosed by the earth's orbit is approximately ____ $\times10^{22}\,\mathrm{m^2}$.

(09 – Jan – 2020 (M)) (D)

Key: (6.94)

Sol: Areal velocity of a planet around the Sun is constant and is given by

$\frac{dA}{dt}=\frac{L}{2m}\Rightarrow dA=\frac{L}{2m}dt$

Integrating both sides $\int dA=\frac{L}{2m}\int dt\Rightarrow A=\frac{L}{2m}T$

L is angular momentum of planet (earth)

m is mass of plane (earth)

$$A = \frac{1}{2} \times 4.4 \times 10^{15} \times 365 \times 24 \times 3600 \Rightarrow \text{Area} = 6.94 \times 10^{22}\,\text{m}^2$$

01. The angular momentum of a planet of mass M, moving around the sun in a circular orbit is J, about the center of sun. If its areal velocity is $\frac{J}{xM}$ then value of x is

(09 – Jan – 2019 (M)) (D)

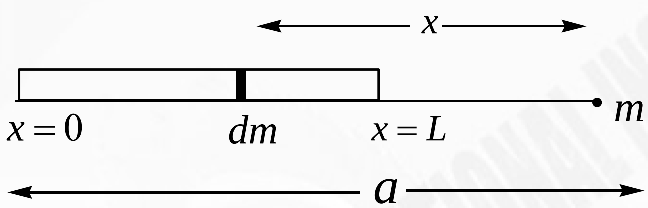
Key: **2**

Sol:

02. A straight rod of length L extend from $x = 0$ to $x = L$. The mass per unit length of the rod is $kx(k \text{ is constant})$. The gravitational force it exerts on a point mass "m" kept at $x = a\ (a > L)$ is **(11 – Jan – 2019 (M)) (D)**

Key:

Sol:



$$dF = \frac{G(dm)m}{x^2}$$

$$= \frac{G(kx)dx\,m}{x^2}$$

$$dF = \frac{G\,km}{x}dx$$

$$F = G\,km \int_{a}^{a-L} \frac{1}{x}dx$$

$$= G\,km(In\ x)_{a}^{a-L}$$

$$= G\,km(In\,(a-L) - In\,a)$$

$$F = G\ km\ In\left(\frac{a-L}{a}\right)$$

03. E_1 is $k \cdot E$ of a satellite in a circular orbit at height h above earth surface and E_2 is energy required to take a satellite to a height "h" above earth surface. If $2E_1 = E_2$ then value of h is $(R = radius\ of\ earth = 6400\ km)$ **(09 – Jan – 2019 (E)) (D)**

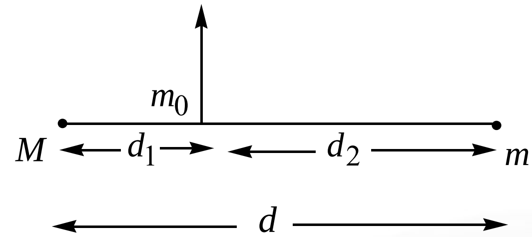
Key: $6400\ km$

Sol:

04. Two stars of masses $M\ km$ and at a distance $'d'$ rotate in a plane about their common center of mass $'O'$. A particle passes through $'O'$ moving perpendicular to star's rotation plane. In order to escape from gravitational field of this binary star, the minimum speed that particle should have at $'O'$ is (G – universal gravitational constant) **(10 – Jan – 2019 (E)) (D)**

Key:

Sol: From law of conservation of energy between 0 & ∞



$$-\frac{GMm_0}{d_1}-\frac{Gmm_0}{d_2}+\frac{1}{2}m_0v^2=0$$

$$\frac{1}{2}v^2=G\left(\frac{M}{d_1}+\frac{m}{d_2}\right)$$

$$d_1=\frac{md}{M+m},\ d_2=\frac{Md}{M+m}$$

$$\frac{1}{2}v^2=G\left(\frac{M(M+m)}{md}+\frac{m(M+m)}{Md}\right)$$

$$\frac{1}{2}v^2=\frac{G(M+m)}{d}\left[\frac{M}{m}+\frac{m}{M}\right]$$

$$=\frac{G(M+m)}{d}\left[\frac{M^2+m^2}{mM}\right]$$

$$v^2=\frac{2G(M+m)\left(M^2+m^2\right)}{mMd}$$

$$v=\sqrt{\frac{2G(M+m)\left(M^2+m^2\right)}{mMd}}$$

11. If $'g'$ is acceleration due to gravity on the surface of earth, having radius R_1, the height at which the acceleration due to gravity reduces to $\frac{g}{2}$ is $\left(\sqrt{x}-1\right)R$ then value of x is ___

(2 – Sep – 2020 (E))

Key: 2

Sol: We know on surface of earth $g=\frac{GM}{R^2}$...1

At height h, $g^1=\frac{GM}{(R+h)^2}$

Given $\left(g^1=\frac{g}{2}\right)$

$$\frac{g}{2}=\frac{GM}{(R+h)^2}...2$$

Eq 1 subin 2

$$\frac{GM}{2R^2}=\frac{GM}{(R+h)^2}$$
$$(R+h)^2=2R^2$$
$$R+h=\sqrt{2}R$$
$$h=\left(\sqrt{2}-1\right)R$$

12. If R is the radius of earth, the height at which the weight of a body becomes 1/4th its weight on the surface of the earth is NR, then value of N **(5 – Sep – 2020 (M))**

Key: 1

Sol: Given $W_h=\frac{1}{4}W_{(Surface)}$

$$mg_h=\frac{1}{4}mg$$
$$g\left[\frac{R}{R+h}\right]^2=\frac{1}{4}g$$
$$\frac{R}{R+h}=\frac{1}{2}$$
$$2R=R+h$$
$$h=R=(1)R$$

So the value of N is 1

13. If 'g' on the surface of the earth is $10\,ms^{-2}$ find its value at a depth of 3200 km is__ (radius of the earth=6400 km) **(5 – Sep – 2020 (E))**

Key: 5

Sol: Given $d=3200\,km$

$$g_d=g\left[1-\frac{d}{R}\right]$$
$$=10\left[1-\frac{3200}{6400}\right]=10\left[1-\frac{1}{2}\right]=\frac{10}{2}=5$$

14. The gravitational force between two objects were proportional to $\frac{1}{R}$ and not as $\frac{1}{R^2}$ where R is separation between them, then particle in circular orbit under such a force will have speed proportional to R^x then value of x is _____ **(2 – Sep – 2020 (M))**

Key: 0

Sol: given $f\,\alpha\,\frac{1}{R}$

$$f=\frac{K}{R}...1$$
$$F_c=\frac{mv^2}{R}...2$$
$$\frac{mv^2}{R}=\frac{K}{R}$$
$$v^2=\frac{K}{m}$$

$v=\sqrt{\frac{K}{m}}$ Which is independent of R

$\therefore v \alpha R^0$

15. A body is released from height 5R where R is radius of the earth then that body reaches the ground with a velocity equal to $\sqrt{\frac{xgR}{3}}$ then value of x is _____('g' is acceleration due to gravity on the surface) **(3 – Sep – 2020 (M))**

Key: 5

Sol: From conservation of energy

$(PE+KE)$ On surface of the earth $= PE$ at height $5R$

$$\frac{-GMm}{R}+\frac{1}{2}mv^2=\frac{-GMm}{(5+5R)}$$

$$\frac{1}{2}mv^2=\frac{GMm}{R}-\frac{GMm}{6R}$$

$$\frac{1}{2}mv^2=\frac{GMm}{R}\left(1-\frac{1}{6}\right)$$

$$v^2=\frac{5GM}{3R} \quad \because g=\frac{GM}{R^2}; GM=gR^2$$

$$v^2=\frac{5gR^2}{3R}$$

$$v=\sqrt{\frac{5gR}{3}}$$

16. The escape velocity from the surface of a planet of mass $5\times10^{24}\,kg$ and mean radius 6670 km is $10^x\,ms^{-1}$ $\left(take\, G=6.67\times10^{-11}\,Nm^2kg^{-2}\right)$ then the value of x is **(4 – Sep – 2020 (E))**

Key: 4

Sol: $V_e=\sqrt{\frac{2\times6.67\times10^{-11}\times5\times10^{24}}{6670\times10^3}}=10^4\,ms^{-1}$

So the value of x is 4

17. A satellite of mass $\frac{M}{64}$ is revolving round the planet of mass M in a circular orbit of radius R, then its angular momentum is $\frac{\sqrt{GM^3R}}{N}$ then value of $'N'$ is _______

(6 – Sep – 2020 (M))

Key: 8

Sol: Angular momentum $L=\sqrt{GMm^2r}$

Here $m=\frac{M}{64}$ and $r=R$

$$L=\sqrt{GM\left(\frac{M^2}{64}\right)R}=\sqrt{\frac{GM^3R}{64}}=\sqrt{\frac{GM^3R}{64}}$$

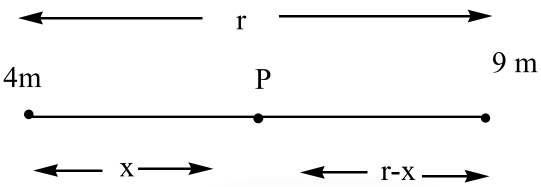
$$L=\frac{\sqrt{GM^3R}}{8}$$

Then the value of $'N'=8$

18. Two bodies of masses $4m$ and $9m$ are separated by a distance r. The gravitational potential at a point on this line joining them where the gravitational field becomes zero is $\frac{-NGm}{r}$ then value of N is ___ **(3 – Sep – 2020 (E))**

Key: 25

Sol: Let the point be at x distance from $4m$ and $(r-x)$ distance from $(9m)$



$$\frac{4Gm}{x^2}=\frac{G(9m)}{(r-x)^2}$$

$$x=\frac{2r}{5}$$

Potential $V=\dfrac{-4Gm}{\left(\dfrac{2r}{5}\right)}-\dfrac{G(9m)}{r-\left(\dfrac{2r}{5}\right)}$

$$V=-25G\frac{m}{r}$$

Then value of N is 25

19. The gravitational field due to the two bodies of masses 900 kg and 1600 kg placed at distance 10 m is zero. The corresponding distance of the point from the larger mass is $\frac{N}{7}m$ then value of N is __ **(4 – Sep – 2020 (M))**

Key: 40

Sol: $\dfrac{Gm_1}{(d-x)^2}=\dfrac{Gm_2}{x^2}$

$$\frac{G(900)}{(10-x)^2}=\frac{G(1600)}{x^2}$$

$$\frac{9}{(10-x)^2}=\frac{16}{x^2}$$

$$\left(\frac{x}{10-x}\right)^2=\frac{16}{9}$$

$$\frac{x}{10-x}=\frac{4}{3};\ 3x=40-4x;\ 7x=40;\ x=\frac{40}{7}$$

Then the value of N is 40

2. The distance between two stars of masses $3M_S$ and $6M_S$ is $9R$. Here R is the mean distance between the centre of the earth and the sun, and M_S is the mass of the sun. The two stars orbit around their common centre of mass in circular orbits with period nT, where T is the period of earths revolution around the sun. The value of $'n'$ is **(24 – Feb – 2021(M)) (D)**

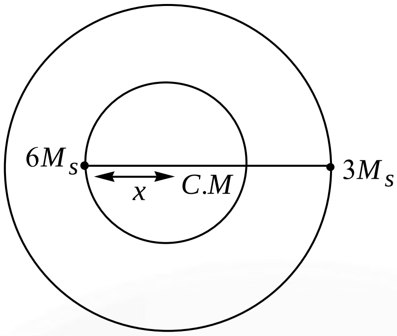
Key: 1

Sol: Both will revolve about common centre of mass.

$$x=\frac{3M_S}{6M_S+3M_S}\times 9R=3R\quad ------(1)$$

$$\frac{6M_S\times 3M_S}{(9R)^2}=6M_S\left(w^2x\right)\quad ------(2)$$

Solving equation (1) and (2) we get



$$w^2 = \frac{GM_s}{81R^3}$$

$$T^{12} = \frac{4\pi^2 R^3}{GM_s} \times 81 \quad -----(3)$$

For the motion of earth around the sun

$$T^2 = \frac{4\pi^2 R^3}{GM_s} \quad -----(4)$$

From (3) and (4)

$$T^{12} = 81T^2$$

$$T^1 = 9T$$

33. Maximum and minimum distance of a comet from sun are $0.4 \times 10^{12}\, m$ and $2 \times 10^{10}\, m$, if the speed of comet at nearest point is $1.5 \times 10^4\, met/\sec$, speed at farther point is $x \times 0.015 m/s$ find x **(16 – March – 2021 (M)) (D)**

Key: 3

Sol: $mV_1r_1 = mV_2r_2 \quad \Rightarrow V_1 = \frac{V_2r_2}{r_1} \quad \Rightarrow V_1 = \frac{1.5 \times 10^4 \times 2 \times 10^{10}}{0.4 \times 10^{12}} = 0.075$

$0.075 = x \times 0.015$

$3 = x$

34. A geostationary satellite is orbiting around an arbitrary point P at a height 23R above surface of P, time period of another satellite in hours at a height of 2R from surface of P is $\frac{x}{\sqrt{8}}$ find x? **(17 – March – 2021 (E)) (D)**

Key: 3

Sol: $T \propto R^{3/2}$

$$\frac{24}{T} \propto \left[\frac{23R + R}{2R + R}\right]^{3/2}$$

$$\frac{24}{T} \propto \left[\frac{24R}{2R + R}\right]^{3/2}$$

$$\frac{24}{T} \propto 8^{3/2}$$

$$\frac{24}{8\sqrt{8}} = T$$

$$\frac{24}{8\sqrt{8}} = \frac{x}{\sqrt{8}} \Rightarrow x = 3$$

35. Time period of a satellite in a circular orbit of radius R is T, the period of another satellite in a circular orbit of radius 25R is $x \times 67.5T$ find x **(18 – March – 2021 (M)) (D)**

Key: 2

Sol: $\left(\frac{T'}{T}\right)^2 = \left(\frac{25R}{R}\right)^3 \Rightarrow T' = 125T$

$125T = x \times 67.5T$

$2 = x$

36. If the satellite is travelling in the same direction as the time rotation of earth, west to east, the interval between successive times at which it will appear vertically over head to an observer at a fixed point on the equator, if the time period of satellite $= 683 \sec ond$ is $x \times 3700 \sec ond$ find x **(18 – March – 2021 (E)) (D)**

Key: 2

Sol: Relative angular velocity $= w_{satellite} - w_{earth} = w_s - w_e$

Relative time period $= \frac{2\pi}{w_s - w_e}$

$$= \frac{2\pi}{\frac{2\pi}{T_s} - \frac{2\pi}{T_e}} = \frac{T_s T_e}{T_e - T_s} = 7417$$

$7417 = x \times 3700$

$2.01 = \frac{7417}{3700} = x$

37. A planet of mass m is in elliptical orbit about the sun mass is M; $m <<< M_{sun}$ with an orbital period, if A be the area of orbit, then angular momentum is $\frac{xmA}{T}$ find x

(18 – March – 2021 (E)) (D)

Key: 2

Sol: Swept area $\frac{dA}{dt} = \frac{1}{2}(r)\left(r\frac{d\theta}{dt}\right) = \frac{r^2 w}{2} = \frac{rV}{2}$

$2\frac{dA}{dt} = rV \; - - - - - (1)$

angular momentum $dL = m\left(\bar{r} \times \bar{V}\right)$ substitute (1)

$dL = m2\frac{dA}{dt}$

$\Rightarrow L = \frac{2mA}{T}$

38. If one wants to remove all the mass of earth to infinity in order to break it up completely, amount of energy that needs to supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is ____

(16 – March – 2021 (E))

Key: 3

Sol: Energy given $= u_f - u_i = 0 - \left(-\frac{3GM^2}{5R}\right) = \frac{3}{5}\frac{GM^2}{R}$

$x = 3$

39. Gravitational potential difference between surface of a planet and a point $20\,m$ above it is $16\,J/kg$, calculate the work done in moving a $4\,kg$ body by $8\,m$ on a slope of 60^0 from the horizontal? in Joule. **(16 – March – 2021 (E)) (D)**

Key: 22.16 Joule

Sol: $h = 20\,m$

Gravitational potential difference $= 16\,J/kg$

vertical distance through which the body has to be raised $= 8\sin 60^0$

$$= 8\frac{\sqrt{3}}{2}$$
$$= 4\sqrt{3}$$

since gravitational potential difference for a dist of $20\,m$ is $16\frac{J}{kg}$, hence the potential difference for a distance of $4\sqrt{3}\,meter$ for a body of mass $4\,kg$ is $= \frac{16}{20}\times 4\sqrt{3}\,\frac{J}{kg}$

Work done in lifting a $4\,kg$ body through a vertical height of $4\sqrt{3}\,m$

$$= \frac{16}{20}\times 4\sqrt{3}\times 4 = \frac{64\times 1.732}{5} = 22.16\,J$$

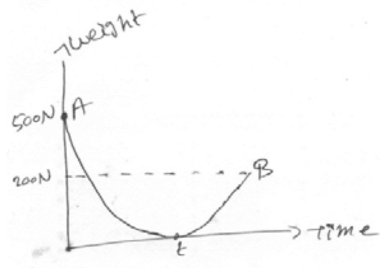
40. Radius in kilometer to which $R_{Earth} = 6400\,km$ to be compressed so that the escape velocity is raised to 10 time is ____ **(17 – March – 2021 (M))**

Key: $64\,km$

Sol: $$\frac{V_{escape} = \sqrt{\frac{2GM}{R}}}{10V_{escape} = \sqrt{\frac{2GM}{R'}}} \Rightarrow 10 = \sqrt{\frac{R}{R'}}$$

$$\Rightarrow R' = \frac{R}{100} = \frac{6400}{100} = 64\,km$$

41. A person of mass $50\,kg$ travels from earth to mars in spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of earth and mars as $10\,m/s^2$ and $4\,m/s^2$ respectively. Find net gravitational field strength after ‘t’ seconds from the following graph. **(20 – July – 2021(M)) (D)**



Key: **0**

Sol: There will be a neutral point where weight will be zero

$\therefore$ Net gravitational field is zero at 't' seconds

42. Consider a planet in some solar system which was a mass four times of mass of earth and density equal to the average density of earth. If the weight of an object on earth is ω and on the planet is $4^{x/3}\omega$ then the value of $'x'$ is

Key: **1**

Sol: $d_E = d_P$

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{4M}{\frac{4}{3}\pi R_P^3} \Rightarrow R_P = 4^{1/3}\cdot R$$

$$g = \frac{GM}{R^2}$$

$$g_P = \frac{G(4M)}{R_P^2} = \frac{4GM}{4^{2/3}\cdot R^2}$$

$$\frac{g_P}{g} = 4^{1/3} \Rightarrow g_P = 4^{1/3} g$$

$$mg_P = 4^{1/3} mg$$

$$\omega_P = 4^{1/3}\,\omega\ (\because \omega = mg)$$

$$\therefore x = 1$$

43. A satellite launched into a circular orbit of radius 'R' around earth while a second satellite is launched into a circular orbit of radius $1.001R$. the percentage difference in the time period of the two satellites is $x\%$ then find the value of $'x'$ **(20 – July – 2021(E)) (D)**

Key: **0.15**

Sol: $T^2 \propto R^3$

$$2\frac{dT}{T} = 3\frac{dT}{R}$$

$$2\frac{dT}{T}\times 100\% = 3\frac{(1.001R - R)}{R}\times 100\%$$

$$\frac{dT}{T}\times 100\% = 1.5\times 0.1\%$$

$$= 0.15\%$$

44. Two stars each of 1 solar mass $\left(= 2\times 10^{30}\,kg\right)$ are approaching each other for a head on collision. When they are at a distance $10^9\,km$, their speeds are negligible the speed with

which they collide is $x \times 10^6$. The radius of each star is $10^4\,km$. Assume the stars to remain undisturbed until they collide then value of $'x'$ is **(20 – July – 2021(E)) (D)**

Key: 2.583

Sol: Here, mass of each star, $M = 2\times 10^{30}\,kg$. Initial distance between two stars,

$r = 10^9\,km = 10^{12}\,m$; Initial potential energy of the system $= -\dfrac{GM^2}{r}$

Total K.E. of the stars $= \dfrac{1}{2}Mv^2 + \dfrac{1}{2}Mv^2 = Mv^2$

where v is the speed of stars with which they collide. When the stars are about to collide, the distance between their centres, $r' = 2R$.

$\therefore$ Final potential energy of the stars $= -\dfrac{GM^2}{2R}$.

Since gain in K.E. is at the cost of loss in P.E.

$$\therefore Mv^2 = -\frac{GM^2}{r} - \left(-\frac{GM^2}{2R}\right) = -\frac{GM^2}{r} + \frac{GM^2}{2R}$$

$$\text{(or) } 2\times 10^{30} v^2 = -\frac{6.67\times 10^{-11}\times\left(2\times 10^{30}\right)^2}{10^{12}} + \frac{6.67\times 10^{-11}\times\left(2\times 10^{30}\right)^2}{2\times 10^7}$$

$$= -2.668\times 10^{38} + 1.334\times 10^{43} = 1.334\times 10^{43}\,J$$

$$\therefore v = \sqrt{\frac{1.334\times 10^{43}}{2\times 10^{30}}} = 2.583\times 10^6\,ms^{-1}$$

45. The minimum and maximum distance of a planet revolving around the sun are $5\times 10^7\,km$ and $3\times 10^8\,km$. If the minimum speed of the planet on its trajectory is $'V_0'$ and maximum speed is $'xV_0'$ then the value of $'x'$ is **(25 – July – 2021(M)) (D)**

Key: 6

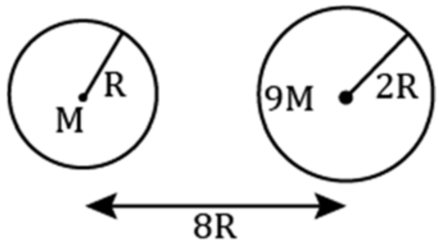
Sol: $mv_0 \times 3\times 10^8 = mv \times 5\times 10^7 \; (\because mvr = constant)$

$\Rightarrow v = 6v_0$

$\therefore x = 6$

46. Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9M respectively have a centre to centre separation 8R as shown in the figure. A satellite of mass $'m'$ is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed $'v'$ required for the satellite to reach the surface of the second planet is $\sqrt{\dfrac{a}{7}\dfrac{GM}{R}}$ then the value of 'a' is ____

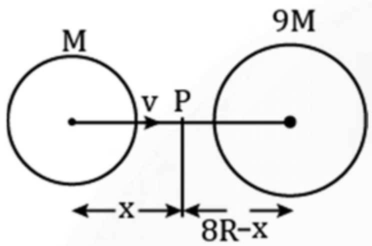
[Given: The two planets are fixed in their position]



(27 – July – 2021(M))

Key: 4

Sol:



Acceleration due to gravity will be zero at P therefore,

$$\frac{GM}{x^2} = \frac{G9M}{(8R-x)^2}$$

$$8R - x = 3x$$

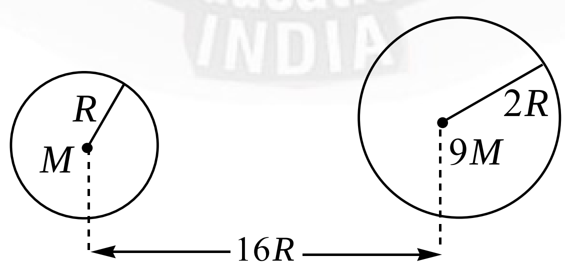
$$x = 2R$$

Apply conservation of energy and consider velocity at P is zero.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{G9Mm}{7R} = 0 - \frac{GMm}{2R} - \frac{G9Mm}{6R}$$

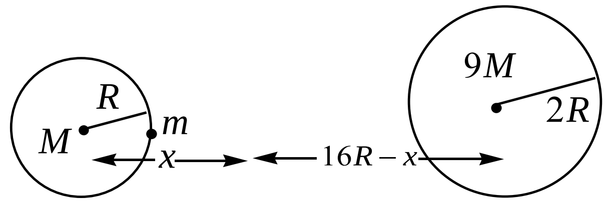
$$\therefore V = \sqrt{\frac{4}{7}\frac{GM}{R}}$$

47. Suppose two planets of radii R and 2R, but mass M and 9M respectively have a centre to centre separation 16R as shown in the figure. A satellite of mass $'m'$ is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed $'v'$ required for the satellite to reach the surface of the second planet is $\sqrt{\frac{6}{x'}\frac{GM}{R}}$ then the value of x' is ____ **(27 – July – 2021(M)) (D)**



Key: 5

Sol: Let gravitational force on $'m'$ is zero at a distance $'x'$ from centre of planet of mass 'M'.



$$\frac{GmM}{x^2}=\frac{Gm(9M)}{(16R-x)^2}$$

$$\left(\frac{16R-x}{x}\right)^2=9 \Rightarrow \frac{16R-x}{x}=3 \Rightarrow x=4R$$

By conservation of energy

$$-\frac{GmM}{R}-\frac{Gm(M9)}{15R}+\frac{1}{2}mV^2=-\frac{GmM}{4R}-\frac{Gm(9M)}{12R}$$

$$-\frac{GmM}{R}\left(1+\frac{3}{5}\right)+\frac{1}{2}mV^2=-\frac{GmM}{R}\left(\frac{1}{4}+\frac{3}{4}\right)$$

$$\frac{1}{2}mV^2=\frac{8}{5}\frac{GmM}{R}-\frac{GmM}{R}$$

$$\frac{V^2}{2}=\frac{3}{5}\frac{GM}{R} \Rightarrow V=\sqrt{\frac{6}{5}\frac{GM}{R}}$$

$$\therefore x^1=5$$

48. Two identical particles of mass $8\,kg$ each go round a circle of radius 'R' under the action of their mutual gravitational attraction. The angular speed of each particle is $\sqrt{\frac{xG}{R^3}}$ then the value of $'x'$ is **(27 – July – 2021(E)) (D)**

Key: 2

Sol: $$\frac{Gmm}{(2R)^2}=mR\omega^2$$

$$\omega=\sqrt{\frac{Gm}{4r^3}}=\sqrt{\frac{G\times 8}{4R^3}}=\sqrt{\frac{2G}{R^3}}$$

$$\therefore\ x=2$$

49. The planet mars has two moons, if one of them has a period 16 hours and an orbital radius of $8\times10^3\,kg$. The mass of mars is $x\times10^{22}\,kg$ then value of $'x'$ is

$$\left(\frac{4\pi^2}{G}=6\times10^{11}N^{-1}m^{-2}kg^2\right)$$ **(27 – July – 2021(E)) (D)**

Key: 9.26

Sol: $$V_0=\sqrt{\frac{GM}{r}} \Rightarrow r\cdot\frac{2\pi}{T}=\sqrt{\frac{GM}{r}}$$

$$\Rightarrow \frac{r^2 4\pi^2}{T^2} = \frac{GM}{r} \Rightarrow M = \frac{4\pi^2}{G}\frac{r^3}{T^2}$$

$$M = 6\times 10^{11}\frac{\left(8\times 10^6\right)^3}{(16\times 60\times 60)^2}$$

$$M = 6\times 10^{11}\times\frac{8\times 8\times 8\times 10^{18}}{(16\times 60\times 60)^2}$$

$$M = 9.26\times 10^{22}\ kg$$

50. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height '4h' is $\frac{2}{3}\sqrt{\frac{R^3}{xGM}}\left[\left(1+\frac{4h}{R}\right)^{3/2}-1\right]$ then the value of 'x' is **(22 – July – 2021(M)) (D)**

Key: 2

Sol: $$-\frac{GmM}{R}+\frac{1}{2}m\left(\sqrt{\frac{2GM}{R}}\right)^2 = \frac{1}{2}mV^2 - \frac{GmM}{R+4h}$$

$$\frac{1}{2}mV^2 = \frac{GmM}{R+4h} \Rightarrow v = \sqrt{\frac{2GM}{R+4h}}$$

$$V = \sqrt{\frac{2GM}{r}}\ \ (\because r = R+4h)$$

$$\frac{dr}{dt} = \sqrt{\frac{2GM}{r}} \Rightarrow dt = \sqrt{\frac{r}{2GM}}dr$$

$$\int dt = \int_R^{R+4h}\frac{r^{1/2}}{\sqrt{2GM}}dr = \frac{2}{3}\frac{1}{\sqrt{2GM}}\left[(R+4h)^{3/2}-R^{3/2}\right]$$

$$t = \frac{2}{3}\frac{1}{\sqrt{2GM}}\left[R^{3/2}\left[\left(1+\frac{4h}{R}\right)^{3/2}-1\right]\right]$$

$$t = \frac{2}{3}\sqrt{\frac{R^3}{2GM}}\left[\left(1+\frac{4h}{R}\right)^{3/2}-1\right]$$

$$\therefore x = 2$$

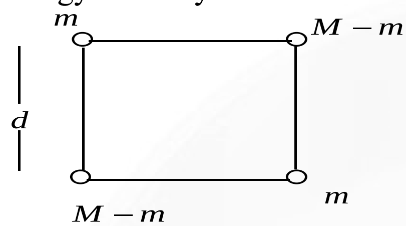
51. The escape speed from earths is $11\,km/s$. A certain planet has a radius twice that of earth but its mean density is the same as that of the earth. Find the value of the escape speed from be planet______. (in km/s)

Key: 22

Sol: $$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\times\frac{4}{3}\pi R^3\rho} = \sqrt{\frac{8\pi G\rho R^2}{3}}$$

$$V_e = \sqrt{\frac{8\pi G\rho(2R)^2}{3}} = 2\sqrt{\frac{8\pi G\rho R^2}{3}} = 2V_e = 2\times 11 = 22\,km/s$$

52. A body of mass $(2M)$ splits into masses $\{m, M-m, m, M-m\}$. Which are rearranged to form a square as shown in the figure. The ration of $\frac{M}{m}$ for which the gravitational potential energy of the system becomes maximum is $x:1$. The value of x is



(27th August 2021, Shift-2)

Key: 2

Sol: Energy maximum when mass is split equally to $\frac{M}{m} = 2$

53. The radius of the earth is about $6400\,km$ and that of mass is about $3200\,km$. The mass of the earth is about 10 times of the mass of mass. An object weights $200\ N$ on the surface of the earth. Its weight on the surface of mass would be ____(in N)

Key: 80 N

Sol: $g = \frac{Gm}{R^2}$

$$g_m = \frac{Gm'}{R'^2} = \frac{\frac{GM}{10}}{\frac{R^2}{4}} = \frac{2}{5}\frac{GM}{R^2} = \frac{2}{5}gE$$

$$W_m = W_E\frac{2}{5} = 200\times\frac{2}{5} = 80\,N$$

54. On a planet whose size is the same and mass four times as that of our earth, find the amount of work done to lift $3\,kg$ mass vertically upwards through $3\,m$ distance on the planet. The value of g on the surface of earth is $10\,m/s^2$.

Key: 360 J

Sol: Here $R_p = \mathrm{R_e}, M_p = 4M_e$

$M = 3\,kg, h = 3m, g_e = 10\,m/s^2$

$$g_e = \frac{GM_e}{\mathrm{R}_e^2}$$

$$g_p = \frac{GM_p}{\mathrm{R}_p^2} = 4\frac{GM_e}{\mathrm{R}_e^2} = 4g_e$$

Work done = $mg_p h = 3\times 4\times 10\times 3 = 360\,J$

55. A body weighs 63N on the surface of earth where acceleration due to gravity is g. The gravitational force on it due to the earth at a height equal to half the radius of earth is ___

(24 – JUNE – 2022(M)) (D)

Key: **28 N**

Sol: $w = 63\,N$ $\qquad h = \dfrac{R}{2}$

$$g_h = g\left[\frac{R}{R+h}\right]^2$$

$$g_h = g\left[\frac{R}{R+\frac{R}{2}}\right]^2$$

$$g_h = g \times \frac{4}{9}$$

$$w' = Mg_h = mg\frac{4}{9}$$

$$w' = 63 \times \frac{4}{9}$$

$$w' = 28\,N$$

56. The height above the surface of the earth where acceleration due to gravity is $\frac{1}{64}$ of its value at surface of the earth is ___ $\times 10^6\,m$ **(24 – JUNE – 2022(M)) (D)**

Key: $44.8 \times 10^6\,m$

Sol: $g_h = \dfrac{9}{64}$

$$g_h = g\left[\frac{R}{R+h}\right]^2$$

$$g_h = \frac{9}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g_h}{g} = \left(\frac{1}{1+\frac{h}{R}}\right)^2 = \frac{1}{64}$$

$$64 = \left(1+\frac{h}{R}\right)^2$$

$$1+\frac{h}{R} = 8$$

$\dfrac{h}{R} = 7$ $\qquad h = 7R$

$$h = 7 \times 6400\,km$$

$h = 44.8 \times 10^6\ m$

57. A particle hanging from a spring stretches it by $1cm$ at earth's surface. Radius of earth is $6400\,km$ at a place $800\,km$ above the earth's surface, the same particle will stretch the spring by ___ cm find x? **(25 – JUNE – 2022(M)) (D)**

Key: 0.79

Sol: $x_1 = 1cm$

$kx_1 = mg$

$k = \frac{mg}{x_1} = 100\,mg$

At height $h = 800\,km$

$g_h = g\left[1 - \frac{2h}{R}\right]$

$g_h = g_0\left[1 - \frac{2 \times 800}{6400}\right]$

$g_h = \frac{3g_0}{4}$

Now

$kx_2 = mg_h$

$100\,mg\,x_2 = m\frac{3g}{4}$

$x_2 = \frac{3}{4}cm$

$x_2 = 0.79\,cm$

58. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = acceleration due to gravity on the surface of the earth) in terms of R the radius of the earth is ___ **(25 – JUNE – 2022(M)) (D)**

Key:

Sol: $g_h = \frac{g}{9}$

$g_h = g\left[\frac{R}{R+h}\right]^2$

$\frac{g}{9} = g\left[\frac{R}{R+h}\right]^2$

$\frac{1}{3} = \frac{R}{R+h}$

$3R = R + h$

$2R = h$

59. The distance of Neptune and Saturn from the sun are respectively 10^{13} and 10^{12} meters and their periodic times are respectively T_n and T_s. if their orbits are circular, then the value of T_n/T_s is $x\sqrt{10}$ find x **(24 – JUNE – 2022(E)) (D)**

Key: **10**

Sol: $R_n = 10^{13}\, m \;\Rightarrow T_n$

$R_s = 10^{12}\, m \;\Rightarrow T_s$

$T^2 \;\alpha\; R^3$

$T \;\alpha\; R^{3/2}$

$$\frac{T_n}{T_s} = \left(\frac{R_n}{R_s}\right)^{3/2}$$

$$\frac{T_n}{T_s} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2}$$

$$\frac{T_n}{T_s} = 10\sqrt{10}$$

$\therefore x = 10$

60. If the mean distance of Jupiter from sun is about $5\,AU$ to complete one revolution. Time taken by Jupiter is ____ [$1\,AU$ = Mean distance of earth from the sun]

(24 – JUNE – 2022(E)) (D)

Key:

Sol: $R = 5\,AU$

$T^2 \,\alpha\, R^3$

$T \,\alpha\, R^{3/2}$

$T \,\alpha\, 5^{3/2}$ *years*

61. The distance between earth and sun becomes four times, then time period becomes ___ times. **(28 – JUNE – 2022(M)) (D)**

Key: **8**

Sol: $T^2 \propto R^3$

$R_1 = R \qquad\qquad R_2 = 4R$

$T_1 = T \qquad\qquad T_2 = ?$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{4R}{R}\right)^3$$

$T_2 = 8T_1$

62. A satellite is launched in a circular orbit of radius R around the earth. A second satellite is launched into a orbit of radius 1.01 R. The period of second satellite is longer than the first one approximately by x% find x **(28 – JUNE – 2022(M)) (D)**

Key: 1.5%

Sol: $T^2 \propto R^3$

$R_1 = R \qquad R_2 = 1.01R$

$\Delta R = 0.01R$

$\frac{\Delta T}{T} \times 100 = ?$

$T^2 \propto R^3$

$2\frac{\Delta T}{T} \times 100 = 3\frac{\Delta R}{R} \times 100$

$\frac{\Delta T}{T} \times 100 = \frac{3}{2} \times \frac{0.01R}{R} \times 100$

$\frac{\Delta T}{T} \times 100 = 1.5\%$

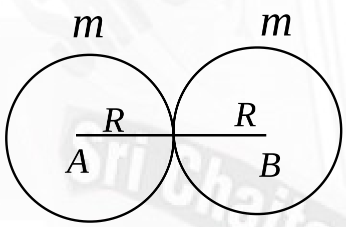
$x = 1.5\%$

63. Two balls each of radius $2R$ equal mass and denfitiy are placed in contact, then force of gravitation between them is proportional to R^x. Find x **(28 – JUNE – 2022(E)) (D)**

Key: 4

Sol: $F = \frac{Gmm}{(2R)^2}$

$F = \frac{Gm^2}{4R^2} = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2}$

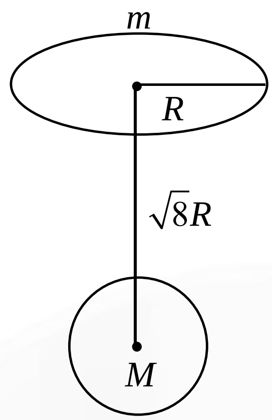


$F = \frac{G4\pi R^6 \rho}{3 \times 4R^2}$

$F \propto R^4$

$x = 4$

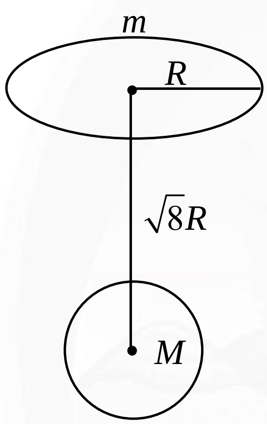
64. The centres of a ring of mass m and sphere of mass M of equal radius R are at a distance $\sqrt{8}R$ apart as shown in fig. The force of attraction between the ring and the sphere is



(28 – JUNE – 2022(E)) (D)

Key:

Sol:



Gravitation field due to ring at $\sqrt{8}R$ distance is

$$E_G = \frac{Gmx}{\left(r^2 + x^2\right)^{3/2}}$$

$x = \sqrt{8}R \qquad r = R$

$$E_G = \frac{Gm\sqrt{8}R}{\left(R^2 + 8R^2\right)^{3/2}} = \frac{2\sqrt{2}Gm}{27R^2}$$

Force of attraction between sphere and ring is

$F = ME_G$

$$F = \frac{2\sqrt{2}GmM}{27R^2}$$

65. The period of revolution of an earth's satellite close to surface of earth is $90\,\text{min}$. The time period of another satellite in an orbit at a distance of 4 times the radius of earth from its surface will be _____ **(29 – JUNE – 2022(E)) (D)**

Key: 720 min

Sol: $T_1 = 90\,\text{min}$

$T_2 = ?$

$R_1 = R$

$R_2 = 3R + R$

$R_2 = 4R$

$T^2 \propto R^3$

$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2}$

$\frac{T_2}{90} = \left(\frac{4R}{R}\right)^{3/2}$

$T_2 = (64)^{1/2} \times 90$

$= 90 \times 8 = 720\,\text{min}$

$T_2 = 720\,\text{min}$

66. The period of revolution of planet A round the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun is ____

(29 – JUNE – 2022(E)) (D)

Key: 4

Sol: $T_A = 8T_B$

$R_A = xR_B \qquad x = ?$

$T^2 \propto R^3$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$

$\frac{R_A}{R_B} = \left(\frac{T_A}{T_B}\right)^{2/3}$

$= \left(\frac{8T_B}{T_B}\right)^{\frac{2}{3}}$

$R_A = 4R_B$

67. Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius $R_1 = 3200\,km$ and $R_2 = 800\,km$ respectively. The ratio of speed of satellite S_1 to the speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where $x =$ ____ **(25 – JUNE – 2022(E))**

Key: 2

Sol: $V \alpha \frac{1}{\sqrt{R}}$

$R_1 = 3200\,km$

$R_2 = 800\,km$

$\frac{V_1}{V_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{800}{3200}}$

$\frac{V_1}{V_2} = \frac{1}{2}$

$x = 2$

68. The mass of the earth is 81 times that of the moon and radius is nearly 4 times that of the moon. If orbital velocity of a particle very close to the surface of earth is V_0, the corresponding value very close to the surface of moon is $\frac{x}{9}V_0$ find x value

(25 – JUNE – 2022(E)) (D)

Key: 2

Sol: $M_e = 81 M_m$

$R_e = 4 R_m$

V_0 earth

V_m moon

$V_e = \sqrt{\frac{GM_e}{R_e}}$ $\quad V_m = \sqrt{\frac{GM_m}{R_m}}$

$V_m = \sqrt{\frac{GM_e \times 4}{81 R_e}}$

$V_m = \frac{2}{9}\sqrt{\frac{GM_e}{R_e}}$

$V_m = \frac{2}{9}V_e$

$x = 2$

69. The mean radius of the orbit of a satellite is 4 times as great as that of a parking orbit of the earth. Then its period of revolution around the earth is ____ days

(25 – JUNE – 2022(E)) (D)

Key: 8

Sol: $T^2 \alpha R^3$ $\quad T_1 = 1 day$

$R_2 = 4R$ $\quad R_1 = R$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$

$\left(\frac{1}{T_2}\right)^2 = \left(\frac{R}{4R}\right)^3$

$T_2 = 4^{3/2} = 8 days$

70. The mass of the earth is 9 times that of mars. The radius of the earth is twice that of mars. The escape velocity of the earth is $12 km/s$. The escape velocity on mars is ___$km/$sec

(29 – JUNE – 2022(M)) (D)

Key:

Sol: $M_e = 9 M_m$

$R_e = 2 R_m$

$V_e = 12 km/s$

$V_M = ?$

$V_e = \sqrt{2\frac{GM}{R}}$

$V_e \propto \sqrt{\frac{M}{R}}$

$\frac{V_M}{V_e} = \sqrt{\frac{M_m}{M_e}} \times \sqrt{\frac{R_e}{R_M}}$

$= \sqrt{\frac{M_m}{9M_m}} \times \sqrt{\frac{2R_m}{R_m}}$

$\frac{V_m}{12} \times \frac{1}{3} \times \sqrt{2}$

$V_m = \frac{12}{3} \times \sqrt{2}$

$V_m = 4\sqrt{2}$

71. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $2/3$ times that of the earth. If the escape speed on the surface of the earth is taken to be $11 km/s$. The escape speed on the surface of the planet in kms^{-1} will be ___ **(29 – JUNE – 2022(M)) (D)**

Key: 3

Sol: $g_p = \frac{\sqrt{6}}{11}g$

$\rho_p = \frac{2}{3}\rho$

$V_e = 11 km/s$

$V_p = ?$

$g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2}$

$g \propto PR$

$R = \frac{g}{\rho}$

$V_e = \sqrt{2gR}$

$V_e \propto \sqrt{gR}$

$V_e \propto \sqrt{g\frac{g}{\rho}} = \sqrt{\frac{g^2}{\rho}}$

$V_e \propto \frac{g}{\sqrt{\rho}}$

$$\frac{V_p}{V_e}=\frac{g_p}{g_e}\times\sqrt{\frac{\rho_e}{\rho_p}}$$

$$=\frac{\sqrt{6}g}{11g}\times\sqrt{\frac{3\rho}{2\rho}}$$

$$\frac{V_p}{V_e}=\frac{\sqrt{6}}{11}\times\sqrt{\frac{3}{2}}$$

$$\frac{V_p}{V_e}=\sqrt{\frac{6^3\times 3}{121\times 2}}=\frac{3}{11}$$

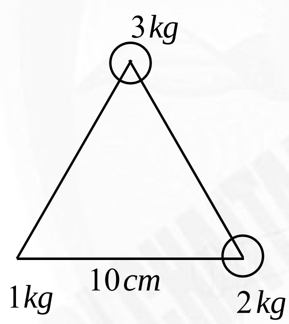
$$\frac{V_p}{V_e}=\frac{3}{11}$$

$$V_p=\frac{3}{11}\times 11=3km/s$$

72. The PE of three objects of masses $1kg, 2kg$ and $3kg$ placed at the three vertices of an equilateral triangle of side $10cm$ is $-xG$ find x? **(27 – JUNE – 2022(E)) (D)**

Key: 110

Sol:



$$U=U_1+U_2+U_3$$

$$U=-\frac{G}{10\times 10^{-2}}[1\times 2+2\times 3+3\times 1]$$

$$=-G\times 10^1[2+6+3]$$

$$=-11G\times 10$$

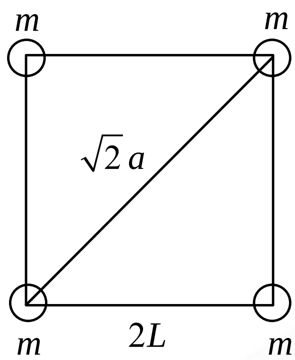
$$U=-110G$$

$$x=110$$

73. The potential energy of a system of $4p$ articles placed at the vertices of a square of side $2l$ is $\frac{Gm^2}{l}\left(2+\frac{1}{\sqrt{x}}\right)$ find x. **(27 – JUNE – 2022(E)) (D)**

Key: 2

Sol:



$U = 4U_1 + 2U_2$

$$U = -\frac{4Gm^2}{2L} - \frac{2Gm^2}{2L\sqrt{2}}$$

$$= -\frac{2Gm^2}{2L}\left[2 + \frac{1}{\sqrt{2}}\right]$$

$$= -\frac{Gm^2}{L}\left[2 + \frac{1}{\sqrt{2}}\right]$$

$\therefore\ x = 2$

74. Gravitational force between two point masses m and M separated by a distance is F. Now if point mass 3m is placed next to m, the total force on M will be ____F where $x =$

(29 – Jan – 2023(M)) (D)

Key: **4**

Sol: $F = \frac{GMm}{r^2}$

$$F' = \frac{GM(m+3m)}{r^2}$$

$$F' = \frac{GMm \times 4}{r^2}$$

75. A satellite of mass $'m'$ revolves around earth of mass M in a circular orbit of radius $'r'$ with angular velocity ω. If radius of orbit becomes $9r$, then angular velocity of this orbit is $\frac{\omega}{x}$ where x is ___

Key: **27**

Sol: $T^2 \propto r^3$

$$\omega^2 \propto \frac{1}{r^3}$$

$$\frac{\omega_2^2}{\omega_2^2} = \frac{r_2^3}{r_1^3}$$

$$\frac{\omega^2}{\omega_1^2} = \frac{(9r)^3}{r^3}$$

$\omega_1 = \frac{\omega}{27}$

76. A man weight 'W' on the surface of earth and his weight at a height 'R' from surface of earth is (R is Radius) $\frac{w}{y}$ where $y =$ **(31 – Jan – 2023(E)) (D)**

Key: 4

Sol: $w = mg$

$w' = mg\left[\frac{R}{R+h}\right]^2$

$= mg\left[\frac{R}{R+R}\right]^2$

$w' = \frac{mg}{4} = \frac{w}{4}$

77. If 'g' on the surface of earth is $9.8\,m/s^2$, then its value at a height of $6400\,km$ is $\left(in\ ms^{-2}\right)$ **(31 – Jan – 2023(E)) (D)**

Key: 2.45

Sol: $g' = g\left[\frac{R}{R+h}\right]^2 \quad h = 6400$

$= 9.8\left[\frac{1}{2}\right]^2$

$= \frac{9.8}{4} = 2.45\,m/s^2$

78. The height from the surface of earth, where the total energy of satellite is equal to its potential energy at a height 2R from the surface of earth (R is radius of earth) is $\frac{R}{x}$ where $x =$ ___ **(30 – Jan – 2023(M)) (D)**

Key: 2

Sol: $-\frac{GMm}{2r} = -\frac{GMm}{3R}$

$r = R + h$

$-\frac{GMm}{2(R+h)} = -\frac{GMm}{3R}$

Solving $h = \frac{R}{2}$

79. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that escape velocity from earth is $11\,km/s$. The escape velocity from the surface of the planet is ____ km/s **(25 – Jan – 2023(M)) (D)**

Key: 110

Sol: $V_e = \sqrt{\dfrac{2GM}{R}}$ $\quad V_p = \sqrt{\dfrac{2GM_p}{R_p}}$

$M_p = 10M \qquad R_p = 0.1R$

$V_p = \sqrt{100} \times V_e = 10 \times 11 = 110\, km/s$

EXERCISE – IV

ASSERTION – REASON QUESTIONS

Directions: Each of these questions contain two statements, Assertion (A) and reason (R). Each of these questions also has four alternative choice, only one of which is the correct answer

You have to select one of the codes (1), (2), (3) and (4) given below.

(1) Both A and R are correct and R is the correct explanation of A

(2) Both A and R are correct but R is NOT the correct explanation of A

(3) A is correct but R is not correct

(4) A is not correct but R is correct

STATEMENT – I & STATEMENT - II QUESTIONS

Directions: Each of these questions contain two statements, Statement - I and Statement - II. Each of these questions also has four alternative choice, only one of which is the correct answer

You have to select one of the codes (1), (2), (3) and (4) given below.

(1) Statement – I is true, Statement – II is false

(2) Statement – II is true Statement – I is false

(3) Both Statement – I, Statement – II are true

(4) Both Statement – I, Statement – II are false

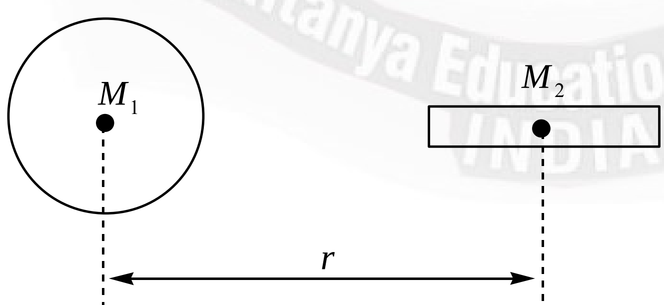
1. Gravitational force and laws

a) Universal law of gravitation

05. **Statement – I:** The force of gravitation between a sphere and a rod of mass M_2 is $= \dfrac{GM_1M_2}{r}$

Statement – II: Newton's law of gravitation holds correct for point masses

(27 – May – 2019) (D)



Key: 2

Sol: We can take sphere as a point mass lying at its centre, but the rod will not be taken as point mass lying as its centre of mass.

b) Principle of super position-Gravitational force

19. **Assertion:** The principle of super position is not valid for gravitational force.
Reason: Gravitational force is a conservative force **(24 – FEB – 2021(M)) (D)**

Key: 4

Sol: The total gravitational force on one particle due to number of particles is the resultant force of attraction exerted on the given particle due to individual particles

i.e. $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + ----$

Principle of superposition is valid

c) Kepler's laws

20. **Assertion:** Smaller the orbit of the planet around the sun, shorter is the time it takes to complete one revolution.
Reason: According to kepler's 3rd law of planetary motion, $T^2 \propto r^3$
(26 – FEB – 2021(M)) (D)

Key: A

Sol: If r is small then T will also be small.

2. Gravitational field intensity

d) Field due to common shapes

07. Given below are the two statements are labelled as Assertion A and the other is labelled as Reason R.
Assertion A: The force of gravitation between a sphere of mass M_1 and another sphere of mass M_2 is $\frac{GM_1M_2}{r}$.
Reason R: Newton's law of gravitation holds correct for point masses.
In the above statements, choose the most appropriate answer from the options given below
(08 – Jan – 2020 (M)) (D)

Key: 4

Sol: We can take the sphere as a point mass lying at its center, but the rod will not be taken as point mass lying at its center of mass.

28. **Assertion:** If earth were a hollow sphere, gravitational field intensity at any point inside the earth would be zero.
Reason: Net force on a body inside the sphere is zero **(25 – FEB – 2021(M)) (D)**

Key: A

Sol: Intensity inside a hollow sphere is zero, so force is also equal to zero
$\vec{F} = m\vec{E}$

29. **Assertion:** Gravitational field intensity at the centre of the ring is zero.
Reason: Net force at the centre of ring is zero **(26 – FEB – 2021(M)) (D)**

Key: A

Sol: Intensity inside a hallow sphere is zero.

3. Acceleration due to gravity and its variation

a) Acceleration due to gravity on the surface of earth

26. **Assertion:** The difference in the value of 'g' at pole and equator is proportional to square of angular velocity of earth
Reason: The value of accelaration due to gravity is minimum at the poles and maximum at equator. **(24 – FEB – 2021(E)) (D)**

Key:
Sol: A correct R wrong

02. **Statement – I:** Weight of an object on the Earth is more in mid –night than what it is at the noon.
Statement – II: At noon gravitational pull on the object by the sun and that by the earth are oppositely directed, and in the mid – night they are in the same direction.

Key:
Sol: Conceptual

47. **Statement – I:** If the earth suddenly stops rotating about its axis, then the acceleration due to gravity will become the same at all the places.
Statement – II: The value of acceleration due to gravity is independent of rotation of the earth

Key:
Sol: Conceptual

b) Variation due to shape of the earth

06. Given below are the two statements are labelled as Assertion A and the other is labelled as Reason R.
Assertion A: The value of acceleration due to gravity does not depend upon the mass of the body.
Reason R: Acceleration due to gravity is a constant quantity.
In the above statements, choose the most appropriate answer from the options given below **(07 – Jan – 2020 (E)) (D)**

Key: 3

Sol: Acceleration due to gravity is given by $g = \frac{GM}{R^2}$. Thus, it does not depend on the mass of the body on which it is acting. Also, it is not a constant quantity. It changes with changes in value of both M and R (distance between two bodies)

c) Variation with height from surface of earth

27. **Assertion:** At height h from ground and at depth h below ground, where h is approximately equal to 0.62R. The value of 'g' acceleration due to gravity is same.
Reason: value of 'g' decreases both sides, in going up and down. **(26 – FEB – 2021(E)) (D)**

Key: A

Sol: Inside the earth's surface, gravitational field varies as $g = \frac{GM}{R^2}\frac{r}{R} = \frac{GMr}{R^3}$

Out side the earths surface

$$g = \frac{GM}{r^2}, dt\ height$$

$$g = \frac{GM}{(R+h)^2}$$

$$h = 0.62R$$

5. Motion of planets and satellites

b) Escape speed

23. **Assertion:** Two different planets have same escape velocity

Reason: The division of their mass and radius must be same $m_1R_2 = m_2R_1$

Key: 1

Sol: $V_e = \sqrt{\dfrac{2GM}{R}}$

$$\frac{M_1}{R_1} = \frac{M_2}{R_2}$$

$$M_1R_2 = M_2R_1$$

24. **Statement - I:** Escape velocity directly proportional to the square root of the radius of the earth

Statement - II: If an object is projected will a velocity equal or above, the object escapes the gravity of earth and will not return back. **(25 – FEB – 2021(E)) (D)**

Key: 3

Sol:

25. **Assertion:** The escape velocities of planet A and B are same/ But A and B are of unequal mass.

Reason: The product of their mass and radius must be same, $M_1R_1 = M_2R_2$

(25 – FEB – 2021(M))

Key: 2

Sol: $V_e = \sqrt{\dfrac{2GM}{R}}$

$$\frac{M_1}{R_1} = \frac{M_2}{R_2}$$

$$M_1R_2 = M_2R_1$$

Hence reason R is not correct

g) Energy of a satellite in circular orbit

05. Given below are the two statements are labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Consider a satellite moving in an elliptical orbit around the earth. As the satellite moves, the work done by the gravitational force of the earth on the satellite for any small part of the orbit is zero.

Reason R: Kinetic energy of the satellite in the above described case is not constant as it moves around the earth.

In the above statements, choose the most appropriate answer from the options given below **(07 – Jan – 2020 (M)) (D)**

Key: 4

Sol: For Assertion A; Gravitational force is not perpendicular to velocity of the satellite. So for any small part of the orbit work done is not zero, although when satellite is at perihelion or aphelion position then work done by gravitational force for small part would be zero.

i) Time period of satellite

21. **Statement – I:** Time period of a satellite around earth is directly proportional to its angular speed
Statement – II: If two satellites angular speed ratio is 1: 2 then the ratio of its time periods around earth is 2: 1

Key: **2**

Sol: Time period $T = \frac{2\pi}{\omega}$

$T \propto \frac{1}{\omega}$

$\frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{2}{1}$

22. **Assertion:** For a satellite revolving very near to earth surface the time period of revolution is given by 1 hour 24 minutes
Reason: The period of revolution of a satellite depends only upon its height above the earths surface
(25 – FEB – 2021(M)) (D)

Key: **1**

Sol: $T = 2\pi\sqrt{\frac{R}{g}}$

$= 84 \min$

$1 hr\, 24 \min$

03. **Statement – I:** The time period of Geostationary satellite is 24 hours.
Statement – II: Geostationary satellite must have the same time period as the time taken by the earth to complete one revolution about its axis.

Key:

Sol: Conceptual

j) Geostationary satellite

18. **Assertion:** The time period of geostationary satellite is 24 hrs.
Reason: Geostationary satellite must have the same time period as the time taken by the earth to complete one revolution about it's axis **(24 – FEB – 2021(M)) (D)**

Key: **1**

Sol: As the geostationary satellite is established in an orbit in the plane of equation at a particular place so it move in the same as the earth and hence its time period of revolution is equal to 24 hrs

01. Which of the following statements regarding keplar's law are true.
A: All planets move in elliptical orbits with the sun situated of one of the foci of ellipse
B: The line that joins any planet to the sun sweeps equal areas in equal intervals of time.
C: The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse trace out by the planet. **(09 – JAN – 2019 (M)) (D)**
1) Only A is true 2) Only B is true 3) Only C is true 4) All A, B, C are true

Key: **4**

Sol:

04. **Statement – I:** If earth is suddenly stops rotating about its axis, then the value of acceleration due to gravity will become same at all the places.

Statement – II: The value of acceleration due to gravity is independent of rotation of earth.

Key:

Sol:

08. **Assertion:** Value of g is maximum at the surface of earth

Reason: $'g'$ decreases with height above and depth below the surface of earth

(2 – Sep – 2020 (E)) (D)

Key: 1

Sol: Conceptual

09. **Assertion:** On earth's satellite we feel weightlessness. Moon is also satellite of earth but on the surface of moon, we do not feel weightlessness

Reason: Gravitational force by earth on us on the surface of moon is zero. But gravitational force by moon on us on its surface is non zero. **(5 – Sep – 2020 (M)) (D)**

Key: 3

Sol: Gravitational force by earth is utilized in providing the necessary centripetal force for rotating round the earth. That's why this force is not felt to us but gravitational force by moon is unutilized that's why it is felt

10. **Assertion:** An object when thrown upward reaches a certain height and falls downward

Reason: Object which through upward get float on the sky **(5 – Sep – 2020 (E)) (D)**

Key: 3

Sol: **conceptual**

11. **Assertion:** A time period of pendulum on a satellite orbiting the Earth is infinity

Reason: Time period of a pendulum is inversely proportional to $\sqrt{g}$

(2 – Sep – 2020 (M)) (D)

Key: 1

Sol: Time period of pendulum in a satellite is infinity it means it may not oscillate as apparent value of 'g' is zero.

So time period being $\alpha \dfrac{1}{\sqrt{g}} \alpha \sqrt{\infty}$

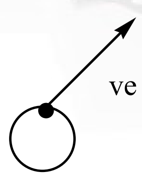
12. **Assertion:** If a particle is projected from the surface of earth with velocity equal to escape velocity, then total mechanical energy is zero

Reason: Total mechanical energy of any closed system is always negative

(3 – Sep – 2020 (M)) (D)

Key: 2

Sol:



Given when a body projected from surface of earth with velocity equal to escape velocity.

$PE = KE$

$\left|\dfrac{-GMm}{r}\right| = \dfrac{1}{2}mve^2 \quad TE = K + U = O$

And when a satellite of mass 'm' moving around the earth with velocity in an orbit of radius 'r' due to gravitational pull of earth

The potential energy of satellite $U = \dfrac{-GMm}{r} \ldots 1$

The kinetic energy of satellite $k = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left[\frac{GM}{r}\right]$

$k = \frac{GMm}{2r}$...2

Total energy $TE = K + U$

$TE = \frac{GMm}{2r} - \frac{GMm}{r} = \frac{-GMm}{2r}$

TE Is constant and negative

13. **Assertion:** Orbital velocity of a satellite is greater than its escape velocity
Reason: Orbit of a satellite is within the gravitation field of earth whereas escaping is beyond the gravitational field of earth

(4 – Sep – 2020 (E)) (D)

Key: 4

Sol: Escape velocity $= \sqrt{2}$ **times of orbital velocity**

14. **Assertion:** The binding energy of a satellite does not depend upon the mass of the satellite
Reason: Binding energy is the negative value of total energy of satellite

(6 – Sep – 2020 (E)) (D)

Key: 4

Sol: The binding energy of satellite does not depend upon the mass of the satellite. This statement is not true we have $Binding\ energy = \frac{GMm}{2r}$

$Total\ energy\ of\ satellite = \frac{-GMm}{2r}$ From the above equation, binding energy depends on mass of satellite. So, both energy terms are not equal binding energy is the negative value of the total energy of the satellite.

15. **Assertion:** Angular momentum of a planet is constant about any point
Reason: Force acting on the planet is a central force

(6 – Sep – 2020 (M)) (D)

Key: 4

Sol: Angular momentum is constant about the centre of sun and force acting on the planet is a central force

16. **Assertion:** If the product of surface area and density is same for two planets, escape velocities will be same for both
Reason: Product of surface area and density is proportional to the mass of the planet per unit radius of the planet **(3 – Sep – 2020 (E)) (D)**

Key: 1

Sol: $Ve = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

$Ve\alpha\sqrt{\frac{M}{R}}$

$\left(4\pi R^2\right).\delta = \left(4\pi R^2\right)\frac{M}{\left(\frac{4}{3}\right)\pi R^3}\alpha\frac{M}{R}$

17. **Assertion:** Gravitational potential and gravitational potential energy both are related to the work done by gravitational force in the gravitational field
Reason: Gravitational field strength is related to the gravitational force in gravitational field

(4 – Sep – 2020 (M)) (D)

Key: 2

Sol: conceptual

30. **Assertion:** Kepler's 2nd law can be understood by conservation of angular momentum principle.

Reason: Kepler's 2nd law is related with areal velocity which can further be proved to be based on conservation of angular momentum $\frac{dA}{dt}=\frac{r^2w}{2}$ **(16 – March – 2021 (M)) (D)**

Key: 1

Sol: Both A and R are correct but R is the correct explanation of A

$$\frac{dA}{dt}=\frac{L}{2m}=\frac{I\omega}{2m}=\frac{mr^2\omega}{2m}=\frac{r^2\omega}{2}$$

31. **Assertion:** If time period of a satellite revolving in circular orbit in equatorial plane is 24 hours, then it must be a geostationary satellite.

Reason: Time period of a geostationary satellite is 24 hours

(17 – March – 2021 (E)) (D)

Key: 2

Sol: Both statements are true and statement – II correctly explains statement - I

32. **Assertion:** For the planets orbiting around sun, angular speed, linear speed and kinetic energy change with time, but angular momentum remains constant.

Reason: No torque is acting on the rotating planet, so its angular momentum is constant

(18 – March – 2021 (E)) (D)

Key: 1

Sol: Both A and R are correct but R is the correct explanation of A

Potential energy of a system of particles

33. **Assertion:** The magnitude of the gravitational potential at the surface of solid sphere is less than that of the centre of sphere

Reason: Due to solid sphere, the gravitational potential is the same within sphere

(16 – March – 2021 (E)) (D)

Key: 3

Sol: Gravitational potential $=V_{in}=\frac{GM}{2R^3}\left[3R^2-r^2\right]$

At surface, $V_{surface}=\frac{GM}{R}$ at $r=R$

$$V_{in}=\frac{3}{2}V_{surface}$$

34. **Assertion:** Value of escape velocity from surface of earth at 30^0 and 60^0 is $V_1=2V_e$; $V_2=\frac{2}{3}V_e$

Reason: Value of escape velocity is independent of angle of projection.

(17 – March – 2021 (M)) (D)

Key: 4

Sol: Value of escape velocity is derived from the method of conservation of total mechanical energy and energy is independent of direction.

35. **Assertion:** The value of acceleration due to gravity does not depend on mass of body

Reason: Acceteration due to gravity is a constant quantity **(18 – March – 2021 (M)) (D)**

Key: 3

Sol: g depends on M, R

$$g = \frac{GM}{R^2}$$

36. **Assertion:** If earth suddenly stops rotating about its axis then acceleration due to gravity will become the same at all the places

Reason: The value of acceleration due to gravity is independent of rotation of earth

(18 – March – 2021 (E)) (D)

Key: 3

Sol: A is correct but R is not correct

37. **Assertion:** The weight of an object between two massive objects in space is zero at a particular point

Reason: At that point net gravitational field is zero due to two massive objects.

(20 – July – 2021(M)) (D)

Key: 1

Sol: Net gravitational field on the line joining of two masses is zero

38. **Assertion:** When a satellite is shifted from higher circular orbit to lower circular orbit it's time period decreases

Reason: Square of the period of revolution of a planet is directly proportional to cube of radius of the orbit **(20 – July – 2021(E)) (D)**

Key: 1

Sol: $T^2 \propto R^3$

39. **Assertion:** An object will experience more weight on a planet with larger mass if planets have equal mean densities.

Reason: When densities are equal planet with larger mass will have more radius

(25 – July – 2021(E)) (D)

Key: 1

Sol: $d = \frac{M}{vol} = \frac{M}{\frac{4}{3}\pi R^3} \Rightarrow M \propto R^3$

$$\omega = mg = m\frac{GM}{R^2}$$

40. **Assertion:** In a binary star system the time periods of two stars revolving about their common centre of mass is same

Reason: The angular velocity of two stars is same **(20 – July – 2021(E)) (D)**

Key: 1

Sol: $T_1 = \frac{2\pi}{\omega}$

$$T_2 = \frac{2\pi}{\omega}$$

41. **Assertion:** The speed of the planet is maximum when it is close to the sun and minimum when it is at maximum distance from the sun

Reason: Conservation of angular momentum **(25 – July – 2021(M)) (D)**

Key: 1

Sol: Conservation of angular momentum

$$mvr = constant \Rightarrow V \propto \frac{1}{r}$$

42. **Assertion:** When an object projected with sufficient velocity from the surface of larger planet to the surface of smaller planet (Densities of planets equal) then it's velocity decreases first then increases

Reason: Conservation of angular momentum **(27 – July – 2021(M)) (D)**

Key: 3

Sol: From conservation of energy. Velocity decreases first then increases

43. **Assertion:** When two equal masses rotates about their common centre of mass then necessary centripetal force is provided by gravitational force.

Reason: For unidentical masses gravitational force is not equal to centripetal force when two objects are rotating about their common centre of mass.

(27 – July – 2021(E)) (D)

Key: 3

Sol: Gravitational force = Centripetal force

44. **Assertion:** Body projected from the surface of a planet with a velocity greater than escape velocity then body reaches to the infinity

Reason: Sum of gravitational potential energy and kinetic energy of the object (body) is greater than zero **(22 – July – 2021(M)) (D)**

Key: 1

Sol: GPE + KE > 0

45. **Assertion:** Orbital velocity of a satellite changes when mass of a revolving satellite is suddenly reduced

Reason: Orbital velocity is inversely proportional to radius of the path traced by satellite

(27 – July – 2021(E)) (D)

Key: 3

Sol: by conservation of energy and angular momentum

46. **Statement – I:** The smaller the orbit of a planet around the sun. The shorter is the time it takes to complete.

Statement – II: According the Kepler's third law of planetary motion, square of time period is proportional to cube of mean distance from sun.

Key:

Sol: Conceptual

48. **Assertion:** value of g is maximum at the surface of earth

Reason: g decreases with height above and depth below the surface of earth.

(24 – June – 2022(M)) (D)

Key: 1

Sol: Both A and R are correct and R is correct explanation of A

49. Consider the following two statements I and II and identify the correct answer

Statement - I: Value of g changes with height

Statement - II: Decrease in g at small heights is more than decrease in g at larger heights.

(25 – June – 2022(M)) (D)

Key: 3

Sol: Both statements I and II are true

50. Given below are 2 statements one is labeled as Assertion A and the other is labeled as Reason R

Assertion: If we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the centre of earth without any variation in its magnitude.

Reason: At equator, the direction of acceleration due to the gravity is towards the centre of earth **(26 – June – 2022(E))**

Key: 4

Sol: A is not correct but R is correct

51. Read the following statements and choose the correct option

Statement - I: The acceleration due to gravity at the equator is less than that at poles due to the shape of the earth as well as due to rotation of earth

Statement - II: The value of acceleration due to gravity at the poles decreases if the speed of rotation of earth increases. **(26 – June – 2022(E)) (D)**

Key: 1

Sol: Only Statement – I is true

52. **Assertion:** Geostationary satellites may be set up in equatorial plane in orbits of my radius more than earth's radius.

Reason: Geostationary satellites have period of revolution of 24 hrs

(24 – June – 2022(E)) (D)

Key: 4

Sol: A is not correct but R is correct

53. **Statement - I:** The law of gravitation holds good for any pair of bodies in the universe

Statement - II: The weight of any person becomes zero when the person is at the centre of the earth. In the light of the above statements choose the correct answer from the options given below **(27 – June – 2022(M))**

Key: 3

Sol: Both statement – I and Statement – II are true

At centre g is zero

54. **Statement - I:** When a heavier and a lighter body are released from the same height, heavier body reaches the ground quickly.

Statement - II: Gravitational force is proportional to the mass of an object.

(27 – June – 2022(M)) (D)

Key: 2

Sol: Statement – I is false, Statement – II is true

$$g = \frac{GM}{R^2}$$

where M is mass of earth and g independent on mass of body.

55. **Assertion:** If radius of earth suddenly shrinks to half its present without changing it's mass value then the period of an earth's satellite will not change.

Reason: Time period of a satellite does not upon the mass of earth.

(28 – June – 2022(M)) (D)

Key: 3

Sol: Assertion is correct but reason is not correct

56. **Assertion:** Gravitational force between two masses in air is F. If they are immerse in water force will remain F

Reason: Gravitational force does not depend on the medium between the masses.

(28 – June – 2022(E)) (D)

Key: 1

Sol: Assertion and reason both are true

56. **Statement - I:** The square of period of revolution of a planet around sun is proportional to cube of average distance of planet

Statement - II: Kepler's third law explains **(29 – June – 2022(E)) (D)**

Key: 3

Sol: Statement – I & II are true

57. **Statement - I:** The speed of satellite always remains constant in an orbit

Statement - II: The speed of a satellite depends on its path. **(25 – June – 2022(E)) (D)**

Key: 2

Sol: Statement – I is false, Statement – II is true

58. **Statement - I:** Escape velocity changes with the mass of the projected body

Statement - II: Escape velocity depends on mass and density of planet but not depend on mass of projected object. **(29 – June – 2022(M)) (D)**

Key: **2**

Sol: Statement – I false, Statement – II true

59. **Assertion:** Gravitational potential energy of earth is always negative.

Reason: Every body on earth is bond by the attraction of earth. **(27 – June – 2022(E)) (D)**

Key: **1**

Sol: Assertion and Reason both are correct

60. Every planet revolves around the sun in an elliptical orbit **(24 – Jan – 2023(E)) (D)**

A) The force acting on a planet is inversely proportional to square of distance from sun

B) Force acting on planet is inversely proportional to product of masses of planet and the sun

C) The centripetal force acting on the planet is directed away from the sun

D) The square of time period of revolution of planet around sun is directly proportional to cube of semi major axis of elliptical orbit

Choose correct options

1) A and D only 2) C and D only 3) B and C only 4) A and C only

Key: **1**

Sol:

61. Identify the correct statements among the following

A) The angular momentum under a central force is constant

B) According to kepler's second law, the areal velocity of the lines joining planet to sun is constant

C) Gravitational force is attractive and repulsive in nature

D) A planet is moving around sun in an elliptical orbit. Then from perihelion to aphelion point speed of the planet continuously increases.

1) A and B only 2) A and D only 3) C and D only 4) B and C only

Key: **1**

Sol:

62. Given below are two statements

Statement - I: Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.

Statement - II: Acceleration due to earth's gravity is same at a height 'h' and depth 'd' from earth's surface if $h = d$.

In light of above statements choose most appropriate answers **(24 – Jan – 2023(M)) (D)**

Key: **1**

Sol: $$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$g' = g\left[1-\frac{d}{R}\right]$$

63. **Statement - I:** A body becomes weightless at centre of earth

Statement - II: As the distance from centre of earth decreases acceleration due to gravity increases **(24 – Jan – 2023(M)) (D)**

Key: **1**

Sol:

64. **Statement - I:** In a free fall, weight of body becomes effectively zero.

Statement - II: Acceleration due to gravity acting on a body having free fall is zero. **(24 – Jan – 2023(M)) (D)**

Key: 1

Sol:

65. Given below are two statements

Statement - I: Acceleration due to gravity is different at different places on the surface of earth

Statement - II: Acceleration due to gravity increases as we go down below the earth's surface **(01 –Feb – 2023(M))**

Key: 1

Sol:

66. **Statement - I:** Gravitational potential energy is not an absolute quantity

Statement - II: Change in gravitational potential energy depends on reference.

In light of above statements choose most appropriate **(30 – Jan – 2023(M)) (D)**

Key: 3

Sol:

<u>EXERCISE – V</u>

(MATRIX MATCHING & PARAGRAPH QUESTIONS)

1. Gravitational force and laws

a) Universal law of gravitation

07.

Column-I	Column-II
A) Force between Sun & Moon	**P)** $F = \dfrac{Gm_1m_2}{d^2}$
B) Gravitational constant G	**Q) Used to determing 'g'**
C) Simple pendulumn	**R) Depends on masses**
D) Newtons law of gravitation	**S) Is universal**

(26 – FEB – 2021(M)) (D)

1) $A-P, B-S, C-Q, D-R$
2) $A-S, B-R, C-P, D-Q$
3) $A-Q, B-R, C-P, D-S$
4) $A-S, B-Q, C-P, D-R$

Key: 1

Sol: $A-P, B-S, C-Q, D-R$

03. A satellite is revolving around the earth in a circular orbit of radius a with a velocity v_0. A particle is projected from the satellite in forward direction with relative velocity $V = \left[\left(\dfrac{\sqrt{5}}{4}\right) - 1\right] v_0$. During the subsequent motion of the particle match the following

(27 – May – 2019) (D)

Column – I	Column – II
I) Total energy of particle	a) $\dfrac{3GM_em}{a}$
II) Minimum distance of particle from the earth	b) $\dfrac{5GM_em}{a}$
III) Maximum distance of particle from the earth	c) $\dfrac{5a}{3}$
	d) $2a$
	e) $\dfrac{2a}{3}$
	f) a

1) $I \rightarrow a, II \rightarrow c, III \rightarrow b$
2) $I \rightarrow c, II \rightarrow a, III \rightarrow d$

3) $I \to d, II \to a, III \to e$ 4) $I \to d, II \to f, III \to e$

Key: 1

Sol: Angular momentum of the particle = $m(v_0 + v)a = \sqrt{\frac{5}{4}} m v_0 a \left(v_0 = \sqrt{\frac{GM_e}{a}} \right)$

Total energy of the particle = $\frac{1}{2}m(v_0+v)^2 - \frac{GM_e m}{a} = \frac{5}{8}\frac{GM_e m}{a} - \frac{GM_e m}{a} = -\frac{3GM_e m}{a}$

At any distance r, T. E = $\frac{1}{2}mu^2 - \frac{GM_e m}{a}$

But angular momentum conservation gives

$mur = \sqrt{\frac{5GM_e}{4a}} \Rightarrow u = \sqrt{\frac{5}{4}\frac{GM_e a}{r^2}}a$

T. E at any distance $r = \frac{1}{2}m\frac{5}{4}\frac{GM_e a}{r^2} - \frac{GM_e m}{r}$

But through conservation of total energy, we have

$\frac{1}{2}m\frac{5}{4}\frac{GM_e a}{r^2} - \frac{GM_e m}{r} = -\frac{3GM_e m}{a}$

On solving we get $3r^2 - 8ar + 5a^2 = 0$

$(r-a)(3r-5a) = 0$

$R = a, r = \frac{5a}{3}$

Minimum distance = a

Maximum distance = $\frac{5a}{3}$

c) Kepler's laws

01. A planet of mass m revolving around the sun in an elliptical orbit of semi-major axis a semi-minor axis b, T is time period i.e. eccentrcity **(24 – Feb – 2021(M)) (D)**

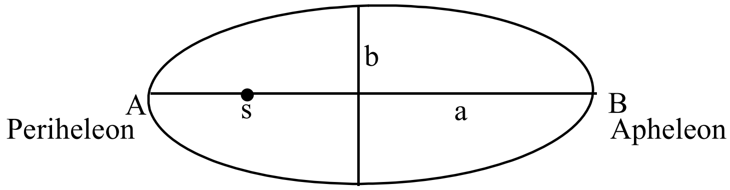
Column – I	Column - II
A) areal velocity is constant	P) Keplers 3rd law
B) $T^2 \propto a^3$	Q) Keplers 2nd law
C) A planet is very close to sun	R) Apiheleon
D) A planet is very far way from sun	S) Perihelion

1) $A-Q, B-P, C-S, D-R$ 2) $A-P, B-Q, C-S, D-R$

3) $A-Q, B-S, C-P, D-R$ 4) $A-R, B-P, C-S, D-Q$

Key: **1**

Sol:



According to Keplers second law

Areal velocity $\frac{dA}{dt} = cons\tan t$

3rd law $T^2 \propto a^3$ (a semi major axis)

3. Acceleration due to gravity and its variation

a) Acceleration due to gravity on the surface of earth

04.

Column – I	Column - II
A) g_h	P) g minimum
B) g_d	Q) $g\left(1-\frac{2h}{R}\right)$
C) g_{euator}	R) $g\left(1-\frac{d}{R}\right)$
D) g_{poles}	S) Maximum

(24 – FEB – 2021(M)) (D)

1) $A-Q, B-R, C-P, D-S$
2) $A-P, B-R, C-P, D-Q$
3) $A-Q, B-R, C-P, D-S$
4) $A-S, B-R, C-P, D-Q$

Key:

Sol: $A-Q, B-R, C-P, D-S$

05. On the surface of earth acceleration due to gravity is 'g' and gravitational potential is V. Match the following **(24 – FEB – 2021(M)) (D)**

Column – I	Column - II
A) At height $h = R$ value of g	p) decreases by factor $\frac{1}{4}$
B) At depth $h = \frac{R}{2}$, value of g	q) decreases by factor $\frac{1}{2}$
C) At height $h = \frac{R}{2}$, value of g	r) decreases by factor $\frac{3}{4}$
D) At depth $h = \frac{R}{4}$, value of g	s) decreases by factor $\frac{2}{3}$
	t) None

1) $A-q, B-q, C-s, D-r$
2) $A-q, B-r, C-s, D-p$

3) $A-p, B-q, C-s, D-r$ 4) $A-q, B-r, C-s, D-p$

Key: **1**

Sol: $g=\frac{GM}{h^2}$

At height $h=R, g^1=\frac{g}{1+\frac{h}{R}}=\frac{g}{2}$

Similarly $h=\frac{R}{2}\Rightarrow g^1=\frac{2}{3}g$

At depth $h=\frac{R}{2}, g^1=g\left(1-\frac{h}{R}\right)=\frac{g}{2}$

Similarly at $h=\frac{R}{4}, g^1=\frac{3}{4}g$

5. Motion of planets and satellites

b) Escape speed

c) Orbital speed of a satellite in circular orbit

06.

Column – I	**Column - II**
A) Orbital velocity	P) $9.8\ m/s^2$
B) Escape velocity	Q) $1.67\ m/s^2$
C) g_{earth}	R) $\sqrt{2gR}$
d) g_{moon}	S) $\sqrt{gR}$

1) $A-S, B-R, C-P, D-Q$ 2) $A-R, B-S, C-Q, D-P$

3) $A-P, B-Q, C-R, D-S$ 4) $A-Q, B-P, C-R, D-S$

Key: $A-S, B-R, C-P, D-Q$

Sol: $g=9.8\ m/\sec^2, g_{moon}=\frac{g_{earth}}{g}=1.67\ m/s^2$, $V_e=\sqrt{2gR}$, $V_0=\sqrt{gR}$

g) Energy of a satellite in circular orbit

02. For a satellite **(24 – FEB – 2021(M)) (D)**

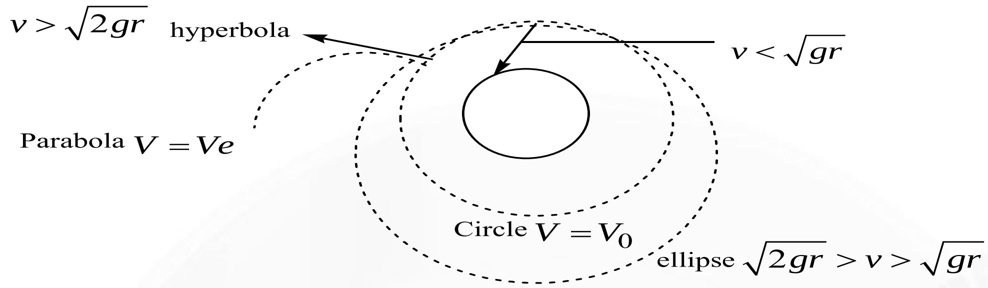
Column I	**Column II**
A) $V=V_0$	P) $TE<0,$ ellipical orbit
B) $V<V_0$	Q) $TE<0,$ circular orbit
C) $V>V_e$	R) $TE=0,$ Parabolic path
D) $V_0>V<V_e$	S) $TE>0,$ Verticity at inbinding

1) $A-Q, B-R, C-S, D-P$ 2) $A-R, B-Q, C-P, D-S$

3) $A-P, B-Q, C-R, D-S$ 4) $A-S, B-Q, C-R, D-P$

Key: 1

Sol:



03. For an orbitary satellite

Column – I	**Column - II**
A) Potential Energy	**P)** $\dfrac{GMm}{2(R+h)}$
B) Kinetic Energy	**Q)** $-\dfrac{GMm}{R+h}$
C) Orbital angular Velocity	**R)** $2\pi\sqrt{\dfrac{(R+h)^3}{GM}}$
D) Time period	**S)** $2\pi\sqrt{\dfrac{GM}{(R+h)^3}}$

(25 – FEB – 2021(M)) (D)

1) $A-Q, B-P, C-S, D-R$ 2) $A-S, B-P, C-Q, D-R$

3) $A-R, B-P, C-Q, D-S$ 4) $A-S, B-R, C-Q, D-P$

Key: 1

Sol:

08. Considering earth to be homogeneous sphere but keeping in mind its spin match the following **(17 – March – 2021 (M)) (D)**

Column – I		**Column – II**	
1)	Acceleration due to gravity	A)	May change from point to point
2)	Orbital angular momentum of the earth as seen from a distant star	B)	Doesnot depend on direction of projection
3)	Escape velocity from Earth	C)	Remains constant
4)	Gravitational potential dure to earth at a particular point	D)	Depend on direction of projection

1) $1-A,B, 2-A,C, 3-C, 4-A,D$ 2) $1-A,B, 2-C, 3-A,B 4-C$

3) $1-A,B, 2-C,D, 3-A,B, 4-C$ 4) $1-A,C, 2-C,D, 3-A,B, 4-C$

Key: 2

Sol: 1) Acceleration due to gravity is different at different points and it doesnot depend on

direction of projection
2) Angular momentum of earth remains constant about the sun
3) Escape velocity depends on acceleration due to gravity and is independent of direction of projection
4) Gravitational potential will remain constant.

09. An artificial satellite is in circular orbit around the earth, one of the rockets of satellite is momentarily fired, the direction of firing of rocket is mentioned in column – I and corresponding change are given in column – II match the matrix

Column – I		**Column - II**	
1)	Toward earth centre	A)	Orbit changes and becomes elliptical
2)	Away from earth centre	B)	Orbit plane changes
3)	At right angle to the plane of orbit	C)	Semi major axis of orbit increases
4)	In forward direction	D)	Energy of earth satellite system increases

(18 – March – 2021 (E)) (D)

1) $1-A,2-A,B,C,3-A,B,C,D,4-A,C,D$
2) $1-A,B,2-A,B,C,3-A,B,C,D,4-A,C,D$
3) $1-A,B,2-A,B,3-A,B,C,D,4-A,C$
4) $1-A,B,2-A,B,3-A,C,D,4-A,C,D$

Key:
Sol: $1-A,2-A,B,C,3-A,B,C,D,4-A,C,D$

10. Match the following

Column – I	Column – II
i) Gravitational potential	a) on the surface of planets with density ratio 1 : 2
ii) Escape velocity	b) conservation of angular momentum
iii) Ratio of the acceleration due to gravity 1: 3	c) varies with the reference point
iv) Orbiting satellite	d) does not depend on the angle
	e) Similar to an atom

1) $i \to a,b;ii \to c;iii \to c,d;iv \to b,c$ 2) $i \to c;ii \to c,d;iii \to d;iv \to e$
3) $i \to c,d;ii \to d;iii \to a;iv \to b,c$ 4) $i \to a,b;ii \to b,c;iii \to c,d;iv \to e$

Key:
Sol: Conceptual

11.

Column – I		**Column – II**	
a)	Acceleration due to gravity at north pole of earth when earth rotates with speed ‘w’	p)	g
b)	Acceleration due to gravity at height ‘x’ from surface of earth	q)	$g\left[1-\frac{x}{R}\right]$
c)	Acceleration due to gravity at depth x	r)	$g\left[1-\frac{2x}{R}\right]$
d)	Acceleration due to gravity at equator due to rotation with speed ‘w’	s)	$g-Rw^2$

(31 – Jan – 2023(E)) (D)

1) $a \to q,b \to p,c \to r,d \to s$ 2) $a \to p,b \to r,c \to q,d \to s$
3) $a \to p,b \to q,c \to r,d \to s$ 4) $a \to r,b \to q,c \to s,d \to p$

Key: **2**
Sol: Conceptual