MATHEMATICS

Max. Marks: 60

SECTION 1

- This section contains **Four (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
- *Full Marks* : +3 If ONLY the correct option is chosen;
- *Zero Marks* : 0 If the none of the options is chosen (i.e. the question is unanswered);
- *Negative Marks* : -1 In all other cases.

39. Tangents are drawn from $P(1,8)$ to the circle $x^2+y^2-6x-4y-11=0$ touch the circle at points A and B, respectively. The radius of the circle which passes through the points of intersection of circles $x^2+y^2-2x-6y+6=0$ and $x^2+y^2+2x-6y+6=0$, and intersects the circum circle of the ΔPAB orthogonally is equal to

A) $\frac{\sqrt{73}}{4}$ **B)** $\frac{\sqrt{71}}{2}$ **C)** 3 **D)** 2

40. From points on the line $3x-4y+12=0$, tangents are drawn to circle $x^2+y^2=4$. Then the chords of contact pass through a fixed point. The slope of the chord of the circle having this fixed point as its midpoint is

A) $\frac{4}{3}$ **B)** $\frac{1}{2}$ **C)** $\frac{1}{3}$ **D)** $\frac{3}{4}$

41. let L = 0, be a line passing through (1, 2) and the point of intersection of this line with $x+y=4$ is at a distance of $\frac{\sqrt{6}}{3}$ units from (1, 2). Then the square of distance of the point (1, 0) from the line $x-y+1=0$, measured in the direction of L = 0.

A) 6 **B)** 8 **C)** 3 **D)** 2

42. Given $A\equiv(1,1)$ and AB is any line through it cutting x-axis in B. If AC is perpendicular to AB and meet the y-axis in C, then the locus of mid point P of BC satisfies the equation ________

A) $x+y=1$ **B)** $x+y=4$ **C)** $x+y=2xy$ **D)** $2x+3y=xy$

SECTION 2

- This section contains **THREE (03)** questions stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
- ***Full Marks: +2*** If ONLY the correct numerical value is entered at the designated place;
- ***Zero Marks: 0*** In all other cases.

Question Stem for Question Nos. 43 and 44

Question Stem

Let $L_1: 4x-3y+13=0$, $L_2: 4x-3y=37$, $L_3: 3x+4y=34$ are three lines in xy plane and $L_4: (1+\lambda)x+(1-\lambda)y=24$ is a variable line. $P(a,b)$ is centre of circle which touches lines L_1, L_2 and L_3

On the basis of above information, answer the following questions:

43. Maximum value of a + b is_____

44. If L_1, L_2, L_3 and L_4 form a quadrilateral, then the value of λ for which slope of line L_4 takes least positive integral value is

Question Stem for Question Nos. 45 and 46

Question Stem

Let S_1 and S_2 be two fixed externally tangent circles with radius 2 and 3 respectively. Let S_3 be a variable circle internally tangent to both S_1 and S_2 at point A and B respectively. The tangents to S_3 at A and B meet at T and given TA = 4.

45. The square of the radius of circle S_3 is

46. If the area of circle circumscribing ΔTAB is $k\pi$, then k is

Question Stem for Question Nos. 47 and 48

Question Stem

Let L_1 be a lien $5x-7y=35$ which cuts x and y axis at A & B respectively. Variable line L_2, which is perpendicular to L_1 cuts x and y axis at C & D respectively. Locus of point of intersection of lines joining AD and BC is the curve S

47. Area enclosed by curve S is $\frac{K\pi}{2}$ then the value of K is_____

48. Coordinates of a point P, which is farthest from origin, on S is (a,b) then the value of $a-b$ is_________

SECTION 3

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
- ***Full Marks*** ***: +4*** If only (all) the correct option(s) is (are) chosen;
- ***Partial Marks*** ***: +3*** If all the four options are correct but **ONLY** three options are chosen,
- ***Partial Marks*** ***: +2*** If three or more options are correct but **ONLY** two options are chosen, both of which are correct:
- ***Partial Marks*** ***: +1*** If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
- ***Zero Marks*** ***: 0*** If unanswered;
- ***Negative Marks: -2*** In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to the correct answer, then

Choosing ONLY (A), (B) and (D) will get +4 marks;
Choosing ONLY (A), will get +1 mark;
Choosing ONLY (B), will get +1 mark;
Choosing ONLY (D), will get +1 mark;
Choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
Choosing any other option(s) will get -2 marks.

49. O, A, C, B are the vertices of a square taken in anticlockwise order on x-y plane where O is the origin, A is on positive x-axis. Line through point A intersect the diagonal OC at D internally, side OB at E internally. Given that $AD:DE=4:3$, $AD=5$ units and the square lies completely in the first quadrant. Which of the following is/are true?

A) Area of square OACB is 49 square units

B) If O' be the reflection of O in the line AE then coordinate of circumcentre of the triangle $AO'E$ is $\left(\frac{7}{2},\frac{21}{8}\right)$.

C) Area of square OACB is 64 square units

D) If O' be the reflection of O in the line AE then coordinate of circumcentre of the triangle $AO'E$ is $\left(\frac{901}{25},\frac{371}{100}\right)$.

50. Let ABCD be a square such that vertices A, B, C, D lie on circles

$x^2+y^2-2x-2y+1=0$, $x^2+y^2+2x-2y+1=0$, $x^2+y^2+2x+2y+1=0$ and $x^2+y^2-2x+2y+1=0$ respectively with centre of square being origin and sides are parallel to coordinate axes. The length of side of such square can be

A) $2-\sqrt{2}$ **B)** $2+\sqrt{2}$ **C)** $3-\sqrt{3}$ **D)** $3+\sqrt{3}$

51. Consider three distinct lines

$x+\lambda y+6=0$

$2x+y-3=0$

$\lambda x+2y+5=0$

let m denotes number of possible value of λ for which given lines are concurrent and n denotes number of possible values of λ for which given lines do not form a triangle, then

A) m=2 **B)** m = 3 **C)** n = 6 **D)** n = 7

52. If line $L:(3x-4y-25=0)$ touches the circle $S:(x^2+y^2-25=0)$ at P and L is common tangent of circles $S=0$ and $S_1=0$ at P and $S_1=0$ passes through $(5,-6)$, then

A) Centre of $S_1=0$ is $\left(\frac{27}{7},\frac{-36}{7}\right)$

B) Length of tangent from origin to $S_1=0$ is $\sqrt{\frac{275}{7}}$

C) Centre of $S_1=0$ is $\left(\frac{27}{7},-\frac{16}{7}\right)$

D) Length of tangent from origin to $S_1=0$ is $\sqrt{\frac{375}{7}}$.

53. Consider two circles $S_1=0$ and $S_2=0$ each of radius 1 unit touching internally the sides of ΔOAB and ΔABC respectively where $O=(0,0)$; $A=(0,4)$ and B, C are points on positive x – axis such that $OB<OC$, then which of the following is/are CORRECT

A) If the angle subtended by $S_1 = 0$ at the point A is θ then $\cos\theta$ is equal to 4/5

B) Length of tangent front A to the circle $S_2 = 0$ is 9/2

C) If one of the diameter of $S_2 = 0$ is chord of the circle $S_3 = 0$ whose centre is $\left(\frac{3}{2}, 3\right)$ and radius of $S_3 = 0$ is r then $\frac{3}{4}r$ is 9/4

D) Radius of the smallest circle that contains both the circles $S_1 = 0$ and $S_2 = 0$ is 9/4

54. Let $E = \left\{(x, y) : x^2 + y^2 - 2y - 39 = 0\right\} - \{(-2,7),(2,-5),(6,3),(-6,3)\}$.

Let F be the set of all straight line segments joining a pair of distinct points of E and passing through $R(1,1)$. E' be set of midpoints of the line segments in the set F. Then, which of the following is/are INCORRECT?

A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ doesnot lies in E'

B) $\exists$ atleast one line segment of set of F which lies on $6x + y = 7$

C) $\exists$ atleast one line segment of set of F which lies on $2x - 5y = -3$

D) $(1,1)$ does not lie in E'.

SECTION 4

- This section contains **THREE (03)** question.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer the using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
- ***Full Marks*** : ***+4*** If ONLY the correct integer is entered;
- ***Zero Marks*** : ***0*** In all other cases.

55. In a $\Delta^{le}\ ABC$, $x + y + 2 = 0$ is the perpendicular bisector of side AB and it meets AB at $(-1,-1)$. If $x - y - 1 = 0$ is $\perp^{lar}$ bisector of side AC and it meets AC at $(2,1)$, P is mid point of BC and distance of P from orthcentre of Δ^{le} ABC is k. Then $k^2 - 9$ is

____

56. Let line $4x+5y=20$ intersect x-axis at A and the y-axis at B. A line L intersect AB and OA at points C and D respectively. The least value of CD^2 for which line L divides the area of ΔOAB into two regions of equal area is $a\sqrt{41}-b$ where $a,b\in N$, then $\frac{b}{a}$ is equal to

57. In a right angled triangle BC = 5, AB = 4, AC = 3, Let S be the circum circle. Let S_1 be the circle touching both sides AB and AC and circle S internally. Let S_2 be the circle touching the sides AB and AC(extended sides) of triangle ABC, and touching the circle S externally if r_1, r_2 are radii of circle S_1 and S_2 respectively, then $\sqrt{1+r_1r_2}$ is

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Sec: Sr.S60_NUCLEUS & STERLING-BT | 2021_P1 | Date: 02-10-2022

Time: 09.00Am to 12.00Noon | PTA-04 | Max. Marks: 180

KEY SHEET

PHYSICS

1	**A**	2	**B**	3	**B**	4	**A**	5	**8.94**	6	**35.77 to 35.78**
7	**2.00**	8	**7.00**	9	**8.00**	10	**5.00**	11	**A,B,D**	12	**A,C**
13	**B,C,D**	14	**B,D**	15	**B**	16	**B,C,D**	17	**8**	18	**5**
19	**9**										

CHEMISTRY

20	**B**	21	**C**	22	**C**	23	**C**	24	**3**	25	**7**
26	**101 to 102**	27	**24**	28	**77 to 78**	29	**112 to 114**	30	**B,D**	31	**C,D**
32	**A,B,C**	33	**B,C**	34	**A,B,C,D**	35	**C,D**	36	**5**	37	**8**
38	**3**										

MATHEMATICS

39	**A**	40	**D**	41	**B**	42	**A**	43	**17**	44	**1.4**
45	**64**	46	**20**	47	**37**	48	**12**	49	**A**	50	**A,B**
51	**A,C**	52	**A,B**	53	**A,B,C,D**	54	**A,B,C,D**	55	**4**	56	**5**
57	**5**										

MATHEMATICS

39. Equation of circum circle of ΔPAB is

$(x-1)(x-3)+(y-8)(y-2)=0 \Rightarrow x^2+y^2-4x-10y+19=0$ given that the circle of form $x^2+y^2-2x-6y+6+\lambda(x^2+y^2+2x-6y+6)=0$ cuts the circle

$x^2+y^2-4x-10y+19=0$ orthogonally. $-2\left(\frac{2\lambda-2}{\lambda+1}\right)-5(-6)=19+6 \Rightarrow \lambda=-9$

$\therefore$ required circle is $x^2+y^2+\frac{5x}{2}-6y+6=0$ $\therefore$ radius $=\frac{\sqrt{73}}{4}$.

40. Let $P\left(h,\frac{3h+12}{4}\right)$ be any point on the line, whose chord of contact is

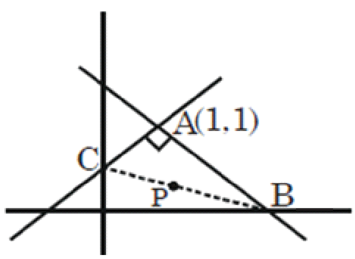
$xh+\left(\frac{3h+12}{4}\right)y-4=0 \Rightarrow h(4x+3y)+(12y-16)=0$ but for all values of $h \in R$, this

chard always passes through $Q(-1,4/3)$ $\therefore$ slope of $OQ=\frac{-4}{3}$;

Hence the slope of chord with point Q as midpoint is $\frac{3}{4}$.

41. $(1+\frac{\sqrt{6}}{3}\cos\theta, 2+\frac{\sqrt{6}}{3}\sin\theta)$ lies on $x+y=4 \Rightarrow \tan\theta=2\pm\sqrt{3}$, now $(1+r\cos\theta, r\sin\theta)$ lies on $x-y+1=0 \Rightarrow r^2=8$.

42.



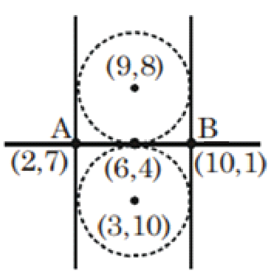
Let $P\equiv(h,k)$, P is the circumcentre of ΔABC

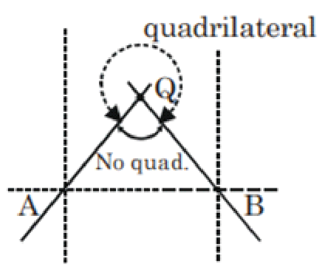
$B\equiv(2h,0)$, $C\equiv(0,2k)$

Now $M_{AC}.M_{AB}=-1$ $\left(\frac{1-2k}{1}\right)\left(\frac{1}{1-2h}\right)=-1$

$1-2k=2h-1 \Rightarrow h+k=1 \Rightarrow x+y=1$

43.

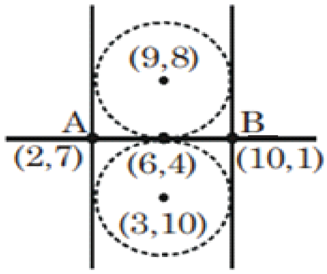


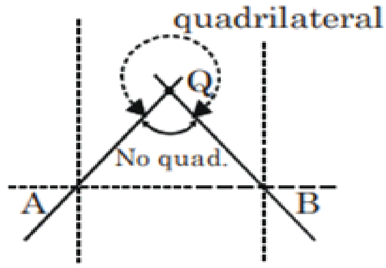


$a+b=13$ or 17

L_4 passes through $Q(12,12)$

44.





$a+b=13$ or 17 $\quad$ L_4 passes through Q(12,12)

45. Draw the figure: AT and BT are radical axis to C_3 and C_1 and C_3 and C_2

$$r_3 = TA\tan\left(\frac{\theta_1+\theta_2}{2}\right) = 4\left(\frac{\frac{1}{2}+\frac{3}{4}}{1-\frac{1}{2}\cdot\frac{3}{4}}\right) = 8$$

$TA = TB = TD = 41$ T is radical centre $\tan\left(\frac{\theta_1}{2}\right)=\frac{1}{2};\tan\left(\frac{\theta_2}{2}\right)=\frac{3}{4}$

TC_3 will be the diameter of circumcircle of ΔTAB

$$TC_3 = \frac{TA}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)} = 4\sqrt{5} \qquad \therefore 20\pi.$$

46. Draw the figure: AT and BT are radical axis to C_3 and C_1 and C_3 and C_2

$$r_3 = TA\tan\left(\frac{\theta_1+\theta_2}{2}\right) = 4\left(\frac{\frac{1}{2}+\frac{3}{4}}{1-\frac{1}{2}\cdot\frac{3}{4}}\right) = 8$$

$TA = TB = TD = 41$ T is radical centre $\tan\left(\frac{\theta_1}{2}\right)=\frac{1}{2};\tan\left(\frac{\theta_2}{2}\right)=\frac{3}{4}$

TC_3 will be the diameter of circumcircle of ΔTAB

$$TC_3 = \frac{TA}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)} = 4\sqrt{5} \qquad \therefore 20\pi.$$

47. $m_{L_1} = \frac{5}{7} \Rightarrow m_{L_2} = \frac{7}{5}$ $\qquad$ Let $L_2: 7x+57=c \Rightarrow C\left(\frac{c}{7},0\right);D\left(0,\frac{c}{5}\right)$

equation of $AD: \frac{x}{7}+\frac{5y}{c}=1$; $\qquad$ equation of BC: $\frac{7x}{c}-\frac{y}{c}=1$

eliminating c, we get locus: $\quad x^2+y^2-7x+5y=0$

This is a circle with center $\left(\frac{7}{2},\frac{5}{2}\right)$ and radius $\sqrt{\frac{37}{2}}$

Area $=\frac{\pi.37}{2}$ sq.units., Also circle passes through origin

farthest point: (7, -5)

48. $m_{L_1}=\frac{5}{7}\Rightarrow m_{L_2}=\frac{7}{5}$ Let $L_2: 7x+57=c\Rightarrow C\left(\frac{c}{7},0\right); D\left(0,\frac{c}{5}\right)$

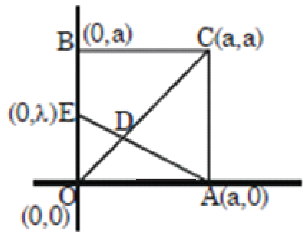
equation of AD: $\frac{x}{7}+\frac{5y}{c}=1$; equation of BC: $\frac{7x}{c}-\frac{y}{c}=1$

eliminating c, we get locus: $x^2+y^2-7x+5y=0$

This is a circle with center $\left(\frac{7}{2},\frac{5}{2}\right)$ and radius $\sqrt{\frac{37}{2}}$ Area $=\frac{\pi.37}{2}$ sq.units.

Also circle passes through origin farthest point: (7, -5)

49.



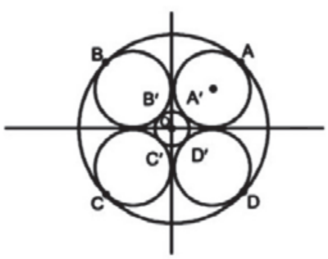
ΔODE is similar to ΔADC $\frac{3}{4}=\frac{\lambda}{a}\Rightarrow\lambda=\frac{3a}{4}$

$E\equiv\left(0,\frac{3a}{4}\right)$, $AE=\sqrt{a^2+\frac{9a^2}{16}}=\frac{5a}{4}$, Now $AD=\frac{4}{7}AE\Rightarrow 5=\frac{4}{7}\times\frac{5a}{4}$ $\Rightarrow a=7$

area of square OACB = 49 Equation of AE is $3x+4y=21$ $O'\equiv\left(\frac{126}{25},\frac{168}{25}\right)$

$\Delta AO'E$ will be right angle at O' so circumcentre of $\Delta AO'E$ is $\left(\frac{7}{2},\frac{21}{8}\right)$

50.



$OA=\sqrt{2}+1=\frac{d}{2}$ $d=2\sqrt{2}+2$

Side $=\frac{2\sqrt{2}+2}{\sqrt{2}}=2+\sqrt{2}$ $OA'=\sqrt{2}-1=\frac{d'}{2}$

$d'=2\sqrt{2}-2$ side $=\frac{2\sqrt{2}-2}{\sqrt{2}}=2-\sqrt{2}$.

51.

: For concurrency $\begin{vmatrix}1 & \lambda & 6\\ 2 & 1 & -3\\ \lambda & 2 & 5\end{vmatrix}=0$

$\Rightarrow 3\lambda^2+16\lambda-35=0\Rightarrow(\lambda+7)(3\lambda-5)=0$ $\Rightarrow\lambda=-7,\frac{5}{3}\Rightarrow m=2$

For not forming triangles either lines are parallel or concurrent

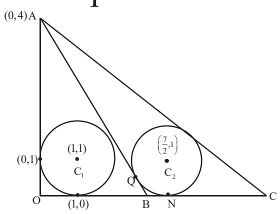
For concurrency $\begin{vmatrix} 1 & \lambda & 6 \\ 2 & 1 & -3 \\ \lambda & 2 & 5 \end{vmatrix} = 0 \quad \Rightarrow 3\lambda^2 + 16\lambda - 35 = 0 \Rightarrow (\lambda+7)(3\lambda-5) = 0 \Rightarrow = -7, \frac{5}{3} \Rightarrow m = 2$

For not forming triangles either lines are parallel or concurrent

$$\left.\begin{array}{l} \frac{1}{2} = \frac{\lambda}{1} \Rightarrow \lambda = 2, \frac{2}{\lambda} = \frac{1}{2} \Rightarrow \lambda = 4 \\ \frac{1}{\lambda} = \frac{\lambda}{2} \Rightarrow \lambda^2 = 2 \Rightarrow \lambda = \pm\sqrt{2} \end{array}\right\} n = 6$$

52. Conceptual.

53.



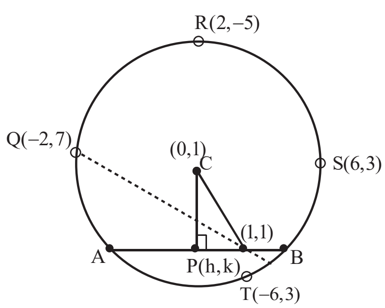
$AC_1 = \sqrt{10}$; $AT = 3\cos\frac{\theta}{2} = \frac{3}{\sqrt{10}}$ $\quad\therefore \cos\theta = \frac{4}{5}$

$\angle ABC = \frac{\pi}{2} + \theta$ from ΔBQC_2 $\quad BQ = \cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{1}{2}$;

from ΔOAB $\tan\theta = \frac{OB}{OA} \Rightarrow OB = 3 \quad AB = 5 \Rightarrow AQ = \frac{9}{2}$;

$BN = \frac{1}{2}$ also $r = 3$ $\quad$ Radius of smallest circle $\frac{1}{2}\left(2 + \frac{5}{2}\right) = \frac{9}{4}$.

54.



AB is variable line segment and element of set F

Locus of P is $(x)(x-1) + (y+1)(y-1) = 0$

$x^2 + y^2 = 2$ .... (1)

When A coincides with $Q(-2,7)$ then foot of $\perp$:

equation of AB $\frac{y-1}{x-1} = \frac{7-3}{-2+6} \Rightarrow \frac{y-1}{x-1} = \frac{4}{4} \Rightarrow y - 1 = x \Rightarrow x - y = 0$

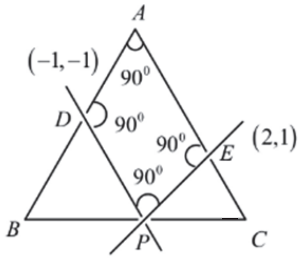
$P : \frac{x-0}{1} = \frac{y-1}{-1} = \frac{-(-1)}{2} \Rightarrow \frac{x}{1} = \frac{y-1}{-1} = \frac{1}{2} \quad x = \frac{1}{2}; y = -\frac{1}{2} + 1 \Rightarrow y = \frac{1}{2}$

If A or B coincide with R; Equation of AB : $\frac{y-1}{x-1} = \frac{-5-1}{2-1} \Rightarrow y - 1 = -6x + 6$

$6x + y = 7$. If A or B coincide with S (6, 3) ; Equation of AB :

$\frac{y-1}{x-1}=\frac{6-1}{3-1}\Rightarrow 2y-2=5x-5 \quad 5x-2y=3$.

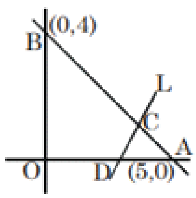
55. Clearly $x+y+2=0$ $x-y-1=0$ are perpendicular to each other $\therefore \angle BAC=90°$.



$\therefore$ A is the ortho centre of $\Delta^{le}ABC$

Mid point of $BC=P$ = Circum centre $=\left(\frac{-1}{2},\frac{-3}{2}\right)$. $\therefore PA^2=DE^2=\sqrt{9+4}=\sqrt{13}$.

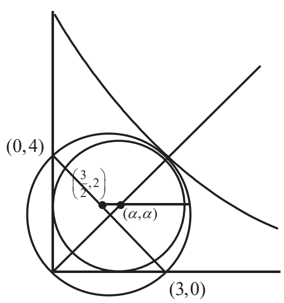
56. $C\left(\frac{20-5t}{4},t\right)$ $D(5,0)$



$\Delta=\frac{1}{2}(5-s)t=5 \qquad CD^2=\left(\frac{20-5t}{4}-s\right)+t^2$

$=\left[\left(\frac{10}{t}\right)-\frac{5t}{4}\right]^2+t^2=\frac{100}{t^2}+\frac{41}{16}t^2-25\geq 5\sqrt{41}-25$

57.



$$\left(\alpha-\frac{3}{2}\right)^2+(\alpha-2)^2=\left(\frac{5}{2}-\alpha\right)^2$$

$$\alpha^2-3\alpha+\frac{9}{4}+\alpha^2-4\alpha+4=\frac{25}{4}-5\alpha+\alpha^2$$

$$\alpha^2-2\alpha=0 \qquad \alpha=0,2 \quad \alpha=2=r_1=2$$

$$\left(\alpha'-\frac{3}{2}\right)^2+(\alpha'-2)^2=\left(\frac{5}{2}+\alpha'\right)^2$$

$$\alpha^2-3\alpha+\frac{9}{4}+\alpha^2-4\alpha+4=\frac{25}{4}+\alpha^2+5\alpha$$

$$\alpha'^2-12\alpha'=0 \qquad \alpha'=0,12 \qquad \alpha'=12=r_2 \Rightarrow \sqrt{1+r_1r_2}=5 \quad \text{Ans}$$