Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec:Sr.Super60_STERLING_BT | Paper -2(Adv-2022-P2-Model) | Date: 03-09-2023

Time: 02.00Pm to 05.00Pm | CTA-04 | Max. Marks: 180

KEY SHEET

MATHEMATICS

1	**9**	2	**2**	3	**5**	4	**6**	5	**8**	6	**4**
7	**2**	8	**4**	9	**AB**	10	**AB**	11	**ABCD**	12	**AB**
13	**ABD**	14	**AB**	15	**D**	16	**B**	17	**A**	18	**B**

PHYSICS

19	**4**	20	**3**	21	**5**	22	**2**	23	**6**	24	**3**
25	**3**	26	**5**	27	**BD**	28	**ACD**	29	**BD**	30	**AC**
31	**BC**	32	**AD**	33	**C**	34	**D**	35	**B**	36	**D**

CHEMISTRY

37	**3**	38	**4**	39	**4**	40	**9**	41	**8**	42	**3**
43	**3**	44	**3**	45	**BCD**	46	**BD**	47	**ABC**	48	**BD**
49	**ACD**	50	**BC**	51	**B**	52	**A**	53	**C**	54	**D**

SOLUTIONS
MATHEMATICS

1.

$$I=\int_0^{\frac{\pi}{12}}\ln(\tan 3x)\,dx \quad =\int_0^{\frac{\pi}{12}}\ln\left\{\tan x.\tan\left(\frac{\pi}{3}+x\right)+\tan\left(\frac{\pi}{3}-x\right)\right\}dx$$

$$=\int_0^{\frac{\pi}{12}}\ln\tan x\,dx+\int_0^{\frac{\pi}{12}}\ln\tan\left(\frac{\pi}{3}+x\right)dx+\int_0^{\frac{\pi}{12}}\ln\tan\left(\frac{\pi}{3}-x\right)dx$$

$$=I_1+\int_{\frac{\pi}{3}}^{\frac{5\pi}{12}}\ln(\tan x)\,dx-\int_{-\frac{\pi}{3}}^{-\frac{\pi}{4}}\ln\tan(-x)\,dx \qquad =I_1+\int_{\frac{\pi}{3}}^{\frac{5\pi}{12}}\ln(\tan x)\,dx+\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\ln(\tan x)\,dx$$

$$=I_1+\int_{\frac{\pi}{4}}^{\frac{5\pi}{12}}\ln(\tan x)\,dx \qquad x=\frac{\pi}{2}-t \quad =I_1+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\ln(\tan t)\,dx$$

$$=I_t+\int_{\frac{\pi}{4}}^{0}\ln(\tan t)\,dx+\int_0^{\frac{\pi}{4}}\ln(\tan t)\,dx$$

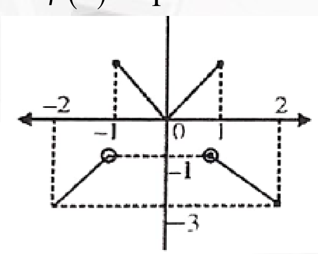
$$=I_1-\int_0^{\pi/4}\ln(\tan t)\,dx+I_t=2I_t-3I\Rightarrow 4I=2I_t\Rightarrow I=\frac{1}{2}I_t\Rightarrow K=\frac{1}{2}$$

2 $$\int_4^8\frac{f'(x)}{(f(x))^2}dx=\left.\frac{1}{f(x)}\right]_4^8=2 \qquad \int_4^8\left(\left(\frac{f'(x)}{(f(x))^2}\right)^2-\frac{f'(x)}{(f(x))^2}+\frac{1}{4}\right)=dx=0$$

$$\int_4^8\left(\frac{f'(x)}{(f(x))^2}-\frac{1}{2}\right)^2=0\Rightarrow\frac{f'(x)}{(f(x))^2}=\frac{1}{2} \qquad \Rightarrow\frac{-1}{f(x)}=\frac{x}{2}-6$$

3. $f(3+x)=f(1-x)\Rightarrow f(3+x)=f(x-1)\Rightarrow f(x+4)=f(x)\,\forall x\in R$

$\Rightarrow f(x)$ is periodic with period 4 and also even



$$f(x)=\begin{cases}x & :0\le x\le 1\\ 1-2x & :1<x\le 2\end{cases}$$

$$\int_0^{100}f(x)\,dx=25\int_{-2}^{2}f(x)\,dx=50\int_0^2 f(x)\,dx \qquad f(x)=\begin{cases}|x| & :0\le x<1\\ 1-2|x| & :1<x\le 2\end{cases}$$

$$=50\left[\left(\frac{1}{2}\times1\times1\right)-\left(\frac{1}{2}(1+3)1\right)\right] \quad \mathrm{I}=-\frac{150}{2}=-75$$

$$\mathrm{D}=25\times2=50 \qquad 2\mathrm{D}+\mathrm{I}=100-75=25$$

4. $$\lim_{x\to 0}\frac{(x-\sin x)\left(2x-\frac{\sin 2x}{2}\right)\left(3x-\frac{\sin 3x}{3}\right)........\left(nx-\frac{\sin nx}{n}\right)}{x^m}=20$$

$$\lim_{x\to 0}\frac{\left(\frac{x-\sin x}{x^3}\right)\left(2x-\frac{\sin 2x}{2}\right)\left(3x-\frac{\sin 3x}{3}\right)........\left(n-\frac{\sin nx}{nx}\right)}{x^{m-(n+2)}}=20$$

For limit to be exist

m must be equal to n+2

$$\frac{1}{6}(2-1)(3-1).......(n-1)=20$$

$$(n-1)!=120\Rightarrow n-1=5\Rightarrow n=6$$

5. $$\int\left(2x^6+15x^4+2x^2+3\right)\cos 2x\,dx$$

$$=\int\left(\underbrace{2x^6+2x^2}_{I}\right)\underbrace{\cos 2x\,dx}_{II}+\int\underbrace{\left(15x^4+3\right)}_{II}\underbrace{\cos 2x\,dx}_{I}$$

$$=\left(2x^6+2x^2\right)\frac{\sin 2x}{2}-\int\left(12x^5+4x\right)\frac{\sin 2x}{2}dx+$$

$$\cos 2x\left(\frac{15x^5}{5}+3x\right)+\int 2\sin 2x\left(\frac{15x^5}{5}+3x\right)dx$$

$$=\left(x^6+x^2\right)\sin 2x-\int\left(6x^5+2x\right)\sin 2x\,dx+$$

$$\left(3x^5+3x\right)\cos 2x+\int 6\sin 2x\left(x^5+x\right)dx$$

$$=\left(x^6+x^2\right)\sin 2x+3\left(x^5+x\right)\cos 2x+4\int x\sin 2x\,dx$$

$$=\left(x^6+x^2\right)\sin 2x+3\left(x^5+x\right)\cos 2x+4\left[x\left(\frac{-\cos 2x}{2}\right)+\int\frac{\cos 2x}{2}dx\right]$$

$$=\underbrace{\left(x^6+x^2+1\right)}_{f(x)}\sin 2x+\underbrace{\left(3x^5+x\right)}_{g(x)}\cos 2x+k$$

6. $$f(u)=\frac{1}{u^3-6u^2+11u-6}=\frac{1}{(u-1)(u-2)(u-3)},\text{ as } u(x)=\frac{1}{x},$$

$$So,\quad f=\frac{1}{\left(\frac{1}{x}-1\right)\left(\frac{1}{x}-2\right)\left(\frac{1}{x}-3\right)}$$

$$\therefore\ f \text{ is discontinuous at } x=1,\frac{1}{2},\frac{1}{3},0.$$

7. $$\left(x^2+1\right)e^x=t$$

8. $$f(x)=k(x+1)^2(x-1)^2(x-5)^2+4\Rightarrow f(2)=81k+4=85\Rightarrow k=1$$

$$f(x)=(x+1)^2(x-1)^2(x-5)^2+4\Rightarrow f(x)=2(x+1)(x-1)(x-5)\left(3x^2-10x-1\right)$$

$$\alpha+\beta=\frac{10}{3}\Rightarrow\lim_{x\to\frac{10}{3}}\left(\frac{x^2-9}{x^2-5x+6}\right)=\frac{19}{4}$$

9. $$f'(x)=-\sin x\cos(\cos x)-\cos x\sin(\sin x)>0$$

$$f\left(-\frac{\pi}{2}\right)=\cos 1,\ f(0)=1+\sin 1 \qquad f(0)-f\left(-\frac{\pi}{2}\right)=\sin 1-\cos 1+1>1$$

Now $f\left(0^{+}\right)<f(0)$ for maxima at $x=0 \Rightarrow \sin 1+1>\sin a+1 \Rightarrow \sin a<\sin 1$

Hence $a=1,3$

10.

$$f(x)=\sin^{-1}\left[\sqrt{1-\sqrt{1-x^2}}\right] \qquad x\in[-1,1], f(x)\in\left[0,\frac{\pi}{2}\right]$$

$$f(x)=\begin{cases}\frac{\pi}{2}, & x=-1\\ 0, & x\in(-1,1)\\ \frac{\pi}{2}, & x=1\end{cases}$$

$f(x)$ is discontinuous at x=1,-1 and hence non-differentiable at x=1,-1.

11. Given Curve is $ax^3-y+b=0 \Rightarrow \quad y=ax^3+b$

Let point P_1 be $\left(t_1, at_1^3+b\right)$

Slope of tangent $=\left.\frac{dy}{dx}\right|_{P_1}=3\,at_1^2$

$\therefore$ Equation of tangent is $y-\left(at_1^3+b\right)=3at_1^2\left(x-t_1\right)$

$\therefore$ Tangent meets curve at $P_2\left(t_2, at_2^3+b\right)$

$\therefore \left(at_2^3+b\right)-\left(at_1^3+b\right)=3at_1^2\left(t_2-t_1\right)$

$\Rightarrow \quad a\left(t_2^3-t_1^3\right)=3at_1^2\left(t_2-t_1\right)$

$\Rightarrow \quad t_2^2+t_1^2+t_2t_1=3t_1^2 \qquad (\because t_1\neq t_2)$

$\Rightarrow \quad t_2^2+t_2t_1-2t_1^2=0$

$\Rightarrow \quad \left(t_2+2t_1\right)\left(t_2-t_1\right)=0$

$\Rightarrow \quad t_2=-2t_1$

Similarly, $\quad t_3=-2t_2$

$\therefore$ abscissae are in G.P. for all values of a and b.

12. Taking log both the sides we get

$$\ln\beta=\lim_{x\to\infty}\left[\frac{\ln\{(1!)(2!)\ldots\ldots\ldots(n\ldots.)\}}{n^2}-\alpha\ln(n)\right]$$

$$=\lim_{x\to\infty}\left[\frac{\ln 1+(\ln 1+\ln 2)+(\ln 1+\ln 2+\ln 3)+\ldots\ldots.}{n^2}-\alpha\ln(n)\right]$$

$$=\lim_{x\to\infty}\left[\frac{n\ln 1+(n-1)\ln 2+(n-2)\ln 3+\ldots\ldots}{n^2}-\alpha\ln(n)\right]$$

$$=\lim_{x\to\infty}\left[\frac{1}{8}\sum_{r=1}^{\infty}\left(\frac{n+1-r}{n}\right)\ln\left(\frac{r}{n}\right)-\alpha\ln(n)+\frac{n(n+1)}{2n^2}\ln(n)\right]$$

$$=\lim_{x\to\infty}\left[\frac{1}{n}\sum_{r=1}^{n}\left(\frac{n+1-r}{n}\right)\ln\left(\frac{r}{n}\right)+\left(\frac{1}{2}+\frac{1}{2n}-\alpha\right)\ln n\right]$$

As limit exists $\quad \alpha=\frac{1}{2}$ and $\ln\beta=\int_0^1(1-x)\ln x\,dx=-\frac{3}{4} \qquad \Rightarrow \beta=e^{-\frac{3}{4}}$

13. a) $f(x)$ being twice differentiable it is continuous but can't be constant throughout the domain .

Hence we can find $x \in (r,s)$ such that $f(x)$is one one. (a0 is true

b) By lagrang's Mean value theorem for $f(x)$ in $[-4.0]$ there exists

$x_0 \in (-4,0)$ such that $f'(x_0) = \dfrac{f(0)-f(-4)}{0-(-4)}$

$\Rightarrow |f(x_0)| = \left|\dfrac{f(0)-f(-4)}{4}\right|$

$-2 \le f(x) \le 2, \therefore -4 \le f(0)-f(-4) \le 4$

$\Rightarrow |f'(0)| \le 1, \therefore (b)$ is true

c)If we consider $f(x) = \sin\left(\sqrt{85}x\right)$ then $f(x)$ satisfies the given condition

$[f(0)]^2 + [f(0)]^2 = 1$ but $\lim\limits_{x\to\infty}\left(\sin\sqrt{85}x\right)$ does not exist (c) is false

d) Let us consider $g(x) = [f(x)]^2 + [f'(x)]^2$ By lagrange's Mean Value theorem $|f(x) \le 1|$

Also $|f(x_1)| \le 2$ as $f(x) \in [-2,2]$ $\therefore g(x_1) < 5,$ for $x_1 \in (-4,0)$

similarly $g(x_2) \le 5,$ for $x_2 \in (0,4)$ Also $g(0) = 85$

Hence $g(x)$ has maxima in (x_1, x_2) say at α such that $g'(\alpha) - 0$ and $g(\alpha) \ge 85$

$g'(\alpha) = 0 \Rightarrow 2f(\alpha)f'(\alpha) + 2f'(\alpha)f''(\alpha) = 0 \Rightarrow 2f'(\alpha)[f(\alpha) + f''(\alpha)] = 0$

If $f'(\alpha) = 0 \Rightarrow g(\alpha) = [f(\alpha)]^2$ and $[f(\alpha)]^2 \le 4$

$\therefore g(\alpha) \ge 85$ (is not possible)$\Rightarrow f(\alpha) = f'(\alpha) = 0$ for $\alpha \in (x_1, x_2) \in (-4,4)$

Hence (d) is true

14.

$(f(x)+f(y)-3)\dfrac{f(x)+f(y)}{x-y} = \dfrac{1}{\sqrt{x}-\sqrt{y}}$

Apply $\underset{x\to y}{Lt}$ on B.S

$\Rightarrow (2f(y)-3)f'(y) = \dfrac{1}{2\sqrt{y}} \Rightarrow \int (2f(y)-3)f'(x)dx = \int \dfrac{1}{2\sqrt{x}}dx$

$\Rightarrow (f(x))^2 - 3f(x) = \sqrt{x} + c$

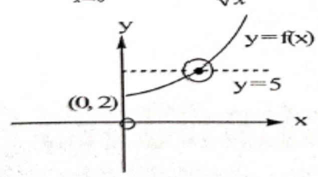
Now $f(0) = 2 \Rightarrow c = -2 \Rightarrow (f(x))^2 - 3f(x) = \sqrt{x} - 2$

$\Rightarrow (f(x))^2 - 3f(x) + (2-\sqrt{x}) = 0$

$\Rightarrow f(x) = \dfrac{3 \pm \sqrt{9 - 4(2-\sqrt{x})}}{2} = \dfrac{3 \pm \sqrt{1+4\sqrt{x}}}{2}$

$f(x) = \dfrac{3+\sqrt{1+4\sqrt{x}}}{2}$

$G.L = \underset{x\to 0^+}{Lt} \dfrac{2+\sqrt{1+4\sqrt{x}} - 2e^{\sqrt{x}}}{\sqrt{x}} = \underset{x\to 0^+}{Lt} \dfrac{2\left(\sqrt{1+4\sqrt{x}} - 2e^{\sqrt{x}}\right)}{\sqrt{x}} = 2(1) = 2$



Also $f'(x) > o, f''(x) > 0 \Rightarrow f(x)$ one one, $f(x) = 5$ has one solution

15. $f(2+x) = f(2-x) \Rightarrow f(4-x) = f(x)$.........(1)

From $f(7+x) = f(7-x) \Rightarrow f(14-x) = f(x)$.........(2)

From equation (1) and (2) we get $f(x+10)=f(x)$ so period is 10

Now replacing x by $x+10$ and then $x-10$ is equations (3) continuity in thus ways $f(x+10n)=f(x)....(4)$

For $\pm1,\pm2,\pm3,.....$

Since $f(0)=0$ equation (4)

$f(\pm10)=f(\pm20)......=f(\pm1000)=0$ so total 201

Now $x=0$ in equation $(1)\, f(x)=f(0)=o$ from equation (1) we have 200 more roots

16. $\because y=\dfrac{x}{x^2+y}\Rightarrow y^2+x^2y-x=0$

$$\because \frac{dy}{dx}=\frac{-(2xy-1)}{(x^2+2y)}\Rightarrow \frac{dy}{dx}=\frac{-\left(2x\dfrac{x}{x^2+y}-1\right)}{(x^2+y+y)}=-\frac{-(2x^2-x^2-y)/x^2y}{\left(\dfrac{x}{y}+y\right)}$$

$$=\frac{-y(x^2-y)}{(x^2+y)(x+y^2)}\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{(y-x^2)}{(x^2+y)(x+y^2)}$$

$$=\frac{d}{dx}(\ln y)=\frac{(y-x^2)}{(x^2+y)(x+y^2)}\Rightarrow \ln y+c\int\frac{(y-x^2)dx}{(x^2+y)(x+y^2)}$$

17. $$\lim_{n\to e}\left(\sum_{r=1}^{n}\left(\frac{n^2}{n^2+r^2}\left(\lim_{m\to e}\sum\left(\frac{1}{n^2+\dfrac{k^2}{m^2}}\right)\frac{1}{m}\right)\right)\right)=\lim_{n\to\infty}\left(\sum_{n=1}^{n}\frac{n^2}{n^2+r^2}\int_0^r\frac{1}{n^2+x^2}dx\right)$$

$$=\lim_{n\to\infty}\left(\sum_{r=1}^{n}\frac{n^2}{n^2+r^2}\tan^{-1}\left(\frac{r}{n}\right)\right)=\int_0^1\frac{\tan^{-1}x}{1+x^2}dx=\frac{\pi^2}{32}$$

18. $[x]\{x\}=ax^2$

Case-I $[x].\{x\}<0$ where $x<0$ while $ax^2\ge0\forall x\in R$ and $x=0$ is one solution but that does not affect the sum of the solution

Thus we need to look at (+ve) solutions for $n\le x<n+1$

$$ax^2-nx+n^2=0\Rightarrow x=\frac{n\pm\sqrt{n^2-4an^2}}{2a}=n\left(\frac{1\pm\sqrt{1-4a}}{2a}\right)$$

For (+Ve) roots we consider $x=\dfrac{n}{2a}\left(1-\sqrt{1-4a}\right)$

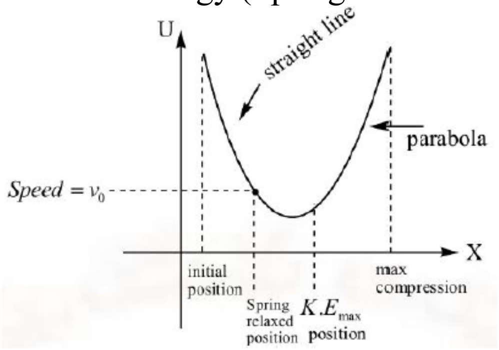
Since , sum of all solutions is 420 $\Rightarrow\sum\dfrac{n}{2a}\left(1-\sqrt{1-4a}\right)=420\Rightarrow\left(\dfrac{1-\sqrt{1-4a}}{2a}\right)\sum n=420$

$$\Rightarrow\left(\frac{1-\sqrt{1-4a}}{2a}\right)\left(\frac{n(n+1)}{2}\right)=420\Rightarrow\left(\frac{1-\sqrt{1-4a}}{2a}\right)406=420$$

(Considering n is 28 I,e $\dfrac{n(n+1)}{2}$ = 406 at $n=28$ which is nearer to 420) $\Rightarrow a=\dfrac{29}{900}$

PHYSICS

19. Potential energy (Spring + Gravitational) with position is as shown



The above graph is traversed twice (back and forth) so, v_0 is achieved four times.

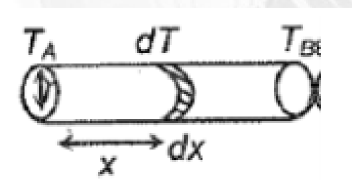
20. Radius of curvature $R=\frac{\left[1+(dy/dx)^2\right]^{3/2}}{d^2y/dx^2}=\frac{\left[1+x^2\right]^{3/2}}{1}$

$$R_m=1(at\ x=0)\ R_{min}=1(at\ x=0)$$

$$\mu mg=mv_{max}^2/R_{min}\Rightarrow v_{max}\sqrt{\left(\frac{9}{10}\right)(10)1}=3m/s$$

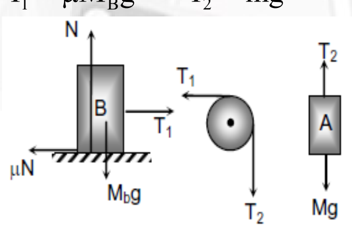
21. Intensity of the source at the cross-section A $\Rightarrow I=\frac{P}{4\pi(50r)^2}=\frac{1.25\times10^3}{4\pi(50r)^2}$

Power absorbed by the end A $T_B=\frac{0.003}{100000\times10^{-10}}=300K$



$$\int_0^j\frac{0.1T_A.dx}{4\times10^{-4}}=-\int_{T_A}^{T_B}TdT\quad\Rightarrow T_A=500\,K$$

22. $T_1=\mu M_Bg\qquad T_2=mg$



Due to friction on pull in limiting case, when block B is about to slide $T_2=T_1e^{\mu\frac{\pi}{2}}$

Solving equation m=2kg

23. $\Rightarrow\frac{1}{v}=\frac{1}{24}-\frac{1}{x}\qquad\frac{1}{v}=-\frac{1}{\left(\frac{24-x}{24x}\right)}$

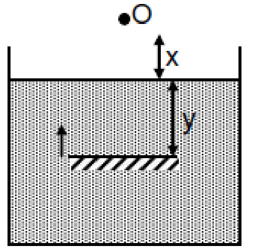
Object distance for silvered lens is $(14-x)+\frac{24x}{(24-x)}$ for image to be an object O, this distance must be equal to equivalent radius of mirror.

For (Reflecting lens is effectively mirror)

$$-\frac{2}{R_{eq}}=2\left(\frac{3}{2}-1\right)\left(\frac{1}{32}-\frac{1}{-32}\right)-\frac{3}{-32}\Rightarrow R_{eq}=-16\text{cm}\ \therefore 16=(14-x)+\frac{24x}{(24-x)}$$

$\Rightarrow x=6\,\text{cm}$

24. Apparent distance of mirror from O



$$=x+\frac{y}{\mu}$$

Distance of final image from O $=2\left(x+\frac{y}{\mu}\right)$

Velocity of image

$$=2\left(\frac{dx}{dt}+\frac{1}{\mu}\frac{dy}{dt}\right)=\frac{2}{\mu}\times 8=\frac{2}{4/3}\times 8=12\,\text{cm/s}$$

25. For $S_1S_2=2.5\lambda$ max path different = 2.5λ

Min path different =0

Between 2.5λ and 0 lie 2λ and $\lambda\Rightarrow$ two circular bright fringes

$n_1=2$

For $S_1S_2=5.7\lambda$ max path different = 5.7λ

Min path different =0

Between 5.7λ and 0 lie $5\lambda, 4\lambda, 3\lambda, 2\lambda, \lambda\Rightarrow$ Five circular bright fringes

$\Rightarrow n_2=5 \qquad \therefore n_2-n_1=5-2=3$

26. $\overline{PQ}=\sqrt{15^2+20^2}=25\,\text{m}$

$$25=\frac{V_P^2\sin 2(45)}{g}\overline{P}V_p=5\sqrt{10}\text{m/s}$$

$$\therefore V_A^2\sin^2\theta-2\times 10\times 12.5-\left(5\sqrt{5}\right)^2\Rightarrow V_A\sin\theta-5\sqrt{15}\text{m/s}$$

$$V_A\cos\theta=V_P\cos 45^\circ=5\sqrt{5}\text{m/s and }\theta=60^\circ$$

$$\therefore\overline{AB}=\frac{V_A^2\sin 2\theta}{g}=25\sqrt{3}$$

27. $$\left\{\begin{aligned}&P_1\left(A\frac{\ell}{2}\right)=2RT,\\&P_1\left(A\left(\frac{\ell}{2}+x\right)\right)=2RT,\end{aligned}\right\}\quad\ldots\ldots.(1)$$

$$\left\{\begin{aligned}&P_1A=K\frac{\ell}{2}\\&P_1A=K\left(\frac{\ell}{2}+x\right)\end{aligned}\right\}\quad\ldots\ldots.(2)$$

Now

$$\begin{Bmatrix} K\left(\frac{\ell}{2}\right)^2 = 2RT, \\ K\left(\frac{\ell}{2}+x\right)^2 = 2RT \end{Bmatrix} \qquad .... (3)$$

Work done by gas.

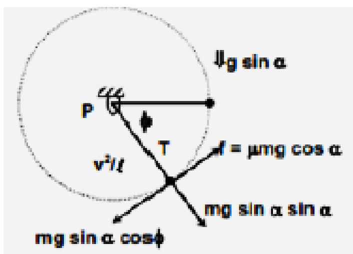
$$dw = pdy - (ky)dy \quad W - K\int_{\ell/2}^{(\ell/2+x)} ydy = \frac{K}{2}\left[\left\{\frac{\ell}{2}+x\right\}^2 - \left\{\frac{\ell}{2}\right\}^2\right]$$

Change in internal energy

$$\Delta U = nC_v\Delta T = 2.\frac{5}{2}R\left(T_1 - T_1\right)$$

$$\Delta Q = W + \Delta U = 6R\Delta T \qquad C = \frac{\Delta Q}{\Delta T} = 6R$$

28.



$$T - mg\sin\alpha\sin\phi = \frac{mv^2}{l} \qquad ....(i)$$

$$mg\sin\alpha\cos\phi - \mu mg\cos\alpha = m\frac{dv}{dt} \qquad ....(ii)$$

work energy theorem

$$(mg\sin\alpha)l\sin\alpha - (\mu mg\cos\alpha)l\phi = \frac{1}{2}mv^2 \quad ....(iii)$$

From (i) and (ii)

$$\Rightarrow T - mg\sin\alpha\sin\phi = a\,mg\sin\alpha\sin\phi - 2(\mu mg\cos\alpha)\phi$$

$$\Rightarrow T = mg(3\sin a\sin\phi - 2\mu\phi\cos\alpha)$$

$$\frac{dT}{d\phi} = 0 = 3\sin\alpha\cos\phi - 2\mu\cos\alpha$$

29.

B, D

$$\frac{\mu_4}{v} - \frac{\mu_1}{u} = \left(\frac{\mu_2-\mu_1}{R_1}\right) + \left(\frac{\mu_3-\mu_2}{R_2}\right) + \left(\frac{\mu_4-\mu_3}{R_3}\right)$$

$$\frac{1.8}{v} - \frac{1}{-25} = \left(\frac{1.2-1}{10}\right) + \left(\frac{1.5-1.2}{-30}\right) + \left(\frac{1.8-1.5}{\infty}\right)$$

$$\frac{1.8}{v}+\frac{1}{25}=\frac{0.2}{10}-\frac{0.3}{30}+0$$

$$\frac{1.8}{v}+\frac{1}{25}=\frac{1}{50}-\frac{1}{100}$$

$$\frac{1.8}{v}=\frac{1}{100}-\frac{1}{25}$$

$$\Rightarrow v=-60\text{ cm}$$

$$\text{Lateral Magnification } m=\left(\frac{\mu_1}{\mu_4}\frac{v}{u}\right)=\frac{1}{1.8}\left(\frac{-60}{-25}\right)$$

$$m=+\frac{4}{3}$$

$$\text{The size of image formed }=\frac{4}{3}x0.3=0.40\text{cm}.$$

30.

$h=0.8\sin 30^0=0.4m$

$\therefore \quad v^2=2gh$

(a) Just before,

$$T_1-mg\sin 30^0=\frac{mv^2}{R_1}(R_1=0.8m)$$

$$T_1=\frac{mg}{2}+\frac{m(2g)(0.4)}{0.8}=\frac{3mg}{2}$$

(b) Just after,

$$T_2-mg\sin 30^0=\frac{mv^2}{R_2}(R_2=0.4m)$$

$$T_2=\frac{mg}{2}+\frac{m(2g)(0.4)}{0.4} \quad \text{or} \quad T_2=\frac{5mg}{2}$$

31.

Initial tension in spring $kx=\frac{4mg}{7}$

FBD of block just after string is cut Since, $v = 0$

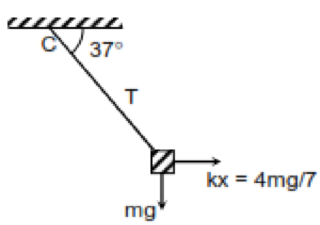
$\therefore$ acceleration along string is zero

$$T=k\cos 37^0+mg\sin 37^0=\frac{37}{7}N$$

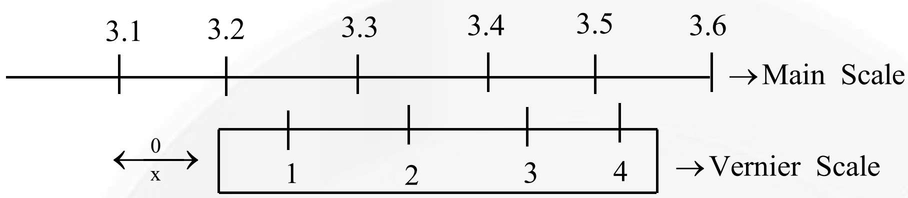
Acceleration is only perpendicular to spring

$$ma=mg\cos 37^0-kx\sin 37^0$$

$$\Rightarrow a=\frac{32}{7}m/s^2$$

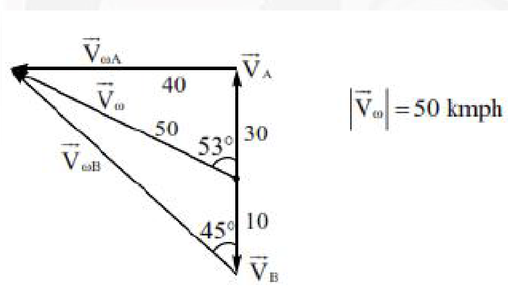


32.
AD
$\because$ Friction is static in nature.
f = 10N
Normal contact force = mg

33.


$x+4=(vsd)=x+0.95+0.90+0.85+0.80=x+3.5\,mm$

31 + 3.5 mm = 34.5 mm
34.5 < 34.5 + x < 35.5
34.5 + x = 35 mm
x = 0.5 mm
Reading = 3.1 cm + 0.5 mm = 3.15 cm

34.


35.
$\frac{\Delta Q}{\Delta t}=\frac{\Delta w}{\Delta t}=$ work done per unit time $=\frac{ka\,\theta}{L}$

Power = F x Velocity = $PAV'=\frac{nRT}{V}AV'$

where $V \to$ volume, $V' \to$ velocity

$\Rightarrow \frac{0.5R(300)}{V}AV'=\frac{ka\theta}{L}$

$\Rightarrow \frac{0.5R(300)}{A.\frac{L}{2}}AV'=\frac{ka\theta}{L}$

$\Rightarrow V'=\frac{ka}{R}\left(\frac{27}{300}\right)=\frac{k}{100R}$

36. if F=0

Then assuming no relative motion acceleration of $A+B=\frac{300}{15}=20\,m/s^2$

$\phi 20\,m/s^2>\mu g$ where $\mu=0.5$ and $g=10\,m/s^2$

$\therefore$ relative motion shall exist Hence $F=0N$

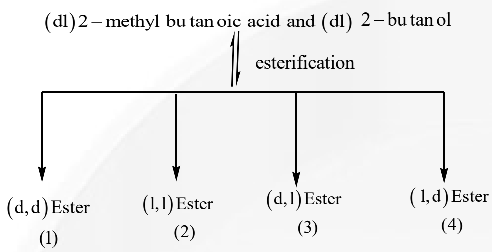
CHEMISTRY

37. 2.68 gm of (A) gives 14.08 gm of AgI

134 gm of (A) gives $\frac{14.08 \times 134}{2.68}$

$= 704 \text{ gm of AgI} \qquad = \frac{704}{235} = \text{mol of AgI} \qquad = 3(\text{OMe}) \text{ groups}$

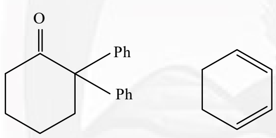
38.



Ester consists of two stereocenters. Chiral center during whole reaction are not effected, that's why all esters are optical active.

39. Diazomethane is used for methylating acidic groups, compound IV has enolic – OH group, hence it can also be methylated by CH_2N_2.

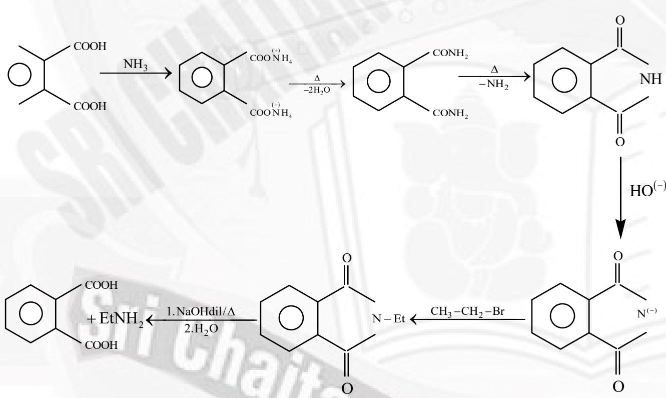
40.

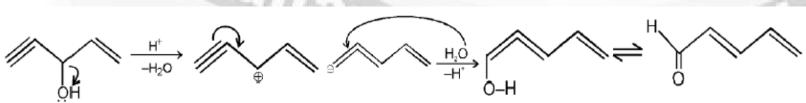


41. Conceptual

42. Conceptual

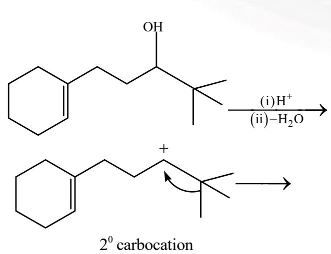
43.





44.

45.



46. The combination $C_6H_5Br + CH_3CH_2OH$ has non-reactive C_6H_5Br, while in the combination $C_6H_5OH + Me_3CBr, Me_3CBr$ being tert-halide will undergo elimination reaction rather than substitution. Hence, only combinations (a) and (c) can be used for preparing ether.

$C_6H_5OH + (CH_3)_2SO_4 \xrightarrow{SN} C_6H_5OCH_3;$

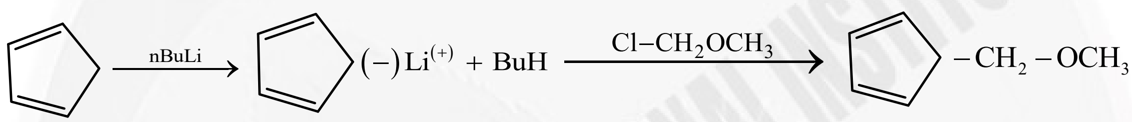
$p-NO_2C_6H_4Br + CH_3CH_2OH \xrightarrow{ArSN} p-NO_2C_6H_4OCH_2CH_3$

47.

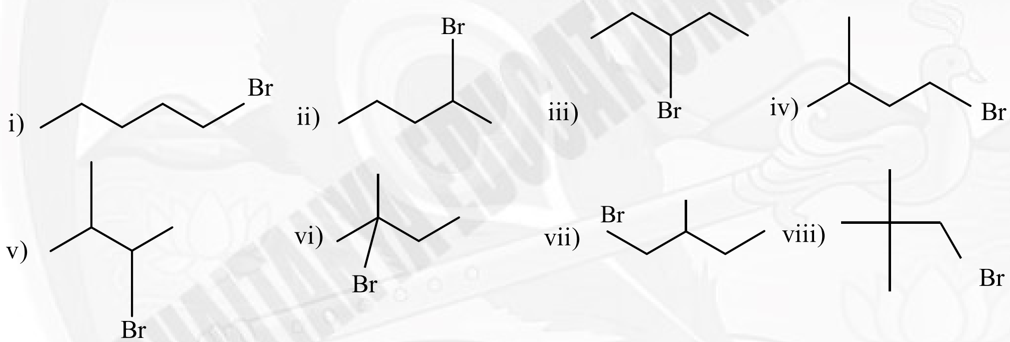
$$C_6H_5NH_2 + AlCl_3 \rightarrow C_6H_5\overset{+}{N}H_2\!-\!AlCl_3^{(-)} \xrightarrow{CH_3CL} \text{No reaction}$$

Substituent $(-\overset{+}{N}H_2AlCl_3^-)$
a meta director

48.

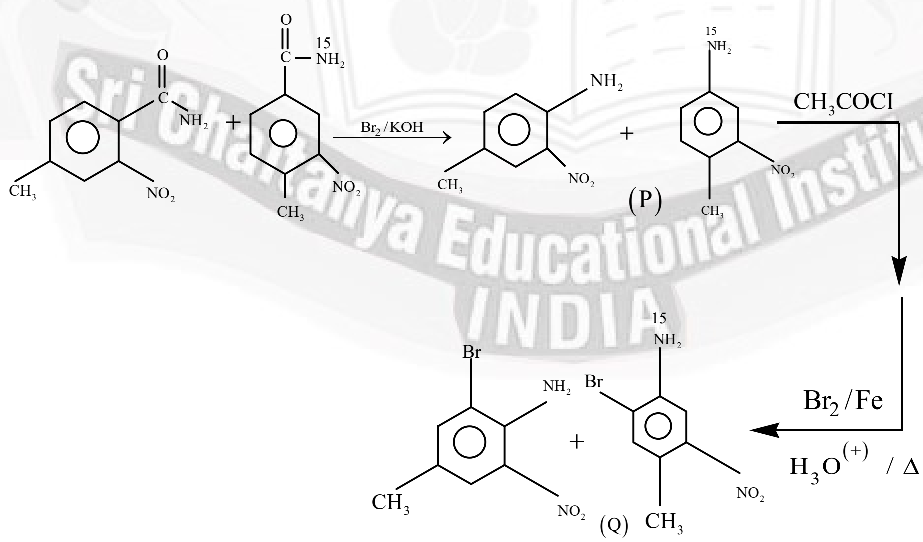


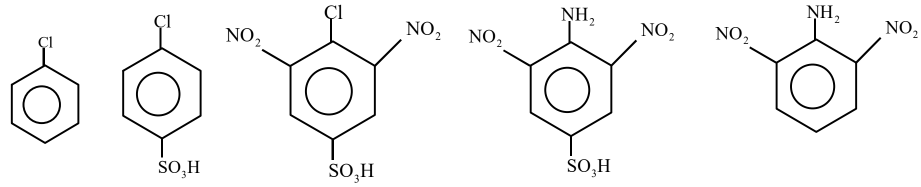
49.



Total 8 structural isomers.
(Viii) is inert towards E – 2
(ii) gives three alkenes in E – 2

50.



51. 

52.

$$CH_3CH_2OCH(CH_3)_2 \xrightarrow{\text{Cone.Hl}}$$
(A)

$$CH_3CH_2I + (CH_3)_2CHI$$

$$\downarrow NaOH \qquad \downarrow NaOH$$

$$\underset{(B)}{CH_3CH_2OH} + \underset{(C)}{(CH_3)_2CHOH}$$

$$\downarrow [O] \qquad \downarrow [O]$$

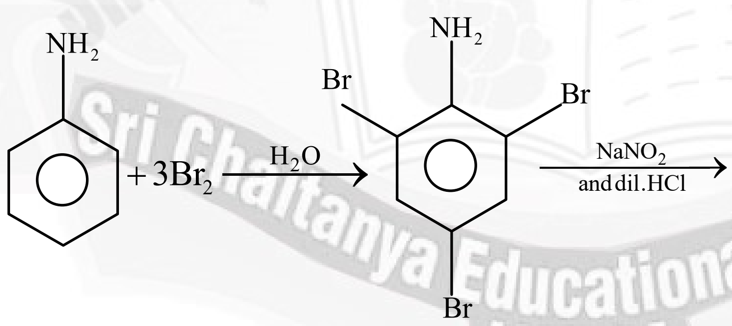
$$CH_3COOH \qquad (CH_3)_2CO$$

Hence the IUPAC name of ether is

$$CH_3-CH_2-O-\overset{2}{C}H-\overset{1}{C}H_3$$
$$|$$
$$3CH_3$$

2 – ethoxypropane

53. The C – O bond lenth in alcohols is 142 pm and in Phenol it is 136 pm. The C – O bond length in phenol is shorter than that in methanol due to the conjugation of unshraed pair of electrons on oxygen with the ring, which imparts double bond character to the C – O bond.

54. 

(2,4,6 – tribromoaniline)