Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.S60, ELITE ,TARGET & LIIT JEE-MAIN Date: 25-05-2022

Time: 09.00Am to 12.00Pm GTM-15 Max. Marks: 300

KEY SHEET

PHYSICS

1)	2	2)	2	3)	1	4)	3	5)	2
6)	2	7)	4	8)	1	9)	3	10)	3
11)	4	12)	2	13)	3	14)	4	15)	2
16)	1	17)	1	18)	3	19)	4	20)	1
21)	2	22)	20	23)	4	24)	35	25)	3
26)	4	27)	19	28)	1	29)	-113	30)	40

CHEMISTRY

31)	2	32)	2	33)	3	34)	1	35)	1
36)	2	37)	4	38)	1	39)	2	40)	4
41)	3	42)	2	43)	4	44)	4	45)	4
46)	2	47)	4	48)	3	49)	3	50)	1
51)	3	52)	8	53)	462	54)	6	55)	4
56)	3	57)	30	58)	2	59)	3	60)	8

MATHEMATICS

61)	3	62)	3	63)	3	64)	1	65)	4
66)	3	67)	2	68)	1	69)	1	70)	1
71)	2	72)	4	73)	3	74)	1	75)	3
76)	3	77)	3	78)	3	79)	4	80)	4
81)	1	82)	5	83)	3	84)	6	85)	1
86)	0	87)	13	88)	1	89)	11	90)	10

SOLUTIONS

PHYSICS

1. Conceptual

2. Conceptual

3. $t=\sqrt{\frac{L}{5}+\frac{L}{300}}\ dt=\frac{1}{\sqrt{5}}\frac{1}{2}L^{-1/2}dL+\left(\frac{1}{300}dL\right)\ dt=\frac{1}{2\sqrt{5}}\ \frac{1}{\sqrt{20}}dL+\frac{dL}{300}=0.01$

$dL\left(\frac{1}{20}+\frac{1}{300}\right)=0.01\ dL\left[\frac{15}{300}\right]=0.01\ dL=\frac{3}{16}$

$\frac{dL}{L}\times 100=\frac{3}{16}\times\frac{1}{20}\times 100=\frac{15}{16}\simeq 1\%$

4. Given, $g=10m/s^2$

Equation of trajectory of the projectile. $y=2x-9x^2$ ...(i)

In projectile motion, equation of trajectory is given by

$y=x\tan\theta_0-\frac{gx^2}{2v_0^2\cos^2\theta_0}$ ...(ii)

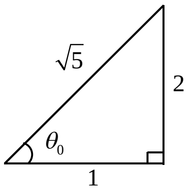
By comparison of Eqs. (i) and (ii), we get

$\tan\theta_0=2$ ...(iii)

and $\frac{g}{2v_0^2\cos^2\theta_0}=9$ or $v_0^2=\frac{g}{9\times 2\cos^2\theta_0}$ ...(iv)

From Eq. (iii), we can get value of $\cos\theta$ and $\sin\theta$

$\cos\theta_0=\frac{1}{\sqrt{5}}$ and $\sin\theta_0=\frac{2}{\sqrt{5}}$ (v)



Using value of $\cos\theta_0$ from Eq. (v) to Eq. (iv), we get

$v_0^2=\frac{10\times\left(\sqrt{5}\right)^2}{2\times(1)^2\times 9}=\frac{10\times 5}{2\times 9}\Rightarrow v_0^2=\frac{25}{9}\ or\ v_0=\frac{5}{3}m/s$ ...(vi)

From Eq. (v), we get $\theta_0=\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

5. Components of velocity at an instant of times t of a body projected at an angle θ is

$v_x=u\cos\theta+g_xt$ and $v_y=u\sin\theta+g_yt$

Here, components of velocity at $t=1s$, is

$v_x=u\cos 60^0+0$ $(\text{as } g_x=0)$ $=10\times\frac{1}{2}=5m/s$ and $v_y=u\sin 60^0+(-10)\times(1)$

$=5\sqrt{3}-10\Rightarrow |V_y|=|10-5\sqrt{3}|m/s$

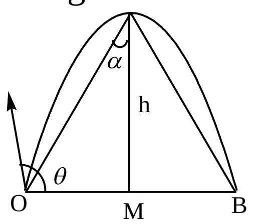
Now, angle made by the velocity vector at time of $t=1s$

$$|\tan\alpha| = \left|\frac{V_y}{V_x}\right| = \frac{|10-5\sqrt{3}|}{5} \quad \Rightarrow \tan\alpha = |2-\sqrt{3}| \text{ or } \alpha = 15^0$$

$\therefore$ Radius of curvature of the trajectory of the projected body, $R = v^2/g\cos\alpha$

$$= \frac{(5)^2 + (10-5\sqrt{3})^2}{10\times 0.97} \left(\therefore v^2 = v_x^2 + v_y^2 \text{ and } \cos 15^0 = 0.97\right) \Rightarrow R = 2.77\ m \approx 2.8m$$

6. The given situation is shown below.



Here, range, $OB = 2h\tan\alpha = \dfrac{u^2\sin 2\theta}{g}$

and $h = \dfrac{u^2\sin^2\theta}{2g} \Rightarrow \dfrac{2h\tan\alpha}{h} = \dfrac{\dfrac{u^2\sin 2\theta}{g}}{\dfrac{u^2\sin^2\theta}{2g}} \Rightarrow \tan\theta = 2\cot\alpha$

$\Rightarrow \sin\theta = \dfrac{2\cos\alpha}{\sqrt{1+\cot^2\alpha}} \Rightarrow h = \dfrac{u^2\sin^2\theta}{2g}$

or $\dfrac{u^2}{2g} = \dfrac{h}{\sin^2\theta} = \dfrac{h}{4\cot^2\alpha}\left(1+4\cot^2\alpha\right) = \dfrac{\tan^2\alpha}{2}+1$

$\Rightarrow \dfrac{u^2}{gh} = \dfrac{\tan^2\alpha}{2}+2 \quad \Rightarrow \dfrac{u^2}{gh} - \dfrac{\tan^2\alpha}{2} = 2$

7. Conceptual

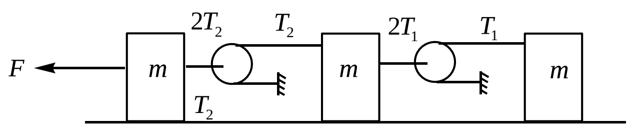
8. Conceptual

9. Let acceleration of mass m relative to wedge down the plane is a_r. Its absolute acceleration in horizontal direction is $a_r\cos 60^0 - a$ (towards right.) Hence, let N be the normal reaction between the mass and the wedge. Then

$N\sin\theta = Ma = m(a_r\cos 60^0 - a)$

or $a_r = \dfrac{(M+m)a}{m\cos 60^0} = \dfrac{2(M+m)a}{m}$

10. $T_1 \geq \mu mg$



$T_2 - 2T_1 \geq \mu mg$

or $T_2 - 2\mu mg \geq \mu\ mg$ (Putting $T_1 = \mu\ mg$)

or $T_2 \geq 3\mu\ mg$

Now, $F - 2\ T_2 \geq \mu mg$

or $F - 6\mu mg \geq \mu mg$ (Putting $T_2 = 3\mu mg$)

or $F \geq 7\mu mg$

11. Conceptual

12. Let, N_1= normal reaction at A and N_2= normal reaction at B

$$\frac{\sqrt{3}N_1}{2}-\frac{mg}{2}-\frac{N_2\sqrt{3}}{2}=ma \quad \frac{\sqrt{3}N_1}{2}-\frac{\sqrt{3}N_2}{2}=ma+\frac{mg}{2} \Rightarrow \quad \frac{N_1+N_2}{2}=\frac{mg\sqrt{3}}{2}$$

$$\Rightarrow \frac{\sqrt{3}N_1}{2}+\frac{\sqrt{3}N_2}{2}=\frac{3mg}{2}$$

From Eqs. (i) and (ii) , we have $\sqrt{3}N_1=2mg+ma,\quad \sqrt{3}N_2=mg-ma$

Since cylinder does not loose contact at B, so

$N_2 \ge 0 \Rightarrow \quad a \le g \Rightarrow \quad a_{\max}=g$

13. Conceptual

14. $F \infty s^{-1/3}$

i.e. acceleration $a \infty s^{-1/3}$

$or \quad v\frac{dv}{ds}=Ks^{-1/3} \quad or \ v^2 \infty \ s^{2/3}$

or $\quad v \infty s^{1/3}$

Now, $P=F.v$

or $\quad P \infty s^{-1/3}, s^{1/3} \quad or \ P \infty s^0$

i.e. power is independent of s.

15. Conceptual

16.

$$a=\frac{F_{net}}{m}=\frac{mg\sin\theta-(\mu_0 x)(mg\cos\theta)}{m}=g\sin\theta-(\mu_0 g\cos\theta)x$$

$$\therefore \quad v.\frac{dv}{dx}=g\sin\theta-(\mu_0 g\cos\theta)x$$

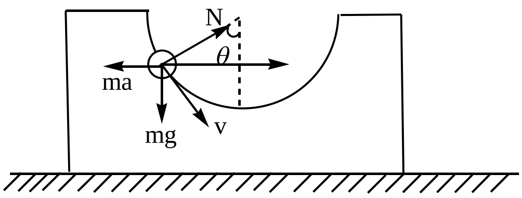
$$\therefore \int_0^0 vdv=\int_0^{x_{\max}}\left[g\sin\theta-(\mu_0 g\cos\theta)x\right]dx$$

Solving this integration, we get

$$x_{\max}=\frac{2\tan\theta}{\mu_0}$$

17. From work energy theorem,

$$W_g+W_{psudo\ force}=K_f-K_i$$



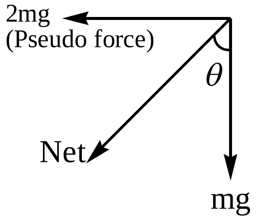
$$mgR\cos\theta-maR(1-\sin\theta)=\frac{1}{2}mv^2 \quad \Rightarrow \quad v=\sqrt{2gR(\sin\theta+\cos\theta-1)}$$

v is maximum when $\frac{dv}{d\theta}=0 \Rightarrow \theta=45^0$

18. Apply work energy theorem

19. Apply work energy theorem

20. In equilibrium, (with respect to cylinder)



$\tan\theta = \frac{2mg}{mg}$

or $\theta = \tan^{-1}(2)$

$\therefore$ Maximum angular displacement $= 2\theta = 2\tan^{-1}(2)$

21. Conceptual

22. Conceptual

23.

$$E_Q = \frac{G\left(\rho.\frac{4}{3}\pi R^3\right)}{(2R)^2} + \frac{G\left[2\rho\left\{\frac{4}{3}\pi(2R)^3 - \frac{4}{3}\pi(R)^3\right\}\right]}{(2R)^2}$$

$$E_P = \frac{G\left(\rho\frac{4}{3}\pi R^3\right)}{R^2} \qquad E_Q / E_P = 15/4 = 3.75$$

24. Density of wire, $d = 9\times10^{-3}\frac{kg}{cm^3} = 9\times10^3 kg/m^3$

Strain in the wire, $\varepsilon = 4.9\times10^{-4}$

Young's modulus of wire is $Y = 9\times10^{10}\frac{N}{m^2}$

Lowest frequency of vibration in wire will be $f = \frac{1}{2L}\sqrt{\frac{T}{(M/L)}}$

Now, $\frac{T}{M/L} = \frac{T}{V\rho/L} = \frac{T}{\left(\frac{LA.\rho}{L}\right)} = \frac{T}{A\rho}$

But $\frac{T}{A}$=stress $Y\times strain = Y\times\varepsilon \Rightarrow \frac{T}{M/L} = \frac{Y\times\varepsilon}{\rho}$

So, from Eq (i), frequency will be

$$f = \frac{1}{2L}\sqrt{\frac{T}{(M/L)}} = \frac{1}{2L}\sqrt{\frac{Y\times\varepsilon}{\rho}} = \frac{1}{2\times1}\times\sqrt{\frac{9\times10^{10}\times49\times10^{-4}}{9\times10^3}} = 35Hz$$

25. Conceptual

26. Conceptual

27. Conceptual

28. Conceptual

29. Conceptual

30. Conceptual

CHEMISTRY

31. Formation of NaCl is an exothermic reaction.

32. $\dfrac{\lambda_1}{\lambda_2}=\dfrac{(mv)_2}{(mv)_1}$

33. $[ClF_2O]^+ - sp^3, [ClF_4O]^- - sp^3d^2$

34. $C_2H_5OH + 2Cr_2O_7^{2-} \rightarrow 2CO_2 + 4Cr^{3+}$

Eq. of $Cr_2O_7^{2-}$ = eq of C_2H_5OH

$\dfrac{8.0\times.05\times6}{1000}=\dfrac{10}{46/12}$ $W=\dfrac{8.0\times.05\times46}{1000\times12}\times6=.0092$ $\%=\dfrac{0.0092}{10}\times100=0.092\%$

35. As per Le Chatelier's principle, equilibrium moves in forward direction.

36. $C(graphite)+\frac{1}{2}O_2(g)\rightarrow CO(g);\Delta H=-ve$ Is not a combustion reaction

37. $\Delta S = 2.303nR\log\dfrac{V_2}{V_1}$

38. De-excitation of electron from 5th orbit to 4th orbit gives first line in Brackett series.

39. $HI_n \rightleftharpoons H^+ + I_n^-$

Acidic basic

Red Blue

$K_a=\dfrac{[H^+][I_n^-]}{[HI_n]}$

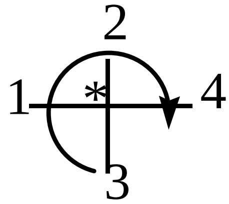
Red from 75% blue from 25% $\therefore [H^+]=3\times10^{-5}\times\dfrac{75}{25}=9\times10^{-5}$

When 75% blue and 25% red, then

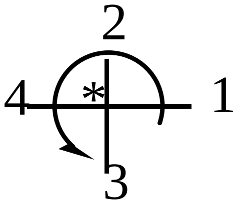
$[H^+]=3\times10^{-5}\times\dfrac{25}{75}=1\times10^{-5}$ $\therefore$ change is $\therefore [H^+]=9\times10^{-5}-1\times10^{-5}$ $=8\times10^{-5}$

40. Localised electron pair on nitrogen.

41.



-2S and



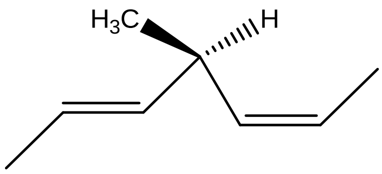
-3R

42. Most stable alkene is the major product.

43. BeF_2 Soluble in water due to grater hydration energy of Be^{+2}, as compared to L.E

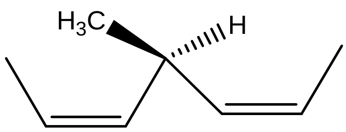
44. $A\ell^{+3}$ ion further can not lose electrons so it cannot act as reducing agent.

45.



(X)

optically active.



(Y)

optically inactive.

46. For lead, (+2) is more stable due to inert pair effect

47. As BOD values increases, pollution increase.

48. As the electronegativity of central atom increases, bond angle increases.

49. Friedel Crafts alkylation is generally not preferred for the preparation of alkyl benzenes because
electrophile undergoes rearrangement giving mixture of products and poly alkylation takes place
as alkyl group is activating

50. More hyper conjugations.

51. I, III and IV

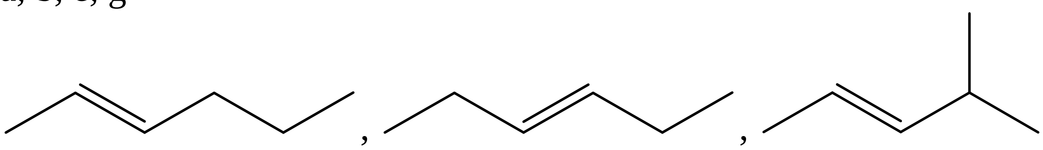
52. $2S_4N_4 \rightarrow 4N_2(g) + S_8(g)$

53. $\Delta G = O$ at equilibrium hence $\Delta H = T\Delta S$ and $\therefore T = \dfrac{\Delta H}{\Delta S}$

54. Borax- $Na_2B_4O_7 \cdot 10H_2O$ and Kernite- $Na_2B_4O_7 \cdot 4H_2O$

55. a, b, e, g

56.



57. $P_1V_1 = P_2V_2$

58. $q = ms\Delta T$

59. i, ii , iii.

60.

KCN	HCN
Volume = 'x' ml	volume = 10ml
Molarity = 5	Molarity = 2m

$\therefore P^H = -\log Ka + \log \dfrac{[salt]}{[Acid]}$

$9.6020 = -\log 5 \times 10^{-10} + \log \dfrac{5x/(10+x)}{20/(10+x)}$

$9.6020 = 9.3010 + \log \dfrac{x}{4}$

$\log \dfrac{x}{4} = 0.3010 = \log 2$

$x = 8ml$

MATHEMATICS

61. Lines are $y-1=\tan\theta(x-1)$ and $(y-1)=\cot\theta(x-1)$ multiply and compare

$\sin 2\theta=\frac{2}{a+2}$

62. $\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(r^2+r\right)-\tan^{-1}\left(r^2-r\right)=\frac{\pi}{2}$

63.

Let $g(x)=e^{-x}f(x)$ As $g''(x)>0$ so $g'(x)$ is increasing

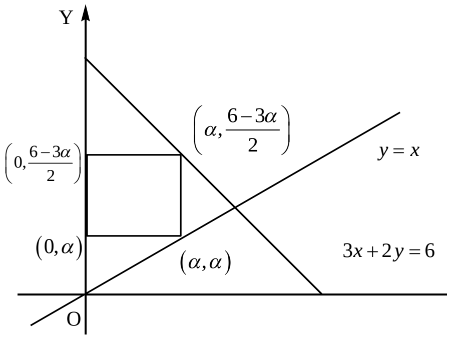
So, for $x<1/4, g'(x)<g'(1/4)=0$

$\Rightarrow \left(f(x)-f(x)\right)e^{-x}<0 \Rightarrow f(x)<f(x)\text{in}(0,1/4)$

64. $ST^2=AS^2+AT^2-2AS.AT.\cos\alpha$ and $AS.AT=\frac{1}{2}bc$

65. Use half angle formulae and Heron's formula

66.



Area of rectangle $=\alpha\left(\frac{6-3\alpha}{2}-\alpha\right)$ $=\alpha\left(\frac{6-5\alpha}{2}\right)$ $\frac{dA}{d\alpha}=0$ $\Rightarrow\alpha=\frac{3}{5}$

$A_{Max}=\frac{3}{5}\times\left(\frac{3}{2}\right)=\frac{9}{10}$ Sq units

67. Since $f''(x)>0$

$\Rightarrow f'(x)$ is always increasing.

$g'(x)=2f'\left(2x^3-3x^2\right)\times\left(6x^2-6x\right)+f'\left(6x^2-4x^3-3\right)\left(12x-12x^2\right)$

$=12\left(x^2-x\right).\left(f'\left(2x^3-3x^2\right)-f'\left(6x^2-4x^3-3\right)\right)$

$=12x(x-1)\left[.f'\left(2x^3-3x^2\right)-f'\left(6x^2-4x^3-3\right)\right]$

For increasing $g'(x)>0$

$\Rightarrow 2x^3-3x^2>6x^2-4x^3-3\left\{\because f'(x)\text{ is incresing}\right\}$

$\Rightarrow (x-1)^2\left(x+\frac{1}{2}\right)>0\to x>-\frac{1}{2}\therefore x\in\left(-\frac{1}{2},0\right)\cup(1,\infty)$

Case-II If $0<x<1$

$f'\left(2x^3-3x^2\right)<f'\left(6x^2-4x^3-3\right)$

$(x-1)^2\left(x+\frac{1}{2}\right)<0$ $x<-\frac{1}{2}$, so there is no solution

$\Rightarrow$ Hence the values are $x\in\left(-\frac{1}{2},0\right)\cup(1,\infty)$

68. $$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0 \Rightarrow \alpha = 1,4,5$$

69. $$\lambda = \frac{m_{21}-m_{11}-m_{22}}{3^{2012}} = \frac{2012\times 3^{2012}-3^{2012}-3^{2012}}{3^{2012}} = 2010$$

70. Given $y = 1+\frac{1}{x-1}+\frac{2x}{(x-1)(x-2)}+\frac{3x^2}{(x-1)(x-2)(x-3)}$

$= \left(1+\frac{1}{x-1}\right)+\frac{2x}{(x-1)(x-2)}+\frac{3x^2}{(x-1)(x-2)(x-3)}$

y$=\frac{x^3}{(x-1)(x-2)(x-3)}$

log y=log$\left(\frac{x}{x-1}\right)+\log\left(\frac{x}{x-2}\right)+\log\left(\frac{x}{x-3}\right)$

Diff. both sides w.r. t x, we get

$\frac{dy}{dx}=\frac{y}{x}\left[\frac{1}{1-x}+\frac{2}{2-x}+\frac{3}{3-x}\right] \therefore\ -9y'(4) = 104$

71. $m\sin 2\alpha = m\sin\{(\alpha+\beta)+(\alpha-\beta)\} = \cos^2(\alpha-\beta)+m\cos(\alpha+\beta)\sin(\alpha-\beta)$

$m\sin 2\beta = m\sin\{(\alpha+\beta)-(\alpha-\beta)\} = \cos^2(\alpha-\beta)-m\cos(\alpha+\beta)\sin(\alpha-\beta)$

Now, $\frac{1}{1-m\sin 2\alpha}+\frac{1}{1-m\sin 2\beta}$

$$\frac{1}{\sin^2(\alpha-\beta)-m\cos(\alpha+\beta)\sin(\alpha-\beta)}+\frac{1}{\sin^2(\alpha-\beta)+m\cos(\alpha+\beta)\sin(\alpha-\beta)}=\frac{2}{1-m^2}$$

72. Continuous at $x=1$ $\quad p+q=-2+\frac{\pi}{4}$

differentiable at $x=1$ $\quad 3p=\frac{1}{1+1}=\frac{1}{2}$ $\quad p=\frac{1}{6};q=\frac{\pi}{4}-\frac{13}{6}$

73. $(1+y)\cos^2 2x+4\cos 2x+3y-1=0$

Since $\cos 2x$ is real, $16-4(3y-1)(1+y)\geq 0$ Or $3y^2+2y-5\leq 0$

But $y=1\Rightarrow \cos 2x=-1$ i.e. $x=\frac{\pi}{2}$ which is not permissible.

74. Total disks in A_n is $S_n=3n(n+1)$ and length of side of hexagon is

$1=2r_n(n-1)+\frac{2r_n}{\sqrt{3}}\Rightarrow r_n=\frac{1}{2n-2+\frac{2}{\sqrt{3}}}$

75. $\bar{d}_1.\bar{d}_2=0\Rightarrow(\bar{a}+2\bar{b}).(2\bar{a}-\bar{b})=0\Rightarrow 2|\bar{a}|^2+3\bar{a}.\bar{b}-2|\bar{b}|^2=0$

$\bar{d}_3.\bar{d}_4=0\Rightarrow(2\bar{a}+\bar{b}).(\bar{a}-2\bar{b})=0\Rightarrow 2|\bar{a}|^2-3\bar{a}.\bar{b}-2|\bar{b}|^2=0$

$\Rightarrow \bar{a}.\bar{b}=0$ $\quad$ Now $[\bar{a}\ \bar{b}\ \bar{a}\times 2\bar{b}]=(\bar{a}\times\bar{b}).(\bar{a}\times 2\bar{b})=2|\bar{a}|^2|\bar{b}|^2$

76. $$A=\begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}(\sin t+\cos t) & e^{-t}(\cos t-\sin t) \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

$$|A| = e^t.e^{-2t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\sin t - \cos t & \cos t - \sin t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t - 2\cos t & \cos t - 2\sin t \\ 0 & 2\sin t - \cos t & -2\cos t - \sin t \end{vmatrix}$$

$$|A| = e^{-t}\left(\sin^2 t + 4\cos^2 t + 4\sin t\cos t - 4\cos t + 4\sin^2 t + \cos^2 t\right)$$

$$|A| = e^{-t}\left(5\sin^2 t + 5.\cos^2 t\right) = 5.e^{-t} \neq o\forall t \in R$$

77. $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} \Rightarrow A^2 = A.A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow A^4 = A^2.A^2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3^2 & 0 \\ 0 & 3^2 \end{bmatrix}$

$\Rightarrow A^8 = \begin{bmatrix} 3^4 & 0 \\ 0 & 3^4 \end{bmatrix}$ and $A^6 = A^4.A^2 = \begin{bmatrix} 3^2 & 0 \\ 0 & 3^2 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3^3 & 0 \\ 0 & 3^3 \end{bmatrix}$

Let $V = \begin{bmatrix} x \\ y \end{bmatrix}$

$$A^8 + A^6 + A^4 + A^2 + I$$

$$\begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix} + \begin{bmatrix} 27 & 0 \\ 0 & 27 \end{bmatrix} + \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 121 & 0 \\ 0 & 121 \end{bmatrix}$$

$$\left(A^8 + A^6 + A^4 + A^2 + I\right)V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 121 & 0 \\ 0 & 121 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 121x \\ 121y \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$$

78. Let $y = \dfrac{\cos^3\theta\sin^3 + 1}{(\sin\theta + \cos\theta + 1)^2} = \dfrac{(\cos\theta + \sin\theta)(1 - \sin\theta\cos\theta) + 1}{(\sin\theta + \cos\theta + 1)^2}$

Let $\cos\theta + \sin\theta = x$ $\quad y = \dfrac{x\left(1 - \dfrac{x^2 - 1}{2}\right) + 1}{(x+1)^2} = \dfrac{2 + 3x - x^3}{2(x+1)^2} = \dfrac{(2-x)(x+1)^2}{2(x+1)^2} = 1 - \dfrac{x}{2}$

$\therefore y = 1 - \dfrac{\cos\theta + \sin\theta}{2} \geq 1 - \dfrac{\sqrt{2}}{2}$

79. The lines are $\dfrac{x}{1} = \dfrac{y}{1} = \dfrac{z-1}{0}, \dfrac{x}{1} = \dfrac{y}{-1} = \dfrac{z+1}{0}$

Let $C = (\alpha, \beta, \gamma)$

The distance of C from $\vec{r} = \vec{a} + s\vec{b}$ is $\dfrac{\left|(\vec{c} - \vec{a}) \times \vec{b}\right|}{\left|\vec{b}\right|}$

Distance of C from the first line is $\sqrt{(\gamma - 1)^2 + \dfrac{(\alpha - \beta)^2}{2}}$ $\quad\quad ....(i)$

The distance of C from the second line is $\sqrt{(\gamma+1)^2+\frac{(\alpha+\beta)^2}{2}}$ ....(ii)

$(i),(ii)\Rightarrow(\gamma+1)^2=(\gamma-1)^2+\frac{(\alpha-\beta)^2}{2}4\gamma=-2\alpha\beta.$ Locus of C is $xy=-2z.$

80. $f'(x)=2\sin(x+\alpha)\cos(x+\alpha)+2\sin(x+\beta)\cos(x+\beta)-2\cos$

$(\alpha-\beta)\cos(x+\alpha)\sin(x+\beta)-2\cos(\alpha-\beta)\sin(x+\beta)\cos(x+\beta)$

$=\sin(2x+2\alpha)+\sin(2x+2\beta)-2\cos(\alpha-\beta)\sin(2x+\alpha+\beta)$

$=2\sin(2x+\alpha+\beta)\cos(\alpha-\beta)-2\cos(\alpha-\beta)\sin(2x+\alpha+\beta)=0$

$\therefore f(x)$ is a constant function.

81. Let equation of plane be $x+2y+3z-7+\lambda x=0$

Since $(-1,0,2)$ lies on it $\therefore \lambda=-2$

Equation of plane is $-x+2y+3z-7=0$

$|\cos\theta|=\frac{6}{7}=\frac{p}{q}$

82. Plane is $-x+z=1\Rightarrow d=\frac{3}{\sqrt{2}}$

83. $f(x+y)=f(x)f(y)$

$f(2)=(f(1))^2,\ f(3)=(f(1))^3$ and so on

$\sum_{r=1}^{n} f(a+r)=f(a)f(1)\left(1+f(1)+(f(1))^2+.....+(f(1))^{n-1}\right)$

$=f(a)\frac{3.(3^n-1)}{2}$

84. If $f(\theta)=0\Rightarrow \sin\theta=0$ or $\sin 2\theta=0$ or $\cos 3\theta=0$ or $\cos 4\theta=0$ or $\cos 5\theta=0$

85. $(t+1)x+8y-4t=0$

$tx+(t+3)y-(3t-1)=0$

$\frac{t+1}{t}=\frac{8}{t+3}=\frac{-4t}{-(3t-1)}$

$t=3$

86.

$$\lim_{x\to\frac{\pi}{4}}\frac{4\sqrt{2}-(\cos x+\sin x)^5-10\cos x(1-\sin 2x)}{1-\sin 2x}=\lim_{x\to\frac{\pi}{4}}\left(\frac{4\sqrt{2}-\left(\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)\right)^5}{1-\sin 2x}\right)-\lim_{x\to\frac{\pi}{4}}10\cos x$$

$$=\lim_{x\to\frac{\pi}{4}}4\sqrt{2}\left(\frac{1-\cos^5\left(x-\frac{\pi}{4}\right)}{1-\sin 2x}\right)-\frac{10}{\sqrt{2}}=\lim_{h\to 0}\frac{4\sqrt{2}(1-\cos^5 h)}{1-\cos 2h}-5\sqrt{2}$$

$$=\lim_{x\to 0}\frac{4\sqrt{2}(1+\cosh+\cos^2 h+\cos^3 h+\cos^4 h)}{2(1+\cosh)}-5\sqrt{2}=5\sqrt{2}-5\sqrt{2}=0$$

87. Taking dot product with $\bar{a},\bar{b},\bar{c}$, we get $5[\bar{a}\ \bar{b}\ \bar{c}]=\bar{a}.\bar{a}$, $3[\bar{a}\ \bar{b}\ \bar{c}]=\bar{a}.\bar{c}$ and $\bar{a}.\bar{b}=0$

So. $\dfrac{\left(2\bar{a}+\bar{c}+3\bar{d}\right).\bar{a}}{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}=13$ as $\bar{d}.\ \bar{a}=\left(\lambda_1\bar{b}+\lambda_2\bar{a}\times\bar{b}\right).\bar{a}=0$

88. Check the continuity and differentiability at integral points. At x = 1, it is not differentiable $f(x)=x\sin\pi(x-1)=-x\sin\pi x\ \ x>1=x\sin\pi x$

It is continuous but not differentiable

89. Let P and Q be points on the lines, then

$P(-2+2s,-6+3s,34-10s),Q(-6+4t,7-3t,7-2t)$

The $d.r's$ of PQ are $(-4+4t-2s,13-3t-3s,-27-2t+10s)$

PQ is perpendicular to the two lines with $d.r's\,2,3,-10$ and $4,-3,-2$.

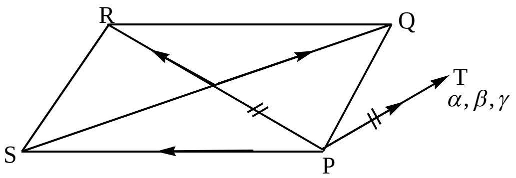
$\therefore 2(-4+4t-2s)+3(13-3t-3s)-10(-27-2t+10s)=0$

$4(-4+4t-2s)-3(13-3t-3s)-2(-27-2t+10s)=0$

$\Rightarrow 113s-19t=301,29-19s=1$

Solving $s=3,t=2,P=(4,3,4)=(a,b,c)\Rightarrow a+b+c=11$

90.



Area of base (PQRS) $=\dfrac{1}{2}\left|\overrightarrow{PR}\times\overrightarrow{SQ}\right|=\dfrac{1}{2}\begin{bmatrix}\hat{i}&\hat{j}&\hat{k}\\3&1&-2\\1&-3&-4\end{bmatrix}$

$=\dfrac{1}{2}\left|-10\hat{i}+10\hat{j}-10\hat{k}\right|=5\left|\hat{i}-\hat{j}+\hat{k}\right|=5\sqrt{3}$

Height = proj. of PT on $\hat{i}-\hat{j}+\hat{k}=\left|\dfrac{1-2+3}{\sqrt{3}}\right|=\dfrac{2}{\sqrt{3}}$

volume $=\left(5\sqrt{3}\right)\left(\dfrac{2}{\sqrt{3}}\right)=10$ cu. Units