Sri Chaitanya IIT Academy.,India.

A.P ✪ T.S ✪ KARNATAKA ✪ TAMILNADU ✪ MAHARASTRA ✪ DELHI ✪ RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 30-09-2023
Time: 09.00Am to 12.00Pm RPTM-09 Max. Marks: 300

KEY SHEET

PHYSICS

1)	1	2)	1	3)	4	4)	1	5)	3
6)	1	7)	2	8)	3	9)	2	10)	3
11)	3	12)	4	13)	2	14)	4	15)	3
16)	3	17)	3	18)	3	19)	1	20)	4
21)	2	22)	20	23)	600	24)	125	25)	2
26)	5	27)	17	28)	50	29)	38	30)	6

CHEMISTRY

31)	1	32)	3	33)	3	34)	1	35)	3
36)	2	37)	2	38)	2	39)	4	40)	1
41)	1	42)	2	43)	2	44)	1	45)	4
46)	1	47)	2	48)	1	49)	2	50)	1
51)	18	52)	2	53)	2	54)	20	55)	5
56)	2	57)	5	58)	4	59)	4	60)	12

MATHEMATICS

61)	4	62)	3	63)	3	64)	4	65)	1
66)	3	67)	4	68)	4	69)	1	70)	2
71)	2	72)	1	73)	3	74)	1	75)	1
76)	2	77)	3	78)	2	79)	1	80)	1
81)	41	82)	14	83)	5	84)	7	85)	1
86)	1600	87)	11	88)	3	89)	8	90)	34

SOLUTIONS
PHYSICS

1. The pressure at the surface of the water is equal to the atmospheric pressure p_0. The pressure at the bottom is $p = p_0 + hd\ g$

$p = 1.01\times10^5 Pa + (0.50m)(1000kg/m^3)(10\ m/s^2)$

$p = 1.01\times10^5 Pa + 0.05\times10^5 Pa$

$p = 1.06\times10^5 Pa$

The area of the bottom $= \pi r^2 \quad = 314\times10^{-3} \quad = \frac{314}{1000} = 0.314$

The force on the bottom is, therefore,

$$F = P\times\pi r^2$$
$$F = (1.06\times10^5 Pa)(0.0314\ m^2)$$
$$F = 3328\ N$$

2. Let ρ_S & ρ_L be the densities of silver and liquid respectively, and m and V be the mass and volume, respectively, of the silver block. Therefore,
Tension in the string = mg – buoyant force$\Rightarrow T = \rho_s Vg - \rho_L Vg = (\rho_S - \rho_L)Vg$

Also, $V = \frac{m}{\rho s}$

$\therefore\ T = \left(\frac{\rho s - \rho L}{\rho s}\right)mg = \frac{(10-0.72)\times10^3}{10\times10^3}\times5\times10 = 9.28\times5 \Rightarrow 46.40\ N$

3. $v_1 = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh} \quad ....(i)$

From Bernoulli's theorem, $\rho gh + 2\rho g\left(\frac{h}{2}\right) = \frac{1}{2}(2\rho)v_2^2$

$$2\rho gh = \rho v_2^2$$
$$v_2 = \sqrt{2gh}$$
$$\Rightarrow v_2 = \sqrt{2gh} \quad ....(ii)$$
$$\therefore \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$$

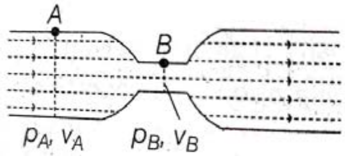
4. Initially, the water flowing out will be less than that flowing into it. Hence, the water level will go on rising when the water level is 'h' the velocity of efflux is $\sqrt{2gh}$, when this becomes equal to the velocity of inflow, the level will become steady as the area of cross section of the filling tube and area of cross section of the hole are equal. This height is given by torricelli's theorem,

$V = \sqrt{2gh}\ (or)\ h = \frac{v^2}{2g}$

There after the water level will not rise.

5. By equation of continuity for sections A and B, we have

$$A_A V_A = A_B V_B$$



$\Rightarrow \quad 40\,V_A = 20V_B$

$\Rightarrow \quad 2\,V_A = V_B \qquad \ldots\ldots\ldots\ldots\ldots(i)$

Now, using Bernoulli's equation (for horizontal tube), we have

$$P_A + \frac{1}{2}\rho v_A^2 = \rho_B + \frac{1}{2}\rho v_B^2$$

$$\Rightarrow \; P_A - P_B = \frac{1}{2}\rho\left(v_B^2 - v_A^2\right)$$

Here, $P_A - P_B = 500\,Nm^{-2}$

And $\qquad \rho = 1000\,kg\,m^{-3}$

$$\Rightarrow \quad 500 = \frac{1}{2}\times 1000\left(v_B^2 - v_A^2\right)$$

$$\Rightarrow \quad v_B^2 - v_A^2 = 1 \qquad \ldots\ldots\ldots..(ii)$$

From Eqs. (i) and (ii), we have

$3v_A^2 = 1$

$$\Rightarrow \; V_A = \frac{1}{\sqrt{3}} = \frac{1}{1.732}$$

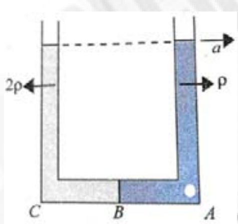
$\therefore \; V_A = 0.577 m/s = 57.7 cm/s$

So, volume flow rate of water $= A_A.V_A$

$= 40\times 57.7 = 2308\,cm^3/s$

6. As the vessel is falling freely, the pressure at all the points in the liquid is same and equal to the atmospheric pressure and hence buoyancy becomes zero.

7.



For the given situation, liquid of density 2ρ should be behind that of ρ.

From the right limb, $P_A = P_{atm} + \rho gh$

$$P_B = P_A + \rho a\frac{1}{2} = P_{atm} + \rho gh + \rho a\frac{1}{2}$$

$$P_C = P_B + (2\rho)a\frac{1}{2} = P_{atm} + \rho gh + \frac{3}{2}\rho al \qquad \ldots\ldots.(i)$$

But from the left limb, $P_C = P_{atm} + (2\rho)gh \qquad \ldots\ldots.(ii)$

From Eqs. (i) and (ii),

$$P_{atm} + \rho gh + \frac{3}{2}\rho al = P_{atm} + 2\rho gh \qquad \Rightarrow h = \frac{3al}{2g}$$

8. For a fluid mass, pressure at all points on same level is equal Also by Pascal's Law, pressure is applied any where in a fluid is equally transmitted is all directions

9. In case of mixture,

$$\rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2}$$

When equal volumes are mixed,

$$4 = \frac{V\rho_1 + V\rho_2}{V + V} = \frac{\rho_1 + \rho_2}{2} \qquad ....(1)$$

When equal masses are mixed,

$$3 = \frac{m + m}{\frac{m}{\rho_1} + \frac{m}{\rho_2}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \qquad ....(2)$$

$\therefore$ From (1) & (2) specific gravity of the metals are z and 6

10. When the vessel is stationary:

Weight = upthrust

i.e., $V\rho_w g = V_i \rho_L g$

($\rho_w =$ density of wood and $\rho_L =$ density of liquid)

Or $\frac{V_i}{V} = \frac{\rho_w}{\rho_L}$ ....(i)

When the vessel moves upwards:

Upthrust – weight = (mass) (acceleration)

Or $V'_i \rho_L (g + g/2) - V_{\rho w} g = \frac{V\rho_w g}{2}$

Or $\frac{V'_i}{V} = \frac{\rho_w}{\rho_L}$ ....(ii)

From Eqs. (i) and (ii), we see that $\frac{V_i}{V} = \frac{V'}{V}$

i.e., fraction (or percentage) or volume immersed in liquid remains unchanged.

11. Volume of ball V=m/ρ acceleration of ball inside the liquid is

$$a = \frac{F_{net}}{m} \qquad a = \frac{\text{uptrust-weight}}{\text{m}} \qquad a = \frac{\left(\frac{m}{\rho}\right)(3\rho)(g) - mg}{m} = 2g\uparrow$$

Velocity of ball while reacting at surface is

$V = \sqrt{2ah} \qquad V = \sqrt{2 \times 2gh} = \sqrt{4gh}$

The ball will jump to a height $H = \frac{V^2}{2g} = \frac{4gh}{2g} = 2h$

12. Given, $l_x, l_y and\ l_z$ be the moment of inertial of disc about $X, Y\ and\ Z -$ axes. By using perpendicular axis theorem, $I_z = I_x + I_y$

Where, I_z is the moment of inertia about its centre of mass $= \frac{MR^2}{2}$

And in symmetrical body, $I_x = I_y$ (Disc in given case)

$\therefore \qquad I_z = 2I_x$

$\Rightarrow \qquad I_x = I_y = \frac{I_z}{2} = \frac{MR^2}{4}$

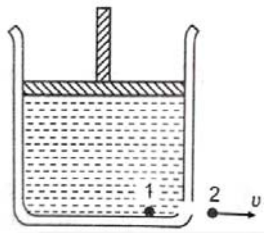
Now, $I_x = I_y \neq I_z$

As, radius of gyration, $k=\sqrt{I/M}$
So, radii of gyration about all axis will not be the same.
$\therefore$ Assertion is not correct but Reason is correct because in a rigid body making rotational motion has fixed body making rotational motion has fixed mass M and radius $R\left(I_x=I_y\right)$

13. Solution: Applying Bernoulli's theorem at 1 and 2 : difference in pressure energy between 1 and 2 = difference in kinetic energy between 1 and 2

Or $\rho gh+\dfrac{mg}{A}=\dfrac{1}{2}\rho v^2$

Or $v=\sqrt{2gh+\dfrac{2mg}{\rho A}}=\sqrt{2\left(gh+\dfrac{mg}{\rho A}\right)}$



14. According to the question.
One division of main scale reading = a cm
n^{th} vernier scale division
$=(n-1)th$ main scale divison
$\therefore$ One division of vernier scale reading
$=\dfrac{(n-1)a}{n}$ ....(i)
We know that, Least count (LC)=
[1main scale division – 1 vernier scale division]
cm
$=a-\dfrac{(n-1)a}{n}$ [using Eq. (i)]
$=\dfrac{a(n-n+1)}{n}=\dfrac{a}{n}cm=\dfrac{a}{n}\times 10\ mm$
$\Rightarrow LC=\dfrac{10a}{n}mm$

15. 15. When an object has two velocities V_1 and V_2, then magnitude of resultant velocity is
$V=\sqrt{V_1^2+V_2^2+2V_1V_2\cos\theta}$
Statement I is correct
Clearly $|\text{displacement}|\le$ distance
Statement II is correct
Instantaneous acceleration
$=\lim\limits_{\Delta t\to 0}\dfrac{\Delta V}{\Delta t}=\lim\limits_{\Delta t\to 0}a$
Statement III is correct

16. Assertion(A): is false, because if we take the example of a ball thrown vertically upwards, then at the topmost point of its motion the ball momentarily comes to rest but acceleration due to gravity keeps on acting on the ball.

Reason: is true whenever any object reverses its direction of motion then the sign of velocity changes because it comes to rest for a moment at the point where it changes its direction of motion. The same example of motion of ball given above can be used to explain it as the ball changes the direction of motion after reaching the topmost point when its speed becomes zero for a moment

17. Intensity at the centre will be zero if path difference is $\lambda/2$. That is,

$$(\mu-1)t=\frac{\lambda}{2} \qquad \text{Or} \qquad t=\frac{\lambda}{2(\mu-1)}$$

18. In an elastic collision of two billiard balls, kinetic energy and linear momentum remain conserved.
In elastic collision, momentum is conserved but total energy of any of ball is not conserved i.e. exchange of energy takes place
Hence, A is true but R is false

19. $u_1=\frac{p}{m}, u_2=0, v_1=\frac{(p-J)}{m}, v_2=\frac{J}{m}$

Now apply $e=\frac{v_2-v_1}{u_1-u_2}=\frac{\frac{J}{m}-\frac{p-J}{m}}{p/m}=\frac{J-p+J}{p}=\frac{2J-p}{p}$

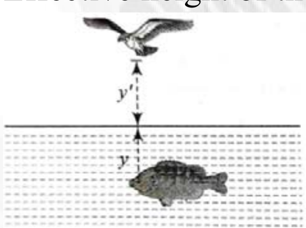
20. In thermodynamic system, heat and work are not state variables because they depend on the path of connecting states thus, heat and work are treated as a path functions in thermodynamics. According to first law of thermodynamics

$Q=\Delta U+W$

$Q-W=\Delta U$

Thus, heat Q and work w are modes of energy transfer to a system resulting in change in its internal energy.

21. Effective height of the bird as seen by the fish, $Y=y+\mu y'$



$\frac{dY}{dt}=\frac{dy}{dt}+\mu\frac{dy'}{dt} \qquad 6=3+\frac{4}{3}\frac{dy'}{dt}$

Or $\qquad \frac{dy'}{dt}=\frac{3\times 3}{4}=\frac{9}{4}=2.25\ m/s$

Therefore, actual velocity of bird = 2.25 m/s

22. Let a be the size of each side of the cube. Then,

$$800\times g=(2)\times(a^2)\times 1\times g$$

$\therefore \qquad a=20\,cm$

23. Taking torque about the attachment point for W, we get

$-T_1(0,4L)+T_2(0.3L)+200(0.2L)=0$

$T=400N, where T_1=T_2=T$

$\sum F_y=0\Rightarrow 2T-W-200=0$

$\Rightarrow 800-200=W \quad \therefore W=600\ N$

24. Applying conservation of angular momentum,

$(5+5)\omega' = 5\times10+5\times20=150$

Or $\omega' = 15\ rad\ s^{-1}$

Initial kinetic energy $= \frac{1}{2}\times5\times10\times10+\frac{1}{2}\times5\times20\times20$

$=250+1000=1250\ J$

Final kinetic energy $=\frac{1}{2}\times10\times15\times15=1125\ J$

Loss of kinetic energy $=(1250-1125)\ J = 125\ J$

25. Heat lost by m grams of steam is gained by calorimeter and water in it. (Steam at 100^0C)

$\xrightarrow{\text{Condensation}}$((Water at 100^0C $\xrightarrow{\text{Cooling}}$(Water at 31^0C)

Heat lost by steam (Colorimeter and water at 25^0C) $\xrightarrow{\text{Heating}}$ calorimeter and water at 31^0C

$=mL_1+m\times S_w\times(100-31)$

$=m\left[L_1+S_w\times(100-31)\right]$

$=m\left[540+1(100-31)\right]=m\times609$

Heat gained by calorimeter and water in calorimeter

$=(M+W)\times S\times(31-25)$

$=(180+20)\times1\times6=200\times6$

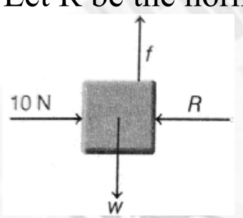
As, heat lost = heat gained

$\Rightarrow m\times609=200\times6$

$\Rightarrow \quad m=\frac{1200}{609}=1.97=2g$

Hence, correct option is (a)

26. Let R be the normal contact force by wall on the block.



$R=10\ N$

$f_L = w \text{ and } f_L=\mu R$

$\therefore\ \mu R = w \text{ or } w=0.5\times10=5\ N$

27. $mg = w_{ice}=F_B \Rightarrow 0.5xg+x^3\left(\rho_{Ice}\right)g$

$0.5\times10^3+x^3(0.9)=x^3$

$mg+w_{ice}=F_B$

$0.5g+x^3(0.9)10^3g \quad = x^3\left(10^3\right)g$

$0.5+900x^3 \quad =1000x^3$

$0.5=100x^3$

$5\times10^{-3}=x^3$

$5^{1/3}\times10^{-1}m=x \qquad \therefore\ x=5^{1/3}\times10cm$

$x=1.7\times10cm$

$x = 17cm$

28. Solution: Given, pressure $(\rho)\,\alpha\, kV^3$

$T_1 = 100^0 C, T_2 = 300^0 C$

$\Rightarrow\ \Delta T = T_2 - T_1 = 300 - 100 = 200^0 C$

By using ideal gas equation,

$$\rho V = nRT$$

$\Rightarrow\quad kV^3.V = nRT \Rightarrow kV^4 = nRT$

On differentiating both side w.r.t

Temperature, we get

$4kV^3 \dfrac{dV}{dT} = nR$

$\Rightarrow\quad 4kV^3 dV = nRdT$

$\Rightarrow\quad kV^3 dV = nRdT/4$

$\Rightarrow\quad pdV = nRT/4$

As, work done (W) = $= pdV = nRdT/4$

$$= \frac{nR}{4}\Delta T = \frac{nR}{4}\times 200 = 50nR$$

29. In equilibrium, $\dfrac{600\times 10}{800\times 10^{-4}} = \dfrac{F}{25\times 10^{-4}} + h\rho g$

$\Rightarrow \dfrac{F}{25\times 10^{-4}} = \dfrac{60}{8}\times 10^4 - 8\times\left(0.75\times 10^3\right)\times 10$

$\dfrac{F}{25\times 10^{-4}} = 1.5\times 10^4$

$\Rightarrow\ F = 37.5\ N$

30. Given, $s = 54\ m, v = 0$

$$u = 150km/h = 150\times\frac{5}{18} = \frac{125}{3}m/s$$

Using $v^2 - u^2 = 2as$, we get

$0 - \dfrac{125^2}{9} = 2\times a\times 54$

$\Rightarrow\quad a = \dfrac{-125^2}{2\times 54\times 9} m/s^2$

Now again using $v^2 - u^2 = 2as$, we get

$0 - \dfrac{125^2}{9^2} = 2\times\left(\dfrac{-125^2}{2\times 27\times 9}\right)\times s$

$\Rightarrow\quad s = \dfrac{125^2}{9^2}\times\dfrac{54\times 9}{125^2} \Rightarrow s = 6m$

CHEMISTRY

31. Ca^{2+} radius is 100 P.M

32. $B_3N_3H_6 + 9H_2O \rightarrow 3H_3BO_3 + 3NH_3 + 3H_2$

33. 40 K.J/mole

34. Diamond has a giant molecular perfect tetrahydron structure. And a 3D solid network of strong covalent bonds. Dut to this strong covalent bonding, it requires very high energy to separate the atoms

35. NCERT

36. The electro positivity of Alkali metals increases with increase in Atomic number
$Li^+ < Na^+ < K^+ < Rb^+ < Cs^+$

37. As we increase in concentration, the association of salvation electrons get started and paramagnetism decreases & colour changes to bronze.

38. Conceptual

39. Size $\alpha \dfrac{1}{Thermal\ \ stability}$ size is inversely proportional to thermal stability

$LiBH_4$	LiA/H_4	$LiGaH_4$	$CaH_2 > SrH_2 > BaH_2$
$2s-1s$	$3s-1s$	$4s-1s$	$4s-1s\ \ 5s-1s\ 6s-1s$
$2p$	$3p$	$4p$	

Similar sized are combine over lapping is good thermal stability is more.

40. $C_4 \rightarrow -C \equiv C- \ \rightarrow\ C_2^{\ 2-}$

$1\sigma,\ \ 2\pi$ bonds is present

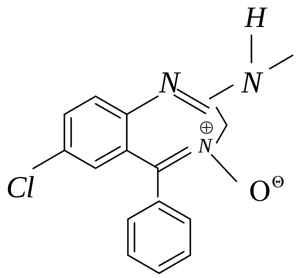
$\underset{1s}{\sigma^2}\ \underset{1s}{\sigma^{2*}}\ \underset{2s}{\sigma^2}\ \underset{2s}{\sigma^{2*}}\ \begin{matrix}\pi 2px^2\\ \pi 2py^2\end{matrix}\ \underset{2p_z}{\sigma}\ \begin{matrix}\pi 2px^*\\ \pi 2py^*\end{matrix}\ \underset{2p_z^{\ *}}{\sigma}$

$\frac{1}{2}(B.O - A.B.O) \qquad \frac{1}{2}(8-4) = 4/2 = 2$

41. $BeCl_2$ is covalent compound $CaCl_2$ & $MgCl_2$ is ionic compound
$\left[Be(H_2O)_4\right]^{2+}$ is an acid. The hydrated beryllium ions is are strongly acidic. The small beryllium ion centre attracts at the electrons in the bonds towards it self and that makes the hydrogen atoms in the water even more positive than the usually rare.

42. 1,2 – glycosidic linkage b present in sucrose.
α-D – gluco pyranose + β-D Fructo Furanose
β-D – gluco pyranose + α-D fructo Furanose
D is reading sugar + 1 is non reducing sugar
1 Reducing unit + No reducing unit

43. SN^2 reaction mechanism it is follow
Nu^- attacks at back side

44. 

45. Resonance energy = Calculated Heete of – practiced heel – hydrogenation of hydrogenation $=7\times 98.6-116.2$ $=84\ K.cal/more$

46. 1) $SN^{1}\rightarrow$ Reaction $\rightarrow$ product is racemisation of product
50% R + 50 % s
2) Pyramidal free radical which undergo inversion

47.

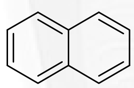
$$CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-Cl+\xrightarrow[Dryether]{Zn} 27\text{dimethyl octane}$$

$$\downarrow$$

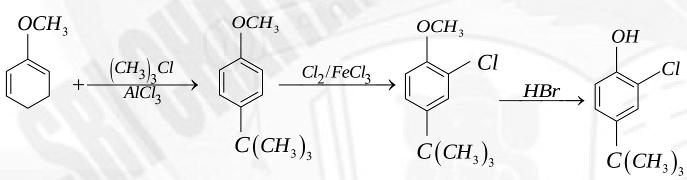
$$CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-Cl-Li$$

$$\downarrow H_2O$$

$$CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_3$$

48.  Benzenoid compounds

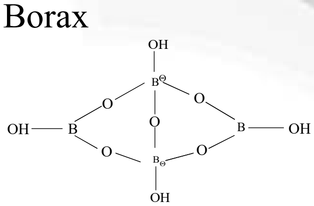
49.



50. $A\xrightarrow[Feel_3]{Cl_2}B\xrightarrow{H_2O}C$ $\downarrow$ +ve. Test with Tollen's Reagent.

51. $B_2H_6+6C_2H_5OH\rightarrow 2B(OC_2H_5)_3+6H_2$

52. Borax



Boran $\rightarrow 2Na^{+}\left[B_4O_5(OH)_4\right]^{2-}.8H_2O$

Sp^2 Hybridised boron atom only participate in .

$P^{\pi}-p^{\pi}$ back bonding No of such boran atoms Is 2

53. Li_2CO_3, $MgCO_3$

54. Conceptual

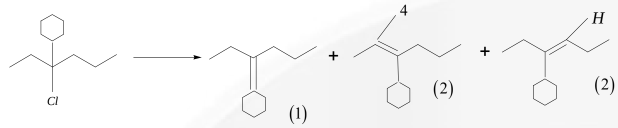
55. $Al = +3 \quad Si = +4, \quad Pb = +2$

$B = +3 \ (or) \ -3$

56. $PHBV, Nylon\ 2 - Nylon\ 6$ is biodegrades

Nylon 6, 6, ----- Teflon is non biodegrades

57.



58. c,d,e,f $\rightarrow$ will the iodoform test have their $\rightarrow -\overset{O}{\overset{\|}{C}}-CH_3 - (or) - \underset{OH}{\underset{|}{CH}}$ groups

59. $CH_3^{\oplus}, BF_3, CH_3^{\cdot}, SO_3 \rightarrow Planar$

Sp^2 Hybridised atoms

60. 4 Aldol product is given

$CH_3CHO + CH_3CH_2^{\ominus}CHO \rightarrow 4 \ product$

1) $CH_2 - \underset{H}{\underset{|}{C}} = \underset{CH_3}{\underset{|}{CH}} - \overset{O}{\overset{\|}{C}} - H$ 2) $CH_3 - \underset{H}{\underset{|}{C}} = CH - \overset{O}{\overset{\|}{C}} - H$ 3) $CH_3 - \underset{\underset{H}{\underset{|}{H_3C - C - H}}}{\underset{\|}{CH}} - \overset{O}{\overset{\|}{C}} - H$

MATHEMATICS

61. $\begin{vmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ \alpha & \beta & 7 & 3 \end{vmatrix}$ by using $R_2 - R_1, R_3 - R_1$ it become

$\begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & \beta-\alpha & 7-\alpha & 3-6\alpha \end{vmatrix}$. Then by using $R_3 - (\beta-\alpha)R_2$ it become

$\begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & \alpha-2\beta+7 & 2\alpha-8\beta+3 \end{vmatrix}$

62. For $P = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ we get $P^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2P$

$A = iP \Rightarrow A^2 = -2P \Rightarrow A^4 = 4(2P) \Rightarrow A^8 = 64(2P)$ $\quad A^8 = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

63. II-4(I) gives $5y+16z=8$, $III-2(II)$ gives $2y+(\lambda-8)z = -\mu-1$

Infinetly many solutions $\Rightarrow \frac{5}{2} = \frac{16}{\lambda-8} = \frac{8}{-\mu-1}$

64. $|A| = e^t.e^{-t}.e^{-t}\begin{vmatrix} 1 & \sin t - 2\cos t & -2\sin t - \cos t \\ 1 & 2\sin t + \cos t & \sin t - 2\cos t \\ 1 & \cos t & \sin t \end{vmatrix}$

$= e^{-t}\begin{vmatrix} 1 & \sin t - 2\cos t & -2\sin t - \cos t \\ 0 & \sin t + 3\cos t & 3\sin t - \cos t \\ 0 & -\sin t + 3\cot & 3\sin t + \cos t \end{vmatrix} = e^{-t}\begin{vmatrix} 1 & \sin t - 2\cos t & -2\sin t \\ 0 & \sin t + 3\cos t & 3\sin t - \cos t \\ 0 & 6\cos t & 6\sin t \end{vmatrix}$

65. A satisfies its characteristic equation

66. $A^T = BCD \to A.A^T = ABCD = S$ $\quad S^3 = (ABCD)(ABCD)(ABCD) = (ABC)(DAB)(CDA)(BCD)$

$= D^T C^T B^T A^T = (BCD)^T A^T = AA^T = S$

67. Take trans pose of 2nd determinant then have same C_1, C_2 with different C_3

Make it as one det by adding both C_3 Then $C_1 = -C_3 \Rightarrow$ always zero

68. Conceptual

69. Put $\lambda = 0$ and get A, similarly put -1 and 1 then get B, C

70. $\frac{K+1}{K} = \frac{8}{K+3} = \frac{4k}{3K-1}$

71. Put $\frac{1}{\sin^2 t} = y$ then $\lim_{y\to\infty}\left(1^y + 2^y + \text{-----------} n^y\right)^{\frac{1}{y}}$

$= \lim_{y\to\infty}(n^y)^{\frac{1}{y}}\left[\left(\frac{1}{n}\right)^y + \left(\frac{2}{n}\right)^y + \text{-------}\left(\frac{n-1}{n}\right)^y + 1\right] = n(1)$

72. $\bar{c}.\bar{a} = ((\bar{a} \times \bar{c}) + \bar{b}).\bar{a} = \bar{b}.\bar{a}$ $\quad \bar{b} \times \bar{c} = (\bar{b}.\bar{c})\bar{a} - (\bar{a}.\bar{b})\bar{c}$

$\therefore [\bar{a}\,\bar{b}\,\bar{c}] = \bar{b}.\bar{c} - (\bar{a}.\bar{b})(\bar{a}.\bar{c}) = \bar{b}.\bar{c} - (\bar{a}.\bar{b})^2$

Also $\bar{c}.\bar{b}=1-[\bar{a}\,\bar{b}\,\bar{c}]$ $\therefore 2\ [\bar{a}\,\bar{b}\,\bar{c}]=1-(\bar{a}.\bar{b})^2\le 1 \therefore [\bar{a}\,\bar{b}\,\bar{c}]\le\frac{1}{2}$

73. $x=\frac{4-y^2}{4},\quad x=\frac{y-2}{2}$ then $f(y)-g(y)$ gives $\frac{1}{4}(y^2+2y-8)$ then use $\left(\frac{\Delta^{3/2}}{6a^2}\right).\frac{1}{4}$

74. $l_1+l_2=20,\quad A_1=\left(\frac{l_1}{4}\right)^2,\ A_2=\pi\left(\frac{l_2}{2\pi}\right)^2\ 2A_1+3A_2=\frac{l_1^2}{8}+\frac{3l_2^2}{4\pi}$ *Then put* $l_2=20-l_1$ *and*

Differentiate it $\Rightarrow l_1=\frac{120}{\pi+6},\ l_2=\frac{20\pi}{\pi+6}$

75. Conceptual

76. $g(x)=\begin{cases} x^3;\ x<1\ \ has\ \ range\ \ (-\infty,1) \\ 3x-2;\ x\ge 1\ \ has\ \ range\ \ [1,\infty) \end{cases}$ (fog) $(x)=\begin{cases} x^3+2 & ; \quad x<0 \\ x^6 & ; \quad 0\le x<1 \\ (3x-2)^2 & ; \quad x\ge 1 \end{cases}$

It is continuous & differentiable at $x=0$

Discontinuous, so not differential at $x=1$

77. Use $f(0)=0, f'(0)=-8, f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\left(\frac{f(h)}{h}+6x(x+h)\right)=f'(0)+6x^2 \qquad \therefore f'(x)=6x^2-8$

78. $\left(\frac{y^4}{x^4}+\frac{2y^2}{8x^2}\right)\frac{dy}{dx}=\frac{x^2+4x^2}{2x}$ simplify and get $y^2dy=2x^3$

79. $\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}=0 \ \Rightarrow\ x-y-z=0$ is plane

80. Solution: $f^1(x)=4x^3-12x^2+24x+1$ and $f^{11}(x)=12x^2-24x+24=12\left[(x-1)^2+1\right]>0\ f^1(x)$ is

increasing only. So $f^1(x)=0, 3^{rd}$ degree polynomial equation, has only one root

f (x) is concave up only and $f(-1)>0, f(0)<0, f(1)>0$

81. Conceptual

82. $\begin{vmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ \alpha & \beta & 7 & 3 \end{vmatrix}\sim\begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & \beta-\alpha & 7-\alpha & 3-6\alpha \end{vmatrix}$ *by* R_2-R_1 *and* $R_3-\alpha R_1$

$\sim\begin{vmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & \alpha-2\beta+7 & 2\alpha-8\beta+3 \end{vmatrix}$ *by* $R_3-(\beta-\alpha)R_2$

Infinitely many solutions $\alpha-2\beta+7=0$ *and* $2\alpha-8\beta+3=0$

83. $A^2+5I=\begin{bmatrix} a^2+bc+5 & ab+bd \\ ac+cd & d^2+bc+5 \end{bmatrix}=O_{2x2}$

$b(a+d)=c(a+d)=0 \rightarrow a+d=0$ so $d=-a, d^2=-ad \rightarrow |A|=5$

84. Solution: Using $R_1-(\sec x)R_3$ *we get* $f(x)=(\sec^2 x+\cot x.\operatorname{cosec} x-\cos x)(\cos^4 x-\cos^2 x)$

Simplification gives $f(x)=-\sin^2 x-\cos^5 x$

$$\int_0^{\pi/2} f(x)dx=-\frac{1}{2}\frac{\pi}{2}-\frac{4}{5}\frac{2}{3}(1)=-\frac{\pi}{4}-\frac{8}{15}$$

85. use R_2-R_1, R_3-R_1 then put $q=p+d, r=p+2d, s=p+3d \Rightarrow f(x)=$ det expansion $=-2d^2$

86. $\int_0^{40\pi}|\sin x|[\sin x]=20\int_0^{2\pi}|\sin x|[\sin x]=20\left[\int_0^{\pi}(\sin x)(0)+\int_{\pi}^{2\pi}(-\sin x)(-1)dx\right]$

87. $I=\int\frac{dx}{(\cos x-\sin x)(1+\sin x\cos x)}=\int\frac{(\cos x-\sin x)dx}{(\cos x-\sin x)^2(1+\sin x\cos x)}=\int\frac{dx}{(1-2\cos x\sin x)(1+\sin x\cos x)}$

Take $\sin x+\cos x=t, \quad \sin x.\cos x=(t^2-1)/2$

$$I=\int\frac{2dt}{(2-t^2)(1+t^2)}=\int\frac{2/3}{t^2+1}+\frac{2/3}{2-t^2}$$

88. $f^1(x)=20x^4-100x^3=20x^3(x-5)=0,$ So $x=0,5$ are stationary points

$f^1(-1)>0$ *and* $f^1(1)<0,$ so f changes from Increase to Decrease at x = 0

So x=0 in mixima Similarly x = 5 is minima So n = 2

$f^{11}(x)=80x^3-300x^2=20x^2(4x-15)=0,$ *at* $x=0,15/4$

$f^{11}(-1)<0$ *and* $f^{11}(1)<0,$ so only concave down exists before and after $x=0$, not inflection point

But $x=15/4$ is a point of inflection. Hence m = 1

89. Equation i, j, k components for both lines we get

$2+t=-3-s; \quad 9+2t=7+2s; \quad 13+3t=p-3s$

$\Rightarrow \quad s+t=-5; \quad s-t=1; \quad p=13+3s+3t$

$\Rightarrow \quad s=-2, \quad t=-3$ *and* $p=-2$ Hence intersection point is $(-1,3,4)$

90. Derive $\frac{dy/dt}{dx/dt}$ *and* $t=-1$