KEY SHEET

1.	B	**2.**	A	**3.**	B	**4.**	A	**5.**	A
6.	D	**7.**	B	**8.**	A	**9.**	A	**10.**	A
11.	B	**12.**	A	**13.**	B	**14.**	B	**15.**	C
16.	D	**17.**	A	**18.**	D	**19.**	D	**20.**	A
21.	A	**22.**	B	**23.**	A	**24.**	A	**25.**	C

HINTS & SOLUTIONS

1. Since $y = e^x$ *and* $y = \log_e x$ are inverse to each other.

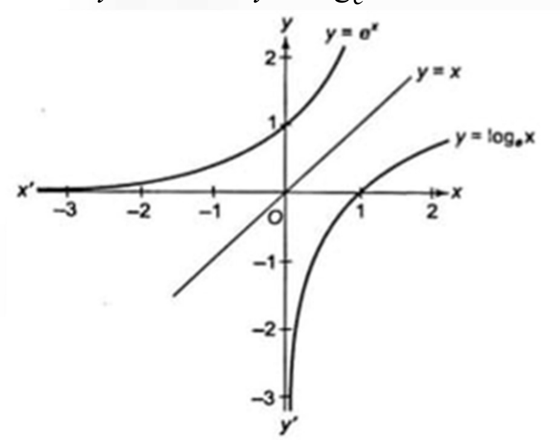
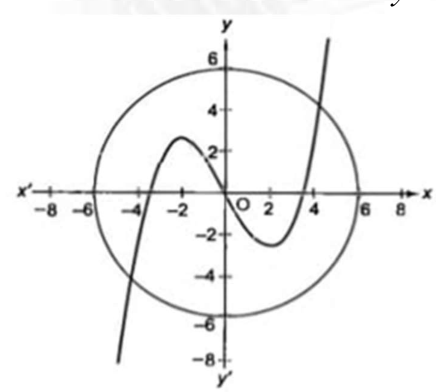


Fig. S-9.61

2. Statement – 2 is correct as $y = f(x)$ is odd and hence statement – 1 is correct.

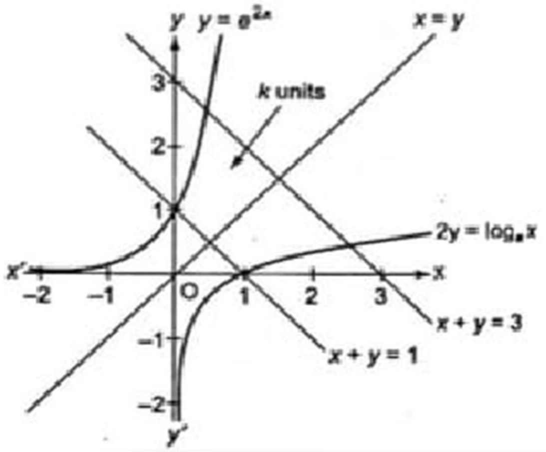


3. $$Area = \int_1^3 -\left(x^2 - 4x + 3\right)dx - \left(\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right)\Big|_1^3 = \frac{4}{3} sq.units$$

Therefore, statement – 1 is time.
Obviously, statement – 2is true, but does not explain statement – 1

4. Given curves are $y^2 - 2y + 4x + 5 = 0$ *and* $x^2 + 2x - y + 2 = 0$ or
$(y-1)^2 = -4(x+1)$ *and* $(x+1)^2 = y - 1$.
Shifting origin to (-1, 1), equation of given curves changes to $y^2 = -4x$ *and* $X^2 = Y$
Hence, statement -1 is true and statement-2 is correct explanation of statement -1.

5. $y = e^{2x}$ *and* $2y = \log_e x$ ae inverse of each other.



The shaded area is given as *k* sq.units.
Thus, the required are is *2k* sq.units

6. R_1 : Points $P(x, y)$ is nearer to (1, 0) than to $x = -1$

$\Rightarrow \sqrt{(x-1)^2 + y^2 < |x+1|} \Rightarrow y^2 < 4x$

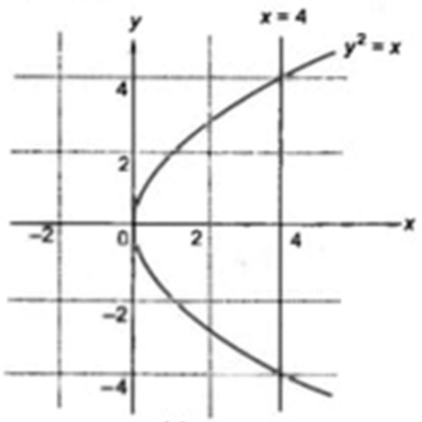
Hence, point P lies inside parabola $y^2 = 4x$.

P_2: Point $P(x, y)$ is nearer to (0, 0) than to (8, 0)

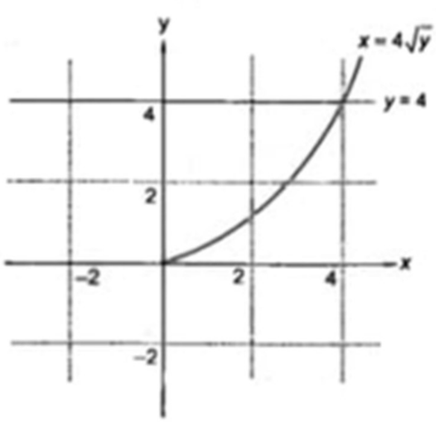
$\Rightarrow |x| < |x-8|$ *or* $x^2 < x^2 - 16x + 64$ *or* $x < 4$

Hence, point P is towards left side of line $x = 4$

The area of common region of R_1 and R_2 is the area bounded by $x = 4$ *and* $y^2 = 4x$



(a)



There area is twice the area bounded by $x = 4\sqrt{y}$ *and* $y = 4$, Now, the area bounded by $x = 4\sqrt{y}$ *and* $y = 4$ is

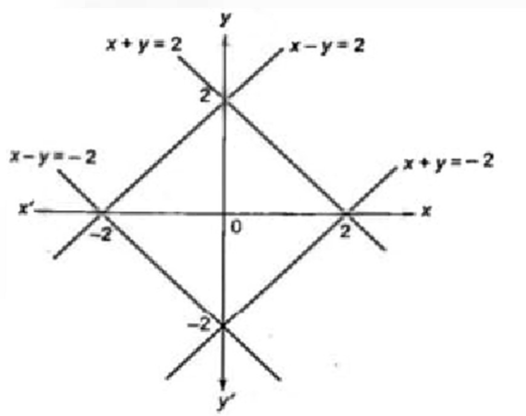
$$A = \int_0^4 \left(4 - \frac{x^2}{4}\right)dx = \left[4x - \frac{x^3}{12}\right]_0^4 = \left[16 - \frac{64}{12}\right] = \frac{32}{3} sq.units$$

Hence, the area bounded by R_1 *and* R_2 is $\frac{64}{3} sq.units$

Thus, statement – 1 is false but statement – 2 is true.

7. $2 \geq \max.\{|x-y|, |x+y|\}$

$\Rightarrow |x-y| \leq 2$ *and* $|x+y| \leq 2$, which forms a square of diagonal length 4 units.



Hence, the area of the region is $\frac{1}{2} \times 4 \times 4 = 8 sq.units$

This is equal to the area of the square of side length $2\sqrt{2}$

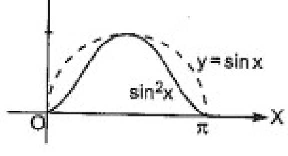
8. $\because \; y = e^{x^3}$

$\therefore \frac{dy}{dx} = e^{x^3}.\, 3x^2 > 0$

$\Rightarrow y$ is an increasing function and area enclosed by the curve $y = e^{x^3}$ between the lines $x = a, x = b$ *and* $x - axis$ *is* $\int_a^b e^{x^3} dx$

9. For $0 < t < 1$

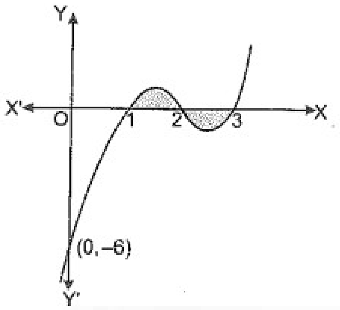
$t^2 < t$



$\therefore\ \sin^2 x < \sin x \Rightarrow \int_0^{\pi} \sin^2 x\, dx < \int_0^{\pi} \sin x\, dx$

10. It is a clear from the figure for $x \in [2.2, 2.8] \Rightarrow (x-1)(x-2)(x-3) \le 0$

$\therefore$ Required area $= \left|\int_{2.2}^{2.8} f(x)dx\right| = \left|\int_{2.2}^{2.8} (x-1)(x-2)(x-3)\,dx\right|$



11. Area $= \int_1^3 -\left(x^2-4x+3\right)dx = -\left[\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right]_1^3 = \frac{4}{3} sq.units$

Therefore, Assertion is true

Obviously, Reason is true but does not explain Assertion.

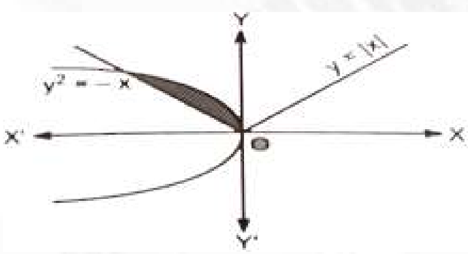
12. Given curves are $y^2 - 2y + 4x + 5 = 0$ and $x^2 + 2x - y + 2 = 0$ or

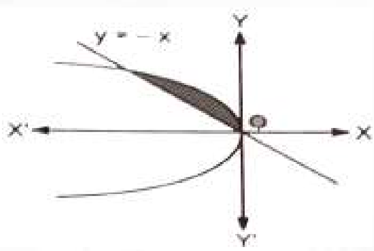
$(y-1)^2 = -4(x+1)$ and $(x+1)^2 = y-1$

Shifting origin to (-1, 1) equation of given curves change to $Y^2 = -4x$ and $X^2 = Y$.

Hence, Assertion is true and Reason is correct explanation of Assertion.

13.

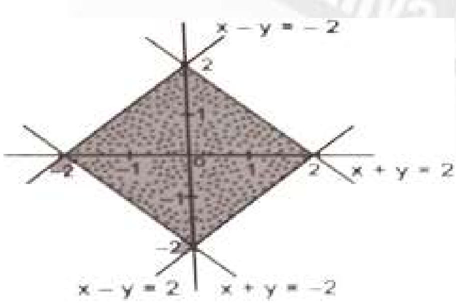




It is clear from the above figures the area between the curve $y^2 = -x$ and $y = |x|$ is the same and between $y^2 = -x$ and $y = -x$.

$\therefore$ Statement -1 is true but Statement-2 is not a correct explanation of Statement-1

14.



$\because$ max. $\{|x-y|, |x+y|\} \le 2 \Rightarrow |x-y| \le 2$ and $|x+y| \le 2$

Which forms a square of diagonal length 4 units

$\Rightarrow$ The area of the bounded region $=\frac{1}{2}\times 4\times 4 = 8\ sq.units$

The is equal to the area of the square of side length $2\sqrt{2}$.

Here both S-1 are true but statement-2 is not correct explanation of S-1

24. **S-1 :**

$$\int_0^3 \left|x^3-4x^2+3x\right|dx = \int_0^1 (x^3-4x^2+3x)dx - \int_1^3 (x^3-4x^2+3x)dx$$

$$=\left[\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}\right]_0^1 - \left[\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}\right]_1^3$$

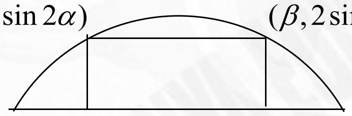
$$=\left(\frac{1}{4}-\frac{4}{3}+\frac{3}{2}\right)-\left[\left(\frac{81}{4}-\frac{4}{3}\times 3^3+\frac{27}{2}\right)-\left(\frac{1}{4}-\frac{4}{3}+\frac{3}{2}\right)\right]$$

$$=\frac{3-16+18}{12}-\left[\frac{81}{4}-\frac{108}{3}+\frac{27}{2}\right]+\left(\frac{3-16+18}{12}\right)$$

$$=\frac{5}{12}-\left(\frac{-27}{12}\right)+\frac{5}{12}$$

$$=\frac{37}{12}$$

S-2 : g(x) is not differentiable at 0,1,2



$$0 \le y \le 2\sin 2x, 0 < x < \frac{\pi}{2}$$

For rectangle $2\sin 2\alpha = 2\sin 2\beta \Rightarrow \alpha+\beta=\frac{\pi}{2}$

$$Perimeter\ = 2(\beta-\alpha)+4\sin 2\beta = 2\left(\beta-\left(\frac{\pi}{2}-\beta\right)\right)+4\sin 2\beta$$

$$=4\beta-\pi+4\sin 2\beta$$

$$\frac{dp}{d\beta}=4\cos 2\beta.2+4$$

$$\cos 2\beta = -\frac{1}{2}, \beta=\frac{\pi}{3}$$

$$\frac{d^2p}{d\beta^2}=-16\sin 2\beta<0\ so\ \ p\,is\ \max$$

So, area of rectangle = $(\beta-\alpha)2\sin 2\beta$

$$=\left(\frac{\pi}{3}-\frac{\pi}{6}\right)2\sin 2\left(\frac{\pi}{3}\right)$$

$$=\frac{\pi}{6}\times 2\times\frac{\sqrt{3}}{2}=\frac{\pi}{2\sqrt{3}}$$

25. **S-1 :** $F(x) = \sin x \int_0^x \cos t\, dt + 2\int_0^x t dt - x^2 + \cos^2 x$

$= \sin x(\sin t)_0^x + 2\left(\frac{t^2}{2}\right)_0^x - x^2 + \cos^2 x = \sin^2 x + x^2 - x^2 + \cos^2 x = 1$

$A = \int_0^5 xF(x)dx = \int_0^5 (x)(1)dx = \left[\frac{x^2}{2}\right]_0^5 = \frac{25}{2}$

S-2 : Required area = $= 2\int_0^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right)dx = 12\sqrt{3} sq.units$