Exercise-5

01. Math the following (adv 2013)

$P_1 : 7x + y + 2z = 3, P_2 : 3x + 5y - 6z = 4$

Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}, L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and The planes $P_1 : 7x + y + 2z = 3, P_2 : 3x + 5y - 6z = 4$ let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines $L_1\, and\, L_2$ and perpendicular to planes $P_1\, and\, P_2$

Match List I with List II and select the correct answer using the code given below the list

	List-I		List-II
P	$a =$	1	13
Q	$b =$	2	-3
R	$c =$	3	1
S	$d =$	4	-2

	P	Q	R	S
(1)	3	2	4	1
(2)	1	3	4	2
(3)	3	2	1	4
(4)	2	4	1	3

Key : 1

Sol :Let any point on L_1 is $(2\lambda + 1, -\lambda, \lambda - 3)$ and that of L_2 is $(\mu + 4, \mu - 3, 2\mu - 3)$ for point of intersection of $L_1\, and\, L_2$ $2\lambda + 1 = \mu + 4,\ -\lambda = \mu - 3,\ \lambda - 3 = 2\mu = 3\ \lambda = 2, \mu = 1$

Intersection point at $L_1\ and\ L_2$ is $(5,-2,-1)$ equation of planed passing through

$(5,-2,-1)\, and \perp er\ to\ P_1\ \& P_2$ is given $\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$

$x - 3y - 2z = 13,\ a = 1, b = -3, c = -2, d = 13$

02. Consider the ling $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}, L_2 : \frac{x-4}{5} = \frac{y+3}{5} = \frac{z+3}{10}$ of the planes: (duplicite)

$P_1 : 7x + y + 2z = 3, P_2 : 3x + 5y - 6z = 4$ let $ax + by + cz =$ of be the equation of the plane passing through the point of intersection of lines L_1 *and* L_2 and perpendicular to planes P_1 *and* P_2 Match List-I with list II select the correct answer the code given below the lists

	List-I		List-II
P	$a =$	1	1744
Q	$b =$	2	-48
R	$c =$	3	16
S	$d =$	4	-32

	P	Q	R	S
(1)	1	3	4	2
(2)	3	2	4	1
(3)	3	2	1	4
(4)	2	4	1	3

Key : 2

Sol : Let any point on L_1 *is* $(2\lambda+1, 3\lambda+2, 4\lambda+3)$ and that of L_2 *is* $(5\mu+4,\ 5\mu-3, 10\mu-3)$ for point of intersection of L_1 *and* L_2 $2\lambda+1 = 5\mu+4, 3\lambda+2 = 5\mu-3, 4\lambda+3 = 10\mu-3$

$\lambda = -8,\ \mu = \frac{-19}{5}$

Intersection of L_1 *and* L_2 *is* $(-15,-22,-29)$ intersection of a plane passing through $(-65,-22,-29)$ *and perpendicular to* P_1 & P_2 given by

$$\begin{vmatrix} x+15 & y+22 & z+29 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 8$$

$16x - 48y - 32z = 1744$

$a = 16, b = -14, c = -32, d = 1744$

03. Match the statements/expenses given, in Column-I with the values given in Column-II (Adv 2006)

	Column-I		Column-II
A	Value(s) of K for which the planes $kx+4y+z=0,$ $4x+ky+2z=0$ and $2x+2y+z=0$ intersect in a straight lines	P	6
B	Two rays $x+y=\|a\|$ and $ax-y=1$ intersects, each other in the first quadrant in the internal at $(a_0, 1_{00})$ the value of a_0 is	Q	2
C	Point (α,β,γ) lines on the plane $x+y+z=2$ let $\bar{a}=\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k},\ \bar{k}\times(\bar{k}\times\bar{a})=0$ $then\ \gamma=$	R	4
D	A line from the origin meets lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-8/3}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively if length $PQ=d$ then d^2	S	1

Key : $A\to Q,R, B\to S, C\to Q, D\to P$

Sol : (A) Since given planes intersect in straight line

$$\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix}=0$$

$$K(K-4)-4(4-Q)+(8-2K)=0$$

$$K^2-4K+8-2K=0$$

$$K=2\,or\,4$$

(B) $x+y=|a|$

$$\frac{ax-y=1}{(1+a)x=1+|a|}$$

$$x=\frac{1+|a|}{1+a}, y=\frac{a|a|-1}{a+1}$$

Rays intersect each other in Q_1 ie $x>0, y\geq 0$

$$\Rightarrow a+1>0\ and\ a|a|-1>0 \Rightarrow a>1$$

$$\therefore a_0=1$$

(c). Given that (α,β,γ) lies the plane $x+y+z=2 \Rightarrow \alpha+\beta+\gamma=2$

Also $\bar{k}\times(\bar{k}\times\bar{a})=(\bar{k}.\bar{a})\bar{k}-(\bar{k}.\bar{k})\bar{a}$

$$r\bar{k}-\alpha\bar{i}-\beta\bar{j}-r\bar{k}=0 \Rightarrow \alpha\bar{i}+\beta\bar{j}=0\ (\therefore \alpha+\beta+\gamma)$$

$$\alpha=0, \beta=0, \gamma=2$$

Let the line though given be $L:\frac{x}{a}=\frac{y}{b}=\frac{z}{c}---(i)$

Since line L intersects $L_1:\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}....(ii)$

And $L_2:\frac{x-8/3}{2}=\frac{y+3}{-1}=\frac{z-1}{1}---(iii)\, at\ P and\, Q$

$\therefore$ Line L and L_1 are coplanar

$$\therefore \_\_\_\_ \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\begin{vmatrix} 2 & 1 & -1 \\ a & b & c \\ 1 & -2 & 1 \end{vmatrix}=0 \Rightarrow a+3b+5c=0--(iv)$$

Also L and L_2 Coplanar

$$\begin{vmatrix} 8/3 & -3 & 1 \\ a & b & c \\ 2 & -1 & 1 \end{vmatrix}=0 \Rightarrow 3a+b-5c=0--(v)$$

Solving (iv) and (v) $\frac{a}{-15-5}=\frac{b}{15+5}=\frac{c}{1-9}\,(or)\frac{a}{5}=\frac{b}{-5}=\frac{c}{2}$

Hence ______ (1) become $\frac{x}{5}=\frac{y}{-5}=\frac{z}{2}=\lambda$

Any point on L, $P(5\lambda,-5\lambda,2\lambda)$ which lines on (ii) also

$$\frac{5\lambda-2}{1}=\frac{-5\lambda-1}{-2}=\frac{2\lambda+1}{1}\Rightarrow \lambda=1$$

$P=(5,-5,2)$

Also any point on $L\ Q(5\lambda,-5\lambda,2\lambda)$

Which lise on (ii) also $\frac{5\lambda-8\backslash 3}{1}=\frac{-5\lambda+3}{-1}=\frac{2\lambda-1}{1}\Rightarrow \lambda=2\backslash 3$

$P=(5,-5,2)$

Also any point on $L\ Q(5\lambda,-5\lambda,2\lambda)$

Which lies on (iii) also $Q\left(\frac{10}{3},\frac{-10}{3},\frac{4}{3}\right)$

Hence $d^2=PQ^2=\left(\frac{25}{9}+\frac{25}{9}+\frac{4}{9}\right)=6$

04. Mach the fallowing (duplicite)

	Column-I		Column-II
A	Values of K for which the planes $kx+2y+z=0$ $2x+ky+z=0$ and $2x+2y+z=0$ intersect in a straight lines	P	2
B	Two rays $x+y=\|b\|$ and $bx-y=1$ intersect each other in the first quadrant in the internal $b\in(b_0,\infty)$ then the value of b_0	Q	3
C	Point (α,β,γ) lines on the plane $x+y+z=2$ let $\bar{a}=\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k}, \bar{j}\times(\bar{j}\times\bar{a})=0$ then $\beta=$	R	1

Key : $A\to P, B\to R, C\to P$

Sol (A) $\begin{vmatrix} k & 2 & 1 \\ 2 & k & 1 \\ 2 & 2 & 1 \end{vmatrix}=0$

$K(k-2)-2(2-2)+1(4-2k)=6$

$K^2-2k+4-2k=0$

$k^2-4k+4=0 \Rightarrow (k-2)^2=0, K=2$

(B) $x+y=|b|$

$bx-y=1$

$(1+9)x=1(b)$

$x=\dfrac{1+(b)}{1+b}, y=\dfrac{b(b)-1}{b+1}$

Ray intersect each other in Q_1 ie $x>0, y>0$

$b+1>0$ and $b|b|-1>0 \Rightarrow b>1$

$\therefore b_0=1$

(C) Given the $(\alpha,\beta,\gamma)=(\bar{j}.\bar{a})\bar{j}-(\bar{j}.\bar{j})\bar{a}$

$\beta\bar{j}-\alpha\bar{i}-\beta\bar{j}-\gamma\bar{k}=0$

$\alpha\bar{i}+r\bar{k}=0$

$\alpha=0, \lambda=0, \beta=2$

05. Consider the following linear equation $ax+by+cz=0; bx+cy+az=0; cx+ay+bz=0$ Match the conditions/expression in Column-I with statements in Column-II and indicate your answer by darking the appropriate bubbles in 4×4 matrix given the ORS

(adv 2007)

	Column-I		Column-II
A	$a+b+c\neq0$ and $a^2+b^2+c^2=ab+bc+ca$	P	The equation represent planes meeting only at a single point
B	$a+b+c=0$ and $a^2+b^2+c^2\neq ab+bc+ca$	Q	The equation represent the line $x=y=z$
C	$a+b+c\neq0$ and $a^2+b^2+c^2\neq ab+bc+ca$	R	The equation represent identical plane
D	$a+b+c=0$ $a^2+b^2+c^2=ab+bc+ca$	S	The equation represent to whole of three dimensional space

Key : $A\to R, B\to Q, C\to P, D\to S$

Sol : The determinant of the coefficient matrix of given equation as

$$\Delta=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=-(a+bc)\left(a^2+b^2+c^2-ab-bc-ca\right)$$

$$=-\frac{1}{2}(a+b+c)(a-b)^2+(b-c)^2+(c-a)^2$$

(A) When $a+b+c\neq0$ and $a^2+b^2+c^2-ab-bc-ca=0$, $(a-b)^2+(b-c)^2+(c-a)^2=0$

$a=b=c(but\neq0\,as\,a+b+c\neq0)$

(B) when $a+b+c=0$ and $a^2+b^2+c^2-ab-bc-ca\neq0\Rightarrow\Delta=0$ and a,b,c are not all equal

All equation are not identical but have infinite many solutions

$ax+by=(a+b)z-(1)$

$bx+cy=(b+c)z--(2)$

On solving (1) and (2) we get $\left(b^2-ac\right)y=\left(b^2-ac\right)z\Rightarrow y=z$

$\Rightarrow ax+by+cy=0\Rightarrow ax=ay\Rightarrow x=y,\quad x=y=z$

The equation represent the line $x=y=z$

(C) When $a+b+c\neq0$ and $a^2+b^2+c^2-ab-bc-ca\neq0$

$\Rightarrow\Delta\neq0\Rightarrow$ Equation have only trivial solution i.e $x=y=z=0$

$\therefore$ The equation represents the three planes meeting at a singe point namely origin

(D) When $a+b+c=0$ and $a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow a=b=c \text{ and } \Delta=0 \Rightarrow a=b=c=0$

All equations are satisfied by all x, y, z The equations represent the whole of the three dimensional space

06. Consider the following linear equations $lx+my+nz=0, mx+ny+lz=0, nx+ly+mz=0$

Match the conditions/expression in column I with statement in column II and indicate your answer (duplicite)

	Column-I		Column-II
A	$l+m+n \neq 0 \text{ and } l^2+m^2+n^2=lm+mn+nl$	P	The equation represent the planes meeting only at singe point
B	$l+m+n=0 \text{ and } l^2+m^2+n^2 \neq lm+mn+nl$	Q	The equation represent the line $x=y=z$
C	$l+m+n \neq 0 \text{ and } l^2+m^2+n^2 \neq lm+mn+nl$	R	The equations represent Identical planes
D	$l+m+n=0 \text{ and } l^2+m^2+n^2=lm+mn+nl$	S	The equations represent the whole of the three dimensional space

Key : $A \to R, B \to Q, C \to P, D \to S$

Sol : $\begin{vmatrix} l & m & n \\ m & n & l \\ n & l & m \end{vmatrix} = -(l+m+n)(l^2+m^2+n^2-lm-mn-nl)$

$$= \frac{-1}{2}(l+m+n)\left((l-m)^2+(m-n)^2+(n-l)^2\right)$$

(A) When $l+m+n \neq 0$ and $l^2+m^2+n^2-lm-mn-nl=0$

$\Rightarrow (l-m)^2+(m-n)^2+(n-l)^2=0$

$\Rightarrow l=m=n\,(\text{but} \neq 0 \text{ as } l+m+n \neq 0)$

This equation represent identical planes

(B) When $l+m+n=0$ and $l^2+m^2+n^2-lm-mn-nl \neq 0$

$\Rightarrow \Delta=0$ and l, m, n are not all equal all equations are not identical but have infinite many solutions

$lx+my=(l+m)z --(1)$

$mx+ny=(m+n)z --(2)$

On solving (1) & (2) we get $(m^2-l\,n)y=(m^2-\ln)z \Rightarrow y=z$

$\Rightarrow lx+my+ny=0 \Rightarrow lx=ly \Rightarrow x=y, x=y=z,$ The equations represent the line $x=y=z$

(C) When $l+m+n\neq 0$ and $l^2+m^2+n^2-lm-mn-nl\neq 0\Rightarrow\Delta\neq 0\Rightarrow$ Equation have only trivial solution ie $x=y=z=0$

$\therefore$ The equation represents the three planes meeting at a single point namely origin

(D) When $l+m+n=0\, and\ l^2+m^2+n^2-\ln-mn-nl=0\Rightarrow l=m=n\ and\ \Delta=0\Rightarrow l=m=n=0$

All equations are satisfied by all x,y,z the equation represent the whole of the three dimensional space

PASSAGE-I

07. Consider the lines (adv 2008)

$$L_1:\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}, L_2:\frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$$

01. The unit vector perpendicular to both $L_1\ and\ L_2$

1. $\frac{-\bar{i}+7\bar{j}+7\bar{k}}{\sqrt{99}}$ 2. $\frac{-\bar{i}-7\bar{j}+5\bar{k}}{5\sqrt{3}}$ 3. $\frac{-\bar{i}+7\bar{j}+\bar{k}}{5\sqrt{3}}$ 4. $\frac{7\bar{i}-7\bar{j}-\bar{k}}{\sqrt{99}}$

02. The shortest distance between $L_1\ and\ L_2$ is

1. 0 2. $\frac{17}{\sqrt{3}}$ 3. $\frac{41}{5\sqrt{3}}$ 4. $\frac{17}{5\sqrt{3}}$

03. The distance of the point $(1,1,1)$ from the plane passing through the point $(-1,-2,-1)$ and whose normal is perpendicular to both the lines $L_1\ and\ L_2$

1. $\frac{2}{\sqrt{75}}$ 2. $\frac{7}{\sqrt{75}}$ 3. $\frac{13}{\sqrt{75}}$ 4. $\frac{23}{\sqrt{75}}$

Sol (1) Vector in the direction of $L_1=\overline{b_1}=3\bar{i}+\bar{j}+2\bar{k}$

Vector in the direction of $L_2=\overline{b_2}=\bar{i}+2\bar{j}+3\bar{k}$

Vector perpendicular to both $L_1\ and\ L_2$

$$\overline{b_1}\times\overline{b_2}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 3 & 1 & 2\\ 1 & 2 & 3\end{vmatrix}=-\bar{i}-7\bar{j}+5\bar{k}$$

Required unit vector $\hat{b}=\frac{-\hat{i}-7\hat{j}+5\hat{k}}{\sqrt{+49+25}}=\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

Key : 2

2 sol: The shortest distance between $L_1\ and\ L_2=\frac{(\overline{a_2}-\overline{a_1}).\overline{b_1}\times\overline{b_2}}{|\overline{b_1}\times\overline{b_2}|}=(\overline{a_2}-\overline{a_1})\hat{b}$

Since $\overline{a_1}=-\bar{i}-2\bar{j}-\bar{k},\overline{a_2}=2\bar{i}-2\bar{j}+3\bar{k}$

$\overline{a_2}-\overline{a_1}=3\bar{i}+4\bar{k}$

$$\therefore(\overline{a_2}-\overline{a_1}).\hat{b}=(3\bar{i}+4\bar{E}).\frac{-\bar{i}-7\bar{j}+5\bar{k}}{5\sqrt{3}}=\frac{-3+20}{5\sqrt{3}}=\frac{17}{5\sqrt{3}}$$

Key : 4

3Sol : The plane passing through $(-1,-2,-1)$ and having normal along

$\bar{b}$ *is* $-(x+1)-7(y+2)+5(z+1)=0, x+7y-5z+10=0$

Distance of point $(1,1,1)$ from the above plane is $=\left|\frac{1+7\times1-5x+10}{\sqrt{1+49+25}}\right|=\frac{13}{5\sqrt{3}}$

Key : 3

PASSAGE-2

08. Consider the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ *and* $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-4}{4}$ (duplicite)

01. The unit vector perpendicular to both L_1 *and* L_2 *is*

1. $\frac{4\bar{i}+4\bar{j}-5\bar{k}}{\sqrt{57}}$ 2. $\frac{4\bar{i}+4\bar{j}-4\bar{k}}{\sqrt{48}}$ 3. $\frac{-4\bar{i}+4\bar{j}-5\bar{k}}{\sqrt{57}}$ 4. $\frac{-4\bar{i}-4\bar{j}-4\bar{k}}{\sqrt{48}}$

02. Shortest distance between L_1 *and* L_2 *is*

1. $\frac{10}{\sqrt{99}}$ 2. $\frac{8}{\sqrt{57}}$ 3. $\frac{3}{\sqrt{57}}$ 4. $\frac{5}{\sqrt{57}}$

03. The distance of the point $(1,1,1)$ from the plane passing through the point $(-1,-2,-1)$ and whose normal is perpendicular both the lines L_1 *and* L_2

1. $\frac{2}{\sqrt{75}}$ 2. $\frac{3}{\sqrt{57}}$ 3. $\frac{13}{\sqrt{57}}$ 4. $\frac{10}{\sqrt{57}}$

1sol : Vector in the direction of $L_1=\overline{b_1}=2\bar{i}+3\bar{j}+4\bar{k}$

Vector in the direction of $L_2=\overline{b_2}=3\bar{i}+2\bar{j}+4\bar{k}$

Vector perpendicular to both L_1 *and* L_2 *is* $\bar{b}=\overline{b_1}\times\overline{b_2}$

$$=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 2 & 3 & 4\\ 3 & 2 & 4\end{vmatrix}$$

$=\bar{i}(12-8)-\bar{j}(8-12)+\bar{k}(4-9)=4\bar{i}+4\bar{j}-5\bar{k}$

$\left|\overline{b_1}\times\overline{b_1}\right|=\sqrt{16+16+25}=\sqrt{57}$

The unit vector perpendicular to both L_1 *and* L_2 *is* $\hat{b}=\frac{\overline{b_1}\times\overline{b_2}}{\left|\overline{b_1}\times\overline{b_2}\right|}=\left(\overline{a_2}-\overline{a_1}\right).\bar{b}$

Hence $\overline{a_1}=\bar{i}+2\bar{j}+3\bar{k}, \overline{a_2}=2\bar{i}+3\bar{j}+4\bar{k}$

$\overline{a_2}-\overline{a_1}=\bar{i}+\bar{j}+\bar{k}$

$\therefore\left(\overline{a_2}-\overline{a_1}\right).\hat{b}=\left(\bar{i}+\bar{j}+\bar{k}\right).\frac{4\bar{i}+4\bar{j}-5\bar{k}}{\sqrt{57}}=\frac{4+4-5}{\sqrt{57}},=\frac{3}{\sqrt{57}}$

Key : 3

3sol : The plane passing through $(-1,-2,-1)$ and having normal along $\bar{b}$ is $\bar{b}$

$$4(x+1)+4(y+2)-5(z+1)=0$$
$$4x+4+4y+8-5z-5=0$$
$$4x+4y-5z+7=0$$

Distance of point $(1,1,1)$ from the above plane

$$=\frac{4\times1+4\times1-5\times1+7}{\sqrt{16+16+25}}$$
$$=\frac{4+4-5+7}{\sqrt{57}}=\frac{10}{\sqrt{57}}$$

Key : 4

09. Statement type Questions (adv 2008)

Consider three planes

$P_1 : x-y+z=1$

$P_2 : x+y-z=-1$

$P_3 : x-3y+3z=2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 *and* P_3, P_3 *and* P_1, P_1 *and* P_2 respectively

Statement –I: At least two of the lines L_1, L_2 *and* L_3 are non-parallel

Statement_II: The three planes do not have a common point

(1) Statement _1_ is true, statement_2 is true, statement -2 is a correct explanation for Statement__1

(2) Statement -1 is true, statement -2 is true, statement -2 is Not a correct explanation for Statement-1

(3) Statement -1 is true, statement-2, false

(4) Statement -1 is false, statement -2, is true

Key : 4

Sol : The given planes are

$P_1 : x-y+z=1---(1)$

$P_2 : x+y-z=-1--(2)$

$P_3 : x-3y+3z=2---(3)$

Since line L_1 is intersection of planes P_2 *and* P_3

$\therefore$ L_1 is parallel to the vector

$$=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 1 & -1\\ 1 & -3 & 3\end{vmatrix}=-4\bar{j}-4\bar{k}$$

Line L_2 is intersection of P_3 *and* P_1

$\therefore$ L_3 is parallel to the vector $= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -2\bar{j} - 2\bar{k}$

and line L_3 is intersection of P_1 and P_2

$\therefore L_3$ is parallel to the vector $= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2\bar{j} + 2\bar{k}$ Clearly lines L_1, L_2 and L_3 are parallel

to each other

$\therefore$ Statement -1 is false

Also family of planes passing through the intersection of P_1 and P_2 is $P_1 + \lambda P_2 = 0$

$x(1+\lambda) + y(\lambda - 1) + z(1-\lambda) + \lambda - 1 = 0$

The three planes have a common point

$\frac{1+\lambda}{1} = \frac{\lambda-1}{-3} = \frac{1-\lambda}{3} = \frac{1-\lambda}{2}$

$1+\lambda = \frac{\lambda-1}{-3}$ we get $\lambda = \frac{-2}{3}, \frac{1+\lambda}{1} = \frac{1-\lambda}{2}$ we get $= \frac{-1}{3}$ Hence there no value of λ which satisfies equation (1)

$\therefore$ The three planes do not have a common point

10. Consider three planes $P_1; x - y + z = 1$ (duplicite)

$P_2; 2x + y + z = 2$

$P_3; x - 2y + 3z = 4$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2 respectively STATEMENT-1 at least two of the lines L_1, L_2, L_3 are non parallel

STATEMENT -2 the three planes do not have a common point

A. STATEMENT-1 is true, STATEMNT -2 is true, STATEMNT -2 is a correct explanation for STATEMNT -1

B. STATEMNT -1 is true, STATEMNT -2 is true, STATEMENT -2 is not a correct explanation for STATEMNT -1

C. STATEMENT -1 is true, STATEMNT -1 is false

D. STATEMNT is False, STATEMENT -2 is true

Key : B

Given planes are.

$P_1 : x + y + z = 1 - -(1)$

$P_2 : 2x + y + z = 2 - -(2)$

$P_3 : x - 2y + 3z = 4 - -(3)$

Since L_1 is intersection planes P_2 and P_3

$\therefore L_1$ is parallel to the vector $=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$

$=\bar{i}(3+2)-\bar{j}(6-1)+\bar{k}(-4-1)$

$=5\bar{i}-5\bar{j}-5\bar{k}$

L_2 is intersection of planes P_1 *and* P_3

$\therefore$ L_2 is parallel to the vector $=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$

$=\bar{i}(-3+2)-\bar{j}(3-1)+\bar{k}(-2+1)$

$=-\bar{i}-2\bar{j}-\bar{k}$

L_3 is intersection of planes P_1 *and* P_2 $\therefore L_3$ is parallel to the vector

$=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$

$=\bar{i}(-1,-1)-\bar{j}(1-2)+\bar{k}(1+2)$

$=\bar{i}(-1-1)-\bar{j}(1-2)+\bar{k}(1+2)$

$=-2\bar{i}+\bar{j}+3\bar{k}$

Lies L_1, L_2, L_3 are non –parallel STATEMNT -1 is true

Also family at planes passing through intersection of P_1 *and* P_2 *is* $P_1+\lambda P_2=0$

$(x-y+z-1)+\lambda(2x+y+z-2)=0$

$(2\lambda+1)x+(\lambda-1)y+(\lambda+1)z-2\lambda-1=0$

The three planes have a common point

$\frac{2\lambda+1}{1}=\frac{\lambda-1}{-2}=\frac{\lambda+1}{3}=\frac{2\lambda+1}{4}$

$2\lambda+1=\frac{\lambda-1}{-2}\quad \frac{\lambda+1}{3}=\frac{2\lambda+1}{4}$

$-4\lambda-2=\lambda-1\quad 4\lambda+4=6\lambda+3$

$5\lambda=-1\ 2\lambda=1$

$\left(\lambda=-\frac{1}{5}\right)\ \left(\lambda=\frac{1}{2}\right)$

Hence there is no values of λ which satisfies (1) $\therefore$ The three planes do not have a common point STATEMNT -2 is true

11. Consider the planes $3x-6y-2z=15$ and $2x+y-2z=5$ (adv 2007)

STATEMENT -1 : The parametric equations of the lines of intersection of the given planes are $x=3+14t, y=1+2t, z=15t$

STATEMENT -2: the vector $14\bar{i}+2\bar{j}+15\bar{k}$ is parallel to the line of intersection of given planes

A. STATEMENT-1 is true, STATEMNT -2 is true, STATEMNT -2 is a correct explanation for STATEMNT -1

B. STATEMNT -1 is true, STATEMNT -2 is true, STATEMENT -2 is not a correct explanation for STATEMNT -1

C. STATEMENT -1 is true, STATEMNT -1 is false

D. STATEMNT is False, STATEMENT -2 is true

Key : D

Sol : The line of intersection of given plane is $3x-6y-2z-15=0=2x+y-2z-5$ for

For $z=0$ we get $x=3$ and $y=-1$ line passes through $(3,-1,0)$

Direction vectors of line is $\bar{b}=\overline{x_i}\times\overline{x_2}$

$$=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix}=14\bar{i}+2\bar{j}+15\bar{E}$$

Equation of line is $\frac{x-3}{14}=\frac{y+1}{2}=\frac{z}{15}=t$

Whose parametric form is $x=3+14t$

$y=2t-1$

$z=15t$

$\therefore$ statement -1 is false, statement -2 is true

12. Consider the planes $x-2y-2z=5$ and $2x+y-2z=8$ (duplicite)

STATEMENT -1, the parametric equations of the line of intersection of the given plans are $x=5t+23, y=-2t-\frac{2}{5}, 3=5t$

STATEMENT -2 the vector $6\bar{i}-2\bar{j}+5\bar{k}$ is parallel to the line of intersection of given planes.

A. STATEMENT-1 is true, STATEMNT -2 is true, STATEMNT -2 is a correct explanation for STATEMNT -1

B. STATEMNT -1 is true, STATEMNT -2 is true, STATEMENT -2 is not a correct explanation for STATEMNT -1

C. STATEMENT -1 is true, STATEMNT -1 is false

D. STATEMNT is False, STATEMENT -2 is true

Key : D

Sol : The line intersection of given plane is

$x-2y-2z-5=02x+y-2z-8$ for $z=0$ we get $x=\frac{21}{5}, y=\frac{-2}{5}$ line passes through $\left(\frac{21}{5},\frac{-2}{5},0\right)$

Direction vector of a line is $\bar{b}=\overline{x_i}\times\overline{x_2}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & -2 & -2\\ 2 & 1 & 2\end{vmatrix}$

$=\bar{i}(4+2)-\bar{j}(-2+4)+\bar{K}(1+4)$

$=6\bar{I}-2\bar{J}+5\bar{K}$

Equation of line $\frac{x-\frac{21}{5}}{6}=\frac{y+\frac{2}{5}}{-2}=\frac{z}{5}=t$

Whose parametric form $x=6t+\frac{21}{5}$

$y=-2t\frac{-2}{5}$

$z=5t$

$\therefore$ Statement -1 is false, statement -2 is true

1. 2. 3. 4