Sri Chaitanya IIT Academy., India.

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

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Central Office, Bangalore

AREA UNDER THE CURVE

EXERCISE - II

NUMERICAL/INTEGER ANSWER TYPE QUESTIONS

Area bounded by curve and axes :

1. Area of the region bounded by the curve $y=(x-1)(x-2)(x-3)$ lying between the ordinates $x=0$ *and* $x=3$ is ____

PRACTICE QUESTIONS

2. The area bounded by the curve $y=\sin^{-1}x$ and the line $x=0,|y|=\frac{\pi}{2}$ is … sq. units

Area bounded by curve and line :

3. If the area of the region bounded by the curves $y^2-2y=-x,\ x+y=0$ is A, then 8A is equal to **[Mains_2023]**

PRACTICE QUESTIONS

4. If the area between the curves $y=x-x^2$ and $y=mx$ is $\frac{9}{2}$, then the sum of all values of m is **[Adv. 1993]**

Area bounded between standard geometrical figures :

5. Let $f(x)=\max\{|x-1|,|x+2|,....,|x+5|\}$. Then $\int_{-6}^{0} f(x)dx$ is equal to **[Main 2022]**

6. For real numbers a, b (a > b > 0), let

   Area $\left\{(x,y):x^2+y^2\le a^2\ and\ \frac{x^2}{a^2}+\frac{y^2}{b^2}\ge 1\right\}=30\pi$ and

   Area $\left\{(x,y):x^2+y^2\ge b^2\ and\ \frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\right\}=18\pi$

   Then the value of $(a-b)^2$ is equal to ____ **(Main June 29, 2022)**

PRACTICE QUESTIONS

7. If the area enclosed by the curve $|x|+|y|\le k$ is $f(k)$ sq. unit, then the value of $\frac{f(10)}{100}$ must be

8. The area of the region bounded by $1-y^2=|x|$ and $|x|+|y|=1$ is … sq. units

Area bounded by two or more curves :

9. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of area R_1 *and* R_2. If max $\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to **(Main 2022)**

10. Let the area enclosed by the lines $x + y = 2,\ y = 0,\ x = 0$ and the curve $f(x) = \min\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$ where [x] denotes the greatest integer $\le x$, be A. Then the value of 12A is _____ **[Main 2023]**

11. If A is the area in the first quadrant enclosed by the curve $C: 2x^2 - y + 1 = 0$, the tangent to C at the point (1, 3) and the line $x + y = 1$, then the value of 60A is **[M- 2023]**

12. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ *and* $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to ____

13. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ *and* $x = 2$. Then 12A is equal to _____ **[Main 2023]**

14. The area of the region enclosed between the curves $x = y^2 - 1$ and $x = |y|\sqrt{1-y^2}$ is …. Sq. units

15. If the area of the region enclosed by the curves $y = x\log x$ and $y = 2x - 2x^2$ is K. then $[K]$…….([.] denotes GIF)

16. Consider two curves $C_1 : y^2 = 4\left[\sqrt{y}\right]x$ and $C_2 : x^2 = 4\left[\sqrt{x}\right]y$, where [.] denotes the greatest integer function. The area of region enclosed by these two curves within the square formed by the lines $x = 1, y = 1, x = 4, y = 4$ is l then $3l - 11$ is

PRACTICE QUESTIONS

17. Find the area bounded by the x-axis, part of the curve $y = \left(1 + \frac{8}{x^2}\right)$ and the ordinate at x = 2 and x = 4. If the ordinate at x = a divides the area into two equal parts, find a **(Adv.1983)**

18. If the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = \alpha x, \alpha > 0$, then α^3 is equal to _____ **[Main 2023]**

19. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3 - 3y^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at the point (-2, 3) be A. Then 8A is equal to ____ **(Main 2023)**

20. The area bounded by the curves $y = -\sqrt{4-x^2}$, $x^2 = -\sqrt{2}y$ and $x = y$ is ℓ then $[\ell]$ (where [.] denotes G.I.F) =

21. If the area included between the two parabolas $y^2 = 4a(x+a)$ *and* $y^2 = 4b(b-x)$ is $\frac{16}{3}$. Then the product of A.M. and the G.M. of a *and* b is

22. If the line $y = mx$ bisects the area enclosed by the lines $x=0, y=0, x=\frac{3}{2}$ and the curve $y = 1+4x-x^2$. Then the value of $m = \dots$ Sq. units.

Area bounded by two or more cure inequalities :

23. If for some $\alpha > 0$, the area of the region $\{(x,y): |x+\alpha| \le y \le 2-|x|\}$ is equal to $\frac{3}{2}$, then the area of the region $\{(x,y): 0 \le y \le x+2\alpha, |x| \le 1\}$ is equal to **[Main 2022]**

24. Let $y = p(x)$ be the parabola passing through the points (-1, 0), (0, 1) and (1, 0). If the area of the region $\left\{(x, y): (x+1)^2 + (y+1)^2 \le 1, y \le p(x)\right\}$ is A, then $12(\pi - 4A)$ is equal to _______ **[Main 2023]**

25. If the area of region $\left\{(x, y): \left|x^2 - 2\right| \le y \le x\right\}$ is A, then $6A + 16\sqrt{2}$ is equal to _____ **[Main 2023]**

26. Let A be the area of the region $\left\{(x, y): y \ge x^2, y \ge (1-x)^2, y \le 2x(1-x)\right\}$ then 540 A is equal to **[Main 2023]**

27. Let for $x \in R$; $f(x) = \frac{x+|x|}{2}$ *and* $g(x) = \begin{cases} x, & x < 0 \\ x^2 & x \ge 0 \end{cases}$.
Then area bounded by the curve $y = (fog)(x)$ and the lines $y = 0, 2y - x = 15$ is equal to ____ **[Main 2023]**

PRACTICE QUESTIONS

28. If the area of the region $S = \left\{(x, y): 2y - y^2 \le x^2 \le 2y, x \ge y\right\}$ is equal to $\frac{n+2}{n+1} - \frac{\pi}{n-1}$, then the natural number n is equation _____ **[Main 2023]**

29. The area of region for which $0 < y < 3 - 2x - x^2$ and $x > 0$ is … sq. units

30. Let the area of the region $\left\{(x,y): |2x-1| \le y \le \left|x^2 - x\right|, 0 \le x \le 1\right\}$ be A. Then $(6A+11)^2$ is equal to _______ **[Main 2023]**

31. The area of the region given by $A = \{(x, y): x^2 \le y \le \min\{x+2, 4-3x\}\}$ is : **[Main 2022]**

32. A farmer F_1 has a land in the shape of a triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curved the form $y = x^n (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is **(Adv.2018)**

33. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ *and* $x = 2$. Then 12A is equal to ______ **[Main 2023]**

34. Area of region defined by $1 \le |x| + |y|$ and $x^2 - 2x + 1 \le 1 - y^2$ is $k\pi$ sq. units, then $k = \dots$

35. Consider the functions $f,g:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^2+\frac{5}{12}$ and

$$g(x)=\begin{cases}2\left(1-\frac{4|x|}{3}\right), & |x|\le\frac{3}{4}\\ 0, & |x|>\frac{3}{4}\end{cases}$$ If α is the area of the region

$\left\{(x,y)\in\mathbb{R}\times\mathbb{R}:|x|\le\frac{3}{4},0\le y\le\min\{f(x),g(x)\}\right\}$, then the value of 9α is ____

Miscelleneous based on area bounded by two or more curves :

36. Let $F(x)=\int_{x}^{x^2+\frac{\pi}{6}}2\cos^2 t(dt)$ for all $x\in R$ and $f:\left[0,\frac{1}{2}\right]\to[0,\infty)$ be a continuous function. For $a\in\left[0,\frac{1}{2}\right]$, if $F'(a)+2$ is the area of the region bounded by x = 0, y = 0, $y=f(x)$ and $x=a$, then $f(0)$ is **(Adv.2015)**

37. If the area bounded by the curve $2y^2=3x$, lines $x+y=3$, $y=0$ and outside the circle $(x-3)^2+y^2=2$ is A, then $4(\pi+4A)$ is equal to _______ **[Main 2023]**

38. A point *P* moves in *xy* plane in such a way that [|*x*|] + [|y|] = 1, where [.] denotes the greatest integer function. Area of the region representing all possible positions of the point *P* is equal to

39. The area of the figure bounded by $x=-1, x=2$ and $y=\begin{cases}-x^2+2, & x\le 1\\ 2x-1, & x>1\end{cases}$ then $[\lambda]=$ _____ (where $[.]$ is G.I.F)

40. A curve passing through (2, 3) and satisfying $\int_0^x t\ y(t)dt=x^2y(x);\ x>0$ is $xy=\lambda$ then λ

$\lambda=$ _____

41. If A is the approximate area of the region bounded by $f(x)=e^{-x^2}$ with y-axis the positive x-axis and $B=\int_0^\infty e^{-x^2}\cos(2020x)dx$ and $\sqrt{\log\left(\frac{A}{B}\right)-1002}=k$, then the value of k is equal to

42. If the area bounded by $y=f(x),\ x=\frac{1}{2},\ x=\frac{\sqrt{3}}{2}$ and *x*-axis is A sq. units where $f(x)=x+\frac{2}{3}x^3+\frac{2}{3}\frac{4}{5}x^5+\frac{2}{3}\frac{4}{5}\frac{6}{7}x^7+.....\infty,|x|<1$; then the value of $[4A]$ is (where $[.]$ is G.I.F)

43. If the area bounded by the curves $y=-x^2+6x-5, y=-x^2+4x-3$ and the line $y=3x-15$ is $\frac{73}{\lambda}$, then the value of λ is

44. A point $P(x, y)$ moves is such a way that $[x+y+1]=[x]$ (where [.] greatest integer function) and $x \in (0, 2)$. Then the area representing all the possible positions of P equals.

PRACTICE QUESTIONS

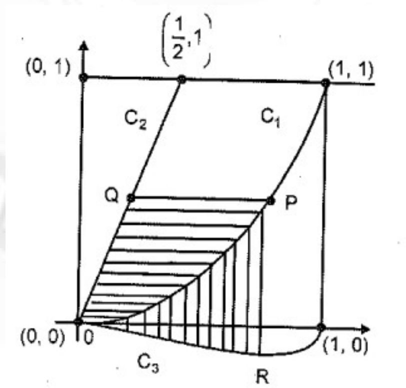
45. Let a and b respectively be the points of local maximum and local minimum of the function $f(x)=2x^3-3x^2-12x$. If A is the total area of the region bounded by $y=f(x)$ the x-axis and the lines $x=a$ and $x=b$, then 4A is equal to ____ **[M-2021]**

46. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix}=\begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y=f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by f(x) and chord AB. **[Adv.2005]**

47. Let $f(x)$ be a continuous function given by $f(x)=\begin{Bmatrix} 2x, & |x|\le 1 \\ x^2+ax+b, & |x|>1 \end{Bmatrix}$

Find the area of the region in the third quadrant bounded by the curves $x=-2y^2$ and $y=f(x)$ lying on the left of the line $8x+1=0$ **(Adv.1999)**

48. Let C_1 and C_2 be the graphs of the functions $y=x^2$ and $y=2x, 0\le x\le 1$ respectively. Let C_3 be the graph of a function $y=f(x), 0\le x\le 1, f(0)=0$. For a point P on C_1, let the lines through P, parallel to the axes, meet C_2 and C_3 at Q and R respectively (see figure). If for every position of P (on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function by $f(x)$ then $f(1)$ is **(Adv.1998)**



49. Let f(x) = Maximum $\{x^2,(1+x)^2,2x(1-x)\}$, *where* $0\le x\le 1$. **(Adv.1997)**

Determine the area of the region bounded by the curves $y=f(x), x-axis, x=0$ *and* $x=1$

50. The area of the figure enclosed by the curve $5x^2+6xy+2y^2+7x+6y+6=0$ is k then $[k]=$ (where [.] denotes G.I.F) is equal to

Area bounded by curves involving standard algebraic function :

51. If A is the area enclosed by the curve $[x+y]+[x-y]=5$ for $x>y, \forall\ x,y>0$ then $4A$ is

52. $P(x,y)$ be a point which moves in $xy-$plane such that $2[y]=3[x].-2<x<5,-3<y<7$, where $[.]$ denotes GIF. If $'\lambda'$ is the area of the region containing the point $P(x,y)$ then the value of $\sqrt{\lambda\sqrt{\lambda\sqrt{\lambda.........\infty}}}$ must be

53. The area of the region of the point $P(x,\ y)$ which satisfy $|x|+|y|+|x+y|\le 1$ is λ. Then 4λ =

54. A curve passing through (2, 3) and satisfying $\int_0^x t\ y(t)dt = x^2 y(x);\ x>0$ is $xy=\lambda$ then $\lambda =$ _____

PRACTICE QUESTIONS

55. The area bounded by the curves $y=\ln x,\ y=|\ln x|,\ y=|\ln|x||$ is

56. Let A_k be the finite area bounded by the line $y=kx+k$ and the parabola $y=x^2$, where k is a positive real number. The value of $\lim\limits_{k\to\infty}\dfrac{A_k}{k^3}$ equals :

57. Let a function $f(x)$ be defined in [-2, 2] as $f(x)=\begin{cases}\{x\}, & -2\le x<-1\\ |\operatorname{sgn} x|, & -1\le x\le 1,\\ \{-x\}, & 1<x\le 2\end{cases}$
where {x} denotes fractional part, then area bounded by graph of $f(x)$ and x-axis is :

58. If $f(x)=\begin{cases}\sqrt{\{x\}}, & x\in Z\\ 1, & x\in Z\end{cases}$ and $g(x)=\{x\}^2$, (where $\{.\}$ denotes fractional part of x), then area bounded by $f(x)$ and $g(x)$ for $x\in[0,10]$ is

59. The area enclosed by $\left[\dfrac{|3x+4y|}{5}\right]+\left[\dfrac{|4x-3y|}{5}\right]=3$ is ……where ([.]denotes GIF)

Area bounded by a function and its Trignometic function :

60. The area bounded by the curves $y=\ln x,\ y=\sin^4 \pi x\ and\ x=0$ is k, then $8k-9$ is equal to

61. The area bounded by $y=\tan^{-1}x, y=\cot^{1}x$ and y-axis in 1st quadrant is … sq. units

Area bounded by parametric curves and lines :

62. Area bounded by the curves $y^2(2a-x)=x^3$ and the line $x=2a$ is $k\pi a^2$ then [k] is (where [.] denotes fractional part of x)

63. For any real t, $x=\dfrac{e^t+e^{-t}}{2}, y=\dfrac{e^t-e^{-t}}{2}$ is a point on the hyperbola $x^2-y^2=1$. Show that the area bounded by this hyperbola and the lines joining its centre to the points corresponding to t_1 and $-t_1$ is Kt_1. Find K. **(Adv. 1982)**