Exercise-3

INTEGER TYPE/ NUMARICAL VALUE QUESTIONS

3D-CO-ORDINATE SYSTEM:-

01a. $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$ and $D(4,5,0)$ $(|\lambda| \le 5)$ are the vertices of a Quadrilateral ABCD. If the area of Quadrilateral is 18 sq.units, then $5-6\lambda=$ ___ (01-02-2023 M)

Key : 11

Sol : Area $=\frac{1}{2}\left|\overline{AC}\times\overline{BD}\right|=18$

$$\overline{AC}\times\overline{BD}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 0 & -3 & -3\\ 8 & 5 & -\lambda\end{vmatrix}=\bar{i}(3\lambda+15)-\bar{j}(24)+\bar{k}(24)$$

$$\left|\overline{AC}\times\overline{BD}\right|=\sqrt{(3\lambda+15)^2+576+576}$$

$$\Delta=18$$

$$\frac{1}{2}\sqrt{(3\lambda+15)^2+576+576}=18$$

$$9(\lambda+5)^2+2(576)=4(324)$$

$$\lambda^2+10\lambda+9=0$$

$$\lambda=-1 \quad \lambda=-9$$

Since $|\lambda|\le 5 \Rightarrow \lambda=-1$

$$\therefore 5-6\lambda=5+6=11$$

01b. The area of the Quadrilateral ABCD, where $A(0,4,1), B(2,3,-1), C(4,5,0)\, and\, D(2,6,2)$ is

Key : 9

Sole : $\overline{AC}\times\overline{BD}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 4 & 1 & -1\\ 0 & 3 & 3\end{vmatrix}=\bar{i}(6)-\bar{j}(12)+\bar{k}(12)$

$$=6\left(i-2\bar{j}+2\bar{k}\right)$$

$$\left|\overline{AC}\times\overline{AB}\right|=6\sqrt{1+4+4}=6(3)=18$$

Area of quadrilateral $=\frac{1}{2}\left|\overline{AC}\times\overline{BD}\right|$

$$=\frac{1}{2}(18)$$

$= 9$

DC'S &DR'S:-

02a. Let $P(-2,-1,1)$ and $Q\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$ be the vertices of the rhombus PRQS. If the direction ratio of the diagonal RS are $\alpha,-1,\beta$ where α,β are integers of minimum absolute values then $\alpha^2+\beta^2=$ ___ (28-07-2022 M)

Key : 450

Sol Dr's of $\overline{PQ}$ are $(a_1,b_1,c_1)=\left(\frac{56}{17}+2,\frac{43}{17}+1,\frac{111}{17}-1\right)$

$$=\left(\frac{90}{17},\frac{60}{17},\frac{94}{17}\right)=(90,60,94)$$

$$=(45,30,47)$$

Dr's of $\overline{RS}$ are $(a_2,b_2,c_2,)=(\alpha,-1,\beta)$

$\therefore a_1a_2+b_1b_2+c_1c_2=0$

$45\alpha+30+47\beta=0$

$\beta=\frac{30-45\alpha}{47}=-15\frac{(3\alpha-2)}{47}$

$\frac{\beta}{-15}=\frac{3\alpha-2}{47}=-1$

$\beta=-15, 3\alpha-2=-47\Rightarrow 3\alpha=-45\Rightarrow\alpha=-15$

$\therefore \alpha^2+\beta^2=225+225=450$

02b. Let $A(2,9,12)$ and $C(-2,11,8)$ are vertices of a square ABCD. If the d.r's of $\overline{BD}$ are $\alpha,\beta,2$ then $|2\alpha-\beta|=$ __

Key : 4

Sol : The d.r's of $\overline{AC}$ are $(a_1,b_1,c_1)=(-4,2,-4)$

The d.r's of $\overline{BD}$ are $(a_2,b_2,c_2)=(\alpha,\beta,2)$

$\therefore a_1a_2+b_1b_2+c_1c_2=0$

$\therefore -4\alpha+2\beta-8=0$

$-4\alpha+2\beta=8$

$2\alpha-\beta=-4$

$\therefore |2\alpha-\beta|=4$

PLANE & 3D-LINES:-

03a. The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ (24-01-2023 M)

Key :14

Sol : $\frac{x-2}{3}=\frac{y-(-1)}{2}=\frac{z-6}{2},\frac{x-6}{3}=\frac{y-1}{-2}=\frac{z-(-8)}{0}$

$$S.D=\frac{\left|\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}$$

$$=\frac{\begin{vmatrix}4 & 2 & -14\\ 3 & 2 & 2\\ 3 & -2 & 0\end{vmatrix}}{\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 3 & 2 & 2\\ 3 & -2 & 0\end{vmatrix}}$$

$\bar{a}=(2,-,1,6)$
$\bar{c}=(6,1,-8)$
$\bar{b}=(3,2,2)$
$\bar{d}=(3,-2,0)$

$$=\frac{\left|4(0+4)-2(0-6)-14(-6-0)\right|}{\left|4\bar{i}+6\bar{j}-12\bar{k}\right|}=\frac{196}{\sqrt{16+36+144}}$$

$$=\frac{196}{\sqrt{196}}=\frac{196}{14}=14$$

03b. The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is d then

$6d^2=$ ____

Key : 1

Sol : $d=\frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\sqrt{\Sigma\left(b_1c_2-b_2c_2\right)^2}}$

$$=\frac{\begin{vmatrix}1 & 2 & 2\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}}{\sqrt{1+4+1}}=\frac{\left|1(-1)-2(-2)+2(-1)\right|}{\sqrt{6}}=\frac{1}{\sqrt{6}}$$

$$\therefore 6d^2=6.\frac{1}{6}=1$$

04a. If the shortest distance between the lines

$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6 then sum of sum of all passive values of λ is

(24-01-2023 A)

Key : 384

Sol : Give lines are $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$

$$S.D\frac{\left|\left[\bar{a}-\bar{c}.\bar{b}\bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}=6$$

$$\frac{\begin{vmatrix} \lambda+\sqrt{6} & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}}{\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}} = \pm 6$$

$\bar{a} = \left(-\sqrt{6}, \sqrt{6}, \sqrt{6}\right)$

$\bar{c} = \left(\lambda, 2\sqrt{6} - 2\sqrt{6}\right)$

$\bar{b} = (2,3,4)$

$\bar{d} = (3,4,5)$

$$\Rightarrow \frac{\left(\lambda+\sqrt{6}\right)(-1) - \sqrt{6}(-2) - 3\sqrt{6}(-1)}{\left|\hat{i}(-1) - \hat{j}(-2) - \hat{k}(-1)\right|} = \pm 6$$

$$\frac{-\lambda - 4\sqrt{6} + 2\sqrt{6} + 3\sqrt{6}}{\sqrt{1+4+1}} = \pm 6$$

$-\lambda + 4\sqrt{6} = \pm 6\sqrt{6}$

$-\lambda + 4\sqrt{6} = 6\sqrt{6} \qquad -\lambda + 4\sqrt{6} = -6\sqrt{6}$

$-\lambda = 2\sqrt{6} \qquad -\lambda = -10\sqrt{6}$

$\lambda = -2\sqrt{6} \qquad \lambda = 10\sqrt{6}$

Quantity sum of the value of $\lambda = \left(-2\sqrt{6} + 10\sqrt{6}\right)^2$

$= \left(8\sqrt{6}\right)^2$

$= 384$

04b. If the lines $\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$ are intersecting on a plane then $\lambda^2 =$

Key : 96

Sol : $\left[\bar{a} - \bar{c}\ \bar{b}\ \bar{d}\right] = 0$

$$\begin{vmatrix} \lambda+\sqrt{6} & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$\bar{a} = \left(-\sqrt{6}, \sqrt{6}, \sqrt{6}\right)$

$\bar{a} = \left(\lambda, 2\sqrt{6}, -2\sqrt{6}\right)$

$\bar{b} = (2,3,4)$

$\bar{d} = (3,4,5)$

$\left(\lambda+\sqrt{6}\right)(-1) - \sqrt{6}(-2) - 3\sqrt{6}(-1) = 0$

$-\lambda - \sqrt{6} + 2\sqrt{6} + 3\sqrt{6} = 0$

$-\lambda = -4\sqrt{6} \Rightarrow = 4\sqrt{6}$

$\lambda^2 = \left(4\sqrt{6}\right)^2 = 16(6) = 96$

05a. Let the equation of the plane passing through the line $x-2y-z-5=0=x+y+3z-5$ and parallel to the line $x+y+2z-7=0=2z+3y+z-2$ be $ax+by+cz=65$ then distance of the point (a,b,c) from the plane $2x+2y-z+16=0$ is (25-01-2023 M)

Key : 9

Sol : Req. equation of the plane is

$x-2y-z-5+\lambda(x+y+3z-5)=0$

$x(1+\lambda)+y(-2-\lambda)+z(-1+3\lambda)-5-5\lambda=0---(1)$

Is Parallel to $x+y+2z-7=0=2x+3y+z-2$

$$\therefore \begin{vmatrix} 1+\lambda & -2+\lambda & -1+3\lambda \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}=0$$

$(1+\lambda)(-5)-(-2+\lambda)(-3)+(-1+3\lambda)(1)=0$

$-5-5\lambda-6+3\lambda-1+3\lambda=0$

$\lambda=12$

$\therefore$ Plane is $13x+10y+35z-65=0$

Distance from $(13,10,35)$ to plane $2x+2y-z+16=0 is$

$$=\left|\frac{26+20-35+16}{\sqrt{4+4+1}}\right|=\frac{27}{3}=9$$

05b. The equation of the plane passing through the line $x-2y-z-5=0=x+y+3z-5$ and parallel to the line whose d.r's are $(-5,3,1)$ is $ax+by+cz=65$ then $a+b+c=$ ____

Key : 58

Sol : Req. Equation of the line is

$x-2y-z-5+\lambda(x+y+3z-5)=0$

$\Rightarrow(1+\lambda)x+(-2+\lambda)y+(-1+3\lambda)z-5-5\lambda=0\rightarrow(1)$

(1) is parallel to the line whose dr's are $(-5,3,1)$

$(1+\lambda)(-5)+(-2+\lambda)(3)+(-1+3\lambda)(1)=0$

$-5-5\lambda-6+3\lambda-1+3\lambda=0$

$\lambda=12$

$\therefore(1)\Rightarrow 13x+10y+35z=65$

$\therefore a+b+c=13+10+35$

$=58$

06a. If the shortest distance between the line joining the points $(1,2,3)\, and\, (2,3,4)$ and the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{0}$ is α then $28\alpha^2$ ____ (25-01-2023 A)

Key : 18

Sol : Equation of the line is $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}--(1)$

$$\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{0}----(2)$$

$\bar{r}=\left(\bar{i}+2\bar{j}+3\bar{k}\right)+t\left(\bar{i}+\bar{j}+\bar{k}\right)$ $\qquad\qquad$ $\bar{r}=\bar{a}+t\bar{b}$

$\bar{r}=\left(\bar{i}-\bar{j}+2\bar{k}\right)+s\left(2\bar{i}-\bar{j}+0.\bar{k}\right)$ $\qquad\qquad$ $r=\bar{c}+s\bar{d}$

$$S.D=\frac{\left|\left[\bar{a}-\bar{c},\bar{b},\bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}=\frac{\begin{vmatrix}0 & -3 & -1\\ 1 & 1 & 1\\ 2 & -1 & 0\end{vmatrix}}{\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 1 & 1\\ 2 & -1 & 0\end{vmatrix}}$$

$$=\frac{\left|0+3(-2)-1(-3)\right|}{\left|\bar{i}\,(1)-\bar{j}\,(-2)+\bar{k}\,(-3)\right|}=\frac{3}{\sqrt{1+4+9}}=\frac{3}{\sqrt{14}}$$

$$\alpha=\frac{3}{\sqrt{14}}$$

$$\therefore 28\alpha^2=28.\frac{9}{14}=18$$

06b. The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}\,and\,\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is d the

$d^2=$ ___

Key : 270

Sol : $S.D\dfrac{\begin{vmatrix}x_1-x_2 & y_1-y_2 & z_1-z_2\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\sqrt{\Sigma\left(b_1c_2-b_2c_1\right)^2}}$

$\left(x_1,y_1,z_1\right)=(3,8,3)$

$\left(x_2,y_2,z_2\right)=(-3,-7,6)$

$\left(a_1,b_1,c_2\right)=(3,-1,1)$

$\left(a_2,b_2,c_2\right)=(-3,2,4)$

$$d=\frac{\begin{vmatrix}6 & 15 & -3\\ 3 & -1 & 1\\ -3 & 2 & 4\end{vmatrix}}{\sqrt{36+225+9}}=\frac{\left|6(-6)-15(15)-3(3)\right|}{\sqrt{270}}$$

$$=\frac{270}{\sqrt{270}}=\sqrt{270}$$

$$\therefore d^2=270$$

07a. Let the equation of the plane P containing the line $x+10=\frac{8-y}{2}=z$ be $ax+by+3z=2(a+b)$ and the distance of the plane P from the point $(1,27,7)$ be c then $a^2+b^2+c^2=$ ____ (29-01-2023 M)

Key : 355

Sol : Given line is $\frac{x+10}{1}=\frac{y-8}{-z}=\frac{z}{1}$ $A(-10,8,0)$ $dr's(1,-,2,1)$

Plane is $ax+by+3z=2(a+b)$

$\therefore a-2b+3=0$

$a-2b=-3\to(1)$

plane passing then $A(-10,8,0)$

$\therefore -10a+8b=2(a+b)$

$-12a=-6b$

$2a=b\to(2)$

Solving (1) and (2)

$a=1\ b=2$

$\therefore$ Plane is $x+2y+3x-6=0$

The distance from $(1,27,7)$ to the plane

$x+2y+3z-6=0$ is $c=\frac{|1+54+21-6|}{\sqrt{1+4+9}}$

$C=\frac{70}{\sqrt{14}}=5\sqrt{14}$

$\therefore a^2+b^2+c^2=1+4+350=355$

07b. Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$ and the distance from $P(0,1,6)$ to the plane is γ then $\alpha^2+\beta^2+\gamma^2=$___

Key : 131

Sol : $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$

$\therefore\ ax_1+by_1+cz_1+d=0$ and $al+bm+cm=0$

$2+3+2\alpha+\beta=0$

$2\alpha+\beta=-5-(1)$ $3-15-2\alpha=0$

$-2\alpha=12$

$\therefore -12+\beta=-5$ $\alpha=-6-(2)$

$\beta=7$

$\therefore$ Plane is $x+3y+6z+7=0$

The distance from P$(0,1,6)$ to the plane is

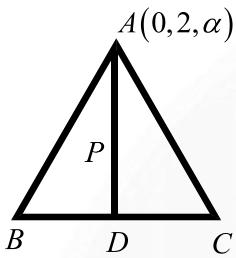
$\gamma=\frac{|0+3+36+7|}{\sqrt{1+9+36}}=\frac{46}{\sqrt{46}}=\sqrt{46}$

$\alpha^2+\beta^2+\gamma^2=36+49+46=131$

08a. Let the coordinates of one values of ΔABC be $A(0,z,\alpha)$ and the other two vertices like on the line $\frac{x+\alpha}{5}=\frac{4-1}{3}$ for $\alpha \in z$ .If the area of ΔABC is 21 sq.units with and the line segment BC has length $2\sqrt{2}$ units, to $\alpha^2 =$ ___ (29-01-2023 M)

Key : 9

Sol :



$$\frac{x+\alpha}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$$

$$P=\frac{\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \alpha & 1 & \alpha+4 \\ 5 & 2 & 3 \end{vmatrix}}{\sqrt{25+4+9}}$$

$$=\left|\bar{i}(3-2\alpha-8)-\bar{j}(3\alpha-5\alpha-20)+\bar{k}(2\alpha-5)\right|\sqrt{38}$$

$$=\frac{\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2}}{\sqrt{38}}$$

$$\Delta = 21$$

$$\frac{1}{2}.\frac{2\sqrt{21}.\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2}}{\sqrt{38}}=21$$

$$\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2}=\sqrt{38}.\sqrt{21}$$

$$4\alpha^2+25+20\alpha+4\alpha^2+400+80\alpha+4\alpha^2+25-20\alpha=38(2)$$

$$12\alpha^2+80\alpha+450=798$$

$$12\alpha^2+80\alpha-3480=0$$

$$3\alpha^{20}+29\alpha-90-87=0$$

$$\alpha(3\alpha+29)-3(3\alpha+29)=0$$

$$\alpha=3$$

$$\therefore \alpha^2=9$$

08b. The vertices B and C of a ΔABC line the line $\frac{x+2}{3}=\frac{y-1}{0}=\frac{Z}{4}$ such that $BC=5$ given that $A(1,-1,2)$ The angle of ΔABC is Δ then $\Delta^2 =$ ____

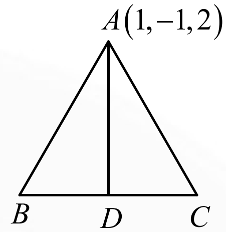
Key : 34

Sol : Any point D on $\overline{BC}$ is

$D=(3\lambda-2,1,4\lambda)$

Dr's of $\overline{AB}\, are(a_1,b_1,c_1)=$

$(3\lambda-3,2,4\lambda-2)$



$\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}=\lambda$

Dr's of $\overline{BC}$ are $(a_2,b_2,c_2)=(3,0,4)$

$\therefore a_1a_2+b_1b_2+c_1c_2=0$

$9\lambda-9+0+16\lambda-8=0 \Rightarrow \lambda=\frac{17}{25}$

Dr's of $AD=\left(\frac{-24}{25},2,\frac{18}{25}\right)$

$\therefore$ Lens $AD=\sqrt{\frac{576}{625}+4+\frac{324}{625}}=\sqrt{\frac{3400}{625}}=\frac{\sqrt{34}}{25}(10)$

$=\frac{2}{5}\sqrt{34}$

ΔABC is $\Delta=\frac{1}{2}.5\frac{2}{5}\sqrt{34}=34$

$\therefore \Delta^2=34$

09a. The equation of the plane passing through the Point $(1,1,2)$ and perpendicular to the line $x-3y-2z-1=0=4x-y+z$ is $Ax+By+Cz=1$ then $140(C-B+A)=$ ___ (30-01-2023 M)

Key : 15

Sol : Line is $x-3y+2z-1=0=4x-y+z$

$$\overline{n_1}\times\overline{n_2}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & -3 & 2\\ 4 & -1 & 1\end{vmatrix}$$

$=-\bar{i}+7\bar{j}+11\bar{k}$

$\therefore$ D.C's uniform to the plane $(a,b,c)=(-1,7,11)$

Equation of the plane is $-1(x-1)+7(y-1)+1,(z-2)=0$

$-x+7y+11z=28$

$-\frac{x}{28}+\frac{7}{28}y+\frac{11}{28}z=1$

$$\therefore 140(C-B+A)=140\left(\frac{11}{28}-\frac{7}{28}-\frac{1}{28}\right)$$

$$=140.\frac{3}{28}=15$$

09b. The equation of the plane pass through $P(1,1,1)$ and perpendicular to the line $x+y-z-3=0=2x+3y+z+4$ is $ax+by+cz=1$ then $4\left(a^2+b^2+c^2\right)=$ ____

Key : 26

Sol : line $x+y-z-3=0=2x+3y+z+4$

Dr's of the line are given by

$$\frac{a}{4}=\frac{-b}{3}=\frac{c}{1}$$

$$(a,b,c)=(4,-3,1)$$

$\therefore$ Equation of the plane is $4(x-1)-3(y-1)+(z-1)=0$

$$4x-3y+z-2=0$$

$$\Rightarrow 2x-\frac{3}{2}y+\frac{1}{2}z=1$$

$$\therefore 4\left(a^2+b^2+c^2\right)=4\left(4+\frac{9}{4}+\frac{1}{4}\right)=16+9+1=26$$

10a. Let a line L passes through the point $P(2,3,1)$ and parallel to the line $x+3y-2z-2=0=x-y+2z.$ If the distance of L from the point $(5,3,8)$ is α then $3\alpha^2=$ ___ (30-01-2023 A)

Key : 158

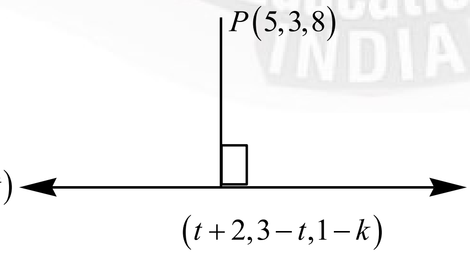
Sol : $\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 3 & -2 \\ 1 & -2 & 2 \end{vmatrix}=4\bar{i}-4\bar{j}-4\bar{k}$

$\therefore$ Equation of line is $\dfrac{x-2}{4}=\dfrac{y-3}{-4}=\dfrac{z-1}{-4}$

$$\Rightarrow \frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-1}{-4}$$

Any point on the line is

$=(t+2,3-t,1-t)$



D'r's of are $(a,b,c)=(t-3,-t-t-7)$

$$\therefore a_1a_2+b_1b_2+c_1c_2=0$$

$t-3+t+t+7=0 \Rightarrow 3t=-4$

$t=\dfrac{-4}{3}$

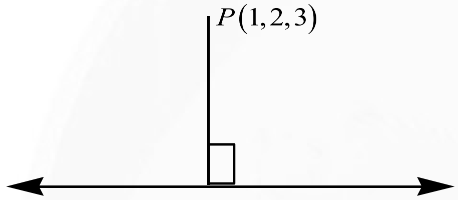
$\therefore Q=\left(\dfrac{-4}{3}+2\,,\,3+\dfrac{4}{3}\,,\,1+\dfrac{4}{3}\right)$

$PQ^2=\dfrac{169}{9}+\dfrac{16}{9}+\dfrac{289}{9}=\dfrac{474}{9}$

$\therefore 3\alpha^2=158$

10b. Find the perpendicular distance from $(1,2,3)$ to the line $\dfrac{x-6}{3}=\dfrac{y-7}{2}=\dfrac{z-7}{-2}$ is

Key : 7



$\dfrac{x-6}{3}=\dfrac{y-7}{3}=\dfrac{z-7}{-2}=\lambda$

Let any point Q on the line is

$=(3\lambda+6,2\lambda+7,-2\lambda+7)$

Dr's of $\overline{PQ}(a,b,c)=(3\lambda+5,2\lambda+5,-2\lambda+4)$

$\overline{PQ}\perp line$

$\therefore a_1a_2+b_1b_2+c_1c_2=0$

$(3\lambda+5)3+(2\lambda+5)(2)+(-2\lambda+4)(-2)=0$

$9\lambda+15-4\lambda+10+4\lambda-8=0$

$17\lambda=-17 \Rightarrow \lambda-1$

$\therefore Q=(3,5,9),\ P(1,2,3)$

$\therefore PQ\sqrt{4+9+36}=\sqrt{49}=7$

11a. Let the line $L:\dfrac{x-1}{2}=\dfrac{y+1}{-1}=\dfrac{z-3}{1}$ intersect the plane $2x+y+3z=16$ at the point P. Let the point Q be to foot of perpendicular from the point $R(1,-1,-3)$ on the line L. If α is the area of the triangle PQR then $\alpha^2=$ ___ (31-01-2023 M)

Key : 180

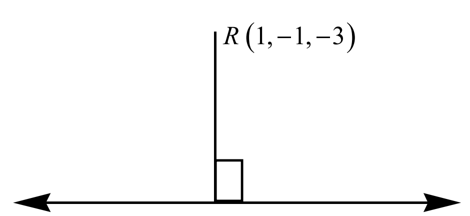
Sol : $\dfrac{x-1}{2}=\dfrac{y+1}{-1}=\dfrac{z-3}{1}=\lambda$

Any point on the line L is $P(2\lambda+1,-\lambda-1,\ \lambda+3)$

Sub: P in the plane $2x+y+3z=16$

$4\lambda+2-\lambda-1+3\lambda+9=16$

$6\lambda=6 \Rightarrow \lambda=1$



$P=(3,-,2,4)$

D.r’s RQ are

$(a_1,b_1,c_1)=(2\lambda,-\lambda,\lambda+6)$

$\therefore a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 4\lambda+\lambda+\lambda+6=0$

$6\lambda=-6\Rightarrow\lambda=-1$

$\therefore Q=(-1,0,2)$

$\therefore P(3,-,2,4)Q(-1,0,2)R(1,-1,-3)$

Area $\Delta PQR=\frac{1}{2}\left|\overline{PQ}\times\overline{PR}\right|$

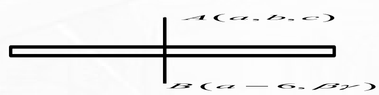
$$\overline{PQ}\times\overline{PR}=\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\-4&2&-2\\-2&1&-7\end{vmatrix}=\bar{i}(-12)-\bar{j}(24)+\bar{k}(0)$$

$=12\left(-\bar{i}-2\bar{j}\right)$

Area $=\frac{1}{2}12\sqrt{1+44}=6\sqrt{5}=\alpha$

$\alpha^2=36(5)=180$

11b. Let the line $L:\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}$ intersection the plane $4x+5y+3z-5=0$ at the point P.

If $Q(1,2,3)$ and area of ΔOPQ is d then 

Key : 4

Sol : Line $L:\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}=\lambda$

Any point P on the line L is $P=(3\lambda-3,-2\lambda+2,\lambda-1)$

Sub: P in the plane $4x+5y+3z-5=0$

$4(3\lambda-3)+5(-2\lambda+2)+3(\lambda-1)-5=0$

$5\lambda=0\Rightarrow\lambda=2$

$\therefore P=(3,-2,1)$

$P=(3,-2,1),Q=(1,2,3)\ O=(0,0,0)$

$$\overline{OP}\times\overline{OQ}=\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\3&-2&1\\1&2&3\end{vmatrix}=\bar{i}(-8)-\bar{j}(8)+\bar{k}(8)=8\left(-\bar{i}-\bar{j}+\bar{k}\right)$$

Area of $OPQ=\frac{1}{2}\left|\overline{OP}\times\overline{OQ}\right|=\frac{1}{2}8.\sqrt{1+1+1}=4\sqrt{3}$

$\therefore d=4\sqrt{3}$

$$\frac{d}{\sqrt{3}}=4$$

12a. The point of intersection 'C' if the plane $8x+y+2z=0$ and the line joining the points $A(-3,-6,1)$ and $B(2,4,-3)$ divides the line segment $\overline{AB}$ internally in the ratio $K:1$. If a,b,c ($|a|,|b|,|c|$ are co-prime) are direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$ then $|a+b+c|=$ ___ (01-02-2023 A)

Key : 10

Sol : Plane $8x+y-2z=0----(1)$

Line $\overline{AB}$ is $\frac{x-2}{5}=\frac{y-4}{10}=\frac{z+3}{-4}=\lambda---(2)$

Any point on the line is $C=(5\lambda+2,10\lambda+4-4\lambda-3)$

Sub: P in (1)

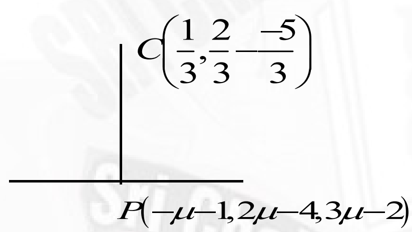
$$8(5\lambda+2)+10\lambda+4+2(-4\lambda-3)=0$$

$$\lambda=\frac{-1}{3}$$

$$C=\left(\frac{1}{3}.\frac{2}{3}\frac{-5}{3}\right)$$

Given line $L:\frac{x-1}{7}=\frac{y+4}{2}=\frac{z+2}{3}=\mu$

$$P(-\mu+1,2\mu-4,3\mu-2)$$



D'r of $\overline{CP}=\left(-\mu+\frac{2}{3},2\mu-\frac{14}{3},3\mu-\frac{1}{3}\right)$

$$\therefore a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow \mu-\frac{2}{3}+4\mu-\frac{28}{3}+9\mu-1=0$$

$$14\mu=11$$

$$\mu=\frac{11}{14}$$

$$\therefore P=\left(\frac{-11}{14}+1,\frac{22}{14}-4,\frac{33}{14}-2\right)$$

$$=\left(\frac{3}{14},\frac{-34}{14},\frac{5}{14}\right)$$

Dr's of $\overline{CP}=(a,b,c)=\left(\frac{3}{14}-\frac{1}{3},\frac{-34}{14}-\frac{-2}{3},\frac{5}{14}+\frac{5}{3}\right)$

$$=\left(\frac{-5}{42},\frac{-130}{42},\frac{85}{42}\right)$$

$$=(-1,.-26,17)$$

$$\therefore |c+b+c|=|-1-26+17|=10$$

12b. The point of intersection of the line $\frac{x+1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}$ and the plane $2x-y+3z-1=0$ is C. If (a,b,c) are dr's of the perpendicular line from the point C on the line $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$ and $|c||b||c|$ are co primer than $|a+b+c|=$ ___

Key : 122

Sol : :Let $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}=\lambda$

Let $C=(3\lambda+1,4\lambda-2,-2\lambda+3)$

Sub : C in the plane $2x-y+3z-1=0$

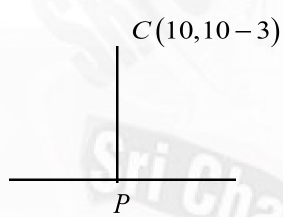
$$\Rightarrow 6\lambda^2-4\lambda+2-6\lambda+9-1=0$$

$$-4\lambda=-12$$

$$\lambda=3$$

$$\therefore C=(10,10,-3)$$

Given line is $\frac{x-1}{1}=\frac{y+4}{2}=\frac{z+2}{3}=t$



$$P=(-t+1,2t-4,3t-2)$$

Dr's of $\overline{PC}$ are $(a_1,b_1,c_1)=(-t-9,2t-14,3t+1)$

Dr's given line $(a_2,b_2,c_2)=(-1,2,3)$

$$\therefore a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow t+9+4t-28+9t+3=0$$

$$14t=16\Rightarrow t=\frac{8}{7}$$

$\therefore$ Dr's $\overline{PC}$ are $\left(\frac{-8}{7}-9,\frac{16}{7}-14,\frac{24}{7}+1\right)$

$$(a,b,c)=\left(\frac{-71}{7},\frac{-82}{7},\frac{31}{7}\right)$$

Dr's $(a,b,c)=(-71,-82,31)$

$\therefore |a+b+c|=|-71-82+31|=122$

13a. If the shortest between two lines $\frac{x+\lambda}{3}=\frac{y-6}{-1}=\frac{z-3}{2},\frac{x-\lambda}{1}=\frac{y+6}{2}=\frac{z-1}{3}$ is $\sqrt{3}$ then $\sum 80\lambda=$ ___ (15-04-2023 M)

Key : 800

Sol : $$\text{S.D}=\frac{\left|\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}=\frac{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{z\left(a_1,b_2-a_2,b_1\right)^2}}$$

$(x_1,y_1,z_1)=(-\lambda,6,3)\ (a,b,c)=(3,-1,2)$

$(x_2,y_2,z_2)=(\lambda,-6,1)(a_2,b_2,c_2)=(1,2,3)$

$$\frac{\begin{vmatrix} 2\lambda & -12 & -2 \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}}{\sqrt{49+49+49}}=\sqrt{3}$$

$|2\lambda(-7)+12(7)-2(7)|=\sqrt{3}.7\sqrt{3}$

$-14\lambda+84-14=\pm 21$

$-14\lambda+70=\pm 21$

$-2\lambda+10=\pm 3$

$-2\lambda+10=3\quad -2\lambda+10=-3$

$-2\lambda=-7\quad -2\lambda=-13$

$\lambda=\frac{7}{2}\quad \lambda=\frac{13}{2}$

$\therefore \sum 80\lambda=80\left(\frac{7}{2}+\frac{13}{2}\right)=80(10)=800$

13b. The shortest distance between the lines $\frac{x-\lambda}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+\lambda}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is $\sqrt{270}$ then $|\sum\lambda|=$

Key : 39

Sol : $$S.D=\frac{\left|\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}=\sqrt{270}$$

$\bar{a}=(\lambda,8,3,)\bar{c}=(-\lambda-7,6)$

$\bar{b}=(3,-,1,1)\bar{d}=(-3,2,4)$

$$\frac{\begin{vmatrix} 2\lambda & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{36+225+9}} = \sqrt{270}$$

$$\left|2\lambda(-6)-15(15)-3(3)\right| = 270$$

$$12\lambda + 234 = \pm 270$$

$$12\lambda + 234 = 270 \qquad 12\lambda + 234 = -270$$

$$12\lambda = 36 \qquad 12\lambda = -504$$

$$\lambda = 3 \qquad \lambda = -42$$

$$\therefore \left|\sum \lambda\right| = |3-42| = 39$$

14a. If (α,β,γ) is the image of $(1,2,6)$ in the plane containing the points $(1,4,0),(1,5,1)$ and $(0,4,1)$ then $\alpha^2+\beta^2+\gamma^2 =$ ____ (10-04-2023 A)

Key :73

Sol : Equation of the plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

$$\begin{vmatrix} x-1 & y-4 & z \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(1)-(y-4)(1)+z(1)=0$$

$$x-1-y+4+z=0$$

$$x-y+z+3=0$$

Image of $(1,2,6)$ is given by

$$\frac{\alpha-1}{2} = \frac{\beta-2}{-1} = \frac{\gamma-6}{1} = \frac{-2(1-2+6+3)}{1+1+1}$$

$$= -2\left(\frac{8}{3}\right) = \frac{-16}{3}$$

Image $(\alpha,\beta,\gamma) = \left(\frac{-13}{3},\frac{22}{3},\frac{2}{3}\right)$

$$\therefore \alpha^2+\beta^2+\gamma^2 = \frac{169+484+4}{9} = \frac{657}{9} = 73$$

14b. If (h,k,l) is the image $(-1,1,6)$ in the plane containing the points $(2,2,-1),(3,4,2),(7,0,6)$ then $h+k+l =$ ___

Key : 14

Sol : Equation of the one is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$

$(x-2)(20)-(y-2)(-8)+(z+1)(-12)=0$

$5x+2y-3z-17=0$

Image of $P(-1,1,6)$ is $Q(h,k,l)$ then

$$\frac{h+1}{5}=\frac{k-1}{2}=\frac{l-6}{-3}=-2\frac{(-5+2-18-17)}{25+4+9}$$

$$=\frac{-2(-38)}{38}$$

$$=2$$

$h=9,k=5,l=0$

$\therefore h+k+l=9+5+0=14$

15a. Let Q and R be two points on the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z-1}{2}$ at a distance $\sqrt{26}$ from $P(4,2,7)$ then the square of the area of ΔPQR ____ (26-07-2022 M)

Key : 153

Sol : Given line is $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z-1}{2}=\lambda$

Let $Q=(2\lambda-1,3\lambda-2,2\lambda+1)$, $P(4,2,7)$

$PQ=\sqrt{26}\Rightarrow PQ^2=26$

$\Rightarrow(2\lambda-5)^2+(3\lambda-4)^2+(2\lambda-6)^2=26$

$4\lambda^2+25-20\lambda+9\lambda^2+16-24\lambda+4\lambda^2+36-24\lambda=26$

$17\lambda^2-16\lambda+51=0$

$\lambda^2-4\lambda+3=0\Rightarrow\lambda=1,\lambda=3$

$\therefore Q=(1,1,3)\,R=(5,7,7)$

Area of $\Delta PQR=\frac{1}{2}\left|\overline{PQ}\times\overline{PR}\right|$

$$\overline{PQ}\times\overline{PR}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ -3 & -1 & -4\\ 1 & 5 & 0\end{vmatrix}=\bar{i}(20)-\bar{j}(4)+\bar{k}(-14)$$

$\Delta=\frac{1}{2}\sqrt{400+16+196}$

$=\sqrt{100+4+49}=\sqrt{153}$

$\Delta^2=153$

15b. Let Q and R be tow pints on the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z-1}{2}$ at a distance $\sqrt{26}$ from $P(4,2,7)$ then $QR^2=$ ___

Key : 68

Sol : Given line is $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z-1}{2}=\lambda$

Let $Q=(2y-1,3\lambda-2,2\lambda+1),P(4,2,7)$

$PQ=\sqrt{26}\Rightarrow PQ^2=26$

$\Rightarrow(2\lambda-5)^2+(3\lambda-4)^2+(2\lambda-6)^2=26$

$4\lambda^2+25-20\lambda+9\lambda^2+16-24\lambda+4\lambda^2+36-24\lambda=26$

$17\lambda^2-68\lambda+51=0$

$\lambda^2-4\lambda+3=0\Rightarrow\lambda=1,\lambda=3$

$\therefore Q=(1,1,3)\ \ R=(5,7,7)$

Are of $\Delta PQR=\frac{1}{2}\left|\overline{PQ}\times\overline{PR}\right|$

$$\overline{PQ}\times\overline{PR}=\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\-3&-1&-4\\1&5&0\end{vmatrix}$$

$\Delta=\frac{1}{2}\sqrt{400+16+196}$

$=\sqrt{100+4+49}=\sqrt{153}$

$\Delta^2=153$

$Q=(1,1,3)R_2(5,7,7)$

$QR^2=16+36+16=68$

16a. Consider a triangle ABC whose vertices are $A(0,\alpha,\alpha)B(\alpha,0,\alpha)\,and\,(\alpha,\alpha,0)\alpha>0$ Let D be a point moving on the line $x+z-3=0=y$ and G be the centroid of $\Delta ABC.$ If the minimum length of GD is $\sqrt{\frac{57}{2}}$, then $\alpha=$___ (30-06-2022 M)

Key : $\alpha=6$

Sol : $G=\left(\frac{2\alpha}{3},\frac{2\alpha}{3},\frac{2\alpha}{3}\right)$

Given line $x+z-3=0=y$

Let D on the line is $D=(\lambda,0,-\lambda+3)$

$\therefore GD^2=\left(\lambda-\frac{2\alpha}{3}\right)^2+\left(-\frac{2\alpha}{3}\right)^2+\left(-\lambda+3-\frac{2\alpha}{3}\right)^2$

Let $f(\lambda)=GD^2=\left(\lambda-\frac{2a}{3}\right)^2+\left(-\frac{2\alpha}{3}\right)^2+\left(-\lambda+3-\frac{2\alpha}{3}\right)^2$

$f^{'}(\lambda)=2\left(\lambda-\frac{2\alpha}{3}\right)+0+2\left(\lambda+3-\frac{2\alpha}{3}\right)(-1)$

For minimum $f^1(\lambda)=0$

$2\left(\lambda-\frac{2\alpha}{3}\right)-2\left(-\lambda+3-\frac{2\alpha}{3}\right)=0$

$2\lambda-3=0$

$\lambda = 3/2$

Minimum $GD = \sqrt{\frac{57}{2}}$

$GD^2 = \frac{57}{2}$

$\Rightarrow \left(\frac{3}{2} - \frac{2\alpha}{3}\right)^2 + \frac{4\alpha^2}{9} + \left(-\frac{3}{2} + 3 - \frac{2\alpha}{3}\right)^2 = \frac{57}{2}$

$\left(\frac{3}{2} - \frac{2\alpha}{3}\right)^2 + \frac{4\alpha^2}{9} + \left(\frac{3}{2} - \frac{2\alpha}{3}\right)^2 = \frac{57}{2}$

$2\left(\frac{9}{4} + \frac{4\alpha^2}{9} - 2\alpha\right) + \frac{4\alpha^2}{9} = \frac{57}{2}$

$\frac{9}{2} + \frac{8\alpha^2}{9} - 4\alpha + \frac{4\alpha^2}{9} = \frac{57}{2}$

$\frac{12}{9}\alpha^2 - 4\alpha + \frac{9}{2} - \frac{57}{2} = 0$

$\frac{4}{3}\alpha^2 - 4\alpha - 24 = 0$

$\alpha^2 - 3\alpha - 18 = 0 \Rightarrow (\alpha - 6)(\alpha + 3) = 0$

$\therefore \alpha = 6, \alpha = -3$

$\therefore \alpha = 6$

16b. Consider a triangle ABC whose vertices are $A(K,K,O), B(K,O,K), C(O,K,K)$ and let D be a point moving on the line $x + z - 3 = 0 = y$ and G be the centroid of the ΔABC. If minimum length GD is $\frac{\sqrt{57}}{2}$ then $\sum k^2 =$ __

Key : 45

Sol : $G = \left(\frac{2k}{3}, \frac{2k}{3}, \frac{2k}{3}\right)$

Give line is $x + z - 3 = 0 = y$

Let D on the line is $D = (\lambda, o, -\lambda + 3)$

$GD^2 = \left(\lambda - \frac{2k}{3}\right)^2 + \left(\frac{2k}{3}\right)^2 + \left(-\lambda + 3 - \frac{2k}{3}\right)^2$

Let $f(\lambda) = GD^2 = \left(\lambda - \frac{2k}{3}\right)^2 + \left(\frac{2k}{3}\right)^2 + \left(-\lambda + 3 - \frac{2k}{3}\right)^2$

$f^1(\lambda) = 2\left(\lambda - \frac{2k}{3}\right) + 0 + 2\left(-\lambda + 3 - \frac{2k}{3}\right)(-1)$

For minimum $f^1(\lambda) = 0$

$\Rightarrow \lambda = \frac{3}{2}$

Minimum $GD=\frac{\sqrt{57}}{2}$

$GD^2=\frac{57}{2}$

Simplify $k=6,-3$

$\sum k^2=36+9=45$

17a. Let a line with direction ratios $(a,-4a,-7)$ be perpendicular to the lines with dr's $(3,-1,2b)$ and $(b,a,-2)$. It the point of intersection of the line $\frac{x+1}{a^2+b^2}=\frac{y-2}{a^2-b^2}=\frac{z}{1}$ and the plane $x-y+z=0$ is (α,β,γ) then $\alpha+\beta+\gamma=$ ____ (29-07-2022 M)

Key : 10

Sol : $\therefore 3a+4a-14b=0$

$7a=14b\Rightarrow a=2b-(1)$

$ab-4a^2+14=0$ ___ (2)

Solving (1) and (2)

$a^2=4 \quad b^2=1$

$\therefore$ line is $\frac{x+1}{5}=\frac{y-2}{3}=\frac{z}{1}=\lambda$

Let any point on the line is

$P=(5\lambda-1,3\lambda+2\lambda)$

Sub: P in the plane $x-y+z=0$

$5\lambda-1-3\lambda-2+\lambda=0$

$3\lambda=3$

$\lambda=1$

$\therefore P=(4,5,1)$

$\therefore \alpha+\beta+\gamma=4+5+1=10$

17b. The line $\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}$ and the plane $4x+5y+3z-5=0$ intersection at (α,β,γ) then The distance from (α,β,γ) to the plane $2x-24+z=5$

Key : 2

Sol : Given line is $\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}=\lambda$

Any point P on the line is $P=(3\lambda-3,-2\lambda+2,\lambda-1)$

Sub: P In the plane $4x+5y+3z-5=0$

$4(3\lambda-3)+5(-2\lambda+2)+3(\lambda-1)-5=0$

$5\lambda=10\Rightarrow\lambda=2$

$\therefore P=(3,-2,1)$

$\therefore$ The distance from $P(3,-2,1)$ to the plane $2x-2y+z-5=0$ is

$$d=\left|\frac{6+4+1-5}{\sqrt{4+4+1}}\right|=\frac{6}{3}=2$$

18a. The plane passing through the line $L: lx-y+3(1-l)z=1,\ x+2y-z=2$ and Perpendicular to the plane $3x+2y+z=6$ is $3x-8y+7z=4$. If $'\theta'$ is the acute angle between the line L and Y-axis, then $415\cos^2\theta=$ (26-07-2022 M)

Key : 125

Sol : The line L: $lx-y+3(1-l)z=1,\quad x+2y-z=2$

d.r's of normal given by

$$\begin{array}{ccc} a & b & c \\ l & -1 & 3(1-l) \\ 1 & 2 & -1 \end{array}$$

$$\frac{a}{6l-5}=\frac{b}{l+3-3l}=\frac{c}{2l+1}$$

$$\frac{a}{6l-5}=\frac{b}{-2l+3}=\frac{c}{2l+1}$$

$$(a,b,c)=(6l-5,-2l+3,2l+1)$$

Normal and line the perpendicular

$$\therefore(6l-5)3+(-2l+3)(-8)+(2l+1)7=0$$

$$18l-15+16l-24+14l+7=0$$

$$48l=32\Rightarrow l=\frac{2}{3}$$

$\therefore D'rs(a,b,c)=\left(-1,\frac{5}{3},\frac{7}{3}\right)$ dr's of $y-\text{axis}\ (0,1,0)$

$$\therefore GB=\frac{\frac{5}{3}}{\sqrt{1+\frac{25}{9}+\frac{49}{9}}}=\frac{\frac{5}{3}}{\frac{\sqrt{83}}{3}}=\frac{5}{\sqrt{83}}$$

$$\therefore 415G^2-6=415.\frac{25}{83}=125$$

18b. If θ is the angle between X-axis and line of intersection of the planes is $x+2y+3z+k_1=0\ \ 3x+3y+z+k_2=0$ then $122\cos^2\theta=$ ___

Key : 49.

Sol : Let (a,b,c) are dr's of the required line then

$a+2b+3c=0$ and $3a+3b+c=0$

$$\frac{a}{-7}=\frac{-b}{-8}=\frac{c}{-3}$$

$\therefore(a,b,c)=(-7,8,-3)$ dr's of X-axis $(1,0,0)$

$\therefore \cos\theta = \frac{|7+0+0|}{\sqrt{49+64+9}\sqrt{1}} = \frac{7}{\sqrt{122}}$

$\therefore 122\cos^2\theta = 122.\frac{49}{122} = 49$

19a. Let $P_1 : \bar{r}\left(2\bar{i} + \bar{j} - 3\bar{k}\right) = 4$ be a plane, let P_2 be another plane passing through the points $(2,-3,2),(2,-2,-3)$ and $(1,-4,2)$. If the d.r's of the line of intersection of $P_1 \& P_2$ be $(16,\alpha\ \beta)$ then the values of $\alpha+\beta$

Key : 28

Sol : $P_1 : 2x+y-3z=4 \to (1)$

$$P_2 : \begin{vmatrix} x-2 & y+3 & z-2 \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(-5)-(y+3)(-5)+(z-2)(1)=0$

$-5x+10+5y+15+z-2=0$

$-5x+5y+z+23=0 \to (2)$

Let (a,b,c) are d.r's of the line then

$2a+b-3c=0 \to (3)$

and $-5a+5b+c=0 \to (4)$

$\therefore \frac{a}{16} = \frac{-b}{-13} = \frac{c}{15}$

$\frac{a}{16} = \frac{b}{13} = \frac{c}{15} = \lambda$

$\therefore (a,b,c) = (16\lambda, 13\lambda, 15\lambda)$

Given that $(16,\alpha,\beta)$

$\therefore \alpha = 13; \beta = 15$

$\therefore \alpha+\beta = 28$

19b. Let $P_1 : x+y+z-1=0$ be a plane. Let P_2 be the another plane passing through the line of intersection of the planes $x-2y+3z-1=0, 2x+y+z-2=0$ and the point $(1,2,3)$. If the d'r's of the line of intersection of $P_1\ and\ P_2$ be $(-5,\alpha,\beta)$ then $\alpha+\beta=$ (29-06-2022 M)

Key : 5

Sol : $P_1 : x+y+z-1=0 --(1)$

P_2 is $x-2y+3z-1+\lambda(2x+y+z-2)=0 ---(2)$

It's passing through $(1,2,3)$

$:1-4+9-1+\lambda(2+2+3-2)=0$

$5+5\lambda=0\Rightarrow\lambda=-1$

$\therefore(2)\Rightarrow x-2y+3z-1-1(2x+y+z-2)=0$

$\therefore\Rightarrow x-2y+3z-1-2x-y-z+2=0$

$-x-3y+2z+1=0$

$\therefore P_2:x+3y-2z-1=0--(3)$

Let the dr’s of lien of intersection of $P_1\,and\,P_2$ are (a,b,c) then

$a+b+c=0---(4)$

$a+3b-2c=0--(5)$

$\frac{a}{-5}=\frac{-b}{-3}=\frac{c}{2}=\lambda$

$(a,b,c)=(-5\lambda,3\lambda,2\lambda)$

Given that dr’s are $(-5,3,2)$

$\therefore\alpha=3,\beta=2$

$\therefore\alpha+\beta=5$

20a. Let d be the distance between the foot of perpendicular of the points $P(1,3,-1)\,and\,Q(2,-,1,3)$ on the plane $-x+y+z=1$ then $d^2\,is$ ___ (29-06-2022 M)

Key : 26

Sol: Given plane is $-x+y+z-1=0$

Foot of the perpendicular of $P(1,2,-,1)$ us given by

$\frac{h-1}{-1}=\frac{k-2}{1}=\frac{l+1}{1}=-\frac{(-1+2-1-1)}{1+1+1}=\frac{1}{3}$

$h=\frac{-1}{3}+1,K=2+\frac{1}{3},l=\frac{1}{3}-1$

$h=\frac{2}{3}\;\;k=\frac{7}{3}\;\;l=\frac{-2}{3}$

$A\left(\frac{2}{3},\frac{7}{3},\frac{-2}{3}\right)$

Foot of the perpendicular of $Q(2,-1,3)$ is given by $\frac{h-2}{-1}=\frac{k+1}{1}=\frac{l-3}{1}=\frac{(-2-1+3-1)}{1+1+1}=\frac{1}{3}$

$h=-\frac{1}{3}+2\;\;k=\frac{1}{3}-1\;\;l=\frac{1}{3}+3$

$B\left(\frac{5}{3}\;\frac{-2}{3}\;\frac{10}{3}\right)$

$d=AB=\sqrt{1+9+16}=\sqrt{26}$

$d^2=26$

20b. Let d be the distance between the foot of the perpendiculars of the points $P(0,1,0)$ *and* $Q(0,0,1)$ on the plane $x+y+z=3$ then $d^2 =$

Key : 2

Sol : foot of perpendicular of $P(0,1,0)$ is given by $\frac{h-0}{1}=\frac{k-1}{1}=\frac{l-0}{1}=-\frac{(0+1+0-3)}{1+1+1}$

$h=k-1=l=\frac{2}{3}:A\left(\frac{2}{3},\frac{5}{3},\frac{2}{3}\right)$

Foot of perpendicular of $Q(0,0,1)$ is given by

$\frac{h-0}{1}=\frac{\kappa-0}{1}=\frac{l-1}{1}=-\frac{(0+0+1-3)}{1+1+1}$

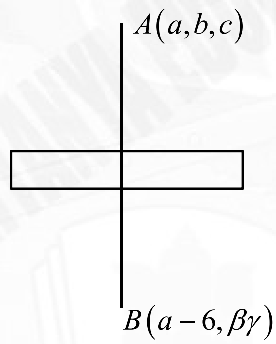
$h=k=l-1=\frac{2}{3}\ ;B\left(\frac{2}{3}\ \frac{2}{3}\ \frac{5}{3}\right)$

$\therefore d=AB=\sqrt{0+1+1}=\sqrt{2}=d^2=2$

21a. Let the mirror image of the Point (a,b,c) with respect to the plane $3x-4y+12z+19=0$ be $(a-6,\beta,\gamma)$. If $a+b+c=5$ *then* $7\beta-9\gamma=$ ___ (27-06-2022 M)

Key : 137

Sol :



Let $A(a,b,c)\,B(a-6,\beta,\gamma)$

D'r's of $\overline{AB}$ all $(6,b-\beta,C-\gamma)$

Dr's of normal to the palne are $(3,-,4,12)$

$\therefore \frac{6}{3}=\frac{b-\beta}{-4}=\frac{c-\gamma}{12}$

$2=\frac{b-\beta}{-4}=\frac{c-\gamma}{12}$

$\therefore b=-8+\beta$, $c=24+\gamma$

Give that $a+b+c=5$

$\Rightarrow a-8+\beta+24+\gamma=5$

$a=-11-\beta-\gamma$

Midpoint of $\overline{AB}$ is $M=\left(a-3,\frac{b+\beta}{2},\frac{c+\gamma}{2}\right)$

Line on $3x-4y+12z+19=0$

$3a-9-2b-2\beta+6c+6\gamma+19=0$

$3a-2b+6c-2\beta+6\gamma+10=0--(1)$

Sub : $a=-11-\beta-\gamma, b=-8+\beta, c=24+\gamma$ in eq(1)

$(1)\Rightarrow -33-3\beta-3\gamma+16-2\beta+144+6\gamma-2\beta+6\gamma+10=0$

$-7\beta+9\gamma=-137$

$\therefore 7\beta-9\gamma=137$

21b. The image of the point $(-1,3,4)$ in the plane $x-2y=0$ is (α,β,γ) then $[\alpha+\beta+\gamma]=$ (where denotes G.I.F)

Key : 3

Sol : Image (α,β,γ) given by

$$\frac{\alpha+1}{1}=\frac{\beta-3}{-2}=\frac{\gamma-4}{0}=-2\frac{(-1-6+0+0)}{1+4}=\frac{14}{5}$$

$$\alpha=\frac{14}{5}-1 \quad \beta=\frac{-28}{5}+3 \quad \gamma=0+4$$

$$\alpha+\beta+\gamma=\frac{14}{5}-1-\frac{28}{5}+3+4$$

$$=\frac{-14}{4}+6=\frac{16}{5}=3.2$$

$$\therefore[\alpha+\beta+\gamma]=3$$

22a. let S be the mirror image of the point $Q(1,3,4)$ with resects to the plane $2x-y+z+3=0$ and let $R(3,5,\gamma)$ be a point on this plane then square of the length of the line segment SR is__ (00-00-2000 M)

Key : 72

Sol : image of $Q(1,3,4)$ with respect to $2x-y+z+3=0$ is given by

$$\frac{h-1}{2}=\frac{k-3}{-1}=\frac{l-4}{1}=-2\frac{(2-3+4+3)}{4+1+1}=-2$$

$h=-3\ k=5, l=2$

$\therefore S=(-3,5,2)$

$R(3,5,\gamma)$ lies on the plane $2x+-y+z+3=0$

$\therefore 6-5+\gamma+3=0\Rightarrow\gamma=-4$

$\therefore SR^2=36+0+36=72$

22b. Let S be the mirror image of the point Q $(3,2,1)$ with respect to the plane $2x-y+3z=7$ and let $R(1,-2,k)$ be a point on this plane. Then the square of the length of the line segment SR is____

Key : 20

Sol : Image of $Q(3,2,1)$ is given by $\frac{h-3}{2}=\frac{k-2}{-1}=\frac{l-1}{3}=\frac{2[6-2+3-7]}{4+1+9}=0$

$\therefore h=3, k=2, l=1$

$\therefore S=(3,2,1)$

$R(1,-2,K)$ liens on the plane $2x-y+3z=7$

$\therefore 2+2+3k=7$

$k=1$

$\therefore R(1,-2,1), S=(3,2,1)$

$\therefore SR^2=4+16+0=20$

23a. Three lines are given by $\bar{r}=\lambda\bar{i}, \lambda\in R, \bar{r}=\mu(\bar{i}+\bar{j})\ \mu\in R, \bar{r}=\mu(\bar{i}+\bar{j}+\bar{k}), \gamma\in R$ Let the lines cuts the plane $x+y+z=1$ at the points A,B,C if the area of the $\Delta ABC\, is\, \Delta$ then $(6\Delta^2)=$ ___

(ADV-2009)

Key : $0.75(=1)$

Sol : Given lines are $\bar{r}=\lambda\bar{i}---(1)$

$\bar{r}=\mu(\bar{i}+\bar{j})---(2)$

$\bar{r}=\gamma(\bar{i}+\bar{j}+\bar{k})---(3)$

These liens are cut the plane $x+y+z=1$ at the point $A(\lambda,0,0)B(\mu,\mu,0)C(\gamma,\gamma,\gamma)$

respectively A,B,C lines on the plane $:\lambda=1, \mu=\frac{1}{2}, \gamma=\frac{1}{3}$

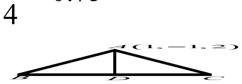
$\therefore A=(1,0,0)B\left(\frac{1}{2},\frac{1}{2},0\right)C\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

$$\overline{AB}\times\overline{AC}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ -\frac{1}{2} & \frac{1}{2} & 0\\ \frac{-2}{3} & \frac{1}{3} & \frac{1}{3}\end{vmatrix}=\bar{i}\left(\frac{1}{6}\right)-\bar{j}\left(\frac{-1}{6}\right)+\bar{k}\left(\frac{-1}{6}+\frac{2}{6}\right)$$

$$=\frac{\bar{i}}{6}+\frac{\bar{j}}{6}+\frac{\bar{k}}{6}$$

Area of $\Delta ABC=\frac{1}{2}\sqrt{\frac{1}{36}+\frac{1}{36}+\frac{1}{36}}=\frac{1}{2}.\frac{\sqrt{3}}{6}=\frac{\sqrt{3}}{12}$

$(6\Delta)^2=36\Delta^2=36.\frac{3}{144}=\frac{3}{4}=0.75$

23b. Let the three liens are  meets the Plane $x+y+z=3$ at A, B and C. If the area of ΔABC is Δ then $16\Delta^2=$

Key : 27

Sol : Given lines are $x=y=z=\lambda$ ___(1)

$$x = y = \mu \_\_\_ (2)$$

$$x = \gamma - -(3)$$

Let these lines cuts the plane $x + y + z = 3$ at $A(\lambda,\lambda,\lambda), B(\mu,\mu,0), C(\gamma,0,0)$ respectively

$\therefore$ These are lie on the plane

$$\therefore \lambda = 1, \mu = \frac{3}{2}, \gamma = 3$$

$$\therefore A = (1,1,1),\ B = \left(\frac{3}{2},\frac{3}{2},0\right)\ C = (3,0,0)$$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{1}{2} & \frac{1}{2} & -1 \\ 2 & -1 & -1 \end{vmatrix} = \bar{i}\left(-\frac{3}{2}\right) - \bar{j}\left(\frac{3}{2}\right) + \bar{k}\left(\frac{-3}{2}\right)$$

Area of ΔABC is $\Delta = \frac{1}{2}\left|\overline{AB} \times \overline{AC}\right|$

$$= \frac{1}{2}\sqrt{\frac{9}{4} + \frac{9}{4} + \frac{9}{4}}$$

$$= \frac{1}{2}\sqrt{\frac{27}{4}}$$

$$\Delta^2 = \frac{27}{16}$$

$$16\Delta^2 = 27$$

24a. If the distance of the point $(1,-2,3)$ from the plane $x + 2y - 3z + 10 = 0$ measured parallel to the line $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ *is* $\sqrt{\frac{7}{2}}$ then the value of $|m| = \_\_\_$ (16-03-2021 A)

Key : 2

Sol : $P(1,-,2,3)$ Plane : $x + 2y - 3z + 10 = 0$

Line : $\frac{x-1}{3} = \frac{y-2}{-m} = \frac{z-(-3)}{1}$

D'rs of the line $(a,b,c) = (3,-m,1)$

Dc's of the line $(l,m,n) = \left(\frac{3}{\sqrt{10+m^2}} \frac{-m}{\sqrt{10+m^2}} \frac{1}{\sqrt{10+m^2}}\right)$

$\therefore$ Required distance $= \left|\frac{ax_1 + by_1 + cz_1 + d}{al + bm + cn}\right| = \sqrt{\frac{7}{2}}$

$$\Rightarrow \frac{|1-4-9+10|}{\left|\frac{3+2m-3}{\sqrt{10+m^2}}\right|} = \sqrt{\frac{7}{2}}$$

$$\Rightarrow \left|\frac{2\sqrt{10+m^2}}{2m}\right| = \sqrt{\frac{7}{2}}$$

$2\left(10+m^2\right)=7m^2 \Rightarrow 20=5m^2$

$\dfrac{a \quad b \quad c}{\begin{matrix}1 & 2 & 3\\ 2 & 3 & 1\end{matrix}}$

$|m|=2$

24b. The distance of the point $(1,2,3)$ from the plane $x+y+z=11$ measured parallel to the line $\dfrac{x-1}{1}=\dfrac{y-2}{-2}=\dfrac{z-3}{k}$ is 15 and $(k>1)$ then $k=$ ___

Key: 2

Sol : $P(1,2,3)$ plane $x+y+z-11=0$

$line:\dfrac{x-1}{1}=\dfrac{y-2}{-2}=\dfrac{z-3}{k}$

d.r's line $(a_1,b_1,c_1)=(1,-,2,k)$

d.c's line $(l_1,m_2,n)=\left(\dfrac{1}{\sqrt{5+k^2}}\dfrac{-2}{\sqrt{5+k^2}}\dfrac{k}{\sqrt{5+k^2}}\right)$

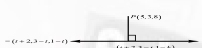
Request distance $=\dfrac{|ax_1+by_1+cz_1+d|}{|al+bm+n|}=15$

$\Rightarrow\dfrac{|1+2+3-11|}{\dfrac{|1-2+k|}{\sqrt{5+k^2}}}=5$

$\Rightarrow\dfrac{5\sqrt{5+k^2}}{|k-1|}=15$

$5+k^2=9\left(k^2-2k+1\right)$

$8k^2-18k+4=0$



$4k(k-2)-1(k-2)=0$

$(k-2)(4k-1)=0$

$\therefore k=2$ or $k=\dfrac{1}{4}$

25a. If the line $\dfrac{x-k}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{3}$ and $\dfrac{x+1}{3}=\dfrac{y+2}{2}=\dfrac{z+3}{1}$ are coplanar then the value of $k=$ ___ (25-07-2021 M)

Key : 1

Sol : Given lines are $\dfrac{x-k}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{3}$

$\dfrac{x-(-1)}{3}=\dfrac{y-(-2)}{2}=\dfrac{z-(-3)}{1}$

$\therefore A(x_1, y_1, z_1) = (k, 2, 3)$

$C(x_2, y_2, z_2) = (-1, -2, -3)$

$b(a_1, b_1, c_1) = (1, 2, 3)$

$d(a_2, b_2, c_2) = (3, 2, 1)$

Lines are coplanar $\left[\overline{a} - \overline{c}, \overline{b}\ \overline{d}\right] = 0$

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$(k+1)(-4)(-8) + 6(-4) = 0$

$k + 1 - 8 + 6 = 0$

$k = 1$

25b. If the liens are $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ *and* $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ are intersection then $2k =$ ___

Key : 9

Sol : $A(x_1, y_1, z_1) = (1, -1, 1)$ $c = (x_2, y_2, z_2) = (3, k, 0)$

$b(a_1, b_1, c_1) = (2, 3, 4)$ $d(c_2, b_2, c_2) = (1, 2, 1)$

Lines are intersecting

$\therefore$ lines are coplanar

$$\therefore \begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -2 & -1-k & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow -2(-5) - (-1-k)(-2) + 1(1) = 0$

$10 - 2 - 2k + 1 = 0$

$9 = 2k$

$\therefore 2k = 9$

26a. Let $(\lambda, 2, 1)$ be point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the points $(-2, -21, 29)$ and $(-1, -16, 23)$

then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{1} - 4 =$ __ (26-02-2021 M)

Key : 8

Sol : d'rs of the line joining the points $(-2,-21,29)$ and $(-1,-16,23)$ are $(a,b,c)=(1,5,-6)$

Equation of the plane is $a(x-y_1)+b(y-y_1)+c(z-z_1)=0$

$\Rightarrow 1(x-4)+5(y+2)-6(z-2)=0$

$x+5y-6z+18=0$

$(\lambda,2,1)$ lines on the plane

$\therefore \lambda+10-6+18=0$

$\lambda=-22$

$\therefore \left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4=\left(\frac{-22}{11}\right)^2-4\left(\frac{-22}{11}\right)-4,=4+8-4=8$

26b. If the point $(\lambda,0,-1)$ lies the plane which is passing through $(-1,6,2)$ and perpendicular to the line joining the points $(1,2,3),(-2,3,4)$ then $\lambda^2+\lambda+4=$__

Key : 16

Sol : Dr's of the line joining the points $(1,2,3)\,and\,(-2,3,4)$ are $(a,b,c)=(-3,1,1)$

Equation of the plane is $-3(x+1)+(y-6)+(z-21)=0$

$-3x-3+y-6+z-2=0$

$3x-y-2z+11=0$

The point $(\lambda,0,-1)$ line on the plane

$\therefore 3\lambda-0+1+11=0$

$3\lambda=-12 \Rightarrow \lambda=-4$

$\therefore \lambda^2+\lambda+4=16-4+4=16$

27a. The square of the distance of the point of intersection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and the plane $2x-y+z=6$ from the point $(-1,-1,2)$ is___ (31-08-2021 M)

Key : 61

Sol : Given line is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}=\lambda$

Any point on the line is $P=(2\lambda+1,3\lambda+2,6\lambda-1)$

Sub: P in the plane $2x-y+z=6$

$\Rightarrow 2(2\lambda+1)-(3\lambda+2)+(6\lambda-1)=6$

$4\lambda+2-3\lambda-2+6\lambda-1=6$

$7\lambda=7 \Rightarrow \lambda=1$

$\therefore P(3,5,5);Q=(-1,-1,2)$

$\therefore PQ^2=16+36+9=61$

27b. If the line $\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}$ and the plane $4x+5y+3z-5=0$ interest at P the $(OP)^2=$____ where O is origin

Key : 14

Sol : Given line is $\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}=\lambda$

Any point on the lines is $P=(3\lambda-3,-2\lambda+2,\lambda-1)$

Sub: P in the plane $4x+5y+3z-5=0$

$4(3\lambda-3)+5(-2\lambda+2)+3(\lambda-1)-5=0$

$12\lambda-12-10\lambda+10+3\lambda-3-5=0$

$5\lambda=10$

$\lambda=2$

$\therefore P=(3,-,2\ \ 1)$

$OP^2=9+4+1$

$=14$

28a. Suppose the line $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ lies on the plane $x+3y-2z+\beta=0$ then $\alpha+\beta=$ ___

(31-08-2021 A)

Key : 7

Sol : Line passing through $P(2,2,-2)$

$\therefore$ Plane passing through $P(2,2,-2)$

$\therefore 2+6+4+\beta=0$

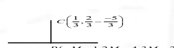
$\therefore \beta=-12$

$al+bm+cn=0$

$\alpha-15-4=0$

$\alpha=19$

$\alpha+\beta=19-12=7$

28b. If the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane  then $4l^2+6m^2=$ ___

Key : 10

Sol : Line passing through $P(3,-2,4)$

$\therefore$ Plane passing through $P(3,-2,-4)$

$\therefore 3l-2m+4=9$

$3l-2m=5---(1)$

$al+bm+cn=0$

$2l-m-3=0$

$2l-m=3---(2)$

$Solving\ (1)\ and\ (2)$

$l=1;\ m=-1$

$\therefore 4l^2+6m^2=4+6=10$

29.a. Let Q be the foot of the perpendicular from the point $P(7,-2,13)$ on the plane containing the lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$ then $(PQ)^2$

(26-08-2021 A)

Key : 96

Sol : Equation of the plane containing the line is

$$\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}=0$$

$$\Rightarrow (x+1)(9)-(y-1)(18)+(z-3)(9)=0$$

$$x-2y+z=0$$

The distance from $P(7,-,2,13)$ to the plane is

$$d=PQ=\left|\frac{7+4+13}{\sqrt{1+4+1}}\right|=\frac{24}{\sqrt{6}}$$

$$(PQ)^2=\frac{24\times 24}{6}=24(4)$$

$$=96$$

29b. The distance form origin to the plane containing the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{4}\ and\ \frac{x-2}{3}=\frac{y-3}{2}=\frac{z-2}{3}$ is d' then $62d^2=$ ___

Key : 100

Sol : The equation of the plane containing the line is

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{vmatrix}=0$$

$$\Rightarrow (x-3)(1)-(y-2)(-6)+(z-1)(-5)=0$$

$$x-3+6y-12-5z+5=0$$

$$x+6y-5z-10=0$$

$\therefore$ The distance from origin to the plane is

$$d=\frac{|10|}{\sqrt{1+36+25}}=\frac{10}{\sqrt{62}}$$

$$\therefore 62d^2=62\frac{(100)}{62}=100$$

30a. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-3}{-3}=\frac{z+5}{7}$ If the point $(1,-1,\alpha)$ lies on the plane P, then the value of $|5\alpha|=$

(18-03-2021 A)

Key : 38

Sol : The equation the plane containing the first line and parallel to the second line is

$$\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$$

$\Rightarrow (x-1)(34)-(y+6)(13)+(z+5)(-25)=0$

$34x-34-13y-78-25z-125=0$

$34x-13y-25z-237=0$

$(1,-1,\alpha)$ line on the plane

$34+13-25\alpha-237=0$

$-25\alpha=190$

$5\alpha=-38$

$\therefore |5\alpha|=38$

30b. A plane containing to the point $(3,2,0)$ and line $\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ is in the form $ax+by+cz=23$ then $|a+b+c|=$ ____

Key : 14

Sol : Let $P(3,2,0)$, $A(1,2,3)$

Required equation of the plane is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ -2 & 0 & 3 \\ 1 & 5 & 4 \end{vmatrix} = 0$$

$(x-1)(-15)-(y-2)(-11)+(z-3)(-10)=0$

$15x-15-11y+22+10z-30=0$

$15x-11y+10z-23=0$

$\Rightarrow 15x-11y+10z=23$

$\therefore |a+b+c|=|15-11+10|=14$

31a. The distance of the point $(-1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{6}$ is (04-09-2021 M)

Key : 1

Sol : d'r's of the line are $(a,b,c)=(2,3,-6)$

d.c's of the line are $(l,m,n)=\left(\frac{2}{7},\frac{3}{7},\frac{-6}{7}\right)$

The req distance $=\frac{|ax_1+by_1+cz_1+d|}{|al+bm+cn|}$

$$= \frac{|1+2+3-5|}{\left|\frac{2}{7}-\frac{3}{7}-\frac{6}{7}\right|} = \frac{1}{|-1|} = 1$$

31b. The distance of the pint $(1,2,3)$ from the plane $x++y+z=11$ measured parallel to the line

$\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z-3}{2}$ is

Key : 15

Sol : d.r's the line are $(a,b,c)=(1,-,2,2)$ d.c's of the normal are $(l,m,n)=\left(\frac{1}{3}\frac{-2}{3}\frac{2}{3}\right)$

Required distance is $d=\frac{|ax_1+by_1+cz_1+d|}{|al+bm+cn|}=\frac{|1+2+3-11|}{\left|\frac{1}{3}\frac{-2}{3}+\frac{2}{3}\right|}=\frac{5}{\frac{1}{3}}=15$