Sri Chaitanya IIT Academy., India.

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Central Office, Bangalore

DIFFERENTIAL EQUATIONS

EXERCISE - V

ASSERTION & RESONING

Reasoning Type Questions :

Each question has four choices a, b, c and d, out of which only one is correct. Each question contains STATEMENT- 1 and STATEMENT- 2.
A) If both the statements are TRUE and Statement-2 is the correct explanation of Statement-1.
B) If both the statements are TRUE and Statement-2 is NOT the correct explanation of Statement-1.
C) If Statement-1 is TRUE and Statement-2 is FALSE
D) If Statement-1 is FALSE and Statement-2 is TRUE

1. **Statement-1**: Curve satisfying the differential equation $y^1 = y/2x$ passing through $(2,1)$ is a parabola with focus $(1/4,0)$

   Statement-2: The differential equation $y^1 = y/2x$ is of variable separable.

2. **Statement-1**: Order of the differential equation formed from $y = c_1x + c_2e^x + c_4e^{-x} + e^{-c_3x}$ where c_1, c_2, c_4 are arbitrary constants is 3

   Statement-2 : Order of the differential equation is equal to the number of arbitrary constants involved in the given algebraic equation.

3. **Statement-1**: If 'P' is a differentiable function of 'x' and $\frac{dP}{dx} - 3P \le 6 \ \forall \ x \ge 0$ and $P(0) = 4$ then $P \le 6e^{3x} - 2 \ \forall \ x \ge 0$

   Statement 2: $\frac{dP}{dx} - 3P - 6 = e^{3x}\frac{d}{dx}\left[(P+2)e^{-3x}\right]$

4. **Statement -1** : The degree of the differential equation $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \log_e\left(\frac{dy}{dx}\right)$ is 2.

   Statement-2 : The degree of a differential equation which can be written as polynomial in the derivatives is the degree of the derivative of the highest order occuring in it.

5. **Statement-1**: If a,b,c$\in$R and 2a+3b+6c=0,then the equation $ax^2+bx+c=0$ has at least one root in (0,1)

   Statement-2: If a continuous function f defined on R assumes both positive and negative values then it vanishes at least once.

6. **Statement-1**: Order of the differential equation of a family of circle of constant radius is 2.

Statement-2: We required two parameters to fix the centre of the circle

7. **Statement-1**: Curve satisfying the differential equation $y' = \frac{y}{2x}$ passing through (2, 1) is a parabola with focus $\left(\frac{1}{4}, 0\right)$.

Statement-2: The differential equation $y' = \frac{y}{2x}$ is of variable separable.

8. **Statement-1**: The differential equation of all circles in a plane must be of order 3.

Statement-2: There is only one circle passing through three non collinear points

9. **Statement-1**: A differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ can be solved by finding.

I.F. = $e^{\int Pdx} = e^{\int 1/x dx} = e^{\log x} = x$ then solution $y.x = \int x^3 dx + c$ because

Statement-2: Since the given differential equation in of the form dy/dx + py = ϕ wherep, ϕ are function of x

10. **Statement-1**: The degree of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \ln\left(\frac{d^2y}{dx^2}\right)$ is 2.

Statement-2: The degree of a differential equation which can be written as polynomial in the derivatives is the degree of the derivative of the highest order in it.

11. **Statement-1**: The differential equation $y^3 dy + (x + y^2)\,dx = 0$ becomes homogeneous if we put $y^2 = t$

Statement-2: All differential equations of first order and first degree becomes homogeneous if we put $y = tx$.

12 **Statement - 1 :** The order of the differential equation formed by the family of curve $y = C_1 e^x + (C_2 + C_3)e^{x+c_4}\ is$ '1' here C_1, C_2, C_3, C_4 are arbitrary constants

Statement - 2 : The order of the differential equation formed by any family of curves is equal to the number of constants present in it

13. **Statement-1** : The differential equation $x(x^2 + y^2 + 1)\,dx + y(x^2 - y^2 + 1)dy = 0$ becomes homogeneous only by putting x^2 = u, y^2 = v

Statement-2 : The differential equation $\frac{dv}{du} = \frac{u+v+1}{u-v+1}$ is reducible to homogeneous differential equation

14. **Statement-1** : Let a solution $y = y(x)$ of the differential equation $y\sin x + y'\cos x = 1$ Satisfying $y(0)=1$ then $y(x) = \sin\left(x + \frac{\pi}{4}\right)$

Statement-2 : The integrating factor of given differential equation is Sec x

15. **Statement-1** : The degree of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \ln\left(\frac{d^2y}{dx^2}\right)$ is 2.

Statement-2 : The degree of a differential equation which can be written as polynomial in the derivatives is the degree of the derivative of the highest order occurring in it.

16. **Statement-1**: Let a curve $y = f(x)$ pass through the point (1, 1)). At any point $P(x, y)$ on the curve tangent and normal are drawn to cut the y-axis at Q and R respectively. If QR = 2, then $f(x) = 1 + \ln\left(1 \pm \sqrt{1-x^2}\right) \pm \sqrt{1-x^2}$

Statement-2: $QR = x\left(\frac{dy}{dx} + \frac{dx}{dy}\right)$

17. Consider the family of curves $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1, \lambda \in R$

Statement-1: The family of curves is self orthogonal

Statement-2: The differential equation obtained by eliminating λ is invariant when $\frac{dy}{dx}$ is replaced by $-1\Big/\frac{dy}{dx}$

18. **Statement-1**: The solution curves of the differential equation $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{1-x^2-y^2}{x^2+y^2}}$ are circles of radius $\frac{1}{2}$

Statement-2 :The substitution $x = r\cos\theta, y = r\sin\theta$ makes the differential equation separable

19. **Statement-1**: The differential equation of parabolas having their vertices at the origin and foci on the x-axis is an equation whose variables are separable

Statement-2: The differential equation of the straight lines which are at a fixed distance p from the origin is an equation of degree 2

Statement-3: The differential equation of all conics whose both axes coincide with the axes of coordinates is an equation of order 2

20. **Statement-1** : The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 2\left(\frac{dy}{dx}\right)^4 + y = 0 \ is\, 2$.

Statement-2 : The order of the higher derivative involved in an ordinary differential equation is equal to the order of the differential equation.

A) TTT B) TTF C) TFT D) FFF

21. **Statement-1** : The number of arbitrary constants in the general solution of the differential equation $x^2\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}+y=0$ *is* 2

Statement-2 : The number of arbitrary constants in the general solution of a differential equation is equal to the order of the differential equation.

22. **Statement-1** : Integrating factor of $\dfrac{dy}{dx}+y=x^2$ *is* e^x

Statement-2 : Integrating factor of $\dfrac{dy}{dx}+P(x)y=Q(x)$ *is* $e^{\int P(x)dx}$

23. **Statement-1** : Integrating factor of $(x+2y^3)\dfrac{dy}{dx}=y$ *is* $\dfrac{1}{y}$

Statement-2 : Integrating factor of $\dfrac{dx}{dy}+P(y)x=Q(y)$ *is* $e^{\int P(y)dy}$

KEY SHEET

1.	D	**2.**	C	**3.**	A	**4.**	D	**5.**	B
6.	A	**7.**	D	**8.**	A	**9.**	A	**10.**	D
11.	C	**12.**	C	**13.**	D	**14.**	D	**15.**	D
16.	A	**17.**	A	**18.**	A	**19.**	A	**20.**	A
21.	A	**22.**	C	**23.**	C				

HINTS & SOLUTIONS

1. $\frac{dy}{dx}=\frac{y}{2x}\Rightarrow\frac{2dy}{y}=\frac{dx}{x}$

$\Rightarrow \log y^2=\log x+\text{cosnt}\Rightarrow y^2=Cx,$ this passes through $(2,1)$ if $C=1/2.$ Thus $y^2=1/2x$ which represents a parabola with focus $(1/8,0)$.

2. Order of the differential equation is equal to the number of *independent* arbitrary constants involved in the given algebraic equation. So statement II is false.

3. Conceptual

4. The given equation represents ellipse $\frac{x^2}{32}+\frac{(y-12)^2}{64}=1$. The maximum value of $\sqrt{x^2+y^2}$ is the distance between (0, 0), (0, 20).

5. $\frac{a}{3}+\frac{b}{2}+c=0$

$f(x)=\frac{ax^3}{3}+\frac{bx^2}{2}+cx$

f(0)=f(1)=0

Apply Rolle's theorem.

6. Conceptual

7. $\frac{dy}{dx}=\frac{y}{2x}\Rightarrow 2\frac{dy}{y}=\frac{dx}{x}$

$\Rightarrow$ $\log y^2=\log x$ + constant $\Rightarrow$ $y^2 = Cx$, this passes through (2, 1) if C = ½.

Thus $y^2=\frac{1}{2}x$ which represents a parabola with focus $\left(\frac{1}{8},0\right)$.

8. Conceptual

9. dy/dx + y/x = x^2 ... (1)
This is term of linear differential equation dy/dx + py = ϕ ... (2)
from (1) and (2) p = –1/x , $\phi = x^2$
I.f. $e^{\int Pdx}=e^{\int 1/x dx = x}$ e
y.I.f = ∫x×I.fd + c
yx = $\int x^3\,dx + c$.

10. Conceptual

11. Statement 2 is false since $\frac{dx}{dx}=\frac{x+y^2}{y+x^2}$ cannot be made Homogeneous

by putting $y=tx$ But if we put $y^2=$t in the differential equation in statement 1

then $2y\frac{dy}{dx}=\frac{dt}{dx}$ And differential equation becomes Homogeneous

12. Since order of differential equation = No. of in dependent arbitrary constants

Given equation can be reduced to $y=c_6\cos(x+c_3)+c_7e^x$

where $c_6=c_1+c_2$ and $c_7=c_4e^{c_5}$ So order of equation is 3

Hence the correct choice is (C)

13. By putting $x^2=u, y^2=v,$ we get $(u+v+1)du+(u-v+1)dv=0$ $\frac{du}{dv}=-\frac{u+v+1}{u-v+1}$

Which is not homogeneous but is reducible to homogeneous form by putting

u =U+α , v = V+β and choose α and β such that $\alpha+\beta$+1=0, α - β+1=0.

Thus statement 1 is false, statement 2 is true

14. It is a linear equation with I.F $e^{\int \tan x dx}=\sec x$

Required solution is y= sinx+cosx $\Rightarrow$ $y=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

15.

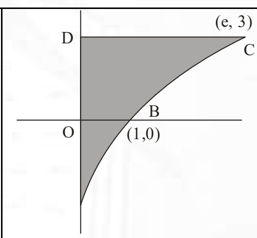
Given,

$$f\left(\frac{x}{y}\right)=f(x)-f(y)$$

Putting x = y, then

$\Rightarrow$ f(1) = 0

$\therefore$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$



$$=\lim_{h\to 0}\frac{f\left(1+\frac{h}{x}\right)}{h}=\frac{3}{x}\left\{\because \lim_{x\to 0}\frac{f(1+x)}{x}=3\right\}$$

$\therefore$ f(x) = 3 ln x + c

Putting x = 1, then

f(1) = 0 + c = 0 $\Rightarrow$f(x) = 3 lnx = y (say)

$\therefore$ x = $e^{y/3}$

$\therefore$ Required area $= \int_{-\infty}^{3} x\,dy = \int_{-\infty}^{3} e^{y/3}dy$

$= 3(e^{y/3})_{-\infty}^{3} = 3(e-0) = 3e$ sq. unit.

$\because$ $f''(x) = -\frac{3}{x^2} < 0 \Rightarrow f(x)$ is concave down.

16. Tangent at P is $Y - y = \frac{dy}{dx}(X - x)$

$\therefore\ Q = (0,\ y - xy'),\ y' = \frac{dy}{dx}$

Normal at P is $(Y-y)y' + X - x = 0$

$\therefore\ R = \left(0,\ y + \frac{x}{y'}\right)$

$QR = 2 \to y' + \frac{1}{y'} = \frac{2}{x}$

$(y')^2 - \frac{2}{x}y' + 1 = 0 \to y' = \frac{1 \pm \sqrt{1-x^2}}{x}$

$dy = \left(1 \pm \sqrt{1-x^2}\right)\frac{dx}{x},\ x = \sin\theta$

$y + c = \int (1 \pm \cos\theta)\frac{\cos\theta}{\sin\theta}d\theta = \ln\sin\theta \pm \int(\cos\theta - \sin\theta)d\theta$

$= \ln\sin\alpha + \ln(\cos ec\theta \pm \cot\theta) \pm \cos\theta = \ln(1 \pm \cos\theta) \pm \cos\theta$

$= \ln\left(1 \pm \sqrt{1-x^2}\right) \pm \sqrt{1-x^2}$

$x = 1,\ y = 1 \to c = -1$

$\therefore\ y = 1 + \ln\left(1 \pm \sqrt{1-x^2}\right) \pm \sqrt{1-x^2}$

17. Differentiating $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, we get $= \frac{x}{a^2+\lambda} + \frac{yy_1}{b^2+\lambda} = 0,\ y_1 = \frac{dy}{dx}$

$\to \frac{x}{a^2+\lambda} = -\frac{yy_1}{b^2+\lambda} = \frac{x+yy_1}{a^2-b^2}$

$\to \frac{x^2}{a^2+\lambda} = \frac{x(x+yy_1)}{a^2-b^2}$ ….(i)

$\to \frac{y^2}{b^2+\lambda} = \frac{y(x+yy_1)}{y_1\ a^2-b^2}$ ….(ii)

(i) + (ii) $\to\ 1 = \frac{\left(x - \frac{y}{y_1}\right)(x+yy_1)}{a^2-b^2}$

$\left(x - \frac{y}{y_1}\right)(x+yy_1) = a^2 - b^2$ which is invariant when y_1 is replaced by $-\frac{1}{y_1}$

18. $x^2 + y^2 = r^2 \to x\,dx + y\,dy = r\ dr$

$xdy - ydx = r\cos\theta(dr\ \sin\theta + r\cos\theta\, d\theta) - r\sin\theta(dr\cos\theta - r\sin\theta\, d\theta) = r^2 d\theta$.

The D.E. reduces to $\frac{rdr}{r^2 d\theta} = \sqrt{\frac{1-r^2}{r^2}}$

$\frac{dr}{\sqrt{1-r^2}} = d\theta$

Integrating, $\sin^{-1} r = \theta + \alpha$

$r = \sin(\theta+\alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha \;=\; \frac{y}{r}\cos\alpha + \frac{x}{r}\sin\alpha$

$x^2 + y^2 - x\sin\alpha - y\cos\alpha = 0$

Which represents the family of circles through the origin with radius $\sqrt{\frac{\sin^2\alpha}{4} + \frac{\cos^2\alpha}{4}} = \frac{1}{2}$

19. S_1 - Equation of parabola is $y^2 = \pm 4ax$

$2y\frac{dy}{dx} = \pm 4a$

D.E of parabola $\Rightarrow y^2 = 2yx\frac{dy}{dx}$

$2\frac{dy}{y} = \frac{dx}{x}$

Which is variable seperable

S_2 - Equation of line which is fixed distance. P from origin can be equation of tangent to circle $x^2 + y^2 = p^2$

Line is $y = mx + p\sqrt{1+m^2}$ $\quad\left(m = \frac{dy}{dx}\right)$

$\left(y - x\frac{dy}{dx}\right)^2 = P\left(1 + \left(\frac{dy}{dx}\right)^2\right)$

So, degree is 2

S_3 - Equation of conic whose both axis co-inside with co-ordinate axis is $ax^2 + by^2 = 1$

As there are two constants, so order of D.E is 2

20-23. Conceptual