PHYSICS-KINEMATICS

RECTILINEAR MOTION

1) **Rectilinear Motion**

i) Distance

1. **Assertion:** Displacement can be decrease with time, but distance can never decrease with time.

Reason: Distance can be many valued function, but displacement can be single valued function.

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false

D. Both A and R are false

Key: B

ii) Displacement

1. **Assertion:** The displacement of a body may be zero though its distance can be infinite.

Reason: If a body moves such that finally it arrives at initial point, then displacement is zero while distance infinite.

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false

D. Both A and R are true

Key: B

2. **Assertion:** A body can have zero distance but non zero displacement.

Reason: Distance is scalar displacement is vector

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false

D. A is false R is True

Key: D

3. **Assertion:** If $\vec{r}_1$ and $\vec{r}_2$ be the initial and final displacement in time t, then

$$\vec{v}_{avg} = \frac{\vec{r}_2 - \vec{r}_1}{t}$$

Reason: The average velocity of a particle having initial and final velocity $\vec{v}$ and $\vec{v}_2$ is $\frac{\vec{v}_1 + \vec{v}_2}{2}$ for uniform acceleration.

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false
D. Both A and R are False

Key: B

(IV) Velocity

A) Instantaneous velocity

Assertion: The instantaneous velocity does not depend on instantaneous position of vector.
Reason: The instantaneous velocity and average velocity of a particle are always same.
A. Both A and R are true and R is the correct explanation of A
B. Both A and R true and R is not the correct explanation of A
C. A is true but R is false
D. Both A and R are False Key: C

V) Acceleration:

A) Instantaneous acceleration

1. **Assertion:** Acceleration is the rate of change of velocity
Reason: A body having nonzero acceleration can have a constant velocity
A. Both A and R are true and R is the correct explanation of A
B. Both A and R true and R is not the correct explanation of A
C. A is true but R is false
D. Both A and R are False

Key: C

2. **Assertion:** A body can have acceleration even if its velocity is zero at a given instant of time
Reason: A body is momentarily at rest when it reverses its direction is of motion
A. Both A and R are true and R is the correct explanation of A
B. Both A and R true and R is not the correct explanation of A
C. A is true but R is false
D. Both A and R are true

Key: A

3. **Assertion:** Always $\left|\frac{d\vec{v}}{dt}\right| = \frac{d}{dt}|\vec{v}|$ where $\vec{v}$ is velocity
Reason: Acceleration is rate of change of speed.
A. Both A and R are true and R is the correct explanation of A
B. Both A and R true and R is not the correct explanation of A
C. A is true but R is false
D. Both A and R are False

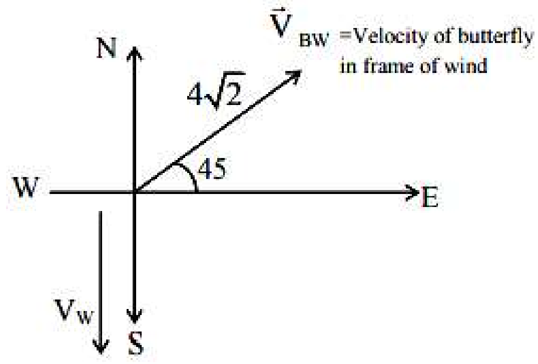
Key: D

II) Displacement

1. A butterfly is flying with a velocity $4\sqrt{2}\ m/s$ in North-East direction. Wind is slowly blowing at $1 m/s$ from North to South. The resultant displacement of the butterfly in 3 seconds is :

1) 3m 2) 20m 3) $20\sqrt{2}$ m 4) 15m

Sol:



$\vec{V}_{BW} = 4\sqrt{2}\cos 45\hat{i} + 4\sqrt{2}\sin 45\hat{j}$

$= 4\hat{i} + 4\hat{j}$

$\vec{V}_w = -\hat{j}$

$\vec{V}_B = \vec{V}_{BW} + \vec{V}_W = 4\hat{i} + 3\hat{j}$

$\vec{S}_B = \vec{V}_B \times t = \left(4\hat{i} + 3\hat{j}\right) \times 3 = 12\hat{i} + 9\hat{j}$

$\left|\vec{S}_B\right| = \sqrt{(12)^2 + (9)^2} = 15m$

III) SPEED

B) Average speed-speed

1. **Assertion:** Average speed is equal to total distance travelled divided by total time taken.

Reason: The average speed of an object may be equal to arithmetic mean of individual speed.

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false

D. Both A and R are False

Key: B

2. **Assertion:** A body may have uniform speed and non-uniform acceleration

Reason: A body may have uniform velocity and nonzero acceleration

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false

D. Both A and R are False

Key: C

V) ACCELERATION

A) Instantaneous speed

1. **Assertion:** A negative acceleration of a body is associated with a slowing down of a body.

Reason: Acceleration is vector quantity

A. Both A and R are true and R is the correct explanation of A
B. Both A and R true and R is not the correct explanation of A
C. A is true but R is false
D. Both A and R are False

Key: B

VI) CALCULUS BASED COMPLEX NUMBERS

1. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$ .If the acceleration of the particle depends on x as x^{-n},where n is an integer the value of n is ___________________

Sol: $x^2 = at^2 + 2bt + C$

$2xv = 2ar + 2b$

$xv = at + b$

$v^2 + ax = a$

$ax = a - \left(\frac{at+b}{x}\right)^2$

$a = \frac{a\left(at^2 + 2bt + c\right) - \left(at+b\right)^2}{x^3}$

$a = \frac{ac - b^2}{x^3}$

$a \propto x^{-3}$

2. A small ball of mass is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2 where v is its speed. The maximum height attained by the ball is:

1) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$ 2) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$ 3) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{2g}$ 4) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{2g}\right)$

Sol: $|a| = g + kv^2$

$\Rightarrow -\frac{vdv}{dh} = g + kv^2$

$\Rightarrow \int_u^0 \frac{vdv}{g+kv^2} = \int_0^{H_{\max}} -dh$

On solving

$H_{\max} = \frac{1}{2k} In\left(1 + \frac{ku^2}{g}\right)$

3. A particle is projected with velocity v_0 along x-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin i.e., $ma = -\alpha x^2$. The distance at which the particle stops:

1) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$ 2) $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$ 3) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$ 4) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Sol: $a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=v\frac{dv}{dx}=-dx^2$

Or $\int_{v_0}^{0} v\,dv=-\alpha\int_0^s x^2dx$

Or $\left[\frac{v^2}{2}\right]_{v_o}^{0}=-\alpha\left[\frac{n^2}{3}\right]_0^s$

Or $\frac{v_0^2}{2}=\frac{\alpha s^3}{3}$

$\therefore S=\left[\frac{3v_0^2}{2d}\right]$

4. The velocity of a particle is $v=v_0+gt+Ft^2$. Its position is x = 0 at t = 0 ; then its displacement after time (t = 1) is :

1) 1) v_0+g+F 2) $v_0+\frac{g}{2}+\frac{F}{3}$ 3) $v_0+\frac{g}{2}+F$ 4) $v_0+2g+3F$

Sol: Given velocity $v=v_0+gt+ft^2$

$\therefore v=\frac{dx}{dt}\ or\ \int_0^x dx=\int_0^t vdt$

$or\ x=\int_0^t\left(v_0+gt+ft^2\right)dt$

$x=v_0t+\frac{gt^2}{2}+\frac{ft^3}{3}+C$

Where C is the constant of integration

Given: $x=0,t=0.\therefore C=0$

Or $x=v_0t+\frac{gt^2}{2}+\frac{ft^3}{3}$

At $t=1\text{sec}$

$\therefore x=v_0+\frac{g}{2}+\frac{f}{3}.$

5. The relation between time **t** and distance x for a moving body is given as $t=mx^2+nx$, where m and n are constants. The retardation of the motion is :

(When v stands for velocity)

1) $2mv^3$ 2) $2mnv^3$ 3) $2nv^3$ 4) $2n^2v^3$

Sol: Let t, x, v, and a are time, distance, velocity and acceleration. m and n are constant.

Relation between t and x is $t=mx^2+nx$

$\frac{dx}{dt}=v$

$\frac{dv}{dt}=a$

Differentiating w.r.t. x, we get

$$\frac{dt}{dx}=\frac{1}{v}=2mx+n$$

$$v=\frac{1}{2mx+n}........(1)$$

Again differentiating w.r.t. t, we get

$$a=\frac{dv}{dt}=-\frac{2m}{(2mx+n)^2}\left(\frac{dx}{dt}\right)$$

Substituting (1) in above equation,

$$a=-(2m)v^3$$

So, the retardation will be,

$$-a=2mv^3$$

6. The instantaneous velocity of a particle moving in a straight line is given as $v=\alpha t+\beta t^2$, where α and β are constants. The distance travelled by the particle between 1s and 2s is :

1) $3\alpha+7\beta$ 2) $\frac{3}{2}\alpha+\frac{7}{3}\beta$ 3) $\frac{\alpha}{2}+\frac{\beta}{3}$ 4) $\frac{3}{2}\alpha+\frac{7}{2}\beta$

Sol: $V=at+\beta t^2$

$$\frac{ds}{dt}=\alpha t+\beta t^2$$

$$\int_{S_1}^{S_2} ds=\int_1^2\left(\alpha t+\beta t^2\right)dt$$

$$S_2-S_1=\left[\frac{\alpha t^2}{2}+\frac{\beta t^3}{3}\right]_1^2$$

As particle is not changing direction So distance = displacement

Distance $=\left[\frac{\alpha[4-1]}{2}+\frac{\beta[8-1]}{3}\right]$

$$=\frac{3\alpha}{2}+\frac{7\beta}{3}$$

7. If the velocity of a body related to displacement x is given by $v=\sqrt{5000+24x}\ m/s$, then the acceleration of the body is ...... m/s^2.

Sol: $V=\sqrt{5000+24x}$

$$v=\left(\sqrt{5000}+24x\right)$$

$$\frac{dv}{dx}=\frac{1}{2\sqrt{5000+24x}}\times 24=\frac{12}{\sqrt{5000+24x}}$$

Now $a=V\frac{dv}{dx}$

$$= \sqrt{5000+24x} \times \frac{12}{\sqrt{5000+24x}}$$

$a = 12m/s^2$

8. The velocity of a particle is $v = v_0 + gt + Ft^2$.Its position is $X = 0$ at $t = 0$; then its displacement after time $(t=1)$ is:

1. $v_0 + \frac{g}{2} + \frac{F}{3}$ 2. $v_0 + g + F$ 3. $v_0 + \frac{g}{2} + F$ 4. $v_0 + 2g + 3F$

Key: 1

Sol: $v = v_0 + gt + Ft^2 \Rightarrow \int ds = \int_0^1 \left(v_0 + gt + Ft^2\right) dt$ $\Rightarrow v_0 + \frac{g}{2} + \frac{F}{3}$

9. A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the time in time ‘t’ is proportional to:

1. t 2. $t^{2/3}$ 3. $t^{3/2}$ 4. $t^{1/2}$

Key: 3

Sol: $P = mav$ $P = m \times \frac{dv}{dt} v$

$P = ma \times \frac{dv}{dx} v$ $Pdt = m.dv.v$

$P\int dx = m\int v^2.dv$ $P\int dt = m\int v.dv$

$P.x = m\frac{v^3}{3}$ $pt = \frac{mv^2}{2}$

$x\alpha v^3$ $t\alpha v^3$

10. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time ‘t’ is proportional to:

1. $t^{3/2}$ 2. $t^{1/4}$ 3. $t^{3/4}$ 4. $t^{1/2}$

Key: 1

Sol: $P = FV; P = maV = mV\frac{dv}{dt}$

$$\frac{V^2}{2} = \frac{P}{m}t \Rightarrow V = \sqrt{\frac{2p}{m}} t^{1/2} dt$$

$$\int_0^s ds = \sqrt{\frac{2p}{m}} \int_0^t t^{1/2} dt$$

$$s = \sqrt{\frac{2p}{m}} \frac{t^{3/2}}{(3/2)} \Rightarrow s = \sqrt{\frac{8p}{9m}} t^{3/2} \quad \therefore s\alpha t^{3/2}$$

11. The displacement x of a particle moving in one dimension, under action of a constant force is related to the time t by equation $t = \sqrt{x} + 3$. Where x in metres and t is in second. Find the displacement (in metres) of the particles when it’s velocity is zero.

Key: 0

Sol: $t=\sqrt{x}+3$

$x=(t-3)^2$

$v=(t-3)$

12. A particle moves along a straight line such that its displacement at any time t is given by $x=t^3-6t^2+3t+4$. What is the magnitude of velocity of the particle when its acceleration is zero.

Key: 9

Sol: $v=\alpha\sqrt{x}$

$\int_0^x \frac{dx}{\sqrt{x}}=\int_0^t dt \qquad \frac{x^{-1/2+1}}{1/2}=\alpha t \qquad 2\sqrt{x}=\alpha t \qquad x=\frac{\alpha^2}{4}t^2$

13. The motion of a body falling from rest in a resistive medium is described by the equation $\frac{dv}{dt}=2(2-v)$.The velocity of the particle at any time is given by the equation $x\left(1-e^{-2t}\right)$, then final out the value of x(assume the proper unit)

Key: 2

$\frac{dv}{dt}=2(2v)$

$\frac{dv}{2-v}=dt$

Sol: $\int_0^V \frac{dv}{2-v}=2\int_0^t dt$

$\frac{\ln(2-v)_0^v}{-1}=2t$

$\ln\frac{2}{2-v}=2t$

14. A particle located at $x=0$ at time $t=0$, starts moving along the positive x direction with a velocity $v$that varies as $v=\alpha\sqrt{x}$. The displacement of the particle varies with time as $t^{n/2}$. Find the value of n.

Key: 4

Sol:

$V=\alpha\sqrt{x}$

$\frac{dx}{dt}=\alpha\sqrt{x}$

$\frac{dx}{\sqrt{x}}=\alpha dt$

$\int\frac{dx}{\sqrt{x}}=\alpha\int dt$

15. The motion of a body falling from rest in a resisting medium is described by the equation $\frac{d\vec{v}}{dt} = \vec{A} - B\vec{v}$ where A and B are constant

A) The initial acceleration is A

B) The initial acceleration is B

C) The velocity at which acceleration becomes zero is A/B

D) The velocity at any time is $v = \frac{A}{B}\left(1 - e^{-Bt}\right)$

Key: A, C, D

Sol: $\frac{d\bar{v}}{dt} = \bar{A} - B\bar{v}$

At, t=0 , v=0

$\therefore \frac{d\,\bar{v}}{dt} = \bar{A}$

Also when $\frac{dv}{dt} = 0$

$v = \frac{A}{B}$

Again $\frac{dv}{dt} = (A - Bv)$

Or $\int_0^v \frac{dv}{(A - Bv)} = \int_0^t dt$

$\left|\ln\left(\frac{A - Bv}{-B}\right)\right|_0^v = |t|_0^t$

$\therefore v = \frac{A}{B}\left(1 - e^{-Bt}\right)$

Paragraph questions:

A motor boat of mass m moves along a lake with velocity v_0. At the moment t=0 the engine of the boat is shut down. Assuming the resistance of water is proportional to the velocity of the boat $\vec{F} = -r\vec{v}$, where r is a constant

16. Find the time duration of the motor boat moved with the shutdown engine;

A) r/v B) r/v_0 C) $r/2v_0$ D) ∞

17. The velocity of the motor boat as a function of the distance covered with the shutdown engine is

A) $v = v_0 - \frac{rs}{m}$ B) $v = \frac{v_0 r}{m}$ C) $v = \frac{v_0(\eta - 1)}{\eta \ln \eta}$ D) $v = v_0\left(1 - e^{\frac{-rs}{m}}\right)$

18. The mean velocity of the motor boat over the time interval (beginning with the moment t=0), during which its velocity decreases η times is

A) $\frac{v_0}{2}$ B) $\frac{\ln v_0}{2\eta}$ C) $\frac{v_0(n-1)}{\eta \ln \eta}$ D) $\frac{v_0}{\ln \eta}$

Key: 16.D, 17. A, 18. C

Sol: **Passage-I**

16 $f = -rv$

$a = \frac{-r}{m}V$

$\frac{dv}{dt} = \frac{-r}{m}V$

$v = v_0\, e^{-rt/m}$

17 $v\frac{dv}{ds} = \frac{-rv}{m}$

$v = v_0 - \frac{rs}{m}$

18 $v = v_0 - \frac{rs}{m}$

$V\ avg = \frac{\int vdt}{\int dt}$

19. Column II gives the rules according to which a particle moving in one dimension (along x–axis) moves. Column I gives some possible event in the subsequent motion of the particle. Match then (x,u,a are the position, velocity and acceleration of the particle respectively 't' is time)

	Column I		Column II
A	The particle is at x=o at some time (I,e particle passes through origin)	(P)	v = -4x + 5 and particle starts with an initial velocity of $5ms^{-1}$
B	The particle reverses its Direction of motion at least once	(Q)	a = -4v+5 and particle starts from rest from x = 1
C	The particle attains terminal velocity (i.e after some time the velocity of the particle becomes constant)	(R)	a = -4x+5 and particle starts from rest at x =1
D	Both speeding up and slowing down take place During the motion,	(S)	v = $-4t^2$+ 5 at t = 0, particles at x = 1
		(T)	x = $-4t^2+5$

Key: $A \to P,S,T;\ B \to P,R,S;$
$C \to P,Q;\ D \to R,S;$

Sol:

$v = \frac{dx}{dt}$

$a = v\frac{dv}{dx}$

$a = \frac{dv}{dt}$

VII) Kinematic equations for uniformly accelerated motion

1. A particle starts from the origin at t=0 with an initial velocity of 3 $\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $\left(6.0\hat{i}+4.0\hat{j}\right)m/s^2$. The x-coordinate of the particle at the instant when its y-coordinate is 32m is D meters. The value of D is:-

1)50 2)32 3)60 4)40

Sol: Formula used: $s-ut+\frac{1}{2}at^2$

Given, initial velocity is only in x− direction,

$v_y=0$

And acceleration in y− direction,

$a_y=4m/s^2$

And displacement in y− direction,

$y=32m$

Using, $y=u_yt+\frac{1}{2}a_yt^2$

$32=0+\frac{1}{2}(4)t^2$

$t=4s$

2.Find D.

Formula used : $s=ut+\frac{1}{2}at^2$

Given, x− coordinate of the particle at 4 s,

=D meter

Initial velocity of the particle

(Along x− axis), $u_x=3m/s$

Initial acceleration of the particle (along x− axis),

$a_y=6m/s^2$

Using, $x=u_xt+\frac{1}{2}a_xt^2$

$D=(3\times4)+\frac{1}{2}(6)(4)^2$

$D=60m$

2. Starting from the origin at time t = 0, with initial velocity $5\hat{j}ms^{-1}$, a particle moves in the x-y plane with a constant acceleration of $\left(10\hat{i}+5\hat{j}\right)ms^{-2}$. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 , are respectively:

1) 4s and 52 m 2) 2s and 24 m 3) 2s and 18 m 4) 5s and 25 m

Sol: Using equation of motion along x-axis

$x=u_xt+\frac{1}{2}a_xt^2$

$[\because u_x = 0]$

$\Rightarrow 20 = 0 + \frac{1}{2} \times 10 \times t^2$

$\Rightarrow t = 2s$

Using equation of motion along y-axis,

$y = u_y xt + \frac{1}{2} a_y t^2$

$y = y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 18m$

3. An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is:

1) $\sqrt{\frac{v^2 + u^2}{2}}$ 2) $\frac{v - u}{2}$ 3) $\frac{u + v}{2}$ 4) $\sqrt{\frac{v^2 - u^2}{2}}$

Sol: The correct option is $C \frac{\sqrt{v^2 + u^2}}{2}$

Suppose the length of the train us l

So, $v^2 - u^2 = 2al$

$\Rightarrow l = \frac{v^2 - u^2}{2a}$........(1)

Also, let the speed of the middle point of the train is v_m.

Then, $v_m^2 - u^2 = 2a \times \frac{1}{2} = al$

$\Rightarrow v_m^2 - u^2 = a \times \frac{v^2 - u^2}{2a}$

[From (1)]

$\Rightarrow v_m = \frac{\sqrt{v^2 + u^2}}{2}$

4. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be

1) $\frac{a_1 + a_2}{a_2}$ 2) $\frac{a_2}{a_1}$ 3) $\frac{a_1}{a_2}$ 4) $\frac{a_1 + a_2}{a_1}$

Sol: In first case $V = 0 + a_1 t_1$

In the second case $0 = V - a_2 t_2$

$a_1 t_1 = a_2 t_2 \Rightarrow \frac{t_1}{t_2} = \frac{a_2}{a_1}$

5. A mosquito is moving with a velocity $\vec{v}-=0.5t^2\hat{i}+3t\hat{j}+9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s?

1) $\tan^{-1}\left(\frac{2}{3}\right)$ *from x-axis*
2) $\tan^{-1}\left(\frac{2}{3}\right)$ *from y-axis*
3) $\tan^{-1}\left(\frac{5}{2}\right)$ *from y-axis*
4) $\tan^{-1}\left(\frac{5}{2}\right)$ *from x-axis*

Sol: $\vec{v}=0.5t^2\hat{i}+3t\hat{j}+9\hat{k}$

$\vec{v}_{at\,t=2}=2\hat{i}+6\hat{j}+9\hat{k}$

$\therefore$ Angle made by direction of motion of mosquito will be

$$\cos^{-1}\frac{2}{11}(From\,x-axis)=\tan^{-1}\frac{\sqrt{117}}{2}$$

$$\cos^{-1}\frac{6}{11}(From\ y-axis)=\tan^{-1}\frac{\sqrt{85}}{6}$$

$$\cos^{-1}\frac{9}{11}(From\ z-axis)=\tan^{-1}\frac{\sqrt{40}}{9}$$

None of the option is matching.

6. A car accelerates from rest at a constant rate a for some time after which it decelerates at a constant rate b to come to rest. If the total time elapsed is t seconds, the total distance travelled is :

1) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$
2) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$
3) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$
4) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$

Sol: Let the car accelerate for time t_1 travelling distance S_1 and acquiring maximum velocity v.

Then, $S_1=\frac{1}{2}\alpha t_1^2$ [from the equation, $S=ut+\frac{1}{2}at^2$]

And $v=at_1$ [from the equation, $v=u+at$]

$$\Rightarrow S_1=\frac{1}{2}\alpha\times\frac{v^2}{a^2}$$

$$=\frac{v^2}{2\alpha}..........(i)$$

After this car decelerates for time t_2 to come to rest

Hence, $S_2=vt_2-\frac{1}{2}\beta t_2^2$

[From the equation, $S=ut-\frac{1}{2}at^2$] and $0=v-\beta t$

[From the equation, $u=v-at$]

$$\Rightarrow S_2=\frac{v^2}{\beta}-\frac{v^2}{2\beta}=\frac{v^2}{2\beta}........(ii)$$

Now, $t_1+t_2=t$

$\Rightarrow \frac{v}{\alpha} + \frac{v}{\beta} = t$

$\Rightarrow v = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$

Also, total distance travelled,

$S = S_1 + S_2$

$= \frac{v^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$= \frac{1}{2}\left(\frac{\alpha\beta t}{\alpha+\beta}\right)^2\left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$= \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$

7. If the velocity of a body related to displacement x is given by $v = \sqrt{5000+24x}\ m/s$, then the acceleration of the body is ...... m/s^2.

Sol: $V = \sqrt{5000+24x}$

$v = \left(\sqrt{5000} + 24x\right)$

$\frac{dv}{dx} = \frac{1}{2\sqrt{5000+24x}} \times 24 = \frac{12}{\sqrt{5000+24x}}$

Now $a = V\frac{dv}{dx}$

$= \sqrt{5000+24x} \times \frac{12}{\sqrt{5000+24x}}$

$a = 12m/s^2$

8. The velocity of a particle is $v = v_0 + gt + Ft^2$ .Its position is $X = 0$ at $t = 0$; then its displacement after time $(t=1)$ is:

1. $v_0 + \frac{g}{2} + \frac{F}{3}$ 2. $v_0 + g + F$ 3. $v_0 + \frac{g}{2} + F$ 4. $v_0 + 2g + 3F$

Key: 1

Sol: $v = v_0 + gt + Ft^2 \Rightarrow \int ds = \int_0^1 \left(v_0 + gt + Ft^2\right)dt$ $\Rightarrow v_0 + \frac{g}{2} + \frac{F}{3}$

9. A force $\vec{F} = \left(40\hat{i}6 + 10\hat{j}\right)N$ acts on a body of mass 5 kg. If the body starts from rest, its position vector $\vec{r}$ at time t = 10 s, will be :

1) $\left(100\hat{i} + 400\hat{j}\right)m$ 2) $\left(100\hat{i} + 100\hat{j}\right)m$ 3) $\left(400\hat{i} + 100\hat{j}\right)m$ 4) $\left(400\hat{i} + 400\hat{j}\right)m$

Sol: $\frac{d\vec{v}}{dt} = \vec{a} = \frac{\vec{F}}{m} = \left(8\hat{i} + 2\hat{j}\right)m/s^2$

$$\frac{d\vec{r}}{dt}=\vec{v}=\left(8t\,\hat{i}+2t\,\hat{j}\right)m/s$$

$$\vec{r}=\left(8\hat{i}+2\hat{j}\right)\frac{t^2}{2}m$$

At $t=10\text{sec}$

$$\vec{r}=\left[\left(8\hat{i}+2\hat{j}\right)50\right]m$$

$$\Rightarrow\vec{r}=\left(400\hat{i}+100\hat{j}\right)m$$

IX) Equations of motion under gravity

A) Released from some height

1. Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

1) 4.18 m 2) 2.94 m 3) 2.45 m 4) 7.35 m

Sol: Time taken by the first drop to reach the ground is :

$$t=\sqrt{\frac{2h}{g}}=\sqrt{2}s$$

When the first drop reaches the ground, the third drop begins to fall. Therefore, the time interval between the drops is :

$$\Delta t=\frac{t}{2}=\frac{1}{\sqrt{2}}$$

Therefore, the distance covered by the second drop is :

$$s=\frac{1}{2}g\Delta t^2$$

$$=\frac{1}{2}\times 9.8\times\frac{1}{2}=2.45m$$

Therefore, the height of the second drop is :

$$h=H-s=9.8-2.45=7.35m$$

2. A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is :

1) 35 m 2) 45m 3) 50 m 4) 25m

Sol: displacement of the first stone when second is dropped $=5$ thus time difference in dropping of the stones is $t=\frac{2h}{g}=1s$

$$h=\frac{1}{2}gt^2=\frac{1}{2}g\left(t-1\right)^2+25\Rightarrow t=3s$$

Thus, $h=45m$

3. Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4th second after its fall to the next droplet x is 34.3 m. At what rate the droplets are coming from the tap? (Take g = 9.8 m/s^2)

1) 3 drops/2 seconds
2) 2 drops/second
3) 1 drop/second
4) 1 drop/7 seconds

Sol: The distance traveled by a freely falling drop is

$$h = ut + \frac{1}{2}at^2$$

Initial velocity is u = 0

In 4 sec, 1st drop will travel.

$$h_1 = \frac{1}{2} \times 9.8 \times 4^2$$

$$= 78.4$$

2nd drop would have travel,

$$h_2 = 78.4 - 34.3$$

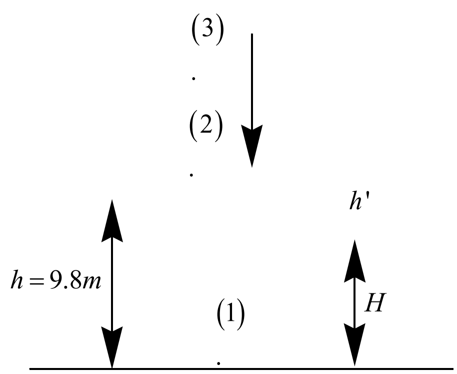
4. Water drops are falling from a nozzle of a shower on to the floor, from a height of 9.8m.The drops fall at a regular interval of time. When the first drop strikes the floor, at the instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor

1. 4.18 m 2. 2.45 m 3. 7.35 m 4. 2.94 m

Key: 3

Sol: for 1st drop; $h = \frac{1}{2}g(2n)^2$

For 2nd drop ; $h' = \frac{1}{2}g(n)^2$



$$\frac{h}{h'} = \frac{4}{1} \Rightarrow h' = \frac{h}{4} = \frac{9.8}{4}$$

So height of 2nd drop

$$H = h - h' = 9.8 - \frac{9.8}{4} = \frac{3}{4} \times 9.8 = 7.35m$$

5. A particle 'p' is dropped from a height 'h' and simultaneously another particle 'Q' is projected vertically upwards with initial of velocity 'u' from the ground. Both the particles move along the same vertical line, then

a) The particles collide if $u > \frac{\sqrt{gh}}{2}$

b) The particles collide which traveling in opposite direction if $u > \sqrt{gh}$

c) The particles collide white traveling in same direction if $\frac{\sqrt{gh}}{2} < u < \sqrt{gh}$

d) If at all the particles collide they collide after a time $\frac{h}{u}$

Key: A, B, C, D

Sol: Conceptual

6. A body dropped from a tower of height h travels $(1/4)^{th}$ of the total distance in the last half-second of its free fall. h =

a) $\frac{1}{2}g\left(2-\sqrt{3}\right)^2$ b) $\frac{1}{2}g\left(\sqrt{3}+2\right)^2$ c) 3g d) $\frac{1}{2}g\left(\sqrt{3}-1\right)^2$

Key: B

$$h = \frac{1}{2}gT^2$$

Sol: $\frac{h}{4} = \frac{1}{2}gT^2 - \frac{1}{2}g(T - \frac{1}{2})^2$

$$T^2 - 4T + 1 = 0$$

$$\Rightarrow T = 2 + \sqrt{3}$$

Passage-II :

A ball is thrown straight up from the edge of the roof of a building with a velocity v_0. A second ball is dropped from the roof one second later. Ignore air resistance. Height of building is *h*. (g = 10ms^{-2})

7. If the height of the building is 20m, the initial speed v_0 of the first ball if the two balls are to hit the ground at the same time will be

a) $2.82\,ms^{-1}$ b) $3.26\,ms^{-1}$ c) $4.12\,ms^{-1}$ d) $8.33\,ms^{-1}$

8. If v_0 is greater than some value $V_{\max}$, a value of h does not exist that allows both balls to hit the ground at the same time, $V_{\max}$ =

a) $10\,ms^{-1}$ b) $20\,ms^{-1}$ c) $5\,ms^{-1}$ d) $15\,ms^{-1}$

9. If v_0 is less than some value $v_{\min}$, again a value of h does not exist that allows both the balls to hit the ground at the same time $v_{\min}$ is

a) $10\,ms^{-1}$ b) $20\,ms^{-1}$ c) $5\,ms^{-1}$ d) $15\,ms^{-1}$

Key: 6. D, 7.A, 8.C

Sol: 6. $h = -v_0 t - \frac{1}{2}gt^2 ....(1)$

$h=\frac{1}{2}g(t-1)^2$ ....(2)

7 & 8.

h can take value from 0 to ∞

If $h=0 \Rightarrow (t-1)=0$ (From (2))

$\Rightarrow + = 1s$

From (1) $v_0 = 2$

If $h=\infty \Rightarrow t=\infty$

Solving for 't' eliminating 'h'

$t=\frac{g}{2(g-v_0)} \Rightarrow v_{\max}=g$

$(v_0)\min = \frac{g}{2}$

10. **Assertion:** The body falls freely, when acceleration of body is equal to acceleration due to gravity.

Reason: A body falling freely will have constant velocity.

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false

D. Both A and R are False

Key: C

B) Vertical projection from ground

1. A ball is thrown up with a certain velocity so that it reaches a height 'h'. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions.

1) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ 2) $\frac{1}{3}$ 3) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ 4) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Sol: *At maximum height, velocity is zero so* $0=u^2-2gh$ ***Or*** $u=\sqrt{2gh}$

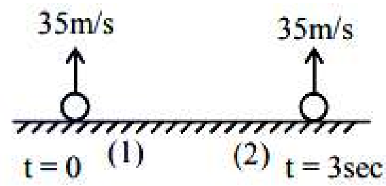
As $s=ut+\frac{1}{2}at^2 so, \frac{h}{3}=\sqrt{2ght}-\frac{1}{2}gt^2$

$$gt^2-2\sqrt{2ght}+\frac{2h}{3}=0 \; or \; t=\frac{2\sqrt{2gh}\pm\sqrt{8gh-\frac{8gh}{3}}}{2g}$$

$$\frac{t_1}{t_2}=\frac{2\sqrt{2gh}-\sqrt{8gh-\frac{8gh}{3}}}{2\sqrt{2gh}+\sqrt{8gh-\frac{8gh}{3}}}=\frac{\sqrt{3}-\sqrt{3-1}}{\sqrt{3}+\sqrt{R-1}}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

2. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of .... m. (Take g = 10 m/s^2)

Sol:



When both balls will collied

$y_1 = y_2$

$35t - \frac{1}{2}\times 10\times t^2 = 35(t-3) - \frac{1}{2}\times 10\times (t-3)^2$

$35t - \frac{1}{2}\times 10\times t^2 = 35t - 105 - \frac{1}{2}\times 10\times t^2$

$-\frac{1}{2}\times 10\times 3^2 + \frac{1}{2}\times 10\times 6t$

$0 = 150 - 30t$

$t = 5\,\text{sec}$

$\therefore$ Height at which both balls will collied

$h = 35t - \frac{1}{2}\times 10\times t^2$

$= 35\times 5 - \frac{1}{2}\times 10\times 5^2$

$h = 50m$

3. A An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding fore of 10 N throughout the motion. The ratio of time of ascent to the time of descent will be equal to:

1) 1:1 2) $\sqrt{2}:\sqrt{3}$ 3) $\sqrt{3}:\sqrt{2}$ 4) 2:3

Key: 2

Sol: $v = u + at$

$0 = u + -(g+2)t$

$t = \frac{u}{g+2}$

$0 - u^2 = 2as$

$-u^2 = 2\times -(g+2)s$

$s = \frac{1}{2}aT^2$

$\frac{u^2}{2(g+2)} = \frac{1}{2}\times (g-2)T^2$

$$\frac{u}{\sqrt{(g^2-4)}}=T$$

$$\frac{t}{T}=\frac{\sqrt{g-2}}{\sqrt{g+2}}=\sqrt{\frac{8}{12}}=\sqrt{\frac{2}{3}}$$

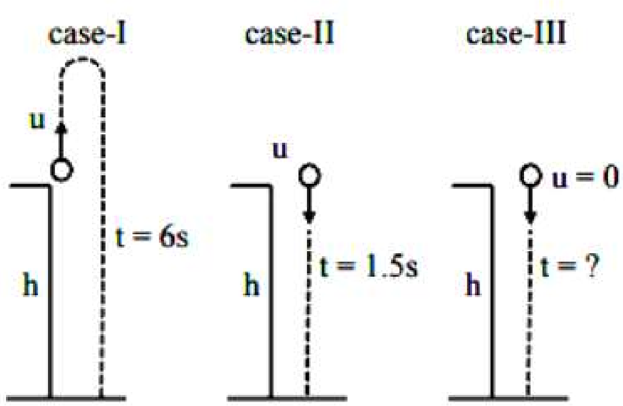
4. **Assertion:** If a particle is thrown upward then distance traveled in last second of upward journey is independent of the velocity projection.
Reason: A vertically projected body covers a distance of 4.9 m in the last second of its upward journey on the earth if air resistance ignored.
A. Both A and R are true and R is the correct explanation of A
B. Both A and R true and R is not the correct explanation of A
C. A is true but R is false
D. Both A and R are False
Key: A

5. Assertion: Two balls of different masses are thrown vertically upward with same speed, they will pass through their point of projection in the downward direction with the same speed (neglect air resistance)
Reason: the maximum height and downward velocity attained at the point of projection are independent of the mass of the ball.
A. Both A and R are true and R is the correct explanation of A
B. Both A and R true and R is not the correct explanation of A
C. A is true but R is false
D. Both A and R are False

Key: A

C) Vertical projection from some height

1. From the top of a tower, a ball is thrown vertically upward which reaches the ground in 6 s. A second ball thrown vertically downward from the same position with the same speed reaches the ground in 1.5 s. A third ball released, from the rest from the same location, will reach the ground in _________s.
Sol: Let height of tower be h and speed of projection in first two cases be u.



For Case:I: 2nd equation $s=ut+1/2at^2$

$h = -u(6) + \frac{1}{2}g(6)^2$

$H = -6u + 18g.......(i)$

For Case:II: $h = u(1.5) + \frac{1}{2}g(1.5)^2$

$h = 1.5u + \frac{2.25g}{2}......(ii)$

Multiplying equation (ii) by 4 we get

$4h = 6u + 4.5g.......(iii)$

$equation(i) + equation(iii) we \ \ get \ \ 5h = 22.5g$

$h = 4.5g$ ……….(iv)

For Case-III:

$h = 0 + \frac{1}{2}gt^2........(v)$

Using equation (4) & Equation (5)

$4.5g = \frac{1}{2}gt^2$

$t^2 = 9 \Rightarrow t = 3s$

2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is:

1) $gH = (n-2)u^2$ 2) $2gH = n^2u^2$

3) $gH = (n-2)^2 u^2$ 4) $2gH = nu^2(n-2)$

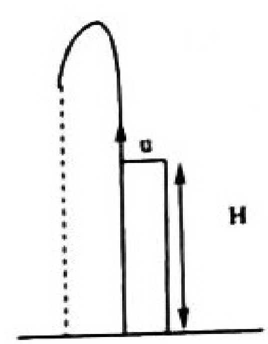
Key: 4

Sol: Let t_1 is the time taken to reach max height $t_1 = \frac{u}{g}$

Let t_2 is the time taken to reach ground from tower equation

$H = -ut_2 + \frac{1}{2}gt_2^2$

$t_2 = \frac{2u \pm \sqrt{4u^2 + 8gH}}{2g} = \frac{u}{g} \pm \sqrt{\frac{u^2 + 2gh}{g^2}}$



Given that $t_2 = nt_1$

$\Rightarrow 2gH = u^2\left[(n-1)^2 - 1\right]$

$\Rightarrow 2gH = u^2\left[n^2 - 2n\right]$ $\Rightarrow 2gH = u^2n[n-2]$

2. A balloon was moving upwards with a uniform velocity of 10m/s. An object of finite mass is dropped from the balloon when it was at a height of 75m from the ground level. The height of the balloon from the ground when object strikes the ground was around: (takes the value of g as $10m/s^2$)

1. 200 m 2)300 m 3) 125 m 4) 250 m

Key: 3

Sol: Body thrown vertically upwards from top of a tower

$$h=-ut+\frac{1}{2}gt^2; u=10m/\sec; h=75m$$

$$g=10m/\sec^2$$

$$\Rightarrow 75=-10t+St^2 \Rightarrow t^2-2t-15=0$$

Solve t=5sec.

In 5 sec balloon will go to further height

$$h'=ut=10\times 5=50m$$

Hence distance between balloon and object after 5 sec is $75+50=125$

4. A stone is projected vertically upwards from an elevated point A. when the stone reaches a distance h below A, its velocity is double of what it was at a height h above A. Then

a) The greatest height attained by store from A is $\frac{4h}{3}$

b) The greatest height attained by store from A is $\frac{5h}{3}$

c) The time to pass again the point A from the instant of projection is $\sqrt{\frac{20h}{3g}}$

d) The time to pass again the point A from the instant of projection is $4\sqrt{\frac{h}{3g}}$

Key: BC

Sol: $v^2=u^2+2gh$

$4v^2=u^2-2gh$

Solving we will get $u^2=\frac{10gh}{3}$

5. A balloon is rising with a constant acceleration of $\frac{g}{8}$ from the ground from rest. After it reaches a height h a stone is released form it at t = 0.

a) The time 'T' at which it strikes the ground is $4\sqrt{\frac{h}{g}}$

b) The displacement of the stone from t = 0 to t = T is $\frac{9h}{8}$

c) The distance covered by the stone from t = 0 to t = T is $\frac{5h}{4}$

d) Maximum height reached by the stone above the ground is $\frac{9h}{8}$

Key: C, D

Sol: Velocity at height h

$$y=\sqrt{2as}=\sqrt{2\frac{9}{8}\times h}=\frac{\sqrt{gh}}{2}$$

$$h=\frac{-\sqrt{gh}}{2}T+\frac{1}{2}gT^{2}$$

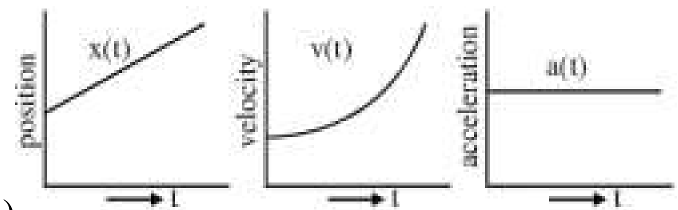
Solving $T=2\sqrt{\frac{h}{g}}$

3. Graphical representation-Motion in 1 d

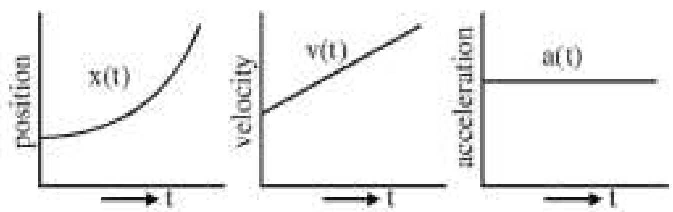
I. Graphs-Motion in 1 d

A) Displacement-time graph

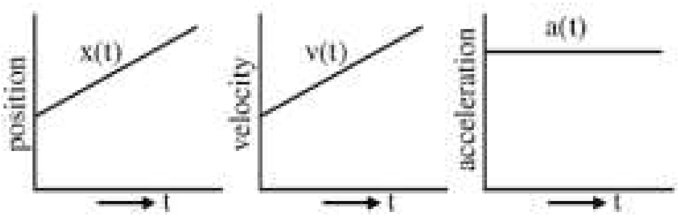
1. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by :



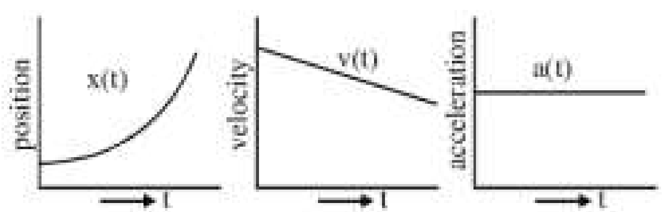
1)



2)



3)



4)

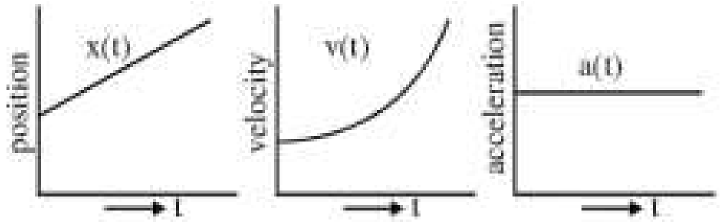
Sol: The velocity-time graph fpr a particle moving at constant speed should be a straight line with a straight line with a positive slope.

The $x-t$ graph should be an opening upward parabola.

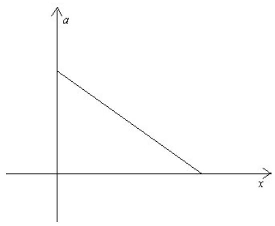
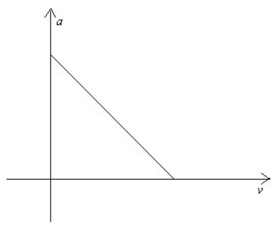
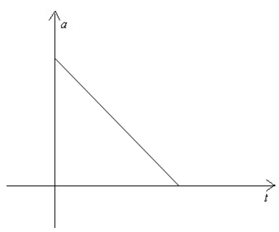
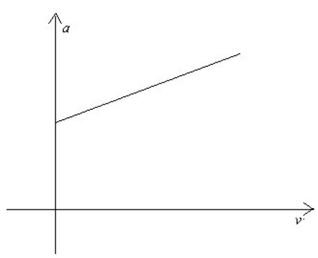
Therefore the acceleration is constant. That is $a=$constant. For an acceleration-time graph.

$v\alpha t=$staraght line graph

$x\alpha t^{2}$ = parabolic graph.



2. A particle is moving on a straight line where different parameters regarding the motions are x=displacement, a=acceleration, v=velocity, t=time.

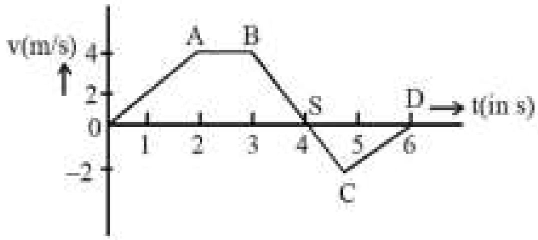
Column-I	Column-II
A. 	p) Non-uniform accelerated motion
B. 	Q) Speed continuously decreasing
C. 	R) Speed continuously increasing
D. 	S) Uniform accelerated motion
	T) Motion is not possible

Key: A-P, R;B-P, R;C-P,R ;D-P, R

Sol: Apply Slope concept for graphs.

(B) Velocity-time graph

1. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6s is:

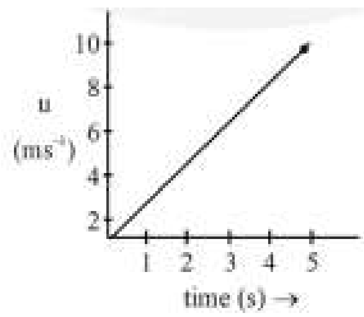


1) 12m 2) $\frac{49}{4}m$ 3) 11m 4) $\frac{37}{3}m$

Sol: Distance $=|A_1|+|A_2|$

$$=\frac{1}{2}\left(1+\frac{13}{3}\right)\times 4+\frac{1}{2}\times\frac{5}{3}\times 2 \;=\frac{37}{3}m$$

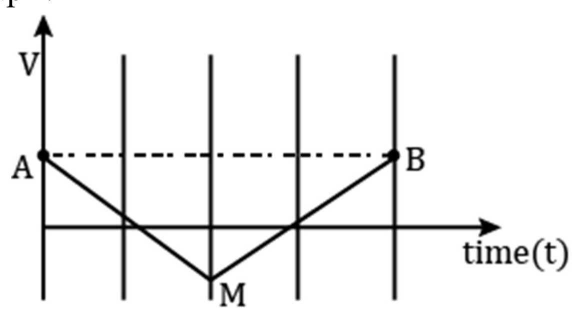
2. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5s will be ______:

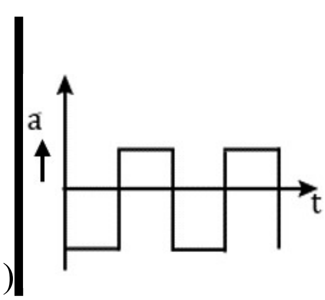


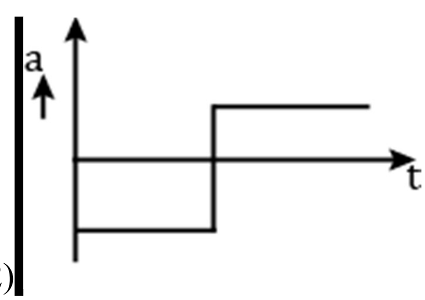
Sol: Distance travelled = Area of speed-time graph

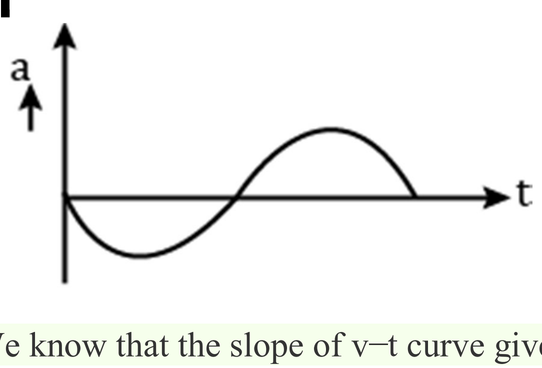
$$=\frac{1}{2}\times 5\times 8=20$$

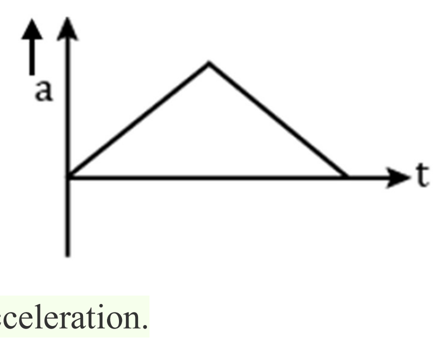
3. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph?



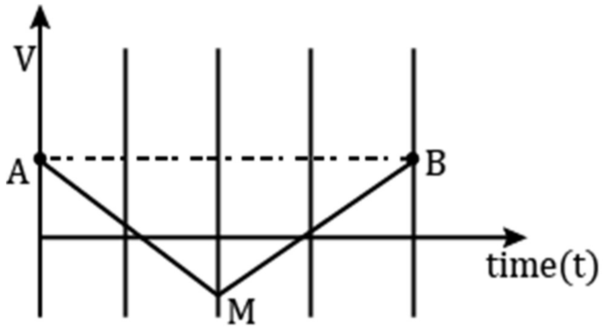
1) 

2) 

3) 

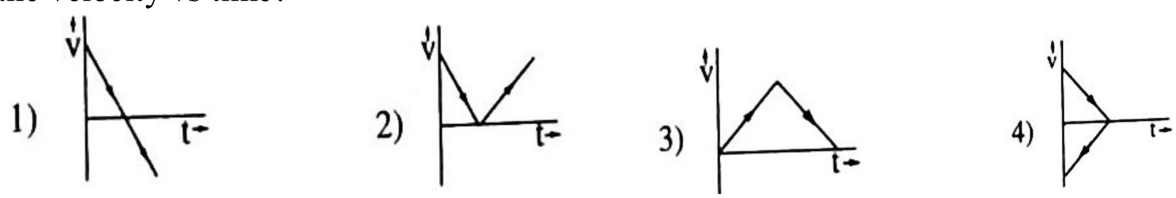
4) 

Sol: We know that the slope of v−t curve gives acceleration.



For AM, slope is constant and negative. Therefore, acceleration is constant and negative. Similarly, for MB, slope is constant and positive. Therefore, acceleration is constant and positive.

4. A body is thrown vertically upwards. Which one of the following graphs correctly represents the velocity vs time?



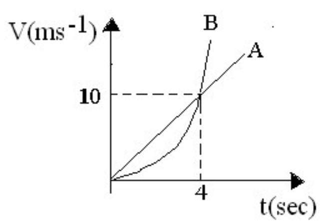
Key: 1

Sol: For upward motion equation is $V = u - gt$

For downward motion equation is $V = gt$

While going up velocity decreases in magnitude. After reaching maximum height, it comes down with velocity increases in magnitude

5. The velocity time graph of two particles A and B are shown. (Where we have time intervals i.e. for C and D match the options which are true over part of interval also)



	COLUMN – I		COLUMN - II
A	At t = 0	P	Acceleration of A > acceleration of B
B	At t = 4s	Q	Acceleration of A < acceleration of B
C	At t < 4s	R	Acceleration of A = acceleration of B
D	At t > 4s	S	Displacement of A > displacement of B
		T	Velocity of A = velocity of B

Key: $A \to P,T; B \to Q,S,T; C \to P,Q,R,S;\ D \to Q,S;$

Sol: Acceleration= slope of v-t graph.

7. **Assertion:** Velocity-time graph for an object in uniform motion along a straight path is a straight line parallel to the time axis

Reason: In uniform motion of an object velocity increases as the square of time elapsed.

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false

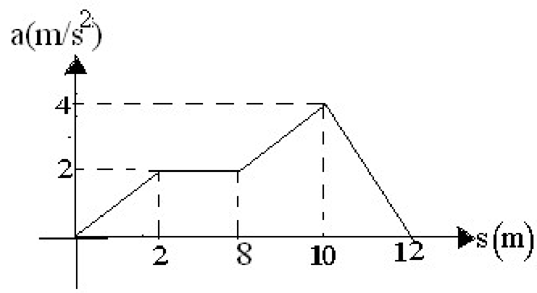
D. Both A and R are False.

Key: 3

C) Distance-time graph

E) Acceleration time graph

1. The acceleration-displacement graph of a particle moving in a straight line is as shown Alongside. Initial velocity of particle is $1ms^{-1}$. Find the velocity of the particle when displacement of the particle is, s = 12m.



Key: 7

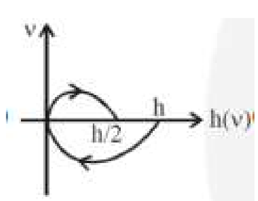
Sol: $\int_0^v v.dv = \int ads$

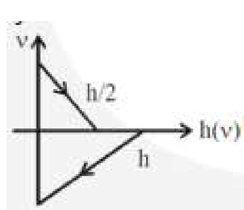
$\frac{v^2}{2}$ area under a-s graph

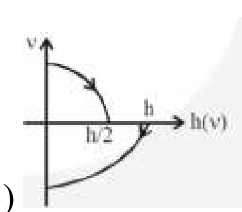
$v = \sqrt{2(area\ under\ a - s\ graph)}$

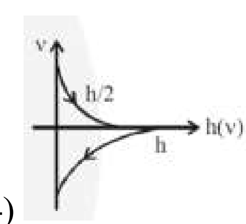
F) Velocity displacement graph-graphs-motion in 1 d

1. A tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by :

1) 

2) 

3) 

4) 

Key: 3

Sol: Velocity at ground (means zero height) is non-zero therefore one is incorrect and velocity versus height is non-linear therefore two is also incorrect.

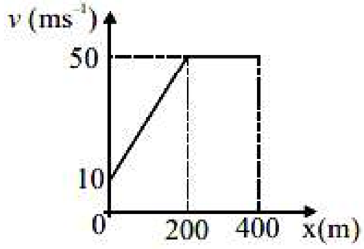
$v^2 = 2gh$

$$v\frac{dv}{dh} = 2g = const.$$

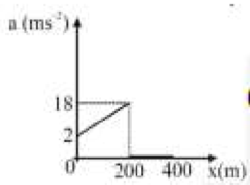
$$\frac{dv}{dh} = \frac{\text{constant}}{v}$$

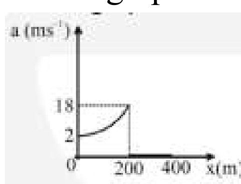
Here we can see slope is very high when velocity is low therefore at maximum height the slope should be very large which is in option 3 and as velocity increases slope must decrease therefor option 3 is correct

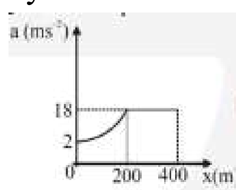
2. The velocity-displacement graph describing the motion of a bicycle is shown in the figure

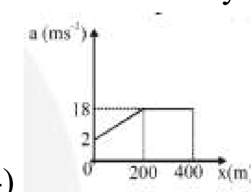


The acceleration-displacement graph of the bicycle's motion is best described by:

1) 

2) 

3) 

4) 

Sol: For $0 \le x \le 200$

$v = mx + c$

$v = \frac{1}{5}x + 10$

$a = \frac{vdv}{dx} = \left(\frac{x}{5} + 10\right)\left(\frac{1}{5}\right)$

$a = \frac{x}{25} + 2$

$\Rightarrow$ Straight line till $x = 200$

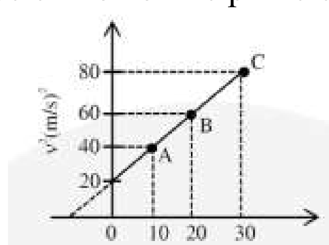
For $x > 200$

$v =$ Constant

$\Rightarrow a = 0$

Hence most approprite options will be (1),otherwise it would be BONUS

3. A particle is moving with constant acceleration 'a'. Following graph shows v^2 versus x(displacement) plot. The acceleration of the particle is___m/s^2 .



Sol: Slope of the graph is,

$$\frac{d\left(v^2\right)}{dx}=\frac{vdv}{dx}$$

which is acceleration of the particle

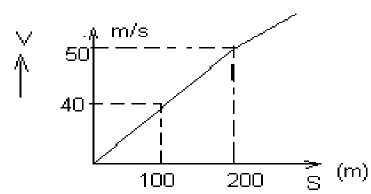
Two point formula for slope of the straight line is $\frac{y_2-y_1}{x_2-x_1}$

Here $\left(x_1,y_1\right)=\left(10,40\right)$

$\left(x_2,y_2\right)=\left(20,60\right)$

Hence, $a=\frac{60-40}{20-10}=2ms^{-2}$

4. The v-s graph for an air plane traveling on a straight way is shown determine acceleration at s=50m



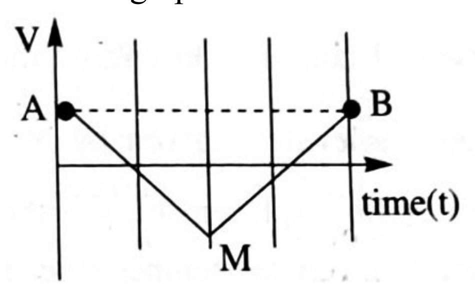
Key: 8

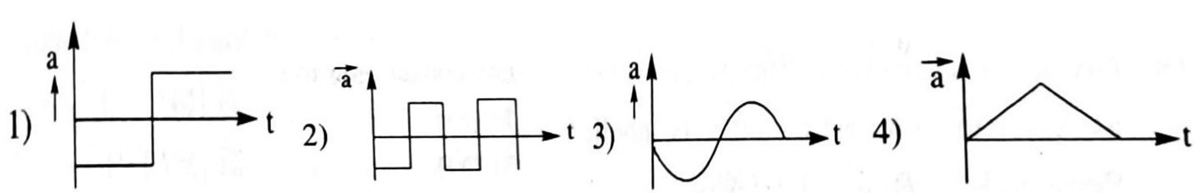
Sol: $a=\frac{vdv}{ds}$

II) Interpretation of graphs

Construction of a-t graph from v-t graph

1. If the velocity time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph?





Key: 1

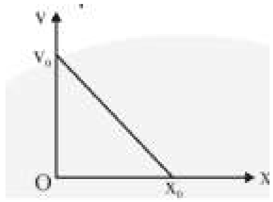
Sol:

$$v=-mt+C;\frac{dv}{dt}=-m$$

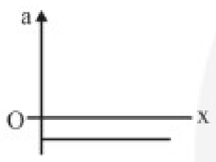
$$\left|\begin{array}{l} v=mt-C \\ \frac{dv}{dt}=m \end{array}\right|$$

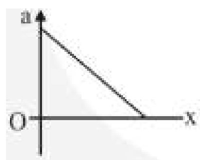
F) Construction of a-s graph from v-s graph

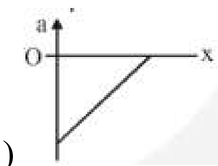
1. The velocity-displacement graph of a particle is shown in the figure.

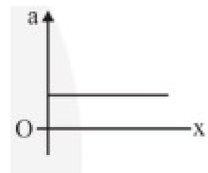


The acceleration-displacement graph of the same particle is represented by :

1) 

2) 

3) 

4) 

Sol: Equation of $5m/v = -\left(\frac{v_0}{x_0}\right)x + v_0$..........(1)

Differentiating both sides

$$\frac{dv}{dt} = -\left(\frac{v_0}{x_0}\right)\frac{dx}{dt}$$

$$\Rightarrow a = -\left(\frac{v_0}{x_0}\right)(v) \qquad \left[v = \frac{dx}{dt}\right]$$

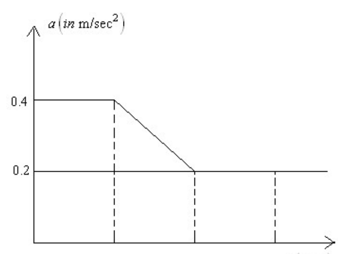
$$\Rightarrow a = -\frac{v_0}{x_0}\left[-\left(\frac{v_0}{x_0}\right)x + v_0\right] \qquad \text{[From equation 1}$$

$$\Rightarrow a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

$\Rightarrow a - x$ Graph will be a straight line with positive slope

$$\left[m = \left(\frac{v_0}{x_0}\right)^2 and - ve\ y - intercept\left[C = -\frac{v_0^2}{x_0}\right]\right]$$

2. The acceleration of a particle which moves along the positive x-axis varies with it's position as shown. If the velocity of the particle is 0.8 m/sec at $x = 0$, the velocity of the particle at $x = 1.4m$ in m/sec is

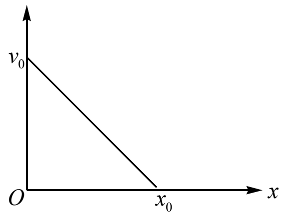


a) 1.6 b) 1.2 c) 1.4 d) none

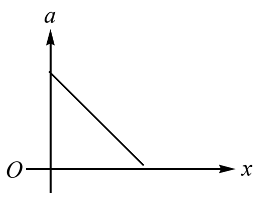
Key: B

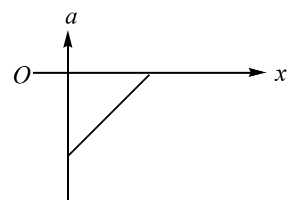
Sol: $a = v\frac{dv}{dx}$

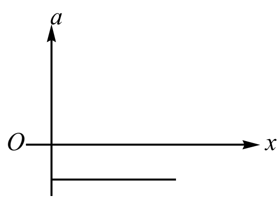
3. The velocity-dispalcement graph of a particle is as shown in the figure.

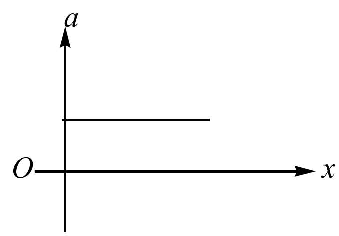


The acceleration-displacement graph of the same particle is represented by

1. 

2. 

3. 

4. 

Key: 2

Sol: From the graph $V = \dfrac{-V_0}{X_0}x + V_0$

Differentiating w.r.t, fine

$$a = \frac{dV}{dt} = \left(\frac{-V_0}{x_0}\right)\left(\frac{dx}{dt}\right) = \left(\frac{-V_0}{x_0}\right)(V)$$

$$\Rightarrow a = \left(\frac{-V_0}{x_0}\right)\left[\frac{-V_0}{x_0}x + V_0\right]$$

$$\Rightarrow a = \left(\frac{V_0}{x_0}\right)^2 x - \frac{V_0^2}{x}$$

So, the graph is a st.line with negative intercept

4. **Relative velocity in 1-d**

I) Definition of relative velocity

1. A child in danger of drowning in a river being carried downstream by a current that flows uniformly at a speed of 2.5 km/h. The child is 0.6km from shore and 0.8 km upstream of a boat landing point on the shore, when a rescue boat sets out. If the boat proceeds at its maximum speed of 20km/h with respect to the water, how long will it take boat to reach the child.(in minutes).

Key: 3

Sol:
$$t = \frac{d}{v}$$
$$= \frac{d}{\left(\overline{v} + \overline{v}_w\right)}$$

2. A plane is to fly due north. The speed of the plane relative to the air is 200 km/h, and the wind is blowing from west to east at 90 km/h

A) The plane should head in a direction given by $\theta = \sin^{-1}(0.65)$ north of west

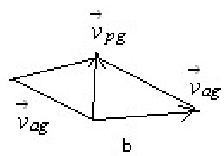
B) The plane should head in a direction given by $\theta = \sin^{-1}(0.45)$ west of north

C) The velocity of plane relative to the ground is 179 km/h in magnitude

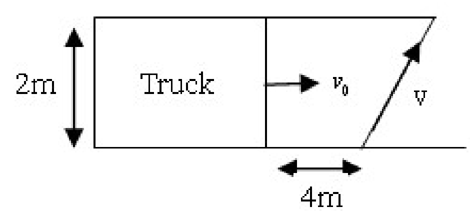
D) The velocity of plane relative to the ground is 159 km/h in magnitude

Key: B,C

Sol: Since the wind is blowing towards the east, the plane must head west of north as shown in figure. The velocity of the plane relative to the ground $\bar{v}_{pg}$ will be the sum of the velocity of the plane relative to the air $\bar{v}_{pa}$ and the velocity of the air relative to the ground $\bar{v}_{ag}$



3. A 2m wide truck is moving with a uniform speed $v_0 = 8m/s$ along a straight horizontal road. A pedestrian starts to cross the road with a uniform speed v when the truck is 4m away from him. The minimum value of v so that he can cross the road safely is



A) 2.62 m/s B) 4.6 m/s C) 3.57 m/s D) 1.414 m/s

Key: C

Sol: Let the man starts crossing the road at an angle θ as shown in figure. For safe crossing the condition is that the man must cross the road by the time the trunk describes the distance $4 + AC \ or \ 4 + 2\cot\theta$

$$\therefore \frac{4+2\cot\theta}{8} = \frac{2/\sin\theta}{v} or \ v = \frac{8}{\sin\theta + \cos\theta} ......(1)$$

For, minimum $v, \frac{dv}{d\theta} = 0$

4. Three particles are located at the Vertices of an equilateral triangle of side 'a'. They all start moving simultaneous with velocity 'V', constant in magnitude, with the first particle heading continuously for the second, the second for the third, and the third for the first. Then

a) The three particles meet at the centroid of the triangle

b) They meet after a time of $\frac{a}{3V}$

c) Each particle travels a distance of 2a/3 till they meet each other.

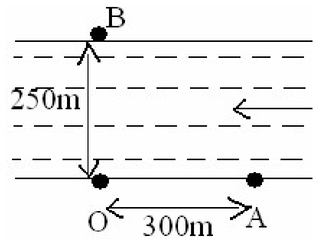
d) Each particle makes two rounds of the meeting point before they meet.

Key: A.C

Sol: $t = \frac{d_{rel}}{v_{rel}}$

Passage-I :

In the figure shown A and B are two cities situated on the two sides of a river of width 250m. A man who can swim at $10ms^{-1}$ in still water and can ran at $10ms^{-1}$ on; the sand beside the river wishes to go from A to B. Water flows in the river at $10ms^{-1}$ from right to left. OA = 300m and the B – side of the river is not walkable.



5 Shortest possible time required to go from A to B is

a) 55s b) 30s c) 25s d) 5s

6. Ratio of the distances covered by man on land and water to reach B in shortest time is

a) $5\sqrt{2}$ b) $\frac{1}{5\sqrt{2}}$ c) $\frac{6}{5}$ d) $\frac{5}{6}$

7. The angle between the direction of velocity of man and direction of river at the starting point A is

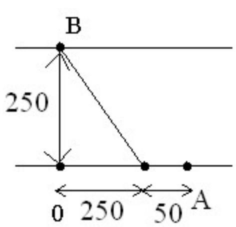
a) 0^0 b) 45^0 c) 90^0 d) 180^0

Key: 5. B 6. B 7. A

Sol:

5. $\frac{50}{10}+\frac{250}{10}=30s$

6. $\frac{50}{250\sqrt{2}}=\frac{1}{5\sqrt{2}}$

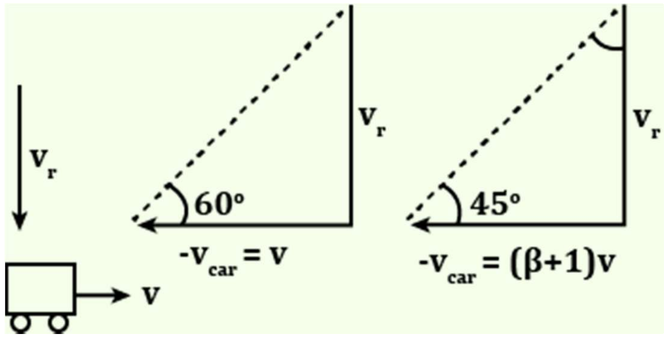


A) When two objects are moving in the same direction1.

1. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1+\beta)$v , this angle changes to 45°. The value of β is close to:

A) 0.41 B) 0.50 C) 0.37 D) 0.73

Sol:



When car is moving with speed v,

$\tan 60^0 = \frac{v_r}{v} ....(i)$

When car us moving with speed $(1+\beta)v$

$\tan 45^0 = \frac{v_r}{(\beta+1)v} ....(ii)$

Dividing (i) by (ii) we get,

$\sqrt{3} = (\beta + 1)$

$\Rightarrow \beta = \sqrt{3} - 1 = 0.732$

2. **Assertion:** The relative velocity between any two bodies moving in opposite direction is equal to sum of the magnitudes of velocities of two bodies.

Reason: Sometimes relative velocity between any two bodies is equal to difference in magnitudes of velocities of the two bodies

A. Both A and R are true and R is the correct explanation of A

B. Both A and R true and R is not the correct explanation of A

C. A is true but R is false

D. Both A and R are False

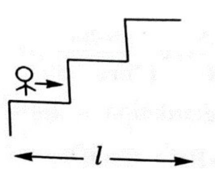
Key: B

3. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up the time t_2. The time taken by him to walk up on the moving escalator will be:

1. $\frac{t_1 t_2}{t_2 - t_1}$ 2. $t_2 - t_1$ 3. $\frac{t_1 t_2}{t_2 + t_1}$ 4. $\frac{t_1 + t_2}{t_2}$

Key: 3

Sol:



$V_{ME} = V_M - V_E \Rightarrow V_M = V_{ME} + V_E$

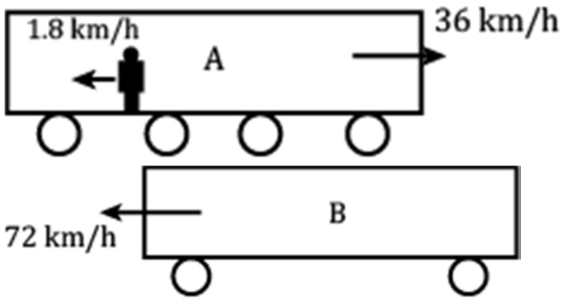
$\frac{l}{t} = \frac{l}{t_1} + \frac{l}{t_2} \Rightarrow t = \frac{t_1 t_2}{t_1 + t_2}$

B) When two objects are moving in the opposite direction

1. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/ hours and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hr. Speed (in ms^{-1}) of this person as observed from train B will be close to: (take the distance between the tracks as negligible)

1) $30.5ms^{-1}$ 2) $29.5ms^{-1}$ 3) $31.5ms^{-1}$ 4) $28.5ms^{-1}$

Sol: According to the question, train A and B are running on parallel tracks in the opposite direction.



$V_A = 36km\,h^{-1} = 10ms^{-1}$

$V_B = -72km\,h^{-1} = -20ms^{-1}$

$V_{man.A} = -1.8km\ h^{-1} = -0.5ms^{-1}$

$V_{man.B} = V_{man.A} + V_{A.B}$

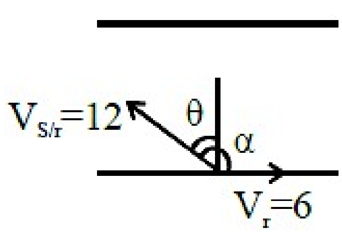
$\Rightarrow V_{man.B} = V_{man.A} + V_A - V_B$

$\Rightarrow V_{man.B} = -0.5 + 10 - (-20)$

$\Rightarrow V_{man.B} = -0.5 + 30 = 29.5ms^{-1}$

2. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is _________°. (Round off to the Nearest Integer) (find the angle in degree)

Sol:



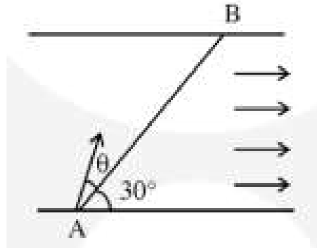
$12\sin\theta = v_r$

$\sin\theta = \frac{1}{2}$

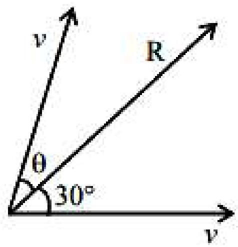
$\theta = 30^0$

$\therefore \alpha = 120^0$

3. A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be ____°, so that the swimmer reaches point B.



Sol:



Both velocity vectors are of same magnitude therefore resultant would pass exactly midway through them

$\theta = 30^0$