# KINEMATICS-1D

- Kinematics in which we deal with the motion i.e. change in position with respect to a either fixed system or a moving system (frame of reference) without taking into account of its cause or force.

- For analysing motion of a particle, there are some physical quantities as its parameters
  (i) Position or position vector - which is the location of the particle with respect to system of co-ordinate axes.
  (ii) Displacement - the change in position vector.
  (iii) Velocity - The time rate of change of displacement
  (iv) Acceleration - The time rate of change of velocity.

Motion (Translational)

Absolute Motion (Frame of reference is a fixed body or system)

Relative Motion (Frame of reference is in motion)

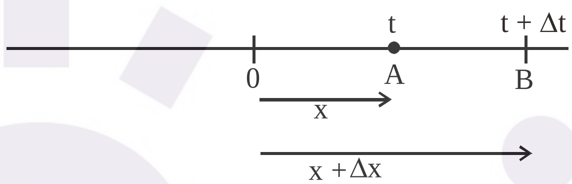


figure (i)

In figure (i) motion is along positive x-direction $\vec{AB}$ = displacement in time $\Delta t$ = $\Delta \vec{x}$

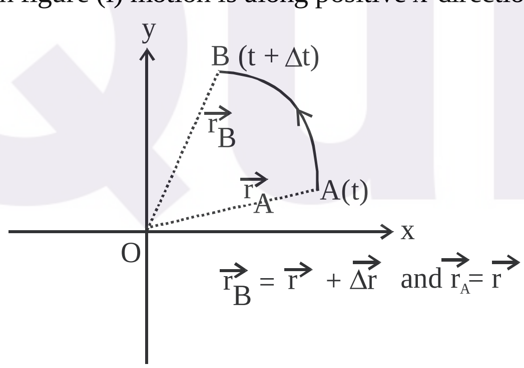


figure (ii)

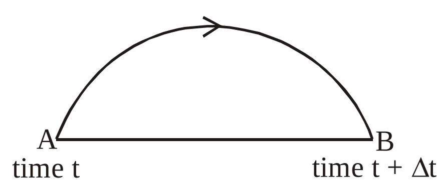
In fig (ii) displacement in time $\Delta t$ , $\vec{AB} = \vec{OB} - \vec{OA} = \Delta\vec{r}$ = change in position vector.

In any kind of motion, the magnitude of displacement is less than or equal to distance (actual path traversed).

- $AB = \Delta S$ = Distance covered in time $\Delta t$ .
  $\vec{AB}$ = Displacement is time $\Delta t = \Delta\vec{r}$ or path length

  $\frac{\Delta S}{\Delta t}$ = Average speed in time $\Delta t$ = $\frac{\text{Distance covered}}{\text{Time elapsed}}$

  $\frac{\Delta\vec{r}}{\Delta t}$ = Average velocity in time $\Delta t$ = $\frac{\text{Displacement}}{\text{Time elapsed}}$




If for different interval the particle has same average velocity then motion is said to be in uniform motion or otherwise motion is said to be in non-uniform motion and if particle is in non-uniform motion then velocity at a point is defined as $\lim\limits_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \vec{v}$ called instantaneous velocity, for one dimension $\vec{v} = \frac{d\vec{x}}{dt}$. The instantaneous velocity at any print is equal to average velocity in any interval for uniform motion i.e., $\vec{v} =$ constant.

As $\quad \Delta S \geq |\Delta \vec{r}|$

$$\frac{\Delta S}{\Delta t} \geq \left|\frac{\Delta \vec{r}}{\Delta t}\right| \text{ i.e. Av. speed } \geq |\text{ Av. velocity}|.$$

Equality sign holds when motion is in one-dimensional and unidirectional.

But for $\Delta t \to 0$, $\frac{\Delta s}{\Delta t} = \frac{|\Delta \vec{r}|}{\Delta t} \Rightarrow \frac{ds}{dt} = \frac{|dr|}{dt}$ i.e. magnitude of instantaneous velocity is the speed at a point.

- If force is in the direction of motion then motion is accelerated and if force is opposite to direction of motion then it is deaccelerated or retared. Rate of change of velocity w.r. to time is same for any interval and it is equal to instantaneous acceleration. Motion is said to be uniformly accelerated otherwise non-uniformly accelerated.

$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} =$ Average acceleration in time interval $\Delta t$.

$\lim\limits_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} =$ acceleration at a point = constant for uniformally accelerated

$\frac{d\vec{v}}{dt} = f(t)$ or $f(s)$ of f(v) then motion is said to be non-uniformly accelerated.

- Translational kinematical equation of motion for uniformly accelerated motion.

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}.t^2 = \frac{1}{2}(\vec{u} + \vec{v})t$$

$$v^2 = u^2 + 2\vec{a}.\vec{s}$$

$$\vec{v} = \vec{u} + \vec{a}.t$$

$$\vec{S}_n = \vec{u} + \frac{1}{2}\vec{a}(2n-1) \quad (S_n = \text{ Displacement is in last one second.})$$

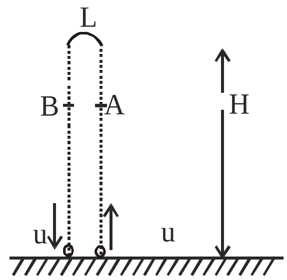
- Equation for non uniformly accelerated motion.

$$\frac{d\vec{x}}{dt} = \vec{v} \qquad \frac{d\vec{v}}{dt} = f(t) \text{ or } f(s) \text{ or } f(v) \text{ and } \frac{d\vec{v}}{d\vec{s}}.\vec{v} = f(s)$$

- Direction of velocity is in direction of change of displacement while direction of acceleration is in direction of change in velocity.

- Vertical motion under gravity - Motion of a particle under the effect of gravity - This is motion in one dimension with constant acceleration a = g = 9.8 $ms^{-2}$ = 32 ft $sec^2$.

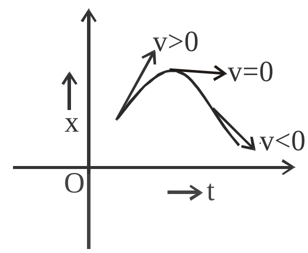
If a particle is projected veritcally up with velocity u then time of ascent = $\frac{u}{g}$ and during which height attained $H = \frac{u^2}{2g}$. Time of descent = $\frac{u}{g}$ $\qquad |\vec{v}_A|(\uparrow) = |\vec{v}_B|(\downarrow)$




$t_{AL} = t_{LB}$ where L is the point up to which particle ascends. where A and B are at same height from surface of earth.

- Effect of air resistance on motion of a particle in vertical motion under gravity - Air resistance is a force that acts on the body opposite to direction of motion and its value depends on the speed.

## KINEMATICAL GRAPH -



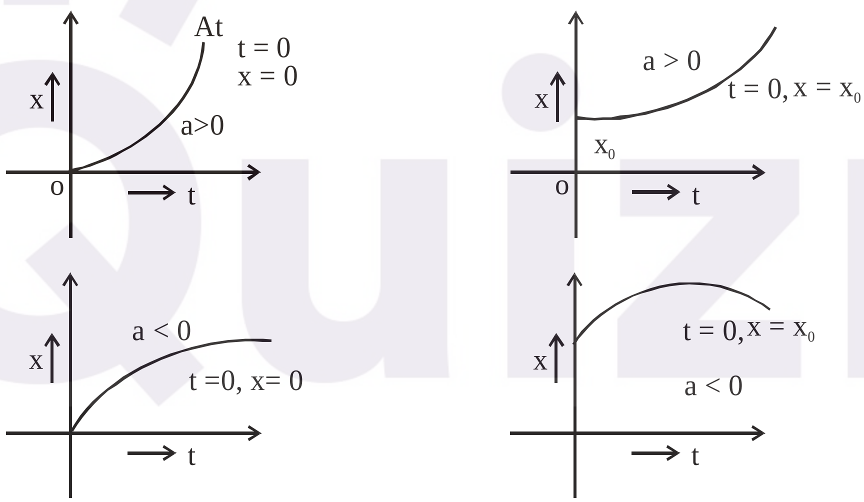

Displacement time graph

From displacement time graph we find

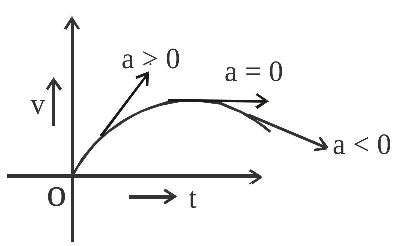
(i) slope at a point gives the instantaneous velocity

(ii) slope of the chord gives the average velocity in the given interval

x-t curve for constant acceleration.




## VELOCITY TIME CURVE -




From velocity time graph we have

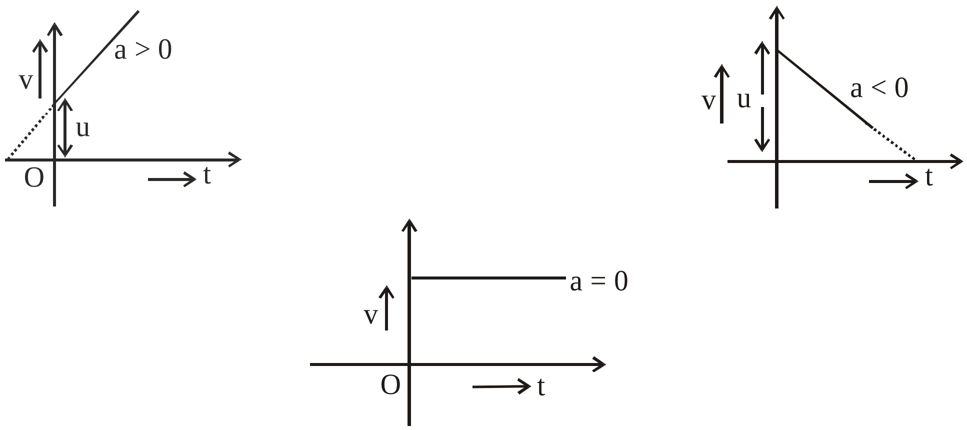
(i) slope of the tangent at a point gives the acceleration at a point.

(ii) slope of the chord gives the average acceleration in a given interval.

(iii) Area under the v-t curve gives displacement, taking area positive for upper of time axis and area negative for lower of time axis.

(iv) Area under the v-t curve gives distance when all the areas either above or below t-axis taken to be positive.

Velocity time curve for uniformly accelerated motions.




Acceleration time curve -

It is  of very physical significance in which area under the curve in the given interval is the change in velocity in that interval.

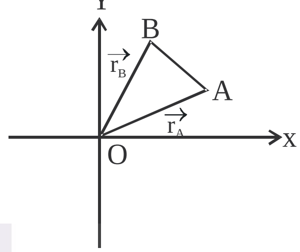
# KINEMATICS-2D

- When motion of a body/particle is analysed by a moving system, then motion is said to be a relative motion.

- Relative velocity of A w.r. to B is defined as the time rate of change of relative displacement of A w.r. to B, which is given by

$$\vec{V}_{AB} = \frac{d\vec{r}_{AB}}{dt} = \frac{d\overrightarrow{BA}}{dt} = \frac{d}{dt}(\overrightarrow{OA} - \overrightarrow{OB}) = \frac{d\vec{r}_A}{dt} - \frac{d\vec{r}_B}{dt}$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B \quad \text{or} \quad \vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$




$\vec{r}_A =$ position vector of A at time t

$\vec{r}_B =$ position vector of B at time t

Relative velocity is simply the vector difference of two velocities.

- For one dimension $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

(i) $\longrightarrow A$ ; $|\vec{V}_{AB}| = |V_A - V_B|$ when motions are along parallel lines

$\longrightarrow B$

(ii) $\longleftarrow B$ ; $|\vec{V}_{AB}| = V_A + V_B$ when motion are along antiparallel lines.

$\longrightarrow A$

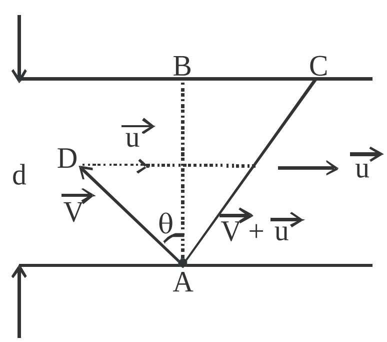
## SWIMMER'S PROBLEMS

When boat/swimmer heads in the river to cross from one bank to another. Then motion of boat/swimmer in the direction of resultant of velocity of flow in the river and velocity of boat/swimmer in still water.

$\vec{V}s,g = \vec{V}s,w + \vec{V}w,g$ ; $\vec{V}s,g$ = velocity of swimmer w.r to ground.

Let $\vec{V}s,w = \vec{V}$ = velocity of swimmer in still water

$\vec{V}w,g = \vec{u}$ velocity of water flow.



Swimmer heads along AD making angle $\theta$ with vertical in the direction of upstream so as while it crosses the river it has less drift along the direction of river flow.

- Time to cross the opposite bank = $\dfrac{d}{V\cos\theta}$

Minimum time to cross the river $=\dfrac{d}{v}$ for which $\theta = 0^0$ i.e. For minimum time to cross the river swimmer should head perpendicular to flow of stream.

- Time to reach just. opposite back (only for v > u)

$u = v\sin\theta$

i.e. $\theta = \sin^{-1}\dfrac{u}{v}$ and time to reach opposite bank $=\dfrac{d}{V\sqrt{1-\left(\dfrac{u}{v}\right)^2}} = \dfrac{d}{\sqrt{V^2-u^2}}$

- For v < u then swimmer heads to reach the opposite bank for minimum drift or through shortest path and hence

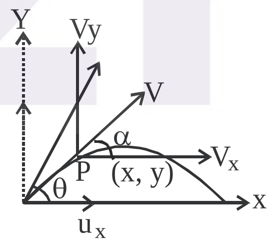
$\dfrac{dBC}{d\theta}=0$ where BC = $(u - V\sin\theta).\dfrac{d}{V\cos\theta}$

$\Rightarrow \sin\theta = \dfrac{V}{u}$ or $\theta = \sin^{-1}\left(\dfrac{V}{u}\right)$

Time to reach the opposite bank through shortest path = $\dfrac{d}{v\sqrt{1-\left(\dfrac{V}{u}\right)^2}} = \dfrac{du}{v\sqrt{u^2-v^2}}$

## PROJECTILE MOTION

An oblique projection of a body from surface of earth the following motion of the body is said to be projectile motion and body itself is called projectile $\theta$ is the angle of projection u is velocity of projection. After time t the projectile reaches at P with velocity V.




Then from equation of projectile

$\vec{a}_x = \dfrac{d^2\vec{x}}{dt^2} = 0$ and $\vec{a}_y = \dfrac{d^2\vec{y}}{dt^2} = g(-\hat{j})$

We have $v_x = u_x = u\cos\theta$ and $v_y = u_y - gt = u\sin\theta - gt$

Hence $v = \sqrt{u_x^2 + v_y^2} = \sqrt{u^2 - 2u\sin 2\theta gt + g^2t^2}$

and $\alpha = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\left(\dfrac{u\sin\theta - gt}{u\cos\theta}\right)$

Equation of trajectory or path of projectile is given by $x = u\cos\theta.t$ and $y = u\cos\theta.t - \dfrac{1}{2}gt^2$

Hence we have the equation by eliminating t.

$y = x\tan\theta - \dfrac{1}{2}g\dfrac{x^2}{u^2\cos^2\theta}$. Hence trajectory is a parabolic path.

Range is the horizontal distance from point of projection to the point in the same plane where projectile strikes which is given by

$R = u\cos\theta \times T;\ T = \text{time of flight}$

Since $T = \dfrac{2u\sin\theta}{g}$ (Sy = 0 = $u_y$ $T - \dfrac{1}{2}.gT^2$ $\Rightarrow u\sin\theta.T - \dfrac{1}{2}gT^2 = 0 \Rightarrow T = \dfrac{2u\sin\theta}{g}$

$R = \dfrac{u^2 \sin 2\theta}{g}$. If $\theta$ is replaced by $90^0 - \theta$. R does not change.

Hence for given initial velocity R remains the same for two possible values of angle of projections if one is $\theta$ then other is $\pi/2 - \theta$.

- Equation of trajectory interms of range $y = x\tan\theta(1 - x/R)$
- Time of ascent = time of descent = $\dfrac{u\sin\theta}{g} = \dfrac{u_y}{g}$
- Maximum height – attained by the projectile from plane from where projectile is projected.

$H = \dfrac{u^2 \sin^2\theta}{g} = \dfrac{u_y^2}{2}$ (At maximum height $v_y^2 = 0 = u_y^2 - 2gH \Rightarrow (u\sin\theta)^2 - 2gH = 0$)

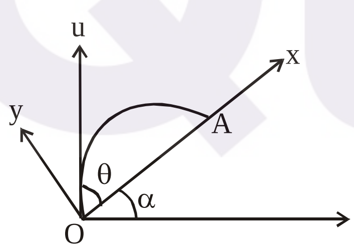
- Along motion of projectile path horizontal velocity remains the same and at hightest point it directs horizontally as no vertical velocity at highest point.
- Every elementary section of projectile path is considered as on curve and the necessary centripetal force required to keep a body on the curve path is pointed along radial direction towards centre of elementary curve path, which is provided by component of weight.
- Time after which the velocity of projectile becomes perpendicular to initial velocity.

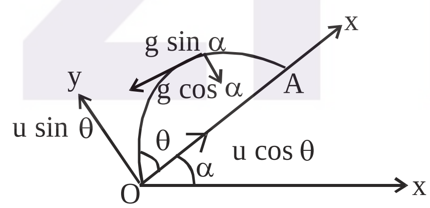
$\vec{u}.\vec{v} = 0 \Rightarrow (u\cos\theta\hat{i} + u\sin\theta\hat{j}).[u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}] = 0$

$\Rightarrow u^2 - u\sin\theta\, gt = 0$ or $t = \dfrac{u}{g\sin\theta}$

## Projectile Motion on the inclined plane

(i) Projectile Motion up the plane




Taking x-axis along inclined plane and y-axis perepndicular to it at point O.

$\vec{a}_x =$ acceleration along x-axis $= g\sin\alpha(-\hat{i})$

$\vec{a}_y = g\cos\alpha(-\hat{j})$

The time of flight is the time taken for projectile travel from O to A

$\therefore$ From $S_y = u_y t + \dfrac{1}{2} a y t^2$ for O to A, $S_y$ = O

$\therefore \Rightarrow \quad t = \dfrac{2u\sin\theta}{g\cos\alpha}$

As at t = 0, Projectile is at O.

Time of flight $= \dfrac{2u\sin\theta}{g\cos\alpha}$ ; Range = OA = R is given

by $\quad S_x = u_x.t + \dfrac{1}{2}a_x t^2$

$$S_x = R = u\cos\theta.\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha.\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

$$= \frac{2u^2\sin\theta}{g\cos^2\alpha}\left[\cos\theta.\cos\alpha - \sin\theta.\sin\alpha\right] \qquad = \frac{2u^2\sin\theta}{g\cos^2\alpha}\left[\cos(\theta+\alpha)\right]$$

$$R = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta+\alpha) - \sin\alpha\right]$$

For the maximum-range $\sin(2\theta+\alpha) = 1$; $\qquad \theta = 45^0 - \alpha/2$

$R_{max}$ for projection inclined up to plane is $R_{max} = \dfrac{u^2}{g(1+\sin\alpha)}$

(ii) Projectile Motion down the inclined plane

The equation of projectile


$\vec{a}_x = g\sin\alpha\,\hat{i}$ & $\vec{a}_y = g\cos\alpha(-\hat{j})$; Range down the plane $= \dfrac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$

Time of flight = $\dfrac{2u\sin\theta}{g\cos\alpha}$

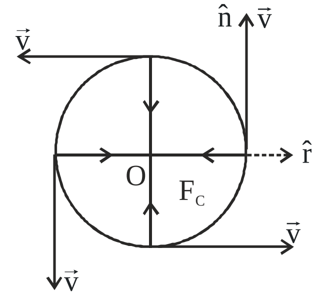
$R_{max}$ down the plane = $\dfrac{u^2(1+\sin\alpha)}{g\cos^2\alpha} = \dfrac{u^2}{g(1-\sin\alpha)}$

It occurs when direction of projection bisects the angle between the vertical and downward slope of the plane.

## CIRCULAR MOTION

Motion along a circular path : when a body is moving along a circular path with constant speed called uniform circular motion, when speed is not constant, motion is said to be non-uniform circular motion.

$$\vec{F}_c = \frac{mv^2}{r}(-\hat{r}) \ ; \ \hat{r} = \text{unit vector along radially outward}$$

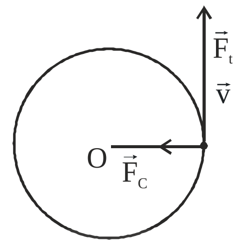



A force required to keep of body on circular path always acts in radially inward direction called centripetal force whose magnitude is $\frac{mv^2}{r}$.

For non-uniform circular motion

$$\vec{F} = F_c(-\hat{r}) + F_t(\hat{n})$$

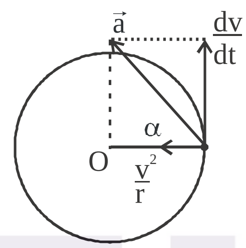
$\vec{n}$ = unit vector along direction of motion or velocity




$$F_c = \frac{mv^2}{r} \text{ and } F_t = m.\frac{dv}{dt}$$

$$\vec{a} = \text{Resultant or net acceleration} = a_c(-\hat{r}) + a_t(\hat{n})$$

$$= \frac{v^2}{r}(-\hat{r}) + \frac{dv}{dt}(\hat{n})$$




$$|\vec{a}| = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

$$\alpha = \tan^{-1}\left(\frac{dv/dt}{v^2/r}\right)$$

Angle of banking: Angle by which an outer edge of circular track is raised to provide the necessary centripetal force through the horizontal component of normal reaction.

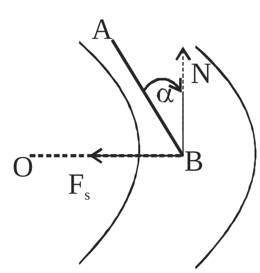
angle of banking $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Motion of vehicle on a horizontal circular track:

$\frac{mv^2}{r}$ is being provided by force of static friction i.e., $F_s = u_s N$ and N=mg $\Rightarrow v^2 = u_s rg$ or $v = \sqrt{u_s rg}$

Condition for no skidding on circular track

$$F_s \geq \frac{mv^2}{r} \text{ or } u_s mg \geq \frac{mv^2}{r} \text{ or } v \leq \sqrt{u_s rg}$$



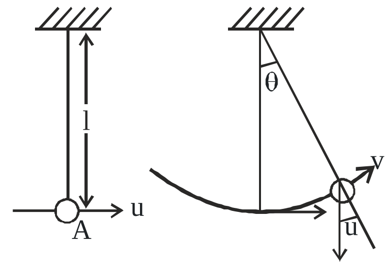

Angle of bending of a cyclist on a rough horizontal circular track to move on is given

$$\tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$F_s$ provides necessary centripetal force $\frac{mv^2}{r}$ and N=mg. For safe turn there is a rotational equilibrium hence no torque about A (Centre of gravity of cycle and cyclist).

Vertical Cricular Motion

u is the velocity imparted at the bottom of the vertical circle. At P, equation of motion




$$T - mg\cos\theta = \frac{mv^2}{r} \qquad ...(i)$$

and from mechanical energy conservation principle,

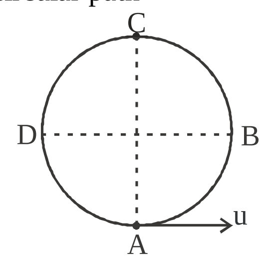
$$\frac{1}{2}mu^2 = mgl(1-\cos\theta) \Rightarrow v^2 = u^2 - 2gl(1-\cos\theta) \qquad ...(ii)$$

from (i) and (ii) $T = mg\cos\theta + \frac{m}{l}\left[u^2 - 2gl(1-\cos\theta)\right]$

$$= \frac{m}{l}\left[u^2 - 2gl + 3gl\cos\theta\right] \qquad ...(iii)$$

from (ii) and (iii) we have velocity and tension at any point on the verticle circular path

For just to complete the verticle circle



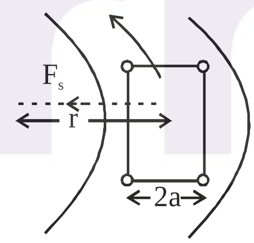

$u = \sqrt{5gl} = \text{velocity}$

At A, $v_A = \sqrt{5gl}$ ; $T_A = 6mg = \text{tension in string when block is at A}$

At B, $V_B = \sqrt{3gl}$ ; $T_B = 3\ mg$

At C, $V_C = \sqrt{gl}$ ; $T_C = mg$

For no toppling of the automobile on horizontal circular track




$F_s.h \le mga$ ;

$\frac{mv^2}{r}.h \le mga$

h is the height of centre of gravity of automobile from surface of road.

$v \le \sqrt{\frac{arg}{h}}$

While toppding wheels nearer to centre of track loose the contact.

# LAWS OF MOTION & FRICTION

## Dynamics :

Motion or state of the body under the given force

## Newton's Law of Motion

First Law: Every body tries to continue in a state of rest or in a uniform motion unless the force is impressed upon it. If the body or system has no net force i.e., in equilibrium, then body is either in a sate of rest or in a uniform motion (Motion with constant speed along straightline).

## Inferences of First Law

(a) Every body has tendency not to change the state of rest or uniform motion, measure of this tendency called intertia so first law often called law of inertia. Inertia measures the tendency not to change the state either from state of rest or in uniform motion.

(b) Definition of force also comes from frist law and it is defined as the cause that changes the state either from state of rest or uniform motion.

(c) Frame of references are of two kinds

  (i) Inertial frame of reference : The frame of reference which is either at rest or moving in uniform motion. Newton's laws are valid only in inertial frame of reference. So it is also called Newtonian frame of reference.

  (ii) Non-inertial frame : The frame of reference which is in accelerated motion. Newton's laws are not valid in non-intertial frame.

(d) There are only two natural states of body (i) At rest and (ii) in uniform motion.

## Newton's Second Law of Motion:

The time rate of change of linear momentum of a body is directly proportional to force experienced and direction of force is in the direction of change in linear momentum.

Linear Momentum of a Particle is the quantity of motion and mathematically defined as the product of mass and velocity, $\vec{p} = m\vec{v}$.

From Newton's 2nd Law

$$\frac{d\vec{p}}{dt} \propto \vec{F} \Rightarrow \frac{d\vec{p}}{dt} = \kappa\vec{F} \quad \text{from definition of one newton, } \kappa = 1$$

1 newton is the force experienced by a body when the rate of change of momentum is unity.

$$\therefore \frac{d\vec{p}}{dt} = \vec{F} \quad \text{...(i) and} \quad \frac{d(m\vec{v})}{dt} = \vec{F} \Rightarrow \vec{F} = m\vec{a} \quad \text{...(ii)}$$

It $\vec{a} = 0$ then $\vec{F} = 0$, i.e., no force means unaccelerated motion.

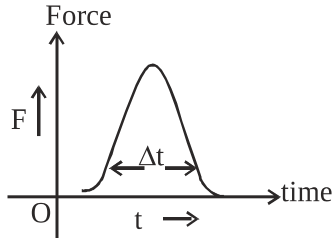
$$d\vec{p} = \vec{F} \cdot dt \Rightarrow \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_0^{\Delta t} \vec{F} \cdot dt \Rightarrow \vec{p}_f - \vec{p}_i = \vec{F}.\Delta t$$

$\vec{F}$ is the force that acts for relatively small time with very large value which is called impulsive force.

Where $\vec{F}.\Delta t$ is the impulse.

Change in linear momentum is equal to the impulse imparted.

Impulse is sudden jerk which causes to change the linear momentum.

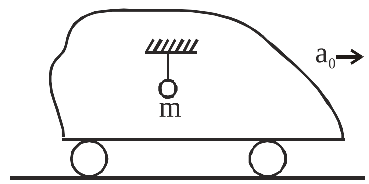



Area under the force vs time curve for impulse imparted gives the change in linear momentum.

Pseudoforce : A force on a body as a consequence of non inertial frame of reference it is equal to the product of mass of the body and acceleration of the frame with respect to an a accelerated car and it is directed oppsite to direction of acceleration of frame of reference.

Equation of motion for a simple pendulum w.r.t. to ground $\vec{T}+\vec{w}=m\vec{a}_0$ ...(i)

Equation of Motion for simple pendulum with respect to a car.




$\vec{T}+\vec{w}=0$ ...(ii)

but it is incorrect

Since frame of reference is accelerated

To write correct equation for (ii) we use (i) $\vec{T}+\vec{w}=m\vec{a}_0$

Now equation of mass m w.r.t to car be written correctly as $\vec{T}+\vec{w}-m\vec{a}_0=0$ ....(iii)

This is the correct equation of motion with respect to accelerated frame. Where $-m\vec{a}_0$ called pseudo force

$\vec{a}_0$ is the acceleration of frame.

## Kind of forces:

(i) Normal reaction: Force on a body due to one another perpendicular to the line of contact.

(ii) Weight : Force by which a body is being attracted towards earth.

(iii) Tension :An elastic force that comes within string to oppose the tendency of deformation, it acts away from system along the string. For massless string tension at every point is same in magnitude.

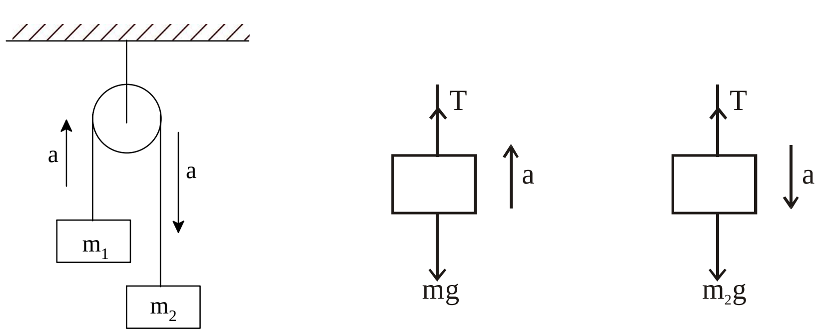
Newton's Third Law of Motion: For a system there are equal and opposite forces on a pair of body due to one another. The one force called action while another called reaction. Total internal forces on a system is always zero.

## Finding acceleration through free body diagram

## Motion of connected bodies

Illustrations 1 :

In Atwood machine find the acceleration of $m_1$ and $m_2$ where pulley and string are massless and pulley is smooth. a = acceleration of $m_1$ or $m_2$.



Solution

Equation of motion for $m_1$

$T - m_1 = m_1 a$ ...(1)

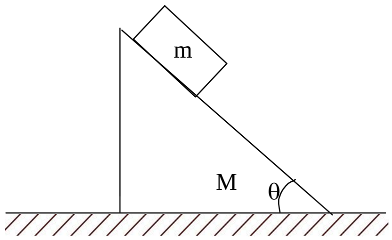
Equation of mass $m_2$

$m_2 g - T = m_2 a$ ...(2)

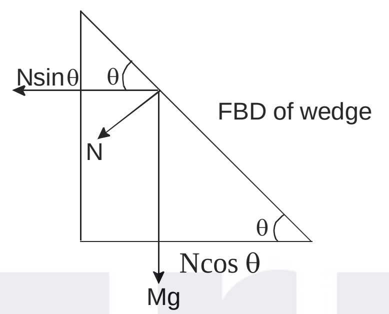
From (1) an (2) we have $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2}$

Illustrations : 2

All the surfaces are smooth for a system of wedge and block. Acceleration of wedge after system is released. Then acceleration of wedge is given by as follows,




Equation of motion for wedge M




$N \sin\theta = Ma$ ...(i)

Let $a_r$ be the acceleration of block w.r.t wedge

Equation for mass m

$mg\sin\theta + mg\cos\theta = m.a_r$ ...(ii)

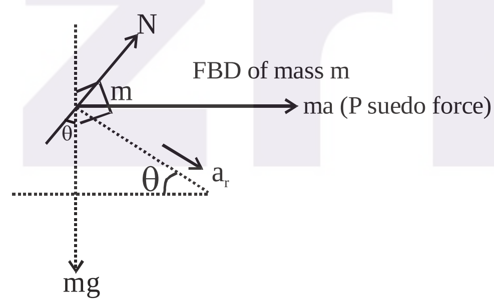
and $N + ma\sin\theta = mg\cos\theta$ ...(iii)

From equation (1) and (iii)

$(Mg\cos\theta - ma\sin\theta)\cdot\sin\theta = Ma$

$\Rightarrow \quad a(M + m\sin^2\theta) = Mg\cos\theta\cdot\sin\theta$

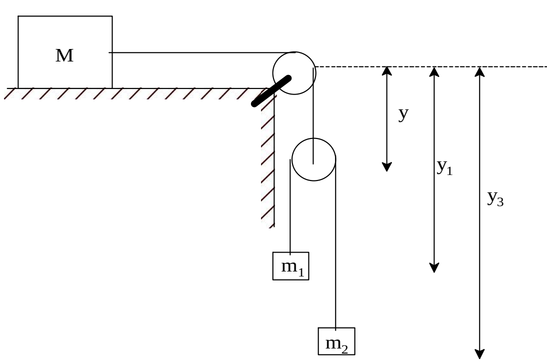
$$a = \frac{Mg\cos\theta\cdot\sin\theta}{M + m\sin^2\theta}$$




## EQUATION OF CONSTRAINTS

Equation involving the acceleration of connecting bodies is the equation of constraints

Illustrations 3 :



L = length of spring which is on the movable pulley.

$$(y_2 - y) + (y_1 - y) + \pi R = l \qquad \ldots(1)$$

R = radius of pulley

Differentiating equation (1) w.r.t time twice

$$\frac{d^2y_1}{dt^2} + \frac{d^2y_2}{dt^2} = 2\frac{d^2y_1}{dt^2}$$

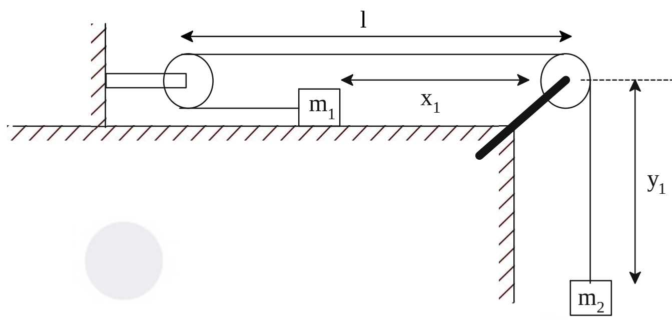
$$\Rightarrow \qquad a_1 + a_2 = 2 \cdot a \Rightarrow a = \frac{a_1 + a_2}{2}$$

a = acceleration of block of mass M

$a_1$ = acceleration of block of mass $m_1$

$a_2$ = acceleration of block of mass $m_2$

Illustrations 4 :




$a_1$ = acceleration of $m_1$

$a_2$ = acceleration of $m_1$

L = length of spring

$= l - x_1 + y_1 + l$

$= 2l - x_1 + y_1$

Differentiating w.r.t. time time-twice

$$\frac{d^2x_1}{dt^2} = \frac{d^2y_1}{dt^2} \Rightarrow a_1 = a_2$$

$a_1$ = acceleration of $m_1$

$a_2$ = acceleration of $m_1$

Note :

Weight shown in the spring is the tension in the spring and weight shown in the weighing machine is the normal reaction acts on the weighing machine.

## FRICTION

When two bodies are either having a tendency of relative motion or a relative motion, a force comes into play along the line of motion to oppose the tendency of relative motion or relative motion. Such phenomenon is called friction and force is called frictional force.

Static friction : Force of friction to oppose the tendency of relative motion called static frictional force which is directly proportional to normal reaction and independent of the area of contact.

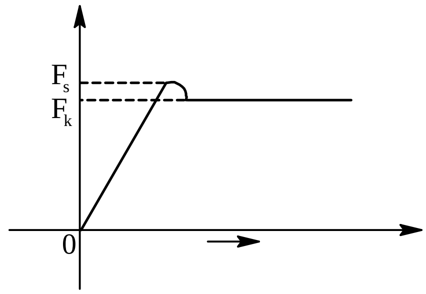
i.e., $F_s \propto N$ or $F_s = \mu_s N$

where $\mu_s$ is the co-efficient of static friction.

### Kinetic friction:

Force of kinetic friction is force of friction to oppose the relative motion and which is given by $F_K$ and $F_K$ is directly proportional to normal reaction and also independent of the area of contact.

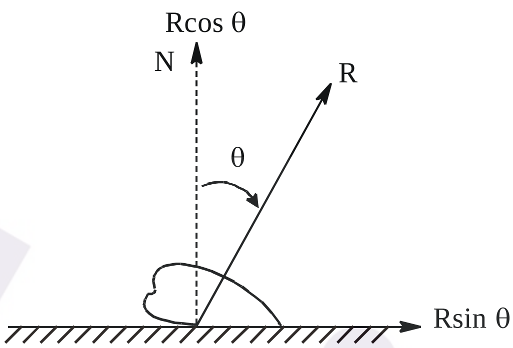
i.e., $F_K \propto N$ or $F_K = \mu_K . N$




$F_s$ (static friction) is a self-adjusting force of friction, maximum value of it is the limiting friction.

$F_s \le \mu_s N$. Direction of force of friction is in opposite to direction of either tendency of relative motion or relative motion.

## Angle of friction




When two rough bodies are is contact then there is a force of equal magnitude but opposite in direction are another called contact force (R) Horizontal component of R provides the force of friction while its normal component is the normal reaction ; R is contact force on a body due to surface.

$$F_s = R\sin\theta \quad \text{and} \quad N = R\cos\theta$$

$$\therefore \quad \frac{F_s}{N} = \tan\theta \quad \Rightarrow \quad \tan\theta = \mu_s \quad \text{or} \quad \theta = \tan^{-1}(\mu_s) < 45^o \text{ As } \mu_s < 1$$

$\theta$ is the angle of friction

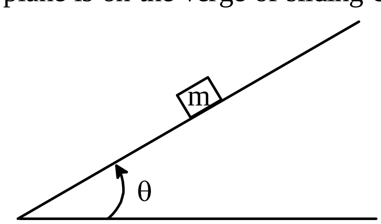
## Angle of Repose :

The maximum angle of inclination for which the block on rough inclined plane is on the verge of sliding called angle of repose.

Here $\quad mg\sin\theta = \mu_s N$

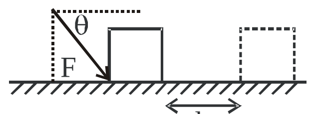
$= \mu_s\ mg\cos\theta$

$\Rightarrow \tan\theta = \mu_s$ or $\theta = \tan^{-1}(\mu_s)$; $\theta$ is the angle of repose.



## WORK POWER AND ENERGY

Work : Work is done by the force if point of application of force is displaced.




Work done by the force is the dot product of force and displacement or work done is the product of mganitude of displacement and component of force along displacement.

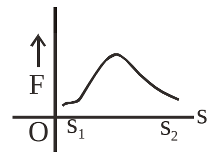
$$dw = \vec{F}\cdot\vec{ds} = F ds \cos\theta = (F\cos\theta) ds$$

$$dw > 0 \text{ if } 0 \le \theta < 90^o$$

$$dw < 0 \text{ if } 90^o < \theta \le 180^o$$

$$dw = 0 \text{ if } \theta = 90^o$$

Work done by variable force for displacement $S_1$ to $S_2$ is given by




$$w = \int_{S_1}^{S_2} F.ds$$ ; F is force along displacement

Work done by the variable force is the area below the curve drawn between component of force along displacement and magnitude of displacement.

The S.I. unit of work is joule others are erg, ev, kwh.

a. 1 joule = $10^7$ erg
b. 1 erg = $10^{-7}$ joule
c. 1 ev = $1.6 \times 10^{-19}$J
d. 1 kwh = $3.6\times10^6$ joule
e. 1 joule = $6.25\times10^{12}$ Mev

Power : The rate of doing work is power

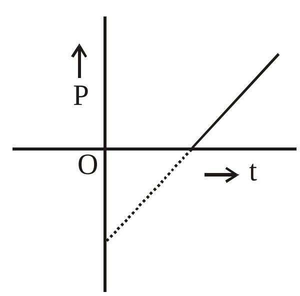
$$P = \frac{\vec{F}.\vec{dx}}{dt} = \frac{dw}{dt} = \frac{dk}{dt}$$

$$P = \vec{F}.\vec{v} = \text{instantaneous power delivered}$$

Unit of power is joule $sec^{-1}$ or watt.

1 hp = 746 watt.

Instantaneous power delivered in case of projectile is give by




$$P = -mg(\hat{j}).[u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}]$$

$$= -mg[u\sin\theta - gt]$$

$$= mg^2t + (-mgu\sin\theta)$$

Power delivered at the maxium height of projectile is zero.

Energy: The ability of doing work is represented by a quantity of motion, called energy.

Mechanical Energy

(i) Kinetic energy (ii) Potential energy

Kinetic energy : Kinetic energy of a body is the energy by virtue of its motion and it is equal to $\frac{1}{2}mv^2$; m = mass of body; v = speed of the body.

Potential energy : When work is done on a system and the system preserves this work in such a way that it can be subsequently recovered back in form of some type of energy, the system is capable of possessing potential energy. This energy is possessed by virtue of position or configuration of body or system.

Conservative and non-conservative forces: Those forces under the action of which, work done depends upon initial and final positions only and not on path followed , are known as conservative forces.

e.g. gravitational force, spring force, electrostatic force, magnetostatic force etc.

Those forces under the action of which, work done depends upon path followed are known as non-conservative forces. e.g. frictional force, viscous force, drag force etc.

Work done by conservative forces on round trip is always zero, while the work done by non-conservative force in round trip is non-zero.

Conservative forces and Potential energy: When, conservative force does positive work, the potential energy of the system decreases. Work done by conservative force is

$$W(x) = -\Delta U$$

or $$F(x).\Delta x = -\Delta U$$

or $$F(x) = -\frac{\Delta U}{\Delta x}$$

or, $$F(x) = -\frac{du}{dx}$$

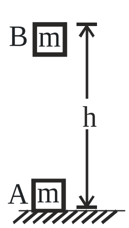
Integrating both sides, for a displacement, x = a to x = b,

We have : $$U_b - U_a = -\int_a^b F(x)dx$$

## TYPES OF POTENTIAL ENERGY

### (i) Gravitational potential energy

If we lift a block through some height (h) from A to B, work is done against the gravity.



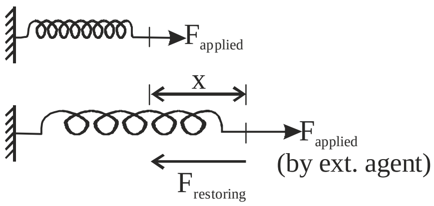

This work done is stored in the form of gravitational potential energy by external agent of the block-earth system. We can write, work done by external agent in raising the block = (mg)h.

(a) If the centre of a body of mass m is raised by a height h, increase in GPE = + mgh

(b) If the centre of a body of mass m is lowered by a distance h, decrease in GPE = mgh

(ii) Elastic potential energy

When a spring is elongated (or compressed), work is done by external agent against the restoring force of the spring. This work is stored in the spring as elastic potential energy. Work done in stretching or compressing spring by a distance x is given by $=\frac{1}{2}kx^2$ and therefore elastic potential energy stored in a spring $=\frac{1}{2}kx^2$.




Work Energy Theorem

Work done by all the forces (conservative, and non-conservative, external and internal) acting on a particle or an System is equal to the change in kinetic energy.

$$W_{net/all} = \Delta K = K_f - K_i$$

If the net work done is positive, K.E. increases and if the net work done is negative, K.E. decreases.

Law of conservation of mechanical energy under the only conservative force on the system total mechanical energy is conserved i.e. $U_i + K_i = U_f + K_f$ if forces are only conservative.

As $W_{all} = \Delta K$ (work energy theorem)

$W_{ext} + W_c + W_{nc} = \Delta K$ For $F_{ext} = 0$ and $F_{nc} = 0$

We have $W_c = K_f - K_i$

$-(U_f - U_i) = K_f - K_i$

$\Rightarrow U_i + K_i = U_f + K_f$ Mechanical Energy conserved

$W_{ext} + W_{nc} = (K_f + U_f) - (K_i + U_i)$ = changing mechanical energy

Law of conservation of linear momentum

For a system if there is no external force then linear momentum remains conserve.

Even if there is an external force but linear momentum is conserved along the line perpendicular to the total external force.

# Centre of Mass, Collision

## CENTRE OF MASS

- Centre of mass is a hypothetical point where all the masses are assumed to be concentrated under the given force.
- Position of centre of mass for n particles system is given

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + ... + m_n\vec{r}_n}{m_1 + ... + m_n} = \frac{\text{Total linear moment of system}}{\text{Total mass}}$$

$$x_{CM} = \frac{m_1x_1 + ... + m_nx_n}{m_1 + ... + m_n} = \frac{\text{Total linear moment along x} - \text{direction}}{\text{Total mass}}$$

$$y_{CM} = \frac{m_1y_1 + ... + m_ny_n}{m_1 + ... + m_n} = \frac{\text{Total linear moment along y} - \text{axis}}{\text{Total mass}}$$

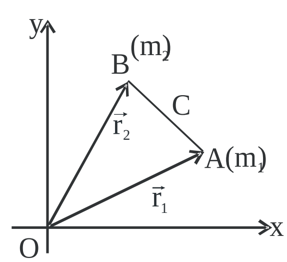
Position of Centre of mass for a rigid body is given by

$$\vec{r}_{CM} = \frac{\int \vec{r}dm}{\int dm} = \frac{\text{Total linear moment of the body}}{\text{Total mass}}$$

$$x_{CM} = \frac{\int xdm}{\int dm} = \frac{\text{Total linear moment along x} - \text{axis}}{\text{Total mass}}$$

$$y_{CM} = \frac{\int ydm}{\int dm} = \frac{\text{Total linear moment along y} - \text{axis}}{\text{Total mass}}$$

- Centre of mass (Mass Centre) is a point which always exist and it is unique. It might lie either inside or outside the body. If all the external forces are applied at the centre of mass, then there is only translational motion.
- Centre of mass of two particles system lie between the line joining of two masses.




$$\overrightarrow{OC} = \overrightarrow{r_{CM}} = \frac{m\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \vec{r}_2 - \frac{m\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

and $$\overrightarrow{CA} = -\frac{m_2\left(\vec{r}_2 - \vec{r}_1\right)}{m_1 + m_2}$$

$$\frac{\overrightarrow{CA}}{\overrightarrow{CB}} = \frac{m_2}{m_1} \Rightarrow m_1\overrightarrow{CA} + m_2\overrightarrow{CB} = 0 \text{ and } \left|\frac{\overrightarrow{CA}}{\overrightarrow{CB}}\right| = \frac{m_2}{m_1}$$

and if $m_1 > m_{2,}$ $\left|\overrightarrow{CA}\right| < \left|\overrightarrow{CB}\right|$ , i.e. centre of mass lies closer to more mass.

Center of mass is the point about which total linear moment is zero

## CENTRE OF MASS AND CENTRE OF GRAVITY

Centre of mass and centre of gravity are the same point if gravitational field is uniform, centre of gravity is a point in a body where the gravitational force acts on it, where body is balanced in all orientation.

Motion of Centre of mass : Consider n particles system of masses $m_1$, ....., $m_n$ and position vectors $\vec{r}_1, ......, \vec{r}_n$ respectively.

$$M.\vec{r}_{CM} = m_1\vec{r}_1 + ... + m_n\vec{r}_n \quad ...(i) \qquad M = m_1 + .... + m_n = \text{Total mass of the system}$$

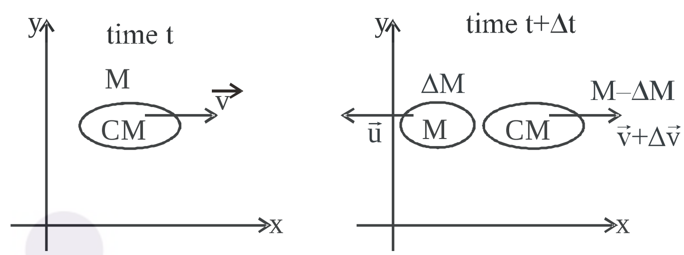
$$M.\vec{v}_{CM} = m_1\vec{v}_1 + ... + m_n\vec{v}_n \quad ...(ii)$$

$$M.\vec{a}_{CM} = m_1\vec{a}_1 + ... + m_n\vec{a}_n = \vec{F}_{Ext}^{(Total)} = M\frac{d\vec{v}_{CM}}{dt} \quad ...(iii)$$

For any system if there is no external force then $\vec{a}_{CM} = 0$ i.e. $\vec{v}_{CM} =$ constant.Hence for a system if system is at rest initially under no external force it means position of centre of mass remains the same during whole observation.

## SYSTEM OF VARIABLE MASS SYSTEM

$$\vec{F}_{Ext} = \lim_{\Delta t \to 0}\frac{\Delta\vec{P}}{\Delta t} = \lim_{\Delta t \to 0}\frac{\vec{P}_f - \vec{P}_i}{\Delta t}$$




$$\vec{F}_{Ext} = \frac{\left[(M - \Delta M)\left(\vec{v} + \Delta\vec{v}\right) + \Delta M\vec{u}\right] - M\vec{v}}{\Delta t}$$

$$= M.\frac{\Delta\vec{v}}{\Delta t} + \left[\vec{u} - \left(\vec{v} + \Delta\vec{v}\right)\right].\frac{\Delta M}{\Delta t}$$

on $\Delta t \to 0$

$$\vec{F}_{Ext} = M.\frac{d\vec{v}}{dt} + (\vec{u} - \vec{v}).\left(-\frac{dM}{dt}\right); \qquad \text{(As mass is decreasing with time).}$$

$$\Rightarrow M.\frac{d\vec{v}}{dt} = \vec{F}_{Ext} + \vec{u}_r\frac{dM}{dt}; \quad F_{thrust} = \vec{u}_r . \frac{d\mu}{dt}$$

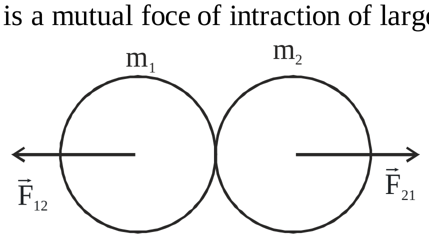
$$M.\frac{d\vec{v}}{dt} = \vec{F}_{Ext} + \vec{F}_{thrust}$$

$\vec{F}_{Ext}$ taken to be weight in case of rocket motion that can be neglecgted in comparision of thrust imparted to he rocket.

$\vec{F}_{thrust} = u_r\frac{dM}{dt} =$ This is momentum transfered by the ejected gas to the system per second.

## COLLISION

A process in which there is a mutual foce of intraction of large magnitude for relatively small time.




$$\Delta\vec{P}_1 = \vec{F}_{12}\Delta t \text{ and } \Delta\vec{P}_2 = \vec{F}_{21}\Delta t$$

So, $\Delta\vec{P}_1 + \Delta\vec{P}_2 = \vec{F}_{12}\Delta t + \vec{F}_{21}\Delta t = 0 =$ Total impulse imparted to the system of colliding bodies and it implies total linear momentum remains conserved.If there is no external force acting on the system, the linear momentum of system is not changed by the collision.

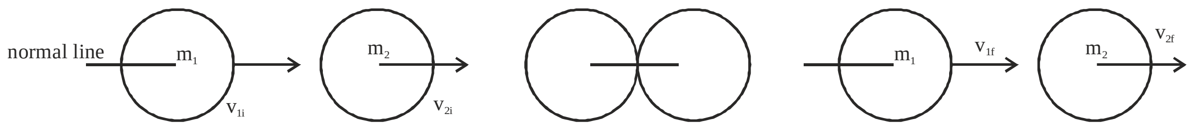
### Elastic and inelastic collision

After collision colliding bodies come to their original shape and size and so no fraction of energy stored as potential energy hence kinetic energy remains conserved before and after collision.

### Head on / oblique collision:

If direction of line of motion is along the line along which the impulse imparted, then collision is head-on otherwise oblique.

### Head on elastic Collision




Before Collision    At Collision    After Collision

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \quad ; \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

Case I: If $m_1 = m_2$, then $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$

After head-on elastic collision they simply exchange their velocities.

Case II: If $m_1 >> m_2$, $v_{2i} = 0$

then $v_{1f} = v_{1i}$ and $v_{2f} = 2v_{1i}$

Case III: If $m_1 << m_2$, $v_{2i} = 0$

then $v_{1f} = -v_{1i}$ and $v_{2f} = 0$

### Newtons Experimental law of restitution

$$-e = \frac{\vec{v}_{2f} - \vec{v}_{if}}{\vec{v}_{2i} - \vec{v}_{1i}} = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

After perfectly inelastic collision colliding bodies stick together or move together after collision.

$0 \le e \le 1$; e is the coefficient of restitution.

e = 0 for perfectly inelastic

e = 1 for perfectly elastic

### For inelastic collision

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad \text{...(i)}$$

$$v_{2f} - v_{1f} = -e\left(v_{2i} - v_{1i}\right) \qquad \text{...(ii)}$$

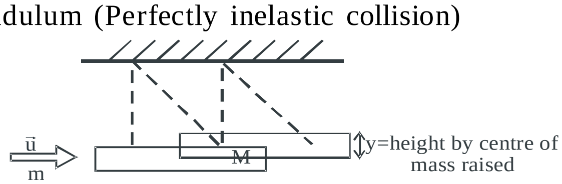
Solving, we have,

$$v_{1f} = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 + em_2}{m_1 + m_2}\right)v_{2i}\,; \qquad v_{2f} = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)v_{2i} + \left(\frac{m_1 + em_1}{m_1 + m_2}\right)v_{1i}$$

If $m_1 = m_2$ and $v_{2i} = 0$

then $v_{1f} = \left(\frac{1-e}{2}\right)v_{1i}$ and $v_{2f} = \left(\frac{1+e}{2}\right)v_{1i}$

### Ballistic Pendulum (Perfectly inelastic collision)



u is the muzzle speed of bullet.

$\frac{1}{2}mu^2 = \text{K.E. of bullet before collision}$

$(M+m)v = mu \Rightarrow \text{vel. of plank and bullet after collision} = \frac{mu}{M+m}$

and $\frac{1}{2}(M+m)v^2 = (M+m)g.y$ (Nelecting friction during the time bulled stops in plank)

or $\frac{1}{2}(M+m)\left(\frac{mu}{M+m}\right)^2 = (M+m)g.y$

$$\therefore \frac{m^2u^2}{2(M+m)} = (M+m)gy; \ u = \frac{M+m}{m}\sqrt{2gy}$$

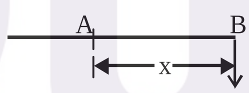
Fractional loss in K.E. in perfectly inelastic collision

$$\frac{\Delta K}{K} = \frac{\frac{1}{2}mu^2 - \frac{1}{2}(M+m)v^2}{\frac{1}{2}mu^2}$$

$$= \frac{\frac{1}{2}mu^2\left[1 - \frac{m}{M+m}\right]}{\frac{1}{2}mu^2} = \frac{M}{m+M}$$

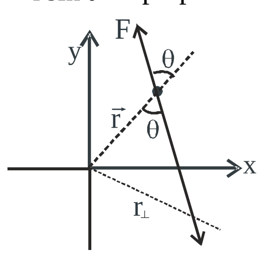
## MOMENT OF THE FORCE/ TORQUE

Moment of the force about a point is the measure of rotation which the product of the applied force and the normal (i.e. perpendicular) distance of line of application of this force from this point. It is also called torque.




Moment of force F about point O is given by $\vec{r} \times \vec{p}$ which indicates the rotating tendency of the force about a point. The anticlockwise forces are generally taken as positive whereas clockwise forces are taken as negative. Torque is another name for moment of the force. Mathematically, torque $\vec{\tau} = \vec{r} \times \vec{F}$, $\theta|\vec{\tau}| = (r\sin\theta)F$

where F is applied force and $r\sin\theta$ is perpendicular to the line of force called moment area.




Torque is a vector. Its direction can be found out from right handed screw rule.

SI Unit of torque is Nm. Dimensional formula of torque is $[ML^2T^{-2}]$.

## RELATIONS OF TORQUE IN DIFFERENT FORMS

In Cartesian co-ordinate system

$$\tau_x = yF_z - zF_y; \ \tau_y = yF_x - xF_z \ \text{ and } \ \tau_z = xF_y - yF_x$$

In Polar co-ordinate system

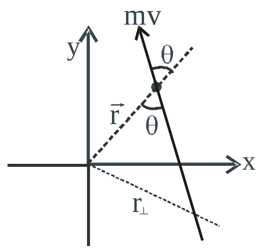
$$\tau = (F\sin\theta)r,$$

where $\theta$ is the angle between the force F and the position vector r

In vector form

$$\vec{\tau} = \left(\hat{i}x + \hat{j}y + \hat{k}z\right) \times \left(\hat{i}F_x + \hat{j}F_y + \hat{k}F_z\right) = \vec{r} \times \vec{F}$$

## Angular Momentum




Angular momentum of a rotating body about a point is given by

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

## WORK DONE IN ROTATIONAL MOTION IN TERMS OF TORQUE

Work done in rotational motion , $\sum\left(\vec{r}_i \times \vec{F}_i\right)\Delta\theta$

where $\sum \vec{r}_i \times \vec{F}_i$ is the algebraic sum of moment of force and $\Delta\theta$ is the angle through which a body is rotated.

$\therefore \quad \Delta W = \text{Total torque} \times \text{angular displacement} \Rightarrow d\omega = \tau d\theta$

$\therefore \quad \omega = \int d\omega = \int \tau . d\theta$, $\tau$ is the instantaneous torque.

## EQUATIONS OF ROTATIONAL MOTION AS COMPARED TO LINEAR MOTION

(i) $\omega^2 - \omega_0^2 = 2\alpha\theta$ comparable to $v^2 - u^2 = 2as$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ comparable to $s = ut + \frac{1}{2}at^2$

(iii) $\omega = \omega_0 + \alpha t$ comparable to $v = u + at$

where $\omega_0$ is initial angular velocity, $\omega$ is final angular velocity, $\alpha$ is angular acceleration and $\theta$ is angular displacement.

## MOMENT OF INERTIA IS A MEASURE OF ROTATIONAL INERTIA OF A BODY

Moment of inertia of a system about an axis of rotation is the sum of the product of massess of its particles and squares of their normal distances from the axis i.e.,

$$I = \sum_{i=1}^{i=n} m_i r_i^2 \quad \text{and} \quad I = \int r^2 dm \text{ for rigid body.}$$

Higher the value of moment of inertia, more difficult is the change of state of rotation. Moment of inertia is neither a scalar nor a vector but it is a tensor.

S.I. unit of moment of inertia is kg $m^2$. Dimensional formula of moment of inertia is $[ML^2T^0]$.

## FACTORS ON WHICH MOMENT OF INERTIA DEPEND

Moment of inertia depend upon

(i) distribution of mass of the body about the axis of the rotation

(ii) shape and size of the body

(iii) position and orientation of the axis of rotation

### Kinetic enery of rotation

The energy of a body due to its rotational motion is called kinetic rotational energy.

$$K.E._{rot} = \frac{1}{2}I\omega^2 \qquad I = \text{M.I. about axis of rotation.}; \ \omega = \text{angular speed about axis.}$$

Moment of inertia in terms of radius of gyration

Radius of gyration (K) : It is the perpendicular distance of which a point mass equal to mass of system for which moment of inertia of the point mass as same as moment of inertia of the system about the axis. The distance is called radius of gyration of the system.

$$I = MK^2 \qquad \text{and} \qquad K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + .... + r_n^2}{n}}$$

Moment of inertia is the product of the mass of a body and the square of the radius of gyration.

Relation between moment of inertia and torque

As $\quad d\omega = \tau d\theta$

$$\frac{d\omega}{dt} = \frac{d\tau d\theta}{dt} = \tau\omega = \frac{d\left(\frac{1}{2}I\omega^2\right)}{dt} = (I\alpha)\omega$$

$$\Rightarrow \tau = I\alpha$$

∴ Moment of inertia of a body an axis is numerically equal to the torque acting on it when the body is rotating with a unit angular acceleration.

Vectorially, $\quad \vec{\tau} = I\vec{\alpha}$

The above relation is called Law of rotation or basic equation of rotation

Relation between moment of inertia and angular momentum

Angular momentum of a particle is given by the product of linear momentum and perpendicular distance of the paticle from the axis of rotation. $L = I\omega$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \text{ and } \vec{\tau} = I\vec{\alpha} = \frac{d}{dt}(I\vec{\omega})$$

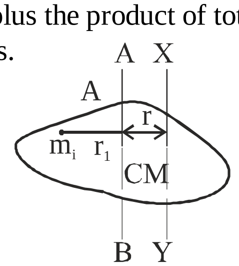
$$\Rightarrow \vec{L} = I\vec{\omega}$$

∴ Moment of inertia of a body about an axis of rotation is equal to the angular momentum of the body about the some axis rotating with unit angular velocity.



Theorem of parallel axes

This theorem states that moment of inertia of a rigid body about an axis ($I_{11}$) parallel to an axis passing through centre of mass ($I_{CM}$) of the body is equal to moment of inertia about this axis plus the product of total mass M of the body and square of perpendicular distance (r) between these parallel axes.




i.e., $\quad I_{||} = I_{CM} + Mr^2$

Theorem of perependicular axes

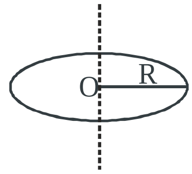
The sum of moments of inertia along two mutually perpendicular axes in a plane is equal to the moment of inertia along the axes perpendicular to this plane

i.e. $\quad I_z = I_x + I_y$

This theorem is only applicable for plane lamina (Mass is distributed along two dimension)

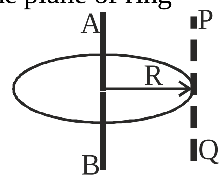
Moment of inertia of a thin uniform ring

(a) Moment of inertia of a thin uniform ring about an axis passing through centre of mass and perpendicular to the plane of the ring is $I = MR^2$.



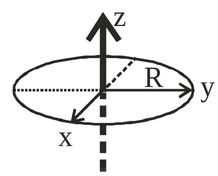

(b) Moment of inertia of a uniform ring about a tangent normal to the plane of ring

$$I_{PQ} = 2MR^2$$



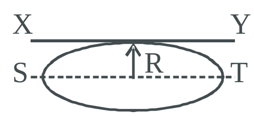

(c) Moment of inertia of a uniform ring abou any diameter of the ring

$$I_x = \frac{MR^2}{2}$$




(d) Moment of inertia of a uniform ring about a tangent in the plane of the ring.

$$I_{XY} = \frac{3}{2}MR^2$$




## Moment of Inertia of a uniform disc

Moment of inertia of a disc about line perpendicular at centre of disc $= \frac{MR^2}{2}$

Moment of inertia of about any diameter of disc $= \frac{MR^2}{4}$

## Moment of inertia of a uniform cylinder

Moment of inertia of a uniform solid cylinder about its axis $= \frac{MR^2}{2}$

Moment of inertia of a hollow cylinder about its axis = $MR^2$

## Moment of Inertia of a sphere about any diameter

Consider a sphere of mass M and radius R. Moment of inertia of the whole sphere about any diameter

$$I = \frac{2}{5}MR^2$$

Moment of inertia of a hollow sphere about any diameter = $\frac{2}{3}MR^2$

## Moment of inertia of a thin rod about an axis passing

Moment of inertia of a thin rod of mass M and length l about a line perpendicular at one of its end $= \frac{ML^2}{3}$

Uniform rectangular lamina of length a and breadth b about perpendicular axis to the plane of lamina and through its centre.

$$= M\left(\frac{a^2 + b^2}{12}\right)$$

## Acceleration of the centre of mass

Acceleration, $\vec{a} = \frac{d^2\vec{r}_{CM}}{dt^2}$, where $\vec{r}_{CM}$ is position vector of centre of mass whose magnitude is given by

$$\vec{r}_{CM} = \left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}\right)$$

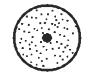
Expression for external force in terms of acceleration of centre of mass

External force, $\vec{F} = \dfrac{Md^2\vec{r}_{CM}}{dt^2}$, where $\vec{r}_{CM}$ is position vector of centre of mass.

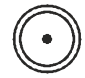
Position of centre of mass in the case of following rigid bodies

A sphere, a ring, a cylinder, a cone and a cube

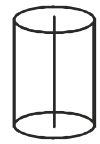
A sphere – centre of the sphere



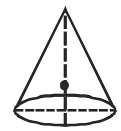
A ring – centre of the ring



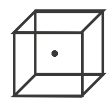
A cylinder – Middle point onthe axis of the cylinder



A solid cone – At a point distant $\frac{1}{4}$th of height of the cone from the base of the axis of the cone



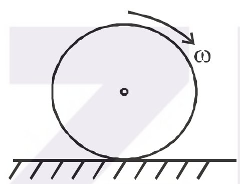
A cube – Point of intersection of diagonals



## ROLLING MOTION ON HORIZONTAL ROUGH SURFACE

Pure rolling motion can be considered as a pure rotation about instantaneous axis of rotation.

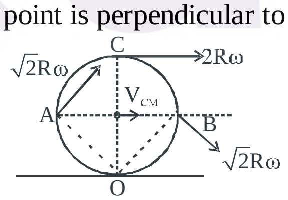
Linear velocity at top point = $2R\omega$



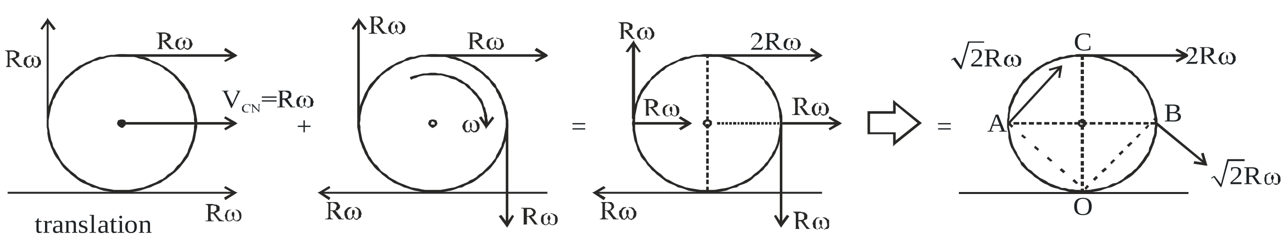

$V_A = V_B = \sqrt{2}R\omega$

$V_0 = 0$,

Direction of velocity at a point is perpendicular to the chord joining point of contact to the point.




Rolling motion is considered as the combnation of translation of centre of mass and rotation about centre of mass.




$V_C = 2R\omega$ $\qquad$ $V_B = V_A = \sqrt{2}R\omega$

$V_0 = 0$, hence point of contact has rolling motion on the inclined plane.

Cylinder rolling down without slipping down a rough inclinde plane.

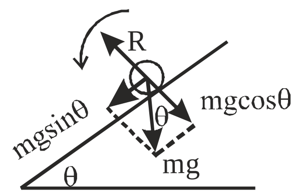
Acceleration, $\boxed{a = \dfrac{mg\sin\theta}{m + I/r^2} = \dfrac{g\sin\theta}{1+\left(\dfrac{k}{r}\right)^2}}$

where $mK^2 = I$, K = radius gyration, r = radius of body.

Hence for $\dfrac{k}{r}$ lesser, then acceleration 'a' down the plane is more so time required to come down is also more.

For pure rolling down the plane, $F_S \le u_S N \Rightarrow I_{CM}\dfrac{a}{R^2} \le u_S N$

$$\Rightarrow \frac{I_{CM}}{R^2}\frac{g\sin\theta}{1+\left(\frac{k}{r}\right)^2} \le \mu_S mg\cos\theta$$




$$\Rightarrow \quad \mu_S \ge \frac{\tan\theta}{1+\left(\frac{r}{k}\right)^2}$$

For pure rolling $\mu_S \ge \dfrac{\tan\theta}{1+\left(\dfrac{r}{k}\right)^2}$ condition for pure rolling down the plane. For disc, $\left(\dfrac{r}{k}\right)^2 = 2 \;\therefore\; \mu_S > \dfrac{1}{3}\tan\theta$

When body either rolls up or down the inclined plane the friction (static) acts on the body at the point of contact always up the plane. However, no power is dissipated by friction hence mechanical energy is conserved.

## CONSERVATION OF ANGULAR MOMENTUM

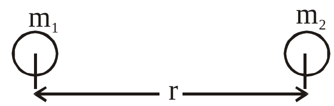
As $\vec{\tau}_E \text{total} = \dfrac{d\vec{L}}{dt}$ i.e. rate of change of angular momentum is equal to the total internal torque on the system, where direction of torque is in the direction of change of angular momentum. If there is no torque about a point or line then there is angular momentum is conserved about that point.

In case when body rolls or rolls with slipping then about point of contact there is no torque hence total angular momentum is conserved.

# Gravitation

- Newton's law of gravitation : Newton in 1665 formulated that the force of attraction between two masses $m_1$ and $m_2$ as

$$F = \frac{Gm_1m_2}{r^2}$$




where $G = 6.67 \times 10^{-11}$. $Nm^{-2}$ and is called universal gravitational constant.

- Gravitational field Intensity : Gravitational force per unit mass placed at a point is called gravitational field intensity at that point. Gravitational field intensity of earth is 'g'

$$\vec{I} = \frac{\vec{F}}{m}$$ where test mass m is very very small.

- Gravitational potential ($V_g$) : Gravitational potential at a point is the amount of work done to bring a unit mass from infinity to that point under the influence of gravitational field of a given mass M, $V_g = -\frac{GM}{r}$

- Gravitational potential and field due to system of discrete mass distribution.

$$V = V_1 + V_2 + V_3 + ......$$ i.e. $$V = \sum_{i=1}^{N} V_i$$

$$\vec{I} = \vec{I}_1 + \vec{I}_2 + \vec{I}_3 + ...$$ i.e. $$\vec{I} = \sum_{i=1}^{N} \vec{I}_i$$

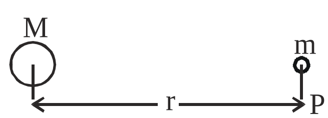
- Gravitational potential and field due to system of continuous mass distribution.

$V = \int dV$ where dV is potential due to elementary mass dM.

$\vec{I} = \int d\vec{I}$ where $d\vec{I}$ is field intensity due to elementary mass dM.

- Gravitational potential energy of two mass system is the amount of work done to bring a mass m from infinity to the point P under the influence of gravitational field of a given mass M. $U_g = -\frac{GMm}{r}$

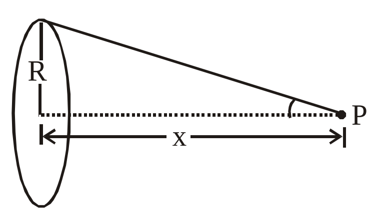
where, $U_g$ is G.P.E. of two mass system. Note that $U_g = mV_g$




- In general, gravitational potential energy of a system is work done against gravitational force in assembling the system from its reference configuration. Infinite mutual separation is reference configuration for mass-system.

- Gravitational field intensity due to a ring of radius R, mass M at any point on the axial line at a distance x from the centre of the ring is

$$E_g = \frac{GM.x}{(R^2 + x^2)^{3/2}}$$




The field is directed towards the centre. At the centre of the ring $E_g$ is minimum (= 0) and $E_g$ is maximum at

$$x = \frac{R}{\sqrt{2}}$$

- Relation between Field and potential : $I = \frac{-dV}{dr} \Rightarrow \vec{I} = \frac{-\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

  $dV = -\vec{I}.\vec{dr}$

- Work done against gravitational force in changing the configuration of a system

  = P.E. in final configuration – P.E. in initial configuration.

  i.e. $\text{Work done} = U_2 - U_1 = W_{\text{Against gravitational force}} = -W_{\text{by gravitational force}}$

- Variation of g with height

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} \text{ if } h > \frac{R}{10}$$

$$g' = g\left(1 - \frac{2h}{R}\right) \text{ if } h < \frac{R}{10}$$

  Note g never becomes zero with height, that is, $g \to 0$ if $h \to \infty$

- Variation of g with depth (d)

$$g' = g\left(1 - \frac{d}{R}\right);$$ where g is acceleration due to gravity at earth surface.

- Variation of g with rotation of earth / latitude

$$g' = g\left(1 - \frac{R\omega^2}{g}\cos^2\lambda\right)$$

  that is, g is maximum at the poles and minimum at the equator

- Escape velocity $\nu_e = \sqrt{\frac{2GM}{R}}$ ;

  Escape velocity is the minimum velocity required to escape a mass from the surface of the earth/ planet from its gravitational. If velocity provided is greater than or equal to escape velocity, the mass will never come back to the earth/planet.

- Planetry motion

  Oribit velocity $\nu_o = \sqrt{\frac{GM}{r}}$ from the fact $\frac{GMm}{r^2} = \frac{mV^2}{r} = \text{Required Centripetal force}$

  where $\nu_o$ is speed with which a planet or a satellite moves in its orbit and r is the radius of the orbit.

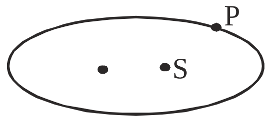
  Time period $T = \frac{2\pi r}{\nu_o}$ or $\boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$ ; where $v_0$ = orbital velocity = $\sqrt{\frac{GM}{r}}$

  Kinetic Energy $KE = \frac{1}{2}mv_o^2 = \frac{GMm}{2r}$, Potential Energy $PE = -\frac{GMm}{r}$

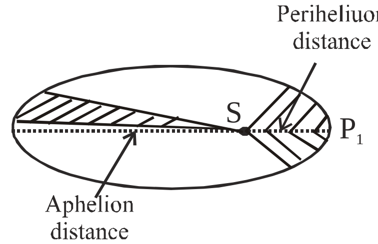
  Net energy $E = KE + PE = -\frac{GMm}{2r}$

- Kepler's Laws

  First Law : The planets revolve around the sun in the elliptical orbits with sun at one of the focus.




Second Law : The radial line sweeps out equal area in equal interval of time. This law may be derived from law of conservation of angular momentum.




Areal velocity $=\dfrac{dA}{dt} = = \dfrac{L}{2m} =$ constant

$\because$ Torque about axis of rotation is zero so angular moment is constant i.e. $I_1\omega_1 = I_2\omega_2$

$\Rightarrow (mr_1)(v_1) = (mr_2)(v_2) \Rightarrow \dfrac{v_1}{v_2} = \dfrac{r_2}{r_1}$

Thus $\dfrac{\nu_1}{\nu_2} = \dfrac{r_2}{r_1}$ or $\dfrac{\nu_{perihelion}}{\nu_{aphelion}} = \dfrac{r_{aphelion}}{r_{perihelion}}$ that is, when the planet is closer to the sun it moves fast.

Third Law: The square of the time period of a planet is proportional to he cube of a semimajor axis

$$T^2 \propto a^3 \text{ or } T^2 \propto r^3$$

If eccentricity of the orbit is e then $\dfrac{r_{aphelion}}{r_{perihelion}} = \dfrac{r_{max}}{r_{min.}} = \dfrac{a+ae}{a-ae} = \dfrac{1+e}{1-e}$

- Weightlessness in a satellite :

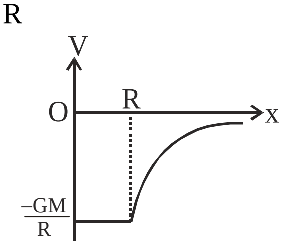
  Net force towards centre $= F_C = ma_c \Rightarrow \left(\dfrac{GMm}{r^2} - N\right) = m\dfrac{V^2}{r}$ where N is contact force by the surface

  $\Rightarrow \dfrac{GMm}{r^2} - N = m\left(\dfrac{GM}{r^2}\right)$ or $N = 0$ that is, the surface of satellite does not exert any force on the body and hence its apparent weight is zero.

- Gravitational potenial due to a ring at any point on its axis, assuming mass of the ring is uniformly or nonuniformly distributed is

$$V = \frac{-GM}{\sqrt{R^2 + x^2}} \quad \text{; potential at the centre is } \frac{-GM}{R}$$

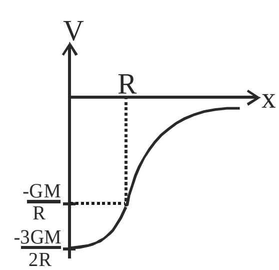
- Graviational potential due do a shell




$$V_{in} = V_{sur} = \frac{-GM}{R}; \quad V_{out} = \frac{-GM}{x}$$

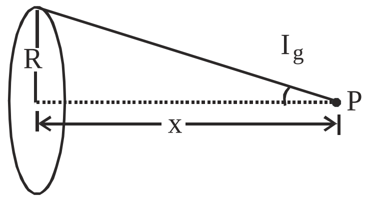
- Gravitational potential due to a solid sphere of radius R

$$V_{in} = \frac{-GM}{2R^3}\left(3R^2 - x^2\right) \quad \text{for} \quad 0 \le x \le R$$




$$V_{sur} = -\frac{GM}{R} \text{ for } x = R; \quad V_{out} = \frac{-GM}{x} \text{ for } x > R$$

- Gravitational field intensity due to a disc

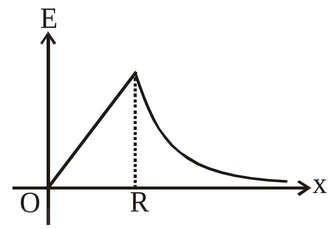



$$E = \frac{2GM}{R^2}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right] = \frac{2GM}{R^2}[1 - \cos\theta]$$

- Gravitational field intensity due to a solid sphere

$$E_{in} = \frac{GMx}{R^3} \text{ for } x < R$$

$$E_{sur} = \frac{GM}{R^2},\ E_{out} = \frac{GM}{x^2} \quad x \geq R$$

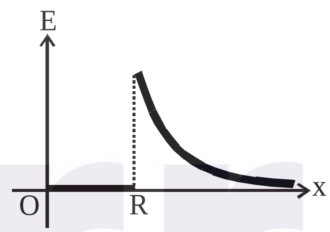



- Gravitational field intensity due to a hollow sphere

$E_{in} = 0$ ; $x < R$

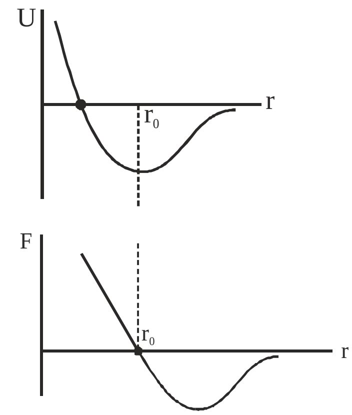
$E_{surface} = \dfrac{GM}{R^2}$ ; $x = R$

$E_{out} = \dfrac{GM}{x^2}$; $x > R$



# ELASTICITY

## INTERMOLECULAR / INTERATOMIC FORCES :




In a solid in equilibrium the separation between atom or molecules is equal to $r_0$. At this separation the force between atoms or molecules is zero and their potential energy is minimum. If atoms or molecules are brought closer than $r_0$, their PE increases and repulsive force comes into play.

If atoms/molecules are taken farther than $r_0$, their PE increases and attractive force comes into play. Molecules/Atoms in a solid therefore behave as if they are joined by stiff springs.

If separation between atom/molecules is changed from $r_0$, a restoring force comes into play which tends to bring them back to their equilibrium separation.

## SOLID :

The state of matter which has s definite shape and size. The constituent atoms/molecules are so closely packed that interatomic forces maintain the relative separation between atoms/molecules thereby giving it a definite shape and size.

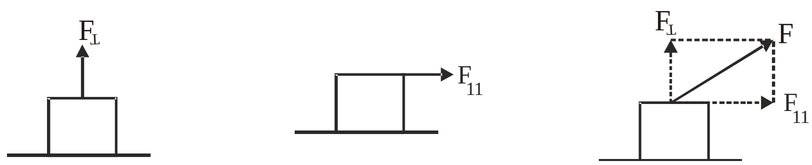
## ELASTICITY :

When a solid is acted upon by a system of external forces in equilibrium, the intermolecular separations change giving rise to internal restoring forces.

- The change in intermolecular separation results in change in shape or size of body i.e., deformation of body.
- The system of external forces, which tend to change shape or size of body is called deforming forces.
- The system of internal forces, which tend to bring the body back to original shape or size is called restoring force.
- If the body regains its original shape or size on removal of deforming forces (i.e., internal restoring forces bring body back to original shape or size) the body is said to be elastic. Otherwise the body is said to be plastic.

## STRESS :

The restoring force set up per unit cross sectional area of body is called stress. For small and slow deformation deforming force is equal to restoring force.




$$\text{Stress} = \frac{\text{Restoring force}}{\text{Cross sectional area}} = \frac{\text{Deforming force}}{\text{Cross sectional area}} = \frac{F}{A}$$

Units and Dimensions of stress are :

(i) $Nm^{-2}$

(ii) $ML^{-1}T^{-2}$

- Stress is not a vector
- Deforming force acting perpendicular to cross sectional area gives rise to normal stress.

$$\text{Normal stress} = \frac{F_{\perp}}{A}$$

- Deforming force acting the parallel to cross sectional area produces tangential stress.

$$\text{Tangential stress} = \frac{F_{||}}{A}.$$

## STRAIN :

A body subjected to stress suffers change in dimensions or shape. The fractional or relative change in dimension or shape is called strain.

$$\text{Strain} = \frac{\text{change in dimension/shape/size}}{\text{original dimension/shape/size}}$$

- Strain has no unit and is a scalar.

(i) Longitudinal Strain

$$= \frac{\text{Change in linear dimension}}{\text{Original linear dimension}}$$

$$= \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$

Longitudinal strain is produced by normal stress in solids only.

(ii) Volumetric Strain :

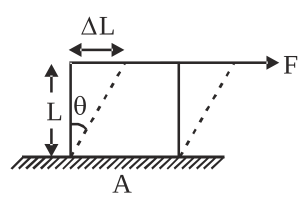
$$= \frac{\text{Change in volume}}{\text{Original volume}}$$

$$= \frac{\Delta V}{V_0} = \frac{V - V_0}{V_0}$$

It is produced by normal stress in all states of matter.

(iii) Shearing Strain :

$$= \frac{\text{Displacement of free surace of body over which tangential deforming force is applied}}{\text{Distance of this free surface from fixed surface}}$$




$$= \frac{\Delta L}{L} = \tan\theta \approx \theta.$$

It is also called the angle of shear.

It is produced by tangential stress in solids only. It refers to change in shape of body without any change in volume.

## HOOKE'S LAW :

For small deformations, the stress developed in a body is directly proportional to strain produced.

$$\text{Stress} \propto \text{Strain}$$

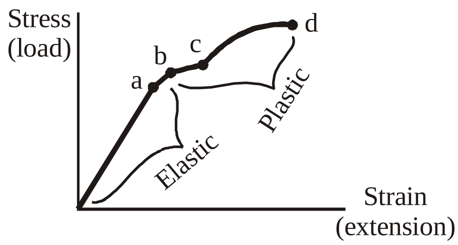
$$\text{Stress} = E \times \text{Strain}$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \text{Modulus of elasticity.}$$

Hooke's Law can also be expressed as : The tension developed in a body is directly proportional to extension produced. (i.e., Load is directly proportional to extension). Modulus of elasticity depends on the nature of stress and strain, material of body and state of body. It has units $Nm^{-2}$ and is a scalar.

The range of deforming forces over which Hooke's Law holds is called proportional range. The limit beyond which body does not remain elastic is called elastic limit.

## Variation of Stress and Strain




| | | |
|---|---|---|
| a : | Proportionate limit | (Hooke's law is valid) |
| b : | Ealstic limit | (body remains elastic) |
| c : | Fracture point | (body breaks) |

If large deformation takes place between the elastic limit and fracture point, the material is ductile. If the material breaks soon after the elastic limit is crossed, it is brittle.

The stress corresponding to fracture point is called breaking stress.

## MODULII OF ELASTICITY

Young's Modulus (Y) : When a normal stress produces a longitudinal strain in a solid, the modulus of elasticity is called Young's modulus.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta L}$$

Area perpendicular to L or F is taken to be the area of cross section.

$$\Delta L = \frac{FL}{YA}$$

$$F = YA\left(\frac{\Delta L}{L}\right) = \frac{YA}{L}\Delta L$$

Comparing with $K\Delta x$ yields

$$K = \frac{YA}{L} \qquad \Rightarrow \qquad K \propto \frac{1}{L}.$$

Bulk Modulus (B) : When normal stress produces volumetric strain modulus of elasticity is called Bulk Modulus.

$$B = \frac{\text{Normal stress}}{\text{Volume strain}}$$

$$= -\frac{F/A}{\Delta V/V} = -\frac{\Delta p}{\Delta V/V} = -\frac{V\Delta p}{\Delta V}.$$

Negative sign is put to keep B positive (volume decreases on increasing pressure).

Modulus of Rigidity $(\eta)$ : When tangential stress produces change in shape without change in volume in solid the modulus of elasticity is called modulus of rigidity.

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$= \frac{F/A}{\Delta L/L} = \frac{F/A}{\theta}.$$

Compressibility (C) : The reciprocal of bulk modulus of a material is called its compressibility.

Poisson's Ratio ($\sigma$) :

The extension in length is called longitudinal strain.

When length of a wire is increased it's area of cross section or radius decreases i.e. Extension in length is accompanied by contraction in radius or area of cross section Contraction in area of cross sectionor radius gives rise to lateral (or perpendicular) strain.

The ratio of lateral strain to longitudinal strain is called Poisson's Ratio.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\Delta D/D}{\Delta L/L} = -\frac{\Delta r/r}{\Delta L/L}.$$

Negative sign is put to keep $\sigma$ positive as radius decreases on increasing length.

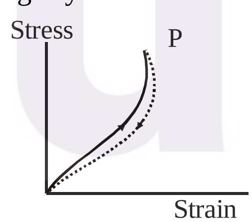
Value of $\sigma$ lies between 0.1 and 0.3 for many common materials.

$Y, B, \eta$ and $\sigma$ are related as :

$$Y = 3B(1-2\sigma) = 2\eta(1+\sigma).$$

## ELASTIC HYSTERSIS :

For many materials the lack of retraceability of stress and strain curve for increasing and decreasing stress is called elastic hystersts. The area enclosed by hystersis loop is proportional to the energy density dissipated in the material. Materials having large hysterisis loss are used as vibration absorbers in machines.




## ELASTIC POTENTIAL ENERGY :

In a uniform wire of length L and cross sectional area A, the restoring force acting when extension in it is x is

$$F = \frac{YA}{L}x.$$

Work done in a further very small extension dx.

$$dW = Fdx = \frac{YA}{L}xdx.$$

Total work done for extension from O to x.

$$W = \frac{1}{2}\frac{YA}{L}x^2 = \frac{1}{2}\times\frac{YAx}{L}\times x = \frac{1}{2}\times\frac{Yx}{L}\times\frac{x}{L}\times AL = \frac{1}{2}Yx.\left(\frac{x}{L}\right)^2\times AL$$

$$= \frac{1}{2}\times \text{ stretching force} \times \text{stretch.}$$

This work is stored in the wire as Elastic potential energy

$$U = W = \frac{1}{2} \times \text{ stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times Y \times \text{ strain}^2 \times \text{volume.}$$

Energy density in stretched wire

$$u = \frac{U}{V} = \frac{1}{2} \times \text{ stress} \times \text{strain}$$

$$= \frac{1}{2} \times \text{ } Y \times \text{strain}^2.$$

- Variation of density with increase in pressure,

$$\rho = \rho_0 (1 + C\,\Delta P)$$

- In case of a rod of length L and radius r fixed at one end , the angle of shear $\phi$ is related to angle of twist $\theta$ by the relation, $L\phi = r\,\theta$
- In case of twisting of a cylinder of length L and radius r, elastic restoring couple per unit twist is given by,

$$C = \frac{\pi \eta r^4}{2L}$$

- In case of bending of a beam of length L, breadth b and thickness d, by a load Mg at the middle, the depression

$$\delta = \frac{MgL^3}{4bd^3Y}$$

and for a beam of circular cross section of radius r and length L.

$$\delta = \frac{MgL^3}{12\pi r^3 Y}$$

# FLUID - DYNAMICS

Liquids and gases are together classified as fluids. They are the substances, which can flow. They cannot withstand shearing or tangential stress. They begin to flow when subjected to such stress.

## PRESSURE:

A fluid at rest exerts an outward force on the inner surface of the container holding it. The force exerted per unit area on the surface of the container by the fluid is called pressure.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{F}{A}$$

$$P = \lim_{\Delta A \to 0}\left[\frac{\Delta F}{\Delta A}\right]$$

Pressure also exists in the interior of the fluid. Fluid within any region exerts an outward force on the fluid around it and fluid outside a region exerts an inward force on the fluid it surrounds. Fluid pressure can therefore be defined as the force exerted per unit area by a fluid on any surface in contact with it.

Pressure is measured in $Nm^{-2}$ or Pascal or mm Hg or bar or torr. or atmosphere.

1 $Nm^{-2}$ = 1 Pascal

1 bar = $10^5$ Pa

1 torr = 1 mm Hg

1 atm = 1.01325 × $10^5$ Pa = 76 cm of Hg = 760 torr

Pressure is a scalar quantity.

## Density:

The mass per unit volume of a fluid is called its density

$$\rho = \frac{M}{V}$$

$$\rho = \lim_{\Delta V \to 0}\left[\frac{\Delta M}{\Delta V}\right]$$

Relative density or specific gravity is defined as

$$S = \frac{\text{density of substance}}{\text{Density of water at } 4°c}$$

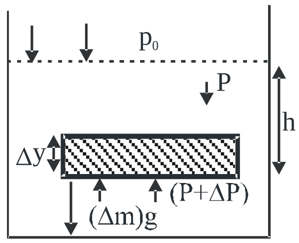
Density varies with temperature $\rho = \rho_0\left(1 - \gamma\Delta T\right)$

Variation of density with change in pressure

$$\rho = \rho_0\left(1 + \frac{\Delta p}{B}\right) = \rho_0\left(1 + C\Delta p\right)$$

## PRESSURE DUE TO A LIQUID COLUMN

Consider a liquid at rest in a container. Let us select a cylindrical portion of the liquid of cross sectional area A and height $\Delta y$. Since the liquid cylinder is in equilibrium,




$$(P+\Delta P)A - PA = \Delta mg$$

$$= \rho Ag\Delta y$$

$$\boxed{\frac{\Delta P}{\Delta y}=\frac{dP}{dy}=\rho g}$$

If the density of the liquid is uniform, the pressure at depth h.

$$\int_{p_0}^{P} dP = \int_0^h \rho g dy$$

$$P = p_0 + \rho gh$$

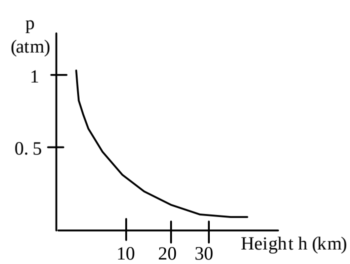
- The atmospheric pressure varies with height from surface of earth as

  $P = P_0 e^{-ha}$

  where $a = \frac{\rho_0 g}{P_0} = 8.55\,\text{km}^{-1} = 8.55$ Km.

  Atmospheric pressure decreases with height exponentially.

The constant a gives the difference in altitude over which the pressure drops by a factor of e. The pressure drops by a factor of 10 when the height increases by 20 km. Thus at the heights h = 20 km, 40 km above the sea level, the pressure would be 0.1 atm, 0.01 atm and so on.




- The pressure at a depth h in a liquid

  $p = p_0 + \rho gh$ is called hydrostatic pressure. The difference between hydrostatic and atmospheric pressure is called guage pressure.

  $P_g = p - p_0 = \rho gh$

- At a point inside the fluid, pressure acts in all directions and has the same value.

- If the liquid container is accelerated in the horizontal direction $\frac{dP}{dx} = -\rho a$

$$\frac{dp}{dx} = -\rho g$$

- The free surface of the liquid is inclined at an angle $\theta$ given by

$$\tan\theta = \frac{a}{g}$$

Force exerted by a fluid on the base of the container due to guage pressure

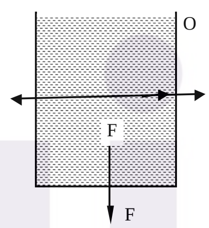
$$F = \rho g V = \text{Weight of liquid in container}$$

- Force exerted by liquid on the vertical walls of the container due to guage pressure.

$$F = \rho g b \int_0^H h\,dh$$

where b = width of container ; H = height of liquid in container.

$$F = \frac{1}{2}\rho g b H^2$$



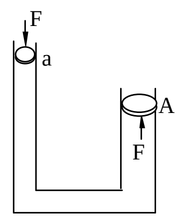

- The moment of this force about O
free surface of liquid,

$$M = \int_0^H h\,dF = F \times \frac{2H}{3}$$

Therefore total force acts at a point which is at a depth $\frac{2H}{3}$ from the free surface of liquid.

## PASCAL LAW

Any change in pressure at one point of a confined fluid at rest is transmitted undiminished to every portion of the fluid and the walls of the container. An important application of this law is hydraulic lift. A small force f is exerted on the piston with small cross sectional area a. The resulting increase in pressure is $\Delta p = \frac{f}{a}$.



This change in pressure is transmitted through the liquid to a larger piston of area A. The force on the larger piston therefore.

$$F = \frac{f}{a}A. = \frac{A}{a}f$$

For a small force exerted on smaller piston a large force acts on larger piston. Hydraulic lift is a force multiplying device with a multiplication factor equal to the ratio of the cross sectional areas of the two pistons.

## Buoyancy : Archimedes Principle

When a body is wholly or partly immersed in a fluid. It experiences an upward thrust/force equal to the weight of the fluid displaced by it. This force is called force of buoyancy or buoyant force.

$$B = M_{\text{displaced fluid}} \times g = \text{Weight of fluid displaced}$$

This principle is Archimedes principle.

- There is a loss in weight of a body inside fluid due to buoyant force on body. Apparent weight of body in a fluid

$$W' = \text{True weight of body} - \text{Buoyant force}$$

$$W' = W - B$$

$$= V_b\rho_b g - V_l\rho_l g$$

$V_l$ = volume of liquid displaced

If the body is fully submerged

$$W' = V(\rho_0 - \rho_l)g$$

- When a body floats in a liquid, the weight of the body is equal to the weight of the liquid displaced ($W' = 0$).
- The centre of gravity of the displaced liquid (called centre of buoyancy) lies vertically above or blow the centre of gravity of the body in equilibrium.

If –

(i) $\rho_b = \rho_l$ – body will float completely immersed in fluid any where in the fluid .

(ii) $\rho_b > \rho_l$ – body will sink to the bottom of liquid

(iii) $\rho_b < \rho_l$ – body will float partially immersed on the surface of fluid.

## FLUID DYNAMICS

When the fluid velocity at any given point is constant in time the fluid motion is said to be steady. Flow of fluids is steady, usually at slow speeds.

If the element of fluids at each point has no net angular velocity about that point, the fluid flow is irrotational. Fluid flow is said to be unsteady or turbutant when the velocities of the fluid particles at any point changes erractically both in magnitude and direction with time.

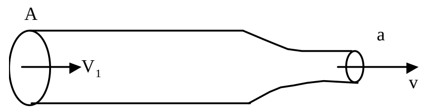
During steady flow the trajectories of the fluid particles are in general curved paths know as streamlines. A streamline is a line drawn in the fluid such that the tangent to the streamline at any point is parallel to the fluid velocity at that point. The fluid velocity can vary from point to point along a streamline, but at any given point, the velocity remains constant in time. Steady flow is also called streamlined flow.

## EQUATION OF CONTINUITY

If there are no sinks or sources of fluid in the tube, the mass of fluid flowing into the tube in a given time must be equal to mass of liquid flowing out of the tube in the same time. Therefore, mass flux is constant.

Mass flux $= \rho_1 Av = \rho_2 av = \text{constant}$

$= \rho VA = \text{constant}$




If the liquid is compressible $\rho_1 = \rho_2$

AV = av

$\boxed{AV = \text{constant}}$

This is equation of continuity

## BERNOULLIS THEOREM

It is principle of conservation of energy applied to fluid flow. For streamline flow of an ideal fluid (nonviscous, incompressible) the sum of pressure energy, kinetic energy and gravitational potential energy per unit mass remains constant.

Pressure energy = PV

Pressure energy per unit mass = $\frac{P}{\rho}$

Kinetic energy per unit mass = $\frac{1}{2}V^2$

Gravitational potential energy per unit mass = gh

So $\frac{P}{\rho} + \frac{V^2}{2} + gh = \text{constant}$

$\frac{1}{2}\rho V^2 + \rho gh = \text{constant.}$

$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant.}$

## APPLICATIONS OF BERNOULLI'S THEOREM

(i) Dynamic lift: Neglecting gravitational effects, velocity is higher at wings points where pressure is lower and vice versa of aeroplanes are so designed that velocity of wind on the upper part of the wings is higher compared to lower part. This results in a pressure deference with lower part having higher pressure. This pressure difference provides a dynamic lift.

(ii) Velocity of efflux : Consider a vessel containing a liquid upto some height H. A small hole is punched in the wall at a depth h below the free surface. The speed of liquid coming out of the hole is called the velocity of efflux of the liquid.

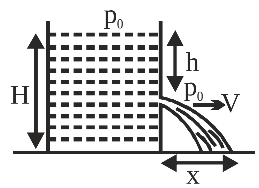
$$p_a + h\rho g + 0 = p_0 + \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gh}$$

It equals the speed of an object fallen from rest through a height h.

Time taken by liqud to reach base levels,

$$t = \sqrt{\frac{2(H-h)}{g}}$$




The distance at which liquid hits the ground,

$$x = z\sqrt{h(H-h)}$$

## VISCOCITY:

A fluid flows in layers. This is called laminar flow of fluid. Also, the liquid layer in contact with stationary surface of container remains at rest. As we move away from stationary surface the speed of liquid layers goes on increasing. There is therefore a relative motion between the layers of fluid.

When a layer of fluid slips or tends to slip on another layer in contact an internal resistance comes into play between layers which tends to destroy their relative motion. This internal resistance acting between layers of fluid is called viscous drag. This viscous drag acting between layers of fluid is

$$F = -\eta A \frac{dv}{dx}$$

where, A = Area of fluid layers;

$\frac{dv}{dx}$ = velocity gradient between liquid layers

$\eta$ = constant of proportionality called coefficient of viscocity depending on the nature of fluid

negative sign indicates opposition to relative motion.

$\eta$ is measured in $Ns/m^2$ or dyne-s/$cm^2$ or poise

## FLOW OF FLUID THROUGH NARROW TUBE: POISEUILLE'S FORMULA

The rate of flow of a fluid of coefficient of viscocity $\eta$ through a tube of length l and radius of cross section r when subjected to a perssure difference P is,

$$\frac{V}{t} = \frac{\pi P r^4}{8\eta l}$$

### Stokes Law:

When a spherical body of radius r falls through a fluid of viscocity $\eta$ with a velocity v a viscous drag acts on it given by,

$$F = 6\pi\eta r v$$

### Terminal Velocity:

When a body falls freely through a fluid, its velocity goes on increasing and so does the viscous drag acting on it. When the viscous drag acting on the body equals, apparent weight of body, it begin to fall with a constant velocity called terminal velocity,

$$F = W - B$$

$$6\pi\eta r V_T = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$V_T = \frac{2r^2(\rho-\sigma)g}{9\eta}$$

## Critical Velocity and Reynolds number:

At slow speeds, flow of fluid is streamlined. As velocity of fluid increases, the flow of fluid becomes turbulent. The largest velocity which allows a steady flow is called critical velocity.

The quantity,

$$N = \frac{\rho v D}{\eta}$$

is called the Reynolds number and determines the nature of flow of fluid.

| | | | |
|---|---|---|---|
| If | $N < 2000$ | – | Flow is steady |
| | $N > 3000$ | – | Flow is turbulent |
| | $2000 < N < 3000$ | – | Flow is unstable |

This relation can be used to calculate critical velocity.

# HEAT AND THERMODYNAMICS

## (THERMAL EXPANSION, CALORIMETRTY AND HEAT TRANSFER)

### Temperature

(i) Temperature is a Macroscopic, physical quantity, basically a measure of degree of hotness or coldness of a body.

(ii) The natural flow of heat is from higher temperature to lower temperature.

(iii) The part of internal energy being transfered due to temperature difference is called heat energy.

(iii) Temperature of a body is directly proportional to the kinetic energy of the random motion of the molecules or atoms of the substance.

(iv) Two bodies are said to be a Thermal Equilibrium with each other, when no heat flows from one body of the other i.e., when both the bodies are at the same temperature.

$$\frac{\text{Reading on any scale} - \text{Melting point of water}}{\text{Boiling point of water} - \text{Melting point of water}} = \text{Constant for all scale}$$

$$\boxed{\frac{t^0C - 0}{100 - 0^0} = \frac{t^0F - 32}{212^0 - 32^0} = \frac{t^0R - 0}{80^0 - 0} = \frac{tK - 273.15}{373.15 - 273.15}}$$

### Thermal Expansion

(i) When heat is supplied to matter and its state does not change, then it usually get expanded. There are also some such substances which contract on heating. Rubber is a very good example of it.

(ii) Thermal expansion is minimum in case of Solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.

(iii) Solids can have all the three types of thermal expansion i.e., one dimensional (Linear expansion), two dimensional expansion (superficial expansion) and three dimensional (volume expansion) while fluids (liquids and gases) usually posses only volume expansion.

Coefficient of Linear Expansion : Coefficient of linear expansion is defined as the increase in length $(\Delta L)$ per unit length (L) per unit rise in temperature. It is usually very small and hence may be expressed without much error by the equation

$$\alpha = \frac{\Delta L}{L(\Delta T)} \quad \text{So,} \quad L_{new} = L + \Delta L = L(1 + \alpha\Delta T)$$

Coefficient of Superficial / Areal Expansion : Coefficient of superficial expansion is defined as the increase in area $(\Delta A)$ per unit area (A) per unit rise in temperature. It is usually very small and hence may be expressed without much error by the equation.

$$\beta = \frac{\Delta A}{A(\Delta T)} \quad \text{So,} \quad A_{new} = A + \Delta A = A(1 + \beta\Delta T)$$

Coefficient of Cubical / Volumetric Expansion : Coefficient of cubical expansion is defined as the increase in volume $(\Delta V)$ per unit volume (V) per unit rise in temperature It is usually very small and hence may be expressed without much error by the equation

$$\gamma = \frac{\Delta V}{V(\Delta T)} \quad \text{So,} \quad V_{new} = V + \Delta V = V(1 + \gamma\Delta T)$$

Relation between $\alpha, \beta, \gamma$ : $\quad \beta = 2\alpha,\ \gamma = 3\alpha$

Anomalous expansion of water : Generally on heating matter expands and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°C. In the range 0°C to 4°C, water contracts on heating and expands on cooling i.e., $\gamma$ is negative. This behaviour of water in the range from 0°C to 4°C is called Anomalous Expansion.

Specific heat capacity c : $c = \frac{\Delta Q}{m\Delta T}$ ; Amount of energy exchange per unit mass per unit change in temperature

Molar Specific heat C : $C = \frac{\Delta Q}{n(\Delta T)}$ ; Amount of energy exchange per unit mole per unit change in temperature

Unit of Heat Energy : 1 cal = 4.18 J also W = JH where J = Mechanical equivalent of Heat

## Water –Equivalent

(i) If thermal capacity of a body is expressed in terms of mass of water, it is called water – Equivalent of the body, i.e., water equivalent of a body is the mass of water which when given same amount of heat as to the body, changes the temperature of water through same range as that of body, i.e.,

W = mc gram where c = specific heat of substance of body.

(ii) The unit of water equivalent W is gm while the dimensions [M]

## Latent Heat, L

The heat of transformation during melting / freezing / vaporisation/condensation is called latent heat.The state of matter changes without changing temperature in such process. For $H_2O$ (water),

$L_{ice} = L_{fusion} = L_{melting} = 80$ Cal/gram and $L_{steam} = L_{vaporisation} = 536$ Cal/gram $= L_{condensation}$

## Principle of Calorimetry

When two bodies (one solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body(s) at higher temperature to body(s) at lower temperature till both acquire same temperature. The body at higher temperature releases heat while that at lower temperature absorbs heat so that:

$$\boxed{\text{Heat lost} = \text{Heat gained}}$$

i.e. principle of calorimetry is an alternative form of the law of conservation of energy.

## Important Conclusions

(i) Temperature of mixture (T) is always between lower temperature and higher temperature i.e, $T_L \le T \le T_H$

(ii) When temperature of a body changes, the body releases heat if its temperature falls and absorbs heat when its temperature rises. The heat released or absorbed by a body of mass m is given by:

$\boxed{Q = mc\Delta T}$ when c = specific heat of the substance of body.

(iii) When state of a body changes, change of state takes place at constant temperature (MP or BP) and heat released or absorbed is given by:

$\boxed{Q = mL}$ where L = Latent heat for the associated change of state.

(a) Heat is absorbed, if solid converts into liquid (at MP) or liquid converts into vapours (at BP) and

(b) Heat is released, if liquid converts into solid or vapours converts into liquid

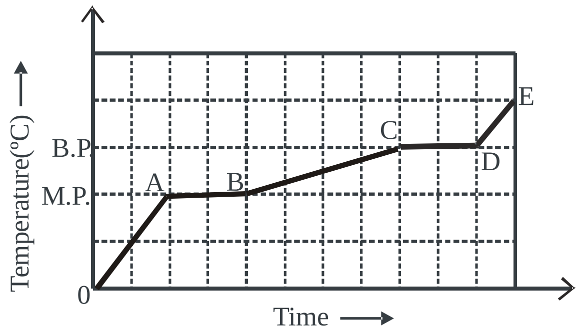
## Heating Curve of a Solid

(i) If a given mass (m) of a solid is supplied heat continuously at a constant rate P and a graph is plotted between temperature and time, the graph so obtained is shown in the adjoining figure and is called as Heating Curve.

(ii) In the regions OA, the temperature of the solid is changing with time, so $\Delta Q = m \times c_{solid} \times \Delta T = P \times \Delta T$

In the region AB the solid is melting into liquid, So temperature does not change. Here $\Delta Q = m \times L$

In the region BC the temperature of the liquid is increasing and $\Delta Q = m \times c_{liquid} \times \Delta T$. Greater is the slope of BC, lesser will be the specific heat of the liquid because slope = $\frac{\Delta T}{\Delta t} = \frac{P}{mc}$




In region CD, vaporisation of liquid takes place and $\Delta Q = mL_{steam}$. In region DE, steam temperature is in increasing and $\Delta Q = m.c_{steam}.\Delta T$, In general, slope of BC is less than slope of OA and DE line.

## HEAT TRANSFER

### Conduction

(i) In conduction the molecules of the body transfer heat from a place at higher temperature to a place at lower temperature without actually moving in the body. i.e. energy transfer without bulk motion of material.

(ii) In steady state heat passing through a bar of length L and cross section A and time t when its ends are at temperature $\theta_1$ and $\theta_2$ is given by: $Q = \frac{KA(\theta_1 - \theta_2)t}{L}$

and rate of flow of heat will be $\boxed{\frac{dQ}{dt} = KA\frac{d\theta}{dx}}$, Here K = coefficient of thermal conductivity.

(iii) Flow of heat through multiple slabs: Suppose a compound slab consists of two rods of lengths $L_1$ and $L_2$ in series with common surface area A. Let $K_1$ and $K_2$ be the coefficient of thermal conductivities respectively. Let the ends of the slab be maintained at a temperature difference of $(\theta_1 - \theta_2)$. In the steady state, in such a case,

the junction temperature is given by $\theta = \frac{K_1L_2\theta_1 + K_2L_1\theta_2}{K_1L_2 + K_2L_1}$ and rate of flow of heat $\frac{\Delta Q}{\Delta t} = \frac{A(\theta_1 - \theta_2)}{\left(\frac{L_1}{K_1} + \frac{L_2}{K_2}\right)}$

### Applications

(a) Rods in series : Total equivalent thermal resistance (R) is equal to sum of individual thermal resistances i.e.,

$$R = R_1 + R_2 + .... = \left(\frac{1}{K_1} \times \frac{L_1}{A_1}\right) + \left(\frac{1}{K_2} \times \frac{L_2}{A_2}\right) + \left(\frac{1}{K_3} \times \frac{L_3}{A_3}\right) + .......$$

(b) Rods in parallel : In parallel, the total thermal resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ....$$

### Convection

(i) Convection requires medium and it is the process in which heat is transferred from one place to the other by actual movement of heated substance (usually fluid)

(ii) The type of convection which results from difference in densities is called natural convection (for example, a fluid in a container heated through its bottom). However, if a heated fluid is forced to move (by a blower, fan or pump), the process is called forced convection.

## Radiation

(i) The process through which heat is transferred directly from one body to another, without requiring any medium is called radiation. Heat from the Sun reaches the earth by radiation passing through several vacuums and transparent mediums.

(ii) Radiation is the fastest mode of heat transfer from one place to another as in this mode heat energy is propagated at speed of light in the form of electromagnetic wave.

(iii) All the bodies radiate energy at all temperatures and at all time. Radiation from a body can never be stopped but can be minimized.

(iv) Radiation does not affect the medium through which it passes.

(v) Rough and dark (i.e. black) surfaces are good absorbers while shining and smooth surfaces are good reflectors of heat radiations.

(vi) Heat radiations are invisible and like light, travel in straight lines, cast shadow, affect photographic plates and can be reflected by mirrors and refracted by lenses.

## Black Body

(i) A body which absorbs all the radiations incident on it is called a perfectly black body.

(ii) A perfectly black body maintained at a suitable temperature emits radiations of all wavelengths.

(iii) A perfectly black body neither reflects nor transmits any radiation, it will always appear black whatever be the colour of the incident radiation.

## Absorptive Power (a)

(i) Absorptive power of a surface is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same time. $a = \frac{(\Delta Q)_{\text{absorbed}}}{(\Delta Q)_{\text{incident}}}$

(ii) For a perfectly black body, absorptive power is maximum and it is unity.

(iii) It has no units and dimensions.

## Emissive Power (e or R)

(i) For a given surface it is defined as the radiant energy emitted per sec per unit area of the surface.

(ii) Emissive power of a surface depends on its nature and temperature. (as given by stafan's law)

## Kirchhoff's Law

(i) According to Kirchhoff's law, the ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

(ii) If a and e represent absorptive and emissive power of a given surface while A and E for a perfectly black body then $\frac{e}{a} = \frac{E}{A}$ = constant for all surface

(iii) For a perfectly black body, $A = 1$ So, for any surface $\frac{e}{a} = E$.

For the radiation of a particular wavelength, $\frac{e_\lambda}{a_\lambda} = \frac{E_\lambda}{A_\lambda} \Rightarrow \frac{e_\lambda}{a_\lambda} = E_\lambda$

Since $E_\lambda$ is constant at a given temperature, hence according to this law, if a surface is a good absorber of a particular wavelength, it is also a good emitter of that wavelength. Similarly, bad absorber are bad emitter.

## Stefan's Law

(i) According to it, the radiant energy emitted by a perfectly black body per unit area per sec (i.e., emissive power of radiancy or intensity of black body radiation) is directly proportional to the fourth power of its absolute temperature, i.e,. $e = \frac{\Delta Q}{A.\Delta t} = R \propto T^4$ or $R = \sigma T^4$ where $\sigma$ is called Stefan's constant.

(ii) If a body is not a perfectly black body $R = \frac{\Delta Q}{A(\Delta t)} = \in \sigma T^4$ and hence $\frac{\Delta Q}{\Delta t} = e\sigma A T^4 = \in \sigma A T^4$

where $e = \in$ is called emissivity or relative emittance has value $0 < e < 1$ depending on the nature of surface. It has no units and dimensions. Emissivity is different from emissive power (represented by R).

## Cooling by Radiation

Rate of cooling of a body at temperature T placed in an environment of temperature $T_0 < (T)$ is given by $\frac{\Delta Q}{\Delta t} = \sigma Ae\left(T^4 - T_0^4\right)$ where $T = (T_0 + \Delta T)$ and $\Delta T$ is the temperature difference between body and surrounding.

## Newton's Law of Cooling

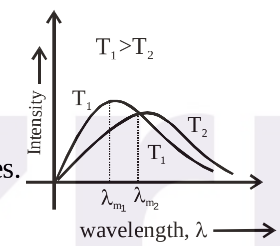
According to Newton's law of cooling, Rate of fall of temperature of a body with time is given by

$$\frac{dT}{dt} = K(T - T_0) \text{ with } K = \frac{eA\sigma}{mc} 4T_0^3$$

where T = temperature of body and $T_0$ = temperature of surrounding.

i.e., rate of cooling of a hot body is directly proportional to temperature difference between the body and its surroundings provided the temperature of the body is not very different from the surroundings.

## Wein's Displacement Law




(i) The quantity of energy radiated out by a body is not uniformly distributed over all the wavelengths emitted by it. It is maximum for a particular wavelength , which is different at different temperatures. As the temperature is increased, the value of wavelength which carries maximum energy is decreased.

(ii) According to this law, wavelength corresponding to maximum energy is inversely proportional to the absolute temperature of the body (i.e., $\lambda_m \propto 1/T$) or $\boxed{\lambda_m T = b}$, b = wein's constant = $2.89 \times 10^{-3}$ mK.

# HEAT AND THERMODYNAMICS (PART - II)

## (KINETIC THEORY OF GAS & THERMODYNAMICS)

### • ASSUMPTIONS OF KINETIC THEORY OF GASES

(i) A gas consists of a large number of identical, tiny spherical, neutral and elastic particles called molecules.

(ii) In a gas, molecules are moving in all possible directions with all possible speeds in accordance with Maxwell's distribution law.

(iii) The space occupied by the molecules is much smaller than the volume of the gas.

(iv) There is no force of attraction among the molecules.

(v) The pressure of a gas is due to elastic collision of gas molecules with the walls of the container.

(vi) The time of contact of a moving molecule with the walls of the container is negligible as compared to the time interval between two successive collisions.

### PRESSURE EXERTED BY AN IDEAL GAS AND ROOT MEAN SQUARE SPEED

(i) Pressure P exerted by an ideal gas is given by

$$P = \frac{1}{3}\rho C^2 \qquad \Rightarrow P = \frac{mN}{3V}C^2$$

where $C^2$ is the mean square velocity , m, mass of each molecule and N, the total number of molecules in the vessel having volume V.

(ii) We define root mean square (rms) speed as

$$C = v_{rms} = \sqrt{\left(C^2\right)} = \sqrt{\frac{(v_1^2 + v_2^2 + .....)}{N}}$$

then equation (1) can be written as

$$v_{rms} = \sqrt{(3P/\rho)} \qquad \text{i.e. } v_{rms} \propto (1/\sqrt{\rho})\text{ ; Hence } v_{rms} \text{ or rate of diffusion } \alpha \frac{1}{\sqrt{\rho}}$$

which is called Graham's Law of Diffusion

(iii) Also translational KE/volume $\quad E = \frac{1}{2}\frac{mN}{V}C^2 = \frac{1}{2}\rho C^2 = \frac{3}{2}P$

hence eq. (1) can be written as $P = \frac{2}{3}\left(\frac{1}{2}\rho C^2\right) = \frac{2}{3}E$

i.e. Pressure of a gas is numerically equal to (2/3) of its K.E. per unit volume.

(iv) From eq. (1) $\quad PV = \frac{1}{3}mN(v_{rms})^2$

$$v_{rms} = \sqrt{\frac{3PV}{mN}} = \sqrt{\frac{3PV}{\text{total mass of gas}}}$$

(v) If M be the molecule weight of gas.

mass of gas = $\mu M$ and $PV = \mu RT$

$$v_{rms} = \sqrt{\frac{3\mu RT}{\mu M}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(N_A K)T}{N_A m}} = \sqrt{\frac{3KT}{m}}$$

- Most probable speed : It is the speed which maximum number of molecules in a gas have. For a gas of molecular weight M at temperature T is given by

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2}{3}}\, v_{rms} = 0.816\, v_{rms}$$

- Average speed : It is the arithmatic mean of the speed of molecules in a gas at a given temperature, i.e.

$$v_{av} = \frac{v_1 + v_2 + v_3 + ......v_n}{N}$$ and according to Kinetic theory of gases,

$$v_{av} = \sqrt{\frac{8RT}{\pi M}} = \left(\sqrt{\frac{8}{3\pi}}\right) v_{rms} = 0.92\, v_{rms}$$ It is thus, evident that $v_{rms} > v_{av} > v_{mp}$

## Kinetic Interpretation of Temperature: Mean Kinetic Energy

According to kinetic theory of gases,

$$PV = \frac{1}{3} mNC^2 \quad \text{or} \quad PV = \mu RT$$

Translational KE of a molecule = $\frac{1}{2} mC^2 = \frac{3}{2} kT$

Further, mean kinetic energy per gm mole is given by

$$E_{mole} = \left(\frac{1}{2} mC^2\right) N_A = \frac{3}{2} kTN_A = \frac{3}{2} RT = \frac{3}{2}(PV) \Rightarrow \text{Energy per unit volume} = \frac{E_{mole}}{V} = \frac{3}{2} P$$

Note that, Average translational KE of a gas molecule depends only on its temperature and its independent of its nature i.e., molecules of different gases say He, $H_2$, and $O_2$ etc., at same temperature will have same translational kinetic energy though their rms speed are different.

## Degrees of Freedom

(i) The term degrees of freedom of a system refers to the possible independent motions, a system can have or number of possible independent modes in which a system can have energy.

(ii) The independent motions of a system can be translational, rotational or vibrational or any combination of these.

(iv) If instead of particle, we consider a molecule of a monoatomic gas (like He, Ar, etc.), which consists of a single atom, the translational motion can take place in any direction in space i.e., it can be resolved along three co-ordinate axes and can have three independent motions and 3 degrees of freedom all translational. A monoatomic molecule can also rotate but due to its small moment of inertia, rotational KE is not significant. Therefore, it does not possess rotational degrees of freedom.

(v) The molecules of a diatomic gas such as ($H_2$, $O_2$ etc) are made of two atoms joined rigidly to one another through a bond. Not only this can move bodily, but also rotate about two of the three co-ordinate axes. However, its moment of inertia about the axis joining the two atoms is negligible compared to that about the other two axes. Hence, it can have only 2 rotational motions. Thus, a diatomic molecule has 5 degrees of freedom: 3 translational and 2 rotational. If vibration is effective then degree of freedom is 7.

## Law of Equipartition of Energy

(i) According to law of equipartititon of energy, energy of a gas molecule is equally distributed among its various degrees of freedom and each degree of freedom is associated with energy $\frac{1}{2}$KT, where K is Boltzmann constant and T temperature of the gas in Kelvin.

(ii) According to equipartitition theorem, the mean internal energy of an ideal monoatomic gas molecule will be

(3/2)kT as it has 3 degrees of freedom. So energy per unit mole = $\left(\frac{3}{2}KT\right).N_A = \frac{3}{2}RT$

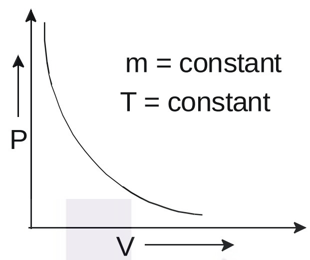
In general, the internal energy of $\mu$ moles of a gas in which each molecule has f degrees of freedom will be

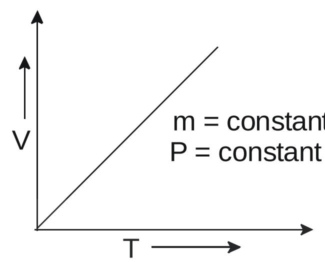
$\boxed{U = \frac{1}{2}\mu fRT}$ For example, for diatomic gas, $f = 5$, so $U = \frac{5}{2}\mu RT$

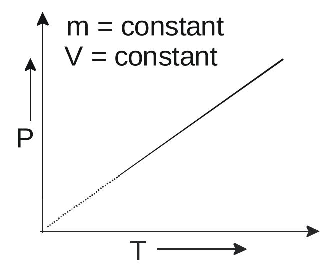
## Gas-Laws

(i) Boyle's Law: According to it for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure i.e.,

$V \propto \frac{1}{P}$ if mass of gas and T are constant ; Graphical forms of the law are as follows:




(ii) Charle's Law: According to it for a given mass of an ideal gas at constant pressure, volume of a gas its directly proportional to its absolute temperature i.e, $V \propto T$ if m and T = constant

(iii) Gay-Lussac's Law: According to it, for a given mass of an ideal gas at constant volume, pressure of a gas is directly proportional to its absolute temperature, i.e., $P \propto T$ if m and V = constant

(iv) Avogadro's Law: According to it, at same temperature and pressure equal volumes of all the gases contain equal number molecules, i.e., $N_1 = N_2$ if P, V and T are same

(v) Dalton's Law: According to it, the pressure exerted by a gaseous mixture is equal to the sum of partial pressure of each component present in the mixture, i.e., $P = P_1 + P_2 + .... P_n$

## THERMODYNAMICS

(i) A system whose state is completely defined by the variables like pressure (P), volume (V), temperature (T) internal energy(U),is called thermodynamic system.

(ii) The variables P, V, T, U whose knowledge specifies the state of a thermodynamical system, are called as thermodynamic variables/parameters.

(iii) All the variables P, V, T, U are not independent as relations like $PV = \mu RT$, $U = (f/2)\mu RT$, connecting these variables exist.

(iv) A P-V diagram for a system is called an indicator diagram. Each dot in a P-V diagram represents a possible state of the system.

(v) A curve drawn between two points on the indicator diagram shows a thermodynamic process obeying some rule.

(vi) The area under a curve on P-V diagram shows work done ON or BY the system.

(vii) Work done ON or BY a gas or system and heat exchange by a system depends upon both the initial state, final state and the path adopted between these two states.

(viii) Change in internal energy of a gas depends only upon initial and the final state but not on the path. It is a unique function of the point on the indicator diagram.So during isothermal process change in internal energy is zero.

## Different Thermodynamical Processes

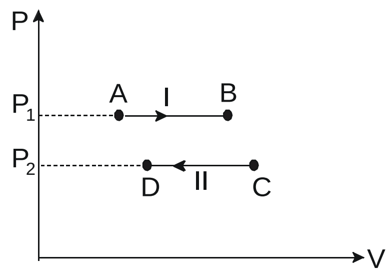
## (i) Isobaric Process

(a) It is thermodynamic process in which pressure is kept constant. i.e. $\frac{V}{T} = \text{constant}$

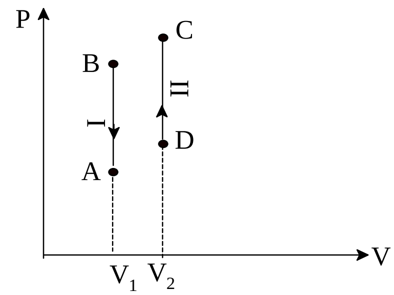
(b) The amount of heat energy transferred is given by $\Delta Q = \mu C_P \Delta T$ ($\mu$ = number of moles)

(c) In the adjoining figure, graphs I and II represent isobaric expansion and compression respectively.

Isobaric Process




Isochoric




## ii) Isochoric (isometric) process

(a) It is thermodynamic process in which the volume of the system is kept constant i.e. (P/T) = constant.

(b) For increasing (decreasing) the pressure of a gas at constant volume, its temperature must be increased (decreased) by adding (taking out) heat energy into (from) the system.

(c) For isochoric process:

(d) In the adjoining figure, graphs I and II represents decrease in pressure at volume $V_1$ and isometric increase in pressure at volume $V_2$ respectively.

## (iii) Isothermal Process

(a) It is a thermodynamic process in which the temperature of the system remains constant. (P.V = constant)

(b) During isothermal expansion of a gas, its volume increases while pressure decreases, while in isothermal compression, the volume decreases while pressure increases.

(c) The isothermal curve on P-V diagram is like a Hyperbola and the slope is given by: $\frac{dP}{dV} = -\left(\frac{P}{V}\right)$

(d) elasticity constant for isothermal process is B = P = Pressure

## (iv) Adiabatic Process

(a) A process in which heat exchange between the system and the surroundings is zero ( $PV^{\gamma} = \text{constant}$ )

(b) The adiabatic curve on the PV diagram is shown in the figure. The slope of the curve at any point is given

by: $\frac{dP}{dV} = -\gamma\left(\frac{P}{V}\right)$ where $\gamma = \frac{C_P}{C_v}$ Hence $(\text{Slop})_{\text{Adiabatic}} = \gamma(\text{slope})_{\text{Isothermal}}$

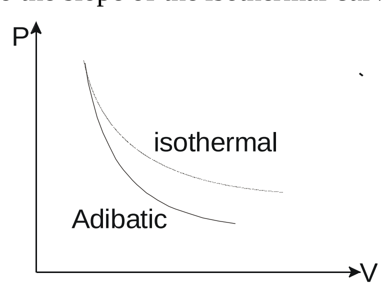
Slope of adiabatic curve is more in magnitude in comparison to the slope of the isothermal curve.

(e) Equation of state of the adiabatic process can be any of the following three types:

(i) $PV^{\gamma}$ = constant

(ii) $TV^{\gamma-1}$ = constant,

(iii) $P^{1-\gamma}T^{\gamma}$ = constant




(f) In adiabatic expansion, temperature decreases while in adiabatic compression, temperature increases.

(g) Isothermal and adiabatic Bulk modulus of a gas are given by $B_{isothermal} = P$ and $B_{adiabatic} = \gamma P$

## Work done in Thermodynamic Process

(i) Total work done by a thermodynamic process in going from some initial state to final state is: $W = \int_{V_i}^{V_f} PdV$

This is equal to area under the curve on P-V diagram.

(ii) Work done by as gas during expansion is taken as positive (as is positive) while during compression it is taken as negative (as is negative).

(iii) Work done in various Thermodynamic processes

(a) Isobaric process: $W = P(V_f - V_1)$

(b) Isometric process: $W = O$ $(\because \Delta V = 0)$

(c) Isothermal process: $W = \mu RT \log_e\left(\frac{V_f}{V_i}\right) = \mu RT \log_e\left(\frac{P_i}{P_f}\right) = P_i V_i \log_e\left(\frac{V_f}{V_i}\right)$

(d) Adiabatic process: $W = \frac{P_i V_i - P_f V_f}{\gamma - 1} = \frac{\mu R(T_i - T_f)}{\gamma - 1} = \mu C_V (T_i - T_f)$ $\quad \because \quad C_V = \frac{R}{\gamma - 1}$

(e) Cyclic process: W = area enclosed in the cycle. This work done is positive when the cycle is traced out in clockwise direction and work is negative when the cycle is traced out in anti-clockwise direction.

## Internal Energy of a Gas

In practice, we ignore potential energies of the molecules and neglect intermolecular forces. Thus, the total KE of all the molecules of the gas is equal to the internal energy of the gas. So, $U = \frac{f}{2} nRT$

## Zeroth Law of Thermodynamics

When two bodies A and B are in thermal equilibrium with a third body C, then A and B are in thermal equilibrium mutually i.e, if $T_A = T_C$ and $T_B = T_C$, then $T_A = T_B$

## First Law of Thermodynamics

(i) First law of thermodynamics is equivalent to law of conservation of energy.

(ii) According to this law, if heat $\Delta Q$ is added to a system then it will be used either as change in internal energy $\Delta U$ of the system and/or as work $\Delta W$ performed by the system i.e., $\Delta Q = \Delta U + \Delta W$

## Specific Heats of a Gas

(i) Gases have infinite number of specific heats but two are mainly used,

(a) Specific heat of a gas at constant volume $(C_v) = \frac{(dQ)_v}{\mu dT}$ ; $dQ = \mu C_v dT = dU$ $(\because dW = PdV = 0)$

(b) Specific heat of a gas at constant pressure $(C_p) = \frac{(dQ)_p}{\mu dT}$ ; $dQ = \mu C_p dT = dU + dW = \mu C_v dT + nRdT$

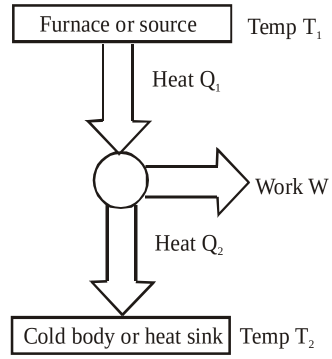
hence $C_p = C_v + R$ $\Rightarrow$ $\boxed{C_p - C_v = R \text{ and } \frac{C_p}{C_v} = \gamma}$

(ii) For any gas: $\gamma = 1 + (2/f) = \frac{f+2}{f}$ where f represents the degrees of freedom)

## Second Law of Thermodynamics

(i) Kelvin's statement: it is impossible for an engine operating in a cyclic process to extract heat from a reservoir and convert it completely into work.

(ii) Clausius statement: It is impossible for a self acting machine unaided by any external agency to transfer heat from a cold to hot reservoir i.e, heat by itself cannot pass from a colder to hotter body.




## Heat Engine

(i) It is a device which converts heat into mechanical work continuously through a cyclic process.

(ii) In a heat engine working substance absorbs heat from the source at a higher temperature $T_1$ converts a part of it into useful work (motion of piston) and rejects the rest to the sink (usually atmosphere) at a lower temperature $T_2$ and comes back to its initial state (thus works in cyclic process).

(iii) Efficiency of a heat engine

$$\eta = \frac{\text{work done}}{\text{heat absorbed}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1};\ \text{as} \frac{Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{T_{low}}{T_{High}}$$

(iv) A perfect heat engine is one which converts all heat into work i.e., $W = Q_1$ so that $Q_2 = 0$ and hence for it $\eta = 1$

## Refrigerator or Heat Pump

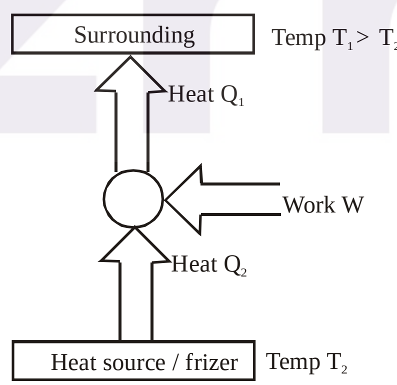
(i) A refrigerator or heat pump is basically a heat engine running in reverse direction. In it working substances takes heat $Q_2$ from a body at lower temperature $T_2$ has a net amount of work done on it by an external agent (usually compressor) and gives out a larger amount of heat $Q_1 (= Q_2 + W)$ to a hot body at temperature $T_1$ (usually atmosphere).

(ii) A refrigerator or heat pump transfers heat from a cold to a hot body at the expense of mechanical energy supplied to it by an external agent. The working substance here is called Refrigerant and works in cyclic process.

(iii) The coefficient of performance of a refrigerator is defined as

$$\beta = \frac{\text{heat extracted from the reservoir at low temperature } T_2}{\text{Work done to transfer the heat}}$$

$$= \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \Rightarrow \frac{1}{\beta} = \frac{Q_1}{Q_2} - 1 = \frac{1}{1-n} - 1$$




(iv) A perfect refrigerator is one which transfers heat from cold to hot body without doing any work i.e., W = 0 so that $Q_1 = Q_2$ and hence for it $\beta \to \infty$

## Carnot Heat Engine

(i) Engine: It consists of four parts (a) A cylinder with perfectly insulating walls and a perfectly conducting base containing a perfect gas as working substance and fitted with a insulating frictionless piston; (b) A source of infinite thermal capacity maintained at a constant higher temperature $T_H$; (c) A sink of infinite thermal capacity maintained at constant lower temperature $T_L$ and (d) A perfectly non-conducting stand for cylinder.

(ii) Carnot Cycle: It consist of four operations in succession: (a) isothermal expansion at higher temperature $T_H$ (b) adiabatic expansion between temperatures $T_H$ and $T_L$ (c) isothermal compression at constant lower temperature $T_L$ and (d) adiabatic compression between temperatures $T_L$ and $T_H$.

(iii) Efficiency of the engine: $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{Q_2}{Q_1}$

# Simple Harmonic Motion

## INTRODUCTION

A particle has oscillatory (vibrational) motion when it moves periodically about a stable equilibrium position. The motion of a pendulum is oscillatory. A weight attached to a stretched spring, once it is released, starts oscillating.Of all the oscillatory motions, most important is called simple harmonic motion (S.H.M.). In this type of oscillatory motion, displacement, velocity, acceleration and force all vary (w.r.t to time) in a way that can be described by either the sine or the cosine function collectively called sinusoids.

In S.H.M. the restoring force acting on the particle is directly proportional to its displacment from the equilibrium position.

$$F \propto x$$

$$F = -Kx$$

K is the constant of proportionality and (–ve) sign shows that the force is always directed toward the mean position.

Using newtons second law SHM is dessembed by,

$$F = ma = \frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where, $\omega = \sqrt{\frac{k}{m}}$ = Angular frequency of SHM.

## KINEMATICS OF S.H.M.

A particle has S.H.M. along the axis OX when its displacement x relative to the origin of coordinate system is given as a function of time by the relation.

$$x = A\sin(wt + \phi)$$

The quantity $(wt + \phi)$ is called the phase angle of the SHM and $\phi$ is called the initial phase i.e. phase at $t = 0$ . The maximum displacement from the origin A, is called the amplitude of the SHM.

Period : $T = \frac{2\pi}{\omega}$

Frequency : No. of oscillations per unit time

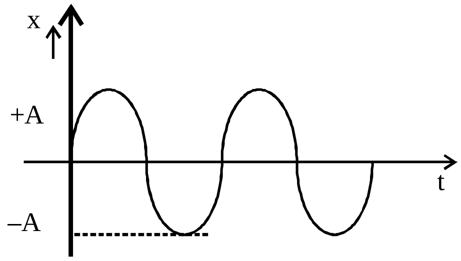
Angular fequency : $\omega = \frac{2\pi}{T} = 2\pi\nu$

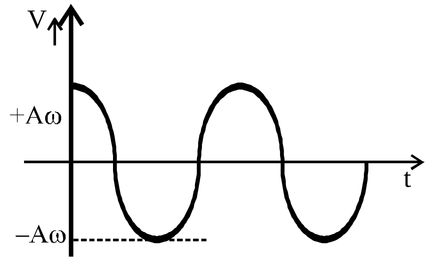
Velocity: $v = \frac{dx}{dt} = -A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$

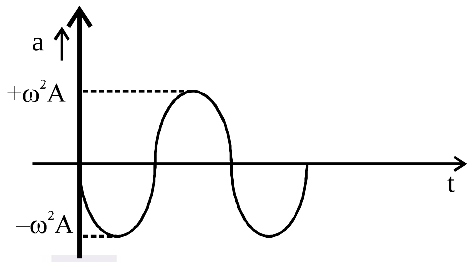
which varies periodically between the values $+\omega A$ and $-\omega A$

Acceleration: $a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t + \phi) = -\omega^2 x$

and varies periodically between the values $+\omega^2 A$ and $-\omega^2 A$




## Velocity and Acceleration as a Function of Displacement

By defintion, the acceleration of a body in S.H.M. is proportional to the displacement. i.e.

$$a = -\omega^2 x$$

$$a = \frac{dv}{dt} = -\omega^2 x$$

$$\Rightarrow \quad \frac{dv}{dx}.\frac{dx}{dt} = -\omega^2 x$$

$$\Rightarrow \quad v.\frac{dv}{dx} = -\omega^2 x \Rightarrow vdv = -\omega^2 x.dx$$

Integrating both sides

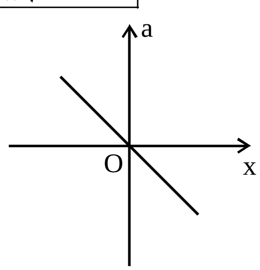
$$\int vdv = -\omega^2 \int x.dx$$

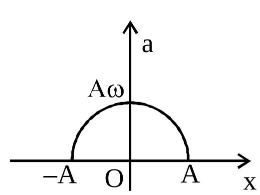
$$\frac{v^2}{2} = -\omega^2 \frac{x^2}{2} + c$$

when x = 0, v = $v_{max}$ = $A\omega$

$$\therefore \quad C = \frac{A^2\omega^2}{2}$$

$$\therefore \quad \boxed{v = \omega\sqrt{A^2 - x^2}}$$



(a) Slop of the line $= -\omega^2$ (b) Variation of v with x is ellipse

## KINETIC ENERGY

The kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$$

and using $x = A\sin(\omega t + \phi)$, we can also express K.E. as

$$K = \frac{1}{2}m\omega^2A^2\left(1 - \sin^2(\omega t + \phi)\right)$$

$$\boxed{K = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)}$$

At $x = \pm A$, $K = K_{min} = 0$

At $x = 0$, $K = K_{max} = \frac{1}{2}m\omega^2A^2$

## POTENTIAL ENERGY

To obtain the potential energy, we use the relation.

$$U = -\int_0^x F.dx = -\int_0^x (-Kx)dx$$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$$

At $x = \pm A$, $U = U_{max} = \frac{1}{2}m\omega^2x^2$

At $x = 0$, $U = K_{min} = 0$

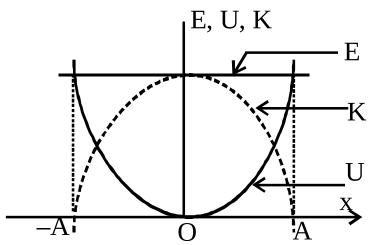
## TOTAL ENERGY

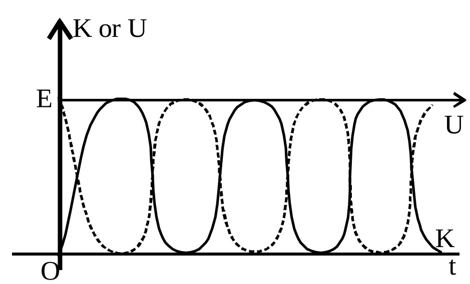
Total Energy E

$$E = K + U = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) + \frac{1}{2}m\omega^2x^2$$

$$\boxed{E = \frac{1}{2}m\omega^2A^2}$$

which is a constant quantity. This was to be expected since the force is conservative.



## TIME PERIOD AND FREQUENCY OF S.H.M.

| Linear S.H.M. | Angular SHM |
|---|---|
| $F = -kx$ | $\tau = -k\theta$ |
| $ma = -kx$ | $I\alpha = -k\theta$ |
| $a = -\frac{k}{m}x$ | $\alpha = -\frac{k}{I}\theta$ |
| $= \omega^2 x$ | $\alpha = -\omega^2\theta$ |
| $\therefore \quad \omega = \sqrt{\frac{k}{m}}$ | $\therefore \quad \omega = \sqrt{\frac{k}{I}}$ |
| $\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$ | $\frac{2\pi}{T} = \sqrt{\frac{k}{I}}$ |
| $\therefore \quad T = 2\pi\sqrt{\frac{m}{k}}$ | $\therefore \quad T = 2\pi\sqrt{\frac{I}{k}}$ |

General Formula therefore for Time period is $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Force factor}}}$

## SPRING MASS SYSTEM

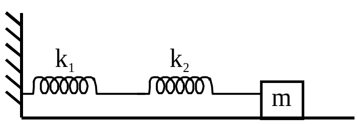
### (I) Series Combination

Two springs are said to be series when both are stretched with the same force F and the total displacement is the sum of individual deformation of each spring i.e.

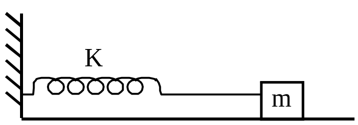
$$F = k_1 x_1 = k_2 x_2$$
$$x = x_1 + x_2$$

where $x_1$ and $x_2$ are deformations in the springs of constants $k_1$ and $k_2$ respectively.
Equivalent stiffness of the spring is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$



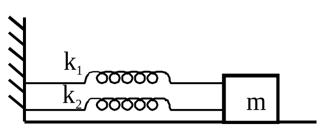

and $\quad T = 2\pi\sqrt{\frac{m}{k}}$




### (II) Parallel Combination

Two springs are said to be in parallel when both are stretched to the same deformation x and the total force F they exert on the block is equal to the sum of the individual forces. i.e.

$$x_1 = x_2 = x$$
$$F = F_1 + F_2 = (k_1 + k_2)x$$
$$\therefore \quad K = (k_1 + k_2)$$
$$T = 2\pi\sqrt{\frac{m}{K}}$$



## RELATION BETWEEN STIFFNESS AND LENGTH OF THE SPRING

The stiffness K of a spring is inversely proportional to its length l.

$$K \propto \frac{1}{l}$$

If a spring of stiffness K and length l is cut into two parts of length $l_1$ and $l_2$, then the stiffness $k_1$ and $k_2$ of each part are given as

$$\frac{k_1}{k_2} = \frac{l_2}{l_1}$$

$$k_1 = \left(\frac{l_1 + l_2}{l_1}\right)K$$

$$k_2 = \left(\frac{l_1 + l_2}{l_2}\right)K$$

## PENDULUMS

(i) Simple Pendulum: For small oscillations, the time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The ime period is independent of mass.

(ii) If the time period of a simple pendulum is 2 seconds, it is called seconds pendulum.

(iii) If the length of the pendulum is large, g no longer remain vertical but will be directed towards the centre of the earth and then time period.

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$$

R = radius of the earth

(a) If l <<R, $\frac{1}{l} >> \frac{1}{R}$ and $T = 2\pi\sqrt{\frac{l}{g}}$

(b) If l >> R, $\frac{1}{l} << \frac{1}{R}$ and $T = 2\pi\sqrt{\frac{l}{g}}$ = 84.6 min

(iv) Time period of a simple pendulum depends on acceleration due to gravity. Then

$$T = 2\pi\sqrt{\frac{l}{|\vec{g}_{eff}|}} \qquad \text{where } \vec{g}_{eff} = \vec{g} - \vec{a}$$

(a) If a simple pendulum is in a carriage which is accelerating with acceleration $\vec{a}$, upwards, then

$$g_{eff} = g + a$$

$$T = 2\pi\sqrt{\frac{l}{a+g}}$$

(b) If the carriage is moving downwards.

$$g_{eff} = g - a$$

$$T = 2\pi\sqrt{\frac{l}{g-a}}$$

(c) If the carriage is in horizontal direction, then

$$g_{eff} = \sqrt{a^2 + g^2}$$

$$T = 2\pi\sqrt{\frac{l}{\sqrt{a^2+g^2}}}$$

(d) In a freely falling lift $g_{eff} = 0$, $T = \infty$, i..e. pendulum will not oscillate.

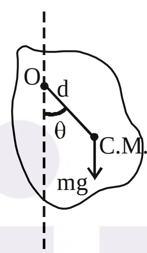
(e) If in addition to gravity, one additional force $\vec{F}$ (e.g. electrostatic force $\vec{F}_e$) is also acting on the bob, then in that case, $\vec{g}_{eff} = \vec{g} + \frac{\vec{F}}{m}$

## PHYSICAL PENDULUM

A physical pendulum is an extended body pivoted about point O, which is at a distance d from its centre of mass. For small angular displacement $\theta$,the restoring torque is given by

$$\tau = -mgd\sin\theta = -mgd\theta$$

Using Newton's second law




$$I\frac{d^2\theta}{dt^2} = -mgd.\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = \frac{-mgd}{I}.\theta$$

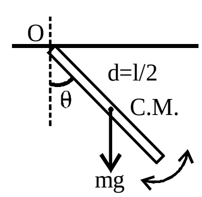
Time period

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

### Some Special Cases

(a) A rod of mass m and length l suspended about its its end.

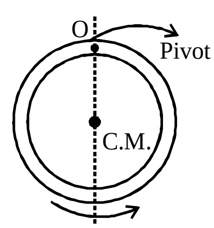
Hence, $d = \frac{l}{2}$, $\quad I = \frac{ml^2}{3}$




$$\therefore \quad T = 2\pi\sqrt{\frac{2l}{3g}}$$

(b) Fig. shows a ring of mass m and radius R, pivoted at a point O on its periphery. It is free to rotate about an axis perpendicular to its plane.

Here, d = R and
I = $2mR^2$




$$\therefore \quad T = 2\pi\sqrt{\frac{2R}{g}}$$

# Wave Motion

## WAVE

It is a process by which transfer of energy and momentum takes from one portion of medium to another protion of medium, without any actual motion of particles of medium.

## WAVE FUNCTION

Any function of space and time which obeys

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \quad \text{represents a wave.}$$

## TRAVELLING WAVE OR PROGRESSIVE WAVE

Any wave equation which is in the form of $y = f(\omega t \pm kx)$ or $y = f(x \pm vt)$ represents a progressive wave.

(a) If t and x are of opposite sign, wave is propagating along positive x-axis.

(b) If t and x are of same sign, the wave is propagating along negative x-axis.

(c) Wave speed $c = \frac{\omega}{k}$

(d) If $\omega t - kx$ = cosntant, then the slope of wave remains constant.

(e) Particle velocity $\quad v_P = \frac{dy}{dt}$

(f) $\text{slope} = \frac{dy}{dx}$

(g) For a wave, $\quad v_P = -v(\text{slope})$

## PLANE HARMONIC PROGRESSIVE OR TRAVELLING WAVE

(a) The equation of plane harmonic progressive wave travelling along (+ve) x-axis is

$$y = A\sin(\omega t - kx)$$

$$y = A\sin(\omega t - kx + \phi)$$

(b) Wave moving along (–ve) x-axis

$$y = A\sin(\omega t + kx)$$

$$y = A\sin(\omega t + kx + \phi)$$

## SPEED OF TRANSVERSE WAVE ON A STRETCHED STRING

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}$$

where $\quad$ T = tension in string

$\mu =$ mass per unit length

$\rho =$ density

$A =$ area of cross section

## Stationary or Standing Wave

The superposition of two identical waves travelling in opposite directions along the same line gives rise to stationary waves. If two waves

$$y_1 = a\sin(\omega t - kx), \qquad y_2 = a\sin(\omega t + kx)$$

These two waves superpose to form stationary wave.

$$y = y_1 + y_2 = 2a\cos kx.\sin\omega t$$

## Some important points regarding Standing Waves

1. Every particle of the medium vibrates in the same manner but amplitude depends on its position

   Amplitude in a starting wave
2. The point of medium with zero amplitude is a point of node and the point of medium with maximum amplitude is a point of antinode.
3. The particle of medium at node remains permanently at rest. Also nodes divide the medium into loops. All particle of a medium lying in a loop vibrate in the same phase with different amplitude.
4. Total energy of a loop remains constant.
5. At node, displacement is zero but pressure is maximum.
6. At anitnode, displacement is maximum but pressure is minimum.
7. The distance between nearest node and antinode is $\frac{\lambda}{4}$
8. The distance between successive nodes or antinodes is $\frac{\lambda}{2}$
9. Frequency of vibration f is

   $$f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$ , where l = length of string, $\mu$ = linear mass density, T= tension
10. If the string vibrates in P loops, $p^{th}$ harmonic is produced, then frequency of $p^{th}$ harmonic is

    $$f_p = \frac{P}{2l}\sqrt{\frac{T}{m}}$$
11. The average power transmitted by a wave is, $P = \frac{1}{2}\mu A^2\omega^2 c$
12. The instensity of wave is, $I = \frac{1}{2}\rho\omega^2 A^2 c$

# SOUND WAVES

## Longitudinal Wave

If a longitudinal wave is passing through a medium, the particles of medium oscillate about their mean position along the direction of propagation of wave. The propagation of transverse wave takes place in the form of crest and trough. But the propagation of longitudinal wave takes place in the form of rarefaction and compression.

## Points

1. Mechanical transverese wave is not possible in gaseous and liquid medium. But longitudinal wave is possible in solid, liquid and gas.

2. In liquid and gas, sound is longitudinal wave
3. The velocity of longitudinal wave

$$\nu = \sqrt{\frac{E}{\rho}}$$ , where E = modulus of elasticity, $\rho$ = density of medium

## Velocity of Sound

(a) In solid, $V = \sqrt{\frac{Y}{\rho}}$, Y = Young's modulus of elasticity

(b) In a fluid (gas or liquid) $V = \sqrt{\frac{B}{\rho}}$, B = Bulk modulus

## SPEED OF SOUND IN AIR

Speed of sound in air is adiabatic. Rarefactors and compressions are so rapid that there is no exchange of heat. Modulus of elasticity involved would then be adiabatic back modulus.

$$E = B_{adiabatic} = \gamma p$$

The speed of sound in therefore,

$$= \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} = 331.3 ms^{-1}$$

## Points

1. The speed of sound does not change due to variation of pressure.
2. $$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}}$$
3. Due to change of temperature by 1°C, the speed of sound is changed by 0.61 $ms^{-1}$.
4. For small variation of temperature.
$$V_t = (V_0 + 0.61t) ms^{-1}$$
5. The speed of sound increase due to increase of humidity.

## DISPLACEMENT WAVE AND PRESSURE WAVE

If the displacement wave equation is

$y = A\sin(\omega t - kx)$, then the pressure wave is

$$P = P_0 \cos(wt - kx)$$

$$P_0 = BAk$$

where, k = angular wave number

A = displacement amplitude

B = Bulk modulus of elasticity

## ENERGY OF SOUND

The kinetic energy per unit volume of medium is $\frac{1}{2}\rho\omega^2 a^2 \cos^2(wt - kx)$,

$\rho$ = density of medium

a = displacement amplitude

Energy density = $(K.E.)_{max} = \frac{1}{2}\rho a^2\omega^2$

## POWER

It is defined as rate of transmission of energy

$$\overline{P} = \frac{1}{2}\rho v w^2 a^2 A$$

## INTENSITY

$$I = \frac{1}{2}\rho\omega^2 a^2 v = \frac{\overline{P}}{A} = \frac{P_0^2}{2\rho c}$$

Loudness = Intensity level = L

$$= 10\log_{10}\left(\frac{I}{I_0}\right) dB$$

Here, $I_0 = 10^{-12}$ w $m^{-2}$ Threshold of hearing

For zero level sound, $I = I_0$

## ORGAN PIPE

It is a cylindrical tube of uniform cross section.

(a) Closed End Organ Pipe : It's one end is closed

$$l = (2n-1)\frac{\lambda}{4}, \; f = \frac{(2n-1)}{4l}V$$

closed end pipe

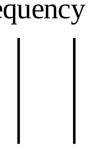
where, V = speed of wave, l = length of tube, n = 1, 2, 3, ....

### Points:

1. The closed end is always a point of displacement node and pressure antinode.
2. Open end of the closed end organ pipe is always a point of displacement antinode and pressure node.
3. The maximum possible wavelength is 4 l.
4. The fundamental frequency is $f_1 = \frac{V}{4l}$
5. Present harmonics: 1st, 3rd, 5th and so on.
   Present overtones: Fundamental, 1st, 2nd, 3rd and so on.
6. The modes of vibration of closed end organ pipe are similar to the modes of vibration of rod fixed or clamped at one end.

(b) Open End Organ Pipe: Its both ends are open. The frequency of vibration is

$$f = \frac{nv}{2l}, \qquad n = 1, 2, 3, ......$$



open end pipe

## POINTS:

1. All harmonics are present
2. Open ends are points of displacement antinode and pressure node.
3. Possible harmonics: 1, 2, 3, 4, 5,
   Possible overtones: fundamental, 1, 2, 3, 4,
4. The maximum possible wavelength is 2l.
5. Fundamental frequency $f_1 = \frac{v}{2l}$
6. When an open end organ pipe is submerged in water upto half of its length it behaves as a closed end organ pipe. But frequency remains unchanged.
7. If the diameter of organ pipe decreases, frequency increases.

## RESONATING AIR COLUMN EXPERIMENT

(a) At first resonance

$$L_1 + e = \frac{\lambda}{4}$$

At second resonance

$$L_2 + e = \frac{3\lambda}{4}$$

so, $L_2 - L_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$

or, $\lambda = 2(L_2 - L_1)$

(b) End correction $e = \frac{(L_2 - 3L_1)}{2}$

## Beats

The superposition of two waves of small difference in frequency in same direction produces beats.

If $y_1 = a\sin\omega_1 t$ and $y_2 = a\sin\omega_2 t$

then $y = 2a\cos\omega t.\sin\omega_{av} t$

Here $\omega = \frac{\omega_1 - \omega_2}{2} = 2\pi\left(\frac{f_1 - f_2}{2}\right)$

$$\omega_{av} = \frac{\omega_1 + \omega_2}{2} = 2\pi\left(\frac{f_1 + f_2}{2}\right)$$

## POINTS

1. The beat frequency = number of beats per second = $|f_1 - f_2|$
2. In the case of beats, the intensity at a point varies periodically
3. Due to waxing of tunning fork, frequency decreases.
4. Due to filing a tunning fork, frequency increases.

## DOPPLER'S EFFECT OF SOUND

$$n' = \left(\frac{V - V_0}{V - V_s}\right)n$$

$V =$ velocity of sound in medium, $V_0 =$ velocity of observer in the medium

$V_s =$ velocity of source in the medium, $n' =$ apparent frequency, n = frequency of source

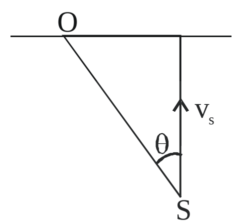
(a) When source moves towards stationary observer $n' = \left(\frac{V}{V - V_s}\right)n$

(b) When source moves away from observer. $n' = \left(\frac{V}{V + V_s}\right)n$

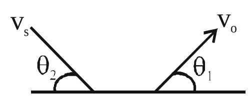
(c) When observer moves towards the stationary source $n' = \left(\frac{V + V_0}{V}\right)n$

(d) When observer moves away from the stationary source $n' = \left(\frac{V - V_0}{V}\right)n$

(e) $n' = \left(\frac{V}{V - V_s \cos\theta}\right)n$

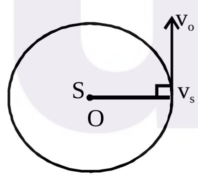



(f) $n' = \left(\frac{V - V_0 \cos\theta_1}{V - V_s \cos\theta_2}\right)n$




(g) When source is at centre and observer is moving on the circular path.

$n' = n$



# Electrostatics
## (Electric Field & Potenial)

### ELECTROSTATICS / FIELD

Electrostatics is a branch of physics which deals with charge at rest charge is the origin of electromagnetic force. When charge is at rest then region around it called electric field while the region around the moving charge called magnetic field.

### PROPERTIES OF CHARGE :

1. For a given closed system the total charge remains conserved.
2. Charge is always quantised i.e. $Q = \pm nc$ ; $n \in I$
   Charge on any body is always in the integral multiple of one electronic charge ($e = 1.6 \times 10^{-19}$ coul).
3. Charge is relativistically invariant.

### TYPES MATERIAL

There are three kinds of material on the basis of conductivity.

(a) Conductor : Having large number of mobile electrons. It is approximately $10^{21}$ electrons/c.c.

(b) Bad conductor : Having very small number of free electrons, it is approximately $10^{7}$ electrons c.c

(c) Semi conductor : Conductivity lies between conductor and insulator. Number of free electrons is approximately $10^{4}$ electron/c.c.

### COULOMB'S LAW :

When two point charges $q_1$ and $q_2$ are separated by a distance r then force of mutual intraction F is given by

$$F \propto q_1 q_2 \quad \text{when } r = \text{constant.}$$

and $$F \propto \frac{1}{r^2} \quad \text{when } q_1 q_2 = \text{constant}$$

Hence $$F \propto \frac{q_1 q_2}{r^2} \text{ or } F = k.\frac{q_1 q_2}{r^2}$$ ; K is proportionality constant

In SI units, $$F = \frac{1}{4\pi \in_0}.\frac{q_1 q_2}{r^2}; \qquad \in_0 = 8.85 \times 10^{-12} F_m \text{ and } \frac{1}{4\pi \in_0} = 9 \times 10^{9} \text{ nt m}^2 \text{ coul}^{-2}$$

In cgs units, $$F = \frac{q_1 q_2}{r^2} \quad ; \quad \in_0 \text{ is per mittivity of the vacuum.}$$

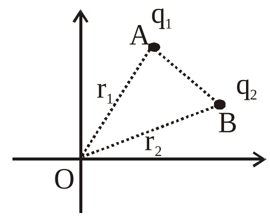
When there is a medium in the intervining region of two charges then

$$F = \frac{1}{4\pi \in_0 \in_r} \cdot \frac{q_1 q_2}{r^2}$$ ( Î$_r$ is the relative permitivity of medium)

Î$_r$ is the relative permitivity which is the dimensionless qunatity that gives the factor by which force is reduced compared to vaccum.

$\in_r = \frac{F_0}{F_m}$, $F_0$ is the force in vacuum and $F_m$ is force in medium.

$\in = \in_0 \in_r =$ Absolute permittivity of the medium.




$$\vec{F}_{AB} = \frac{1}{4\pi \in_0} \cdot \frac{q_1 q_2}{|\vec{BA}|^2} \cdot \frac{\vec{BA}}{|\vec{BA}|}$$

$$\vec{F}_{AB} = \frac{q_1 q_2}{4\pi \in_0 |\vec{BA}|^3} \cdot \vec{BA}$$

where $\vec{BA} = \vec{OA} - \vec{OB} - = \vec{r}_1 - \vec{r}_2$

$$\vec{F}_{AB} = \frac{q_1 q_2}{4\pi \in_0 |\vec{r}_1 - \vec{r}_2|^3} \cdot (\vec{r}_1 - \vec{r}_2)$$

when $q_1 q_2 > 0$, Force is attractive

and $q_1 q_2 < 0$, Force is repulsive

## Process of Charging :

(1) By rubbing or friction - when two bodies are rubbed together there is transfer of electrons from body, which is surplus in electron to another body which is surplus in electrons and get positive charge of equal amount to negative charge.

(2) By conduction - when a charged body is in contact with another uncharged one threre is redistribution of charges on entire are is of both bodies followed by mechanical separation. The amount of charge redistribution on body depends on surface area.

(3) By induction - when an uncharged body is brought near charged on the charge opposite nature induced over the uncharged one.

The induced charge is always less than or equal to inducing charge. Induction is always followed by attraction, but attraction is not the surest test of induction.

If q be inducing charge, then charge induced on a body having dielectric constant K is given by

$q' = -q\left(1 - \frac{1}{K}\right)$, if charge is induced on the surface of a conductor then induced charge is

$q' = -q$ (As k is infinity for a conductor)

## Distribution of Charges :

(a) Linear charge distribution : - If charge gets appeared on a body of linear dimension.

Linear charge density $(\lambda) =$ charge per unit length.

(b) Surface charge distrbution :- If charge gets appeared on a body having two dimensions.

Surface charge density $(\sigma) =$ charge per unit area

(c) Volume charge density : If charge is enclosed in a volume;

Volume charge density=charge per unit volume

Electric field : The site around the charge at rest called electric field.

Electric field strength or Electric intensity is defined as the force experienced by a unit positive charge.

$\vec{E} = \lim_{q_0 \to 0} \frac{F}{q_0}$ where $q_0$ is the positive test charge.

Unit of $\vec{E}$ is newton coulmob and dimension is $\left[MLT^{-3}A^{-1}\right]$. The resultant electric field at any point is equal to vector sum of electric field at that point due to various charges i.e. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + .....$

Resultant of two electric fields which are at an angle $\theta$ is given by

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta} \text{ and } \tan\alpha = \frac{E_2\sin\theta}{E_1 + E_2\cos\theta}$$

$\alpha$ is the angle of $\vec{E}$ with $\vec{E}_1$

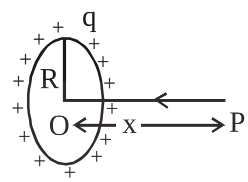
Electric field intensity $(\vec{E})$ for some body having uniformly continuous charge distribution.

1. Electric field strength due to a point charge at a distance r is given by

$$\vec{E}_1 = \frac{1}{4\pi\in_0}\frac{q}{r^2}.\hat{r}$$

2. Electric field strength due to a uniformly charged ring at a distance x from centre of the axis of ring..

$$\vec{E} = \frac{1}{4\pi\in_0}\frac{qx}{(x^2+R^2)^{3/2}}$$




E becomes max at $x = \frac{R}{\sqrt{2}}$: Direction of $\vec{E}$ is away from centre along axis.

3. Electric field strength due to uniformally charged rod of length l at a distance r along perpendicular line from centre and linear charge density is $\lambda$.

$$E \neq \frac{q}{4\pi\in_0 r\sqrt{\frac{l^2}{4}+r^2}}$$

where $\lambda$ = charge per unit length when l $\rightarrow \infty$

$$E = \frac{\lambda}{2\pi\in_0 r}$$

4. Electric field strength due to uniformally charged spherical shell of radius R at a distance r from, centre of shell

$E = 0$, if $r < R$

$E = \frac{Q}{4\pi\in_0 R^2} = \frac{\sigma}{\in_0}$; if $r \geq R$ where Q is charged on shell or $\sigma$ is surface charge density.

5. Electric field strength due to uniformally charged solid sphere of radius R at a distance r from centre of sphere.

$E = \frac{Q}{4\pi\in_0 R^3}r = \frac{\rho}{3\in_0}r$; r < R; where Q is charged in solid sphere enclosed ;

$\rho$ is volume charge density and $E = \frac{Q}{4\pi\in r^2}$ where $r \geq R$

## Electric Dipole :

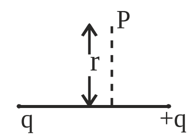
A system of two equal and opposite charges separated by a small distance called dipole. The dipole moment of dipole is defined as the product of charge and distance of separation and direction of dipole moment is from –ve charge to positive charge.

$p = q\Delta l$; Direction of dipole moment $(\vec{p})$ is from the charge to positive charge.

$\vec{E}$ due to dipole at a distance r from centre of dipole along axis of dipole, $\vec{E} = \dfrac{2\vec{P}}{4\pi \in_0 r^3}$.

$\vec{E}$ (Electric field strength) due to dipole at a distance r from centre of dipole along perpendicular bisector of line (Along equational line)

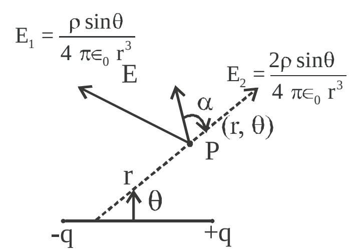
$$\vec{E} = \frac{-\vec{P}}{4\pi \in_0 r^3}$$




$\vec{E}$ (Electric field strength) at a point $(r, \theta)$ from centre of dipole.

$$E = \frac{P}{4\pi \in_0 r^3}\sqrt{1 + 3\cos^2\theta}$$

and $\alpha$ = angle made by $\vec{E}$ with $\vec{r} = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$




## DIPOLE IN AN EXTERNAL ELECTRIC FIELD :

When dipole of dipole moment $\vec{P}$ is placed in an external electric field of field strength $\vec{E}$ then torque and potential energy at orientation $\theta$ are given by $\vec{\tau} = \vec{P} \times \vec{E}$ and $U = -\vec{P}.\vec{E}$ respectively. when dipole is placed in non uniform electric field then there no translational equilibrium and no rational equilibrium while for uniform field there is always translational equilibrium but no rational.

## ELECTRIC FLUX :

In the region around the charge at rest there exists hypothetical electric lines, which measures the electric field strength at a point around charge at rest.

## ELECTRIC FLUX DENSITY :

It is the number of electric lines cross through unit area held normally to the electric lines.

$$\phi_E = \int \vec{E}.d\vec{s} = \int E\cos\theta\, ds = \text{Electric flux through the area ds}$$

$\theta$ is the angle between $\vec{E}$ and area vector.

Gauss' law : The total electric flux through a closed loop is always equal to $\dfrac{1}{\in_0}$ lines the total charge enclosed.

$$\phi_{\in} = \oint \vec{E}\, d\vec{s} = \frac{1}{\in_0}(\text{charge enclosed})$$

## Application of Gaus's law :

(a) The charge given to the conductor always get apeared on outside of the conductor.

(b) The electric field strength at a perpendicular bisector on uniformally charged rod of infinite length at a distance r from centre of rod

$$E = \frac{\lambda}{2\pi \in_0 r}.$$

(c) The electrostatic potential energy per unit volume (Energy density) = $\frac{1}{2}\in_0 E^2$ in air or vacuum.

$$= \frac{1}{2}\in_0 \in_r E^2 \text{ in medium of relative}$$

permitivity or dielectric constant is $\in_r$ or $k$.

(d) The electric field strength $\vec{E}$ near the surface of uniformly conducting sheet of surface charge density

$$E = \frac{\sigma}{\in_0}$$

(e) The electric field strength E near the surface of infinite long plane thin non-conducting sheet of charge of surface charge density $\sigma$ is $E = \frac{\sigma}{2\in_0}$

## Properties of electric lines :

1. Electric lines are continuous curves that emanate from positive charge and terminate to negative charge.
2. Number of electric lines per unit area at a point gives the magnitudeof electric field strength while direction of electric field strength is given by the tangent drawn at the point along the direction of electric lines.
3. Direction and magnitude $\vec{E}$ at a point are unique.
4. No two electriclines intersect at a point.

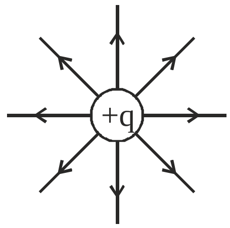
## Dipole Interaction

When two dipoles are placed along axial line having same direction of dipole moments then force of mutual attraction is $\frac{1}{4\pi}\frac{6P_1P_2}{r^4}$ where $P_1$ and $P_2$ are respective dipole moments while mutual potential energy is $\frac{1}{4\pi}\frac{2P_1P_2}{r^3}$; r = distance between centre S of dipole S when two dipoles are held parallel at a distance r where direction of dipole moments are same then mutual force (repulsion) and potential energy are given by $\frac{1}{4\pi}\frac{3P_1P_2}{r^4}$ and $\frac{1}{4\pi}\frac{P_1P_2}{r^3}$ respectively.

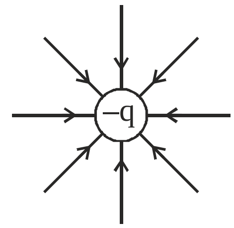
## Electric lines

The region around charge at rest in which there hypothetically exists continuous lines called electric lines

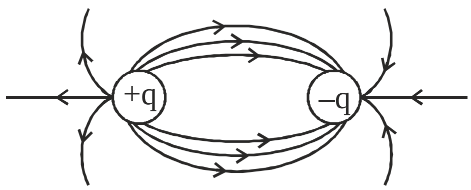
Electric line is an imaginary line along which a positive charge will move if left free.

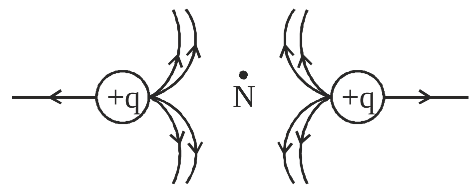



Radially outward




Radially inward




Electric lines never formed closed loop and meet the equipotential surface or conducting surface perpendicularly.

## Electrostatic Potential Energy :

When there is a system of charges (assembing by two or more charges) there exists potential energy in the system. Electrostatic potential energy is the work done against electrostatic force to create a system of charges. When two charges $q_1$, $q_2$ are separated by a distance r then mutual potential energy

$U = \dfrac{1}{4\pi \epsilon_0} \dfrac{q_1 q_2}{r}$ Electric potential at a point.

## Electric Potential :

Electric potential at a point defined as the work required by an external agent to displace a unit positive charge from infinity to the point or also defined as work done by electrostatic force to displace from point to infinity.

$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

## Electric Potential and Electric Field Strength :

Negative rate of change of electric potential is the electric field strength along the line there is change in potential

$$-\frac{dv}{dr} = E \text{ or } \vec{E} = E_x \hat{i} + Ey\,\hat{j} + E_z \hat{k}$$

where $$E_x = -\frac{\partial v}{\partial x};\ Ey = -\frac{\partial v}{\partial y};\ E_z = -\frac{\partial v}{\partial z}$$

Negative sign gives the direction of $\vec{E}$ along the line in which increase in distance causes the decrease in potential.

## Equipotential surface :

Every point on a surface in at same potential called equipotential surface. Electric lines always join the equipotential surface perpendicularly.

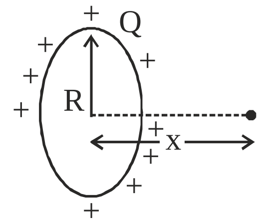
## ELECTRIC POTENTIAL DUE TO SOME CHARGE DISTRIBUTIONS :

1. Electric Potential at a distance r due to a point charge q

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

2. Electric potential due to uniformally charged circular ring at a distance x from centre of ring along axis of ring

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{\sqrt{x^2 + R^2}}$$

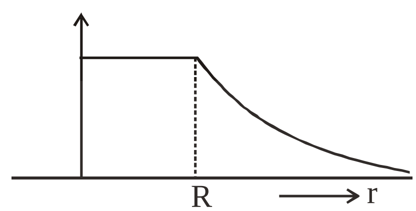



3. Electric potential due to uniformally charged hollow sphere or shell at a distance r

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R};\ r \le R$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2};\ r > R$$

R = Radius of the hollow sphere




4. Electric potential due to uniformally charged sphere of radius R and volume charge density $(\rho)$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} = \frac{\rho R^3}{3\varepsilon_o r};\left(\rho = \frac{Q}{\frac{4}{3}\pi R^3}\right) \quad ;\ r \ge R$$

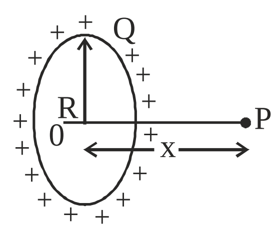
$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{3R^2 - r^2}{2R^3}\right] = \frac{\rho\left(3R^2 - r^2\right)}{6\varepsilon_o} \quad ;\ r < R$$

at centre of informally charged solid sphere,

$$V = \frac{3}{2}\times\frac{1}{4\pi\varepsilon_o}\frac{Q}{R} = \frac{3Q}{8\pi\varepsilon_o R}$$

5. Electrical potential at a distance x from uniformally charged disc

$$V = \frac{\sigma}{2\varepsilon_o}\left[\sqrt{x^2 + R^2} - x\right]$$




$\sigma$ = surface charge density.

6. Electric Potential due to dipole

At axial point

$$V = \frac{1}{4\pi\varepsilon_o}\frac{P}{r^2};\ P = \text{Dipole moment,}$$

At equatorial point

$$V = 0$$

At general point (r, $\theta$)

$$V = \frac{1}{4\pi\varepsilon_o}\frac{P\cos\theta}{r^2}$$

## CAPACITOR

A conductor has a limited capacity to hold charge called capacitor some of suitable examples of capacitors are

(i) parallel plate capacitor (ii) spherical capacitor (iii) cylindrical capacitors

Capacitance of a capacitor is defined as the amount of charge required to raise the potential by unity. Experimentally, charge given to the conductor directly proportional to potential of the conductor.

i.e., $Q \propto v \Rightarrow Q = cv$; c is proportionality constant called capacitance and it does depend upon the shape and size of the capacitor. is capacitence of conductor unit of capacitor is forced (coulomb volt$^{-1}$).

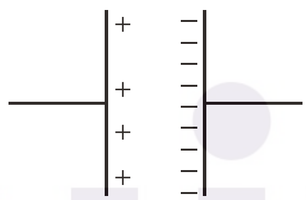
### FACTORS AFFECTING THE CAPACITENCE OF THE CAPACITOR :

1. The presence of another uncharged conductor near the charged one raises the capacitence of charged conductor.
2. Increase in surface area increase the capacitance.
3. Presence of dielectric increases the capacitance.

### PARALLEL PLATE CAPACITOR :

It consists two parallel metallic plates separated by a small distance having equal and opposite charges..

Electric field strength $\vec{E}$ is directed from positive plate to negative plate.



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

$\sigma$ = surface charge density; Q = Charge appeared on each plate; A = Area of the plate

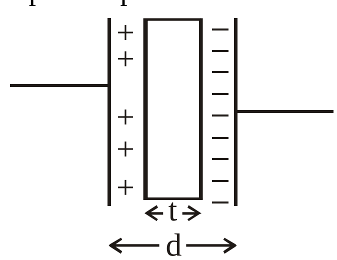
C = Capacitance of parallel plate capacitor

$$C = \frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

When dielectric of thickness t is inserted between parallel plate capacitor

$$V = \frac{Qd}{A\varepsilon_0}$$

$$C = \frac{A\varepsilon_0}{d - t + \frac{t}{k}}$$




As V = potential difference

$$= \frac{\sigma}{\varepsilon_0}(d - t) + \frac{\sigma}{\varepsilon_0 K} \cdot t$$

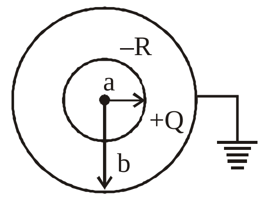
when length of dielectric state t = d (separation of two parallel plates)

Then $$C = \left(\frac{A\varepsilon_0}{d}\right)K.$$

**Spherical Capacitor :** It consists of two concentric conducting spheres of radii a and b (a < b). Inner sphere is given charge +Q. while outer sphere is carthed.

Potential Difference

$$V = Q\left(\frac{1}{4\pi\varepsilon_o} - \frac{1}{4\pi\varepsilon_o b}\right)$$

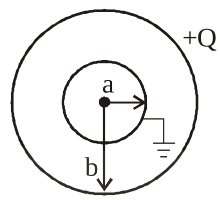



and capacitance $c = 4\pi\varepsilon_o \frac{ab}{b-a}$.

In presence of dielectric medium between spheres capacitence, $c = 4\pi\varepsilon_o K \frac{ab}{a-b}$

If outer sphre is given charge +Q while sphere is carthed inner then induced charge on inner sphere

$Q^1 = -\frac{a}{b}Q$ and capacitence is given an

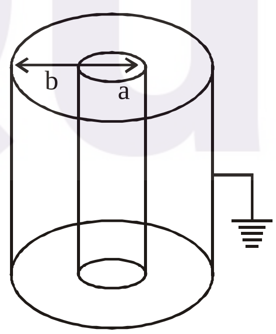



$$c = 4\pi\varepsilon_o \frac{b^2}{b-a} = 4\pi\varepsilon_o \frac{ab}{a-b} + 4\pi\varepsilon_o b$$

**Cylindrical Capacitor :**

It consists of two concentric cylinders of radii a and b (a < b), inner cylinder is given charge +Q while outer cylinder is earthed length of cylinders is l then

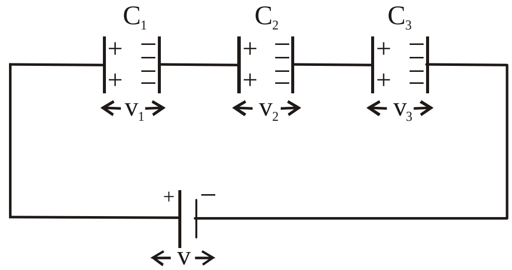
$$c = \frac{2\pi\varepsilon_o l}{\log_e(b/a)}$$




**Grouping of Capacitor**

**1. Series grouping :**

Two or more capacitors are said to be in series arrangement if there is same charge distributed on r each while sum of potential difference across and the capacitors is the potential difference across system of capacitors.




$v = v_1 + v_2 + v_3$ ...(i) and $c_1v_1 = c_2v_2 = c_3v_3$ ...(ii)

$$\therefore \quad \frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$$

In series combination potential difference and energy distributes in the inverse ratio of capacitence i.e.

$$v = \alpha \frac{1}{c} \text{ and } U \alpha \frac{1}{c}.$$

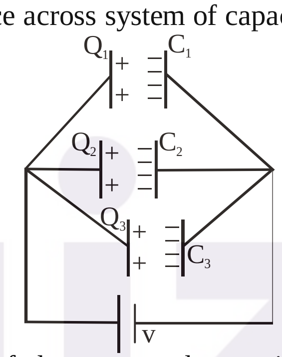
## 2. Parallel grouping :

Two are more capacitors are said to be in parallel arrangement if there is same potential difference across each and which is equal to applied potential difference across system of capacitors.

$$Q = Q_1 + Q_2 + Q_3$$

$$v_1 = \frac{Q_1}{C_1} = v_2 = \frac{Q_2}{C_2} = v_3 = \frac{Q_3}{C_3}$$

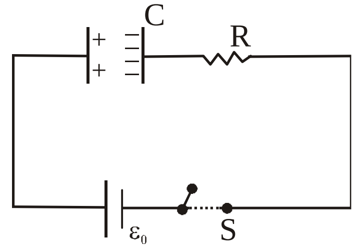
$$\Rightarrow C_{eq} = C_1 + C_2 + C_3$$




and charge drawn from source is equal to sum of charge on each capacitor. In parallel combination charge and energy distributes in ratio of capacitence i.e. $Q \alpha l$ and $U \alpha C$.

## RC-Circuit

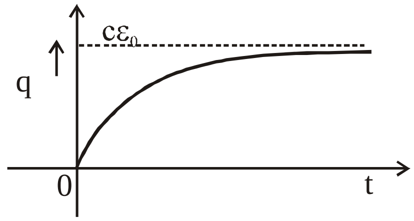
## 1. For charging




when S (switch) is closed capacitor starts charging, in the transient state of charging, potential difference appears across capacitor as well as resistor. When capacitor gets fully charged the entire potential difference appeared across the capacitor. The charge on capacitor at the t

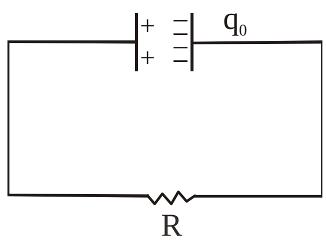
$$q(t) = c\varepsilon_0\left(1 - e^{-}\frac{t}{RC}\right) \text{ and } i(t) = \frac{\varepsilon_0}{R}e^{t/Rc}$$

and $$v(t) = \varepsilon_o\left(1 - c^{-}t/Rc\right)$$




The quantity RC is called one time constant after one time constant the charge appears is 0.63 ( $c\varepsilon_0$ ) i.e. 63% of equilibrium charge

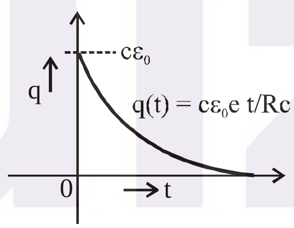
2. For discharging




After completion of charging, if battery is removed, capacitor starts discharging. In transient state charge on capacitor at any instant

$$q(t) = c\varepsilon_o.e^{-t/Rc} = q_o e^{-t/Rc}$$

and potential difference across capacitor $v(t) = \varepsilon_o e^{-t/Rc}$




value of Rc is one time constant and during discharging it is time during which charge on a capacitor falls to 0.37 times of initial charge i.e., 37% of initial charge.

Energy stored in capacitor :

Capacitor is a device that stores electric energy

$U = \frac{1}{2}CV^2$; where C is the capacitance of capacitor and V be potential

$= \frac{1}{2}QV = \frac{Q^2}{2C}$; Q = charge appeared on the each plate of capacitor.

Dielectric :

Dielectrics are insulating (non-conducting) materials which transmits electric effect without conduction of charge. Dielectric are of two types :

1. Polar dielectrics :

A polar molecule has permanent electric dipole moment $(\vec{P})$ in absence of electric field to. But a polar dielectric has no-net dipolemoment in absence of electric field because polar molecules are randomly oriented. In the presence of electric field polar molecules tends to line up in direction of electric field, and the substance has finite dipole moment. e.g. water $CO_2$, HCl etc. are made of polar atoms/molecules.

2. Non polar dielectric :

In non-polar molecules, each molecule has zero dipole moment in its normal state. When electric field is applied, molecules become induced electric dipole e.g., $N_2$, $O_2$, benzene etc. are made of non-polar atoms/ molecules.

In general, any non-conducting material can be called as a dielectric but broadly non-conducting material having non-polar molecules referred to on dielectric.

Polarisation of dielectric Slab :

It is the process of inducing equal and opposite charges on the two faces of dielectric on the application of electric field.

Dielectric const of dielectric medium (K). = $\dfrac{\text{Electric field between plates with air}}{\text{Electric field between plates with medium}}$

If a very high electric field is created in a dielectric, then the dielectric behaves like a conductor. The phenomenon is dielectric break down.

# Current Electricity
# (Electric Current, Thermal and Chemical Effect of Current)

## ELECTRIC CURRENT

Consider the ends of the conductor be connected to a battery, i.e., an electric field is maintained within the conductor. Now the field acts on the electrons and gives them a resultant motion in the direction of $-\vec{E}$ because a free charge in electric field experiences a force. The flow of electrons constitutes an electric current.

The time rate of flow of charge through any cross section is called current. If a charge $\Delta q$ passes through an area in time $\Delta t$, then the average electric current through the area in this time is defined as

$$i_{av} = \frac{\Delta q}{\Delta t} \qquad ...(1)$$

Now, the instantaneous current is given by

$$i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} \qquad ...(2)$$

The SI unit of current is ampere. If one coulomb of charge crosses an area in one second, the current is one ampere. For transient currnet $i = \frac{dq}{dt}$ while for steady current $i = \frac{q}{t}$

The conventional current is in opposite direction to the direction of movment of electrons.

## CURRENT DENSITY

The current density j at a point is defined as a vector having magnitude equal to current-per unit area surrounding that point and normal to the direction of charge flow, i.e., direction in which current passes through that point.

If $\overrightarrow{\Delta S}$ be the area vector corresponding to area $\Delta S$, then $\Delta i = \vec{j}.\overrightarrow{\Delta S}$

The total current through finite surface area S is

$i = \int_s \vec{j}.\overrightarrow{\Delta S}$, If current i is uniformally distributed over an areas and perpendicular to it then $i = \int \vec{j}.\overrightarrow{\Delta S}$

## DRIFT VELOCITY

we know that a conductor contains a large number of free electrons or conduction electrons. When electrons leave their atoms and become free, the atoms of the conductor become positively charged and are called positive ions. So, the remaining material is a collection of relatively positive ions known as lattice.

In the absence of any external electric field, the electric current through this area is zero, otherwise the conductor will not remain equipotential.

When an electric field is established between the two ends of the conductor, the free electrons experience an electric force opposite to the field. Due to this force, the motion of electrons is accelerated.

The field does not give an accelerated motion to the electrons but it simply gives them a small constant velocity along the conductor which is superimposed on the random motion of the electrons. So, the electrons drift slowly opposite to the applied field. The net transfer of electrons across a cross section results in current. If the electron drifts a distance l in a long time t. we define drift velocity as

$$v_d = \frac{l}{d} \qquad ...(1)$$

The drift velocity is the average uniform velocity by free electrons inside a conductor by the application of an electric field.

where e is charge of electron with mass m.

[$\because$ force on electron due to electric field, F = e E and acceleration , a = F/m = (e E/m)]

$$\therefore \qquad v_d = \frac{eE}{m}.\tau \qquad \tau \text{ in time between two successive collision.} \qquad ...(3)$$

## RELATIONSHIP BETWEEN CURRENT DENSITY AND DRIFT

An electric field is maintained between the two ends of a conductor towards the left. The electrons move towards the right. Let the drift velocity of the electrons be $v_d$. Suppose there are n charge carriers per unit volume and each charge carrier has a charge e. In time dt, the electron advance a distance l which is given by

$$l = v_d \, dt$$

Now calculate the number of electrons crossing the length l of the conductor in time dt. This will be equal to the number of electrons contained in a volume Al, i.e. $Av_d$ dt.

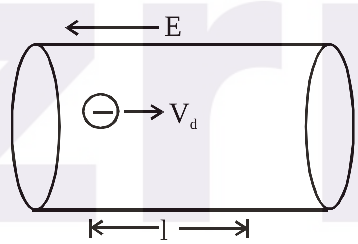
$\therefore$ number of electrons = volume × number of electrons per unit volume

$= A\, v_d\, dt \times n$

Hence charge crossing in time dt

= number of electrons × charge on the electron

$= Av_d\, dt\, n\, e$




Further, current $i = \dfrac{\text{charge crossing in time dt}}{\text{time dt}}$

$$i = \frac{Av_d dt\, n\, e}{dt} = Av_d ne$$

So $\boxed{i = neAv_d}$ ...(1)

The current density is given by

$$j = \frac{i}{A} = \frac{neAv_d}{A}$$

or $\boxed{j = n\, e\, v_d}$ ...(2)

Mobility of free electron is defined in a conductor is drift velocity acquired per unit electric field strength.

applied across the conductor, $\mu = \dfrac{Vd}{E} \Rightarrow Vd = \mu E$

or $i = \mu(neA)E$

## OHM'S LAW

The current flowing through a conductor is always directly proportional to the potential difference across its two ends.

$$V \propto i \quad \text{or} \quad V = Ri$$

where R is a constant of proportionality and is called as resistance of the conductor. So, the reistance of a conductor is defined as the ratio of the potential difference applied across the conductor to the current flowing through it, i.e. R = V/i. The value of resistance depends upon the nature of conductor, its dimensions and physical conditions.

We know that drift velocity $v_d$ is given by

$$v_d = \left(\frac{eE}{m}\right)\tau$$

$$= \left(\frac{eV}{ml}\right)\tau \qquad \left(\because E = \frac{V}{l}\right) \qquad ...(1)$$

We also know that relation betwen current i and drift velocity $v_d$ is given by

$$i = neAv_d \qquad ...(2)$$

Substituting the value of $v_d$ in eq. (2) from eq. (1), we have

$$i = neA\left(\frac{eV}{ml}\right)\tau = \left(\frac{ne^2A\tau}{ml}\right)V$$

or $$\frac{V}{i} = \frac{ml}{ne^2A\tau} = R \qquad \text{a constant}$$

R is constant for a given conductor, known as resistance of the conductor. Therefore,

$$V = R\,i$$

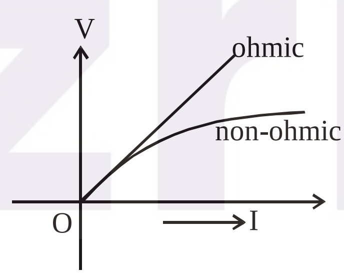
We know that

$$j = n\,e\,v_d$$

Further, $$v_d = \left(\frac{eE}{m}\right)\tau$$

$\therefore$ $$j = ne\left[\left(\frac{eE}{m}\right)\tau\right] = \frac{ne^2\tau}{m}E$$

or $$j = \sigma E \qquad \text{where } \sigma = \frac{ne^2\tau}{m}$$




The constant $\sigma$ is called as electrical conductivity and is temperature dependent. So we have

. $$\boxed{j = \sigma E}$$

This equation is known as Ohm's law.

V-I line is not a straight line.

## RESISTIVITY AND CONDUCTIVITY

The reistance of a conductor is directly proportional to its length l and inversely proportional to the area of cross section A, i.e.

$$R \propto l \qquad \text{and} \qquad R \propto \frac{1}{A}$$

$$R \propto \frac{l}{A} \qquad \text{or} \qquad R = \rho\left(\frac{l}{A}\right)$$

If l = 1 and A = 1 , then $R = \rho$

Therefore, specific resistance of the material of a conductor is equal to the resistance offered by the wire of unit length and unit area of corss section of the material of wire. Its unit is ohm-metre. This is constant for a material.

The reciprocal of resistivity of the material of a conductor is called as conductivity

$$\sigma = \frac{1}{\rho} = \frac{j}{E}$$

The unit of conductivity is ohm$^{-1}$ metre $^{-1}$ $(\Omega m)^{-1}$. Good conductors of electricity have large conductivity than insulators.

## FACTORS AFFECTING ELECTRICAL RESISTIVITY

The drift velocity $v_d$ in magnitude of electrons is given by

$$v_d = \left(\frac{eE}{m}\right)\tau \qquad ...(1)$$

The current flowing through the conductor due to drift of electrons is given by

$$i = n\,A\,e\,v_d$$

$$= nAe\left(\frac{eE}{m}\right)\tau = \frac{nAe^2E}{m}\tau \qquad ...(2)$$

If V be the potential difference applied across the two ends of the conductor, then

$$E = \frac{V}{l} \qquad ...(3)$$

From eqs. (2) and (3) we get,

$$i = \frac{nAe^2V}{ml}\tau$$

$$\text{or} \quad R = \frac{V}{i} = \frac{m}{ne^2\tau}\left(\frac{l}{A}\right) \quad \text{or} \quad R = \rho\left(\frac{l}{A}\right) \qquad ...(4)$$

where $\rho = \text{resistivity} = \dfrac{m}{ne^2\tau}$

The resistivity $\rho$ of the material of a conductor depends upon the following factors:

(i) It is inversely proportional to the number of free electrons per unit volume n of the conductor, i.e., depends on the nature of material.

(ii) It is inversely proportional to the average relaxation time $\tau$ of free electrons in the conductor. As $\tau$ is a function of temperature and hence the resistivity of a conductor depends on its temperature. The resistivity increases with the increase in temperature of conductor.

## TEMPERATURE DEPENDENCE OF RESISTIVITY

Small temperature variations, the variation of resistivity can be expressed as

$$\rho(T) = \rho(T_0)\left[1 + \alpha(T - T_0)\right]$$

where $\rho(T)$ and $\rho(T_0)$ are the resistivities at temperature T and $T_0$ respectively and $\alpha$ is temperature coefficient of resistivity.

The resistance of a conductor is given by

$$R = \rho\left(\frac{l}{A}\right)$$

$\rho(T)$ = Resistivity at temperature T

$\rho(T_0)$ = Resistivity at temperature of $T_0$.

The resistance depends on the length and area of cross section besides resistivity. When the temperature increases, the length and area of cross section also increases are quite small and the factor (l/A) may be treated as constant. Therefore,

$$R \propto \rho$$

$R(T) =$ Restance at temperature T.

Now, $R(T) = T(T_0)\left[1 + \alpha(T - T_0)\right]$ $\quad$ R(T) = Resistance at temperature $T_0$.

where $\alpha$ is known as temperature coefficient of resistance.

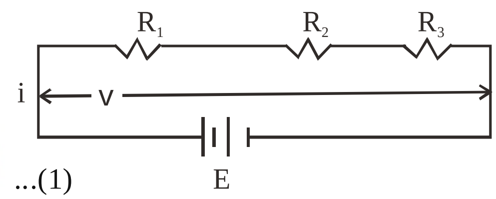
## Grouping of Resistance

### (A) Resistances in Series

In the shown figure. shows the series combination of three resistors having resistance$R_1$ and $R_3$ and. A battery of e.m.f. E is connected across this combination. In this combination, the same current i is flowing through each resistor.

Let $V_1$ , $V_2$ and $V_3$ be the potential differences across $R_1$ , $R_2$ and $R_3$ respectively. Now according to Ohn's

$V_1 = i R_1$, $V_2 = i R_2$ and $V_3 = i R_3$




Further $\quad V = V_1+V_2+V_3$

$$= iR_1 + iR_2 + iR_3$$

$$= i(R_1 + R_2 + R_3) \quad ...(1)$$

If $R_2$ be the equivalent resistance of series combination, then the potential difference V across the combination will be

$$V = i R_S \quad ...(2)$$

Compairing eqs. (1) and 2), we get

$$R_S = R_1 + R_2 + R_3 \quad ...(3)$$

In series combination, the following points should be remembered

(i) The current is same in every part of the circuit

(ii) The total reistance of the circuit is equqal to the sum of indivd resistances connected in the circuit.

(iii) The total resistance of series combination is more than the greatest resistance of the circuit.

(iv) The potential difference across any resistor is proportional to its resistance, i.e. $v_1 : v_2 : v_3 = R_1 : R_2 : R_3$

### (B) Resistance in Parallel

Fig. shows a parallel combination of three resistors having resistance $R_1$, $R_2$ and $R_3$ battery of e.m.f. E is connected points A and B. Let i be the current from the battery and $i_1$, $i_2$ and $i_3$ be the currents through resistance $R_1$ , $R_2$ and $R_3$ respectively. Then

$$i = i_1 + i_2 + i_3 \quad ...(5)$$

As shown in the figure, the potential difference across each resistance is V. Applying Ohm's law, we have

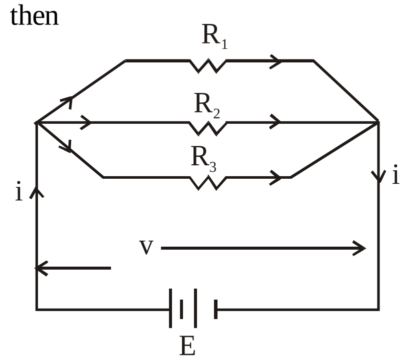
$$V = i_1R_1 = i_2 R_2 = i_3 R_3$$

or $\quad i_1 = \dfrac{V}{R_1}$ , $i_2 = \dfrac{V}{R_2}$ and $i_3 = \dfrac{V}{R_3}$

Substituting these values in eq. (5), we get

$$i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \qquad \ldots(6)$$

Let $R_p$ be the equivalent resistance of the parallel combination, then




$$V = i R_p \quad \text{or} \quad i = \frac{V}{R_p} \qquad \ldots(7)$$

Comparing eqs. (6) and (7), we get

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or $$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad \ldots(8)$$

The reciprocal of equivalent resistance of parallel combination is equal to the sum of the reciprocals of the individual resistances.

The following points should be remembered in case of parallel combination:

(i) The potential difference across each resistance is the same

(ii) The current is different in different resistances. The sum of the currents in different resistances is equal to the main currents in the circuit, i.e.,

$$i = i_1 + i_2 + i_3 \qquad \ldots(9)$$

(iii) The current through any registor is inversely proportional to its resistance.

(iv) The total resistance in parallel combination is less than the least resistance used in the circuit.

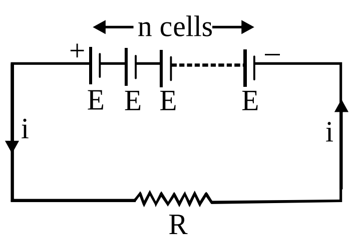
## BATTERY AND ELECTROMOTIVE FORCE (E.M.F.)

A battery is a device which maintains a constant potential difference between its two terminals.

The potential difference between the two terminals. The potential difference between two terminals provides an electrostatic field Ee between tw terminals the emf is defined as the work done while by cell a unit positive charge flows from –ve plate to +ve plate.

## GROUPING OF CELLS

(1) Sereis grouping : Fig shows a series combination of n cells each of e.m.f. E and internal resistance r.




$\therefore$ current through the circuit $= \dfrac{\text{total emf}}{\text{total resistance}}$

or $$i = \frac{nE}{(R + nr)} \qquad \ldots(1)$$

(i) If R>>r, r i.e., the effective internal resistance is as far far less than external resistance r can be neglected in comparison of R, then

$$i = \frac{nE}{R} = \text{n times the current drawn from single cell.} \qquad \ldots(2)$$

(ii) If r >> R, i.e., the effective internal resistance is far far greater than external resistance, then R can be neglected in comparison to hr, then

$$i = \frac{nE}{nR} = \frac{E}{r} \qquad ...(3)$$

The current in the circuit is the same as due to a single cell, so n of useful

(iii) If in series grouping of n cells, s cells are reversed, then

$$E_{eq} = (n-s)E - sE = (n-2s)E$$

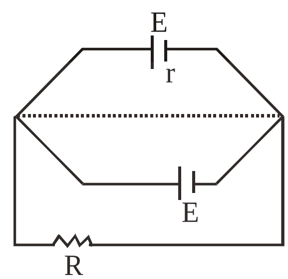
Total resistance of the circuit = (R+n r)

$$\therefore \qquad i = \frac{(n-2s)E}{(R+nr)} \qquad ...(4)$$

(2) Parallel grouping

$\therefore$ Total Resistance of the circuit $= \left[R + (r/n)\right]$ [$\because$ R and (r/n) are in series]

Now, current in the cirut, $i = \dfrac{E}{R + \left(\dfrac{r}{n}\right)}$ ...(5)




(i) If R>> (r/n), i.e. (r/n) can be neglected in comparison to R, then

$$i = \frac{E}{R} \qquad ...(6)$$

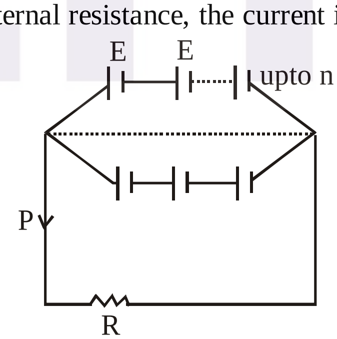
Therefore, the current in the circuit is equal to the circuit current due to a single cell.

(ii) If (r/n) >> R, i.e., R can be neglected in comarison to (r/n), then

$$i = \frac{nE}{r} \qquad ...(7)$$

Therefore, if the effective internal resistance is greater than the external resistance, the current in the circuit is equal to n time the circuit current due to a single cell.

(3) Mixed Grouping




Total resistance of circuit $= R + \left(\dfrac{nr}{m}\right)$

The current i in the circuit is given by

$$i = \frac{nE}{R + (nr/m)} = \frac{nmE}{mR + nr} = \frac{NE}{mR + nr}$$

The current i in the circuit will be maximum when the factor ( m R+ n r) in the denominator is minimum. The denominator is minimum when mR = n r

$$\therefore \qquad R = \left(\frac{nr}{m}\right)$$

Hence current will be maximum when external resistance is equal to the total internal resistance of all the cells.

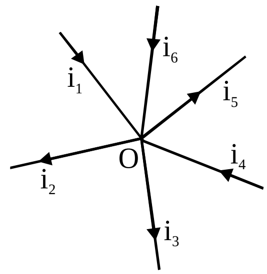
## KIRCHHOFF’S LAW

Ohmm;s law is unable to give current in complicated cirucit. Kirchhoff’s in1842, gave two general laws which are extremely useful in electrical circuits. There are.

(i) The algebraic sum of the currents at any junction in a circuit is zero, i.e.

$$\sum i = 0$$

This means that there is no accumulation of electric charge at any point in the circuit.




(ii) In any closed circuit,the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf in the circuit i.e.,

$$\sum iR = \sum E$$

The product of current and resistance is taken as positive when we traverse in the direction of current. The emf is taken positive when we traverese from negative to positive electrode through electrolyte.

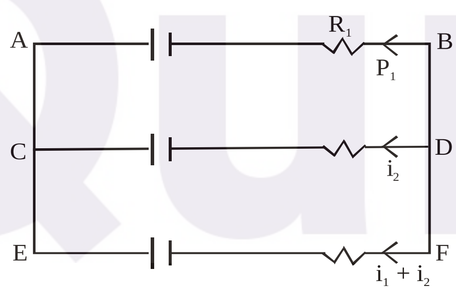
Let us apply Kirchhoff's second law to figure shown

For the mesh ACDBA,

$$i_1R_1 - i_2R_2 = E_1 - E_2 \qquad ...(i)$$

For the mesh EFDCE

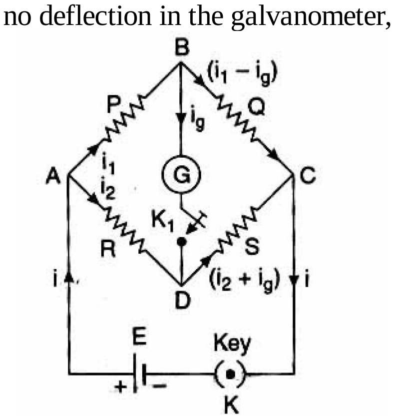
$$i_2R_2 + (i_1 + i_2)\,R_3 = E_2 \qquad ...(ii)$$




From FFBAE, $i_1 R_1 + (i_1 + i_2)\,R_3 = E_1$ ...(iii)

## CONDITION OF BALANCE IN WHEATSTONE'S BRIDGE

When there is no deflection in the galvanometer, the bridge is known as balanced. The condition of balance is given by

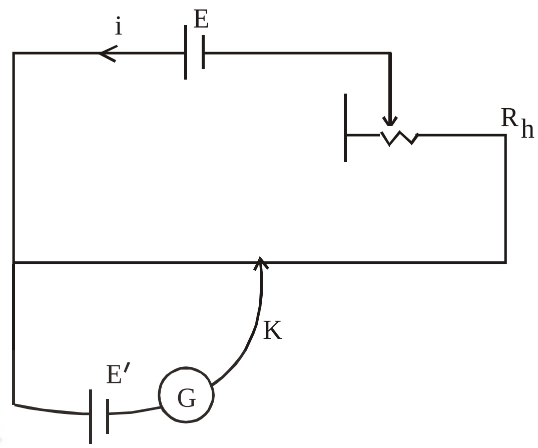


$$\frac{P}{Q}=\frac{R}{S} \quad \text{or} \quad \frac{P}{R}=\frac{Q}{S}$$

When galvanometer and battery are exchanged then still the galvanometer shows no reflection.

## POTENTIOMETER

Potentiometer is a device which is used to meosure the potential difference more accurately then an ideal voltmeter. The potentiometer does not any current from source. Hence it is equivalent to an draw ideal voltmeter.




Let v be potential difference across certain portion of wire. Let i be current through portion of stretched wire

$$v = iR \qquad \text{--- (i)} \qquad R=\rho.\frac{l}{A}$$

$$v = i\rho\left(\frac{l}{A}\right)$$ For i = constant through wire of uniform cross-section

If L = total length of potentiometer wire

$\varepsilon =$ emf of driving cell or standard cell.

$$K=\frac{\varepsilon}{L}$$

$$V = K.l \Rightarrow v=\frac{\varepsilon}{L}.l$$

Comparision of emfs of two cells can found by potentiometer $\frac{\varepsilon_1}{\varepsilon_2}=\frac{l_1}{l_2}$ where $l_1$ and $l_2$ are balancing lengths while cells of $\varepsilon_1$ and $\varepsilon_2$ are attached respectively.

The internal resistance r of a cell is given by $r=\left(\frac{i_1}{i_2}-1\right)R;$ where $l_1$ and $l_2$ are balancing lengths and R is external resistance

## HEATING EFFECT OF CURRENT

The phenomenon in which heat energy is produced in a conductor due to flow of electric current (flow of electrons) is known as heating effect of current.

Consider a resistor of resistance R. Let a potential difference V is maintained at its ends and a current i is flowing through it for a time t. If the charge q flows through it in time t, then

$$q = i\,t \text{ [charge = current × time]}$$

Now, the workdone by electric field on free electrons in time t is given by

$$= V\,(i\,t) \text{ joule}$$

$$= (i\,R)\,(i\,t) = i^2 R\,t \text{ joule}$$

The workdone by electric field is converted in thermal energy of resistor through the collisions with ions or atoms. The thermal energy is generally referred to as heat produced in resistor. So, the amount of heat produced (H) is given by $\boxed{H = W = i^2Rt \text{ joule}}$

In calorie, the heat produced is given by

$$H = \frac{i^2Rt}{4.18} \text{ calorie} \quad \text{This is expression for joule's law of heating...(4)}$$

## JOULE'S LAWS OF HEATING

Joule's laws :

(a) The heat produced in a given resistor in a given time is proportional to the square of current flowing in it, i.e.,

$$H \propto i^2 \qquad ...(1)$$

(b) The heat produced in a given resistor in a given time by a given current is directly proportional to the resistance, i.e.,

$$H \propto R \qquad ...(2)$$

(c) The heat produced in a given resistor by a given current is proportional to time t for which the current is passed, i.e.,

$$H \propto t \qquad ...(3)$$

## ELECTRIC POWER

The electric power is defined as the rate at which work is done by the source of e.m.f. in maintaining the current in an electric circuit.

If an amount of work W is done in maintaining electric current in a circuit for a time t, then electric power is given by

$$P = \frac{W}{t} \qquad ...(1)$$

Let a current i ampere flows through a conductor for a time t second under a potential difference V volt. The workdone for maintaining the current is given by

$$W = V\,i\,t \text{ joule} \quad ...(2)$$

So, the power of an electric circuit is one watt when one ampere current flows through it under a potential difference of one watt.

1 watt = 1 joule/sec.

The bigger units of electric power are

1 kW = $10^3$ W and 1 MW = $10^6$ W

Commercial unit of power is horse power (HP).

1 HP = 746 watt.

Other expression for power are :

$$P = i^2R \text{ and } P = \frac{V^2}{R}$$

$$\boxed{P = Vi = i^2R = \frac{V^2}{R}} \quad ...(3)$$

(i) When resistances are connected in series.

In this case, the current in each resistance will be the same. Hence from eq. (3), we have

$$P \propto V \quad \text{and} \quad p \propto R$$

This shows that in series connections, the potential difference and power consumed will be more in larger resistance.

(ii) When resistances are connected in parallel.

In this case, the potential difference V across each resistance is same. Hence from eq. (3), we have

$$P \propto \left(\frac{1}{R}\right) \text{ and } i \propto \left(\frac{1}{R}\right)$$

This shows that in parallel connections, the current and power consumed will be more in smaller resistance.

## APPLICATIONS OF HEATING EFFECT OF CURRENT

### (1) Series combination of bulbs :

Consider a series combination of three bulbs of powers $P_1$, $P_2$ and $P_3$ which are manufactured for working on a supply of V volt. The resistances of these bulbs are respectively.

$$R_1 = \frac{V^2}{P_1}, R_2 = \frac{V^2}{P_2} \text{ and } R_3 = \frac{V^2}{P_3} \quad ...(1)$$

$\therefore$ total resistance, $R = R_1 + R_2 + R_3$ ...(2)

Effective power $\dfrac{V^2}{P} = \dfrac{V^2}{P_1} + \dfrac{V^2}{P_2} + \dfrac{V^2}{P_3}$

or $$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} \quad ...(3)$$

Current through each bulb

$$i = \frac{V}{R_1 + R_2 + R_3} \quad ...(4)$$

The brightness of these bulbs are

$$H_1 = i^2 R_1,\ H_2 = i^2 R_2 \text{ and } H_3 = i^2 R_3 \ ...(5)$$

This shows that the bulb with highest resistance will glow with maximum brightness. Further $R \propto \frac{1}{P}$, therefore, the bulb of lowest power or wattage will have highest resistance and will glow with maximum brightness.

### (2) Parallel combination of bulbs :

Consider a parallel combination of three bulbs of powers $P_1$, $P_2$ and $P_3$ respectively which are manufactured for working on a supply voltage V volt. In this case, we have

$$R_1 = \frac{V^2}{P_1}, R_2 = \frac{V^2}{P^2} \text{ and } R_3 = \frac{V^2}{P_3} \ ...(6)$$

Now $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ ...(7) (where R is effective resistance of the circuit)

From eqs. (6) and (7), we get

$$P = P_1 + P_2 + P_3 \qquad ...(8)$$

The brightness of three bulbs will be respectively

$$H_1 = \frac{V^2}{R_1},\ H_2 = \frac{V^2}{R_2} \text{ and } H_3 = \frac{V^2}{R_3}$$

The resistance of highest wattage (power) bulb is minimum and hence the bulb of maximum wattage will glow with maximum brightness.

## SEEBECK EFFECT

Seeback discovered that if two dissimilar metals (say bars or wires of copper and iron) are joined in series to form a closed circuit, and their two junctions are maintained at different temperatures, an e.m.f. is developed.

The current produced in this way without the use of a cell or a battery is known as thermoelectric current and the e.m.f. responsible for thermoelectric current is known as thermo e.m.f. This effect is known as Seebeck effect. The arrangement of wires is known as thermocouple.

Seebeck observed that the magnitude and direction of thermo e.m.f. depends on

(i) the nature of metals forming the thermocouple.

(ii) differnece in temperatures of two junctions.

Seebeck also observed that if the hot and cold junctions are interchanged then the direction of thermoelectric current is also reversed. This shows that seebeck effect is reversible effect.

### Thermoelectric Series

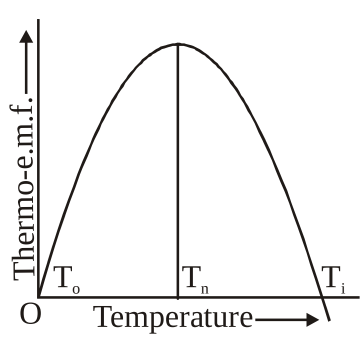
Seebeck arranged a large number of metals in a series such that when any two of these metals form a thermocouple, the current at the cold junction is from the metal occurring earlier in the series to the metal occurring later in the series. The series is known as themoelectric series. The series is as follows :

Antimony, nichrome, iron, zinc, copper, gold, silver, lead, aluminium, mercury, platinum, nickel constantan, bismuth.

### Variation of thermo e.m.f. with temperature

(Neutral temperature and temperature of inversion) :

If a graph is plotted between the temperature of the hot junction and the thermo e.m.f. e, the cold junction being kept at 0°C, a parabolic curve is obtained as shown in fig. The thermo e.m.f.




increases with the temperature of hot junction and becomes maximum at a particular temperature. The temiperature of the hot junction at which thermo e.m.f., in a thermocouple is maximum is known as neutral temperature $T_n$ for that couple.

Thus the temperature at which the thermo e.m.f. is zero is known as inversion temperature or temperature of inversion.

Beyond this temperature the e.m.f. again increases but in the reverse direction.

The temperature of inversion depends upon

(i) the nature of materials forming the thermocouple

(ii) the temperature of cold junction.

The thermo e.m.f., e varies with temperature according to the following equation.

$$e = aT + bT^2 \qquad ...(1)$$

$$\frac{de}{dT} = a + 2bT$$

at $T = T_n$, e is maximum, i.e., $\frac{de}{dT} = 0$. Thus

$$0 = a + 2bT_n$$

or $$T_n = -\frac{a}{2b} \qquad ...(2)$$

Further at $T = T_i, e = 0$. Thus from equation (1)

$$0 = aT_i + bT_i^2$$

$$T_i = -\frac{a}{b} \qquad ...(3)$$

From equations (2) and (3)

$$T_i = 2T_n$$

Thus the inversion temperature $T_i$ is as much above the neutral temperature as the temperature of the cold junction (0°C) is below it. $T_i$ is therefore not a constant for the given thermocouple but depends upon the temperature of the cold junction.

If $T_0$ be the temperature of cold junction, then

$$T_i - T_n = T_n - T_0 \quad \text{or} \quad T_i = 2T_n - T_0$$

$$\therefore \quad \boxed{T_n = \frac{T_i + T_0}{2}}$$

## Peltier's Effect :

Peltier discovered an effect which is the converse of Seebeck effect. When a current is passed across the junction of two dissimilar metals, heat is evolved at one junction and absorbed at the other, i.e., one junction is heated and the other is cooled. This effect is known as Peltier effect.

## Peltier Coefficient :

The amount of heat (in joules) absorbed or evolved at a junction of two different metals when one coulomb of charge flows at the junction is called the Peltier coefficient. It is denoted by

$$\pi = \frac{\Delta H}{\Delta Q} = \frac{\text{Peltier heat}}{\text{charge flowing}}$$

This coefficient is not constant but varies as the absolute temperature of the junction. It also depends on the metal used.

If a charge q coulomb passes across a junction having a peltier coefficient $\pi$ volt, then the energy absorbed or evolved at the junction = $\pi q$ joule.

If V be the junctional P.D. in volt, then

energy absorbed or evolved = Vq joule

$$\therefore \quad \pi q = Vq$$

$$\pi = V$$

Hence the Peltier coefficient expressed in joule per coulomb is numerically equal to the junctional P.D. in volt.

### Thomson effect :

Thomson observed that when two parts of a single conductor are maintained at different temperatures and a current is passed through it, heat may be absorbed or evolved in different sections of may be absorbed or evolved in different sections of the conductor. This effect is called Thomson effect.

According to Thomson effect, heat is absorbed or evolved in excess of Joule heat when a current is passed through an unequally heated conductor.

### Thomson coefficient :

Thomson coefficient is defined as the amount of heat evolved or absorbed when a unit positive charge is passed through a part of the wire whose ends are maintained at a unit temperature difference. This is denoted by $\sigma$.

Let a charge $\Delta Q$ is passed through a small part of the wire having a temperature difference $\Delta T$ between the ends. Thomson heat is

$$\Delta H = \sigma(\Delta Q)(\Delta T)$$

or

$$\sigma = \frac{\Delta H}{(\Delta Q)(\Delta T)}$$

## CHEMICAL EFFECT OF ELECTRIC CURRENT

It has been observed that some liquids allow the passage of current through them while some do not show such behaviour. On the basis of their electrical behaviour liquids can be divided into the following three categories :

(i) The liquids which do not allow the current to pass through them. For example distilled water, vegetable oil etc.

(ii) The liquids which allow the current to pass through them but do not dissociate into ions. For example, mercury.

(iii) The liquids which allow current to pass through them and also dissociate into ions. For example salt solutions, acid and bases. Such liquids are called electrolytes.

Thus when a current is passed through an electrolyte, it dissociates into ions. This is known as chemical effect of current.

## FARADAY'S LAWS OF ELECTROLYSIS

The relation between quantity of electric charge passed and the amount of ion deposited at the electrode is given by Faraday's laws of electrolysis. There are two laws :

### Faraday's first law :

According to Faraday's first law, the mass of the substance deposited or liberated in electrolysis is directly proportional to the charge passed through the electrolyte.

Let m be the mass of a substance deposited or liberated at an electrode when a charge q is passed through the electrolyte. Thus

$$m \propto q \quad \text{or} \quad m = Zq \qquad ...(1)$$

where Z is constant of proportionality and is known as electrochemical equivalent (E.C.E.) of the substance.

If i be the current passed through the electrolyte for a time t, then

$$q = i\,t \qquad ...(2)$$

From eqs. (1) and (2)

$$m = Z\,i\,t \qquad ...(3)$$

If q = 1 coulomb, then Z = m

Thus the electrochemical equivalent (E.C.E.) of a substance may be defined as the mass of the substance liberated or deposited on an electrode during electrolysis when one coulomb of charge is passed through the electrolyte.

The S.I. unit of E.C.E. is kg/coulomb. But generally this is expressed in gram/coulomb ($gC^{-1}$). The value of E.C.E. of copper and silver are $3294 \times 10^{-7}$ $gC^{-1}$ and $11180 \times 10^{-7}$ $gC^{-1}$ respectively.

### Faraday's second law :

According to Faraday's second law, when the same amount of charge is passed through different electrodes, the masses of different substances deposited or libarated at the electrodes are proportional to their chemical equivalents.

If $m_1$ and $m_2$ be the masses of the substances deposited or liberated and $E_1$ and $E_2$ be their respective chemical equivalent, then

$$\frac{m_1}{m_2} = \frac{E_1}{E_2} \quad \text{or} \quad \frac{Z_1 i t}{Z_2 i t} = \frac{E_1}{E_2} \quad \text{or} \quad \frac{Z_1}{Z_2} = \frac{E_1}{E_2}$$

The chemical equivalent of the substance is defined as the ratio of atomic weight to the valency. Thus

$$E = \frac{\text{atomic weight}}{\text{valency}}$$

The atomic weight of silver is 108 and its valency is 1. Therefore, its chemical equivalent is 108. Similarly, the chemical equivalent of copper is 31.75.

## FARADAY CONSTANT

From Faraday's second law

$$\frac{Z_1}{Z_2} = \frac{E_1}{E_2} \quad \text{or} \quad \frac{E_1}{Z_1} = \frac{E_2}{Z_2}$$

$$\therefore \quad \frac{E}{Z} = \text{a constant} = F \text{ (Faraday constant)}$$

Thus the ratio of $\left(\frac{E}{Z}\right)$ is same for all substances and is called as Faraday constant.

Now, $$F = \frac{E}{Z} = \frac{E}{\left(\frac{m}{q}\right)} = \frac{Eq}{m}$$

So, the Faraday constant is equal to the charge required to liberate one gram equivalent of substance at an electrode during electrolysis. Its value is 96500 C/gram equivalent.

In case of copper, E.C.E. = 0.0003294 $gC^{-1}$ and E = 31.75 g

$\therefore$ Faraday constant = $\dfrac{31.75}{0.0003294}$

= 96500 C/gram equivalent.

The charge of 1 mole of electrons is called one faraday. So

one faraday = $N_A \times e$

$= (6.023 \times 10^{23}) \times (1.602 \times 10^{-19}$ C)

= 96500 C.

Therefore, faraday is unit of charge (1 faraday = 96500 C) while the quantity charge per mole of electrons is called Faraday constant (F = 96500 C/mole or 1 faraday).

# Magnetic Effect of Current & Magnetism

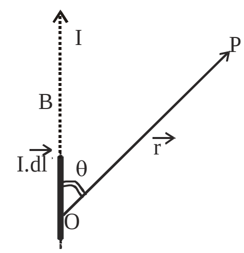
(i) Oesterd experimently discovered a magnetic field around a conductor carrying electric current.

(a) A magnet at rest or charge in motion produces a magnetic field around it while an electric charge at rest produces an electric field around it.

(b) A current carrying conductor has a magnetic field and not an electric field around it. On the other hand, a charge moving with a uniform velocity has an electric as well as a magnetic field around it.

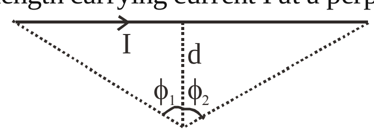
<u>Biot-Savart's law :</u> The magnetic induction $d\vec{B}$ at a point P due to an infinitesimal element of current (length dl and current I) at a distance r is given by :

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{(d\vec{l} \times \vec{r})}{r^3}$$




For $\theta = 0$ or $\theta = \pi$, $\sin\theta = 0$ thus field at a point on the line of the wire is zero.

(ii) The magnetic induction B due to a straight wire of finite length carrying current I at a perpendicular distance d is given by $\boxed{B = \frac{\mu_0}{4\pi} \times \frac{I}{d}(\sin\phi_1 + \sin\phi_2)}$

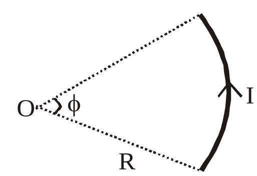



where $\phi_1$ and $\phi_2$ are the angles made by upper and lower ends of the wire with the perpendicular distance d at the point of observation.

(iii) If the wire is infinitely long, from both sides then $\phi_1 = \phi_2 = 90^0$. So, magnetic field at perpendicular distance d is given by $B = \frac{\mu_0}{4\pi} \times \frac{2I}{d} = \frac{\mu_0 I}{2\pi d}$

- Magnetic field due to a part of circular current carrying loop (arc) subtending angle $\phi$ at the centre O is :

$$\boxed{B = \frac{\mu_0 I \phi}{4\pi R}} \; ; \quad \text{where } \phi \text{ is in radian.}$$




So, Magnetic field at the centre of semicircular current carrying loop is : $B = \frac{\mu_0 I}{4R}$

- The magnetic induction along the axis of a long current carrying solenoid at the centre part. $B = \mu_0 nI$ where I = current flowing through solenoid, n = (N/l) = number of turns per unit length of solenoid. Magnetic induction at the ends of the solenoid. $B' = (\mu_0 nI / 2)$

## 2. Lorentz force on a charged particle in uniform constant magnetic field :

(i) When a charge q moves in a magnetic field of induction B with a velocity $\upsilon$ then it experience a sideway deflecting force F, given by $\vec{F} = q(\vec{\upsilon} \times \vec{B})$

Thus, the force $\vec{F}$ is always perpendicular to $\vec{\upsilon}$ and $\vec{B}$. So, no workdone and hence no change in kinetic energy.

(ii) If charge is at rest inside the magnetic field no force will act on it, hence the particle remains at rest.

(iii) If charge is moving parallel to magnetic field $(\theta = 0)$ no force acts on it. Thus, a charged particle initially moving parallel to magnetic field will continue to move with same constant velocity.

Case A When charged particle enters the magnetic field at right angles i.e $\boxed{\vec{V} \perp \vec{B}}$

(i) Since the force is perpendicular to velocity vector $\vec{\upsilon}$, it provides the required centripital force for circular motion.

(ii) (a) The force equation towards centre is $\dfrac{m\upsilon^2}{r} = q\upsilon B$

(b) The radius of circular path is $\boxed{r = \dfrac{m\upsilon}{qB}}$

where $m\upsilon = p = \sqrt{2mK}$ = momentum of the particle .

(c) Time period of revolution is $\boxed{T = \dfrac{2\pi r}{\upsilon} = \dfrac{2\pi m}{qB}}$

(d) The frequency is $\boxed{f = \dfrac{1}{T} = \dfrac{qB}{2\pi m}}$

(e) The angular frequency is $\omega = 2\pi f = \dfrac{qB}{m}$. This is often called cyclotron frequency.

Case B : When the particle enters the magnetic field at an inclination (i.e. $\vec{\upsilon}$ is not perpendicular to B).

(i) In this case, the path is helical.

(ii) It is a superposition of vertical drift and horizontal circular motion.

(iii) Due toc omponent of $\upsilon$ perpendicular to $\vec{B}$ i.e. $\upsilon_\perp = \upsilon \sin\theta$, the particle describes a circular path of radius r, such that

$$\frac{m\upsilon_\perp^2}{r} = q\upsilon_\perp B \qquad \text{or} \qquad r = \frac{m\upsilon_\perp}{qB} = \frac{m\upsilon\sin\theta}{qB}$$

(iv) The time period, frequency and angular frequency are :

(a) $T = \dfrac{2\pi m}{qB}$ (b) $f = \dfrac{qB}{2\pi m}$ (c) $\omega = \dfrac{qB}{m}$

(v) The pitch of the helical path is

$$p = \upsilon_\parallel \times T \quad = \upsilon\cos\theta \times T \quad = \frac{2\pi m\upsilon}{qB}\cos\theta \quad = \frac{2\pi r}{\tan\theta}$$

Ampere's law :

(i) The line integral of magnetic field around any closed path is equal to $\mu_0$ times the total current passing through the closed circuit, i.e. $\oint \vec{B}.\vec{dl} = \mu_0 I$

(ii) For a long solid metal rod of radius R carrying a current I

If $r < R$, $B = \left(\dfrac{\mu_0 I}{2\pi R^2}\right) r$, i.e. $B \propto r$

But If $r \geq R$; $B = \dfrac{\mu_0 I}{2\pi r}$ ; for both solid and hollow metallic rod (pipe).

(iii) For a hollow metallic rod carrying a uniform current, for points inside the rod, the magnetic field is zero.

## 5. Force on current carrying wire in a magnetic field :

(i) Force on a current element of length dl placed in a magnetic field B is : $d\vec{F} = I(d\vec{l} \times \vec{B})$

In special case of a straight wire of length l in a uniform magnetic field $\vec{B}$, the force is :

$\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$ or $F = IlB \sin\theta$ where $\theta =$ angle between of current flow and magnetic flight.

(ii) Force between two parallel current carrying conductors :

(a) Two parallel wires carrying currents in the same direction attract each other, while those carrying currents in the opposite direction repel each other.

(b) The force of attraction or repulsion per unit length between two parallel conductors carrying current $I_1$ and $I_2$ is given by $\dfrac{F}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi d}$

## Magnetic field produced by a moving charge :

The magnetic field produced by a moving charge q, at point P is : $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q(\vec{\upsilon} \times \hat{r})}{r^2}$ tesla

# MAGNETISM

## 1. Torque on a current carrying loop suspended freely in a magnetic field:

(i) When a loop is suspended freely in a magnetic field and a current is passed through i, we find that the resultant force on the loop is zero but the resultant torque is not zero (in general)

(ii) If normal to the plane of the loop makes an angle $\theta$ with the direction of uniform magnetic field B, then torque acting on the loop is given by : $\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{M} \times \vec{B}$

where $\vec{M} = NI\vec{A}$ = magnetic dipole moment of the current carrying coil.

## 2. Moving coil galvanometer:

(i) It is used for the measurement of current

(ii) The current to be measured is conducted to the coil through the suspension wire. The current deflects the coil in the radial magnetic field between the soft iron cylinder and concave pole pieces. The amount of deflection serves as a measure of current.

(iii) The current I is directly proportional to deflection $\phi$

At equilibrium, $\tau_{\text{deflecting}} = \tau_{\text{restoring}} \Rightarrow NIAB = C\phi \quad \Rightarrow \phi = \left(\dfrac{NAB}{C}\right)(I)$

where C= elastic torsional constant of the suspension wire, N = number of turns in the coil, A = Area per turn of the coil and B = magnetic induction of radial magnetic field.

(iv) The current sensitivity = $S_I = \dfrac{\phi}{I} = \dfrac{1}{K} = \dfrac{NAB}{C}$ ; The voltage sensitivity = $S_V = \dfrac{\phi}{V} = \dfrac{S_I}{R}$

## 3. Ammeter:

(i) An ammeter is used to measure the current in a circuit. It is connected in series with the ciruit to avoid division of current, but when placed in series with the circuit, it increases the resistance and decreases the current being measured by it. Hence, an ideal ammeter has a zero resistance.

(ii) The resistance of a milliammeter is more than that of an ammeter

(iii) To convert a galvanometer which gives full scale deflection for a current $I_g$ so that it may be used to read a current I, the value of the shunt required is given by: $S = \left[I_g G/\left(I - I_g\right)\right]$

where G = galvanometer resistance

## 4. Voltmeter:

(i) When a high resistance R is connected in series with a galvanometer of resistance G, it becomes a voltmeter. If $I_g$ represents the minimum current for full scale deflection of the galvanometer, then yhe minimum difference $V_g$ across the terminals of the galvanometer for full scale deflection is given by: $V_g = I_g G$

(ii) Now, the poential difference V across the terminal of the series combination of R and G is given by:

$$V = I_g\left(R + G\right) \qquad \text{So,} \qquad \frac{V}{V_g} = \frac{R+G}{G}$$

(iii) To measure potential difference across any element of the circuit we use a voltmeter. A voltmeter is connected in parallel with the element to avoid division of voltage, but when placed in parallel with the element it shares current from the element and decreases the potential difference across the element before measuring it. Hence, an ideal voltmeter has an infinite resistance so that it may not change the current in the element.

## 5. Magnet and Magnetism

(i) Magnetic poles exists in pairs, i.e., an isolated magnetic pole does not exist.

(ii) The force between magnetic poles obeys inverse square law.

(iii) A freely suspended current carrying solenoid behaves just like a bar magnet.

## 6. Magnetic lines of force:

(i) The magnetic lines of force are the curves such that the tangent drawn on it at any point indicates the direction of magnetic field.

(ii) The magnetic lines of force form closed curves, emerging from the north pole and entering the south pole.

(iii) These lines of force also never cross each other.

(iv) The intensity of magnetic field at any point in the field is defined as the number of lines of force passing per unit area perpendicular to the lines of force.

## 7. Other important points concerning a magnet :

(i) When a magnet of length 2l and pole strength m is placed in a magnetic field B, then the couple acting on the bar magnet is given be, $\tau = MB\sin\theta$, where M = m(2l)= magnetic moment of the magnet and $\theta$ is the angle between the bar magnet and direction of magnetic field.

(ii) The work done in deflecting the magnet through an angle $\theta$ from equilibrium position is given by:

$$W = MB(1-\cos\theta)$$

(iii) (a) If a bar magnet of moment M and pole strength m is cut into two equal halves along its axial line, then pole strength of each part is m/2 and the magnetic moment of each part is M/2.

(b) If a bar magnet of magnetic moment M and pole strength m is cut into two equal halves, along its equatorial line, the pole strength of each part is m and magnetic moment is M/2

(iv) (a) The magnetic induction on the axial line (end position) of a bar magnet is given by:

$$B_{axial} = \frac{\mu_0}{4\pi} \times \frac{2Md}{\left(d^2 - l^2\right)^2} \qquad (\text{along } S \rightarrow N)$$

where B = magnetic induction, d= distance between the centre of the magnet and the given point on the axial line, 2l = length of the magnet. For a short magnet,

$$\boxed{B_{axial} = \frac{\mu_0}{4\pi} \times \frac{2M}{d^3}}$$

(b) The magnetic induction on the equatorial line (broad side on position) is given by:

$$B_{equatorial} = \frac{\mu_0}{4\pi} \times \frac{M}{\left(d^2 + l^2\right)^{3/2}} \qquad (\text{parallel to } \overrightarrow{NS})$$

For a short magnet, $\boxed{B_{equatorial} = \frac{\mu_0}{4\pi} \times \frac{M}{d^3}}$ Thus, for a short magnet $\frac{B_{axial}}{B_{equatorial}} = \frac{2}{1}$

(v) The magnetic field induction due to a short bar magnet at a point distant d from the centre of the magnet is given by $\boxed{B = \frac{\mu_0}{4\pi} \times \frac{M}{d^3}\sqrt{1+3\cos^3\theta}}$

## 8. Properties of magnetic materials:

(i) Magnetising force or intensity of magnetisign field H:

(a) The intensity of magnetising field is defined as the force experienced by a unit north pole placed at a point in the field.

(b) The direction of $\vec{H}$ is the same as the direction of $\vec{B} = \mu\vec{H}$, where $\mu$ is called the magnetic permeability.

(ii) Intensity of magnetisiation (I)

(a) When a magnetic material is placed in a magnetic field, it is magnetised and it acquires a magnetic dipole moment M. The intensity of magnetisation is defined as the magnetic dipole moment per unit volume, i.e., $I = \frac{M}{V} = \frac{2ml}{A(2l)} = \frac{m}{A}$

where A is the area of cross-section of the material. So intensity of magnetisation may also be defined as pole strength per unit area of cross section.

(iii) Magnetic susceptibility $(\chi)$:

(a) Magnetic susceptibility indicates the ease with which a substance can be magnetised.

(b) The susceptibility is defined as the ratio of the intensiy of magnetisation to the magnetising field H in which the material is placed, i.e.,

$\chi = (I/H)$ It has no units.

(iv) Magnetic permeability $(\mu)$: The permeability is defined as the ratio of magnetic induciton (B) to the magnetising force (H), i.e. $\mu = (B/H)$

(v) Magnetic induction or flux density (B): The flux density is the total number of lines of force per unit area due to the flux density $B_0$ in vaccum produced by that magnetising field and flux density $B_m$ due to magnetisation of the material. Thus $B = B_0 + B_m$

## 9. Diamagnetic mateials:

(i) Materials which are repelled by magnets are known as diamagnetic maerials.

Example : bismuth, zinc, copper, silver, gold, diamond, NaCl, water, nitrogen, hydrogen, etc.

(ii) These material get magnetised in a direction opposite to that of the magnetic field.

(iii) In a non-uniform magnetic field, they move from regions of higher concentration field, they move from regions of higher concentration to regions of lower concentration.

(iv) Relative permeability of these materials is less than one but positive.

## 10. Paramagnetic material

(i) Materials which are feebly attracted by magnets are known as paramagnetic materials. Examples: aluminium, sodium, platanium, manganese, $CuCl_2$, $FeCl_3$, oxygen, etc.

(ii) These materials get magnetised in the direction of the magnetic field.

(iii) A paramagnetic rod suspended in a uniform magnetic field becomes parallel to the direction of the field.

(iv) In a non-uniform magnetic field, they move towards region of higher field.

(v) Relative permeability of these materials is just greater than one and positive

(vi) The magnetic susceptibility is inversely poportional to absolute temperature. This is called Curie law, $\chi = (C/T)$, where C is called Curie's constant.

## 11. Ferromagnetic materials:

(i) Subsances which are strongly attracted by magnets are known as feromagnetic substances. Examples: iron, nickel, cobalt, gadolinium.

(ii) A ferromagnetic rod, when suspended in a uniform magnetic field, aligns along the direction of the field.

(iii) In a non-uniform magnetic field, a ferromagnetic material moves towards regions of higher magnetic field.

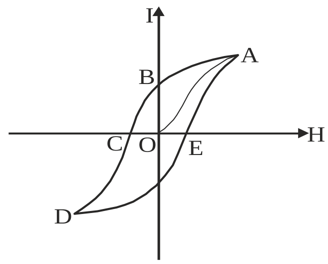
(iv) The relative permeability of these materials is very large ($10^2$ to $10^6$)

(v) The magnetic susceptibiliy of these materials is positive and very high ($10^2$ to $10^6$)

(vi) Ferromagnetism is due to the existence of magnetic domains. Ferromagnetic materials exhibit hysteresis.

## 12. Hysteresis loop

(i) Hysteresis loop or cycle is a plot a intensity of magnetisation (I) against magnetising field (H) over closed loop ABCDEA.




(ii) Retentivity: The residual magnetism present inside the speciment even when the external magnetising foce is made zero is called retentivity, or Retentivity is the capacity of the material to retain its magnetism when the magnetising force is removed. The intercept OB is a measure of retentivity.

(iii) Coercivity: Coercivity is the capacity of the material to reain its magnetism inspite of any demagnetising process. The intercept OC is a measure of retentivity.

(iv) The area of the hysteresis loop is a measure of work done or energy dissipation or hysteresis loop.

(v) (a) For soft iron: Coercivity is less, reentivity is more, hysteresis loss is less, susceptibility is more and permeability is more

(b) For steel: Coercivity is more, retentivity is less, hystersis loss is more, susceptibility is less and permeability is less.

(vi) Soft iron is used in transformers, moving coil galvanometers, electromagnets, etc., while steel is used for permanent magnets.

## 13. Earth's magnetic field:

(i) An imaginary vertical plane passing through magnetic north and magnetic north and magnetic south of a freely suspended magnet is called the magnetic meridian.

(ii) An imaginary vertical plane passing through north and south poles of the earth at a place is called as geographical meridian.

(iii) Declination: $(\theta)$

(a) Declination is the angle between magnetic meridian and geographical meridian at a given place.

(b) The value of declination at equator is $17°$ . Declination varies from place to place.

(c) The lines joining the places of equal declination are called isogonal lines

(d) The lines joining the places of zero declination are called agonal lines.

(v) Dip or inclination $(\delta)$ :

(a) The angle made by the earth's magnetic field with the horizontal at a place is called dip or inclination at that place.

(b) Dip circle is the instrument used to measure the dip

(c) It varies beween 0° and 90°. At the magnetic equator it is zero and 90° at poles

(d) The lines joining the places of equal dip are called isoclinal lines.

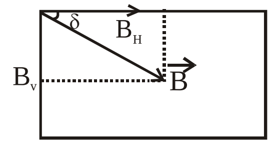
(e) The lines joining the places of zero dip are called aclinal lines.

(vi) Horizontal component $(B_H)$

(a) The component of the total induction of the earth's magnetic field (B) along the horizontal direction is called the horizontal component.

(b) The horizontal component can be measured with the help of a deflection magnetometer.

(c) If $\theta$ is the dip, i.e., the angle between total magnetic induction of the earth's magnetic field B and horizontal component $B_H$ then




(I) horizontal component $B_H = B\cos\delta$

(II) vertical component $B_V = B\sin\delta$

(III) $B = \sqrt{B_H^2 + B_V^2}$ and $\tan\delta = (B_V / B_H)$

(d) $B_H$ is zero at magnetic poles and maximum at the magnetic equator.

## 14. Vibration magnetometer

(i) If a magnet of dipole moment M oscillates in a uniform induction field B, then the time period of vibration magnetometer is :

$T = 2\pi\sqrt{\frac{I}{MB}}$ where $I = m\left[\frac{l^2 + b^2}{12}\right]$ is the moment of inertia of the magnet abou the axis of oscillation.

(ii) If two magnets of dipole moments $M_1$ and $M_2$ of same dimensions and same mass are oscillating in the same field, then $\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}$

(iii) A magnet is oscillating in a magnetic field and its time period is T sec. If another identical magnet is placed over that magnet with similar poles togehter, then the time period remains unchagned $(\because I' = 2I \,\&\, M' = 2M)$

(iv) Two magnents of magnetic moments $M_1$ and $M_2$ ($M_1 > M_2$) are placed one over the other parallel. If $T_1$ is the time period when like poles touch each other and $T_2$ is the time period when unlike poles touch each other, then $\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$

# EMI

## Magnetic Flux:

(i) The magnetic flux through a small area $d\vec{A}$ placed in a magnetic field $\vec{B}$ is defined as:

$$d\phi = \vec{B}.d\vec{A} = B(dA)\cos\theta$$

(ii) The magnetic flux can be positive, negative or zero depending on the angle $\theta$. For $\theta = 90^o$ and $\phi = 0$. Thus, whenever the angle between area vector, and magnetic field is 90°, the flux is zero, i.e., whenever the plane of the surface is parallel to $\vec{B}$, the flux is zero. The flux is positive for $0^o \le \theta \le 90^o$ and negative for $180^o \ge \theta \ge 90^o$.

(iii) The magnetic flux through a closed surface is always zero, i.e.,

$$\phi = \oint \vec{B}.d\vec{A} = 0 \quad ;$$ This equation suggests, there is no existance of monopoles.

## Laws of electromagnetic inducîton:

(i) First law: Whenever there occurs a change in the magnetic flux linked with a coil, there is produced an induced e.m.f. in the coil. The induced e.m.f. lasts so long as the change in flux is taking place. There is an induced current only when coil circuit is complete.

(ii) Second Law : The magnitude of induced e.m.f. is equal to the rate of change in the magnetic flux, i.e. $e \propto (d\phi/dt)$. For N turns, $e \propto N(d\phi/dt)$

## Lenz's Law :

The direction of the induced current is such that it tends to oppose the cause of change in magnetic flux.

(a) Combining with Faradays law of EMI, we have $e = -N.\dfrac{d\phi}{dt}$ for N number of turns.

(b) Lenz's law is based on law of conservation of energy.

## Some other important points:

(i) The induced e.m.f. in a circuit does not depend on the resistance of the circuit as $e = -\dfrac{d\phi}{dt}$. However, the induced current in the circuit does depend on the resistance.

$$I = \frac{e}{R} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right)$$

(ii) The induced charge that flows in the circuit depends on the change of flux only and not on how fast or slow the flux changes.

$$\frac{dq}{dt} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \quad \text{or} \quad dq = \frac{d\phi}{R}$$

On integrating, the total charge that flows in the circuit is found to be:

$$q = \frac{(\phi_1 - \phi_2)}{R}$$

## Induced E.M.F. across a conducting rod:

(i) Conducting rod moving in a uniform magnetic field: When a conducting rod of length l moves with a velocity $\upsilon$ in a uniform magnetic field of induction B such that the plane containing $\vec{\upsilon}$ and l makes an angle $\theta$ with $\vec{B}$ thent he magnitude of the average induced e.m.f. |e| is given by : $|e| = \upsilon Bl\sin\theta$

(ii) Conducting rod rotating with angular velocity $\omega$ in a uniform magnetic field : When a rod of length l rotates with angular velocity $\omega$ in a uniform magnetic field B, then induced e.m.f. across the ends of the rotating rod is : $e = (1/2)B\omega l^2 = B\pi f l^2 = BAf$

where $A = \pi l^2$ = area swept by the rod in one rotation and f is the frequency of rotation.

## Self-inductance :

(i) When a current I flows through a coil, it produces a magnetid flux $\phi$ through it. Then $\phi \propto I$ or $\phi = LI$, where L is constant, called the coefficient of self-in-duction or self-inductance of the coil.

(ii) Further, $$e = -(d\phi/dt) = -\frac{d}{dt}(LI) = -L(dI/dt)$$

(iii) Self-inductance L of a solenoid ofN turns, length l, area of cross-section A, with a core material of relative permeability $\mu_r$ is given by : $L = \mu_r\left(\frac{\mu_0}{4\pi}\right)\frac{4\pi N^2 A}{l}$

## Mutual inductance :

(i) When a current I flowing in the primary coil produces a magnetic flux $\phi$ in the secondary coil, then $\phi \propto I$ or $\phi = MI$, where M is a constant, called the coefficient of mutual induction or mutual inductance.

(ii) $$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI) = -M\left(\frac{dI}{dt}\right)$$

(iii) Mutual inductance M of two coaxial solenoid is given by : $$M = \mu_r\left(\frac{\mu_0}{4\pi}\right)\frac{4\pi N_1 N_2 A}{l}$$

where $N_1$ and $N_2$ represent the total number of turns in the primary coil and the secondary coil.

## Series and parallel combination of inductances :

(i) Two inductors of self-inductances $L_1$ and $L_2$ are kept so far apart that their mutual inductance is zero. These are connected in series. Then the equivalent inductance is : $L = L_1 + L_2$

(ii) Two inductors of self-inductances $L_1$ and $L_2$ are connected in series and they have mutual inductance M. Then the equivalent inductance of the combination is : $L = L_1 + L_2 \pm 2M$

The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.

(iii) Two inductors of self-inductances $L_1$ and $L_2$ are connected in parallel. The inductors are so far apart that their mutual inductance is negligible. Then their equivalent inductance is :

$$L = \frac{L_1 L_2}{L_1 + L_2} \quad \text{or} \quad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

(iv) If two coils of self-inductances $L_1$ and $L_2$ are wound over each other, the mutual inductance is given by: $M = K\sqrt{L_1 L_2}$ (where K is called coupling constant). It is equal to zero if there is no coupling. It is equal to zero if there is no coupling. It is equal to 1 for maximum coupling. The maximum coupling occurs when the two coils are wound over each other, over a ferromagnetic core.

## Growth and decay of current in LR circuit :

(i) When a switch in an LR circuit is closed, the current does not become maximum immediately but it takes some time, i.e. there is a time lag.

(ii) If R be the resistance present in the circuit, then current I at any instant is given by : $E - L\,(dI/dt) = IR$

(a) At start, I = 0, so (dI/dt) is maximum and $(dI/dt)_{max.} = E/L$.

(b) Finally, (dI/dt) = 0, therefore I is maximum and $I_{max.} = E/R$ i.e. final current in the circuit is independent of inductance L.

(iv) The instantaneous current in the circuit during its growth is given by : $I = \frac{E}{R}(1 - e^{-\frac{R}{L}t})$

Here, (L/R) = time constant of LR circuit. The time constant is the time in which current rises to 0.6321 times the maximum current whichis equal to (E/R).

(v) When the switch in an LR circuit is opened, the instantaneous current I is given by $I = \left(\frac{E}{R}\right)e^{-\frac{R}{L}t}$

Hence, the time consant of an LR circuit may also be defined as the time in which the current falls to 0.3679 times of its initial current.

(vi) Decay or growth of current in LR circuit is fast when L/R is small and slow when (L/R) is large.

## Transformer :

(i) The transformer was inverted by Henry. It works on the principle of mutual induction and is used in AC only. It suitably changes AC voltage.

(ii) A transformer consists of a (a) primary coil of turns $N_p$, (b) seconcdary coil of turns $N_s$ and (c) a laminated soft iron core.

(iii) If $V_p$ and $V_s$ denote the voltage across the primary coil and the secondary coilrespectively. then $(V_s/V_p) = (N_s/N_p)$.

(iv) In an actual transformer,

Output power $\leq$ input power but in an ideal transformer

Output power = input power i.e. $V_sI_s = V_pI_p$

($I_p$ and $I_s$ are the current in primary and secondary coils respectively).

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

(v) There are two types of transformers :

(a) Step-up trnasformer : Here, $N_s > N_p$, so $V_s > V_p$ and $I_s < I_p$.

(b) Step-down transfomer : Here, $N_s < N_p$, so $V_s < V_p$ and $I_s > I_p$.

## ALTERNATING CURRENT

The instantaneous value of an AC is given by : $I = I_0 \sin \omega t$

and the alternaing voltage is given by : $E = E_0 \sin \omega t$

Here, $\omega$ is the angular frequency of Ac and $(\omega / 2\pi)$ is the frequency of Ac. $(2\pi / \omega)$ represents the time period of AC.The frequency of AC represents the number of cycles of AC completed in one second. AC supplied in India has a frequency of 50Hz.

### Mean Value :

The mean value of AC represented by the equation, $I = I_0 \sin \omega t$, is zero over one complete cycle and is meaningless. In practice, mean value of alternating current refers to its average value over half cycle.

$$I_{mean} = \frac{2I_0}{\pi}$$

- A moving coil galvanometer, connected to an AC source of 50 Hz AC, shows a steady zero reading of the pointer. If the frequency is 2 Hz, the pointer oscillates with equal amplitude on either side of zero position.

  RMS or Virtual value : The RMS value of defined as the square root of the mean of square of the instantaneous value of current over the complete cycle. It may also be defined as the direct current which produces the same heating effect in a resistor as the actual AC in the same time.

$$I_V = \frac{I_0}{\sqrt{2}} = I_{rms}$$

- AC ammeter or voltmeter measures virtual current or virtual voltage (these are hot wire instrument)
- Form Factor $= \frac{\text{Virtual value}}{\text{Mean value}} = \frac{e_0 / \sqrt{2}}{2e_0 / \pi} = \frac{\pi}{2\sqrt{2}} = 1.1$

### (A) AC through pure resistor R :

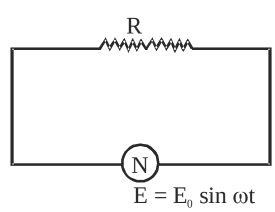
(i) Alternating e.m.f. of the source : $E = E_0 \sin \omega t$

(ii) A resistance oppose current but does not oppose a change in current. Hence, current is in phase with e.m.f.

(iii) The instantaneous value of the current is given by :

$$I = \frac{E_0 \sin \omega t}{R}$$

The virtual value of current $I_V$ is given by $I_V = \frac{E_V}{R}$




### (B) AC through pure inductor of inductance L :

(i) Alternating e.m.f. of the source : $E = E_0 \sin \omega t$

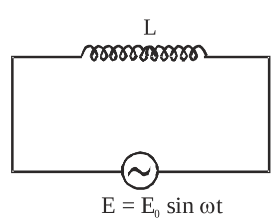
(ii) An inductor does not opposte current but opposes a change in current.

(iii) Since the voltage changes continuously, Hence, the current reacts to the change and inductive reactance

$$X_L = \omega L$$

(iv) The current lags behind the voltage by $\pi / 2$

(v) The instantaneous current is given by : $I = \left(\frac{E_0}{\omega L}\right) \sin\left(\omega t - \frac{\pi}{2}\right)$




The virtual value of the current is given by $I_V = E_V / \omega L$

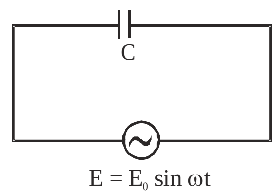
## (C) AC through a pure capacitor of capacitance C :

(i) Alternating e.m.f. of the source. $E = E_0 \sin \omega t$ .

(ii) A capacitor has infinite resistance for a DC source. With an AC source, voltage changes, hence charge ont he plates of the capacitor changes with time i.e. there is a current. The current leads the voltage by $\pi / 2$ .

(iii) The capacitor has a capacitive reactang $X_C$ in AC circuit, given by : $X_C = (1/\omega C)$

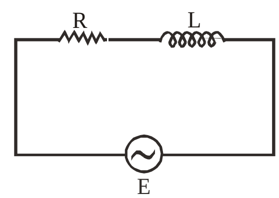
(iv) The instantaneous valeu of current is




$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The virtual value of current $I_V$ is given by : $I_V = \frac{E_V}{(1/\omega C)}$

## (D) AC through RL circuit :

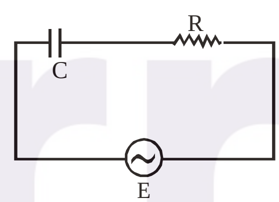



(i) In this case, we have impedance.

$$Z = \sqrt{R^2 + (X_L)^2} \quad \text{and} \quad I = \frac{E_0}{z} \sin(\omega t - \phi)$$

(ii) Phase angle, $\tan \phi = (X_L / R) = (\omega L / R)$

## (E) AC through CR circuit :

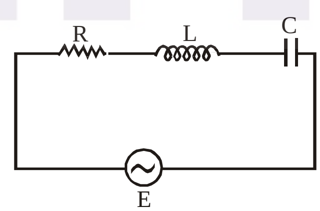



(i) In this case, we have impedence

$$Z = \sqrt{R^2 + (X_C)^2} \quad \text{and} \quad I = \frac{E_0}{z} \sin(\omega t + \phi)$$

(ii) Phase angle, $\tan \phi = (X_C / R) = (I / \omega CR)$

## (F) AC source connected to resistor R, an inductor L and a capacitor C in series :




(i) The virtual current $I_V$ is given by :

$$I_V = \frac{E_V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

(ii) The current and voltage have a phase difference $\phi$ given by :

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} \quad \text{and So, current} \quad I = \frac{E_0}{z} \sin(\omega t + \phi)$$

(iii) The impedance which represents the effective resistance of the circuit to AC source is represented by Z. The impedance Z is the vector sum of resistance and reactances in AC series circuit.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(iv) In AC series combination, virtual voltage are added vectorially i.e.

$$Ev = \sqrt{V_R^2 + (V_L - V_C)^2}$$

where $V_R = I_V R$, $V_C = I_V \, (1/C\omega)$ and $V_L = I_V (\omega L)$

## Power in an AC circuit :

(i) The power in an electric circuit is the rate at which electric energy is consumed in the circuit.

$$\text{Average power, } \langle P \rangle = E_{rms} \times I_{rms} \times \cos\phi$$

So, the product of rms value of voltage and current when multiplied by $\cos\phi$ gives the power dessipated.

(ii) $\cos\phi$ is known as power factor. For a LCR series circuit.

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{R}{z}$$

(iii) Wattless current : we know that the average power in a circuit is given by :

$\langle P \rangle = E_{rms} \times I_{rms} \times \cos\phi$; Here $(I_{rms}\cos\phi)$ is called watt less current which does not contribute in power dessipation.

## Resonance in Series R-L-C circuit :

(i) A particular frequency of AC at which impedance of a series LCR circuit becomes minimum or the current becomes maximum is called the resonant frequency and the circuit is called as series resonance circuit.

(ii) At resonance frequency

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(iii) At resonance $\tan\phi = \dfrac{\omega_0 L - \frac{1}{\omega_0 C}}{R} = \text{zero}$ i.e. $\phi = 0$ i.e. phase difference is zero. So, $\cos\phi = +1$, i.e. power factor is maximum.

(iv) At resonance, $I_{max} = \dfrac{E_V}{Z_{min}} = \dfrac{E_V}{R}$

In this case, $V_L = I_V \omega_0 L$, $\quad V_C = I_V\ (1/\omega_0 C)$

$$\because V_L = V_C \quad \text{and} \quad E_V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad \text{hence,} \quad E_V = V_R$$

i.e. at resonance with L and C in series, the current is maximum through the L and C combination but potential difference across the combination is zero.

(v) $Q = \dfrac{\omega L}{R}$ or $\dfrac{1}{\omega CR}$ i.e. the quality factor may be defined as the ratio of reactance of either inductance or capacitance at resonance frequency to the resistance of the circuit.

## Choke coil :

(i) We know that in purely resistive circuit, the power loss is maximum because power factor $\cos\phi = (R/Z) = 1$ $(\because Z = R)$. Hence, the use of resistance is avoided in AC circuits to control current.

(ii) A choke coil is a coil which has high inductance and negligible resistance. Thus, the power factor is almost zero. So a choke coil controls the alternating current without an apreciable energy loss. This is used with fluorescent tubes to control the current.

# ELECTROMAGNETIC WAVES

## Maxwell's Equations

(a) $\oint \vec{E}.d\vec{s} = \dfrac{q}{\epsilon_0}$ (Gauss's theorem in electrostatics)

(b) $\oint \vec{B}.d\vec{s} = 0$ (Gauss's law in magnetism)

(c) $\oint \vec{E}..d\vec{s} = -\dfrac{d}{dt}\int \vec{B}.d\vec{s}$ (Faraday's law of electromagnetic induction)

## Important Features of Electromagnetic waves

- E.M. waves are transverse waves in which there are sinusoidal variations of electric and magnetic fields. These two fields exist at right angles to each other as well as at right angles to the direction of wave propagation.
- Both these fields vary with time and space and have the same frequency of variation.
- These waves can travel through vacuum also, hence these waves are non-mechanical.
- Velocity of electromagnetic wave in free space (vaccum) is constant and given by

$$c = \frac{1}{\sqrt{\mu_0 \in_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} = 3 \times 10^8\, ms^{-1}$$

- Direction of wave propagative is given by the direction of $\vec{E} \times \vec{B}$.
- Examples of electromagnetic waves are radio waves, microwaves, infrared rays, light waves, ultraviolet rays, X-rays and $\gamma -$rays.
- The amplitudes of electric and magnetic fields in free space, in electromagnetic waves are related by $E_0 = cB_0$

## Energy Density of Electromagnetic Eave

- The average energy density of electric field is, $U_E = \dfrac{1}{2}\in_0 E^2 = \dfrac{1}{2}\in_0 \left(E_0/\sqrt{2}\right)^2 = \dfrac{1}{4}\in_0 E_0^2$
- The average density of magnetic field is, $U_B = \dfrac{B^2}{2\mu_0} = \dfrac{\left(B_0/\sqrt{2}\right)^2}{2\mu_0} = \dfrac{B_0^2}{4\mu_0}$
- In EM waves the average energy density due to either field are equal i.e. $U_E = U_B$

## Momentum of Electromagnetic wave

- The electromagnetic wave has linear momentum associated with it. The linear momentum p carried by the portion of wave having energy U is given by $p = \dfrac{U}{c}$. As per plank, $p = \dfrac{hv}{c} = \dfrac{h}{\lambda}$

## Production of Electromagnetic Waves

- An electromagnetic wave is emitted when an electron orbiting in higher stationary orbit of an atom jumps to one of the lower stationary orbits of that atom.
- Accelerated charge (e.g. LC oscillator) produces EM waves.
- Some electromagnetic waves (i.e. X-rays) are also produced when fast moving electrons are suddenly stopped by a metal surface having high atomic number.

## Electromagnetic Spectrum

The major components spectrum with their wavelength $\lambda$ ranges in incrasing order are

1. Gamma rays $[\lambda = 6 \times 10^{-19}\, m\, to\, 10^{-11}\, m]$

2. X-rays $[\lambda = 6\times10^{-19}\,\text{m to } 3\times10^{-8}\,\text{m}]$
3. Ultraviolet $[\lambda = 6\times10^{-10}\,\text{m to } 4\times10^{-7}\,\text{m}]$
4. Visible light $[\lambda = 4\times10^{-7}\,\text{m to } 8\times10^{-7}\,\text{m}]$
5. Infra red $[\lambda = 8\times10^{-7}\,\text{m to } 3\times10^{-5}\,\text{m}]$
6. Heat radiations $[\lambda = 8\times10^{-5}\,\text{m to } 10^{-1}\,\text{m}]$
7. Micro waves $[\lambda = 10^{-3}\,\text{m to } 0.03\,\text{m}]$
8. Ultra high frequency $[\lambda = 10^{-1}\,\text{m to } 1\,\text{m}]$
9. Very high ratio frequency $[\lambda = 1\,\text{m to } 10\,\text{m}]$
10. Radio frequencies $[\lambda = 10\,\text{m to } 10^{4}\,\text{m}]$

# Optics
# (Ray Optics)

## REFLECTION :

Light is a form of energy which is propagated as electromagnetic waves. It does not require a medium for its propagation. Its speed in free space (i.e., vacuum) is $3 \times 10^8$ m/s.

As in visible region the possible number of different wavelengths (or frequencies) between 4000 Å and 7000 Å are infinite and different frequencies produce the sensation of different colours, the number of colours in visible region is infinite. However, our eye can distinguish only following six colours.

Eye is most sensitive to yellow-green light, light of wavelength 5550 Å.

Persistence of eye is 1/10 s, i.e., if time interval between two successive light pulses is lesser than 0.1 s, eye cannot distinguish them separately. Also the resolving limit of the eye is one minute, two objects separated by distance d will not be distinctly visible to the eye if the angle, $\theta$ subtended by them at the eye,

$$0 < 0°1' \quad \text{i.e.,} \quad \frac{d}{D} < \frac{\pi}{(180 \times 60)}$$

When light passes from one medium to the other, velocity and wavelength m change, amplitude may decrease or remain constant, but frequency and colour of light do not change, i.e., colour of light is determined by its frequency (a not wavelength), e.g., if red light passes from air to water (or glass) its velocity and wavelength in water (or glass) will be different from that in air frequency and colour remain the same.

If the velocity of light in a medium (such as water or glass) is same in all directions, the medium is called isotropic. However, if the velocity of light is different in different directions (e.g., in calcite or quartz, etc.), the medium is said to be anisotropic for that light.

### Principle of Reversibility of Light:

If a light ray is reversed, it always retraces its path. Object and image positions are interchangeable. The points corresponding to object and image are called conjugate points.

### Optical Path :

It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium. If light travels a path length d in a medium at speed v, the time taken by it will be (d/v). So optical path length.

$$L = c \times \left[\frac{d}{v}\right] = \mu d \; ; \; \left[\text{as} \frac{c}{v} = \mu\right]$$

As for all media $\mu > 1,$ optical path length is always greater than actual path length.

## REFLECTION:

### Laws of Reflection :

The incident-ray, reflected-ray and normal to the reflecting surface at the point of incidence all lie in the same plane. The angle of reflection is equal to the angle of incidence, i.e., $\angle i = \angle r$.

## Reflection from Plane-surfaces

The image is always erect, virtual and of same size as the object. It is at the same distance behind the mirror as the object is in front of it.

- Some points regarding plane reflecting surface.
- If the rays after refraction or reflection actually converge at a point, the image's said to be real and it is not only to be seen but also obtained on a suitably placed screen. However, if the rays do not actually converge but appear to do so and actually diverge, the image is said to be virtual. A virtual image can only be seen and cannot be obtained on a screen at the position of the image.
- If an object moves towards a plane mirror at speed v, the image will also approach (or recede) at same speed v, the speed of image relative to object will be v – (–v) = 2v. Similarly if the mirror is moved towards or (away from) the object with speed v the image will move towards (or away from) the object with speed 2v.
- Deviation $\delta$ is defined as the angle between directions of incident ray and emergent ray. So if light is incident at an angle of incidence i, $\delta = 180 - (\angle i + \angle r) = (180 - 2i)$

  If keeping the incident ray fixed, the mirror is rotated by an angle $\theta,$ about an axis in the plane of mirror, the reflected ray is rotated through an angle $2\theta.$
- The image formed by a plane mirror suffers lateral-inversion in the image formed by a plane mirror left is turned into right and vice-versa with respect to object as.
- To see his full image in a plane mirror a person requires a mirror of at least half of his height.
- To see a complete wall behind himself a person requires a mirror of at least (1/3) the height of wall and he must be in the middle of wall and mirror.
- If there are two plane mirrors inclined to each other at an angle $\theta,$ the number of images of a point object formed are determined as follows:
- If $(360/\theta)$ is even integer (say m) number of images formed

  n = (m – 1), for all positions of object
- If $(360/\theta)$ is odd integer (say m) number of images formed

  n = m, if the object is not on the bisector of mirrors

  n = (m – 1), if the object is on the bisector of mirrors
- If $(360/\theta)$ is a fraction, the number of images formed will be equal to its integral part.

## Reflection at Spherical Surface

- The focal length of a spherical mirror of radius R is given by f = (R/2)
- The power of a mirror is defined as $P = -\dfrac{1}{f\,(\text{in m})} = -\dfrac{100}{f\,(\text{in cm})}$
- If a thin object of linear size O is situated vertically on the axis of a mirror at a distance u from the pole and its image of size I is formed at a distance v (from the pole), magnification (transverse) is defined as :

$$m = \left[\frac{I}{O}\right] = -\left[\frac{v}{u}\right]$$

–ve magnification implies that image is inverted with respect to object* +ve magnification means that image is erect with respect to object.

- If an object is placed at a distance u from the pole of a mirror and its image is formed at a distance v

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

- If the 1-D object is placed with its length along the principle axis, the so called longitudinal magnification becomes

$$m_L=\frac{I}{O}=-\frac{(v_2-v_1)}{(u_2-u_1)}=-\frac{dv}{du}=\left[\frac{v}{u}\right]^2=m^2$$

## Newton's formula:

- In case of spherical mirrors if object distance ($x_1$) and image distance ($x_2$) are measured from focus instead of pole u (f + $x_1$) and v = (f + $x_2$), the mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ reduces to } \frac{1}{(f+x_2)}+\frac{1}{(f+x_1)}=\frac{1}{f}$$

which on simplification gives $x_1x_2 = f^2$

In case of spherical mirrors if we plot a graph between.

(1/u) and (1/v), it will be a straight line with intercept (1/f) with each axis as $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ becomes y + x = c with $c=\frac{1}{f}$.

Graph between u and v will be a hyperbola, as for $u = f, v=\infty$ and for $u=\infty$, $v = f$. A line u = v will cut this hyperbola at (2f, 2f).

Concave mirror behaves as convex lens (both convergent) while convex mirror behaves as concave lens (both divergent).

## Usasge of Convex / Concave Mirrors:

As convex mirror gives erect, virtual and diminished image. In convex mirror, the field of view is increased as compared to plane mirror. This is why it is used as rear-view mirror in vehicles. Concave mirrors give enlarged, erect and virtual image (if object is between F and P), so these are used by dentists for examining teeth. Further due to their converging property concave mirrors are also used as reflectors in automobile head lights and search lights and by ENT surgeons in ophthalmoscope.

## Refraction :

### Law of Refraction:

1. Incident ray, Refracted ray and normal at a point lie in same plane.
2. Product of refractive index and the sine of angle made by light ray with the normal remains constant when light travels from one medium to another.

i.e. $\mu \times \sin i = \text{constant}$

$$\mu_1 \times \sin i = \mu_2 \sin r$$

If light passes from rarer to denser medium $\mu_1=\mu_R$ and $\mu_2=\mu_D$ so that in passing from rarer to denser medium, the ray bends towards the normal.

If light passes from denser to rarer medium $\dfrac{\sin i}{\sin r} = \dfrac{\mu_R}{\mu_D} < 1$

in passing from denser to rarer medium, the ray bends away from the normal.

## Apparent Depth and Normal Shift

$$\frac{d_{Ac}}{d_{Ap}} = \frac{\mu_1}{\mu_2}$$

Object in a denser medium is seen from a rerer medium.

The distance between object and its image, called normal shift and with $d_{Ac}$ = t, will be

$$x = d_{Ac} - d_{Ap} = t - (t/\mu) = t[1 - (1/\mu)]$$

Object in a rarer medium is seen from a denser medium

$$x = d_{Ap} - d_{Ac} = [(\mu - 1)]t$$

Further in passing through a medium of thickness t and refractive index $\mu$, a ray incident at a small angle $\theta$ is displaced parallel to itself by 'y' called lateral displacement.

$$y = \left[\frac{\mu - 1}{\mu}\right]t$$

If there are number of liquids of different depths, one over the other

$$d_{Ac} = d_1 + d_2 + d_3 \ldots \text{ and } \qquad d_{Ap} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + \ldots$$

$$\mu = \frac{d_{Ac}}{d_{Ap}} = \frac{d_1 + d_2 + \ldots}{(d_1/\mu_1) + (d_2/\mu_2) + \ldots} = \frac{\Sigma d_i}{\Sigma(d_i/\mu_i)}$$

$$\mu = \left[\frac{2\mu_1\mu_2}{\mu_1 + \mu_2}\right] = \text{Harmonic mean.}$$

## Total Internal Reflection :

If light is passing from denser to rarer medium through a plane boundary, then $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$; so with $\mu = (\mu_D/\mu_R)$,

$$\sin i = \frac{\mu_R}{\mu_D}\sin r \quad \text{i.e.,} \quad \sin i = \frac{\sin r}{\mu}$$

$$\sin i \propto \sin r \quad \text{with} \quad (\angle i) < (\angle r)$$

So as angle of incidence i increases angle of refraction r will also increase and for certain value of i (<90°) r will become 90°. The value of angle of incidence for which r = 90° is called critical angle and is denoted by $\theta_C$ and in the light will be given by

$$\sin\theta_C = \frac{\sin 90}{\mu} \quad \text{i.e.,} \quad \sin\theta_C = \frac{1}{\mu}$$

the total light incident on the boundary will be reflected back into the same medium from the boundary. This phenomenon is called total internal reflection. Here it is worthy to note that :

For total internal reflection to take place light must be propagating from denser to rarer medium.

In case of total internal reflection, as all (i.e., 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirrors or refraction from lenses there is some loss of intensity as all light can never be reflected or refracted.

From Snell's law, $\theta_C = \sin^{-1}(1/\mu)$

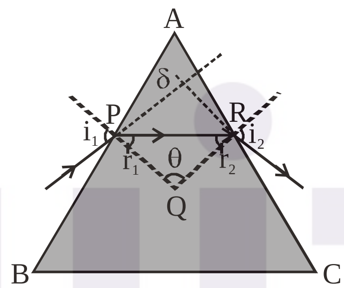
## Prism-Theory:

Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which light emerges are plane and non-parallel.

Angle of prism or refracting angle of prism means the angle between the faces on which light is incident and from which it emerges.

Angle of deviation means the angle between emergent and incident rays.

The angle of deviation $\delta$ will be




$$\delta = (i_1 - r_1) + (i_2 - r_2) \qquad \left[\begin{array}{l} r_1 + r_2 + \theta = 180^o \Rightarrow A + 90^o + \theta + 90^o = 360 \Rightarrow A + \theta = 180^o \\ r_1 + r_2 + \theta = A + \theta \Rightarrow r_1 + r_2 = A \end{array}\right]$$

$$\delta = (i_1 + i_2) - (r_1 + r_2)$$

$$\delta = [i_1 + i_2 - A] \qquad (r_1 + r_2) = A$$

This is the required result and holds good if emergent ray exists.

If angle of prism A is small, $r_1$ and $r_2$ (as $r_1 + r_2 = A$) and hence $i_1$ and $i_2$ will also be small. Since for small angles sin $\theta = \theta$. Snell's law at first and second surfaces of prism gives respectively:

$$i_1 = \mu r_1 \quad \text{and} \quad \mu r_2 = i_2$$

$$(i_1 + i_2) = \mu(r_1 + r_2) = \mu A$$

So for small angle of prism.

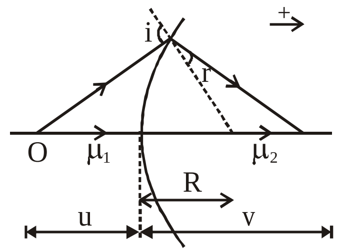
Deviation will be maximum when angle of incidence $i_1$ is maximum

$$\delta_{max} = 90 + i_2 - A$$

## Minimum deviation :

Minimum deviation of a ray through the prism is given by: $\mu = \dfrac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$

## Refraction at Spherical Surfaces




Refraction at single spherical surface is shown in the figure above.

Here light rays starting form an object O, placed in a medium of R.I. $\mu_1$, move into another medium of R.I. $\mu_2$ after being refracted at a curved surface of radius R. Object distances (u) and image distance (v) as shown in the figure, are measured from pole. Radius of curvature (R) is also measured from the pole, as shown. Then

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For refraction at single surface and for real objects,

$\mu$ is always –ve

v is +ve if image forms on otherside, ie in next medium.

v is –ve if image forms on same side, ie in first medium

R is +ve if surface is convex as seen from the object

The image is real if formed inside the second medium, because such image is due to actual meeting of rays after refraction. If formed on the same side, there is no actual meeting of rays, hence the image is virtual.

Lateral magnification, $m = \frac{\text{Image size}}{\text{Object size}} = \frac{\mu_1 v}{\mu_2 v}$

## Refraction Through Lens

### Lens maker's formula for thin lens

Let $R_1$ and $R_2$ be radii of curvature of the first and second spherical surface. Let f be the focal length of the lens and $\mu$ be the refractive index of lens material w.r.t. the medium in which the lens is placed. Then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{or} \quad P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ where P is power of the lens

### Linear Magnification for a lens

Magnification $m = \frac{v}{u}$

We know that $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{v}{v} - \frac{v}{u} = \frac{v}{f}$ $\Rightarrow 1 - \frac{v}{u} = \frac{v}{f}$ $\Rightarrow \frac{v}{u} = \frac{f - v}{f}$ $\Rightarrow m = \frac{f - v}{f}$

We can also express m in terms of u and f.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{u}{v}-\frac{u}{u}=\frac{u}{f} \quad \Rightarrow \quad \frac{u}{v}=\frac{u+f}{f} \quad \Rightarrow \quad m=\frac{f}{f+u}$$

## Power of a Lens

$$P=\frac{1}{f}$$

Since focal length of a convex lens or a converging lens is positive, therefore its power is positive. Similarly, the power of a concave lens or a diverging lens is negative.

Opticians express the power of a lens in terms of a unit called the DIPOTRE. It is regarded as the SI unit of optical power. The power of a lens is said to be one dioptre if the focal length of the lens is 1 metre.

When focal length is in cm, $P=\frac{100}{f}$ dioptre.

## Lens Formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

This equation holds good for both convex and concave lenses, whether the image formed is real or virtual.

For real image, v is +ve because real image always forms or other side of the lens. For virtual image v is –ve.

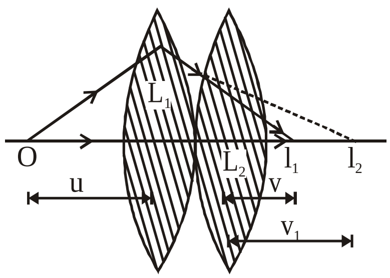
Focal length f is +ve for convex lens and –ve for concave lens. Object distance u is always -ve for a real object and u is always +ve for a virtual object.

Convex Lens

| Position of object | Position of image | Real/Virtual | Inverted/erect | Magnification and size of image |
|---|---|---|---|---|
| at infinity | at focus | real | inverted | m < 1 greatly diminished |
| beyond 2f | between f and 2f | real | inverted | m = 1 same size |
| at 2f | at 2f | real | inverted | m = 1 same size |
| between f and 2f | beyond 2f | real | inverted | m > 1 magnified |
| at f | at inifinity | real | inverted | $m = \infty$ magnified |
| between optical centre and focus | at a distance greater than the object distance and on the same side object | virtual | erect | m> 1 magnified |
| Concave Lens | | | | |
| at inifinity | at focus (v = f) | virtual | erect | m < 1 diminished |
| between infinity and optical centre | between optical centre and focus | virtual | erect | m < 1 diminished |

## FOCAL LENGTH OF COMBINATION OF TWO THIN LENSES IN CONTACT

Let two lenses be in contact, as shown in the figure. Focal length of the combination is given by




$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

If P is the power of the combination, then $P = P_1 + P_2$ where $P_1$ and $P_2$ are the powers of the individual lenses. If two lenses are separated by distance d then focal length of system of two lenses is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

## DISPLACEMENT METHOD FOR FINDING THE FOCAL LENGTH OF A CONVEX LENS

In this method, the distance between the object and the screen must be greater than 4f, where f is the focal length of the convex lens. The image on the screen can be formed corresponding to two different positions of the lens. Figure (i) shows the magnified image of size $I_1$ for the position $L_1$ of the lens.

$$m_1 = \frac{I_1}{O} = \frac{v}{u}$$

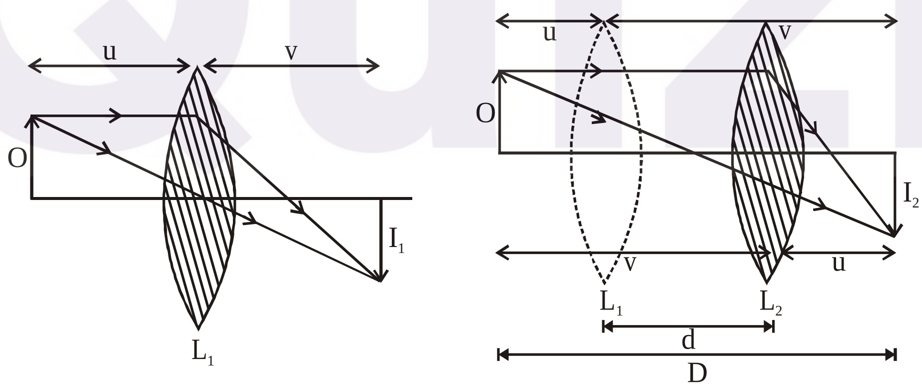



Figure (ii) shows the diminished image of size $I_2$ for the position $L_2$ of the lens.

$$m_2 = \frac{I_2}{O} = \frac{v}{u}$$

From (1) and (2),

$$\frac{I_1 I_2}{O^2} = \frac{v}{u} \times \frac{u}{v} = 1$$

or $\quad O = \sqrt{I_1 I_2} \Rightarrow$ object size = $\sqrt{(\text{1st image size}) \times (\text{2nd image size})}$

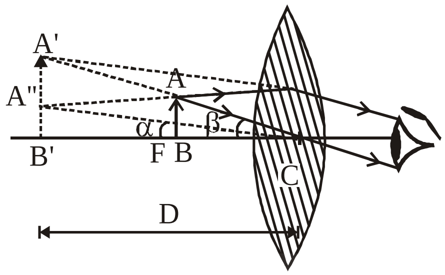
# OPTICAL INSTRUMENTS

## Simple Microscope

It is just one lens of short focal length. It is also known as 'simple magnifier' or magnifying glass'.

Magnifying power of a microscope is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the eye, when both are placed at least distance of distinct vision (D). D is usually taken as 25 cm for a normal person.

$$M = 1 + \frac{D}{f} \qquad ...(1)$$




In case the eye is placed behind the lens at a distance 'a', then the above relation is modified as follows:

$$M = 1 + \frac{D - a}{f}$$

But usually the eye is placed very close to the lens. Hence the equation (1) is applicable. So, smaller the focal length of the lens, greater will be the magnifying power. The simple microscope may be used in such a way that the image is formed at infinity. Then,

$$M = \frac{D}{f} \qquad ...(2)$$

It is clear from comparison of equations (1) and (2) that

(i) The maximum angular magnification is obtained when the image is at the near point, ie at least distance of distinct vision (D) and

(ii) The minimum angular magnification is obtained when the image is at infinity.

## Compound Microscope

The magnification produced by the compound microscope is the product of the magnification produced by the eyepiece and objective.

$$M = M_e \times M_o$$

where $M_e$ and $M_o$ are the magnifying powers of the eyepiece lens and objective lens respectively.

$$M_e = 1 + \frac{D}{f_e}$$ where $f_e$ is the focal length of the eyepiece.

$$M_o = \frac{v_o}{u_o}$$

where $v_o$ is the distance of intermediate image from the objective lens and $u_o$ is the distance of the object from the objective lens. Then

$$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

The object is placed very close to the principla focus of the objective lens. Therefore $u_o$ is nearly equal to $f_o$.

Moreover, the focal length of the eyepiece is small. So, the intermediate image is formed very close to the eyepiece. Hence $v_o$ is nearly equal to the length L of the microscope tube. Here L is the separation between the two lenses.

$$M = \frac{L}{f_o}\left(1+\frac{D}{f_e}\right)$$

It is clearly from the above equation that smaller the focal lengths of the objective and eyepiece, larger is the magnifying power.

If the image forms at infinity the expression for magnifying power is further simplified as below. In this case, the microscope is said to be in normal adjustment.

$$\therefore \qquad M = M_o \times M_e = -\frac{L}{f_o} \times \frac{D}{f_e}$$

If the object is not placed close to the focus of objective lens, then we can get value of $\frac{v_o}{u_o}$ as shown below.

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \text{ or } \frac{v_o}{v_o} - \frac{v_o}{u_o} = \frac{v_o}{f_o} \qquad \text{or } -\frac{v_o}{u_o} = -1\frac{v_o}{f_o} \text{ or } \frac{v_o}{u_o} = 1 - \frac{v_o}{f_o}$$

## ASTRONOMICAL REFRACTION TELESCOPE

(a) When the image is formed at infinity, the magnifying power is $M = -\frac{f_o}{f_e}$

Hence, for high angular magnification, the objective should have a large local length and the eyepiece a small focal length. It may be noted that the separation of the lenses is $f_o + f_e$.

(b) When the final image is formed at the least distance of distinct vision, the magnifying power is modified as follows.

$$M = \frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

## TERRESTRIAL TELESCOPE

For observing erect image of terrestrial objects, an inverting lens of focal length f is placed between the objective and the eyepiece. This lens increases length by the telescope of 4f, but does not cause any change in magnification. Length of telescope becomes $f_o + 4f + f_e$

The adjustment of the telescope is called normal adjustment, when image forms at infinity. Then, Magnifying power $M = \frac{f_o}{f_e}$.

If the final image is formed at the least distance of distinct vision, then magnifying power is

$$M = \frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

## GALILEAN TELESCOPE

Here the eye piece is a concave lens.

Length of telescope tube is $f_o - f_e$, where $f_o$ and $f_e$ represent the focal lengths of the objective (convex) and eyepiece (concave) respectively. Magnifying power is the ratio of the focal lengths of the objective and eyepiece as in other cases.

The main disadvantage of the Galilean telescope is that the field of view of this telescope is small as compared to the other terrestrial telescope and the astronomical telescope. However the advantage is smaller length of the telescope.

# WAVE OPTICS

## NATURE OF LIGHT :

Light is an electromagnetic wave which is sinusoidally varying electric and magnetic fields propagated from one part to another part. The electric and the magnetic field are given by

$$E_y = E_0 \sin(kx \pm wt)$$

$$B_z = B_0 \sin(kx \pm wt)$$

It propagates as transverse non mechanical wave in a medium at a speed given by $V = \frac{1}{\sqrt{\mu \in}}$;

The electric and magnetic fields are related as $E_0 = VB_0$

## REFRACTIVE INDEX OF A MEDIUM :

Refractive index of a medium is defined as

$$\mu = \frac{\text{speed of light in vaccum}}{\text{speed of light in med}} \Rightarrow \mu = \frac{c}{v}$$

## INTERFERENCE :

The modification in the distribution of intensity of light in the region of superposition of two coherent light waves is called interference. At some points the waves superpose in such a way that the resultant intensity is greater than the sum of the intensities due to separate waves (constructive interference) while at some other points intensity is less than the sum of the separate intensities (destructive interference).

## YOUNG' DOUBLE SLIT EXPERIEMENT :

Let the two waves each of angular frequency $\omega$ from sources $S_1$ and $S_2$ reach the point P. Equations are given by $y_1 = A_1 \sin[\omega t - kx]$ and, $y_2 = A_2 \sin[\omega t - k(x + \Delta x)]$

So the resultant wave at P by principle of superposition will be

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx - \phi);$$ where $\phi$ is initail phase difference

$$y = A \sin(\omega t - kx - \alpha)$$ where, $A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$

and, $\alpha = \tan^{1}[A_2 \sin\phi / (A_1 + A_2 \cos\phi)]$

Superposition of two waves of equal frequencies propagating almost in the same direction, results in harmonic wave of same frequency $\omega$ and wavelength $(\lambda = 2\pi / k)$ but amplitude A. The intensity of resultant wave

$$I = K\left[A_1^2 + A_2^2 + 2A_1A_2 \cos\phi\right]$$

$$I = I_1 + I_2 + 2\left(\sqrt{I_1 I_2}\right)\cos\phi$$

The resultant intensity at P is not just the sum of intensities due to separate waves $(I_1 + I_2)$ but different and depends on phase difference $\phi$ and the position of the point P.

## Condition for Interference

**(a) Intensity will be maximum when :**

$$\cos\phi = \max. = 1$$

or $\quad \phi = \pm 2\pi n$ with n = 0, 1, 2

$$\frac{2\pi}{\lambda}(\Delta x) = \pm 2\pi n$$

$$\Delta x = \pm n\lambda$$

$$I_{max} = \left(I_1 + I_2 + 2\sqrt{I_1 I_2}\right)$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \propto (A_1 + A_2)^2$$

Intensity will be maximum at those points where path difference is an integral multiple of wavelength $\lambda$ and maximum intensity is greater than the sum of two intensities ($I_1 + I_2$). These points are called points of constructive interference or interference maxima.

**(b) Intensity will be minimum when :**

$$\cos\phi = \min. = -1;\ \phi = \pm\pi, \pm 3\pi, \pm 5\pi;\ \phi = \pm(2n-1)\pi;\ \frac{2\pi}{\lambda}(\Delta x) = \pm(2n-1)\pi;\ \Delta x = \pm(2n-1)\lambda/2$$

$$I_{min} = \left(I_1 + I_2 - 2\sqrt{I_1 I_2}\right) \Rightarrow I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \propto (A_1 - A_2)^2$$

Intensity will be minimum at those points where path difference is an odd integral multiple of $(\lambda/2)$ and minimum intensity is less than the sum of two intensities ($I_1 + I_2$). These points are called points of destructive interference or interference minima.

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \qquad I_{min} = \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}\ ;\ \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$$

- All maxima are equally spaced (as path difference between two consecutive maxima is $\lambda$) and equally bright the two waves from $S_1$ and $S_2$ have same frequency and start in the same phase at P they have a constant phase difference $\phi = (2\pi/\lambda)\Delta x$, developed due to different paths traversed by them. Such waves are said to be 'Coherent' and produce sustained interference effects.

If d is the separation between the slits and D (>>d) is the distance of screen from the plane of slits as

$$\Delta x = d\sin\theta \quad \sin\theta = (\Delta x/d)$$

for small $\theta$, $\sin\theta = \tan\theta = \theta = (y/D)$

$$\frac{y}{D} = \frac{\Delta x}{d} \qquad y = \frac{D}{d}(\Delta x)$$

- If the point P is nth bright fringe, $\Delta x = n\lambda$ and hence

$$\left(y_n\right)_{Bright} = \frac{D}{d}(n\lambda) \qquad n = 0, 1, 2, \text{etc.}$$

- If the point P is nth minima

$$\left(y_n\right)_{Dark} = \frac{D}{d}(2n-1)\frac{\lambda}{2} \qquad n = 1, 2, \text{.... etc.}$$

- Fringe-width $\beta$ is defined as the distance between two consecutive maxima (or minima) on the screen

$$\beta = \Delta y \qquad\qquad \Delta x = \lambda$$

$$\beta = \frac{D}{d}(\lambda)$$

- As linear position y is related to the angular position $\theta$ by $\theta = (y/D)$, i.e., $\Delta\theta = (\Delta y/D)$, the so called angular fringe-width

$$\theta_0 = \frac{\beta}{D} = \frac{\lambda}{d}$$ Fringe-width is independent of n.

- If the transparent sheet of refractive index $\mu$ and thickness t is introduced in one of the paths of interfering waves, optical path will become $\mu t$ instead of t for the portion in which glass is inserted so the optical path will increase by $(\mu-1)t$. Due to this, a given fringe from its present position y will shift to a new position y', the lateral shift of the fringe is

$$y_0 = y' - y = \frac{D}{d}(\mu-1)t = \frac{\beta}{\lambda}(\mu-1)t$$

entire fringe-pattern is displaced by $y_0$ towards the side in which the thin sheet is introduced without any change in fringe width.

## DIFFRACTION :

The flaring out or encroachment of light in the shadow zone as it passes around obstacles or through small apertures is called diffraction.

As a result of diffraction, the edges of the shadow do not remain well defined and sharp but become blurred and fringed. If the width of the aperture is comparable to the wavelength of light, most of the incident wavefront will be obstructed and in accordance with Huygens' wave theory a cylindrical (or spherical) wavefront depending on the aperture (slit or hole) will originate from it as the direction of wave motion is normal to the wavefront, after passing through the aperture light will flare out. This flaring out or spreading of light is the so called diffraction.

In case of diffraction at single slit theory shows that intensity at a point on the screen is given by:

$$I_{(\theta)} = I_m\left[\frac{\sin\alpha}{\alpha}\right]^2 ; \alpha = \frac{\phi}{2} = \frac{\pi}{\lambda}(d\sin\theta)$$

From this it is clear that I will be minimum when for $\alpha \neq 0$,

$$\sin\alpha = 0, \quad \alpha = n\pi \qquad n = 1, 2, \ldots$$

- Angular position of minima in case of diffraction at single slit is given by:

$$\frac{\pi}{\lambda}(d\sin\theta) = n\pi \qquad\qquad d\sin\theta = n\lambda$$

And as central maximum extends between first minima on either side, for small $\theta$, the angular width of central maximum will be: $\theta_0 = 2\theta_1 = (2\lambda / d)$

- At centre as $\theta = 0$ and hence $(\sin\alpha/\alpha) \to 1$. This in turn means that intensity at centre is always maximum and equal to $I_m$. This maximum of intensity is called central maximum.
- At the position of a minima, wavelets from the two ends of the slit reach in phase differeing by an integral multiple of $2\pi$ as in this situation path difference $(d\sin\theta)$ condition of minima is $n\lambda$.
- Subsidiary maxima are approximately midway between two consecutive minima and of decreasing inteisty. The position of nth subsidiary maxima will therefore be given by:

$$[d\sin\theta]_{max} = \frac{n\lambda + (n+1)\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda$$

- The angular width of subsidiary maximum $\theta_S = (\theta_n - \theta_{n-1}) = (\lambda / d)$ is half that of central maximum $[\theta_0 = 2(\lambda / d)]$.
- Due to diffraction at a circular aperture, a converging lens can never form a point image of an object but it produces a bright disc called Airy disc surrounded by dark and bright concentric rings. The minimum radius of the image disc is given by

$$r = 1.22\frac{\lambda}{d}(f)$$

- Diffraction limits the ability of optical instruments to form distinct images of objects when they are close to each other. According to Rayleigh (called rayleigh's criterion), two objects of equal intensity will be just resolved (i.e., distinctly visible) by an optical instrument if the central diffraction maximum of one lies at the first minimum of the other. So the angular limit of resolution will be equal to the angular separation between the centre of central maximum and first minimum, which for a single slit will be

$$\theta_R = \frac{\lambda}{d} - 0, \qquad \theta_R = \frac{\lambda}{d}$$

for circular aperture such as lens, $\theta_R$ is found to be $1.22(\lambda / d)$; so two objects at a distance D with separation y will be distinctly visible only if

$$\theta \geq \theta_R, \qquad (y/D) \geq 1.22(y/d)$$

- A diffraction-grating consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the direction of principal maxima (PM) is given by

$$d\sin\theta = n\lambda$$

where d is the distance between two consecutive slits and is called grating element and n order of principle maxima.
- The dispersive and resolving power of a grating are given by

$$DP = \frac{d\theta}{d\lambda} = \frac{n}{d\cos\theta} \qquad RP = \frac{\lambda}{d\lambda} = nN$$

closely spaced lines on a grating give greater dispersion while greater number of lines increase its resolving power.

## POLARISATION :

An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$ Because all directions of vibrations of $\vec{E}$ are equally probable the resultant electromagnetic wave is a superposition of waves produced by the individual atomic sources. This resultant wave is called unpolarised light and is symmetrical about the direction of wave propagation. If somehow (say using polaroids or Nicol-prism) we confine the vibrations of electric vector in one direction perpendicular to the direction of wave motion the light is said to be plane polarised or linearly-polarised and the phenomenon of confining the vibrations of a wave in a specific direction perpendicular to the direction of wave motion is called polarisation. The plane containing the direction of vibration and wave motion is called plane of polarisation.

All the vibrations of an unpolarised light at a given instant can be resolved in two mutually perpendicular directions and hence, an unpolarised light is equivalent to the superposition of two mutually perpendicular identical plane polarised lights.

- If in case of unpolarised light, electric vector in some plane is either more or less, than in its perpendicular plane, the light is said to be 'partially polarised'
- If an unpolarised light is converted into plane polarised light, its intensity reduces to half.
- A part from partially polarised and plane (i.e., linearly) polarised, light can also be circularly or elliptically polarised, that too left-handed or right handed. Elliptically and circularly polarised lights result due to superposition of two mutually perpendicular plane polarised lights differing in phase by $(\pi/2)$ with.

## Methods of Obtaining Plane Polarised Light :

### By Reflection :

Brewster discovered that when light is incident at a particular angle on a transparent substance, the reflected light is completely plane polarised with vibrations in a plane perpendicular to the plane of incidence. This specific angle of incidence is called polarising angle $\theta_p$ and is related to the refractive index $\mu$ of the material through the relation: $\tan\theta_p = \mu$. This is known as Brewster's law.

### By Scattering :

When light is incident on atoms and molecules, the electrons absorb the incident light and re-radiate it in all directions. This process is called scattering. it is found that scattered light in directions perpendicular to the direction of incident light is completely plane polarised while transmitted light is unpolarised. Light in all other directions is partially polarised.

- If plane polarised light of intensity $I_0\left(= KA^2\right)$ is incident on a polaroid and its vibrations of amplitude A make an angle $\theta$ with the transmission axis, then the component of vibrations parallel to transmission axis will be A cos $\theta$ while perpendicular to it A sin $\theta$. Now, as polaroid will pass only those vibrations which are parallel to its transmission axis, i.e., A cos $\theta$, so the intensity of emergent light will be

$$I = K\left(A\cos\theta\right)^2 = KA^2\cos^2\theta$$

$$I = I_0\cos^2\theta$$

This law is called Malus law.

- If an unpolarised light is converted into plane polarised light (say by passing it through a polaroid or a Nicol-prism), its intensity becomes half.
- If light of intensity $I_1$ emerging from one polaroid called polariser is incident on a second polaroid (usually called analyser) the intensity of the light emerging from the second polaroid in accordance

with Malus law will be given by $I_2 = I_1 \cos^2 \theta'$

where $\theta'$ is the angle between the transmission axis of the two polaroids.

- Optically activity of a substance is measured with the help of polarimeter in terms of 'specific rotation' which is defined as the rotation produced by a solution of length 10 cm (1 dm) and of unit concentration (i.e., 1 g/cc) for a given wavelength of light at a given temperature,

$$[\alpha]_{t°C}^{\lambda} = \frac{\theta}{L \times C}$$

where $\theta$ is the rotation in length L at concentration C.

# Modern Physics

## Rutherford Seattening :

Rutherford seattering experiments helped in understanding structure of atom. Rutherford bombarded a narrow beam of $\alpha$ – particles on a gold foil and observed visible light scintillations on a zinc surplide screen. Rutherford observed that :

(1) Most of the $\alpha$ – particles were either undeflected or deflected through small angles of the order of 1°.

(2) A few $\alpha$ – particles were deflected through angles as large as 90° or more.

## Bohr Model of Hyrogen atom

Bohr proposed his theory of structure of atom on the basis of following assumptions :

(1) Electrons move in circular orbits about proton under the influence of coulomb force of attraction. These orbits are stationary states in which electrons do not continuously radiate electromagnetic energy.

(2) The emission or absorption of electron takes place only when there is a transition of electrons between two stationary states.

(3) The angular momentum of this system in a stationary state is an integral multiple of $\frac{h}{2\pi}(=\hbar)$.

On the basis of these assumptions :

(a) Radius of nth Bohr's orbit

$$r_n = \frac{4\pi \in n^2h^2}{me^2} = n^2 a_0$$

(b) Velocity of electron

$$V_n = \frac{1}{n}\frac{e^2}{4\pi \in_0 \hbar} = \frac{V_0}{n} = \frac{1}{n}\frac{\hbar}{ma_0}$$

(c) Energy of electron in nth orbit,

$$E = -\frac{e^2}{8\pi \in_0 r_n} = -\frac{e^2}{8\pi \in_0 a_0 n^2} = -\frac{E_0}{n^2} = \frac{13.6\,\text{eV}}{n^2}$$

A quanta of light is emitted when an atom in excited state decays to a lower energy state.

$$h\upsilon = E_f - E_i$$

(d) Frequency, wavelength, wave number of transitions

$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{E_f - E_i}{hc}$$

$$= -\frac{E_0}{hc}\left[\frac{1}{n_i^2} - \frac{1}{n_i^2}\right] \qquad = R\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

Where $R = \frac{E_0}{hc}$ = Rydberg constant = $1.097373 \times 10^7\ m^{-1}$

## X-RAYS

These are electromagnetic wave whose wavelengths typically range from 0.01 to 1 nm.

When an electron strikes a metallic target, before stopping it makes several collisions with atoms. Electrons may interact with the atom in either of the two ways :

(i) Due to strong nuclear electric field, electron is decelerated. In the process it radiates electromagnetic energy. Electron emits a series of photons with varying energy. These photons are x-rays. The x-rays produced in this process are called continuous x-rays.

(ii) When the high energy electron collides with one of the lower shell K electrons in a target atom, if enough energy can be transfered to this electron, the atom may be ionised. An electron from one of the higher shells will change its state and fill the inner shell vacancy at lower energy emitting radiation. The emitted radiation in heavy atoms is x-ray. Photons emitted in this way is called characteristic x-ray.

### De Broglie or Matter Waves :

De Broglie proposed that material particles also have both wave and particle properties.

The wavelength to be associated with a particle is given by Planck's constant divided by the particle's momentum.

$$\lambda = \frac{h}{p}$$

This relation for photons was extended to all particles by De Broglie. Waves associated with particles are called matter waves, and the wavelength is called the de Broglie wavelength of a particle.

- The DeBroglie wavelength of the electron is large enough to be observed. Because of their small mass, electrons can have a small momentum and in turn a large wavelength.
- If m is the mass and v the velocity of the material particle, then

$$p = mv$$

$$\lambda = \frac{h}{mv}$$

- If E is the kinetic energy of the material particle, then

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

Therefore, the de Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2mE}}$

- For electrons ($m_e = 9.1 \times 10^{-31}$ kg)

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}V}}\,m = \frac{12.27}{\sqrt{V}}\,\mathring{A}$$

### Bohr's Quantisation Condition :

The angular momentum of the electron in this orbit is L = rp. Using the above relation,

$$L = rp = \frac{nh}{2\pi} = n\hbar$$ which is Bohr's quantisation condition.

### Atomic Nucleus

Nuclear size is of the order of femtometre (1 fm = $10^{-15}$ m).

The radius of a nucleus is given by $R = R_0 A^{1/3}$

The value of $R_0$ is, $R_0 \approx 1.2\times10^{-15}$ m $\approx 1.2$ fm

## Radioactivity

1. Radioactive decay is a statistical process; we cannot precisely predict the timing of a particular radioactivity of a particular nucleus. The nucleus can disintegrate immediately or it may take infinite time. We can predict the probability of the number of nuclei disintegrating at an instant.
2. Radioactivity is independent of all the external conditions. We caannot induce radioactivity by applying strong electrical field, magnetic field, high temperature, high pressure, etc.
3. The energy liberated during radioactive decay comes from within individual nuclei.
4. When a nucleus undergoes alpha or beta decay, its atomic number changes and it transforms into a new element. Thus elements can be transformed from one to another.
5. The rate, at which a particular decay process occurs in a radioactive sample, is proportional to the number of radioactive nuclei present (i.e., those nuclei that have not decayed). If N is the number of radioactive nuclei present at some instant, the rate of change of N is,

$$\frac{dN}{dt} = -\lambda N$$

where $\lambda$ is called decay constant. The minus sign indicates that $\frac{dN}{dt}$ is negative.

$$N = N_0 e^{-\lambda t}$$

where the constant $N_0$ represents the number of nuclei or radioactive nuclei at $t = 0$.

- Half life of a radioactive substance is the time it takes half of a given number of radioactive nuclei to decay. Setting $N = N_0/2$ and $t = T_{1/2}$ we get

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

Writing the above equation in the form $e^{\lambda T_{1/2}} = 2$ and taking natural logarithm of both sides, we get

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- Mean life (average life) $\tau$ is defined as the average time the nucleus survives before it decays.

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$$

## PHOTOELECTRIC EFFECT

The phenomenon of emission of electrons from a metallic surface by the use of light (or radiant energy) of certain minimum frequency (or maximum wavelength) is called photoelectric effect.

The incident photon interacts with a single electron and loses its energy in two parts.

(a) Firstly, in getting the electron released from the bondage of the nucleus.

(b) Secondaly, to importing kinetic energy to emitted electron.

Accordingly, if hν is the energy of incident photon, then

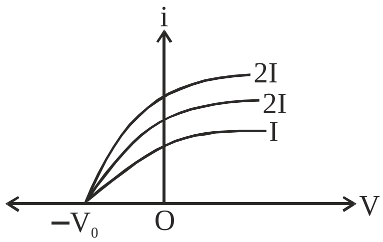
$$h\nu = \phi_0 + E_{max}$$

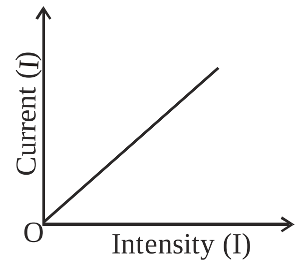
$$h\nu = h\nu_0 + K_{max.} = h\nu = e\nu_0$$

This is Einstein's photoelectric equation, where $\phi_0$ is work function and $E_{max} = \frac{1}{2}mv_{max}^2 = eV_s$ is the maximum kinetic energy of photo-electrons emitted. $\nu_0$ is the reverse potential differenic required to stop the electron from ejecting, called stopping potential.

## Effect of Intensity

For a given frequency, if intensity of incident light is increased, the photoelectric current increases but the stopping potential remains the same. In photoelectric effect current (i) is directly proportional to intensity(I) of incident light.

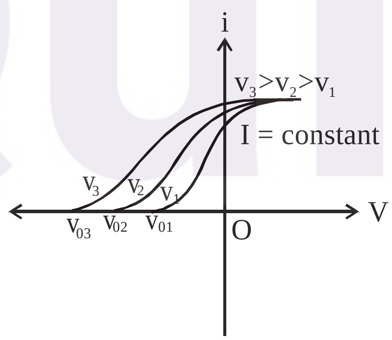



The intensity of incident light affects the photoelectric current but leaves the maximum kinetic energy of photoelectrons unchanged.

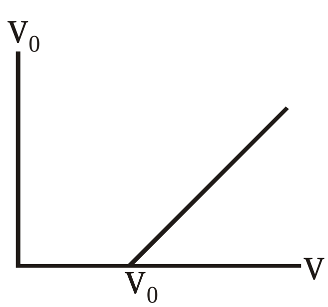
## Effect of Frequency

When the intensity of incident light is kept fixed and frequency is increased, the photoelectric current remains the same but the stopping potential increases. If the frequency is decreased, the stopping potential decreases and at a particular frequency of incident light, the stopping potential becomes zero. This value of frequency of incident light for which the stopping potential is zero is called threshold frequency $v_0$, No photoelectric emission takes place below this frequency. Thus the increase of frequency increases maximum kinetic energyof photoelectrons but leaves the photoelectric current unchanged.




## Effect of Photo-Metal

- When frequency and intensity of incident light are kept fixed and photo-metal is changed, we observe that stopping potential ($V_0$) versus frequency ($\nu$) graphs are similar, cutting, frequency axis at different points. This shows that threshold frequency are different for different metals, the slope $\left(\frac{\nu_0}{\nu}\right)$ for all the metal is same and hence a universal constant.



## Effect of Time

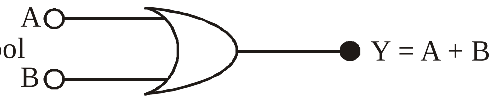
There is no time lag between incidence of light and the emission of photo-electrons.

# LOGIC GATES

## OR Gate :

Boolean expression $Y = A + B$

Symbol



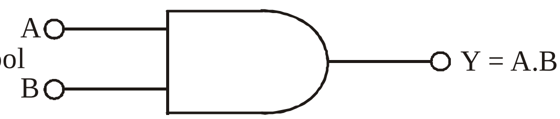

Truth table

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

## AND gate

Boolean expression $Y = A \cdot B$

Symbol



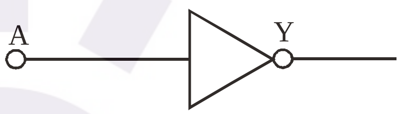

Truth table

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

## NOT gate

Boolean expression $Y = \overline{A}$

Symbol



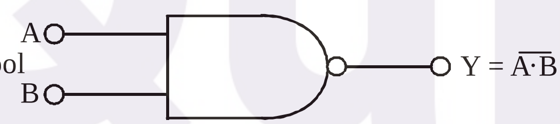

Truth table

| A | Y |
|---|---|
| 0 | 1 |
| 1 | 0 |

## NAND gate

Boolean expression $\overline{A\text{g}B}$

Symbol



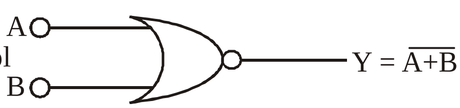

Truth table

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

## NOR gate

Boolean expression $Y = \overline{A + B}$

Symbol



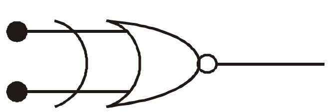

Truth table

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

## XOR gate or Ex–OR gate

Boolean expression $Y = A\overline{B} + \overline{A}B$

Symbol

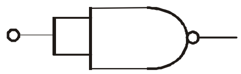


Truth table

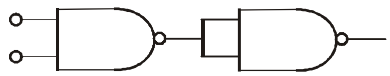
| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

## NAND and NOR work as basic building blocks

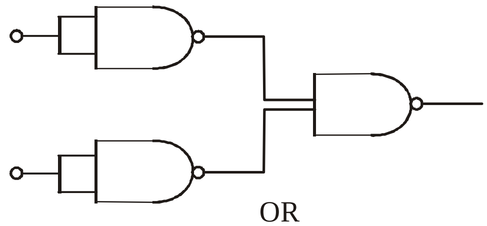
Any logic gate can be realised by using only NAND gates or only NOR gates. Therefore these two gates are called the building blocks NAND / NOR are also universal gate.



NOT



AND



OR

## SEMICONDUCTORS

Semiconductors : Electrical properties of which lie between conductor and insulators.

Semiconductors are materials that have a small energy gap of the order of 1eV. At 0K (absolute zero), the semiconductors behave like insulators.

Intrinsic Semiconductors (pure) : Semiconductor which are free from inpurity.

Intrinsic Semiconductors have an equal number of electrons in conduction band and holes in valence band

$n_e = n_h$

where $n_e$ = number of electrons per unit volume

$n_h$ = number of holes per unit volume

$n_e \times n_h = n_i^2$

$n_i$ = intrinsic charge carrier density or intrinsic charge carrier concentration

Doped or Extrinsic Semiconductors : Semi conductors doped or added with certian impurity to increase its conductivity.

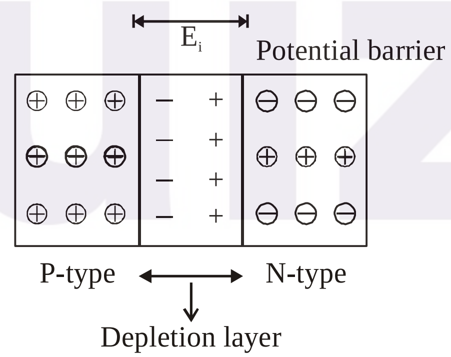
N-Type : In N type of semiconductor electrons are majority charge carriers and holes are minority charge carriers.

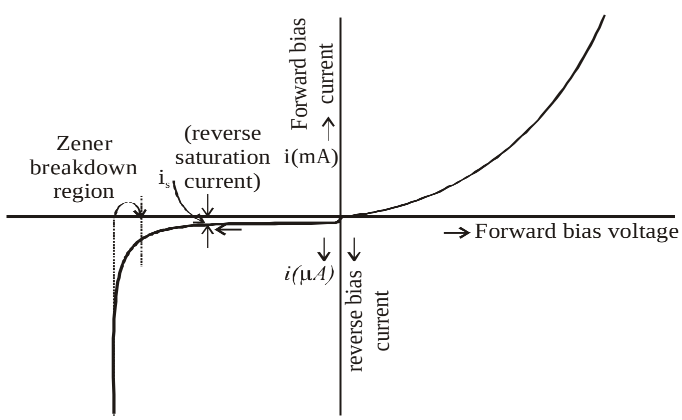
P Type : In P type semiconductors holes are majority charge carriers whereas electrons are minority charge carriers.

## Semiconductor Devices

The P-N junction Diode :

P side of P-N junction has holes as a majority charge carriers and electrons as a minority charge carriers whereas N side has electrons as a majority charge carriers and holes as a minority charge carriers. holes diffuse from P side to N side whereas electrons diffuse from N side to P side.




P-N Junction Characteristics

Dynamic Resistance :

$$R = \frac{\Delta V}{\Delta i}$$

Where $\Delta V$ denotes a small change in the applied potential difference and $\Delta i$ denotes corresponding small change in current.

Dynamic Resistance is equal to the reciprocal of the slope of the i-V characteristic.

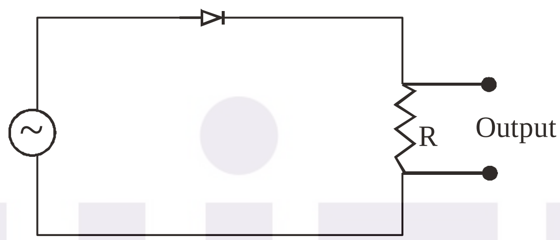
Photodiode : When a light of proper wavelength falls on the junction, new electron-hole pairs are created. The number of charge carriers increases and hence the conductivity of the junction increases. If the junction is connected in some circuit, the current in the circuit is controlled by the intensity of the incident light.

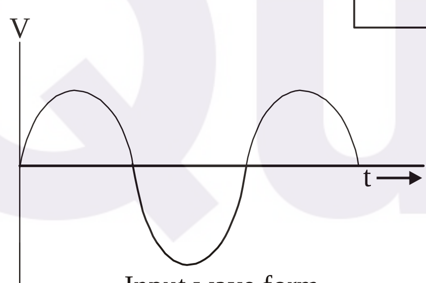
Light-emitting Diode (LED) : When a conduction electron makes a transition to the valence band to fill up a hole in P-N junction, the extra energy is emitted as a photon. If the wavelength of this photon is in the visible range one can see the emitted light. Such a P-N junction is known as light emitting diode (LED).

Zener diode : A diode operated in Zener break down mode is called Zener diode. In this type of mode of operation current increases rapidly but voltage remains almost constant. Thus it is used to obtain constant voltage output.

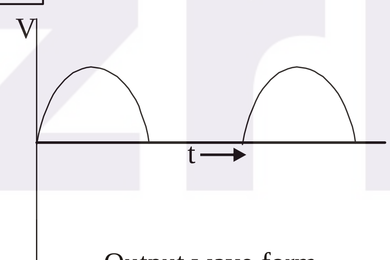
P-N Junction as a Rectifier : PN Junction can be used to convert A.C into unidirectional current. (DC)

(a) Half wave rectifier




Input wave form



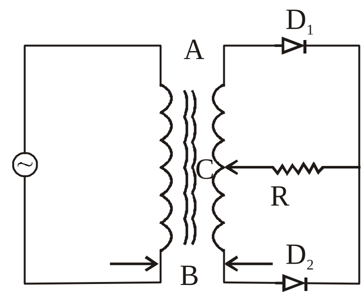

Output wave form

Average out put current = $\frac{I_o}{\pi}$

where $I_o$ = amplitude of the input current

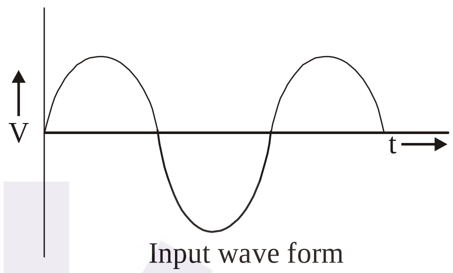
R.M.S. Value of output current = $\frac{I_o}{2}$

(b) Full Wave rectifier

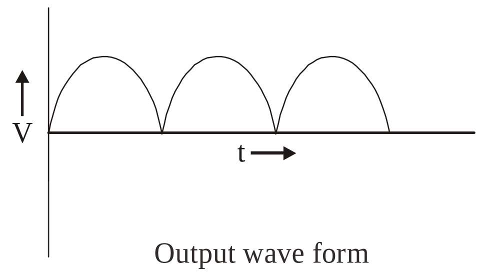



If $V_A > V_C > V_B$ $D_1$ Conducts

If $V_B > V_C > V_A$ $D_2$ Conducts




Input wave form




Output wave form

Average output current = $\dfrac{2I_o}{\pi}$

R.M.S. Value of output current = $\dfrac{I_o}{\sqrt{2}}$

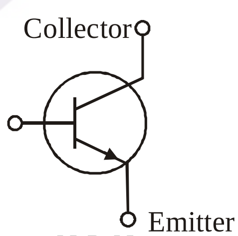
<u>Junction transistor</u>

It has three terminals

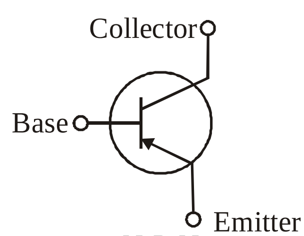
(a) Emitter (b) Base (c) Collector

Emitter is heavily doped, collector is moderately doped and base is thin and lightly doped.

Symbol

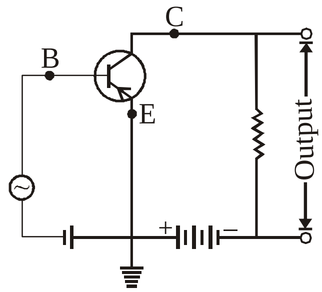



N-P-N

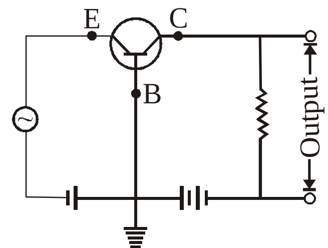



N-P-N

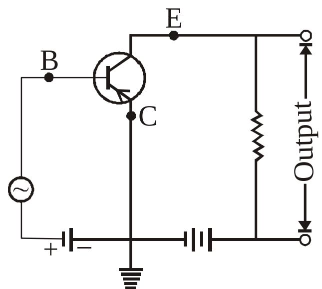
A transistor can be operated in three different modes. Common-emitter, common collector and common base.




Common Emitter




Common Base




Common Collector

Transistor as an Amplifier

1. Common base :

   In this type amplifier, base to emitter junction is forward biased whereas base to collector is reverse biased

   Transistor parameter

   Current again $= \alpha = \dfrac{I_0}{I_E}$; AC current gain $= \dfrac{\Delta I_c}{\Delta I_e}$

   Voltage gain $= A_v = \dfrac{\Delta v_0}{\Delta v_i} = \dfrac{I_0 R_{out}}{I_E R_{in}}$ = Current gain × Resistance gain $= \alpha \times \dfrac{R_0}{R_i}$

   Where $R_o$ = Resistance of the output circuit

   $R_i$ = Resistance of the input circuit

   Power gain $= \alpha^2 \times$ Resistance gain

2. Common emitter amplifier

   Current gain $\beta = \dfrac{\Delta I_c}{\Delta I_b}$

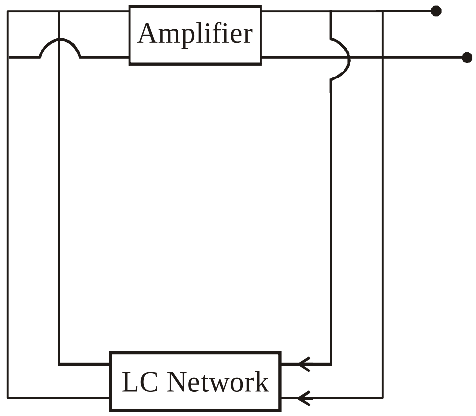
   Voltage gain ($A_v$) = Current gain × Resistance gain = $\beta \times A_v$

   Power gain $= \beta^2 \times$ Resistance gain

   Resistance gain $= \dfrac{R_o}{R_i}$

Transistor used as an oscillator converts D.C. into A.C.

Amplifier section is just a transistor used in common-emitter mode.




$$f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Part of output energy is sent back in phase to input circuit. This is also called positive feed back.