Sri Chaitanya IIT Academy.,India.

A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 09-09-2023
Time: 09.00Am to 12.00Pm RPTM-06 Max. Marks: 300

KEY SHEET

PHYSICS

1)	3	2)	2	3)	3	4)	4	5)	1
6)	1	7)	3	8)	3	9)	4	10)	3
11)	4	12)	3	13)	4	14)	4	15)	2
16)	1	17)	2	18)	3	19)	3	20)	1
21)	2250	22)	412	23)	55	24)	8	25)	1600
26)	3	27)	16	28)	2	29)	1	30)	9

CHEMISTRY

31)	3	32)	2	33)	4	34)	4	35)	1
36)	3	37)	1	38)	1	39)	1	40)	1
41)	3	42)	3	43)	4	44)	1	45)	2
46)	4	47)	3	48)	2	49)	4	50)	1
51)	7	52)	3	53)	3	54)	5	55)	3
56)	6	57)	4	58)	5	59)	4	60)	3

MATHEMATICS

61)	3	62)	1	63)	2	64)	4	65)	4
66)	3	67)	3	68)	2	69)	3	70)	2
71)	1	72)	4	73)	1	74)	1	75)	2
76)	3	77)	1	78)	3	79)	1	80)	4
81)	4	82)	9	83)	256	84)	0	85)	4
86)	5	87)	125	88)	0	89)	6	90)	4

SOLUTIONS

PHYSICS

1. At the time of maximum compression, the speeds of blocks will be the same. Let that speed be v and maximum compression be x.
Applying conservation of momentum,
$(m_1+m_2)v = m_1v_1 + m_2v_2$
$\Rightarrow v = 4m/s$
Applying conservation of mechanical energy
$\frac{1}{2}kx^2 + \frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
Solving, we get $x = 0.02m$

2. The initial velocity of CM is upward. The acceleration of the CM is 'g' downward.

3. During contact apart of kinetic energy appears as potential energy due to deformation of shape of bodies

4. Since the collision is elastic, there is no loss in KE

5. Let us adopt the vector approach. Let the mass of each particle be 'm' and let they be denoted by A,B and C. Before collision,
velocity of $A = 10\hat{i}$
velocity of $B = 20\left(\cos 30^0\hat{i} + \sin 30^0\hat{j}\right)$
$= 10\left(\sqrt{3}\hat{i} + \hat{j}\right)$
Celocity of $C = 30\hat{j}$
Initial momentum, $\vec{P} = m\left[10\hat{i} + 10\left(\sqrt{3}\hat{i} + \hat{j}\right) + 30\hat{j}\right]$
$= 10m\left[\left(1+\sqrt{3}\right)\hat{i} + 4\hat{j}\right]$........(1)
Let the final velocity be $\vec{v}$ then final momentum
$\overrightarrow{p_2} = 3m\vec{v}$
According to the law of conservation of linear momentum, we have initial momentum = final momentum
i.e. $10m\left[\left(1+\sqrt{3}\right)\hat{i} + 2\hat{j}\right] = 3m\vec{v}$
$\vec{v} = \frac{10}{3}\left[\left(1+\sqrt{3}\right)\hat{i} + 4\hat{j}\right]$
Therefore, magnitude of $\vec{v}$ i.e.,
$$|\vec{v}| = \frac{10}{3}\sqrt{\left(1+\sqrt{3}\right)^2 + 4^2} = \frac{10}{3}\sqrt{20+2\sqrt{3}}m/s$$

6. U= velocity of sphere A before impact. As the spheres are identical, the triangle ABC formed by joining their centres is equilateral. The spheres B and C will move in direction AB and AC after impact making an angle of 30^0 with the original lines of motion of ball A.
Let v be the speed of the ball B and C after impact
Momentum conservation gives

$$\left(\frac{m}{2}\right)u = mv\cos 30^0 + mv\cos 30^0$$

$$u = 2\sqrt{3}v \Rightarrow v = \frac{u}{2\sqrt{3}}(i)$$

From Newton's experimental law, for an oblique collision, we have to take components along normal, i.e., along AB for balls A and B

$$v_B - v_A = -e(u_B - u_A)$$

$$v - 0 = -(0 - u\cos 30^0)$$

$$v = eu\cos 30^0$$

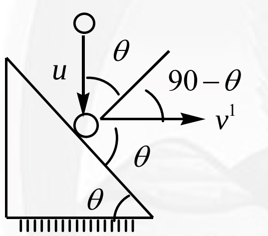
Combining Eqs. (i) and (ii), we get $e = 1/3$.

7. The acceleration of the centre of mass is $\alpha_{CM} = \frac{F}{2M}$

The acceleration of the centre of mass at time t will be

$$x = \frac{1}{2}\alpha_{CM}t^2 = \frac{Ft^2}{4m}$$

8.



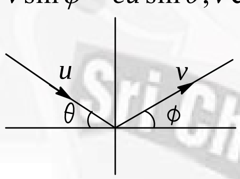
$$e = \frac{v'\cos(90-\theta)}{u\cos\theta} \qquad = \frac{v'}{u}\tan\theta........(1)$$

$$u\sin\theta = v'\cos\theta \qquad = \frac{v'}{u}\tan\theta$$

From Eqs. (i) and (ii), $e = \tan^2\theta$

9. Net external force is zero. There fore, no acceleration for CM. So, CM may move with a constant velocity

10. $v\sin\varphi = eu\sin\theta, v\cos\varphi = u\cos\theta$



$$v = u\sqrt{\cos^2\theta + e^2\sin^2\theta} = u\sqrt{1-\sin^2\theta + e^2\sin^2\theta}$$

$$v = u\sqrt{1-(1-e^2)\sin^2\theta}$$

$$I = m(v\sin\varphi + u\sin\theta) = mu\sin\theta(1+e)$$

$$Ratio\ of\ KE = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \cos^2\theta + e^2\sin^2\theta$$

11. Using conservation of linear momentum, we have

$$m_2v_0 = (m_1+m_2)v \Rightarrow v = \frac{m_2}{m_1+m_2}v_0$$

From work–energy conservation,

$$\frac{1}{2}m_2v_0^2-\frac{1}{2}(m_1+m_2)\frac{m_2^2v_0^2}{(m_1+m_2)^2}=\frac{1}{2}kx^2$$

$$x=v_0\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}$$

12. When the two balls collide with each other, as the mass of the two balls is equal, they exchange their velocities on colliding elastically. Let the speed of the ball B when it reaches back to the initial position be v. Then

$$4mgh=\frac{1}{2}mv^2+mgh\Rightarrow v=\sqrt{6gh}$$

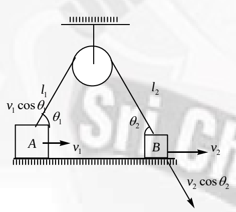
Height reaches by particle B (from highest point on the incline) is

$H_B=\frac{v^2\sin^2 60^0}{2g}=\frac{9h}{4}$; total height $=h+\frac{9h}{4}=\frac{13h}{4}$

After collision the particle A reaches the maximum height =h

Ratio $=\frac{H_A}{H_B}=\frac{4}{13}$

13. A part of kinetic energy of colliding particles may be converted into potential energy during the collision and hence may not be conserved
14. Resolving power is directly proportional to diameter of objective
15. Adiabatic curve is steeper than isothermal curve. Therefore, area under adiabatic curve is smaller than the area under isothermal; curve ie, work done by the gas in adiabatic expansion is smaller than the work done by the gas in isothermal expansion. The reverse is also true. Reason is true. Reason is also true but
 Reason does not explain Assertion
16. For any angle between two plane mirrors locus of all images is circle
17. With respect to observer the radial for distant objects is very large hence their angular velocity is almost zero
18. From figure $l_1+l_2=C \;\; or \;\; \frac{dl_1}{dt}+\frac{dl_2}{dt}=0$



$$-v_1\cos\theta_1+v_2\cos\theta_2=0 \;\; \text{(or)} \;\; \frac{v_1}{v_2}=\frac{\cos\theta_2}{\cos\theta_1}$$

19. From $s=ut+\frac{1}{2}at^2=0+\frac{1}{2}at^2, t=\sqrt{\frac{2s}{a}}$

From smooth plane $a=g\sin\theta$ For rough plane, $a'=g(\sin\theta-\mu\cos\theta)$

$$\therefore t'=nt\Rightarrow\sqrt{\frac{2s}{g(\sin\theta-\mu\cos\theta)}}=n\sqrt{\frac{2s}{g\sin\theta}}\;\therefore n^2g(\sin\theta-\mu\cos\theta)=g\sin\theta$$

When $\theta=45^0, \sin\theta=\cos\theta=\frac{1}{\sqrt{2}}$ Solving we get $\mu=\left(1-\frac{1}{n^2}\right)$

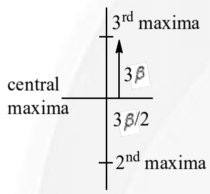
20. Critical angel $\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

Wavelength increases in the sequence of VIBGYOR. According to Cauchy's formula refractive index (μ) decreases as the wavelength increases. Hence, the refractive index will increase in the sequence of ROYGBIV. The critical angle θ_C will thus increase in the same order VIBGYOR. For green light the incidence angle is just equal to the critical angle. For yellow, orange and red the critical angle will be greater than the incidence angel. So these colours will emerge from the glass air interface.

21. d=0.5 mm and D=0.5 m

Separation $= 3\beta + 1.5\beta = 4.5\beta$

$= 4.5 \times \frac{\lambda D}{d} = 2.25mm$



22.

$1MSD = 1mm,$

$10VSD = 9MSD \Rightarrow 1VSD = 0.9mm$

$LC = 1MSD - 1VSD = 0.1mm = 0.01cm$

$Zero\,error = +4divisions$

$MSR = 4.1cm, VC = 6$

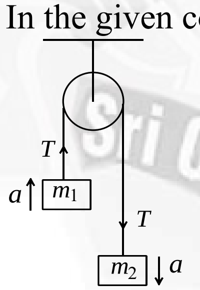
$Diameter = MSR + (VC - zero\,error)LC$

$= 4.1 + (6-4)\times 0.01 = 4.12cm = 412\times 10^{-2}cm$

23. Area under acceleration-time graph gives the change in velocity. Hence,

$v_{\max} = \frac{1}{2}\times 10\times 11 = 55ms^{-1}$

24. In the given condition tension in the string



$T = \frac{2m_1m_2}{m_1+m_2}g = \frac{2\times 0.36\times 0.72}{1.08}\times 10$

$T = 4.8N$

And acceleration of each block

$a = \left(\frac{m_2 - m_1}{m_1+m_2}\right), g = \left(\frac{0.72-0.36}{0.72+0.36}\right)g = \frac{10}{3}m/s^2$

Let 'S' is the distance covered by block of mass $0.36Kg$ n first sec

$S = ut + \frac{1}{2}at^2 \Rightarrow S = 0 + \frac{1}{2}\left(\frac{10}{3}\right)\times 1^2 = \frac{10}{6}m$

$\therefore$ Work done by the string $W = TS = 4.8 \times \frac{10}{6} \quad \Rightarrow W = 8\,Joule$

25.

$\frac{W}{Q} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = 0.5$

$\therefore Q = 2W$

$= 2 \times 800 = 1600J$

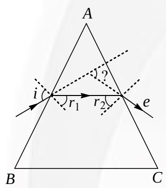
26. Given $i = 60^0, \delta = 30^0$ & $A = 30^0$

We have $\delta = i + e - A \ldots\ldots.(1)$

From Eq. (i), we get

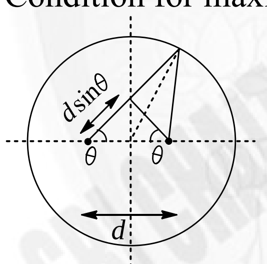
$30^0 = 60^0 + e - 30^0 \ (or)\ e = 0$

So r_2 is also zero, then $r_1 = A = 30^0$



So $\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60^0}{\sin 30^0} = \sqrt{3}$ Hence the value of $a = 3$

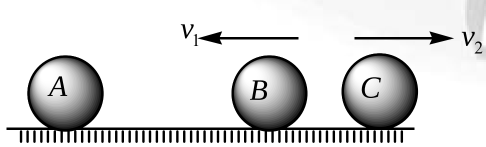
27. Condition for maxima is



$d\sin\theta = n\lambda \quad \sin\theta = \frac{n\lambda}{d} = n\left(\frac{0.50}{2.0}\right) = 0.25m$

As $\sin\theta$ lies between -1 and 1, so we wish to find all values of n for which $|0.25n| \le 1$ These values are $-4, -3, -2, -1, 0, +1, +2, +3, +4$ For each of these, there are two different values of θexcept for -4 and $+4$. A single value of $\theta, -90^0$ and $+90^0$ is associated with $n = -4$ and $n = +4$ respectively. Thus, there are 16 different angles in all and therefore 16 maxima.

28. For the first collision, $e = 1, v = v_1 + v_2$



$\Rightarrow v_2 = v - v_1 \ldots\ldots(i)$

By momentum conservation

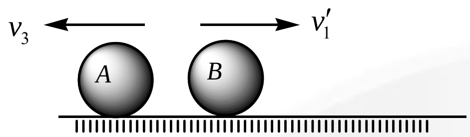
$m_B v = -m_B v_1 + m_C v_2$

$m_B v = -m_B v_1 + 4m_B v_2$

$v_2 = \frac{v_1 + v}{4}$........(ii)

From Eqs. (i) and (ii), $v_1 = \frac{3}{5}v$ and $v_2 = \frac{2}{5}v$

For the second collision, $e = 1$



$v_1 = v_1' + v_3 \Rightarrow v_3 = v_1 - v_1'$.....$(iii)$

By momentum conservation, $-m_B v_1 = m_B v'_1 - m_A v_3$

Or $-m_B v_1 = m_B v'_1 - 4m_B v_3 \left(\because m_A = 4m_B\right)$

$v_3 = \frac{v_1' + v_1}{4}$.........$(iv)$

From Eqs. (iii) and (iv), $v_1' = \frac{3}{5}v_1 = \frac{3}{5}\left(\frac{3}{5}v\right) = \frac{9}{25}v$

Clearly $\frac{9}{25}v < \frac{2}{5}v$

Therefore, 'B' cannot collide with 'C' for the second time

Hence, the total number of collisions is 2

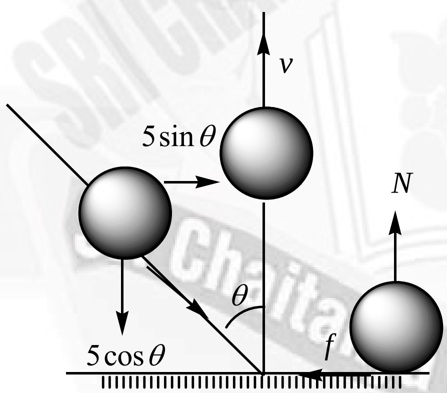
29. From impulse–momentum theorem,

$\int Ndt = m\left(v + 5\cos\theta\right)$.........$(i)$

$\int fdt = m5\sin\theta$

$\mu\int Ndt = m5\sin\theta$.........$(ii)$

$\Rightarrow \mu m\left(v + 5\cos\theta\right) = m5\sin\theta$



According to Newton's law of restitution,

$v = e5\cos\theta$

Solve to get $\mu = 1$

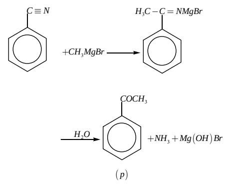
30. When object is placed at the focus of the lens, i.e., at 22 cm from the lens, image will be formed at infinity. Shift in the position of object:

$25 - 22 = \left(1 - \frac{1}{\mu}\right)t \Rightarrow 3 = \left[1 - \frac{1}{1.5}\right]t$

$t = \frac{(3)(1.5)}{0.5} = 9cm$

CHEMISTRY

31.



This is methyl ketone which gives iodoform test.

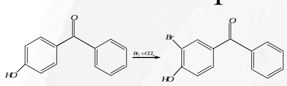
32. Reactivity order of NA reaction :

HCHO>RCHO>Aromatic aldehyde> $R-\overset{O}{\overset{\|}{C}}-CH_3 > R-\overset{O}{\overset{\|}{C}}-C_2H_5$

Rate of NA reaction is directly proportional –I and –M effect, greater is –I and –M effect, more is the reactivity .

Butanone< Propanone <PhCHO< C_2H_5CHO.

33.



-OH is ring activating group and is o,p-director but paraposition is blocked

So –Br will attach ortho to –OH group.

34. (1) $CH_3CH_2-\underset{Cl}{\overset{Cl}{c}}-Cl+O^-H \longrightarrow \underset{un\ stable}{CH_3-CH_2-\underset{OH}{\overset{OH}{c}}-OH} \xrightarrow{H_3O^+} CH_3CH_2COOH+H_2O$

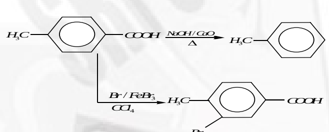
(2) Iodoform reaction

$CH_3-CH_2-\overset{O}{\overset{\|}{C}}-CH_3+IO^- \longrightarrow CHI_3+CH_3CH_2COO^- \xrightarrow{H_3O^+} CH_3CH_2COOH$

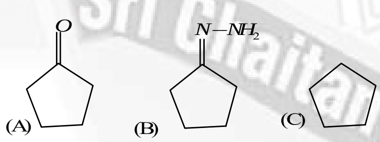
(3) $CH_3CH_2CH_2OH \xrightarrow[H_3O^+]{KMnO_4} CH_3CH_2COOH$ S

(4) $CH_3CH_2CH_2Br+Mg->CH_3CH_2CH_2MgBr \xrightarrow{CO_2} CH_3CH_2CH_2COOMgBr \xrightarrow{H_3O^+} CH_3CH_2CH_2COOH$

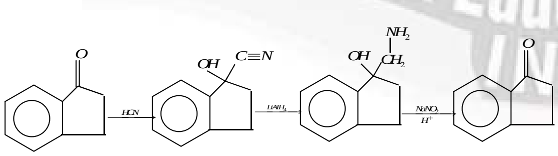
35.



36.



37.



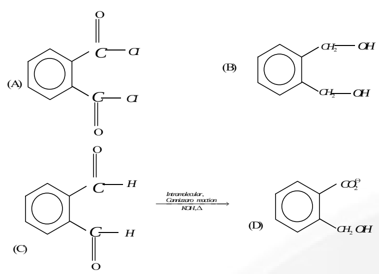
38. In esterification alcohol and carboxylic acid reacts to form ester

$\underset{\text{alcohol acts as nucleophile}}{R-OH}+R-\overset{O}{\overset{\|}{C}}-OH \underset{\text{nucleophilic acyl substitution}}{\rightarrow} R-O-\overset{O}{\overset{\|}{C}}-R$

Electron withdrawing groups on carboxylic acid will increase the rate of esterification.

39. In presence of NaOH intramolecular aldol condensation take place.

40.



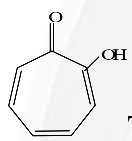
41. $I - aromatic\ II - aliphatic - III - anti\ aromatic$

42. conceptual

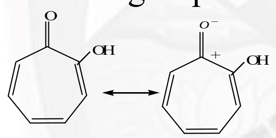
43. conceptual

44. Gives allyl alcohols

45.



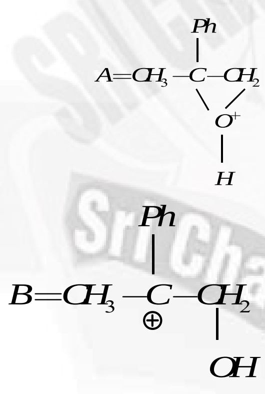
Tropolone is an aromatic compound and has 8π electrons $\left(6\pi e^{-}\ are\ endocyclic\ and\ 2\pi e^{-} are\ exocyclic\right)$ and π electrons of $>C=O$ group in troplone is not involved in aromaticity.



aromatic compound$(6\pi e^{-})$

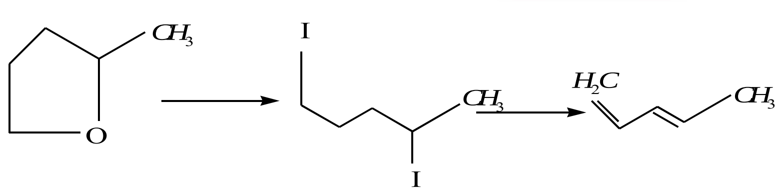
46. Both are acidic to release CO_2 gas by reacting with $NaHCO_3$

47.

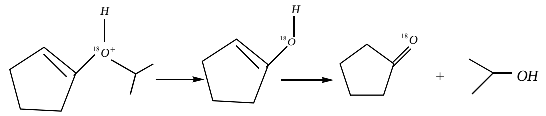


$C = H_3C - \underset{OCH_3}{\overset{Ph}{C}} - CH_2 - OH$

48.

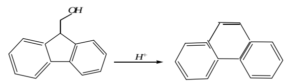


49.



50. conceptual

51.



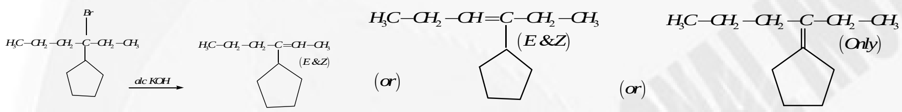
52. $= \dfrac{404-92}{104} = \dfrac{312}{104} = 3$

53.

$$\left.\begin{array}{r} -OH \\ For -C\equiv CH \\ -COOH \end{array}\right\} 3CH_3\ MgBr$$

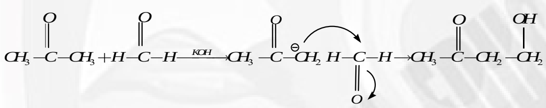
54.

Total no. of alkenes will be 5



55. $A, D, E:-$ Compounds having benzylic 'H' gives oxidation with $KMnO_4$

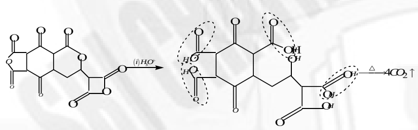
56.



One mole of HCHO gives 1-CH_2-HO group. Total CH_2-HO groups added in products =6

Hence moles of HCHO consumed=6

57.



58. b , c, d, e, f, j

59. Moles of $X = \dfrac{10\,mg}{80} = 0.125\ m\ mol.$

$Moles\,consumed\ of\ H_2 = \dfrac{8.4}{22.4} = 0.375\ m\ mol.$

$\dfrac{^{n}H_2}{n_X} = \dfrac{0.375}{0.125} = 3$

So, the compound X have 3 double bonds. Ozonolysis of the compound yields only formaldehyde and dialdehyde, so the compound should be

$$H_2C=CH-CH=CH-\overset{H}{\overset{|}{C}}=CH_2 \rightarrow \underset{4\ molecules}{\underline{2\,HCHO+2C_2O_2H_2}}$$

$Molecular\ \ mass = (12\times 6)+1\times 8 = 72+8 = 80\,amu$

$Ozonolysis\ form:$

60.

PBr_3, KCN, H_3O^+

MATHEMATICS

61.

Tangent at (1,1) is $y-1=3(x-1)$ meets $x-axis$ at $(\frac{2}{3},0)$

$\therefore$ Area $=\int_0^1 x^3 dx - Area of \Delta^{le}$ $\qquad = \frac{1}{4}-\frac{1}{2}.\frac{1}{3}.1=\frac{1}{12}sq.units.$

62.

$|y|=e^{-|x|}-\frac{1}{2}$, $y=0 \Rightarrow x=\log_e 2$

$\therefore$ Area $=4\int_0^{\log 2}(e^{-x}-\frac{1}{2})dx=2(1-\log 2)$

63.

$-8<x<8 \Rightarrow y=2$

Area $=\frac{1}{2}(1+3)\times 2=4sq.units.$

64.

$3y^2+(4x)y+(3x^2-1)=0$

$$y=\frac{-2x\pm\sqrt{3-5x^2}}{3}$$

$3-5x^2 \ge 0 \Rightarrow x \in [-\sqrt{\frac{3}{5}},\sqrt{\frac{3}{5}}]$

$$\text{Area}=\int_{-\sqrt{\frac{3}{5}}}^{\sqrt{\frac{3}{5}}}(y_1-y_2)dx=\frac{4\sqrt{5}}{3}\int_0^{\sqrt{\frac{3}{5}}}\sqrt{\frac{3}{5}-x^2}\,dx=\frac{\pi}{\sqrt{5}}$$

65. Area$=\frac{1}{2}.2\pi.\pi=\pi^2$

66.

$y_1=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}-\frac{A}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}.y_1=2\sin^{-1}x-A$ Again differentiate

67.

$\frac{dy}{dx}=\frac{y\sqrt{y^2-1}}{x\sqrt{x^2-1}} \Rightarrow \sec^{-1}y=\sec^{-1}x+c$

$x=2, y=\frac{2}{\sqrt{3}} \Rightarrow c=-\frac{\pi}{6}$

$\therefore y=\sec\left(\sec^{-1}x-\frac{\pi}{6}\right)$

$now\frac{1}{y}=\cos\left(\cos^{-1}\frac{1}{x}-\cos^{-1}\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{24}+\frac{1}{2}\sqrt{1-\frac{1}{x^2}}$

68. x+y=t

69.

: 1 already there

$$\frac{2}{3}x^{\frac{-1}{3}}+\frac{2}{3}y^{\frac{-1}{3}}.\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{-x^{\frac{-1}{3}}}{y^{\frac{-1}{3}}}$$

2: $\therefore\frac{dx}{dy}=\frac{x^{\frac{-1}{3}}}{y^{\frac{-1}{3}}}\Rightarrow\int x^{\frac{1}{3}}dx=\int y^{\frac{1}{3}}\,dy$

$x^{\frac{4}{3}}-y^{\frac{4}{3}}=cons\tan t$

70. : $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$, put $y=ux$ $\therefore$ sol is $x^2+y^2=cx$

Put(1,1) $\Rightarrow x^2+y^2=2x$.

$f(x)=\operatorname{sgn}\left((\sin x+1)^2-4\right)$ *and* $(1+\sin x)^2-4\le 0$

$\therefore f(x)=0$ *when* $(1+\sin x)^2-4=0$

$\therefore m=0 \quad \therefore g(x)<m$

71. $\Rightarrow x^2+2(m+3)x+(4m+12)<0$ *for same* x,

$\Delta>0$

$4(m+3)^2-4(4m+12)>0\Rightarrow m\in(-\alpha,-3)\cup(1,\alpha)$

$\therefore m$ *can not be* -3

72. For 0<x<1,[x]=0 f is defind.

For $1\le x<2,[x]=1\Rightarrow f(x)=\frac{1}{\sqrt{2}}$

For $2\le x<3,[x]=2\Rightarrow f(x)=\frac{1}{\sqrt{2}}$

$\therefore$ 'f' is continuous at $x=\frac{3}{2}$

discontinuous at $x=2$, derivable at $\frac{4}{3}$.

73.

$g(f(x)) = x \Rightarrow g^1(f(x))f^1(x) = 1$

$f(x) = 9 \Rightarrow x^3 + x - 1 = 9 \Rightarrow x = 2$

$\therefore g^1(f(2)).f^1(2) = 1$

$\Rightarrow g^1(9)13 = 1 \Rightarrow \dfrac{g(9)}{g^1(9)} = 26$

74.

$2yy^1 = 1$ and $y^1 = 2 \Rightarrow y = \frac{1}{4} \therefore x = \frac{33}{16}$

dist. from $\left(\frac{33}{16}, \frac{1}{4}\right)$ to $y = 2x$ is $\left|\dfrac{\frac{33}{8} - \frac{1}{4}}{\sqrt{2^2 + 1^2}}\right| = \dfrac{31}{8\sqrt{5}}$

75.

$Let\ H(x) = \begin{vmatrix} f(a) & f(x) \\ g(a) & g(x) \end{vmatrix}$

$H(x)$ satisfies conditions ofLMVT

$\therefore H^1(c) = \dfrac{H(b) - H(a)}{b - a} = \dfrac{1}{b - a}\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix}$

$\therefore \begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b - a).H^1(c)$

76.

$I = \displaystyle\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{1 + x^{2024}} \cdot \frac{1}{1 + x^2} dx$

$Put\ x = \frac{1}{t} \therefore I = \displaystyle\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{t^{2024}}{t^{2024} + 1} \cdot \frac{t^2}{1 + t^2}\left(\frac{-1}{t^2}\right) dt$

$\therefore 2I = \tan^{-1} x \Big|_{1/\sqrt{3}}^{\sqrt{3}} \Rightarrow I = \frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{12}$

77

In $\left[0, \frac{1}{2}\right], 1 - x^2 \le 1 - x^m$

.

$\therefore \dfrac{1}{\sqrt{1 - x^2}} \ge \dfrac{1}{\sqrt{1 - x^m}}$

Inte. Btwn $0, \frac{1}{2}$

78. $I_1 = \frac{\pi}{4}, I_2 = \frac{\pi}{8}, I_3 = \frac{\pi}{16}$

$\therefore$ G.P.

79.

: $\int \frac{x^6\left(2x+3x^{-4}\right)}{x^6\left(x^4 - \frac{2}{x} + x^6\right)} dx$

$= \int \frac{2x+3x^{-4}}{\left(x^2 - x^{-3}\right)^2} dx = \frac{-1}{x^2 - x^{-3}} + 1$

80.

$x.f^1(x) + f(x).1 = 6\ f(x) f^1(x) \Rightarrow f(x) = (6f(x)-x).f(x)$

$\int \frac{2x(x-6f(x)) + (6+f(x)-x).f^1(x)}{(6f(x)-x)(x^2-f(x))^2} dx \quad = -\int \frac{2x - f^1(x)}{(x^2 - f(x))^2} dx \ = \frac{1}{x^2 - f(x)} + c$

81.

Area $= 4\int_{-\infty}^{0} e^y dy$

$= 4e^y \big|_{-\infty}^{0} = 4 sq.unit$

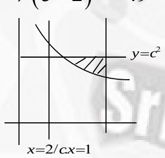
82.

$y = \frac{4}{x^2} = c^2 \Rightarrow x = \frac{2}{c}$

$\int_{\frac{2}{c}}^{1} \left(c^2 - \frac{4}{x^2}\right) dx = 49$

$c^2x + \frac{4}{x}\Big]_{\frac{2}{c}}^{1} = 49$

$\Rightarrow (c-2)^2 = 49 \Rightarrow c = 9$



83.

differentiating bothsides

$x.\phi(x) + \phi(x).1 = 3x^2 - 2\phi^1(x)$

$(x+2)\ \phi^1(x) + \phi(x).1 = 3x^2$

$\Rightarrow \phi^1(x) + \frac{\phi(x)}{x+2} = \frac{3x^2}{x+2}$

$\therefore$ *solution is* $\phi(x)(x+2) = x^3 + c$

$\phi(0) = 4 \Rightarrow c = 8$

$\therefore \phi(x) = \frac{x^3+8}{x+2}, \phi(2) = 4$

84.

$$\int \frac{1}{y+1} dy = -\int \frac{\cos x}{2+\sin x} dx$$
$$\Rightarrow (y+1)(2+\sin x) = c$$
$$y(0) = 1 \Rightarrow c = 4$$
$$\therefore y(\pi/2) = 1/3$$

85.

Let $y = f(x)$ *be the curve*

Normal at $p(x,y)$ *is* $Y - y = -\frac{dx}{dy}(X - x)$

$$NQ = Y.\frac{dy}{dx} = \frac{x(1+y^2)}{1+x^2} \Rightarrow \frac{x+X}{1+x^2} = \frac{ydy}{1+y^2}$$

$\therefore$ *curve is* $1+x^2 = c(1+y^2)$, $c = 5$

$$\therefore \Rightarrow x^2 - 5y^2 = 4$$

86.

$$4\left|\tan\left(\frac{\pi}{4} - x\right)\right|$$

function defined only when $\cos x > 0, \ \sin x > 0$

$$\therefore 0 \le x \le \frac{\pi}{4}$$

f(x)= $\therefore -\frac{\pi}{4} \le \frac{\pi}{4} - x \le \frac{\pi}{4}$

$$\therefore -1 \le \tan\left(\frac{\pi}{4} - x\right) \le 1$$

$$\therefore -4 \le 4\tan\left(\frac{\pi}{4} - x\right) \le 4$$

$$\therefore \le \left|4\tan\left(\frac{\pi}{4} - x\right)\right| \le 4$$

87.

$$\lim_{x\to\infty} (1+f(x))^3 = \lim_{x\to\infty} (1+4)^3 = 125$$

$$\therefore \lim_{x\to\infty} \left(\frac{2x^2+2x+1}{x^2+x+2}\right)^{\frac{6x+1}{3x+2}} = 2^2 = 4$$

88.

$$\lim_{x\to\infty} \left(1+\frac{f(x)}{x^2}\right) = 3 \ \lim_{x\to\infty} \frac{f(x)}{x^2} = 2$$

$\therefore f(x)$ *must have* x^2 *as factor*

$\therefore$ *Let* $f(x) = ax^2(x-1)(x-2)$

simplify. $f(2) = 0$

89.

$$I=\int_0^{\frac{\pi}{2}}\frac{2024\sin^{2023}x-2020.\cos^{2023}x}{\sin^{2023}x+\cos^{2023}x}\,dx$$

$$I=\int_0^{\frac{\pi}{2}}\frac{2024\cos^{2023}x-2020.\sin^{2023}x}{\cos^{2023}x+\sin^{2023}x}\,dx \qquad 2I=\int_0^{\frac{\pi}{2}}\left(2024-2020\right).1\,dx=4\frac{\pi}{2}=2\pi$$

$$\Rightarrow I=\pi\ \therefore [2\lambda]=6$$

90.

$$f(x)=\frac{1}{4}(4x^3-6x^2+4x+1)\ =\frac{1}{4}[x^4-(1-x)^4]+\frac{2}{4}$$

$$f(x)+f(1-x)=1\to 1$$

$$\therefore f(f(x))+f(1-f(x))=1$$

$$Let\ I=\int_{\frac{1}{4}}^{\frac{3}{4}} f(f(x))\,dx\to 2$$

$$also\ I=\int_{\frac{1}{4}}^{\frac{3}{4}} f(f(1-x))dx=\int_{\frac{1}{4}}^{\frac{3}{4}} f(1-f(x))dx\ \ form\ 1$$

$$\therefore 2I=\int_{\frac{1}{4}}^{\frac{3}{4}} f(f(x))+f(1-f(x))\,dx$$

$$2I=\int_{\frac{1}{4}}^{\frac{3}{4}} 1\frac{6}{4}\ \Rightarrow I=\frac{1}{4}$$