ANSWER KEYS

1. (3)	**2.** (3)	**3.** (4)	**4.** (8)	**5.** (1)	**6.** (4)	**7.** (2)	**8.** (1)
9. (21)	**10.** (3)	**11.** (3)	**12.** (1)	**13.** (4)	**14.** (2)	**15.** (2)	**16.** (3)
17. (4)	**18.** (1)	**19.** (2)	**20.** (4)	**21.** (3)	**22.** (1)	**23.** (4)	**24.** (1.5)
25. (4)	**26.** (3)	**27.** (6)	**28.** (2)	**29.** (2)	**30.** (2)		

1. **(3)** $\alpha+\beta=-\mathrm{a}$ and $\alpha\beta=1$

Let S and P be the sum and product of the roots of the required equation. Then,

$\mathrm{S}=-\alpha-\frac{1}{\beta}-\frac{1}{\alpha}-\beta=-\left(\alpha+\beta\right)-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

$=-\left(\alpha+\beta\right)-\left(\frac{\alpha+\beta}{\alpha\beta}\right)=-\left(-\mathrm{a}\right)-\left(\frac{-\mathrm{a}}{1}\right)=2\mathrm{a}$

$\mathrm{P}=-\left(\alpha+\frac{1}{\beta}\right)\left(-\left(\frac{1}{\alpha}+\beta\right)\right)$

$=1+\alpha\beta+\frac{1}{\alpha\beta}+1=1+1+1+1=4$

So, the required equation is

$\mathrm{x}^2-\mathrm{Sx}+\mathrm{P}=0$

i.e. $\mathrm{x}^2-2\mathrm{ax}+4=0$

2. **(3)**

We have $\frac{k+1}{k}+\frac{k+2}{k+1}=\frac{-b}{a}\ldots.(i)$

and $\frac{k+1}{k}\cdot\frac{k+2}{k+1}=\frac{c}{a}$

$\Rightarrow\frac{k+2}{k}=\frac{c}{a}$

or $\frac{2}{k}=\frac{c}{a}-1=\frac{c-a}{a}$

or $\mathrm{k}=\frac{2\mathrm{a}}{\mathrm{c}-\mathrm{a}}$ .....(ii)

Now eliminate k putting the value of k in 1^{st} relation, we get

$\frac{c+a}{2a}+\frac{2c}{c+a}=\frac{-b}{a}$

$\Rightarrow\left(c+a\right)^2+4ac=-2b\left(a+c\right)$

$\Rightarrow\left(a+c\right)^2+2b\left(a+c\right)=-4ac$

Adding b^2 on both sides,

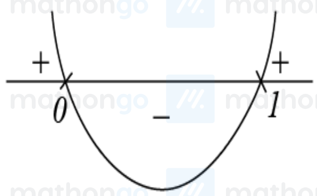
$\left(a+b+c\right)^2=b^2-4ac$

3. **(4)** $D=(2n-1)^2-4n(n-1)=4n^2+1-4n-4n^2+4n=1>0$

Product of roots $=\frac{n-1}{n}<0$

Solving this by wavy-curve method,we get-

$\Rightarrow n\in(0,1)$



4. **(8)**

$x^2+2x-n=0;\ n\in[5,\ 100]$

In order for the given equation to have integral roots, D must be a perfect square,

$D=4+4n=4(1+n)$

$\Rightarrow 1+n$ is a perfect square

$\Rightarrow 1+n=9,\ 16,\ 25,\ 36,\ 49,\ 64,\ 81,\ 100$

$\Rightarrow n=8,\ 15,\ 24,\ 35,\ 48,\ 63,\ 80,\ 99$

Therefore, 8 values are possible.

5. **(1)**

Given equation has more than two roots if it is an identity

$\Rightarrow \cos 3\theta+1=0$; $2\cos 2\theta-1=0$ and $1-2\cos\theta=0$

$\Rightarrow \cos 3\theta=-1 \Rightarrow \theta=\pm\frac{\pi}{3}$ which does not satisfy $2\cos 2\theta-1=0$

Hence, no value possible

6. **(4)** $1, \alpha+\beta, \alpha\beta$ are in A.P. $\Rightarrow 1, \frac{-b}{a}, \frac{c}{a}$ are in A.P.

$\Rightarrow 1+\frac{c}{a}=\frac{-2b}{a} \Rightarrow a+c+2b=0 \ldots(1)$

$\frac{1}{\alpha}, \frac{1}{2}, \frac{1}{\beta}$ are in A.P. $\Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}=1 \Rightarrow \alpha+\beta=\alpha\beta$

$\Rightarrow \frac{-b}{a}=\frac{c}{a} \Rightarrow b+c=0 \ldots(2)$

From (1) & (2) we get,

$a=-b=c$

$\Rightarrow \alpha, \beta$ are roots of equation $x^2-x+1=0$

Now, $\frac{\alpha^2+\beta^2-2\alpha^2\beta^2}{2\left(\alpha^2+\beta^2\right)}=\frac{1}{2}-\frac{(\alpha\beta)^2}{(\alpha+\beta)^2-2\alpha\beta}$

$=\frac{1}{2}-\frac{(1)^2}{(1)^2-2(1)}=\frac{1}{2}+1=1.5$

7. **(2)** Let a, β be the roots of a quadratic equation and a^2, β^2 be the roots of another quadratic, since the quadratic remains the same, we have $a+\beta=a^2+\beta^2$

and $\quad \alpha\beta=\alpha^2\beta^2$

Now, $a\beta=a^2\beta^2 \Rightarrow a\beta*(a\beta-1)=0$

$\Rightarrow a=0$ or $\beta=0$ or $a\beta=1$

If $a=0$, then $\beta=\beta^2$ [putting $a=0$ in(i)]

$\Rightarrow \beta(1-\beta)=0 \Rightarrow \beta=0, \beta=1$

Thus, we get two sets of values of a and β viz. $\alpha=0, \beta=0$ and $\alpha=0, \beta=1$. Now if $a\beta=1$, then $a+\frac{1}{a}=a^2+\frac{1}{a^2}$ [putting $\beta=\frac{1}{a}$ in (i)]

$\Rightarrow \quad a+\frac{1}{a}=\left(a+\frac{1}{a}\right)^2-2$

$\Rightarrow \quad \left(a+\frac{1}{a}\right)^2-\left(a+\frac{1}{a}\right)-2=0$

$\Rightarrow \quad a+\frac{1}{a}=2$ or $a+\frac{1}{a}=-1$

$\Rightarrow \quad a=1$ or $a=\omega, \omega^2$

Putting $a=1$, in $\alpha\beta=1$, we get $\beta=1$, and putting $a=\omega$ in $\alpha\beta=1$, we get $\beta=\omega^2$

Putting $a=\omega^2$ in $\alpha\beta=1$, we get $\beta=\omega$, thus, we get four sets of values of a, β viz.,

$a=0, \beta=0; a=0, \beta=1; \alpha=\omega, \beta=\omega^2; \alpha=1, \beta=1$.

Thus, there are four quadratics which remain unchanged by squaring their roots.

8. **(1)** Given, $x^2+5\sqrt{2}x+10=0$

and $P_n=\alpha^n-\beta^n$

Now $\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}=\frac{P_{17}\left(P_{20}+5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19}+5\sqrt{2}P_{18}\right)}$

$\frac{P_{17}\left(\alpha^{20}-\beta^{20}+5\sqrt{2}(\alpha^{19}-\beta^{19})\right)}{P_{18}\left(\alpha^{19}-\beta^{19}+5\sqrt{2}(\alpha^{18}-\beta^{18})\right)}$

$\frac{P_{17}\left(\alpha^{19}(\alpha+5\sqrt{2})-\beta^{19}(\beta+5\sqrt{2})\right)}{P_{18}\left(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2})\right)}$

Since $\alpha+5\sqrt{2}=-10/\alpha$ ... (1)

and $\beta+5\sqrt{2}=-10/\beta$ ... (2)

Now put there values in above expression

$\frac{P_{17}\left(\alpha^{19}(\alpha+5\sqrt{2})-\beta^{19}(\beta+5\sqrt{2})\right)}{P_{18}\left(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2})\right)}=-\frac{10P_{17}P_{18}}{-10P_{18}P_{17}}=1$

9. **(21)** Given α,β are the roots of the quadratic equation $2x^2-5x+1=0$

Let us find an equation with roots α^2 and β^2, let $y=x^2$, so $x=\sqrt{y}$

$2y-5\sqrt{y}+1=0$

$\Rightarrow 2y+1=5\sqrt{y}$

$\Rightarrow 4y^2+4y+1=25y$

$\Rightarrow 4y^2-21y+1=0\left\langle{}^{c}_{d}\right.$

Put $\alpha^2=c$ and $\beta^2=d$

Now, $S_n=(c)^n+(d)^n$

Consider

$4S_{2021}+S_{2019}=4\left(c^{2021}+d^{2021}\right)+c^{2019}+d^{2019}$

$=c^{2019}\left(4c^2+1\right)+d^{2019}\left(4d^2+1\right)$

$=c^{2019}(21c)+d^{2019}(21d)$

$=21S_{2020}$

Hence, $\frac{4S_{2021}+S_{2019}}{S_{2020}}=21$

10. **(3)** Consider $x^2-47x+k=0$

For real roots, $47^2-4k\geq 0\Rightarrow k\leq 552$

$\therefore\quad k=1,2,3\ldots,552$

Product of real roots $=1\times 2\times 3\times 4\times\ldots\ldots\times 552=552!$

11. (3)

$-3<\frac{x^2-\lambda x-2}{x^2+x+1}<2$

$\Rightarrow -3x^2-3x-3<x^2-\lambda x-2<2x^2+2x+2$ (since $x^2+x+1>0,\ \forall x\in R$)

$\Rightarrow 4x^2+x(3-\lambda)+1>0, x^2+x(2+\lambda)+4>0$

$(i)\ 4x^2-x(\lambda-3)+1>0$

$\Rightarrow D<0\Rightarrow(\lambda-3)^2-4\times4\times1<0$

$\Rightarrow(\lambda-3+4)(\lambda-3-4)<0$

$\Rightarrow(\lambda+1)(\lambda-7)<0\Rightarrow\lambda\in(-1,7)$

$(ii)\ x^2+x(\lambda+2)+4>0$

$\Rightarrow D<0\Rightarrow(\lambda+2)^2-4\times4<0$

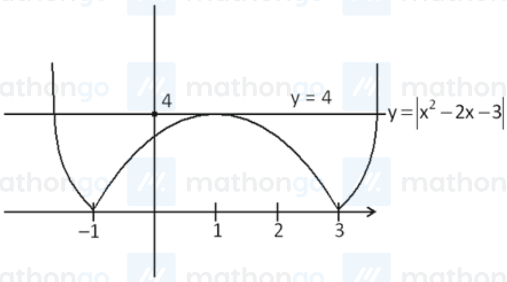
$(\lambda+2-4)(\lambda+2+4)<0$

$(\lambda-2)(\lambda+6)<0\Rightarrow\lambda\in(-6,2)$

Taking the intersection of the solutions of (i) and (ii), we get,

$\lambda\in(-1,2)$

12.



(1)

$b<0\rightarrow$ no solution

$b=0\rightarrow$ two solutions

$0<b<4\rightarrow$ four solutions

$b=4\rightarrow$ three solutions

13. (4) $D=25b^2-4\times3a\times7c$

$=25(-a-c)^2-84ac$ (given that $a+b+c=0$)

$=25(a^2+c^2+2ac)-84ac$

$=25(a^2+c^2)-34ac$

$=17(a^2+c^2-2ac)+8(a^2+c^2)$

$=17(a-c)^2+8(a^2+c^2)$

$D>0\Rightarrow$ Roots are real and distinct

14. **(2)** As $ax^2 + 2bx - 5c = 0$ has non -real roots,

$D < 0$

$\rightarrow (2b)^2 - 4a(-5c) < 0$

$\rightarrow 4b^2 + 20ac < 0$

$\rightarrow b^2 + 5ac < 0$

$\rightarrow 5ac < -b^2 \cdots$ (iii)

Which means (ac) is always less than 0

Now, for $ac < 0$, either (i) $a < 0, c > 0$ (ii) $a > 0, c < 0$

Lets assume (i) $a < 0, c > 0$

$a + b + c > \frac{9c}{4}$

$\rightarrow 4(a + b) > 5c$

$\rightarrow 4a(a + b) < 5ac$ [Sign changes as $a < 0$ here]

$\rightarrow 4a(a + b) < -b^2$ [by using (iii)]

$\rightarrow 4a^2 + 4ab < -b^2$

$\rightarrow 4a^2 + 4ab + b^2 < 0$

$\rightarrow (2a + b)^2 < 0$ which is not possible so, our assumption is wrong

Correct answer is (2)

15. **(2)** $D = B^2 - 4AC$

$= (2(a + b - 2c))^2 - 4(a - b)^2$

$= 4\left\{(a + b - 2c)^2 - (a - b)^2\right\}$

$= 4(a + b - 2c - a + b)(a + b - 2c + a - b)$

$= 4(2b - 2c)(2a - 2c)$

$= 16(b - c)(a - c)$

$= 16(c - b)(c - a)$

If c lies between a and b, then D is negative. Hence, the roots will be imaginary and the graph will be entirely above the x-axis as the coefficient of x^2 is positive.

16. **(3)** Let, α be the common root and the other roots of the equations be 4β and 3β respectively. Then,

$\alpha + 4\beta = 6,\ \alpha(4\beta) = a$

$\alpha + 3\beta = c,\ \alpha(3\beta) = 6 \Rightarrow \frac{4}{3} = \frac{a}{6} \Rightarrow a = 8$

The first equation is $x^2 - 6x + 8 = 0$

Whose roots are 2 and 4

If $\alpha = 2 \Rightarrow \beta = 1$

So, roots of first equation is 2, 4 and that of second equation is 2, 3

If $\alpha = 4 \Rightarrow \beta = \frac{1}{2} \Rightarrow 3\beta = \frac{3}{2}, 4\beta = 2$

Here the roots are not integers $\Rightarrow \alpha = 2$

17. **(4)** Let, $f(x)=4x^2-20kx+\left(25k^2+15k-66\right)=0\ldots\ldots\ldots\ldots\ldots\ldots(i)$

Let the roots of $f(x)=0$ be $\alpha,\ \beta$

Since $\alpha,\ \beta$ are real.

$\therefore\ D\geq 0$

$\Rightarrow 400k^2-4.4\left(25k^2+15k-66\right)\geq 0$

$\Rightarrow\ -15k+66\geq 0\quad\Rightarrow\quad k\leq\frac{22}{5}\ldots\ldots\ldots\ldots\ldots(ii)$

We have $\alpha,\ \beta<2$

$\therefore\ \alpha+\beta<4$

$\Rightarrow\ -\frac{(-20k)}{4}<4\ \Rightarrow k<\frac{4}{5}\ldots\ldots\ldots\ldots\ldots(iii)$

$f(x)=4(x-\alpha)(x-\beta)$

$\therefore\quad f(2)=4(2-\alpha)(2-\beta)=4(+)(+)=+\text{ve}$

$\therefore\ f(2)=16-40k+\left(25k^2+15k-66\right)>0$

$\Rightarrow 25k^2-25k-50>0\quad\Rightarrow\quad k^2-k-2>0$

$\Rightarrow\ (k+1)(k-2)>0\quad\Rightarrow\quad k<-1$ or $k>2\ldots\ldots\ldots\ldots(\text{iv})$

Combining $(ii),\ (iii)$ & $(iv),$ we get $k\in(-\infty,-1)$

18. Clearly, $f(-1)>0, f(2)<0$

since, $f(0)=-4<0$

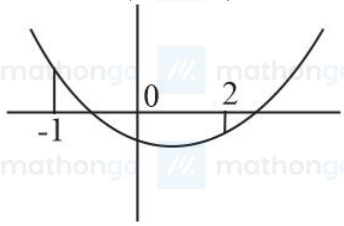
$\Rightarrow f(-1)=1-a-4>0\Rightarrow -a-3>0\Rightarrow -a>3$

(1) or $a<-3$ and $f(2)=4+2a-4<0$

$\Rightarrow a<0$

Combining the conditions for $f(-1)$ and $f(2)$, we get

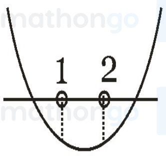
$\Rightarrow a\in(-\infty,-3)$.



19. (2)

The roots of $f(x) - x = 0$ are 1, 2 and 3.
So, we get,
$f(x) - x = (x-1)(x-2)(x-3)(x-a)$
For $x = -1$, we get,
$f(-1) + 1 = (-2)(-3)(-4)(-1-a) = 24(1+a)$
For $x = 5$, we get,
$f(5) - 5 = 4 \cdot 3 \cdot 2(5-a) = 24(5-a)$
$f(-1) + f(5) = (23 + 24a) + (125 - 24a) = 148$
For $x = 0$, we get,
$f(0) - 0 = (-1)(-2)(-3)(-a) = 6a$
For $x = 4$, we get,
$f(4) - 4 = 3 \cdot 2 \cdot 1 \cdot (4-a) = 24 - 6a$
$f(0) + f(4) = 28$
$\left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right] = \left[\frac{148}{28}\right]$
$\left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right] = 5$

20. (4) Let, $f(x) = x^2 + ax + a^2 + 6a$
$\therefore\ f(1) \le 0$



$\Rightarrow\quad a^2 + 7a + 1 < 0$
or $\quad \frac{-7-3\sqrt{5}}{2} < a < \frac{-7+3\sqrt{5}}{2}$ ....(i)
$f(2) \le 0$
$\Rightarrow\quad a^2 + 8a + 4 < 0$
or $\ -4 - 2\sqrt{3} < a < -4 + 2\sqrt{3}$ ....(ii)
and $\quad D > 0$
$\Rightarrow a^2 - 4 \cdot 1\left(a^2 + 6a\right) > 0$
$\Rightarrow a^2 + 8a < 0$
or $\quad -8 < a < 0$ ....(iii)
From Eqs. (i), (ii) and (iii), we get,
$\frac{-7-3\sqrt{5}}{2} \le a \le -4 + 2\sqrt{3}$
Hence, integral values of a are $-6, -5, -4, -3, -2, -1$
Required Sum
$= (-6)^2 + (-5)^2 + (-4)^2 + (-3)^2 + (-2)^2 + (-1)^2$
$= 91.$

21. (3)

Let α be the common root

$\Rightarrow k\alpha^2+\alpha+k=0$

$k\alpha^2+k\alpha+1=0$

On solving, we get,

$\frac{\alpha^2}{1-k^2}=\frac{\alpha}{k^2-k}=\frac{1}{k^2-k}$

$\frac{\alpha^2}{\alpha}=\frac{1-k^2}{k^2-k}$ and $\frac{\alpha}{1}=\frac{k^2-k}{k^2-k}$

$\Rightarrow \alpha=\frac{1-k^2}{k^2-k}=1 \Rightarrow k^2-k=1-k^2$

$\Rightarrow 2k^2-k-1=0 \Rightarrow k=-\frac{1}{2}, 1$

For $k=1$, equations are identical, thus not possible

Hence, $k=-\frac{1}{2}$

22. (1)

$(6k+2)x^2+rx+3k-1=0$

$(12k+4)x^2+px+6k-2=0$ have

both roots common.

So,

$\frac{6k+2}{12k+4}=\frac{r}{p}=\frac{3k-1}{6k-2}$

$\Rightarrow \frac{r}{p}=\frac{1}{2} \Rightarrow 2r-p=0$

23. (4) Let, the roots be $\alpha, \beta, \alpha+2$.

$S_1=\alpha+\beta+\alpha+2=2\alpha+\beta+2=13 \Rightarrow 2\alpha+\beta=11 \Rightarrow \beta=11-2\alpha$

$S_2=\alpha\beta+\beta(\alpha+2)+(\alpha+2)\alpha=15$

$\Rightarrow \beta(\alpha+\alpha+2)+\alpha(\alpha+2)=15$

$\Rightarrow (11-2\alpha)(2\alpha+2)+\alpha(\alpha+2)=15$

$\Rightarrow 22\alpha+22-4\alpha^2-4\alpha+\alpha^2+2\alpha=15$

$\Rightarrow 3\alpha^2-20\alpha-7=0 \Rightarrow (\alpha-7)(3\alpha+1)=0$

$\Rightarrow \alpha=7$ or $-\frac{1}{3}$.

$\alpha=7, \beta=11-2\alpha=11-14=-3, \gamma=\alpha+2=9$

$\alpha=-\frac{1}{3}, \beta=11-2\alpha=11+\frac{2}{3}=\frac{35}{3}, \gamma=\alpha+2=\frac{5}{3}$.

Since, $\alpha\beta\gamma=-189$, hence we will take the first case.

$|\alpha|+|\beta|+|\gamma|=|7|+|-3|+|9|=19$

24. (1.5) $x^3+3x^2+5x+3=0$ has one root $x=-1$

$\therefore x^3+3x^2+5x+3=(x+1)(x^2+2x+3)$

$\Rightarrow a=2,\ b=3$

Now, value of $\left(\frac{b}{a}\right)=\left(\frac{3}{2}\right)=1.5$ is the answer

25. (4) Given equation,

$4\left(x^2+\frac{1}{x^2}\right)+16\left(x+\frac{1}{x}\right)-57=0$

Let, $x+\frac{1}{x}=y$; $x^2+\frac{1}{x^2}=y^2-2$

$\Rightarrow 4y^2+16y-65=0$

$\Rightarrow y=\frac{13}{2}$ or $\frac{5}{2}$

When, $y=\frac{5}{2}$

$x+\frac{1}{x}=\frac{5}{2}\Rightarrow x=2$ or $\frac{1}{2}$

When, $y=-\frac{13}{2}$

$\Rightarrow x+\frac{1}{x}=-13/2$

$\Rightarrow 2x^2+13x+2=0$

$\Rightarrow x=\frac{-13\pm\sqrt{153}}{4}$

Since x is rational, $x=2$ or $\frac{1}{2}$

product of rational roots is 1

26. (3) The given equation will be true in 3 cases.

Case 1: when $x^2-5x+5=1$

$\Rightarrow x^2-5x+4=0$

$\Rightarrow x=1,\ 4$

Case 2: when $x^2+4x-60=0$

$\Rightarrow x=6,\ -10$

Case 3: when $x^2-5x+5=-1$ and $x^2+4x-60\in$ even integers

Now, $x^2-5x+5=-1$

$\Rightarrow x=2,\ 3$

Only $x=2$ satisfies the given condition,

Hence, sum of all real values of x is $1+4+6-10+2=3$

27. (6)

Let $\log_x 10=t$

$\therefore t^3-t^2-6t=0$

$\Rightarrow t\left(t^2-t-6\right)=0$

$\Rightarrow t=0,-2,\ 3$

$\Rightarrow \log_x 10=0,\ -2,\ 3$

$\Rightarrow 10=x^0,x^{-2},x^3$

$\Rightarrow x=10^{-\frac{1}{2}},\ 10^{\frac{1}{3}}$

Let $\alpha=10^{-\frac{1}{2}}$ and $\beta=10^{\frac{1}{3}}$

Now, $\left|\frac{1}{\log_{10}\alpha\beta}\right|=\left|\frac{1}{\log_{10}10^{-\frac{1}{6}}}\right|$

$\Rightarrow\left|\frac{1}{\log_{10}\alpha\beta}\right|=\left|\frac{-6}{\log_{10}10}\right|=6$

28. **(2)** Let, $t = 2^{11x}$

$\Rightarrow \frac{\left(2^{11x}\right)^3}{2^2} + 2^{11x}.2^2 = \left(2^{11x}\right)^2.2 + 1$

$\Rightarrow \frac{t^3}{4} + 4t = 2t^2 + 1$

$\Rightarrow t^3 - 8t^2 + 16t - 4 = 0$

Cubic in t has roots t_1, t_2, t_3

i.e. $t_1t_2t_3 = 4 \Rightarrow 2^{11x_1}.2^{11x_2}.2^{11x_3} = 4$

$\Rightarrow 2^{11(x_1+x_2+x_3)} = 2^2$

$\Rightarrow 11(x_1 + x_2 + x_3) = 2 \Rightarrow x_1 + x_2 + x_3 = \frac{2}{11}$

29. **(2)** We have, $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$

Let $e^x = t$

$t^4 - t^3 - 4t^2 - t + 1 = 0$

Dividing the while equation by t^2

$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$

$\left\{t^2 + \left(\frac{1}{t^2}\right)\right\} - \left\{t + \left(\frac{1}{t}\right)\right\} - 4 = 0 \left\{t + \left(\frac{1}{t}\right)\right\}^2 - \left\{t + \left(\frac{1}{t}\right)\right\} - 6 = 0$ let $t + \left(\frac{1}{t}\right)$ be α

$\Rightarrow \alpha^2 - \alpha - 6 = 0, \alpha = t + \frac{1}{t} \geq 2$

$\Rightarrow \alpha = 3, -2$ (reject)

$\Rightarrow t + \frac{1}{t} = 3$

$\Rightarrow$ The number of real roots $= 2$.

30. **(2)** Let $2^{\frac{\pi}{cos^{-1}x}}$ be t

$\Rightarrow \quad t \geq 2$

equation becomes $t^2 - \left(a + \frac{1}{2}\right)t - a^2 = 0$

has one roots 2 or greater than 2 and other root less than 2, $f(2) \leq 0$

$\Rightarrow 4 - \left(a + \frac{1}{2}\right)2 - a^2 \leq 0$

$a^2 + 2a - 3 \geq 0$

$(a + 3)(a - 1) \geq 0$

$a \leq -3$ or $a \geq 1$