ANSWER KEYS

1. (6)	**2.** (2)	**3.** (3)	**4.** (4)	**5.** (1)	**6.** (2)	**7.** (1)	**8.** (2)
9. (4)	**10.** (1)	**11.** (3)	**12.** (2)	**13.** (6)	**14.** (1680)	**15.** (2)	**16.** (3)
17. (2)	**18.** (13)	**19.** (2)	**20.** (0)	**21.** (40)	**22.** (24)	**23.** (4)	**24.** (4)
25. (2)	**26.** (2)	**27.** (6)	**28.** (3)	**29.** (26)	**30.** (3)		

1. (6)

$\frac{(2i)^n}{(1-i)^{n-2}}$

$= \frac{(2i)^n}{\left(-2i\right)^{\frac{n-2}{2}}}$

$= \frac{\left(2i\right)^{\frac{n+2}{2}}}{\left(-1\right)^{\frac{n-2}{2}}}$

$= \frac{(2)^{\frac{n+2}{2}} i^{\frac{n+2}{2}}}{\left(-1\right)^{\frac{n-2}{2}}}$ ...(i)

Clearly n must be even

Now, we need to check (i) by substituting even values one by one. $n = 2, 4$ are rejected because it does not give a positive integer

So the least positive integer possible is $n = 6$

2. (2)

Let,

$z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$\because$ Re$(z_1 z_2)$= 0

$x_1 x_2 - y_1 y_2 = 0 \ldots\ldots(i)$

$\because$ Re$(z_1 + z_2)$= 0

$x_1 + x_2 = 0 \ldots\ldots(ii)$

From equations (i) and (ii) we get

$x_1{}^2 + y_1 y_2 = 0$

$\Rightarrow y_1 y_2 = -x_1{}^2$

Therefore $Im(z_1)$ and $Im(z_2)$ are of opposite sign

3. **(3)** Let $z_1=\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right)$

Let $z=3+iy$

$\bar{z}=3-iy$

$z_1=\dfrac{2iy+(9+y^2)}{2-3(3+iy)+5(3-iy)}$

$=\dfrac{9+y^2+i(2y)}{8-8iy}$

$=\dfrac{(9+y^2)+i(2y)}{8(1-iy)}$

$\operatorname{Re}(z_1)=\dfrac{(9+y^2)-2y^2}{8(1+y^2)}$

$=\dfrac{9-y^2}{8(1+y^2)}$

$=\dfrac{1}{8}\left[\dfrac{10-(1+y^2)}{(1+y^2)}\right]$

$=\dfrac{1}{8}\left[\dfrac{10}{1+y^2}-1\right]$

$1+y^2\in[1,\infty]$

$\dfrac{1}{1+y^2}\in(0,1]$

$\dfrac{10}{1+y^2}\in(0,10)$

$\dfrac{10}{1+y^2}-1\in(-1,9]$

$\operatorname{Re}(z_1)\in\left(\dfrac{-1}{8},\dfrac{9}{8}\right]$

$\alpha=\dfrac{-1}{8},\beta=\dfrac{9}{8}$

$24(\beta-\alpha)=24\left(\dfrac{9}{8}+\dfrac{1}{8}\right)=30$

4. **(4)**

$u=\frac{2(x+iy)+i}{(x+iy)-ki}=\frac{2x+(2y+1)i}{x+(y-k)i}\times\frac{x-(y-k)i}{x-(y-k)i}$

Real part of $u=Re(u)=\frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2}$

Imaginary part of $u=\operatorname{Im}(u)\frac{x(2y+1)-2x(y-k)}{x^2+(y-k)^2}$

Now $\operatorname{Re}(u)+\operatorname{Im}(u)=1$

$\frac{2x^2+(2y+1)(y-k)+x(2y+1)-2x(y-k)}{x^2+(y-k)^2}=1$

for y-axis put $x=0$

$\Rightarrow\frac{(2y+1)(y-k)}{(y-k)^2}=1$

$\Rightarrow(y-k)(y+1+k)=0$

$y=k,\ -(1+k)$

Now point $P(0,\ k),\ Q(0,\ -(1+k))$

$PQ=|2K+1|=5$

$2k+1=\pm5$

$2k=4,-6$

$k=2,-3$

Hence, $k=2\quad(k>0)$.

5. **(1)**

The given system of equations has more than one solution, then it must have infinitely many solutions.

So, $\frac{4i}{8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)}=\frac{1+i}{\alpha+i\beta}$ $\left(\because \begin{aligned} a &= \alpha-i\beta \\ \Rightarrow \bar{a} &= \alpha+i\beta \end{aligned}\right)$

$\Rightarrow \frac{4i}{8\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)}=\frac{1+i}{\alpha+i\beta}$

$\Rightarrow \alpha i-\beta=-1-i+\sqrt{3}i-\sqrt{3}$

$\Rightarrow -\beta+\alpha i=\left(-1-\sqrt{3}\right)+\left(-1+\sqrt{3}\right)i$

$\Rightarrow \alpha=-1+\sqrt{3}$ & $-\beta=-1-\sqrt{3}$

$\therefore \frac{\alpha}{\beta}=\frac{-1+\sqrt{3}}{1+\sqrt{3}}=\frac{-1+\sqrt{3}}{1+\sqrt{3}}\times\frac{1-\sqrt{3}}{1-\sqrt{3}}$

$=\frac{-\left(1-\sqrt{3}\right)^2}{1-3}=\frac{1+3-2\sqrt{3}}{2}=2-\sqrt{3}$

6. **(2)**

$\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right)\leq 2$

$\Rightarrow \frac{|z|+11}{(|z|-1)^2}\geq\frac{1}{2}$, $\{\because \log_a(b)\leq c\Rightarrow b\geq a^c \; if \; 0<a<1\}$

$\Rightarrow 2|z|+22\geq(|z|-1)^2$, $\left\{\because |z|\neq 1 \;\&\; (|z|-1)^2>0\right\}$

$\Rightarrow 2|z|+22\geq|z|^2+1-2|z|$

$\Rightarrow |z|^2-4|z|-21\leq 0$

$\Rightarrow (|z|-7)(|z|+3)\leq 0$

$\Rightarrow |z|\leq 7$

$\therefore$ Largest value of $|z|$ is 7

7. **(1)** Let $\alpha=\frac{\sqrt{3}}{2}+\frac{i}{2}$, then $z=\alpha^5+(\bar{\alpha})^5$

We know if $z_n=\bar{z}^n$, then, $z=2\,\mathrm{Re}\left(\alpha^5\right)$

Hence, $I(z)=0$.

8. **(2)** Let $Z=x+iy, x\in R, y\in R$

$x-iy=i\left(x^2-y^2+(2xy)i+x\right)$

$x=-2xy \quad \ldots(1)$

$-y=-y^2+x^2+x$

$\Rightarrow x=0, y=-\frac{1}{2}(\text{from (1)})$

If $x\neq 0$, then $y=0,1$

If $y=-\frac{1}{2}$, then $x=\frac{1}{2},-\frac{3}{2}$

$Z=0+i0, 0+i, \frac{1}{2}-\frac{i}{2}, -\frac{3}{2}-\frac{i}{2}$

Summation of $|Z|^2$ of all the complex numbers mentioned above are 4

9. **(4)**

Given $\left|z-\frac{1}{z}\right|=2$

We know that $\left||z|-\frac{1}{|z|}\right|\leq\left|z-\frac{1}{z}\right|\leq|z|+\frac{1}{|z|}$ Let $|z|=r$

i.e. $\left|r-\frac{1}{r}\right|\leq 2$ and

$r+\frac{1}{r}\geq 2$ which is always true.

Now $\left|r-\frac{1}{r}\right|\leq 2$

$\Rightarrow r-\frac{1}{r}\geq -2$ & $r-\frac{1}{r}\leq 2$

$\Rightarrow r^2-1\leq 2r$

$\Rightarrow (r-1)^2\leq 2$

$\Rightarrow r-1\leq\sqrt{2}$

$\therefore |z|_{\max}=1+\sqrt{2}$

10. (1)

Given, the minimum value v_0 of $v = |z|^2 + |z-3|^2 + |z-6i|^2$, $z \in \mathbb{C}$ is attained at $z = z_0$,

Let $z = x + iy$

$v = x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-6)^2$

$= (3x^2 - 6x + 9) + (3y^2 - 12y + 36)$

$= 3(x^2 + y^2 - 2x - 4y + 15)$

$= 3\left[(x-1)^2 + (y-2)^2 + 10\right]$

$V_{\min}$ at $z = 1 + 2i = z_0$ and $v_0 = 30$

So, $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$

$= \left|2(1+2i)^2 - (1-2i)^3 + 3\right|^2 + 900$

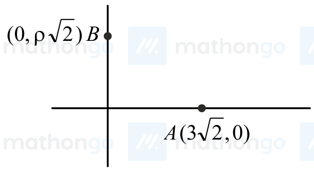
$= |-6 + 8i - (1 + 8i - 6i - 12) + 3|^2 + 900$

$= |8 + 6i|^2 + 900 = 1000$

11. (3)

Note that $|z - 3\sqrt{2}|$ is the distance between the point z and $\left(3\sqrt{2}, 0\right)$ on argand plane.

Note that $|z - p\sqrt{2}i|$ is the distance between the point z and $\left(0, p\sqrt{2}\right)$ on argand plane.



$|z - 3\sqrt{2}| + |z - p\sqrt{2}i|$ is sum of distance of z from $A\left(3\sqrt{2}, 0\right)$ and $B\left(0, p\sqrt{2}\right)$

For minimising the sum, z should lie on AB and AB should be equal to $5\sqrt{2}$

Now, $(AB)^2 = 18 + 2p^2$

$\Rightarrow 18 + 2p^2 = \left(5\sqrt{2}\right)^2$

$\Rightarrow p^2 = 16$

i.e. $p = \pm 4$

12. (2)

Given $z^2+3\bar{z}=0$

Put $z=x+iy$, then we know that $\bar{z}=x-iy$, hence, we have

$(x+iy)^2+3(x-iy)=0$

$\Rightarrow\ x^2+i^2y^2+2ixy+3x-3iy=0$

We also, know that $i^2=-1$, hence, we get

$\Rightarrow\ x^2-y^2+2ixy+3x-3iy=0$

$\Rightarrow\ \left(x^2-y^2+3x\right)+i\left(2xy-3y\right)=0+i0$

On comparing the real and imaginary parts, we get

$x^2-y^2+3x=0\quad\ldots(1)$

And $2xy-3y=0\quad\ldots(2)$

$\Rightarrow\ y(2x-3)=0$

$\Rightarrow x=\frac{3}{2},\ y=0$

Put $x=\frac{3}{2}$ in equation (1), we get $\frac{9}{4}-y^2+\frac{9}{2}=0$

$\Rightarrow\ y^2=\frac{27}{4}$

$\Rightarrow\ y=\pm\frac{3\sqrt{3}}{2}$.

$\Rightarrow\ (x,\ y)=\left(\frac{3}{2},\ \frac{3\sqrt{3}}{2}\right),\ \left(\frac{3}{2},\ \frac{-3\sqrt{3}}{2}\right)$.

Now, put $y=0$, in the equation (1), we get $x^2+3x=0$

$\Rightarrow x=0,\ -3$.

$\therefore\ (x,\ y)=(0,\ 0),\ (-3,\ 0)$.

$\therefore$ No of solutions$=n=4$

Now, $\sum_{k=0}^{\infty}\left(\frac{1}{n^k}\right)=\sum_{k=0}^{\infty}\left(\frac{1}{4^k}\right)$

$=\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\ldots$

The above progression is a geometric progression, with first term $a=1$ and common ratio $r=\frac{1}{4}$ and the sum of infinite terms of the geometric progression is $\frac{a}{1-r}$

Thus, $\sum_{k=0}^{\infty}\left(\frac{1}{n^k}\right)=\frac{1}{\left(1-\frac{1}{4}\right)}$

$=\frac{1}{\left(\frac{3}{4}\right)}=\frac{4}{3}$.

13. (6)

Given,

$z^2=\bar{z}\cdot 2^{1-|z|}\quad\cdots(1)$

On taking modulus both side we get,

$\Rightarrow |z|^2=|\overline{z}|\cdot 2^{1-|z|}$

$\Rightarrow |z|=2^{1-|z|},\quad\because\ b\neq 0\Rightarrow |z|\neq 0$

So comparing both side we get $|z|=1\quad\cdots(2)$

Now putting $z=a+ib$ then $\sqrt{a^2+b^2}=1\quad\cdots(3)$

Now again from equation (1), equation (2), equation (3) we get:

$a^2-b^2+i2ab=(a-ib)2^0=a-ib$

Now on comaparing imagenary and real part we get,

$\therefore\ a^2-b^2=a$ and $2ab=-b$

Now solving we get, $a=-\frac{1}{2}$ and $b=\pm\frac{\sqrt{3}}{2}$

So, $z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ or $=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

Now solving $z^n=(z+1)^n\Rightarrow\left(\frac{z+1}{z}\right)^n=1$

$\Rightarrow\ \left(1+\frac{1}{z}\right)^n=1$

$\Rightarrow\ \left(\frac{1+\sqrt{3}i}{2}\right)^n=1$

$\Rightarrow\ \left(-\omega^2\right)^n=1$, then minimum value of n is 6

14. **(1680)** $\left(\frac{z^2+8iz-15}{z^2-3iz-2}\right) \in R$

$\Rightarrow 1+\frac{(11iz-13)}{(z^2-3iz-2)} \in R$

Put $z=\alpha-\frac{13}{11}i$

$\Rightarrow \left(z^2-3iz-2\right)$ is imaginary

Put $z=x+iy$

$\Rightarrow \left(x^2-y^2+2xyi-3ix+3y-2\right) \in$ Imaginary

$\Rightarrow \operatorname{Re}\left(x^2-y^2+3y-2+(2xy-3x)i\right)=0$

$\Rightarrow x^2-y^2+3y-2=0$

$x^2=y^2-3y+2$

$x^2=(y-1)(y-2)$

Put $x=\alpha, y=\frac{-13}{11}$ $\left\{\text{as } z=\alpha-\frac{13i}{11}\right\}$

$$\alpha^2=\left(\frac{-13}{11}-1\right)\left(\frac{-13}{11}-2\right)$$

$$\alpha^2=\frac{(24\times 35)}{121}$$

$242\alpha^2=48\times 35=1680$

15. **(2)**

Given:

$\left(1-\sqrt{3}i\right)^{200}=2^{199}\left(p+iq\right)$

$\Rightarrow \left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{200}=\frac{1}{2}(p+iq)$

$\Rightarrow \left(\cos\left(\frac{\pi}{3}\right)-i\sin\left(\frac{\pi}{3}\right)\right)^{200}=\frac{1}{2}(p+iq)$

$\Rightarrow \left(\cos\left(\frac{200\pi}{3}\right)-i\sin\left(\frac{200\pi}{3}\right)\right)=\frac{1}{2}(p+iq)$

$\Rightarrow \left(\cos\left(201\pi-\frac{\pi}{3}\right)-i\sin\left(201\pi-\frac{\pi}{3}\right)\right)=\frac{1}{2}(p+iq)$

$\Rightarrow \left(-\cos\left(\frac{\pi}{3}\right)-i\sin\left(\frac{\pi}{3}\right)\right)=\frac{1}{2}(p+iq)$

$\Rightarrow 2\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)=p+iq$

$\Rightarrow -1-i\sqrt{3}=p+iq$

So,

$p=-1,\ q=-\sqrt{3}$

$\alpha=p+q+q^2=2-\sqrt{3}$

$\beta=p-q+q^2=2+\sqrt{3}$

So,

$\alpha+\beta=4$

$\alpha\cdot\beta=1$

Required equation is

$x^2-4x+1=0$

16. **(3)**

Given,

$\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$

Now let $z=\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}$,

So, $\bar{z}=\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}=\frac{1}{z}$

So, $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$

$=\left(\frac{1+z}{1+\bar{z}}\right)^3$

$=\left(\frac{1+z}{1+\frac{1}{z}}\right)^3$

$=z^3\left(\frac{1+z}{1+z}\right)^3$

$=z^3$

$=\left(\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}\right)^3$

$=i^3\left(\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}\right)^3$

$=-i\left(\cos\left(3\times\frac{2\pi}{9}\right)-i\sin\left(3\times\frac{2\pi}{9}\right)\right)$

$=-i\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)$

$=-i\left(\frac{-1}{2}-i\frac{\sqrt{3}}{2}\right)$

$=-\frac{1}{2}\left(\sqrt{3}-i\right)$

17. **(2)** As

$|z\omega|=1$

If $|z|=r$, then $|\omega|=\frac{1}{r}$

Let $\arg(z)=\theta$

$\therefore\ \arg(\omega)=\left(\theta-\frac{3\pi}{2}\right)$

So,

$z=re^{i\theta}$

$\Rightarrow \bar{z}=re^{i(-\theta)}$

$\Rightarrow \omega=\frac{1}{r}e^{i\left(\theta-\frac{3\pi}{2}\right)}$

Now, consider

$\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}=\frac{1-2e^{i\left(-\frac{3\pi}{2}\right)}}{1+3e^{i\left(-\frac{3\pi}{2}\right)}}=\left(\frac{1-2i}{1+3i}\right)$

$=\frac{(1-2i)(1-3i)}{(1+3i)(1-3i)}=\frac{1-5i+6i^2}{10}=\frac{-5+5i}{10}=-\frac{1}{2}(1+i)$

Then,

principal $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

$=$ principal $\arg\left(-\frac{1}{2}(1+i)\right)$

$=-\left(\pi-\frac{\pi}{4}\right)=\frac{-3\pi}{4}$

18. (13)

Given:

$z=\frac{1-i\sqrt{3}}{2}$

$\Rightarrow z=-\left(\frac{-1+i\sqrt{3}}{2}\right)$

$\Rightarrow z=-\omega$

where, ω is the cube root of unity. So,

$1+\omega+\omega^2=0$

$\omega^3=1$

Now, let

$A=\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+\ldots+\left(z^{21}+\frac{1}{z^{21}}\right)^3$

$\Rightarrow A=\left(-\omega-\frac{1}{\omega}\right)^3+\left(\omega^2+\frac{1}{\omega^2}\right)^3+\left(-\omega^3-\frac{1}{\omega^3}\right)^3+\ldots.+\left(-\omega^{21}-\frac{1}{\omega^{21}}\right)^3$

$\Rightarrow A=-\left(\frac{\omega^2+1}{\omega}\right)^3+\left(\frac{\omega^4+1}{\omega^2}\right)^3+\left(-1-\frac{1}{1}\right)^3+\ldots.+\left(-\omega^{21}-\frac{1}{\omega^{21}}\right)^3$

$\Rightarrow A=-\left(\frac{-\omega}{\omega}\right)^3+\left(\frac{-\omega^2}{\omega^2}\right)^3+(-1-1)^3+\ldots.+\left(-\omega^{21}-\frac{1}{\omega^{21}}\right)^3$

$\Rightarrow A=1+(-1)+(-1-1)^3+1+(-1)+(-1-1)^3\ldots.+(-1-1)^3$

$\Rightarrow A=3[1+(-1)]+(-1-1)^3$

$\Rightarrow A=-8$

Then,

$21+A=21-8$

$=13$

19. (2)

Given, $z^2+z+1=0\Rightarrow z=w,w^2$

Now, $\left|\sum_{n=1}^{15}\left(z^{\mathrm{n}}+(-1)\frac{1}{z^n}\right)^2\right|=\left|\sum_{n=1}^{15}\left(z^{2n}+\frac{1}{z^{2n}}+2(-1)^n\right)\right|$

$=\left|\sum_{n=1}^{15}w^{2n}+\frac{1}{w^{2n}}+2(-1)^n\right|$

$=\left|\frac{w^2\left(1-w^{30}\right)}{1-w^2}+\frac{\frac{1}{w^2}\left(1-\frac{1}{w^{30}}\right)}{1-\frac{1}{w^2}}+2(-1)\right|$

$=\left|\frac{w^2(1-1)}{1-w^2}+\frac{\frac{1}{w^2}(1-1)}{1-\frac{1}{w^2}}-2\right|$

$=|0+0-2|=2$

20. (0)

$P(x)=f\left(x^3\right)+xg\left(x^3\right)$

$P(1)=f(1)+g(1)\quad\ldots(1)$

Now $P(x)$ is divisible by x^2+x+1

$\Rightarrow P(x)=Q(x)\left(x^2+x+1\right)$

$P(\omega)=0=P\left(\omega^2\right)$ where ω,ω^2 are non-real cube roots of units

$P(x)=f\left(x^3\right)+xg\left(x^3\right)$

$P(\omega)=f\left(\omega^3\right)+\omega g\left(\omega^3\right)=0$

$f(1)+\omega g(1)=0\quad\ldots(2)$

$P\left(\omega^2\right)=f\left(\omega^6\right)+\omega^2 g\left(\omega^6\right)=0$

$f(1)+\omega^2 g(1)=0\quad\ldots(3)$

Now, $(2)+(3)$

$\Rightarrow 2f(1)+\left(\omega+\omega^2\right)g(1)=0$

$2f(1)=g(1)\quad\ldots(4)$

Now, $(2)-(3)$

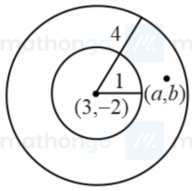
$\Rightarrow\left(\omega-\omega^2\right)g(1)=0$

$g(1)=0=f(1)\quad$ from (4)

from (1), $P(1)=f(1)+g(1)=0$

21. (40)

We know that $|z-(3-2i)|=r$ represents a circle with centre $(3,-2)$ and radius r.



$1<|z-3+2i|<4$
$1<(a-3)^2+(b+2)^2<16$

The ordered pairs of (a,b) satisfying the above inequality are $(0,\pm2)$,$(\pm2,0)$,$(\pm1,\pm2)$,$(\pm2,\pm1)$

$(\pm2,\pm3)$,$(3\pm,\pm2)$,$(\pm1,\pm1)$,$(2\pm,\pm2)$

$(\pm3,0)$,$(0,\pm3)$,$(\pm3\pm1)$,$(\pm1,\pm3)$

i.e. total 40 elements are present in the given set

22. $\omega=z\bar{z}+k_1z+k_2iz+\lambda(1+i)$

$\operatorname{Re}(w)=x^2+y^2+k_1x-k_2y+\lambda=0$

$$\text{Centre } \equiv\left(\frac{-k_1}{2},\frac{k_2}{2}\right)\equiv(1,2)$$

$\Rightarrow k_1=-2, k_2=4$

radius $=1\Rightarrow\lambda=4$

(24) $\operatorname{Im}=k_1y+k_2x+\lambda=0$

$\therefore 2x-y+2=0$

$$d=\frac{2}{\sqrt{5}}$$

$$\frac{1^2}{4}=1-\frac{4}{5}=\frac{1}{5}$$

$\therefore 301^2=24$

23. (4)

Given:

$\left|\frac{z-2}{z-3}\right|=2$

Let $z=x+iy$, then we have

$\left|\frac{x-2+iy}{x-3+iy}\right|=2$

$\Rightarrow\sqrt{(x-2)^2+y^2}=2\sqrt{(x-3)^2+y^2}$

$\Rightarrow x^2+y^2-4x+4=4x^2+4y^2-24x+36$

$\Rightarrow 3x^2+3y^2-20x+32=0$

$\Rightarrow x^2+y^2-\frac{20}{3}x+\frac{32}{3}=0$

So, $(\alpha,\beta)\equiv\left(\frac{10}{3},0\right)$

And,

$\gamma=\sqrt{\frac{100}{9}-\frac{32}{3}}=\sqrt{\frac{4}{9}}=\frac{2}{3}$

Therefore,

$3(\alpha+\beta+\gamma)=3\left(\frac{10}{3}+\frac{2}{3}\right)=12$

24. (4)

Here $S_n:|z-(3-2i)|=\frac{n}{4}$ represents a circle with center $C_1(3,-2)$ and radius $\frac{n}{4}$

and $T_n:|z-(2-3i)|=\frac{1}{n}$ represents a circle with center $C_2(2,-3)$ and radius $\frac{1}{n}$

For $S_n\cap T_n=\phi$, both circles do not intersect each other.

When $C_1C_2>\frac{n}{4}+\frac{1}{n}$

i.e. $\sqrt{2}>\frac{n}{4}+\frac{1}{n}$

then possible values of $n=1,2,3,4$

Hence, there are total four values possible.

25. (2)

$\bar{z}=iz^2 \Rightarrow (x-iy)=i(x+iy)^2$

$\Rightarrow\ x-iy=(x^2-y^2)i-2xy$

i.e., $x=-2xy$ and $-y=x^2-y^2$

$\Rightarrow\ x=0, y=-\frac{1}{2}$

When $x=0;\ y=0,1$

When $y=-\frac{1}{2};\ x=\pm\frac{\sqrt{3}}{2}$

$(0,0)$ will be rejected as vertices would be non-real roots.

So, the vertices will be $(0,1)$, $\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$ and $\left(-\frac{\sqrt{3}}{2},-\frac{-1}{2}\right)$

Hence, area of $\Delta=\frac{1}{2}\times\sqrt{3}\times\frac{3}{2}=\frac{3\sqrt{3}}{4}$

26. (2)

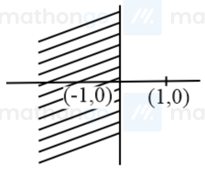
Set A is $\left\{z\in C:\left|\frac{z+1}{z-1}\right|<1\right\}$

$\Rightarrow\left|\frac{\bar{z}+1}{\bar{z}-1}\right|<1$

$\Rightarrow|\bar{z}+1|<|\bar{z}-1|$

$\Rightarrow\ (x+1)^2+y^2<(x-1)^2+y^2$

$\Rightarrow\ x<0$



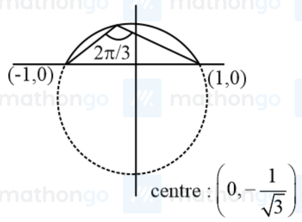
Set B is $\left\{z\in C:\arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\right\}$

$\Rightarrow\arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}$

$\Rightarrow\tan^{-1}\left(\frac{y}{x-1}\right)-\tan^{-1}\left(\frac{y}{x+1}\right)=\frac{2\pi}{3}$

$\Rightarrow x^2+y^2+\frac{2y}{\sqrt{3}}-1=0$

$\Rightarrow$ Centre $\left(0,-\frac{1}{\sqrt{3}}\right)$



Hence, $A\cap B$ will represent a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

27. (6)

$z^2+az+12=0$

$z_1+z_2=-a$ and $z_1z_2=12$

If $0,\ z_1,\ z_2$ are vertices of equilateral triangles

$z_2=z_1e^{\frac{i\pi}{3}}$

$z_2=z_1\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$

$2z_2-z_1=\sqrt{3}iz_1$

squaring both sides

$\Rightarrow 4z_2^2+z_1^2-4z_1z_2=-3z_1^2$

$\Rightarrow (z_1+z_2)^2=3z_1z_2$

$\Rightarrow \mathrm{a}^2=3\times12$

$\Rightarrow|a|=6$

28. (3)

Here $|z_1 - 3| = \frac{1}{2}$ represents a circle on argand plane with centre $(3, 0)$ and radius $\frac{1}{2}$

Given $|z_2 + |z_2 - 1||^2 = |z_2 - |z_2 + 1||^2$

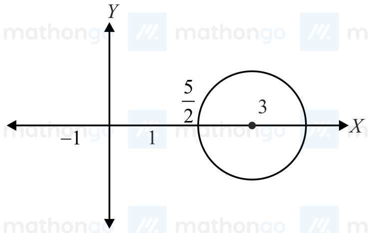
$\Rightarrow (z_2 + |z_2 - 1|)(\bar{z}_2 + |z_2 - 1|) = (z_2 - |z_2 + 1|)(\bar{z}_2 - |z_2 + 1|)$

$\Rightarrow z_2(|z_2 - 1| + |z_2 + 1|) + \bar{z}_2(|z_2 - 1| + |z_2 + 1|) = |z_2 + 1|^2 - |z_2 - 1|^2$

$\Rightarrow (z_2 + \bar{z}_2)(|z_2 + 1| + |z_2 - 1|) = 2(z_2 + \bar{z}_2)$

$\Rightarrow$ Either $z_2 + \bar{z}_2 = 0$ or $|z_2 + 1| + |z_2 - 1| = 2$

i.e. z_2 lies on imaginary axis or it lies on the line segment joining $(-1, 0)$ and $(1, 0)$



So, the minimum distance between z_1 & z_2 will be the distance between the points $(1, 0)$ & $\left(\frac{5}{2}, 0\right)$

Hence, $|z_1 - z_2|_{\text{min}} = \frac{3}{2}$

29. (26)

Given $|z - 2| \leq 1$

$(x - 2)^2 + y^2 \leq 1 \quad \cdots(i)$ represents interior region of a circle with centre $(2, 0)$ and radius 1

Now $z(1 + i) + \bar{z}(1 - i) \leq 2$

Puting $z = x + iy$, we get

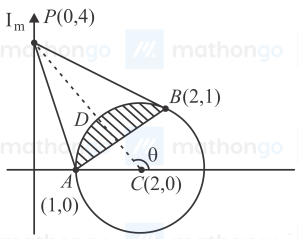
$(x + iy)(1 + i) + (x - iy)(1 - i) \leq 2$

$\Rightarrow x - y + i(x + y) + x - y - i(x + y) \leq 2$

$\therefore\ x - y \leq 1 \quad \ldots(i)$

Let $A(1, 0)$ and $B(2, 1)$ be the points on the line $x - y = 1$ and the circle $(x - 2)^2 + y^2 = 1$

$|z - 4i|$ represents the distance from $P(0, 4)$ to any point in the region S



Here $PA = \sqrt{17},\ PB = \sqrt{13}$

So $A(1, 0)$ is the point representing z_2

Let $D(2 + \cos\theta, 0 + \sin\theta)$ be the point representing z_1

$\therefore\ m_{CP} = \tan\theta = -2$

$\cos\theta = -\frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore\ D \equiv \left(2 - \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

$\Rightarrow z_1 = \left(2 - \frac{1}{\sqrt{5}}\right) + \frac{2i}{\sqrt{5}}$

So $|z_1| = \frac{25 - 4\sqrt{5}}{5}$ & $z_2 = 1$

$\therefore\ |z_2|^2 = 1$

$\therefore\ 5\left(|z_1|^2 + |z_2|^2\right) = 30 - 4\sqrt{5}$

$\therefore \alpha = 30$

$\beta = -4$

Hence $\alpha + \beta = 26$

30. (3)

We know that the complex number z satisfying $|z - z_0| = r$ represents a circle with centre z_0 and radius r units.

Hence, for $S_1 = |z - 1| \le \sqrt{2}$, z lies on and inside the circle of radius $\sqrt{2}$ units and centre $(1, 0)$.

For S_2, let $z = x + iy$

Now, $(1 - i)(z) = (1 - i)(x + i\,y)$

$\Rightarrow (1 - i)(z) = x + iy - ix - i^2y$

$\Rightarrow (1 - i)(z) = x + iy - ix + y$

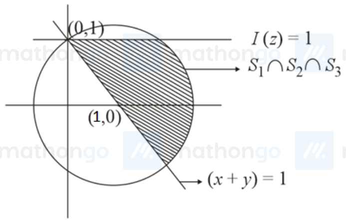
$\Rightarrow Re((1 - i)z) = x + y$

$\Rightarrow x + y \ge 1.$

And, for S_3, again let $z = x + iy$,

$\Rightarrow y \le 1.$

Plotting all the inequalities on the graph, we get



Now, the common part is shown in the shaded part, hence, we get infinite points in the shaded region.

$\Rightarrow S_1 \cap S_2 \cap S_3$ has infinitely many elements.