## ANSWER KEYS

**1.** (2) **2.** (3) **3.** (3) **4.** (-1) **5.** (2) **6.** (1) **7.** (4) **8.** (2)
**9.** (1) **10.** (2)

**1. (2)**

Since, $5x - 1 < x^2 + 2x + 1$

$\Rightarrow \quad x^2 - 3x + 2 > 0$

$\Rightarrow \quad (x-1)(x-2) > 0$

$\therefore \quad x < 1$ or $x > 2 \quad \ldots(1)$

Again, $x^2 + 2x + 1 < 7x - 3$

$\Rightarrow \quad x^2 - 5x + 4 < 0$

$\Rightarrow \quad (x-1)(x-4) < 0$

$\Rightarrow 1 < x < 4 \quad \ldots(2)$

From Eqs. (1) and (2), we get

$2 < x < 4$

Since, $x$ is integral, then the required value is $x = 3$.

**2. (3)**

We have,

$(x-2)^4(x-3)^3(x-4)^2(1-x) \leq 0$

$\Rightarrow (x-2)^4(x-3)^3(x-4)^2(x-1) \geq 0$

Using wavy-curve method, we get

$x \in (-\infty, 1] \cup [3, \infty)$

**3. (3)**

$\frac{x+1}{2x-1} \geq 1$ and $\frac{x+1}{2x-1} - 2 < 0$

$\Rightarrow x \in \left(\frac{1}{2}, 2\right]$ and $x \in \left(-\infty, \frac{1}{2}\right) \cup \left(1, \infty\right) \Rightarrow x \in \left(1, 2\right]$

**4. (-1)** $\frac{1}{x-2} - \frac{1}{x} \leq \frac{2}{x+2}$

$\frac{2}{x(x-2)} \leq \frac{2}{x+2}$

$\frac{(x+2)-x(x-2)}{x(x-2)(x+2)} \leq 0$

$\frac{-x^2+3x+2}{x(x-2)(x+2)} \leq 0 \Rightarrow \frac{x^2-3x-2}{x(x-2)(x+2)} \geq 0$

$\left(-2, \frac{3-\sqrt{17}}{2}\right] \cup \left(0, 2\right) \cup \left[\frac{3+\sqrt{17}}{2}, \infty\right)$

**5.** **(2)**
$\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \leq 0$
$\Rightarrow x = 0, \frac{4}{3}, 2, 3x - 4 < 0, x - 5 > 0$
or $3x - 4 > 0, x - 5 < 0$
$\left[\because x^2, (x-2)^4, (2x-7)^6 > 0\right]$
$\Rightarrow x = 0, \frac{4}{3}, 2, \ x < \frac{4}{3}, \ x > 5$ or $x\frac{4}{3}, \ x < 5$
$\Rightarrow \ x = 0, 2$ and integral value between
$\frac{4}{3} < x < 5$ i.e., $x = 2, 3, 4$.
Hence, positive integral solutions are $2, 3, 4$.
So, three positive integral solutions are possible.

**6.** **(1)**
The given expression is $7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$
$\Rightarrow \log\left(\frac{16}{15}\right)^7 + \log\left(\frac{25}{24}\right)^5 + \log\left(\frac{81}{80}\right)^3$
Since bases of every logarithmic terms in addition are equal. So, we can follow the following property of logarithm-
$\log_m(x)+\log_m(y)=\log_m(xy)$ So, by property of logarithm, we have $\Rightarrow \log\left[\left(\frac{16}{15}\right)^7 \times \left(\frac{25}{24}\right)^5 \times \left(\frac{81}{80}\right)^3\right] = \log 2$Hence, required value is $\log 2$.

**7.** **(4)**
Here, $5^{2\log_{10}x} = 5 + 4 \times 5^{\log_5 x^{\log_{10}5}}$ $\{\because a = b^{\log_b a}\}$
$= 5 + 4 \times 5^{\log_{10} 5^{\log_5 x}} = 5 + 4 \times 5^{\log_{10} x}$
**Note :** Our purpose was to write $x^{\log_{10}(5)}$ in terms of an exponential with the common base 5.
So, the equation becomes
$\left(5^{\log_{10}x}\right)^2 - 4\left(5^{\log_{10}x}\right) - 5 = 0$
$\Rightarrow \left(5^{\log_{10}x} - 5\right)\left(5^{\log_{10}x} + 1\right) = 0$
But $5^{\log_{10}x} + 1 \neq 0$ $\left(\because 5^{\log_{10}x} = +ve\right)$
$\therefore 5^{\log_{10}x} - 5 = 0$
$\therefore 5^{\log_{10}x} = 5$ ; or $\log_{10}x = 1$
Hence, $x = 10$

**8.** **(2)** $abc + 1 = \frac{\log 12}{\log 24} \times \frac{\log 34}{\log 36} \times \frac{\log 36}{\log 48} + 1$
$= \frac{\log(48 \times 12)}{\log 48}$
$= \frac{2\log 24}{\log 48} = 2bc$
Hence, $(B)$ is correct.

**9.** $\log_x 2 \log_{2x} 2 = \log_{4x} 2$
$\Rightarrow \quad (\log_x)(\log_2 2x) = \log_2 4x$
Let $\log_\tau x = t$
**(1)** $\Rightarrow t(1 + t) = 2 + t$
$\Rightarrow t^2 = 2$
$\Rightarrow t = \pm\sqrt{2}$
$\Rightarrow \quad x = 2^{\pm}\sqrt{2}$
Hence, (A) is correct.

**10. (2)** $\log_{10}(7x-9)^2+\log_{10}(3x-4)^2=2$

$\Rightarrow \quad (7x-9)^2(3x-4)^2=(10)^2$

$\Rightarrow \quad (7x-9)(3x-4)=\pm 10$

$\Rightarrow \quad \left(21x^2-55x+36\right)=\pm 10$

Either $21x^2-55x+46=0$ or $21x^2-55x+26=0$

$21x^2-55x+46=0$ has no real root.

and $21x^2-55x+26=0$ has 2 real roots

Hence, (B) is correct.