Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_(NUCLEUS,STERLING)&LIIT _BT **JEE-MAIN** **Date: 18-01-2023**

Time: 09.00Am to 12.00Pm **GTM-08** **Max. Marks: 300**

KEY SHEET

PHYSICS

1)	**4**	2)	**4**	3)	**4**	4)	**1**	5)	**4**
6)	**2**	7)	**3**	8)	**1**	9)	**1**	10)	**4**
11)	**2**	12)	**2**	13)	**4**	14)	**4**	15)	**3**
16)	**4**	17)	**2**	18)	**1**	19)	**1**	20)	**2**
21)	**150**	22)	**100**	23)	**40**	24)	**48**	25)	**3**
26)	**4**	27)	**25**	28)	**2**	29)	**30**	30)	**500**

CHEMISTRY

31)	**3**	32)	**1**	33)	**2**	34)	**1**	35)	**1**
36)	**1**	37)	**4**	38)	**3**	39)	**1**	40)	**3**
41)	**3**	42)	**4**	43)	**4**	44)	**2**	45)	**1**
46)	**1**	47)	**3**	48)	**3**	49)	**4**	50)	**3**
51)	**16**	52)	**32**	53)	**6**	54)	**536**	55)	**6**
56)	**55**	57)	**9**	58)	**6**	59)	**7**	60)	**7**

MATHEMATICS

61)	**4**	62)	**2**	63)	**1**	64)	**1**	65)	**3**
66)	**2**	67)	**2**	68)	**2**	69)	**3**	70)	**2**
71)	**4**	72)	**3**	73)	**3**	74)	**2**	75)	**2**
76)	**4**	77)	**3**	78)	**3**	79)	**4**	80)	**4**
81)	**17**	82)	**9**	83)	**6**	84)	**21**	85)	**2**
86)	**8**	87)	**7**	88)	**191**	89)	**6**	90)	**8**

SOLUTIONS

PHYSICS

1. Independent of R.

The radiation pressure acting on sphere $= \dfrac{Intensity\ of\ radiation\ at\ sphere}{speed\ of\ light}$

$= \dfrac{P_o}{4\pi x^2 c}$ where P_0 is the power radiated by sun

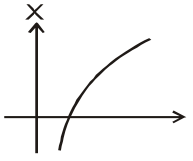
Force on sphere due to radiation pressure $= \dfrac{P_o}{4\pi x^2 c}\pi R^2$

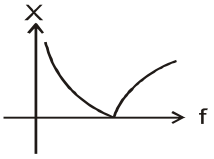
Gravitational pull on sphere $= \dfrac{GMm}{x^2}$

The gravitational force and the force due to light pressure both decrease with the square of the distance from the Sun. If a ball of radius R floats, it must be in neutral equilibrium and will float at any height above the sun. This is quite independent of the radius of the sphere.

2. The plot shows a linear relationship between x and log y, which means that y is an exponential in x. Since y decreases as x increases, the answer must be (4).

3. Gravity exerts a force mg downward on the block, which means that the wedge must exert a force mg upward on the block. Thus, the block exerts a force mg downward on the wedge, and gravity also exerts a force mg downward on the wedge. Since these forces have no horizontal components, no friction with the ground is necessary to keep the wedge static.

4. $X = \left(\omega L - \dfrac{1}{\omega C}\right)$. Hence the graph of X vs. f is 

. Therefore the plot of magnitude of X, that is |X| vs. f is 

5. The gravitational force vanishes at the midway point between the planets, so the rocket only needs to have enough energy to get there. The initial and final gravitational potential energies are

$$U_i = -\frac{GMm}{R} - \frac{GMm}{3R} = -\frac{4GMm}{3R} \text{ and } U_f = -\frac{2GMm}{2R} = -\frac{GMm}{R}$$

Thus, the initial kinetic energy needed is

$$\frac{1}{2}mv^2 = U_f - U_i = \frac{GMm}{3R}$$

which implies $v = \sqrt{\dfrac{2GM}{3R}}$

6. Momnetum of system is zero.

7. $\frac{\tan\delta}{\tan\delta'} = \cos\alpha$

$\tan\delta' = \frac{\tan\delta}{\cos\alpha}$

since $\cos\alpha < 1$

$\therefore \tan\delta' > \tan\delta$

$\delta' > \delta$

8. Sky wave signals are used for long distance communication. The sky wave signals are less stable than ground wave signals because of the variation of state of ionosphere.

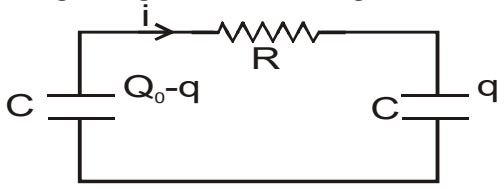
9. The potential difference across the inductor is e = E– iR.
Hence the plot of e versus i is a straight line with negative slope.

10. Since both charged particles move along same straight line, the magnetic field due to one particle at location of other is zero. Hence there is no magnetic interaction amongst the charged particles.

11. At any time t, the charge on right capacitor be q. Applying Kirchoff's law

$\frac{Q_0 - q}{C} = iR + \frac{q}{C}$ $\quad\therefore\quad$ $\frac{Q_0 - 2q}{2CR} = \frac{dq}{dt}$

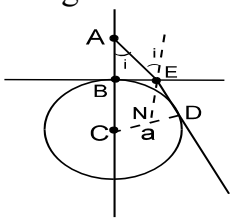
integrating and evaluating the constant we get



Hence $q = \frac{Q_0}{2}(1 - e^{-\frac{2t}{2RC}})$ or $i = \frac{dq}{dt} = \frac{Q_0}{RC}e^{-\frac{2t}{2RC}}$

12. From symmetry of induced charges on sphere, the net electrostatic force F_e on induced charges on sphere due to point charge is along line joining centre of sphere and point charge as shown.

13.



Given $\angle NED = 30°$

$\therefore \angle BED = 120°$

BCDE is cyclic quadrilateral

$\therefore \angle BCD = 60°$

The line CE will be angle bisector of $\angle BCD$

$\therefore$ BE = a tan30° $= \frac{a}{\sqrt{3}}$ $\quad\therefore\quad$ now $\tan i = \frac{BE}{AB} = \frac{a\sqrt{3}}{a/2} = \frac{2}{\sqrt{3}}$

$\therefore \sin i = \frac{2}{\sqrt{7}}$ now by snell's law $1 \times \sin i = n \sin r \Rightarrow \frac{2}{\sqrt{7}} = n \times \frac{1}{2}$ $\Rightarrow n = \frac{4}{\sqrt{7}}$

14. Net heat absorbed by one mole of diatomic gas in going from A $\rightarrow$ B (isochoric process) and B $\rightarrow$ C (isobaric process) is

$$\Delta Q = C_V \Delta T + C_P \Delta T = \frac{5}{2}RT_0 + \frac{7}{2}RT_0 \quad \Delta Q = 6RT_0$$

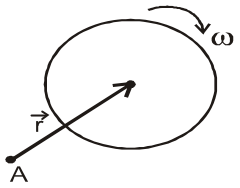
15. $P_0 + \rho_1 gh - \rho_2 gh + \frac{2T}{r} = P_0 \qquad = T = \frac{r}{2}(\rho_2 - \rho_1)gh$

16. From thermodynamics second law, the relation between COP of a Carnot refrigerator and heat pump is given as

$$(\text{COP})_{\text{heat pump}} = (\text{COP})_{\text{REF}} + 1$$

17. The angular momentum of disc about point A is

$$\vec{L}_A = I_{cm}\vec{\omega} + m\vec{r} \times \vec{v}_{cm}$$



$\vec{v}_{cm}$ = velocity of centre of mass of disc = 0. $\qquad \therefore \quad L = I_{cm}\omega = \frac{1}{2}MR^2\omega$

18. From archimedes principle statement-2 is correct explanation of statement-1.

19. $\lambda_m T = const.(Fu;r) \qquad \ln \lambda_m = \ln T = C$

$$\frac{d\lambda_m}{\lambda_m} + \frac{dT}{T} = 0 \qquad \therefore \quad \frac{d\lambda_m}{\lambda_m} = -\frac{dT}{T}$$

\ =

Now $\frac{d\lambda_m}{\lambda_m} = -1\% = -\frac{1}{100}$ (–ve sign indicates decrease)

dT = 1 (given) $\qquad \therefore \quad$ T = 100 K.

20. The magnitude of phase difference between the points separated by distance 10 metres $= k \times 10 = [10\pi \times 0.] \times 10 = \pi$

21. $\frac{1}{2}mV_m^2 = 15 \times 10^{-3} \quad V_m = \sqrt{0.150}\ m/s$

$$A_\omega = \sqrt{0.150}\ m/s \quad Lq_m = \sqrt{\frac{g}{L}} = \sqrt{0.150}\ m/s$$

$$\sqrt{gL} = \frac{\sqrt{0.150}}{100 \times 10^{-3}} \Rightarrow \frac{0.150}{0.1} = 1.5m$$

22. We have $K_\alpha = \frac{m_y}{m_y + m_\alpha}.Q \Rightarrow K_\alpha = \frac{A-4}{A}.Q \Rightarrow 48 = \frac{A-4}{A}.50 \Rightarrow A = 100$

23. Strain $(\varepsilon) = \frac{\Delta\ell}{\ell} = \alpha\Delta T = (10^{-5})(200) = 2 \times 10^{-3}$

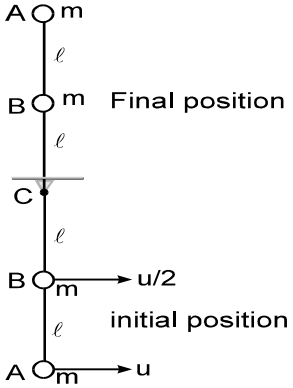
Stress = Y (strain)

Stress = $10^{11} \times 2 \times 10^{-3} = 2 \times 10^8$ N/m^2

$\Rightarrow$ Required force = stress × Area = $(2 \times 10^8)(2 \times 10^{-6}) = 4 \times 10^2 = 400$ N

$\therefore$ Mass to be attached $= \dfrac{400}{g} = 40$ kg

24. Let the initial velocity given to the mass at A be u.



Then the velocity of mass at B is u/2

As the system moves from initial the final position

Increase in potential energy is = 4 mg$l$ + 2mg$l$

Decrease in kinetic energy $= \dfrac{1}{2}mu^2 + \dfrac{1}{2}m\left(\dfrac{u}{2}\right)^2 = \dfrac{5}{8}mu^2$

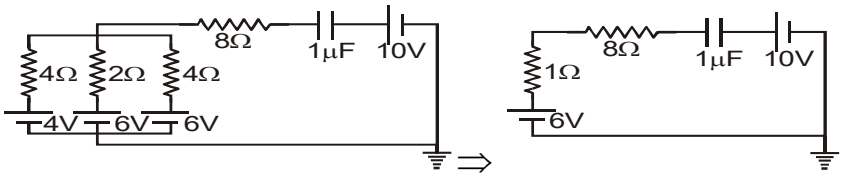
From conservation of energy

$\dfrac{5}{8}mu^2 = 6\ mgl$ or $u = \sqrt{\dfrac{48}{5}g\ell}$

25. $n\lambda = t(\mu - 1)$

$30 \times 600 \times 10^{-9} = 36 \times 10^{-6} \times (\mu\text{-}1) = \dfrac{18 \times 10^{-6}}{36 \times 10^{-6}} = .5$

$\Rightarrow \mu - 1 = .5 \Rightarrow \mu = 1.5$



26. (Moderate)

$E = \dfrac{\frac{8}{4} + \frac{6}{2} + \frac{4}{4}}{\frac{1}{4} + \frac{1}{2} + \frac{1}{4}} = 6V;\ Q = 4V \times 1\mu F = 4\mu C$

27. $e = (\vec{v} \times \vec{B}).\ell$

$e = [\hat{i} \times (3\hat{i} + 4\hat{j} + 5\hat{k})].5\hat{j}$

$\Rightarrow e = 25 volt$

28. $I_{av} = \dfrac{c\varepsilon_0 E_0^2}{2} = \dfrac{3 \times 10^8 \times 8.85 \times 10^{-12} \times 36^2}{2}$

$= 1.72 \text{W/m}^2$

29. $V_{in} = V + V_Z$

30. $n = \dfrac{\text{band width}}{\text{signal band width}}$

CHEMISTRY

31. Conceptual
32. Conceptual
33. Conceptual
34. Negative pole of water molecules will attract the positive charged rod and positive pole of water can attract negative charge of rod. Hexane is non polar.
35. Water sample is reported to be highly polluted if BOD value of sample is more than 17 ppm
36. Due to steric hindrance $BrCl_3$ cannot be formed.
37. Cyclopentadienyl anion carry a negative charge. Since the complex contains three negative groups(2 cyclopentadienyl anion ligands and one BF_4^- counter ion) the oxidation state of iron is +3
38. I) Incorrect. As the reaction is exothermic, increase in temperature decreases K_c.
    II) Correct. Addition of an inert gas at constant pressure drives the reaction in the direction of increasing number of moles.
    III) Correct. Increase in pressure moves the reaction in the direction of decreasing number of moles.
    IV) Incorrect. Value of K_c does not provide any information on rate of the reaction and hence regarding catalyst.
39. Refer NCERT
40. Conceptual
41. The colloidal particle formed is $AgI|I^-$, thus Pb^{2+} will be the most effective in coagulating it.
42. Refer NCERT
43. Diamnine and dicarboxylic acid must give a polyamide. The connectivity must be -NH-ring-NH-CO-ring-CO- or the reverse at meta positions of both the rings. Only 4 satisfies this criterion
44. Proline does not have primary amino group while histidine has. With $NaNO_2$/HCl, histidine gives effervescence of nitrogen.
    Serine has an alcohol group and hence gives red colouration with ceric ammonium nitrate.
    Tyrosine has phenolic group and hence would give characteristic colouration with $FeCl_3$.
    As both Lysine and glutamic acids both have acidic groups, they both would evolve CO_2 with $NaHCO_3$ and hence can't be differentiated.
45. E_2 elimination. Zaitsev product
46. Equanil is a tranquilizer
47. Wolf-Kishner reduction selectively reduces carbonyl group.
48. The carbon atom of the carbonyl group of benzaldehyde is less electrophilic than carbon atom of the carbonyl group present in propanal. The polarity of the carbonyl group is reduced in benzaldehyde due to resonance as shown below and hence it is less reactive than propanal.
49. Refer NCERT
50. The gausche form of N_2H_4 is stable due decrease in setric repulsion of lone pairs and nitrogen atoms
51. Conceptual

52. $\cos\theta = \frac{s}{s-1}$

$-0.47 = \frac{s}{s-0.47}$

53. Except I_2Cl_6 the remaining are angular with oxygen bridge.

54. As the gas behaves non-ideally, Final density $= \frac{\text{Initial Mass}}{\text{Final Volume}} = \frac{1.43\times1.5}{0.004} = 536\,g/L$

55.

At the first nearest position there are 8 atoms while at the second nearest position there are 6 atoms.

56. $MnO_2 + C_2O_4^{2-} + 4H^+ \longrightarrow 2Mn^{2+} + 2CO_2 + 2H_2O$

$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

$2\times n_{C_2O_4^{2-}}(\text{unused}) = 5\times n_{MnO_4^-} \Rightarrow 2\times x = 5\times\frac{0.1\times30.0}{1000} \Rightarrow x = 7.5\times10^{-3}\,\text{mol}$

$n_{C_2O_4^{2-}}\text{used}) = \frac{1.89}{126} - 7.5\times10^{-3} = 7.5\times10^{-3} = n_{Mn}(\text{in pyrolusite})$

$\Rightarrow \%\ \text{of Mn} = \frac{7.5\times10^{-3}\times55}{0.75}\times100 = 55\%$

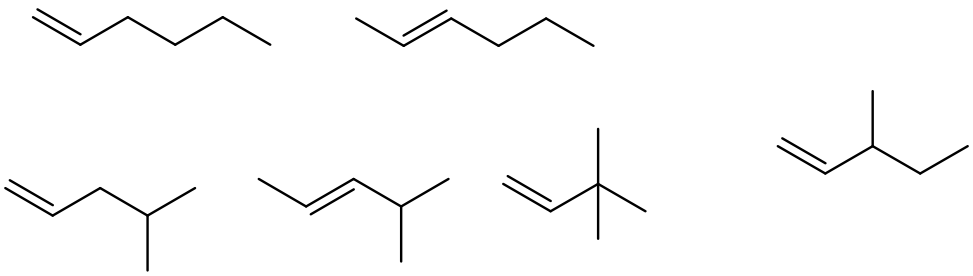
57.

$CH_4 + 10\ OH^- \longrightarrow CO_3^{2-} + 7H_2O + 8e^-$

$\text{No. of Faradays required} = \frac{1}{8}\times\frac{80\times3600\times1}{96500}$

$V_{CH_4} = \frac{1}{8}\times\frac{80\times3600\times1}{96500}\times24\,L = 9L$

58.



59. A dodecapeptide is made up of 12 amino acid units. On hydrolysis they take up 12 moles of water.

$\frac{89\times x}{980+11\times18}\times100 = 52.9 \Rightarrow x = 7$

60. Primary amines give carbylamine test.

MATHEMATICS

61. $g(n)=\int_0^{n^2+n+1} e^{\{x/2\}}\left\{\frac{x}{2}\right\}d\{x\}$ $=(n^2+n+1)\int_0^1 e^{\{x/2\}}\left\{\frac{x}{2}\right\}dx$

$=(n^2+n+1)\int_0^1 e^{x/2}\left(\frac{x}{2}\right)dx$ $=n^2+n+1\left[4-2e^{1/2}\right]$

So, minimum value is $12-6\sqrt{e}$

62. $f'(t)=\frac{e^{t^2}t^5}{\left(t^4+2t^2+2\right)^2}\Rightarrow f'(1)=\frac{e}{25}$

Let $x^2=v\Rightarrow\frac{dv}{dx}=2x\Rightarrow\frac{1}{2}\int\frac{e^v v^2}{\left(v^2+2v+2\right)^2}dv$

$\Rightarrow\frac{1}{2}\int e^v\left(\frac{1}{v^2+2v+2}-\frac{2v+2}{\left(v^2+2v+2\right)^2}\right)=\frac{1}{2}\frac{e^v}{v^2+2v+2}$ $=\frac{1}{2}\frac{e^{x^2}}{x^4+2x^2+2}$

$\therefore\ f(t)=\frac{1}{2}\frac{e^{x^2}}{x^4+2x^2+2}\Big|_0^t=\left(\frac{e^{t^2}}{t^4+2t^2+2}-\frac{1}{2}\right)\frac{1}{2}$ $\therefore\ f(1)=\frac{e}{10}-\frac{1}{4}$

$\therefore\ f(1)+f'(1)=\frac{e}{10}+\frac{e}{25}-\frac{1}{4}=\frac{7e}{50}-\frac{1}{4}$

63. Let $x^2=t\Rightarrow\frac{dt}{dx}=2x\Rightarrow I=\frac{1}{2}\int_0^{2\pi}\frac{t\sin^{2022}t\,dt}{\cos^{2022}t+\sin^{2022}t}$

$\Rightarrow I=\frac{1}{2}\int_0^{2\pi}\frac{(2\pi-t)\sin^{2022}t}{\cos^{2022}t+\sin^{2022}t}\Rightarrow 2I=\int_0^{2\pi}\frac{\pi\sin^{2022}t}{\cos^{2022}t+\sin^{2022}t}$

$\Rightarrow 2I=4\int_0^{2\pi}\frac{\pi\sin^{2022}t}{\cos^{2022}t+\sin^{2022}t}$ $\Rightarrow 2I=4\int_0^{2\pi}\frac{\pi\cos^{2022}t}{\cos^{2022}t+\sin^{2022}t}$

$\Rightarrow 4I=4\int_0^{2\pi}\pi\,dt\Rightarrow I=\frac{\pi^2}{2}.$

64. $\frac{dy}{dx}=\frac{(x-2)+(y-2)}{(x-2)-(y-2)}$

Put $x-2=h, y-2=k$

$\frac{dk}{dh}=\frac{h+k}{h-k}$ put $k=vh$ $\Rightarrow v+h\frac{dv}{dh}=\frac{1+v}{1-v}$

$\Rightarrow\int\frac{1-v}{1+v^2}dv=\int\frac{dh}{h}\Rightarrow\tan^{-1}v=\frac{1}{2}\ln\left(1+v^2\right)+\ln h+c$

$\Rightarrow\tan^{-1}\left(\frac{y-2}{x-2}\right)=\frac{1}{2}\ln\left(1+\frac{(y-2)^2}{(x-2)^2}\right)+\ln(x-2)+c$

$\because$ it passes through (3, 2)

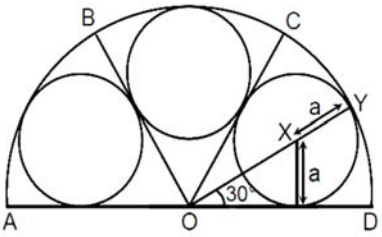
$\Rightarrow\tan^{-1}0=\frac{1}{2}\ln 1+\ln 1+c\Rightarrow c=0$

$\because$ it also passes through (p + 2, 3)

$\tan^{-1}\left(\frac{1}{p}\right)=\frac{1}{2}\ln\left(1+\frac{1}{p^2}\right)+\ln p$ $\Rightarrow 2\tan^{-1}\left(\frac{1}{p}\right)=\ln\left(1+p^2\right)$

65. From the diagram, $\angle AOB = \angle BOC = \angle COD = 60°$

$\Rightarrow \angle YOD = \frac{\pi}{6}$



$\Rightarrow$ Let X be the centre of right hand circle, $OX \sin 30° = a$ $\Rightarrow OX = 2a$

Now $r = OY = 2a + a \quad a = \frac{r}{3}$

66. $a^2 = \frac{1}{4}, b^2 = \frac{1}{9}$ *let* P *be* (x_1, y_1)

Equation normal $\frac{1x}{4x_1} - \frac{1y}{9y_1} = \frac{5}{36}$

It cuts y-axis at $\left(0, -\frac{5y_1}{4}\right) \Rightarrow \frac{-5y_1}{4} = \frac{-5}{8\sqrt{3}} \Rightarrow y_1 = \frac{1}{2\sqrt{3}}$ $\therefore x_1 = \frac{1}{4}$

Thus $\alpha = \frac{5}{36}$

$Area\ of\ \Delta OAB = \frac{1}{2} \times \frac{5}{36} \times \frac{5}{8\sqrt{3}} = \frac{25}{576\sqrt{3}}$

67. All the coefficient of odd powered terms is an odd number of odd degree terms multiplied together and all coefficient of even powered terms has an even number of odd degree terms multiplied together. Since every odd degree term is negative and every even degree is positive so we sum is just $Q(-1) = P(-1)^5 = \frac{243}{32}$.

68. $A_3 = 4$

$G_2 = \sqrt{5}$

$H_1 = \frac{5}{4}$

So equation is $(x - A_3)(x - G_2)(x - H_1) = 0$

69. Conceptual

70. $Sum = {}^{51}C_1 + {}^{52}C_2 + {}^{53}C_3 + ... + {}^{99}C_{49}$

$= \text{coefficient of } x \text{ in} \left[(1+x)^{51} + x^{-1}(1+x)^{52} + x^{-2}(1+x)^{53} + ... + x^{-48}(1+x)^{99}\right]$

$= \text{coefficient of } x \text{ in} \dfrac{(1+x)^{51}\left[\left(\frac{1+x}{x}\right)^{49} - 1\right]}{\left(\frac{1+x}{x} - 1\right)}$

$= \text{coefficient of } x \text{ in} \left[x^{-48}(1+x)^{100} - x(1+x)^{51}\right] \quad = {}^{100}C_{49} - 1 = {}^{100}C_{51} - 1$

71. $f(-x) \neq f(x)$ for the condition given in option (1)

$f(-x) \neq -f(x)$ for the condition given in option (2)

$f(x+T) \neq f(x)$ for the condition given in option (3)

Hence (4) is the correct option.

72. Combined mean,

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2} = \frac{200\times 25 + 300\times 10}{500} = 16$$

Let $d_1 = \bar{x}_1 - \bar{x} = 25-16 = 9$ and $d_2 = \bar{x}_2 - \bar{x} = 10-16 = -6$

$$\sigma^2 = \frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1+n_2} = \frac{200(9+81)+300(16+38)}{500} = \frac{33600}{500} = 67.2$$

73. $$\lim_{m\to\infty}(a_m)^{\frac{1}{m}} = \left(\left(\left(1+\frac{1}{m}\right)^{m+1} - \frac{m+1}{m}\right)^{-m}\right)^{\frac{1}{m}} = \frac{1}{e-1}$$

74. In $\Delta ECD, \tan 3\alpha = \frac{h}{CD}$

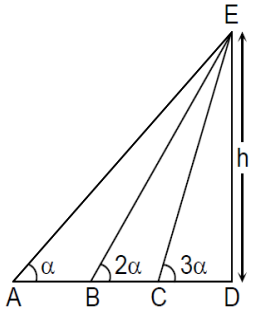
$\Rightarrow CD = h\cot 3\alpha$ _____ (1)

In $\Delta EBD, \tan 2\alpha = \frac{h}{BD}$

$\Rightarrow BD = h\cot 2\alpha$ _____ (2)

In $\Delta EAD, \tan\alpha = \frac{h}{AD}$

$\Rightarrow AD = h\cot\alpha$ _____ (3)



From equation (2) and (3), we have $AD - BD = h\cot\alpha - h\cot 2\alpha$

$\Rightarrow AB = h(\cot\alpha - \cot 2\alpha)$ _____ (4)

From equation (1) and (4), we have $BD - CD = h\cot 2\alpha - h\cot 3\alpha$

$\Rightarrow BC = h(\cot 2\alpha - \cot 3\alpha)$ _____ (5)

From equation (4) and (5), we have $\frac{AB}{BC} = \frac{h(\cot\alpha - \cot 2\alpha)}{h(\cot 2\alpha - \cot 3\alpha)} = \frac{\frac{\cos\alpha}{\sin\alpha} - \frac{\cos 2\alpha}{\sin 2\alpha}}{\frac{\cos 2\alpha}{\sin 2\alpha} - \frac{\cos 3\alpha}{\sin 3\alpha}} = \frac{\frac{\sin(2\alpha-\alpha)}{\sin\alpha\sin 2\alpha}}{\frac{\sin(3\alpha-2\alpha)}{\sin 2\alpha\sin 3\alpha}}$

$= \frac{\sin 3\alpha}{\sin\alpha} = 3 - 4\sin^2\alpha = 3 - 2(1-\cos 2\alpha) = 1 + 2\cos 2\alpha$

75. Define A as the number of the elements in S, we have

$$A = \binom{2006}{1}9^{2005} + \binom{2006}{3}9^{2003} + \ldots\ldots + \binom{2006}{2005}9$$

On the other hand, $(9+1)^{2006} = \sum_{k=0}^{2006}\binom{2006}{k}9^{2006-k}$ and $(9-1)^{2006} = \sum_{k=0}^{2006}\binom{2006}{k}(-1)^k 9^{2006-k}$

So, $A=\binom{2006}{1}9^{2005}+\binom{2006}{3}9^{2003}+.....+\binom{2006}{2005}9=\frac{1}{2}10^{2006}-8^{2006}$

76. Since each has equally 9 different possible results for A and B to draw a ball from the packet independently, the total number of possible events is $9^2=81$. From $a-2b+10>0$ we get $2b<a+10$. We find that when $b=1,2,3,4,5$ a can take any value in $1,2,3,.....,9$ to make the inequality hold. Then we have $9\times5=45$ admissible events
When b = 6, a can 3, 4, ….., 9 and there are 7 admissible events
When b = 7, a can 5, 6, 7, 8, 9 and there are 5 admissible events
When b = 8, a can 7, 8, 9 and there are 3 admissible events
When b = 9, a can 9 and there are 1 admissible events
So, the required probability is $\frac{45+7+5+3+1}{81}=\frac{61}{81}$

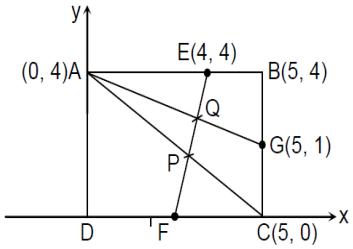
77. $(a,a)\notin R$
If $(a,b)\in R\Rightarrow(b,a)\in R$
If $(a,b)\in R,(b,c)\in R\Rightarrow(a,c)\notin R$

78. Differential equation can be written as, $(p-x)(p-2\sin x)(2p+\cos x)=0$ which has solution as $(2y-x^2-c)(y+2\cos x-c)(2y+\sin x-c)=0$

79. $P\left(\frac{\bar{A}}{\bar{B}}\right)=\frac{1-P(A\cup B)}{P(\bar{B})}=\frac{1-x}{x^2}\le1\Rightarrow\quad x^2+x-1\ge0$

$\Rightarrow\quad x\ge\frac{\sqrt{5}-1}{2}\ or\ x\le\frac{-1-\sqrt{5}}{2}\ \therefore\ x$ is positive $\qquad x\ge\frac{\sqrt{5}-1}{2}$

80. Equation of AG
$y-4=\frac{4-1}{0-5}(x-0);y-4=-\frac{3}{5}(x)$
$5y-20=-3x;y=-\frac{3}{5}x+4$



Equation AC $\qquad y=-\frac{4}{5}x+4$

Equation EF, $y=2x-7\Rightarrow\frac{PQ}{EF}=\frac{10}{91}$

81. Conceptual

82. $\because\ \lim_{x\to0}\left(3-2\cos x\sqrt{\cos2x}\right)^{\left(\frac{x+3}{x^2}\right)}$ *is of the form* $1^\infty\qquad\therefore$ it is equal to $e^{\lim_{x\to0}\left(\frac{2-2\cos x\sqrt{\cos2x}}{x^2}\right)\times(x+3)}$

Now, $\lim_{x\to0}\frac{2-2\cos x\sqrt{\cos2x}}{x^2}$

$$=\lim_{x\to 0}\frac{\frac{-2\cos x}{2\sqrt{\cos 2x}}\times-\sin 2x\times 2-\sqrt{\cos 2x}\left(-\sin x\right)}{2x}$$

$$=\lim_{x\to 0}\frac{\frac{2\cos x\sin 2x}{\sqrt{\cos 2x}}+2\sin x\sqrt{\cos 2x}}{2x}=2+1=3 \quad \therefore \quad e^{\lim_{x\to 0}\left(\frac{2-2\cos x\sqrt{\cos 2x}}{x^2}\right)(x+3)}=e^{3\times 3}=e^9 \quad \therefore\ a=9$$

83. $\therefore\ C_1: x^2+y^2=a^2-b^2 \Rightarrow C_2: x^2+y^2=\frac{a^2+b^2}{3}$

Director circle of C_1 is $x^2+y^2=2\left(a^2-b^2\right)\Rightarrow 2\left(a^2-b^2\right)=\frac{a^2+b^2}{3}$

$\Rightarrow 6a^2-6b^2=a^2+b^2\Rightarrow 5a^2=7b^2$

$5a^2=7a^2\left(1-e_1^2\right)\Rightarrow e_1^2=\frac{2}{7}$ *and* $5a^2=7a^2\left(e_2^2-1\right)\Rightarrow e_1^2=\frac{12}{7}$

84. Asymptotes $3(x-1)-2(y+3)=0$ and $3(x-1)+2(y+3)=0$

$3x-2y-9=0$ and $3x+2y+3=0$

It touches $x^2+y^2-2x+k/13=0$

$\therefore\ A\left(-\frac{5}{13},-\frac{12}{13}\right)$ *and* $B\left(\frac{31}{13},-\frac{12}{13}\right)$

$\therefore\ k=-23$

85. On each roll, the probability that Kiran decides to go to Russia is $\frac{3}{2}$ times the probability she decides to go to Ukraine, so the total probability that she goes to Russia is $\frac{3}{2}$ times the total probability that she goes to Ukraine. Since the total probability is 1(they are the only two eventual outcomes) Kiran goes to Ukraine with probability $\frac{2}{5}$ and Russia with probability $\frac{3}{5}$.

86. Let $Z=m+\sqrt{n}i$

$\bar{Z}=m-\sqrt{n}i$

i.e., x^3+ax+b has roots -20, $Z,\bar{Z}$ and x^3+cx^2+b has roots -21, $Z,\bar{Z}$

$\Rightarrow -20+Z+\bar{Z}=0$

$m=10$

$-21Z-21\bar{Z}+Z\bar{Z}=0$

$-21\left(Z+\bar{Z}\right)+Z\bar{Z}=0 \quad \Rightarrow n=320$

87. Equation reduces to $\left(\cos x+\cos 4x\right)+\left(\cos 2x+\cos 3x\right)=0$

$\Rightarrow\ 2\cos\frac{5x}{2}\left[2\cos x\cos\frac{x}{2}\right]=0\Rightarrow\cos\frac{5x}{2}=0, or\ \cos x=0\ or\ \cos\frac{x}{2}=0$

$\Rightarrow\ x=\frac{(2n+1)\pi}{5}\ or\ x=\frac{(2n+1)\pi}{5}=0\ or\ x=(2n+1)\pi, n\in z$

$\Rightarrow x=\left\{\frac{\pi}{5},\frac{3\pi}{5},\pi,\frac{7\pi}{5},\frac{9\pi}{5},\frac{\pi}{2},\frac{3\pi}{2}\right\}$. Hence, number of solution 7

88. $\sum_{k=0}^{10}\frac{{}^{10}C_k\,{}^{10+k}C_{10}}{(-2)^k}$

Coefficient of x^{10} in $\left[{}^{10}C_0(1+x)^{10}-{}^{10}C_1\frac{(1+x)^{11}}{2}+{}^{10}C_2\frac{(1+x)^{12}}{2^2}+.....+{}^{10}C_{10}\frac{(1+x)^{20}}{2^{10}}\right]$

Coefficient of x^{10} in $(1+x)^{10}\left[{}^{10}C_0-{}^{10}C_1\left(\frac{1+x}{2}\right)+{}^{10}C_2\left(\frac{1+x}{2}\right)^2-{}^{10}C_3\left(\frac{1+x}{2}\right)^3+.....-{}^{10}C_{10}\left(\frac{1+x}{2}\right)^{10}\right]$

Coefficient of x^{10} in $(1+x)^{10}\left(1-\left(\frac{1+x}{2}\right)\right)^{10}$

Coefficient of x^{10} in $\frac{(1-x^2)^{10}}{2^{10}}=\frac{{}^{10}C_5}{2^{10}}=\frac{252}{1024}=\frac{63}{128}$

89. $f\left(e^{-\frac{i\pi}{3}}\right)=(a+c)\omega-b+c;\omega=-\frac{1+i\sqrt{3}}{2}$

$\Rightarrow (a+c)\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)-b+c=4038\sqrt{3}i-4032$

$\Rightarrow a+c=8076;\left(\frac{c-a}{2}\right)-b=-4032$ $\Rightarrow a=c=4038$ and $b=4032$

90. $A=\{3,4,5,....\};B=\{....,-7,-6\}$