Sec:Sr.Super60_ STERLING_BT | Paper -2(Adv-2021-P2-Model) | Date: 17-09-2023
Time: 02.00Pm to 05.00Pm | CTA-06 | Max. Marks: 180

KEY SHEET

PHYSICS

1	**ABC**	2	**D**	3	**ABC**	4	**ABCD**	5	**AC**	6	**BD**
7	**80**	8	**60**	9	**25**	10	**5**	11	**32.18**	12	**0**
13	**A**	14	**C**	15	**A**	16	**B**	17	**2**	18	**9**
19	**1**										

CHEMISTRY

20	**BD**	21	**CD**	22	**AC**	23	**ACD**	24	**CD**	25	**ACD**
26	**12.5**	27	**13.88**	28	**2.46**	29	**7.14**	30	**45.37**	31	**13.95**
32	**C**	33	**D**	34	**C**	35	**C**	36	**5**	37	**4**
38	**3**										

MATHEMATICS

39	**ABD**	40	**ABC**	41	**AB**	42	**AD**	43	**ABC**	44	**AD**
45	**0**	46	**7**	47	**9**	48	**1**	49	**50**	50	**100**
51	**C**	52	**A**	53	**C**	54	**B**	55	**2**	56	**9**
57	**1**										

SOLUTIONS

PHYSICS

1. Velocity = length time, acceleration=length(time)^2

$$\Rightarrow Length = \frac{(Velocity)^2}{acceleration} \quad i.e,\ L' = \frac{v'^2}{a'}, L = \frac{v^2}{a}$$

$$\Rightarrow \frac{L'}{L} = \left(\frac{v'}{v}\right)^2 \left(\frac{a}{a'}\right) = \left(\frac{\alpha^2}{\beta}\right)\frac{1}{\alpha\beta} = \alpha^3/\beta^3$$

Now $m' = \frac{F'}{a'}, m = \frac{F}{a}$ $\qquad \frac{m'}{m} = \frac{F'}{F}\frac{a}{a'} = \frac{1}{\alpha\beta} \times \frac{1}{\alpha\beta} = \frac{1}{\alpha^2\beta^2}$

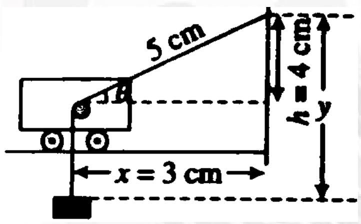
Time = velocity/acceleration, i.e.,

$T' = \frac{v'}{a'}$ and $T = \frac{v}{a}\ \frac{T'}{T} = \frac{v'}{v}\frac{a}{a'} = \frac{\alpha^2}{\beta}\frac{1}{\alpha\beta} = \frac{\alpha}{\beta^2}$

Momentum = mass $\times$ velocity

$$p' = m'v', P = mv, \frac{p'}{p} = \frac{m'}{m}\frac{v'}{v} = \frac{1}{\alpha^2\beta^2}\frac{\alpha^2}{\beta}\frac{\alpha^2}{\beta} = \frac{1}{\beta^3}$$

2.



$$(y-h) + \sqrt{x^2+h^2} = l \quad \text{or} \quad \frac{dy}{dt} + \frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{3}{5}(-v_A)$$

$$v_B = \frac{3}{5}v_A \quad \frac{d^2y}{dt^2} = \frac{v_A^2h^2}{(x^2+h^2)^{\frac{3}{2}}} \Rightarrow a_B = v_A^2\frac{16}{(5)^3} = \frac{16}{125}v_A^2$$

3. The work done by friction on the block is equal to change in KE

$$W_1 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{mv_0}{m+M}\right) - \frac{1}{2}mv_0^2 = -ve$$

So, choice (1) is correct. The work done by friction on the plank

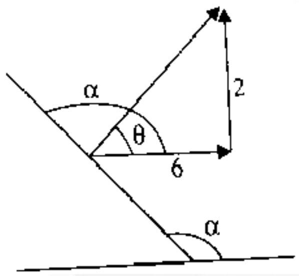
$W_2 = \frac{1}{2}mv^2 - 0 = +ve$, So choice (2) is correct. Net work done by friction

4. $\frac{\overrightarrow{dv}}{dt} =$ is total acceleration $= \vec{g}$

Modulus of $\frac{d|\vec{v}|}{dt}$ is the tangential acceleration $g\sin\alpha$

Normal acceleration, $a_{normal} = g\cos\alpha$

5. Impulse is change in momentum. Hence, impulse $= 1(\vec{v}_2 - \vec{v}_1) = (3\hat{i} + \hat{j})$



As impulse is in the normal direction of colliding surface

$\tan\theta = \frac{1}{3}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right)$ $\alpha = 90^0 + \tan^{-1}\left(\frac{1}{3}\right)$

6. For mono atomic gas, $C_p = \frac{5}{2}R$ for polyatomic gas $C_v = 3R$, [J;,K'M';

And $C_v = \frac{3}{2}R$

For diatomic gas, $C_p = \frac{7}{2}R$ And $C_v = \frac{5}{2}R$

7. $T = 8g = 80N$

8. $T\sin\theta = 80\sin 30^0 + w$

$80\sin\theta = 40 + 30$ $\sin\theta = \frac{7}{8} = 0.86$ $\theta \approx 60^0$

9. $h = -ut + \frac{1}{2}gt^2$ $h = -20(5) + \frac{1}{2}(10)25$ $= 25\ m$

10. $V_{avg} = \frac{(u+v)}{2} = 5\ ms^{-1}$

11. $\frac{d\theta}{dt} = -KA.\frac{dT}{dx}$ $= 2.05 \times 4 \times 10^{-4} \times 15.7\cos(15.7x) \times 25 \times 10^2$

At one end, $x = o$ $\therefore \frac{d\theta}{dt} = \frac{96.55}{3}w$ $= 32.18W$

12. At center of rod $x = \frac{l}{2} = 10cm$

$\therefore \frac{d\theta}{dt} = KA\frac{dt}{dx} = 2 \times 5 \times 4 \times 10^{-4}\ 25\cos(1.57) \times 15.7 = 0$

13. (1) Let us be the velocity of ball A is $2.0m$. $2.0(u\sin\theta)t - \frac{1}{2}gt^2$

For bullet C, vertical distance travelled $= u\sin 30^0 t + \frac{1}{2}gt^2 ......(i)$

$$d = u^{'}t - \frac{1}{2}gt^2 \qquad ......(ii)$$

From equations (i) and (ii),

$u\sin 30^0 t + u^{'}t = 1500$ $\qquad 50t + 100t = 1500$ Which gives $t = 10\sec$

14. $CO =$ greatest height for bullet A

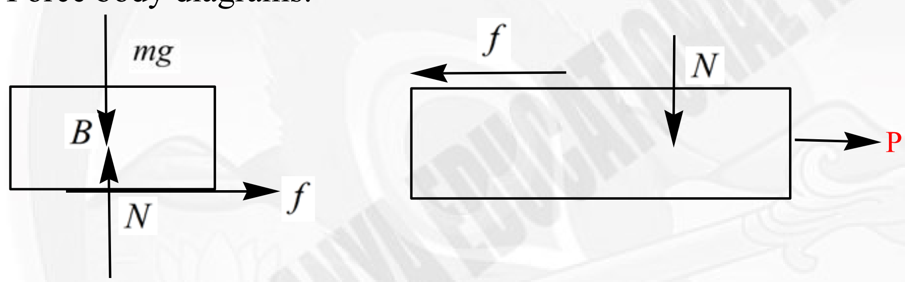
$$CO = u^{'}t - \frac{1}{2}gt^2 = 100(10) - \frac{1}{2}(10)^3 = 500$$

$AC = \frac{1}{2}$ (Range for bullet at A)

$$AC = u\cos 30^0 t = 500\sqrt{3}$$

$$\tan\phi = \frac{CO}{AC} = \frac{1}{\sqrt{3}} \Rightarrow \phi = 30^0$$

15. Force body diagrams:



Let both the blocks move together.

Acceleration of blocks, $a = \frac{P}{(m+M)}$

$$f = m\left(\frac{P}{m+M}\right)$$

If both the blocks moves together, $f \le \mu mg$

$$\frac{mP}{(m+M)} \le \mu mg$$

$$P \le \mu(m+M)g$$

16. If the cube begins to slide then, $P = \mu(m+M)g$

$a_m = \frac{f}{m} = \mu g$ (towards $+x$ direction)

$a_M = \frac{P - \mu mg}{M}$ (towards $+x$ direction)

$$\vec{a}_m, M = \vec{a}_m - \vec{a}_M = \mu g - \left(\frac{P - \mu mg}{M}\right) \qquad = \frac{\mu(m+M)g - P}{M}$$

If the cube falls from the plank, it will cover a distance l relative to plank

$$-l=\frac{1}{2}a_mM^{t^2}\Rightarrow t=\sqrt{\frac{2l}{a_{M,m}}}=\sqrt{\frac{2lM}{P-\mu(m+M)g}}$$

17.

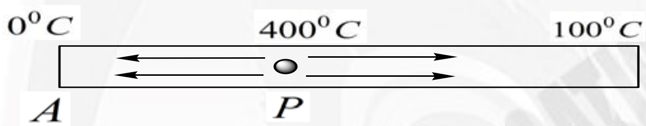
$$\mu(S_2P)-S_1P=m\lambda\Rightarrow\mu\sqrt{d^2+x^2}-\sqrt{d^2+x^2}=m\lambda$$

$$\Rightarrow(\mu-1)\sqrt{d^2+x^2}=m\lambda\Rightarrow\left(\frac{3}{2}-1\right)\sqrt{d^2+x^2}=m\lambda$$

Or Squaring this equation we get,

$$x^2=4m^2\lambda^2-d^2\quad\Rightarrow\quad P^2=4\text{ or }P=2$$

18.



Heat will flow both sides from point P. $\Rightarrow\left(\frac{dQ}{dt}\right)_{calorimetry}=\left(\frac{dQ}{dt}\right)_{conduction}$

$$L_1\frac{dm_1}{dt}=\left(\frac{Temperature\ \ difference}{Thermal\ \ resistance}\right)_1=\frac{400}{(\lambda x)/KA}$$

Similarly, $L_1\frac{dm_2}{dt}=\frac{400-100}{(100-\lambda)x/KA}$

In above tow equations, $\frac{dm_1}{dt}=\frac{dm_2}{dt}(given)$ $L_1=80\,cal g^{-1}$ and $L_2=540\,cal g^{-1}$

Solving these two equations, we get $\lambda=9$.

19. Equivalent capacity in case of series connection of

Capacitors, $\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}\Rightarrow C_{eq}=\frac{C_1C_2}{C_1+C_2}\ C_{eq}=\frac{C_1C_2}{C_1+C_2}=\frac{2000\times3000}{2000+3000}=1200\mu F$

Also, $\frac{\Delta C_{eq}}{C_{eq}^2}=\frac{\Delta C_1}{C_1^2}+\frac{\Delta C_2}{C_2^2}\Rightarrow\frac{\Delta C_{eq}}{C_{eq}}=\left(\frac{\Delta C_1}{C_1^2}+\frac{\Delta C_2}{C_2^2}\right).C_{eq}$

$$\Rightarrow\frac{\Delta C_{eq}}{C_{eq}}=\left\{\frac{10}{(2000)^2}+\frac{15}{(3000)^2}\right\}\times1200=5\times10^{-3}$$

Energy stored in this combination of capacitors

$$U=\frac{1}{2}C_{eq}V^2\Rightarrow\frac{\Delta U}{U}=\frac{\Delta C_{eq}}{C_{eq}}+\frac{2\Delta V}{V}\qquad\Rightarrow\frac{\Delta U}{U}=5\times10^{-3}+\frac{2(0.02)}{5}=13\times10^{-3}$$

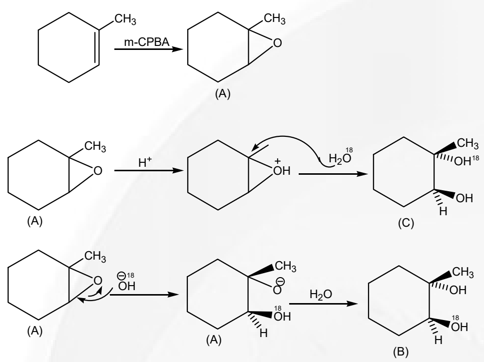
Hence % error = 1.3

CHEMISTRY

20. A and B are diastereomers
B and C are diastereomers

21. The correct order of basicity is
$(IV) > (III) > (II) > (I)$
A compound is more basic, if it's conjugate acid can be stabilized through resonance.

22.



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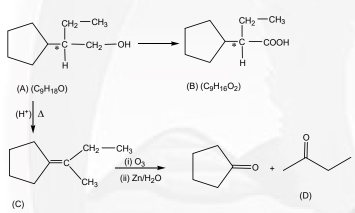
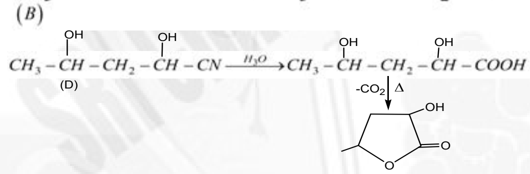
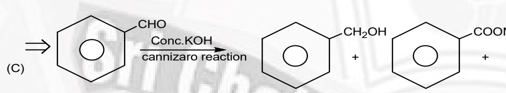


24. $(A) \Rightarrow Ph-CH=CH-CH_3 \xrightarrow{O_3} \underset{(C)}{Ph-CHO} + \underset{(B)}{CH_3-CHO}$

$$\underset{(B)}{CH_3-CHO} \xrightarrow{dil.KOH} CH_3-\overset{OH}{\overset{|}{CH}}-CH_2-\overset{O}{\overset{||}{C}}-H \xrightarrow{HCN} \underset{(D)}{CH_3-\overset{OH}{\overset{|}{CH}}-CH_2-\overset{OH}{\overset{|}{CH}}-CN}$$





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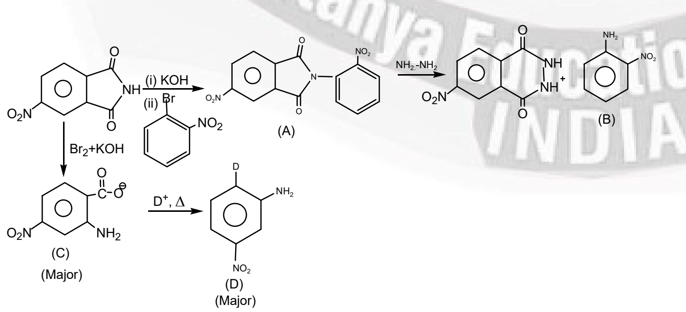


26. 20ml dilute unreacted acid solution required $= 35 ml\ of\ \frac{N}{10} NaOH$ solution

$\Rightarrow$ 500 ml of dilute unreacted acid solution required

$=\frac{35}{20}\times 500ml\,of\,\frac{N}{10}\,NaOH$ solution $\quad =\frac{35}{20}\times\frac{500}{10}ml\,of\,'N'NaOH$ solution

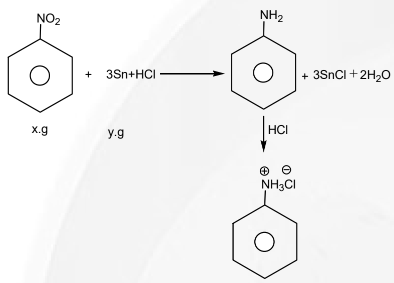
$=87.5ml\,of\,'N'NaOH\ =87.5ml\,of\,'N'NaOH=87.5ml\,of\,'N'H_2SO_4$

Acid used for the neutralization of $NH_3=100-87.5=12.5ml\,'N'H_2SO_4$

27. Percentage of nitrogen = $1.4\times N\times\frac{V}{W}$

$N=1, V=12.5 W=1.26$ $\therefore$ % of Nitrogen in the given compound$=1.4\times 1\times\frac{12.5}{1.26}=13.88$

28.



Organic salt (Anilinium ion)(2.58g)
Mass of organic salt produced = 2.58g

Molar mass of Anilinium ion = 129 g/mol $\therefore$ moles of organic = $\frac{2.58}{129}=0.02mol$

1 mole of organic salt produced by 1 mole of nitrobenzene

$\Rightarrow$ 0.02 moles of organic salt produced by

$\frac{0.02\times 1}{1}=0.02$ moles of nitrogenzene

Mass of nitrobenzene (x)=no.of moles $\times$molar mass

$=0.02\times 123=2.46g$ $\therefore$ The value of $(x)=2.46g$

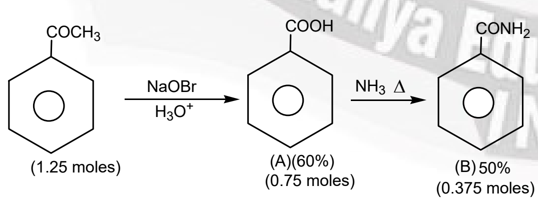
29. 1 mole of organic salt produced by '3' moles of 'Sn'

$\Rightarrow$ 0.02 moles of organic salt produced by

$\frac{0.02\times 3}{1}=0.06 mole\,of\,'Sn'$ $\therefore$ mass of $S_n(y)$=no.of moles of $'Sn'$X molar mass

$=0.06\times 119\,g/mol=7.14g$ $\qquad\therefore$ Value of $'y'$ = 7.14g

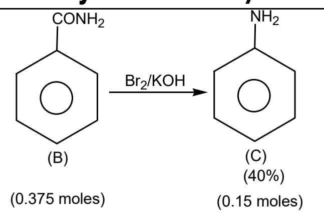
30. 150g of Acetophenone = 1.25 moles, (no.of moles = $\frac{150}{molar\ mass}=\frac{150}{120}=1.25 molar$)



i..e 1.25 moles of Acetophenone gives 0.375 moles of benzamide

$\therefore$ Amount of benzanide (x) = $0.375\times molar\ mass$ $\qquad =0.375\times 121=45.37g$

31.

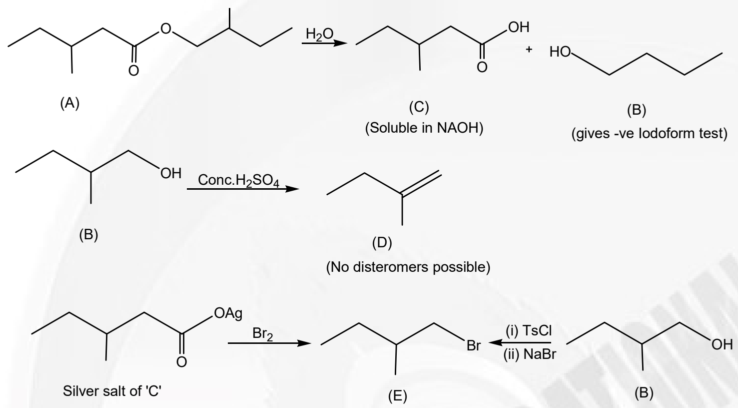


0.375 moles of benzamide (B) gives 0.15 moles of Aniline

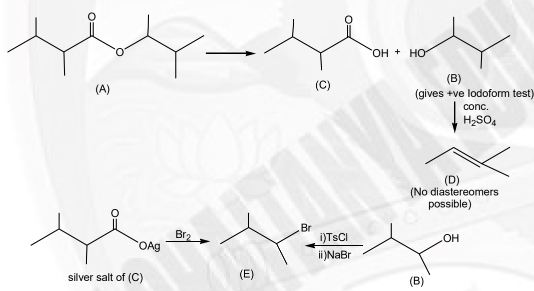
$\therefore$ Amount of Aniline (y) = $0.15 \times molar\ mass = 0.15 \times 93 = 13.95g$

Amount of Aniline (y)=13.95g

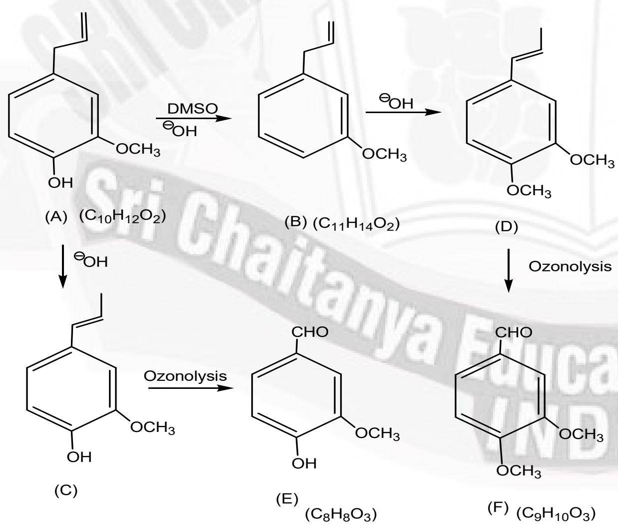
32.



33.



34,35



36. Statements (i), (ii), (iii), (v) and (vi) are correct

37. $PHBV$, $Nylon-2-nylon-6$, $Cellulose$, $Dextron$

38. (c) – N.G.P, (e) – Two consecutive SN^2 reactions gives retention

(g) - SN^i

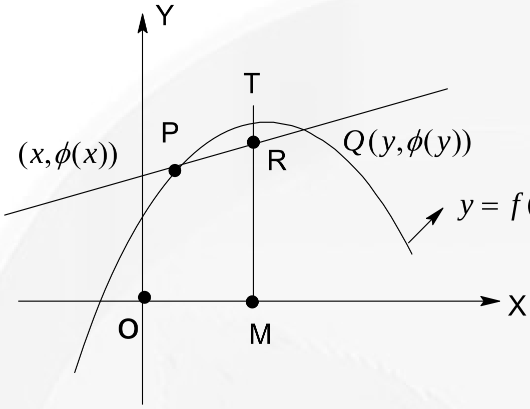
MATHEMATICS

39. Period of $|\sin 2x|+|\cos 2x|$ is $\pi/4$ but $f(x)=In([|\sin 2x|+|\cos 2x|])$

Max. value of $|\sin 2x|+|\cos 2x|=\sqrt{2}$

f (x) is many one and into function

40.



Take $P=(x,\Phi(x));Q=(y,\Phi(y))$ be any two points the curve $y=\Phi(x)$

Let 'R' divides the line segment $\overline{PQ}$ in the ratio 2:1 then R= $\left(\frac{x+2y}{3},\frac{\Phi(x)+2\Phi(y)}{3}\right)$

Clearly $TM > RM$ $\Rightarrow \Phi\left(\frac{x+2y}{3}\right)>\frac{\Phi(x)+2\Phi(y)}{3}$

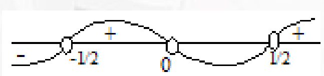
Equality holds iff $\Phi(x)$ is a linear function. $\therefore \Phi(x)=ax+b$

$\because \Phi'(0)=1 \Rightarrow a=1$ $\because \Phi(0)=2$ $b=2 \because \Phi(x)=x+2$

41. Clearly $g(x)$ is even hence $f(x)$ is odd as $f(0)=0$

$g'(0)=0; f^1(0)=g(0)$ (obvious)

42. $\because f(x)=2x^2-In|x|$ $\therefore$ $f'(x)=4x-\frac{1}{x}=\frac{(2x+1)(2x-1)}{x}$



For increasing, $f'(x)>0 \therefore$ $x\in\left(-\frac{1}{2},0\right)\cup\left(\frac{1}{2},\infty\right)$

And for decreasing, $f'(x)<0$ $\therefore$ $x\in\left(-\infty,-\frac{1}{2}\right)\cup\left(0,\frac{1}{2}\right)$

43. $0<x<\frac{\pi}{2}\Rightarrow\frac{\sin x}{x}$ is decreasing and $\sin x<x<\tan x$

$\Rightarrow\frac{\sin(\sin x)}{\sin x}>\frac{\sin x}{x}>\frac{\sin(\tan x)}{\tan x}\Rightarrow I_1>I_2>I_3$

44. If (x,y) is any point on the curve, the sub tangent at $(x,y)=y\frac{dx}{dy}$

$\therefore y\frac{dx}{dy} = nx(given)$ or $n\frac{dy}{y} = \frac{dx}{x}$

Integrating $n\log y = log\, x + \log c$ or $\log y^n = \log cx$

or $y^n = cx.......(i)$

which is the required equation of the family of curves.
Putting $x = 2,\ y = 3$ in

(i), we have $3^n = 2c$ or $c = \frac{3^n}{2}$ Putting this value of c in

(i) $y^n = \frac{3^n}{2}x$ or $2y^n = 3^n x$

(ii) which is the particular curve passing through the point (2,3)
Putting $n = 1$ in
(ii), we have $2y = 3x$ which is a straight line putting $n = 2$ in
(ii) we have $2y^2 = 9x$ which is a parabola.

45. $f(x) = 2^x + 2^{|x|}$ For $x \ge 0$, $f(x) = 2.2^x$ And for $x < 0; f(x) = 2^x + 2^{-x}$

Since in $\pi < 2$ so, the equation $2^x + 2^{|x|} = ln\,\pi$ has no solution

46. Now the required area $= \int_{-1}^{0}\left(2^x + 2^{-x}\right)dx + \int_0^1 2.2^x\,dx = \frac{7}{2In2} = \frac{7}{2}.\log_2 e = \log_2\left(e^{7/12}\right)$

47. $\frac{\left[1^2 x^x\right] + \left[2^2 x^x\right] + ...+ \left[n^2 x^x\right]}{n^3} = \frac{\Sigma n^2\left(x^x\right)}{n^3}$

$= \underset{x\to 0}{Lt}\ \underset{n\to\infty}{Lt}\ \frac{\Sigma n^2\left(x^x\right)}{n^3} = \frac{1}{3} = p$ then $p^{-2} = 9$

48. $\underset{x\to 0}{Lt}\left(\left[\frac{\sin x}{x}\right] + \left[\frac{\tan x}{x}\right]\right) = 1$ since $\sin x < x < \tan x$ for $0 < x < \frac{\pi}{2}$

49. $f_n(x) = \sum_{r=1}^{n} \frac{\sin^2 x}{\cos^2\left(\frac{x}{2}\right) - \cos^2\left(\frac{2r+1}{2}\right)x}$

$f_n(x) = \sin x \sum_{n=1}^{n} \frac{\sin\{(n+1)(x - nx)\}}{\sin(n+1)x.\sin(nx)}$

$g_n(x) = f_1(x), f_2(x), f_3(x)......f_n(x) = \frac{\sin(x)}{\sin(n+1)x}$

$I_{n+2} - I = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x}dx = 2\int_0^{\pi}\cos(n+1)x dx = 0,\qquad I_{n+2} = I_n$

And $\quad I_1 = I_3 = I_5 = ... = \pi \qquad \sum_{k=1}^{100} I_n = \left(I_1 + I_2 + I_3..... + I_{100}\right)$

50. $\lim\limits_{x\to 0}\int_0^x \frac{9dt}{xf_9(t)g_9(t)}=\lim\limits_{x\to 0}\frac{9}{x}\int_0^x \frac{\sin^2(10)t}{\sin(9t)\sin t}dt\left(\frac{0}{0}\right)$

$=\lim\limits_{x\to 0}\frac{9\sin^2 10x}{\sin 9x\sin x}=100$

51. At $x=1,-1$ it is not differentiable

52. At $x=4$ its not differentiable

53. $f''(x)>0\Rightarrow f'(x)$ is an increasing function

$f'(x_1)>f'(x_2)\Rightarrow x_1>x_2, f'(x_1)=f'(x_2)\Rightarrow x_1=x_2$

$h'(x)=\sin 2x\left(f'(\sin^2 x)-f'(\cos^2 x)\right)$

$f'(x)=0\Rightarrow \sin 2x=0\Rightarrow x=0$

or $f'(\sin^2 x)=f'(\cos^2 x)\Rightarrow \sin^2 x=\cos^2\Rightarrow \tan^2 x=1\Rightarrow x=\pm\frac{\pi}{4}$

54. $h(x)$ is increasing $\Rightarrow h'(x)>0$

Case (I)

$$\left.\begin{array}{l}(i)\sin 2x>0\Rightarrow x\in\left(0,\frac{\pi}{2}\right)\\(ii)\, f'(\sin^2 x)>f'(\cos^2 x)\Rightarrow \tan^2 x>1\end{array}\right\}\Rightarrow x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$$

Case (II)

$$\left.\begin{array}{l}\sin 2x<0\Rightarrow x\in\left(-\frac{\pi}{2},0\right)\\ f'(\sin^2 x)<f'(\cos^2 x)\Rightarrow \tan^2 x<1\end{array}\right\}\Rightarrow x\in\left(\frac{-\pi}{2},-\frac{\pi}{4}\right)$$

55. Putting $\tan x=t$, we get $f(t)$ is a polynomial function of the form $f(t)=\pm t^n+1$

When $t=2, f(2)=9\Rightarrow n=3$ $\quad\therefore f(t)=t^3+1, f'(2)=12\therefore\frac{f'(2)}{6}=2$

56. Clearly $f(1)=\frac{1}{2}, f(2)=1, f(3)=\frac{3}{2}, f(4)=2....$

$f(1), f(2), f(3), f(4).....$ are in AP with C.D $=\frac{1}{2}$ first term $=\frac{1}{2}$

$f(15)=\frac{15}{2}, f(3)=\frac{3}{2}, f(12)=\frac{12}{2}, f(10)=\frac{10}{2}$

Required Ans. $\dfrac{\frac{15+3}{2}}{\frac{12-10}{2}}=\frac{18}{2}=9$

57. $f(x)=\frac{1}{2}\sec x\Rightarrow f(0)=\frac{1}{2}, f'(0)=0, f''(0)=\frac{1}{2};\therefore f(0)+f'(0)+f''(0)=1$