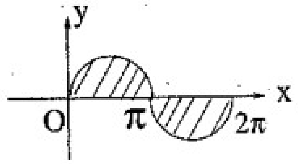
KEY SHEET

1.	C	**2.**	C	**3.**	B	**4.**	B	**5.**	C
6.	B	**7.**	C	**8.**	D	**9.**	C	**10.**	B
11.	C	**12.**	A	**13.**	A	**14.**	A	**15.**	C
16.	D	**17.**	A	**18.**	B	**19.**	C	**20.**	C
21.	D	**22.**	D	**23.**	A	**24.**	A	**25.**	C
26.	B	**27.**	D	**28.**	A	**29.**	A	**30.**	C
31.	B	**32.**	B	**33.**	C	**34.**	A	**35.**	B
36.	A	**37.**	B	**38.**	A	**39.**	A	**40.**	A
41.	C	**42.**	C	**43.**	B	**44.**	A	**45.**	C
46.	C	**47.**	D	**48.**	A	**49.**	A	**50.**	B
51.	A	**52.**	C	**53.**	A	**54.**	A	**55.**	D
56.	D	**57.**	D	**58.**	B	**59.**	D	**60.**	A
61.	D	**62.**	A	**63.**	B	**64.**	D	**65.**	C
66.	A	**67.**	A	**68.**	C	69.	A	70.	C
71.	A	**72.**	D	**73.**	A	74.	A	75.	B
76.	B	**77.**	C	**78.**	D	79.	B	80.	A
81.	B	**82.**	B	**83.**	D	84.	D	85.	D
86.	C	**87.**	B	**88.**	D	89.	C	90.	A
91.	C	**92.**	D	**93.**	C	94.	D	95.	B
96.	C	**97.**	D	**98.**	C	99.	B	100	B
101.	A	**102.**	C	**103**	C	104.	A	105	D
106.	C	**107.**	A	**108**	A	109.	A	110	A
111.	A	**112.**	A						

HINTS & SOLUTIONS

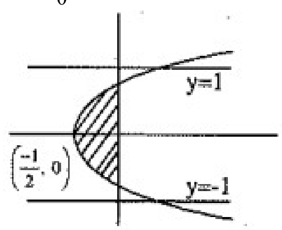
1. $y = x\sin x \; in[0,2\pi]$

$$\int_0^{\pi} x\sin x dx + \int_{\pi}^{2\pi} -x\sin x dx = 4\pi \; sq.units$$

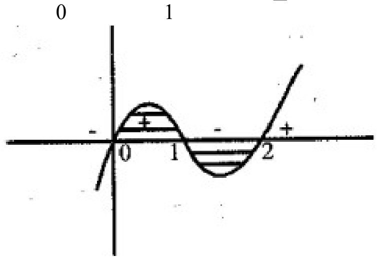


2. $$A = 2\int_0^{\frac{\pi}{2}} \cos^2 x dx = \int_0^{\frac{\pi}{2}} (1-\cos 2x)dx = x - \frac{\sin 2x}{2}\Bigg]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

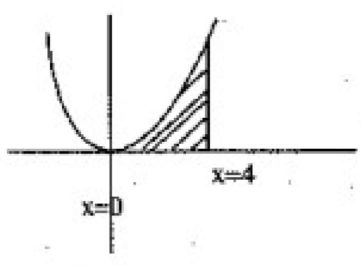
3. $$A = 2\int_0^1 \left(\frac{y^2-1}{2}\right)dy$$



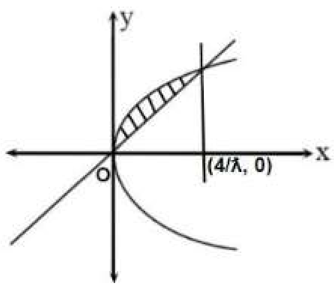
4. $$A = \int_0^1 ydx - \int_1^2 ydx = \frac{1}{2}$$



5. $$A = \int_0^4 \frac{x^2}{8} dx = \left[\frac{x^3}{24}\right]_0^4$$



6.



$$A = \int_0^{4/\lambda} \left(\sqrt{4\lambda x} - \lambda x\right) dx = \frac{1}{9}$$

$\lambda = 24$

7. $\therefore$ Coordinates of point P are (2, 3)

Given equation of parabola is $(y-2)^2 = (x-1)$

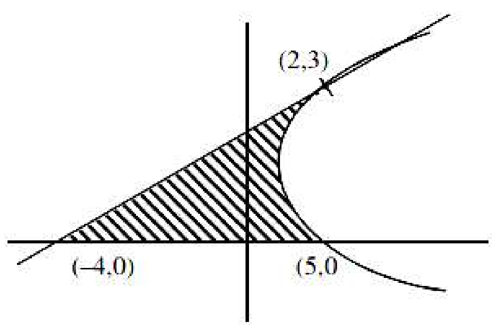
Differentiating above w.r.t. *x,* we get $2(y-2)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2(y-2)} \qquad \therefore \left[\frac{dy}{dx}\right]_{At(2,3)} = \frac{1}{2}$

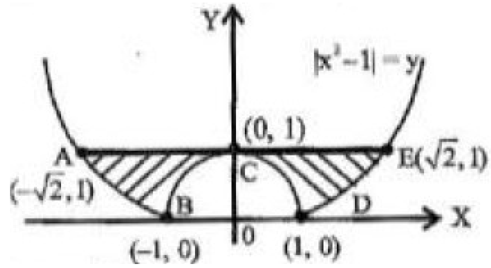
So, equation of tangent at P(2, 3) is $y - 3 = \frac{1}{2}(x-2) \Rightarrow x - 2y + 4 = 0$

$\therefore$ Required area $= \int_0^3 \left[(y-2)^2 + 1 - (2y-4)\right] dy = \int_0^3 \left(y^2 - 6y + 9\right) dy$

$= \left[\frac{y^3}{3} - 3y^2 + 9y\right]_0^3 = 9$ sq.units.



8.

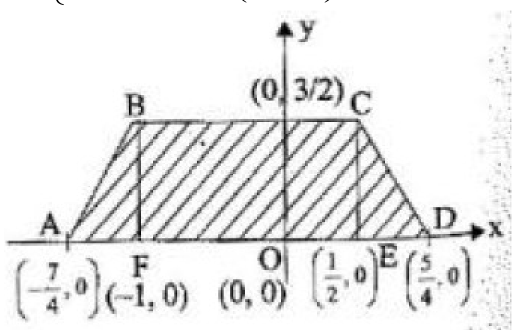


Area = ABCDEA

$$=2\left(\int_0^1\left(1-\left(1-x^2\right)\right)dx+\int_1^{\sqrt{2}}\left(1-\left(x^2-1\right)\right)dx\right)=\frac{8}{3}\left(\sqrt{2}-1\right)$$

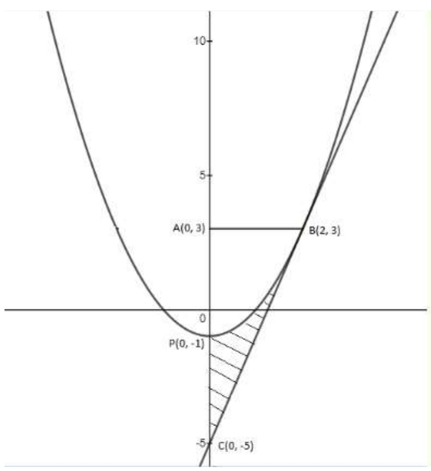
9. From the given curve

$$y=\begin{cases}3+(x+1)+\left(x-\frac{1}{2}\right), & x<-1\\ 3+(x+1)+\left(x-\frac{1}{2}\right), & -1\le x<\frac{1}{2}\\ 3-(x+1)-\left(x-\frac{1}{2}\right), & \frac{1}{2}\le x\end{cases}\Rightarrow y=\begin{cases}\frac{7}{2}+2x, & x<-1\\ \frac{3}{2}, & -1\le x<\frac{1}{2}\\ \frac{5}{2}-2x, & \frac{1}{2}\le x\end{cases}$$



$\therefore$ Area bounded = $\displaystyle\int_{\frac{7}{4}}^{-1}2x+\frac{7}{2}+\int_{1}^{1/2}\frac{3}{2}dx+\int_{1/2}^{5/4}\left(\frac{5}{2}-2x\right)dx=\frac{27}{8}sq.units$

10.



Sol :

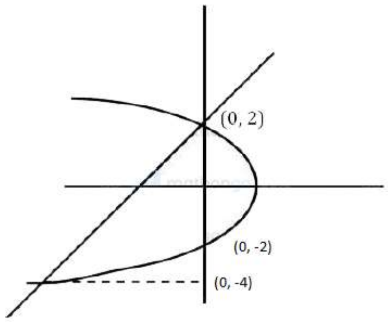
Equation of the tangent at (2, 3) is $y-3=4(x-2)$

Required area = area (ΔABC)-$area(OABP)$

$$=\frac{13}{2}\times 8\times 2-\int_{-1}^{3}\sqrt{y+1}\,dy$$

$$=\frac{8}{3}sq.units$$

11.



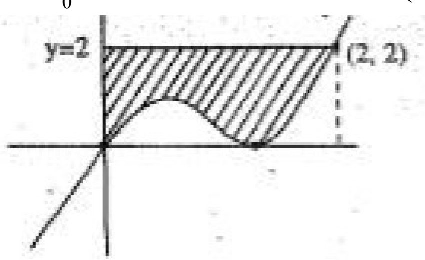
$$y^2 + 4x = 4$$

$$y^2 = -4(x-1)$$

$$A = \int_{-4}^{2} \left(\frac{4-y^2}{4} - \frac{y-2}{2} \right) dy = 9$$

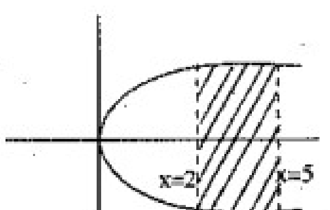
12.

$$A = \int_{0}^{2} \left[2 - (x^3 - 2x^2 + x) \right] dx = \left(2x - \frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} \right)_0^2 = \frac{10}{3}$$



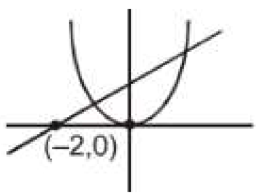
13.

$$A = 2\int_{2}^{5} 2\sqrt{x}\,dx = 4\left[\frac{x^{3/2}}{3/2} \right]_2^5$$



14. $f(x) = \dfrac{d}{dx}(xe^x) = x.e^x + e^x = e^x(x+1)$

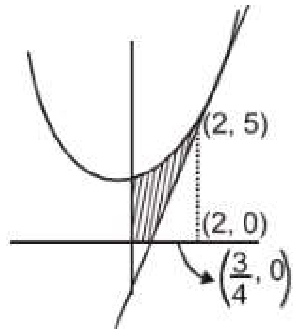
15.



P.O.I. of both curves we get $x = 2, -1$

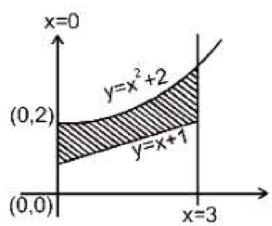
So $\int_{-1}^{2}\left(\frac{x+2}{4}-\frac{x^2}{4}\right)dx=\frac{9}{8}$

16.



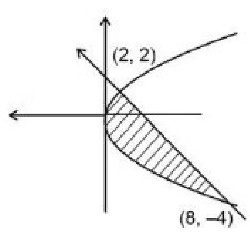
$$A=\int_{0}^{2}\left(x^2-1\right)dx-\frac{1}{2}.\frac{5}{4}.5=\frac{37}{24}$$

17.

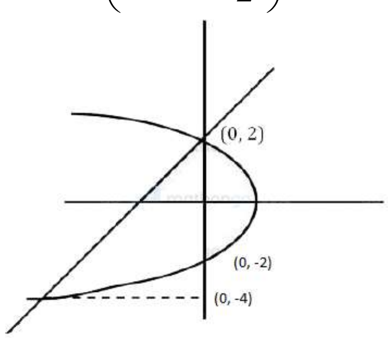


$$Area=\int_{0}^{3}\left(\left(x^2+2\right)-\left(x+1\right)\right)dx=\left(\frac{15}{2}\right)$$

18.



$$A=\int_{-4}^{2}\left(4-y-\frac{y^2}{2}\right)dy=18\,sq.units$$

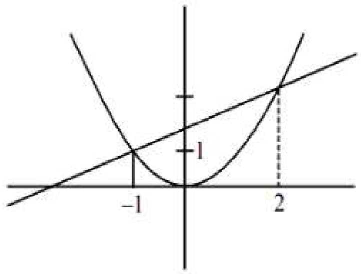


19.

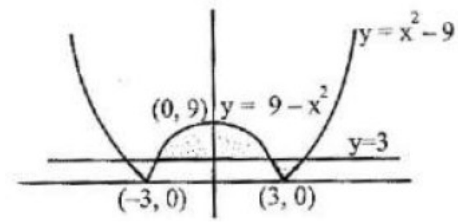
$$y^2 + 4x = 4$$

$$y^2 = -4(x-1)$$

$$A = \int_{-4}^{2} \left(\frac{4-y^2}{4} - \frac{y-2}{2} \right) dy = 9$$

20. 

$$Area = \int_{-1}^{2} \left(2 + x - x^2\right) = \frac{9}{2}$$

21. 

$\because\ y = \left|x^2 - 9\right| \Rightarrow y = x^2 - 9, x \geq 3$

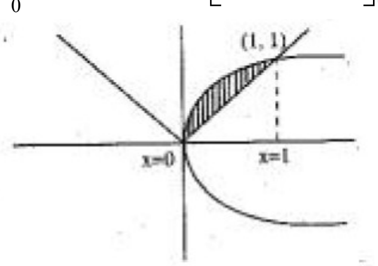
$$\text{Area of shaded region} = 2\int_0^3 \left(\sqrt{9+y} - \sqrt{9-y}\right) dy + 2\int_3^9 \left(\sqrt{9-y}\right) dy$$

$$= 2\left[\int_0^3 (9+y)^{1/2} dy - \int_0^3 (9-y)^{1/2} dy + \int_3^9 (9-y)^{1/2} dy\right]$$

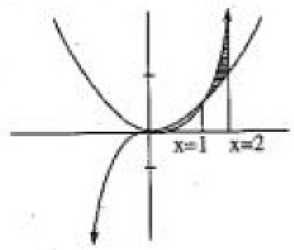
$$= 2\left[\frac{2}{3}\left[(9+y)^{3/2}\right]^3 + \frac{2}{3}\left[(9-y)^{3/2}\right]_0^3 - \frac{2}{3}\left[(9-y)^{3/2}\right]_3^9\right]$$

$$= \frac{4}{3}\left[12\sqrt{2} - 27 + 6\sqrt{6} - 27 - (0 - 6\sqrt{6})\right] = \frac{4}{3}\left[24\sqrt{3} + 12\sqrt{6} - 54\right] = 8\left(4\sqrt{3} + 2\sqrt{6} - 9\right)$$

22. $A\int_0^1 \left(\sqrt{x} - x\right) dx = \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$



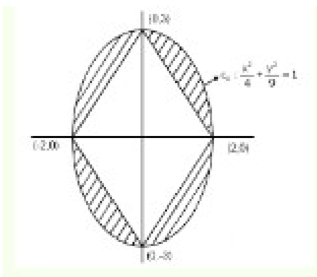
23. $A = \int_1^2 x^3 - x^2 dx = \left(\frac{x^4}{4} - \frac{x^3}{3}\right)_1^2$



24. Equation of tangent : $y+1=3(x+1)$ i.e.. $y=3x+2$

Point of intersection with curve (2, 8)

So Area = $\int_{-1}^{2}\left((3x+2)-x^3\right)dx = \frac{27}{4}$



25.

Given that $\frac{x^2}{4}+\frac{y^2}{9}=1, \frac{x^2}{4}+\frac{y^2}{9}=1 \Rightarrow a=2, b=3$

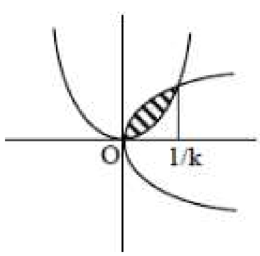
Now, area of ellipse = $\pi ab \ = 6\pi$

Step Required area = Area of ellipse – Area of quadrilateral

$= \pi \times 2 \times 3 - \frac{1}{2} \times 6 \times 4 = 6(\pi - 2)$

Area bounded by the given curves is $6(\pi - 2)$

26.



$y = kx^2$

$x = ky^2 \Rightarrow x = k(kx^2)^2 \Rightarrow x = k^3x^4 \Rightarrow x = 0$

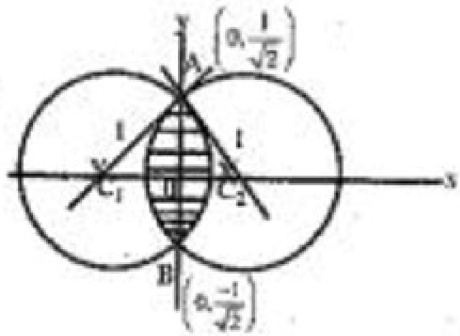
$x = \frac{1}{k}$

$$\text{Area} = \int_0^{1/k} \sqrt{\frac{x}{k}} - kx^2 dx$$

$$\frac{1}{\sqrt{k}} \frac{x^{3/2}}{3/2} - k\frac{x^3}{3}\Bigg|_0^{1/k} = 1 \Rightarrow \frac{2}{3k^2} - \frac{1}{3k^2} = 1$$

$$k^2 = \frac{1}{3} \Rightarrow k = \frac{1}{\sqrt{3}}$$

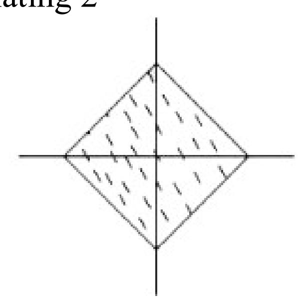
27. $\Delta = \frac{1}{2}bh = \frac{1}{2}(4)(2) = 4$ (OR) R.A. Area of sector $ABC_1 +$ Area of sector $BC_2A -$ Area of square AC_1BC_2



28. Here it is given that $|x - y| \le 2$ ……(1)

And $|x + 4| \le 2$ …..(2)

Combinating 2



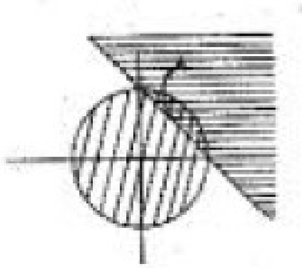
Square of side length $2\sqrt{2}$

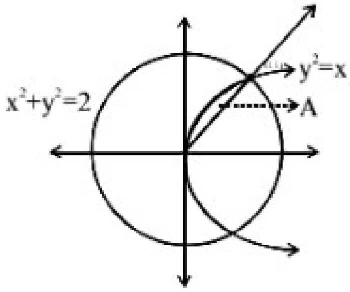
29. $y = ax^2$ and $x = ay^2$

Points of intersection are O(0, 0) and $A\left(\frac{1}{a}, \frac{1}{a}\right)$

$$\Rightarrow \int_0^{1/a} \left(\frac{\sqrt{x}}{a} - ax^2\right) dx = 1 \Rightarrow \frac{2}{3a^2} - \frac{1}{3a^2} = 1 \Rightarrow \frac{1}{3a^2} = 1. \quad \therefore a = \pm\frac{1}{\sqrt{3}}$$

30. $A = \int_0^1 \left(\sqrt{1 - x^2} - (1 - x)\right) dx$



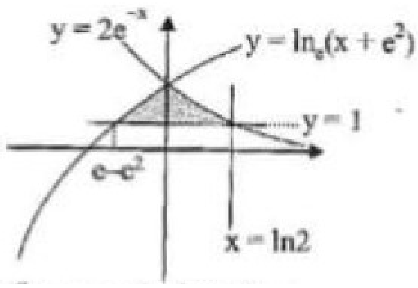


31.

$$A=\int_0^1\left(\sqrt{x}-x\right)dx=\left[\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]=\frac{1}{6}$$

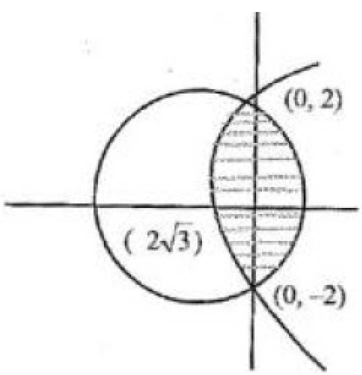
Area = $\pi r^2-\frac{1}{6}=\frac{1}{6}(12\pi-1)$

32.



From figure required area is $=\int\limits_{e-e^2}^{0}\ln(x+e^2)-1dx+\int\limits_{0}^{\ell n2}2e^{-x}-1\,dx=1+e=\ell n2$

33.



$x^2+y^2+4\sqrt{3}x-4=0$

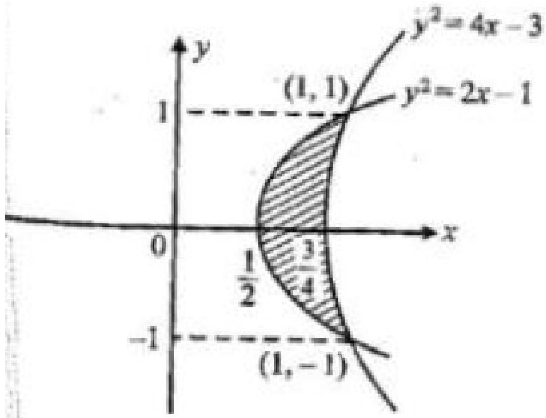
$y^2=8x+4$

On solving both the equations
Point of intersections are (0, 2) and (0, -2)
Both are symmetric about x-axis

Area = $2\int_0^2 \left(\sqrt{16-y^2}-2\sqrt{3}\right)-\left(\frac{y^2-4}{8}\right)dy$

After solving we get, Area = $\frac{1}{3}\left[8\pi+4-12\sqrt{3}\right]$

34. Given parabola's $y^2=2\left(x-\frac{1}{2}\right) and\ y^2=4\left(x-\frac{3}{4}\right)$



Required area = $2\int_0^1 \left(\frac{y^2+3}{4}-\frac{y^2+1}{2}\right)dy = 2\int_0^1 \frac{1-y^2}{4}dy = \frac{1}{2}\left|y-\frac{y^3}{3}\right|_0^1 = \frac{1}{3}$

35. $A_1=\int_0^{\pi/4}(\cos x-\sin x)dx=\sqrt{2}-1$

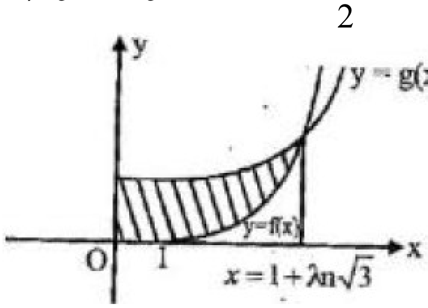
$A_2=\int_0^{\pi/4}\sin x dx+\int_{\pi/4}^{\pi/2}\cos x\ dx=\sqrt{2}\left(\sqrt{2}-1\right)$

36. $\because f(x)=e^{x-1}-e^{|x-1|}$

$\therefore\ f(x)=\begin{cases}0 & x\le 1\\ e^{x-1}-e^{1-x} & x\ge 1\end{cases}$ and $g(x)=\frac{1}{2}\left(e^{x-1}+e^{1-x}\right)$

If $f(x)=g(x)$

$\Rightarrow e^{x-1}-e^{-(x-1)}=\frac{e^{x-1}+e^{1-x}}{2}\Rightarrow e^{2(x-1)}=3\Rightarrow x=\frac{1}{2}\ell n3+1\Rightarrow x=1+\ell n\sqrt{3}$

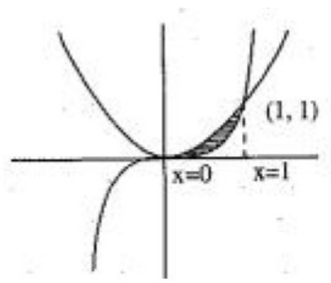


So bounded area = $\int_{0}^{\frac{1}{2}\ell n3+1} g(x)dx - \int_{1}^{\frac{1}{2}\ell n3+1} f(x)dx$

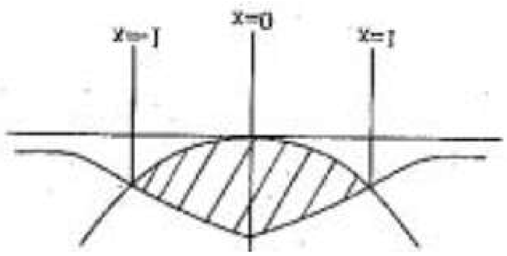
$$= \frac{1}{2}\left[e^{x-1} - e^{1-x}\right]_{0}^{\frac{1}{2}\ell n3+1} - \left[e^{x-1} + e^{1-x}\right]_{1}^{\frac{1}{2}\ell n3+1} = 2 - \sqrt{3} + \frac{1}{2}\left(e - \frac{1}{e}\right)$$

37. Area = $\int_{0}^{\sqrt{2}-1} \left(\sqrt{\dfrac{1+\dfrac{2t}{1+t^2}}{\dfrac{1-t^2}{1+t^2}}} - \sqrt{\dfrac{1+\dfrac{2t}{1+t^2}}{\dfrac{1-t^2}{1+t^2}}} \right) \dfrac{2dt}{1+t^2}$

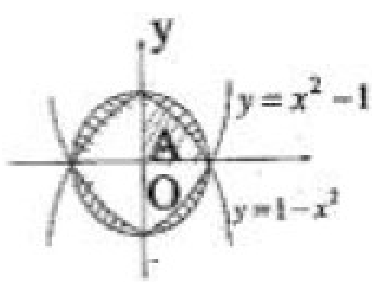
38. $A = \int_{a}^{1} (x^2 - x^3)dx = \left(\dfrac{x^3}{3} - \dfrac{x^4}{4}\right)_0^1$



39. $R.A = 2\int_{0}^{1} \left(\dfrac{2}{1+x^2} - x^2\right)dx = \pi - \dfrac{2}{3} sq.units$



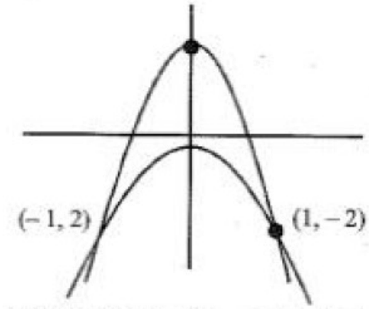
40.



$A = \int_{0}^{1} (1 - x^2)dx = \dfrac{2}{3}$

Required area = area of circle $-4A \Rightarrow \pi - \dfrac{8}{3} = \dfrac{3\pi - 8}{3}$

41. Solving $y+2x^2=0$; $y+3x^2=1$

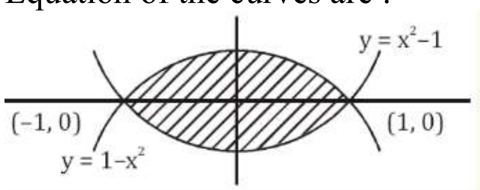


Point of intersection (1, -2) and (-1, -2)

Area = $2\int_0^1 \left\{\left(1-3x^2\right)-\left(-2x^2\right)\right\} dx$

$2\int_0^1 \left(1-x^2\right)dx = 2\left(x-\frac{x^3}{3}\right)_0^1 = \frac{4}{3} = 15-6=9\ sq.units$

42. Equation of the curves are :



$y=x^2-1$

$y=1-x^2$

On solving both the equations $\Rightarrow 1-x^2=x^2-1 \Rightarrow 2x^2-2=0$

$\Rightarrow x^2-1=0 \Rightarrow x=1,-1$

Points of the intersection are (0, 1) and (-1, 0)

Now, the area of the shaded portion is, $\Rightarrow A=\int_{-1}^{1}\left(\left(1-x^2\right)-\left(x^2-1\right)\right)dx$

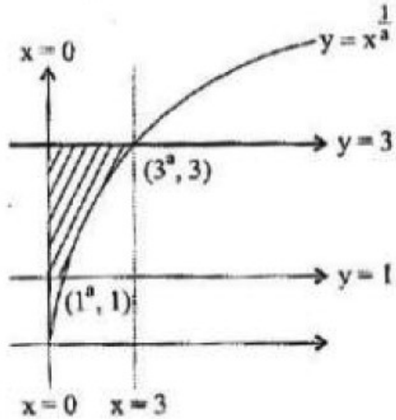
$\Rightarrow A=\int_{-1}^{1}\left(2-2x^2\right)dx \Rightarrow A=\left[2x-\frac{2}{3}x^3\right]_{-1}^{1}$

$\Rightarrow A=4-\frac{4}{3} \Rightarrow A=\frac{8}{3}sq.units$

43. Given curves are $y=3,\ y=1,\ x=0\ \&\ x=y^a$ and area rigion is $\frac{364}{3}$ sq.units.

Take $y^a=x \Rightarrow y=x^{1/a}$

Required area = $\int_0^3 x dy$



$$= \int_1^3 y^a dy = \left[\frac{y^{a+1}}{a+1}\right]_1^3$$

Apply limit $\Rightarrow \dfrac{3^{a+1}}{a+1} = \dfrac{364}{3}$

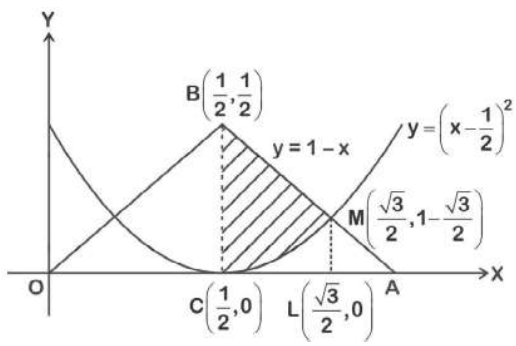
Put a = 5, then $\dfrac{3^6}{6} = \dfrac{364}{3}$

$3^6 = 729 = 3^6$

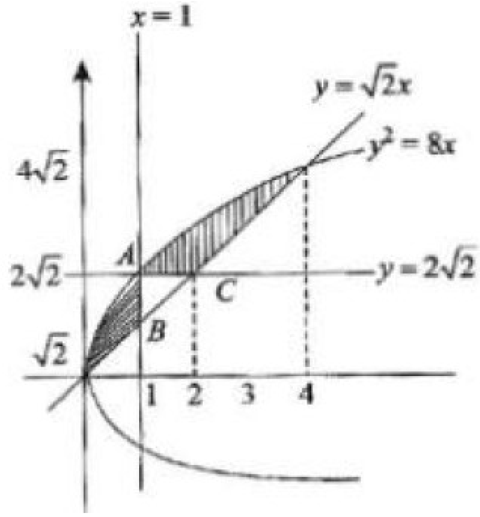
Therefore LHS = RHS

Thus, a = 5 is the correct value.

44.



$$A = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left[(1-x) - \left(x - \frac{1}{2}\right)^2\right] dx = \frac{\sqrt{3}}{4} - \frac{1}{3}$$

45. 

Here $A(1, 2\sqrt{2}), B(1, \sqrt{2}), C(2, 2\sqrt{2})$

Area of $\Delta ABC = \frac{1}{2}(\sqrt{2}).1 = \frac{\sqrt{2}}{2}$

So required Area = $\int_0^4 \left(\sqrt{8x} - \sqrt{2x}\right)dx - \frac{\sqrt{2}}{2} = \frac{32\sqrt{2}}{3} - 8\sqrt{2} - \frac{\sqrt{2}}{2} = \frac{13\sqrt{2}}{6}$

46. Given $y^2 = 8x$, and $y^2 = 16(3-x) \quad \Rightarrow y^2 = -16(x-3)$

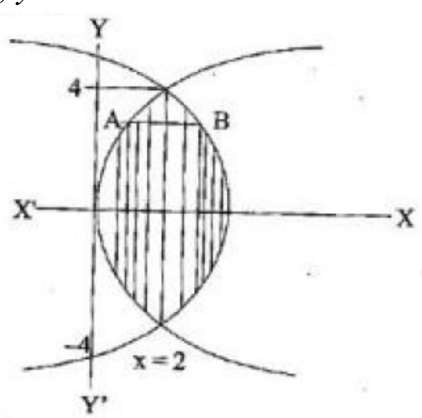
On solving both the curves we get point of intersection.

$y^2 = 8x$ & $y^2 = -16(x-3)$

$8x = -16x + 48$

$24x = 48$

$x = 2;\ y = \pm 4$

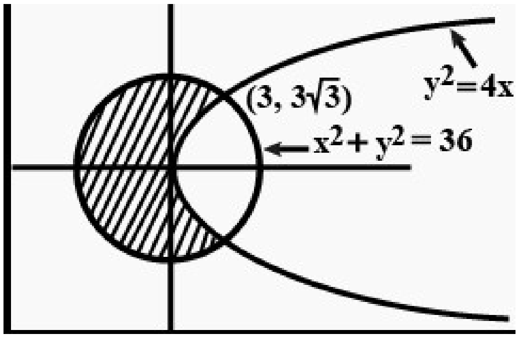


$$A = 2.\int_0^4 \left(x_R - x_L\right)dy$$

Required Area = $2.\int_0^4 \left(3 - \underbrace{\frac{y^2}{16}}_{A} - \underbrace{\frac{y^2}{8}}_{B}\right)dy = 2\left(3y - \frac{y^3}{3\times 16} - \frac{y^3}{3\times 8}\right)_0^4$

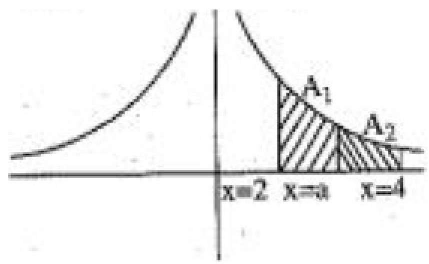
$$=2\left(3\times4-\frac{4\times4\times4}{3\times16}-\frac{4\times4\times4\times2}{3\times8\times2}\right)=2\left(12-\frac{4}{3}-\frac{8}{3}\right)=2\times12\left(1-\frac{1}{3}\right)=2\times12\times\frac{2}{3}=16$$

47.

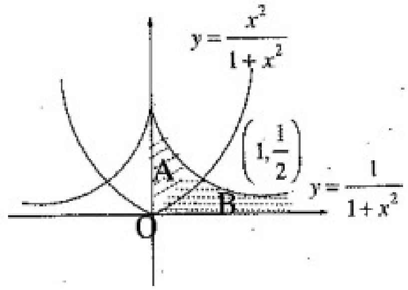


Required Area = $=\pi(6)^2-2\int_0^3\sqrt{9x}dx-\int_3^6\sqrt{36-x^2}dx=24\pi-3\sqrt{3}$

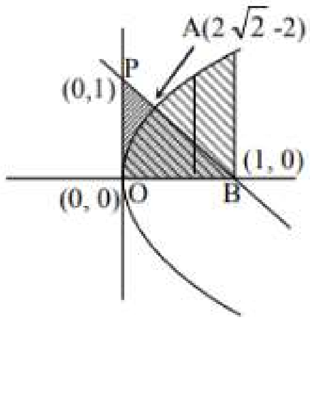
48. $A_1=A_2\Rightarrow\int_2^a\left(1+\frac{8}{x^2}\right)dx=\int_a^4\left(1+\frac{8}{x^2}\right)dx$



49. $A=\int_0^1\left(\frac{1}{1+x^2}-\frac{x^2}{1+x^2}\right)dx=\frac{\pi}{2}-1\Rightarrow A+B=\int_0^\infty\frac{1}{1+x^2}dx=\frac{\pi}{2}$



50. $C_1:y^2\le4x$; $C_2:x+y\le1$
$x\ge0; y\ge0$



Area : shaded region of curve OAB

A= Area of Δ_{OBP} - Area of region OAP

$$\Delta_{OBP} = \frac{1}{2}\times 1\times 1 = \frac{1}{2}$$

Area of OAP = $\int_{0}^{2\sqrt{2}-2} \frac{y^2}{4}dy + \int_{2\sqrt{2}-2}^{1} (1-y)dy = \frac{1}{12}\left[y^3\right]_0^{2\sqrt{2}-2} + \left[y-\frac{y^2}{2}\right]_{2\sqrt{2}-2}^{1}$

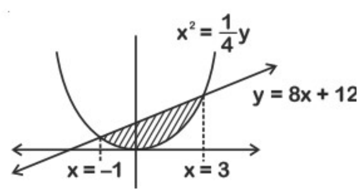
$$= \frac{23}{6} - \frac{8}{3}\sqrt{2}$$

$$A = \frac{1}{2} - \frac{23}{6} + \frac{8\sqrt{2}}{3}$$

$$a = \frac{8}{3}, b = -\frac{20}{6}$$

$$\therefore\ a-b = 6$$

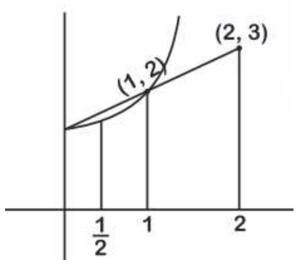
51.



Area = $\int_{-1}^{3}\left(8x+12-4x^2\right)dx = 4\left(2.\frac{x^2}{2}+3x-\frac{x^3}{3}\right)_{-1}^{3}$

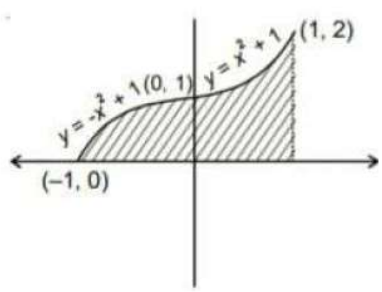
$\frac{128}{3} sq.units$

52.



Area = $\int_{\frac{1}{2}}^{1} x^2+1)dx + \int_{1}^{2}(x+1)dx = \frac{79}{24}$

53. $A = \{(x,y): 0 \le x\,|\,x\,|+1\ and -1 \le x \le 1\}$

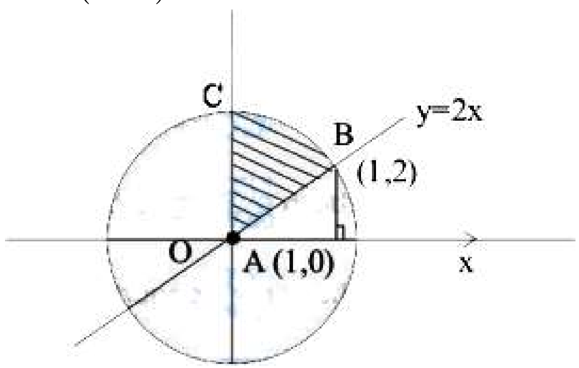


$\therefore$ Area of shaded region = $\int_{-1}^{0}\left(-x^2+1\right)dx+\int_{0}^{1}\left(x^2+1\right)dx$

$$=\left(-\frac{x^3}{3}+x\right)_{-1}^{0}+\left(\frac{x^3}{3}+x\right)_{0}^{1}=0-\left(\frac{1}{3}-1\right)+\left(\frac{1}{3}+1\right)-(0+0)$$

$$=\frac{2}{3}+\frac{4}{3}=\frac{6}{3}=2 \text{ square units.}$$

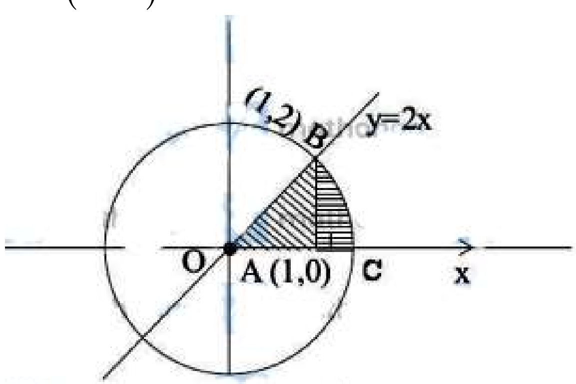
54. $y^2+\left(x-1\right)^2=4$



Shaded portion = circular (OABC)

$$-Ar(\Delta OAB)=\frac{\pi(4)}{4}-\frac{1}{2}(2)(1)$$

$A=\left(\pi-1\right)$

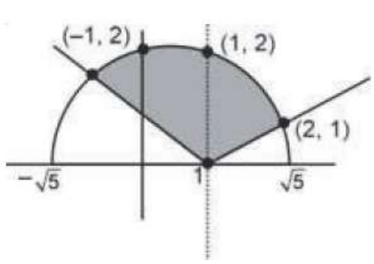


Area OBC = $Ar\left(\Delta AOB\right)$ + Area of arc of circle (ABC)

$$=\frac{1}{2}(1)\left(2\right)+\frac{\pi(2)^2}{4}=\pi+1$$

$$\frac{A}{B}=\frac{\pi-1}{\pi+1}$$

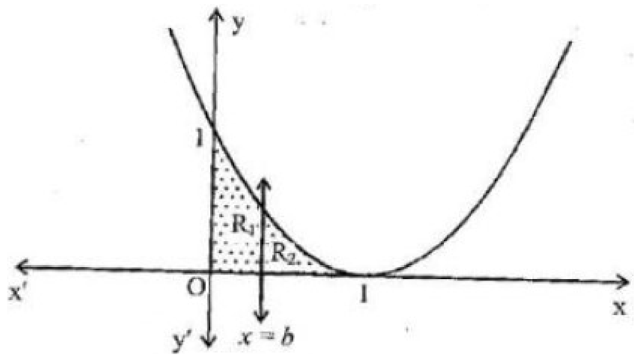
55.



$$A=\int_{-1}^{1}\left(\sqrt{5-x^2}-(1-x)\right)dx = =\left(\frac{5\pi}{4}-\frac{1}{2}\right) \text{ sq.units}$$

56. $$R_1-\int_0^b (x-1)^2\,dx=\left[\frac{(x-1)^3}{3}\right]_0^b=\frac{(b-1)^3+1}{3}$$

$$R_2=\int_b^1 (x-1)^2\,dx=\left[\frac{(x-1)^3}{3}\right]_b^1=\frac{(b-1)^3}{3}$$



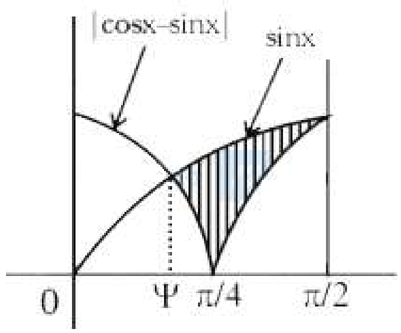
$$\because R_1-R_2=\frac{1}{4}\Rightarrow\frac{2(b-1)^3}{3}+\frac{1}{3}=\frac{1}{4}\Rightarrow (b-1)^3=-\frac{1}{8}\Rightarrow b-1=\frac{-1}{2}\ \therefore\ b=\frac{1}{2}$$

57. $$|\cos x-\sin x|\le y\le \sin x$$

Intersection point of $\cos x-\sin x=\sin x \Rightarrow\ \tan x=\dfrac{1}{2}$

Let $\psi=\tan^{-1}\dfrac{1}{2}$

So, $\tan\psi=\dfrac{1}{2},\ \sin\psi=\dfrac{1}{\sqrt{5}},\cos\psi=\dfrac{2}{\sqrt{5}}$



$$Area=\int_{v}^{\pi/2}(\sin x-|\cos x-\sin x|)dx$$

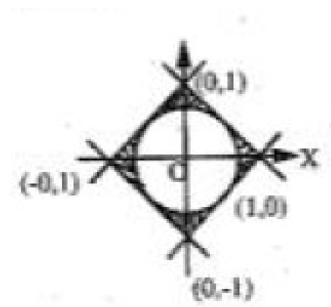
$$=\int_{v}^{\pi/2}(\sin x-(\cos x-\sin x)dx+\int_{\pi/4}^{\pi/2}(\sin x(\sin x-\cos x))dx$$

$$= \int_{\psi}^{\pi/4} (2\sin x - \cos x)dx + \int_{\pi/4}^{\pi/4} \cos x \, dx = [-2\cos x - \sin x]_{\psi}^{\pi/4} + [\sin x]_{\pi/4}^{\pi/2}$$

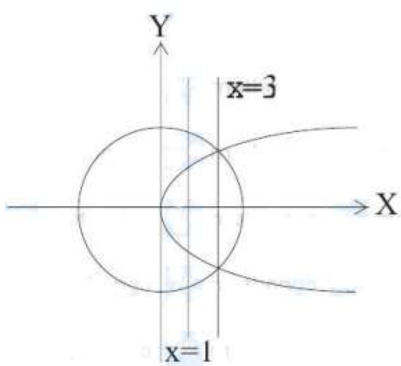
$$= -\sqrt{2} - \frac{1}{\sqrt{2}} + 2\cos\psi + \sin\psi + \left(1 - \frac{1}{\sqrt{2}}\right) = -\sqrt{2} - \frac{1}{\sqrt{2}} + 2\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{1}{\sqrt{5}}\right) + 1 - \frac{1}{\sqrt{2}}$$

$$= \sqrt{5} - 2\sqrt{2} + 1$$

58. Required Area = Area of square – area of circle $= 4.\frac{1}{2}1.1 - \pi\left(\frac{1}{\sqrt{2}}\right)^2 = 2 - \frac{\pi}{2}$



59.



Area $= \int_{1}^{3} 2\sqrt{x} \, dx + \int_{3}^{\sqrt{21}} \sqrt{21 - x^2} \, dx$

$$\Delta = \frac{8}{3}\left(3\sqrt{3} - 1\right) + 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right) - 6\sqrt{3}$$

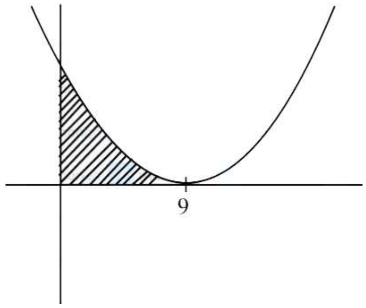
$$\frac{1}{2}\left(\Delta - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right) = \frac{2\sqrt{3} - \frac{8}{3}}{2} = \sqrt{3} - \frac{4}{3}$$

60. $x^2 - px + \frac{5p}{4} = 0$

$D = p^2 - 5p = p(p-5)$

$\therefore q = 9$

$0 \le y \le (x-9)^2$

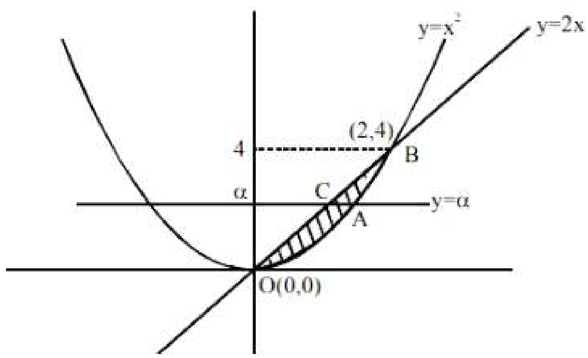


$$\text{Area} = \int_0^9 (x-9)^2\,dx = 243$$

61. $$\text{Area} = \left(\int_{-3}^{1} (3-2x-x^2)\right)dx$$

$$12-(x^2)_{-3}^{1} - \frac{1}{3}(x^3)_{-3}^{1} = 20 - \frac{28}{3} = 11 - \frac{1}{3} = \frac{32}{3}$$

62.



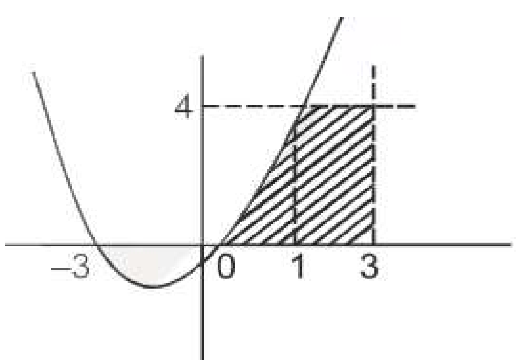
$y \ge x^2 \Rightarrow$ upper region of $y = x^2$

$y \le 2x \Rightarrow$ lower region of $y = 2x$

According to ques, area of OABC = 2 area of OAC

$$\Rightarrow \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)dy = 2 \Rightarrow \int_0^{\alpha} \left(\sqrt{y} - \frac{y}{2}\right)dy \Rightarrow \boxed{3\alpha^2 - 8\alpha^{3/2} + 8 = 0}$$

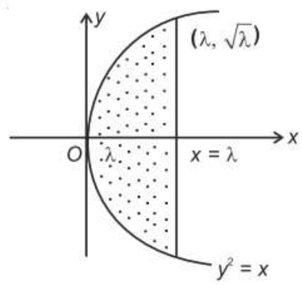
63.



$$\text{Area of the required region} = \int_0^1 \left(x^2+3x\right)dx + \int_1^3 4.dx$$

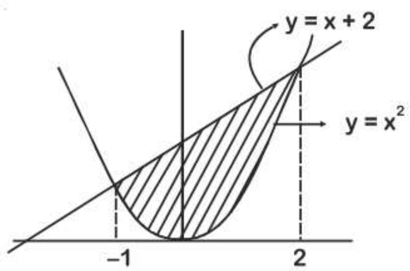
$$\frac{1}{3} + \frac{3}{2} + 8 = \frac{59}{6}$$

64.



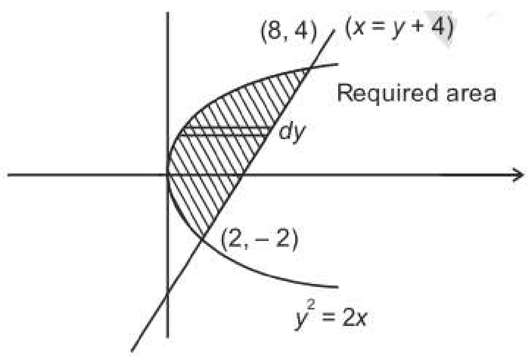
$$A(\lambda)=2\times\frac{2}{5}\left(\lambda\times\sqrt{\lambda}\right)=\frac{4}{3}\lambda^{3/2}\Rightarrow\frac{\mathrm{A}(\lambda)}{A(4)}=\frac{2}{5}\ \Rightarrow\lambda=4.\left(\frac{4}{25}\right)^{1/3}$$

65.



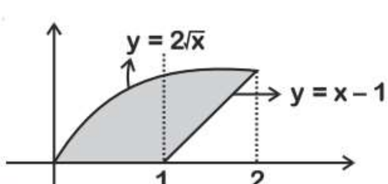
$$\text{Area}=\int_{-1}^{2}\left(\left(x+2\right)-x^2\right)dx=\frac{9}{2}$$

66.



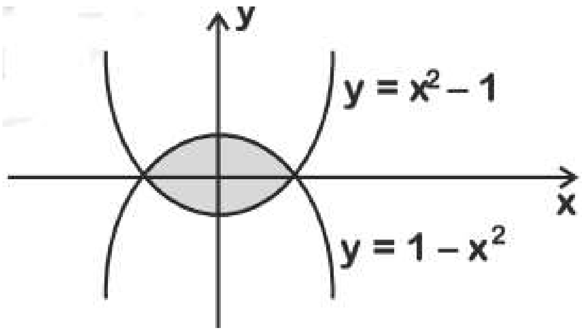
$$Area=\int_{-2}^{4}xdy\ \ =18$$

67.



$$A=\int_{0}^{1}2\sqrt{x}dx+\int_{1}^{2}\left(2\sqrt{x}-\left(x-1\right)\right)dx=\frac{8\sqrt{2}}{3}-\frac{1}{2}$$

68.



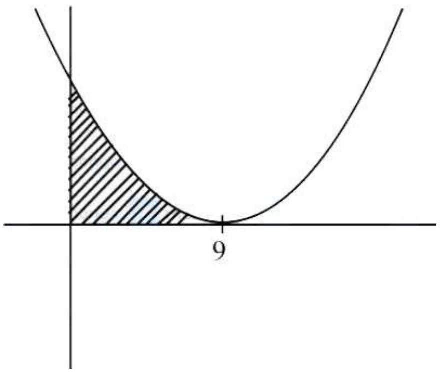
$$Area = 2\int_0^1 \left(\left(1-x^2\right)-\left(x^2-1\right)\right)dx = \frac{8}{3}$$

69. $x^2 - px + \frac{5p}{4} = 0$

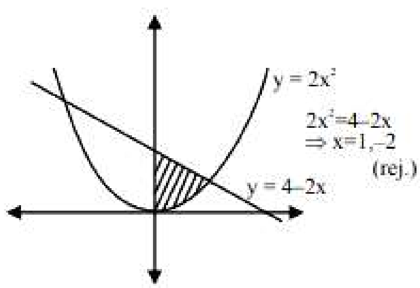
$D = p^2 - 5p = p(p-5)$

$\therefore q = 9$

$0 \le y \le (x-9)^2$



Area = $\int_0^9 (x-9)^2\,dx = 243$



70.

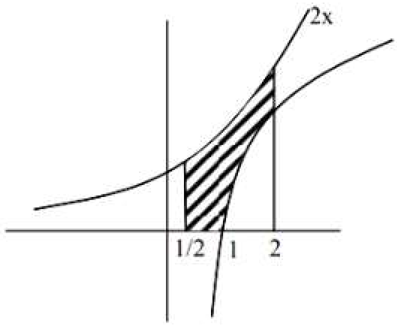
The given curves are $\left\{(x,y)\in R \middle| x\ge 0, 2x^2 \le y \le 4-2x\right\}$

$$y = \begin{cases} 2x^2 & ....(i) \\ 4-2x & ....(ii) \end{cases}$$

Solving (i) and (ii), we get the point of intersection as (1, 2) and (02, 8).

$\therefore$ Required area $= \int_0^1 (4-2x)\,dx - \int_0^1 2x^2\,dx = \left[4x - x^2 - f\dfrac{2x^3}{3}\right]_0^1 = 3 - \dfrac{2}{3} = \dfrac{7}{3}$

71.



For $\dfrac{1}{2} \le x \le 1$, we have $0 \le y \le 2^x$ and for $1 \le x \le 2$, we have $\log_e x \le y \le 2^x$

Required area = Area of shaded region

$$= \int_{1/2}^{1} 2^x\,dxc + \int_{1}^{2} \left(2^x - \log_e x\right)dx = \left.\frac{2^x}{\log_e 2}\right|_{1/2}^{1} + \left[\frac{2^x}{\log_e 2} + \left(x.\log_e x - x\right)\right]_{1/2}^{2}$$

$$= \frac{2^1}{\log_e 2} - \frac{2^{1/2}}{\log_e 2} + \left\{\frac{2^2}{\log_e 2} - 2.\log_e 2 = 2\right\} - \left\{\frac{2}{\log_e 2} + 1\right\}$$

$$= \left(\log_e 2\right)^{-1}\left[2^{1/2} + 2^2 - 2\right] - 2\log_e 2 + 2 - 1 = \left(\log_e 2\right)^{-1}\left(4 - \sqrt{2}\right) - 2l\,og_e 2 + 1$$

$\Rightarrow \alpha = 4 - \sqrt{2},\ \beta = -2\ and\ \gamma = 1$

Now, $\left(\alpha + \beta - 2\gamma\right)^2 = \left(4 - \sqrt{2} - 2\right)^2 = \left(-\sqrt{2}\right)^2 = 2$

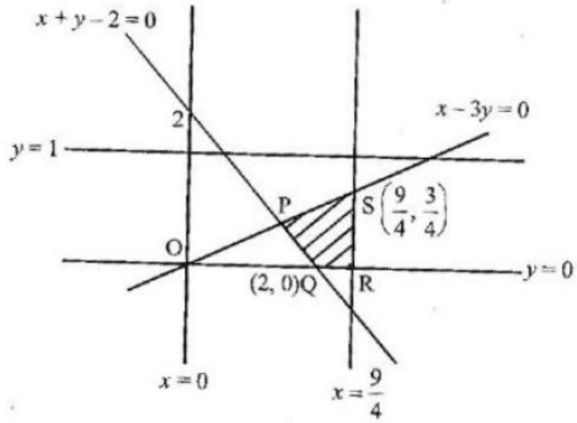
72. Required area = $= 2\int_0^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right)dx = 12\sqrt{3}sq.units$

73. Area of ABCD = Area of $AA'CD -$ Area of $AA'B$

$= \dfrac{1}{2}\left(\dfrac{1}{2} + \dfrac{3}{4}\right).\left(\dfrac{9}{4} - \dfrac{3}{2}\right) - \dfrac{1}{2}.\dfrac{1}{2}.\dfrac{1}{2} = \dfrac{11}{32}$

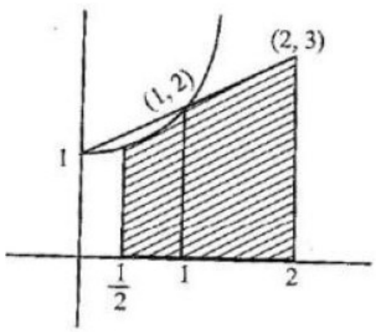
74. Let $x + y - 2 = 0,\ x = 3y,\ y = 1\ and\ x = \dfrac{9}{4}$

On solving, we get $P\left(\dfrac{3}{2}, \dfrac{1}{2}\right);\ Q(2,0);\ R\left(\dfrac{9}{4}, 0\right);\ S\left(\dfrac{9}{3}, \dfrac{3}{4}\right)$



Area = $\frac{1}{3}\int_{3/2}^{9/4} x\,dx - \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{6}\left[x^2\right]_{3/2}^{9/4} - \frac{1}{8} = \frac{1}{6}\times\frac{45}{16} - \frac{1}{8} = \frac{11}{32}$

75.

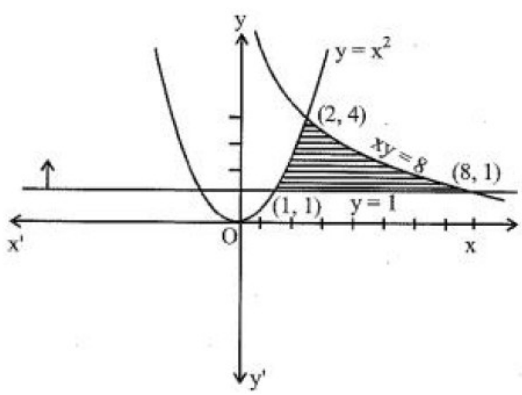


Required area = $\int_{\frac{1}{2}}^{1}(x^2+1)dx + \int_{1}^{2}(x+1)dx = \left[\frac{x^3}{3}+x\right]_{\frac{1}{2}}^{1} + \left[\frac{x^2}{2}+x\right]_{1}^{2}$

$= \left[\frac{4}{3} - \frac{13}{24}\right] + \frac{5}{2} = \frac{79}{24}.\ c$

76. $xy \le 8, 1 \le y \le x^2$

Intersection points of $xy = 8$ *and* $y = 1$ is (8, 1); $xy = 8$ and $y = x^2$ is (2, 4) and $y = x^2$ and $y = 1$ is (1, 1)



Required area $= \int_1^2 x^2 dx + \int_2^8 dx - \int_1^8 1 dx$

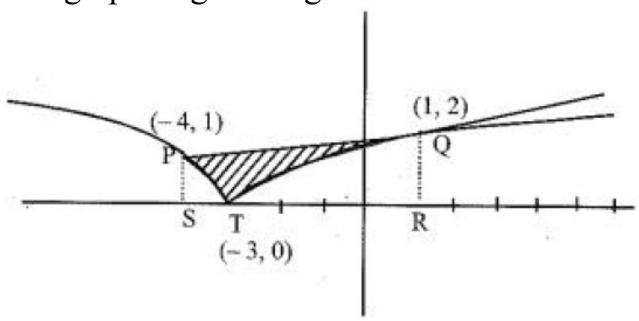
$$= \left[\frac{x^3}{3}\right]_1^2 + [8\ln x]_2^8 - [x]_1^8 = \frac{8}{3} - \frac{1}{3} + 8\ln 8 - 8\ln 2 - (8-1)$$

$$\frac{7}{3} + 24\ln 2 - 8\ln 2 - 7 = 16\ln 2 - \frac{14}{3}$$

77. $y \geq \sqrt{|x+3|} \Rightarrow y^2 = |x+3| \Rightarrow y^2 = \begin{cases} -(x+3) \text{ if } x < -3 \\ (x+3) \text{ if } x \geq -3 \end{cases}$ …..(i)

Also $y \leq \dfrac{x+9}{5}$ *and* $x \leq 6$ ….(ii)

Solving (i) and (Ii), we get intersection points as (1, 2), (6, 3), (-4, 1), (-39, -6)
The graph of given region is as follows

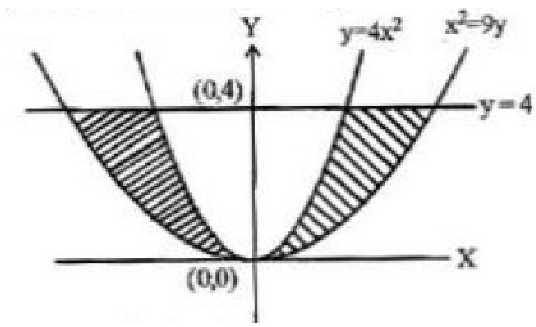


Required area = Area (trap PQRS) – Area (PST + TQR)

$$= \frac{1}{2} \times (1+2) \times 5 - \left[\int_{-4}^{-3} \sqrt{-x-3}\, dx + \int_{-3}^{1} \sqrt{x+3}\, dx\right]$$

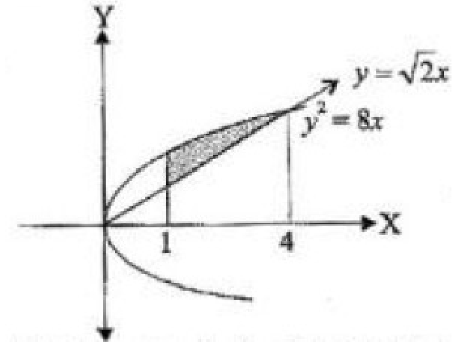
$$= \frac{15}{2}\left[\left(\frac{2(-x-3)^{3/2}}{-3}\right)_{-4}^{3} + \left(\frac{2(x+3)^{3/2}}{3}\right)_{-3}^{1}\right] = \frac{15}{2} - \left[\frac{2}{3} + \frac{16}{3}\right] = \frac{15}{2} - 6 = \frac{3}{2} \text{ sq.units}$$

78. Now, draw the required region



Area of required region $= 2\int_0^4 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy = 2\int_0^4 \frac{5}{2}\sqrt{y}\ dy = \frac{80}{3}$

79. Given equations are $y^2 = 8x$ and $y = \sqrt{2}x$



Put the value of y in other equation $\Rightarrow 8x = 2x^2,\ 2x^2 - 8x = 0$

$2x(x-4)=0 \Rightarrow x=0\ \&\ 4$

Area : $\int_1^4 \left(2\sqrt{2}\sqrt{x} - \sqrt{2}x\right)dx = 2\sqrt{2}\left(\frac{x^{3/2}}{3/2}\right)^3 - \sqrt{2}\left(\frac{x^2}{2}\right)_1^4$

Apply the limit, $= \frac{4\sqrt{2}}{3}(8-1) - \frac{\sqrt{2}}{3}(16-1) = \frac{28\sqrt{2}}{3} - \frac{15\sqrt{2}}{2} = \frac{11\sqrt{2}}{6}$

80. $x^2 + (y-2)^2 \le 2^2$ *and* $x^2 \ge 2y$

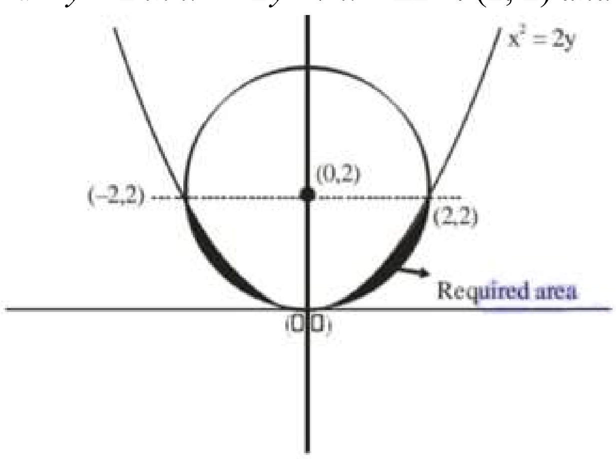
Solving circle and parabola simultaneously :

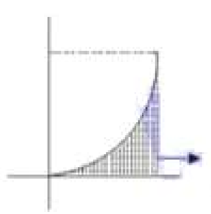
$2y + y^2 - 4y + 4 = 4$

$y^2 - 2y = 0$

$y = 0, 2$

Put $y = 2$ *in* $x^2 = 2y \rightarrow x = \pm 2 \Rightarrow (2, 2)$ *and* $(-2, 2)$





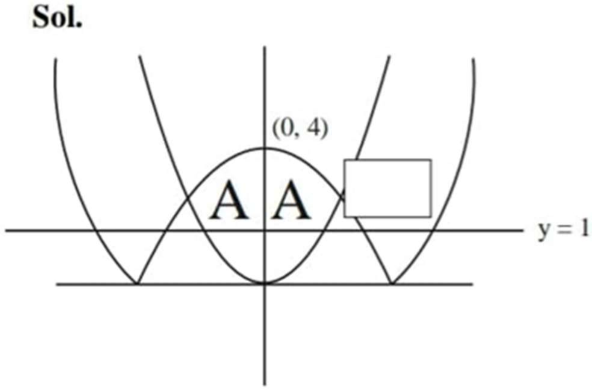
$= 2\times 2 - \frac{1}{4}\cdot\pi\cdot 2^2 = 4 - \pi$

Required $= 2\left[\int_0^2 \frac{x^2}{2}dx - (4-\pi)\right] = 2\left[\frac{x^3}{6}\Big|_0^2 - 4 + \pi\right] = 2\left[\frac{4}{3} + \pi - 4\right]$

$$= 2\left[\pi - \frac{8}{3}\right] = 2\pi - \frac{16}{6}$$

81. Key : B

Sol :



Required Area = $2\left[\int_1^2 \sqrt{y} + \int_2^4 \sqrt{4-y}\,dy\right] = \frac{4}{3}\left[4\sqrt{2}-1\right]$

82. $Area = \int_0^1 (x^2 - 2x^3 + x^2 + 3)dx = \left(\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} + 3x\right)_0^1$

83. $(y-2)dy + (x+a)dx = 0$

$(x+a)^2 + (y-2)^2 = (a+1)^2 + 4;\ x = 2 \Rightarrow a = -1$

$C:\ (x-1)^2 + (y-2)^2 = 2^2$

$y - 2 = \sqrt{3}(x-1);\ y = 0;\ x = 1 - \frac{2}{\sqrt{3}}$

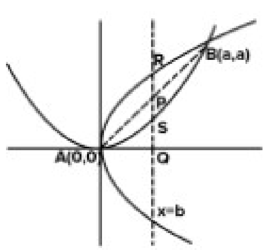
$RS = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

84. Here, $18x^2 - 9\pi x + \pi^2 = 0 \Rightarrow (3x-\pi)(6x-\pi) = 0 \Rightarrow \alpha = \frac{\pi}{6},\ \beta = \frac{\pi}{3}$

Also, gof(x) = cosx

$\therefore$ Required area = $\int_{\pi/6}^{\pi/3} \cos x\,dx = \frac{\sqrt{3}-1}{2}$

85.



Area between $y^2 = ax$ and $x^2 = ay$ is $\frac{a^2}{3}$

$$\therefore \int_0^a \left(\sqrt{ax} - \frac{x^2}{a} \right) dx = \frac{a^2}{6} \quad \ldots.(i)$$

Equation of AB is $y = x$

Given $\Delta OQR = \frac{1}{2} \Rightarrow \frac{1}{2} \times b \times b = \frac{1}{2} \Rightarrow b = 1$

Now, according to the question

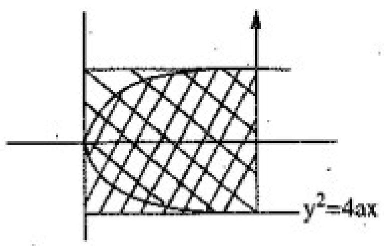
$$\int_0^1 \left(\sqrt{ax} - \frac{x^2}{a} \right) dx = \frac{1}{2} \int_0^a \left(\sqrt{ax} - \frac{x^2}{a} \right) dx \ = \ \frac{a^2}{6} \Rightarrow a^6 - 12a^3 + 4 = 0$$

86. $R_1 = \int_{-1}^{2} xf(x)dx = \int_{-1}^{2} (1-x) f(1-x)dx \quad \left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right]$

$$\Rightarrow R_1 = \int_{-1}^{2} (1-x) f(x)dx \quad [\because f(x) = f(1-x) \ on \ [-1, 2]]$$

Now, $R_1 + R_1 = \int_{-1}^{2} xf(x)dx + \int_{-1}^{2} (1-x) f(x)dx \Rightarrow 2R_1 = \int_{-1}^{2} f(x)dx = R_2$

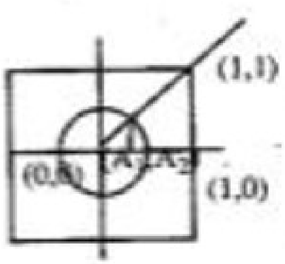
87. $A_1 = 2\int_0^a 2\sqrt{ax}dx = 4\sqrt{a}\left[\frac{x^{3/2}}{3/2} \right] = \frac{8a^2}{3}$



$A_2 = a \times 4a = 4a^2$

88. $A_1 =$ Area of triangle $= \frac{(\sqrt{2}-1)^2}{2}$

$$A_2 = \int_{\sqrt{2}}^{1/2} \sqrt{1-2x}dx = \left[-\frac{2}{3} \frac{(1-2x)^{\frac{3}{2}}}{2} \right]_{\sqrt{2}-1}^{\frac{1}{2}} = \frac{(\sqrt{2}-1)^3}{3}$$



89. Tangent at $\left(\sqrt{5}, \frac{4}{3}\right)$ is $\sqrt{5}x + 3y = 9$

Directrix of hyperbola is $x = 1$

$\therefore\ A = \left(1, \frac{9-\sqrt{5}}{3}\right)$ $and\ B = (0, 3)$

Required area = $4 \times \left\{\left(1 \times \frac{9-\sqrt{5}}{3}\right) + \frac{1}{2} \times 1 \times \left(3 - \frac{9-\sqrt{5}}{3}\right)\right\} = 10.5$

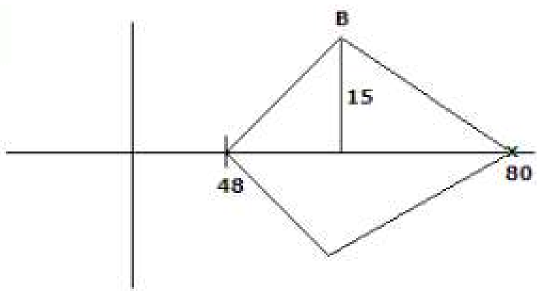
90. Required area is bounded between two ellipse

$$\frac{(x-1)}{1} + \frac{y^2}{\frac{3}{4}} = 1 \qquad \frac{(x-2)}{1} + \frac{y^2}{\frac{3}{4}} = 1$$

$$\sqrt{3}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

$\therefore a = 3, b = 4 \Rightarrow a + b = 7$

91.



Required area = $\frac{30 \times 32}{2} = 480\ sq.units$

92. We have, $y^2 = \frac{x^2}{4}(4-x)(x-2)$

$$|y| = \frac{|x|}{2}\sqrt{(4-x)(x-2)}$$

Let $y_1 = \frac{x}{2}\sqrt{(4-x)(x-2)}$ and $y_2 = -\frac{x}{2}\sqrt{(4-x)(x-2)}$

$\therefore$ Required area, $A = 2\int_2^4 y_1\, dx = \int_2^4 x\sqrt{(4-x)(x-2)}\, dx$ ….(i)

Also, $A = \int_2^4 (6-x)\sqrt{(4-x)(x-2)}\, dx$

Adding (i) and (ii), we get $2A = 6\int_2^4 \sqrt{(4-x)(x-2)}\, dx$

$$A = 3\int_2^4 \sqrt{1-(x-3)^2}\, dx = \frac{3\pi}{2}$$

93. Given condition is $A_1 = 2A_2$

Given graph is arectangle then the required area

$A_1 + A_2 = xy - 8$

Now, Put $A_1 = 2A_2$ in the above eq.$\Rightarrow \frac{3}{2}A_1 = xy - 8 \Rightarrow A_1 = \frac{2}{3}xy - \frac{16}{3}$

Now, take $\Rightarrow f(x) = \frac{2}{3}\left(x\frac{dy}{dx} + y\right) \Rightarrow \frac{2}{3}x\frac{dy}{dx} = \frac{y}{3}$

Take integral both sides$\Rightarrow 2\int \frac{dy}{y} = \int \frac{dy}{y} \Rightarrow 2\ln y = \ln x + \ln c \quad \Rightarrow y^2 = cx$ …..(i)

From given graph $f(4) = 2 \Rightarrow c = 1$

From (i)

So $y^2 = x$

Slope of normal = -6 $\Rightarrow y = -6(x) + 3 + 54 \Rightarrow y + 6x = 57$

Put coordinate (10 – 4) in above equation$\Rightarrow -4 + 60 = 56 = 57$

94. Given, $\int_{\pi/4}^{\beta} f(x)dx = \beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$

On differentiating with respect to β on both sides, we get

$f(\beta) = \sin\beta + \beta\cos\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$ (by Leibnitz rule)

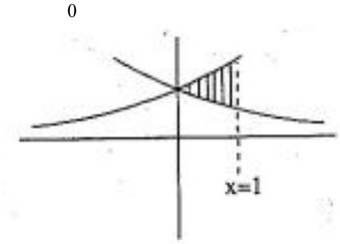
Put $\beta = \frac{\pi}{2}$

Then, $f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2} - \frac{\pi}{4}\sin\frac{\pi}{2} + \sqrt{2} = 1 + 0 - \frac{\pi}{4} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$

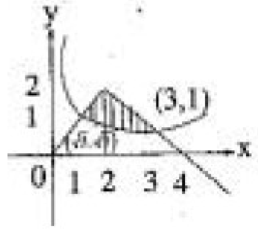
95. Let $k = \int_0^1 f(x)dx \Rightarrow g(x) = x - k \Rightarrow f(x) = 1 + \frac{3x^2}{2}$

96. $a = 4, c - b = 4\pi, b + c = \frac{9\pi}{2}$

97. $A = \int_0^1 (e^x - e^{-x})dx = (e^x + e^{-x})_0^1$



98. Area $\int_{\sqrt{3}}^{2}\left(x - \frac{3}{x}\right)dx + \int_{2}^{3}\left(4 - x - \frac{3}{x}\right)dx$

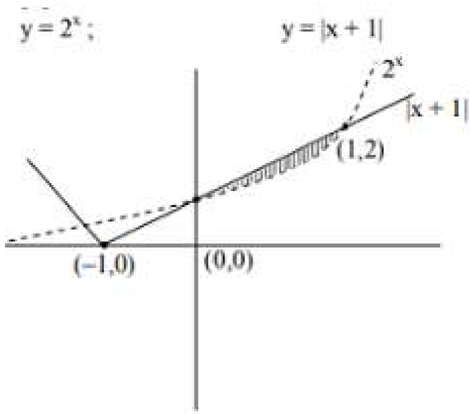


$$= \frac{4 - 3\log 3}{2} \, sq.units$$

99. Required Area $= \int_1^e [\log_e x - (\log_e x)^2] dx = \int_1^e \log_e x dx - \int_1^e (\log_e x)^2 .1.dx$

100. Hence the area of the given region $= \int_{\log_5 2}^{\log_5 6} \left\{ 8.5^x + 4 - (25^x + 16) \right\} dx$

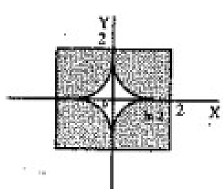
101.



$$Area = \int_0^1 \left((x+1) - 2^x \right) dx$$

$$\left(\frac{x^2}{2} + x - \frac{2^x}{\log_e 2} \right)_0^1 = \frac{3}{2} - \frac{1}{\log_e 2}$$

102.



$$S = 16 - 4\int_0^{\ln 2} \left(e^{-x} - \frac{1}{2} \right) dx = 16 + 4\left[e^{-x} + \frac{x}{2} \right]_0^{\ln 2}$$

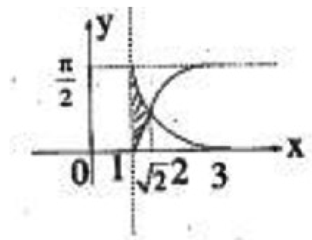
$$= 16 + 4\left(e^{-\ln 2} + \frac{1}{2}\ln 2 - 1 \right) \; = 16 + 4\left(\frac{1}{2}\ln 2 - \frac{1}{2} \right) = 14 + 2\ln 2$$

103. $A=\frac{1}{4}\pi r^2=\frac{1}{4}\pi(4)=\pi \Rightarrow B=\int\limits_0^{\sqrt{2}}\left(\sqrt{4-x^2}-\sqrt{2}\sin\frac{\pi x}{2\sqrt{2}}\right)dx=\frac{2\pi+\pi^2-8}{2\pi}$;

$\frac{A}{B}=\frac{2\pi^2}{2\pi+\pi^2-8}$

104. Area = $\int\limits_1^{\sqrt{2}}(\operatorname{cosec}^{-1}x-\sec^{-1}x)dx$

$=\log(3+2\sqrt{2})-\frac{\pi}{2}sq.units$

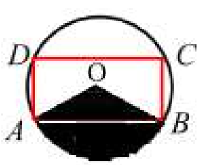


105. $f(x)=\begin{cases}\cos x,\ 0\le x\le\frac{\pi}{4}\\ \sin x,\ \frac{\pi}{4}\le x\le\frac{5\pi}{6}\\ \frac{1}{2},\ \frac{5\pi}{6}\le x\le\frac{5\pi}{3}\end{cases}$

Required area = $\int\limits_0^{\frac{5\pi}{3}}f(x)dx=\int\limits_0^{\frac{\pi}{4}}\cos x\,dx+\int\limits_{\frac{\pi}{4}}^{\frac{5\pi}{6}}\sin x+\int\limits_{\frac{5\pi}{6}}^{\frac{5\pi}{3}}\frac{1}{2}dx$

106. $A=\int\limits_0^1 e^y\sin\pi y\,dy=\left(\frac{e^y}{\pi^2+1}(\sin\pi y-\pi\cos\pi y)\right)_0^1;\ \frac{(e+1)}{\pi^2+1}$

107. Shaded area is the required region $=\frac{\pi r^2}{4}=\frac{\pi(4)^2}{4}=4\pi\ sq.units$



108. $A=\int_0^{\pi/2}\left((\sin x+\cos x)-|\cos x-\sin x|\right)dx$

$A=\int_0^{\pi/4}\left((\sin x+\cos x)-(\cos x-\sin x)\right)dx$

$+\int_{\pi/4}^{\pi/2}\left((\sin x+\cos x)-(\sin x-\cos x)\right)dx$

$A=2\int_0^{\pi/4}\sin x\,dx+2\int_{\pi/4}^{\pi/2}\cos x\ dx$

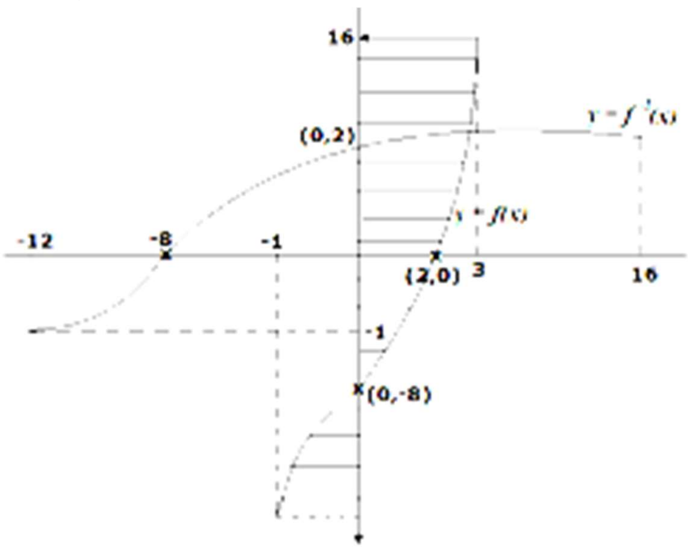
$A=-2\left(\frac{1}{\sqrt{2}}-1\right)+2\left(1-\frac{1}{\sqrt{2}}\right)$

$A = 4 - 2\sqrt{2} = 2\sqrt{2}\left(\sqrt{2}-1\right)$

109. Required area = Area bounded by $y = x^3 - x^2 + 2x - 8$ between $y = -12,\ y = 16$ and y-axis

$y = x^3 - x^2 + 2x - 8 = 0$ crosses x-axis at $x = 2$

y-axis is $y = -8$



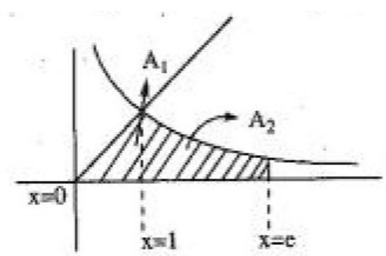
When $y = -12,\ x^3 - x^2 + 12x - 8 = -12 \Rightarrow x = -1$

$y = 16 \Rightarrow x^3 - x^2 + 2x - 8 = 16 \Rightarrow x = 3$

Required area = $\displaystyle\int_{-1}^{0}\left(x^3 - x^2 + 2x - 8 + 12\right)dx + \int_{0}^{3}\left(16\left(x^3 - x^2 + 2x - 8\right)\right)dx = \frac{325}{6} sq.units$

110. $RA = \displaystyle\int_{-3}^{1} g(x)dx + \int_{1}^{5} g(x)dx = \int_{-3}^{1} f^{-1}(x)dx = \int_{1}^{5} f^{-1}(x)dx$

Put $x = f(t) \Rightarrow [\Delta] = 4$



111. $A_1 = \dfrac{1}{2}(1)(1) = \dfrac{1}{2} sq.unit;\ A_2 = \displaystyle\int_{1}^{e}\frac{1}{x}dx = (\log x)_1^e$

112. $A_n = \displaystyle\int_{0}^{1}(x - x^n)dx = \frac{n-1}{2(n+1)}$

$(A_2).(A_3).(A_4)....(A_n) = \dfrac{1}{2^{n-1}}\left(\dfrac{12}{n(n+1)}\right) = \dfrac{1}{(n^2+n)2^{n-2}} \Rightarrow a + b + c = 0$