DPP 9.1

Matrices and Its Properties (Level 1)

Single Correct Answer Type

1. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, then $(A + B)^2 =$

   (a) A (b) B (c) I (d) $A^2 + B^2$

2. If the value of $\prod_{k=1}^{50} \begin{bmatrix} 1 & 2k-1 \\ 0 & 1 \end{bmatrix}$ is equal to $\begin{bmatrix} 1 & r \\ 0 & 1 \end{bmatrix}$ then r is equal to

   (a) 62500 (b) 2500 (c) 1250 (d) 12500

3. A square matrix P satisfies $P^2 = I - P$ where I is identity matrix. If $P^n = 5I - 8P$, then n is

   (a) 4 (b) 5 (c) 6 (d) 7

4. A and B are two square matrices such that $A^2B = BA$ and if $(AB)^{10} = A^k B^{10}$, then k is

   (a) 1001 (b) 1023 (c) 1042 (d) none of these

5. If matrix $A = [a_{ij}]_{3\times3}$, matrix $B = [b_{ij}]_{3\times3}$, where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0 \ \forall\ i, j$, then $A^4 \cdot B^3$ is

   (a) Singular (b) Zero matrix
   (c) Symmetric (d) Skew-Symmetric matrix

6. If $A\begin{pmatrix} 1 & 3 & 4 \\ 3 & -1 & 5 \\ -2 & 4 & -3 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 5 \\ 1 & 3 & 4 \\ +4 & -8 & 6 \end{pmatrix}$, then $A =$

   (a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

7. Let $A = \begin{bmatrix} -5 & -8 & -7 \\ 3 & 5 & 4 \\ 2 & 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$. If AB is a scalar multiple of B, then the value of $x + y$ is

   (a) -1 (b) -2 (c) 1 (d) 2

8. $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ and $MA = A^{2m}$, $m \in N$ for some matrix M, then which one of the following is correct?

   (a) $M = \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$

(b) $M = (a^2 + b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $M = (a^m + b^m) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $M = (a^2 + b^2)^{m-1} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

9. If $A = [a_{ij}]_{m \times n}$ and $a_{ij} = (i^2 + j^2 - ij)(j - i)$, n is odd, then which of the following is not the value of $Tr(A)$

(a) 0 (b) $|A|$ (c) $2|A|$ (d) none of these

10. $|A - B| \neq 0$, $A^4 = B^4$, $C^3A = C^3B$, $B^3A = A^3B$, then $|A^3 + B^3 + C^3| =$

(a) 0 (b) 1 (c) $3|A|^3$ (d) 6

11. If $AB + BA = O$, then which of the following is equivalent to $A^3 - B^3$

(a) $(A - B)(A^2 + AB + B^2)$ (b) $(A - B)(A^2 - AB - B^2)$

(c) $(A + B)(A^2 - AB - B^2)$ (d) $(A + B)(A^2 + AB - B^2)$

12. A, B, C are three matrices of the same order such that any two are symmetric and the 3[rd] one is skew symmetric. If $X = ABC + CBA$ and $Y = ABC - CBA$, then $(XY)^T$ is

(a) symmetric (b) skew symmetric

(c) $I - XY$ (d) $-YX$

13. If A and P are different matrices of order n satisfying $A^3 = P^3$ and $A^2P = P^2A$ (where $|A - P| \neq 0$) then $|A^2 + P^2|$ is equal to

(a) n (b) 0 (c) $|A||P|$ (d) $|A + P|$

Answers Key

Single Correct Answer Type

1. (d)	2. (b)	3. (c)	4. (b)	5. (a)
6. (d)	7. (b)	8. (d)	9. (d)	10. (a)
11. (c)	12. (d)	13. (b)		

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Single Correct Answer Type

1. (d) $AB = BA = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

2. (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 7 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 16 \\ 0 & 1 \end{bmatrix}$

If n is no. of matrices that are multiplied, then product is

$\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$.

$\therefore \quad r = 2500$

3. (c) $P^3 = P(I - P) = P - P^2 = P - (I - P) = 2P - 1$

$P^4 = P(2P - I) = 2(I - P) - P = 2I - 3P$

$P^5 = P(2I - 3P) = 2P - 3(IP) = 5P - 3I$

$P^6 = P(5P - 3I) = 5(I - P) - 3P = 5I - 8P$

So $n = 6$

4. (b) $(AB).(AB) = A(BA)B$

$= A^3B^2$

$(AB)(AB)(AB) = A^7B^3$

so $(AB)^n = A^{2^n - 1} B^n$

so $k = 2^{10} = 1 = 1023$

5. (a) Here $a_{ij} + a_{ji} = 0 \Rightarrow A^T = -A$
and $b_{ij} - b_{ji} = 0 \Rightarrow B^T = B$
and A, B are 3×3 matrices,
Hence $|A| = 0, \Rightarrow |A^4 B^3| = 0$
$\Rightarrow A^4 B^3$ is singular matrix.

6. (d) By observation its obvious that R_1 and R_2 are interchanged and R_3 of R.H.S -2 times R_3 of L.H.S.

$$\text{i.e.,} \quad R_1 \to R_2, R_3 \to -2R_3 \Rightarrow A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

7. (b) $AB = \lambda B$, where λ is non-zero scalar.

$$\begin{bmatrix} -5x - 8y - 7 \\ 3x + 5y + 4 \\ 2x + 3y + 3 \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \\ 2 \end{bmatrix}$$

i.e., $-5x - 8y - 7 = \lambda x$
$3x + 5y + 4 = \lambda y$
$2x + 3y + 3 = 2\lambda$

Adding $0 = \lambda(x + y + 2)$

$\lambda \neq 0 \Rightarrow x + y + 2 = 0$

$x + y = -2$

8. (d) Clearly option (d) satisfies the given conditions.

9. (d) As $a_{ij} = (i^2 + j^2 - ij)(j - i)$

$a_{ji} = (j^2 + i^2 - ji)(i - j) = -a_{ij}$

$\Rightarrow A$ is skew symmetric $\Rightarrow T_r(A) = 0$.

Also $|A| = 0$.

10. (a) $(A^3 + B^3 + C^3)(A - B) = A^4 - A^3B + B^3A - B^4 + C^3A - C^3B = 0$

$\Rightarrow \quad |(A^3 + B^3 + C^3)(A - B)| = 0$

$\Rightarrow \quad |(A^3 + B^3 + C^3)| = 0$, since $|(A - B)| \neq 0$.

11. (c) $(A + B)(A^2 - AB - B^2) = A^3 - A^2B - AB^2 + BA^2 - B\,AB - B^3$

$= A^3 - B^3 - A^2B - AB^2 - ABA + AB^2$

$(\because AB = -BA)$

$= A^3 - B^3 - A^2B + A^2B$

$= A^3 - B^3$

12. (d) $(XY)^T = Y^TX^T$

$Y^T = (ABC - CBA)^T$

$= C^TB^TA^T - A^TB^TC^T$

$= -CBA + ABC = Y$

$X^T = (ABC + CBA)^T$

$= C^TB^TA^T + A^TB^TC^T$

$= -CBA - ABC = -X$

13. (b) $(A^2 + P^2)(A - P) = A^3 - A^2P + P^2A - P^3$

$= (A^3 - P^3) + (P^2A - A^2P)$

$= 0$

$\therefore \quad |(A^2 + P^2)(A - P)| = 0$

$\therefore \quad |A^2 + P^2| = 0 \quad (\because |A - P| \neq 0)$

DPP 9.2

Matrices and Its Properties (Level 2)

Single Correct Answer Type

1. Let A, B are square matrices of same order satisfying $AB = A$ and $BA = B$ then $(A^{2010} + B^{2010})^{2011}$ equals.
   (a) $A + B$ (b) $2010(A + B)$
   (c) $2011(A + B)$ (d) $2^{2011}(A + B)$

2. The number of 2×2 matrices A, that are there with the elements as real numbers satisfying $A + A^T = I$ and $AA^T = I$ is
   (a) zero (b) one (c) two (d) infinite

3. If the orthogonal square matrices A and B of same size satisfy $\det A + \det B = 0$ then the value of $\det(A + B)$
   (a) -1 (b) 1
   (c) 0 (d) none of these

4. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}$, $C=ABA^T$, then A^TC^nA equals to $(n \in I^+)$
   (a) $\begin{bmatrix}-n & 1\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & -n\\ 0 & 1\end{bmatrix}$
   (c) $\begin{bmatrix}0 & 1\\ 1 & -n\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ -n & 1\end{bmatrix}$

5. Let A be a 3×3 matrix given by $A = (a_{ij})_{3\times 3}$. If for every column vector X satisfies $X'AX = 0$ and $a_{12} = 2008$, $a_{13} = 1010$ and $a_{23} = -2012$. Then the value of $a_{21} + a_{31} + a_{32} =$
   (a) -6 (b) 2006 (c) -2006 (d) 0

6. A and B are two non-singular matrices such that $A^6 = I$ and $AB^2 = BA (B \neq I)$. A value of k so that $B^k = I$
   (a) 31 (b) 32 (c) 64 (d) 63

7. Let A be a 2×3 matrix, whereas B be a 3×2 matrix. If det. $(AB) = 4$, then the value of det. (BA) is
   (a) -4 (b) 2 (c) -2 (d) 0

8. Let A be a square matrix of order 3 so that sum of elements of each row is 1. Then the sum elements of matrix A^2 is
   (a) 1 (b) 3 (c) 0 (d) 6

9. A and B be 3×3 matrices such that $AB + A + B = 0$, then
   (a) $(A + B)^2 = A^2 + 2AB + B^2$
   (b) $|A| = |B|$
   (c) $A^2 = B^2$
   (d) none of these

10. If $(A + B)^2 = A^2 + B^2$ and $|A| \neq 0$, then $|B| =$ (where A and B are matrices of odd order)
    (a) 2 (b) -2 (c) 1 (d) 0

Multiple Correct Answers Type

11. If $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$, then
    (a) $A^3 - A^2 = A - I$ (b) $\mathrm{Det}(A^{2010} - I) = 0$
    (c) $A^{50}=\begin{bmatrix}1 & 0 & 0\\ 25 & 1 & 0\\ 25 & 0 & 1\end{bmatrix}$ (d) $A^{50}=\begin{bmatrix}1 & 1 & 0\\ 25 & 1 & 0\\ 25 & 0 & 1\end{bmatrix}$

12. If the elements of a matrix A are real positive and distinct such that $\det(A + A^T)^T = 0$ then
    (a) $\det A > 0$ (b) $\det A \geq 0$
    (c) $\det(A - A^T) > 0$ (d) $\det(AA^T) > 0$

13. If $A = \begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$ and X is a non zero column matrix such that $AX = \lambda X$, where λ is a scalar, then values of λ can be
    (a) 3 (b) 6
    (c) 12 (d) 15

14. If A, B are two square matrices of same order such that $A + B = AB$ and I is identity matrix of order same as that of A, B, then
    (a) $AB = BA$ (b) $|A - I| = 0$
    (c) $|B - I| \neq 0$ (d) $|A - B| = 0$

Answers Key

Single Correct Answer Type

1. (d) 2. (c) 3. (c) 4. (d) 5. (c)
6. (d) 7. (d) 8. (b) 9. (a) 10. (d)

Multiple Correct Answers Type

11. (a, b, c) 12. (a, c, d)
13. (a, d) 14. (a, c)

DPP 9.2

Single Correct Answer Type

1. (d) Given $AB = A$ and $BA = B$

$$\Rightarrow \begin{matrix} A^2 = A \\ B^2 = B \end{matrix}$$

$$\Rightarrow \begin{matrix} A^n = A \\ B^n = B \end{matrix}$$

$$\Rightarrow \quad (A^{2010} + B^{2010})^{2011} = (A + B)^{2011}$$

Now $(A + B)^2 = A^2 + B^2 + AB + BA$

$$= 2(A + B)$$

$$\Rightarrow \quad (A + B)^k = 2^k(A + B)$$

$$\Rightarrow \quad (A^{2010} + B^{2010})^{2011} = (A + B)^{2011} = 2^{2011}(A + B)$$

2. (c) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$A + A^T = I$

$$\Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow 2a = 1, b + c = 0, 2d = 1$$

$$\Rightarrow a = \frac{1}{2}, c = -b, d = \frac{1}{2}$$

$$\Rightarrow A = \begin{pmatrix} \frac{1}{2} & b \\ -b & \frac{1}{2} \end{pmatrix}$$

Now $AA^T = I$

$$\Rightarrow \begin{pmatrix} \frac{1}{2} & b \\ -b & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -b \\ b & \frac{1}{2} \end{pmatrix} = I$$

$$\Rightarrow \begin{pmatrix} \frac{1}{4} + b^2 & 0 \\ 0 & b^2 + \frac{1}{4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow b^2 + \frac{1}{4} = 1 \Rightarrow b = \pm\frac{\sqrt{3}}{2}$$

$$\therefore \quad A = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \text{ and } \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

No. of matrices = 2

3. (c) Since A and B are orthogonal, $\det A = -1$, $\det B = 1$, or the other way round

Now $A^T(A + B)B^T = (A + B)^T$

We have on taking determinants on both sides

$(\det A) \det(A + B) \det B = \det(A + B)$

$\Rightarrow -\det(A + B) = \det(A + B)$

$\Rightarrow \det(A + B) = 0$

4. (d) $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

$\therefore \quad AA^T = I$ (i)

Now, $C = ABA^T$

$\Rightarrow \quad A^TC = BA^T$ (ii)

Now $A^TC^nA = A^TCC^{n-1}A = BA^TC^{n-1}A$ (from (ii))

$= BA^TCC^{n-2}A = B^2A^TC^{n-2}A = \ldots$

$$= B^{n-1}A^TCA = B^{n-1}BA^TA = B^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$$

5. (c) Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$,

$$(x_1 \quad x_2 \quad x_3)\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + (a_{12} + a_{21})x_1x_2 + (a_{13} + a_{31})x_1x_3 +$$

$$(a_{23} + a_{32})x_2x_3 = 0$$

It is true for every x_1, x_2, x_3,

then $a_{11} = a_{22} = a_{33} = 0$, $a_{12} + a_{21} = 0$, $a_{13} + a_{31} = 0$, $a_{23} + a_{32} = 0$

$\therefore$ A is a skew symmetric matrix

$a_{21} = -2008$

$a_{31} = -2010$

$a_{32} = 2012$

6. (d) $A^6 = I \Rightarrow BA^6 = B$

$\Rightarrow \quad (BA)\, A^5 = B$

$\Rightarrow \quad AB^2 A^5 = B$

$\Rightarrow \quad AB\, (AB^2)\, A^4 = B$

$\Rightarrow \quad A^2B^4 A^4 = B$

Proceeding like this we get

$A^6B^{64} = B \Rightarrow B^{64} = B$

$\Rightarrow \quad B^{63} = I$

$\Rightarrow \quad k = 63$

7. (d) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}$, $B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \\ b_5 & b_6 \end{bmatrix}$

$$\text{So, } \det(BA) = \begin{vmatrix} b_1a_1 + b_2a_4 & b_1a_2 + b_2a_5 & b_1a_3 + b_2a_6 \\ b_3a_1 + b_4a_4 & b_3a_2 + b_4a_5 & b_3a_3 + b_4a_6 \\ b_5a_1 + b_6a_4 & b_5a_2 + b_6a_5 & b_5a_3 + b_6a_6 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ 0 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} b_1 & b_2 & 0 \\ b_3 & b_4 & 0 \\ b_5 & b_6 & 0 \end{vmatrix} = 0 \text{ (column by row)}$$

8. (b) Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$

Given $a + b + c = 1$

$p + q + r = 1$

$x + y + z = 1$

$$\Rightarrow \quad A^2 = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + bp + cx & ab + bq + cy & ac + br + cz \\ pa + qp + rx & pb + q^2 + ry & pc + qr + rz \\ xa + yp + zx & xb + yq + zy & xc + yr + z^2 \end{bmatrix}$$

Sum of elements of

$R_1 = a^2 + bp + cx + ab + bq + cy + ac + br + cz$

$= a(a + b + c) + b(p + q + r) + c(x + y + z)$

$= a + b + c = 1$

Similarly sum of elements of

$R_2 = p(a + b + c) + q(p + q + r) + r(x + y + z)$

$q + r = 1$

$R_3 = x(a + b + c) + y(p + q + r) + z(x + y + z)$

$= x + y + z = 1$

$\therefore$ sum of elements of A^2 is 3.

9. (a) Given $AB + A + B = O$

$\Rightarrow \quad AB + A + B + I = I$

$\Rightarrow \quad A(B + I) + (B + I) = I$

$\Rightarrow \quad (A + I)(B + I) = I$

$\Rightarrow \quad (A + I)$ and $(B + I)$ are inverse of each other

$\Rightarrow \quad (A+I)(B+I) = (B+I)(A+I)$

$\Rightarrow \quad AB = BA$

Thus A and B are commutative

$\Rightarrow \quad (A+B)^2 = A^2 + 2AB + B^2$

10. (d) $(A+B)^2 = A^2 + B^2$

$\Rightarrow \quad AB + BA = O$

$\Rightarrow \quad AB = -BA$

$\Rightarrow \quad |AB| = |-BA|$

$\Rightarrow \quad |A||B| = -|B||A|$ (A and B are odd ordered matrices)

$\Rightarrow \quad |B| = -|B|$ ($|A| = 2$)

$\Rightarrow \quad |B| = 0$

Multiple Correct Answers Type

11. (a, b, c)

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A^3 - A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \text{ and } A - I = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$\Rightarrow \quad A^3 - A^2 = A - I$ and $\det(A - I) = 0$

$\Rightarrow \quad \text{Det } |A^n - I| = \text{Det } ((A-I)(1 + A + A^2 + \ldots + A^{n-1}))$

$= \text{Det } (A - I) \text{ Det } (1 + A + A^2 + \ldots + A^{n-1}) = 0$

$A^3 - A^2 = A - I$ (i)

$\Rightarrow \quad A^4 - A^3 = A^2 - A$ (ii)

$\Rightarrow \quad A^5 - A^4 = A^3 - A^2 = A - I$ (Using (1))

If n is even $A^n - A^{n-1} = A^2 - A$ (iii)

If n is odd $A^n - A^{n-1} = A - I$ (iv)

Consider n is even

$\therefore \quad A^n - A^{n-1} = A^2 - A$ (Using (iii))

$A^{n-1} - A^{n-2} = A - I$ (Using (iv))

On adding, we get

$A^n - A^{n-2} = A^2 - I$

$\Rightarrow \quad A^n = A^{n-2} + A^2 - I$

$= (A^{n-4} + A^2 - I) + A^2 - I$

$= (A^{n-6} + A^2 - I) + 2(^2 - I)$

$$= (A^2) + \frac{n-2}{2}(A^2 - I)$$

$$A^n = \left(\frac{n}{2}\right)A^2 - \left(\frac{n-2}{2}\right)I$$

$\therefore \quad A^{50} = 25A^2 - 24I$

$$= 25\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} - 24\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$$

12. (a, c, d)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad a \neq b \neq c \neq d > 0$$

$$A + A^T = \begin{pmatrix} 2a & b+c \\ b+c & 2d \end{pmatrix}$$

$|A + A^T| = 4ad - (b+c)^2 = 0 \Rightarrow b + c = 2\sqrt{ad}$

$\Rightarrow \quad b + c = 2\sqrt{ad} > 2\sqrt{bc}$ (A.M > G.M)

$\Rightarrow \quad ad > bc$

$\Rightarrow \quad ad - bc > 0$ (as $a \neq b \neq c \neq d > 0$)

$\Rightarrow \quad \det A > 0$

$$|A - A^T| = \left|\begin{pmatrix} 0 & b-c \\ c-b & 0 \end{pmatrix}\right| = 0 + (b-c)^2 > 0$$

$$|AA^T| = |A||A^T| = |A|^2 = (\det A)^2 > 0$$

13. (a, d)

$$\text{Let } X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\therefore \quad AX = \lambda X \Rightarrow \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \lambda a \\ \lambda b \\ \lambda c \end{bmatrix}$$

$\Rightarrow \quad (8 - \lambda)a - 6b + 2c = 0$

$-6a + (7 - \lambda)b - 4c = 0$

and $2a - 4b + (3 - \lambda)c = 0$

$$\text{For non-zero solution } \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda = 0, 3, 15$

14. (a, c)

$A + B = AB$

$\Rightarrow \quad I - (A + B - AB) = I$

$\Rightarrow \quad (I - A)(I - B) = I$

$\Rightarrow \quad |I - A||I - B| = I$

$\Rightarrow \quad |I - A|, |I - B|$ are non zero

Also $(I - B)(I - A) = I$

$\Rightarrow \quad I - B - A + BA = I$

$\Rightarrow \quad A + B = B + A$

$\Rightarrow \quad AB = BA$

$\Rightarrow$ (a) and (c) are correct.

DPP 9.3

Single Correct Answer Type

1. (c) Adj $(4A) = 4^2$ Adj(A) $= 16$Adj (A)

$\Rightarrow \quad |\text{Adj } 4A| = 16^3\, |\text{Adj } A| = 16^3 \cdot 5^2$

2. (a) $AB = BA$

Pre and post multiplying both sides by A^{-1}

$\Rightarrow \quad A^{-1}(AB)A^{-1} = A^{-1}(BA)A^{-1}$

$\Rightarrow \quad (A^{-1}A)BA^{-1} = A^{-1}B(AA^{-1})$

$\Rightarrow \quad BA^{-1} = A^{-1}B$

$\Rightarrow (BA^{-1})^T = (A^{-1})^T B^T = A^{-1}B$ (since A is symmetric, $\therefore A^{-1}$ is also symmetric)

Thus $(A^{-1}B)^T = A^{-1}B$

$\Rightarrow \quad A^{-1}B$ is symmetric

$(A^{-1}B^{-1})^T = ((BA)^{-1})^T$

$= ((AB)^{-1})^T$

$= ((AB)^T)^{-1}$

$= (B^TA^T)^{-1}$

$= (BA)^{-1}$

$= A^{-1}B^{-1}$

$\Rightarrow A^{-1}B^{-1}$ is also symmetric

DPP 9.3

Adjoint and Inverse of Matrix

Single Correct Answer Type

1. If A is a square matrix of order 3 such that $|A| = 5$, then $|\text{Adj}\,(4A)| =$
   (a) $5^3 \times 4^2$ (b) $5^2 \times 4^3$ (c) $5^2 \times 16^3$ (d) $5^3 \times 16^2$

2. If A and B are two non singular matrices and both are symmetric and commute each other, then
   (a) Both $A^{-1}B$ and $A^{-1}B^{-1}$ are symmetric.
   (b) $A^{-1}B$ is symmetric but $A^{-1}B^{-1}$ is not symmetric
   (c) $A^{-1}B^{-1}$ is symmetric but $A^{-1}B$ is not symmetric
   (d) Neither $A^{-1}B$ nor $A^{-1}B^{-1}$ are symmetric

3. If A is a square matrix of order 3 such that $|A| = 2$, then $|(\text{adj}\,A^{-1})^{-1}|$ is
   (a) 1 (b) 2 (c) 4 (d) 8

4. Let matrix $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, where $x, y, z \in N$. If $|\text{adj}(\text{adj}(\text{adj}(\text{adj}\,A)))| = 4^8 \cdot 5^{16}$, then the number of such (x, y, z) are
   (a) 28 (b) 36 (c) 45 (d) 55

5. A be a square matrix of order 2 with $|A| \neq 0$ such that $|A + |A|\,\text{adj}\,(A)| = 0$, where adj($A$) is a adjoint of matrix A, then the value of $|A - |A|\,\text{adj}\,(A)|$ is
   (a) 1 (b) 2 (c) 3 (d) 4

6. If A is a skew symmetric matrix, then $B = (I - A)(I + A)^{-1}$ is (where I is an identity matrix of same order as of A)
   (a) idempotent matrix (b) symmetric matrix
   (c) orthogonal matrix (d) none of these

7. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then the trace of the matrix Adj(Adj A) is
   (a) 1 (b) 2 (c) 3 (d) 4

8. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = (\text{adj}\,A)$ and $C = 5A$, then find the value of $\dfrac{|\text{adj}\,B|}{|C|}$
   (a) 25 (b) 2 (c) 1 (d) 5

9. Let A and B be two non-singular square matrices such that $B \neq I$ and $AB^2 = BA$. If $A^3 = B^{-1}A^3B^n$, then value of n is
   (a) 4 (b) 5 (c) 6 (d) 7

10. If A is an idempotent matrix satisfying $(I - 0.4A)^{-1} = I - \alpha A$ where I is the unit matrix of the same order as that of A then the value of α is
    (a) $-1/3$ (b) $1/3$ (c) $-2/3$ (d) $2/3$

11. If A and B are two non-singular matrices which commute, then $(A(A + B)^{-1} B)^{-1} (AB) =$
    (a) $A + B$ (b) $A^{-1} + B^{-1}$
    (c) $A^{-1} + B$ (d) none of these

Multiple Correct Answers Type

12. If A is a non-singular matrix of order $n \times n$ such that $3ABA^{-1} + A = 2A^{-1}BA$, then
    (a) A and B both are identity matrices
    (b) $|A + B| = 0$
    (c) $|ABA^{-1} - A^{-1}BA| = 0$
    (d) $A + B$ is not a singular matrix

13. If the matrix A and B are of 3×3 and $(I - AB)$ is invertible, then which of the following statement is/are correct?
    (a) $I - BA$ is not invertible
    (b) $I - BA$ is invertible
    (c) $I - BA$ has for its inverse $I + B(I - AB)^{-1} A$
    (d) $I - BA$ has for its inverse $I + A(I - BA)^{-1} B$

14. If A is a square matrix such that $A.(\text{Adj}\,A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then
    (a) $|A| = 4$ (b) $|\text{adj}\,A| = 16$
    (c) $\dfrac{|\text{adj}(\text{adj}\,A)|}{|\text{adj}\,A|} = 16$ (d) $|adj\;2A| = 128$

Answers Key

Single Correct Answer Type

1. (c) 2. (a) 3. (c) 4. (b) 5. (d)
6. (c) 7. (a) 8. (c) 9. (c) 10. (c)
11. (a)

Multiple Correct Answers Type

12. (b, c) 13. (b, c)
14. (a, b, c)

DPP 9.3

Single Correct Answer Type

1. (c) $\text{Adj}(4A) = 4^2\,\text{Adj}(A) = 16\text{Adj}(A)$

$\Rightarrow \quad |\text{Adj}\ 4A| = 16^3\,|\text{Adj}\ A| = 16^3 \cdot 5^2$

2. (a) $AB = BA$

Pre and post multiplying both sides by A^{-1}

$\Rightarrow \quad A^{-1}(AB)A^{-1} = A^{-1}(BA)A^{-1}$

$\Rightarrow \quad (A^{-1}A)BA^{-1} = A^{-1}B(AA^{-1})$

$\Rightarrow \quad BA^{-1} = A^{-1}B$

$\Rightarrow (BA^{-1})^T = (A^{-1})^T B^T = A^{-1}B$ (since A is symmetric, $\therefore A^{-1}$ is also symmetric)

Thus $(A^{-1}B)^T = A^{-1}B$

$\Rightarrow \quad A^{-1}B$ is symmetric

$$\begin{aligned}(A^{-1}B^{-1})^T &= ((BA)^{-1})^T \\ &= ((AB)^{-1})^T \\ &= ((AB)^T)^{-1} \\ &= (B^TA^T)^{-1} \\ &= (BA)^{-1} \\ &= A^{-1}B^{-1}\end{aligned}$$

$\Rightarrow A^{-1}B^{-1}$ is also symmetric

3. (c) $|adj\ A^{-1}| = |A^{-1}|^2 = \dfrac{1}{|A|^2}$

$|(adj\ A^{-1})^{-1}| = \dfrac{1}{|adj\ A^{-1}|} = |A|^2 = 2^2 = 4$

4. (b) $|adj(adj(adj(adjA)))| = |A|^{16} = 4^8 \cdot 5^{16}$

$\Rightarrow\ |A| = 10$

$\Rightarrow\ x + y + z = 10$

Number of positive integral solutions is ${}^9C_2 = 36$

5. (d) Let $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, adj (a) $= \begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

Let $|A| = d = mq - np$

$$|A + d\ adjA| = \begin{vmatrix} m+qd & n(1-d) \\ p(1-d) & q+md \end{vmatrix} = 0$$

$\Rightarrow\ mq + m^2d + q^2d + mqd^2 - np + 2npd - npd^2 = 0$

$\Rightarrow\ (mq - np) + (mq - np)d^2 + m^2d + q^2d + 2mqd - 2d^2 = 0$

$\Rightarrow\ (d + d^3 - 2d^2) + d(m^2 + q^2 + 2mq) = 0$

$\Rightarrow\ d[(d-1)^2 + (m+q)^2] = 0 \Rightarrow d = 1, m + q = 0$

Now, $|A - d\ adj\ |A|| = -(m+q)^2 + 4(mq - np) = 4d = 4$

6. (c) $B = (I - A)(I + A)^{-1}$

$\Rightarrow\ B^T = (I + A^T)^{-1}(I - A^T)$

$= (I - A)^{-1}(I + A),$

$\Rightarrow\ BB^T = (I - A)(I + A)^{-1}(I - A)^{-1}(I + A)$

$= (I - A)(I - A)^{-1}(I + A)^{-1}(I + A)$

$= I$

(As $(I - A).(I + A) = (I + A)(I - A)$)

7. (a) Here $|A| = 1$

$\Rightarrow$ Adj (Adj A) $= |A|^{3-2} \cdot A = A$

(Where trace of a matrix is the sum of the elements in the principal diagonal)

8. (c) $\dfrac{|adj\ B|}{|C|} = \dfrac{|adj(adj\ A)|}{|5A|} = \dfrac{|A|^{(3-1)^2}}{5^3|A|} = \dfrac{|A|^3}{125}$

Now $|A| = 5$

$\therefore\ \dfrac{|adj\ B|}{|C|} = 1$

9. (c) $BA = AB^2$

$\Rightarrow\ BA = AB^2$

$\Rightarrow\ A = B^{-1}AB^2$

$\Rightarrow\ A^2 = (B^{-1}AB^2)(B^{-1}AB^2)$

$= B^{-1}A(BA)B^2$

$= B^{-1}A\,AB^2B^2$

$= B^{-1}A^2B^4$

$\therefore\ A^3 = B^{-1}A^3B^6$

10. (c) Given $A^2 = A$

$\Rightarrow\ I = (I - 0.4A)(I - \alpha A)$

$= I - I\alpha A - 0.4AI + 0.4\alpha A^2$

$= I - A\alpha - 0.4A + 0.4\alpha A$

$= I - A(0.4 + \alpha) + 0.4\alpha A$

$\Rightarrow\ 0.4\alpha = 0.4 + \alpha$

$\Rightarrow\ \alpha = \dfrac{-3}{2}$

11. (a)

$(A(A + B)^{-1}B)^{-1}(AB)$

$= B^{-1}((A + B)^{-1})^{-1}A^{-1}(AB)$

$= B^{-1}(A + B)A^{-1}(AB)$

$= (B^{-1}A + I)A^{-1}(AB)$

$= (B^{-1}AA^{-1} + A^{-1})(AB)$

$= (B^{-1} + A^{-1})(AB)$

$= B^{-1}AB + A^{-1}AB$

$= B^{-1}BA + A^{-1}AB$

$= A + B$

Multiple Correct Answers Type

12. (b, c)

$3ABA^{-1} + A = 2A^{-1}BA$

$\Rightarrow\ 3ABA^{-1} + A + 2A = 2A^{-1}BA + 2A$

$\Rightarrow\ 3A(BA^{-1} + I) = 2(A^{-1}B + I)A$

$\Rightarrow\ 3A(B + IA)A^{-1} = 2A^{-1}(B + AI)A$

$\Rightarrow\ 3A(B + A)A^{-1} = 2A^{-1}(B + A)A$

Let $B + A = X$

$\Rightarrow\ 3AXA^{-1} = 2A^{-1}XA$

$\Rightarrow\ 3^n|A||X||A^{-1}| = 2^n|A^{-1}||X||A|$

$\Rightarrow\ 3^n|X| = 2^n|X|$ (as $|A| \neq 0$)

$\Rightarrow\ |X| = 0$ or $|A + B| = 0$ (i)

Let $M = ABA^{-1} - A^{-1}BA$

$\therefore\ AM = A^2BA^{-1} - BA \Rightarrow BA = A^2BA^{-1} - AM$

Now $3ABA^{-1} + A = 2A^{-1}BA$

$= 2A^{-1}(A^2BA^{-1} - AM)$

$= 2ABA^{-1} - 2M$

$\Rightarrow\ ABA^{-1} + A = -2M$

$\Rightarrow\ A(BA^{-1} + I) = -2M$

$A(A + B)A^{-1} = -2M$

Taking determinants both sides we get

$|-2M| = |A||A + B||A^{-1}| = 0$

$\Rightarrow\ |ABA^{-1} - AB^{-1}A| = 0$

13. (b, c)

Let $(I - AB)^{-1} = P$

$\Rightarrow\ P(I - AB) = I$

$\Rightarrow\ P - PAB = I$

$\Rightarrow\ PB^{-1} - PA = B^{-1}$

$\Rightarrow\ BPB^{-1} - BPA = I$

$\Rightarrow\ BPB^{-1} = I + BPA$

Now $BPB^{-1} = B(I - AB)^{-1}B^{-1}$

$= B(B(I - AB))^{-1}$

$= (B^{-1})^{-1}(B(I - AB))^{-1}$

$= (B(I - AB)B^{-1})^{-1}$

$= ((B - BAB)B^{-1})^{-1}$

$= (I - BA)^{-1}$

14. (a, b, c)

$A(adj\ A) = |A|I = 4I$

$\therefore\ |A| = 4$

$|adj\ A| = |A|^{3-1} = 16$

$adj\ KA = K^{3-1}\ adj\ A$

$|adj(adj\ A)| = |A|^{(3-1)^2} = |A|^4 = 256$

$\therefore\ |adj\ KA| = (K^{3-1})^3\ |adj\ A|$

$\therefore\ |adj\ 2A| = 2^6 \cdot 16$