



A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60\_NUCLEUS&STERLING\_BT JEE-MAIN Date: 12-08-2023 Time: 09.00Am to 12.00Pm RPTM-02 Max. Marks: 300

#### **KEY SHEET**

#### **PHYSICS**

1)	4	2)	2	3)	4	4)	4	5)	1
6)	1	7)	2	8)	1	9)	1	10)	1
11)	1	12)	4	13)	3	14)	4	15)	4
16)	2	17)	2	18)	2	19)	3	20)	1
21)	15	22)	40	23)	50	24)	375	25)	625
26)	9	27)	5	28)	12	29)	48	30)	0

#### **CHEMISTRY**

31	4	32	3	33	2	34	4	35	1
36	4	37	3	38	2	39	3	40	2
41	2	42	1	43	3	44	1	45	4
46	4	47	3	48	2	49	4	50	4
51	4	52	4	53	4	54	11	55	8
56	6	57	8	58	4	59	5	60	4

### **MATHEMATICS**

THE	MAT	ICS					Tail	into	ns
61)	2	62)	3	63)	3	64)	I	65)	4
66)	3	67)	1	68)	<b>序注</b> []	69)	4	70)	2
71)	2	72)	3	73)	11	74)	2	75)	1
76)	3	77)	4	78)	2	79)	3	80)	1
81)	9	82)	2	83)	12	84)	2	85)	3
86)	7	87)	3	88)	8	89)	3	90)	5

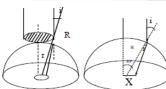
#### SOLUTIONS PHYSICS

- 1. Conceptual
- 2. Let m is mass evoperated  $m \times 540 = (150 m) \ 80 \ m = \frac{600}{31} \cong 20 \ gm$
- 3. Acc to Jouls law  $\frac{1}{2}kx^2 = J[ms\Delta t + Ms\Delta t]$
- If '\theta' Junction temp  $\frac{3kA(\theta_2 \theta)}{d} = \frac{kA(\theta \theta_1)}{3d}$
- 5.  $5M_1 + M_1L = 50M_2$
- **6.** Apply first law of thermodynamics
- 7. Conceptual
- 8.  $\theta_1:\theta_2:\theta_3=R_1:R_2:R_3 = \frac{1}{2}:\frac{3}{5}:\frac{1}{2} = 5:6:5 \quad \theta_2=\frac{6}{16}\times120^0=45^0C$
- 9. Energy Flux  $\frac{\theta}{At} = \frac{k(\theta_1 \theta_2)}{l}$
- 10. Principle of Calorimetry
- 11. According to principle of calorimetry 10m + (m-20)80 = 2000
- 12. According to Thermometry  $\frac{t-0}{100-0} = \frac{\frac{x_0}{2} \frac{x_0}{3}}{x_0 x_0 / 3}$
- 13. V actual of bird =  $V \cos 37^{0} \hat{i} + V \sin 37^{0} \hat{j}$ No change in horizontal component

$$\stackrel{-}{V}$$
 app w.r.t fish =  $V \cos 37^{0} \hat{i} + \mu V \sin 37^{0} \hat{j}$ 

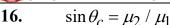
$$V \cos 37^{0} \hat{i} + \frac{4}{3} \times V \frac{3}{5} \hat{j} = \frac{4}{5} V \hat{i} + \frac{4}{5} V \hat{j}$$

- $\therefore$  Bird appears to fly at  $45^0$  to horizontal.
- Using Snell's law:  $\sin i = \frac{R}{2R} \Rightarrow i = 30^{\circ} \sin i = \mu \sin r \quad \sin r = \frac{1}{2\sqrt{2}}$



$$\frac{x}{\sin r} = \frac{R}{\sin(120 - r)} \qquad x = \frac{R}{10} \left(\sqrt{21} - 1\right)$$

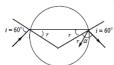
Normal shift produced by slab is independent of object distance from slab  $\therefore V_{app} = V_{actual} = 0$ 



$$1 - \cos^2 \theta_c = \left(\frac{\mu_2}{\mu_1}\right)^2 1 - \left(\hat{n}\hat{p}\right)^2 = \left(\frac{\mu_2}{\mu_1}\right)^2 1 - \left[\frac{4}{\sqrt{25}}\right]^2 = \left(\frac{\mu_2}{\mu_1}\right) \Rightarrow \frac{\mu_2}{\mu_1} = \frac{3}{5} \Rightarrow \mu_2 = \frac{3\sqrt{3}}{5}$$

17. From figure it is clear  $\alpha = 180 - (i + r)$  and  $\mu = \frac{\sin i}{\sin r} \Rightarrow \sqrt{3} = \frac{\sin 60^{\circ}}{\sin r}$   $r = 30^{\circ}$ 

$$\alpha = 90^0$$



Let x be the depth of point P from surface 18.

App. depth of point P from surface 
$$x = \frac{x}{\mu}$$

App. depth of image of P from surface =  $\frac{x+2h}{x}$ 

So, separation between two =  $\frac{x+2h}{\mu} - \frac{x}{\mu}$   $\Rightarrow \frac{2h}{\mu}$ 

- $\delta = A(\mu 1)$  Slope of line will be A. 19.
- **20.**  $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$   $\frac{1}{10} = \frac{1}{v} - \frac{1}{-15}$   $\frac{1}{v} = \frac{1}{10} - \frac{1}{-15} = \frac{1}{30}$  v=-30 cm If rays strike at pole of the mirror u = 0

 $\therefore$  for  $2^{nd}$  refraction at lens object at pole of mirror gives image at the position of object  $\therefore d = 30cm$ 

- 21. Conceptual
- Principal of calorimetry M(540+60) = 200(80 + 40)22.
- Let  $\theta_1, \theta_2, \theta_3$  be temp of water in containers  $c_1, c_2, c_3$ 23.

$$\theta_1 - 60 = 2(60 - \theta_2)$$

$$\Rightarrow \theta_1 + 2\theta_2 = 180$$

$$\begin{array}{ll}
\theta_{1} - 60 = 2(60 - \theta_{2}) & \Rightarrow \theta_{1} + 2\theta_{2} = 180 \\
2(\theta_{1} - 60) = 1(60 - \theta_{3}) & \Rightarrow 2\theta_{1} + \theta_{3} = 180 \\
1(\theta_{2} - 30) = 2(30 - \theta_{3}) & \Rightarrow \theta_{2} + 2\theta_{3} = 90 \\
\text{Solution: } \theta_{1} = 80 & \theta_{2} = 50 & \theta_{3} = 20
\end{array}$$

$$\Rightarrow 2\theta_1 + \theta_3 = 180$$

$$1(\theta_2 - 30) = 2(30 - \theta_3)$$

$$\Rightarrow \theta_2 + 2\theta_3 = 90$$

Solution: 
$$\theta_1 = 80$$

$$\theta_2 = 50$$

$$\theta_3 = 20$$

$$1(80-\theta) = (\theta-50) + (\theta-20)$$

$$\Rightarrow 3\theta = 150$$

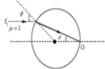
 $1(80-\theta) = (\theta - 50) + (\theta - 20) \qquad \Rightarrow 3\theta = 150$   $\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} + \frac{1}{R}\right) = \frac{1}{2R}2R = f_1 \Rightarrow R = \frac{f_1}{2} = \frac{25}{4}cm$ 24.

$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right) \left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{1}{3R}f_2 = -3R = \frac{75}{4}cm$$

NOW, 
$$-\frac{1}{F} = \frac{2}{f_1} + \frac{2}{f_2} - \frac{1}{f_m} - \frac{1}{F} = \frac{4}{25} - \frac{8}{75} - \frac{1}{\infty} \Rightarrow F = -\frac{75}{4}cm$$

The point object is placed at a distance,  $d = \frac{75}{2} = 37.50cm$ 

25.  $\sin \phi = \mu \sin \beta \cos \beta = \mu / 2 = 5 / 8 \left( \sin ce\phi / 2 = \beta \right) \therefore 100COS\beta = 100 \times \frac{5}{8} = 62.5$ 



26. 
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{24} = \left[ \left( \frac{\mu_1}{\frac{4}{3}} - 1 \right) \left( \frac{1}{20} + \frac{1}{20} \right) + \left( \frac{\mu_2}{\frac{4}{3}} - 1 \right) \left( \frac{1}{-20} - \frac{1}{20} \right) \right] \Rightarrow$$

$$(\mu_1 - \mu_2) = \left(\frac{5}{9}\right)$$

27.

For upper part of lens 
$$u = -\frac{3f}{2}$$
,  $h_0 = 2cm$   $m = \frac{h_i}{h_0} = \frac{f}{f + \left(\frac{-3f}{2}\right)} = 0.5$ 

$$h_i = 2 \times 0.5 = 1cm$$

i.e image is formed at a height of 3 cm (i.e 2+1) from main principal axis. (above principal axis) Similarly, for lower part, image is formed 3cm below main principal axis. Hence distance between image = 3+3=6 cm.

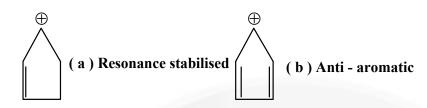
$$\frac{3}{2v_1} - \frac{1}{\infty} = \frac{(0.5)}{-20} \frac{2}{f} - \frac{3}{2v_1} = \frac{0.5}{-20} \frac{2}{f} = \frac{1}{2} \left(\frac{1}{20} + \frac{1}{30}\right) f = \frac{4 \times 20 \times 30}{50} = -48cm$$

Total angular dispersion  $= (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A' = 0$ 



#### **CHEMISTRY**

31.

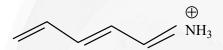




(c) Hyperconjugative stablised

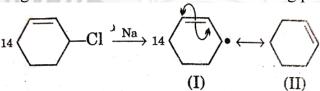
a>c>b

**32.** 



( N can never be pentavalent)

- 33.  $-CH_3$  is better +H group than  $-CD_3$
- 34.  $\begin{pmatrix} 0 & 0 & 0 \\ \parallel & \parallel & \parallel \\ CH_3 C CH_2 C CH_2 CH_3 & \text{having less acidic Hydrogen.} \end{pmatrix}$
- 35. Aromatic
- 36. N is better donor than O.
- **37.** Resonance energy is difference of most stable resonating energy and energy of real molecule.
- **38.** Distance increasing, I-effect decreases.
- **39.** Conjugate Acid of Pyrrole is non-aromatic
- 40.  $SP^3$  carbon
- 41. Bonds increased; bond length decreases.
- **42.** E D G increasing basic character.
- 43. The given reaction is Wurtz which is taking place through free radical formation



Possible products obtained are

(A) (I) + (I) 
$$\longrightarrow$$
 14  $\longleftrightarrow$  two chiral centre

:. Number of products = 3 (Symmetrical structure)



(B) (II) + (II) 
$$\longrightarrow$$
  $\underbrace{\qquad \qquad}_{14}$  3 products

(C) (I) + (II) 
$$\longrightarrow$$
 14

Two chiral center with number of symmetry in the molecule.

 $\therefore$  Number of products = 4

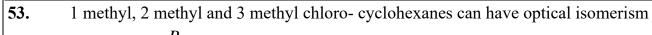
Total number of products = 3+3+4=10

44. Less stability, more heat of combustion.

- **46.** Functional group more priority
- 47. Restricted rotation molecule only two stereo isomers possible
- 48. : 60%-40%=20% exaticmatric across  $20 = \frac{\left[\alpha\right]_{mtx}}{-154} \times 100 = -31$  60%-40%=20%

- **50.** Carbon surrounded by four different groups.
- 51.  $\begin{array}{c}
   & \text{NBS} \\
   & \text{NBS} \\
   & \text{NBS}
  \end{array}$   $\begin{array}{c}
   & \text{Br} \\
   & \text{R+S}
  \end{array}$   $\begin{array}{c}
   & \text{E+Z} \\
   & \text{Br}_{2}
  \end{array}$

52. 
$$\begin{array}{c} CH_3 \\ CH_3 \\ CH_3 \end{array} \begin{array}{c} CH_2 \\ CH_3 \end{array} \begin{array}{c} CH_3 \\ CH_3 \end{array}$$



So P=12 and 
$$\frac{P}{3} = 4$$

54. Chiral carbon

55.

Number of stereo centres =  $2^n = 2^3 = 8$ 

Number of Alpha acidic Hydrogens. **56.** 

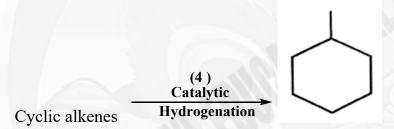
57. Compound

$$H_3C - CH = CH - CH = CH - CH = C = C = C - CH_3$$

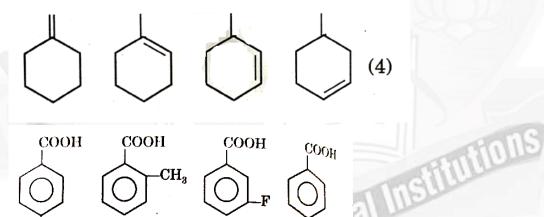
Has three geometrical centre and is unsymmetrical thus, total

$$G.I. = 2^3 = 8.$$

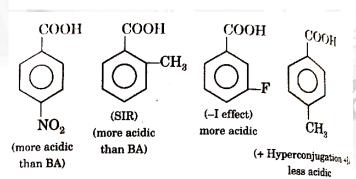
**58.** 

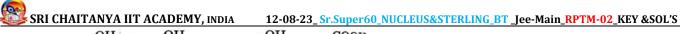


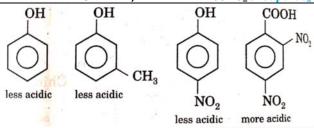
Possible cyclic alkenes



**59.** 





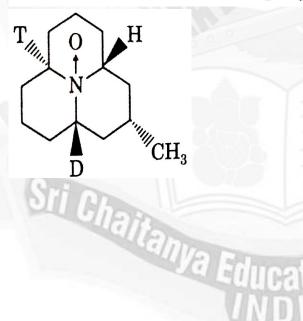


Acidic strength order:

$$H-C-OH > Ph-C-OH > CH_3-COOH$$
O
O
-I < +M
slightly

tional Institutions

**60.** 





# 

**61.** 
$$P(4t_1^2+1,8t_1^3-1)$$
  $Q(4t_2^2+1,8t_2^3-1)$ 

$$\frac{dy}{dx} = \frac{24t^2}{8t} = 3t$$

Slope of tangent at P is  $m_1 = 3t_1$ 

Slope of tangent at Q is  $m_2 = 3t_2$ 

$$m_1 m_2 = -1$$

$$9t_1t_2 = -1$$
 ----(1)

Eq. of tangent at *P* is  $3t_1x - y = 4t_1^3 + 3t_1 + 1$ 

If passes through Q

$$\therefore \left(\frac{t_1}{t_2}\right)^3 - 3\left(\frac{t_1}{t_2}\right) + 2 = 0$$

$$\therefore \frac{t_1}{t_2} = -2 \qquad ----(2)$$

From (1) & (2)
$$t_1 = -\frac{\sqrt{2}}{3}$$

The normal at Q is tangent at the point P

$$\therefore 27(\sqrt{2}x + y) = 35\sqrt{2} - 27$$

Let 
$$y = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$$

$$\ln y = \sin^2 x \cdot \ln \left( \frac{\sqrt{3e}}{2\sin x} \right)$$

$$\frac{1}{y}y' = \ln\left(\frac{\sqrt{3e}}{2\sin x}\right) 2\sin x \cos x + \sin^2 x \frac{2\sin x}{\sqrt{3e}} \frac{\sqrt{3e}}{2} \left(-\cos ecx \cot x\right)$$

$$\frac{dy}{dx} = 0 \Rightarrow \ln\left(\frac{\sqrt{3e}}{2\sin x}\right) 2\sin x \cos x - \sin x \cos x = 0$$

$$\Rightarrow \sin x \cos x \left[ 2 \ln \left( \frac{\sqrt{3e}}{2 \sin x} \right) - 1 \right] = 0$$

$$\Rightarrow \sin x \cos x \left[ 2\ln \left( \frac{\sqrt{3}e}{2\sin x} \right) - 1 \right] = 0$$

$$\Rightarrow \ln \left( \frac{3e}{4\sin^2 x} \right) = 1 \Rightarrow \frac{3e}{4\sin^2 x} = e \Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \sin x = \frac{\sqrt{3}}{2} \left( as \ x \in \left( 0, \frac{\pi}{2} \right) \right)$$

$$\Rightarrow \sin x = \frac{\sqrt{3}}{2} \left( as \ x \in \left(0, \frac{\pi}{2}\right) \right)$$

$$\Rightarrow local \max value = \left(\frac{\sqrt{3e}}{\sqrt{3}}\right)^{3/4} = e^{3/8} = \frac{k}{e}$$

$$\Rightarrow k^8 = e^{11} \Rightarrow \left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8 = e^3 + e^6 + e^{11}$$



## SRI CHAITANYA IIT ACADEMY, INDIA

$$h = 3R \frac{dh}{dt} = 3\frac{dR}{dt}$$

$$V = \frac{1}{3}\pi R^2 \cdot 2R + \frac{2}{3}\pi R^3 \Rightarrow V = \frac{4}{3}\pi R^3 \Rightarrow \frac{dv}{dt} = 2\pi R^2 \frac{dR}{dt} \frac{dv}{dt} = 4\pi \cdot \frac{h^2}{9} \times \frac{1}{3} \frac{dh}{dt}$$

$$=4\pi \frac{81}{27} \times \frac{dh}{dt} = 12\pi$$

**64.** 
$$F(x) = \frac{x^{101}}{2} - 23x^{101} - \frac{45x^2}{2} + 1035x$$

$$F(x) = \frac{x}{2}(x^{100} - 45)(x - 46)$$

**65.** I: 
$$f(x) = x$$
 and  $g(x) = -x^2$  on R

II: 
$$f(x) = \frac{1}{x}, x > 0$$

66. II: 
$$f(x) = \frac{1}{x}, x > 0$$

$$f\left(c\right) \begin{cases} \frac{1}{x;} & \text{if } x^2 > 1 \qquad \Rightarrow x < -1 \qquad or \ x > 1 \\ ax^3 + bx^2; & \text{if } 0 \le x^2 < 1 \qquad \Rightarrow -1 < x < 1 \\ \frac{1/x + ax^3 + bx^2}{2}; & \text{if } x^2 = 1 \end{cases}$$

$$\therefore$$
 f is continuous

$$\therefore$$
 at  $x = 1$ 

and at 
$$x = -1$$

$$1 = a + b$$

$$\therefore b = 0 \frac{-1 = -a + b}{and \quad a = 1} \qquad \dots (1)$$

$$\therefore \quad \text{point A and B are} = (-1,3) \text{ and } (1,1).$$

$$\therefore g'(x) = \lambda (x-1)(x+1)$$

$$g(x) = \lambda \left(\frac{x^3}{3} - x\right) + c$$

$$g\left(-1\right) = \frac{2\lambda}{3} + c = 3$$

$$g(-1) = \frac{2\lambda}{3} + c = 3$$

$$\frac{g(1) = -\frac{2\lambda}{3} + c = -1}{c = 1 \text{ and}} \quad \lambda = 3 \qquad \dots (3)$$

$$\therefore \qquad g(x) = x^3 - 3x + 1$$

$$\therefore \qquad g(2) = 3$$

$$\therefore \qquad g(x) = x^3 - 3x +$$

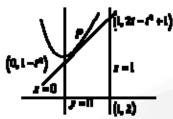
$$\therefore g(2) = 3$$

**67.** 
$$P(t,t^2+1)$$

Tangent at *P* is 
$$y = 2tx + 1 - t^2$$

Area of trapezium = 
$$\frac{1}{2} |2 + 2t - 2t^2| = |1 + t - t^2|$$

Max area if  $t = \frac{1}{2}$ 



**68.** 
$$y = \frac{2}{3}x^3 + \frac{1}{2}x^2$$
  $\therefore \frac{dy}{dx} = \frac{2}{3}3x^2 + \frac{1}{2}2x = 2x^2 + x$ 

Since the tangent makes equal angles with the axes.

$$\Rightarrow \frac{dy}{dx} = \pm 1 \Rightarrow 2x^2 + x = \pm \Rightarrow 2x^2 + x - 1 = 0 \quad (2x^2 + x + 1) = 0 \text{ has no real roots}$$

$$\Rightarrow (2x - 1)(x + 1) = 0 \quad \Rightarrow x = \frac{1}{2} \text{ or } x = -1$$

**69.** 
$$y^2 = x^2 + (150)^2$$

When  $y=250 \Rightarrow x=200$  meters

$$2y \cdot \frac{dy}{dt} = 2x \cdot \frac{dx}{dt} \quad \frac{dy}{dt} = \frac{x}{y} \cdot \frac{dx}{dt} = \frac{200}{250} \times 10 = 8 \text{ meters / sec}$$

70. 
$$(i)4x^2 + 11x + 6 > 0 \ x \in (-\infty, -2) \cup \left(\frac{-3}{4}, \infty\right)$$

$$(ii)$$
4 $x$  + 3  $\in$  [-1,1]  $x$   $\in$  [-1,-1/2]

$$\left(iii\right)\frac{10x+6}{3} \in \left[-1,1\right]$$

$$x \in \left[\frac{-9}{10}, \frac{-3}{10}\right] x \in \left(\frac{-3}{4}, \frac{-1}{2}\right] \quad \alpha = \frac{-3}{4}, \beta = \frac{-1}{2}$$

$$\alpha + \beta = \frac{-5}{4} 36 \left| \alpha + \beta \right| = 45$$

71. 
$$l = Lt \frac{\sin 3x}{x} = 3, \ m = Lt \frac{2 \tan x}{x(1 + \tan^2 x)} = 2$$

The quadratic equation whose roots area 3, 2 is

$$x^{2} - (3+2)x + 3.2 = 0 \Rightarrow x^{2} - 5x + 6 = 0$$

72. 
$$h(x)$$
 graph has sharp edge at  $x=x_0$  if  $f(x_0) = g(x_0)$ 

73. 
$$f(g(x)) = \frac{x^4}{(1-x^2)(1-2x^2)}$$

Dis. Cont. at 
$$x = \pm \frac{1}{\sqrt{2}}, 0, \pm 1$$
.

$$\lim_{h\to 0} \frac{f(c+h) - f(c) + f(c) - f(c-h)}{h}$$



# SRI CH<u>AITANYA IIT ACADEMY, india</u> $\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} + \lim_{h \to 0} \frac{f(c-h) - f(c)}{h}$

$$f^2(c)+f^2(c)$$

(2) is false

75. (a): 
$$f(x)$$

fourth derivative is also odd.

$$f(0) = 0$$

76. 
$$f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6(x - 3)(x - 2)$$

f'(x) = 0 at x = 2,3, so that this is not one – one.

Range of f(x) is [1,29], this is into.

For  $\alpha > 0$ , f(x) will into if; 77.

$$2.2 + \alpha^2 > \alpha \cdot \frac{2}{2} + 10$$
  $\Rightarrow$   $\alpha^2 - \alpha - 6 > 0$ 

$$\Rightarrow (\alpha - 3)9\alpha + 2 > 0 \Rightarrow \alpha^2 - \alpha - 6 > 0$$

$$\alpha_{\min} = 4$$

78. Let 
$$g(x) = (x^{2023} + 1)|(x-5)(x+1)|$$
 not derivable at x=5

Let 
$$h(x) = \sin(|x|)$$
 not derivable at  $x=0$ 

And let  $s(x) = \cos(|x-1|)$  derivable everywhere

$$f(x) = g(x) + h(x) + s(x)$$
 not derivable at 0 and 5 (two point)

**79.** CONCEPTUAL

**80.** Let 
$$f^{-1}(x) = h(x)$$

$$\therefore \qquad g'(x) = \frac{-1}{h^2(x)} h'(x)$$

$$\Rightarrow g'(4) = \frac{-1}{h^2(4)}.h'(4) \text{ where h'}(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow h'(4) = \frac{1}{f'(3)} = \frac{3}{4} = \frac{-1}{9} \cdot \frac{3}{4} = \frac{-1}{12}$$

$$h^{2}(4) \qquad f'(x)$$

$$\Rightarrow h'(4) = \frac{1}{f'(3)} = \frac{3}{4} = \frac{-1}{9} \cdot \frac{3}{4} = \frac{-1}{12}$$
81. 
$$f\left(\frac{3}{2}\right) = 0 \Rightarrow \lim_{x \to \frac{3}{2}} |x^{2} - 3x| + a \le 0$$

$$a \le -\frac{9}{4}$$
Hence  $|4t| = 0$ 

$$a \le -\frac{9}{4}$$

Hence, 
$$|4k| = 9$$

**82.** Hint: 
$$f(x) = 30 - 2x - x^3$$

$$f(x) = -2 - 3x^2 < 0 \Rightarrow f(x)$$
 is decreasing function

Hence 
$$f(f(f(x))) > f(f(-x)) \Rightarrow f(f(x)) < f(-x)$$

$$\Rightarrow f(x) > -x$$

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$$\Rightarrow 30 - 2x - x^3 > -x \Rightarrow x^3 + x - 30 < 0 \Rightarrow (x - 3)(x^2 + 3x + 10) < 0$$

$$\Rightarrow x < 3$$

**83.** 
$$(x^2+1)(y-3) = x \implies y = 3 + \frac{x}{x^2+1}$$

$$m = \frac{dy}{dx} = \frac{1 - x^2}{\left(1 + x^2\right)^2} \qquad \Rightarrow \frac{dm}{dx} = \frac{-2x\left(3 - x^2\right)}{\left(1 + x^2\right)^2}$$

For extremum f'(x) = 0; x=0,  $x = \pm \sqrt{3}$  ; x=0, y=3

$$x = \sqrt{3}, y = 3 + \frac{\sqrt{3}}{4}$$
  $\left(\sqrt{3}, 3 + \frac{\sqrt{3}}{4}\right)$ 

$$x = -\sqrt{3}, y = 3 - \frac{\sqrt{3}}{4}$$
  $\left(\sqrt{3}, 3 - \frac{\sqrt{3}}{4}\right)$ 

84. 
$$\frac{dy}{dx} = \frac{1 - \cos x}{2\sqrt{x + \sin x}} = 0 \Rightarrow \cos x = 1 \Rightarrow x = 0, \pm 2\pi, \pm 4\pi...$$

$$\therefore x \in [-10,10] \Rightarrow x = 0, \pm 2\pi \quad ; \text{ For } x = 0, \ y = 0; \\ x = 2\pi, \ y = \pm \sqrt{2}\pi \end{cases} \\ \text{they satisfy} \quad -3 \leq y \leq 3$$

$$x=-2\pi, y^2=-2\pi$$

Also, slope at (0,0) is undefined hence points are  $(2\pi,2\pi)$  and  $(2\pi,-\sqrt{2\pi})$ 

**85.** 
$$f(1) = f(2) \Rightarrow 1 + m + n = 8 + 4m + 2n \Rightarrow 3m + n + 7 = 0.$$

$$f^{1}(C) = 0 \Rightarrow 3C^{2} + 2mC + n = 0 \Rightarrow \frac{16}{3} + \frac{8m}{3} + n = 0(C = \frac{4}{3})$$

$$\Rightarrow$$
 8m + 3n + 16 = 0 on solving we get m = -5, n = 8 Hence m + n = 3

87. 
$$g(x) = \begin{cases} x^2 + 2x + 5, & x < 0 \\ 2x^2 - 6x + 4, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \end{cases}$$

**88.** 
$$f(x) = \frac{1}{3}(x^2 - 4x + 3)$$

**89.** 
$$g(x) = x^2 \text{ (but } y = 0)$$

90. 
$$f(x) = 2\sin\left(x - \frac{\pi}{2}\right) + 2$$
$$\Rightarrow [\alpha, \beta] = [3, 4] \text{ and } n = 6, m = 2$$