Sri Chaitanya IIT Academy.,India.
A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI
A right Choice for the Real Aspirant
ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS & STERLING_BT | Paper -1(Adv-2021-P1-Model | Date: 03-09-2023
Time: 09.00Am to 12.00Pm | RPTA-05 | Max. Marks: 180

KEY SHEET

PHYSICS

1	**C**	2	**C**	3	**A**	4	**A**	5	**2**	6	**1.1 to 1.2**
7	**3.6 to 3.7**	8	**78 to 79**	9	**2.2 to 2.3**	10	**2.7 to 2.8**	11	**ABCD**	12	**ABD**
13	**ABCD**	14	**CD**	15	**BC**	16	**BD**	17	**7**	18	**4**
19	**9**										

CHEMISTRY

20	**C**	21	**B**	22	**B**	23	**D**	24	**98**	25	**15**
26	**22**	27	**0**	28	**5**	29	**4**	30	**ABD**	31	**AB**
32	**ACD**	33	**BD**	34	**BD**	35	**A**	36	**4**	37	**1**
38	**4**										

MATHEMATICS

39	**C**	40	**A**	41	**A**	42	**B**	43	**1.77**	44	**0.88 to 0.89**
45	**0.48**	46	**0**	47	**0.82**	48	**1.45 to 1.46**	49	**BCD**	50	**ABD**
51	**ACD**	52	**BCD**	53	**ABCD**	54	**ABCD**	55	**9**	56	**1**
57	**4**										

SOLUTIONS

PHYSICS

1. Fig (a) shows component of tension (acting A) along horizontal and vertical. Fig. (b) Shows forces along the line of greatest slope (XX') and perpendicular to the incline. In equilibrium $T = mg$ [considering block B] -------- (1)

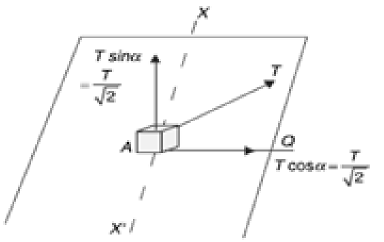


Fig (a)

$$N = \frac{mg}{\sqrt{2}} - \frac{T}{2} = Mg\left[\frac{\sqrt{2}-1}{2}\right] \qquad \text{...............(2)}$$

Forgot about the friction for a moment

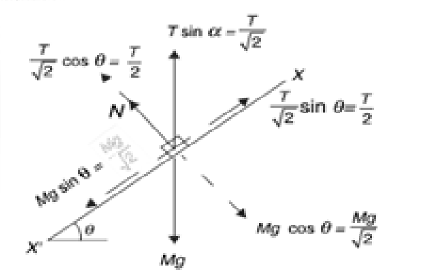


Fig (b)

Unbalanced force along $XX' =$

$$N = \frac{mg}{\sqrt{2}} - \frac{T}{2} = Mg\left[\frac{\sqrt{2}-1}{2}\right] \qquad \text{..........(3)}$$

And there is a force along $AQ = \frac{T}{\sqrt{2}} = \frac{mg}{\sqrt{2}}$ ---------- (4)

The friction will balance the resultant of the force given by (3) and (4)

The resultant has been shown in fig. (C)

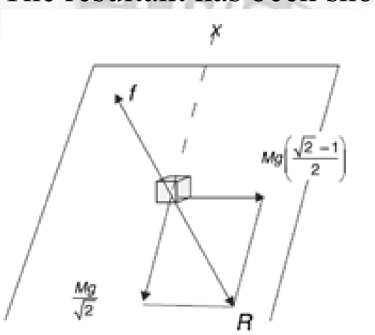


Fig (c)

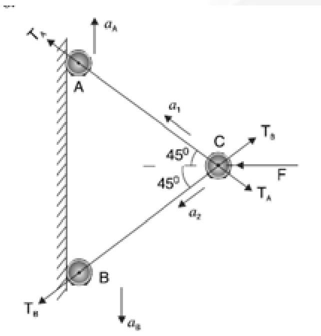
$$\therefore f = Mg\sqrt{\left(\frac{\sqrt{2}-1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$=\frac{\sqrt{5-2\sqrt{2}}}{2}Mg\ \ f \le \mu N$$

$$=\frac{\sqrt{5-2\sqrt{2}}}{2}Mg \le \mu\left(\frac{\sqrt{2}-1}{2}\right)mg$$

$$\Rightarrow \frac{\sqrt{5-2\sqrt{2}}}{\sqrt{2}-1} \le \mu$$

2.



Component of acceleration of C can be assumed along the two roads (the rods are……)as a_1 *and* a_2

since rods are rigid and $a_1 = \frac{a_A}{\sqrt{2}}$ $a_2 = \frac{a_B}{\sqrt{2}}$

For $A\frac{T_A}{\sqrt{2}} = ma_A$……….. (1)

For $B\frac{T_B}{\sqrt{2}} = ma_B$………. (2)

For $C\frac{F}{\sqrt{2}} - T_A = ma_1$………(3)

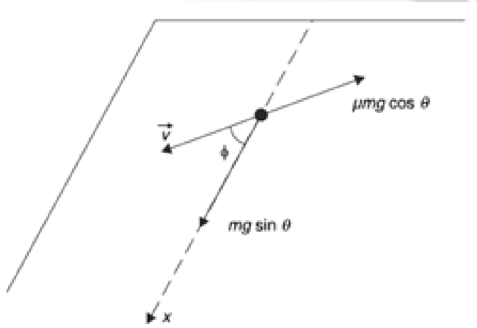
And $\frac{F}{\sqrt{2}} - T_B = ma_2$………(4)

Using (2) and (4)

$$\frac{F}{\sqrt{2}} - 2\sqrt{2}ma_B = ma_2\ \frac{F}{\sqrt{2}} - 2\sqrt{2}ma_B = m = \frac{a_B}{\sqrt{2}}\left(\because a_2 = \frac{a_B}{\sqrt{2}}\right)$$

$$\therefore \frac{F}{\sqrt{2}} = ma_B\left(\frac{1}{\sqrt{2}} + 2\sqrt{2}\right) \qquad \frac{F}{5m} = a_B$$

3. Normal reaction on the particle is always $N = mg\cos\theta$



Friction force on the particle is $f = \mu mg\cos\theta$ and is always opposite of instantaneous velocity. Another force on the particle in the plane of the incline is $mg\sin\theta$ which is always directed along x direction. Force on the particle in tangential direction (i.e., in direction of its velocity) is

$F_t = mg\sin\theta.\cos\phi - \mu mg\cos\theta$

Acceleration in tangential direction is

$a_t = g\sin\theta(\cos\phi - 1)[\because \mu = \tan\theta]$............(a)

Similarly, acceleration of disc along x-direction is

$a_x = g\sin\theta - (\mu g\cos\theta)\cos\phi$

$= g\sin\theta(1-\cos\phi)$..........(b)

$\therefore a_t + a_x = 0$ [Adding (a) and (b)]

Integrating, we get

$v + v_x = c$ [c is constant]

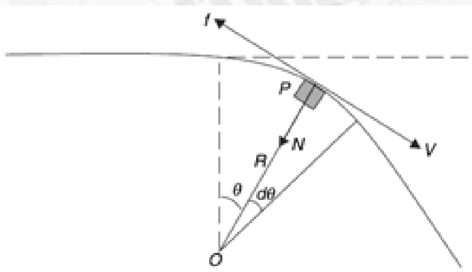
$But\ v_x = v\cos\phi \quad \therefore v + v\cos\phi = c$

Initially $\phi = \frac{\pi}{2}; v = v_0$

$\therefore c = v_0$

$\therefore v = \frac{v_0}{1+\cos\phi}$

4. Consider the object at point P on the curve. Let its velocity at this point be V making an angle θ with the original direction. Let the centre curvature of the path at this point be O and the radius of curvature be R.



The normal force is $N = \frac{mv^2}{R}$

Kinetic friction force on the object is $f = \mu N = \frac{\mu mv^2}{R}$ work done by the friction in small angular displacement $d\theta$ of the block is $dW_f = -f(Rd\theta) = -\mu mv^2 d\theta$ from work energy theorem, work done by the friction is equal to change in KE of the object

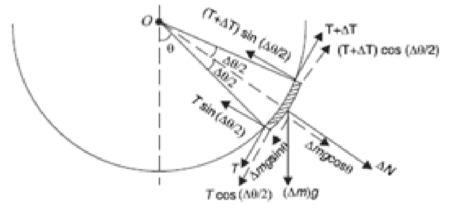
$K = \frac{1}{2}mv^2\ dK = mv\ dv$

$$\therefore mvdv = -\mu mv^2 d\theta \Rightarrow \frac{dv}{v} = -\mu d\theta \Rightarrow \int_u^v \frac{dv}{v} = -\mu\int_0^{\theta_0} d\theta$$

$$\Rightarrow \ln v - \ln u = -\mu\theta_0 \Rightarrow \ln\left(\frac{v}{u}\right) = -\mu\theta_0 \Rightarrow \frac{v}{u} = e^{-\mu\theta_0} \Rightarrow v = ue^{-\mu\theta_0}$$

Answer does not shape of the curve.

5. Consider an element of the rope having angular width $\Delta\theta$.



Length of the element $= R\Delta\theta$

Mass of element $= \lambda R\Delta\theta$

Tension at lower end of the element $= T$

For tangential equilibrium of the element

$$(T+\Delta T)\cos\left(\frac{\Delta\theta}{2}\right) - T\cos\left(\frac{\Delta\theta}{2}\right) = \pi R\Delta\theta.g\sin\theta$$

For small angle $\Delta\theta, \cos\left(\frac{\Delta\theta}{2}\right) = 1 \therefore T + \Delta T - T = \lambda Rg\Delta\theta\sin\theta$

$\Delta T = \lambda Rg.\Delta\theta.\sin\theta$ For $\Delta\theta \to 0$ $dT = \lambda Rg\sin\theta d\theta$ ……….. (1)

For equilibrium in radial direction

$$\Delta N + \lambda Rg\Delta\theta\cos\theta = T\sin\left(\frac{\Delta\theta}{2}\right) + (T+\Delta T)\sin\left(\frac{\Delta\theta}{2}\right)$$

$$\Delta N = T\frac{\Delta\theta}{2} + (T+\Delta T)\frac{\Delta\theta}{2} - \lambda Rg\Delta\theta.\cos\theta$$

$$\left[\because \textit{ for small}\Delta\theta, \sin\left(\frac{\Delta\theta}{2}\right) \to \frac{\Delta\theta}{2}\right]$$

$\Delta T\Delta\theta = 0$ [Product very small quantities]

$\Delta N = T\Delta\theta = -\lambda Rg\Delta\theta.\cos\theta$

$\Delta N \geq 0$ At all points $\Rightarrow T \geq \lambda Rg\cos\theta$ At all points

At bottom (i.e., at $\theta = 0^o$)

$T_1 \geq \lambda Rg$ ……. (2)

From equation (1)

$dT = \lambda Rg\sin\theta d\theta$

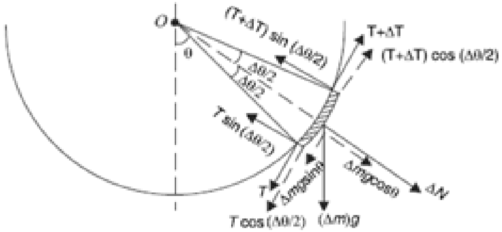
$$\therefore \int_{T_1}^{T_0} dT = \lambda Rg\int_0^{\pi/2}\sin\theta d\theta$$

$$T_0 - T_1 = \lambda Rg$$

$$\therefore T_0 = T_1 + \lambda Rg$$

$$\therefore T_0 \geq 2\lambda Rg\left[u\sin g(2)\right]$$

6. Consider an element of the rope having angular width $\Delta\theta$.



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$\Delta T = \lambda Rg.\Delta\theta.\sin\theta$

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$$\therefore \int_{T_1}^{T_0} dT = \lambda Rg\int_0^{\pi/2}\sin\theta d\theta$$

$T_0 - T_1 = \lambda Rg$

$\therefore T_0 = T_1 + \lambda Rg$

$\therefore T_0 \geq 2\lambda Rg[u\sin g(2)]$

Maximum chance of loosing contact is at lowest point where Tension is λgR

$dT = \lambda Rg\ \sin\theta\ d\theta,$ ($\because$ where $\lambda = \dfrac{dm}{dx}$)

$T - T_{lowest} = \lambda gR\left[1-\cos 30^0\right]$

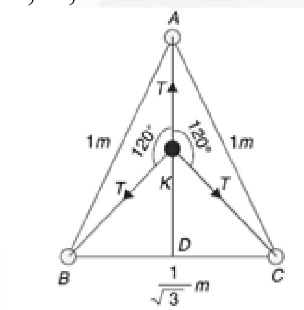
$= \lambda gR\left[1-\dfrac{\sqrt{3}}{2}\right]$

$T = \lambda gR + 0.1339\lambda gR$

$1.1339\lambda gR$

7. If m is mass of each particle, tension in each string is $T = mg$.

For equilibrium of the knot it is necessary that the three strings from120^o angle with each other. The situation has been shows in figure.
A, B, and C are holes and K is the knot.



$$KD = \frac{1}{2\sqrt{3}}\tan 30^o = \frac{1}{6}m$$

$$AD = \sqrt{1^2 - \left(\frac{1}{2\sqrt{3}}\right)^2} = \sqrt{\frac{11}{12}} = \frac{1}{2}\sqrt{\frac{11}{3}}m$$

$$\therefore AK = \frac{1}{2}\sqrt{\frac{11}{3}} - \frac{1}{6}$$

And $BK = CK = \dfrac{1}{2\sqrt{3}\sin 60^o} = \dfrac{1}{3}m$

$\therefore$Length of string on the table is

$$BK + CK + AK = \frac{1}{3} + \frac{1}{2}\sqrt{\frac{11}{3}} - \frac{1}{6} + \frac{1}{3} = \frac{1}{2}\left[1+\sqrt{\frac{11}{3}}\right]m$$

$$PE = -mgh$$

$$= 3\times 10\left(9-\left(AK+BK+CK\right)_{\min}\right)$$

$$= -255\left(1-\sqrt{\frac{11}{867}}\right)$$

$s/r = 78.8$

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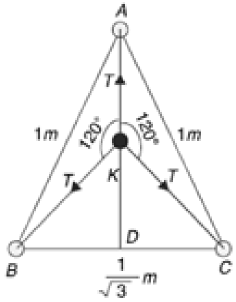

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$$PE = -mgh$$

$$= 3\times 10\left(9 - \left(AK + BK + CK\right)_{\min}\right)$$

$$= -255\left(1 - \sqrt{\frac{11}{867}}\right)$$

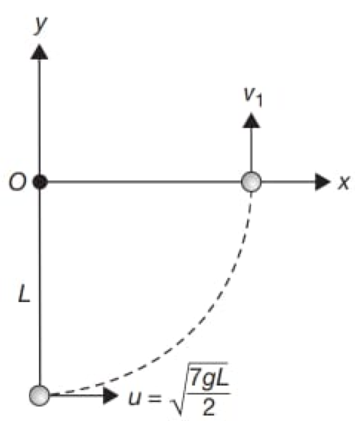
$$s/r = 8.87$$

9. $W_{A to C} = KE_C - KE_A$

$$(mg)(2L) - \mu mg(8L) - 2\mu mg(4L) = \frac{1}{2}mV_C^2$$

$$2mgL - 16\mu mgL = \frac{1}{2}mv_c^2 \; V_C = \sqrt{\frac{7}{2}gL}$$

By law of conservation of energy, we get



$\frac{1}{2}m\frac{7gL}{2} = mgL + \frac{1}{2}mv_1^2 \Rightarrow 7gL = 4gL + 2v_1^2$

$\Rightarrow v_1^2 = \frac{3}{2}gL \quad \Delta\vec{v} = \vec{v}_1 - \vec{u}$

$\Rightarrow \Delta\vec{v} = \left(\sqrt{\frac{3}{2}gL}\right)\hat{j} - \left(\sqrt{\frac{7}{2}gL}\right)\hat{i}$

$\Rightarrow |\Delta\vec{v}| = \sqrt{\frac{3}{2}gL + \frac{2gL}{2}} \Rightarrow |\Delta\vec{v}| = \sqrt{5gL}$

10. $v^2 = gL\sin\left(30^o\right) = \frac{gL}{2}$

$\Rightarrow \vec{v} = -v\sin\theta\hat{i} + v\cos\theta\hat{j} \quad where\ \theta = 30^0$

$\Rightarrow \vec{v} = \sqrt{\frac{gL}{8}}\left(-1\hat{i} + \sqrt{3}\hat{j}\right)$

$p + q = 1.73 + 1 = 2.73$

11. Since the balloon is ascending at a constant rate.

So, $v_0 = \frac{dy}{dt} \Rightarrow dy = v_0 dt \Rightarrow y = v_0 t$

Also, we have

$v_x = \frac{dx}{dt} = ky$

$\Rightarrow dx = kydt = kv_0 tdt \qquad \{\because y = v_0 t\}$

Integrating, we get

$x = kv_0\left(\frac{t^2}{2}\right)$

$\Rightarrow x = k\frac{v_0}{2}\left(\frac{y}{v_0}\right)^2$

$\Rightarrow x = \frac{1}{2}\frac{ky^2}{v_0}$

A) For finding the tangential and normal accelerations, we require an expression for the velocity as a function of height y. so we have

$v_y = v_0 and\ v_x = ky$

$\Rightarrow v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + k^2y^2}$

Therefore tangential acceleration,

$a_T = \frac{dv}{dt} = \frac{k^2y}{\sqrt{v_0^2 + k^2y^2}}\frac{dy}{dt} = \frac{k^2yv_0}{\sqrt{v_0^2 + k^2y^2}}$

$\Rightarrow a = \frac{dv_x}{dt} = k\frac{dy}{dt} = kv_0 \qquad \left\{\because \frac{dv_y}{dt} = 0\right\}$

B) To find the normal acceleration, since we know that

$$a^2 = a_N^2 + a_T^2$$

$$\Rightarrow a_N = \sqrt{a^2 - a_T^2} = \frac{kv_0}{\sqrt{1+\left(\frac{ky}{v_0}\right)^2}}$$

C) $R_C = \frac{v^2}{a_c} = \frac{v_0}{k}\left[1+\left(\frac{ky}{v_0}\right)^2\right]^{3/2}$

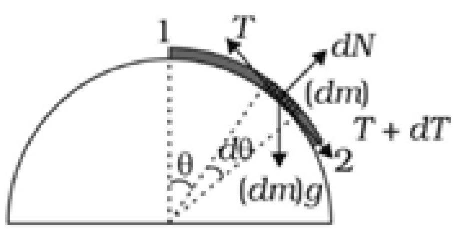
D) Curvature $\alpha \frac{1}{R_c}$

As $y\uparrow$ increases, $R_c \uparrow$, $Curvature \downarrow$

12. I) Initial tangential acceleration of all elements will be same for the element of mass dm:

$$(T+dT)+(dm)g\sin\theta - T = (dm)a_t \Rightarrow dT + \left(\frac{m}{l}rd\theta\right)g\sin\theta = \frac{m}{l}rd\theta a_t$$

Integrating between 1 and 2



$$\Rightarrow \int_{T_1}^{T_2} dT + \frac{m}{l}rg\int_o^{l/r}\sin\theta d\theta = \frac{m}{l}ra_t\int_o^{l/r} d\theta$$

$$\Rightarrow (T_2 - T_1) + \frac{m}{l}rg\left(1-\cos\frac{l}{r}\right) = \frac{m}{l}ra_t\frac{l}{r}$$

But at free ends tension is zero

$\therefore T_1 = T_2 = 0$ $\therefore a_t = \frac{rg}{l}\left(1-\cos\frac{l}{r}\right)$

II) Integrating between 1 and θ

$$(T-O)+\frac{m}{l}rg(1-\cos\theta) = \frac{mr}{l}\left[\frac{rg}{l}\left(1-\cos\frac{l}{r}\right)\right]\theta$$

Differentiating w.r.t θ:

$$\frac{dT}{d\theta}+O+\frac{mr}{l}\left[\frac{rg}{l}\left(1-\cos\frac{l}{r}\right)\right]$$

For $T_{max'}\frac{dT}{d\theta} = o$ $\Rightarrow \sin\theta = \frac{r}{l}\left(1-\cos\frac{l}{r}\right)$

$$\alpha = 90-\theta \Rightarrow \alpha\cos^{-1}\left(\frac{r}{\ell}\left(1-\frac{\cos\ell}{r}\right)\right)$$

D) To prevent slipping

$F_{friction} = F_{weight}$

$\mu\left(\frac{m}{l}\right)g(r)\sin\alpha = \left(\frac{m}{l}\right)(r)(1-\cos\alpha)g$

$\mu = \frac{1-\cos\alpha}{\sin\alpha}$ $\left[\alpha = \frac{l}{r}\right]$

$= \tan\left(\frac{\alpha}{2}\right)$

13. Conceptual

14. A) Only vertical force present in the system. So vertical momentum does not change.

B) Also horizontal velocity will remain same.

Let's work from CM frame, in this frame disk moves in circular path. For circular motion

$mg\cos\theta - ma_0\sin\theta - N = \frac{mV_1^2}{R}$ ……..(i)

$'V_1'$ is velocity of disk relative to the hemisphere

According to conservation of linear momentum $m(V_1\cos\theta - V_2) - mV_2$

$\Rightarrow V_2 = \frac{V_1\cos\theta}{2}$ ………(ii)

WE theorem (for whole system)

$mgR(1-\cos\theta) = \frac{1}{2}mV_2^2 + \frac{1}{2}m\left\{(V_1\cos - V_2)^2 + V_1^2\sin^2\theta\right\}$ ……(iii)

At the time of breakoff $N \to 0 \Rightarrow a_0 \to 0\left(Because\ N\sin\theta = ma_0\right)$

So from (i)

$V_1^2 = gR\cos\theta$ ……..(iv)

Now from (i) and (iii)

$2gR(1-\cos\theta) = \frac{(V_1\cos\theta)^2}{4} + \frac{(V_1\cos\theta)^2}{4} + V_1^2\sin^2\theta$

$= \frac{V_1^2\cos^2\theta}{2} + V_1^2(1-\cos^2\theta)$

$2gR(1-\cos\theta) = V_1^2 - V_1^2\frac{\cos^2\theta}{2}$

From (iv),

$2gR(1-\cos\theta) = gR\cos\theta - \frac{gR\cos\theta.\cos^2\theta}{2}$

$2 - 2\cos\theta = \cos\theta - \frac{\cos^3\theta}{2}; 2(2-2\cos\theta) = 2\cos\theta - \cos 5\theta$

$\Rightarrow \cos^3\theta - 6\cos\theta + 4 = 0 \Rightarrow \left(\cos\theta = \sqrt{3}-1\right)$

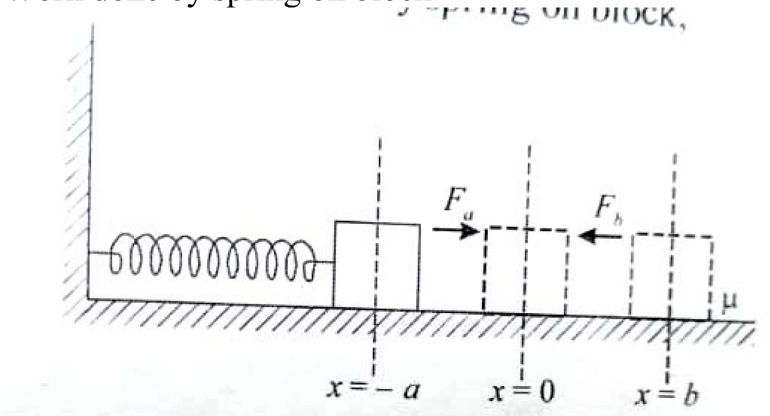
$(\cos\theta - 2)(\cos^2\theta + 2\cos\theta - 2) = 0$

$\cos\theta = \frac{-2 \pm \sqrt{4+8}}{2}$

$= -1 \pm \sqrt{3}$

D) Disc will be accelerating only not bowl.

15. Work done by spring on block



$$w_s = \frac{1}{2}ka^2 - \frac{1}{2}kb^2 = \frac{1}{2}k\left(a^2 - b^2\right)$$

By work energy theorem,

$$K_i + w = K_f$$

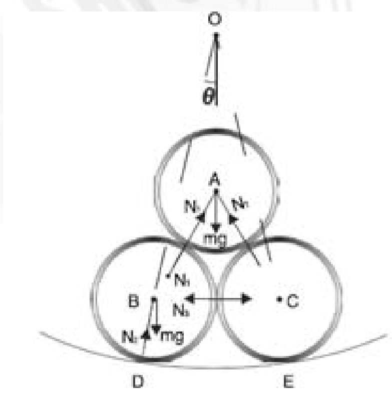
$$0 + \frac{1}{2}ka^2 - \mu mg\left(a+b\right) - \frac{1}{2}kb^2 = 0$$

$$\mu mg\left(a+b\right) = \frac{1}{2}k\left(a^2 - b^2\right)$$

$$\mu = \frac{k\left(a-b\right)}{2mg}$$

16. Force applied by D is more but velocity of string in his hands is less in the same proportion. Hence the power is same.

17.



$O =$ Centre of larger cylinder.

DO will pass through centre of lower cylinder (B)

$N_1 =$ Contact force between A and B (and A and C)

$N_2 =$ Contact force between B and the large cylinder

$N_3 =$ Contact force between B and C.

<u>For the vertical equilibrium of A</u>

$$2N_1 \cos 30^o = mg \Rightarrow \sqrt{3}N_1 = mg \text{ ....(1)}$$

For horizontal equilibrium of B

$N_1 \sin 30^o = N_2 \sin\theta$

$\frac{mg}{2\sqrt{3}} = N_2 \sin\theta$ ....(2)

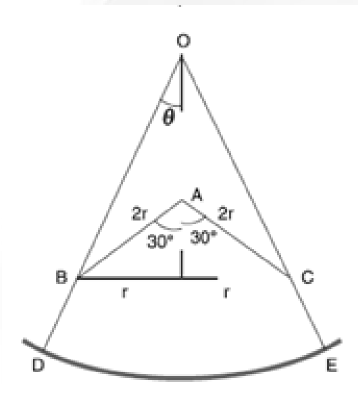
For vertical equilibrium of whole system

$2N_2 \cos\theta = 3mg$

$\therefore N_2 \cos\theta = \frac{3}{2}mg$ ....(3)

$(2) \div (3)$

$\tan\theta = \frac{1}{3\sqrt{3}} \Rightarrow \frac{r}{\sqrt{(R-r)^2 - r^2}} = \frac{1}{3\sqrt{3}}$



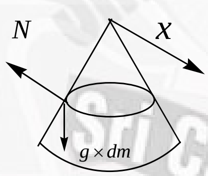
$\Rightarrow 27r^2 = (R-r)^2 - r^2$

$\Rightarrow (R-r)^2 = 28r^2$

$R - r = 2\sqrt{7}r$

$\therefore R = r(1+2\sqrt{7})$

18.



Consider the potential energy of the band as a function of R. To be in equilibrium, the band must be at a position of minimum potential energy. Let the elastic potential energy be$U_i = \frac{1}{2}k(2\pi R - 2\pi R)^2$ and the gravitational potential energy measured with respect to the vertex of the code$U_i = -mg(R-r)\cot\theta$

The total potential energy is $U = 2k\pi^2(R-r)^2 - mg(R-r)\cot\theta$

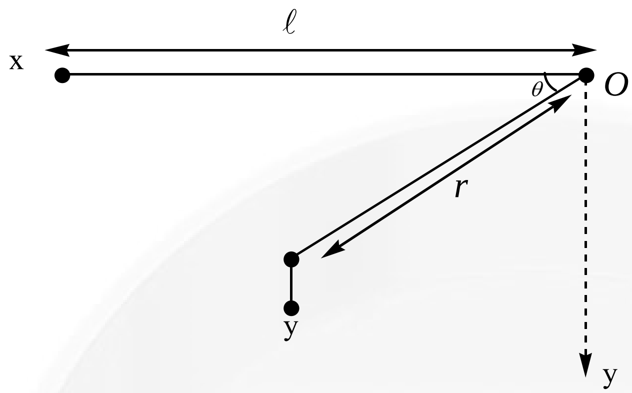
At the minimum position,

$\frac{dU}{dR} = 4\pi^2 k(R-r) - mg\cot\theta = 0$

Which has solution

$$R = r + \frac{mg}{4\pi^2 k}\cot\theta$$

19.



Let the second nail be located as a distance r from the first nail and angle θ below horizontal as shown. Now speed of the bob at its lowest position can be found using conservation of energy.

$$\frac{1}{2}mv^2 = mg\left[r\sin\theta + \ell - r\right]$$

$$\Rightarrow v^2 = 2g\left(r\ \sin\theta + \ell - r\right) \qquad \text{.......(i)}$$

The particle will complete circle about the second nail if $v^2 = 5g\left(\ell - r\right)$ ......(ii)

From (i) and (ii)

$$2\left(r\sin\theta + \ell - r\right) = 5\left(\ell - r\right)$$

$$\Rightarrow 3r + 2r\sin\theta = 3\ell \qquad \text{.......(iii)}$$

Now coordinates of the nail are

$x = r\cos\theta, y = r\sin\theta$

Using these in (iii), we get

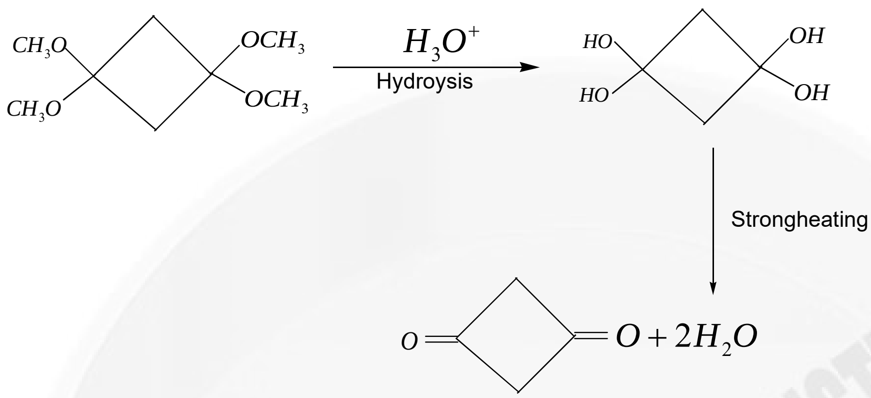
$$3\sqrt{x^2 + y^2} + 2y = 3\ell$$

$$\text{Or}\, 9\left(x^2 + y^2\right) = \left(3\ell - 2y\right)^2$$

CHEMISTRY

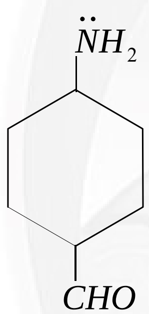
20. **Reason-I:** Intermolecular H-bonding and intramolecular H-bonding
Reason-II: Molecular weight

21.



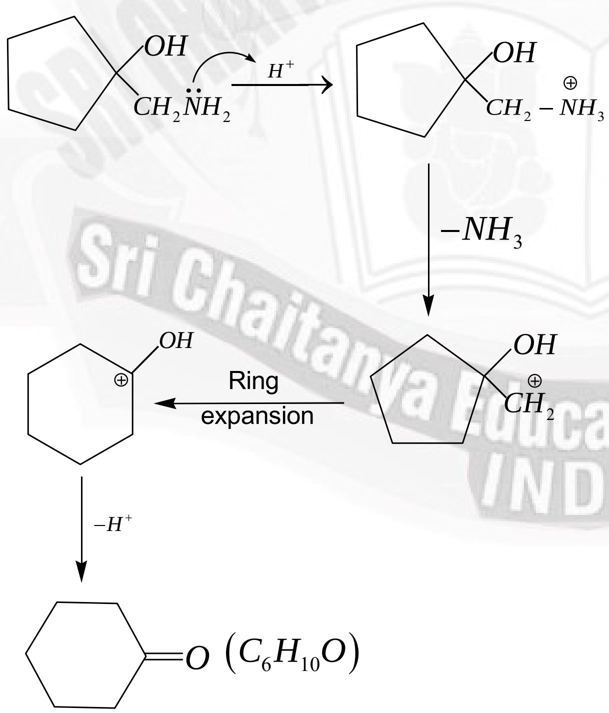
22. Tertiary Carbocation is more stable in option (B)

23.



N- atom have Sp^3 hybridization So, %S character is less and Lone Pairs are localized

24.



25. $CH_3-CH_2-NH_2 \xrightarrow{HNO_2} CH_3-CH_2-CH_2-OH$

now, $59gm.\xrightarrow{will\ give}60gm$

So, $14.75gm.\xrightarrow{will\ give}\dfrac{14.75\times60}{59}=15gm$

26.

$$CH_3-CH=\underset{\substack{|\\C_2H_5(X)}}{C}-CH_2-CH_3\xrightarrow[H_2O_2]{HBr}(CH_3-CH_2)_2-CH-\overset{\substack{Br\\|}}{C}H-CH_3(P)$$

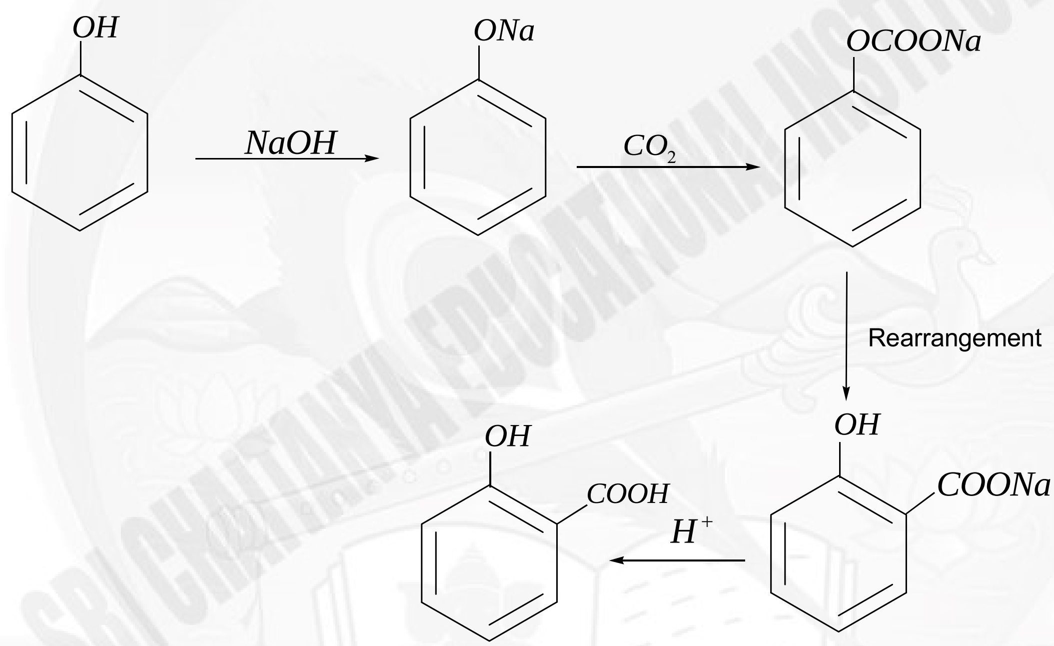
NO.of σ-bonds=22 in product (P)

27.

$$CH_3-CH=\underset{\substack{|\\C_2H_5}}{C}-\underset{(X)}{CH_2CH_3}\xrightarrow[NaBH_4]{Hg(ACO)_2/H_20}\underset{(Y)}{(C_2H_5)_3C-OH}$$

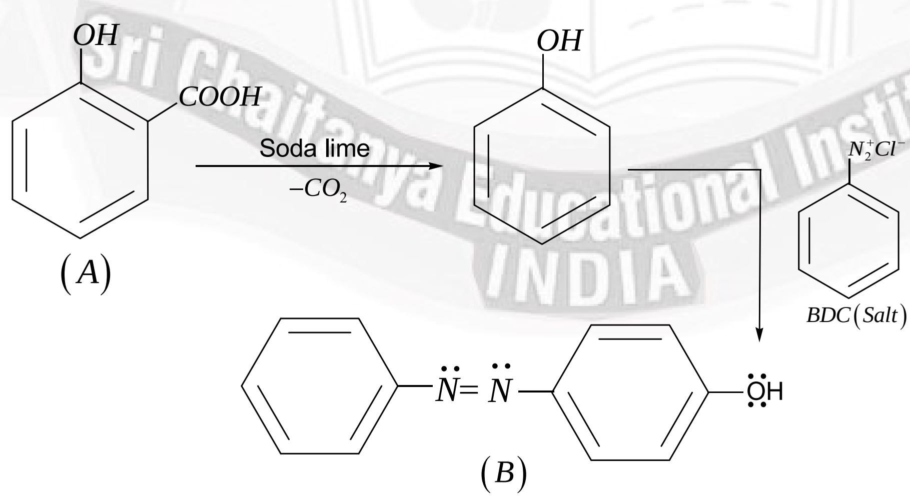
Oximercuration demerculation gives Markovnikov's product without any rearrangement.

28.



$C_7H_6O_3(A)$

29.

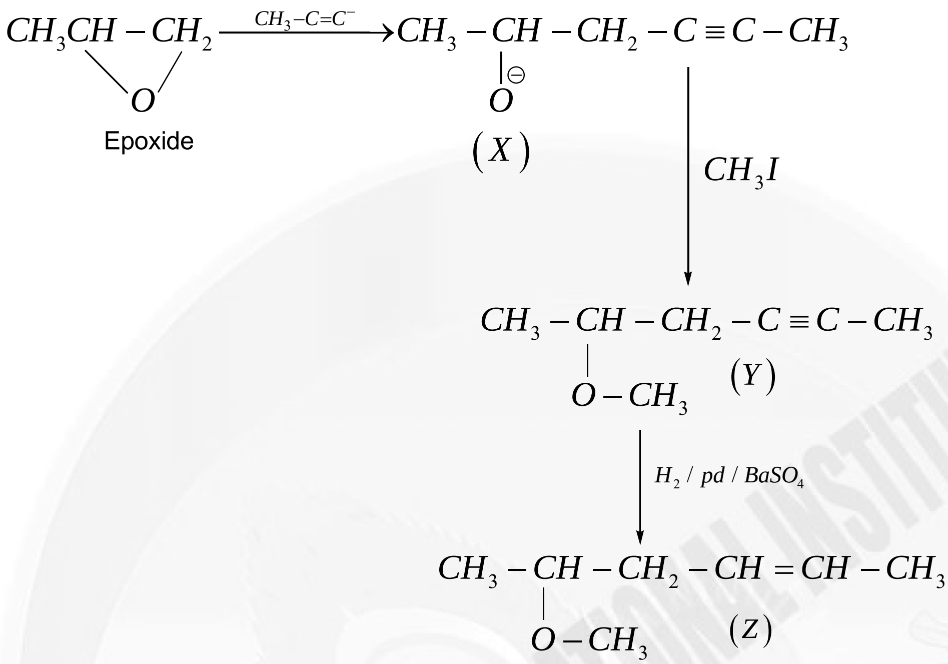


There are 4-lone pairs

30. Conceptual

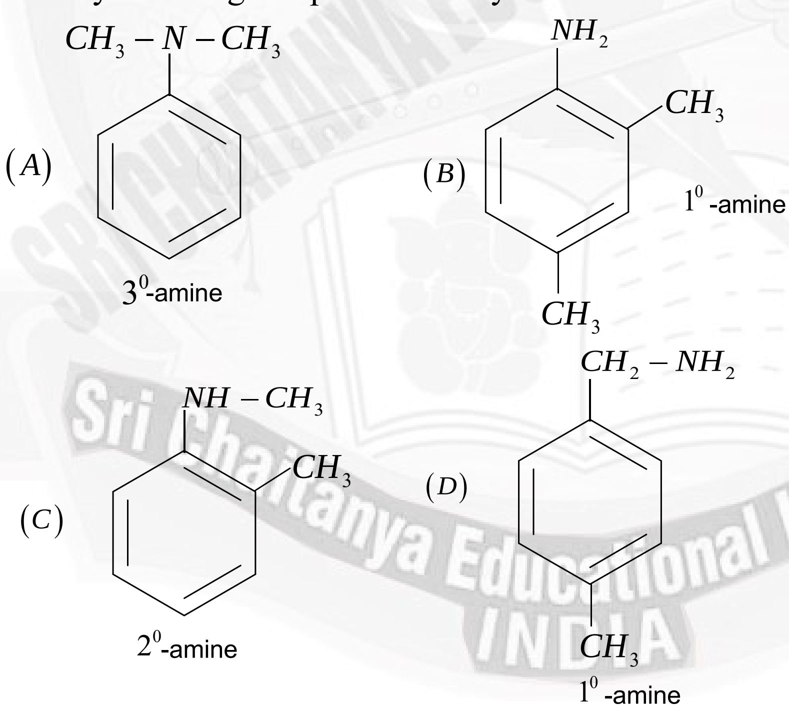
31. $CH_3CH_2OH \xrightarrow{(CH_3CO)_2O} CH_3COOH + CH_3COOC_2H_5$

32.

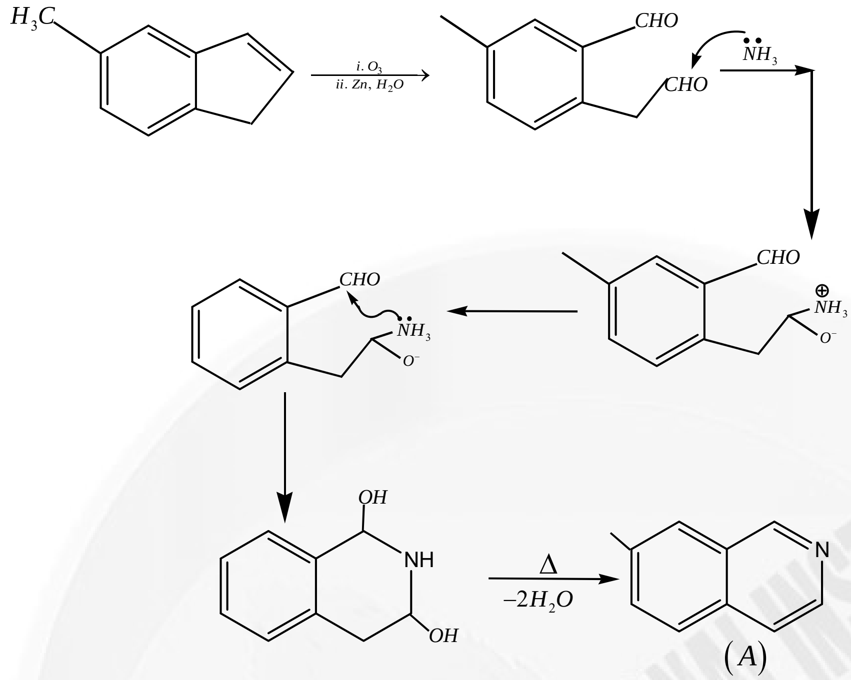


33. Phenol gives violet colored complex with neutral $Fecl_3$ and benzoic acid gives brisk effervescence with $NaHCO_3$

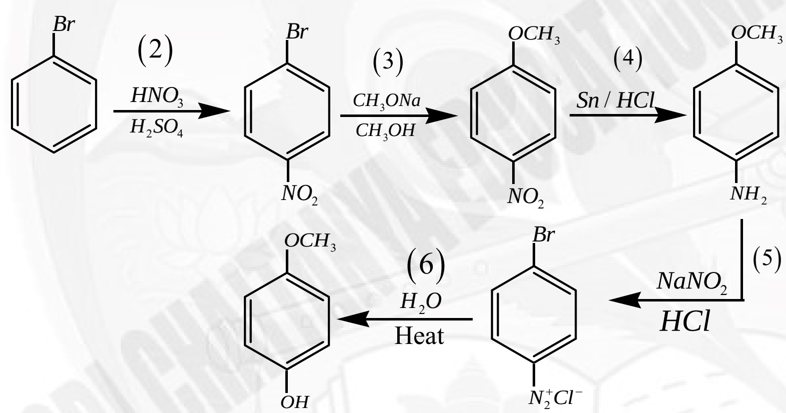
34. Primary amines gives positive carbylamines test



35.

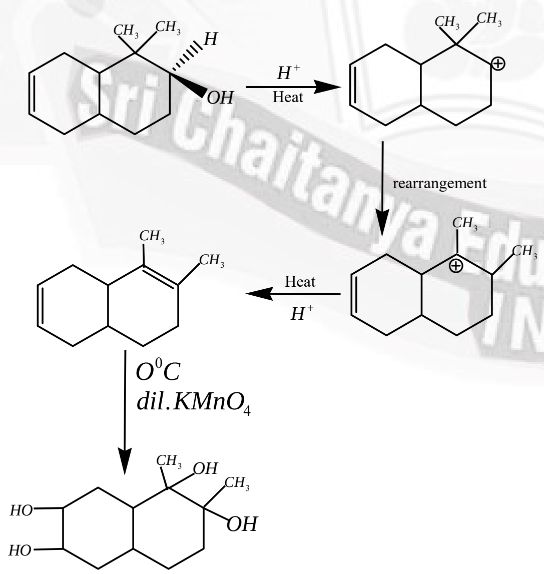


36.



37. More stable carbocation is possible on $C-1$

38.



MATHEMATICS

39. $\left(\frac{\{x\}}{1+\{x\}}\right)=\frac{y}{1+y}\leq\frac{\pi}{4}$

Angle of $\tan\left(\frac{\{x\}}{1+\{x\}}\right)$ is less than $\frac{\pi}{4}$.

$\left[\tan\left(\frac{\{x\}}{1+\{x\}}\right)\right]=0$

$x/4\in(0,1)$

$\left\{\frac{x}{4}\right\}=\frac{x}{4}$ $\int_0^4\frac{x}{4}dx=\left|\frac{x^2}{8}\right|_0^4=2$

40. $(x^4+8x^3+18x^2+16x+5)=(x+1)^4+4(x+1)^3$

$81e^2-1$

41. $=\int_{-3/2}^{-1/2}((x+1)^5-2x^2-4x-1)dx$

$=\int_{-3/2}^{-1/2}((x+1)^5-2(x+1)^2+1)dx=\int_{-1/2}^{1/2}(y^5-2y^2+1)dy$

$=0-\frac{4}{3}\times\frac{1}{8}+1=\frac{5}{6}$

42.

$\int_2^{12}\frac{1+\sqrt{1+4x}}{2x}dx$

$\int_2^{12}\frac{1}{2x}dx+\int_2^{12}\frac{\sqrt{1+4x}}{2x}dx$

$\frac{1}{2}\ln x\Big|_2^{12}+\int_3^7\frac{x^2}{x^2-1}dx$

$\frac{1}{2}\ln 6+\int_3^7\frac{x^2}{x^2-1}dx$

43. $I=2e^4\int_0^\infty e^{-x^2-4/x^2}\,dx.$

$x\mapsto\frac{2}{x}$

$I=2e^4\int_0^\infty\frac{2}{x^2}e^{-4/x^2-x^2}\,dx.$ N

Adding these then gives

$I=e^4\int_0^\infty\left(1+\frac{2}{x^2}\right)e^{-x^2-4/x^2}\,dx=e^4\frac{1}{e^4}\int_0^\infty\left(1+\frac{2}{x^2}\right)e^{-(x-2/x)^2}\,dx.$

$u=x-\frac{2}{x}$ $I=\int_{-\infty}^{\infty}e^{-u^2}\,du=\sqrt{\pi}=1.77$

44. Substitute $x\leftrightarrow\sqrt{n}x$ $\lim_{n\to\infty}\int_{-\frac{\sqrt{n}}{2}}^{\frac{\sqrt{n}}{2}}\left(1-\frac{4x^2}{n}+\frac{x^4}{n^2}\right)^n dx$

$=\int_{-\infty}^{\infty}e^{-4x^2}\,dx$

$=\frac{1}{2}\int_{-\infty}^{\infty}e^{-x^2}\,dx$

$=\int_0^\infty e^{-x^2}\,dx=0.88$

45. $\int_0^1 \binom{207}{7} x^{200}(1-x)^7\,dx = \frac{1}{208}$

46. $= \frac{1}{2025!}\int_0^{2024} x(x-1)(x-2)......................(x-2024)dx$

$= \frac{1}{2025!}\int_{-2012}^{2012} (x-2012)(x-2011).......(x-1)x$

$(x+1)............(x+2011)(x+2012)dx$

$= \frac{1}{2025!}\int_{-2}^{2} x(x^2-1^2)(x^2-2^2)..............(x^2-2012^2)dx$

$=0$

47. $\sum_1^\infty \int_{1/(n+1)}^{1/n} nx dx = \sum_1^\infty \frac{n}{2}\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right) = \frac{1}{2}\sum_1^\infty\left(\frac{1}{n}-\frac{n+1-1}{(n+1)^2}\right)$

$= \frac{1}{2}\sum_1^\infty\left(\frac{1}{n}-\frac{1}{n+1}+\frac{1}{(n+1)^2}\right) \quad = \frac{1}{2}\left(1+\sum_2^\infty \frac{1}{n^2}\right) = \frac{\pi^2}{12} = 0.82$

48. $\int_0^1 \frac{[(x+1)+x^2-x^2]\log(x)}{x^3-1}\mathrm{d}x = \int_0^1 \frac{\log x}{x-1}\mathrm{d}x - \int_0^1 \frac{x^2\log x}{x^3-1}\mathrm{d}x.$

$\int_0^1 \frac{x^2\log x}{x^3-1}\mathrm{d}x = \frac{1}{3}\int_0^1 \frac{\log u^{1/3}}{u-1}\mathrm{d}u = \frac{1}{9}\int_0^1 \frac{\log x}{x-1}\mathrm{d}x.$

$I_0 = \frac{8}{9}\int_0^1 \frac{\log x}{x-1}\mathrm{d}x.\ I_0 = (-1)\frac{8}{9}\int_0^1 \sum_{n=0}^\infty x^n\cdot\log x\mathrm{d}x = (-1)\frac{8}{9}\sum_{n=0}^\infty \int_0^1 x^n\cdot\log x\mathrm{d}x.$

$I_0 = \frac{8}{9}\sum_{n=0}^\infty \frac{1}{(1+n)^2} = \frac{4\pi^2}{27} = 1.45$

49.

$\int_0^{\frac{\pi}{2}} x\tan 2x\ln(\tan x)dx,\ x \mapsto \frac{\pi}{2}-x$

$I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \tan 2x\ln(\tan x)dx, \quad \tan x = \frac{\sin 2x}{1+\cos 2x}$

$I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}}\left[\frac{\sin 2x}{\cos 2x}\ln(\sin 2x) - \frac{\sin 2x}{\cos 2x}\ln(1+\cos 2x)\right]dx$

$J \overset{x\to\pi-x}{=} -J \Rightarrow J=0, \qquad I = -\frac{\pi}{8}K.$

$K \overset{y=\cos x}{=} \int_{-1}^{1}\frac{\ln(1+y)}{y}dy$

50.

$I = \int_0^\infty \frac{\arctan(1/x)}{\sqrt{x}(1+x)}\mathrm{d}x$

$= \frac{1}{2}\int_0^\infty \frac{\arctan(x)+\arctan(1/x)}{\sqrt{x}(1+x)}\mathrm{d}x$

$= \frac{\pi}{4}\int_0^\infty \frac{\mathrm{d}x}{\sqrt{x}(1+x)} = \frac{\pi^2}{4}$

51. $= \int_0^\infty \frac{x}{x^8-1} - \int_0^\infty \frac{1}{x^8-1} = \int_0^\infty \frac{1/2}{x^4-1} - \int_0^\infty \frac{1}{x^8-1}$

$$=\int_0^{\infty}\frac{1/2(x^4+1)}{x^8-1}-\int_0^{\infty}\frac{1}{x^8-1}=\int_0^{\infty}\frac{1/2(x^4-1)}{x^8-1}=\frac{1}{2}\int_0^{\infty}\frac{1}{x^4+1}\quad=\frac{\pi}{4\sqrt{2}}$$

52.
$$=\int_0^{\infty}\frac{1}{(1+x^{-2023})(1+x^{-2})}\frac{1}{x^2}dx=\int_0^{\infty}\frac{x^{2023}}{(1+x^{2023})(1+x^2)}dx$$

$$2I=\int_0^{\infty}\frac{1}{(1+x^{2023})(1+x^2)}dx+\int_0^{\infty}\frac{x^{2023}}{(1+x^{2023})(1+x^2)}dx$$

$$=\int_0^{\infty}\frac{1+x^{2023}}{(1+x^{2023})(1+x^2)}dx=\int_0^{\infty}\frac{1}{1+x^2}dx=\frac{\pi}{2}\qquad I=\frac{\pi}{4}$$

53
$$\int_{1/2}^{2}\log\log\left(x+\frac{1}{x}\right)dx$$

$$=\int_0^{3/2}\log\log\sqrt{x^2+4}\,dx=\int_{1/2}^{2}\log\log\sqrt{x^2-x+\frac{17}{4}}\,dx$$

$$\int_{\frac{1}{2}}^{2}\log\left(\frac{\log(x+\frac{1}{x})}{\log(x^2-x+\frac{17}{4})}\right)dx$$

$$=\int_{1/2}^{2}\left(\log\log\left(x+\frac{1}{x}\right)-\log\log\sqrt{x^2-x+\frac{17}{4}}-\log 2\right)dx$$

$$=-\int_{1/2}^{2}\log 2dx=-\frac{3}{2}\log 2$$

54.
$$=\int_0^{\pi/6}\sqrt{\tan(\theta)+\sec(\theta)}\,\mathrm{d}\tan(\theta)\quad=\int_0^{\pi/6}\sqrt{\tan(\theta)+\sec(\theta)}\sec^2(\theta)\mathrm{d}\theta$$

$$=\frac{1}{\sqrt[4]{3}}-\int_0^{\pi/6}\tan(\theta)\frac{\sec^2(\theta)+\tan(\theta)\sec(\theta)}{2\sqrt{\tan(\theta)+\sec(\theta)}}\mathrm{d}\theta$$

$$=\frac{1}{\sqrt[4]{3}}-\frac{1}{2}\int_0^{\pi/6}\sqrt{\tan(\theta)+\sec(\theta)}\,\mathrm{d}\sec(\theta)=\frac{1}{2}+\frac{1}{2}\int_0^{\pi/6}\sec(\theta)\frac{\sec^2(\theta)+\tan(\theta)\sec(\theta)}{2\sqrt{\tan(\theta)+\sec(\theta)}}\mathrm{d}\theta$$

$$=\frac{1}{2}+\frac{1}{4}\int_0^{\pi/6}\sqrt{\tan(\theta)+\sec(\theta)}\sec^2(\theta)\mathrm{d}\theta\qquad=\frac{2}{3}$$

55.

When plotted on graph we can see that these
create triangles of area $1/2$ between any two intervals $[n,n+1]$
The value of this integral is

56. $\lim_{n\to\infty}\int_0^{\infty}\left(1+\frac{t}{n}\right)^{-n}\cdot\cos\left(\frac{t}{n}\right)dt=\int_0^{\infty}e^{-t}dt=1$

57. In each interval
$[1,2],[2,3],[3,4]$

The area is $\frac{1}{2}$ which means that the answer is $\frac{8}{2}=4$