Sri Chaitanya IIT Academy.,India.

A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS &ALL_BT JEE-MAIN Date: 17-01-2023

Time: 09.00Am to 12.00Pm GTM-07 Max. Marks: 300

KEY SHEET

PHYSICS

1)	**1**	2)	**4**	3)	**4**	4)	**3**	5)	**3**
6)	**2**	7)	**1**	8)	**2**	9)	**1**	10)	**2**
11)	**3**	12)	**1**	13)	**3**	14)	**1**	15)	**2**
16)	**2**	17)	**4**	18)	**2**	19)	**2**	20)	**1**
21)	**18**	22)	**1**	23)	**16**	24)	**48**	25)	**136**
26)	**1**	27)	**2**	28)	**3**	29)	**2400**	30)	**412**

CHEMISTRY

31)	**1**	32)	**3**	33)	**1**	34)	**3**	35)	**1**
36)	**2**	37)	**2**	38)	**1**	39)	**2**	40)	**3**
41)	**4**	42)	**4**	43)	**1**	44)	**3**	45)	**2**
46)	**4**	47)	**1**	48)	**3**	49)	**2**	50)	**1**
51)	**42**	52)	**30**	53)	**1**	54)	**1**	55)	**3**
56)	**0**	57)	**4**	58)	**2**	59)	**24**	60)	**2**

MATHEMATICS

61)	**4**	62)	**3**	63)	**4**	64)	**3**	65)	**3**
66)	**3**	67)	**2**	68)	**3**	69)	**4**	70)	**4**
71)	**3**	72)	**1**	73)	**2**	74)	**3**	75)	**1**
76)	**2**	77)	**1**	78)	**4**	79)	**4**	80)	**4**
81)	**75**	82)	**0**	83)	**1062**	84)	**5376**	85)	**1**
86)	**2**	87)	**12**	88)	**1552**	89)	**12**	90)	**2**

SOLUTIONS
PHYSICS

1.

0	0	0
1	1	1
0	1	0
1	0	0

From the truth table
Y = A.B
So it is "AND" gate

2. $d=\sqrt{2Rh}$ $\Rightarrow d\propto\sqrt{h}\Rightarrow\frac{h^1}{h}=\left(\frac{d^1}{d}\right)^2$ $\Rightarrow h^1=900\,m$

3. Given, $\lambda=\frac{h}{mv_0}$

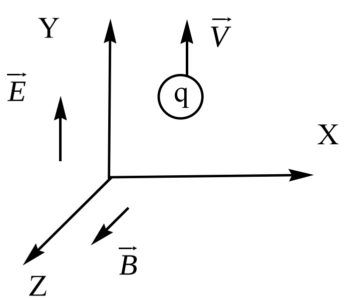
Velocity of an electron after time 't'

$V=V_0-\frac{E(-e)}{m}t=V_0+\frac{Ee}{m}t$ $\therefore$ Wave length

$\lambda=\frac{h}{mv}=\frac{h}{m\left[v_0+\frac{Ee}{m}t\right]}$ $=\frac{\lambda_0}{1+\frac{eE_0}{mv_0}t}$

4. $A=A_0e^{-\lambda t}$

Then $\frac{A}{A_0}=\frac{1}{(2)^{t/T}}\Rightarrow 4=\frac{t}{T}$ $\therefore T=\frac{30}{4}=7.5$



5.

$C=\frac{E_0}{B_0}$

$F_E=E_0q$

$F_B=q\vartheta B_0$

$\frac{F_E}{F_B}=\frac{C}{V}=10$

6. $R.P=\frac{2\mu\tan\beta}{1.22\lambda}\propto\mu$

$\frac{R.P_{(medium)}}{R.P_{(air)}}=\frac{\mu_m}{\mu_{air}}=\frac{2}{1}$

7. $\tan\phi^1=\frac{1}{\cos\theta}\times\tan\phi$

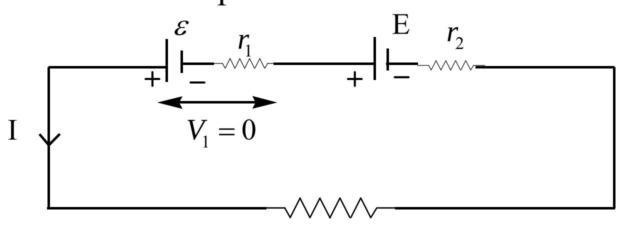
$$\tan 60 = \frac{1}{\cos 45^0} \times \tan\phi$$

Actual dip $\phi = \tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$

8. $H_{(DC)} = I_{DC}^2 \times R = (4)^2 \times 3 = 16 \times 3 \ J$

$H_{(AC)} = (I_{rms})^2 \times R^1 = (4/\sqrt{2})^2 \times 2 = 16 \ J$

$\therefore$ ratio of heat produces is 3:1



9.

$I = \frac{\varepsilon + \varepsilon}{R + r_1 + r_2} = \frac{2\varepsilon}{R + r_1 + r_2}$ ____(1)

But $V_1 = \varepsilon - Ir_1$ _____(2)

Given $V_0 = 0$ _____(3)

From (1),(2) and (3), $R = r_1 - r_2$

10. $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$$\frac{M_1}{M_2} = \frac{I_1}{I_2} \times \left(\frac{T_2}{T_1}\right)^2 = \frac{8}{3}$$

11. A. RMS velocity $= \sqrt{\frac{3RT}{M}}$

T is same & M is same so, RMS velocities will be same

So, A is correct

B. $n_1 : n_2 = 1:4$

$p_1 : p_2 = \frac{n_1RT_1}{V_1} : \frac{n_2RT_2}{V_2}$

$n_1 : n_2 = 1:4$

$[T_1 = T_2, V_1 = V_2]$

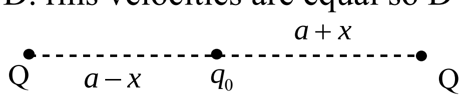
So, B is correct

C. $P_1 : P_2 = 1:4$ and not 1 : 1

So, C is wrong

D. rms velocities are equal so D is wrong

12.



$$F_{net} = \frac{KQq_0}{(a-x)^2} - \frac{KQq_0}{(a+x)^2} \text{ towards right}$$

When displacement is x towards left

Sq $$F_{net} = -\frac{KQq_0}{(a-x)^2} + \frac{KQq_0}{(a+x)^2}x$$

$$= -KQq_0 \frac{\left[(a+x)^2 - (a-x)^2\right]}{\left(a^2 - x^2\right)^2} = -KQq_0 \frac{4ax}{a^4}$$

$$= -\frac{KQq_0 4ax}{4\pi\varepsilon_0 a^3} = \frac{-Qq_0}{\pi\varepsilon_0 a^3}x$$

$$a = \frac{F_{net}}{m} = -\frac{Qq_0}{\pi\varepsilon_0 m a^3}x$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\pi\varepsilon_0 m a^3}{Qq_0}} \qquad = \sqrt{\frac{4\pi^3\varepsilon_0 m a^3}{Qq_0}}$$

13. $R_1 : R_2 = \frac{L_1}{K_1A_1} : \frac{L_2}{K_2A_2} \quad = \frac{L_1}{L_2}.\frac{K_2}{K_1}.\frac{A_2}{A_1} = 2.9.\frac{1}{2} = 9$

Let junction temperature be T

Then , $\frac{T-450}{R_1} + \frac{T-0}{R_2} = 0$

$$\Rightarrow T = \frac{R_1R_2}{R_1+R_2}\left(\frac{450}{R_1} + \frac{0}{R_2}\right)$$

$$\Rightarrow T = \frac{1}{1+R_2/R_2}\left(\frac{450R_2}{R_1}\right) \qquad = \frac{9}{10}\times\frac{450}{9} = 45^0C$$

14. A. Small temperature difference allows use of newton's law of cooling

$$\frac{dQ}{dt} = -kA(\theta-\theta_0)$$

$(\theta-\theta_0)$ is doubled $\Rightarrow \frac{dQ}{dt}$ doubled

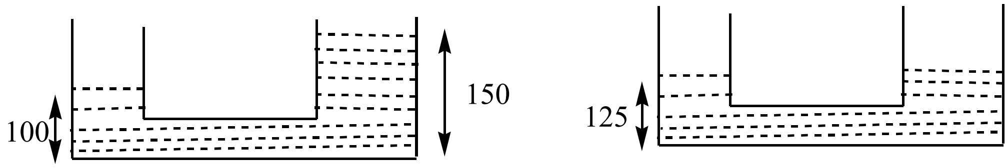
B. $\left.\frac{dQ}{dt}\right|_P : \frac{dQ}{dt_Q} = T_A^4 : T_B^4 \qquad = 283^4 : 293^4$

$$= 1:\left(\frac{293}{283}\right)^4 \qquad = 1:\left(1+\frac{10}{283}\right)^4$$

$$\approx 1:1+\frac{40}{283} \qquad = 1 : 1.15$$

[considering same emissivities]

15.



Initial

Effectively , 25 cm column of water from top of right vessel entered the left $\omega_G = mgh$ (h is height reduced of the COM)

$$= (16)(25)10^{-3}g(25)\times10^{-2} = 1J$$

16. Total mechanical energy $= -\frac{1}{2}$(potential energy)

[for circular orbits under central forces]

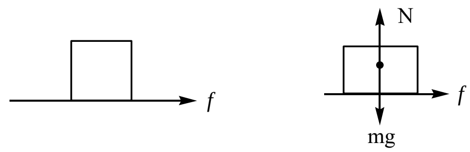
SQ $T.M.E_A : T.M.E_B$

$$= \frac{-GMm_1}{2r_1} : -\frac{GMm_2}{2r_2}$$

$= m_1 r_1 : M_2 r_2$ $\quad = (4m)(4r) : (3m)(3r)$ $\quad = 16 : 9$

17. Force needed $= \frac{dm}{dt}\vartheta = (0.5)5 = 2.5$

Power needed $= F\vartheta = 2.5(5) = 12.5W$



18.

N – mg = 0, $\quad f = \mu N$

f = ma $\quad \Rightarrow a = \mu g = 4\,m/s^2$

u = 0, $\vartheta = 2$ $\quad \vartheta^2 - u^2 = 2as \Rightarrow s = \frac{2^2 - 0^2}{2\times 4} = 0.5m$

19. $[\tau] = ML^2T^{-2}$

$\frac{\Delta\tau}{\tau} = \frac{\Delta M}{M} + 2\frac{\Delta L}{L} - \frac{2\Delta T}{T} \Rightarrow$ % error = 5% + 2(5%) – 2 (-5%)= 25%

20. Consider downward –ve

Then, u = - 100 $\quad$ a = -10

$\vartheta = -100 - 10t$ and after collision , the velocity becomes zero from – 200 almost suddenly so, option A is correct

21. $F = 12t - 3t^2; \tau = 1.5(12t - 3t^2)$

$\alpha = \frac{1.5(12t - 3t^2)}{4.5} = 4t - t^2$ $\qquad \frac{d\omega}{dt} = (4t - t^2) \Rightarrow \omega = 2t^2 - \frac{t^3}{3}$

To change the direction of motion, pulley need to come to rest momentarily,

$2t^2 - \frac{t^3}{3} = 0 \Rightarrow t = 6\,\text{sec}$

$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3} \Rightarrow \theta = \frac{2t^3}{3} - \frac{t^4}{12}$ $\qquad \therefore (\theta)_{t=6\sec} = 36\ rad = \frac{18}{\pi} rev$

22. Given, $T_1 = T_2 \Rightarrow \frac{2u_1}{g} = \frac{2u_2\sin\theta}{g}$

$u_1 = u_2\sin\theta$ $\qquad \frac{H_1}{H_2} = \frac{u_1^2}{2g} \times \frac{2g}{u_2^2\sin^2\theta} = 1$

23. Initial maximum velocity at mean position, $\qquad v_1 = A_1\omega_1, \omega_1 = \sqrt{\frac{K}{m_1}}$

By LCLM, $m_1v_1 = (m_1 + m_2)v_2$

$$V_2 = \frac{m_1v_1}{(m_1 + m_2)} = A_2\omega_2, \omega_2 = \sqrt{\frac{K}{(m_1 + m_2)}}$$

$$\frac{v_1}{v_2}=\frac{A_1\omega_1}{A_2\omega_2}\Rightarrow\frac{A_1}{A_2}=\frac{v_1(m_1+m_2)}{m_1v_1}\times\sqrt{\frac{m_1}{m_1+m_2}}\quad\therefore\frac{A_1}{A_2}=\sqrt{\frac{m_1+m_2}{m_1}}=\sqrt{\frac{1.024}{0.9}}=\sqrt{\frac{10.24}{9}}$$

$$\frac{A_1}{A_2}=\frac{3.2}{3}=\frac{\alpha}{\alpha-1}\Rightarrow 3.2\alpha-3.2=3\alpha\qquad\Rightarrow\alpha=16$$

24. $$\eta=\frac{F}{A\theta},\theta=\frac{x}{l}\Rightarrow 25\times10^9=\frac{18\times10^4\times60\times10^{-2}}{15\times60\times10^{-4}\times x}\Rightarrow x=\frac{18\times10^{-3}}{15\times25}=48\times10^{-6}m=48\mu m$$

25. $$E=\rho J=\frac{\rho i}{A}\quad;F=Ee\ \therefore F=\frac{\rho i}{A}e=\frac{1.7\times10^{-8}\times1}{2\times10^{-6}}\times1.6\times10^{-19}\ =136\times10^{-23}N$$

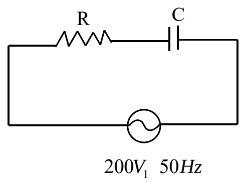
26. $$\int\vec{E}.\vec{ds}=\frac{q}{\epsilon_0}\Rightarrow E.2\pi xl=\frac{\rho\pi x^2l}{\epsilon_0}$$

$$E=\frac{x\rho}{2\epsilon_0};\ \text{Given, } x=\frac{2\epsilon_0}{\rho}\qquad\therefore E=1V/m$$

27. $$I_A=I+4I+2\sqrt{4I^2}\cos\frac{\pi}{3}=5I$$

$$I_B=I+4I+2\sqrt{4I^2}+\cos\frac{\pi}{3}=7I\qquad\therefore I_B-I_A=7I-5I=2I$$

28. $$P=\frac{V^2}{R}\Rightarrow R=\frac{100\times100}{50}=200\Omega\qquad i^2R=50\Rightarrow i^2=\frac{50}{200}\Rightarrow i=\frac{1}{2}A$$



$$x_C=\frac{1}{\omega C}=\frac{1}{100\pi C}\qquad i=\frac{1}{2}=\frac{V}{Z}=\frac{200}{\sqrt{X_C^2+R^2}}$$

$$(400)^2=X_C^2+200^2\Rightarrow X_C=100\sqrt{12}$$

$$\frac{1}{100\pi C}=100\sqrt{12}\Rightarrow C=\frac{100}{\pi\sqrt{12}}\mu F=\frac{50}{\pi\sqrt{3}}\mu F$$

29. $$\frac{S}{R}=\frac{l}{100-l}\Rightarrow\frac{S}{5600}=\frac{50}{700}\Rightarrow S=2400\Omega$$

30. 1 MSD = 1 mm,
10VSD = 9 MSD $\Rightarrow$ 1VSD = 0.9 mm
LC = 1MSD – 1VSD = 0.1 mm = 0.01 cm
Zero error = +4 divisions
MSR = 4.1 cm, VC = 6
Diameter = MSR + (VC – zero error) LC
$=4.1+(6-4)\times0.01\quad=4.12cm=412\times10^{-2}cm$

CHEMISTRY

31. $\% Br = \frac{80}{188} \times \frac{0.36}{0.45} \times 100 = 34.04$

32. $\rightarrow PhSO_2Cl \rightarrow$ Hinsberg reagent

$\longrightarrow Ph-\underset{\underset{O}{\|}}{C}-NH_2 \longrightarrow Ph-N=C=O$

Hoffmann bromamide reaction

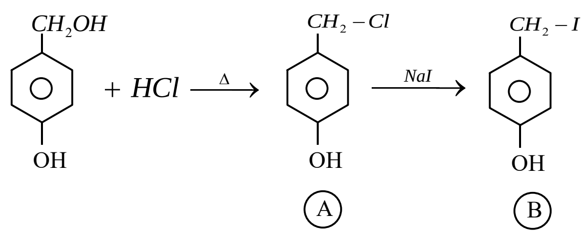
$\rightarrow \underset{i\text{ amine}}{R-NH_2} \rightarrow$ carbyl amine reaction

$\rightarrow$ Saytzeff product x hoff man product

33. Aldoses give Seliwanoff test slowly and furfural has 5-carbon atoms
So aldopentose

34. Morphine is narcotic drug. Saccharin is 550 times sweeter than sucrose chloroxylenol is antiseptic phenelizine in antidepressant

35.

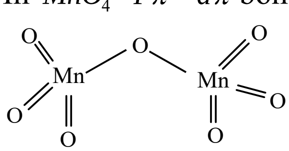


36. Novolac $\rightarrow$ PhOH + HCHO
Glyptal $\rightarrow$ Glycol + phthalic Acid
Buna –s $\rightarrow$ Butadiene + Styrene
Dacron (or) terylene $\rightarrow$ Terphthalic acid + ethylene glycol

37. Excess sulphates in water have laxative effect
$NO_3^- \rightarrow$ methemoglobinemia
$Pb^{+2} \rightarrow$ Kidneydamage
$F^- \rightarrow$ Brown mottling of teeth

38. Cis –(10)- annulene is not aromatic due to lack of planarity

39. In neutral solutions I^- is oxidized to IO_3^- by $KMnO_4$
So statement –I is false
In MnO_4^{-2} $P\pi - d\pi$ bonding is present



40.

41. $BeCl_2$ & $AlCl_3$ acts as lewis acids and $Be(OH)_2, Al(OH)_3$ are amphoteric

42. Pyrophosphoric acid $H_4P_2O_7$

43. Calcination and leaching are used in concentration of ore not in the purification of metal

44. H_2O_2 is used as OA & RA in both medium and $dH_2O_2 : 1.44\, gm/cc, d_{D_2O} = 1.1059\, gm/cc$

45. Same B.pt means same concentration

$$\frac{2}{M_A} \times \frac{1000}{100} = \frac{8}{M_B} \times \frac{1000}{100}$$

$\frac{M_B}{M_A}=4 \qquad M_B = 4\,M_A$

46. IE_1 of Zn is more than Ga due to stable E.C

47. With increase in nuclear charge orbitals come closer to nucleus and their energy decreases

48. Gases with greater inter molecular attractions are easily liquifiable have higher T_C and are readily adsorbed

49. amount of 'C' in solutions $=\frac{250\times10.8}{100} = 2.5\times10.8 = 27\,gm$ of 'C'

180 gm of glucose has 72 gm of 'C'

Amount of glucose with 27 gm of 'C' $=\frac{27\times180}{72}=67.5$

$m=\frac{67.5}{180}\times\frac{1000}{(250-67.5)}=\frac{67.5\times1000}{180\times182.5}=2.0548gm$

50. O_2, Cu^{+2}, Fe^{+3} are paramagnetic NaCl, H_2O are diamagnetic

So both a statements are correct

51. % purity $=\frac{12.6}{30}\times100=\frac{126}{3}=42$

52. Overall yield $=\frac{60}{100}\times\frac{50}{100}\times100 \quad = 30\%$

53. 1:1 electrolyte means

$\left[CO(NH_3)_4Cl_2\right]Cl$

Primary valency is 3 $\quad (CO^{+3})$

54. $C_2O_4^{-2}$ is oxidized to CO_2

O.NO changes from 3 to 4

55.

Species	CN^-	NO^+	O_2	O_2^+	O_2^{+2}
B.O	3	3	2	2.5	3

56. $M=\frac{2.34\times10^{-3}}{78}\times\frac{1000}{100}=3\times10^{-4}=$ solubility

K_{sp} of $CaF_2 = 4S^3 = 4\times\left(3\times10^{-4}\right)^3$

$=4\times27\times10^{-12}m^3 \qquad =108\times10^{-12}m^3 \qquad =0.0108\times10^{-8}M$

57. KO_2, NO_2, ClO_2, NO are paramagnetic

58. $\Delta U = nC_v \;\; \Delta T \qquad 5000 = n$ (20.785-8.314) (500-300)

$n=\frac{25}{12.471}=2.0046 \quad \approx 2$

59. $K_2Cr_2O_7 + 6Fe^{+2} \rightarrow 6Fe^{+3} + 2Cr^{+3}$

$(K_2Cr_2O_7)\frac{M_1V_1}{1}=\frac{M_2V_2}{6}(Fe^{+2})$

$\frac{20\times0.02}{1}=\frac{M_2\times10}{6} \qquad M_2 = 0.024M \qquad = 24\times10^{-2}M$

60. From 2 to 1 when initial pressure of 'NO' is doubled by keeping P_{H_2} const, initial rate increases by four times so order w.r.t 'NO' is 2.

MATHEMATICS

61. Circle is $x^2+y^2-2gx+6y-19c=0$

It passes through $(6,1)$ $\therefore 36+1-12g+6-19c=0$

$12g+19c=43\ldots\ldots(1)$

Line $x-2cy=8$ passes through Centre $S(g,-3)$

$\therefore g+6c=8$

By (1) & (2) g = 2, c = 1

$\therefore$ Circle is $x^2+y^2-4x+6y-19=0$

$\therefore$ x-intercept $=2\sqrt{a^2+19c}$ $\quad=2\sqrt{4+19}=2\sqrt{23}$

62. Since $f(x)$ is continues at $x=4$

$\Rightarrow f(4-)=f(4+)$

$$16+4b=\int_0^4(5-|t-3|)dt=\int_0^3(2+t)dt+\int_3^4(8-t)dt$$

$$=\left(2t+\frac{t^2}{2}\right)_0^3+\left(8t-\frac{t^2}{2}\right)_3^4=6+\frac{9}{2}-0+(32-8)-\left(24-\frac{9}{2}\right)$$

$$\Rightarrow 16+4b=15\Rightarrow b=\frac{-1}{4}$$

$$f(x)=\begin{cases}\int_0^x(5-|t-3|)dt, x>4\\ x^2-\frac{x}{4} \qquad , x\le 4\end{cases}$$

$$f'(x)=\begin{cases}5-|x-3|, x>4\\ 2x-\frac{1}{4} \quad , x\le 4\end{cases}=\begin{cases}8-x \quad , x>4\\ 2x-\frac{1}{4} \quad , x\le 4\end{cases}$$

$f(x)$ is decreasing in $\left(-\infty,\frac{1}{8}\right)\cup(8,\infty)$

63. $2x+ky-5z=1$, $\quad 3kx-ky+z=5$ are two perpendicular planes

$6k-k^2-5=0$

$k=1,5$

But $k<3\Rightarrow k=1$

$\therefore \quad 2x+y-5z=1, \quad 3x-y+z=5$

$P=(2x+y-5z-1)+\lambda(3x-y+z-5)=0$

$\Rightarrow \quad x(3\lambda+2)+y(1-\lambda)+z(\lambda-5)=(5\lambda+1)$

x- intercept = 1 $\quad \Rightarrow 5\lambda+1=3\lambda+2 \quad \Rightarrow \lambda=\frac{1}{2}$

$y-$intercept $=\frac{5\lambda+1}{1-\lambda}=7$

64. $A(1,1)$ $B(-4,3)$ $C(-2,-5)$

Area $(\Delta ABC)=18$

Let $P(\alpha,\beta)$ lies on BC

$$\text{Area } \Delta APB=\frac{1}{2}\begin{vmatrix}1 & 1 & 1\\ \alpha & \beta & 1\\ -4 & 3 & 1\end{vmatrix}=\frac{1}{2}|2\alpha+5\beta-7|$$

Given $\dfrac{Area\,\Delta APB}{Area\,\Delta ABC}=\dfrac{4}{7} \quad \Rightarrow |2\alpha+5\beta-7|=\dfrac{144}{7}$

$\Rightarrow 2\alpha+5\beta-7=\pm\dfrac{144}{7}$ ------(1)

Equation of $\overline{AC}$ is $2x-y-1=0$ ----- (2)

It cuts x-axis at $M\left(\dfrac{1}{2},0\right)$

Equation of $\overline{BC}$ $4x+y+13=0$ --- (3)

Solving (1) & (3) we get

$P=\left(\dfrac{-36}{7},\dfrac{53}{7}\right) or \left(\dfrac{-20}{7},\dfrac{-11}{7}\right)$

Since $x-$coordinates of B,C are -4 and -2 respectively

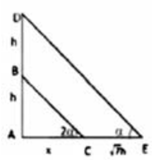
$\Rightarrow P=\left(\dfrac{-20}{7},\dfrac{-11}{7}\right)\left(-4<x-\text{intercept of } P<-2\right)$

Equation $\overline{AP}$ is $2x-3y+1=0 \qquad y=0\Rightarrow x=\dfrac{-1}{2}$

Let $N\left(\dfrac{-1}{2},0\right)$

$$\therefore \quad \text{Area of } \Delta NAM=\frac{1}{2}\begin{vmatrix}1 & 1 & 1\\ -\frac{1}{2} & 0 & 1\\ \frac{1}{2} & 0 & 1\end{vmatrix}=\frac{1}{2}$$

65.



in $\triangle ABC \tan 2\alpha=\dfrac{h}{x}$

in $\triangle ADE \tan\alpha=\dfrac{2h}{x+\sqrt{7}h}$

$$\therefore \tan\alpha = \frac{2h}{\frac{h}{\tan 2\alpha}+\sqrt{7}h} = \frac{2\tan 2\alpha}{1+\sqrt{7}\tan 2\alpha}$$

$$\tan\alpha = t \qquad \Rightarrow t = \frac{\frac{4t}{1-t^2}}{1+\sqrt{7}\frac{2t}{1-t^2}} = \frac{4t}{1-t^2+2\sqrt{7}t}, t \neq 0$$

$$\Rightarrow 1-t^2+2\sqrt{7}t = 4 \qquad \Rightarrow t^2-2\sqrt{7}t+3=0$$

$$t = \frac{2\sqrt{7}\pm\sqrt{28-12}}{2} = \frac{2\sqrt{7}\pm 4}{2} = \sqrt{7}\pm 2 \qquad \therefore t = \sqrt{7}-2$$

66. truth table

P	q	~p	~q	p^~q
T	T	F	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	T	F

$(p \wedge r)$	$\Leftrightarrow$	$(p\wedge \sim q)$	$\cong$	$\sim p$	$(p\wedge q)$
T	F	F	T	T	T
F	T	F	T	F	F
F	F	T	F	T/F	F
F	F	T	F	T/F	F

r is equivalent to q.

67. $\vec{a} = 2\vec{i} - \vec{j} + 5\vec{k}$ $\qquad$ $\vec{b} = \alpha\vec{i} + \beta\vec{j} + 2\vec{k}$

$$(\vec{a}\times\vec{b}) = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 5 \\ \alpha & \beta & 2 \end{pmatrix} = \hat{i}(-2-5\beta) - \hat{j}(4-5\alpha) + \hat{k}(2\beta+\alpha)$$

$$(\vec{a}\times\vec{b})\times i = \begin{vmatrix} i & j & k \\ -2-5\beta & 5\alpha-4 & \alpha+2\beta \\ 1 & 0 & 0 \end{vmatrix} = (\alpha+2\beta)j + (4-5\alpha)k$$

$$[(\vec{a}\times\vec{b})\times\hat{i}]\cdot\hat{k} = \left[(\alpha+2\beta)\hat{j} + (4-5\alpha)\hat{k}\right]\cdot\hat{k}$$

$$\frac{23}{2} = 4-5\alpha \qquad \Rightarrow \alpha = \frac{-3}{2}$$

$$\vec{b}\times 2\vec{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & \beta & 2 \\ 0 & 2 & 0 \end{vmatrix} = -4i + 2\alpha k$$

$$|\vec{b}\times 2\hat{j}| = \sqrt{(-4)^2+(2\alpha)^2} = \sqrt{16+4\alpha^2}$$

$$=\sqrt{16+4\left(\frac{-3}{2}\right)^2}=\sqrt{16+9}=\sqrt{25}=5.$$

68. Terms divisible by 7 between 10000 and 99,999 are $\{10,003;10,010;\ldots 99,995\}$

No. of terms $=12857$

Terms divisible by both 7 and 5 is which are divisible by $\mathrm{LCM}(7,5)=35$

$\Rightarrow$ these are $\{10,010;10045;\ldots 9995\}$

No. of terms $=2572$

$$p=\frac{12857-2572}{90000}=\frac{1.0285}{9}$$

$9p=1.0285$

69. $y^2=8x \qquad 4A=8,\ A=2$

$P(a,b)$ bca Point or the Parabola $\qquad y=mx+\dfrac{A}{m}\Rightarrow y=mx+\dfrac{2}{m}$

$S=x^2+y^2-10x-14y+65=0$

$C(5,7)\,lies\ on\ y=mx+\dfrac{2}{m}$

$7=5m+\dfrac{2}{m}\Rightarrow 5m^2-7m+2=0 \qquad m=1,\dfrac{2}{5}$

$y^2=8x$ diff. both sides w.r.t 'x'

$$2y\frac{dy}{dx}=8\Rightarrow\frac{dy}{dx}=m=\frac{4}{y}$$

$$\frac{dy}{dx}at\ P(a,b)=\frac{4}{b}=1,(or)\frac{2}{5}$$

From (1) $\quad b=4\,or\,10$

$$b^2=8a\Rightarrow a=2,\frac{25}{2}$$

$$A=2\left(\frac{25}{2}\right)=25$$

$B=4\times 10=40 \qquad A+B=65$

70. $\vec{a}=\alpha i+j+\beta k$

$\vec{b}=3\hat{i}-5\hat{j}+4\hat{k}$

$\vec{a}\times\vec{b}=i(4+5\beta)-j(4\alpha-3\beta)+k(-5\alpha-3)$

Given

$\vec{a}\times\vec{b}=-i+9j+12\hat{k}$

By comparison

$4+5\beta=-1 \qquad -5\alpha-3=12$

$5\beta=-5 \qquad -5\alpha=15$

$\boxed{\beta=-1}$ $\boxed{\alpha=-3}$

$\vec{a}=-3i+j-k$

$\vec{b}=3i-5j+4k$

$\vec{b}-2\vec{a}=(3+6)i-7j+6k$

$=9i-7j+6k$

$\vec{b}+\vec{a}=0i-4j+3k$

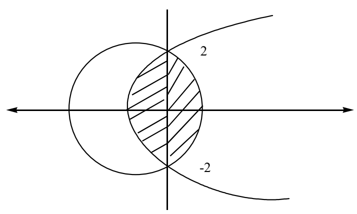
Projection of $(\bar{b}-2\bar{a})on(\bar{b}+\bar{a})$

$$=\frac{(\vec{b}+\vec{a})\cdot(\vec{b}-2\vec{a})}{(\vec{b}+\vec{a})}$$

$$=\frac{28+18}{5}=\frac{46}{5}$$

71. $y^2=8x+4$; $x^2+y^2+4\sqrt{3}x-4=0$

Points of intersection $=(0,\pm 2)$



$x^2+4\sqrt{3}x+y^2=4$ $\quad x^2+22\sqrt{3}x+y^2+12=4+12$

$(x+2\sqrt{3})^2+y^2=4^2$ $\quad y=\sqrt{4^2-(x+2\sqrt{3})^2}$

$(x+2\sqrt{3})=\sqrt{16^2-y^2}$

$$A=\int_{-2}^{2}\left(\sqrt{16-y^2}-2\sqrt{3}-\left(\frac{y^2-4}{8}\right)\right)dy \qquad =\frac{1}{3}\left(4-12\sqrt{3}+8\pi\right)$$

72. $\frac{dy}{dx}-y=x$

$\text{I.F}_1=e^{-\int dx}=e^{-x}$

Solution $y.I.F=\int x.e^{-x}dx$

$y\cdot e^{-x}=\int x\cdot e^{-x}dx=-(1+x)e^{-x}+c$

$y=-1-x+ce^x$ ------ (1)

$y_1(0)=0\Rightarrow 0=-1-0+c\Rightarrow c=1$

$\therefore y=-1-x+e^x$ ------ (2)

$y_2(0)=1\Rightarrow 1=-1-0+c\Rightarrow c=2$

$\Rightarrow y=-1-x+2e^x$ ----- (3)

If (2) and (3) are intersect then

$\Rightarrow -1-x+e^x=-1-x+2e^x$

$e^x=0$ Not possible

$\therefore$ No point of intersection.

73. R.H.L $= \lim_{x\to -1^+} f(x)\text{asin}\left(\frac{-\pi}{2}\right)+2=-a+2$

L.H.L $= \lim_{x\to -1^-} f(x)=0+3=3$

$\lim_{x\to -1} f(x)$ exist $\therefore -a+2=3 \Rightarrow a=-1$

$\therefore f(x)=-\sin\left(\frac{\pi}{2}[x]\right)+2+[-x]$

Now $\int_0^4 f(x)dx=\int_0^1 f(x)dx+\int_1^2 f(x)dx+\int_2^3 f(x)dx+\int_3^4 f(x)dx$

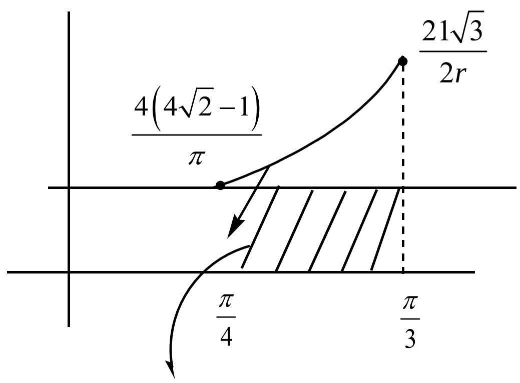
$=\int_0^1(0+2-1)dx+\int_1^2(-1+2-2)dx+\int_2^0(0+2-3)dx+\int_3^4(1+2-4)dx$

$=(1-0)-1(2-1)-(3-2)-(4-3)$

$=1-1-1-1=-2$

74. Given $I=\int_{\pi/4}^{\pi/3}\left(\frac{8\sin x-\sin 2x}{x}\right)dx$

$f(x)=\frac{8\sin x-\sin 2x}{x}$



min $area > \frac{5\pi}{12}$

$f(x)$ is an increasing function

$\int_{\pi/4}^{\pi/3} f(\pi/4)<I<\int_{\pi/4}^{\pi/3} f(\pi/3)$

$f(\pi/4)=\frac{4\sqrt{2}-1}{\pi/4}>\frac{4\sqrt{2}-1}{\pi/4}\ \frac{4\sqrt{2}-1}{(1)}$

$\left(\frac{4\sqrt{2}-1}{1}\right)\left(\frac{\pi}{12}\right)<\int_{\pi/4}^{\pi/3} f(\pi/4)<I<\int_{\pi/4}^{\pi/3} f(r/3)$

$\left(4\sqrt{2}-1\right)\left(\frac{\pi}{12}\right)>\frac{5\pi}{12}$ _____(1)

$I>\frac{5\pi}{12}$

75. $(2023-2)^{2022}+(2023-1)^{2021}$

$\Rightarrow (7k_1-2)^{2022}+(7k_1-1)^{2021}=7N+2^{2022}-1$

Remainder $= 2^{2022} - 1$

$= (8)^{674} - 1 = (7+1)^{674} - 1$

Remainder = 1-1 = 0

76. $S_5 = \frac{5}{2}[2a+4d] = 5a+10d$

$S_9 = \frac{9}{2}[2a+8d] = 9a+36d$

Now $= \frac{S_5}{s_9} = \frac{5}{17} \Rightarrow \frac{5a+10d}{9a+36d} = \frac{5}{17}$

$\Rightarrow 17a+34d = 9a+36d$

$\Rightarrow 8a = 2d \Rightarrow d = 4a$ ---- (1)

Now $a_{15} = a+14d = 57a$

given $110 < a_{15} < 120 \Rightarrow 110 < 57a < 120$

$\Rightarrow 1.929 < a < 2.105$

Now $S_{10} = \frac{10}{2}\{2a+9d\} = 190a$ from (1)

= 380 for a = 2.

77. $V = x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-6)^2$

$= 3(x-1)^2 + 3(y-2)^2 + 30$

V is min at $Z_0 = 1+2i \quad v_0 = 30$

$\left|2Z_0^2 - z_0^3 + 3\right|^2 + v_0^2 = \left(2(-3+4i) - (-11-12i) + 3\right|^2 + 900$

$= |8+6i|^2 + 900 = 100 + 900 = 1000.$

78. $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$ $\alpha, \beta \in R$

$Trace = 1-5 = -4, |A| = -5+4 = -1.$

$A^2 + 4A - I = 0$

$2A^2 + 8A - 2I = 0$

$2A^2 + 8A = 2I.$

$\alpha A^2 + \beta A = 2I \Rightarrow \alpha + \beta = 2+8 = 10.$

79. i) $aR_1b \Leftrightarrow ab \geq 0$

reflexive; let $a \in R$

$aR_1 a \Rightarrow a^2 \geqslant 0$

$\Rightarrow R_1$ is reflexive.

Symmetric; let $a, b \in R$

$aR_1b \Rightarrow ab \geqslant 0$

$\Rightarrow ba \geqslant 0$

$\Rightarrow bR_1a.$

$\Rightarrow R_1$ is symmetric

Transitive let a,b,c $\in R$

$aR_1b \Rightarrow ab \geqslant 0$ *and* $bR_1c \Rightarrow bc \geqslant 0$

$\therefore\ ab \geqslant 0$ *and* $bc \geqslant 0 \Rightarrow ac \geqslant 0$

$\Rightarrow ac \geqslant 0 \Rightarrow aR_1c$

$\Rightarrow R_1$ is transitive $\Rightarrow R_1$ is equivalence

(ii) given $aR_2b \Rightarrow a \geqslant b$

reflexive; let a$\in R$.

$aR_2a \Rightarrow a \geq a$

$\Rightarrow R_2$ is reflexive.

Symmetric; let $a, b \in R$.

$aR_2b \Rightarrow a \geq b$

$\Rightarrow b \geq a$ is need not be true

$\Rightarrow R_2$ is not symmetric

$\Rightarrow R_2$ is not equivalence.

80. Given $f(a) = \alpha$

where $a \in N - \{1\}$ and α is the Maximum of the powers of those primes p such that

for $a = 2$ and $p \neq 2$ then

p^{α} is not divisible by a for any $\alpha \in N$

$\therefore f$ is Not $1-1 \Rightarrow f + g$ is Not $1-1$.

$f(a) = \alpha$, such that p^{α} divides a.

$g(a) = a + 1$ for all $a \in N - \{1\}$

$\therefore \quad (f+g)(a) = \alpha + a + 1$

now, $g(a) = a + 1 \quad g : N - \{1\} \to N$

in co-domain is N.

but is $1 < a < \infty, 2 < a + 1 < \infty$

Range of g is $(2, \infty)$ $\Rightarrow$ range is not equal to co-domain hence g is not onto.

$\therefore f + g$ is not onto.

81. Ellipse $(x+1)^2 + 4(y+1)^2 = \lambda + 5, \lambda \neq -5$

$a^2 = \lambda + 5, b^2 = \dfrac{\lambda + 5}{4}$

Given latus rectum = 4

$\Rightarrow \dfrac{2b^2}{a} = 4 \Rightarrow b^4 = 4a^2$

$\Rightarrow (\lambda + 5)^2 = 64(\lambda + 5)$

$\Rightarrow \lambda + 5 = 64 \quad (\because \lambda \neq -5)$

$\Rightarrow \lambda = 59 \quad a^2 = 64$

$l =$ length of major axis = 2a = 16

$\therefore \lambda + l = 75$

82. $Z^2 + Z = 0 \Rightarrow (x+y)^2 + x - iy = 0$

$x^2-y^2+x+i(2xy-y)=0$

$y(2x-1)=0$ and $x^2-y^2+x=0$

y = 0 (or) 2x-1 = 0

of y = 0, x = 0,-1

of $x=\frac{1}{2}, y=\pm\frac{\sqrt{3}}{2}$

$\sum(\operatorname{Re} Z+\operatorname{Im} Z)$

$Z \in S$

$=(0+0)+(-1+0)+\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)=0$

83. $f(x)=2x^2-x-1$

$|f(x)| \le 800 \Rightarrow -800 \le 2n^2-n-1 \le 800$

$\Rightarrow -19 \le n \le 20$

$\sum_{n\in s} f(x)=\sum_{n=-19}^{20}\left(2n^2-n-1\right)$, replacing with n – 20

$=\sum_{n=1}^{40}\left(2n^2-81n+819\right)$

$=\frac{2\times 40\times 41\times 81}{6}-\frac{81(40\times 41)}{2}+819\times 40=10620$

84. $A=\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_8 \end{bmatrix}, a_i=-1 \;\; or \;\; 1$

Sum of elements of principal diagonal in $A^T A=6$

$\Rightarrow a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2=6$

Number of matrices $=9_{c_6}+2^6$ ($\because$ 6 places, each in 2 ways)

= 5376

85. Put $x^2=t$, then M + m = 2

Given equation $\frac{dy}{dt}+\frac{y}{\sin t\cos t\log\tan t}$

$=\frac{\sin t-\cos t}{\sin t\cos t.\log\tan t}$

I.F $=e^{\int\frac{dt}{\sin t\cos t\log\tan t}}$

$=\log\tan t$

Solutions is $y\log\tan t=\int\frac{\sin t-\cos t}{\sin t\cos t}dt=\int(\sec t-\operatorname{cosec} t)dt$

$=\log\left(\frac{\sec t+\tan t}{\operatorname{cosec} t-\cot t}\right)+C$

$\left(\frac{\pi}{6},1\right)$ is on this $\Rightarrow C=-\log\left[3\left(2+\sqrt{3}\right)\right]$

When $t=\frac{\pi}{3}, y=-1$

$\therefore \left| y\left(\sqrt{\frac{\pi}{3}}\right) \right| = 1$

86. $y^5 - 9xy + 2x = 0$ -----(1)

$4y^4 \frac{dy}{dx} - 9x\frac{dy}{dx} - 9y + 2 = 0 \Rightarrow \frac{dy}{dx} = \frac{9y-2}{5y^4 - 9x}$

Tangent is parallel to x-axis

$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow y = \frac{2}{9}$, not satisfying (1)

Tangent is parallel to y-axis, if $5y^4 - 9x = 0$

From (1), $y^5 - 5y^5 + \frac{10}{9}y^4 = 0$

$\Rightarrow y = 0, \frac{5}{18}$

$\therefore M = 0, N = 2, M + m = 2$

87. Equation y plane passing through the line

$4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3 is$

$4ax - y + 5z - 7a + \lambda(2x - 5y - z - 3) = 0$ ----- (1)

The line $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ lies in this plane

$\therefore$ Substituting $(4,-1,0)$ in ---- (1)

$10d + 9a + 1 = 0$ ----- (2)

$D \cdot R's$ of normal of (1) are $(2\lambda + 4a, -5\lambda - 1, 5 - \lambda)$

$\therefore 1(2\lambda + 4a) - 2(-5\lambda - 1) + 1(5 - \lambda) = 0$

$\Rightarrow 11\lambda + 4a = -7$ ------- (3)

Solving (2), (3) $\lambda = -1, a = 1$

The equation of plate is $2x + 4y + 6z - 4 = 0$

$x + 2y + 3z - 2 = 0 - 4(\because$ from (1))

thy first on the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ is $P(7t+3, 2-t, 3-4t)$, lies on (4), then $t = 2$

$P(\alpha, \beta, \nu) = (17, 0, -5) \Rightarrow \alpha + \beta + \gamma = 12$

88. vertices of hyperbola $= (0, \pm 8)$

For ellipse, $B' = (0,-8), B = (0,8)$

$\therefore 2b = 16, b = 8$

e_1 is eccentricity of hyperbola $= \frac{\sqrt{49+64}}{8}$

e_2 is eccentricity of ellipse

Given $e_1 e_2 = \frac{1}{2}$

$\therefore e_2 = \frac{4}{\sqrt{113}}$

For ellipse $a^2 = b^2\left(1-e_2^2\right) = 64\left(1-\frac{16}{113}\right)$

$d = \frac{2a^2}{b} = \frac{2\times 64\times 97}{113\times 8} = \frac{64\times 97}{113}$

$113d = 1552$

89. $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right) = k, 0 < |x| < \frac{1}{\sqrt{2}}$

$\Rightarrow \sqrt{\frac{1-2x^2}{1-x^2}} = k$

$\Rightarrow \frac{1-2x^2}{1-x^2} = k^2 \Rightarrow 1-2x^2 = k^2 - k^2x^2$

$\Rightarrow \left(k^2-2\right)x^2 = k^2-1$

$x = \pm\sqrt{\frac{k^2-1}{k^2+-2}} \Rightarrow$ sum of the roots $\alpha+\beta = 0 \Rightarrow \frac{\alpha}{\beta} = -1$ ------- (1)

$x^2 - bx - 5 = 0$

Sum of the roots $= \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{\alpha}{\beta} = b$ ---(2)

$\Rightarrow \frac{2}{\alpha^2} - 1 = b$ (from(1))

$\alpha^2 = \frac{2}{b+1}$ -----(3)

Product of roots $= -5$

$\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)\frac{\alpha}{\beta} = -5 \Rightarrow \frac{2}{\alpha^2}\times -1 = -5 \Rightarrow b = 4$ (from(3))

$\alpha^2 = \frac{2}{5} = \left(\frac{k^2-1}{k^2-2}\right)$ $3k^2 = 1 \Rightarrow k^2 = \frac{1}{3}$ $\frac{b}{k^2} = 12$

90. $\overline{x} = 15, M = 15$

$\Sigma x = 150$ $15 = \frac{\sum x_i^2}{10} = 225$

$\sum x_i^2 = 2400$

$\overline{x}$ new $= \frac{x_1 + \cdots x_{n-2} + 15 - 25}{10} = \frac{\sum x_i - 10}{10} = 14$

M new $= \frac{2400 - (25)^2 + (15)^2}{10} - (14)^2$

M new $= \frac{2000}{10} - 196 = 4 \therefore S\cdot D = 2$