



Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: **Sr.Super60_NUCLEUS-BT**

Paper -2(Adv-2020-P2-Model)

Date: 10-09-2023

Time: 02.00Pm to 05.00Pm

CTA-04

Max. Marks: 180

KEY SHEET

PHYSICS

1	5	2	2	3	5	4	2	5	2	6	3
7	ABCD	8	ABC	9	ABC	10	ABC	11	AC	12	ACD
13	60	14	4.2	15	4	16	6	17	3	18	8

CHEMISTRY

19	4	20	5	21	3	22	5	23	5	24	2
25	ABCD	26	ABCD	27	ABCD	28	BC	29	BC	30	ABC
31	0.2 - 0.4	32	0.33	33	3	34	322	35	3	36	9

MATHEMATICS

37	2	38	4	39	8	40	0	41	1	42	6
43	ABCD	44	ABCD	45	AB	46	ABC	47	ABCD	48	ABC
49	1	50	4	51	58	52	9	53	24	54	4.80

SOLUTIONS

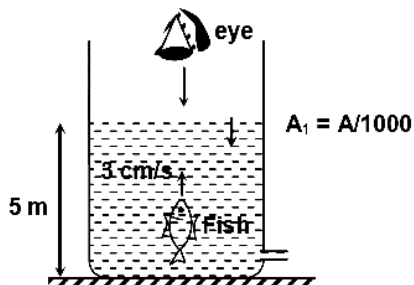
PHYSICS

$$1. \quad \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{2.4}{2r} - \frac{1}{-d} = \frac{2.4 - 1}{r} \frac{1}{d} = \frac{1.4 - 1.2}{r}$$

$$\frac{1}{d} = \frac{0.2}{r} \quad \Rightarrow d = 5r \quad d = 5 \text{ cm}$$

$$2. \quad \text{Velocity of efflux} = v = \sqrt{2g \times 5} = 10 \text{ m/s}$$



$$\text{Velocity of surface} = \frac{10 \times A / 1000}{A} = 1 \text{ cm/s}$$

The velocity of fish relative to the observer is

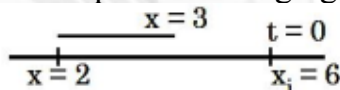
$$v = \frac{d}{dt} \left(\frac{x}{\mu} + y \right)$$

$$= \frac{3}{4} \frac{dx}{dt} + \frac{dy}{dt} = \frac{3}{4} \times (-4) + 1 = -3 + 1 = -2 \text{ cm/s}$$

$$3. \quad x = t^2 - 4t + 6 \quad \text{at } t = 0 \quad x_i = 6$$

$$\text{at } t = 3 \text{ sec } x_f = 3 \quad v = 2t - 4 \quad \text{at } t = 2 \quad v = 0$$

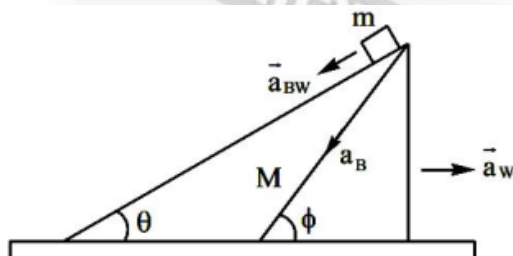
Hence particle changing direction of motion at $t = 2$ sec.



At $t = 2$; $x = 2$

Total distance 5 m

4.



$$\frac{a_w}{\sin(\phi - \phi)} = \frac{a_B}{\sin \theta} \Rightarrow \frac{a_w}{a_B} = \frac{\sin(\phi - \theta)}{\sin \theta}$$

$$ma_B \cos \phi = Ma_w$$

$$\Rightarrow \frac{m}{M} = \frac{a_w}{a_B \cos \phi} = \frac{\sin(\phi - \theta)}{\sin \theta \cdot \cos \phi} = \frac{1 \times 2 \times 2}{2 \times 1 \times 1} = 2$$

5. Length $\propto G^x c^y h^z$

$$L = [M^{-1} L^3 T^{-2}]^x [L T^{-1}]^y [M L^2 T^{-1}]^z$$

By comparing the power of M, L and T in both sides we get

$$-x + z = 0, 3x + y + 2z = 1 \text{ and } -2x - y - z = 0$$

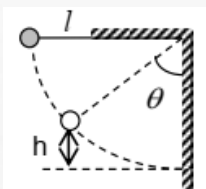
By solving above three equations we get

$$x = \frac{1}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$$

6. Velocity of bob just before collision, $u = \sqrt{2gl}$

The velocity of wall just after collision becomes,

$$v = e\sqrt{2gl}$$



If 'h' is the height attain after first collision, then

$$\frac{1}{2} m (e\sqrt{2gl})^2 = mgh \Rightarrow h = e^2 l$$

Height attain after n^{th} collision $h_n = e^{2n} l$

$$\Rightarrow l(1 - \cos \theta) = e^{2n} l \quad \Rightarrow 1 - \cos \theta = \left(\frac{2}{\sqrt{5}}\right)^{2n}$$

$$\text{for } \theta < 60^\circ, \left(\frac{4}{5}\right)^n < \frac{1}{2} \quad \Rightarrow n = 3$$

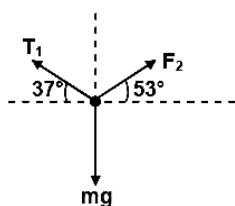
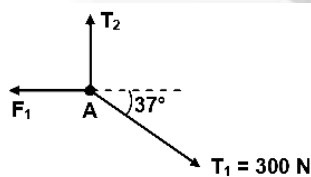
7. $T_2 = T_1 \sin 37^\circ$ (i)

$$F_1 = T_1 \cos 37^\circ$$
(ii)

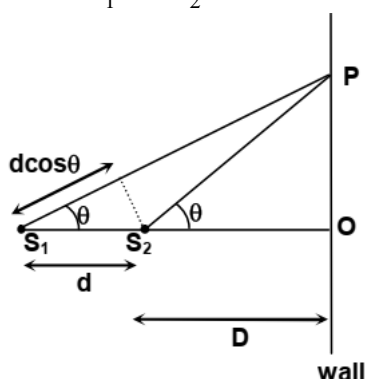
$$F_2 = T_1 \cos 37^\circ / \cos 53^\circ$$
(iii)

$$Mg = T_1 \sin 37^\circ + F_2 \sin 53^\circ$$
(iv)

From the above equation find F_1 , F_2 , T_2 and mass of the



8. $\Delta x = S_1P - S_2P$



$$d \cos \theta = n\lambda \quad \dots\dots (i)$$

$$\Delta x_{\max} = d = 10\lambda$$

$$n\lambda = 10\lambda$$

$$n = 10$$

From equation (i) for the 4th bright ring

$$10\lambda \cos \theta = 6\lambda \quad \theta = 53^\circ$$

$$\cos \theta = \frac{3}{5} = \frac{D}{\sqrt{D^2 + y^2}}$$

$$9D^2 + 9y^2 = 25D^2 \quad y = \frac{8}{3}m$$

For dark ring path difference is

$$\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \frac{9\lambda}{2}, \frac{11\lambda}{2}, \frac{13\lambda}{2}, \frac{15\lambda}{2}, \frac{17\lambda}{2}, \frac{19\lambda}{2}$$

9. Since the process in chamber 2 is adiabatic

$$\therefore P_0 V_0^\gamma = P_2 V_2^\gamma \quad \therefore P_0 V_0^{5/3} = \frac{27}{8} P_0 V_2^{5/3}$$

$$\therefore V_2 = \left(\frac{8}{27}\right)^{3/5} V_0 \quad \therefore \text{Volume of chamber}$$

$$1 = 2V_0 - V_2 = \left[2 - \left(\frac{8}{27}\right)^{3/5}\right] V_0$$

$$P_0^{1-\gamma} T_0^\gamma = C \quad \therefore T_2 = \left(\frac{27}{8}\right)^{2/5} T_0$$

$$\text{Work by the gas} = \frac{P_0 V_0 - P_2 V_2}{\gamma - 1}$$

10. $x_{\text{initial}} = 0; x_{\text{final}} = 5$

$$\text{Displacement} = x_{\text{final}} - x_{\text{initial}} = 5m$$

$$\text{Distance travelled} = 7 + 9 = 9m$$

$$v = \text{slope of } x - t \text{ graph} = \frac{2}{20} = 0.1 m/s$$

$$\langle v \rangle = \frac{\text{total distance}}{\text{total time}} = \frac{9}{80} = 0.11 \text{ m/s}$$



11. $s = ut + \frac{1}{2}at^2$

$$12 = 0 + \frac{1}{2}a(2)^2 \Rightarrow a = 6 \text{ m/s}^2$$

$$a = g \sin \theta - \mu g \cos \theta$$

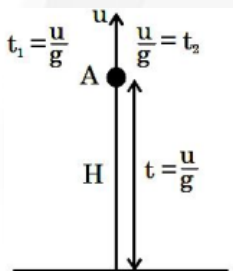
$$6 = 8 - 6\mu \Rightarrow 6\mu = 2$$

$$\mu = \frac{1}{3}$$

$$v = u + at = 0 + (2)$$

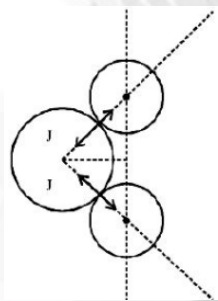
$$v = 12 \text{ m/s}$$

$$s = ut + \frac{1}{2}at^2 = 12(4) + \frac{1}{2}6(4)^2 \quad s = 96 \text{ m}$$

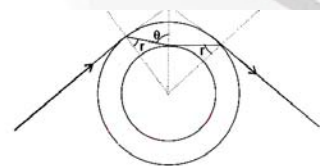


12. 1) $2mv_1 \cos \theta = mv_2$ 2) $\frac{1}{2}m\Omega = 2 \cdot \frac{1}{2}mv_1^2$ 3) $\sin \theta = \frac{nd/2}{d}$ 4) $mv = 2m \cdot v_1 \cos \theta$

$$\theta = 45^\circ \text{ and } n = \sqrt{2} \text{ (for A to stop)}$$



13. $1 \sin 90^\circ = 2 \sin r$



$$r = 30^\circ$$

$$\text{For inner surface } 2 \sin C = 1.5 \sin 90^\circ$$

$$\sin C = 3/4 = 0.75$$

$$\text{Using sine rule } \frac{\sqrt{3}R}{\sin(\pi - \theta)} = \frac{R}{\sin r} \quad \sin \theta = \frac{\sqrt{3}}{2} = 0.865$$

As $\theta > C$ hence total internal reflection occurs. From the geometry angle of deviation is 60°

$$14. \quad LC = \frac{\text{pitch}}{\text{no. of circular divisions}} = \frac{0.75}{50} = 0.015 \text{ mm}$$

Negative zero error.

$$20 \text{ divisions} \times LC \Rightarrow 20 \times 0.015 = (-0.3)$$

Which metal is put

$$\text{Measured value} = 3.75 \text{ mm} + 10 \text{ divisions} \times LC$$

$$= 3.75 \text{ mm} + 10 \times 0.015 = 3.90 \text{ mm}$$

$$TV = MV - (\text{zero error}) = 3.9 - (-0.3) = 4.20 \text{ mm}$$

15. Centre of mass must lie on line L_1

$$\frac{x}{l} + \frac{y}{l} = 1 \quad 2x + 2y = l \quad a + b = 4$$

16. Ring rises \Rightarrow tension is zero

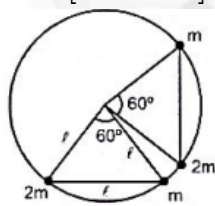
17. The speeds given to $2m$ will also be possessed by ' m '

\therefore K.E in horizontal position gets converted in P.E in vertical position

$$\frac{1}{2} 2mv^2 + \frac{1}{2} mv^2 = \text{change in P.E in vertical position}$$

$$\Delta P.E = 2mg[l \cos 30^\circ - l \cos 60^\circ] + mg\left[l \cos 30^\circ + \frac{l}{2}\right]$$

$$2mg\left[\frac{l\sqrt{3}}{2} - \frac{l}{2}\right] + mg\left[\frac{l\sqrt{3}}{2} + \frac{l}{2}\right]$$



$$\therefore mgl[\sqrt{3} - 1] + mgl\left[\frac{\sqrt{3} + 1}{2}\right]$$

$$\therefore mgl\left[\sqrt{3} - 1 + \frac{\sqrt{3} + 1}{2}\right] = mgl\left[\frac{3\sqrt{3} - 1}{2}\right]$$

$$K.E = \frac{1}{2} 3mv^2 = mgl\left[\frac{3\sqrt{3} - 1}{2}\right] \therefore v = \sqrt{\left(\frac{3\sqrt{3} - 1}{3}\right)gl}$$

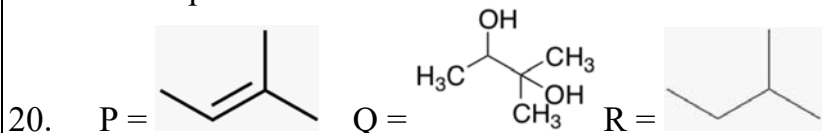
18. 8 m/s

$$\text{Use LCE between points, } \frac{1}{2} mv_1^2 + \frac{1}{2} kx_1^2 = \frac{1}{2} mv_2^2 + \frac{1}{2} kx_2^2 + mgh$$

$$x_1 = 2m, x_2 = 1m \text{ and } h = 3m$$

CHEMISTRY

19. Conceptual



i,ii,iii,iv,vi re correct

21. Acetone

22. 5moles CO₂ gas releases

23. a, c, d, e, f are formed optically inactive

24. Conceptual

25. Conceptual

26. A= Intra molecular aldol condensation

B= Stephen's reduction

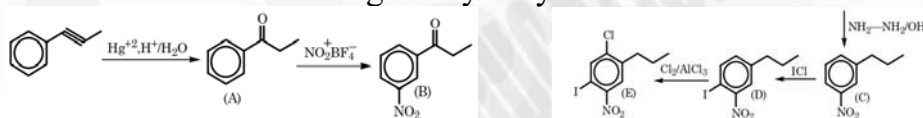
C is correct

D is correct

27. ABCD are correct

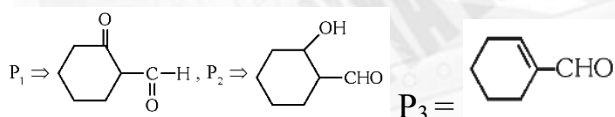
28. B and C are correct

29. Both B & C formed during the hydrolysis of ether



30.

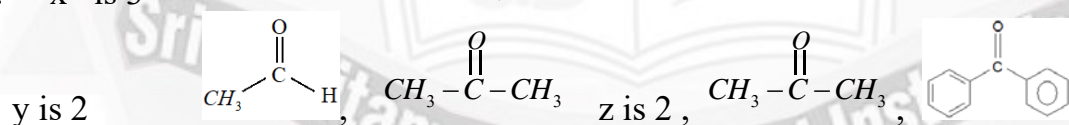
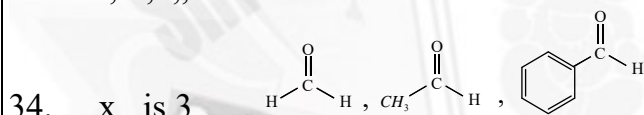
31.



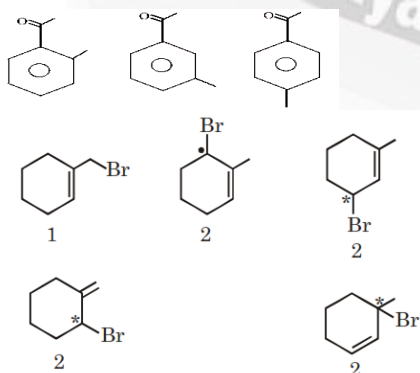
32. Reaction with NaOI gives =A,B,C,D,J,N,O

lucas test gives faster = I, reaction with Cu/300° to give alkene = I

33. i, ii, v,,



35.



36.

MATHEMATICS

$$\begin{aligned}
 37. \quad & \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \sum_{n=0}^{\infty} \int_0^1 x^{2^n k} dx \\
 &= \int_0^1 \sum_{k=1}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{k-1} x^{2^n k}}{k} dx = \int_0^1 \sum_{n=0}^{\infty} \ln(1 + x^{2^n}) dx \\
 &= \int_0^1 \ln((1+x)(1+x^2)(1+x^4)\dots) dx = \int_0^1 \ln\left(\frac{1-0}{1-x}\right) dx \\
 &= -\int_0^1 \ln x dx = -[x \ln x - x]_0^1 = 1
 \end{aligned}$$

$$\begin{aligned}
 38. \quad & y = \sin(x+y) \\
 & y' = \cos(x+y)(1+y') \\
 & y' = \frac{\cos(x+y)}{1-\cos(x+y)}
 \end{aligned}$$

Given

$$\frac{\cos(x+y)}{1-\cos(x+y)} = \frac{1}{\sqrt{2}-1} \Rightarrow \cos(x+y) = \frac{1}{\sqrt{2}}$$

$$-\left(2\pi + \frac{\pi}{4}\right), -\left(2\pi - \frac{\pi}{4}\right), -\left(4\pi - \frac{\pi}{4}\right)$$

pair of (x, y) \equiv

$$\begin{aligned}
 & \left(\frac{\pi}{4} - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(2\pi + \frac{\pi}{4} - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(2\pi - \frac{\pi}{4} + \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(4\pi - \frac{\pi}{4} + \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \\
 & \left(-\frac{\pi}{4} + \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right), \left(-2\pi - \frac{\pi}{4} + \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right), \left(-2\pi - \frac{\pi}{4} - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(-4\pi + \frac{\pi}{4} - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)
 \end{aligned}$$

$$\sum_{k=1}^n |\alpha_k| = 16\pi, \quad \sum_{k=1}^n |\beta_k| = \frac{8}{\sqrt{2}}$$

$$\text{so, } \left\lfloor \frac{16\pi - \frac{8}{\sqrt{2}}}{11} \right\rfloor = \left\lfloor \frac{16\pi - 2\sqrt{2}}{11} \right\rfloor = [4.31] = 4$$

$$\begin{aligned}
 39. \quad & \text{Let } \theta = \frac{\pi}{4} + x \\
 & \Rightarrow d\theta = dx \text{ or } 4\theta = \pi + 4x \Rightarrow \pi - 4\theta = -4x
 \end{aligned}$$

$$= \int_{-\pi/2}^0 \frac{(-4x) \tan\left(\frac{\pi}{4} + x\right)}{1 - \tan\left(\frac{\pi}{4} + x\right)} dx$$

$$\begin{aligned}
 &= -4 \int_{-\pi/2}^0 \frac{x(1+\tan x)}{1-\tan x} dx \\
 &= -4 \int_{-\pi/2}^0 \frac{x(1+\tan x)}{1-\tan x} \cdot \frac{(1-\tan x)}{(-2)\tan x} dx = 2 \int_{-\pi/2}^0 \left(\frac{x}{\tan x} + x \right) dx
 \end{aligned}$$

$$I = -\frac{\pi^2}{4} + 2 \int_0^{\frac{\pi}{2}} \frac{t}{\tan t} dt \quad x = I_1$$

$$I_1 = \int_0^{\frac{\pi}{2}} t \cot t = \frac{\pi}{2} \ln 2$$

$$\text{Hence } 2 \cdot \frac{\pi}{2} \ln 2 - \frac{\pi^2}{4} = \pi \ln 2 - \frac{\pi^2}{4}$$

$$\Rightarrow k = 2, w = 4$$

$$\Rightarrow kw = 8$$

$$40. \quad f(x) = \left[\frac{\tan \pi x^3}{u} \right] \left[\frac{\sqrt{9 + \tan^2 x} + \sqrt[3]{27 + \tan^3 x}}{v} \right] \Rightarrow f'(x) = u'v + uv'$$

$$u = 0 \text{ at } x = -2 \Rightarrow f'(-2) = \sec^2(\pi x^3) 3\pi x^2 \sqrt{9 + \tan^2 \pi x} \sqrt[3]{27 + \tan^3 \pi x} \text{ at } x = -2$$

$$\therefore f'(-2) = 12\pi \cdot 3 \cdot 3 = 108\pi$$

$$g(x) = u \cdot v \Rightarrow g''(x) = u''v + 2u'v' + uv''; u = 1 - \cos x$$

$$\text{at } x = 0; u = 0, u' = 0 \Rightarrow g''(0) = \cos x \cdot \frac{\cos^{-1} x + \tan^{-1} x}{(1+x^2)\cot^{-1} x} \Big|_{x=0}$$

$$= 1 \times \frac{\frac{\pi}{2} + 0}{1 \times \frac{\pi}{2}} = 1$$

$$41. \quad I = \int \frac{\{f'(x)x - f(x)\} dx}{\{f(x) + x\} \sqrt{x(f(x) - x)}}$$

$$\begin{aligned}
 I &= \int \frac{\left\{ \frac{f'(x)x - f(x)}{(x)^2} \right\}}{\left(\frac{f(x)}{x} + 1 \right) \sqrt{\frac{f(x)}{x} - 1}} dx
 \end{aligned}$$

$$\text{Let } \frac{f(x)}{x} - 1 = t^2 \Rightarrow \frac{f'(x)x - f(x)}{(x)^2} dx = 2t dt$$

$$\therefore I = \int \frac{2t dt}{(t^2 + 2)t} = \frac{2}{\sqrt{2}} \tan^{-1} \left(\frac{t}{\sqrt{2}} \right) + c = \sqrt{2} \tan^{-1} \left\{ \sqrt{\frac{f(x)}{2x} - \frac{1}{2}} \right\} + c$$

$$= \sqrt{2} \tan^{-1} \left\{ \sqrt{\frac{f(x) - x}{2x}} \right\} + c$$

$$m = 2, n = 2$$

$$42. \quad 1, 2, 2, 3, 3, 3, \dots$$

$$\therefore a_1 = 1; a_2 = a_3 = 2; a_4 = a_5 = a_6 = 3; \dots$$

$$\text{If } \frac{m(m-1)}{2} + 1 \leq n \leq \frac{m(m+1)}{2}, \text{ then } a_n = m$$

$$\Rightarrow \frac{m}{\sqrt{\frac{m(m+1)}{2}}} \leq \frac{a_n}{\sqrt{n}} \leq \frac{m}{\sqrt{1 + \frac{m(m-1)}{2}}}$$

$$\begin{matrix} m \rightarrow \infty & & m \rightarrow \infty \\ \sqrt{2} & & \sqrt{2} \end{matrix}$$

$$43. \quad \text{On integration and using given condition we get } g(x) \text{ as follows :}$$

$$f(x) = \begin{cases} \tan^{-1} x - \frac{\pi}{6} & \forall x \in [1, \infty) \\ \tan^{-1} x - C & \forall x \in (-\infty, -1] \end{cases}$$

$$\text{Here } f(\sqrt{3}) = \frac{\pi}{6} \text{ can fix only one branch of } f(x), \text{ but does not give any information about}$$

$$\text{other branch of } f(x). \text{ So } f'(-\sqrt{3}) = -\frac{\pi}{3} - C \text{ can be equal to any real number of choosing}$$

$$\text{suitable value of 'C'}$$

$$44. \quad \text{Here the function inside the integration is even function, so}$$

$$I = 2 \int_0^{\infty} \frac{\sin \left(x + \frac{1}{x} \right) \cos \left(x - \frac{1}{x} \right)}{x + \frac{1}{x}} dx = \int_0^{\infty} \frac{\sin 2x + \sin \frac{2}{x}}{x + \frac{1}{x}} dx \dots (1)$$

$$\text{put } x = 1/t, \text{ we get } I = \int_{\infty}^0 \frac{\sin \frac{2}{t} + \sin 2t}{\frac{1}{t} + t} \cdot \frac{-dt}{t^2} = \int_0^{\infty} \frac{\sin 2x + \sin \frac{2}{x}}{x + \frac{1}{x}} \frac{dx}{x^2} \dots (2)$$

$$\text{Add eq(1) and eq(2). } 2I = \int_0^{\infty} \frac{\sin 2x + \sin \frac{2}{x}}{x} dx \dots (3)$$

$$\int_0^{\infty} \frac{\sin 2x}{x} \cdot dx = \int_0^{\infty} \frac{\sin 2x}{2x} \cdot 2dx = \int_0^{\infty} \frac{\sin t}{t} \cdot dt$$

$$\int_0^{\infty} \frac{\sin \frac{2}{x}}{x} dx = \int_0^{\infty} \frac{\sin \frac{2}{x}}{\frac{2}{x}} \cdot \frac{2}{x^2} dx = \int_0^{\infty} \frac{\sin t}{t} dt \quad (\text{using } 2/x = t), \text{ now using these eq(3) we get}$$

$$I = \int_0^{\infty} \frac{\sin \frac{2}{x}}{x} dx = \int_0^{\infty} \frac{\sin x}{x} dt$$

45. Taking log both sides we get $\ln \beta = \lim_{n \rightarrow \infty} \left[\frac{\ln \{(1!)(2!) \dots (n!)\}}{n^2} - \alpha \ln(n) \right]$

$$= \lim_{n \rightarrow \infty} \left[\frac{\ln 1 + (\ln 1 + \ln 2) + (\ln 1 + \ln 2 + \ln 3) + \dots}{n^2} - \alpha \ln(n) \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{n \ln 1 + (n-1) \ln 2 + (n-2) \ln 3 \dots}{n^2} - \alpha \ln(n) \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{r=1}^n \left(\frac{n+1-r}{n} \right) \ln \left(\frac{r}{n} \right) + \left(\frac{1}{2} + \frac{1}{2n} - \alpha \right) \ln n \right]$$

As limit exists

$$\alpha = \frac{1}{2} \text{ and } \ln \beta = \int_0^1 (1-x) \ln x \, dx = -\frac{3}{4}$$

$$\Rightarrow \beta = e^{\frac{3}{4}}$$

46. $f(x) = \sec^2 x + 2 \sec^2 x \tan^2 x$

$$g(x) = \frac{\sin(2nx)}{2 \sin x}$$

A) $\int (1 + 2 \tan^2 x) \sec^2 x \, dx$

$$= \tan x + \frac{2}{3} \tan^3 x + C$$

B) $\lim_{n \rightarrow 0} \frac{(\sec^2 x + 2 \sec^2 x \tan^2 x - 1) \cdot 4}{\sin^2 2nx} = e^{\frac{3}{2}}$

C) $I_{2n+2} - I_{2n} = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{\sin(2n+2)x - \sin 2nx}{\sin x} dx$

$$= \int_0^{\frac{\pi}{2}} \cos(2n+1)x \, dx = \frac{1}{(2n+1)} \sin(2n+1) \frac{\pi}{2}$$

$$= \frac{1}{3} \cdot \frac{1}{5} \cdot \frac{1}{7} \cdot \frac{1}{9} \cdot \frac{1}{11} \cdot \frac{1}{13} \text{ for } n \in [1, 6]$$

$$D) g(x) = 0 \Rightarrow x = \frac{\pi}{2}$$

$$\sin 8x = 0, \sin x \neq 0$$

$$\Rightarrow x = \frac{\pi}{8} \cdot \frac{\pi}{4} \cdot \frac{3\pi}{8} \cdot \frac{\pi}{2} \cdot \frac{5\pi}{8} \cdot \frac{3\pi}{4} \cdot \frac{7\pi}{8}$$

$$47. \quad A) \text{ Let } g(x) = e^{-x} \int_0^x f(t) dt$$

$$g(0) = 0 = g(1) \Rightarrow g'(c) = 0 \Rightarrow e^{-c} \cdot f(c) - e^{-c} \int_0^c f(t) dt = 0$$

$$B) \text{ apply R.T on } g(x) = (1-x) e^x \int_0^x f(x) dx$$

$$C) g(x) = e^{-x^2/2} \int_0^x f(x) dx$$

$$D) g(x) = e^{x(x-1)} \int_0^x f(x) dx$$

$$48. \quad \int \frac{x^4}{(x^4+1)^2} dx = \int x \cdot \frac{x^3}{(x^4+1)^2} dx$$

(now use integration by parts using 'x' as first function)

$$= -\frac{x}{4(x^4+1)} + \frac{1}{4} \int \frac{dx}{x^4+1} = -\frac{x}{4(x^4+1)} + \frac{1}{8} \int \frac{1-x^2+1+x^2}{x^4+1} dx$$

$$= -\frac{x}{4(x^4+1)} + \frac{1}{8} \int \frac{\left(\frac{1}{x^2}-1\right)}{\left(\frac{1}{x^2}+x^2\right)} dx + \frac{1}{8} \int \frac{\left(\frac{1}{x^2}+1\right)}{\left(\frac{1}{x^2}+x^2\right)} dx$$

$$= \frac{-x}{4(x^4+1)} - \frac{1}{16\sqrt{2}} \ln \left(\frac{\left(x + \frac{1}{x}\right) - \sqrt{2}}{\left(x + \frac{1}{x}\right) + \sqrt{2}} \right) + \frac{1}{8\sqrt{2}} \tan^{-1} \left(\frac{x^2-1}{\sqrt{2}x} \right) + C$$

$$= \frac{-x}{4(x^4+1)} - \frac{1}{16\sqrt{2}} \ln \left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} \right) + \frac{1}{8\sqrt{2}} \tan^{-1} \left(\frac{x^2-1}{\sqrt{2}x} \right) + C$$

$$\text{so } A = 4, f(x) = \frac{x^2-1}{\sqrt{2}x}, \text{ and } g(x) = x^2 + \sqrt{2}x + 1$$

49. In equation F_n^3 is effectively 3-element Fibonacci sum. So, $1 + 1 + 2 = 4$, $3 + 5 + 8 = 16$, $13 + 21 + 34 = 68$ and so on. So we have the recurrence $F_n^3 = F_{n+2} + F_{n+1} + F_n$, so rather clearly we have L_3 as

$$L_3 = \lim_{n \rightarrow \infty} \frac{F_{n+2} + F_{n+1} + F_n}{F_{n+1} + F_n + F_{n-1}}$$

Therefore, we have after multiplying with F_{n-4}

$$L_3 = \frac{\lim_{n \rightarrow \infty} \frac{F_{n+2}}{F_{n-4}} + \lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_{n-4}} + \lim_{n \rightarrow \infty} \frac{F_n}{F_{n-4}}}{\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_{n-4}} + \lim_{n \rightarrow \infty} \frac{F_n}{F_{n-4}} + \lim_{n \rightarrow \infty} \frac{F_{n-1}}{F_{n-4}}}$$

Then, we can write L_3 as

$$L_3 = \frac{\phi^6 + \phi^5 + \phi^4}{\phi^3 + \phi^2 + \phi} = \frac{\phi^4 \cdot (\phi^2 + \phi + 1)}{\phi \cdot (\phi^2 + \phi + 1)} = \phi^3$$

$$\text{Where } \phi = \frac{1 + \sqrt{5}}{2}$$

50. $|xy| - r|x| - r|y| + r^2 \leq 0$

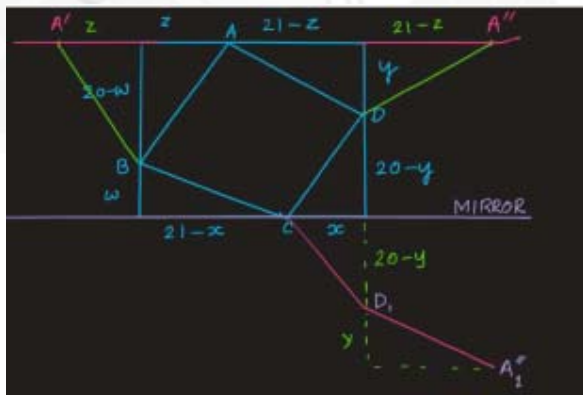
$$\Rightarrow (|x| - r)(|y| - r) \leq 0$$

$$P(r) = (2r+1)^2 \sum_{r=1}^n P(r) = \frac{n}{3} (4n^2 + 12n + 11)$$

$$\lim_{n \rightarrow \infty} \frac{\frac{n}{3} (4n^2 + 12n + 11) - \lambda n^3}{an^2 + bn + c} = \frac{1}{2}$$

$$\Rightarrow \lambda = \frac{4}{3} \text{ and } a = 8$$

51.



Given expression

$$= AB + BC + CD + DA$$

$$= A'B + BC + CD + DA''$$

$$= A'A_1''$$

$$= \sqrt{(A'A'')^2 + (A''A_1'')^2}$$

$$= \sqrt{42^2 + 40^2} = 2 \times 29$$

52. $\sqrt{x+1} + x = t^3 > 2x = t^3 - t^{-3}$

$$\sqrt{x^2+1} - x = t^{-3}$$

$$\int \left(x - \sqrt{x^2+1} \right)^{1/3} + \left(x + \sqrt{x^2+1} \right)^{1/3} dx = \int \left(t - \frac{1}{t} \right) \left(3t^2 + \frac{3}{t^4} \right) \frac{dt}{2}$$

$$= \frac{3}{2} \int \left(t^3 + \frac{1}{t^3} - t - \frac{1}{t^5} \right) dt = \frac{3}{2} \left[\frac{t^4}{4} + \frac{t^{-2}}{-2} - \frac{t^2}{2} - \frac{t^{-4}}{-4} \right]$$

$$= \frac{3}{8} \left[t^4 + t^{-4} - 2(t^2 + t^{-2}) \right]$$

If $x = 7$, then $t^3 = 7 + 5\sqrt{2} \Rightarrow t^{-3} = 5\sqrt{2} - 7$

$$\Rightarrow t^3 - \frac{1}{t^3} = 14 \Rightarrow \left(t - \frac{1}{t} \right)^3 + 3 \left(t - \frac{1}{t} \right) = 14$$

$$\Rightarrow t - \frac{1}{t} = 2 \Rightarrow t^2 + t^{-2} = 6 \Rightarrow t^4 + t^{-4} = 34$$

$$\therefore \int_0^7 f(x) dx = \frac{3}{8} [(34 - 12) - (2 - 4)] = \frac{3}{8} [24] = 9$$

53. If $q - p$ is maximum then $y = \pm 1$ should be tangents

$$\therefore q - p = 4\sqrt{14 + 10\sqrt{2}}$$

54. $\frac{1-a^2+4a}{1+a^2} = \frac{3-4b+3b^2}{1-b^2} \Rightarrow \cos x + 2\sin x = 3\sec y - 2\tan y$

$$\Rightarrow (\cos x + 2\sin x)\cos y = 3 - 2\sin y$$

$$\Rightarrow \cos x \cos y + 2\sin x \cos y + 2\sin y = 3$$

$$\Rightarrow \frac{\cos x \cos y}{1} = \frac{\sin x \cos y}{2} = \frac{\sin y}{2} \Rightarrow \tan x = 2, \tan y = \sin x$$