Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 02-09-2023
Time: 09.00Am to 12.00Pm RPTM-05 Max. Marks: 300

KEY SHEET

PHYSICS

1)	3	2)	3	3)	3	4)	1	5)	2
6)	1	7)	3	8)	2	9)	3	10)	3
11)	1	12)	1	13)	1	14)	3	15)	4
16)	3	17)	2	18)	1	19)	2	20)	2
21)	8	22)	16	23)	2	24)	1	25)	4
26)	2	27)	4800	28)	9	29)	8	30)	6

CHEMISTRY

31)	4	32)	3	33)	2	34)	1	35)	2
36)	3	37)	3	38)	2	39)	1	40)	3
41)	4	42)	3	43)	4	44)	1	45)	3
46)	3	47)	2	48)	1	49)	2	50)	2
51)	5	52)	3	53)	2	54)	7	55)	6
56)	3	57)	5	58)	5	59)	4	60)	5

MATHEMATICS

61)	4	62)	3	63)	4	64)	4	65)	1
66)	2	67)	4	68)	3	69)	1	70)	2
71)	3	72)	2	73)	1	74)	1	75)	1
76)	3	77)	1	78)	4	79)	1	80)	3
81)	15	82)	0	83)	16	84)	5051	85)	3
86)	1	87)	0	88)	1	89)	0	90)	1011

SOLUTIONS

PHYSICS

1. For statement-I

The maximum speed by which cyclist can take a turn on a circular path

$\Rightarrow v \le \sqrt{\mu rg} \le \sqrt{0.2 \times 2 \times 9.8} \Rightarrow v \le \sqrt{3.92}$

Speed of cyclist, $7kmh^{-1} 7 \times \frac{5}{18} = 1.94\, m/s$

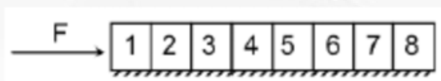
The maximum safe speed on a banked frictional road

$v_{allowable} = \sqrt{rg\frac{(\mu + \tan\theta)}{1 - \mu\tan\theta}} \quad \Rightarrow v = \sqrt{\frac{2 \times 9.8(0.2 + \tan 45°)}{1 - 0.2 \times \tan 45°}} = \sqrt{\frac{2 \times 9.8 \times 1.2}{0.8}} = 5.42$ m/s

Speed of cyclist, $18.5\, kmh^{-1} = 5.13\, m/s$

So, both the statements are true.

2. $F_{81} = ma$



$F_{21} = 7\, ma$

$\frac{F_{21}}{F_{87}} = 7$

4. Let Wedge is moving rightward with acceleration a and mass m has an acceleration A with respect to wedge along the surface of the wedge in upward direction, so $\frac{h}{\sin\alpha} = \frac{1}{2}At^2 \Rightarrow A = \frac{2h}{t^2 \sin\alpha}$ .....(1)

With the help of FBD of mass m in the frame of wedge, we can write

$A = a\cos\alpha - g\sin\alpha - g\sin\alpha$

$\Rightarrow a = g\tan\alpha + \frac{2h}{t^2 \sin\alpha\cos\alpha} = 10 \times \frac{3}{4} + 2 \times 3 \times \frac{5}{3} \times \frac{5}{4} \times \frac{1}{5 \times 5} = 8m/s^2$

5. Using work energy theorem, we get $\sum W_1 = \sum W_2$

$\left[W_g + W_{friction}\right]_1 = \left[W_g + W_{friction}\right]_2$

Since, $|f_1| > |f_2| \Rightarrow [W_{ext}]_1 > [W_{ext}]_2$

6. Case I:

$T_1 - 2mg$

$ma_1 = 2mg - T$

$\Rightarrow a_1 = g \uparrow$

Case II:

$T_2 - mg$

$3ma_2 = 3mg - T_2$

$\Rightarrow a_2 = 2g/3 \downarrow$

Case II:

$T_2 = mg$

$3ma_2 = 3mg - T_2$

$\Rightarrow a_2 = 2g/3 \downarrow$

Case III:

$T_2 = 4mg$

$ma_3 = 4mg$

$\Rightarrow a_3 = 3g \uparrow$

Case IV:

$T_4 = mg$

$2ma_4 = 2mg - T_4$

$\Rightarrow a_4 = g/2 \downarrow$

7. $T_1 \cos\alpha = T_3 = T_2 \cos\beta$

$T_1 \sin\alpha + T_2 \sin\beta = mg$

$$\Rightarrow mg = T_1\sqrt{1-\left(\frac{T_3}{T_1}\right)^2} + T_2\sqrt{1-\left(\frac{T_3}{T_2}\right)^2}$$

$$\Rightarrow m = \frac{\sqrt{T_1^2 - T_3^2} + \sqrt{T_2^2 - T_3^2}}{g}$$

8. From FBD of lift.

$T_1 = T_2 + mg, m =$ mass of lift

$\Rightarrow T_1 - T_2 = mg$

$\Rightarrow T_{net} = mg$

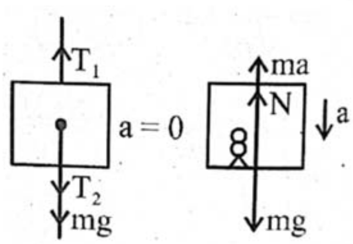
So, (I) is true

From FBD of person,

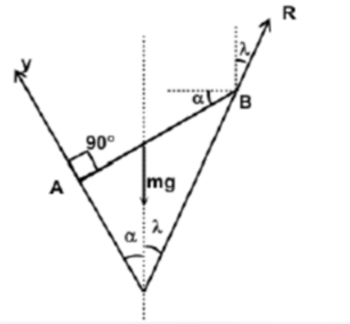
$N + ma = mg$

$N = mg - ma \Rightarrow N < mg$

$So, (II)\ is\ false$



9. Since body is in equilibrium, under the influence of three forces only so they must be concurrent. Using Lami's theorem we can write



$$\tan\alpha = \frac{L}{2x} \text{ and } \tan(\alpha+\lambda) = \frac{L}{x}$$

$$\tan(\alpha+\lambda) = 2\tan\alpha \Rightarrow \frac{\tan\alpha + \tan\lambda}{1-\tan\alpha\tan\lambda} = 2\tan\alpha$$

$$\Rightarrow \tan\alpha + \tan\lambda = 2\tan\alpha - 2\tan^2\alpha\tan\lambda$$

$$\Rightarrow \tan\lambda = \mu = \frac{\tan\alpha}{1+2\tan^2\alpha}$$

10. $v_x = 1 \Rightarrow x = t \text{ and } v_y = 6t \Rightarrow y = 3t^2 \Rightarrow y = 3x^2$

$$\Rightarrow \frac{dy}{dx} = 6x, \frac{d^2y}{dx^2} = 6 \Rightarrow \left.\frac{dy}{dx}\right|_{x=\frac{\sqrt{2}}{3}} = 2\sqrt{2}$$

As we know that $R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{(1+8)^{3/2}}{6} = 4.5m$

11. $8x_B = x_A \Rightarrow 8v_B = v_A \Rightarrow 8a_B = a_A$

12. Area under velocity time graph gives distance of body in given time.

$$v = \frac{dx}{dt} \Rightarrow x = \int v\,dt = Area$$

Area under acceleration time graph gives change in velocity in the given time.

$$a = \frac{dv}{dt} \Rightarrow v = \int a\,dt = Area$$

13. $\overline{a_B} = \overline{a_A} + \left(\overline{\alpha} \times \overline{r_{BA.}}\right) - \omega^2 \overrightarrow{r_{BA}}$

$= -5j + 4\alpha i + 3\alpha j - (12i - 16j)$

a_B along Y-axis should be zero.

$\Rightarrow 11 + 3\alpha = 0$

$\Rightarrow \alpha = -\frac{4}{3}\ rad/s^2$

$a_B = (4\alpha - 12)i = -\frac{80}{3} i\, m/s^2$

14. If you push a cart with some force then according to newton third law the cart will exert an equal and opposite force on you. Sot the cart push you with wame amount of force in opposite direction. So Statement I is correct. The action reaction pairs mentioned in statement one cannot cancel each other because action and reaction forces act on two different bodies. Sp tjeu cammpt camce; each other. So Statement 2 is wrong.

15. According to 1st law of thermodynamics

$\Delta Q = \Delta U + W$

If $\Delta Q > 0, \Delta U < 0\, and\, W > 0$ is also possible.

Hence $\Delta T < 0$, so T decreases.

Statement I is false

$W > 0; \int pdV > 0$

Therefore volume of system must increase during positive work done by the system.
Statement II is true.

16. $\Delta Q_{AB} = nCp\Delta T = 2 \times \frac{5R}{2}(2T_0 - T_0) = 5RT_0$

In the process BC , $PT^{-2} = constant$

$pp^{-2}V^{-2}$ = constant

PV^2 = constant

$\therefore$ molar heat Capacity

$c = c_v + \frac{R}{1-x} = \frac{3R}{2} + \frac{R}{1-2}$

$c = \frac{R}{2}$

$\therefore \Delta Q_{BC} = nC\Delta T = 2 \times \frac{R}{2}(T_0 - 2T_0) = -RT_0$

$\therefore \left|\frac{\Delta Q_{AB}}{\Delta Q_{BC}}\right| = \frac{5RT}{RT_0} = 5$

17. In steady state

$\frac{\Delta Q}{\Delta t} = -KA\frac{dT}{dx}$

$\frac{\Delta Q}{\Delta t} = -\alpha TA\frac{dT}{dx}$

$\frac{\Delta Q}{\Delta t}\int_0^t dx = -aA\int_{3T_0}^{T_0} TdT$

$\frac{\Delta Q}{\Delta t}\ell = 4\alpha AT_0^2 \qquad ...(i)$

Similarly

$\frac{\Delta Q}{\Delta t}\int_0^{\alpha 2} dx = -\alpha A\int_{3T_0}^{T} TdT$

$$\frac{\Delta Q}{\Delta t}\frac{\ell}{2}=\alpha A\frac{\left(9T_0^2-T^2\right)}{2} \qquad ...(ii)$$

Dividing (ii) by (i)

$$9T_0^2-T^2=4T_0^2$$

$$T^2=5T_0^2$$

$$T=\sqrt{5}T_0$$

18. $C=C_v+\alpha T^2$

$$C_v+\frac{RT}{V}\frac{dV}{dT}=C_v+\alpha T^2$$

$$\int\frac{\alpha T}{R}dT=\int\frac{dV}{V}+\ell nk$$

$$\frac{\alpha T^2}{2R}=\ell n(kv)$$

$$kV=e\frac{\alpha T^2}{2R}$$

$$\therefore ve-\left(\frac{\alpha T^2}{2R}\right)=cons\tan t$$

19. $mg=k(2\ell-\ell)=k\ell$

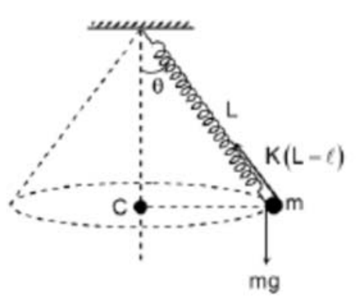
$$k(L-\ell)\cos\theta=mg$$

$$\frac{mg}{\ell}(L-\ell)\frac{\sqrt{L^2-r^2}}{L}=mg \qquad \Rightarrow\sqrt{L^2-r^2}=\frac{L\ell}{L-\ell}$$

$$\Rightarrow L^2-\left(\frac{L\ell}{L-\ell}\right)^2=r^2$$

$$\Rightarrow L^2\left(1-\frac{\ell^2}{(L-\ell)^2}\right)=r^2$$

$$\Rightarrow r^2=L^2\left(\frac{L^2-2L\ell}{(L-\ell)^2}\right) \qquad \Rightarrow r=\frac{L}{\sqrt{L-\ell}}\sqrt{L(L-2\ell)}$$

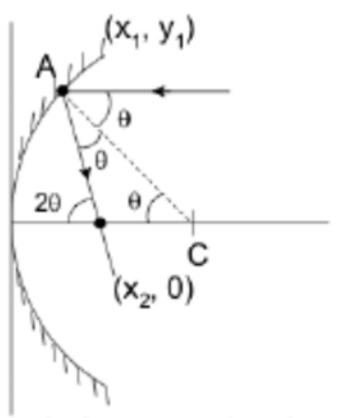


20. After refraction through the quarter cylinder the emergent ray falls on the parabolic feflector parallel to x-axis.

$$\tan(\pi-\theta)=-\frac{1}{\left.\frac{dy}{dx}\right|_{(x_2,x_1)}}=y_1$$

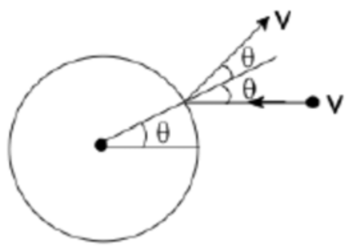
$$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}=\frac{y_1}{x_2-x_1}$$

$$\Rightarrow x_2=\frac{1}{2}m=50\,cm$$



Ac is a normal at A

21. In the frame of the heavy cylinder the particle comes in with speed V and then bounces off.



$$f=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} 2mv\cos^2\theta\left(Rd\cos\theta\right)\ell vn$$

$$\frac{f}{\ell}=\frac{8}{3}mv^2nR$$

22. For reaction only

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R} \qquad ....(i)$$

$$v=-10\,cm$$

Differentiate equation (i) w.r.t.time

$$-\frac{\mu_2}{v^2}\frac{dv}{dt}+\frac{\mu_1}{dt}=0$$

$$\frac{dv}{dt}=\frac{\mu_1}{\mu^2}\frac{v^2}{\mu^2}+\frac{du}{dt}=\frac{4}{1\times3}\times1\times6=8m/s \text{ (towards left)}$$

The velocity of image formed after refraction is 8 m/s (towards left)

The velocity of image formed after reflection and then refraction =8 m/s (towards right).
The relative velocity between two images formed $=16m/s$

23. Path difference at 'O'

$$=(\mu_1-\mu_2)t=4.5\times10^{-5}m$$

Hence phase difference $=\phi\dfrac{2\pi}{\lambda}\Delta r=20\pi$

Now, $l_1=\dfrac{l_0}{16}\,and\,l_2=\dfrac{l_0}{25}$

So, $I=\left(\sqrt{l_1}+\sqrt{\dfrac{1}{2}}\right)^2$

= 1.62 W/m^2

=1.60 W/m^2

24. $I_{C.B.F.}=K\left[A_1^2+A_2^2+2A_1A_2\cos\phi\right]=I_0$

$$KA^2=\frac{I_0}{16},\left(A_1=A,A_2=3A\,and\,\phi=0^\circ\right)$$

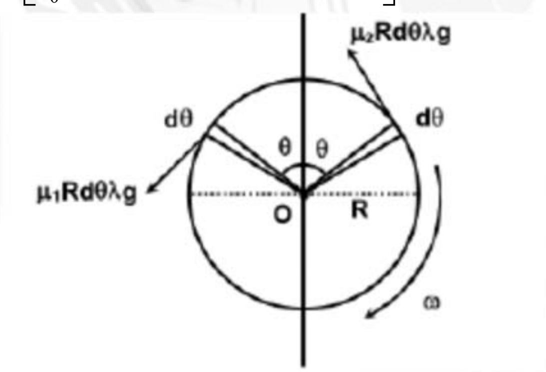
$$I_\theta=\frac{16}{25}K\left[A_1^2+A_2^2+2A_1A_2\cos\phi\right]=1$$

$$\phi=\frac{2\pi}{\lambda}\Delta x=\frac{2\pi}{\lambda}\times d\sin\theta=\frac{2\pi}{\lambda}\times d\frac{3}{5}$$

$So,\phi=\pi$

Hence, $I=\dfrac{I_0}{4}\times\dfrac{16}{25}=\dfrac{5\times16}{100}=0.80\,W/m^2$

25. $2\left[\displaystyle\int_0^{\pi/2}(\mu_1-\mu_2)Rg\lambda\cos\theta d\theta\right]=2(\mu_1-\mu_2)R\lambda g$



$$\therefore a=\frac{2(\mu_1-\mu_2)R\lambda g}{2\pi R\lambda}=4$$

26. $T\propto P^ad^bE^c$

$$\Rightarrow\left[M^0L^0T\right]=\left[ML^{-1}T^{-2}\right]^a\left[ML^{-3}\right]^b\left[ML^2T^{-2}\right]^c$$

Equating exponents of M and T, We get

$a+b+c=0\, and\, -2a-2c=1$

$\Rightarrow a+c=-\frac{1}{2} \qquad \Rightarrow b=\frac{1}{2}$

Hence $a+4b+c=3/2=1.5$

27. $\frac{3}{4}I_{max}=I_{max}\cos^2\frac{\phi}{2}\Rightarrow \cos\frac{\phi}{2}=\frac{\sqrt{3}}{2}\Rightarrow\frac{\phi}{2}=\frac{\pi}{6}\Rightarrow\phi=\pi/3$

$\phi=\frac{2\pi}{\lambda}\Delta x=\frac{\pi}{3}\Rightarrow\Delta x=\frac{\lambda}{6}$

Given $d=200\,\mu m=2\times10^{-4}m$

$$2\left[\sqrt{L^2+\frac{9d^2}{4}}-\sqrt{L^2+\frac{d^2}{4}}\right]=\frac{\lambda}{6}$$

$$\Rightarrow\frac{2d^2}{L}=\frac{\lambda}{6} \qquad \Rightarrow\lambda=\frac{12d^2}{L}=\frac{12\times4\times10^{-8}}{1}=4800A$$

28. $At\, v_{max}$

$Av_{max}^2=F=\frac{km}{v_{max}}$ (where k is a proportionalty constant)

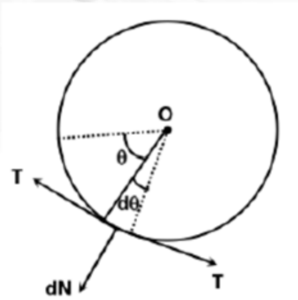
$m^{2/3}v_{max}^2\propto\frac{m}{v_{max}} \qquad \therefore\ v_{max}\propto m^{1/9}$

29. $2T\sin\frac{d\theta}{2}-dN=\lambda Rd\theta\frac{v^2}{R}$ (λ is linear mass density of belt)

$\therefore Td\theta-dN=\lambda v^2d\theta$

$$\therefore\ \text{so total normal}\ =\int_{\frac{\pi}{2}}^{+\frac{\pi}{2}}dN\cos\theta=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}T\cos\theta d\theta-\lambda v^2\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cos\theta d\theta$$

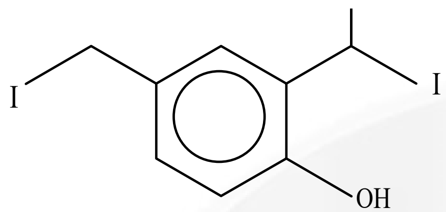
$\therefore\ N=8\,newton$



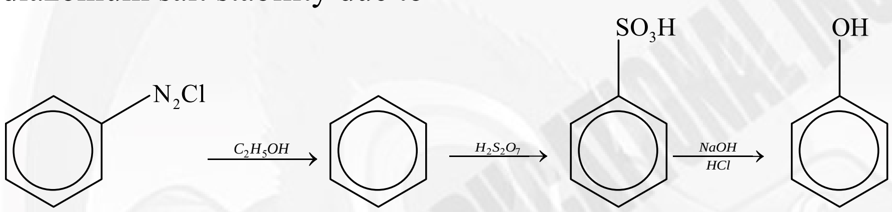
30. $\frac{1}{2}\cdot\frac{m}{2}v_0^2=\frac{1}{2}K\left(\frac{3F}{K}\right)^2+\frac{3F}{2}\cdot\frac{3F}{K}$

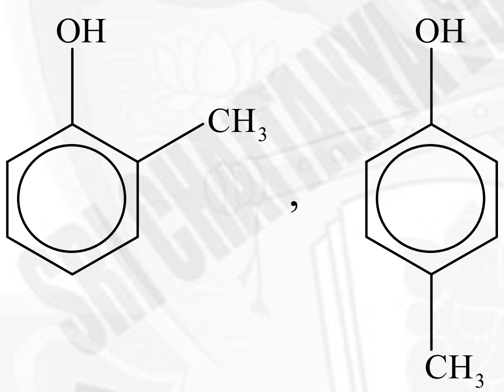
$V_0=6\ m/s$

CHEMISTRY

31. 

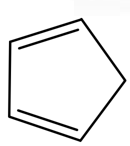
32. Williamson synthesis SN^2 mechanism
33. Acetylene is not possible
34. Similar molecular mass alcohols and ethers have almost same solubility
35. From cumene phenol Industrially prepared
36. diazonium salt stability due to

37. 

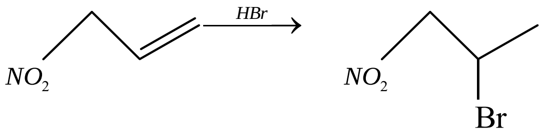
38. 

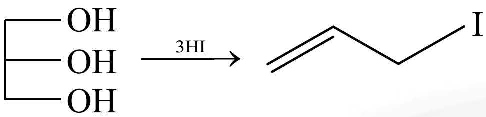
pka=10.2

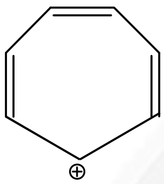
39. Acidic hydrogen in alkylhalide doesnot form grignards reagent
40.

	pka
$HC\equiv CH$	25
	15

41. Phenoxide weaker base than ethoxide
42. ethane, methane are used to prepare chlorofuoro carbons
43. In groups from top to bottom nucleophilicity increases
44. L.P – L.P repulsions decreases bond angle
45. Reaction is not Electrophilic substitution

46. 

47. 

48.  +ve charge delocalized on all carbon atoms

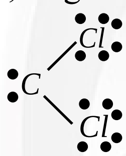
49. Benzene

50. Poly alkylation takes place in friedelcraft reaction

51. Alcohols gives red colour with 'CAN' 2^{o} alcohols gives blue colour in victor mayer test

52. Aromatic amines are less basic

53. i, iii gives white precipitate

54. 

55. Bridgehead and aryl, vinyl halides not involve in SN reactions

56. H.O.H $\alpha \dfrac{1}{\text{stability}}$

57. Five planar atoms in propyne

58. $CH_3-C\equiv CH$

59. 4 moles of $NaNH_2$

60.

$$C-C-C-C-\overset{\overset{O}{\|}}{C}-NH_2,\ C-\underset{\underset{C}{|}}{C}-C-\overset{\overset{O}{\|}}{C}-NH_2$$

$$C-C-\underset{\underset{C}{|}}{C}-\overset{\overset{O}{\|}}{C}-NH_2,\ C-\overset{\overset{C}{|}}{\underset{\underset{C}{|}}{C}}-\overset{\overset{O}{\|}}{C}-NH_2$$

2

MATHEMATICS

61. $f'(x)+f'(2-x)=0$

$f(x)-f(2-x)=\lambda$

Put $x=1$, we get $\lambda=0$

$\therefore f(x)=f(2-x) \therefore f(x+1)=f(1-x)$

$\therefore \ f(1)=4 \therefore \ y=f(x)=(x-1)^2+4$

$$\int_1^3 \frac{dx}{(x-1)^2+4}=\left[\frac{1}{2}\tan^{-1}\frac{x-1}{2}\right]_1^3=\frac{1}{2}\left[\tan^{-1}1-\tan^{-1}0\right]=\frac{\pi}{8}$$

62. $g(x)=f'(x)$

Now, $\int_{-1}^{1} f^2(x)g(x)dx=\int_{-1}^{1} f^2(x)f'(x)dx=\frac{1}{3}\left(f^3\right)(x)\Big|_{-1}^{1}$

$=\frac{1}{3}\left[\left(f(1)\right)^3-\left(f(-1)\right)^3\right]=\frac{1}{3}\left[27-(-27)\right]=18$

63. $\int_1^{f(x)} f^{-1}(t)dt=\frac{1}{3}\left(x^{3/2}-8\right)$

Differentiating both sides w.r.t x, we get

$f^{-1}\left(f(x)\right)f'(x)=\frac{\sqrt{x}}{2}\Rightarrow xf'(x)=\frac{\sqrt{x}}{2}\Rightarrow f'(x)=\frac{1}{2}x^{\frac{-1}{2}}$

Integrate both side w.r.t x, we get

$f(x)=\sqrt{x}+C$

Given $f(1)=0\Rightarrow C=-1$

Hence, $f(x)=\sqrt{x}-1\Rightarrow f(9)=3-1=2$.

64. a) $f(x)=\sin x-x^2+1$

$f'(x)=\cos x-2x$

$\Rightarrow f'(x)<0 \ for \ x>x_0$

$f'(x)>0$ for $x<x_0$

Hence $x=x_0$ is point of maxima

b) $f(x)=x\log_e x-x+e^{-x}$

$f'(x)=\log_e x+1-1-e^{-x}=\log_e x-e^{-x}$

c) $f(x)=-x^3+2x^2-3x+1$

$f'(x)=-3x^2+4x-3$

$D<0$

$\therefore f'(x)<0$

d) $f'(x)=-\pi\sin\pi x+10+6x+3x^2$

$$=3\left((x+1)^2+\frac{7}{3}\right)-\pi\sin\pi x$$

$$\therefore f'(x)>0$$

65. Consider $f'(x)=4ax^2+3bx^2+2cx+d$

$$\Rightarrow f(x)=ax^4+bx^2+cx^2+dx+e$$

$$f(0)=e \text{ and } f(3)=81a+27b+9c+3d+e$$

$$=3(27a+9b+3c+d)+e=e$$

Hence, Rolle's theorem is applicable for $f(x)$

$\Rightarrow$ there exists at least one c in (a,b) such that $f'(c)=0$

66. $$g(x)=\int_x^{\pi/4}\left(f'(t)\sec t+\tan t\sec t\, f(t)\right)dt$$

$$\therefore g(x)=\int_x^{\pi/4}d\left(f(t).\sec t\right)=\left[f(t)\sec t\right]_x^{\pi/4}$$

$$\Rightarrow g(x)=f\left(\frac{\pi}{4}\right)\sec\frac{\pi}{4}-f(x).\sec x=2-\frac{f(x)}{\cos x}$$

$$\therefore \lim_{x\to\left(\frac{\pi}{2}\right)^-}f(x)=2-\lim_{x\to\left(\frac{\pi}{2}\right)^-}\frac{f(x)}{\cos x}$$

$$=2-\lim_{x\to\left(\frac{\pi}{2}\right)^-}\frac{f'(x)}{(-\sin x)}$$

(Using L' Hospital rule)

$$=2+\frac{f'\left(\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}}=2+\frac{1}{1}=3$$

67. $$U_n=\prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^r$$

$$L=\lim_{x\to\infty}\left(U_n\right)^{-4/n^2}$$

$$\therefore \log L=\lim_{n\to\infty}\frac{-4}{n^2}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)^r$$

$$\Rightarrow \log L=-4\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}.\frac{r}{n}\log\left(1+\frac{r^2}{n^2}\right)$$

$$\Rightarrow \log L=-4\int_0^1 x\log\left(1+x^2\right)dx$$

$$Put\, 1+x^2=t$$

$\therefore L = e^2/16$

68. $\int_0^2 \left(f(x) - f(4x)\right)dx = 10.$

Put $x = 2t \Rightarrow dx = 2dt$

$\therefore 2\int_0^1 \left(f(2t) - f(8t)\right)dt = 10$

$\therefore \int_0^1 \left(f(2t) - f(8t)\right)dt = 5$

On adding above two $\int_0^1 \left(f(x) - f(8x)\right)dx = 10$

69. $\frac{dy}{dx} = 12x\left(x^2 - x + 1\right) + a$

$\frac{d^2y}{dx^2} = 12\left(3x^2 - 2x + 1\right) > 0$

$\Rightarrow \frac{dy}{dx}$ is an increasing function

But $\frac{dy}{dx}$ is a polynomial of degree 3 $\Rightarrow$ it has exactly one real root

70. $I_{2k} = \int_0^{\pi} \frac{\sin(2kx)}{\sin x} dx = 0$

$I_{2k+1} - I_{2k-1} = \int_0^{\pi} \frac{\sin(2k+1)x - \sin(2k-1)}{\sin x} dx$

$= \int_0^{\pi} \frac{2\sin x \cos 2kx}{\sin x} dx = 2\frac{\sin(2kx)}{2k}\Big|_0^{\pi} = 0$

$\therefore\ I_{2k+1} = I_{2k-1} \Rightarrow I_{2011} = I_{2009} = \ldots = I_3 = I_1 = \pi$

$f(x) = x^5 + 5x - 1$

71. $f(1) = 5 \Rightarrow f^{-1}(5) = 1$

$f(2) = 41 \Rightarrow f^{-1}(41) = 2$

Put $f^{-1}(x) = t$

$\therefore I = \int_1^2 \frac{f'(t)}{t^5 + 5t} dt = \int_1^2 \frac{5t^4 + 5}{t^5 + 5t} dt$

$= \ln\left(t^5 + 5t\right)_1^2 = \ln 42 - \ln 6 = \ln 7$

72. $s(x)=\int \frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}$

$=\frac{x^7}{2x^7+x^2+1}+c$

73. $\int \frac{\left(x^2+1\right)\sin 2x-2x\sin^2 x}{\left(x^2+1\right)^2}dx$

$=\int f\left(\frac{\sin^2 x}{x^2+1}\right)$

$f(x)=\frac{\sin^2 x}{x^2+1}+c$

$C=0$

$f(x)=\frac{\sin^2 x}{x^2+1}$

$f\left(\frac{\pi}{4}\right)=\frac{\left(1/\sqrt{2}\right)^2}{\frac{\pi^2}{16}+1}=\frac{1/2}{\frac{\pi^2}{16}+1}=\frac{8}{\pi^2+16}$

74. As f is continuous so $f(0)=\lim\limits_{x\to\infty} f(x)$

$\Rightarrow f(0)=\lim\limits_{n\to\infty} f(1/4n)=\lim\limits_{n\to\infty}\left(\left(\sin e^n\right)e^{-n^2}+\frac{1}{1+1/n^2}\right)=0+1=1$

75. Let $y=\sqrt{-3+4x-x^2}\Rightarrow x^2+y^2-4x+3=0$

$\therefore$ point (x,y) lies on this circle $(x-2)^2+y^2=1$

$C(2,0)$ and radius 1

$CP=5,$ then the maximum distance between the point P and any point on the circles is 6

$\Rightarrow$ Maximum value of $\left(\sqrt{-3+4x-x^2}+4\right)^2+(x-5)^2$ is 36

76. $f_1(x)=0$ has mini two solutions in $[0,4]$

$f_2(x)=0$ has mini 3 solutions in $[0,4]$

$f_2'(x)=0$ has mini 2 solutions in $[0,4]$

$f_1(x)\,f_2'(x)=0$ has minimum 4 solutions in $[0,4]$

$\frac{d}{dx}\left(f_1(x)\,f_2'(x)\right)=0$ has minimum 3 solutions in $[0,4]$

77. $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos x}{1+e^x}dx \qquad =\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos x}{1+e^{-x}}dx=I$

$I=\int_0^{\frac{\pi}{2}}\cos x\,dx \qquad =\left[\sin x\right]_0^{\pi/2}=1$

78. $f(x)=\log x$

79. $k = f\left(\frac{0+2}{2}\right) = f(1) = 3$

80. $I = \int_0^x \frac{1+2\sin x}{(2+\sin x)^2}dx$

Multiplying numerator and denominator by $\sec^2 x$, we get

$I = \int \frac{\sec^2 x + 2\sec x \tan x}{(2\sec x + \tan x)^2}dx \; Put(2\sec x + \tan x) = t$

$\Rightarrow (\sec^2 + 2\sec x \tan x)dx = dt$

$I = \int \frac{dt}{t^2} = \frac{-1}{t} = \frac{-1}{2\sec x + \tan x} = \frac{-\cos x}{2+\sin x}\Big]_0^{\pi/2} = \frac{1}{2}$

81. $\int_{-6}^{12} f(x)dx = 9 \Rightarrow \int_{-6}^{0} f(x)dx + \int_0^{12} f(x)dx = 9$

$\Rightarrow \int_{-6}^{0} f(x)dx = -3 \Rightarrow \int_{-6}^{6} f(x)dx = 12 \Rightarrow \int_{-6}^{0} f(x)dx + \int_0^6 f(x)dx = 12 \Rightarrow \int_0^6 f(x)dx = 15$

82. $\int_a^b f(x)\,dx + \int_{f(a)}^{f(b)} f^{-1}(x)\,dx = b\cdot f(b) - a\cdot f(a)$

$\int_0^1 (1-x^7)^{1/4}dx + \int_1^0 (1-x^4)^{1/7}dx = 0$

83. Gives $\int_0^x t^2 \sin(x-t)dt = x^2$

$\Rightarrow \int_0^x (x-t)^2 \sin t\,dt = x^2$

$\Rightarrow x^2\int_0^x \sin t\,dt - 2x\int_0^x t\sin t + \int_0^x t^2 \sin t = x^2$

$\Rightarrow x^2(1-\cos x) - 2x(-x\cos x + \sin x) + (-x^2\cos x + 2x\sin + \cos x - 1) = x^2$

$\Rightarrow \cos x = 1$

So, $x = 2n\pi; n \in I$

Hence, number of values of $0, 2\pi, 4\pi., ..., 30\pi$

84. $I_2 = \int_0^1 (1-x^{50})^{101} dx = \left[(1-x^{50})^{101} x\int_0^1 - \int_0^1 101(1-x^{50})^{100}(-50x^{49})\right]$

$= 5050\int_0^1 (1-x^{50})^{100} x^{50} \quad = 5050\int_0^1 (1-x^{50})^{100}(1-(1-x^{50})$

$= 5050(I_1 - I_2) \quad \therefore 5051 I_2 = 5050 I_1$

85. $\int \frac{f(x)}{x^2(x+1)^2}dx = \int\left(\frac{A}{x}+\frac{B}{x^2}+\frac{c}{x+1}+\frac{D}{(x+1)^2}+\frac{E}{(x+1)^3}\right)dx$

$A = C = 0$

$f(x) = ax^2 + bx + 1 = B(x+1)^3 + Dx^2(x+1)$

$B + D = 0 \ +E\,x^2$

$B = 1 \Rightarrow D = -1$

$f(x) = (x+1)^3 - x^2(x+1) + Ex^2 = 3$

86. $g(x) - |x(2x+1)(2x-1)|\cos \pi x$

Differentiable of $x = 0$

87. $p(x)$ has local minima and maxima at $x = 1$ and $x = -1$ resp.

So, $p'(x) = a(x-1)(x+1) = a(x^2-1)$

$\therefore \ p(x) = a\int(x^2-1)dx + c = a\left(\frac{x^3}{3} - x\right) + c$

Given that $p(-3) = 0$

$\Rightarrow a\left(-\frac{27}{3}+3\right) + c = 0 \Rightarrow -6a + c = 0$

Also $\int_{-1}^{1}\left(a\left(\frac{x^3}{3}-x\right)+c\right)dx = 18$

$\Rightarrow 2c = 18 \ or \ c = 9$

So, from (1), we get a $= \frac{3}{2}$

$\Rightarrow \ p(x) = \frac{3}{2}\left(\frac{x^3}{3} - x\right) + 9$

So, sum of coefficient $= \frac{1}{2} - \frac{3}{2} + 9 = 8$

88. $\lim_{\lambda \to 10} \frac{f(2+h^4) - f(2-h^4)}{h^4}$

Use L – H rule

89. $f(x)$ is constant function

90. $g(x) = \log\left(x + \sqrt{x^2+1}\right)$