Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60 JEE-MAIN Date: 17-09-2022
Time: 09.00Am to 12.00Pm RPTM-02 Max. Marks: 300

KEY SHEET

PHYSICS

1)	2	2)	3	3)	3	4)	4	5)	4
6)	4	7)	3	8)	1	9)	2	10)	2
11)	2	12)	1	13)	2	14)	2	15)	3
16)	2	17)	4	18)	2	19)	3	20)	1
21)	530	22)	375	23)	625	24)	24	25)	6720
26)	7	27)	4	28)	9	29)	5	30)	12

CHEMISTRY

31)	1	32)	3	33)	4	34)	1	35)	1
36)	2	37)	4	38)	3	39)	3	40)	3
41)	1	42)	1	43)	2	44)	3	45)	3
46)	4	47)	1	48)	3	49)	4	50)	3
51)	6	52)	8	53)	4	54)	3	55)	6
56)	4	57)	3	58)	5	59)	9	60)	8

MATHEMATICS

61)	1	62)	1	63)	1	64)	1	65)	1
66)	4	67)	3	68)	4	69)	3	70)	2
71)	2	72)	3	73)	4	74)	3	75)	4
76)	2	77)	4	78)	3	79)	3	80)	3
81)	105	82)	8	83)	5	84)	2	85)	81
86)	3	87)	15	88)	6	89)	2	90)	11

MATHEMATICS

61. $A=\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix} \Rightarrow |A|=(1-a\omega)(1-c\omega)$, For A to be non-singular matrix, none of a and c should be ω^2 So, $a=c=\omega$ While b can take value ω or ω^2 So, the number of distinct matrices in the set 'S' is 2

62. $a_1=(2m+1)^2, a_2=(2n+1)^2 \Rightarrow a_1-a_2=4\left(m(m+1)-n(n+1)\right)=8k$

So, difference of any two odd square is always a multiple of 8 Now apply C_1-C_3 and C_2-C_3 then C_1 and C_2 both become multiple of 8 so Δ always a multiple of 64

63. $3\sin t+4\ge 1, \sin t+3\ge 2$

$$\Rightarrow \begin{vmatrix} 3 & 2 & 1 & 4 \\ 0 & 8 & 4 & 6 \\ 0 & 0 & \ge 1 & \ge 2 \end{vmatrix}$$ $\Rightarrow$ Unique solutions

64. **Case-I:** $1\to 7$ times

And-$1\to 2$ times

Number of possible matrix $=\frac{9!}{7!2!}=36$

Case-II: $1\to 6$ times

$-1\to 1$ Times

and $0\to 2$ times

Number of possible matrix $=\frac{9!}{6!2!}=252$

Case-III: $1\to 5$ times

$0\to 4$ times

Number of possible matrix $=\frac{9!}{5!4!}=126$

Hence total number of all such matrix A=414

65. $A=\begin{pmatrix} 1 & 2 & 2^2 \\ 1/2 & 1 & 2 \\ 1/2^2 & 1/2 & 1 \end{pmatrix}$

$A^2=3A,\ A^3=3^2A,$ ....

$$A^2+A^3+\ldots A^{10}=3A+3^2A+\ldots..3^9A \quad =\frac{3(3^9-1)}{3-1}A=\frac{3^{10}-3}{2}A$$

66.
$$\Delta = \begin{vmatrix} -K & 3 & -14 \\ -15 & 4 & -K \\ -4 & 1 & 3 \end{vmatrix} = 121 - K^2, \Delta \neq 0 \qquad k \in R - \{11, -11\}$$

$$If\ k = -11, \Delta_2 = \begin{vmatrix} 11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$$

No solution

67.
$$X = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, X^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{pmatrix}, Z = \begin{pmatrix} \alpha^2 & -\alpha\beta & \beta^2 - \alpha\gamma \\ 0 & \alpha^2 & -\alpha\beta \\ 0 & 0 & \alpha^2 \end{pmatrix}$$

$$Y.Y^{-1} = I$$

$$\begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{pmatrix} \begin{pmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{\alpha}{5} = 1 \Rightarrow \alpha = 5$$

$$-\frac{2}{5}\alpha + \frac{\beta}{5} = 0 \Rightarrow \beta = 10$$

$$\frac{\alpha}{5} - \frac{2\beta}{5} + \frac{\gamma}{5} = 0$$

$$\Rightarrow \gamma = 15$$

$$\Rightarrow (\alpha + \beta + \gamma)^2 = (5 + 10 + 15)^2 = 900$$

68. $a_{ij} = -a_{ji} \Rightarrow$ A is a skew symmetric of even order

$\therefore |A|$ is a perfect square

69.
$$f^1(0) = \begin{vmatrix} 22 & 44 & 66 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 33 & 66 & 99 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 44 & 88 & 144 \end{vmatrix} = 0$$

$\because$ co-eff of $x = 0$

70.
$$(I + A)^n = I + {}^nC_1A^{n-1} + {}^nC_2 + A^{n-2} + \ldots + {}^nC_nA^n$$

$$= I + ({}^nC_1 + {}^nC_2 + \ldots + {}^nC_n)A$$

$$= I + \left(2^n - 1\right) A$$

71. Non-trivial solutions $\Rightarrow |A| = 0$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow (a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$

$$\Rightarrow a+b+c=0 \; Eq, is \;\; at^2 + bt + c = 0, p.o.r = \frac{c}{a} is - ve$$

72. $$\frac{a + \frac{b}{2} + \frac{b}{2} + \frac{c}{3} + \frac{c}{3} + \frac{c}{3}}{6} \geq \sqrt[6]{a\left(\frac{b}{2}\right)^2 \left(\frac{c}{3}\right)^3}$$

73. $$\min f(x) > \max g(x) \Rightarrow \frac{4\left(2c^2\right) - 4b^2}{4(1)} > \frac{4(-1)b^2 - 4c^2}{4(-1)}$$

$$\Rightarrow 2c^2 - b^2 > b^2 + c^2 \qquad \Rightarrow 2c^2 - c^2 > b^2 + b^2$$

$$\Rightarrow c^2 > 2b^2 \qquad \therefore \left|\frac{c}{b}\right| > \sqrt{2}$$

74. $x^{\log_3 x} > 3$ taking logarithm with base 3

$\left(\log_3 x\right)\left(\log_3 x\right) > 1 \quad \Rightarrow p^2 - 1 > 0$ where $p = \log_3 x$

$p < -1 \;\; or \; p > 1 \quad \Rightarrow x > 3 \;\; or \;\; x < \frac{1}{3}$ and $x > 0$

75. (i) $\frac{-b}{2a} > 5$ (ii) $b^2 - 4ac > 0$ (iii) $f(5) > 0$

76. Let $t = x^2 + x + 1 \;\Rightarrow t \in \left[\frac{3}{4}, \infty\right)$

Hence $(t+1)^2 - (a-3)t(t+1) + (a-4)t^2 = 0$

$$\Rightarrow t^2 + 2t + 1 - (a-3)\left(t^2 + t\right) + (a-4)t^2 = 0$$

$$\Rightarrow t(2 - a + 3) + 1 = 0 \Rightarrow t = \frac{1}{a-5} \quad \Rightarrow \frac{1}{a-5} \geq \frac{3}{4} \qquad \therefore a \in \left(5, \frac{19}{3}\right]$$

77. $$\frac{2\alpha\beta}{\alpha+\beta} = 4 \quad \Rightarrow b = 4 + \sqrt{5}$$

78. A.M of 3rd and 7th A.M's inserted between 36, 1296 = $\left(\frac{36 + 1296}{2}\right) = 666$

[In A.P sum of the terms which are equidistant from the beginning and ending are equal)

79. AM of roots = GM of roots=1then each root =1, a=6, b=-4

80. $$81x^2 + kx + 256 = 0; \; x = \alpha. \; \alpha^3 \;\; \Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$$

Now $-\frac{k}{81} = \alpha + \alpha^3 = \pm\frac{100}{27} \qquad \Rightarrow k = \pm 300$

81. The minimum numbers of zeros $=\dfrac{15\times15-15}{2}=\dfrac{225-15}{2}=\dfrac{210}{2}=105$

82. $\left|A^{-1}.adj\left(B^{-1}\right).adj\left(2A^{-1}\right)\right|=\dfrac{1}{|A|}.\dfrac{1}{|B|^2}.\left|2A^{-1}\right|^2$

$=\dfrac{1}{|A|}.\dfrac{1}{|B|^2}\dfrac{64}{\left|A^2\right|}=\dfrac{64}{|A|^3|B|^2}=\dfrac{64}{8\times9}=\dfrac{8}{9}$

83. $A=\begin{bmatrix}1&0&0\\0&1&1\\0&-2&4\end{bmatrix}\Rightarrow |A-xI|=\begin{vmatrix}1-x&0&0\\0&1-x&1\\0&-2&4-x\end{vmatrix}=(1-x)^2(4-x)+2(1-x)$

$\therefore f(x)=-x^3+6x^2-11x+6\quad f(A)=0\Rightarrow -A^3+6A^2-11A+6I=0$

$\Rightarrow A\left(A^2-6A+11I\right)=6I\ \Rightarrow\dfrac{1}{6}\left(A^2-6A+11I\right)=A^{-1}\ \therefore(\alpha,\beta)=(-6,11)$

84. : $Y=10M+1; Z=10N$ $\quad\Delta=-X-4(-Z)+Y$

$\Delta+1=-X+4Z+Y+1=10k$ $\quad x=40N+10M+1+1-10k=10[4N+M-K]+2$

85. $Q^2=PAP^TPAP^T=PA^2P^T$

$Q^4=PA^2P^TPA^2P^T=PA^4P^T$ $\Rightarrow P^TQ^8P=A^8$

$A^2=\begin{bmatrix}3&-2\sqrt{3}-2\\0&1\end{bmatrix}\quad A^3=\begin{bmatrix}3\sqrt{3}&-\\-&-\end{bmatrix}\quad A^n=\begin{bmatrix}\left(\sqrt{3}\right)^n&-\\-&-\end{bmatrix}\Rightarrow A^8=\begin{bmatrix}\left(\sqrt{3}\right)^8&-\\-&-\end{bmatrix}$

86. $\dfrac{\alpha^{10}-\beta^{10}-2\left(\alpha^8-\beta^8\right)}{2\left(\alpha^9-\beta^9\right)}\Rightarrow\dfrac{\alpha^8\left(\alpha^2-2\right)-\beta^8\left(\beta^2-2\right)}{2\left(\alpha^9-\beta^9\right)}\Rightarrow\dfrac{\alpha^8(6\alpha)-\beta^8(6\beta)}{2\left(\alpha^9-\beta^9\right)}\Rightarrow3$

87. $x^2+ax+12=0$

$x^2+bx+15=0$

Adding $2x^2+(a+b)x+27=0\ \rightarrow(1)$

3rd equation $x^2+(a+b)x+36=0\ \rightarrow(2)$

$(1)-(2)\Rightarrow x^2-9=0\Rightarrow x=\pm3$; +ve root is $x=3$

$a=-7,b=-8\Rightarrow a-b=1,a+b=-15$

88. Clearly $x>0\,and\,x\neq\dfrac{1}{5}$ $\log_{5x}\left(\dfrac{5}{x}\right)=\dfrac{\log_5 5-\log_5 x}{\log_5 5+\log_5 x}$, $put\log_5 x=t$

$(\log_5 x)^2+\log_{5x}\left(\dfrac{5}{x}\right)=1\Rightarrow t^2+\dfrac{1-t}{1+t}=1\Rightarrow t=0,1,-2\Rightarrow x=1,5,5^{-2}$

89. $1+\dfrac{1}{n^2}+\dfrac{1}{(n+1)^2}=\left(1+\dfrac{1}{n}-\dfrac{1}{n+1}\right)^2$

90. $\left(l^2-l-30\right)<0\Rightarrow(l-6)(l+5)<0\Rightarrow-5<l<6\Rightarrow32>x>\dfrac{1}{64}$