Q1 - 24 June - Shift 1

Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then A + C is equal to_____.

(A) 16 (B) 88/5

(C) 72 (D) –8

Space for your notes:

Q2 - 24 June - Shift 1

If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals _____

Space for your notes:

Q3 - 24 June - Shift 2

A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with

(A) length of latus rectum 3

(B) length of latus rectum 6

(C) focus $\left(\frac{4}{3}, 0\right)$

(D) focus $\left(0, \frac{3}{4}\right)$

Space for your notes:

Q4 - 24 June - Shift 2

Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line $x + 2y = 6$. Then the directrix of P_2 is $x + 2y =$ _____.

Space for your notes:

Q5 - 25 June - Shift 1

If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to

(A) $3+4\sqrt{2}$ (B) $-5+6\sqrt{2}$

(C) $-4+3\sqrt{2}$ (D) $7+6\sqrt{2}$

Space for your notes:

Q6 - 25 June - Shift 1

Let $x = 2t$, $y = \frac{t}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that $SA \perp BA$, where A is any point on the conic. If k is the ordinate of the centroid of ΔSAB, then $\lim\limits_{t\to 1} k$ is equal to

(A) $\frac{17}{18}$ (B) $\frac{19}{18}$

(C) $\frac{11}{18}$ (D) $\frac{13}{18}$

Space for your notes:

Q7 - 25 June - Shift 2

If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

(A) $\frac{3}{2}$ (B) $\frac{26}{9}$

(C) $\frac{5}{2}$ (D) $\frac{23}{6}$

Space for your notes:

Q8 - 26 June - Shift 1

Let the normal at the point P on the parabola $y^2 = 6x$ pass through the point (5, –8). If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is :

(A) –3 (B) $-\frac{9}{4}$ (C) $-\frac{5}{2}$ (D) –2

Space for your notes:

Q9 - 27 June - Shift 1

A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$.

Then $(4\alpha - 8)^2$ is equal to _____.

Space for your notes:

Q10 - 27 June - Shift 2

If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x+y-29=0$, is $x^2+ay^2+bxy+cx+dy+k=0$ then

$a + b + c + d + k$ is equal to

(A) 575 (B) –575

(C) 576 (D) –576

Space for your notes:

Q11 - 28 June - Shift 2

If vertex of a parabola is (2, – 1) and the equation of its directrix is 4x – 3y = 21, then the length of its latus rectum is

(A) 2 (B) 8

(C) 12 (D) 16

Space for your notes:

Q12 - 29 June - Shift 2

Let P : $y^2 = 4ax$, $a > 0$ be a parabola with focus S.Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line y = 3x + 5 touch the parabola P at A and B. Then the value of a for which A,B and S are collinear is:

(A) 8 only (B) 2 only

(C) $\frac{1}{4}$ only (D) any $a > 0$

Space for your notes:

Answer Key

Q1 (A) **Q2** (2929) **Q3** (A) **Q4** (10)

Q5 (C) **Q6** (D) **Q7** (C) **Q8** (B)

Q9 (63) **Q10** (D) **Q11** (B) **Q12** (D)

Q1 (A)

$x^2 + y^2 + Ax + By + C = 0$ is passing through (0,6)

$\Rightarrow 6B + C = -36$

The tangent of the parabola $y = x^2$ at (2, 4) is

$4x - y - 4 = 0$ ----(1)

The tangent of circle $x^2 + y^2 + Ax + By + C = 0$ at (2, 4) is

$(4 + A)x + (8 + B)y + 2A + 4B + 2C = 0$ -----(2)

From Equation (1) and (2)

$$\frac{4+A}{4} = \frac{8+B}{-1} = \frac{2A+4B+2C}{-4}$$

$A + 4B = -36$ ---(3)

$3A + 4B + 2C = -4$ ---(4)

From equation (3) and (4)

$A + C = 16$

Q2 (2929)

$\alpha = \frac{1}{5}\cos\theta,\ \beta = \frac{1}{2}\sin\theta$

Equation of tangent to $y^2 = 4x$

$y = mx + \frac{1}{m}$

It passes through (α, β)

$\frac{1}{2}\sin\theta = m\frac{1}{5}\cos\theta + \frac{1}{m}$

$m^2\left(\frac{\cos\theta}{5}\right) - m\left(\frac{1}{2}\sin\theta\right) + 1 = 0$

It has two roots m_1 and m_2 where $m_1 = 4m_2$

$m_1 + m_2 = \frac{\frac{1}{2}\sin\theta}{\frac{\cos\theta}{5}}$

$m_1 m_2 = \frac{5}{\cos\theta}$

After eliminating m_1 and m_2

$\cos\theta = \frac{-5 \pm \sqrt{29}}{2}$

$\alpha = \frac{-5 \pm \sqrt{29}}{10} \Rightarrow 10\alpha + 5 = \pm\sqrt{29}$

$\beta^2 = \frac{1}{4}\sin^2\theta \Rightarrow 16\beta^2 = -50 \pm 10\sqrt{29}$

$(10\alpha + 5)^2 + (16\beta^2 + 50)^2 = 2929$

Q3 (A)

Let Point P(x,y)

$Y - y = y'(X - x)$

$Y = 0 \Rightarrow X = x - \frac{y}{y'}$

$Q\left(x - \frac{y}{y'}, 0\right)$

Mid Point of PQ lies on y axis

$x - \frac{y}{y'} + x = 0$

$y' = \frac{y}{2.x} \Rightarrow 2\frac{dy}{y} = \frac{dx}{x}$

$2\ell ny = \ell nx + \ell nk$

$y^2 = kx$

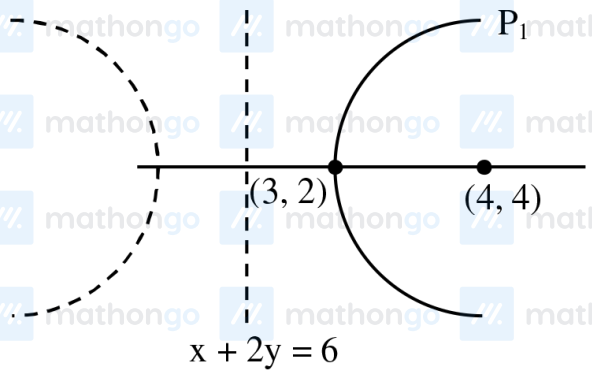
It passes through $(3, 3) \Rightarrow k = 3$

curve c $\Rightarrow y^2 = 3x$

Length of L.R. = 3

Focus = $\left(\frac{3}{4}, 0\right)$ **Ans. (A)**

Q4 (10)



P_1: Directorix :

$x + 2y = k$

$x + 2y - k = 0$

$$\left|\frac{3+4-K}{\sqrt{5}}\right| = \sqrt{5}$$

$|7 - k| = 5$

$7 - K = 5 \quad 7 - K = -5$

$\boxed{k = 2} \quad \boxed{k = 12}$

Accepted Rejected

Passes through focus

$D_1 = x + 2y = 2 \Rightarrow d$

$\ell = x + 2y = 6$

$D_2 = x + 2y = C \Rightarrow d$

$\Rightarrow \boxed{c = 10}$

Q5 (C)

C_1: $x^2 + y^2 = 2$

C_2 : $y^2 = x$

Let tangent to parabola be $y = mx + \frac{1}{4m}$.

It is also a tangent of circle so distance from centre of circle (0, 0) will be $\sqrt{2}$.

$$\left|\frac{\frac{1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2} \quad \Rightarrow \quad 1 = 32m^2 + 32m^4$$

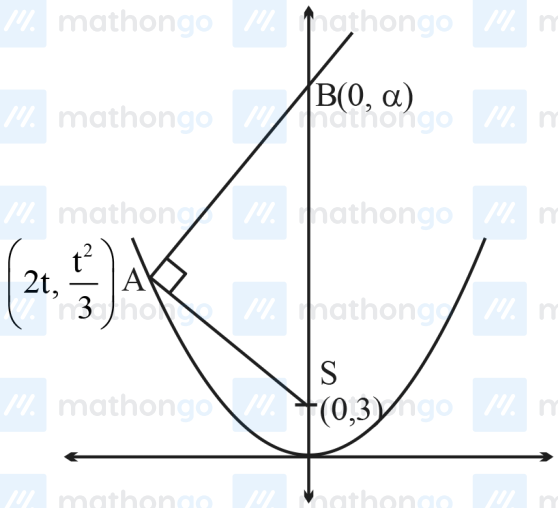
by solving

$m^2 = \frac{3\sqrt{2}-4}{8}$, $m^2 = \frac{-3\sqrt{2}-4}{8}$ (rejected)

$$m = \pm\sqrt{\frac{3\sqrt{2}-4}{8}}$$

so, $8\,|m_1m_2| = 3\sqrt{2}-4$

Q6 (D)



parabola $x^2 = 12y$

$SA \perp SB$

so, $m_{AS} \cdot m_{AB} = -1$

$$\frac{\left(3-\frac{t^2}{3}\right)}{(0-2t)} \cdot \frac{\left(\alpha-\frac{t^2}{3}\right)}{(0-2t)} = -1$$

by solving

$$3\alpha = \frac{27t^2+t^4}{t^2-9}$$

ordinate of centriod of $\Delta SAB = K = \frac{\alpha+\frac{t^2}{3}+3}{3}$

$$k = \frac{9+3\alpha+t^2}{9}$$

$$\lim_{t\to 1} k = \lim_{t\to 1} \frac{1}{9}\left(9+t^2+\frac{27t^2+t^4}{(t^2-9)}\right) = \frac{13}{18}$$

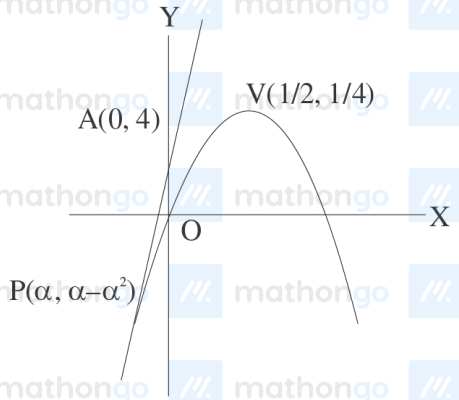
Q7 (C)

Slope of tangent at P = Slope of line AP

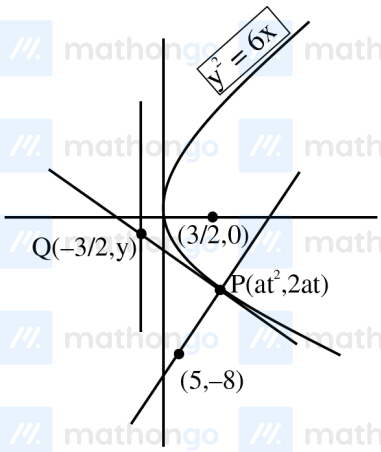
$$y'|_P = 1 - 2\alpha = \frac{\alpha - \alpha^2 - 4}{\alpha}$$

Solving $\alpha = -2 \Rightarrow P(-2,-6)$

Slope of PV = $\frac{5}{2}$



Q8 (B)



Equation of normal : $y = -tx + 2at + at^3$ $\left(a = \frac{3}{2}\right)$

since passing through (5, –8), we get $t = -2$

Co-ordinate of Q : (6, –6)

Equation of tangent at Q : $x + 2y + 6 = 0$

Put $x = \frac{-3}{2}$ to get $R\left(\frac{-3}{2}, \frac{-9}{4}\right)$

Q9 (63)

Vertex and focus of parabola $y^2 = 2x$ are V (0, 0) and $S\left(\frac{1}{2}, 0\right)$ resp.

Let equation of circle be

$(x - h)^2 + (y - k)^2 = 4$

$\because$ Circle passes through (0, 0)

$\Rightarrow h^2 + k^2 = 4$ .....(1)

$\because$ Circle passes through $\left(\frac{1}{2}, 0\right)$

$$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4} \text{.....(2)}$$

On solving (1) and (2)

$$4 - h = \frac{15}{4}$$

$$h = 4 - \frac{15}{4} = \frac{1}{4}$$

$$k = +\frac{\sqrt{63}}{4}$$

$k = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre $\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

$$\left(x - \frac{1}{4}\right)^2 + \left(k - \frac{\sqrt{63}}{4}\right)^2 = 4$$

From figure

$$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8 + \sqrt{63}}{4}$$

$$4\alpha - 8 = \sqrt{63}$$

$$(4\alpha - 8)^2 = 63$$

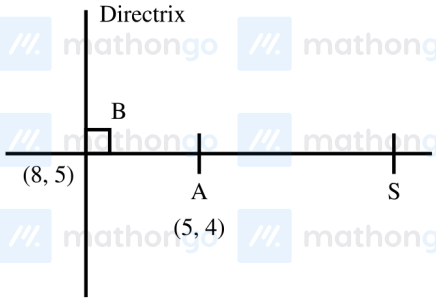
Q10 (D)

Vertex (5,4)

Directrix : $3x + y - 29 = 0$

Co-ordinates of B (foot of directrix)

$$\frac{x-5}{3} = \frac{y-4}{1} = -\left(\frac{15+4-29}{10}\right) = 1$$



$x = 8, y = 5$

S = (2, 3) (focus)

Equation of parabola

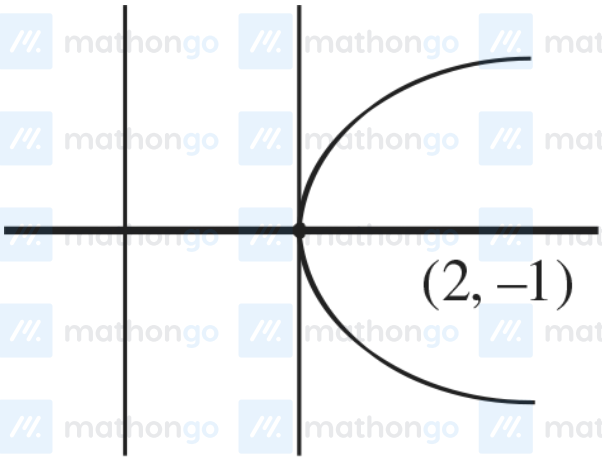
PS = PM

so equation is

$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

$a + b + c + d + k = 9 - 6 + 134 - 2 - 711 = -576$

Q11 (B)



$$4x - 3y = 21$$

$$a = \frac{|8+3-21|}{5} = \frac{10}{5} = 2$$

$$\therefore \text{latus rectum} = 4a = 8$$

Q12 (D)

Lines making angle $\frac{\pi}{4}$ with $y = 3x + 5$ have slope –2 & 1/2.

Which are perpendicular to each-other so, A, S, B are collinear for all $a > 0$.