MATHEMATICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set 'S' is

1) 2 **2)** 6 **3)** 4 **4)** 8

62. If $a_i, i = 1,2,......9$ are perfect odd squares, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is always a multiple of

1) 4 **2)** 5 **3)** 6 **4)** 7

63. The system of equations in 3 unknowns is

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 8 & 4 \\ 0 & 0 & [3\sin\theta + 4] \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ [\sin\theta + 3] \end{bmatrix}$$

Where [.] is GIF and then the system possesses

1) Unique solution, for every θ

2) Infinite number of solutions, for every θ

3) No solutions, for every θ

4) finite number of solutions but not Unique, , for every θ

64. Let A be a 3×3 matrix having entries from the set $\{-1,0,1\}$. The number of all such matrices A, having sum of all the entries equal to 5, is________

1) 414 **2)** 441 **3)** 141 **4)** 144

65. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all $i, j = 1,2,3.$ then the matrix $A^2 + A^3 + ... + A^{10}$ is equal to

1) $\left(\frac{3^{10}-3}{2}\right)A$ **2)** $\left(\frac{3^{10}-1}{2}\right)A$ **3)** $\left(\frac{3^{10}+1}{2}\right)A$ **4)** $\left(\frac{3^{10}+3}{2}\right)A$

66. The system of equations $-kx + 3y - 14z = 25; -15x + 4y - kz = 3; -4x + y + 3z = 4$ is consistent for all k in the set

1) R **2)** $R - \{-11,13\}$ **3)** $R - \{13\}$ **4)** $R - \{-11,11\}$

67. Let $X=\begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix}$, $Y=\alpha I+\beta X+\gamma X^2$ and $Z=\alpha^2 I-\alpha\beta X+\left(\beta^2-\alpha\gamma\right)X^2$,$\alpha,\beta,\gamma\in R$.

If $Y^{-1}=\begin{pmatrix}\frac{1}{5}&\frac{-2}{5}&\frac{1}{5}\\0&\frac{1}{5}&\frac{-2}{5}\\0&0&\frac{1}{5}\end{pmatrix}$ then $(\alpha+\beta+\gamma)^2$ is equal to____

1) 100 **2)** 400 **3)** 900 **4)** 600

68. If, for a square matrix A $=\left[a_{ij}\right]_{n\times n}, a_{ij}=i^2-j^2$ of even order then

1) A is a skew – symmetric and |A| = 0

2) A is symmetric and |A| is a perfect square

3) A is symmetric and |A| = 0

4) A is skew symmetric and |A| is a perfect square

69. The Co-efficient of x in the expansion of $\begin{vmatrix}(1+x)^{22}&(1+x)^{44}&(1+x)^{66}\\(1+x)^{33}&(1+x)^{66}&(1+x)^{99}\\(1+x)^{44}&(1+x)^{88}&(1+x)^{144}\end{vmatrix}$ is

1) 22 **2)** -22 **3)** 0 **4)** 1

70. If A is an idempotent matrix then $(I\ +\ A)^n$ equals $(n\in N)$

1) $I+2^n A$ **2)** $I+\left(2^n-1\right)A$ **3)** $I+\left(2^n-2\right)A$ **4)** $I+2^{n-1}A$

71. If $a>0>b>c$ and the system of equations

$ax+by+cz=0, bx+cy+az=0,\ cx+ay+bz=0$ has a nontrivial solution, then the roots of the quadratic equation $(b+c)t^2+(c+a)t+(a+b)=0$ are necessarily

1) Positive **2)** of opposite sign

3) non real **4)** Negative

72. Let a,b,c$\in R^+$ such that $a+b+c=6$ then the range of ab^2c^3 *is*

1) $(0,\infty)$ **2)** $(0,1)$ **3)** $(0,108]$ **4)** $(1,96]$

73. For non-zero real numbers b and c such that min of $f(x) >$ max of g(x) where $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2, (x \in R)$ then $\left|\frac{c}{b}\right|$ lies in

1) $\left(0,\frac{1}{2}\right)$ 2) $\left(\frac{1}{2},\infty\right)$ 3) $\left[2,\sqrt{5}\right]$ 4) $\left(\sqrt{2},\infty\right)$

74. If $x^{\log_3 x} > 3$, then x belongs to

1) $(1,3)$ 2) $(1,\infty)$ 3) $\left(0,\frac{1}{3}\right)$ 4) $\left(\frac{1}{3},1\right)$

75. If both roots of the equation $x^2-2kx+k^2+k-6=0$ are greater than 5 then k cannot lie in the interval

1) $(0,1)$ 2) $(6,\infty)$ 3) $(0,5)$ 4) All the three

76. If $\left(x^2+x+2\right)^2-(a-3)\left(x^2+x+1\right)\left(x^2+x+2\right)+(a-4)\left(x^2+x+1\right)^2=0$ has at least one root then the complete set of values of a.

1) $(1,5)$ 2) $\left(5,\frac{19}{3}\right]$ 3) $[0,2]$ 4) $(-2,0]$

77. The harmonic mean of the roots of $\left(5+\sqrt{2}\right)x^2-bx+8+2\sqrt{5}=0$ is 4 then the value of b is

1) $\sqrt{2}$ 2) $\sqrt{2}+1$ 3) $3+\sqrt{5}$ 4) $4+\sqrt{5}$

78. If 9 H.Ms are inserted between $\frac{1}{36}$ *and* $\frac{1}{1296}$ then the A.M of the reciprocals of the third and the seventh H.Ms is equal to

1) 630 2) 648 3) 666 4) 672

79. $x^4-4x^3+ax^2+bx+1=0$ has 4 positive roots, then $a+b=$

1) 6 2) -4 3) 2 4) 0

80. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

1) -81 2) 100 3) -300 4) 144

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11**).

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

81. The minimum number of zeros in an upper triangular matrix of order 15×15 is

82. If A and B are square matrices of order 3×3, where $|A|=2$ and $|B|=3$, then

$9\,|(A^{-1}).adj(B^{-1}).adj(2A^{-1})|=$

83. Let $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{bmatrix}$ and $6A^{-1}=A^2+\alpha A+\beta I$, then $|\alpha+\beta|$ is

84. X, Y and Z are positive number such that Y and Z have respectively 1 and 0 at their unit's place and Δ is the determinant $\begin{vmatrix}X & 4 & 1\\ Y & 0 & 1\\ Z & 1 & 0\end{vmatrix}$. If $(\Delta+1)$ is divisible by 10 then X has at its unit's place

85. Let $A=\begin{bmatrix}\sqrt{3} & -2\\ 0 & 1\end{bmatrix}$ and P be any orthogonal matrix of order 2 such that $Q=PAP^T$ and let $R=\left[r_{ij}\right]_{2\times 2}=P^TQ^8P$ Then $r_{11}=$

86. Let α and β be the roots of equation $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$, for $n\geq 1$, then the value of $\dfrac{a_{10}-2a_8}{2a_9}$ is

87. The equations $x^2+ax+12=0$, $x^2+bx+15=0$ and $x^2+(a+b)x+36=0$ have a common positive root then the value of $\left|\dfrac{a+b}{a-b}\right|$ is

88. The sum of all the integral roots of the equation $(\log_5 x)^2+\log_{5x}\left(\dfrac{5}{x}\right)=1$ is equal to__________.

89. If $x=\sqrt{1+\dfrac{1}{1^2}+\dfrac{1}{2^2}}+\sqrt{1+\dfrac{1}{2^2}+\dfrac{1}{3^2}}+\ldots\ldots\sqrt{1+\dfrac{1}{2019^2}+\dfrac{1}{2020^2}}$ then $4040(2020-x)=$

90. If $l=\log_{0.5}x$ and $A=\{x\in N\,/\,x\text{ is prime and } l(l-1)<30\}$ then n(A) =