



# Sri Chaitanya IIT Academy., India. A.P., TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

### A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60 JEE-MAIN Date: 23-03-2022 Time: 09.00Am to 12.00Pm **GTM-01** Max. Marks: 300

#### **KEY SHEET**

#### **PHYSICS**

1	2	2	1	3	3	4	2	5	1
6	1	7	2	8	3	9	4	10	2
11	1	12	1	13	4	14	3	15	4
16	3	17	1	18	3	19	2	20	4
21	20	22	3	23	60	24	175	25	144
26	9	27	5	28	30	29	2	30	200

#### **CHEMISTRY**

31	2	32	4	33	3	34	1	35	4
36	2	37	4	38	3	39	4	40	2
41	4	42	3	43	3	44	1	45	4
46	4	47	1	48	3	49	4	50	1
51	1	52	6	53	4	54	4	55	0
56	4	57	3	58	8	59	2	60	3

#### **MATHEMATICS**

61	3	62	3	63	3	64	3	65	1
66	2	67	3	68	3	69	4	70	1
71	3	72	3	73	3	74	1	75	3
76	4	77	3	78	4	79	4	80	2
81	20	82	2	83	1	84	5	85	0
86	1	87	2	88	6	89	2	90	575



## **SOLUTIONS**

#### **PHYSICS**

1. 
$$P = \frac{\Delta T}{L} \Rightarrow K = \frac{PL}{\Delta TA} \Rightarrow K = \frac{100 \times 3 \times 10^{-3}}{10 \times 1.5} = 2 \times 10^{-2}$$

2. 
$$\overrightarrow{r_f} = \overrightarrow{r_i} + \Delta \overrightarrow{s_1} + \Delta \overrightarrow{s_2} + \dots$$

$$\overrightarrow{r_f} = (2\hat{i} + 3\hat{j}) + 5\hat{i} + 8\hat{j} + (-2\hat{i} + 4\hat{j}) + (-6\hat{j})$$

$$\overrightarrow{r_f} = 5\hat{i} + 9\hat{i} \quad \text{Distance from thrower} = \sqrt{5^2 + 9^2} = \sqrt{106}$$

3. Dimension of z is dimension of time and only option C has dimension of time.

4. 
$$i = \frac{V}{R + G}$$

$$\frac{i}{2} = \frac{V}{\frac{SG}{S + G} + R} \times \frac{S}{S + G}$$

$$\Rightarrow \frac{2S}{SG + RG + RS} = \frac{1}{R + G}$$

$$SG + SR = RG$$

$$\frac{SR}{R - S} = G$$

$$R >> S$$

$$\Rightarrow G \approx S$$

5. WD = 
$$\Delta QV$$
  $\Rightarrow \Delta G = 4\mu C$ 

$$Q_{i} = \frac{2 \times 4}{2 + 4} \times 6 = 8\mu C \quad Q_{f} = 12\mu C = C_{eq} \times 6$$

$$C_{eq} = 2\mu F \Rightarrow C_{1} = C_{2} = 4\mu F \quad k = \frac{C_{1}}{C_{0}} = 2$$

6. 
$$x = \frac{\lambda D}{d} = \frac{700 \times 10^{-9} \times 5}{0.5 \times 10^{-3}} = 7mm$$

$$7. D = \frac{\mu_0 NI}{2\pi R}$$

$$F - N_1 = 0.5a$$
  $N_1 - f_2 = 2a$   
 $f_2 = \mu N_2$   
 $f_1 = \mu_5 N_1 = 5N$   $N_2 = 20 + f_1 = 25N$   
 $N_1 = \frac{1}{5}(25) = 2a$   
 $N_1 = 5 + 2a$ 

$$\mu_s N_1 = mg$$

$$\frac{1}{2} (5 + 2a) = \left(\frac{1}{2}\right) (10)$$

$$2a = 5$$

$$a = 2.5 \text{ m/s}^2$$
  
W = area enclose in the cycle

$$W = \frac{1}{2} 4 \times 10^{-4} \times 2 \times 10^5 = 40J$$



$$LC = \frac{0.5}{50} = 0.01mm$$

zero error = 
$$0.03$$
mm

Reading = 
$$5.0 + 29 \times 0.01$$

$$d = 5.29 - 0.03 = 5.26$$
mm

11. 
$$P_{av} = \frac{W_{Total}}{time} = \frac{\frac{1}{2}mv^2}{time} = \frac{\frac{1}{2}(1550)(26.8)^2}{7.1} / 746$$

$$= 105 \text{ hp}$$

$$12. \quad \frac{1}{f} = \frac{1}{\infty} - \frac{2}{f_L}$$

$$\frac{1}{-6} = \frac{-2}{f_L} f_L = 12cm$$

$$R = 12cm$$

$$\frac{1}{f} = \frac{1}{-12} - \frac{1}{24}$$

$$f = -8$$

$$f_L = 16$$

$$R = 12cm \qquad \frac{1}{f} = \frac{1}{-12} - \frac{1}{24}$$

$$f = -8 \qquad f_L = 16$$

$$\frac{1}{16} = \frac{1}{12} + \frac{1}{f} \qquad f = -48$$

$$f = -48$$

$$\frac{1}{-48} = \frac{\left(\mu - 1\right)}{-12}$$

$$\mu - 1 = \frac{1}{4}$$

$$\mu = \frac{5}{4}$$

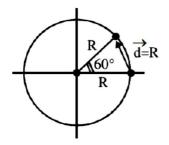


13.

As we know

$$\omega = \frac{\pi t}{3}$$

$$\alpha = \frac{d\omega}{dt} = \frac{\pi}{3} \text{ rad } / \text{ s}^2$$



$$\theta = \frac{\pi}{3} = \frac{1}{2}\alpha t^2$$



$$\frac{\pi}{3} = \frac{1}{2} \left( \pi / 3 \right) t^2$$

Now displacement of particle is given as d = R = 2m

$$V = \frac{2}{\sqrt{2}} = \sqrt{2} \, \text{m/s}$$

14. 
$$f = n\frac{v}{L}$$
 
$$\frac{df}{f} = \frac{dv}{v}$$
 
$$\frac{dv}{v} = \frac{1}{2}\frac{dT}{T}$$
 
$$v = \sqrt{\frac{\gamma RT}{M}}$$
 
$$df = f\frac{dv}{v}$$
 
$$= \frac{f}{2}\frac{dT}{T}$$
 
$$= \frac{10^4}{2} \times \frac{1}{300} = \frac{50}{3} = 16.67HZ$$

16. 
$$v_1 \ell_1 = v_2 \ell_2 v_1 = 20$$
  
 $v_2 = \frac{40}{3}$   $\frac{\ell_1}{\ell_2} = \frac{v_2}{v_1} = \frac{2}{3}$ 

17. 
$$v = u \cos \theta + \frac{\sigma}{2\varepsilon_0} \frac{q}{m} \times \frac{u \sin \theta}{g}$$
$$= u \cos \theta \left( 1 + \frac{q\sigma}{2\varepsilon_0 mg} \tan \theta \right)$$

18. 
$$v = \sqrt{\frac{2gh}{1 - \left(\frac{A_2}{A_1}\right)^2}} = \frac{\sqrt{2\cancel{0} \times 0\cancel{3}}}{\cancel{3}/4}$$

$$2\sqrt{2} R = \frac{u^2 \sin 120}{g}$$

$$= \frac{8 \times \sqrt{3}/2}{10} = \frac{8\sqrt{3}}{20}$$

$$= \frac{2\sqrt{3}}{5}$$

19. 
$$M = \int kr 4\lambda r^2 dr$$
$$= \cancel{A}\pi k \frac{R^4}{\cancel{A}} \qquad v = \sqrt{\frac{GM}{2R}} = \sqrt{\frac{G\lambda kR^4}{2R}}$$
$$= \sqrt{\frac{\pi kR^3G}{2}}$$

20. 
$$(B.E.)_y = (90 \times 1.008 + 138 \times 1.009 - 228.03)u$$
  
= 1.932 u = 1.932 × 931.5MeV  
Hence Binding energy is per Nucleon is



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$$= \frac{1.932 \times 931.5}{228}$$

$$= 7.89 \text{ MeV}$$

$$21. \quad E_x = -\frac{\partial v}{\partial x} = -6\hat{i}$$

$$E_y = -\frac{\partial v}{\partial y} = -8\hat{j} \quad E_z = \frac{\partial v}{\partial z} = -8z\hat{k}$$

$$\overrightarrow{E}_{net} \ at \ orgin = \sqrt{6^2 + 8^2} = 10$$
  
$$\Rightarrow |\overrightarrow{F}| = |q\overrightarrow{E}| = 20N$$

22. Induced field in rod, 
$$E = vB$$
  
electric field on surface of sphere  $= \frac{KQ}{R^2}$ 

$$\frac{KQ}{R^2} = vB \Rightarrow R^2 = \frac{kQ}{vB} = \frac{9 \times 10^9 \times 30}{9 \times 1}$$
$$R = \sqrt{3} \times 10^5 = 1.73 \times 10^5$$

23. For all collision to take place electron has to excite to n = 3.

Perfectly inelastic collision
$$mu + 0 = 5mV \qquad \text{Loss} = \frac{1}{2}mu^2 - \frac{1}{2}5m \times \left(\frac{4}{5}\right)^2 = \frac{4E}{5}$$

$$\frac{4E}{5} = 12.09 \times \left(2\right)^2 eV \implies E = 60.45 \, eV$$

24. 
$$\frac{f_{\text{max}}}{\Delta f_{half \ of \ \text{max } power}} = \text{Quality factor}$$
$$\frac{x_c}{P} = \text{Quality factor}$$

25. Impulse on block = 
$$\left(\frac{IA}{C}\right) \cos^2 53^0 \times (\Delta t)$$
  
=  $\frac{(20)(10 \times 10^{-4})}{3 \times 10^8} \times (0.6)^2 \times 6 \times 10^{-3}$   
=  $\frac{72}{5} \times 10^{-14} kg \, m/s$ 

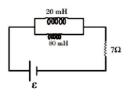
Now we have Impulse = mv 
$$\frac{72}{5} \times 10^{-14} = 1 \times 10^{-9} v$$
  $v = \frac{72}{5} \times 10^{-5} m/s$   
Now we have  $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$ 

$$10^{-5}x^2 = 10^{-9} \times \frac{72}{5} \times 10^{-5} \times \frac{72}{5} \times 10^{-5}$$



$$x = \frac{72}{5} \times 10^{-7} m \qquad N = \frac{7.2}{5} = 1.44$$

26.



$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{16 \times 10^{-3}}{8} = 2ms \qquad i = i_0 \left( 1 - e^{\frac{-t}{\tau}} \right) \qquad \frac{99}{100} i_0 = i_0 \left( 1 - e^{\frac{-t}{\tau}} \right)$$

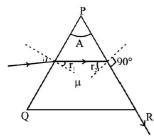
$$i = i_0 \left( 1 - e^{\frac{-t}{\tau}} \right)$$

$$\frac{99}{100}i_0 = i_0 \left(1 - e^{\frac{-t}{\tau}}\right)$$

$$e^{\frac{-t}{\tau}} = 0.01; \frac{-t}{\tau} \ln(0.01) \frac{t}{\tau} = \ln(100)$$
 t = 9.2 ms

27. 
$$\frac{\Delta g}{g} = \frac{\Delta \ell}{L} + \frac{2\Delta T}{T} = \frac{0.1}{95.6} + 2 \times \frac{1}{41}$$
$$= \frac{41 + 956 \times 2}{956 \times 41} \approx \frac{2}{41} \approx 5\%$$

28.



for this condition,  $r_2 = \theta_c$ ,  $r_1 = A - \theta_c$ 

 $1\sin i = \mu \sin r_1 \quad \sin i = \mu \sin (A - \theta_c) = \mu (\sin A \cos \theta_c - \cos A \sin \theta_c)$ 

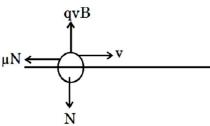
$$= \mu \left[ \sin 60^{0} \frac{\sqrt{\mu^{2} - 1}}{\mu} - \cos 60^{0} \times \frac{1}{\mu} \right]$$

$$= \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} - \frac{1}{2} = \frac{1}{2} \quad i = 30^{0}$$

29. 
$$0^2 = v^2 - 2\mu_k gS_{rel}$$

$$\frac{9}{2 \times 2} = S_{rel} = 2.25 m$$

30.



$$N = qvB - \mu qvB = \frac{mdv}{dt} = mv\frac{dv}{dS} \qquad \mu qBS = mv \quad S = \frac{3 \times 10^{-6} \times 4}{0.3 \times 10^{-6} \times 0.2} = 200m$$



#### **CHEMISTRY**

- 31. Vander Waal's constants
- 32.  $XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$
- 33. The dehydration of secondary and tertiary alcohols to give corresponding ethers is unsuccessful as elimination competes over substitution and therefore, alkenes are easily for the dehydration of secondary and tertiary alcohols to give corresponding ethers is unsuccessful as elimination competes over substitution and therefore, alkenes are easily formed
- 34. Salicylic acid because it stabilizes the corresponding salicylate ion by intramolecular H-bonding
- 35. All are correct statements
- 36. There are three d orbital's or involve
- 37. Mid row effect
- 38.  $ICl_2^- = linear (sp^3d hybridisation), BrF_2^+ (BrF_3 F) = Angular ClF_4^- = square planar (sp^3d^2 hybridisation), AlCl_4^- = tetrahedral (sp^3 hybridisation).$
- 39. In acid medium coupling occurs o/p-to  $-NH_2$  group while in basic medium, it occurs o/p-to OH group. Thus

(i) 
$$H_2N$$
  $\longrightarrow$   $OH + C_6H_5 \stackrel{\dagger}{N} \equiv NC1^- \stackrel{H^+}{\longrightarrow} H_3\stackrel{\dagger}{N} \stackrel{}{\longrightarrow} OH$   $N = N - C_6H_5$  (ii)  $H_2N$   $\longrightarrow$   $OH + C_6H_5\stackrel{\dagger}{N} \equiv NC1^- \stackrel{OH^-}{\longrightarrow} H_2N \stackrel{\alpha}{\longrightarrow} O^-$  The enol-form of acetone on treatment with  $D_2O$  undergoes repeated of

40. The enol-form of acetone on treatment with  $D_2O$  undergoes repeated enolization and deuteration to give ultimately  $CD_3 - CO - CD_3$ 

$$\begin{array}{c|c} OH & OD \\ CH_3-C=CH_2 & OCH_3-C=CH_2 \\ \hline Enoi-form of acetone & O & OH \\ \hline \longrightarrow CH_3-C-CH_2D & OCH_2=C-CH_2D \\ \hline OD & OCH_2-C-CH_2D \\ \hline \longrightarrow CH_2=C-CH_2D & OCH_2-C-CH_2D \\ \hline \longrightarrow CH_2=C-CH_2D & OCH_2-C-CH_2D \\ \hline \longrightarrow D_3C-C-C-CD_3 \\ \hline \end{array}$$

41. Electrophilic addition

42. 
$$CH_{3} - CH - CH_{3} \xrightarrow{HNO_{2}} CH_{3} - CH - CH_{3} \xrightarrow{[O]}$$

$$NH_{2} \qquad OH$$

$$CH_{3} - CO - CH_{3} \xrightarrow{(i) CH_{3}Mgl} A (CH_{3})_{3}COH$$

$$B \xrightarrow{(ii) H^{+}/H_{2}O} \xrightarrow{1,1-Dimethylethanol}$$

- 43.  $2Al + 6HCl(dill) \rightarrow 2AlCl_3 + 3H_2$  $2Al + 2NaOH + 6H_2O \rightarrow 2a[Al(OH)_4 + 3H_2]$
- 44.  $P_4 + 20HNO_3 \rightarrow 4H_3PO_4 + 20NO_2 + 4H_2O$



- Adsorption of sulphide particles on air bubble takes place in froath floatation process. 45.
- 46. Both are not double salts
- In acidic solution, NH<sub>3</sub> forms a bond with H<sup>+</sup> to give NH<sub>4</sub><sup>+</sup> ion which doesnot have a 47. lone pair on N to act a ligand.
- 48. Convert these Newmann projections into open chain structures:

Structures I and II being position isomers are, in fact, structural isomers.

49. 
$$\Delta S = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} = C_v \ln 2 + R \ln \frac{1}{2}$$
  
=  $C_v \ln 2 - R \ln 2 = (C_v - R) \ln 2$ 

- 50. Soap + water is an example of micelles
- 51. Equilibrium is attained
- 52. It is dehydrohalogenation reaction
- 53. Toulene

$$54. \qquad \mu = 4.9 BM = \sqrt{n \left(n+2\right)}$$

Number of unpaired electrons = 4

4 unpaired electrons are present, if the ion is Cr<sup>2+</sup>

In a total of 22 electrons  $\left(1s^22d^22p^63s^23p^63d^4\right)$ ,

13 electrons are with one spin and remaining 9 electrons are with opposite spin.

- 55. Mond's process
- 56. stoichiometric coefficient is 4
- Molecular weight of  $MF_X = 96 + (atomic weight of F) \times x$ 57. =(96+x19)

100 g water contains 
$$\rightarrow \left(\frac{41.2}{96 + x \times 19}\right)_{\text{mole of}} MF_3$$

$$\Rightarrow \frac{412}{96+19x} = \text{molarity}$$

$$M' = \frac{\Delta T}{K_f}$$
  $M' = \frac{1.38}{0.512}....(1)$   
 $\frac{1.38}{0.512} = \frac{412}{96+19x}$   $\Rightarrow x = 3$ 

$$\frac{1.38}{0.512} = \frac{412}{96 + 19x} \implies x = 3$$



58. 
$$p^{H} = \frac{p^{K_a} (HA^{2-}) + p^{K_a} (H_2A^{-})}{2} = \frac{11+5}{2} = \frac{16}{2} = 8$$

59. 
$$2(r^{+}+r^{-})=3.2; r^{+}+r^{-}=1.6; r^{+}=1.6-1.4=0.2A^{0}$$
  
=  $2\times10^{-1}A^{0} \Rightarrow x=2$ 

60. After 2 seconds surface area becomes 1/4 th. Hence radius becomes 1/2 of initial therefore vol will become 1/8 th dissolved vol = 7/8 mass dissolved =  $7/8 \times 1/7 = 1/8$ gm

molarity = 
$$\frac{1}{8 \times 125} = \frac{1}{1000} = 10^{-3}$$
 q = 0.77V.



#### **MATHEMATICS**

Given lines x + 1 = 0, y + 1 = 061.

Pair of angle bisector

$$\left(\frac{x+1}{\sqrt{1}}\right)^2 - \left(\frac{y+1}{\sqrt{1}}\right)^2 = 0 \implies x^2 + 2x + 1 - y^2 - 2y - 1 = 0$$
$$\implies x^2 - y^2 + 2x - 2y = 0$$

62. 
$$C_1 + C_2 \rightarrow C_1$$

$$2 1 + \cos^2 x \cos 2x$$

$$2 \cos^2 x \cos 2x$$

$$\begin{vmatrix} 2 & 1 + \cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$$R_1 - R_2 \rightarrow R_1$$

$$\begin{vmatrix} 2 & \cos^2 x & \cos 2x \end{vmatrix}$$

$$1 \cos^2 x \sin 2x$$

Open w.r.t. 
$$R_1$$
 –(2 sin 2x – cos 2x)

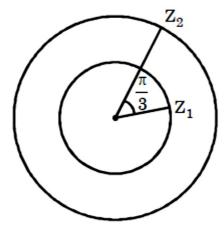
$$\cos 2x - 2 \sin 2x = f(x) |f(x)|_{\max} = \sqrt{1+4} = \sqrt{5}$$

63. 
$$\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$$

$$y = vx$$
  $\frac{dy}{dx} = v + x \frac{dv}{dx}$ 

$$v + x \frac{dv}{dx} = 1 + v + v^2$$
  $\frac{dv}{1 + v^2} = \frac{dx}{x}$ 

$$tan^{-1}(v) = \ell nx + c$$
  $c = \frac{\pi}{4}$   $y = x tan \left(\ell nx + \frac{\pi}{4}\right)$ 



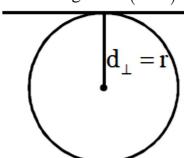
$$|z_1 + z_2| = \sqrt{|z_1|^2 + |z_2|^2 + 2|z_1||z_2|} \cos \theta$$
$$= \sqrt{4 + 9 + 2 \times 2 \times 3 \times \frac{1}{2}} \quad \sqrt{4 + 9 + 6} = \sqrt{19}$$



65. Equation of tangent is

$$\frac{x\cos\phi}{4} + \frac{\sqrt{11}\sin\phi}{16}y = 1$$

is also tangent to  $(x-1)^2 + y^2 = 16$ 



$$\left| \frac{\frac{\cos \phi}{4} - 1}{\frac{\cos^2 \phi}{16} + \frac{11\sin^2 \phi}{256}} \right| = 4 \quad \phi = \pm \frac{\pi}{3}$$

66. Plane passing through (42, 0, 0), (0, 42, 0), (0, 0, 42)

From intercept from, equation of plane is x + y + z = 42

$$\Rightarrow$$
 (x - 11) + (y - 19) + (z - 12) = 0

let 
$$a = x - 11$$
,  $b = y - 19$ ,  $c = z - 12$ 

a + b + c = 0 Now, given expression is

$$3 + \frac{a}{b^2c^2} + \frac{b}{a^2c^2} + \frac{c}{a^2b^2} - \frac{42}{14abc}$$

$$3 + \frac{a^3 + b^3 + c^3 - 3abc}{a^2b^2c^2} \quad \text{If } a + b + c = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc \Rightarrow 3$$

67. 
$$I = \int_{0}^{\infty} \frac{\tan^{-1} x}{(x+1)^{2}} dx$$

Put 
$$x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2}dt$$

$$= \int_{\infty}^{0} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\left(\frac{1}{t}+1\right)^{2}} \left(\frac{-1}{t^{2}}\right) dt = \int_{0}^{\infty} \frac{\cot^{-1} t}{\left(1+t\right)^{2}} dt = I$$

$$2I = \frac{\pi}{2} \int_{0}^{\infty} \frac{1}{(1+t^2)} dt \Longrightarrow I = \frac{\pi}{4}$$

68. Consider all those cases where teacher is making visit to the zoo not taking A (one of the student out of 20). Hence teacher is selecting 2 students from rest 19. Hence  $^{19}C_2$  visits are there which do not include A.

#### Let equation of plane containing line be : $\ell(x-1) + m(y+2) + nz = 0$ then 69.

$$2\ell - 3m + 5n = 0$$
 and  $\ell - m + n = 0$ 

$$\therefore$$
 plane is  $2(x-1) + 3(y+2) + z = 0$ 

i.e., 
$$2x + 3y + z + 4 = 0$$
 :  $a = 2, b = -3, c = 1$ 

$$\Rightarrow \frac{b^2}{(a+c)} = \frac{9}{3} = 3$$

70. 
$$\therefore f^{2}(x) = \int_{0}^{x} \frac{t f(t)}{1+t^{2}}$$

$$\therefore 2f(x)f'(x) = \frac{xf(x)}{1+x^2}; f(x) = 0 \text{ (not possible)}$$

$$\therefore f^{\mid}(x) = \frac{x}{2(1+x^2)} \Longrightarrow f(x) = \frac{1}{4} log(1+x^2) + C$$

$$\therefore f(0) = 0 \Rightarrow C = 0 \therefore f(x) = \frac{1}{4} log(1 + x^2) \Rightarrow f(e^4 - 1) = 1$$

71. 
$$I = \int_{0}^{10} [x] \cdot e^{[x] - x + 1}$$

$$I = \int_{0}^{1} 0 dx + \int_{1}^{2} 1 \cdot e^{2-x} + \int_{2}^{3} 2 \cdot e^{3-x} + \dots + \int_{9}^{10} 9 \cdot e^{10-x} dx$$

$$\Rightarrow I = \sum_{n=0}^{9} \int_{n}^{n+1} n \cdot e^{n+1-x} dx \Rightarrow -\sum_{n=0}^{9} n \left( e^{n+1-x} \right)_{n}^{n+1}$$

$$= -\sum_{n=0}^{9} n \cdot (e^{0} - e^{1}) = (e - 1) \sum_{n=0}^{9} n$$

$$=(e-1)\frac{9.10}{2}$$
 = 45 (e-1)

72. 
$$y' = \frac{-\sin 2x}{\left(2 + \cos^2 x\right)^2} = 0 \Rightarrow \sin 2x = 0 \Rightarrow x = \frac{n\pi}{2}$$

if n odd 
$$x = (2k + 1)\frac{\pi}{2}$$

$$y = \frac{1}{2}$$

if n even 
$$x = (2k)\frac{\pi}{3}$$

$$y = \frac{1}{3}$$

73. Tangent vertex equation 
$$\frac{x}{3} + \frac{y}{4} = 1$$

So are 
$$=\frac{1}{2}.3.4 = 6$$

Locus of the foot of the perpendicular from focus to tangents is tangent at vertex. So tangent at vertex passes through (3, 0) and (0, 4).



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74. We can conclude 
$$-8a + 4b - 2c + 5 = 0$$

$$12a - 4b + c = 0$$

$$c = 3$$
  $a = -\frac{1}{2}$   $b = -\frac{3}{4}$ 

$$c = 3$$

$$c + 2b + a = 3 - \frac{3}{2} - \frac{1}{2} = 1$$

75. 
$$x^2 + y^2 + ax + 2ay + c = 0$$

$$2\sqrt{g^2 - c} = 2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$$

$$\Rightarrow \frac{a^2}{4} - c = 2 \qquad \dots (1)$$

$$2\sqrt{f^2 - c} = 2\sqrt{a^2 - c} = 2\sqrt{5}$$

$$\Rightarrow a^2 - c = 5 \qquad \dots (2)$$

$$\frac{3a^2}{4} = 3 \Rightarrow a = -2(a < 0)$$

$$\therefore c = -1$$

Circle 
$$\Rightarrow$$
  $x^2 + y^2 - 2x - 4y - 1 = 0$ 

$$\Rightarrow (x-1)^2 + (y-2)^2 = 6$$

Given 
$$x + 2y = 0 \Rightarrow m = -\frac{1}{2}$$

$$m_{tan gent} = 2$$
 Equation of tangent

$$\Rightarrow (y-2) = 2(x-1) \pm \sqrt{6}\sqrt{1+4}$$

$$\Rightarrow 2x - y \pm \sqrt{30} = 0$$

Perpendicular distance from 
$$(0, 0) = \left| \frac{\pm \sqrt{30}}{\sqrt{4+1}} \right| = \sqrt{6}$$

76. Given lines in the form 
$$a(1) + b(-2) + c = 0$$
  
Being lines concurrent, triangle will not form.

77. 
$$f(x) = \sum_{n=0}^{\infty} \sin \frac{2x}{3^n} \cdot \sin \frac{x}{3^n}$$

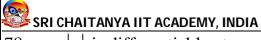
$$= \frac{1}{2} \sum_{n=1}^{\infty} \left( \cos \frac{x}{3^n} - \cos \frac{x}{3^{n-1}} \right) = \frac{1}{2} \lim_{x \to \infty} \sum_{n=1}^{n} \left( \cos \frac{x}{3^n} - \cos \frac{x}{3^{n-1}} \right)$$

$$= \frac{1}{2} \lim_{x \to \infty} \left( \cos \frac{x}{3^n} - \cos x \right) = \frac{1 - \cos x}{2}$$

$$\therefore f(x) = 0 \Rightarrow x = 2n\pi, x \in \mathbb{Z}$$

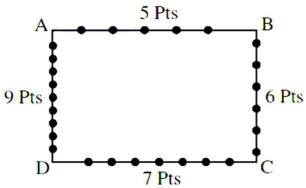
$$628 < 200 \,\pi$$

$$\therefore Sum = 2\pi + 4\pi + \dots + 2.99\pi = \frac{2.99.100}{2}\pi = 9900\pi$$



78.  $\mathbf{x} | \mathbf{x} |$  is differentiable at  $\mathbf{x} = 0$ , hence 1, 3 are points of non differentiability.

79.



 $\alpha$  = Number of triangles

$$\alpha = 5.6.7 + 5.7.9 + 5.6.9 + 6.7.9$$
  
= 210 + 315 + 270 + 378  
= 1173  $\beta$  = Number of quadrilateral

$$\beta = 5.6.7.9 = 1890 \,\beta - \alpha = 1890 - 1173 = 717$$

80. Let 
$$x = a \sec \theta$$
  
 $y = b \csc \theta$ 

$$\sqrt{x^2 + y^2} = \sqrt{a^2 \sec^2 \theta + b^2 \cos \sec^2 \theta} \ge a + b$$

81. 
$$\sqrt{1+\frac{1}{a^2}+\frac{1}{\left(a+1\right)^2}} = \frac{a^2+a+1}{a(a+1)} = 1+\frac{1}{a}-\frac{1}{a+1}$$

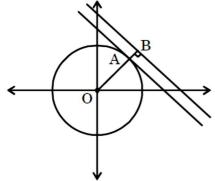
$$\sum_{a=1}^{19} \sqrt{1 + \frac{1}{a^2} + \frac{1}{\left(a+1\right)^2}} = 19 + \frac{1}{1} - \frac{1}{19+1} = 20 - \frac{1}{20} \text{ and}$$

$$\frac{x}{20} = \frac{1 + \left(1 - \frac{1}{2}\right) + \left(1 - \frac{2}{3}\right) + \left(1 - \frac{3}{4}\right) + \dots + \left(1 - \frac{24}{25}\right)}{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{25}}$$

$$\frac{x}{20} = 1 \Rightarrow x = 20$$

82. 
$$|z_1| = 2$$
 and  $(1-i)z_2 + (1-i)\overline{z_2} = 8\sqrt{2}$ 

$$\therefore$$
  $x^2 + y^2 = 4$  and  $x + y = 4\sqrt{2}$ 



$$AB = OB - r$$

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$$=2$$

83. Let ' $\theta$ ' be the angle between  $\overrightarrow{a}$  and  $\overrightarrow{b}$  then

$$\begin{vmatrix} \overrightarrow{a} + \overrightarrow{b} \end{vmatrix}^2 = 1 + 1 + 2\cos\theta = 4\cos^2\theta/2$$
and 
$$\begin{vmatrix} \overrightarrow{a} - \overrightarrow{b} \end{vmatrix}^2 = 1 + 1 - 2\cos\theta = 4\sin^2\theta/2$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2\cos\theta & -2\theta/2 \end{vmatrix} = 1 + 1 - 2\cos\theta = 4\sin^2\theta/2$$

$$\therefore \frac{1}{|\overrightarrow{a}+\overrightarrow{b}|} + \frac{1}{|\overrightarrow{a}-\overrightarrow{b}|} = \frac{1}{4} \left( \cos ec^2\theta / 2 + \sec^2\theta / 2 \right)$$

$$= \frac{1}{4} \left( 2 + \tan^2 \theta / 2 + \cot^2 \theta / 2 \right) \ge 1 \left\{ \text{using AM} \ge GM \right\}$$

84. 
$$\int e^{x^{2}+x} (4x^{3} + 4x^{2} + 5x + 1) dx$$

$$= \int e^{x^{2}+x} ((2x+1)(2x^{2}+x) + (4x+1)) dx$$

$$= \int d(e^{x^{2}+x})(2x^{2}+x) dx + \int (4x+1)(e^{x^{2}+x}) dx$$

$$= (2x^{2}+x) \cdot e^{x^{2}+x} - \int (4x+1)e^{x^{2}+x} dx + \int (4x+1)e^{(x^{2}+x)} dx$$

$$= (2x^{2}+x)e^{x^{2}+x} + c \quad f(x) = 2x^{2}+x \Rightarrow f'(1) = 4(1)+1=5$$

85. Let  $\omega_i$  (i = 1,2,3,4,5) denotes 1, 2, 3, 4, 5 white in bag

$$P(\omega/\omega_i) = \frac{i}{5}, (i = 1, 2, 3, 4, 5)$$
  $P(\omega_i) = \frac{1}{5}, (i = 1, 2, 3, 4, 5)$ 

Now, 
$$P(\omega / \omega_5) = \frac{P(\omega_5) \cdot P(\omega / \omega_5)}{\sum_{i=1}^{5} P(\omega_i) P(\omega / \omega_i)}$$

$$=\frac{\frac{\frac{1}{5}*1}{\frac{1}{5}\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}+\frac{5}{5}\right)}}{\frac{1}{5}\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}+\frac{5}{5}\right)}=\frac{1}{3}$$

86. Let 
$$A = (x-1)(x^2-2).....(x^{20}-20)$$
  

$$= x.x^2.x^3......x^{20}\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x^2}\right)....\left(1-\frac{20}{x^{20}}\right)$$

$$\therefore A = x^{210}\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x^2}\right).....\left(1-\frac{20}{x^{20}}\right)$$

 $\therefore$  Coefficient of  $x^{210}$  in A is 1.

87. 
$$\{\alpha,\beta\} = \{\alpha^2,\beta^2\} \implies \{\alpha,\beta\} = (0,0)(1,0)(1,1)(\omega,\omega^2)$$
  
 $b = 0, 1, 2-1$ 



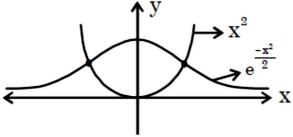
88. For least area, 
$$\lambda = 0$$
 ::  $A_{\min} = 2 \int_{0}^{4} (12 - (x^2 - 4)) dx = \frac{256}{3}$ 

$$\therefore \alpha + 3\beta = 265 \Rightarrow 6$$

$$\therefore \alpha + 3\beta = 265 \Rightarrow 6$$

$$\lim_{m \to \infty} \frac{\lim_{m \to \infty} -\frac{2\sin^2(\frac{x}{2m})}{(\frac{x}{2m})^2} \cdot \frac{x^2}{m} \cdot m^2}{\left(\frac{x}{2m}\right)^2} = e$$

$$f(x) = e^{-\frac{x^2}{2}}$$



 $\therefore$  No. of solutions are 2.

90. 
$$a = 1, a + d = \log_y x, a + 2d = \log_z y$$

$$a + 3d = -15 \log_x z$$

$$\therefore (1+d)(1+2d)(1+3d) = -15$$

$$\Rightarrow 6d^3 + 11d^2 + 6d + 16 = 0 \Rightarrow \boxed{d = -2}$$

$$S_{25} = \frac{25}{2} [2(1) + (24)(-2)] = 25(1 - 24) = -575$$