Exercise-1

TOPIC:-3D COORDINATE SYSTEM:

1A. (2019-Apr) If a point $R(4, y, z)$ lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,-3,4)\, and\, (8,0,10)$, then the distance of R from the origin is

1. $2\sqrt{21}$ 2. $\sqrt{53}$ 3. $2\sqrt{14}$ 4. 6

Key : 3

Sol : Given points are $P(2,-3,4), Q(8,0,10)\, and\, R(4,y,z)$. Now, equation of line passing through points P and Q is $\frac{x-8}{6}=\frac{y-0}{3}=\frac{z-10}{6}$

{since equation of a line passing through two points $A(x_1, y_1, z_1)\, and\, B(x_1, y_2, z_2)$ is given by $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$}

$\Rightarrow \frac{x-8}{2}=\frac{y}{1}=\frac{z-10}{2}.....(i)$

$\because$ Point P, Q and R are collinear, so

$\frac{4-8}{2}=\frac{y}{1}=\frac{z-10}{2} \Rightarrow -2 = y = \frac{z-10}{2}$

$\Rightarrow y=-2\; and\; z=6$

So, point R is $(4,-26)$, therefore the distance of point R from origin is

$OR=\sqrt{16+4+36}=\sqrt{56}=2\sqrt{14}$

1B. If a point $R(4, y, z)$ lies on the lines segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$ then the distance of R from $(1,1,1)$ is

1. $\sqrt{43}$ 2. $\sqrt{42}$ 3. $\sqrt{40}$ 4. $\sqrt{41}$

Key : 1

Sol : Equation of line is $\frac{x-2}{6}=\frac{y+3}{3}=\frac{z-4}{6}, sub(4,y,z)$

$\Rightarrow 1=\frac{y+3}{1}=\frac{z-4}{2} \Rightarrow y=-2, z=6$

$\therefore R=(4,-2,6)\; S=(1,1,1)$

$RS=\sqrt{43}$

D.C'S & D.R'S:-

02 A. (2023-Jan) Let a unit vectors $\widehat{OP}$ make angle α,β,γ with the positive of the co-ordinate axes

OX,OY,OZ respectively, where $\beta \in \left(0,\frac{\pi}{2}\right)$. If $\widehat{OP}$ is perpendicular to the plane though

Points $(1,2,3),(2,3,4)\,and\,(1,5,7)$, then which one of the following is true?

1. $\alpha \in \left(0,\frac{\pi}{2}\right) and\ \gamma \in \left(\frac{\pi}{2},\pi\right)$
2. $\alpha \in \left(0,\frac{\pi}{2}\right) and\ \gamma \in \left(0,\frac{\pi}{2}\right)$
3. $\alpha \in \left(\frac{\pi}{2},\pi\right) and\ \gamma \in \left(0,\frac{\pi}{2}\right)$
4. $\alpha \in \left(\frac{\pi}{2},\pi\right) and\ \gamma \in \left(\frac{\pi}{2},\pi\right)$

Key : 4

Sol : Dc's of OP: $\cos\alpha,\cos\beta,\cos\gamma\,(\cos\beta > 0)$

OP is parallel to the normal of the plane containing $(1,2,3),(2,3,4),(1,5,7)$ is

Dr's of above normal: $1,-4,3$

$$\frac{\cos\alpha}{-1}=\frac{\cos\beta}{4}=\frac{\cos\gamma}{-3} \Rightarrow \cos\alpha<0,\cos\gamma<0$$

02B. A unit vector $\overline{OP}$ make angle α,β,γ with the positive direction of the coordination axes OX,OY,OZ respectively, when $\beta \in \left(1,\frac{\pi}{2}\right)$.If $\overline{OP}$ is perpendicular to the plane through points (1,2,3), (2,3,4) and (1,1,1) then which one of the following is true?

1. $\alpha \in \left(1,\frac{\pi}{2}\right), \gamma \in \left(\frac{\pi}{2},\pi\right)$
2. $\alpha \in \left(0,\frac{\pi}{2}\right), \gamma \in \left(0,\frac{\pi}{2}\right)$
3. $\alpha \in \left(\frac{\pi}{2},\pi\right)$ and $\gamma \in \left(0,\frac{\pi}{2}\right)$
4. $\alpha \in \left(\frac{\pi}{2},\pi\right), \gamma \in \left(\frac{\pi}{2},\pi\right)$

Key:4

Sol: Eq. Of plane is $-x+2y-z=0$

$$\therefore \frac{\cos\alpha}{-1}=\frac{\cos\beta}{2}=\frac{\cos\gamma}{-1}$$

$\therefore \cos\alpha<0,\cos\gamma<0$

3A. (2021-Aug) The angle between the straight lines, whose direction cosines are given by the equations $2l+2m-n=0\ and\ mn+nl+lm=0$, is

1. $\frac{\pi}{3}$
2. $\frac{\pi}{2}$
3. $\cos^{-1}\left(\frac{8}{9}\right)$
4. $\pi-\cos^{-1}\left(\frac{4}{9}\right)$

Key : 2

Sol : Given equations are $2l+2m-n=0.......(i)$

and $mn+nl+lm=0 \Rightarrow lm+n(1+m)=0.....(ii)$

From (i) and (ii), we get

$lm+2(l+m)^2=0 \Rightarrow 2l^2+2m^2+5lm=0$

$\Rightarrow 2\left(\frac{l}{m}\right)^2+2+5\left(\frac{l}{m}\right)=0$

Putting $\frac{l}{m}=t, we\, get$

$2t^2+5t+2=0 \Rightarrow (2t+1)(t+2)=0 \Rightarrow t=-2,-\frac{1}{2}$

Now, when $t=-2, \frac{l}{m}=-2 \Rightarrow l=-2m$

$\therefore$ From $(i), n=-2m$

So, direction cosines are $(-2m,m,-2m) i.e., (-2,1,-2)$ and when $t=-\frac{1}{2}, \frac{l}{m}=\frac{1}{2}=-2l$

$\therefore$ From $(i), n=-2l$

So, direction cosines are $(l,-2l,-2l) i.e., (1,-2,-2)$

$\therefore \cos\theta=(-2)(1)+(1)(-2)+(-2)(-2)=0$

$\therefore$ Lines are perpendicular.

3B. An angle between the lines whose direction cosines are given by the equations $l+3m+5n=0$ and $5lm-2mn+6nl=0$ is

1. $\cos^{-1}\left(\frac{1}{8}\right)$ 2. $\cos^{-1}\left(\frac{1}{3}\right)$ 3. $\cos^{-1}\left(\frac{1}{4}\right)$ 4. $\cos^{-1}\left(\frac{1}{6}\right)$

Key: 4

Sol: $l=-3m-5n$ ....(1), $5lm-2mn+6nl=0$ ....(2)

From (1) and (2) $\Rightarrow m^2+3mn+2n^2=0$

$\Rightarrow m=-n$ (or) $m=-2n$

$\therefore$Dr is are (-2,-1,1) and (1,-2,1)

$\therefore \cos\theta=\frac{-2+2+1}{\sqrt{6}.\sqrt{6}}=\frac{1}{6} \Rightarrow \theta=\cos^{-1}\left(\frac{1}{6}\right)$

4A. (2021-Feb) Let α be the angle between the liens whose direction cosines, satisfy the equations $l+m-n=0$ and $l^2+m^2-n^2=0$. Then the value of $\sin^4\alpha+\cos^4\alpha$ is

1. $3/4$ 2. $5/8$ 3. $3/8$ 4. $1/2$

Key : 2

Sol : we know that $l^2+m^2+n^2=1$ and given that $l^2+m^2-n^2=0$

$\Rightarrow 2n^2=1 \Rightarrow n=\pm\frac{1}{\sqrt{2}}$

Also, $l+m=n \Rightarrow l^2+m^2+2lm=n^2$

$\Rightarrow n^2+2lm=n^2 \Rightarrow lm=0$

If $l=0, then\, m=n=\pm\frac{1}{\sqrt{2}}$

And if $m = 0, then\, l = n = \pm\frac{1}{\sqrt{2}}$

So, direction cosines of two liens are

$\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) and \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

So, $\cos\alpha = \frac{0+0+\frac{1}{2}}{\sqrt{1}\sqrt{1}} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

$\therefore \sin^4\alpha + \cos^4\alpha = \left(\sin\frac{\pi}{3}\right)^4 + \left(\cos\frac{\pi}{3}\right)^4$

$= \left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \frac{9}{16} + \frac{1}{16} = \frac{5}{8}$

4B. The angle between the lines whose direction cosines satisfy the equation $l + m + n = 0$ and $l^2 = m^2 + n^2$ is

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{6}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{3}$

Key: 4

Sol: From (1) and (2)

$\Rightarrow mn = 0$

$\therefore$ Dr;s are (-1,1,0) and (-1,0,1)

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

PLANES:-

05A. (2023-Jan) The distance of the point $(7,-3,-4)$ from the plane passing through the points $(2,-3,1),(-1,1,-2)\, and\, (3,-4,2)\, is$

1. $5\sqrt{2}$ 2. $4\sqrt{2}$ 3. 4 4. 5

Key : 1

Sol : Plane passing through the points $(2,-3,1),(-1,1,-2)\, and\, (3,-4,2)$ is

$$\begin{vmatrix} x-2 & y+3 & z-1 \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow x - z - 1 = 0$

Required distance $= \frac{10}{\sqrt{2}} = 5\sqrt{2}$

05B. The distance from origin to the plane passing through the points (2,-3,1),(-1,1,-2) and (3,-4,2) is

1. $\sqrt{2}$ 2. $2\sqrt{2}$ 3. $\frac{1}{\sqrt{2}}$ 4. $3\sqrt{2}$

Key: 3

Sol: Eq of plane is $\begin{vmatrix} x-2 & y+3 & z-1 \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = 0$

$\Rightarrow x - z - 1 = 0$

Distance $= \frac{1}{\sqrt{2}}$

06A.(2023 -Jan) Let the plane containing the line of intersection of the plane $P1 : x + (\lambda - 4)y - 4y + z = 1$ and $P2 : 2x + y + z = 2$ passes through the point $(0,1,0)$ *and* $(1,0,1)$. Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane $P2$ is

1. $5\sqrt{6}$ 2. $4\sqrt{6}$ 3. $3\sqrt{6}$ 4. $2\sqrt{6}$

Key : 3

Sol : $P_1 : x + (\lambda + 4)y + z - 1 = 0$

$P_2 : 2x + y + z - 2 = 0$

Equation of the plane containing the line of intersection of the plane P1 and P2 is of the from $P_1 + tP_2 = 0$

$\Rightarrow [x + (\lambda + 4)y + z - 1] + t[2x + y + z - 2] = 0......(1)$

Eq. (1) pass through $(0,1,0)$ & $(1,0,1)$

i.e., $[(\lambda + 4) + t] - 1 - 2t = 0$

$\Rightarrow \lambda - 1 + 3 = 0......(2)$

Also $(1 + 2t) + (1 + t) - 1 - 2t = 0 \Rightarrow t = -1....(3)$

Eq. (2) & (3) $\lambda + 1 + 3 = 0 \Rightarrow \lambda = -4$

Now $(2\lambda, \lambda, -\lambda) = (-8, -4, 4)$

Then distance from $(-8, -4, 4)$ to $P2$ is

$= \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

$= \frac{|2(-8) + (-4) + (4) - 2|}{\sqrt{4+1+1}} = \frac{|-16 - 4 + 4 - 2|}{\sqrt{6}} = \frac{18}{\sqrt{6}} = 3\sqrt{6}$

06B. The equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$, for $C \in Z$, if the distance of this plane from the point

$(a,-c,c)$ is $\frac{2}{\sqrt{a}}$ then $a+b+c=$

1. 3 2. 4 3. 5 4) 2

Key: 1

Sol: Eq of plane be $(x+2y+az-2)+\lambda(x-y+z-3)=0$

$\therefore a=3, b=1,\ \lambda=\frac{-7}{2}$

$\therefore$Distamce=$\frac{2}{\sqrt{a}} \Rightarrow c=-1$

$\therefore a+b+c=3$

07 A.(2023-Feb) Let the image of the point $P(2,-1,3)$ in the plane $x+2y-z=0$ be Q. The distance of the plane $3x+2y+z+29=0$ from the point Q is

1. $2\sqrt{14}$ 2. $\frac{22\sqrt{2}}{7}$ 3. $3\sqrt{14}$ 4. $\frac{24\sqrt{2}}{7}$

Key : 3

Sol : Foot of $P(2,-1,3)$ in the plane $x+2y-z=0\, is \left(\frac{5}{2},0,\frac{5}{2}\right)$

Image of $P(2,-1,3)$ in the plane $x+2y-z=0\, is\, (3,1,2)$

Distance of the plane $3x+2y+z+29=0$ of the point $Q\, is = \left|\frac{9+2+2+29}{\sqrt{14}}\right| = 3\sqrt{14}$

07B. The image of the point P(1,2,6) in the plane passing through the points A(1,2,0) B(1,4,1) and C(0,5,1) be Q (α,β,γ) then $\alpha+\beta+\gamma=$

1. 9 2. 10 3. 11 4. 12

Key: 1

Sol: Eq. Of plane is x+y-2z-3=0

$\therefore \frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=-2\left(\frac{1+2-12-3}{1+1+4}\right)$

$\Rightarrow \alpha=5, \beta=6, \gamma=-2$

08 A.(2023-Apr) A plane P contains the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=6\, and\, \vec{r}.(2\hat{i}+3\hat{j}+4\hat{k})=-5$. If Passes through the point $(0,2,-2)$, then the square of distance of the point $(12,12,18)$ form the plane P is

1. 155 2. 260 3. 310 4. 1240

Key : 2

Sol : $\pi_1 \equiv x+y+z-6$

$\pi_2 \equiv 2x+3y+4z+5=0$

Required plane is $\pi_1 + \lambda\pi_2 = 0$

$\Rightarrow (x+y+z-6)+\lambda(2x+3y+4z+5)=0$

Passing through $(0,2,-2)$

$\Rightarrow \lambda = 2$

Required plane is $(x+y+z-6)+2(2x+3y+4z+5)=0$

$\pi = 5x+7y+9z+4=0$

$\perp er$ dis from $(12,12,18)$ to the plane $\pi = 0$

$\Rightarrow \frac{|60+84+162+4|}{\sqrt{24+49+81}}$

$d = dis = \frac{310}{\sqrt{155}}$

$d^2 = \frac{310\times 310}{155} = 620$

08B. A plane contains the line of intersection of the plane $\bar{r}.(\bar{i}+\bar{j}+\bar{k})=6$ and $\bar{r}.(2\bar{i}+3\bar{j}+4\bar{k})=-5$ if passes through the point (0,2,-2), then the distance from(0,0,0) to plane is

1. $\frac{1}{\sqrt{155}}$ 2. $\frac{2}{\sqrt{155}}$ 3. $\frac{3}{\sqrt{155}}$ 4. $\frac{4}{\sqrt{155}}$

Key: 4

Sol: Eq. Of plane is $(x+y+z-6)+\lambda(2x+3y+4z+5)=0$ passing through (0,2,-2)

$\Rightarrow \lambda = 2$

Distance=$\frac{4}{\sqrt{155}}$

09A. (2021-Mar) The equation of the plane which contains the y-axis and passes through the point (1,2,3) is

1. $3x+z=6$ 2. $x+3z=10$ 3. $3x-z=0$ 4. $x+3z=0$

Key : 3

Sol : The equation of any plane containing y-axis is of the from $x+kz=0$

Since, it passes through (1,2,3) $\therefore 1+3k=0 \Rightarrow k=\frac{-1}{3}$

Thus, the required equation of plane is

$x-\frac{1}{3}z=0 \Rightarrow 3x-z=0$

09B. The equation of the plane which contains the X-axis and passing through the point (1,2,3) is

1. $3y-2z=0$ 2. $3y+2z=0$ 3. $2y+3z=0$ 4) $2y-3z=0$

Key: 1

Sol: Eq of plane be $y+kz=0$

Sub(1,2,3) $\Rightarrow K=\dfrac{-2}{3}$

$\therefore$ Eq is $3y-2z=0$

10A. (2021-Feb) Consider the three planes

$P_1: 3x+15y+21z=9, P_2: x-3y-z=5 \; and$

$P_3: 2x+10y+14z=5$

Then, which one of the following is true?

1. P_2 and P_3 are parallel
2. $P_1, P_2 \; and \; P_3$ all are parallel
3. $P_1 \; and \; P_3$ all are parallel
4. $P_1 \; and \; P_2$ are parallel.

Key : 3

Sol : We have, $P_1: x+5y+7z=3$

$P_2, P_3: x+5y+7z=\dfrac{5}{2}$

So, $P_1 \; and \; P_3$ are parallel as direction ratios of normal are same.

10B. Consider the three planes $P_1: 3x+6y+9z=15$, $P_2: x+2y+3z=4$, $P_3: x-3y-z=5$ then which one of the following is true?

1. P_1, P_2 parallel 2. P_1, P_2 perpencular 3. P_2, P_3 are parallel 4. P_1, P_3 are parallel

Key: 1

Sol: P_1, P_2 parallel

11A. (2021-July) Let the plane passing through the point $(-1,0,2)$ and perpendicular to each of the planes $2x+y-z=2$ and $x-y-z=3$ be $ax+by+cz+8=0$. Then the value of $a+b+c$ is equal to

1. 4 2. 8 3. 5 4. 3

Key : 1

Sol : Since, the plane $ax+by+cz+8=0$ is passing through $(-1,0,-2)$

$\therefore -a-2c+8=0$

$\Rightarrow a+2c=8 .....(i)$

Also, $ax+by+cz+8=0$ is perpendicular to the planes

$2x+y-z=2 \; and \; x-y-z=3$

$\therefore 2a+b-c=0 ....(ii), a-b-c=0---(iii)$

Adding (ii) and (iii), we get

$3a-2c=0----(iv)$

From (i) and (iv) we get

$a=2, c=3$

$\therefore$ From (iii), $b=-1$

$\therefore a+b+c=2-1+3=4$

11B. The equation of the plane passing through the point (1,2,-3) and perpendicular to the point $3x+y-2z=5$ and $2x-5y-z=7$ is

1. $6x-5y-2z-2=0$
2. $6x-5y+2z+10=0$
3. $3x-10y-2z+11=0$
4. $11x+y+17z+38=0$

Key: 4

Sol: Normal vector of required plane is $\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\bar{i}-\bar{j}-17\bar{k}$

Eq of plane is $11(x-1)+1(y-2)+17(z+3)=0$

$\Rightarrow 11x+y+17z+38=0$

12A. (2020-Jan) The mirror image of the point $(1,2,3)$ in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane?

1. $(1,-,1,1)$
2. $(-1,-1,1)$
3. $(1,1,1)$
4. $(-1,-1,-1)$

Key : 1

Sol : Dr's of normal to the plane are $\left(\frac{10}{3},\frac{10}{3},\frac{10}{3}\right) i.e (1,1,1)$

Midpoint of given points is $\left(\frac{-2}{3},\frac{1}{3},\frac{4}{3}\right)$.

$\therefore$ Equation plane is $1\left(x+\frac{2}{3}\right)+1\left(y-\frac{1}{3}\right)+1\left(z-\frac{4}{3}\right)=0$

$\Rightarrow x+y+z=1$

From the option, only $(1,-,1,1)$ lies on this plane.

12B. The mirror image of the point (1,2,3) in a plane (-1,2,-3) , which of the following points lies on this plane?

1. (0,1,2)
2. (0,2,0)
3. (0,2,1)
4. (1,-1,1)

Key: 2

Sol: Midpoint=(0,2,0) lies on the plane

13A. (2019-Apr) The equation of a plane containing the line of intersection of the planes $2x-y-4=0$ and $y+2z-4=0$ and passing through the point $(1,1,0)$ is

1. $x-3y-2z=-2$
2. $2x-z=2$
3. $x-y-z=0$
4. $x+3y+z=4$

Key : 3

Sol : The required plane is

$2x-y-4+\lambda(y+2z-4)=0.$ It passes through $(1,1,0)$

$\therefore\ 2-1-4+\lambda(1+0-4)=0 \Rightarrow -3+\lambda(-3)=0$

$\Rightarrow \lambda=-1$

Thus, the equation of plane is $2x-2y-2z=0$

$\Rightarrow x-y-z=0$

13B. The vector equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$ is

1. $x-z+2=0$ 2. $x+y+z=0$ 3. $2x+3y+5=0$ 4. $x-y=0$

Key: 1

Sol: Eq of is $(x+y+z-1)+\lambda(2x+3y+4z-5)=0$

Eq (1) is $\perp$ to $x-y+z=0$

$\Rightarrow \lambda=-\frac{1}{3}$

$\therefore$Eq of plane is $x-z+2=0$

14A. (2019-Apr) A plane which bisects the angle between the two given planes $2x-y+2z-4=0$ *and* $x+2y+2z-2=0,$ passes through the point

1. $(2,4,1)$ 2. $(1,4,-1)$ 3. $(1,-4,1)$ 4. $(2,-4,1)$

Key : 4

Sol : Given planes $2x-y+2z-4=0\, and\, x+2y+2z-2=0$

Equation of angle bisectors are

$\frac{2x-y+2z-4}{\sqrt{4+1+4}}=\pm\left(\frac{x+2y+2z-2}{\sqrt{1+4+4}}\right)$

$\Rightarrow \frac{2x-y+2z-4}{3}=\pm\left(\frac{x+2y+2z-2}{3}\right)$

$\therefore 2x-y+2z-4=x+2y+2z-2$

$\Rightarrow x-3y-2=0........(i)$

or $2x-y+2z-4=-(x+2y+2z-2)$

$\Rightarrow 3x+y+4z-6=0$

Since, point $(2,-4,1)$ satisfies equation (ii).

So, required point is $(2,-4,1)$

14B. The acute angle bisector of the two planes $x-2y-2z+1=0$ and $2x-3y-6z+1=0$ is

1. $13x-23y-32z+10=0$ 2. $x-5y-4z+4=0$

3. $x+y+z+1=0$ 4. $2x+3y-z+5=0$

Key: 1

Sol: Eq of plane bisection the angle between planes is

$$\left|\frac{x-2y-2z+1}{\sqrt{1+4+4}}\right|=\left|\frac{2x\_3y-6z+1}{\sqrt{4+9+36}}\right|\Rightarrow\frac{x-2y-2z+1}{3}=\pm\left(\frac{2x-3y-6z+1}{7}\right)$$

15A.(2019-Jan) The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to $y-axis$ also passes through the point

1. $(-3,0,-1)$ 2. $(3,2,1)$ 3. $(-3,1,1)$ 4. $(3,3,-1)$

Key : 2

Sol : Equation of plane through the intersection of $x+y+z=1$ and $2x+3y-z+4=0$ is

$(x+y+z-1)+\lambda(2x+3y-z+4)=0$

$\Rightarrow(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0.....(i)$

Direction ratios of normal to the plane (i) are $1+2\lambda, 1+3\lambda, 1-\lambda$

Sine (i) is parallel to y-axis

$\therefore 1+3\lambda=0\Rightarrow\lambda=-1/3$

$\therefore$ The equation of plane is $x+4z-7=0$

Clearly, only point $(3,2,1)$ satisfies this equation.

15B. The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to X- axis is

1. $y-3z+6=0$ 2. $x+y+z=2$ 3. $2x+3y+1=0$ 4. $3y+z+3=0$

Key: 1

Sol: Eq of plane is $(x+y+z-1)+\lambda(2x+3y-z+4)=0$

$\Rightarrow(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$

It is parallel to X axis

$\Rightarrow 1+2\lambda=0$

$\Rightarrow\lambda=\frac{-1}{2}$

Eq to plane is $y-3z+6=0$

16A.(2019-Apr) A plane passing through the points $(0,-1,0)$ and $(0,0,1)$ and making an angle $\frac{\pi}{4}$ with the plane $y-z+5=0$, also passes through the point.

1. $(\sqrt{2},1,4)$ 2. $(-\sqrt{2},-1,-4)$ 3. $(-\sqrt{2},1,-4)$ 4. $(\sqrt{2},-1,4)$

Key : 1

Sol : Let $ax+by+cz=d$ be the equation of the plane.

It passes through $(0,-1,0)$ and $(0,0,1)$

$\therefore 0-b+0=d\Rightarrow b=-d$ and $0+0+c=d\Rightarrow c=d$ $\therefore ax-dy+dz=d$

Now, $\cos\theta = \left|\dfrac{\overrightarrow{n_1}.\overrightarrow{n_2}}{\left|\overrightarrow{n_1}\right|.\left|\overrightarrow{n_2}\right|}\right|$

$\Rightarrow \cos\dfrac{\pi}{4} = \left|\dfrac{0-d-d}{\sqrt{a^2+d^2+d^2}.\sqrt{0+1+1}}\right|$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \left|\dfrac{-2d}{\sqrt{a^2+2d^2}.\sqrt{2}}\right| \Rightarrow \dfrac{2d}{\sqrt{a^2+2d^2}} = \pm 1$

$\Rightarrow a^2+2d^2 = 4d^2 \Rightarrow a = \pm\sqrt{2}d$

$\therefore$ Plane is $\pm\sqrt{2}x - y + z = 1$

Clearly $\left(\sqrt{2},1,4\right)$ in option (a) satisfies the above plane.

16B. The direction ration of normal to the plane through the points (0,-1,0) and (0,0,1) and making an angle $\dfrac{\pi}{4}$ with the plane $y-z+5=0$ are

1. $\sqrt{2},2,-1$ 2. 2,-1,1 3. $2,\sqrt{2},-\sqrt{2}$ 4. $2\sqrt{3},1,-1$

Key: 3

Sol: Eq of plane is $a\left(x-0\right)+b\left(y+1\right)+c\left(z-0\right)=0$

$\Rightarrow ax+by+cz+b=0$

If passes through (0,0,1) $\Rightarrow b+c=0$----(1)

$\cos\dfrac{\pi}{4} = \dfrac{b-c}{\sqrt{2}.\sqrt{a^2+b^2+c^2}} \Rightarrow a = \pm\sqrt{2}c$

$\therefore$ Dr's of $\left(a,b,c\right) = \left(\sqrt{2},-1,1\right)$(or) $\left(\sqrt{2},1,-1\right)$(or) $\left(2,\sqrt{2},-\sqrt{2}\right)$

17A. (2020-Sep) The plane passing through the point (3,1,1) contains two lines whose direction rations are $1,-2,2$ and $2,3,-1$ respectively. If this plane also passes through the point $(\alpha,-3,5)$, then α is equal to

1. 10 2. -5 3. 5 4. -10

Key : 3

Sol : Equation of a plane passing through $\left(x_1,y_1,z_1\right)$ and containing lines whose d.r's are $<a_1,b_1,c_1>and<a_2,b_2,c_2>$ is given by $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

$\therefore$ Required equation of plane is given by

$\begin{vmatrix} x-3 & y-1 & z-1 \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = 0$

$\Rightarrow \left(x-3\right)\left(2-6\right)-\left(y-1\right)\left(-1-4\right)+\left(z-1\right)\left(3+4\right)=0$

$\Rightarrow -4x+12+5y-5+7z-7=0$

$\Rightarrow -4x+5y+7z=0$

This plane also passes through $(\alpha,-3,5)$

$\therefore -4\alpha-15+35=0 \Rightarrow 4\alpha=20 \Rightarrow \alpha=5$

17B. A plane passing through the point (3,1,1) contains the lines whose direction ratios are 1,-2,2 and 2,3,-1 respectively. If this plane also passes through the point $(K,1,1)$, then K=

1. 3 2. 4 3. 5 4. 6

Key: 3

Sol: Eq of plane is $\begin{vmatrix} x-3 & y-1 & z-1 \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix}=0 \Rightarrow 4x-5y-7z=0$ sub $(K,1,7)$

$\Rightarrow 4K-5-7=0$

$\Rightarrow K=3$

18A. (2019-Apr) If the plane $2x-y+2z+3=0$ has the distances $\frac{1}{3}$ *and* $\frac{2}{3}$ units from the planes $4x-2y+4z+\lambda=0$ and $2x-y+2z+\mu=0$, respectively, then the maximum value of $\lambda+\mu$ is equal to

1. 5 2. 13 3. 9 4. 15

Key : 2

Sol : The given plane is $2x-y+2z+3=0 \, or \, 4x-2y+4z+6=0.......(i)$

$\Rightarrow$ plane (i) is parallel to the plane $4x-2y+4z+\lambda=0$

$\therefore$ Distance between these two planes is

$\frac{|\lambda-6|}{\sqrt{16+4+16}}=\frac{1}{3} \Rightarrow \frac{|\lambda-6|}{\sqrt{36}}=\frac{1}{3} \Rightarrow |\lambda-6|=\frac{6}{3}=2 \Rightarrow \lambda=8.4$

Again distance between $2x-y+2z+3=0$ *and* $2x-y+2z+\mu=0$ *is*

$\frac{|\mu-3|0}{\sqrt{4+1+4}}=\frac{2}{3} \Rightarrow \frac{|\mu-3|}{\sqrt{9}}=\frac{2}{3}$

$\Rightarrow |\mu-3|=2 \Rightarrow \mu=5,1$

$\therefore$ Maximum value of $\mu+\lambda=13$

18B. If the plane $2x-y+2z+3=0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ untis from the planes $4x-2y+4z+\lambda=0$ and $2x-y+2z+\mu=0$ respectively then minimum values of $\lambda+\mu=$

1. 5 2. 6 3. 7 4. 8

Key: 1

Sol: $\frac{|\lambda-6|}{\sqrt{16+4+16}}=\frac{1}{3} \Rightarrow \lambda=8,4$

$\frac{|\mu-3|}{\sqrt{4+1+4}}=\frac{2}{3} \Rightarrow \mu=1,5$

19A.(2019-Jan) A tetrahedron has vertices $P(1,2,1), Q(2,1,3), R(-1,1,2)$ *and* $(0,0,0)$. The angle between the faces OPQ and PQR is

1. $\cos^{-1}\left(\frac{7}{31}\right)$ 2. $\cos^{-1}\left(\frac{17}{31}\right)$ 3. $\cos^{-1}\left(\frac{19}{35}\right)$ 4. $\cos^{-1}\left(\frac{9}{35}\right)$

Key : 3

Sol : Here, $\overline{OP} \times \overline{OQ} = \left(\hat{i}+2\hat{j}+\hat{k}\right)\times\left(2\hat{i}+\hat{j}+3\hat{k}\right) = 5\hat{i}-\hat{j}-3\hat{k}$

Again, $\overline{PQ} \times \overline{PR} = \left(\hat{i}-\hat{j}+2\hat{k}\right)\times\left(-2\hat{i}-\hat{j}+\hat{k}\right) = \hat{i}-5\hat{j}-3\hat{k}$

Let angle between face OPQ and PRQ is θ

$$\therefore \cos\theta = \frac{5+5+9}{\left(\sqrt{25+9+1}\right)^2} = \frac{19}{35}$$

19B. For any four points 0(0,0,0), P(1,2,1),Q(2,3,0),R(0,1,-1), the angle between the planes OPQ and PQR is

1. $\cos^{-1}\left(\frac{5}{\sqrt{28}}\right)$ 2. $\cos^{-1}\left(\frac{5}{\sqrt{14}}\right)$ 3. $sin^{-1}\left(\frac{5}{\sqrt{28}}\right)$ 4. $sin^{-1}\left(\frac{5}{\sqrt{14}}\right)$

Key: 1

Sol: $\overline{OP}\times\overline{OQ} = -3\bar{i}+2\bar{j}+\bar{k}$, $\overline{PQ}\times\overline{PR} = -3\bar{i}+3\bar{j}$

$$\cos\theta = \frac{9+6}{\sqrt{9+4+1}\sqrt{9+9}} = \frac{5}{\sqrt{28}}$$

20A. (2021-Feb) If $(1,5,35),(7,5,5),(1,\lambda,7)$ *and* $(2\lambda,1,2)$ are coplanar, then the sum of all possible values of λ is

1. $-\frac{39}{5}$ 2. $-\frac{44}{5}$ 3. $\frac{44}{5}$ 4. $\frac{39}{5}$

Key : 3

Sol : Let $A(1,5,35), B(7,5,5), C(1,\lambda,7)$ and For the points to be coplanar,

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$\Rightarrow 6\left[-33(\lambda-5)-112\right]-30\left[-(\lambda-5)(2\lambda-1)\right]=0$

$\Rightarrow -33\lambda+53+5\left(2\lambda^2-11\lambda+5\right)=0$

$\Rightarrow 10\lambda^2-88\lambda+78=0 \Rightarrow 5\lambda^2-44\lambda+39=0$

$\therefore$ Required sum $=44/5$

20B. If the points (0,0,9),(1,1,8),(1,2,7) and ($(2,2,\lambda)$ and corplanar then $\lambda =$

1. 7 2. 8 3. 9 4. 10

Key: 1

Sol: $\left[\overline{AB}\ \overline{AC}\ \overline{AD}\right]=0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ 2 & 2 & (\lambda-9) \end{vmatrix} = 0 \Rightarrow \lambda = 7$$

21A.(2022-July) Let Q be the foot of perpendicular drawn from the point $P(1,2,3)$ to the plane $x+2y+z=14.$ If R is a point on the plane such that $\angle PRQ = 60^0$, then the area of ΔPRQ is equal to

1. $\frac{\sqrt{3}}{2}$ 2. $\sqrt{3}$ 3. $2\sqrt{3}$ 4. 3

Key : 2

Sol : Equation of plane : $x+2y+z=14$ point $P(1,2,3)$ Q is the foot of the perpendicular drawn from P to the plane

Length of perpendicular

$$PQ = \left|\frac{1+2\times 2+3-14}{\sqrt{1^2+2^2+1^2}}\right| = \left|\frac{-6}{\sqrt{6}}\right| = \sqrt{6}$$

Now, in $\Delta PQR, \tan 60^0 = \frac{PQ}{RQ}$

$$\Rightarrow \sqrt{3} = \frac{PQ}{RQ} \Rightarrow RQ = \frac{PQ}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$$

So, area of $\Delta PQR = \frac{1}{2}\times QR \times PQ = \frac{1}{2}\times\sqrt{6}\times\sqrt{2} = \sqrt{3}$

21B. Let Q be the foot of the perpendicular drawn from the point $P(0,0,0)$ to the plane $x+2y+z=14.$ If R is a point on the plane such that $\underline{|PRQ} = 45^0$ then the area of ΔPQR is equal to

1. $\frac{49}{3}$ 2. $\frac{49}{5}$ 3. $\frac{49}{4}$ 4. 50

Key : 1

Sol : $PQ = \frac{14}{\sqrt{1+4+1}} = \frac{14}{\sqrt{6}}$

$\therefore\ \tan 45^0 = \frac{PQ}{RQ} \Rightarrow PQ = \frac{14}{\sqrt{6}}$

$\therefore$ Area $= \frac{1}{2}\times\frac{14}{\sqrt{6}}\times\frac{14}{\sqrt{6}} = \frac{49}{3}$

22A.(2022-June) If the plane $2x+y-5z=0$ is rotated about its line of intersection with the plane $3x-y+4z-7=0$ by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point

1. $(2,-2,0)$ 2. $(-2,2,0)$ 3. $(1,0,2)$ 4. $(-1,0,-2)$

Key : 3

Sol : Given, plane $2x+y-5z=0$ $3x-y+4z=7$

Both the plane are perpendicular to each other.

Let the equation of the plane formed be

$2x+y-5z+\lambda(3x-y+4z-7)=0$

$\Rightarrow 2x+y-5z+3\lambda-\lambda y+4\lambda z-7\lambda=0$

$\Rightarrow (2+3\lambda)x+(1-\lambda)y+(-5+4\lambda)z-7\lambda=0....(i)$

This plane is perpendicular to $2x+y-5z=0$

Then, $a_1a_2+b_1b_2+c_1c_2=0,\ 2(2+3\lambda)+(1-\lambda)-5(-5+4\lambda)=0$

$\Rightarrow 4+6\lambda+1-\lambda+25-20\lambda=0$

$\Rightarrow -15\lambda+30=0$

$\Rightarrow -15\lambda=-30$

$\lambda=2$

Substitute $\lambda=2$ in the Eq. (i),

$(2+6)\times+(1-2y)+(-5+8)z=14$

$\Rightarrow 8x-y+3z=14$

Put $(1,0,2)$ in the given Eq. (ii)..

$\text{LHS}=8-0+6=14=RHS$

22B. If the plane $x+y+z=1$ is rotated through 90^0 about its line of intersection with the plane $x-2y+3z=0$, the new position of the plane is

1. $x-5y+4z=1$ 2. $x-5y+4z=1$ 3. $x-8y+7z+2=0$ 4. $x-8y+7z=1$

Key : 3

Sol : $(x+y+z-1)+\lambda(x-2y+3z)=0--(1)$

It is $\perp$ to $x+y+z-1=0$

$\Rightarrow a_1a_2+b_1b_2+c_1c_2=0 \Rightarrow \lambda=\frac{-3}{2}$

From (1)

$x-8y+7z+2=0$

23A.(2022-June) In the mirror image of the point $(2,4,7)$ in the plane $3x-y+4z=2$ is (a,b,c), then $2a+b+2c$ is equal to

1. 54 2. 50 3. -6 4. -42

Key : 3

Sol : The image or reflection (x, y, z) of a point (x_1, y_1, z_1) in a plane $ax+by+cz+d=0$ is given by $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}==\frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$

$\Rightarrow \frac{a-2}{3}=\frac{b-4}{-1}=\frac{c-7}{4}=\frac{-2(6-4+28-2)}{9+1+16}=-\frac{28}{13}$

$\therefore a=2-\frac{84}{13} \Rightarrow a=-\frac{58}{13}, b=4+\frac{28}{13} \Rightarrow b=\frac{80}{13}$

$c=7-\frac{112}{13} \Rightarrow c=\frac{-21}{13}$

$\because 2a+b+2c=\frac{-116+80-42}{13}$

$=-\frac{78}{13}=-6$

23B. If $Q(0,-1,3)$ is the image of the point P in the plane $3x-y+4z=2$ and R is the point $(3,-1,-2)$ then PR is

1. $\sqrt{10}$ 2. $2\sqrt{10}$ 3. $3\sqrt{10}$ 4. $4\sqrt{10}$

Key : 1

Sol : $P=(3,-2,1), R=(3,-1,-2)$

$PR=\sqrt{10}$

24A. (2022 -June) Let the points on the plane P be equidistant from the points $(-4,2,1)\, and\, (2,-2,3)$. Then, the acute angle between the plane P and the plane $2x+y+3z=1$ is

1. $\frac{\pi}{6}$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{3}$ 4. $\frac{5\pi}{12}$

Key : 3

Sol : Let the point on the plane P be (x, y, z) Given that the points on the plane P be equidistance from the points $(-4,2,1)\, and\, (2,-2,3)$. Thus,

$(x+4)^2+(y-2)^2+(z-1)^2=(x-2)^2+(y+2)^2+(z-3)^2$

$\Rightarrow x^2+16+8x+y^2+4-4y+z^2-2z+1$

$=x^2-4x+8+y^2+4y+4+z^2-6z+9$

$\Rightarrow 12x-8y+4z=4$

$\Rightarrow 3x-2y+z=1....(i)$

Given, that $2x+y+3z=1.....(ii)$

Let θ be an acute angle between the plane 'P' and the plane $2x+y+3z=1$

$\therefore \cos\theta=\left|\frac{3\times 2-2\times 1+3\times}{\sqrt{(3)^2+(-2)^3+1^2}\sqrt{(2)^2+(1)^2+(3)^2}}\right|=\left|\frac{6-2+3}{\sqrt{14}.\sqrt{14}}\right|=\frac{7}{14}=\frac{1}{2}$

Thus, $\cos\theta = \frac{1}{2} \Rightarrow \theta \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

24B. If P denotes the plane consisting of all points that are equidistant from the points $A(-4,2,1)$ and $B(2,-4,3)$ and Q be the plane $x-y+cz=1$ where C in R. If the angle between the planes P and Q is 45^0 then the value of C is

1. -17 2. -2 3. 17 4. $\frac{24}{27}$

Key : 2

Sol : $\overline{n_1} = 3\bar{i} - 3\bar{j} + \bar{k}, \overline{n_2} = \bar{i} - \bar{j} + c\bar{k}$

$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}||\overline{n_2}|}$

25A. (2022- June) Let P be the plane passing through the intersection of the planes $r.\left(\hat{i}+3\hat{j}-\hat{k}\right)=5$ and $r.\left(2\hat{i}-\hat{j}+\hat{k}\right)=3$ and the point $(2,1,-2)$. Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ *and* $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then, the points

1. X and $X+Y$ are on the same side of P
2. Y and Y-X are on the opposite sides of P
3. X and Y are on the opposite sides of P
4. X+Y and $X-Y$ are on the same side to P

Key : 3

Sol : The equation of the plane P is

$r.\left[\left(\hat{i}+3\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)\right]=5+3\lambda$

$r.\left[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(\lambda-1)\hat{k}\right]=5+3\lambda$

The plane P passes through the point

$(2,1-2).\left[2\hat{i}+\hat{j}-2\hat{k}\right].\left[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(\lambda-1)\hat{k}\right]=5+3\lambda$

$\Rightarrow 2(1+2\lambda)+1(3-\lambda)-2(\lambda-1)=5+3\lambda$

$\Rightarrow 2+4\lambda+3-\lambda-2\lambda+2=5+3\lambda$

$\Rightarrow 2\lambda=2$

$\lambda=1$

Equation pale P is

$r.\left(3\hat{i}+2\hat{j}\right)=8$ *or* $r.\left(3\hat{i}+2\hat{j}\right)-8=0$

$P_x:\left(\hat{i}-2\hat{j}+4\hat{k}\right).\left(3\hat{i}+2\hat{j}\right)-8=3-4-8=-9$

$P_\gamma:\left(5\hat{i}-\hat{j}+2\hat{k}\right).\left(3\hat{i}+2\hat{j}\right)-8=15-2-8=5$

$P_{x+\gamma}:\left(6\hat{i}-3\hat{j}+6\hat{k}\right).\left(3\hat{i}+2\hat{j}\right)-8=18-6-8=4$

$P_{x-\gamma}:\left(4\hat{i}+\hat{j}-2\hat{k}\right).\left(3\hat{i}+2\hat{j}\right)-8$

$=12+2-8=6$

$P_{x-\gamma}:\left(4\hat{i}-\hat{j}+2\hat{k}\right).\left(3\hat{i}+2\hat{j}\right)-8$

$=-12-2-8=-22$

25B. Let P be the plane passing through the point $(1,2,3)$ and the line of intersection of the planes $\bar{r}.\left(\bar{i}+\bar{j}+4\bar{k}\right)=16\,and\,\bar{r}.\left(-\bar{i}+\bar{j}+\bar{k}\right)=6$ then which of the following points does not lien on P?

1. $(3,3,2)$ 2. $(6,-6,2)$ 3. $(4,2,2)$ 4. $(-8,8,6)$

Key : 3

Sol : Equation of plane is $P_1+\lambda P_2=0$

$3x+y+7z-26=0$

26A. (2021-Aug) The equation of the plane passing through the line of intersection of the planes $r.\left(\hat{i}+\hat{j}+\hat{k}\right)=1\,and\,r.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the X-axis is

1. $r.\left(\hat{j}-3\hat{k}\right)+6=0$ 2. $r.\left(\hat{i}+3\hat{k}\right)+6=0$ 3. $r.\left(\hat{i}-3\hat{k}\right)+6=0$ 4. $r.\left(\hat{j}-3\hat{k}\right)-6=0$

Key : 1

Sol : Given, equation of planes $r.\left(\hat{i}+\hat{j}+\hat{k}\right)=1......(i)$

$r.\left(2\hat{i}+3.\hat{j}-\hat{k}\right)=0......(ii)$

Equation of pane passing through the intersection of the planes Eqs. (i) and (ii) is given by $(x+y+z-1)+\lambda(2x+3y-z+4)=0\;or\,(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z+(-1+4\lambda)=0......(iii)$

Plane (iii) in parallel to X-axis

$1+2\lambda=0$ {coefficient of $x=0$}

$\Rightarrow\lambda=\frac{-1}{2}$

$\therefore$ From Eq. (ii) becomes

$y-3z+6=0\,or\;r.\left(\hat{j}-3\hat{k}\right)+6=0$

26B. The equation of the plane passing though the line of intersection of the planes $\bar{r}.\left(\bar{i}+\bar{j}+\bar{k}\right)=1$ and $\bar{r}.\left(2\bar{i}+3\bar{j}-\bar{k}\right)+4=0$ and parallel to $y-axis$ is

1. $x+4z-7=0$ 2. $2x+3y=0$ 3. $x-4z=6$ 4. $x+z=7$

Key : 1

Sol : Equation of $P_1+\lambda P_2=0$

$\Rightarrow(x+y+z-1)+\lambda(2x+3y-z+4)=0$

$\Rightarrow(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z+(-1+4\lambda)=0$ It is parallel to $\Rightarrow\lambda=\frac{-1}{3}$

$\therefore$Equation of plane is $x+4z-7=0$

27A.(2019-Apri) Let P be the plane, which contains the line of intersection of the planes, $x+y+z-6=0$ *and* $2x+3y+z+5=0$ and it is perpendicular to the $XY-$plane. Then, the distance of the point $(0,0,256)$ from P is equal to

1. $63\sqrt{5}$ 2. $205\sqrt{5}$ 3. $\frac{11}{\sqrt{5}}$ 4. $\frac{17}{\sqrt{5}}$

Key : 3

Sol : Equation of plane, which contains the line of intersection of the planes

$x+y+z-6=0$ *and* $2x+3y+z+5=0$, is $(x+y+z-9)+\lambda(2x+3y+z+5)=0$

$\Rightarrow(1+2\lambda)+(1+3\lambda)+1(1+\lambda)z+(5\lambda-6)=0....(1)$

$\therefore$ The plane(i) is perpendicular to XY-plane

(as DR's of normal to XY-plane I s(0,0,1)).

$\therefore 0(1+2\lambda)+0(1+3\lambda)+1(1+\lambda)=0$

$\Rightarrow x+2y+11=0......(ii)$

Which of the required equation of the plane.

Now, the distance of the point $(0,0,256)$ from plane $P\,is\,\frac{0+0+11}{\sqrt{1+4}}=\frac{11}{\sqrt{5}}$

{$\therefore$ distance of (x_1,x_1,z_1) from the plane $ax+by+cz-d=0$, is $\left|\frac{ax_1+by_1+cz_1-d}{\sqrt{a^2+b^2+c^2}}\right|$}

27B. Let p be the plane, which contains the line of intersection of the planes $x+y+z-6=0\,and\,2x+3y+z+5=0$ and it is perpendicular to yz plane then the distance of the point $(0,0,0)$ from P is equal to

1. $^{17}/_{\sqrt{2}}$ 2. $^{17}/_{\sqrt{3}}$ 3. $\frac{16}{\sqrt{3}}$ 4. $\frac{1}{\sqrt{2}}$

Key : 1

Sol : Equation of plane is $(x+y+z-6)+\lambda(2x+3y+z+5)=0$

$\Rightarrow(1+2\lambda)x+(1+3\lambda)y+(1+\lambda)z+(5\lambda-6)=0$

It is perpendicular to YZ-plane

$\Rightarrow\lambda=\frac{-1}{2}$

Equation plane is $y-z+17=0$

Distance is $=\frac{17}{\sqrt{2}}$

28A.(2020- Sep) The plane which bisects the line joining the points $(4,-2,3)$ and $(2,4,-1)$ at right angles also passes through the point

1. $(0,-1,1)$ 2. $(4,0,-1)$ 3. $(4,0,1)$ 4. $(0,1,-1)$

Key : 2

Sol : The equation of pane which bisects the line joining the points $P(4,-2,3)$ *and* $Q(2,4,-1)$ passes through the mid-point of P and Q it is given that the plane is perpendicular to PQ.

$\because M\left(\frac{4+2}{2},\frac{-2+4}{2},\frac{3-1}{2}\right)=(3,1,1)$

and DR's PQ is $2,-6,4$

So, the equation of plane is

$2(x-3)-6(y-1)+4(z-1)=0$

$\Rightarrow 2x-6y+4z=4$

Now, from the options, the point $(4,0,-1)$ contained by the plane.

28B. A plane bisects the line segment joining the points $(1,2,3)$ *and* $(-3,4,5)$ at right angles. Then this plane also passes through the point

1. $(1,2,-2)$ 2. $(-1,2,3)$ 3. $(-3,2,1)$ 4. $(3,2,1)$

Key : 3

Sol : Midpoint $=(-1,3,4)$

Equation of plane is $2x-y-z+9=0$

3D LINES:-

29A. (2021-Aug) A plane P contains the line $x+2y+3z+1=0=x-y-z-6,$ and is perpendicular to the plane $-2x+y+z+8=0$. Then which of the following points lie on P?

1. $(2,-1,1)$ 2. $(0,1,1)$ 3. $(-1,1,2)$ 4. $(1,0,1)$

Key : Equation of plane containing the given lines is $x+2y+3z+1+\lambda(x-y-z-6)=0$

$\Rightarrow(1+\lambda)x+(2-\lambda)y+(3-\lambda)z+1-6\lambda=0$

This plane is $\perp$ to plane $-2x+y+z+8=0$

$\Rightarrow -2(1+\lambda)+(1)(2-\lambda)+(1)(3-\lambda)=0$

$\Rightarrow 3-4\lambda=0\Rightarrow\lambda=\frac{3}{4}$

$\therefore$ Equation of the plane is

$\frac{7}{4}x+\frac{5}{4}y+\frac{9}{4}z-\frac{7}{2}=0$

$\Rightarrow 7x+5y+9z-14=0$

Only point (0,1,1) satisfies this equation.

29B. Plane passing through the point of intersection of the planes $x+2y+z-1=0$ and $2x+y+3z-2=0$ and perpendicular to the plane $x+y+z-1=0$ and $x+ky+3z-1=0$ then K=

1. $\frac{5}{2}$ 2. $\frac{5}{3}$ 3. $\frac{2}{3}$ 4. $\frac{3}{2}$

Key: 1

Sol: Eq of plane be $(x+2y+z-1)+\lambda(2x+y+3z-2)=0$ ---(1),

Eq (1) is $\perp$ to $x+y+z-1=0$

$\Rightarrow \lambda=\frac{-2}{3}$

$\therefore$ Eq of plane be $x-4y+3z-1=0$

It is $\perp$er to $x+Ky+3z-1=0$

$\therefore 1-4K+9=0 \Rightarrow K=\frac{5}{2}$

30A. (2020- Sep) The foot of the perpendicular drawn from the point $(4,2,3)$ to the line joining the points $(1,-2,3)$ and $(1,1,0)$ lies on the plane

1. $2x+y-z=1$ 2. $x-y-2z=1$ 3. $x-2y+z=1$ 4. $x+2y-z=1$

Key : 3

Sol : Since, equation of lien joining the point $(1,-2,3)$ *and* $(1,1,0)$ *is* $\frac{x-1}{0}=\frac{y-1}{-3}=\frac{z-0}{3}=k$ (Let)

$\therefore$ Coordinate of general point on the line is $P(1,1-3k,3k)$.

Is Now, Let point $p(1,1-3K,3K)$ is the foot of perpendicular of point $M(4,2,3)$ on the line. So PM is perpendicular to the line, so

$0(4-1)-3(2-1+3k)+3(3-3k)=0$

$\Rightarrow -3-9k+9-9k=0$

$\Rightarrow 18k=6$

$\Rightarrow k=1/3$

$\therefore$ Point $P(1,0,0)$ and according to the options plane $2x+y-z=1$ contains the point $P(1,0,1)$

30B. The foot of the perpendicular from $(1,2,3)$ to the line joining the points $(6,7,7)$ *and* $(9,9,5)$ is

1. $(3,5,9)$ 2. $(3,5,6)$ 3. $(1,2,3)$ 4. $(1,3,4)$

Key : 1

Sol : Equation of line is $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}=k$

$\therefore (x,y,z)=(3k+6,2k+7,-2k+7)$

$\therefore 3(3k+5)+2(2k+5)-2(-2k+7)=0$ $\Rightarrow [K=-1]$ $\therefore (x,y,z)=(3,5,9)$