ANSWER KEYS

1. (2)	**2.** (4)	**3.** (3)	**4.** (1)	**5.** (2)	**6.** (1)	**7.** (2)	**8.** (1)
9. (3)	**10.** (3)	**11.** (1)	**12.** (2)	**13.** (4)	**14.** (2)	**15.** (10)	**16.** (4)
17. (1)	**18.** (4)	**19.** (2)	**20.** (3)	**21.** (3)	**22.** (30)	**23.** (1)	**24.** (3)
25. (3)	**26.** (3)	**27.** (5)	**28.** (3)	**29.** (3)	**30.** (1)		

1. **(2)** As per conditions of logarithms,

$x^2-3>0, 6x-10>0 \Rightarrow x>\sqrt{3}$ (Intersection of both conditions) ...(i)

Also $\log_2\left(\frac{x^2-3}{6x-10}\right)=-1 \Rightarrow \frac{x^2-3}{6x-10}=\frac{1}{2}$

$\Rightarrow \quad x^2-3=3x-5$

$\Rightarrow \quad x^2-3x+2=0 \Rightarrow x=1,2$

But from $(i), x>(\sqrt{3})$

So, $x=2$ is the answer

2. **(4)** The given expression is

$\log_{10}(2^x+1)+x=\log_{10}(6)+x\log_{10}(5)$

We know that $\log_m(x)+\log_m(y)=\log_m(xy)$ & $\log_m(x)-\log_m(y)=\log_m\left(\frac{x}{y}\right)$

$\Rightarrow \log_{10}(2^x+1)=x(\log_{10}(5)-\log_{10}(10))+\log_{10}(6)$

$\Rightarrow \log_{10}(2^x+1)=-\log_{10}(2)^x+\log_{10}(6)$

$\Rightarrow \log_{10}(2^x+1)+\log_{10}(2^x)=\log_{10}(6)$

$\Rightarrow \log_{10}[(2^x)(2^x+1)]=\log_{10}(6)$

Taking antilog on both sides, we get

$\Rightarrow (2^x)(2^x+1)=6$

$\Rightarrow (2^x)^2+2^x-6=0$

$\Rightarrow (2^x-2)(2^x+3)=0$

$2^x=(-3)$ is rejected as value of an exponential function cannot be negative

$\Rightarrow 2^x=2 \Rightarrow x=1$

3. **(3)** As we know that $\log_a(b) > 0$, if $(0 < a < 1)$ and $(0 < b < 1)$.

Given, $\log_{\frac{1}{3}}\left(x^2+2x\right) > 0$

$\Rightarrow 0 < x^2+2x < 1$

Breaking into two cases:

Case I : $x^2+2x > 0$

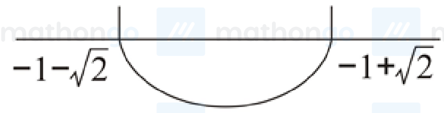
$\Rightarrow x(x+2) > 0$

$\Rightarrow x \in(-\infty,-2) \cup(0, \infty) \quad \ldots(1)$

Case II : $x^2+2\mathrm{x} < 1$

$\Rightarrow x^2+2x-1 < 0$

$\Rightarrow\left(x+1+\sqrt{2}\right)\left(x+1-\sqrt{2}\right) < 0$



$\Rightarrow -1-\sqrt{2} < x < -1+\sqrt{2} \quad \ldots(2)$

From equation (1) and (2),we get

$x \in\left(-1-\sqrt{2},-2\right) \cup\left(0, \sqrt{2}-1\right)$

Thus, $\left(-1-\sqrt{2},-2\right) \cup\left(0, \sqrt{2}-1\right)$ is correct option.

4. $\log_{175} 5x=\log_{343} 7x=k$

(1) $\Rightarrow \frac{5}{7}=\left(\frac{175}{343}\right)^k \Rightarrow k=\frac{1}{2}$

So, $\log 5x_{175}=\left(\frac{1}{2}\right)$ or $5x=\sqrt{175}$ or $5x=5\sqrt{7}$ or $x=\sqrt{7}$

Now,as per question-

$\log\left(x^4-2x^2+7\right)_{42}$

$=\log(49-14+7)_{42}=\log 42_{42}=1$

Therefore, 1 is the answer

5. **(2)**

We have,

$\log_3(x-3)=\log_9(\mathrm{x}-1)$

$\Rightarrow \frac{\log(x-3)}{\log 3}=\frac{\log(\mathrm{x}-1)}{\log 9}$

$\Rightarrow \frac{\log(x-3)}{\log 3}=\frac{\log(\mathrm{x}-1)}{\log 3^2}$

$\Rightarrow \frac{\log(x-3)}{\log 3}=\frac{\log(\mathrm{x}-1)}{2\log 3}$

$\Rightarrow 2\log(x-3)=\log(\mathrm{x}-1)$

$\Rightarrow \log(x-3)^2=\log(\mathrm{x}-1)$

$\Rightarrow (\mathrm{x}-3)^2=(\mathrm{x}-1)$

$\Rightarrow \mathrm{x}^2-7\mathrm{x}+10=0$

$\Rightarrow \mathrm{x}=2,\ 5$

$\mathrm{x}=2$ is not possible as $\log_3(x-3)$ is not defined for $x=2$.

Therefore, $x=5$.

6. **(1)** Let

$$D=\begin{vmatrix} 5^{\sqrt{\log_5 3}} & 5^{\sqrt{\log_5 3}} & 5^{\sqrt{\log_5 3}} \\ 3^{-\log_{1/3}(4)} & (0.1)^{\log_{0.01}(4)} & 7^{\log_7(3)} \\ 7 & 3 & 5 \end{vmatrix}$$

$C_2 \leftrightarrow C_2 - C_1$ and $C_3 \leftrightarrow C_3 - C_1$

$$\Rightarrow D=\begin{vmatrix} 5^{\sqrt{\log_5 3}} & \left[5^{\sqrt{\log_5 3}}-5^{\sqrt{\log_5 3}}\right] & \left[5^{\sqrt{\log_5 3}}-5^{\sqrt{\log_5 3}}\right] \\ 3^{-\log_{1/3}(4)} & \left[\left(0.1\right)^{\log_{0.01}(4)}-3^{-\log_{1/3}(4)}\right] & \left[7^{\log_7(3)}-3^{-\log_{1/3}(4)}\right] \\ 7 & -4 & -2 \end{vmatrix}$$

$$\Rightarrow D=\begin{vmatrix} 5^{\sqrt{\log_5 3}} & 0 & 0 \\ 3^{-\log_{1/3}(4)} & \left[\left(0.1\right)^{\log_{0.01}(4)}-3^{-\log_{1/3}(4)}\right] & \left[7^{\log_7(3)}-3^{-\log_{1/3}(4)}\right] \\ 7 & -4 & -2 \end{vmatrix}$$

$$\Rightarrow D=\begin{vmatrix} 5^{\sqrt{\log_5 3}} & 0 & 0 \\ 3^{-\log_{1/3}(4)} & \left[(4)^{\log_{0.01}(0.1)}-(4)^{-\log_{1/3}(3)}\right] & \left[3^{\log_7(7)}-(4)^{-\log_{1/3}(3)}\right] \\ 7 & -4 & -2 \end{vmatrix}$$

$\left[\because c^{\log_a(b)}=b^{\log_a(c)}\right]$

$$\Rightarrow D=\begin{vmatrix} 5^{\sqrt{\log_5 3}} & 0 & 0 \\ 3^{-\log_{1/3}(4)} & \left[(4)^{\frac{1}{2}}-(4)^{1}\right] & \left[3^{1}-(4)^{1}\right] \\ 7 & -4 & -2 \end{vmatrix}$$

$\left[\because \log_a(b)=\frac{1}{\log_b(a)}\right]$

$$\Rightarrow D\begin{vmatrix} 5\sqrt{\log_5 3} & 0 & 0 \\ 4 & -2 & -1 \\ 7 & -4 & -2 \end{vmatrix}=0$$

7. **(2)** For $x=1$, both parts of the equation vanish, consequently $x=1$ is root of the equation.

For $x\neq 1$ Dividing both sides of the given equation by $\log x_3 \log x_4 \log x_5$, we get-

$$1=\frac{1}{\log_5 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}=\log_x 5+\log_x 3+\log_x 4$$

$=\log_x 60$

$\Rightarrow x=60$. Thus the required set is $\{1,60\}$.

8. **(1)** The given expression is equal to

$\log_n 2+\log_n 3+\ldots+\log_n 53$

$=\log_n(2.3\ldots 53)=\log_n 53!=\dfrac{1}{\log_{53!} n}$

9. **(3)** $\log_2 \log x$ is meaningful if $(\log x>0)$ or $(x>1)$

Since $4^{\log_2 \log x}=2^{2\log_2 \log x}=\left(2^{\log_2 \log x}\right)^2=(\log x)^2$

$\left[a^{\log_a x}=x, a>0, a\neq 1\right]$

So the given equation reduces to

$2(\log x)^2-\log x-1=0$

$\Rightarrow \log x=1, \log x=-1/2$.

But we know that $\log x>0$, therefore $\log x=1$ i.e. $x=e$

10. Let $X = x^{\frac{1}{\ln y}+\frac{1}{\ln z}} \cdot y^{\frac{1}{\ln z}+\frac{1}{\ln x}} \cdot z^{\frac{1}{\ln x}+\frac{1}{\ln y}}$

(3) $\Rightarrow \quad \ln X = \ln x\left(\frac{1}{\ln y}+\frac{1}{\ln z}\right)+(\ln y)\left(\frac{1}{\ln z}+\frac{1}{\ln x}\right)+\left(\frac{1}{\ln x}+\frac{1}{\ln y}\right)(\ln z)$

Now given $\ln x+\ln y+\ln z=0$

$\therefore \quad \frac{\ln x}{\ln y}+\frac{\ln z}{\ln y}=-1$

Similarly $\frac{\ln y}{\ln x}+\frac{\ln z}{\ln x}=-1$ and

$\frac{\ln x}{\ln z}+\frac{\ln y}{\ln z}=-1$

$\therefore \quad$ R.H.S. $=-3$

$\therefore \quad \ln X=-3$

$X=e^{-3}$

11. **(1)** $x \in (2n+1)\pi/2, n\pi$ where $n \in \mathbf{I}$. The given inequality can be written as $\frac{\log_2(x^2-8x+23)}{\log_2|\sin x|} > \frac{3}{\log_2|\sin x|}$

As $\log_2|\sin x|<0$, we get

$\log_2(x^2-8x+23)<3$

$\Rightarrow x^2-8x+23<2^3=8$

$\Rightarrow x^2-8x+15<0$

$\Rightarrow (x-5)(x-3)<0 \Rightarrow 3<x<5$

For $x \in (3,5), x \in \pi, \frac{\pi}{2}, \frac{3\pi}{2}$. Hence

$x \in (3,\pi) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 5\right)$

12. **(2)** Taking log of both the sides with base 3, we have

$\left(\log_3 x^2+(\log_3 x)^2-10\right)(\log_3 x)=-2\log_3 x$

This equation is equivalent to

$\log_3 x=0$ or $2\log_3 x+(\log_3 x)^2-8=0$

$\Rightarrow \quad x=1, \log_3 x=-1\pm 3$ i.e. $\log_3 x=2, \log_3 x=-4$

Hence $x=1, 3^2, 3^{-4}=1, 9, 1/81$

13. Let $A=\log_a x \cdot \log_{10} a \cdot \log_a 5$

(4) $\quad =\log_{10} x \cdot \log_a 5$

$=\log_a(5)^{\log_{10} x}$

Let $\log_{10} x=x$, So, $A=\log_a 5^x$

Let $B=\log_{10}\left(\frac{x}{10}\right)=\log_{10} x-1=(x-1)$

Let $C=\log_{100} x+\log_4 2$

$=\frac{1}{2}\log_{10} x+\frac{1}{2}=\left(\frac{x+1}{2}\right)$

$\therefore 9^c=9^{\frac{x+1}{2}}=3^{x+1}=3^x \cdot 3$

According to question,

$\frac{6}{5}\cdot 5^x-\frac{3^x}{3}=3\cdot 3^x$

$\rightarrow 6\cdot 5^{x-1}=3^x\left(\frac{1}{3}+3\right)$

$\rightarrow 6\cdot 5^{x-1}=3^{x-1}(10)$

$\rightarrow 5^{x-2}=3^{x-2}$

Which is only possible when-

$x=2 \rightarrow \log_{10} x=2 \rightarrow x=10^2=100$

So, (4) is the correct option

14. **(2)** For (1) to hold, we must have

$x>0, x \neq 1$ and $2x^2+x-1>0$

$\Rightarrow x>0, x \neq 1$ and $(2x-1)(x+1)>0$

$\Rightarrow x>1/2, x \neq 1$

We can write (1) as

$\log_x\left(\frac{2x^2+x-1}{2}\right)>-1$ (2)

For $1/2<x<1$, (2) can be written as

$\frac{2x^2+x-1}{2}<\frac{1}{x}$

$\Rightarrow \quad 2x^3+x^2-x<2$

$\Rightarrow \quad 2\left(x^3-1\right)+x(x-1)<0$

$\Rightarrow \quad (x-1)\left(2x^2+3x+2\right)<0$

$\Rightarrow \quad x<1 \quad \left[\because 2x^2+3x+2>0 \forall x>0\right]$

For $x>1$, (2) can be written as

$$\frac{2x^2+x-1}{2}>\frac{1}{x}$$

$\Rightarrow \quad (x-1)\left(2x^2+3x+2\right)>0$

This is true for each $x>1$.

Thus, (1) holds for $1/2<x<1, x>1$.

15. **(10)** $\log_{\sqrt{2}\sin x}\left(1+\cos x\right)=2$

$\sqrt{2}\sin x \neq 1, \quad \sqrt{2}\sin x>0, \quad 1+\cos x>0$

$\Leftrightarrow \sin x \neq \frac{1}{\sqrt{2}}, \sin x>0$ and

$x \neq$ odd multiple of $\pi \Rightarrow x \in \left(0, \pi\right)-\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (feasible region)

(i) $\Leftrightarrow \left(\sqrt{2}\sin x\right)^2=1+\cos x$

$\Leftrightarrow 2\sin^2 x=1+\cos x$

$\Leftrightarrow 2\cos^2 x+\cos x-1=0$

$\Leftrightarrow (2\cos x-1)(\cos x+1)=0$

$\Rightarrow \cos x=\frac{1}{2} \quad \ldots \left[\cos x+1>0\right]$

$\Rightarrow x=\frac{\pi}{3}$

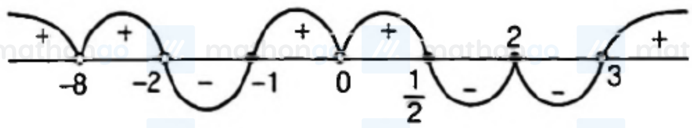
$\Rightarrow p=1, q=3$

$\Rightarrow p^2+q^2=10$

16. **(4)** We have, $\frac{(x-2)^{10000}(x+1)^{253}\left(x-\frac{1}{2}\right)^{971}(x+8)^4}{x^{500}(x-3)^{75}(x+2)^{93}} \geq 0$

The critical points are $(-8)\cdot(-2),(-1), 0, \frac{1}{2}, 2, 3$

$[\because x \neq -2, 0, 3]$



Hence, $x \in (-\infty,-8] \cup [-8,-2) \cup [-1,0) \cup \left(0, \frac{1}{2}\right] \cup (3, \infty)$

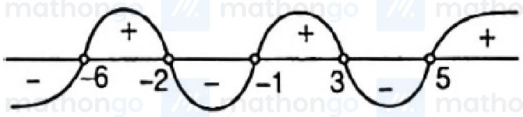
or $x \in (-\infty,-2) \cup [-1,0) \cup \left(0, \frac{1}{2}\right] \cup (3, \infty)$

$\{2\}$ also satisfy the given inequality.

Hence, answer is option 4.

17. **(1)** We have, $f(x)=\frac{(x-3)(x+2)(x+6)}{(x+1)(x-5)}$

The critical points are $(-6),(-2),(-1),3,5$



For $f(x)>0, \forall x \in(-6,-2) \cup(-1,3) \cup(5, \infty)$

For $f(x)<0, \forall x \in(-\infty,-6) \cup(-2,-1) \cup(3,5)$

18. **(4)** This equation has the form $|f(x)|=-f(x)$

when, $f(x)=\frac{x^2-8x+12}{x^2-10x+21}$

such an equation is equivalent to the collection of systems

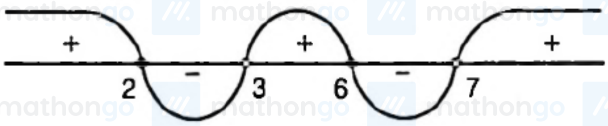
$$\begin{cases} f(x)=-f(x), \text{ if } f(x) \geq 0 \\ f(x)=f(x), \text{ if } f(x)<0 \end{cases}$$

The first system is equivalent to $f(x)=0$ and the second system is equivalent to $f(x)<0$ the combining both systems, we get

$$f(x) \leq 0$$

$$\therefore \frac{x^2-8x+12}{x^2-10x+21} \leq 0$$

$$\Rightarrow \frac{(x-2)(x-6)}{(x-3)(x-7)} \leq 0$$



Hence, by Wavy curve method,

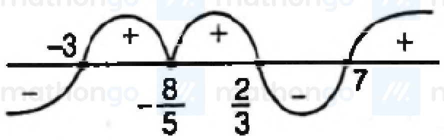
$x \in[2,3) \cup[6,7)$

19. **(2)** We have, $(x+3)(3x-2)^5(7-x)^3(5x+8)^2 \geq 0$

$\Rightarrow -(x+3)(3x-2)^5(x-7)^3(5x+8)^2 \geq 0$

$\Rightarrow (x+3)(3x-2)^5(x-7)^3(5x+8)^2 \leq 0$

[take before $x,+$ ve sign in all brackets]



The critical points are $(-3),\left(-\frac{8}{5}\right), \frac{2}{3}, 7$

Hence, $x \in(-\infty,-3] \cup\left[\frac{2}{3}, 7\right] \cup\left\{-\frac{8}{5}\right\}$

20. $\frac{x|x|}{x+2} \leq 1$

$\frac{x|x|-x-2}{x+2} \leq 0$

(3) Case I $x \in [0, \infty)$

$\frac{x^2-x-2}{x+2} \leq 0$

$\Rightarrow \quad \frac{(x-2)(x+1)}{x+2} \leq 0$

$\Rightarrow x \leq 2$

$\Rightarrow$ integral values $0, 1, 2$

Be careful: x cannot be 0 otherwise in the given inequality denominator will become 0

Case II $x \in (-\infty, 0)$

$\frac{-x^2-x-2}{x+2} \leq 0$

$\Rightarrow \quad x > -2$

$\Rightarrow \quad x = -1$

So 3 integral values

21. (3) We have, $\sqrt{(-x^2+4x-3)} > 6-2x$

This inequation is equivalent to the collection of two systems, of inequations

i.e. $\begin{cases} 6-2x \geq 0 \\ -x^2+4x-3 > (6-2x)^2 \end{cases}$ and $\begin{cases} 6-2x < 0 \\ -x^2+4x-3 \geq 0 \end{cases}$

$\Rightarrow \begin{cases} x \leq 3 \\ (x-3)(5x-13) < 0 \end{cases}$ and $\begin{cases} x > 3 \\ (x-1)(x-3) \leq 0 \end{cases}$

$\Rightarrow \begin{cases} x \leq 3 \\ \frac{13}{5} < x < 3 \end{cases}$ and $\begin{cases} x > 3 \\ 1 \leq x < 3 \end{cases}$

The second system has no solution and the first system has solution in the interval $\left(\frac{13}{5} < x < 3\right\}$

Hence, $x \in \left(\frac{13}{5}, 3\right)$ is the set of solution of the original inequation.

22. (30) We have, $y = 2[x] + 3 = 3[x-2] \ldots$ (i)

$\Rightarrow \quad 2[x] + 3 = 3([x] - 2)$ [from property (i)]

$\Rightarrow \quad 2[x] + 3 = 3[x] - 6$

$\Rightarrow \quad [x] = 9$

From Eq. (i), $y = 2 \times 9 + 3 = 21$

$\therefore \quad [x+y] = [x+21] = [x] + 21 = 9 + 21 = 30$

Hence, the value of $[x+y]$ is 30

23. (1) $[x] + \sum_{r=1}^{2000} \frac{\{x+r\}}{2000} = [x] + \sum_{r=1}^{2000} \frac{\{x\}}{2000}$

[We know, $\{x+$ Integer$\} = \{x\}$

As r takes only integral values here, $\{x+r\} = \{x\}$]

$= [x] + \frac{\{x\}}{2000} \sum_{r=1}^{2000} 1 = [x] + \frac{\{x\}}{2000} \times 2000 = [x] + \{x\} = x$

24. (3) This equation is equivalent to the collection of systems

$$\begin{cases} |x-(4-x)|-2x=4, \text{ if } 4-x \geq 0 \\ |x+(4-x)|-2x=4, \text{ if } 4-x<0 \end{cases}$$

$$\Rightarrow \begin{cases} |2x-4|-2x=4, & \text{if } x \leq 4 \\ 4-2x=4, & \text{if } x>4 \end{cases} \quad \ldots\text{(i)}$$

The second system of this collection

gives $x=0$

but $x>4$

Hence, second system has no solution.

The first system of collection Eq. (i) is equivalent to the system of collection

$$\begin{cases} 2x-4-2x=4, \text{ if } 2x \geq 4 \\ -2x+4-2x=4, \text{ if } 2x<4 \end{cases}$$

$$\Rightarrow \begin{cases} -4=4, \text{ if } x \geq 2 \\ -4x=0, \text{ if } x<2 \end{cases}$$

The first system is failed and second system gives $x=0$.

Hence, $x=0$ is unique solution of the given equation.

25. (3)

As the minimum value of $|x-1|+|x-2|+|x-3|+|x-4|$ is 4.

Hence number of solutions $=0$

26. (3) We have, $\frac{2x}{(2x^2+5x+2)} > \frac{1}{(x+1)}$

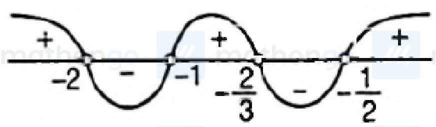
$\Rightarrow \frac{2x}{(x+2)(2x+1)} - \frac{1}{(x+1)} > 0$

$\Rightarrow \frac{(2x^2+2x)-(2x^2+5x+2)}{(x+2)(x+1)(2x+1)} > 0$

$\Rightarrow -\frac{(3x+2)}{(x+2)(x+1)(2x+1)} > 0$

or $\frac{(3x+2)}{(x+2)(x+1)(2x+1)} < 0$

The critical points are $(-2), (-1), \left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)$



Hence, $x \in (-2,-1) \cup \left(-\frac{2}{3}, -\frac{1}{2}\right)$

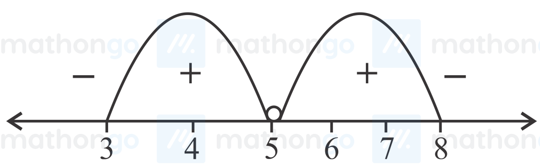
27. (5)

$$\frac{(x^2-2x+8)(e^x+2)(x-3)(x-8)}{(\log_2(x^2+3))(x-5)^2} \leq 0$$

x^2-2x+8, e^x+2 and $\log_2\left(x^2+3\right)$ are positive quantities

Next we have to find condition for $(x-3)$,$(x-5)$ and $(x-8)$

At $x=5$, the denominator $=0$. So $x=5$ is not a solution. Therefore, number of integral solutions will be between 3 and 8 excluding 5 (using wavy curve method)



Thus, we have 5 integral values possible.

28. (3) If $|a+b+c|=|a|+|b|+|c|$ then a, b, c have same sign

$\left|1-\log_{1/6} x\right|+|-\log_2 x|+|2|=|3-\log_{1/6} x-\log_2 x|$

$\therefore 1-\log_{1/6} x \geqslant 0$

$\frac{1}{6} \leqslant x$

$-\log_2 x \geqslant 0$

$x \leqslant 2$

$\therefore x \in\left[\frac{1}{6}, 2\right], a=2$ and $b=12$

$\frac{a+b}{2}=7$

29. (3) The given inequation is equivalent to the collection of systems

$$\left\{\begin{array}{l}\left|1-\frac{x}{1+x}\right| \geq \frac{1}{2}, \text{ if } x \geq 0 \\ \left|1+\frac{x}{1-x}\right| \geq \frac{1}{2}, \text{ if } x<0\end{array} \Rightarrow\left\{\begin{array}{l}\frac{1}{|1+x|} \geq \frac{1}{2}, \text{ if } x \geq 0 \\ \frac{1}{|1-x|} \geq \frac{1}{2}, \text{ if } x<0\end{array}\right.\right.$$

$$\Rightarrow\left\{\begin{array}{l}\frac{1}{1+x} \geq \frac{1}{2}, \text{ if } x \geq 0 \\ \frac{1}{1-x} \geq \frac{1}{2}, \text{ if } x<0\end{array} \Rightarrow\left\{\begin{array}{l}\frac{1-x}{1+x} \geq 0, \text{ if } x \geq 0 \\ \frac{1+x}{1-x} \geq 0, \text{ if } x<0\end{array}\right.\right.$$

$$\Rightarrow\left\{\begin{array}{l}\frac{x-1}{x+1} \leq 0, \text{ if } x \geq 0 \\ \frac{x+1}{x-1} \leq 0, \text{ if } x<0\end{array}\right.$$

For $\frac{x-1}{x+1} \leq 0$, if $x \geq 0$

$\therefore \quad 0 \leq x \leq 1 \quad$ ...(i)

For $\frac{x+1}{x-1} \leq 0$, if $x<0$

$\therefore \quad -1 \leq x<0$ ...(ii)

Hence, from Eqs. (i) and (ii), the solution of the given equation is $x \in[-1,1]$

Aliter

$\left|1-\frac{|x|}{1+|x|}\right| \geq \frac{1}{2} \Rightarrow\left|\frac{1}{1+|x|}\right| \geq \frac{1}{2}$

$\Rightarrow \quad \frac{1}{1+|x|} \geq \frac{1}{2} \Rightarrow 1+|x| \leq 2$ or $|x| \leq 1$

$\therefore -1 \leq x \leq 1$ or $x \Rightarrow[-1,1]$

30. (1)

$[2x]-[x+1]=2x \quad \ldots(1)$

$-[x+1]=\{2x\},\ 0 \leq\{2x\}<1,$

$-[x+1]=0,\ [x+1]=0$

$-1 \leq x<0,\ -2 \leq 2x<0$

$[2x]=-2,\ -1$

from equation (1)

$[2x]-0=2x,\ 2x=-2,\ -1$

$x=-1,-\frac{1}{2}$