Sri Chaitanya IIT Academy.,India.

A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 07-10-2023
Time: 09.00Am to 12.00Pm RPTM-10 Max. Marks: 300

KEY SHEET

PHYSICS

1)	1	2)	2	3)	2	4)	4	5)	3
6)	4	7)	2	8)	3	9)	3	10)	1
11)	1	12)	4	13)	2	14)	4	15)	2
16)	1	17)	3	18)	4	19)	3	20)	3
21)	20	22)	20	23)	6	24)	47	25)	5
26)	48	27)	13	28)	4	29)	40	30)	1250

CHEMISTRY

31)	1	32)	2	33)	3	34)	4	35)	4
36)	4	37)	3	38)	3	39)	3	40)	1
41)	3	42)	1	43)	2	44)	1	45)	3
46)	2	47)	4	48)	1	49)	4	50)	1
51)	4	52)	1	53)	3	54)	1	55)	5
56)	1	57)	1	58)	8	59)	2	60)	8

MATHEMATICS

61)	1	62)	2	63)	4	64)	1	65)	4
66)	1	67)	2	68)	1	69)	1	70)	1
71)	1	72)	2	73)	4	74)	2	75)	3
76)	2	77)	1	78)	4	79)	3	80)	3
81)	216	82)	130	83)	2	84)	2	85)	5
86)	69	87)	1	88)	2	89)	1	90)	2

SOLUTIONS

PHYSICS

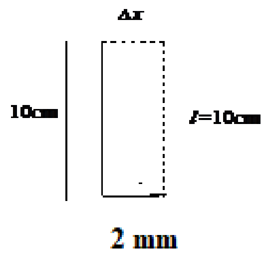
1.
$$\frac{1}{2}F\Delta L = \frac{1}{2}mv^2$$
$$\frac{1}{2}\frac{AY(\Delta L)^2}{L} = \frac{1}{2}mv^2$$
$$\therefore Y = 4.42\times 10^4\ N/m^2$$

2. Let F be the force at distance x from the front end
$$\frac{F}{X} = \frac{2F_0}{L}\ \therefore F = \frac{2F_0X}{L}$$
$$Strain = \frac{d\delta}{dX} = \frac{2F_0X}{YAL} = \frac{strees}{Y}; \int d\delta = \int_0^L \frac{2F_0X}{YSL}dX;$$
$$\delta = \frac{2F_0}{YS}\left[\frac{x^2}{2}\right]^2 = \frac{F_0L}{SY}$$

3.
$$\tan\theta = \frac{dr}{dy} = \frac{r_1 - r_2}{L}\ \therefore dy = \frac{Ldr}{r_1 - r_2}$$
$$de = \frac{F.dy}{\pi r^2 y} = \frac{F}{\pi r^2 y}\frac{Ldr}{(r_1 - r_2)} = \frac{FL}{\pi Y(r_1 - r_2)}.\frac{dr}{r^2}$$
$$\int de = \frac{FL}{\pi Y(r_1 - r_2)}\int_{r_2}^{r_1}\frac{dr}{r^2} = \frac{FL}{\pi Y(r_1 - r_2)}\left(\frac{-1}{r}\right)_{r_2}^{r_1}$$
$$= \frac{FL}{\pi Y(r_1 - r_2)}\left(\frac{-1}{r_1} + \frac{1}{r_2}\right)$$
$$= \frac{FL}{\pi Y(r_1 - r_2)}\frac{r_1 - r_2}{r_1 r_2} \Rightarrow e = \frac{FL}{\pi Y r_1 r_2}$$

4. Assuming Hookes law to be valid.
$$T\alpha(\Delta l)$$
$$T = k(\Delta l)$$
$$Let, l_0 = natural\,length(original\,length)$$
$$\Rightarrow T = k(l - l_0)$$
$$So, T = k(l_1 - l_0)\ and\ 2T = k(l_2 - l_0)$$
$$\Rightarrow \frac{T}{2T} = \frac{l_1 - l_0}{l_2 - l_0}$$
$$\Rightarrow l_0 = 2l_1 - l_2$$

5.



$W = T.\Delta A = T.21\Delta x$

$= 7.2\times10^{-2}\times2\times10\times10^{-2}\times2\times10^{-3}$

$= 28.8\times10^{-6} = 2.88\times10^{-5}\,J$

6.

PV = constant

$$\left(P_0+\frac{4T}{a}\right)\frac{4}{3}\pi a^3+\left(P_0+\frac{4T}{b}\right)\frac{4}{3}\pi b^3=\left(P_0+\frac{4T}{c}\right)\frac{4}{3}\pi c^3$$

$$\Rightarrow P_0(a^3+b^3-c^3)=4T(c^2-a^2-b^2)$$

$$\Rightarrow T=\frac{P_0(a^3+b^3-c^3)}{4(c^2-a^2-b^2)}=\frac{P_0(6^3-5^3-4^3)a}{4(4^2+5^2-6^2)}=\frac{27P_0a}{20}$$

7. If R be the meniscus radius R

$\cos(\theta+\alpha)=b$

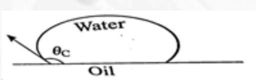
Excess pressure on concave side of meniscus

$$=\frac{2S}{R}$$

$$h\rho g=\frac{2S}{R}=\frac{2S}{b}\cos(\theta+\alpha)$$

$$\Rightarrow h=\frac{2S}{b\rho g}\cos(\theta+\alpha)$$

8.



$\theta_c > 90^\circ$

For water oil interface

9. $V_t \alpha R^2; V_2 = 4V$

10. $$\tau=\int\left[\frac{2\eta r^2 2\omega(2\pi r dr)}{h}\right]=\frac{2\eta\pi r^4\omega}{h}\Rightarrow P=\tau 2\omega=\frac{4\eta\pi r^4\omega^2}{h}$$

11. $$mg\sin\theta=f_v=\eta a^2\frac{v}{t}$$

$$\therefore \eta=\frac{mgt\sin\theta}{a^2v}=\frac{\rho agt\sin\theta}{v}$$

12. Terminal velocity of a spherical body in liquid

$V_t \alpha r^2 \Rightarrow x_f = \frac{\Delta V_t}{V_t} = 2.\frac{\Delta r}{r}$

$\Rightarrow \frac{\Delta V_t}{V_t} \times 100\% = 2\frac{(0.1)}{5} \times 100 = 4\%$

$Also V_t \alpha r^2 V_t \alpha r^2$

$Reason\, R is\ false$

13. When the oil is poured, the fraction of ice in the water decreases, i.e., volume of ice melted into water is greater than volume of water displaced by ice. So water level rises. Overall volume of ice will decrease as it melts. So the upper level of oil falls.

14. $V_A a_A = v_B \times a_B = v_A \times 4 = v_B \times 2 \quad v_B = 2v_A$---------------(i)

Again, $\frac{1}{2}\rho v_A{}^2 + \rho g h_A + p_A = \frac{1}{2}\rho v_B{}^2 + \rho g h_B + p_B \frac{1}{2}\rho v_A{}^2 + p_A = \frac{1}{2}\rho v_B{}^2 + p_B$

$\Rightarrow P_A - P_B = \frac{1}{2}\rho(v_B{}^2 - v_A{}^2) = \frac{1}{2} \times 1 \times \left(4v_A{}^2 - v_A{}^2\right) \Rightarrow 3 \times 1 \times 1000 = \frac{1}{2} \times 1 \times 3 \times v_A{}^2$

$(P_A - P_B = 2cm$ of water column $\quad = 3 \times 1 \times 1000 dyn / cm^2$

$\therefore v_A = \sqrt{\frac{9000}{3}} = 54.77 cm / s$

So the rate of flow $= v_A a_A = 54.77 \times 4 = 219 cm^3 / s$

15. Heat required (Q) = $\int_5^{15} msdt = \int_5^{15} 1 \times (0.2 + 0.14t + 0.023t^2)dt = 41 cal$

16. $\frac{dw}{dQ} = \frac{dQ - dU}{dQ} = 1 - \frac{dU}{dQ} = 1 - \frac{1}{\gamma} = 1 - \frac{3}{5} = \frac{2}{5}$

$dw = \frac{2}{5}dQ; \frac{2}{5}pdt = Fds$

$\Rightarrow \frac{2}{5}P = (P_0 A + mg)\left(\frac{ds}{dt}\right) \Rightarrow v = \frac{2}{5}\frac{P}{(P_0 A + mg)}$

17. Statement-I

$T_1 = -73^o C = 200 K$

$T_2 = 527^o C = 800 K$

$\frac{V_1}{V_2} = \frac{\sqrt{\frac{3RT_1}{M}}}{\sqrt{\frac{3RT_2}{M}}} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{200}{800}} = \frac{1}{2}$

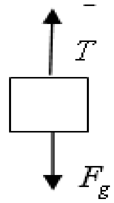
$V_2 = 2V_1$

$Statement - II$

$PV = nRT$

18. **Statement-I**

When elevator is moving with uniform speed $T = F_g$



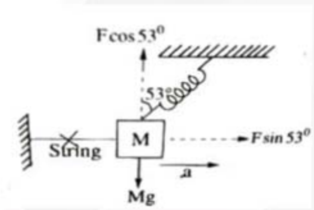
Statement-2

When elevator is going down with decreasing speed, its acceleration is upward.

$$Hence\ W - N = \frac{W}{g} \times -a$$

$$N = W\left(1 + \frac{a}{g}\right) i.e\ more\ than\ weight$$

19.



$$F\cos 53° = Mg$$

$$F\sin 53° = Ma \qquad \Rightarrow a = g\tan 53°$$

20. Initially image is formed at A itself.

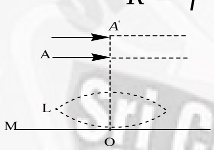
After refraction from lens the rays must be incident normally on the plane mirror.

$$\frac{1}{v} - \frac{1}{-OA} = \frac{1}{f}$$

$$f = OA = 18\,cm$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} - \frac{1}{R}\right)$$

$$R = f = 18\,cm$$



After filling the liquid between lens and mirror and placing the object at A'same thing Occurs.

$$\frac{\mu}{v} - \frac{1}{OA^1} = \frac{\frac{3}{2} - 1}{R} + \left[\frac{\mu_1 - \frac{3}{2}}{-R}\right]$$

$$\Rightarrow \frac{\mu_1}{\infty} + \frac{1}{24} = \frac{1}{36} - \frac{\mu_1 - \frac{3}{2}}{18}$$

$$\Rightarrow \frac{\mu_1 - \frac{3}{2}}{18} = \frac{1}{36} - \frac{1}{24}$$

$$\Rightarrow \frac{\mu_1 - \frac{3}{2}}{18} = \frac{-12}{36 \times 24} \Rightarrow \mu_1 = \frac{3}{2} - \frac{1}{4} = \frac{5}{4}$$

21.
$$|F| = \eta A \frac{\Delta v}{\Delta h} : 0.1 = 5\times10^{-3}\times 0.2\times\frac{v}{20\times10^{-3}}$$
$$v = 0.02\ m/s\ or\ v = 20\times10^{-3}\ m/s$$

22.
$$Slope = \frac{1}{YA}$$
$$\Rightarrow\ Y = \frac{1}{(slope)A}$$
$$Y = \frac{1}{2\times10^{-6}(0.25\times10^{-5})}$$
$$Y = 2\times10^{11}\ N/m^2$$

23. Surface tension =T

R: Radius of bigger drop

R: Radius of smaller drop

Volume will remain same
$$\frac{4}{3}\pi R^3 = 216\times\frac{4}{3}\pi r^3 \Rightarrow R = 6r$$
$$\mu_i = T.4\pi R^2$$
$$\mu_f = T.4\pi r^2\times216$$
$$\frac{\mu_f}{\mu_i} = \frac{216r^2}{R^2} = 6$$

24. For continuity equation
$$av_1 = \frac{a}{2}v_2 \Rightarrow v_2 = 2v_1$$
from Bernoulie's theorem,
$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$
$$P_1 - P_2 = \rho\left[\left(\frac{v_2^2 - v_1^2}{2}\right) + g(h_2 - h_1)\right]$$
$$4100 = 1000\left[\left(\frac{4v_1^2 - v_1^2}{2}\right) + 10\times(0-1)\right]$$
$$\frac{41}{10} + 10 = \frac{3v_1^2}{2}$$
$$\Rightarrow v_1 = \sqrt{\frac{47}{5}} \Rightarrow x = 47$$

25.
Given $s = \frac{t^2}{4}m$, m=10kg
$$\frac{ds}{dt} = \frac{1}{4}2t = \frac{t}{2} = \frac{2}{2} = 1; V = 1m/s$$
$$W = KE = \frac{1}{2}mv^2 = \frac{1}{2}\times10\times1 = 5J$$

26. $\beta_1 = \frac{\lambda_1 D}{d}$ and $\beta_2 = \frac{\lambda_2 D}{d}$

$\beta_2 = 16\,mm$ and $\beta_2 = 12\,mm$

so $LCM\,(\beta_1, \beta_2) = 48\,mm$

So at 48 mm distance both bright fringes will be found.

27. $m = \frac{f}{u+f}$

The magnification in case of lens,

$+m = \frac{f}{-10+f}$ $\quad ------(i)$

$-m = \frac{f}{-20+f}$ $\quad ------(ii)$

$(i)/(ii)$

$-1 = \frac{f-20}{f-10}$

$10 - f = f - 20 \Rightarrow f = 15$

$m = \frac{f}{u+f}$

$+m = \frac{f}{-8+f}$ $\quad ------(i)$

$-m = \frac{f}{-18+f}$ $\quad ------(ii)$

$(i)/(ii)$

$-1 = \frac{f-18}{f-8}$

$8 - f = f - 18 \Rightarrow f = 13$

28. $\frac{\omega_{upper}}{\omega_{lower}} = \frac{(3v-(-v))/2R}{v/2R} = 4$

29. $\frac{K.E_{Translational}}{K.E_{Total}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)} = 60\% \Rightarrow KE_{rotational} == 100 - 60 = 40\%$

30. From principle of calibration of thermometers

$$\frac{X-(LEP)_1}{(UFP)_1-(LEP)_1} = \frac{Y-(LEP)_2}{(UFP)_2-(LEP)_2}$$

$$\Rightarrow \frac{x+125}{500} = \frac{40+70}{40}$$

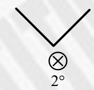
CHEMISTRY

31. In the 1st option bulky Me groups are at anti position. It has least eclipsing as well as least van der-waal strain.

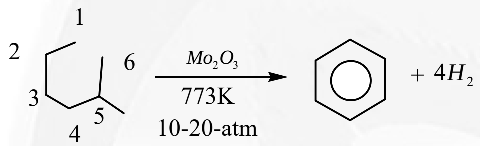
32. In presence of sunlight free radical substitution takes place. To form stable free radical, γ -hydrogen will be removed first.

$$\underset{(E)}{H_3\underset{\delta}{C}-\underset{\gamma}{C}H_2-\underset{\beta}{C}H=\underset{\alpha}{C}H_2} \xrightarrow{Br_2,hv} \underset{\text{Major product}}{H_3C-\overset{\overset{Br}{|}}{CH}-CH=CH_2} + H_3C-CH=CH-\underset{\underset{Br}{|}}{CH_2}$$

Intermediate: $\underset{\text{stable intermediate}}{H_3C-\dot{C}H-CH=CH_2}$

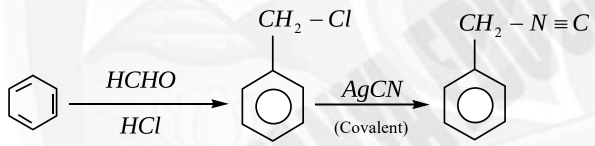
33. $CH_3-CH_2-CH_2-Cl \xrightarrow{AlCl_3} CH_3-CH_2-\underset{1^\circ}{\overset{+}{C}H_2} \xrightarrow{\text{Rearrangement}}$ 

34.



Mo_2O_3 at 773K temperature and 10-20 atm pressure aromatising agent which converts open alkyl chain into aromatic compound

35.



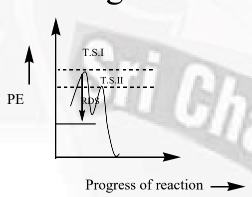
36. S_N1 is two step reaction where in step (1) formation of carbocation takes place which is slow and requires more E_a is RDS.

1 is wrong as step (1) has less E_a i.e., less formation of carbonation.

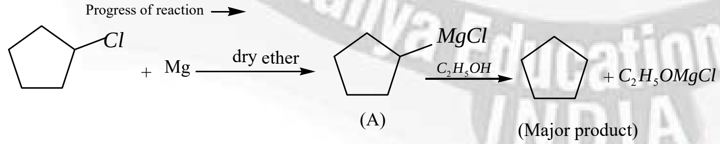
2 is wrong as it is P.E diagram of single step reaction

3 is wrong as both steps have almost same E_a.

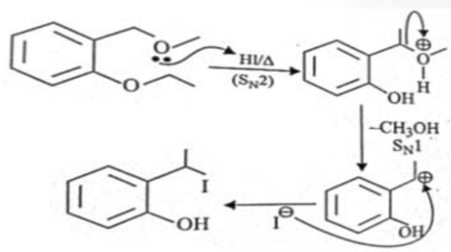
P.E diagram for S_N1



37.

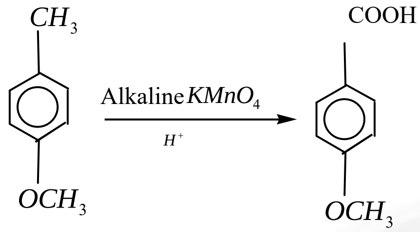


38.



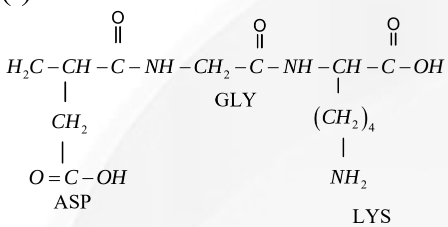
In is nucleophilic substitution reaction and cleavage of ethers with HI

39. Alkaline $KMnO_4$ oxidizes $1°$ benzylic carbon to – COOH group.



(X)

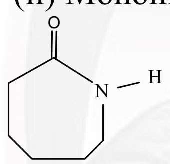
40. (i)



Number of $\overset{O}{\overset{\|}{C}}$ groups =4

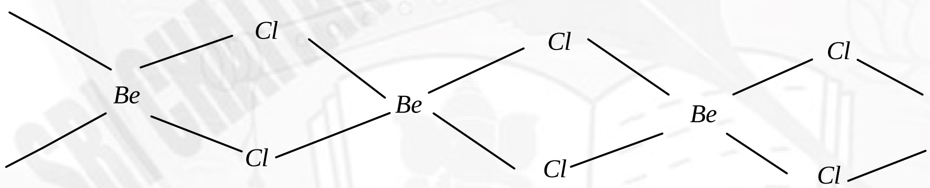
Number of peptide linkages =2 Total =4+2=6

(ii) Monomer of Nylon is caprolactum



'N' - Hetero atom within the cyclic system.
i.e One Hetero atom

41. $BeCl_2$ has a chain structure in solid state



$BeCl_2$ in vapour phase exist as chloro bridged dimer state

42. Concept

43. Concept

44. Conceptual

45. ΔH_{X-X} (KJ/mole)

Bond Dissociation Enthalpy order

$Cl_2 > Br_2 > F_2 > I_2$

242.6 192.8 158.8 151.1

46. Conceptual

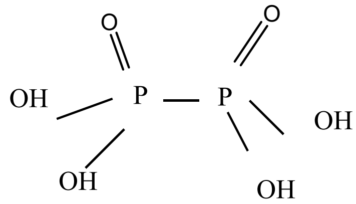
47. Two lone pair electrons at equitorial position give more stability due to minimum repulsion than axial position in BrF_3 molecule

48. Conceptual

49. Moist SO_2 act as reducing agent

50. H_2O to H_2Te thermal stability decreases due to increase in bond length and increase in reducing nature

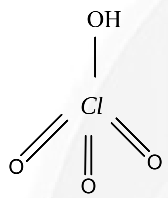
51. Hypophosphoric acid - $H_4P_2O_6$



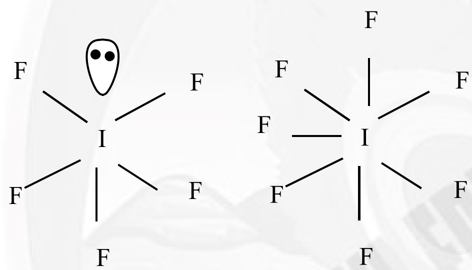
$4\times1+2\times x+6\times(-2)=0 \therefore x=+4$

52. α - Sulphur and β-sulphur are diamagnetic in nature while S_2 is paramagnetic as it has two unpaired e^-s according to molecular orbital theory (MOT) which is similar to O_2

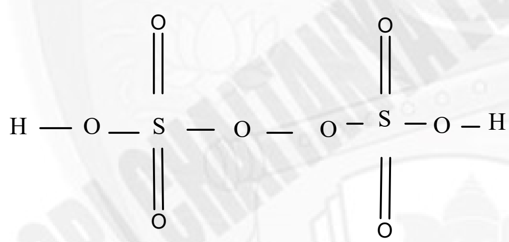
53. Perchloric acid ($HClO_4$) contains 3($Cl=O$) bonds as shown



54.



55.



P= +6

Q= 1

R = 2

P + Q – R = 6 + 1 – 2=5

56. X= $[Fe(H_2O)_5NO]^{+2}$ "Brown ring complex"

$x+5(0)+1(+1)=+2$

$x=+1$

57. hypophosphoric acid = $H_4P_2O_6$ X=0

Pyrophosphoric acid = $H_4P_2O_7$ Y=1

X + Y = 0 + 1 =1

58. H_2SO_4 = moderate oxidising agent OX-state of S = +6 Q = 6

SO_2 has canonical structure P = 2 P + Q = 2+6=8

59. Number of incorrects are ii, v only

60. P = NO_2 ; Q= NO $x.y=4\times2$

x + 2(-2) = 0 y + 1(-2) =0 $=8$

$x=+4$ $y=2$

MATHEMATICS

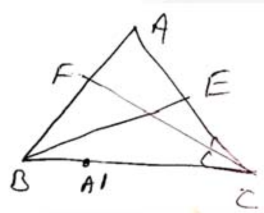
61. a,b,c are In A.P $\Rightarrow 2b = a + b \quad ax + by + 2b - a = 0$

$a(x-1)+b(y+2)=0$ represents family of lines passing through the point $(1,-2)$ the equation of circle with $(1,-2)$ and radius r is $(x-1)^2+(y+2)^2=r^2 \rightarrow 1$

Given circle $x^2+y^2-4x-4y+1=0 \rightarrow 2$ 1 and 2 are orthogonally

62. Let $C=\left(\alpha, \dfrac{10+\alpha}{4}\right) \quad E=\left(\dfrac{\alpha+3}{2}, \dfrac{6+\alpha}{8}\right)$ lies on

$6x+10y-59=0 \Rightarrow \alpha=10 \quad C=(10,5)$



Also reflection of **A** about **CF** lies on **BC** find equation **BC**, solve BC and **BE**

63. Combined equation of pair of tangents $s_1{}^2 = s.s_{11}$ of these lines meets X-axis at A and B

$$AB=\frac{8h}{\sqrt{h^2-16}}$$

$$\text{Area of } \Delta PAB = \frac{4h^2}{\sqrt{h^2-16}} = f(h)$$

f is min $\Rightarrow f^1(h)=0 \Rightarrow h=4\sqrt{2}$

64. Let $P\left(\dfrac{1+a}{2}, \dfrac{1-a}{2}\right)=(2h,2k) \quad x^2+y^2-\left(\dfrac{1+a}{2}\right)x-\left(\dfrac{1-a}{2}\right)y=0$

Centre $=\left(\dfrac{1+a}{4}, \dfrac{1-a}{4}\right)=(h,k) \quad x^2+y^2-2hx-2ky=0$

Let $(\alpha,-\alpha)$ be any point on x+y=0

The equation of the chord whose mid point $(\alpha,-\alpha)$ is $S=S_{11}$

$\alpha x-\alpha y-h(x+\alpha)-k(y-\alpha)=\alpha^2+\alpha^2-2hr+2k\alpha$

$2\alpha^2-3(h-k)\alpha+2h^2+2k^2=0$

$\Delta>0$ since these existy two chords

$9(h-k)^2-8\left(2k^2+2h^2\right)>0,\ 7k^2+7h^2+8hk<0$

$$7\left(\frac{1-a}{4}\right)^2+7\left(\frac{1+a}{4}\right)^2+18\left(\frac{1-a^2}{16}\right)<0,\ a^2>8$$

65. Let eq of AB is $7x-y-4=0 \Rightarrow m_1=7$

Let eq of AC is $x+y+1=0 \Rightarrow m_2=-1$

Slope of BC is m

$$Tan\theta = \left|\frac{m_1 - m}{1 + mm_1}\right| = \left|\left|\frac{m - m_2}{1 + mm_2}\right|\right| \Rightarrow = 1/3, -3$$

66. Orthocentre is (2,-1)
Circumcentre is (1,-2)

67. Use $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

68. $L = \underset{x\to 1}{lt} \frac{f|x|}{x-1} = finite, \therefore f(1) = 0 = L = \underset{x\to 1}{lt} f^1(x) = f^1(1)$

Now $f(x) + f^1(x) + f^{11}(x) = x^5 + 6x \to 1,$

Clearly $f(x)$ is a poly nomial of degree 5

$f^1(x) + f^{11}(x) + f^{111}(x) = 5x^4 \to 2$

$f^{11}(x) + f^{111}(x) + f^1(x) = 20x^3 \to 3$

$f^{111}(x) + f^{1V}(x) + f^v(x) = 60x^2 \to 4$

From 3-4 $\Rightarrow f^{11}(x) - f^V(x) = 20x^3 - 60x^2$

$f^{11}(x) - 120 = 20x^3 - 60x^2 \qquad \left(f^2(x) = 120\right)$

$f^{11}(x) = 20x^3 - 60x^2 + 120$

$x = 1 \Rightarrow f^{11}(1) = -40 + 120 = 80$

From 1 $\qquad f(1) + f^1(1) + f^{11}(1) = 1 + 64$

$0 + f^1(1) + 80 = 65 \quad f^1(1) = -15$

69. $3\sin t + 4 \geq 1, \sin t + 3 \geq 2 \begin{vmatrix} 3 & 2 & 1 & 4 \\ 0 & 8 & 4 & 6 \\ 0 & 0 & \geq 1 & \geq 2 \end{vmatrix}$

Rank A = Rank AB=3 $\qquad \Rightarrow$Unique Solution

70. $\begin{bmatrix} 1 & 2 & 2^2 \\ 1/2 & 1 & 2 \\ 2^{1/2} & 1/2 & 1 \end{bmatrix}$

$A^2 = 3A, \qquad A^3 = 3^2 A$

$A^2 + A^3 + ... + A^{10} = 3A + 3^2 A + ... + 3^9 A = A3\left(\frac{3^9 - 1}{3}\right)$

71. But $x^2 + \frac{2}{x^2} + 1 = t \qquad I = \int \frac{x - \frac{2}{x^3}}{\sqrt{x^2 + x^{\frac{2}{2}} + 1}} \qquad \left(2x - \frac{4}{x^3}\right)dx = dt$

72. We have $y^5 - 9xy + 2x = 0$

$5y^4 \frac{dy}{dx} - 9\left(x\frac{dy}{dx} + y\right) + 2 = 0 \quad \frac{dy}{dx} = \frac{9y-2}{5y^4 - 9x}$

For Horizontal $\Rightarrow 9y - 2 = 0 \qquad \left(\frac{dy}{dx} = 0\right)$

$y = 2/9$

$x = \frac{y^5}{2-9y}$, at y=2/9 not defined

For vertical $5y^4 - 9x = 0$

$x = \frac{5y^4}{9}, \qquad y^5 - 9y\frac{5y^4}{9} + 2.\frac{5y^4}{9} = 0 \qquad \Rightarrow y = 0, \frac{5}{18}$, N=2

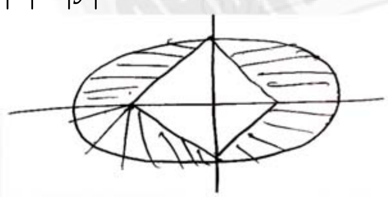
73. $f^1(x) = 3\sin x + 4\cos x, \qquad n\varepsilon\left[\frac{5\pi}{4}, \frac{4\pi}{3}\right] \qquad f^1(x) < 0$

$f(x)$ is decreasing

An min of $f(x)$ is at $x = \frac{4\pi}{3}$

$f\left(\frac{4\pi}{3}\right) = \int_{\frac{5\pi}{4}}^{\frac{4\pi}{3}} (3\sin t + 4\cos t)dt = \frac{3}{2} + \frac{1}{\sqrt{2}} - 2\sqrt{3}$

74. $|x| + |y| \geq 2$



$\frac{x^2}{9} + \frac{y^2}{4} = 1$

Area = Area of ellipse-Area of Square

75. $\bar{a} \times \bar{b} = \bar{c} \Rightarrow \bar{c}$ is perpendicular to $\bar{a} = \bar{b}$

$\bar{b} \times \bar{c} = \bar{a} \Rightarrow \bar{a}$ is perpendicular to $\bar{b} = \bar{c}$

So $\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular

$\bar{a} \times \bar{b} = \bar{c} \Rightarrow |a||b| = |\bar{c}| \Rightarrow |\bar{b}| = \frac{|\bar{c}|}{2}$

$\bar{b} \times \bar{c} = \bar{a} \Rightarrow |\bar{b}||\bar{c}| = |\bar{a}| \Rightarrow |c| = 2$

$|\bar{b}| = 1$

1) $\left[\overline{abc}\right] + \left[\overline{cab}\right] = 2\left[\overline{abc}\right] = 2|a||b||c| = 8$

2) $\bar{a}$ is parall to $\bar{b}\times\bar{c}$

So Projection of $\bar{a}$ on $\bar{b}\times\bar{c} = \dfrac{\bar{a}\left(\bar{b}\times\bar{c}\right)}{\left|\bar{b}\times\bar{c}\right|} = \dfrac{[abc]}{|b||c|} = \dfrac{4}{2} = 2$

3) $\left|3\bar{a}+\bar{b}-2\bar{c}\right|^2 = 9|a|^2+|\bar{b}|^2+4|c|^2 = 53$

76. $\dfrac{dy}{dx} = 2Tanx(\cos x - y) \qquad = 2Tanx\cos x - 2yTanx$

$\dfrac{dy}{dx} + (2Tanx)y = 2Tanx\cos x$

I.F $= e^{\int 2Tanxdx} = \dfrac{1}{\cos^2 x}\; y\sec^2 x = \dfrac{2}{\cos x} + c$

Parsing through $\left(\dfrac{\pi}{4},0\right) \Rightarrow c = -2\sqrt{2}$

$f(x) = 2\cos x - 2\sqrt{2}\cos^2 x$

77. Equation of plane $\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$ =0

78. Let A= (0,0), B= (4,0), c= (0,3)

Circumcentre of circle S is (2,3/2) and circumradius = $\dfrac{5}{2}$

Equation of S is $(x-2)^2+(y-3/2)^2 = \dfrac{25}{4}$------------(1)

If circle S_1 is having radius r_1 and touching AB and AC $\Rightarrow$ Its centre is (r_1, r_2).

S_1 and S touch internally $\Rightarrow \sqrt{(r_1-2)^2+(r_1-3/2)^2} = |r_1-5/2|$

$\Rightarrow r_1^2 - 2r_1 = 0 \Rightarrow r_1 = 2$

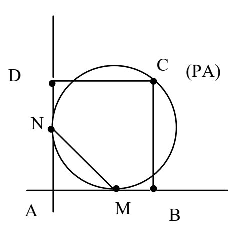
If S_2 is having r_2 and touching AB and AC $\Rightarrow$ centre of $S_2 = (r_1, r_2)$

S_2 touches S externally $\Rightarrow (r_2-2)^2+(r_2-3/2)^2 = (r_2+5/2)^2$

$\Rightarrow r_2^2 - 12r_2 = 0 \Rightarrow r_2 = 12 \Rightarrow r_1 r_2 = 24$

79. Let r be the radius of the circle and A = (0,0) AB is along x-axis and y-axis.

Equation of circle is $(x-r)^2+(y-r)^2 = r^2$--------------(1)



Equation of MN is $x + y = r$

Now $\perp^{lar}$ distance from C(p, q) to the above line is 5

$\Rightarrow \left|\frac{p+q-r}{\sqrt{2}}\right| = 5 \Rightarrow (p+q-r)^2 = 50$-------------(2)

(p, q) lies on circle (1) $\Rightarrow p^2+q^2-2r(p+q)+r^2=0$

$\Rightarrow (p+q-r)^2-2pq=0$

$50-2pq=0 \Rightarrow pq=25$

$\therefore$ Area of rectangle ABCD is 25

80. Clearly $x+y+2=0$,

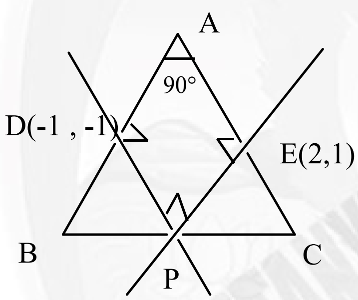
$x-y-1=0$ are perpendicular to each other

$\therefore \angle BAC = 90°$

$\therefore$ A is the ortho centre of $\Delta^{le} ABC$

Mid point of BC = P = circum centre $= \left(\frac{-1}{2}, \frac{-3}{2}\right)$

$\therefore PA^2 = DE^2 = \sqrt{9+4} = \sqrt{13}$



81. Sum of the squares of maximum and minimum distances from $(2,3)$ to circle

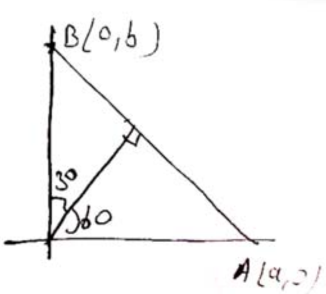
$a+b=(cp+r)^2+(cp-r)^2 \qquad =\left(9+2\sqrt{2}\right)^2+\left(9-2\sqrt{2}\right)^2$

82. Eq of AB is $x\cos\frac{\pi}{3}+y\sin\frac{\pi}{3}=p$, $\qquad \frac{x}{2}+\frac{y\sqrt{3}}{2}=P$

$A(2P,O), B\left(O, \frac{2}{\sqrt{3}}P\right)$

Area $\Delta OAB = \frac{98}{3}\sqrt{3} \qquad \frac{1}{2}(2P)\frac{2}{\sqrt{3}}P = \frac{98}{3}\sqrt{3} \Rightarrow P^2 = 49$

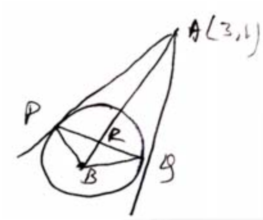
$a^2-b^2 = 4P^2 - \frac{4P^2}{3} = \frac{8}{3}P^2 = \frac{392}{3}$



83.

Eq of chord PQ is 2x+3y=0, $AR = \frac{9}{\sqrt{3}}, BR = \frac{4}{\sqrt{13}}$

$$8\frac{Areaof\,\Delta APQ}{Areaof\,\Delta BPQ} = 8\frac{AR}{BR} = 18$$



84. $x^2 + f^2(x) - 9 = f^1(x) + f^{11}(x)$

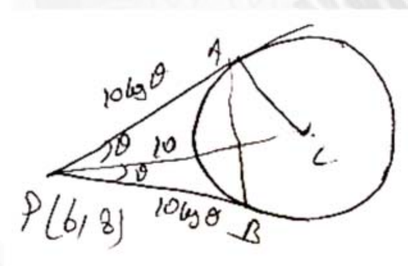
At minima $\Rightarrow f^1(x) = 0 \Rightarrow f^{11}(x) > 0$

$x^2 + f^2(x) - 9 > 0$, $x^2 + y^2 - 9 > 0$

P lies out side the circle $x^2 + y^2 = 8$

85. Area of $\Delta PAB = \frac{1}{2}(10\cos\theta)(10\cos\theta)\sin(2\theta)$ $= 10^2\cos^3\theta\sin\theta$

$$l = \sin^2\theta + \cos^2\theta = \sin^2\theta + \frac{\cos^2\theta}{3} + \frac{\cos^2\theta}{3} + \frac{\cos^2\theta}{3} \geq 4\left(\frac{\cos^6}{3^3}.\sin^2\theta\right)^{\frac{1}{4}}$$



$\cos^3\theta.\sin\theta \leq \frac{3\sqrt{3}}{16}$ $\quad 100\cos^3\theta.\sin\theta = 100.\frac{3\sqrt{3}}{16}$

$$\cos^3\theta.\sin\theta = \frac{3\sqrt{3}}{16} = \left(\frac{\sqrt{3}}{2}\right)^3.\frac{1}{2}$$

$$\sin\theta = \frac{1}{2}, \cos\theta = \frac{\sqrt{3}}{2}\ T\,an\,\theta = \frac{1}{\sqrt{3}}$$

$$Tan\theta\Delta APC \quad Tan\theta = \frac{r}{10\cos\theta}$$

86. Distance between $AC = \sqrt{65}$

87. $L_{11}L_{22} > 0$

$a^2 + ab + 1 > 0$

$\Delta < 0$

$b^2 - 4 < 0$

$b \in (-2, 2)$

88. Equation of any plane containing the general plane is

$x + y + 2z - 3 + \lambda(2x + 3y + 4z - 4) = 0 \rightarrow 1$

If plane 1 is parallel to z-axis $\Rightarrow \lambda = -\frac{1}{2}$

The plane parallel to z-axis is $y + 2 = 0 \rightarrow 2$

89. Apply expansions

90.

$$f(x) = \begin{cases} 1 - 4x^2; \; 0 \le x < \frac{1}{2} \\ 4x^2 - 1; 1/2 \le x < 1 \\ \quad -1 \qquad 1 \le x < 2 \end{cases}$$