

- If the sum of the squares of the reciprocals of the roots  $\alpha$  and  $\beta$  of the equation  $3x^2 + \lambda x - 1 = 0$  is 15, then  $6(\alpha^3 + \beta^3)^2$  is equal to  
 (1) 46 (2) 36  
 (3) 24 (4) 18
- The sum of all integral values of  $k (k \neq 0)$  for which the equation  $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$  in  $x$  has no real roots, is \_\_\_\_\_.
- If  $\alpha, \beta \in \mathbb{R}$  are such that  $1 - 2i$  (here  $i^2 = -1$ ) is a root of  $z^2 + \alpha z + \beta = 0$ , then  $(\alpha - \beta)$  is equal to:  
 (1) -7 (2) 7  
 (3) -3 (4) 3
- The value of  $3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$  is equal to  
 (1)  $1.5 + \sqrt{3}$  (2)  $2 + \sqrt{3}$   
 (3)  $3 + 2\sqrt{3}$  (4)  $4 + \sqrt{3}$
- Let  $p, q \in \mathbb{Q}$ . If  $2 - \sqrt{3}$  is a root of the quadratic equation  $x^2 + px + q = 0$ , then  
 (1)  $p^2 - 4q + 12 = 0$  (2)  $q^2 + 4p + 14 = 0$   
 (3)  $p^2 - 4q - 12 = 0$  (4)  $q^2 - 4p - 16 = 0$
- Let  $\alpha$  be a root of the equation  $1 + x^2 + x^4 = 0$ . Then the value of  $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$  is equal to:  
 (1) 1 (2)  $\alpha$   
 (3)  $1 + \alpha$  (4)  $1 + 2\alpha$
- Let  $\alpha, \beta$  be the roots of the quadratic equation  $x^2 + \sqrt{6}x + 3 = 0$ . Then  $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$  is equal to  
 (1) 81 (2) 9  
 (3) 72 (4) 729
- If  $\alpha$  and  $\beta$  are the roots of the equation  $375x^2 - 25x - 2 = 0$ , then  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \alpha^r + \lim_{n \rightarrow \infty} \sum_{r=1}^n \beta^r$  is equal to:  
 (1)  $\frac{1}{12}$  (2)  $\frac{21}{346}$   
 (3)  $\frac{7}{116}$  (4)  $\frac{29}{358}$
- Let  $\alpha, \beta (\alpha > \beta)$  be the roots of the quadratic equation  $x^2 - x - 4 = 0$ . If  $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$ , then  $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$  is equal to \_\_\_\_\_.
- If  $\alpha, \beta$  are roots of the equation  $x^2 + 5(\sqrt{2})x + 10 = 0, \alpha > \beta$  and  $P_n = \alpha^n - \beta^n$  for each positive integer  $n$ , then the value of  $\left( \frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} \right)$  is equal to
- The sum of all the roots of the equation  $|x^2 - 8x + 15| - 2x + 7 = 0$  is  
 (1)  $9 - \sqrt{3}$  (2)  $9 + \sqrt{3}$   
 (3)  $11 - \sqrt{3}$  (4)  $11 + \sqrt{3}$
- The number of the real roots of the equation  $(x+1)^2 + |x-5| = \frac{27}{4}$  is \_\_\_\_\_.
- Let  $\alpha, \beta$  be the roots of the equation  $x^2 - \sqrt{2}x + \sqrt{6} = 0$  and  $\frac{1}{\alpha^2} + 1, \frac{1}{\beta^2} + 1$  be the roots of the equation  $x^2 + ax + b = 0$ . Then the roots of the equation  $x^2 - (a+b-2)x + (a+b+2) = 0$  are :  
 (1) non-real complex numbers (2) real and both negative  
 (3) real and both positive (4) real and exactly one of them is positive
- Let  $\alpha, \beta$  be the roots of the equation  $x^2 - 4\lambda x + 5 = 0$  and  $\alpha, \gamma$  be the roots of the equation  $x^2 - (3\sqrt{2} + 2\sqrt{3})x + 7 + 3\lambda\sqrt{3} = 0$ . If  $\beta + \gamma = 3\sqrt{2}$ , then  $(\alpha + 2\beta + \gamma)^2$  is equal to

15. Let  $m$  and  $n$  be the numbers of real roots of the quadratic equations  $x^2 - 12x + [x] + 31 = 0$  and  $x^2 - 5|x + 2| - 4 = 0$  respectively, where  $[x]$  denotes the greatest integer  $\leq x$ . Then  $m^2 + mn + n^2$  is equal to
16. If  $x^2 + 9y^2 - 4x + 3 = 0$ ,  $x, y \in R$ , then  $x$  and  $y$  respectively lie in the intervals  
 (1)  $\left[-\frac{1}{3}, \frac{1}{3}\right]$  and  $\left[-\frac{1}{3}, \frac{1}{3}\right]$  (2)  $[1, 3]$  and  $\left[-\frac{1}{3}, \frac{1}{3}\right]$   
 (3)  $\left[-\frac{1}{3}, \frac{1}{3}\right]$  and  $[1, 3]$  (4)  $[1, 3]$  and  $[1, 3]$
17. The minimum value of the sum of the squares of the roots of  $x^2 + (3 - a)x = 2a - 1$  is  
 (1) 6 (2) 4  
 (3) 5 (4) 8
18. The probability of selecting integers  $a \in [-5, 30]$  such that  $x^2 + 2(a + 4)x - 5a + 64 > 0$ , for all  $x \in R$ , is:  
 (1)  $\frac{7}{36}$  (2)  $\frac{2}{9}$   
 (3)  $\frac{1}{6}$  (4)  $\frac{1}{4}$
19. If for some  $p, q, r \in R$ , not all have same sign, one of the roots of the equation  $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$  is also a root of the equation  $x^2 + 2x - 8 = 0$ , then  $\frac{q^2 + r^2}{p^2}$  is equal to-
20. Let  $a, b \in R$  be such that the equation  $ax^2 - 2bx + 15 = 0$  has repeated root  $\alpha$  and if  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - 2bx + 21 = 0$ , then  $\alpha^2 + \beta^2$  is equal to:  
 (1) 37 (2) 58  
 (3) 68 (4) 92
21. Let  $f(x)$  be a quadratic polynomial with leading coefficient 1 such that  $f(0) = p, p \neq 0$ , and  $f(1) = \frac{1}{3}$ . If the equations  $f(x) = 0$  and  $f \circ f \circ f \circ f(x) = 0$  have a common real root, then  $f(-3)$  is equal to \_\_\_\_\_.
22. The set of all real values of  $\lambda$  for which the quadratic equation  $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$  always have exactly one root in the interval  $(0, 1)$  is :  
 (1)  $(-3, -1)$  (2)  $(0, 2)$   
 (3)  $(1, 3]$  (4)  $(2, 4]$
23. The sum of  $162^{th}$  power of the roots of the equation  $x^3 - 2x^2 + 2x - 1 = 0$  is \_\_\_\_\_.
24. Let  $S = \left\{ x : x \in R \text{ and } \left( \sqrt{3} + \sqrt{2} \right)^{x^2 - 4} + \left( \sqrt{3} - \sqrt{2} \right)^{x^2 - 4} = 10 \right\}$ . Then  $n(S)$  is equal to  
 (1) 2 (2) 4  
 (3) 6 (4) 0
25. Let  $S = \left\{ \alpha : \log_2(9^{2\alpha - 4} + 13) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha - 4} + 1\right) = 2 \right\}$ . Then the maximum value of  $\beta$  for which the equation  $x^2 - 2\left(\sum_{\alpha \in S} \alpha\right)x + \sum_{\alpha \in S} (\alpha + 1)^2 \beta = 0$  has real roots, is \_\_\_\_\_.
26. If  $a + b + c = 1, ab + bc + ca = 2$  and  $abc = 3$ , then the value of  $a^4 + b^4 + c^4$  is equal to:
27. The number of points, where the curve  $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in R$  cuts  $x$ -axis, is equal to.....
28. The equation  $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0, x \in R$  has :  
 (1) four solutions two of which are negative (2) two solutions and both are negative  
 (3) no solution (4) two solutions and only one of them is negative
29. Let  $\alpha_1, \alpha_2, \dots, \alpha_7$  be the roots of the equation  $x^7 + 3x^5 - 13x^3 - 15x = 0$  and  $|\alpha_1| \geq |\alpha_2| \geq \dots \geq |\alpha_7|$ . Then,  $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$  is equal to \_\_\_\_\_.

30. The number of pairs  $(a, b)$  of real numbers, such that whenever  $\alpha$  is a root of the equation  $x^2 + ax + b = 0$ ,  $\alpha^2 - 2$  is also a root of this equation, is :

- (1) 6
- (2) 8
- (3) 4
- (4) 2