Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec:Sr.Super60_NUCLEUS & STERLING_BT Paper -2(Adv-2022-P2-Model) Date: 13-08-2023

Time: 02.00Pm to 05.00Pm CTA-01 Max. Marks: 180

KEY SHEET

MATHEMATICS

1	**8**	2	**3**	3	**1**	4	**2**	5	**5**	6	**4**
7	**9**	8	**1**	9	**AB**	10	**ACD**	11	**B**	12	**BCD**
13	**ABC**	14	**BC**	15	**B**	16	**B**	17	**D**	18	**C**

PHYSICS

19	**3**	20	**7**	21	**5**	22	**1**	23	1	24	**9**
25	**3**	26	**7**	27	**BC**	28	**AC**	29	**AB**	30	**ABC**
31	**AC**	32	**BC**	33	**A**	34	**B**	35	**C**	36	**A**

CHEMISTRY

37	**7**	38	**6**	39	**9**	40	**3**	41	**2**	42	**0**
43	**5**	44	**4**	45	**ACD**	46	**AD**	47	**AB**	48	**ABC**
49	**ACD**	50	**BCD**	51	**A**	52	**D**	53	**B**	54	**C**

SOLUTIONS

MATHEMATICS

1. $\sqrt{y}=a, \sqrt{x-y}=b \Rightarrow x=y+x-y=a^2+b^2$

$a^2+b^2-4a=2b \Rightarrow (a-2)^2+(b-1)^2=5\ (a,b\geq 0)$

$\sqrt{a^2+b^2}\in\{0\}\cup\left[2,2\sqrt{5}\right]$ $\qquad x=a^2+b^2\in\{0\}\cup[4,20]$

$p=4,\ q=20 \Rightarrow q-3p=20-12=8$

2. First, note that if $x>2$, then $x^3-3x>4x-3x=x>\sqrt{x+2}$, so all solutions x should satisfy $-2\leq x\leq 2$. Therefore, we can substitute $x=2\cos a$ for some $a\in[0,\pi]$. Then the given equation becomes $2\cos 3a=\sqrt{2(1+\cos a)}=2\cos\frac{a}{2}$, So $2\sin\frac{7a}{4}\sin\frac{5a}{4}=0$,

Meaning that $a=0,\frac{4\pi}{7},\frac{4\pi}{5}$. It follows that the solutions to the original equation are

$x=2, 2\cos\frac{4\pi}{7}, -\frac{1}{2}\left(1+\sqrt{5}\right)$

3. $x^2+2ax+a=\sqrt{a^2+x-\frac{1}{16}-\frac{1}{16}}$

$x^2+2ax+a^2+a-a^2=\sqrt{a^2+x-\frac{1}{6}-\frac{1}{16}}$

$(x+a)^2+\frac{1}{16}-a^2=-a+\sqrt{a^2+x-\frac{1}{16}}$

$f(x)=f^{-1}(x) \quad\Rightarrow\quad f(x)=x \quad\Rightarrow x^2+2ax+\frac{1}{16}=x$

$\Rightarrow\quad x^2+(2a-1)x+\frac{1}{16}=0$

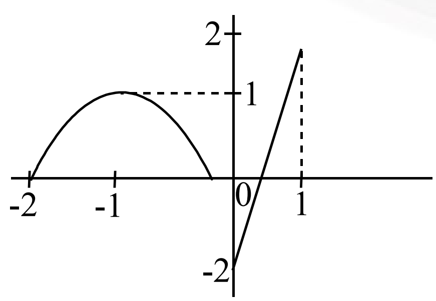
$\Delta>0 \Rightarrow (2a-1)^2-4\left(\frac{1}{16}\right)>0 \Rightarrow \left(2a-1-\frac{1}{2}\right)\left(2a-+\frac{1}{2}\right)=0$

$a>\frac{3}{4}(or)\,a<\frac{1}{4}$

4. At P (x, y), for minima $f'(x)=0$ and

$f''(x)>0 \Rightarrow x^2+f^2(x)-6>0 \quad\Rightarrow x^2+y^2>6, i.e.,$

P lies outside $x^2+y^2=6$.

5. 

$f(x)=t \Rightarrow f(t)=\frac{3}{4}$ $\Rightarrow 4t-2=\frac{3}{4},\ 1-t^2-2x=\frac{3}{4}$

$\Rightarrow t=\frac{11}{16},\ t^2+2t+1=1-\frac{3}{4}=\frac{1}{4}$ $\Rightarrow (t+1)^2=\frac{1}{4} \Rightarrow t=\frac{-1}{2},\frac{-3}{2}$

$f(x)=\frac{11}{16} \to 3$ solutions $f(x)=\frac{-1}{2} \to 1$ solution

$f(x)=\frac{-3}{2} \to 1$ solution $\Rightarrow 5$ solutions are possible

6. $\lim_{x\to\infty} a\,f(x)+f'(x)=b$ $a=\leq \ln\left(\frac{\pi}{10}\right), b=\sec\left(\frac{\pi}{5}\right)$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{e^{ax}f(x)}{e^{ax}}\left(\frac{\infty}{\infty}\ form\right)=\lim_{x\to\infty}\frac{e^{ax}\left(a\,f(x)+f'(x)\right)}{e^{ax}(a)}=\frac{b}{a}$$

$$\frac{b}{a}=\frac{\sec\left(\frac{\pi}{5}\right)}{\sin\left(\frac{\pi}{10}\right)} \qquad =\frac{1}{\sin 18^0 \sin 54^0}=\frac{16}{4}=4$$

7. $$\frac{1}{5^{2^r}-1}-\frac{1}{5^{2^r}+1}=\frac{2}{5^{2^{r+1}}}$$

Multiply with 2r and re arranging $\Rightarrow \frac{2^r}{5^{2^r}+1}=\frac{2^r}{5^{2^r}-1}-\frac{2^{r+1}}{5^{2^{r+1}}-1}$

$$=\phi(r)-\phi(n+1) \qquad \sum_{r=0}^{n}\frac{2^r}{5^{2^r}+1}=\sum_{r=0}^{n}\left(\phi(r)-\phi(r+1)\right)$$

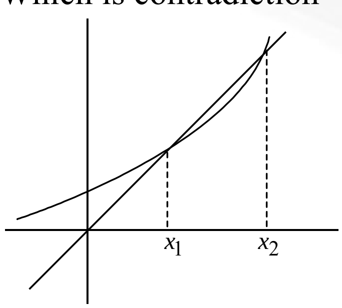
$$=\phi(0)-\phi(n+1) \qquad =\frac{1}{4}-\frac{2^{n+1}}{5^{2^{n+1}}-1}=\frac{1}{4}-\frac{1}{\frac{5^{2^{n+1}}}{2^{n+1}}-\frac{1}{2^{n+1}}} \qquad L=\frac{1}{4}$$

8. Let $2^x>3x \Rightarrow 2^{x-1}>2^x+3x$

$\therefore \quad (x-2)+2\log_2\left(2^x+3x\right)<(x-2)+2\log_2\left(2^{x+1}\right)$

$=x-2+2x+2=3x$ $\therefore \quad 2^x<3x$

Which is contradiction $\therefore \quad 2^x=3x$



$x,\in(0,1), x_2\in(3,4)$

9. $\alpha = \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{2k} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}$ $g(x) = 2^{\frac{x}{3}} + 2^{\frac{1-x}{3}}$

$$\frac{2^{\frac{x}{3}} + 2^{\frac{1-x}{3}}}{2} \geq \left(2^{\frac{x}{3}+\frac{1-x}{3}}\right)^{\frac{1}{2}} \Rightarrow g(x) \geq 2^{\frac{7}{6}}$$

Also g (x) = $1 + 2^{\frac{1}{3}}$ at x = 0, 1

$$g'(x) = 2^{\frac{x}{3}}.\frac{1}{3}.\log_e^2 + 2^{\frac{1-x}{3}}\left(\frac{-1}{3}\right)\log_e^2 \qquad = \frac{1}{3}\log_e^2\left[2^{\frac{x}{3}} - 2^{\frac{1-x}{3}}\right]$$

$$2^{\frac{x}{3}} > 2^{\frac{1-x}{3}} \Rightarrow x > 1-x \Rightarrow x > \frac{1}{2}$$

$$2^{\frac{x}{3}} < 2^{\frac{1-x}{3}} \Rightarrow \qquad x < 1-x \Rightarrow a < \frac{1}{2}$$

$\left(0,\frac{1}{2}\right)$ $g(x)$ is decreasing $\left(\frac{1}{2},1\right)$ $g(x)$ increasing

10. A) Apply LMVT on (0,4)

$$f'(c) = \frac{f(4)-f(0)}{4-0} = \frac{1}{4}\ for\ c \in (0,4)$$

B) Apply LMVT on (0,4) and (4,8)

$$f'(c_1) = \frac{1}{4}\ for\ c_1 \in (0,4)\ and\ f'(c_2) = 0\ for\ c_2 \in (4,8)$$

$f'(x)$ is continuous as f(x) is twice differentiable at some point c, the value of $f'(c) = 1/12$

D) Consider $g(x) = \int_0^{x^3} f(t)dt$

Apply LMVT on (0, 1) and (1, 2) $\quad g^{|}(\alpha) = \frac{g(1)-g(0)}{1-0}, \alpha \in (0,1)$

$$g^{|}(\beta) = \frac{g(2)-g(1)}{2-1}, \beta \in (1,2) \Rightarrow g(2) = g^{|}(\alpha) + g^{|}(\beta) = 3\alpha^2 f(\alpha^3) + 3\beta^2 f(\beta^3)$$

11. $\left(f^{|}(x)\right)^2 - K^2(f(x))^2 \leq 0$

$$\Rightarrow \left(f^{|}(x) - Kf(x)\right)\left(f^{|}(x) + f(x)\right) \leq 0$$

$$\Rightarrow \left(e^{-kx}f(x)\right)^1\left(e^{kx}f(x)\right)^1 \leq 0$$

Let $g_1(x)=e^{-kx}f(x), g_2(x)=e^{kx}f(x)$ $\therefore g_1^1(x).g_2^1(x)\le 0$

$\therefore$ One of $g_1(x), g_2(x)$ is decreasing

Let $g_1(x)$ be increasing but $g_1(0)=0 \Rightarrow g_1(x)\ge 0 \Rightarrow f(x)\ge 0$

Let $g_2(x)$ be decreasing but $g_2(0)=0 \Rightarrow g_2(x)\le 0 \Rightarrow f(x)\le 0$

$\therefore f(x)=0 \forall x\in[0,1]$

12. $f(k)=\int_0^{\pi/2}|k-2\sin t|\cos dt = 2\int_0^1\left|x-\frac{k}{2}\right|dx$

= 1 – k for $k\le 0$ = k – 1 for $k\ge 2$

$\frac{1}{2}\left(k^2-2k+2\right)$ for $0\le k\le 2$

13. Consider $G(x)=e^{-x}\int_0^x f(t)dt$ and apply Rolle's theorem

consider $H(x)=e^x(1-x)\int_0^x f(t)dt$ and apply Rolle's theorem.

$\int_0^1 f(x)dx=0$

$G(x)=e^{-x}\int_0^x f(t)dt$ $\quad G(0)=0,\quad G(1)=0$

$\Rightarrow \quad G'(c)=0$ for some $c\in(0,1)$

$\Rightarrow e^{-c}(f(c))-e^{-c}\int_0^c f(t)dt=0 \quad \Rightarrow \quad \int_0^c f(t)dt=f(c)$

$H(x)=e^x(1-x)\int_0^x f(t)dt$

$H(0)=0, H(1)=0$

$H'(c)=0$ for some $c\in(0,1)$

$\Rightarrow e^c(1-c)(f(c))+e^c(1-c)\int_0^c f(t)dt-e^c\int_0^c f(t)dt=0$

$\Rightarrow \quad e^c(1-c)f(c)-ce^c\int_0^c f(t)dt=0 \Rightarrow \int_0^c f(t)dt=\frac{(1-c)}{c}f(c)$

14. Let sides of rectangular be 8x, 5x and side of removed square be y.

$V=(8x-2y)(5x-2y)y$

$V = 40x^2y - 26xy^2 + 4y^3$

$\frac{dV}{dy} = 40x^2 - 52xy + 12y^2$

$4y^2 = 4 \Rightarrow \boxed{y=1}$

$\frac{dV}{dy} = 0 \Rightarrow x = y, \frac{3}{10}y$

$\frac{d^2V}{dy^2} = -52x + 24y$

$\frac{d^2V}{dy^2} < 0 @ x = y$ Sides are 8y, 5y.

15. $50p(x)p^{||}(x) = (50-1)\left(p^{|}(x)\right)^2 \quad \Rightarrow 50\left[p(x)p^{||}(x) - \left(p^{|}(x)\right)^2\right] = \left(p^{|}(x)\right)^2$

$$\Rightarrow 50\frac{p(x)p^{||}(x) - \left(p^{|}(x)\right)^2}{(p(x))^2} + \left(\frac{p^{|}(x)}{p(x)}\right)^2 = 0$$

$$\Rightarrow 50d\left(\frac{p^{|}(x)}{p(x)}\right) + \left(\frac{p^{|}(x)}{p(x)}\right)^2 = 0 \quad \text{........ (1)}$$

$\ell n\ p(x) = \ell nA + \ell n(x-\alpha_1) \ ......+\ \ell n(x-\alpha_{50})$

$$\frac{p^{|}(x)}{p(x)} = \frac{1}{x-\alpha_1} + \frac{1}{x-\alpha_2} + ... + \frac{1}{x-\alpha_{50}} = \sum_{i=1}^{50}\frac{1}{x-\alpha_i}$$

$$\frac{p^{|}(x)}{p(x)} = \sum_{i=1}^{50}\frac{1}{(x-\alpha_i)} \quad \text{........ (2)}$$

$$\left(\frac{p^{|}(x)}{p(x)}\right)^{|} = -\sum_{i=1}^{50}\frac{1}{(x-\alpha_i)^2} \quad \text{........ (3)}$$

Substitute (2) and (3) in equation (1)

$$50\sum_{i=1}^{50}\frac{1}{(x-\alpha_i)^2} = \left(\sum_{i=1}^{50}\frac{1}{x-\alpha_i}\right)^2$$

If all roots are equal then LHS = RHS but all roots are distinct, so roots are imaginary.

16. The given expression can be interpreted as the square of the distance between the points (tanA, 4cotA) and (cosB, SinB).
The minimum value of this distance is the minimum distance between the curves
$\mathrm{xy} = 4$ and $\mathrm{x}^2 + \mathrm{y}^2 = 1$. $\quad \mathrm{P}\left(\mathrm{t}, \frac{4}{\mathrm{t}}\right), \mathrm{Q}(0,0)$

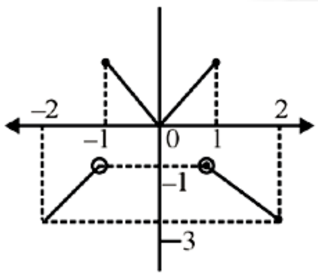
Least distance $= PQ-1=\left(\sqrt{t^2+\frac{16}{t^2}}-1\right)\geq\sqrt{\left(t-\frac{4}{t}\right)^2+8}-1=\sqrt{8}-1=2\sqrt{2}-1$

$\geq 4\sqrt{2}-1$

17. $f(3+x)=f(1-x)\ \Rightarrow f(3+x)=f(x-1)$

$\Rightarrow f(x+4)=f(x)\forall x\in R$

$\Rightarrow f(x)$ is periodic with period 4 and also even.



$$f(x)=\begin{cases} x & ;\ 0\leq x\leq 1 \\ 1-2x & ;\ 1<x\leq 2 \end{cases}$$

$$f(x)=\begin{cases} |x| & ;\ 0\leq x<1 \\ 1-2|x| & ;\ 1<x\leq 2 \end{cases}$$

$$\int_0^{100} f(x)dx=25\int_{-2}^{2} f(x)dx=50\int_0^2 f(x)dx$$

$$=50\left[\left(\frac{1}{2}\times1\times1\right)-\left(\frac{1}{2}(1+3)1\right)\right]$$

$$I=-\frac{150}{2}=-75$$

$D=25\times2=50$

$2D-I=100+75=175$

18. $f^{||}(c_1)\,f^{||}(c_2)<0$ and $f^{|}(c_1)=f^{|}(c_2)=0$

$\Rightarrow f^{||}(c_1)-f^{||}(c_2)>0$

$\Rightarrow f^{||}(c_1)>0$ and $\Rightarrow f^{||}(c_2)<0$

$\Rightarrow f^{|}(x)=0$ atleast four times in $[c_1-1,c_2+1]$

PHYSICS

19. $p_2 = p_1 + \frac{Dx}{A} = 10^5 Pa + \frac{1000N/m}{10^{-2}m^2}.x,$

Where D is the spring constant. Then the volume of the enclosed gas is

$V_2 = A.x = 10^{-2}m^2.x.$, Since temperature is constant, Boyle's law can be applied:

$p_1V_1 = p_2V_2 = \left(p_1 + \frac{Dx}{A}\right)Ax.$. Rearranged by the powers of x : $Dx^2 + p_1Ax - p_1V_1 = 0$,

and the solution of the equation is $x = \frac{-p_1A \pm \sqrt{p_1^2A^2 + 4Dp_1V_1}}{2D}$

Numerically, the equation (1) is $10^3\frac{N}{m}.x^2 + 10^3N.x - 2.10h2Nm = 0$,

Which simplifies to $5x^2 + 5x - 1 = 0$, and the solution is

$$x = \frac{-5 \pm \sqrt{25+20}}{10} = 0.1708m \approx 0.171m.$$

20. If the mercury level sinks by x cm, then the rise in the other arm is also x cm. If we calculate in the Hgcm unit of pressure, then we get a very simple equation for the requested rise in the level. The initial pressure of the enclosed air is $p_0 = 76$ Hgcm, the final pressure is $p_1 = (76 + 2x)$ Hgcm. According to Boyle's law

$p_0h_0A = (p_0 + 2x)(h_0 + \Delta h + x)A$. In our case $\Delta h = -10cm = -h_0/2$. Substituting this and simplifying by A gives $p_0h_0 = (p_0 + 2x)\left(\frac{h_0}{2} + x\right)$, numerically

$76Hgcm.20cm = (76 + 2x)Hgcm.(10 + x)cm.$

From here, (omitting the dimensions) equation $x^2 + 48x - 380 = 0$

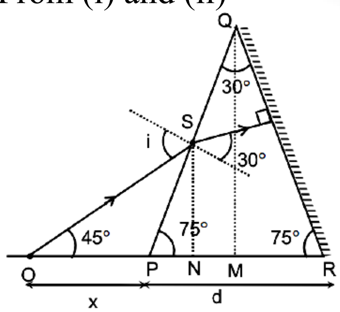
Is acquired, whose positive solution is $x = \frac{-48 + \sqrt{48^2 + 4.380}}{2}cm = 6.9cm.$

21. $1 \times \sin i = \sqrt{3} \times \sin 30^0$ $\quad i = 60^0$

$\tan 75^0 = \frac{SN}{d/4}$ ……(i)

$\tan 45^0 = \frac{SN}{x + \frac{d}{4}}$ …….(ii)

From (i) and (ii)



$x=\frac{d}{4}\left(\sqrt{3}+1\right)$

22. The initial pressure of the enclosed air is $p_1=p_0+mg/A$, while its final (maximum) pressure is $p_2=\left[p_0+\left(m+m_{Hg}\right)g/A\right]=p_0+mg/A+QH_g gx$.

According to Boyle's law: $V_2=\frac{p_1}{p_2}V_1$

Substituting the expressions for p_1, p_2, V_1 and V_2, we get:

$$\left(h+h_1-x\right)A=\frac{p_0+\frac{mg}{A}}{p_0+\frac{mg}{A}+QH_g gx}.Ah.$$

Let us divide the equation by A and multiply by the denominator of the fraction:

$$\left(p_0+\frac{mg}{A}+QH_g gx\right)\left(h+h_1-x\right)=\left(p_0+\frac{mg}{A}\right)h.$$

After rearranging this according to the powers of x, we get:

$$QH_g g\,x^2-\left[QH_g g\left(h_1+h\right)-\left(p_0+mg/A\right)\right]x-\left(p_0+mg/A\right)h_1=0.$$

$$p_0=10\frac{N}{cm^2};\qquad \frac{mg}{A}=\frac{72}{20}\frac{N}{cm^2}=3.6\frac{N}{cm^2};$$

$$QH_g g=13.6.10^{-3}\frac{kg}{cm^3}.10\frac{m}{s^2}=0.136\frac{N}{cm^3},$$

Inserting these into the equation, we obtain:

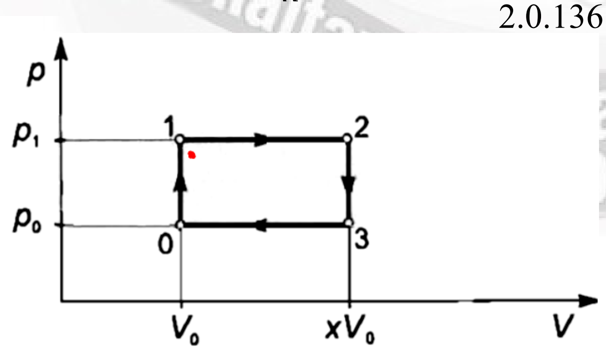
$$0.136\frac{N}{cm^3}.x^2-\left(0.136\frac{N}{cm^3}.40cm-13.6\frac{N}{cm^2}\right).x-13.6\frac{N}{cm^2}.7cm=0.$$

Dividing the equation by unit N/cm^2, we find:

$$0.136\frac{1}{cm}.x^2+8.16.x-95.2cm=0.$$

The solution is :

$$x=\frac{-8.16+\sqrt{8.16^2+4.95.2.0.136}}{2.0.136}cm=10cm.$$

23. 

$$\frac{p_0V_0}{T_0}=\frac{p_1xV_0}{4T_0},$$

from which

$$p_1 = \frac{4p_0}{x}.$$

With this, the area enclosed by the graph is

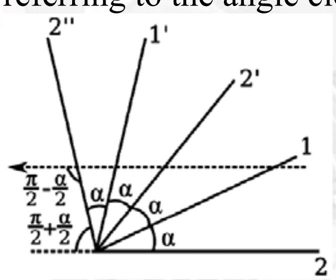
$$W_{useful} = \left(\frac{4p_0}{x} - p_0\right)\left(xV_0 - V_0\right) =$$

$$= p_0V_0\left(\frac{4}{x} - 1\right)\left(x-1\right) = \frac{p_0V_0}{x}\left(4-x\right)\left(x-1\right)$$

$$\eta\left(x\right) = \frac{W_{useful}}{Q_{in}} = \frac{\frac{p_0V_0}{x}\left(4-x\right)\left(x-1\right)}{\frac{p_0V_0}{x}\left(11.5x-4\right)} = \frac{\left(4-x\right)\left(x-1\right)}{11.5x-4} = \frac{x^2-5x+4}{4-11.5x} =$$

$$\eta_{\max} = 0.1059$$

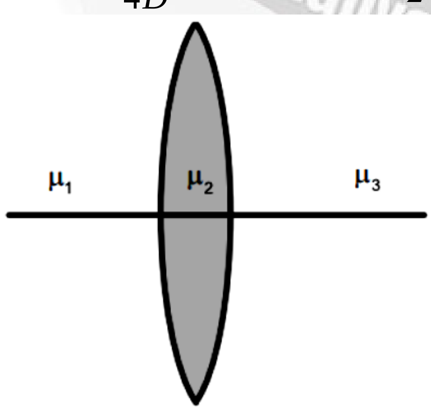
24. Due to the symmetrical nature of the set-up and the reversibility of light rays, the path of the light ray between the 4^{th} reflection (with mirror 2) and the 5^{th} reflection (with mirror 1) must be symmetrical about the symmetry axis of the two mirrors as well. That is, the path must be perpendicular to the symmetry axis. Then, the angle subtended by the ray after emerging from the 4^{th} reflection (with mirror 2) and mirror 2 must $\frac{\pi}{2} - \frac{\alpha}{2}$ (we are referring to the angle closer to the point of connection of the mirrors)



4^{th} reflection is labeled above. We have $4\alpha + \left(\frac{\pi}{2} + \frac{\alpha}{2}\right) = \pi \Rightarrow \alpha = \frac{\pi}{9}$.

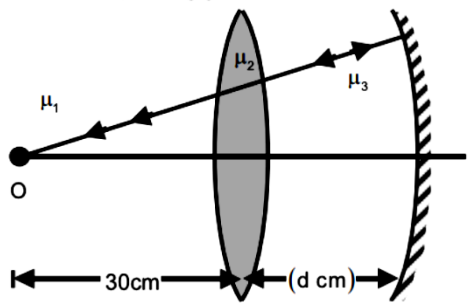
25. $\frac{2R}{D} = 0.01$ $\qquad \frac{\cancel{\sigma}\, \cancel{4\pi} R^2 T_0^4}{\cancel{4\pi} D^2} \cancel{\pi r^2} = \cancel{\sigma} T^4 4 \cancel{\pi r^2}$

$T^4 = T_0^4 \frac{R^2}{4D^2}$ $\qquad T = \frac{6000 \times 10^{-1}}{2} T^4 = \frac{T_0^4}{4} \times \frac{10^{-4}}{4}$ $\qquad = 300K$

26. 

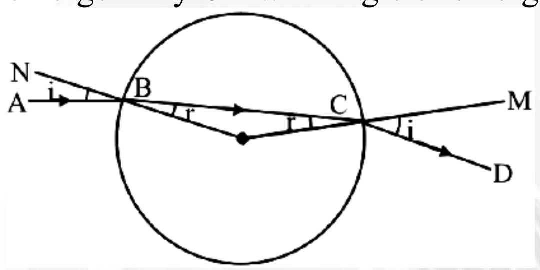
$$\frac{\mu_3}{v}-\frac{\mu_1}{u}=\frac{\mu_3-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2} \qquad \frac{4}{v}-\frac{1}{-30}=\frac{2-1}{10}+\frac{4-2}{-10}$$

$v=-30$



$d+30=100$

27. The diagram shows the path of a ray AB incident at B on the sphere at angle i. The path ABCD shows is refracted ray BC, inside the sphere with angle r of refraction and the emergent ray CD with angle of emergence i.



Obviously $\angle OBC=\angle OCB=r$.
Hence $\angle ABN=\angle DCM=i$ for all values of i.
Also, if $\angle ABN=90^0$, grazing incident at B, then
$\angle DCM=90^0$, grazing emergence at C.
But AB and CD will not be parallel.

28. For first case
Image distance = 3 times object distance.

$$\therefore \quad \frac{1}{\upsilon}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{3\upsilon}-\frac{1}{(-u)}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{3\upsilon}+\frac{1}{u}=\frac{1}{f} \qquad \ldots\ldots\ldots \text{(i)}$$

When distance between screen and lens is increased by 10 cm

$$\upsilon=3u+10 \qquad |u'|=\frac{\upsilon}{5} \quad \therefore \quad \frac{1}{\upsilon}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{3u+10}-\frac{1}{u'}=\frac{1}{f} \Rightarrow \frac{1}{3u+10}+\frac{5}{3u+10}=\frac{1}{f}$$

From equations (i) and (ii)

$$\therefore \quad u=\frac{20}{3}cm \qquad \therefore \quad f=5$$

29. $\mu_1 \sin r_1=\sin i_1 \quad \Rightarrow \quad r_1=\sin^{-1}\left(\frac{\sin i_1}{\mu_1}\right)$

$$\frac{\mu_1}{\mu_2}\sin i_2 = \sin r_2 \qquad \frac{\mu_1}{\mu_2}\frac{R_1}{R_2}\frac{\sin i_1}{\mu_1} = \sin r_2 \quad r_2 = \sin^{-1}\left(\frac{R_1}{\mu_2 R_2}\sin i_1\right)$$

30. The deviation produced by ABCD is zero. Hence the cavity will not have any effect on the deviation. $\mu = \dfrac{\sin\left(\dfrac{\delta_{\min}+A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \dfrac{8}{5}$

$$\sin\left(\frac{\delta_{\min}+A}{2}\right) = \left(\frac{8}{5}\right)\sin\left(\frac{60^0}{2}\right) \qquad \delta = 46^0$$

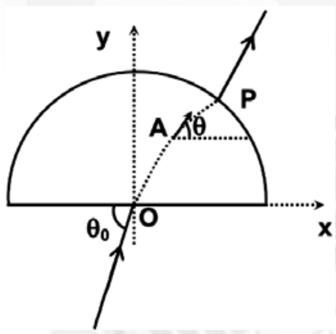
31. Cylinder absorbs energy from left face and radiate from both, So

$$\frac{P}{2}\left(1-\cos 60^0\right) = \sigma A T^4 + \sigma A T'^4 \qquad \Rightarrow P = 68W$$

Heat current = 1 W

$$\frac{kA\Delta T}{\ell} = 1 \qquad \Rightarrow k = 0.057$$

32. Let at any point A of trajectory of ray the tangent to the path of ray makes an angle θ with x–axis. From snell's law



$$1\times\sin 90^0 = n\sin\theta \qquad \Rightarrow \sin\theta = \frac{1}{n} = \frac{24-x}{24}$$

$$\tan\theta = \frac{24-x}{\sqrt{48x-x^2}} \quad \frac{dy}{dx} = \frac{24-x}{\sqrt{48x-x^2}}$$

$\int_0^y dy = \int_0^x \frac{(24-x)\,dx}{\sqrt{48x-x^2}} \Rightarrow x^2+y^2-48x=0$, so the path of the ray is circular.

At P, $x^2+y^2=R^2=144 \Rightarrow 48x=144 \quad \Rightarrow x = 3\,cm$

33. where in our case $pV = p_0V_0$, and so the pressure expressed with the variables of the process is $p = \dfrac{p_0V_0}{V_0+Avt}$, and with it the power as a function of time is $P = \dfrac{p_0V_0Av}{V_0+Avt}$,

Numerically $\quad P = \dfrac{10^5\dfrac{N}{m^2}.1m^3.0.1m^2.10^{-2}\dfrac{m}{s}}{1m^3+0.1m^2.10^{-2}\dfrac{m}{s}.t} = \dfrac{100}{1+10^{-3}\dfrac{1}{s}.t}W.$

34. We must take into account here that the heat transferred per unit time is proportional to the temperature difference. Let us introduce the following notation : T_{out1}, T_{out2} and T_{r1}, T_{r2} are the temperatures outdoors and in the room in the first and second cases respectively. The thermal power dissipated by the radiator in the room is $k_1(T - T_r)$, where k_1 is a certain coefficient. The thermal power dissipated from the room is $k_2(T_r - T_{out})$, where k_2 is another coefficient. In thermal equilibrium, the power dissipated by the radiator is equal to the power dissipated from the room. Therefore, we can write

$$k_1(T - T_{r1}) = k_2(T_{r1} - T_{out1})$$

Similarly, in the second case,

$$k_1(T - T_{r2}) = k_2(T_{r2} - T_{out2})$$

Dividing the first equation by the second, we obtain

$$\frac{T - T_{r1}}{T - T_{r2}} = \frac{T_{r1} - T_{out1}}{T_{r2} - T_{out2}}$$

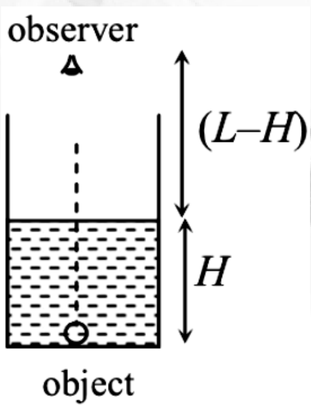
Hence we can determine T:

$$T = \frac{T_{r2}T_{out1} - T_{r1}T_{out2}}{T_{r2} + T_{out1} - T_{out2} - T_{r1}} = 60^0 C$$

35. The number of molecules passing through a hole of cross–sectional area A during a time interval t is Av Nt, where v is the component, perpendicular to the wall, of the average molecular velocity, and N is the number density (the number of particles in unit volume). At equilibrium no. of molecules going out should be equal to the number of molecules going in.

The square of the speed of the molecules (a measure of the internal energy of the gas) is proportional to the gas temperature T, and, from the ideal gas equation, the number density is proportional to the quotient of the pressure p and the temperature.

36. $X_{app} = L - H + \frac{H}{\mu}$ $\qquad \frac{dX_{app}}{dt} = \frac{dH}{dt}\left(\frac{1-\mu}{\mu}\right)$ $\qquad \pi r^3 H = V$



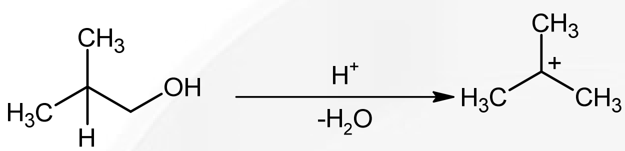
$$\therefore \frac{dH}{dt} = -\frac{2H}{r}\frac{dr}{dt} = \frac{2KH}{r}$$

$$\therefore \frac{dX_{app}}{dt} = \frac{2KV}{\pi r^3}\left(\frac{(1-\mu)}{\mu}\right)$$

CHEMISTRY

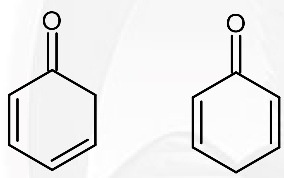
37. –COOH group, all OH groups, SH group and H of terminal alkynes react with MeMgBr to give CH_4.

38. 5 monochloro products (excluding SI as they come out as single fraction with their enantiomers) and left over reactant (20%).
As there are no other organic products formed, 20 % of the reactant must be left unused, which is also a fraction.

39. 

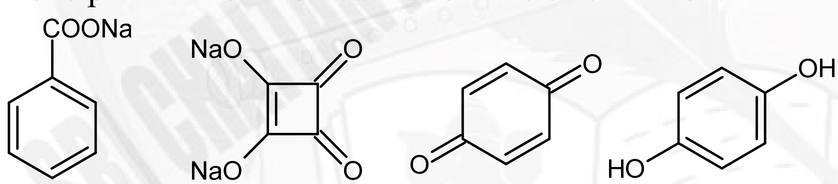
40. 6 carbon atoms will be involved in aromatization (part of benzene ring), rest 3 will be in alkyl group(s).

41. Tautomerism involving both ortho positions give the same keto form (i.e., they are not distinct).



42. Not possible to have POS or COS as the number of carbon atoms in the alkyl residues are not the same.

43. Lone pair in 5 is involved in aromatic stabilization.

44. 

45. A, C and D satisfy the condition for tautomerism.

46. B has same configuration as the given structure.
C is the enantiomer of the given structure.

47. A and B are correct IUPAC names of the given compound.

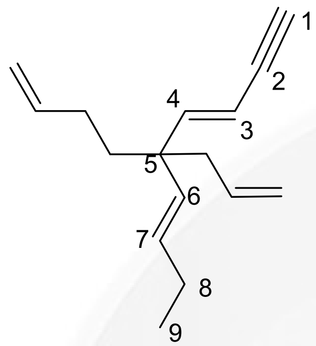
48. A) Intramolecular hydrogen bonding decreases the acidic character of *o*-nitrophenol
B) Conjugate acid of Me_2NH has two hydrogen atoms that can involve in HB with water while that of Me_3N has only one.
C) Conjugate base of *cis* isomer is stabilized by intramolecular HB.
D) No HB in the conjugate base of *o*-isomer.

49. It has 14stereogenic atoms (both carbon atoms in C=C are stereogenic).
DU = 10 (6 rings, three C=Cs and a C=O)
All stereoisomers of optically active (no POS or COS)

50. When X = F, Cl, **P** is major product (probability controlled)

When X = Br, **Q** is major product (bromine is more selective, reacts with more reactive tertiary H)

Iodination is not successful under given conditions as HI reduces back the formed RI to reactant.

51. 

5-(but-3-enyl)-5-(prop-2-enyl)nona-3,6-dien-1-yne

52. (A) has lesser number of C and H. (B), (C) and (D) are isomeric. As branching increases among isomeric alkanes, stability increases and hence heat of combustion decreases.

53. (B) is the least significant contributing structure, as it is against the +R effect of OCH_3 group.

54. I–resonance; III–sp^2 hybridized nitrogen; IV–sp^2 hybridized nitrogen with positive charge on nitrogen.