Exercise-2

3D-LINES

Sub topic:- Equation of line passing through the given point and parallel to a line whose direction ratios are a,b,c

01. Equation of the line which passes through (1,2,3) and parallel to the line $\frac{x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3} is$

1. $\frac{x-1}{1}=\frac{y-2}{7}=\frac{z-3}{\frac{3}{2}}$ 2. $\frac{x-1}{1}=\frac{y-2}{7}=\frac{z-3}{\frac{3}{2}}$ 3. $\frac{x-1}{\frac{3}{2}}=\frac{y-2}{1}=\frac{z-3}{7}$ 4.None

Key : 1

Sol : Line is parallel to given line $\Rightarrow D.R's$ of line is $1,7,\frac{3}{2}$

$$\frac{x-1}{1}=\frac{y-2}{7}=\frac{z-3}{\frac{3}{2}}, \quad \Rightarrow \frac{x-1}{1}=\frac{y-2}{7}=\frac{2z-6}{3}$$

Sub topic:- Equation of line passing through given point and having direction cosines in 3D

02. Equation of the line passing through $(3,2,-1)$ and having D.C's as (4,5,8) is

1. $\frac{x-3}{4}=\frac{y-2}{5}=\frac{z+1}{8}$ 2. $\frac{x-3}{5}=\frac{y-2}{4}=\frac{z+1}{8}$
3. $\frac{x+3}{5}=\frac{y+2}{4}=\frac{z+1}{8}$ 4. None

Key : 1

Sol : The equation of the line passing through (x_1, y_1, z_1) and having D.C's as l,m,n is

$$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$

$\therefore$ Required line is $\frac{x-3}{4}=\frac{y-2}{5}=\frac{z+1}{8}$

Sub topic:- Equation of line passing through two given points in 3D

03. Equation of the line passing through $(-2,1,3)$ and $(1,1,4)$ is

1. $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z-3}{1}$ 2. $\frac{x-2}{3}=\frac{y+1}{0}=\frac{z+3}{1}$
3. $\frac{x+2}{4}=\frac{y+1}{3}=\frac{z-3}{2}$ 4. $\frac{x-3}{1}=\frac{y-1}{1}=\frac{z-2}{1}$

Key : 1

Sol : Use the formula $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

Sub topic:- Reduction from un symmetrical from to symmetrical from

04. Symmetrical from of the equation of the line $x-y+2z-5=0=3x+y+z-6=0$ is

1. $\frac{4x-11}{-3}=\frac{4y+9}{5}=\frac{z}{1}$
2. $\frac{x-11}{-3}=\frac{y+9}{5}=\frac{z}{1}$
3. $\frac{x+11}{-3}=\frac{y-9}{5}=\frac{z}{1}$
4. None

Key :1

Sol : The given equations are

$x-y+2z-5=0---(1)$

$3x+y+z-6=0--(2)$

Let a,b,c be the dr's of the line

Since the line lies in both the given planes. Therefore their normal's are perpendicular to the line

$\therefore a-b+2c=0---(3)$

$3a+b+c=0---(4)$

Salving (2) and (4) we get, $\frac{a}{-3}=-\frac{b}{5}=\frac{c}{4}$

Hence the proportional dr's of the line are $(-3,5,4)$

To find the point on the straight line, put $z=0$

In two given equations

$x-y=5 \, and \, 3x+y=6$

Solving there equation, we get $x=\frac{11}{4}, g=\frac{-9}{4}$ so, one point on the line is $\left(\frac{11}{4},-\frac{9}{4},0\right)$

Hence, the equation of the required line is $\frac{x-11/4}{-3}=\frac{y-\left(-\frac{9}{4}\right)}{5}=\frac{z-0}{4}$

Sub topic:- Symmetrical from of a line in 3D

05. If the equation of line is $x=ay+b; z=cy+d$, then its symmetrical form is

1. $\frac{x-a}{b}=\frac{y}{1}=\frac{z-c}{d}$
2. $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$
3. $\frac{x+a}{b}=\frac{y}{1}=\frac{3+c}{d}$
4. $\frac{x+b}{a}=\frac{y}{1}=\frac{z+d}{c}$

Key : 2

Sol : Given line is $x=ay+b, z=cy+d$

$\Rightarrow x-b=ay \quad z-d=cy \Rightarrow \frac{x-b}{a}=y \quad \frac{z-d}{c}=y$

$\therefore$ symmetrical form of given line is $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$

Sub topic:- Point of intersection of line and the plane in 3D

06a. (2023 Apr) Let P be the point of intersection of the line $\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$ and the plane $x+y+z-2=0$. If the distance of the point P from the plane $3x-4y+12z=32$ is q, then q and 2q are the roots of the equation.

1. $x^2+18x-72=0$ 2. $x^2-18x+72=0$ 3. $x^2-18x-72=0$ 4. $x^2+18x+72=0$

Key : 2

Sol : Given line is $\frac{x+3}{3}=\frac{y+2}{1}=\frac{z-1}{-2}=\lambda(let)---(1)$

Any point on $(1) is\, P=(3\lambda-3,\lambda-2,-2\lambda+1)$

P lies on the plane $x+y+z-2=0$

$\Rightarrow 3\lambda-3+\lambda-2+1-2\lambda-2=0$

$\Rightarrow 2\lambda-6=0 \Rightarrow \lambda=3$

$q=$ The distance from $P(6,1,-5)$ to $3x-4y+12z-32=0$

$$q=\left|\frac{18-4-60-32}{\sqrt{9+16+144}}\right|=6$$

$\therefore$ The Q.E whose roots are $6,12$ is $x^2-(6+12)x+6\times12=0$

06b. Let A be the point of Intersection of the line $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z-1}{2}$ and the plane $x-y+z-3=0$ If the distance of the point A from the plane $x-2y+z-2=0$ is l, then l and 2l are the roots of the equation is

1. $8\sqrt{6}x^2-54x+81=0$ 2. $x^2-54x+81=0$

3. $\sqrt{6}x^2-x+81=0$ 4. $\sqrt{6}x^2-40x+8=0$

Key : 1

Sol : Given line is $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z-1}{2}=\lambda(let)--(1)$

Any point on (1) is $A=(\lambda+3,-\lambda-2,2\lambda+1)$

A lies on the plane $x-y+z-3=0$

$\Rightarrow 4\lambda+3=0 \Rightarrow \lambda=\frac{-3}{4}$

$$\therefore A=\left(-\frac{3}{4}+3,\frac{3}{4}-2,1-\frac{3}{4}\times2\right)$$

$$A=\left(\frac{9}{4},\frac{-5}{4},\frac{-1}{2}\right)$$

$l=$ The distance from A to the Plane $x-2y+z-2=0$

$$l=\frac{\left|\frac{9}{4}-2\left(\frac{-5}{4}\right)+\left(-\frac{1}{2}\right)-2\right|}{\sqrt{1+4+1}}=\frac{\left|\frac{9}{4}+\frac{10}{4}-\frac{1}{2}-2\right|}{\sqrt{6}}$$

$$l=\frac{9}{4\sqrt{6}},2l=\frac{18}{4\sqrt{6}}$$

Required equation is

$$x^2-\left(\frac{9}{4\sqrt{6}}+\frac{18}{4\sqrt{6}}\right)x+\left(\frac{9}{4\sqrt{6}}\right)\left(\frac{18}{4\sqrt{6}}\right)=0$$

$$x^2-\frac{27}{4\sqrt{6}}x+\frac{162}{16\sqrt{6}}x+\frac{162}{16\sqrt{6}}=0$$

$$x^2-\frac{27}{4\sqrt{6}}x+\frac{81}{8\sqrt{6}}=0$$

$$8\sqrt{6}x^2-54x+81=0$$

07a. (2022 July) Let a line with direction ratios $a,-4a,-7$ be perpendicular to the lines with direction ratios $3,-1,2b$ and $b,a-2.$ If the point of intersection of the line $\frac{x+1}{a^2+b^2}=\frac{y-2}{a^2-b^2}=\frac{z}{1}$ and the plane $x-y+z=0$ is $(\alpha,\beta,\gamma),then\,\alpha+\beta+\gamma$ is equal to ……

1. 10 2. 20 3. 30 4. 40

Key : 1

Sol : Line with Dr's $a,-4a,-7$ perpendicular to line with DR's $3,-1,2b$

So, $a_1a_2+b_1b_2+c_1c_2=0$

$3a+4a-14b=0\Rightarrow a=2b$

Also line, with DR's $a,-4a,-7$

Per perpendicular to line with DR's b,a -2

So, $ab-4a^2+14=0$

$\Rightarrow a\left(\frac{a}{2}-4a^2+14=0\right)\quad\left[\because b=\frac{a}{2}\right]$

$\Rightarrow -\frac{7}{2}a^2+14=0$

$\Rightarrow a=\pm 2$

For $a=2,b=1,\,for\,a=-2,b=-1$

Line $L:\frac{x+1}{a^2+b^2}=\frac{y}{a^2-b^2}=\frac{z}{1}$

$L:\frac{x+1}{5}=\frac{y-2}{3}=\frac{z}{1}=\lambda$

So, any point on the line let say $P(5\lambda-1,3\lambda+2,\lambda)$ If line intersects the plane $x-y+z=0$ then, P lies on the plane

$5\lambda-1-3\lambda-2+\lambda=0\Rightarrow\lambda-1=0$

$\Rightarrow\lambda=1$

$P(4,5,1) \equiv (\alpha,\beta,\gamma)$

$\alpha+\beta+\gamma = 4+5+1 = 10$

07b. Let a line with direction ratios $(a,-4a-7)$ be perpendicular to the lines with direction ratios $3,-1,2b.$ and $b,a,-2.$ If the point of intersection of the line $\frac{x+2}{a^2-b^2}=\frac{y-1}{a^2+b^2}=\frac{z-2}{1}$ and the plane $x-y+z=0$ is (α,β,γ), then $\alpha+\beta+\gamma=$

1.8 2. -8 3.10 4.0

Key : 2

Sol : Line with dR's $a-4a,-7$ perpendicular to line with $D'R's\,3,-1,\ 2b.$

$\therefore a_1a_2+b_1b_2+c_1c_2 \Rightarrow 3a+4a-14b=0$

$\Rightarrow 7a=14b \to [a=2b]$

Also line, with DR'S $a,-4a,-7$ perpendicular to the line with $DR's\,b,a,-2$

$\therefore a(b)+a(-4a)+14=0$

$\Rightarrow a\left(\frac{a}{2}\right)-4a^2+14=0 \Rightarrow \frac{a^2}{2}-4a^2+14=0$

$\Rightarrow \frac{-7a^3}{2}+14=0 \Rightarrow 7a^2=28 \Rightarrow a^2=4 \ \Rightarrow a=\pm 2$

For $a=2,b=1,\ for\ \ a=-2,b=-1$

Line $L:\frac{x+2}{4-1}=\frac{y-1}{4+1}=\frac{z-2}{1}$

$L:\frac{x+2}{3}=\frac{y-1}{5}=\frac{z-2}{1}=\lambda$

Any point on the line is $P=(3\lambda-2,5\lambda+1,\lambda+2)$

If line intersects the plane $x-y+z=0$

$\Rightarrow 3\lambda-2-5\lambda-1+\lambda+2=0 \Rightarrow 4\lambda-5\lambda-1=0 \Rightarrow \lambda-1=0 \Rightarrow -1=\lambda$

$\therefore P=(-5,-4,1)=(\alpha,\beta,\gamma)$

$\therefore \alpha+\beta+\gamma=-5-4+1=-8$

08a. (2021 Aug) The square of the distance of the point of intersection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and plane $2x-y+z=6$ from the point $(-1,-1,2)$ is

1. 61 2. 60 3. 40 4. 15

Key : 1

Sol : $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}=\lambda$

$$\begin{cases} x=2\lambda+1 \\ y=3\lambda+2 \\ z=6\lambda-1 \end{cases}$$

Equation of plane is $2x-y+z=6 \Rightarrow 2(2\lambda+1)-(3\lambda+2)+(6\lambda-1)=6$

$7\lambda = 7$

$\lambda = 1$

$P(3,5,5)$

$(\text{Distance})^2 = (3+1) + (5+1)^2 + (5-2)^2 = 16 + 36 + 9 = 61$

08b.The square of the distance of the point of intersection of the line $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z-1}{1}$ and the plane $x + y - z - 6 = 0$ from the point $(1,1,-2)$ is

1. $\frac{258}{4}$ 2. $\frac{152}{7}$ 3.4 4.9

Key : 1

Sol : Let $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z-1}{1} = \lambda - (1)$

$\therefore x = 3\lambda - 1, y = 2\lambda - 2, z = \lambda + 1. \Rightarrow any$ point $on\, the\, line\, 1\, is$

$P = (3\lambda - 1, 2\lambda - 2, \lambda + 1)$

Equation of the plane is $x + y - z - 6 = 0$

$\Rightarrow 3\lambda - 1 + 2\lambda - 2 - \lambda - 1 - 6 = 0 \Rightarrow 4\lambda - 10 = 0 \Rightarrow 2\lambda - 5 = 0 \Rightarrow \lambda = \frac{5}{2}$

$\therefore P = \left(3.\frac{5}{2} - 1, 2\times\frac{5}{2} - 2, \frac{5}{2} + 1\right) = \left(\frac{15-2}{2}, 3, \frac{7}{2}\right) = \left(\frac{13}{2}, 3, \frac{7}{2}\right)$

Let Q $= (1, 1-2)$

$\therefore (PQ)^2 = \left(\frac{13}{2} - 1\right)^2 + (3-1)^2 + \left(\frac{7}{2} + 2\right)^2$

$= \left(\frac{11}{2}\right)^2 + 4 + \left(\frac{11}{2}\right)^2 = \frac{121}{4} + 4 + \frac{121}{4} = \frac{121 + 16 + 121}{4} = \frac{258}{4}$

Sub topic:- Foot of the perpendicular from a point on a line in 3D

09a. (2020 Jan) If the foot of the perpendicular drawn from the point $(1,0,3)$ on a line passing through $(\alpha,7,1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to ……

1. 4 2. 9 3. 11 4. 100

Key : 4

Sol : It is given that point $Q\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ is foot of perpendicular of point $P(1,0,3)$ to a line passes through point $A(\alpha,7,1), so\, PQ \perp AQ.$

$\because dr's$ of line segment PQ is $\left(\frac{2}{3}, \frac{7}{3}, \frac{8}{3}\right)$ and $dr's$ of line AQ$\left(\alpha - \frac{5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$

$\therefore \frac{2}{3}\left(\alpha - \frac{5}{3}\right) + \frac{7}{3}\left(\frac{14}{3}\right) + \frac{8}{3}\left(-\frac{14}{3}\right) = 0 \Rightarrow (3\alpha - 5) + 49 - 56 = 0$

$\Rightarrow 3\alpha - 5 - 7 = 0 \Rightarrow \alpha = 4$

9b. If the foot of the perpendicular drawn from the point $(-1,0,2)$ on a line passing though $(a,1,2)$ is $(1,3,5)$, then 'a' is equal to ….

1. $\frac{17}{2}$ 2. 17 3. 4 4. 2

Key : 1

Sol : d.r's of $PQ=(1+1,3-0,5-2)=\begin{pmatrix}2, & 3, & 3\\ a_1, & b_1, & c_1\end{pmatrix}$

$d.r's\,of\ AQ=(1-a,3-1,5-2)$

$PQ\perp AQ\Rightarrow a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 2(1-a)+3(2)+3(3)=0$

$\Rightarrow 2-2a+6+9=0\Rightarrow 17-2a=0\Rightarrow 17-2a\Rightarrow a=\frac{17}{2}$

10a. (2022 June) Let the foot of the perpendicular from the point $(1,2,4)$ on the line $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}$ be P. Then, the distance of P from the plane $3x+4y+12z+23=0\,is$

1. 5 2. $\frac{50}{13}$ 3. 4 4. $\frac{63}{13}$

Key :1

Sol : Direction ratios of PQ $3-4\lambda,1-2\lambda,5-3\lambda$ Direction ratios of PR 4,2,3

$\because PQ\perp PR.$

$\therefore 4(3-4\lambda)+2(1-2\lambda)+3(5-3\lambda)=0\Rightarrow 29-29\lambda=0\Rightarrow\lambda=1$

$\therefore P\equiv(2,3,2)$

Now, required distance $=\frac{|3\times2+4\times3+12\times2+23|}{\sqrt{3^2+4^2+(12)^2}}=\frac{65}{13}=5$

10b. Let the foot of the perpendicular from the point $(1,3,2)$ on the line $\frac{x+2}{1}=\frac{y-1}{2}=\frac{z+1}{1}\,be\,P.$ Then the distance of P from the plane $6x+3y+3z-2=0\,is$

1. $\frac{11}{\sqrt{54}}$ 2. 11 3. $\sqrt{54}$ 4. $\frac{\sqrt{54}}{3}$

Key : 1

Sol : Given line is $\frac{x+2}{1}=\frac{y-1}{2}=\frac{z+1}{1}=K---(1)$

Any point P on the line (1) is

$P=(K-2,2k+1,K-1)$

Dor;s of $PQ=(k-3,2k-2,k-3)$

Dor's of $(1)=(1,2,1)$

$\therefore PQ\perp L\Rightarrow(k-3)+2(2k-2)+(k-3)=0$

$K-3+4K-4+K-3=0 \Rightarrow 6k-10=0 \Rightarrow K=\frac{10}{6}=\frac{5}{3}$

$\therefore P=\left(\frac{5}{3}-2, 2\left(\frac{5}{3}\right)+1, \frac{5}{3}-1\right)$

$P=\left(\frac{-1}{3}, \frac{13}{3}, \frac{2}{3}\right)$

The perpendicular distance from $P\left(\frac{-1}{3}, \frac{13}{3}, \frac{2}{3}\right) to$

The plane $6x+3y+3z-2=0 is \left|\frac{6\left(\frac{-1}{3}\right)+3\left(\frac{13}{3}\right)+3\left(\frac{2}{3}\right)-2}{\sqrt{36+9+9}}\right|=\left|\frac{-2+13+2-2}{\sqrt{54}}\right|=\frac{11}{\sqrt{54}}$

Sub topic:- Image of a point in a line in 3D

11a. (sep-2020) If (a,b,c) is the image of the point $(1,2,-3)$ in the line $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$, then

$a+b+c=$

1. 3 2. 2 3. -1 4. 1

Key : 2

Sol : Given line is

$\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}=\lambda(Let)--(1)$

Any point on (1) is

$R=(2\lambda-1, -2\lambda+3, -\lambda)$

d.R's of PR $=(2\lambda-2, -2\lambda+1, -\lambda+3)$

d.r.f$(1)=(2,-2,-1)$

$PR \perp$ given line $\Rightarrow a_1a_2+b_1b_1+c_1c_2=0$

$\Rightarrow 2(2\lambda-2)$

$-2(1-2\lambda)-1(3-\lambda)=0$

$4\lambda-4-2+4\lambda-3+\lambda=0$

$9\lambda-9=0$

$\lambda=1$

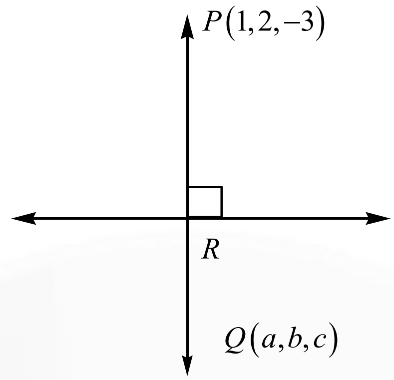
$\therefore R=(1,1,-1)$

$R=$mind point of PQ

$R=\frac{P+Q}{2} \Rightarrow Q=2R-P$

$(a,b,c)=(2,2,-1)-(1,2,-3)=(1,0,1)$

$\therefore a+b+c=2$



11b. If (α,β,γ) is the image of the point $(0,1,-2)$ in the line $\frac{x-1}{-2}=\frac{y-2}{1}=\frac{z+1}{0}$, then $5(\alpha+\beta+\gamma)$ is

equal to

1. 23 2. 20 3. 40 4. 60

Key : 1

Sol : Given line is

$$\frac{x-1}{-2}=\frac{y-2}{1}=\frac{z+1}{0}=\lambda(let)---(1)$$

Any point on the line (1) is R

$$\therefore R=(-2\lambda+1,\lambda+2,-1)$$

d.r's of PR $=(1-2\lambda,\lambda+1,1)$

$$d.r's\, if\,(1)=(-2,1,0)$$

$$PR\perp(1)\Rightarrow a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow -2(1-2\lambda)+(1+\lambda)+0(1)=0\Rightarrow -2+4\lambda+1+\lambda=0\Rightarrow \lambda=\frac{1}{5}$$

$$\therefore R=\left(1-2\left(\frac{1}{5}\right),2+\frac{1}{5},-1\right)$$

$$=\left(\frac{5-2}{5},\frac{10+1}{5},-1\right)=\left(\frac{3}{5},\frac{11}{5},-1\right)$$

$\therefore R=$ mid point of $PQ\,(\because Q\, is\, image\, of\, P)$

$$R=\frac{P+Q}{2}\Rightarrow Q=2R-P$$

$$\Rightarrow Q=\left(\frac{6}{5},\frac{22}{5},-2\right)-(0,1,-2)=\left(\frac{6}{5}-0,\frac{22}{5}-1,-2+2\right)=\left(\frac{6}{5},\frac{22-5}{5},0\right)$$

$$(\alpha,\beta,\gamma)=\left(\frac{6}{5},\frac{17}{5},0\right)$$

$$\alpha+\beta+\gamma=\frac{6}{5}+\frac{17}{5}+0=\frac{23}{5}\Rightarrow 5(\alpha+\beta+\gamma)=23.$$

12a. (2022 june) Let the image of the point $P(1,2,3)$ in the line $L:\frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ be Q Let $R(\alpha,\beta,\gamma)$ be a point that divides internally the line segment PQ in the ratio $1:3$. Then, the value of $22(\alpha+\beta+\gamma)$ is equal to……

1. 125 2. 130 3. 4 4. 120

Key : 1

Sol : The point dividing PQ in the ratio 1:3 will be mid-point of P and foot of perpendicular from P on the line.

$\therefore$ Let a point on line be P^1

$$\Rightarrow \frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda \Rightarrow P^1(3\lambda+6,2\lambda+1,3\lambda+2)$$

As P^1 is foot of perpendicular.

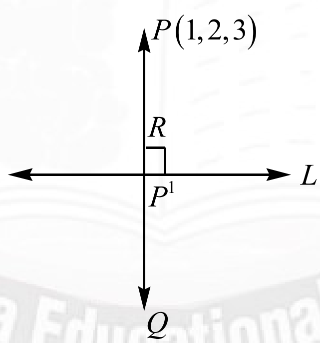
$$\therefore (3\lambda+5)3+(2\lambda-1)2+(3\lambda-1)3=0$$

$$\Rightarrow 22\lambda+15-2-3=0 \Rightarrow \lambda=\frac{-5}{11}$$

$$\therefore P^1\left(\frac{51}{11},\frac{1}{11},\frac{7}{11}\right)$$

Mid point of $PP^1=\left(\frac{\frac{51}{11}+1}{2},\frac{\frac{1}{11}+2}{2},\frac{\frac{7}{11}+3}{2}\right)=\left(\frac{62}{22},\frac{23}{22},\frac{40}{22}\right)\equiv(\alpha,\beta,\gamma)$

$$22(\alpha+\beta+\gamma)=62+23+40=125$$



12b. Let the image of the point $P(1,3,2)$ in the line $L:\frac{x-1}{1}=\frac{y-6}{3}=\frac{z+2}{2}$ $be\,Q$. Let $R(a,b,c)$ be a point that divides internally the line segment PQ in the ratio 1:2. Then the value of $21(a+b+c)$ is equal to

1.100 2. 200 3. 300 4. 106

Key : 4

Sol : Let M be the foot of the perpendicular from P on the line L

$\therefore \frac{x-1}{1}=\frac{y-6}{3}=\frac{z+2}{2}=\lambda \Rightarrow M=(\lambda+1,3\lambda+6,2\lambda-2)$

d.r's PM $(\lambda,3\lambda+3,2\lambda-4)$

$PM \perp L \Rightarrow 1(\lambda)+3(3\lambda+3)+2(2\lambda-4)=0$

$\Rightarrow \lambda+9\lambda+9+4\lambda-8=0 \Rightarrow 14\lambda+1=0 \Rightarrow \lambda=-\frac{1}{14}$

$\therefore M=\left(\frac{-1}{14}+1,\frac{-3}{14}+6,\frac{-2}{14}-2\right)=\left(\frac{13}{14},\frac{84-3}{14},\frac{-30}{14}\right)=\left(\frac{13}{14},\frac{81}{14},\frac{-30}{14}\right)$

$M=\frac{P+Q}{2} \Rightarrow Q=2M-P$

$\Rightarrow Q=\left(\frac{13}{7},\frac{81}{7},\frac{-30}{7}\right)-(1,3,2)=\left(\frac{13}{7}-1,\frac{81}{7}-3,\frac{-30}{7}-2\right)$

$Q=\left(\frac{6}{7},\frac{60}{7},\frac{-44}{7}\right)$

$R=\frac{1Q+2P}{1+2}$ $\quad \overset{\quad 1 \quad\quad 2}{P(1,3,2)\;R\quad\quad Q}$

$R=\frac{\left(\frac{6}{7},\frac{60}{7},\frac{-44}{7}\right)+(2,6,4)}{3}=\frac{1}{3}\left(\frac{6}{7}+2,\frac{60}{7}+6,\frac{-44}{7}+4\right)$

$R=\frac{1}{3}\left(\frac{20}{7},\frac{102}{7},\frac{-16}{7}\right)$

$R=\frac{1}{21}(20,102,-16)=(a,b,c)$

$21(a+b+c)=21\left(\frac{20}{21}+\frac{102}{21}-\frac{16}{21}\right)=20+102-16=106$

13a. (2021 Feb) Let $a,b \in R.$ if the mirror image of the point $P(a,6,9)$ with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9} is (20,b,-a-9)$ then $|a+b|$ is equal to

1. 88 $\quad$ 2. 86 $\quad$ 3. 84 $\quad$ 4. 90

Key : 1

Sol : Given, $P(a,6,9)$

Equation of line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$

Image of point P with respect to line $Q(20,b,-a-9)$ midpoint of P and

$Q=\left(\frac{a+20}{2},\frac{6+b}{2},\frac{-a}{2}\right)$

This point lies on line

$$\therefore \frac{\frac{a+20}{2}-3}{7}=\frac{\frac{6+b}{2}-2}{5}=\frac{\frac{-a}{2}-1}{-9}\Rightarrow \frac{a+14}{14}=\frac{b+2}{10}=\frac{a+2}{18}$$

$$\Rightarrow \frac{a+14}{14}=\frac{a+2}{18}\,and\,\frac{b+2}{10}=\frac{a+2}{18}$$

Solving, we get $a=-56, b=-32$

$a=-56, b=-32$

$\therefore |a+b|=88$

13b. Let $\alpha,\beta \in R.$ If the mirror image of the point $P(\alpha,2,3)$ w.r.t the line $\frac{(x-2)}{3}=\frac{(y-1)}{1}=\frac{(z-3)}{5}$ $is\,(10,\beta-9,\alpha)\,then\,|\alpha+\beta|$ is equal to

1. 15 2. 86 3. 84 4. 90

Key : 1

Sol : M = Midpoint of PQ

$$M=\left(\frac{\alpha+10}{2},\frac{\beta-7}{2},\frac{3+\alpha}{2}\right)$$

$\therefore$ M line on the line L

$$\Rightarrow \frac{\frac{\alpha+10}{2}-2}{3}=\frac{\frac{\beta-7}{2}-1}{1}=\frac{\frac{3+\alpha}{2}-3}{5}$$

$$\Rightarrow \frac{\alpha+6}{6}=\frac{\beta-9}{2}=\frac{\alpha-3}{10}$$

$$\Rightarrow \frac{\alpha+6}{6}=\frac{\alpha-3}{10}$$

$\Rightarrow 5\alpha+30=3\alpha-9$

$\Rightarrow 2\alpha=-39$

$$\Rightarrow \alpha=\frac{-39}{2}$$

$$\frac{\beta-9}{2}=\frac{\alpha-3}{10}$$

$5\beta-45=\alpha-3$

$$5\beta=\frac{-39}{2}-3+45$$

$$5\beta=\frac{-39}{2}+42=\frac{-39+84}{2}=\frac{45}{2}$$

$$5\beta=\frac{45}{2}\Rightarrow \beta=\frac{9}{2}$$

$$\therefore \alpha+\beta=\frac{-39}{2}+\frac{9}{2}=\frac{-30}{2}=-15$$

$|\alpha+\beta|=15$

Sub topic:- perpendicular distance of a point from a line in 3D

14a. (2019 April) The length of the perpendicular from the point $(2,-1,4)$ on the straight line,

$\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is

1. Greater than 3 but less than 4.
2. Less than 2
3. Greater than 2 but less than 3.
4. Greater than 4.

Key : 1

Sol : Equation of given line is

$$\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}=r(let).....(i)$$

Coordinates of a point on line (i) is $P(10r-3,-7r+2,r)$

$dr's\ of\ AP=(10r-5,3-7r,r-4)$

$AP\perp L$

$\Rightarrow 10(10r-5)-7(3-7r)+(r-4)=0$

$\Rightarrow 100r-50-21+49r+r-4=0$

$\Rightarrow 150r=75\Rightarrow r=1/2$

So, that foot of perpendicular is $P\left(2,-\frac{3}{2},\frac{1}{2}\right)$

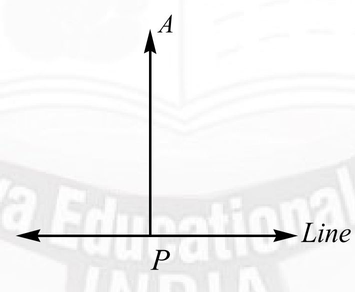
[Put $r=\frac{1}{2}$ in the coordinates of point P]

Now, perpendicular distance of point $A(2,-1,4)$ from the line (i) is

$$PA=\sqrt{(2-2)^2+\left(-\frac{3}{2}+1\right)^2+\left(\frac{1}{2}-4\right)^2}$$

$$=\sqrt{\frac{1}{4}+\frac{49}{4}}=\sqrt{\frac{50}{4}}=\frac{5}{\sqrt{2}}$$

Which lies in $(3,4)$



14b. The length of the perpendicular of from the point $(1,2,-1)$ on the straight line,

$\frac{x-2}{1}=\frac{y-1}{3}=\frac{z}{2}$ is

1. Grater then 3 be less than 4
2. Less than 2
3. Greater than 2 but less than 3
4. Greater than 4

Key : 2

Sol : $L:\frac{x-2}{1}=\frac{y-1}{3}=\frac{z}{2}=r(say)$

Any point of on the line L is

$P=(r+2,3r+1,2r)$

$dr's\,of\;\;AP=(r+1,3r-1,2r+1)$

$AP\perp L\Rightarrow 1(r+1)+3(3r-1)+2(2r+1)=0$

$\Rightarrow r+1+9r-3+4r+2=0$

$\Rightarrow 14r+3-3=0\Rightarrow r=0$

$\therefore P(2,1,0)$

$PA=\sqrt{(2-1)^2+(1-2)^2+(0+1)^2}=\sqrt{1+1+1}=\sqrt{3}$

Sub topic:- Angle between two lines in 3D

15a. (2022 June) If the two lines $l_1:\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-2}{0}\,and\,l_2:\frac{x-1}{1}=\frac{2y+3}{\alpha}=\frac{z+5}{2}$ perpendicular, then an angle between the lines l_2 and $l_3:\frac{1-x}{3}=\frac{2y-1}{-4}=\frac{z}{4}is$

1. $\cos^{-1}\left(\frac{29}{4}\right)$ 2. $\sec^{-1}\left(\frac{29}{4}\right)$ 3. $\cos^{-1}\left(\frac{2}{29}\right)$ 4. $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Key : 2

Sol : $L_1:\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-2}{0}$

$a_1=3,b_1=-2,c_1=0$

$$L_2:\frac{x-1}{1}=\frac{y+\left(\frac{3}{2}\right)}{\left(\frac{\alpha}{2}\right)}=\frac{z+5}{2}$$

$a_2=1,b_2=\frac{\alpha}{2},c_2=2$

Given, that L_1 perpendicular to L_2.

$\therefore a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 3-\alpha+0=0$

$\Rightarrow \alpha=3$

$\Rightarrow a_2=1,b_2=\frac{3}{2},c_2=2$

Now, $L_3:\frac{x-1}{-3}=\frac{y-\left(\frac{1}{2}\right)}{-2}=\frac{z-0}{4}$

$a_3=-3,b_3=-2,c_3=4$

Angle between $L_2\,and\,L_3$ is given by,

$$\cos\theta = \frac{|a_2a_3 + b_2b_3 + c_2c_3|}{\sqrt{a_2^2 + b_2^2 + c_2^2}\sqrt{a_3^2 + b_3^2 + c_3^2}}$$

$$= \frac{|-3-3+8|}{\sqrt{1+\frac{9}{4}+4\sqrt{9+4+16}}}$$

$$\cos\theta = \frac{4}{\sqrt{29}\sqrt{29}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{29}\right)$$

$$\Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

15b. If the two lines $L_1 : \frac{x-2}{3} = \frac{y-1}{2} = \frac{z-0}{-1} \; and \; L_2 : \frac{x+1}{1} = \frac{2y-3}{\alpha} = \frac{z-5}{2}$ are perpendicular, then the angle between the lines $L_2 \; \& \; L_3 : \frac{1-x}{3} = \frac{3y-1}{-6} = \frac{z}{1}$ is

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{6}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{3}$

Key : 3

Sol : For $l_1 : a_1 = 3, b_1 = 2, c_1 = -1 \; For \, l_2 : a_2 = 1, b_2 = \frac{\alpha}{2}, c_2 = 2$

Given that l_1 is perpendicular to l_2

$$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow 3 + 2\left(\frac{\alpha}{2}\right) - 2 = 0$$

$$\Rightarrow \alpha + 120 \Rightarrow \alpha = -1$$

$$\therefore l_2 : \frac{x+1}{1} = \frac{y-\frac{3}{2}}{-\frac{1}{2}} = \frac{z-5}{2}, l_3 : \frac{x-1}{-3} = \frac{y-\frac{1}{3}}{-2} = \frac{z}{1}$$

For $l_3 : a_3 = -3, b_3 = 2, c_3 = 1$

Angle between $l_2 \; and \; l_3$ is given by

$$\cos\theta = \frac{|a_3a_2 + b_3b_2 + c_3c_2|}{\sqrt{a_3^2 + b_3^2 + c_3^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{\left|-3 + \left(-\frac{1}{2}\right)(-2) + 2\right|}{\sqrt{1+\frac{1}{4}+4\sqrt{9+4+1}}}$$

$$= \frac{|-3+1+2|}{\sqrt{5+\frac{1}{4}\sqrt{14}}} = 0 \Rightarrow \theta = \frac{\pi}{2}$$

16a. (2019 Jan) If lines $x = ay + b, z = cy + d \; and x = a^1z + b^1, y = c^1z + d^1$ are perpendicular, then

1. $ab^{'} + bc^{'} + 1 = 0$ 2. $bb^{'} + cc^{'} + cc^{'} + 1 = 0$ 3. $aa^{'} + c^{'} + c^{'} = 0$ 4. $cc^{'} + a + a^{'} = 0$

Key :3

Sol : Let 1st line $x = ay + b, z = cy + d$.

$\Rightarrow \frac{x-b}{a} = y, \frac{z-d}{c} = y$

$\Rightarrow \frac{x-b}{a} = y = \frac{z-d}{c} ----(1)$

Let 2nd line is $x = a^{'}z + b^{'}, y = c^{'}z + d^{'}$

$\Rightarrow \frac{x-b^{'}}{a^{'}} = z, \frac{y-d^{'}}{c^{'}} = z \Rightarrow \frac{x-b^{'}}{a} = \frac{y-d^{'}}{c^{'}} = z ---(2)$

(1)And (2) are perpendicular then $aa^1 + c^1 + c = 0$

16b. If the lines $x = py + q, z = ry + s \; and \; x = p^1z + q^1, y = r^1z + s^1 \; are$ Perpendicular, then

1. $pq^1 + qr^1 + 1 = 0$ 2. $qq^1 + rr^1 + 1 = 0$ 3. $pp^1 + r + r^1 = 0$ 4. $rr^1 + p + p^1 = 0$

Key : 3

Sol : 1st line is $x = py + q, z = ry + s$

$\frac{x-q}{p} = y, \frac{z-s}{r} = y$

$\Rightarrow \frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r} ----(i)$

2nd line is $x = p^1z + q^1, y = r^1z + s^1$

$\frac{x-q^1}{p^1} = z \quad \frac{y-s^1}{r^1} = z$

$\therefore \frac{r-q^1}{p^1} = \frac{y-s^1}{r^1} = \frac{z}{1} ---(2)$

$\therefore d.r'r \; of (i) = (p,1,r)$

$d.r's \; f(2) = (P^1, r^1, 1)$

$\therefore$ The two lines are perpendicular

$\Rightarrow PP^1 + r^1 + r = 0$

Sub topic:- Angle between the line and a plane in 3D

17a. (2019 Jan) If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$

is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ then the value of K is

1. $\sqrt{\frac{5}{3}}$ 2. $\sqrt{\frac{3}{5}}$ 3. $\frac{-3}{5}$ 4. $\frac{-5}{3}$

Key : 1

Sol : d.r's of given line are $\begin{pmatrix} 2,1,-2 \\ a_1,b_1,c_1 \end{pmatrix}$ d.r's of normal to the plane are $\begin{pmatrix} 1,-2,-k \\ a_2,b_2,c_2 \end{pmatrix}$

$\therefore$ Let $'\theta'$ be the angle between line and plane

$$\therefore \sin\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\sin\theta = \frac{|2-2+2k|}{\sqrt{4+1+4}\sqrt{1+4+k^2}} = \frac{|2K|}{\sqrt{9}\sqrt{5+K^2}}$$

$$\sin\theta = \frac{2|K|}{3\sqrt{5+K^2}}$$

$$\therefore \frac{1}{3} = \frac{2|K|}{3\sqrt{5+K^2}}$$

S.O.b.S

$$5 + K^2 = 4K^2$$

$$5 = 3K^2 \Rightarrow K^2 = \frac{5}{3}$$

$$\Rightarrow K = \pm\sqrt{\frac{5}{3}}$$

Given

$$\theta = \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

$$\sin\theta = \sqrt{1-\frac{8}{9}} = \frac{1}{3}$$

17b. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$, the value of λ is

1. $\frac{5}{3}$ 2. $\frac{-3}{5}$ 3. $\frac{3}{4}$ 4. $\frac{-4}{3}$

Key : 1

Sol : d.r's of given line $(1,2,2) = (a_1, b, c_1)$

d.r's normal to the plane are $\left(2,-1,\sqrt{\lambda}\right) = (a_2, b_2, c_2)$

Let $'\theta'$ be the angle between line and the plane

$\therefore \sin\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\frac{1}{3} = \frac{|2 - 2 + 2\sqrt{\lambda}|}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}$

$\frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$

$\frac{1}{9} = \frac{4\lambda}{9(5+\lambda)}$

$5 + \lambda = 4\lambda \Rightarrow 5 = 3\lambda$

$\Rightarrow \lambda = \frac{5}{3}$

18. (2023 Apr) For $a_1, b \in z$ and $|a-b| \le 10,$ let the angle between the plane $P: ax + y - z = b$ and the line $l: x - 1 = a - y = z + 1 be \cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point $(6,-6,4)$ from the plane P is $3\sqrt{6},$ the $a^4 + b^2$ is equal to

1. 25 2. 85 3. 48 4. 32

Key : 4

Sol : Plane P: $ax + y - z = b, line\, l: \frac{x-1}{1} = \frac{y-a}{-1} = \frac{z-(-1)}{1}$

Angle between the plane and line is

$\sin\theta = \frac{|al + bm + cn|}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$ $\quad$ $given\ \cos\theta = \frac{1}{3}$

$\Rightarrow \frac{\sqrt{8}}{3} = \frac{|a-1-1|}{\sqrt{a^2+1+1}\sqrt{1+1+1}}$ $\quad$ $\sin\theta = \sqrt{1 - \frac{1}{9}}$

$\Rightarrow \frac{\sqrt{8}}{3} = \frac{|a-2|}{\sqrt{a^2+2}.\sqrt{3}}$ $\quad$ $= \sqrt{\frac{8}{9}}$

$\Rightarrow \frac{|a-2|}{\sqrt{a^2+2}} = \sqrt{\frac{8}{3}} -----(1)$

The distance of the point $(6,-6,4)$ from the plane $P = 3\sqrt{6}$

$\Rightarrow \frac{|6a - 6 - 4 - b|}{\sqrt{a^2+2}} = 3\sqrt{6}$

$\Rightarrow \frac{|6a - b - 10|}{\sqrt{a^2+2}} = 3\sqrt{6} ---(2)$

From (1) and (2) $a = -2, b = -4$

$\therefore\ a^4 + b^2 = 16 + 6 = 32$

Sub topic:- Equation of the plane through two lines in 3D

19a. (2022 July) Let the lines $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}\,and\,\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then, which of the following points does not lie on P?

1. $(0,-2,-2)$ 2. $(-5,0,-1)$ 3. $(3,-1,0)$ 4. $(0,4,5)$

Key : 4

Sol : Given ,$\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}---(1)\,and\,\frac{x-26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}--(2)$

Given that (1) and (2) coplanar then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0,\Rightarrow \begin{vmatrix} 1+26 & 2+18 & 3+28 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{vmatrix}=0$$

$\Rightarrow 27(\lambda-6)-20(\lambda^2+4)+31(3\lambda+2)=0$

$\Rightarrow 27\lambda-162-20\lambda^2-80+93\lambda+62=0$

$\Rightarrow -20^2\lambda+120\lambda-180=0$

$\Rightarrow \lambda^2-6\lambda+9=0$

$\Rightarrow (\lambda-3)^2=0 \Rightarrow \lambda=3$

Equation of the plane containing (1) and (2) is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{vmatrix}=0$$

$\Rightarrow 3x+13y-11z+4=0$

From the options.

$(0,-2,-2)$:

$3(0)+13(-2)-11(-2)+4=0$

$(3,-1,0):3(3)+13(-1)-11(0)+4=0$

$(0,4,5):3(0)+13(4)-11(5)+4\neq 0$

$\therefore (0,4,5)$ point does not lie in the plane.

19b. Let the lines $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}\,and\,\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ be coplanar. The equation of the plane containing these two lines is

1. $x+y-z+1=0$ 2. $x+y-z+1=0$ 3. $3x+13y-11z+4=0$ 4. $x-2y+z-4=0$

Key : 3

Sol : Given lines

$\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}---(1)$ and $\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda} are---(2)$

Coplanar $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2` & c_2 \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} -27 & -20 & -31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{vmatrix}=0$

$\Rightarrow \lambda=3.$

Equation of the plane counting (1) and (2) is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$

$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{vmatrix}=0$

$(x-1)(-3)-(y-2)(13)+(z-3)(11)=0$

$-3x+3-13y+26+11z-33=0$

$\Rightarrow 3x+13y-11z+4=0$

20a. (2021 Aug) Let Q be the foot the perpendicular from the point $P(7,-2,13)$ on the plane containing the

Lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8} and \frac{x+1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$. Then, $(PQ)^2$ is equal to

1. 96 2. 106 3. 46 4. 90

Key : 96

Sol : Plane containing the lines would be

$\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}=0$

$\Rightarrow (x+1)(49-40)-(y-1)(42-24)+(z-3)(30-21)=0$

$\Rightarrow 9(x+1)-18(y-1)+9(z-3)=0$

$\Rightarrow x-2y+z=0$

Now PQ will be equal to the perpendicular distance of the point $P(7,-2,13)$ from the plane $x-2y+z=0$

$\therefore PQ=\left|\frac{7-2(-2)+13}{\sqrt{1^2+(-2)^2+(1)^2}}\right|$

$=\left|\frac{7+4+13}{\sqrt{1+4+1}}\right|=\left|\frac{24}{\sqrt{6}}\right|=4\sqrt{6}$

$PQ^2 = \left(4\sqrt{6}\right)^2 = 16\times 6 = 96$

20b. Let Q be the foot of the perpendicular from the point $P(1,-,2,3)$ on the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{5}=\frac{z-3}{4}\, and\, \frac{x-0}{1}=\frac{y-1}{3}=\frac{z-2}{1}$ then $27(PQ)^2=$

1.32 2. 40 3. 90 4. 2

Key : 1

Sol : Equation of the plane containing given lines is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 5 & 4 \\ 1 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow (x-1)(5-12)-(y-2)(2-4)+(z-3)(6-5)=0$

$\Rightarrow (x-1)(-7)-(y-2)(-2)+(z-3)(1)=0$

$\Rightarrow -7x+2y+z=0$

$\Rightarrow 7x-2y-z=0$

$PQ =$ Perpendicular distance of the point P from the plane $7x-2y-z=0$

$$\therefore PQ = \left|\frac{7(1)-2(-2)-3}{\sqrt{7^2+(-2)^2+(-1)^2}}\right| = \frac{|7+4-3|}{\sqrt{49+4+1}}$$

$$PQ = \frac{8}{\sqrt{54}}$$

$$\therefore (PQ)^2 = \frac{64}{54} = \frac{32}{27}$$

$$27(PQ)^2 = 32$$

Sub topic:- Shortest distance between two lines in 3D

21a. (june-2022) The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$ is

1. $\frac{18}{\sqrt{5}}$ 2. $\frac{22}{3\sqrt{5}}$ 3. $\frac{46}{3\sqrt{5}}$ 4. $6\sqrt{3}$

Key : 1

Sol : Vectors form of the given equation are

$r=\left(3\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-\hat{k}\right)$ and $r=\left(-3\hat{i}+6\hat{j}+5\hat{k}\right)+\mu\left(2\hat{i}+\hat{j}+3\hat{k}\right)$

$\bar{r}=\bar{a}+\lambda\bar{b}\, and\, \bar{r}=\bar{c}+\mu\bar{d}$

$\therefore$ Shortest distance between the two lines is $= \frac{\left|\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|} = \frac{18}{\sqrt{5}}$

21b. The shortest distance between the lines $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-1}{2}, \frac{x-4}{4}=\frac{y-5}{4}=\frac{z-2}{3} is$

1. $\frac{1}{\sqrt{6}}$ 2. $\frac{7}{\sqrt{6}}$ 2. $\sqrt{6}$ 3. $\frac{11}{\sqrt{6}}$

Key: 1

Sol: Vector form of given equations are

$$\bar{r}=\left(2\bar{i}+3\bar{j}+\bar{k}\right)+\lambda\left(3i+4j+2k\right)\Rightarrow \bar{r}=\bar{a}+\lambda\bar{b}$$

and $\bar{r}=\left(4\bar{i}+5\bar{j}+2\bar{k}\right)+\mu\left(4i+5j+3k\right)\Rightarrow \bar{r}=\bar{c}+\mu\bar{d}$

$\therefore$ Shortest distance between the two lines $=\frac{\left|\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}$

$$\therefore\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]=\begin{vmatrix}-2 & -2 & -1\\ 3 & 4 & 2\\ 4 & 5 & 3\end{vmatrix}=-1$$

$$\bar{b}\times\bar{d}=\begin{vmatrix}i & j & k\\ 3 & 4 & 2\\ 4 & 5 & 3\end{vmatrix}=i\left(2\right)-j\left(1\right)+k\left(-1\right)=2i-j-k$$

$$\therefore\left|\bar{b}\times\bar{d}\right|=\sqrt{4+1+1}=\sqrt{6}$$

$$S.D=\frac{1}{\sqrt{6}}$$

22a. (June -2022) If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda} and \frac{x-2}{2}=\frac{y-4}{4}=\frac{z-5}{5} is \frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is

1. 16 2. 6 3.16 4.15

Key : 1

Sol : $Let\,\bar{a}=\hat{i}+2\hat{j}+3\hat{k}\, and\, \bar{c}=2\hat{i}+4\hat{j}+5\hat{k}$

$\bar{b}=2\hat{i}+3\hat{j}+\lambda\hat{k}, \bar{d}=\hat{i}+4\hat{j}+5\hat{k}$

$\therefore\bar{b}\times\bar{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & \lambda\\ 1 & 4 & 5\end{vmatrix}=\hat{i}\left(15-4\lambda\right)-\hat{j}\left(10-\lambda\right)+5\hat{k}$ and $\bar{a}-\bar{c}=\bar{i}+2\bar{j}+2\hat{k}$

$\therefore$ Shortest distance $\frac{\left|\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}=\frac{1}{\sqrt{3}}$

Given $=\left|\frac{-2\lambda+5}{\sqrt{\left(15-4\lambda\right)^2+\left(10-\lambda\right)^2+25}}\right|=\frac{1}{\sqrt{3}}$

On squaring both sides, we get

$$3(5-2\lambda)^2=(15-4\lambda)^2+(10-\lambda)^2+25\Rightarrow 5\lambda^2-80\lambda+275=0\Rightarrow \lambda^2-16\lambda+55=0$$

Sum of the value of λ is 16.

22b. If the shortest distance between the lines

$$\frac{x-1}{\lambda}=\frac{y-2}{3}=\frac{z-3}{2}\,and\,\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-5}{4}\,is\,\frac{1}{\sqrt{3}}$$

Then sum of all possible values of λ is

1. $\frac{20}{29}$ 2. $\frac{30}{29}$ 3. $\frac{46}{29}$ 4. $\frac{6}{29}$

Key: 3

Sol: Vectors of given equations are

$$\bar{r}=\bar{a}+\lambda\bar{b}\Rightarrow \bar{a}=(1,2,3),\bar{b}=(\lambda,3,2)$$

$$\bar{r}=\bar{c}+\mu\bar{d}=\bar{c}=(2,4,5),\bar{d}=(1,5,4)$$

Now $\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]=\begin{vmatrix}-1 & -2 & -2\\ \lambda & 3 & 2\\ 1 & 5 & 4\end{vmatrix}=-1(12-10)+2(4\lambda-2)-2(5\lambda-3)$

$$=-2+8\lambda-4-10\lambda+6$$

$$=-2\lambda$$

$$\bar{b}\times\bar{d}=\begin{vmatrix}i & j & k\\ \lambda & 3 & 2\\ 1 & 5 & 4\end{vmatrix}=i(2)-j(4\lambda-2)+k(5\lambda-3)=2i-2j(2\lambda-1)+k(5\lambda-3)$$

$$\left|\bar{b}\times\bar{d}\right|=\sqrt{4+4(2\lambda-1)^2+(5\lambda-3)^2}$$

$$=\sqrt{41\lambda^2-46\lambda+17}$$

S.D $=\frac{\left|\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}=\frac{2\lambda}{\sqrt{41\lambda^2-46\lambda+17}}$

$$\therefore \frac{2\lambda}{\sqrt{41\lambda^2-46\lambda+17}}=\frac{1}{\sqrt{3}}(Given)$$

S.O.B.S

$$\frac{4\lambda^2}{41\lambda^2-46\lambda+17}=\frac{1}{3}\Rightarrow 12\lambda^2=41\lambda^2-46\lambda+17$$

$$\Rightarrow 29\lambda^2-46\lambda+17=0$$

23a. (2023 Jan) The line l_1 passes through the point $(2,6,2)$ and is perpendicular to plane $2x+y-2z=10$. Then the shortest distance between the l_1 and the line $\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$ is

1. 7 2. $\frac{13}{3}$ 3. 9 4. $\frac{19}{3}$

Key : 3

Sol : $l_1 : \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

$$S.D = \frac{\begin{vmatrix} 3 & 10 & 2 \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix}}{\sqrt{64+16+64}} = \frac{|-12-80-16|}{12} = \frac{108}{12} = 9.$$

$$S.D = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\sum (b_1c_2-b_2c_1)^2}}$$

23b. The line l_1 passes through the point $(1,2,1)$ and is perpendicular to plane $x+2y-z=5$

Then the shortest distance between the l_1 and the line $l_2 : \frac{x+4}{2} = \frac{y+1}{2} = \frac{z}{-3}$

1.7 2. $\frac{13}{\sqrt{3}}$ 3. $\frac{19}{\sqrt{3}}$ 4. $\frac{19}{\sqrt{21}}$

Key : 4

Sol : $l_1 : \frac{x-1}{1} = \frac{(y-2)}{2} = \frac{z-1}{-1}, l_2 : \frac{x+4}{2} = \frac{y+1}{2} = \frac{z}{-3}$

$$S.D = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\sum (b_1c_2-b_2c_1)^2}}$$

$$= \frac{\begin{vmatrix} -5 & -3 & -1 \\ 1 & 2 & -1 \\ 2 & 2 & -3 \end{vmatrix}}{\sqrt{16+4}}$$

$$= \frac{|-5(-6+2)+3(-3+2)-1(2-4)|}{\sqrt{21}}$$

$$= \frac{20-3+2}{\sqrt{21}} = \frac{19}{\sqrt{21}}$$

From l_1 :

$(x_1, y_1, z_1) = (1,2,1)$

$(a_1, b_1, c_1) = (1,2,-1)$

From l_2 : $(x_2, y_2, z_2) = (-4,-3,0)$

$(a_2, b_2, c_2) = (2,2-3)$

$$\bar{b}\times\bar{d}=\begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 2 & 2 & -3 \end{vmatrix}$$

$$=i(-6+2)-j(-3+2)+k(2-4)$$

$$=i(-4)-j(-1)+k(-2)$$

$$\left|\bar{b}\times\bar{d}\right|=\sqrt{16+1+4}=\sqrt{21}$$

Sub topic:- Coplanar lines

24a. (2023 Apr) The line, that is coplanar to the line $\frac{x+3}{-3}=\frac{y+1}{1}=\frac{z-5}{5}, is$

1. $\frac{x-1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$
2. $\frac{x+1}{1}=\frac{y-2}{2}=\frac{z-5}{5}$
3. $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{4}$
4. $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$

Key : 4

Sol : $\begin{vmatrix} -3+1 & 1-2 & 5-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}=\begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}=0$

The lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$$

$$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}\, are\, coplanar$$

$$\therefore\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

24b. The line, that is coplanar to the line $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-2}{3}$, is

1. $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{4}$
2. $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-1}{2}$
3. $\frac{x-3}{1}=\frac{y-2}{2}=\frac{z-1}{3}$
4. $\frac{x-1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$

Key : 1

Sol : The lines $\frac{x-x_1}{a}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1};\frac{x-x_2}{a_2};\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$

are coplanar $\Rightarrow \begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

$\therefore \begin{vmatrix} -1 & 1 & 1 \\ 3 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = -1(8-9)-1(12-6)+1(9-4) = 1-6+5 = 0$

25a. (2023 Apr) Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{2}$ and $l_2 : 3x+2y+z-2=0=x-3y+2z-13$ be coplanar if the point $P(a,b,c)$ on l_1 is nearest to the point $Q(-4,-,3,2)$, then $|a|+|b|+|c| =$

1.14   2. 10   3. 8   4. 12

Key : 2

Sol : Plane passing through L_2 is of the form

$(3x+2y+z-2)+\lambda(x-3y+2z-13)=0$

L_1 lies on above plane then,

$(3+\lambda)3+(2-3\lambda)(-2)+(-2)(1+2\lambda)=0$

$\Rightarrow \lambda = \frac{9}{4}$

Plane containing L_1, L_2 is $21x-19y+22z-125=0$

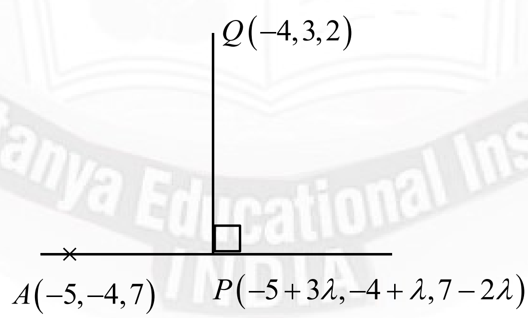
$(-5,-4,\alpha)$ lies on the plane $\Rightarrow \alpha = 7$

DR's of $PQ = (3\lambda-1, \lambda-1, 5-2\lambda)$

$\Rightarrow \lambda = 1, \therefore 3(3\lambda-1)+\lambda(-1)+(-2)(5-2\lambda)=0$

$P=(-2,-3,5)=(a,b,c)$

$\therefore |a|+|b|+|c| = 10$



25b. If the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x-2y+z+5=0=2x+3y+4z-k$ are coplanar then $K =$ _____

1.4   2. 6   3. 5   4. 0

Key : 1

Sol : $L_1 : \frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2} = r\left(Let\right)$

$x = 3r-4, y = 5r-6, z = -2r+1$

$\therefore P = \left(3r-4, 5r-6-2r+1\right)$

P lies on $3x-2y+z+5=0$

$3\left(3r-4\right)-2\left(5r-6\right)-2r+1+5=0$

$-3r=-6 \Rightarrow r=2$

$\therefore P = \left(2,4,-3\right)$

P lies on $2x+3y+4z-k=0$

$2\left(2\right)+3\left(4\right)+4\left(-3\right)=k$

$K=4$

26a. (July 2021) If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3} and \frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar, then the value of k is……

1. 2   2. 1   3. 4   4. 0

Key : 1

Sol : Given lines

$\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3} and \frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are coplanar.

$$so, \begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \left(k+1\right)\left(2-6\right)-4\left(1-9\right)+6\left(2-6\right)=0$

$\Rightarrow -4k+4=0$

$\Rightarrow k=1$

26b. If The lines $\frac{x-a}{1} = \frac{y-2}{3} = \frac{z-3}{2} and \frac{x+1}{1} = \frac{y+2}{2} = \frac{z+3}{3}$ are coplanar, then the value of 'a' is

1.0   2. 1   3. 2   4. 4

Key :1

Sol : Given lines are $\frac{x-a}{1} = \frac{y-2}{3} = \frac{z-3}{2} ---(1)$ $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z+3}{3} --(2)$

From (1) $\left(x_1, y_1, z_1\right) = \left(a_1, 2, 3\right)$ From (2) $\left(x_2, y_2, z_2\right) = \left(-1,-2,-3\right)$

$\left(a_1, b_1, c_1\right) = \left(1,3,2\right)$ $\left(a_2, b_2, c_2\right) = \left(1,2,3\right)$

The lines (1) and (2) are coplanar $\Rightarrow \begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

$$\Rightarrow\begin{vmatrix} a+1 & 4 & 6 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{vmatrix}=0$$

$\Rightarrow (a+1)(9-4)-4(3-2)+6(2-3)=0$

$\Rightarrow 5(a+1)-4(1)+6(-1)=0$

$\Rightarrow 5a+5-4-6=0 \Rightarrow 5a-5=0 \Rightarrow a=1$

27a. **(2020-sep)** If for some $\alpha \in R,$ then lines $L_1 : \frac{x+1}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point

1. $(10,2,2)$ 2. $(2,-10,-2)$ 3. $(10,-2,-2)$ 4. $(-2,10,2)$

Key : 2

Sol : it is given that lines $L_1 : \frac{x+1}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$

And $L_2 : \frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, so

$$\begin{vmatrix} -2+1 & -1-2 & -1-1 \\ \alpha & 5-\alpha & 1 \\ 2 & -1 & 1 \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} -1 & -3 & -2 \\ \alpha & 5-\alpha & 1 \\ 2 & -1 & 1 \end{vmatrix}=0 \Rightarrow -(5-\alpha+1)+3(\alpha-2)-2(-\alpha-10+2\alpha)=0$$

$\Rightarrow 2\alpha+8=0 \Rightarrow \alpha=-4$

$\therefore$ Equation of line $L_2 : \frac{x+2}{-4}=\frac{y+1}{9}=\frac{z+1}{1}$

Now, from the options the point $(2,-10,-2)$

Satisfy the line L_2.

27b. If for some $a \in R,$ the lines $L_1 : \frac{x-1}{-1}=\frac{y-2}{1}=\frac{z+1}{2}$ and $L_2 : \frac{x+2}{5-a}=\frac{y+1}{a}=\frac{z+1}{1}$ are coplanar then the line L_2 Pass through the point

1. $(-2,-,1,1)$ 2. $(-4,-4,-2)$ 3. $(2,2,1)$ 4. $(1,2,1)$

Key : 2

Sole : From $L_1 : (x_1,y_1,z_1)=(1,2-1),(a_1,b_1,c_1)=(-1,1,2)$

For L_2 $(x_2,y_2,z_2)=(-2,-1,-1),(a_2,b_2,c_2)=(5-a,a,1)$

The lines L_1 *and* L_2 are coplanar $\Rightarrow\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$

$$\Rightarrow \begin{vmatrix} 3 & 3 & 0 \\ -1 & 1 & 2 \\ 5-a & a & 1 \end{vmatrix} = 0 \Rightarrow 3 \quad \Rightarrow a = 3$$

$$L_2 : \frac{x+2}{5-a} = \frac{y+1}{a} = \frac{z+1}{1}$$

$$L_2 : \frac{x+2}{2} = \frac{y+1}{3} = \frac{z+1}{1}$$

Now, from the options the point $(-4,-4,-2)$ satisfy the line L_2

Sub topic:- Area of triangle in 3D

28a. (2019 Apr) The Vertices B and C of a ΔABC lie on the line $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that $BC = 5$ units. Then, the area (In square units) of this triangle given that $A = (1,-1,2)$ is

1. $\sqrt{34}$ 2. $2\sqrt{34}$ 3. $5\sqrt{17}$ 4. 6

Key : 1

Sol : Given line is

$$\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4} = \lambda(let) --(1)$$

D lines on (1)

$\therefore D = (3\lambda - 2, 1, 4\lambda)$

Now D.r's of BC $= (3,0,4) = (a_1, b_1, c_1)$

D.R's Of AD $= \underset{a_2}{(3\lambda - 3}, \underset{b_2}{2}, \underset{c_2}{4\lambda - 2)}$

$AD \perp BC \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow 3(3\lambda - 3) + 0 + 4(4\lambda - 2) = 0$

$\Rightarrow 9\lambda - 9 + 16\lambda - 8 = 0$

$\Rightarrow 25\lambda - 17 = 0$

$\Rightarrow \lambda = \frac{17}{25}$

$$\therefore D = \left(3 \times \frac{17}{25} - 2, 1, 4 \times \frac{17}{25}\right) = \left(\frac{1}{25}, 1, \frac{68}{25}\right)$$

$$AD = \sqrt{\left(1 - \frac{1}{25}\right)^2 + (-1-1)^2 + \left(2 - \frac{68}{25}\right)^2}$$

$= \frac{2}{5}\sqrt{34}$

$\therefore$ Area of ΔABC

$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 5 \times \frac{2}{5}\sqrt{34} = \sqrt{34}$

28b. The vertices B and C of $a\ \Delta ABC$ lie on the line $\frac{x-2}{0}=\frac{y+1}{1}=\frac{z}{2}$ such that $BC=4$ units.

Then, the area (in sq.units) of this triangle, given that point $A=(0,1,-1)$ is

1. 4 2. 6 3. 5 4. 2

Key : 2

Sol : Given line is

$\frac{x-2}{0}=\frac{y+1}{1}=\frac{z}{2}=\lambda(let)----(1)$

D lies on BC

$\therefore D=(2,\lambda-1,2\lambda)$

D.r's of BC $(0,1,2)=(a_1,b_1,c_1)$

D.R's of AD $(2,\lambda-2,2\lambda+1)$

$=(a_2,b_2,c_2)$

Since $AD\perp BC \Rightarrow a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 0+1(\lambda-2)+2(2\lambda+1)=0$

$\Rightarrow \lambda-2+4\lambda+2=0$

$\Rightarrow 5\lambda=0 \Rightarrow \lambda=0$

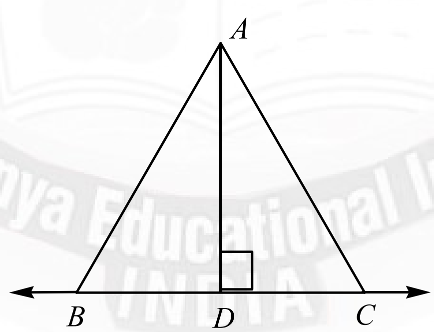
$\therefore D=(2,-1,0)$

Now, $AD=\sqrt{4+4+1}=\sqrt{9}=3$

$\therefore$ Area of $\Delta ABC=\frac{1}{2}BC\times AD$

$=\frac{1}{2}\times 4\times 3$

$=6$ sq.units



LINES AND PLANES

1a. (2023 Apr) If the equation of the plane passing through the line of intersection of the planes $2x-y+z=3,\ 4x-3y+5z+9=0$ and parallel to the line $\frac{x+1}{-2}=\frac{y+3}{4}=\frac{z-2}{5}$ is $ax+by+cz+6=0$, then $a+b+c=$ ____

1. 15 2. 12 3. 13 4. 14

Key : 4

Sol : $\pi_1=2x-y+z-3=0$, $\pi_2=4x-3y+5z+9=0$ Required plane is $\pi_1+\lambda\pi_2=0$

$\Rightarrow(2x-y+z-3)+\lambda(4x-3y+5z+9)=0$

$\Rightarrow(2+4\lambda)x+(-1-3\lambda)y+(1+5\lambda)z+(9\lambda-3)=0$

Given the plane is parallel to the line

$-2(2+4\lambda)+4(-1-3\lambda)+5(1+5\lambda)=0$

$-4-8\lambda-4-12\lambda+5+25\lambda=0$

$5\lambda-3=0\Rightarrow\lambda=\frac{3}{5}$

$\therefore\frac{22}{5}.x-\frac{14}{5}.y+\frac{20}{5}z+\frac{12}{5}=0$

$a=11,b=-7,c=10$

$\therefore a+b+c=14$

1b. If the equation of the plane through the line of intersection of the planes $2x+y+3z-2=0$, $x-y+z+4=0$ and parallel to the line $\frac{x-1}{2}=\frac{y-3}{5}=\frac{z-2}{-4}$ is $\alpha x+\beta y+\gamma z+k=0$, then $\alpha+\beta+\gamma=$

1.39 2. 30 3. 49 4. 20

Key : 1

Sol : $\pi_1:2x+y+3z-2=0$, $\pi_2:x-y+z+4=0$

Required plane is $\pi_1+\lambda\pi_2=0$

$\Rightarrow(2x+y+3z-2)+\lambda(x-y+z+4)=0---(1)$

$\Rightarrow(2+\lambda)x+(1-\lambda)y+(3+\lambda)z+(4\lambda-2)=0$

Given the plane is parallel to the line

$\frac{x-1}{2}=\frac{y-3}{5}=\frac{z+2}{-4}$

$\therefore2(2+\lambda)+5(1-\lambda)-4(3+\lambda)=0$

$4+2\lambda+5-5\lambda-12-4\lambda=0$

$-7\lambda-3=0\Rightarrow-7\lambda=3$

$\Rightarrow\lambda=\frac{-3}{7}$

Sub λ in (1)

$2x+y+3z-2-\frac{3}{7}(x-y+z+4)=0$

$14x+7y+21z-14-3x+3y-3z-12=0$

$11x+10y+18z-26=0$

$\therefore \alpha=11, \beta=10, \gamma=18$

$\alpha+\beta+\gamma=39$

2a. (2023 Apr) Let the line L passes through the point $(0,1,2)$ interest the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane $2x+y-3z=4$ Then the distance of the point $P(1,-9,2)$ from the line L is

1. $\sqrt{54}$ 2. 9 3. $\sqrt{69}$ 4. $\sqrt{74}$

Sol : $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=r(let)$

Any point on the line is $B=(2r+1, 3r+2, 4r+3)$

$A=(0,1,2)$

DR's of $AB=(2r+1, 3r+1, 4r+1)$

$\overleftrightarrow{AB}$ is parallel to the plane $2x+y-3z=4 \Rightarrow al+bm+cn=0$

$\Rightarrow 2(2r+1)+1(3r+1)-3(4r+9)=0 \Rightarrow r=0$

$\therefore B=(1,2,3)$

$L=\left(\frac{x-0}{1}=\frac{y-1}{1}=\frac{z-2}{1}\right)=t \Rightarrow Q=(t, t+1, t+2)$

$P=(1,-9,2)$

$DR's\, t\, PQ=(t-1, t+10, t)$

$\therefore 1(t-1)+1(t+10)+1(t)=0 \Rightarrow t=-3$

$Q=(-3,-2,-1)$

$|\overline{PQ}|=\sqrt{16+49+9}=\sqrt{74}$

2b. Equation of line passing through $A(1,0,3)$ interesting the line $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{1}$ and parallel to the plane $x+y+z=2$ is

1. $\frac{x-\frac{1}{3}}{-\frac{2}{3}}=\frac{y-\frac{3}{2}}{\frac{3}{2}}=\frac{z-\frac{13}{6}}{-\frac{5}{6}}$ 2. $\frac{x-1}{-2}=\frac{y-3}{3}=\frac{z-13}{-5}$

3. $\frac{x+1}{-2}=\frac{y+3}{3}=\frac{z-13}{-5}$ 4. None

Key : 1

Sol : Any point on line $\frac{x}{2}=\frac{y-1}{3}=\frac{3-2}{1}=\lambda$ is

$B=(2\lambda,3\lambda+1,\lambda+2)$ Dr's of $AB=(2\lambda-1,3\lambda+1,\lambda-1)$ and AB is parallel to plane

$x+y+z=2$

$\Rightarrow (2\lambda-1)1+1(1+3\lambda)+(\lambda-1)1=0\ (\because a_1a_2+b_1b_2+c_1c_2=0)$

$6\lambda=1 \Rightarrow \lambda=\frac{1}{6}$

$B=\left(\frac{1}{3},\frac{3}{2},\frac{13}{6}\right)$

$Dr's\ of\ AB=\left(\frac{1}{3}-1,\frac{3}{2}-0,\frac{13}{6}-3\right)$

$=\left(\frac{-2}{3},\frac{3}{2},\frac{-5}{6}\right)$

Then equation of the line $\frac{x-\frac{1}{3}}{-\frac{2}{3}}=\frac{y-\frac{3}{2}}{\frac{3}{2}}=\frac{z-\frac{13}{6}}{-\frac{5}{6}}$

3a. (2023 Apr) Let the line passing through the points $P(2,-1,2)$ and $Q(5,3,4)$ meet the plane $x-y+z=4$ at the point R. Then the distance of the point R from the plane $x+2y+3z+2=0$ measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal.

1. 3 2. $\sqrt{61}$ 3. $\sqrt{189}$ 4. $\sqrt{31}$

Key : 1

Sol : Equation of PQ is $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=t$

Let $R=(3t+2,4t-1,2t+2)$ Lie on $x-y+z=4$

$\Rightarrow 3t+2-4t+1+2t+2-4=0$

$\Rightarrow t+1 \Rightarrow t=-1$

$R=(-1,-5,0)$

Equation of line through R, Parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$

is $\frac{x+1}{2}=\frac{y+5}{2}=\frac{z-0}{1}=s(Let)$

Let $S=(2s-1,2s-5,s)$ lies on $x+2y+3z+2=0$

$\Rightarrow s=1$

$\therefore S=(1,-3,-1)$

$RS=\sqrt{4+4+1}=3$

3b. Let the line passing through the points $P(1,0,-1)\, and\, Q(1,2,3)$ meet the plane $x+y-z=5$ at the point R. Then the distance of the point R from the plane $x-y+2z-2=0$ measured parallel to the line $\frac{x-5}{1}=\frac{y+2}{2}=\frac{z-3}{1}$ is equal.

1. $\sqrt{864}$ 2. $\sqrt{860}$ 3. $\sqrt{60}$ 4. $\sqrt{160}$

Key : 1

Sol : Equation of PQ is $\frac{x-1}{0}=\frac{y-0}{2}=\frac{z+1}{4}=t$

$Let\ R=(1,2t,4t-1)$ lies on $x+y-z=5$

$\Rightarrow 1+2t-4t+1=5$

$\Rightarrow -2t=3 \Rightarrow t=\frac{-3}{2}$

$\therefore R=\left(1,-3,4\times\frac{-3}{2}-1\right)$

$R=(1,-3,-7)$

Equation of the line through R, parallel to

$\frac{x-5}{1}=\frac{y+2}{2}=\frac{z-3}{1}$ is

$\frac{x-1}{1}=\frac{y+3}{2}=\frac{z+7}{1}=s,$

Let $S=(s+1,2s-3,s-7)$ lie on, $x-y+2z-2=0$

$s+1-2s+3+2s-14-2=0$

$\Rightarrow s-12=0 \Rightarrow s=12$

$S=(13,21,5), R=(1,-,3,-7)$

$RS=\sqrt{(13-1)^2+(21+3)^2+(5+7)^2}$

$RS=\sqrt{(13-1)^2+(21+3)^2+(5+7)^2}$

$=\sqrt{144+576+144}=\sqrt{864}$

4a. (2022 June) Let $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}$ lie on the plane $px-qy+z=5,$ for some $p,q\in R.$ The shortest distance of the plane from the origin is

1. $\sqrt{\frac{3}{109}}$ 2. $\sqrt{\frac{5}{142}}$ 3. $\sqrt{\frac{5}{71}}$ 4. $\sqrt{\frac{1}{142}}$

Key : 2

Sol : Line : $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{1}....(i)$

$DR's: 3,-2-1$

$Plane: px-qy+z=5$ ……..(ii)

$\because$ Line is perpendicular to the normal of the plane.

$\therefore 3(p)+(-2)(-q)+(-1)(1)$c

$\Rightarrow 3p+2q-1=0......(iii)$

Point $(2,-1,-3)$ lies on the plane. Therefore, $2p+q-3=5$

$\Rightarrow 2p+q-8=0...(iv)$

On solving Eq.s (iii) and (iv) we get

$p=15, q=-22$

Equation of plane, $15x\times 22y+z-5=0$

It's distance from origin $(0,0,0)$

$$\left|\frac{0+0+0-5}{\sqrt{(15)^2+(22)^2+(1)^2}}\right|$$

$$=\left|\frac{-5}{\sqrt{710}}\right|=\sqrt{\frac{5}{142}}$$

4b. Let $\frac{x-1}{-2}=\frac{y-2}{3}=\frac{z-3}{-1}$ lie on the plane $ax-by+z=2,$ for some $a,b,\in R.$ The shortest distance of the plane from the origin is

1. $\frac{14}{5\sqrt{3}}$ 2. $\frac{14}{\sqrt{3}}$ 3. 14 4. 3

Key : 1

Sol : Given line : $\frac{x-1}{-2}=\frac{y-2}{3}=\frac{z-3}{-1}$

Dr's of line : $(-2,3,-1)$

Plane : $ax-by+z-2=0$

d.r's of normal : $a,-b,1$

$\because$ Line is perpendicular to the normal of the plane.

$\therefore(-2)(a)+3(-b)+(-1)=0$

$-2a-3b-1=0\Rightarrow 2a+3b+1=0--(1)$

Point $(1,2,3)$ lies on the plane

$\therefore a(1)-b(2)+3-2=0$

$a-2b+1=0-(2)$

Solving (1) and (2) we get a,b

$\therefore a=-\frac{5}{7}, b=\frac{1}{7}$

Equation of the plane is

$\frac{-5}{7}x-\frac{1}{7}y+z-2=0$

$-5x-y+7z-14=0$

Its distance from origin $(0,0,0)$ is

$\frac{|-14|}{\sqrt{25+1+49}}=\frac{14}{\sqrt{75}}=\frac{14}{5\sqrt{3}}$

5a. (2020 Sep) The distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to line

$\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is

1.1/7 2. 1 3. 7 4. 7/5

Key : 2

Sol : Equation of the line parallel to the given line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ and passes through point

$A(1,-2,3) is \quad \frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=r(let).....(i)$

$\therefore$ The coordinate of point on line (i) is $P(2r+1,3.r-2,3-6r)$

Let point P on the given plane $x-y+z=5.$ So, $2r+1-3r+2+3-6r=5$

$7r=1$

$\Rightarrow r=\frac{1}{7}$

$\therefore$ Required distance $AP=\sqrt{(2r)^2+(3r)^2+(-6r)^2}=7r=1$

5b. The distance of the point $(1,0,2)$ from the plane $x+y-z+2=0$ measured parallel to the line $\frac{x}{3}=\frac{y}{2}=\frac{3}{1} is$

1. $\frac{\sqrt{14}}{4}$ 2. $\frac{\sqrt{13}}{4}$ 3. $\frac{\sqrt{12}}{4}$ 4. $\sqrt{13}$

Key : 1

Sol : Equation of the line parallel to the given line $\frac{x}{3}=\frac{y}{2}=\frac{z}{1}$ and pass through point

$A(1,0,2)$ is $\frac{x-1}{3}=\frac{y-0}{2}=\frac{z-2}{1}=\lambda(Let)---(i)$

The coordinates of the point on line (i) is

$P(3\lambda+1,2\lambda,\lambda+2)$

P lies on the plane $x+y-z+2=0$

$3\lambda+1+2\lambda-\lambda-2+2=0$

$4\lambda+1=0\Rightarrow\left[\lambda=-\frac{1}{4}\right]$

$\therefore P=\left(3\left(-\frac{1}{4}\right)+1, 2\left(\frac{-1}{4}\right), \frac{-1}{4}+2\right)$

$=\left(\frac{-3+4}{4}, \frac{-1}{2}, \frac{-1+8}{4}\right)$

$P\left(\frac{1}{4}, \frac{-1}{2}, \frac{7}{4}\right), A=(1,0,2)$

$\therefore PA=\sqrt{\left(1-\frac{1}{4}\right)^{2}+\left(O+\frac{1}{2}\right)^{2}+\left(2-\frac{7}{4}\right)^{2}}$

$=\sqrt{\left(\frac{3}{4}\right)^{2}+\left(\frac{1}{4}\right)^{2}+\left(\frac{1}{4}\right)^{2}}=\sqrt{\frac{9}{16}+\frac{1}{4}+\frac{1}{16}}$

$=\sqrt{\frac{9+4+1}{16}}$

$\sqrt{\frac{14}{16}}=\frac{\sqrt{14}}{4}$

6. (2019 Apr) If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z-16=0$ at a point Q. Then $PQ=$

1. 14 2. $\sqrt{14}$ 3. $2\sqrt{7}$ 4. $2\sqrt{14}$

Key : 4

Sol : Given line is $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}=r(let)---(i)$

$\therefore$ Coordinate of general point on (i) is $R=(3r+2, 2r-1, -r+1)$

$\therefore P=(3r_1+2, 2r_1-1, -r_1+1)$ *and*

$Q=(3r_2+2, 2r_2-1, -r_2+1)$

P is point of intersection line (1) and the plane $2x+3y-z+13=0$

$\Rightarrow 2(3r_1+2)+3(2r_1-1)-(-r_1+1)+13=0$

$\Rightarrow r_1=-1$

$\therefore P=(-1,-3,2)$

And simirly for point Q, we get

$3(3r_2+2)+(2r_2+1)+4(-r_2+1)-16=0$

$\Rightarrow r_2=1$

$\therefore Q=(5,1,0)$

Now $PQ=\sqrt{(5+1)^{2}+(1+3)^{2}+(0-2)^{2}}$

$=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

7a.(2021 Feb) The distance of the point $(1,1,9)$ from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane $x+y+z=17$ is

1. $2\sqrt{19}$ 2. $19\sqrt{2}$ 3. $\sqrt{38}$ 4. 38

Key : 3

Sol : Given , $P(1,1,9)$ Equation of plane $x+y+z=17$ Equation of line

$\Rightarrow \frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda(let)----(i)$

$\therefore$ Any point on 1 is

$(\lambda+3, 2\lambda+4, 2\lambda+5)$

$\because$ This point lies on the plane

$x+y+z=17.$

$\therefore \lambda+3+2\lambda+4+2\lambda+5=17$

$\Rightarrow \lambda=1$

$\therefore$ The coordinate of point is $(4,6,7)$

$\therefore$ Required distance between (1,1,9) and $(4,6,7)$ is

$=\sqrt{(4-1)^2+(6-1)^2+(7-9)^2}$

$=\sqrt{9+25+4}=\sqrt{38}$

7b. The distance of the point $(1,-1,0)$ from the point of intersection of the line $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-0}{2}$ and the plane $x-y+z=10$ is

1. $\frac{\sqrt{2102}}{4}$ 2. $\sqrt{2102}$ 3. $\frac{\sqrt{2102}}{3}$ 4. 2021

Key : 1

Sol : Let $P=(1,-1,0)$

Given line is $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z}{2}=\lambda--(1)$

Any point on the line $(1) is (A)=(3\lambda+1, \lambda+2, 2\lambda)$

A lies on the plane $x-y+z-10=0$

$\Rightarrow 3\lambda+1-\lambda-2+2\lambda-10=0$

$\Rightarrow 4\lambda-11=0 \Rightarrow \lambda=\frac{11}{4}$

$\therefore A=\left(3\times\frac{11}{4}+1, \frac{11}{4}+2, 2\times\frac{11}{4}\right)$

$=\left(\frac{33+4}{4}, \frac{11+8}{4}, \frac{11}{2}\right)$

$A=\left(\frac{37}{4},\frac{19}{4},\frac{11}{2}\right)$

Now,

$PA=\sqrt{\left(\frac{37}{4}-1\right)^2+\left(\frac{19}{4}+1\right)^2+\left(\frac{11}{2}\right)^2}$

$=\sqrt{\frac{(33)^2}{16}+\frac{(23)^2}{16}+\frac{(11)^2}{4}}$

$=\sqrt{\frac{1089}{16}+\frac{529}{16}+\frac{121}{4}}$

$=\sqrt{\frac{1089+529+484}{16}}=\frac{\sqrt{2102}}{4}$

8a. (2021 Aug) Suppose the line $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-2z+\beta=0$ then, $\alpha+\beta=$

1. 7 2. 10 3. 20 4. 40

Key : 1

Sol : Given line is $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}---(i)$ and the plane is $x+3y-2z+\beta=0---(ii)$

Line (i) passes through $(2,2,-2)$ which lies on the plane $x+3y-2z+\beta=0$

$\Rightarrow 2+6+4+\beta=0\Rightarrow\beta=-12$

Also given line is perpendicular to normal of the plane

$\Rightarrow\alpha(1)-5(3)+2(-2)=0$

$\Rightarrow\alpha=19$

$\therefore\alpha+\beta=7$

8b. Suppose the line $\frac{x-1}{l}=\frac{y+2}{2}=\frac{z-1}{1}$ lies on the plane $x-y+2z+m=0.$ then $l=$ ___, $m=$ ___,

1. $l=0,m=-5$ 2. $l=1,m=-5$ 3. $l=0,m=2$ 4. $l=-5,m=0$

Key : 1

Sol : Given line is $\frac{x-1}{l}=\frac{y+2}{2}=\frac{z-1}{1}---(i)$ and

The plane is $x-y+2z+m=0---(ii)$

Live (i) passes through $(1,-2,1)$ which lie on the plane $x-y+2z+m=0\Rightarrow 1+2+2+m=0$

$\Rightarrow m=-5$

Also given line is perpendicular to normal the plane

$\Rightarrow l(1)+2(-1)+1(2)=0$

$\Rightarrow l-2+2=0\Rightarrow[l=0]$

9a. (2021 Aug) The distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line, whose direction rations are $2,3,-6$ is

1. 3 2. 5 3. 2 4.1

Key : 4

Sol : Let A be any point on the plane. $x-y+z=5$ and $B=(1,-2,3)$ The equation of the line AB whose direction ratios are $2,3,-6$ is

$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}\lambda(Let)$

$\Rightarrow x=2\lambda+1, y=3\lambda-2, z=-6\lambda+3$

A lines on the plane, $2\lambda+1-3\lambda+2+3-6\lambda=5$

$\therefore A=(2\lambda+1,3\lambda-2,3-6\lambda)$

$-7\lambda=-1\Rightarrow\lambda=\frac{1}{7}$

$\therefore A=\left(\frac{9}{7},\frac{-11}{7},\frac{15}{7}\right)$

Distance $AB=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}=1$

9b. The distance of the point $(1,0,-1)$ from the plane $x+y-z=4$ measured parallel to the line, whose direction rations are $1,1,0$ is

1. $\sqrt{2}$ 2. $\sqrt{3}$ 3. 2 4. $\sqrt{6}$

Key : 1

Sol : Let A be any point on the plane $x+y-z=4\,and\,B=(1,0,-1)$

The equation of the line AB whose d.r's are 1,1,0 is

$\frac{x-1}{1}=\frac{y-0}{1}=\frac{z+1}{0}=\lambda(let)\ x=\lambda+1, y=\lambda, z=-1$

$\therefore A=(\lambda+1,\lambda,-1)$

A lies on the plane $x+y-z=4$

$\Rightarrow\lambda+1+\lambda+1=4$

$\Rightarrow 2\lambda=2\Rightarrow\lambda=1$

$A=(2,1,-1)$

$\therefore AB=\sqrt{2}$

10. (2020 Jan) If the distance between the plane, $23x-10y-2z+48=0$ and the plane containing the lines

$\frac{x+1}{2}=\frac{y-3}{4}=\frac{z-1}{3}$ and $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda\in R)$ is equal to $\frac{k}{\sqrt{633}}, then\, K=$ ____

1. 1 2. 2 3. 3 4. 4

Key : 3

Sol : The distance between the plane $23x-10y-2z+48=0$ and the plane containing the lines $\frac{x+1}{2}=\frac{y-3}{4}=\frac{3+1}{3} and \frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda},(\lambda \in R)$ is same as the perpendicular distgance measure to plane either of points $(-1,3,-1)(or)(-3,-2,1)$

Required distance $=\frac{|23(-1)-10(3)-2(-1)+48|}{\sqrt{(23)^2+(-10)^2+(-2)^2}}$

$=\frac{|-23-30+2+48|}{\sqrt{529+100+4}}=\frac{3}{\sqrt{633}}$

$=\frac{k}{\sqrt{633}}(given)$

$\therefore K=3$

11. (2019) (Jan) The perpendicular distance from the origin to the plane containing the two liens

$\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7} and \frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$ is

1. $11\sqrt{6}$ 2. $\frac{11}{\sqrt{6}}$ 3. 11 4. $8\sqrt{11}$

Key : 2

Sol : Equation of the plane containing given two lines is $\begin{vmatrix} x+2 & y-2 & z+5 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}=0$

$\Rightarrow (x+2)(35-28)-(y-2)(21-7)+(z+5)(12-5)=0$

$\Rightarrow (x+2)(7)-(y-2)(14)+(z+5)(7)=0$

$\Rightarrow (x+2)(1)-(y-2)(2)+(z+5)(11)=10$

$\Rightarrow x+2-2y+4+z+5=0$

$\Rightarrow x-2y+z+11=0$

$\therefore$ The perpendicular distance from $(0,0,0,)$ to the plane $x-2y+z+11=0$ is

$\frac{|11|}{\sqrt{(1)^2+(-2)^2+(1)^2}}$

$=\frac{11}{\sqrt{1+4+1}}=\frac{11}{\sqrt{6}}$

($\therefore$ The perpendicular distance from $(0,0,0)$ to the

plane $ax+by+cz+d=0$ is $\frac{|d|}{\sqrt{a^2+b^2+c^2}}$)

12. (2023 Apr) Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ Intersects the line $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ *and* $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the midpoint of the line segment AB from the plane $2x-2y+z=14$ is

1. 3
2. $\frac{11}{3}$
3. 4
4. $\frac{10}{3}$

Key : 3

Sol : Point of intersection of the line $\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}$ and $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$

Let $\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda \, and \, \frac{x-5}{4}\frac{y-7}{3}=\frac{z+2}{1}=k$

$x=\lambda, y=6-2\lambda, z=5\lambda-8,\ x=4k+5, y=3k+7, z=k-2$

$\therefore \lambda=4k+5, 6-2\lambda=3k+7,\ 5\lambda-8=k-2$

$\Rightarrow k=-1, \lambda=1 \, and \, A=(1,4,-3)$

Pont of intersection of the lines

$\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5} \ and \ \frac{x+3}{6}=\frac{y-3}{-3}=\frac{3-6}{1}=k$

$\frac{x}{1}=\frac{y-6}{-2}=\frac{3+8}{5}=\lambda \ and \ \frac{x+3}{6}=\frac{y-3}{-3}=\frac{3-6}{1}=k$

$x=\lambda, y=-2\lambda+6, z=5\lambda-8\ x=6k-3, y=-3k+3, z=k+6$

$\Rightarrow K=1, \lambda=3 \ and \ B=(3,0,7)$

$\therefore C=$midpoint of $AB=(2,2,2)$

Distance of C from the plane $2x-2y+z=14$ is $\left|\frac{4-4+2-14}{\sqrt{4+4+1}}\right|=4$