Sec: Sr.Super60_NUCLEUS & STERLING_BT Paper -1(Adv-2020-P1-Model Date: 27-08-2023
Time: 09.00Am to 12.00Pm RPTA-04 Max. Marks: 198

KEY SHEET

PHYSICS

1	**B**	2	**B**	3	**B**	4	**B**	5	**A**	6	**C**
7	**ACD**	8	**C**	9	**AB**	10	**BC**	11	**BD**	12	**D**
13	**8**	14	**8**	15	**5**	16	**1.8**	17	**2.83**	18	**20**

CHEMISTRY

19	**C**	20	**B**	21	**C**	22	**B**	23	**B**	24	**C**
25	**ABCD**	26	**BC**	27	**AB**	28	**BD**	29	**AB**	30	**AB**
31	**79.25**	32	**38**	33	**55**	34	**11**	35	**19**	36	**23.24 – 23.28**

MATHEMATICS

37	**A**	38	**C**	39	**B**	40	**C**	41	**D**	42	**C**
43	**D**	44	**AC**	45	**BD**	46	**BC**	47	**BC**	48	**BD**
49	**6**	50	**0.50**	51	**27**	52	**36**	53	**20**	54	**2**

SOLUTIONS

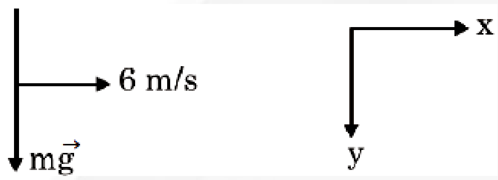
PHYSICS

1. Equation of motion perpendicular to inclination

$10\sin 30^0 - g\cos 30^0 t = 0$ $\qquad t = \frac{10}{10\sqrt{3}} \Rightarrow \left(t = \frac{1}{\sqrt{3}}\right)$

Time of flight T = 2t = $\frac{2}{\sqrt{3}}$ sec

2. $mg = C \times 8$ $\qquad -C \times \vec{v}_{m\omega}$



$-C \times \vec{v}_{m\omega} = mg$

$v_{m\omega}$ should be vertically down. $\qquad \Rightarrow \vec{v}_{m\omega} = 8\ m/s\ \hat{i}$

$\vec{v}_m - \vec{v}_\omega = 8\hat{i}$ $\qquad v_m = 8\hat{i} + 6\hat{j}$

3. $\frac{df}{f^2} = \frac{du}{u^2} + \frac{dv}{v^2}, du = dv$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{u^2} + \frac{1}{v^2} - \frac{2}{uv} = \frac{1}{f^2}$

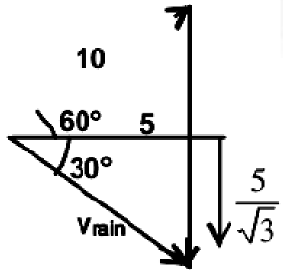
$\frac{df}{f^2} = du\left(\frac{1}{f^2} + \frac{2}{uv}\right) = du\left(\frac{1}{f^2} + \frac{2}{u}\left(\frac{1}{f} + \frac{1}{u}\right)\right)$

$df = du\left(1 + \frac{2f}{u} + \frac{2f^2}{u^2}\right) \Rightarrow$ df is minimum when

$0 = -\frac{2f}{u^2} - \frac{4f^2}{u^3}$ $\qquad$ u = –2f

(for $u = \infty$, 2nd derivative is zero so it is not a minimum)

4. $V_{rain} = \sqrt{5^2 + \left(\frac{5}{\sqrt{3}}\right)^2}$



$= 5\sqrt{1 + \frac{1}{3}} = \frac{5 \times 2}{\sqrt{3}} = \frac{10}{\sqrt{3}}$

5. Error $= \frac{\ell.c.}{N}$

6. $(1) 2.3056 \rightarrow 4$ decimals

$10.138 \rightarrow 3$ decimals

$-7.4671 \rightarrow 4$ decimals

4.9765

Answer should have 3 decimals.

$(2) 2.38 \times 1.0 = 2.38 \rightarrow$ answer should have 2 significant digits.

$(3) \frac{8.05}{3.1} = 2.59 \approx 2.6$ answer should have 2SD.

$(4) 1.11 - 0.1 = 1.01 \approx 1.0$ but is an intermediate step so we keep 1 digit extra.

$1.01 \times 9.0 = 9.09 \approx 9.1 \rightarrow$ both 1.01 and 9.0 have 2SD.

7. The velocity parallel to the plane is unaltered by the impacts, so that the distance described parallel to the plane will be zero at the end of a time t given by :

$$0 = vt\cos(\theta - \alpha) - \frac{(g\sin\alpha)t^2}{2} \qquad \Rightarrow t = \frac{2v\cos(\theta - \alpha)}{g\sin\alpha}$$

Also, since the elasticity is perfect, the velocity perpendicular to the plane is just reversed at each impact. The time of flight for each trajectory is thus twice the time in which the velocity $v\sin(\theta - \alpha)$ is destroyed by $g\cos\alpha$, and thus $t = \frac{2v\sin(\theta - \alpha)}{g\cos\alpha}$

Clearly the particle will return to the point of projection if the first of these is some multiple, n, of the second, i.e., if $\frac{2v\cos(\theta - \alpha)}{g\sin\alpha} = n\frac{2v\sin(\theta - \alpha)}{\cos\alpha}$

i.e., if $\cot\alpha.\cot(\theta - \alpha)$ is an integer.

8. Least count $= \frac{0.5}{100} = 0.005\ mm$

Zero error $= 0 + 0.005 \times 2 = 0.01\ mm$

So, true diameter $= 0.5 \times 8 + 0.005 \times 83 - 0.01 = 4.405\,mm$

9. At the particles reach point P at same time

$$\frac{2v_0\sin\alpha}{g} = \frac{v_0}{g\sin\beta} \Rightarrow 2\sin\alpha = \frac{1}{\sin\beta}$$

Since their horizontal displacement are equal

$$v_0\cos\alpha t = \frac{v_0}{2}t\cos\beta \qquad 2\cos\alpha = \cos\beta$$

10. In series error is $\Delta R_1 + \Delta R_2$

In parallel : $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{\Delta R_{eq}}{R_{eq}^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Delta R_{eq} = \frac{1}{15}\Omega = 0.06\Omega \simeq 0.1\Omega$$

11. $L = \left(M^{-1}L^3T^{-2}\right)^x \left(LT^{-1}\right)^y \left(ML^2T^{-1}\right)^z$

$-x + z = 0$

$-2x - y = z = 0$

$y = -3x$

$3x + y + z \quad z = 1$

$y = \frac{-3}{2} \qquad z = x = \frac{1}{2}$

12. $100 \times \frac{df}{f} = \frac{5\phi}{3}\left(\frac{\cancel{0.1}}{2500} + \frac{\cancel{0.1}}{625}\right) \times 100 \qquad = \frac{1}{3}\%$

13. $\frac{A \; a_1,(t-4)v^{|} + u}{B \; a_2,t,v^{|}}$

$S = \frac{1}{2}a_1(t-4)^2 = \frac{1}{2}a_2t^2$

t = 8s $\qquad v^{|} = a_2t = 8m/s$

$v^{|} + v = a_1(t-4) \quad 8 + v = 4(4) \qquad v = 8m/s$

14. $R = \left(\frac{n-1}{n+1}\right)^2$

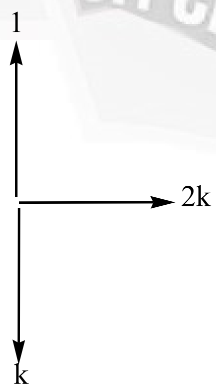
$\log R = 2\log(n-1) - 2\log(n+1)$

Differentiating $\frac{dR}{R} = \frac{2dn}{n-1} - \frac{2dn}{n+1}$

$\frac{DR}{R} = 2\left[\frac{1}{n-1} - \frac{1}{n+1}\right]\Delta n = \left(\frac{4n}{n^2-1}\right)\frac{\Delta n}{n} = \frac{8}{3} \times 3\% = 8\%$

15. w.r.t belt particle will move along straight line

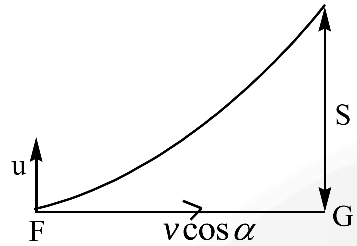
$v^2 = (1-k)^2 + 4k^2$



$\frac{dv^2}{dt} = -2(1-k) + 8k = 0$

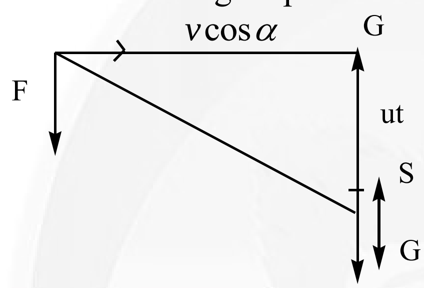
$k=\frac{1}{5}$ $\quad v^2=\frac{16}{25}+\frac{4}{25}$ $\quad v=\frac{2}{\sqrt{5}}$

16. Motion in ground frame



Motion in wind frame

In wind frame goal post shift by ut



$$\frac{u}{v\cos\alpha}=\frac{ut-S}{L}$$

$$\frac{L}{v\cos\alpha}=t-\frac{S}{u}$$

$$t=\frac{S}{u}+\frac{L}{v\cos\alpha}=1.8\sec$$

17. Distance between balls will be minimum when they are along same vertical line

$$AB=\frac{1}{2}g\sin\alpha\left(t_1^2-t_2^2\right)$$

$$\frac{AB}{2}=\frac{1}{2}g\sin\alpha t^2$$

$$t=\sqrt{\frac{t_1^2-t_2^2}{2}}=2.82$$

18. $$\frac{u^2}{g\left(1-\frac{a-c}{b}\right)}=b$$

$$u^2=g(b-a+c)$$

$$v^2=g(b-a+c)+2ga$$

$$=g(a+b+c)$$

$$=400$$

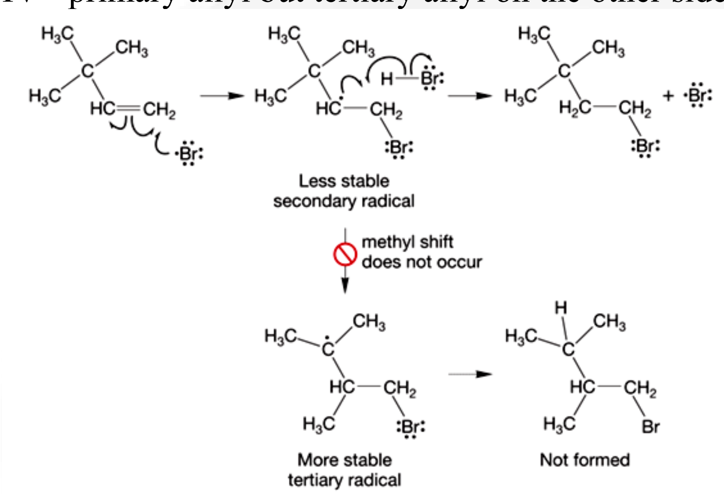
$$v=20m/\sec$$

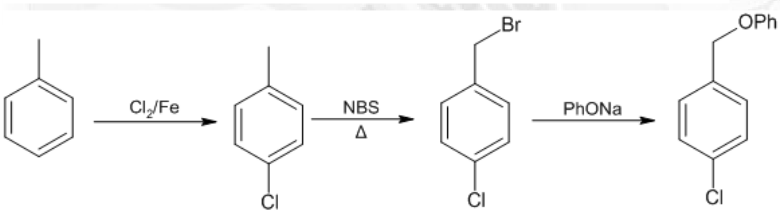
CHEMISTRY

19. With benzene, decarbonylation of acylium ion occurs and hence alkylation product is formed. Anisole, being more reactive, reacts quickly with acylium ion to give normal acylation product as the major product.

20. Lone pair on N activates the 2^{nd} ring.

21. I – tertiary
II – primary allyl
III – primary allyl but secondary allyl on the other side
IV - primary allyl but tertiary allyl on the other side

22. 

23. 

24. $S_N Ar$: A and C
$S_N 2$: B

25. The reaction will mainly proceed by E2, giving mixture all three alkenes. Ether will be formed as minor product by $S_N 2 + S_N 1$

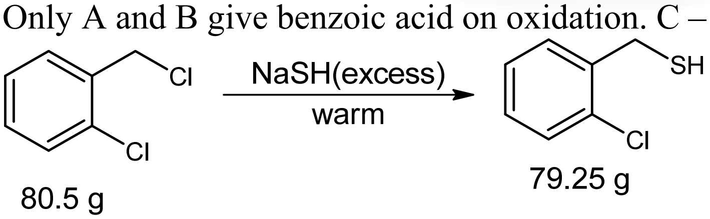
26. Neighbouring group participation.

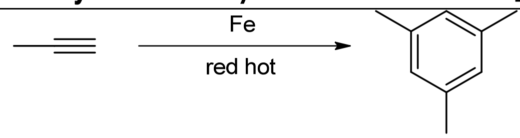
27. ICl splits in presence of $ZnCl_2$ to give iodonium ion.
KI converts BDC to iodobenzene

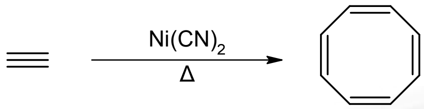
28. NaCl doesn't react.
$HCl - ZnCl_2$ gives rearranged product as the major product

29. As EtONa is a stronger base and hence facilitates E2 giving propene as the main product while EtSNa is a stronger nucleophile and hence gives $S_N 2$ product as the main product.

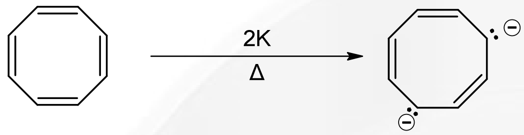
30. Only A and B give benzoic acid on oxidation. C – no reaction, D – phthalic acid

31. 

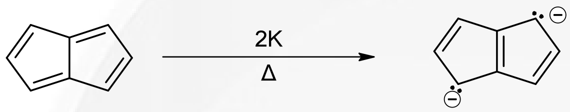




(no resonance as it is non-planar)



(aromatic)



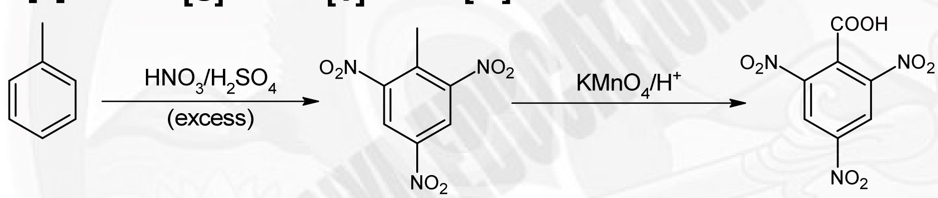
(aromatic)

32.

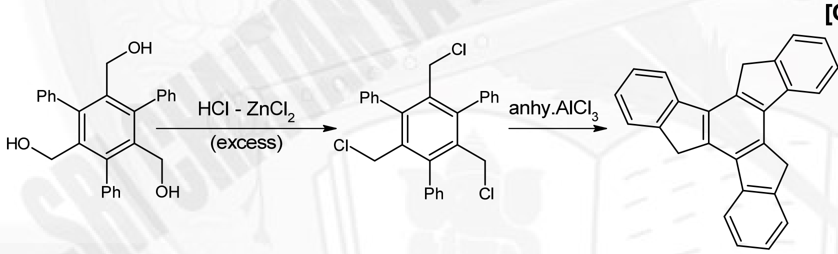
6+8+8+8+8 (COT) = 30

33.

$PhCH_3$	Ph_2CH_2	Ph_3CH	Ph_3CCl
[T]	**[U]**	**[V]**	**[W]**



34. **[Q]**



35. **[R]**

36. Let x be the % of inverted product.

$$\frac{x}{100}\times(43)-\frac{(1-x)}{100}43=10 \Rightarrow x=61.63$$

% of retention product = 38.37

$\Rightarrow$ % of $S_N1 = 76.74$

$\Rightarrow$ % of $S_N2 = 23.26$

MATHEMATICS

37. $f(x)=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ $\quad \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx=\frac{\left(\sin^{-1}x\right)^2}{2}+C$

$g(x)=\frac{\left(\sin^{-1}(x)\right)^2}{2}$ $\quad g\left(\frac{1}{2}\right)=\frac{1}{2}\left(\frac{\pi}{6}\right)^2=\frac{\pi^2}{7^2}$

$g\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{2}\left(\frac{\pi}{4}\right)^2=\frac{\pi^2}{32}$

38. $\int \frac{\left(3x^{10}+2x^8-2\right)\left(x^{10}+x^8+1\right)^{\frac{1}{4}}}{x^6}dx$

$=\int \frac{\left(3x^{10}+2x^8-2\right)\times\left(x^4\right)^{\frac{1}{4}}\left(x^6+x^4+x^{-4}\right)^{\frac{1}{4}}}{x^6}dx$ $\int\left(3x^5+2x^3-2x^{-5}\right)\left(x^6+x^4+x^{-4}\right)^{\frac{1}{4}}dx$

$x^6+x^4+x^{-4}=t^4$ $\left(6x^5+4x^3-4x^{-5}\right)dx=4t^3dt$

$\left(3x^5+2x^3-2x^{-5}\right)dx=2t^3dt$ $\quad =\int(t)\left(2t^3\right)dt=\frac{2t^5}{5}+C$

$=\frac{2}{5}\left[x^6+x^4+x^{-4}\right]^{\frac{5}{4}}+C$

$f(1)=\frac{2}{5}(1+1+1)^{\frac{5}{4}}=\frac{2}{5}(3)^{\frac{5}{4}}=\frac{6.(3)^{\frac{1}{4}}}{5}$

$f(2)=\frac{2}{5}\left[2^6+2^4+2^{-4}\right]^{\frac{5}{4}}=\frac{2}{5}\frac{\left[2^{10}+2^8+1\right]^{\frac{5}{4}}}{25}=\frac{1}{80}[1281]^{\frac{5}{4}}$

39. $\frac{x+\sin x+\cos x}{x-\sin x+\cos x}=t^2$ $\quad \frac{2(1+x\cos x-\sin x)}{(x-\sin x+\cos x)^2}dx=2t\,dt$

$I=\int\frac{t\,dt}{t}=t+C$ $\quad =(x+\sin x+\cos x)^{\frac{1}{2}}(x-\sin x+\cos x)^{\frac{-1}{2}}+C$

40. $\sqrt{1+\sqrt{x}}=\sec 4\theta$

$\tan^{-1}\left(\frac{\sqrt{\sec 2\theta+1}-\sqrt{\sec 2\theta-1}}{\sqrt{\sec 2\theta+1}+\sqrt{\sec 2\theta-1}}\right)=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$=\tan^{-1}\left(\frac{\sqrt{2}.\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)=\tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)$

$=\frac{\pi}{4}-\theta$ $\quad \int \sqrt{x}.\tan\left(2\left(\frac{\pi}{4}-\theta\right)\right)dx=\int \sqrt{x}.(\cot 2\theta)\,dx=\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}}}dx$

$=\int x^{\frac{1}{4}}dx=\frac{4}{5}x^{\frac{5}{4}}+C$ $\quad A=\frac{4}{5},B=\frac{5}{4}\Rightarrow AB=1$

41. $I=\int \frac{\sin\theta.\sin 2\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$

$\Rightarrow I=\int \frac{\sin\theta.2\sin\theta\cos\theta.\sin^2\theta\left(\sin^4\theta+\sin^2\theta+1\right)\left(2\sin^4\theta+3\sin^2\theta+6\right)^{\frac{1}{2}}}{2\sin^2\theta}d\theta$

Let $\sin\theta=t\Rightarrow \cos\theta\, d\theta=dt$

$\therefore I=\int t^2\left(t^4+t^2+1\right)\left(2t^4+3t^2+6\right)^{\frac{1}{2}}dt=\int\left(t^5+t^3+t\right)t\left(2t^4+3t^2+6\right)^{\frac{1}{2}}dt$

$=\int\left(t^5+t^3+t\right)\left(t^2\right)^{\frac{1}{2}}\left(2t^4+3t^2+6\right)^{\frac{1}{2}}dt\ =\int\left(t^5+t^3+t\right)\left(2t^6+3t^4+6t^2\right)^{\frac{1}{2}}dt$

Let $2t^6+3t^4+6t^2=u^2$ $\quad \Rightarrow 12\left(t^5+t^3+t\right)dt=2udu$

$\therefore \quad I=\int\left(u^2\right)^{\frac{1}{2}}.\frac{2udu}{12}=\int\frac{u^2}{6}du=\frac{u^3}{18}+C \quad =\frac{\left(2t^6+3t^4+6t^2\right)^{\frac{3}{2}}}{18}+C$

When t = $\sin\theta$

And $t^2=1-\cos^2\theta$ will give option (4)

42. $\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx=\int\frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx \quad =\int\frac{\sin 3x+\sin 2x}{\sin x}dx$

$=\int\frac{3\sin x-4\sin^3x-2\sin x\cos x}{\sin x}dx \quad =\int\left(3-4\sin^2x+2\cos x\right)dx$

$=\int\left(3-2(1-\cos 2x)+2\cos x\right)dx \quad =\int\left(1+2\cos 2x+2\cos x\right)dx$

$=x+\sin 2x+2\sin x+c$

43. $x^4-8x^3+18x^2-8x+1$

$=x^4-8x^3+16x^2+2x^2-8x+1 \quad =x^2(x-4)+2\left(x^2-4x\right)+1$

$=\left(x^2-4x\right)^2+2\left(x^2-4x\right)+1=\left(x^2-4x+1\right)^2$

$\int\frac{(x-2)dx}{\left(x^2-4x+1\right)}=\frac{1}{2}\int\frac{(2x-4)}{x^2-4x+1} \quad =\frac{1}{2}\ln\left|x^2-4x+1\right|+C$

44. $\int \frac{dx}{\left(x+\sqrt{x(1+x)}\right)^2} = \int \frac{dx}{\left(\sqrt{x^2+x}+x\right)^2}$

Consider $x+\sqrt{x^2+x}=t \qquad \sqrt{x^2+x}=t-x$

$x^2+x=t^2+x^2-2tx \qquad \Rightarrow \quad (2t+1)x=4^2 \qquad \Rightarrow x=\frac{t^2}{2t+1}$

$dx=\left(\frac{(2t+1)(2t)-t^2(2)}{(2t+1)^2}\right)dt=\frac{2t^2+2t}{(2t+1)^2}dt$

$\int \frac{\left(\frac{2t^2+2t}{(2t+1)^2}\right)dt}{t^2}=\int \frac{2dt}{(2t+1)^2}+\int \frac{2}{t(2t+1)^2}dt$

$=2\ln\left(\frac{t}{2t+1}\right)+\frac{1}{2t+1}+C, \qquad t=x+\sqrt{x^2+x}$

45. $\frac{\cos x-\sin x}{(1+\cos x)\cos x+\cos x\sin x+(1+\sin x)\sin x}=\frac{\cos x-\sin x}{1+\sin x+\cos x+\sin x\cos x}$

$=\frac{(1+\cos x)-(1+\sin x)}{(1+\cos x)(1+\sin x)}=\frac{1}{1+\sin x}-\frac{1}{1+\cos x}$

$\int \frac{1}{1+\sin x}dx-\int \frac{1}{1+\cos x}dx=\tan x-\sec x-\tan\frac{x}{2}+C$

$=\frac{\sin x-1}{\cos x}-\tan\frac{x}{2}+\left(C^1+1\right) \qquad =\frac{1-\cot\left(\frac{x}{2}\right)}{1+\cot\left(\frac{x}{2}\right)}+\left(1-\tan\frac{x}{2}\right)+C^1$

$=\frac{1-\cot\frac{x}{2}+1+\cot\frac{x}{2}-1-\tan\frac{x}{2}}{1+\cot\left(\frac{x}{2}\right)} \qquad =\frac{1-\tan\left(\frac{x}{2}\right)}{1+\cot\left(\frac{x}{2}\right)}+C$

46. $\int \frac{x^3+x+1}{x^4+x^2+1}dx=\frac{1}{2}\int \frac{2x^3+2x+2}{x^4+x^2+1}dx \qquad =\frac{1}{2}\left[\int \frac{2x+1}{x^2+x+1}dx+\int \frac{1}{x^2-x+1}dx\right]$

$=\frac{1}{2}\log\left|x^2+x+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+C$

47. $x-y+3=t \qquad x+y=t^2$

$x=\frac{t^2+t-3}{2} \qquad dx=\left(\frac{2t+1}{2}\right)dt$

$$\int \frac{dx}{x+y+6} = \int \frac{\left(\frac{2t+1}{2}\right)dt}{t^2+6} = \frac{1}{2}\left[\int \frac{2t}{t^2+6}dt + \int \frac{dt}{t^2+6}\right]$$

$$= \frac{1}{2}\log\left(t^2++6\right) + \frac{1}{2\sqrt{6}}\tan^{-1}\left(\frac{t}{\sqrt{6}}\right) + C$$

$$= \frac{1}{2}\log\left(x+y+6\right) + \frac{1}{2\sqrt{6}}\tan^{-1}\left(\frac{x-y+3}{\sqrt{6}}\right) + C$$

48. Put tan x = t and $\sec^2 x = dt$

$$I = \int e^t \sqrt{\sqrt{t^2+1}+t}.\left(2+\frac{1}{\sqrt{t^2+1}}\right).dt$$

$$= \int e^t.\left(2\sqrt{\sqrt{t^2+1}+t} + \frac{\sqrt{\sqrt{t^2+1}+t}}{\sqrt{t^2+1}}\right).dt = \int e^t.\left(f(t)+f'(t)\right).dt$$

$$= e^t.f(t)+C = e^t.2\sqrt{\sqrt{t^2+1}+t+C}$$

$$= e^{\tan x}.2\sqrt{\tan x+\sec x}+C$$

So $f(x) = 4(\tan x+\sec x)$

49. $$\int \frac{\left(x^2-1\right)dx}{\left(x^4+3x^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)} + \int \frac{dx}{x^4+3x^2+1}$$

$$\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)} + \frac{1}{2}\int \frac{\left(x^2+1\right)-\left(x^2-1\right)dx}{x^4+3x^2+1}$$

Put $\tan^{-1}\left(x+\frac{1}{x}\right) = t$

$$\int \frac{dt}{t} + \frac{1}{2}\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5} - \frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$$

Put $x-\frac{1}{x} = y, x+\frac{1}{x} = z$

$$\log_e t + \frac{1}{2}\int \frac{dy}{y^2+5} - \frac{1}{2}\int \frac{dz}{z^2+1}$$

$$=\log_e \tan^{-1}\left(x+\frac{1}{x}\right)+\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)$$

$$-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

$$\alpha=1,\beta=\frac{1}{2\sqrt{5}},\gamma=\frac{1}{\sqrt{5}},\delta=\frac{-1}{2} \text{ or } \alpha=1,\beta=\frac{-1}{2\sqrt{5}},\gamma=\frac{-1}{\sqrt{5}},\delta=\frac{-1}{2}$$

$$10(\alpha+\beta\gamma+\delta)=10\left(1+\frac{1}{10}-\frac{1}{2}\right)=6$$

50. $$\int\frac{x^2+4x-1}{\left(x^2+3^2\right)\sqrt{x+3}}dx=\int\frac{2x^2+8x-2}{2\left(x^2+3\right)^2\sqrt{x+3}}dx=\frac{-2\sqrt{x+3}}{3\left(x^2+3\right)}+C$$

$$f(x)=\frac{-2}{3}\frac{\sqrt{x+3}}{x^2+3}\quad f(1)=\frac{-2}{3}.\frac{2}{4}=\frac{-1}{3}$$

$$f(6)=\frac{-2}{3}.\frac{3}{(39)}=\frac{-2}{39}\qquad -39f(6)=2$$

51. $$\int e^x\left(\frac{x^4+4x^3+64}{(x+4)^2}\right)dx=\int e^x\left[\frac{x^3}{x+4}+\frac{64}{(x+4)^2}\right]dx$$

$$=\int e^x\left(\frac{x^3+64-64}{x+4}+\frac{64}{(x+4)^2}\right)dx=\int e^x\left(x^2-4x+16-\frac{64}{x+4}+\frac{64}{(x+4)^2}\right)dx$$

$$=e^x\left[\left(x^2-4x+16-(2x-4)+2\right)-\frac{64}{x+4}\right]+C$$

$$f(x)=x^2-6x+22-\frac{64}{x+4}$$

$$f(0)=22-16=6\quad f(3)=13-\frac{64}{7}=\frac{27}{7}\quad 7f(3)=27$$

52. $$\int\frac{x^4+1}{\left(x^4+2\right)^{\frac{3}{4}}}dx=\frac{1}{2}\int\frac{2x^4+2}{\left(x^4+2\right)^{\frac{3}{4}}}dx$$

$$=\frac{1}{2}\left[\int\frac{x^4}{\left(x^4+2\right)^{\frac{3}{4}}}dx+\int\left(x^4+2\right)^{\frac{1}{4}}dx\right]\quad=\frac{1}{2}\left(x\left(x^4+2\right)^{\frac{1}{4}}\right)+C$$

$$f(x)=\frac{x}{2}\left(x^4+2\right)^{\frac{1}{4}}\qquad f(0)=0$$

$$f\left(\sqrt{2}\right)=\frac{\sqrt{2}}{2}(4+2)^{\frac{1}{4}}=\frac{\sqrt{2}.(6)^{\frac{1}{4}}}{2}\Rightarrow\left(\sqrt{2}f\left(\sqrt{2}\right)\right)^8=\left(6^{\frac{1}{4}}\right)^8=36$$

53. $$\int\frac{5x^2+2}{25x^4-4\sqrt{5}x+3}dx=\frac{1}{2}\int\frac{\left(10x^2+4\right)dx}{25x^4-4\sqrt{5}x+3}$$

$$=\frac{1}{2}\int\frac{\left(10x^4+4\right)dx}{\left(\sqrt{5}x-1\right)^2\left(\left(\sqrt{5}x+1\right)^2+2\right)}=\frac{1}{2}\int\frac{\left(\sqrt{5}x+1\right)^2+2+\left(\sqrt{5}x-1\right)^2}{\left(\sqrt{5}x-1\right)^2\left(\left(\sqrt{5}x+1\right)^2+2\right)}dx$$

$$=\frac{1}{2}\int\frac{dx}{\left(\sqrt{5}x-1\right)^2}+\frac{1}{2}\int\frac{dx}{\left(\sqrt{5}x+1\right)^2+2}$$

$$=\frac{-1}{2\sqrt{5}}.\frac{1}{\left(\sqrt{5}x-1\right)}+\frac{1}{2\sqrt{10}}\tan^{-1}\left(\frac{\sqrt{5}x+1}{\sqrt{2}}\right)+C$$

$$f\left(\frac{\sqrt{2}-1}{\sqrt{5}}\right)=\frac{\pi}{4}\qquad g\left(\frac{2}{\sqrt{5}}\right)=1$$

54. $$\int\frac{\left(x^2+1\right)dx}{x\sqrt{x^2+2x-1}}\sqrt{1-x-x^2}=\int\frac{\frac{x^2+1}{x^2}dx}{\frac{x}{x}.\sqrt{\frac{x^2+2x-1}{x}}\sqrt{\frac{1-x-x^2}{x}}}$$

$$=\int\frac{\left(1+\frac{1}{x^2}\right)dx}{\sqrt{\left(x-\frac{1}{x}+2\right)\left(\frac{1}{x}-x-1\right)}}\qquad x-\frac{1}{x}+2=t^2$$

$$\left(1+\frac{1}{x^2}\right)dx=2t\quad dt\qquad =\int\frac{2t\,dt}{t\sqrt{1-t^2}}=2\sin^{-1}(t)+C$$

$$=2\sin^{-1}\left(\sqrt{x-\frac{1}{x}+2}\right)+C\quad f\left(\frac{1}{2}\right)=2\sin^{-1}\left(\sqrt{\frac{1}{2}-2+2}\right)=2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=2\left(\frac{\pi}{4}\right)=\frac{\pi}{2}$$

$$f(x)=2\sin^{-1}\left(\sqrt{x-\frac{1}{x}+2}\right)$$

$$f\left(\frac{\sqrt{89}-5}{8}\right)=2\sin^{-1}\left(\sqrt{\frac{3}{4}}\right)=2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=2\left(\frac{\pi}{2}\right)=\frac{2\pi}{3}\qquad\left[f\left(\frac{\sqrt{89}-5}{8}\right)\right]=\left[\frac{2\pi}{3}\right]=2$$