ANSWER KEYS

1. (1) **2.** (2) **3.** (4) **4.** (3) **5.** (2) **6.** (3) **7.** (1) **8.** (3)
9. (4) **10.** (4)

1. **(1)** $\alpha = -\omega,\ \beta = -\omega^2$

$\alpha^{2015} = -\omega^2,\ \beta^{2015} = -\omega$

2. **(2)**

As we know if $lx^2 + mx + n = 0$ is identity, then $l = 0,\ m = 0,\ n = 0$

So, $a^2 - 3a + 2 = 0$

$(a-2)\ (a-1) = 0$

$a = 2\ ,\ a = 1$

$a^2 - 5a + 6 = 0$

$(a-3)(a-2) = 0$

$a = 3,\ 2$

$a^2 - 4 = 0$

$(a-2)(a+2) = 0$

$a = \pm 2$

$\therefore \quad a = 2$

3. **(4)** Let $\alpha\ ,\ \beta$ and γ, δ are the roots of the equations

$x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ respectivaly.

$\therefore \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a.$

Given $\alpha - \beta = \gamma - \delta \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$a^2 - 4b = b^2 - 4a \Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow a + b + 4 = 0.\ (\because a \neq b)$

4. (3)

Let α and β be the roots of the given equation. $x^2+(4-\lambda)x+3=\lambda$

$\Rightarrow x^2+(4-\lambda)x+3-\lambda=0$

$\Rightarrow \alpha+\beta=-(4-\lambda)=\lambda-4$ and $\alpha\beta=3-\lambda$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(\lambda-4)^2-2(3-\lambda)$

$=\lambda^2-6\lambda+10$

$=(\lambda-3)^2+1$

For least value $\lambda=3$

5. (2) Since, 4 is a root of $x^2+ax+12=0$

$\therefore \quad 16+4a+12=0 \Rightarrow a=-7$

Let the roots of the equation $x^2+ax+b=0$ be α and α

$\therefore \quad 2\alpha=-a$

$\Rightarrow \quad \alpha=\frac{7}{2}$

And $\quad \alpha.\alpha=b$

$\Rightarrow \quad \left(\frac{7}{2}\right)^2=b$

$\Rightarrow \quad b=\frac{49}{4}$

6. (3) Since the given equation has no real solution.

$\Rightarrow D<0$

$\Rightarrow 4(1+3m)^2-4\times(1+m^2)(1+8m)<0$

$\Rightarrow (1+9m^2+6m)-(1+8m+m^2+8m^3)<0$

$\Rightarrow -2m(2m-1)^2<0$

Clearly, for infinite values of m given condition is true.

7. (1)

Given, $6x^2 - 11x + \alpha = 0$

Roots of the quadratic equation$= \frac{-(-11) \pm \sqrt{11^2 - 4(6)(\alpha)}}{2 \times 6} = \frac{11 \pm \sqrt{121 - 24\alpha}}{12}$

For the roots to be rational, discriminant must be non-negative and a perfect square.

Let $121 - 24\alpha = k^2$

$121 - 24\alpha \geq 0 \Rightarrow \alpha \leq \frac{121}{24}$

Since, we have to find positive integral values for α, range of α is $0 < \alpha < \frac{121}{24}$

The positive integral values in the above range are $1, 2, 3, 4, 5$.

$\alpha = 1 \Rightarrow 121 - 24\alpha = 121 - 24(1) = 97$, not a perfect square.

$\alpha = 2 \Rightarrow 121 - 24\alpha = 121 - 24(2) = 73$, not a perfect square.

$\alpha = 3 \Rightarrow 121 - 24\alpha = 121 - 24(3) = 49$, is a perfect square.

$\alpha = 4 \Rightarrow 121 - 24\alpha = 121 - 24(4) = 25$, is a perfect square.

$\alpha = 5 \Rightarrow 121 - 24\alpha = 121 - 24(5) = 1$, is a perfect square.

So, possible value of $\alpha = 3,\ 4,\ 5$

Hence, there are 3 positive integral values for α for which the roots are rational.

8. (3) Given Equation : $x^2 + |2x - 3| - 4 = 0$

$x < \frac{3}{2} \Rightarrow x^2 - (2x - 3) - 4 = 0$

$\Rightarrow x^2 - 2x - 1 = 0$

Roots $\alpha, \beta = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

root $1 + \sqrt{2} > \frac{3}{2}$, hence rejected

$\Rightarrow$ root accepted $1 - \sqrt{2}$

$x \geq \frac{3}{2} \Rightarrow x^2 + (2x - 3) - 4 = 0$

$\Rightarrow x^2 + 2x - 7 = 0$

Roots $\gamma, \delta = \frac{-2 \pm \sqrt{4+4\times7}}{2} - 1 \pm 2\sqrt{2}$

root $-1 - 2\sqrt{2} < \frac{3}{2}$, hence rejected

$\Rightarrow$ root accepted $-1 + 2\sqrt{2}$

Sum of roots $= \left(1 - \sqrt{2}\right) + \left(-1 + 2\sqrt{2}\right) = \sqrt{2}$

9. **(4)** Let α, β be roots $x^2 - 5x + 3 = 0$.

$\alpha + \beta = 5,\ \alpha\beta = 3$

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{25-6}{3} = \frac{19}{3}$

and $\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$.

So equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$x^2 - \frac{19}{3}x + 1 = 0 \Rightarrow 3x^2 - 19x + 3 = 0$.

10. **(4)** $\frac{a_{10}-2a_8}{2a_9} = \frac{(\alpha^{10}-\beta^{10})-2(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)}$

$= \frac{\alpha^8(\alpha^2-2)-\beta^8(\beta^2-2)}{2(\alpha^9-\beta^9)}$

($\because \alpha$is root of $x^2 - 6x - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$)

($\because$Also, β is root of $x^2 - 6x - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$)

$= \frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{2(\alpha^9-\beta^9)} = \frac{6(\alpha^9-\beta^9)}{2(\alpha^9-\beta^9)} = 3$