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Physics - Waves

Formulae

1. Velocity of sound wave $v=n\lambda$ and frequency $n=(1/T)$

2. Velocity of transverse waves in a string : $v=\sqrt{\frac{T}{m}}=\sqrt{\frac{T}{\pi r^2 d}}$

3. Velocity of longitudinal waves :

   a) in solids: $v=\sqrt{(Y/\rho)}$ (Y-Young's modulus, $\rho=$ density)

   b) In liquids : $v=\sqrt{(B/\rho)}$ (B=Bulk modulus)

   c) In gases : $v=\sqrt{(\gamma P/\rho)}$ (Laplace formula)

4. **Effect of temperature :**

   a) $v=v_0\sqrt{(T/273)}$ or $v=v_0+0.61t$

   b) $(v_{sound}/v_{rms})=\sqrt{(\gamma/3)}$

5. **Wave equation :**

   a) $y=a\sin\frac{2\pi}{\lambda}(vt-x)$

   b) $y=a\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$

   c) $y=a\sin(\omega t-kx)$, where wave velocity $v=\frac{\omega}{k}=n\lambda$

6. **Particle velocity :**

   a) $v_{particle}=(\partial y/dx)=ka\cos(\omega t-kx)$

   b) maximum particle velocity, $(V_{particle})\max=\omega a$

7. **Slope of the wave :**

   a) slope$=(\partial y/dx)=-ka\cos(\omega t-kx)$

   b) $\mathrm{V_{particle}}=$ wave velocity x slope of the wave

8. **Wave equation** : $\frac{\partial^2 y}{\partial t^2}=v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$

9. **Intensity of sound waves :**

   a) $I=(E/At)$

   b) If ρ is the density of the medium; v the velocity of the wave 'n' is the frequency and 'a' the amplitude then $I=2\pi^2\rho n^2 a^2$ i.e., $I\alpha n^2 a^2$

c)Intensity level is decibel : $\beta 10\log\left(I/I_0\right)$. Where, $I_0 =$ Threshold of hearing $=10^{-12}\,watt/m^2$

10. **Principle of superposition** : $y = y_1 + y_2$

11. **Resultant amplitude :** $a=\sqrt{\left(a_1^2+a_2^2+2a_1a_2\cos\phi\right)}$

12. **Resultant amplitude :** $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

a) For constructive interference $\phi = 2n\pi, a_{\max} = a_1 + a_2$ and $I_{\max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

b) For destructive interference : $\phi = (2n-1)\pi, a_{\min} = a_2 - a_2$ and $I_{\min} = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2$

13. a) Beat frequency $= n_1 - n_2$ and beat period $T = \left(T_1T_2/T_2 - T_1\right)$

b) If there are forks in successive order each giving x beat/sec with nearest neighbor, then $\eta_{last} = \eta_{first} + (N-1)x$

14. **Stationary waves :**

a) When the wave is reflected from a free boundary, is :

$$y = +2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi t}{T} = 2a\cos kx\sin\omega t$$

b) When the wave is reflected from a rigid boundary, is :

$$Y = -2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T} = -2a\sin kx\cos\omega t$$

15. **Vibrations of a stretched string :**

a) For fundamental tone : $n_1 = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$

b) For ρ th harmonic : $n_\rho = \frac{p}{\lambda}\sqrt{\frac{T}{m}}$

c) The ratio of successive harmonic frequencies $n_1 : n_2 : n_3 : ..... = 1:2:3:...$

d) Sonometer : $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}\ \left(m = \pi r^2 d\right)$

e) Melde's experiment i) Transverse mode : $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

ii) Longitudinal model : $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

16. **Vibrations of closed organ pipe**

a) For fundamental tone: $n_1 = \left(\frac{v}{4L}\right)$

b) For first overtone (third harmonic): $n_2 = 3n_1$

c) Only odd harmonics are found in the vibrations of a closed organ pipe and $n_1 : n_2 : n_3 : ..... = 1:3:5: .....$

17. **Vibrations of open organ pipe :**

a) For fundamental tone : $n_1 = \left(v/2L\right)$

b) For first overtone (second harmonic) : $n_2 = 2n_1$

c) Both even and odd harmonics are found in the vibrations of an open organ pipe and $n_1 : n_2 : n_3 .... = 1:2:3:....$

18. **End correction :**

a) Closed pipe : $L = L_{pipe} + 0.3d$

b) Open pipe : $L = L_{pipe} + 0.6d$

where d=diameter =2r

19. **Resonance column** :

a) $\ell_1 + e = \frac{\lambda}{4}$; b) $\ell_2 + e = \frac{3\lambda}{4}$

c) $e = \frac{\ell_2 - 3\ell_1}{2}$; d) $n = \frac{v}{2(\ell_2 - \ell_1)}$ $(\lambda = 2(\ell_2 - \ell_1))$

e) Velocity of sound $v = 2n(\ell_2 - \ell_1)$

20. **Kundt's tube :** $\frac{v_{air}}{v_{rod}} = \frac{\lambda_{air}}{\lambda_{rod}}$

21. **Longitudinal vibration of rods :**

a) Both ends open and clamped in middle :

i) Fundamental frequency, $n_1 = (v/2\ell)$

ii) Frequency of first overtone, $n_2 = 3n_1$

iii) Ratio of frequencies, $n_1 : n_2 : n_3 ..... = 1:3:5:....$

b) One end clamped

i) Fundamental frequency, $n_1 = (v/4\ell)$

ii) Frequency of first overtone, $n_2 = 3n_1$

iii) Ratio of frequencies, $n_1 : n_2 : n_3 :.... = 1:3:5:....$

22. **Frequency of a turning fork :** $n = A\sqrt{\frac{Yt}{\rho\ell^2}}$

Where t= thickness, $\ell =$ length of prong, $Y =$ Young's modulus, $A =$ area of cross section and $\rho =$ density

23. **Doppler Effect for Sound :**

a) Observer stationary and source moving :

i) Source approaching : $n' = \frac{v}{v - v_s} \times n$ and $\lambda' = \frac{v - v_s}{v} \times \lambda$

ii) Source receding : $n' = \frac{v}{v + v_s} \times n$ and $\lambda' = \frac{v + v_s}{v} \times \lambda$

b) Source stationary and observe moving :

i) Observer receding away from source : $n' = \frac{v + v_0}{v} \times n$ and $\lambda' = \lambda$

ii) Observer receding away from source : $n'\frac{v-v_0}{v}\times n$ and $\lambda'=\lambda$

c) Source and observe both moving :

i) S and O moving towards each other: $n'=\frac{v+v_0}{v-v_s}\times n$

ii) S and O in moving away from each other : $n'=\frac{v-v_0}{v-v_s}\times n$

iii) S and O in same direction, S behind O: $n'=\frac{v-v_0}{v-v_s}\times n$

iv) S and O in same directions, S ahead of O: $n'=\frac{v+v_0}{v+v_s}\times n$

d) Effect of motion of medium : $n'=\frac{v\pm v_m\pm v_0}{v\pm v_m\pm v_s}$

e) Change in frequency : (i) Moving source passes a stationary observer.

$\Delta n=\frac{2vv_s}{v^2-v_s^2}\times n$

for $v_s << v_,\Delta,=\frac{2v_s}{v}\times n$

ii) Moving observe passes a stationary source: $\Delta n=\frac{2v_0}{v}\times n$

f) Source moving towards or away from hill or wall

i) Source moving towards wall

a) Observer between source and wall

$n'=\frac{v}{v-v_s}\times n$ (for direct waves)

$n'=\frac{v}{v-v_s}\times n$ (for reflected waves)

b) Source between observe and wall

$n'=\frac{v}{v+v_s}\times n$ (for direct waves)

$n'=\frac{v}{v-v_s}\times n$ (for reflected waves)

ii) Source moving away form wall

a) Observer between source and wall

$n'=\frac{v}{v+v_s}\times n$ (for direct waves)

$n'=\frac{v}{v+v_s}\times n$ (for reflected waves)

b) Source between observe and wall

$n'=\frac{v}{v-v_s}\times n$ (for direct waves)

$n' = \frac{v}{v+v_s} \times n$ (for reflected waves)

g)Moving Target :

i) S and O stationary at the same place and target approaching with speed u

$n' = \left(\frac{v+u}{v-u}\right) \times n$ or $n' = \left(1+\frac{2u}{v}\right) \times n$ (for u <<v)

ii) S and O stationary at the same place and target receding with speed u

$n' = \left(\frac{v-u}{v+u}\right) \times n$ or $n' = \left(1-\frac{2u}{v}\right) \times n$ (for u <<v)

h) **SONAR** : $n' = \frac{v \pm v_{sub}}{v \pm v_{sub}} \times n \cong \left(1 \pm \frac{2v_{sub}}{v}\right) \times n$

(upper sign for approaching submarine while lower sign for receding submarine)

i) Transverse Doppler effect : There is no transverse Doppler effect in sound. For velocity component $v_s \cos\theta$

$n' = \frac{v}{v \pm v_s \cos\theta} \times n$ (- sign for approaching and + sign for receding)

24. **Doppler Effect for light**

a) Red shift (When light source is moving away):

$n' = \sqrt{\frac{1-v/c}{1+v/c}} \times n$ or $\lambda' = \sqrt{\frac{1+v/c}{1-v/c}} \times \lambda$

For v<< $c_r \Delta n = -\left(\frac{v}{c} \times n\right)$ or $\Delta\lambda' = \left(\frac{v}{c}\right) \times \lambda$

b) Blue shift (when light source is approaching)

$n' = \sqrt{\frac{1+v/c}{1-v/c}} \times n$ or $\lambda' = \sqrt{\frac{1-v/c}{1+v/c}} \times \lambda$

For v<< $c_r \Delta n = \left(\frac{v}{c}\right) n$ or $\Delta\lambda' = -\left(\frac{v}{c}\right) \lambda$

Exercise : I

(Straight Objective Including PYQ's)

I) Introduction to waves,Travelling wave

Definition of wave, 1-D,2-D,3-D waves, Mechanical and non mechanical waves (Mechanical waves, Non mechanical waves), Longitudinal and transverse waves (Longitudinal wave, Transverse wave), Equation of travelling wave (Equation of plane progressive harmonic wave, Amplitude ;frequency and speed of the wave from the equation), Relation between path difference and phase difference - Equation of plane progressive harmonic wave, Displacement ;velocity and acceleration of the particle, Relation between wave velocity and particle velocity, Speed of the transverse wave in a string, Speed of the transverse wave in solids, Sound wave(Speed of sound wave in solids, Speed of sound wave in liquids and gases, Conversion from displacement variation to pressure variation, Speed of the longitudinal wave), Energy of a progressive wave, Sub-Sonic, Super sonic, Mach number, Audible range, infra sonic and ultra sonic, Intensity of sound wave(Formula for intensity, Variation of intensity with distance, Sound level).

1*. In longitudinal & transverse waves the particles of the medium vibrate respectively are

A) Parallel & perpendicular to direction of propagation

B) Perpendicular & Parallel to direction of propagation

C) Parallel & remains stationary

D) Perpendicular & remains stationary

Key : A

2*. For an E.M wave to propagate

A) a medium with elasticity, inertia & density is required

B) a medium with high pressure & low temperature is required

C) a medium with low pressure & high temperature is required

D) No medium is essential.

Key : D

4*. An observer standing at sea-coast observes 54 waves reaching the coast per minute. If the wavelength of wave is 10m. Find the velocity what type of waves are observed ?

A) $9\,ms^{-1}$ and transverse waves

B) $9\,ms^{-1}$ combined longitudinal and transverse waves

C) $9\,ms^{-1}$ longitudinal waves

D) $9\,ms^{-1}$ Electro magnetic wave

Key : B

Sol : Frequency $n=\frac{54}{60}=\frac{9}{10}Hz$; $\lambda=10m$

$\therefore\ v=n\lambda=10\times\frac{9}{10}=9ms^{-1}$

3. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by

A) $\sqrt{\frac{m_1}{m_2}}$ B) $\sqrt{\frac{m_2}{m_1}}$ C) $\frac{m_1}{m_2}$ D) $\frac{m_2}{m_1}$

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Key : B

Sol : $V=\sqrt{\frac{\gamma RT}{M}} \Rightarrow v\alpha\frac{1}{\sqrt{M}}$

$\frac{V_1}{V_2}=\sqrt{\frac{M_2}{M_1}}=\sqrt{\frac{m_2}{m_1}}$

3a. Two diatomic ideal gases 1 and 2 of molecular masses 28 and 32 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by

A) $\sqrt{\frac{8}{7}}$ B) $\sqrt{\frac{7}{8}}$ C) $\frac{8}{7}$ D) $\frac{7}{8}$

Key : A

Sol : $V=\sqrt{\frac{\gamma RT}{M}} \Rightarrow v\alpha\frac{1}{\sqrt{M}}$

$\frac{V_1}{V_2}=\sqrt{\frac{M_2}{M_1}}=\sqrt{\frac{32}{28}}=\sqrt{\frac{8}{7}}$

4. Travelling wave in a stretched string is described by the equation $y=A\sin(kx-\omega t)$. The maximum particle velocity is **IIT - 1997**

A) $A\omega$ B) v C) $d\omega/dk$ D) x/t

Key : A

Sol : $v=\frac{dy}{dt}=-A\omega\cos(kx-\omega t)$

$\therefore\ v_{\max}=A\omega$

Maximum particle velocity of the particle in SHM, $v_{\max}=A\omega$

4a. Travelling wave in a stretched string is described by the equation $x=40cm\cos(50\pi t-0.02\pi y)$. The maximum particle velocity is in ms^{-1} is

A) 20π B) 10π C) π D) 0

Key : A

Sol : $v_{\max}=A\omega$

Maximum particle velocity of the particle is SHM, $v_{\max}=A\omega$

5. Equation of travelling wave on a stretched string of linear density 5 g/m is y=0.03 $\sin(450t-9x)$ where distance and time are measured is SI units. The tension in the string is **Jee Mains 2019**

A) 10N B) 12.5N C) 7.5N D) 5N

Key : B

Sol: $y=0.03\sin(450t-9x)$; $v=\frac{\omega}{k}=\frac{450}{9}=50m/s$

$v=\sqrt{\frac{T}{\mu}}\Rightarrow\frac{T}{\mu}=2500$

$\Rightarrow T=2500\times5\times10^{-3}=12.5N$

5a. Equation of travelling wave on a stretched string of linear density 10 g/m is y=0.02 $\sin(600t-3x)$ where distance and time are measured is SI units. The tension in the string is

A) 400N B) 120.5N C) 17.5N D) 500N

Key : A

Sol: $v=\omega/k=\frac{600}{3}=200ms^{-1}\Rightarrow 200=\sqrt{\frac{T}{\mu}}\Rightarrow T=400N$

5b. Equation of travelling wave on a stretched string of linear density 5 g/cm is y=0.01 $\sin(50t-x)$ where distance and time are measured is SI units. The tension in the string is

A) 1400N B) 1200.5N C) 1250 N D) 1500N

Key : A

Sol: $v=\omega/k=\frac{50}{1}=50ms^{-1}\quad\Rightarrow 50=\sqrt{\frac{T}{\mu}}\Rightarrow 2500=\frac{T}{5\times10^{-3}/10^{-2}}\Rightarrow T=1250N$

5c. Equation of travelling wave on a stretched string of linear density 0.5 g/cm is y=0.005 $\sin(100t-2x)$ where distance and time are measured is SI units. The tension in the string is

A) 100N B) 125N C) 150 N D) 130N

Key : B

Sol: $v=\omega/k=\frac{100}{2}=50ms^{-1}\quad\Rightarrow 50=\sqrt{\frac{T}{\mu}}\Rightarrow 2500=\frac{T}{0.5\times10^{-3}/10^{-2}}\Rightarrow T=125N$

6. Travelling harmonic wave is represented by the equation $y(x,t)=10^{-3}\sin(50t+2x)$,where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave ? **Jee Mains 2019**

A) Negative x-axis with speed $25ms^{-1}$

B) The wave is propagating along the positive x-axis with speed $25ms^{-1}$

C) The wave is propagating along the positive x-axis with speed $100ms^{-1}$

D) The wave is propagating along the negative x-axis with speed $100ms^{-1}$

Key : A

Sol: $y = a\sin(\omega t + kx)$ represents wave moving along -ve x-axis

$\Rightarrow$ Wave is moving along –ve x-axis with speed $v = \frac{\omega}{K} \Rightarrow v = \frac{50}{2} = 25m/\sec$

6a. Travelling harmonic wave is represented by the equation $y(x,t) = 10^{-3}\sin(100t - 2x)$,where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave ?

A) The wave moves along positive x-axis with a speed $50ms^{-1}$

B) The wave is propagating along the positive x-axis with speed $25ms^{-1}$

C) The wave is propagating along the positive x-axis with speed $100ms^{-1}$

D) The wave is propagating along the negative x-axis with speed $100ms^{-1}$

Key : A

Sol: $y = a\sin(\omega t + kx)$

$\Rightarrow$ Wave is moving along –ve x-axis the standard equation is $a\sin(\omega t + kx)$

$v = \frac{\omega}{K} \Rightarrow v = \frac{100}{2} = 50m/\sec$

6b. Travelling harmonic wave is represented by the equation $y(x,t) = 0.02\sin(50t + 3x)$,where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave ?

A) The wave moves along positive x-axis with a speed $50ms^{-1}$

B) The wave moves along negative x-axis with speed $12.5ms^{-1}$

C) The wave is propagating along the positive x-axis with speed $100ms^{-1}$

D) The wave is propagating along the negative x-axis with speed $100ms^{-1}$

Key : B

Sol: $y = a\sin(\omega t + kx)$

$\Rightarrow$ Wave is moving along –ve x-axis the standard equation is $a\sin(\omega t - kx)$

$v = \frac{\omega}{K} \Rightarrow v = \frac{50}{4} = 12.5m/\sec$

6c. Travelling harmonic wave is represented by the equation $y(x,t) = 0.05\sin(20t - x)$,where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave ?

A) The wave moves along positive x-axis with a speed $50ms^{-1}$

B) The wave moves along negative x-axis with speed $12.5ms^{-1}$

C) The wave is propagating along the positive x-axis with speed $100ms^{-1}$

D) The wave is propagating along the positive x-axis with speed $20ms^{-1}$

Key : D

Sol: $y = a\sin(\omega t + kx)$

$\Rightarrow$ Wave is moving along –ve x-axis the standard equation is $a\sin(\omega t - kx)$

$v = \frac{\omega}{K} \Rightarrow v = \frac{20}{1} = 20m/\sec$

7. The pressure wave $P = 0.01\sin[1000t - 3x]Nm^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0^0C. On some other day when temperature is T, the speed of sound produced by the same balde and at the same frequency is found to be $336ms^{-1}$. Approximate value of T is **Jee Mains 2019**

A) 12^0C B) 11^0C C) 15^0C D) 4^0C

Key : D

Sol: $V_{sound}\alpha\sqrt{T} \Rightarrow \frac{336\times3}{1000} = \sqrt{\frac{T_2}{273}}$

$T_2 = 277K \Rightarrow T_2 = 4^0C$

7a. The pressure wave $P = 0.01\sin[1020t - 2x]Nm^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0^0C. On some other day when temperature is T, the speed of sound produced by the same balde and at the same frequency is found to be $340ms^{-1}$. Approximate value of T is

A) $121.3K$ B) $111.4K$ C) $91.5K$ D) $41K$

Key : C

Sol: $V_{sound}\alpha\sqrt{T} \Rightarrow \frac{340\times2}{1020} = \sqrt{\frac{T_2}{273}}$

$T_2 = \frac{4}{9}\times273 \Rightarrow T_2 = \frac{1092}{9} = 121.3K$

7b. The pressure wave $P = 0.01\sin[1320t - 5x]Nm^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0^0C. On some other day when temperature is T, the speed of sound produced by the same balde and at the same frequency is found to be $330ms^{-1}$. Approximate value of T is

A) 153.6^0C B) $111.4C^0$ C) $91.5K$ D) $41K$

Key : A

Sol: $V_{sound}\alpha\sqrt{T} \Rightarrow \frac{330\times5}{1320} = \sqrt{\frac{T_2}{273}}$

$T_2 = \frac{4}{9}\times273 \Rightarrow T_2 = \frac{25}{16\times273} = 426.6K$

$t_2 = 426.6 - 273 = 153.6^0C$

7c. The pressure wave $P = 0.01\sin[300t - x]Nm^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0^0C. On some other day when temperature is T, the speed of sound produced by the same balde and at the same frequency is found to be $330ms^{-1}$. Approximate value of T is

A) $0K$ B) 0^0C C) $-273K$ D) $41K$

Key : B

Sol: $V_{sound}\alpha\sqrt{T} \Rightarrow \frac{300\times1}{300}=\sqrt{\frac{T_2}{273}}$

$T_2 = 273K \Rightarrow t_2 = 273-273 = 0^0 C$

8. Assume that the displacement(s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement(s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10Pa$, $v \sim 300m/s, p \sim 1kg/m^3$ and $f \sim 1000Hz$, then v will be of the order of (take the multiplicative constant to be 1)

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A) $\frac{3}{100}mm$ B) $10mm$ C) $\frac{1}{10}mm$ D) $1mm$

Key : A

Sol : As we know,

Pressure amplitude, $\Delta P_0 = aKB = S_0KB = S_0 \times \frac{\omega}{V} \times \rho V^2$

$\left[\because K = \frac{\omega}{V}, V = \sqrt{\frac{B}{\rho}}\right]$

$\Rightarrow S_0 = \frac{\Delta P_0}{\rho V\omega} \approx \frac{10}{1\times300\times1000}m = \frac{1}{30}mm \approx \frac{3}{100}mm$

8a. Assume that the displacement(s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement(s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 5Pa$, $v \sim 300m/s, p \sim 1kg/m^3$ and $f \sim 2000Hz$, then v will be of the order of (take the multiplicative constant to be 1)

A) $\frac{1}{120}mm$ B) $\frac{1}{240}mm$ C) $\frac{2}{260}mm$ D) $\frac{3}{260}mm$

Key : A

Sol : As we know,

Pressure amplitude, $\Delta P_0 = aKB = S_0KB = S_0 \times \frac{\omega}{V} \times \rho V^2$

$\left[\because K = \frac{\omega}{V}, V = \sqrt{\frac{B}{\rho}}\right]$

$\Rightarrow S_0 = \frac{\Delta P_0}{\rho V\omega} \approx \frac{5}{1\times300\times2000}m = \frac{5}{600}mm = \frac{1}{120}mm$

8b. Assume that the displacement(s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement(s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 20Pa$, $v \sim 320m/s, p \sim 1kg/m^3$ and $f \sim 1000Hz$, then v will be of the order of (take the multiplicative constant to be 1)

A) $\frac{2}{16}mm$ B) $\frac{4}{20}mm$ C) $\frac{3}{18}mm$ D) $\frac{1}{16}mm$

Key : D

Sol : As we know,

Pressure amplitude, $\Delta P_0 = aKB = S_0 KB = S_0 \times \frac{\omega}{V} \times \rho V^2$

$\left[\because K = \frac{\omega}{V}, V = \sqrt{\frac{B}{\rho}}\right]$

$\Rightarrow S_0 = \frac{\Delta P_0}{\rho V \omega} \approx \frac{20}{1\times 320\times 1000} m = \frac{1}{16} mm$

8c. Assume that the displacement(s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement(s) further depends on the speed of sound (v), density of air (ρ) and the frequency(f). If $\Delta p \sim 15 Pa$, $v \sim 300 m/s, p \sim 1 kg/m^3$ and $f \sim 500 Hz$, then v will be of the order of (take the multiplicative constant to be 1)

A) $\frac{2}{16} mm$ B) $\frac{2}{10} mm$ C) $\frac{1}{10} mm$ D) $\frac{1}{16} mm$

Key : C

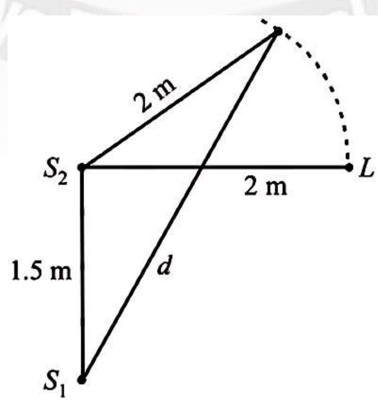
Sol : As we know,

Pressure amplitude, $\Delta P_0 = aKB = S_0 KB = S_0 \times \frac{\omega}{V} \times \rho V^2$

$\left[\because K = \frac{\omega}{V}, V = \sqrt{\frac{B}{\rho}}\right]$

$\Rightarrow S_0 = \frac{\Delta P_0}{\rho V \omega} \approx \frac{15}{1\times 300\times 500} m = \frac{1}{10} mm$

9. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 1m$, in phase. S_1 and S_2 are placed 1.5m apart (see fig.). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2m away from S_2. The listener moves away from S_1 ,keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed then the listener is at distance d from S_1. Then, d is : **Jee Mains 2020 Sep 5th PM**



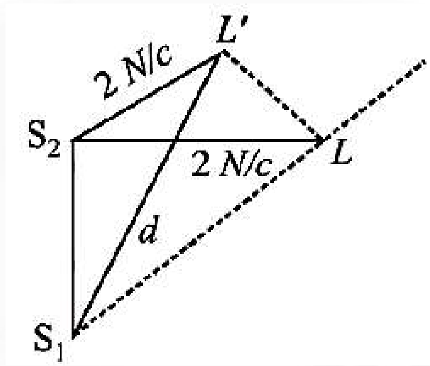
A) 12m B) 5m C) 2m D) 3m

Key : D

Sol : Initially, $S_2L = 2m$

$$S_1L = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \frac{5}{2} = 2.5m$$

Path difference, $\Delta x = S_1L - S_2L = 0.5m = \frac{\lambda}{2}$

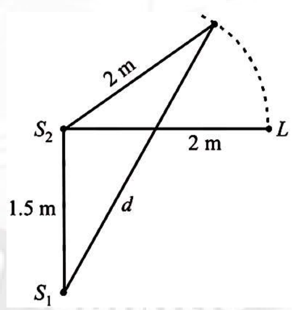


When the listener move from L, first maxima will appear if path difference is integral multiple of wavelength. For example

$\Delta x = n\lambda = 1\lambda$ (n=1 for first maxima)

$\therefore \Delta x = \lambda = S_1L' - S_2L \quad \Rightarrow 1 = d - 2 \Rightarrow d = 3m$

9a. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 2m$, in phase. S_1 and S_2 are placed 1.5m apart (see fig.). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 3m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed then the listener is at distance d from S_1. Then, d is :



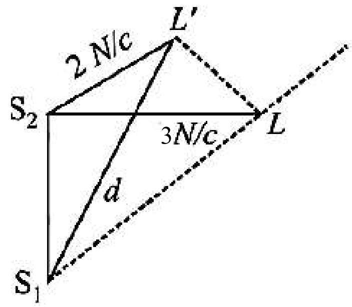
A) 4m B) 2m C) 1m D) 5m

Key : A

Sol : Initially, $S_2L = 2m$

$$S_1L = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \frac{5}{2} = 2.5m$$

Path difference, $\Delta x = S_1L - S_2L = 0.5m = \frac{\lambda}{2}$

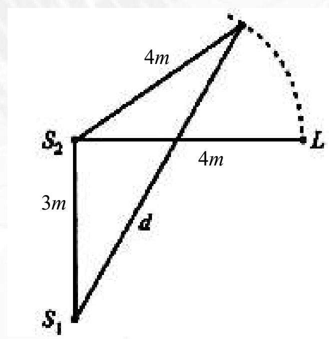


When the listener move from L, first maxima will appear if path difference is integral multiple of wavelength. For example

$\Delta x = n\lambda = 1\lambda$ (n=1 for first maxima)

$\therefore \; \Delta x = \lambda = S_1L' - S_2L \quad \Rightarrow 1 = d - 3 \Rightarrow d = 4m$

9b. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 3m$, in phase. S_1 and S_2 are placed 3m apart (see fig.). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 4m away from S_2. The listener moves away from S_1,keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed then the listener is at distance d from S_1. Then, d is :



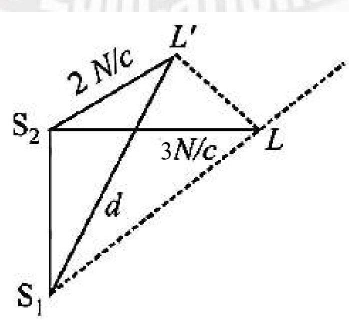
A) 4m B) 6m C) 1m D) 5m

Key : A

Sol : Initially, $S_2L = 4m$

$S_1L = \sqrt{3^2 + 4^2} = 5m$

Path difference, $\Delta x = S_1L - S_2L = 1m = \frac{\lambda}{2}$



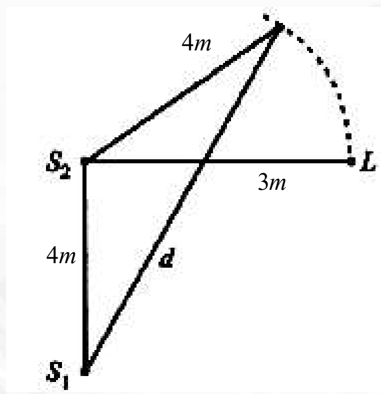
When the listener move from L, first maxima will appear if path difference is integral multiple of wavelength. For example

$\Delta x = n\lambda = 1\lambda$ (n=1 for first maxima)

$\therefore \Delta x = \lambda = S_1L' - S_2L$

$\Rightarrow 1 = d - 4 \Rightarrow d = 5m$

9c. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 2m$, in phase. S_1 and S_2 are placed 4m apart (see fig.). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 3m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed then the listener is at distance d from S_1. Then, d is :



A) 4m B) 6m C) 1m D) 5m

Key : D

Sol : Initially, $S_2L = 4m$

$S_1L = \sqrt{3^2 + 4^2} = 5m$

Path difference, $\Delta x = S_1L - S_2L = 1m = \frac{\lambda}{2}$

When the listener move from L, first maxima will appear if path difference is integral multiple of wavelength. For example

$\Delta x = n\lambda = 1\lambda$ (n=1 for first maxima)

$\therefore \Delta x = \lambda = S_1L' - S_2L$ $\Rightarrow 2 = d - 3 \Rightarrow d = 5m$

10. Which of the following equations represents a travelling wave? **Jee Mains 2021**

A) $y = Ae^{-x^2}(vt + \theta)$ B) $y = A\sin(15x - 2t)$

C) $y = Ae^{x}\cos(\omega t - \theta)$ D) $y = A\sin x \cos \omega t$

Key: B

Sol: $Y = F(x, t)$

For travelling wave y should be linear function of $x \& t$

and they must exist as $(x \pm vt)$

So $y = A\sin(15x - 2t)$

10a. Which of the following equations represents a travelling wave?

A) $y = Ae^{-x^2}(vt+\theta)$ B) y = Asin(30x - 2t)

C) $y = Ae^{x}\cos(\omega t - \theta)$ D) $y = A\sin x \cos \omega t$

Key: B

Sol: Y = F(x, t)

For travelling wave y should be linear function of $x \,\&\, t$

and they must exist as $(x \pm vt)$

So y = Asin(30x - 2t)

10b. Which of the following equations represents a travelling wave?

A) y = Asin(45x - 2t) B) $y = Ae^{-x^2}(vt+\theta)$

C) $y = Ae^{x}\cos(\omega t - \theta)$ D) $y = A\sin x \cos \omega t$

Key: A

Sol: Y = F(x, t)

For travelling wave y should be linear function of $x \,\&\, t$ and they must exist as $(x \pm vt)$

So y = Asin(45x - 2t)

10c. Which of the following equations represents a travelling wave?

A) $y = Ae^{2x}\cos(\omega t - \theta)$ B) $y = Ae^{-x^2}(vt+\theta)$

C) y = Asin(60x - 2t) D) $y = A\sin x \cos \omega t$

Key: C

Sol: Y = F(x, t)

For travelling wave y should be linear function of $x \,\&\, t$

and they must exist as $(x \pm vt)$ So y = Asin(60x - 2t)

11. A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave?

Jee Mains 2021

A) $Y(x,\ t) = 0.03\left[\sin 5.1x - \left(0.2 \times 10^3\right)t\right]$ B) $Y(x,\ t) = 0.06\left[\sin 5.1x - \left(1.5 \times 10^3\right)t\right]$

C) $Y(x,\ t) = 0.06\left[\sin 0.8x - \left(0.5 \times 10^3\right)t\right]$ D) $Y(x,\ t) = 0.03\left[\sin 5.1x - \left(1.5 \times 10^3\right)t\right]$

Key: D

Sol: $\omega = 2\pi f$

$= 1.5 \times 10^3$

$A = \frac{6}{2} = 3cm = 0.03m$

$k = \omega/v = \frac{2\pi \times 245}{300} = 5.12m^{-1}$

11a. A sound wave of frequency 125 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 10 cm. What will be the mathematical expression of this travelling wave?

A) $Y(x, t) = 0.03\left[\sin 5.1x - \left(0.2 \times 10^3\right)t\right]$ B) $Y(x, t) = 0.06\left[\sin 5.1x - \left(1.5\times10^3\right)t\right]$

C) $Y(x, t) = 0.05\left[\sin 2.61x - \left(0.785\times10^3\right)t\right]$ D) $Y(x, t) = 0.03\left[\sin 5.1x - \left(1.5\times10^3\right)t\right]$

Key: C

Sol: $\omega = 2\pi f$

$= 1.5\times10^3$

$A = \frac{10}{2} = 5cm = 0.05m$

k=ω/v=$\frac{2\pi\times125}{300} = 2.61m^{-1}$

11b. A sound wave of frequency 490 Hz travels with the speed of 300 ms^{-1} along the positive
x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What
will be the mathematical expression of this travelling wave?

A) $Y(x, t) = 0.03\left[\sin 10.25x - (3077.2\ t)\right]$ B) $Y(x, t) = 0.06\left[\sin 5.1x - \left(1.5\times10^3\right)t\right]$

C) $Y(x, t) = 0.05\left[\sin 2.61x - \left(0.785\times10^3\right)t\right]$ D) $Y(x, t) = 0.03\left[\sin 5.1x - \left(1.5\times10^3\right)t\right]$

Key: A

Sol: $\omega = 2\pi f$

$= 1.5\times10^3$

$A = \frac{6}{2} = 3cm = 0.03m$

k=ω/v=$\frac{2\pi\times490}{300} = 10.25m^{-1}$

11c. A sound wave of frequency 120 Hz travels with the speed of 300 ms^{-1} along the positive
x-axis. Each point of the wave moves to and fro through a total distance of 10 cm. What
will be the mathematical expression of this travelling wave?

A) $Y(x, t) = 0.03\left[\sin 10.25x - (3077.2\ t)\right]$ B) $Y(x, t) = 0.06\left[\sin 5.1x - \left(1.5\times10^3\right)t\right]$

C) $Y(x, t) = 0.05\left[\sin 2.512x - (753.6)t\right]$ D) $Y(x, t) = 0.03\left[\sin 5.1x - \left(1.5\times10^3\right)t\right]$

Key: C

Sol: $\omega = 2\pi f$ $= 1.5\times10^3$

$A = \frac{10}{2} = 5cm = 0.05m$

$k = \omega / v = \frac{2\pi \times 120}{300} = 2.512 m^{-1}$

12. A longitudinal wave is represented by $y = 10\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$ cm. The maximum particle velocity will be four times the wave velocity then determined value of wavelength is equal to __cm **Jee Main 2022**

A) 2π B) 5π C) π D) $\frac{5\pi}{2}$

Key :B

Sol : $Vp = 4.V_w; \Rightarrow A\omega = 4 \times \omega / K \Rightarrow 10 = 4.\frac{\lambda}{2\pi} \quad 5\pi = \lambda$

12a. A longitudinal wave is represented by $y = 5\sin 10\pi t - (x/\lambda)$ cm. The maximum particle velocity will be four times the wave velocity then determined value of wavelength is equal to ___cm:

A) 2π B) 5π C) π D) $\frac{5\pi}{2}$

Key :D

Sol : $Vp = 4.V_w; V_p = 4\frac{\omega}{k} \Rightarrow 5(10\pi) = 4\left(\frac{10\pi}{k}\right) \Rightarrow k = \frac{4}{5} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{5\pi}{2}$

12b. A longitudinal wave is represented by $y = 10\sin 314t - (x/\lambda)$ cm. The maximum particle velocity will be three times the wave velocity then determined value of wavelength is equal to ___cm nearly

A) 21 B) 5 C) 1 D) 10

Key : A

Sol : $Vp = 4.V_w; V_p = 3\frac{\omega}{k} \Rightarrow 3140 = 3 \times 50 \times \lambda$

$\lambda = \frac{3140}{150} = 20.9 \approx 21 cm$

12c. A longitudinal wave is represented by $y = 0.2\sin\left(\pi t - \frac{x}{\lambda}\right)$ cm. The maximum particle velocity will be two times the wave velocity then determined value of wavelength is equal to ___cm nearly

A) 0.2π B) 0.4π C) 0.8π D) 0.9π

Key : A

Sol : $Vp = 2.V_w; V_p = 2\frac{\omega}{k} \Rightarrow 0.2 \times \pi = 2(1/2)(\lambda) \Rightarrow \lambda = 0.2\pi cm$

13 A transverse wave is represented by $y = 2\sin(\omega t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity will be : **Jee Mains 2022**

A) 4π B) 2π C) π D) 2

Key :A

Sol : Given $n\lambda = A\omega$

$$n\lambda = A\omega \frac{\omega}{2\pi}\lambda = 2\omega \Rightarrow \lambda = 4\pi$$

13a. A transverse wave is represented by $y = 0.2\sin(100t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity will be ___ cm.

A) 4 B) 10 C) π D) 1.26

Key :B

Sol : Given $Vp_{\max} = v_{\omega}$ $n\lambda = A\omega$

$$\Rightarrow 0.2 \times 100 = \frac{100}{2\pi} \times \lambda = 0.4\pi = 0.4 \times 3.14 = 1.256cm$$

13b. A transverse wave is represented by $y = 10\sin(50t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes twice the maximum particle velocity will be ___ cm.

A) 4 B) 10π C) π D) 1.26

Key :B

Sol : Given $Vp_{\max} = 2v_{\omega}$ $n\lambda = A\omega$

$$\Rightarrow 10 \times 50 = 2(50/2\pi)(\lambda) \Rightarrow \lambda = 10\pi$$

13c. A transverse wave is represented by $y = 5\sin 2\pi(5t - x/\lambda)$ cm. The value of wavelength (in cm) for which the wave velocity becomes four times the maximum particle velocity will be ___ cm.

A) 2.5π B) 10π C) π D) 1.26

Key : A

Sol : Given $n\lambda = A\omega$

$$Vp_{\max} = 4v_{\omega} \Rightarrow 5 \times 10\pi = 4(5)(\lambda) \Rightarrow \lambda = 2.5\pi cm$$

14*. In a progressive wave two particles are differing in time by half of time period. The path difference & phase difference between the particles is

A) $\frac{\lambda}{2}$ & $\frac{\pi}{2}$ B) $\frac{\lambda}{2}, \pi$ C) $\lambda, \frac{\pi}{2}$ D) λ, π

Key : B

Sol : For 2π (phase difference) – λ (path difference)- T (Time difference)

$\therefore$? - $T/2$

Path difference $\Delta x = \frac{T}{2} \times \frac{\lambda}{2} = \frac{\lambda}{2}$; $\Delta\phi = \frac{T}{2} \times \frac{2\pi}{T} = \pi rad$

15*. Find the change in volume of 6lit., of alcohol if the pressure is decreased from 200cm of Hg to 75cm of Hg. Velocity sound in alcohol is 1280 m/sec and density of alcohol is 0.81 g/cc density of Hg 13.6 g/cc and g $= 9.8 m/s^{-1}$

A) 1cc B) 0.75cc C) 0.5cc D) 0.25cc

Key : B

Sol : $v=\sqrt{\frac{B}{\rho}}$ $\qquad B=v^2\rho$

But $B=-v\frac{\Delta P}{\Delta V}=v^2\rho$

$\Delta v=\frac{v(-\Delta p)}{\rho v^2}$

$=\frac{-6\times10^{-3}(75-200)\times13.6\times981}{0.81\times(1.28\times10^5)^2}=0.75cc$

16*. A copper bar 1m long is clamped at its middle and set vibrating. Given that $v=\sqrt{\frac{y}{\rho}}$.

Density $\rho=8.9\times10^3 kg/m^3$ and $y=11\times10^{10}N/m^2$. Find the frequency of vibration

A) $1.8kHz$ B) $1.15kHz$ C) $15kHz$ D) $10kHz$

Key : A

Sol: $v=\sqrt{\frac{11\times10^{10}}{8.9\times10^3}}=10^4\sqrt{\frac{11}{89}}=0.351\times10^4$ $\qquad L=\frac{\lambda}{2}\Rightarrow\lambda=2\times1=2m$

$n=\frac{v}{\lambda}=\frac{3510}{2}=1755Hz\approx1.8kHz$

17*. A sound wave of $\lambda=40cm$ travels in air. If the difference between the maximum of minimum pressures at a given point is $1\times10^{-3}Nm^{-2}$. Find the displacement amplitude of vibration of particles of the medium. Given bulk modulus of air is $1.4\times10^5Nm^{-2}$

A) $2.2\times10^{-10}m$ B) $4.4\times10^{-10}m$ C) $3.2\times10^{-10}m$ D) $6.2\times10^{-10}m$

Key : A

Sol : $P_0=\frac{1\times10^{-3}}{2}=0.5\times10^{-3}Nm^{-2}$

Displacement amplitude $S_0=\frac{P_0}{BK}=\frac{P_o\lambda}{B\times2\pi}$

$S_0=\frac{0.5\times10^{-3}\times40\times10^{-2}}{2\times3.14\times1.4\times10^5}=2.2\times10^{-10}m$

18*. The pressure amplitude of a sound wave from a radio receiver is $2\times10^{-2}Nm^{-2}$ and intensity at the point is $5\times10^{-7}Wm^{-2}$. If by tuning the volume knob the pressure amplitude is increased to $2.5\times10^{-2}Nm^2$. Find the intensity

A) $9.8\times10^{-8}Wm^{-2}$ B) $7.8\times10^{-7}Wm^{-2}$

C) $9.8\times10^{-7}Wm^{-2}$ D) $17.8\times10^{-7}Wm^{-2}$

Key : B

Sol: $\frac{I^1}{I}=\left(\frac{P_0^1}{P_0}\right)^2$

$$I^1 = \left(\frac{2.5}{2}\right)^2 \times 5\times10^{-7} = 7.8\times10^{-7} Wm^{-2}$$

19. A source of sound of frequency 600Hz is placed inside water. The speed in water is $1500m/s$ and in air it is 300 m/s. The frequency of sound recorded by an observer is standing in air is **IIT 2004**

A) 200 Hz B) 300 Hz C) 120 Hz D) 600 Hz

Key : D

Sol : The frequency is a characteristic of source. It is independent of the medium.

19a. A source of sound of frequency 220Hz is placed inside water. The speed in water is $1500m/s$. What is the frequency of same sound observed in air, when speed of sound in air is 340 m/s.

A) 240 Hz B) 300 Hz C) 220 Hz D) 600 Hz

Key : C

Sol : The frequency is a characteristic of source. It is independent of the medium.

II) Superposition of waves

Constructive interference - Superposition of waves, Destructive interference - Superposition of waves, Resultant intensity

1. Two coherent sources produces waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio **Jee Mains 2019**

A) 4:1 B) 25:9 C) 16:9 D) 5:3

Key : B

Sol: $\frac{I_{\max}}{I_{\min}} = 16 \Rightarrow \frac{A_{\max}}{A\min} = 4 \Rightarrow \frac{A_1+A_2}{A_1-A_2} = 4A_1 - 4A_2 = A_1 + A_2$

$3A_1 = 5A_2 \Rightarrow \frac{A_1}{A_2} = \frac{5}{3} \Rightarrow \frac{I_1}{I_2} = \frac{25}{9}$

1a. Two coherent sources produces waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 4. The intensity of the waves are in the ratio

A) 4:1 B) 25:9 C) 9:1 D) 5:3

Key : C

Sol: $\frac{I_{\max}}{I_{\min}} = 16 \Rightarrow \frac{A_{\max}}{A\min} = 2 \Rightarrow \frac{A_1+A_2}{A_1-A_2} = 1$

$2A_1 - 2A_2 = A_1 + A_2$

$A_1 = 3A_2 \Rightarrow \frac{A_1}{A_2} = 3 \Rightarrow \frac{I_1}{I_2} = 9:1$

1b. Two coherent sources produces waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 9. The intensity of the waves are in the ratio

A) 4:1 B) 25:9 C) 9:1 D) 5:3

Key : A

Sol: $\frac{I_{max}}{I_{min}} = 16 \Rightarrow \frac{A_{max}}{A\min} = \frac{3}{1} \Rightarrow \frac{A_1 + A_2}{A_1 - A_2}$

$3A_1 - 3A_2 = A_1 + A_2$

$2A_1 = 4A_2 \Rightarrow \frac{A_1}{A_2} = \frac{2}{1} \Rightarrow \frac{I_1}{I_2} = 4:1$

1c. Two coherent sources produces waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 25. The intensity of the waves are in the ratio

A) 4:1 B) 25:9 C) 9:1 D) 9:4

Key : D

Sol: $\frac{I_{max}}{I_{min}} = 25 \Rightarrow \frac{A_{max}}{A\min} = 5 \Rightarrow \frac{A_1 + A_2}{A_1 - A_2}$

$5A_1 - 5A_2 = A_1 + A_2$

$4A_1 = 6A_2 \Rightarrow \frac{A_1}{A_2} = \frac{3}{2} \Rightarrow \frac{I_1}{I_2} = 9:4$

2. A small speaker delivers 2W of audio output. At what distance from the speaker will one detect 120dB intensity sound ? [Given reference intensity of sound as $10^{-12} W/m^1$]

Jee Mains 2019

A) 10cm B) 20cm C) 40cm D) 30cm

Key : C

Sol: $\beta = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 120 = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 10^{12} = \frac{I}{I_0}$

$= 1 = \frac{\rho}{4\pi d^2} \Rightarrow d^2 = \frac{2}{4\pi}$

$\Rightarrow d = \frac{1}{\sqrt{2\pi}} = 0.3989m$

$d \approx 40cm$

2a. A small speaker delivers 5W of audio output. At what distance from the speaker will one detect 100dB intensity sound ? [Given reference intensity of sound as $10^{-12} W/m^1$]

A) 10m B) 6.3m C) 4m D) 3m

Key : B

Sol: $\beta = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 100 = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 10^{10} = \frac{I}{I_0} \Rightarrow I = 10^{-12} \times 10^{10} = 10^{-2}$

$= 1 = \frac{\rho}{4\pi d^2} \Rightarrow d^2 = \frac{5}{4\pi \times 10^{-2}} = \frac{500}{4\pi}$

$\Rightarrow r = \sqrt{\frac{125}{\pi}} = 6.3m$

2b. A small speaker delivers 3W of audio output. At what distance nearly from the speaker will one detect 90dB intensity sound ? [Given reference intensity of sound as $10^{-12} W/m^1$]

A) 10m B) 6.3m C) 16m D) 3m

Key : C

Sol: $\beta = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 90 = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 10^9 = \frac{I}{I_0} \Rightarrow I = 10^{-12} \times 10^9 = 10^{-3}$

$= 1 = \frac{\rho}{4\pi d^2} \Rightarrow d^2 = \frac{3}{4\pi \times 10^{-2}} \Rightarrow r = \sqrt{240} = 15.5$ nearly 16 m

2c. A small speaker delivers 1W of audio output. At what distance nearly from the speaker will one detect 150dB intensity sound ? [Given reference intensity of sound as $10^{-12} W/m^1$]

A) 0.9cm B) 0.3cm C) 16cm D) 3.0cm

Key : A

Sol: $\beta = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 150 = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 10^{15} = \frac{I}{I_0} \Rightarrow I = 10^{-12} \times 10^{15} = 10^3$

$= 1 = \frac{\rho}{4\pi d^2} \Rightarrow d^2 = \frac{1}{4\pi \times 10^3} = \frac{10^{-5}(25)}{3.14} \Rightarrow r = 0.9cm$

3. Three harmonic waves having equal frequency v and same intensity I_0, have phase angles $0, \frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to : **Jee Main 2020**

A) $5.8I_0$ B) $0.2I_0$ C) $3I_0$ D) I_0

Key : A

Sol : $I \alpha A^2$

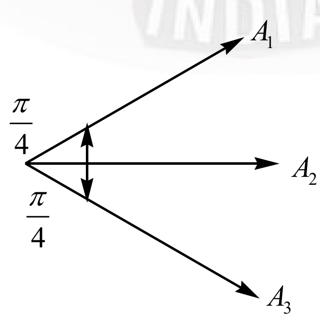
$I = KA^2$

$A = \sqrt{\frac{I}{K}}$

According to the equation

$\therefore \ A_R = (A_1x + A_2x + A_3x)\hat{i} + (A_1y + A_2y + A_3z)$

$= \left(A\cos\frac{\pi}{4} + A + A\cos\frac{\pi}{4}\right)\hat{i} + \left(A\sin\frac{\pi}{x} + 0 - A\sin\frac{\pi}{4}\right)\hat{j}$



4. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$ This expression may be considered to be a result of the superposition of **IIT -1992**

A) two B) three C) four D) five

Key : B

Sol : Given equation, $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$

$$= 2\left(2\cos^2\frac{t}{2}\sin 1000t\right)$$

Or, $= 2\left(2\cos^2\frac{t}{2}\sin 1000t\right)$ …(1)

Or, $y = 2\cos t\sin 1000t + 2\sin 1000t$ ….(2)

Or, $y = \sin 1001t + \sin 999t + 2\sin 1000t$ ….(3)

Hence, for the given periodic motion, three independent harmonic motions are superposed.

5. The ends of stretched wire of length 'L' are fixed at $x = 0$ and $x = L$ in one experiment the displacement of the wire is $y_1 = A\sin(\pi x/L)\sin\omega t$ and energy E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L)\sin 2\omega t$ and energy is E_2. Then **IIT -2001**

A) $E_1 = E_2$ B) $E_2 = 2E_1$ C) $E_2 = 4E_1$ D) $E_2 = 16E_1$

Key : C

Sol : Energy α $(amplitude)^2$ $(frequency)^2$

$$\frac{E_1}{E_2} = \frac{A_1^2}{A_2^2}\cdot\frac{\upsilon_1^2}{\upsilon_2^2} \qquad \frac{E_1}{E_2} = \frac{A^2}{A^2}\times\left(\frac{\upsilon}{2\upsilon}\right)^2 = \frac{1}{4}$$

$\therefore\ E_2 = 4E_1$

5a. The ends of stretched wire of length 'L' are fixed at $x = 0$ and $x = L$ in one experiment the displacement of the wire is $y_1 = A\sin(\pi x/L)\sin\omega t$ and energy E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L)\sin 3\omega t$ and energy is E_2. Then

A) $E_2 = 9E_1$ B) $E_1 = E_2$ C) $E_2 = 4E_1$ D) $E_2 = 16E_1$

Key : A

Sol : Energy α $(amplitude)^2$ $(frequency)^2$

$$\frac{E_1}{E_2} = \frac{A_1^2}{A_2^2}\cdot\frac{\upsilon_1^2}{\upsilon_2^2} \qquad \frac{E_1}{E_2} = \frac{A^2}{A^2}\times\left(\frac{\upsilon}{3\upsilon}\right)^2 = \frac{1}{9}$$

$\therefore\ E_2 = 9E_1$

6. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonance in the fundamental mode, with a tuning fork is 0.1m. When this length is changed to 0.35m, the same tuning fork resonates with the first overtone. Calculate the end correction

IIT 2003

A) 0.012m B) 0.025m C) 0.05m D) 0.024m

Key : B

Sol : $l_1 + x = \frac{\lambda}{4} \Rightarrow \lambda = 4(l_1 + x)$

$(l_2 + x) = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4}{3}(l_2 + x)$

$\therefore \; v_1 = \frac{v}{\lambda_1} = \frac{v}{4(l_1 + x)}$

Given v_1 is equal to v_2. So

$\frac{v}{4(l_1 + x)} = \frac{3v}{4(l_2 + x)}$

$\Rightarrow x = 0.025m$

6a. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonance in the first overtone with the tuning fork is 0.3m. When this length is changed to 0.6m, the same tuning fork resonates with a second overtone. The end correction in cm will be

A) 0.025 B) 0.15 C) 0.1 D) 0.035

Key : B

Sol : $l_1 + x = \frac{3\lambda}{4} \Rightarrow 0.3 + x = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4(0.3 + x)}{3}$

$l_2 + x = \frac{5\lambda}{4} \Rightarrow 0.6 + x = \frac{5\lambda}{4} \Rightarrow \lambda = \frac{4(0.6 + x)}{5}$

$\frac{4(0.3 + x)}{3} = \frac{4(0.6 + x)}{5}$

$5(0.3 + x) = 3(0.6 + x)$

$\therefore x = 0.15cm$

7. In a resonate tube with tuning fork of frequency 512Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7cm. The maximum possible error in the speed of sound is **IIT 2005**

A) 5.12 cm/s B) 102.4 cm/s C) 204.8 cm/s D) 153.6 cm/s

Key : C

Sol : For first resonance,

$l_1 + e = \frac{\lambda}{4}$ but $v = \nu\lambda$

$\therefore \; v = \nu 4(l_1 + e)$

$\Rightarrow l_1 = e = \frac{v}{4v}$

For second resonance,

$l_2 + e = \frac{3\lambda}{4}$

$v = v\frac{4}{3}(l_2 + e)$

$\Rightarrow \ l_2 + e = \frac{3v}{4v}$

From Eqs. (i) and (ii), we get

$v = 2v(l_2 - l_1)$

$\therefore \ \Delta v = 2v(\Delta l_2 + \Delta l_1)$

$= 2 \times 512 \times (0.1\ 0.1) cm/s$

$= 204.8 cm/s$

7a. In a resonate tube with tuning fork of frequency 512Hz, first resonance occurs at water level equal to 33.3 cm and second resonance occurs at 64.3cm. The maximum possible error in the speed of sound is ___ cm/s

A) 204.8 B) 512 C) 153.6 D) 102

Key : A

Sol : $v = 2n(l_2 - l_1)$

$\Delta v = 2n[\Delta l_1 + \Delta l_2]$

$\Delta v = 2 \times 512[0.1 + 0.1]$

$\Delta v = 2 \times 512[0.1 + 0.1] = 204.8$

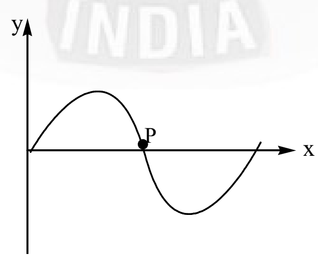
For $L_1 = 33.3$

$\Delta l_1 = 0.1$

For $L_2 = 64.3$

$\Delta l_2 = 0.1$

8. A transverse sinusoidal wave moves along a string in the positive x direction at speed of 10 cm/s. Then wavelength of the wave is 0.5 m and its amplitude is 10cm. At a particular time t, the snapshot of the wave is shown in the figure. The velocity of point P when its displacement is 5 cm is



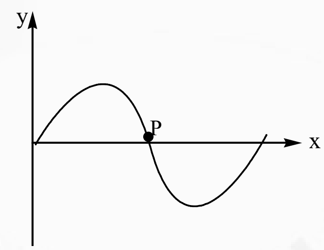
A) $\frac{\sqrt{3}\pi}{50}\hat{j} m/s$ B) $-\frac{\sqrt{3}\pi}{50}\hat{j} m/s$ C) $\frac{\sqrt{3}\pi}{50}\hat{i} m/s$ D) $-\frac{\sqrt{3}\pi}{50}\hat{i} m/s$

Key : A

Sol : Particle velocity, v_p is$\rightarrow$v (Slope of the y-ax graph) Here, v is positive, as the wave is travelling in the positive direction. Slope at P is negative.

There fore, velocity of particle is negative y (or $\hat{j}$) direction

8a. A transverse sinusoidal wave moves along a string in the positive x direction at speed of 20 cm/s. Then wavelength of the wave is 10 cm and its amplitude is 20cm. At a particular time t, the snapshot of the wave is shown in the figure. The velocity of point P when its displacement is 17.32 cm is



A) $0.2\pi\hat{j}$ B) $-0.2\pi\hat{j}$ C) $0.4\pi\hat{j}$ D) $-0.4\pi\hat{j}$

Key : C

Sol : Slope at P negative therefore particle velocity positive

$y = A\sin(kx - \omega t)$

$17.32 = 20\sin(kx - \omega t)$

$\sin(kx - \omega t) = \frac{\sqrt{3}}{2}$

$\therefore \ (kx - \omega t) = 60^0$

$\omega = 2\pi n = 4\pi$

$n = \frac{v}{\lambda} = \frac{20}{10} = 2Hz$

$v = A\omega\cos(kx - \omega t)$

$v = 0.2 \times 4\pi \times \cos 60$

$v = 0.4\pi\hat{j}$

III) Reflection and refraction of waves , Stationary waves

Reflection - Reflection and refraction of waves,(Change in the amplitude and phase of a wave reflected from fixed boundary, Change in the amplitude and phase of a wave reflected from a free boundary) Refraction - Reflection and refraction of waves,(Amplitude of the transmitted wave, Change in the phase of the transmitted wave, Amplitude of the Reflected Wave, Change In the phase of the reflected wave) Equation of stationary wave, Standing waves on string, Standing waves in organ pipes, (Position of nodes for displacement wave, Position of nodes for pressure wave, Position of antinodes displacement wave, Position of antinodes for pressure waves, Stationary waves in stretched string fixed at ends, Stationary longitudinal wave in open organ pipe, Stationary longitudinal wave in closed organ pipe, End correction, Resonance tube).

1. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is **IIT -1995**

A) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ B) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$ C) $300\left(\frac{2\rho}{2\rho-1}\right)$ D) $300\left(\frac{2\rho-1}{2\rho}\right)$

Key : A

Sol : In air : $T = mg = \rho Vg$

$$\therefore\ f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{\rho Vg}{m}} \qquad \ldots.(i)$$

When the object is half immersed in water.

$T = mg -$ upthrust

$$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$$

$$\therefore\ f' = \frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho-\rho_\omega)}{m}} = \frac{1}{2\ell}\sqrt{\frac{Vgp}{m}}\sqrt{\frac{(2\rho-\rho\omega)}{2\rho}}$$

$$\frac{f'}{f} = \sqrt{\frac{2\rho-\rho\omega}{2\rho}} f' = f\left(\frac{2\rho-\rho_\omega}{2\rho}\right)$$

$$= 300\left[\frac{2\rho-1}{2\rho}\right]^{1/2} Hz$$

1a. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 100 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is

A) $100\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ B) $100\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$ C) $100\left(\frac{2\rho}{2\rho-1}\right)$ D) $100\left(\frac{2\rho-1}{2\rho}\right)$

Key : A

Sol : In air : $T = mg = \rho V g$

$$\therefore\ f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{\rho V g}{m}} \qquad \text{....(i)}$$

When the object is half immersed in water.

$T = mg -$ upthrust

$$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$$

$$\therefore\ f' = \frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho-\rho_\omega)}{m}} = \frac{1}{2\ell}\sqrt{\frac{Vgp}{m}}\sqrt{\frac{(2\rho-\rho\omega)}{2\rho}}$$

$$\frac{f'}{f} = \sqrt{\frac{2\rho-\rho\omega}{2\rho}} f' = f\left(\frac{2\rho-\rho_\omega}{2\rho}\right)$$

$$= 100\left[\frac{2\rho-1}{2\rho}\right]^{1/2} Hz$$

2. A cylinder resonance tube open at both ends has fundamental frequency 'f' in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is______ **IIT -1992**

A) f B) 2f C) f/2 D) f/3

Key : A

Sol : If figure (a) both ends of the tube in air.

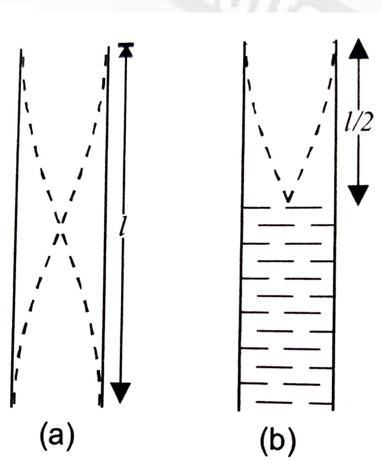
$$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$$

$$\therefore\ f = \frac{c}{\lambda} = \frac{c}{2\ell}$$

In figure (b) when half of the tube vertically dipped in water,

$$\frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell$$

$$\therefore\ f' = \frac{c}{\lambda'} = \frac{c}{2\ell} = f$$



2a. A cylinder resonance tube open at both ends has fundamental frequency 25 in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is______

A) 25 B) 50 C) 75 D) 100

Key : A

Sol : If figure (a) both ends of the tube in air.

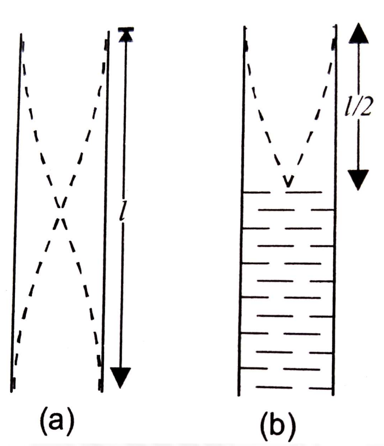
$$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$$

$$\therefore \; f = \frac{c}{\lambda} = \frac{c}{2\ell} = 25$$

In figure (b) when half of the tube vertically dipped in water,

$$\frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell$$

$$\therefore \; f' = \frac{c}{\lambda'} = \frac{c}{2\ell} = f = 25$$



3. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m $(m << M)$. When the car is at rest, the speed of transverse waves in the string is $60 ms^{-1}$. When the car has acceleration a, the wave – speed increases to $60.5 ms^{-1}$. The value of a, in terms of gravitational acceleration g, is closest to **Jee Mains 2019**

A) $\frac{g}{5}$ B) $\frac{g}{20}$ C) $\frac{g}{10}$ D) $\frac{g}{30}$

Key : A

Sol: $60 = \sqrt{\frac{Mg}{\mu}}$

$$60.5 = \sqrt{\frac{M\left(g^2 + a^2\right)^{1/2}}{\mu}} = \frac{60.5}{60}\sqrt{\frac{g^2 + a^2}{g^2}}$$

$$\left(1 + \frac{0.5}{60}\right)^4 = \frac{g^2 + a^2}{g^2} = 1 + \frac{2}{60}$$

$$= g^2 + a^2 = g^2 + g^2 \times \frac{2}{60}$$

$$= a = g\sqrt{\frac{2}{60}} = \frac{g}{\sqrt{30}} = \frac{g}{5.47} = \frac{g}{5}$$

3a. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m $(m << M)$. When the car is at rest, the speed of transverse waves in the string is $50ms^{-1}$. When the car has acceleration a, the wave – speed increases to $50.5ms^{-1}$. The value of a, in terms of gravitational acceleration g, is closest to

A) $\frac{g}{20}$ B) $\frac{g}{5}$ C) $\frac{g}{10}$ D) $\frac{g}{30}$

Key : B

Sol: $50 = \sqrt{\frac{Mg}{\mu}}$

$$50.5 = \sqrt{\frac{M\left(g^2 + a^2\right)^{1/2}}{\mu}} = \frac{50.5}{50}\sqrt{\frac{g^2 + a^2}{g^2}}$$

$$\left(1 + \frac{0.5}{50}\right)^4 = \frac{g^2 + a^2}{g^2} = 1 + \frac{2}{50}$$

$$= g^2 + a^2 = g^2 + g^2 \times \frac{2}{50}$$

$$= a = g\sqrt{\frac{2}{50}} = \frac{g}{\sqrt{25}} = \frac{g}{5}$$

3b. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m $(m << M)$. When the car is at rest, the speed of transverse waves in the string is $100ms^{-1}$. When the car has acceleration a, the wave – speed increases to $100.5ms^{-1}$. The value of a, in terms of gravitational acceleration g, is closest to

A) $\frac{g}{20}$ B) $\frac{g}{5}$ C) $\frac{g}{5\sqrt{2}}$ D) $\frac{g}{30}$

Key : C

Sol: $100 = \sqrt{\frac{Mg}{\mu}}$

$$100.5 = \sqrt{\frac{M\left(g^2 + a^2\right)^{1/2}}{\mu}} = \frac{100.5}{100}\sqrt{\frac{g^2 + a^2}{g^2}}$$

$$\left(1 + \frac{0.5}{100}\right)^4 = \frac{g^2 + a^2}{g^2} = 1 + \frac{2}{100}$$

$$= g^2 + a^2 = g^2 + g^2 \times \frac{2}{100}$$

$$= a = g\sqrt{\frac{2}{100}} = \frac{g}{\sqrt{50}} = \frac{g}{5\sqrt{2}}$$

3c. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m $(m << M)$. When the car is at rest, the speed of transverse waves in the string is $10ms^{-1}$. When the car has acceleration a, the wave – speed increases to $10.5ms^{-1}$. The value of a, in terms of gravitational acceleration g, is closest to

A) $\frac{g}{\sqrt{5}}$ B) $\frac{g}{5}$ C) $\frac{g}{5\sqrt{2}}$ D) $\frac{g}{30}$

Key : A

Sol: $10 = \sqrt{\frac{Mg}{\mu}}$

$$10.5 = \sqrt{\frac{M\left(g^2+a^2\right)^{1/2}}{\mu}} = \frac{10.5}{10}\sqrt{\frac{g^2+a^2}{g^2}}$$

$$\left(1+\frac{0.5}{10}\right)^4 = \frac{g^2+a^2}{g^2} = 1+\frac{2}{10}$$

$$= g^2+a^2 = g^2+g^2\times\frac{2}{10}$$

$$= a = g\sqrt{\frac{2}{10}} = \frac{g}{\sqrt{5}}$$

4. A string of length 1m and mass 5g is fixed at both ends. The tension in the string is 8.0N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to: **Jee Mains 2019**

A) 10.0cm B) 33.3cm C) 16.6cm D) 20.0cm

Key : D

Sol: We speed $= \sqrt{\frac{T}{\mu}}$ $\lambda = v/f = \sqrt{\frac{8}{5\times10^{-3}}} = 40ms^{-1}$

$$\lambda = \frac{40}{100} = 0.4m = 40cm$$

Separation between two successive nodes is $\frac{\lambda}{2} = \frac{40}{2} = 20cm$

4a. A string of length 1m and mass 5.9g is fixed at both ends. The tension in the string is 5 kg.wt. The string is set into vibration using an external vibrator of frequency 1000 Hz. The separation between successive nodes on the string is close to:

A) 10.0cm B) 11.67cm C) 16.6cm D) 20.0cm

Key : B

Sol: We speed $= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{49}{9\times10^{-4}}} = \frac{700}{3}ms^{-1} \Rightarrow \lambda = \frac{v}{n}$

$$= \frac{700}{30} = \frac{7}{30}m : \frac{70}{3}cm$$

Separation between two successive nodes is $\lambda/2 = \frac{70}{3\times2} = 35/3cm = 11.67cm$.

4b. A string of length 1m and mass 10g is fixed at both ends. The tension in the string is 100 N. The string is set into vibration using an external vibrator of frequency 500 Hz. The separation between successive nodes on the string is close to:

A) 10.0cm B) 11.67cm C) 16.6cm D) 20.0cm

Key : A

Sol: We speed $=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{100}{10^{-2}}}=100ms^{-1}$

$=\frac{100}{500}=\frac{1}{5}m:\frac{100}{5}=20cm$

Separation between two successive nodes is $\lambda/2=\frac{20}{2}=10cm$.

4c. A string of length 0.5m and mass 1g is fixed at both ends. The tension in the string is 10 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to:

A) 60.0cm B) 51.67cm C) 16.6cm D) 35.35cm

Key : D

Sol: We speed $=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{10}{2\times10^{-3}}}=50\sqrt{2}$

$=\frac{50\sqrt{2}}{100}=\frac{1}{\sqrt{2}}m:\frac{100}{\sqrt{2}}=70.7cm$

Separation between two successive nodes is $\lambda/2=\frac{70.7}{2}=35.35cm$.

5. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz) **Jee Mains 2019**

A) 6 B) 4 C) 7 D) 5

Key : C

Sol: $(2n-1)f_c=20000$; $f_c=1.5kHZ$; $n=7$

5a. An open organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz)

A) 10 B) 11 C) 13 D) 15

Key : C

Sol: $nf_0=20000; n=\frac{20000}{1500}=13.33$

5b. A closed organ pipe has a fundamental frequency of 2857 Hz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz)

A) 3 B) 4 C) 5 D) 6

Key : B

Sol: $(2n-1)f_c = 20000\,;2n-1=\frac{20000}{2857}=7\,;\ n=4$

5c. An open organ pipe has a fundamental frequency of 5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz)

A) 4 B) 3 C) 5 D) 8

Key : A

Sol: $nf_0 = 20000\,;n=\frac{20000}{5000}=4$

6. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork frequency 256 Hz which produces first resonance when water reaches a mark 27cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to

Jee Mains 2019

A) $322ms^{-1}$ B) $341ms^{-1}$ C) $335ms^{-1}$ D) $328ms^{-1}$

Key : D

Sol: $11+x+e=\frac{\lambda_2}{4}\ldots\ldots(1)$

$27+x+e=\frac{\lambda_2}{4}\ldots\ldots(2)$

$16\times10^{-2}=\frac{v}{4}\left(\frac{1}{256}-\frac{1}{512}\right);v=328m/s$

6a. A tuning fork with frequency 800Hz produces resonance in a resonance air column tube with upper end open and lower end closed by water surface. Successive resonance are observed at lengths 9.75cm, 31.25cm and 52.75cm. The speed of sound in air is ____

A) $272ms^{-1}$ B) $400ms^{-1}$ C) $360ms^{-1}$ D) $344ms^{-1}$

Key : D

Sol : $\frac{\lambda}{4}+e=9.75$

$\frac{3\lambda}{4}+e=31.25$

$\frac{\lambda}{2}=31.25-9.75-=21.5$

$=43cm$

$v=f\lambda=800\times0.43=344$

6b. A tuning fork is used to produce resonance in a glass tube. The length of air column in this tube can be adjusted by variable piston. At room temperature 27^0C two successive resonances are produced at 20cm and 73 cm of column if frequency of tuning fork is 320 Hz, velocity of sound in air at 27^0C is _____

A) $330ms^{-1}$ B) $339ms^{-1}$ C) $300ms^{-1}$ D) $350ms^{-1}$

Key : B

Sol : $V = 2n(l_2 - l_1) = 2\times 320\frac{(73-20)}{100} = 339$

6c. The first second resonating lengths in an air column apparatus are 20 cm and 65 cm respectively. Diameter of the pipe is around ___ cm

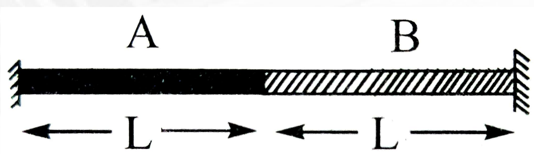
A) 2 B) 4 C) 6 D) 8

Key : D

Sol : $e = \frac{l_2 - 3l_1}{2} = \frac{65-3\times 20}{2} = \frac{5}{2}$

$e = 0.3D \qquad D = \frac{e}{0.3} = \frac{5}{2}\times\frac{10}{3} = \frac{50}{6} = 8$

7. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p:q is **Jee Mains 2019**



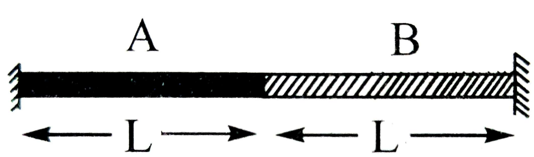
A) 1:2 B) 3:5 C) 1:4 D) 4:9

Key : A

Sol: $n_A\frac{V_A}{2L} = n_B\frac{V_B}{2L}$

$\frac{n_A}{n_B} = \frac{V_B}{V_A} = \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{r^2}{(2r)^2}} = 1:2$

7a. A wire of length 2L, is made by joining two wires I & II of same length but different radii r and 3r and made of the same material. It is vibrating at frequency such that the joint of the two wires forms a node. If the number of antinodes in wire I is p and that in II is q then the ratio I:II is



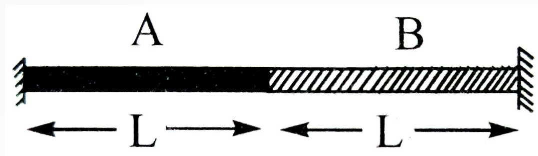
A) 1:2 B) 1:3 C) 1:4 D) 4:9

Key : B

Sol: $p \times \frac{V_I}{2L} = q \times \frac{V_{II}}{2L}$

$$\frac{p}{q} = \frac{V_{II}}{V_I} = \sqrt{\frac{A_I}{A_{II}}} = \sqrt{\frac{r^2}{(3r)^2}} = 1:3$$

7b. A wire of length 2L, is made by joining two wires I & II of same length but different radii r and 5r and made of the same material. It is vibrating at frequency such that the joint of the two wires forms a node. If the number of antinodes in wire I is p and that in II is q then the ratio I:II is



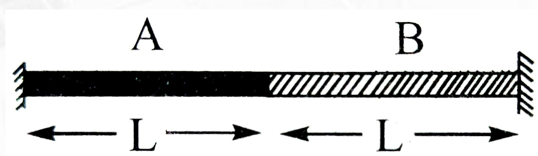
A) 1:2 B) 1:3 C) 1:5 D) 4:9

Key : C

Sol: $p \times \frac{V_I}{2L} = q \times \frac{V_{II}}{2L}$

$$\frac{p}{q} = \frac{V_{II}}{V_I} = \sqrt{\frac{A_I}{A_{II}}} = \sqrt{\frac{r^2}{(5r)^2}} = 1:5$$

7c. A wire of length 2L, is made by joining two wires I & II of same length but different radii r and 7r and made of the same material. It is vibrating at frequency such that the joint of the two wires forms a node. If the number of antinodes in wire I is p and that in II is q then the ratio I:II is



A) 1:2 B) 1:3 C) 1:5 D) 1:7

Key : C

Sol: $p \times \frac{V_I}{2L} = q \times \frac{V_{II}}{2L}$

$$\frac{p}{q} = \frac{V_{II}}{V_I} = \sqrt{\frac{A_I}{A_{II}}} = \sqrt{\frac{r^2}{(7r)^2}} = 1:7$$

8. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is : (All quantities are in SI units.) **Jee Mains 2019 April 9th**

A) 40 m B) 20 m C) 80 m D) 60 m

Key : C

Sol: In 4th harmonic $\ell = \frac{4.\lambda}{2} = 2\lambda$

$$\lambda = \frac{2\pi}{k} = \frac{2\times3.14}{0.157} = 20 = 40m \Rightarrow 2\lambda = \ell = 80m$$

8a. A string is clamped at both the ends and it is vibrating in its 3rd harmonic. The equation of the stationary wave is $Y = 0.3\sin(1.57x)\cos(100\pi t)$. The length of the string is : (All quantities are in SI units.)

A) 4 m B) 2 m C) 8 m D) 6 m

Key : D

Sol: In 3rd harmonic $\ell = \frac{3}{2}\lambda$

$$\lambda = \frac{2\pi}{k} = \frac{2\times3.14}{1.57} = 4m \Rightarrow \ell = 1.5\lambda = 6m$$

8b. A string is clamped at both the ends and it is vibrating in its 2nd harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(50\pi t)$. The length of the string is : (All quantities are in SI units.)

A) 40 m B) 20 m C) 80 m D) 60 m

Key : A

Sol: In 2nd harmonic $\ell = \frac{2}{2}\lambda$

$$\lambda = \frac{2\pi}{k} = \frac{2\times3.14}{0.157} = 40m = \ell$$

8c. A string is clamped at both the ends and it is vibrating in its 1st harmonic. The equation of the stationary wave is $Y = 0.3\sin(\pi x)\cos(200\pi t)$. The length of the string is : (All quantities are in SI units.)

A) 2 m B) 1 m C) 8 m D) 6 m

Key : B

Sol: In 1st harmonic $\ell = \frac{1}{2}\lambda$

$$\lambda = \frac{2\pi}{\pi} = 2m \Rightarrow \ell = 2/2 = 1m$$

9. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is

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A) 320 m/s, 80 Hz B) 180 m/s, 120 Hz C) 320 m/s, 120 Hz D) 180 m/s, 80 Hz

Key : A

Sol: Given string is in 3rd harmonic mode

$$\Rightarrow n = \frac{3v}{2\ell} \Rightarrow 240 = \frac{3(v)}{2(2)}; v = 320m/s$$

Fundamental frequency

$$\Rightarrow n = \frac{v}{2\ell} \Rightarrow \frac{320}{2(2)} = 80Hz$$

9a. A string 1 m long and fixed at its ends is driven by a 500 Hz vibrator. The string vibrates in its second harmonic mode. The speed of the wave and its fundamental frequency is

A) 320 m/s, 80 Hz B) 500 m/s, 250 Hz C) 320 m/s, 120 Hz D) 180 m/s, 80 Hz

Key : B

Sol: Given string is in second harmonic mode

$$\Rightarrow n_2 = \frac{2v}{2\ell} \Rightarrow 500 = \frac{2(v)}{2(1)}; v = 500m/s$$

Fundamental frequency

$$\Rightarrow n_1 = \frac{v}{2\ell} \Rightarrow \frac{500}{2(1)} = 250Hz$$

9b. A string 0.5 m long and fixed at its ends is driven by a 100 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is

A) 30 m/s, 20 Hz B) 50 m/s, 20 Hz C) 32 m/s,10 Hz D) 33.3 m/s, 33.3 Hz

Key : D

Sol: Given string is in third harmonic mode

$$\Rightarrow n_3 = \frac{3v}{2\ell} \Rightarrow 100 = \frac{3(v)}{2(0.5)}; v = 100/3m/s$$

Fundamental frequency

$$\Rightarrow n_1 = \frac{v}{2\ell} \Rightarrow \frac{100}{3(2)\times 0.5} = 33.33Hz$$

9c. A string 100 cm long and fixed at its ends is driven by a 200 Hz vibrator. The string vibrates in its fifth harmonic mode. The speed of the wave and its fundamental frequency is

A) 80 m/s, 40 Hz B) 50 m/s, 20 Hz C) 32 m/s,10 Hz D) 33.3 m/s, 33.3 Hz

Key : A

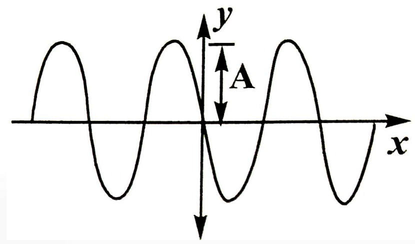
Sol: Given string is in fifth harmonic mode

$$\Rightarrow n_5 = \frac{5v}{2\ell} \Rightarrow 200 = \frac{5(v)}{2(1)}; v = 80ms^{-1}$$

Fundamental frequency

$$\Rightarrow n_1 = \frac{v}{2\ell} \Rightarrow \frac{80}{2} = 40Hz$$

10. A progressive wave travelling along the positive x-direction is represented by $y(x,t)=A\sin(kx-\omega t+\phi)$. Its snapshot at $t=0$ is given in the figure.



For the wave, the phase ϕ is **Jee Mains 2019**

A) π B) $\frac{\pi}{2}$ C) $-\frac{\pi}{2}$ D) 0

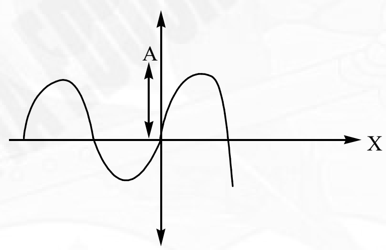
Key : A

Sol: Slope $=\frac{dy}{dx}=Ak\cos(kx-\omega t+\phi)$

At $x=0; t=0,$ slope is –ve

$\therefore \phi=\pi$; slope negative

10a. A progressive wave travelling along the positive x-direction is represented by $y(x,t)=A\sin(kx-\omega t+\phi)$. Its snapshot at $t=0$ is given in the figure.



For the wave, the phase ϕ is

A) π B) $\frac{\pi}{2}$ C) $-\frac{\pi}{2}$ D) 0

Key : D

Sol: Slope $=\frac{dy}{dx}=Ak\cos(kx-\omega t+\phi)$

At $x=0; t=0,$ slope is +ve

$\therefore \phi=0$

11. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound V in air by resonance tube method. Resonance is observed to occur at two successive length of the air column, $l_1=30$ cm and $l_2=70$ cm. Then, V is equal to

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A) $332ms^{-1}$ B) $379ms^{-1}$ C) $338ms^{-1}$ D) $384ms^{-1}$

Key : D

Sol: $V=2n(l_2-l_1)=2\times480(40\times10^{-2})V=384m/s$

11a. In a resonance pipe first and second resonances are obtained at depths 22.7cm and 70.2cm respectively. What will be end correction ?

A) 1.05cm B) 2cm C) 1.5cm D) 2.5cm

Key : A

Sol : $e = \frac{l_2 - 3l_1}{2} = \frac{70.2 - 3\times 22.7}{2} = 1.05cm$

11b. If a resonance tube is sounded with a tuning fork of frequency 256 Hz, resonance occurs at 35 cm and 105 cm. The velocity of sound is about_____

A) $360m/s$ B) $320m/s$ C) $300m/s$ D) $340m/s$

Key : A

Sol : $V = 2n(l_2 - l_1) = 2\times 256\frac{(105-35)}{100} = 358.4m/s$

11c. A long glass tube is held vertically in water. A tuning fork is struck and held over the tube. Resonances are observe at two successive lengths 05.m and 0.84m above the surface of water. If velocity of sound in air 340m/s, then frequency of the tuning fork is ___

A) 400 B) 500 C) 600 D) 700

Key : B

Sol : $v = 2n(l_2 - l_1)$

$$n = \frac{v}{2(l_2 - l_1)} = \frac{340}{2(0.84-0.5)} = \frac{340}{2\times 0.34} = 500Hz$$

12. Two identical strings X and Z made of same material have tension T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_X / T_Z is :

A) 2.25 B) 0.44 C) 1.25 D) 1.5

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Key : A

Sol : Using $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$,

Where, T = tension and $\mu = \frac{\text{mass}}{\text{length}}$

$$\frac{f_x}{f_z} = \frac{450}{300} = \sqrt{\frac{T_x}{T_z}}\quad v = \frac{100\pi}{0.5\pi}$$

$$\therefore \frac{T_x}{T_z} = \frac{9}{4} = 2.25$$

12a. Two identical strings x and y made of same material have tension T_x and T_y in them. If their fundamental frequency are 400 Hz and 300 Hz respectively then the ratio $\frac{T_x}{T_z}$ is

A) 3.1 B) 2.11 C) 4.5 D) 3.45

Key : B

Sol : $\frac{F_x}{F_y}=\frac{400}{300}=\sqrt{\frac{T_x}{T_z}}$

$\frac{16}{9}=\frac{T_x}{12}$

$\frac{T_x}{T_z}=16.9=2.11$

12b. Two identical strings x and y made of same material have tension T_x and T_y in them. If their fundamental frequency are 500 Hz and 200 Hz respectively then the ratio $\frac{T_x}{T_z}$ is

A) 3.1 B) 4.25 C) 6.25 D) 0

Key : C

Sol: $\frac{F_x}{F_y}=\frac{500}{200}=\sqrt{\frac{T_x}{T_y}}$

$\frac{5}{2}=\sqrt{\frac{T_x}{T_y}}$

$\frac{25}{4}=\frac{T_x}{T_y}$

$\frac{T_x}{T_y}=6.25$

12c. Two identical strings x and y made of same material have tension T_x and T_y in them. If their fundamental frequency are 550 Hz and 350 Hz respectively then the ratio $\frac{T_x}{T_z}$ is

A) 1.46 B) 4.00 C) 2.46 D) 1.05

Key : C

Sol: $\frac{F_x}{F_y}=\sqrt{\frac{T_x}{T_y}}$

$\frac{550}{350}=\sqrt{\frac{T_x}{T_y}}$

$\frac{T_x}{T_y}=\left(\frac{11}{7}\right)^2=\frac{121}{49}=2.46$

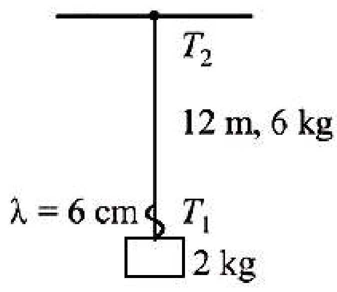
13. A uniform thin rope of length 12m and mass 6 kg hangs vertically from a rigid support and a block of mass 2kg is attached to its free end. A transverse short wave-train of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ? **Jee Main 2020**

A) 3 B) 6 C) 12 D) 9

Key : C

Sol : Using, $V = f\lambda$

$$\frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2} \Rightarrow \lambda_2 = \frac{V_2}{V_1}\lambda_1$$



Again using,

$$n = \frac{V}{\lambda} = \sqrt{\frac{T}{M}}, \lambda_2 = \sqrt{\frac{T_2}{T_1}}\lambda_1 \quad T_2 = 8g(Top)$$

$$= \sqrt{\frac{8g}{2g}}\lambda_1 = 12cm \qquad T_1 = 2g(Bottom)$$

13a. A uniform rope of length 12m and mass 6kg hangs vertically from a rigid support. A block of a mass 2kg is attached to the free end of the rope. A transverse pulse of wave length 0.06m, is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope ?

A) 0.03m B) 0.12m C) 0.09m D) 0.06m

Key : B

Sol : $\frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2}$ $\qquad T_1 = 8g$

$$\lambda_2 = \left(\frac{V_2}{V_1}\right)\lambda_1 \qquad T_1 = 2g$$

$$\lambda_2 = \frac{\sqrt{T_2}}{\sqrt{T_1}}\lambda_1$$

$$\lambda = \sqrt{\frac{8g}{2g}} \times 0.06$$

$$\lambda_2 = 2 \times 0.06 = 0.012m$$

13b. A uniform rope of length 10m and man 15kg hangs vertically from a rigid support. A block of man 5kg is attached to the free end of the rope. A transverse pulse of wavelength 0.08 m is produced at the lower end of the rope. The wavelength of the pulse when is reaches the top of the rope will be

A) 0.08m B) 0.04m C) 0.16m D) 0m

Key : C

Sol : $\mu = \frac{15}{10} = 1.5$

$$T_{lower} = 50N$$

$T_{upp} = 200N$

$V_L = \sqrt{\frac{50}{1.5}} = \sqrt{\frac{500}{15}}$

$V_u = \sqrt{\frac{200}{1.5}}$

$\frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2}$

$\frac{\sqrt{\frac{50}{1.5}}}{0.08} = \frac{\frac{\sqrt{200}}{1.5}}{\lambda_2}$

$\lambda_2 = 0.16m$

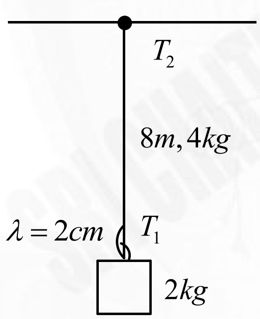
13c. A uniform thin rope of length 8m and mass 4 kg hangs vertically from a rigid support and a block of mass 2kg is attached to its free end. A transverse short wave-train of wavelength 2 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ?

A) 3.4 B) 6.5 C) 12.2 D) 9.4

Key : A

Sol : Using, $V = f\lambda$

$\frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2} \Rightarrow \lambda_2 = \frac{V_2}{V_1}\lambda_1$



Again using,

$n = \frac{V}{\lambda} = \sqrt{\frac{T}{M}}, \lambda_2 = \sqrt{\frac{T_2}{T_1}}\lambda_1$ $T_2 = 6g(Top)$

$= \sqrt{\frac{6g}{2g}} \times 2 = 3.4cm$ $T_1 = 2g(Bottom)$

14. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5m, while the distance between one crest and one trough is 1.5m. The possible wavelength (in m) of the waver are : **Jee Main 2020**

A) 1,3,5,…. B) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, ...$ C) $1, 2, 3, ...$ D) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ...$

Key : B

Sol : Distance between one crest and on tough $= 1.5m$

$=(2n_1+1)\frac{\lambda}{2}$

Distance between two crests =5m = $n_2\lambda$

$\frac{1.5}{5}=\frac{(2n_1+1)}{2n_2}\Rightarrow 3n_2=10n_1+5$

Here n_1 and n_2 are integer.

If $n_1=1, n_2=5$ $\therefore \lambda=1$

$n_1=4, n_2=15$ $\therefore \lambda=1/3$

$n_1=7, n_2=25$ $\therefore \lambda=1/5$

Hence possible wavelengths $\frac{1}{1},\frac{1}{3},\frac{1}{5}$ metre.

14a. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 6m, while the distance between one crest and one trough is 2m. The possible wavelength
(in m) of the wave are :

A) 1,3,5,…. B) $\frac{1}{1},\frac{6}{17},\frac{6}{24},.....$ C) $1,2,3,....$ D) $\frac{1}{2},\frac{1}{4},\frac{1}{6},.....$

Key : B

Sol : Distance between one crest and on tough $=2m$

$=(2n_1+1)\frac{\lambda}{2}$

Distance between two crests =6m = $n_2\lambda$

$\frac{2}{6}=\frac{(2n_1+1)}{2n_2}\Rightarrow n_2=3n_1+3$

Here n_1 and n_2 are integer.

If $n_1=1, n_2=6$ $\therefore \lambda=1$

$n_1=4, n_2=17$ $\therefore \lambda=6/17$

$n_1=7, n_2=24$ $\therefore \lambda=6/24$

Hence possible wavelengths $\frac{1}{1},\frac{6}{17},\frac{6}{24},.....$ metre.

14b. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 3m, while the distance between one crest and one trough is 1.5m. The possible wavelength
(in m) of the wave are :

A) 1,3,5,…. B) $\frac{1}{1},\frac{6}{17},\frac{6}{24},.....$ C) $\frac{1}{2},\frac{1}{6},\frac{3}{28},....$ D) $\frac{1}{2},\frac{1}{4},\frac{1}{6},.....$

Key : B

Sol : Distance between one crest and on tough $=1.5m$

$=(2n_1+1)\frac{\lambda}{2}$

Distance between two crests =3m = $n_2\lambda$

$$\frac{1.5}{3}=\frac{(2n_1+1)}{2n_2}\Rightarrow n_2=4n_1+2$$

Here n_1 and n_2 are integer.

If $n_1=1, n_2=6$ $\quad\therefore\ \lambda=\frac{1}{2}$

$n_1=4, n_2=18$ $\quad\therefore\ \lambda=1/6$

$n_1=7, n_2=28$ $\quad\therefore\ \lambda=3/28$

Hence possible wavelengths $\frac{1}{2},\frac{1}{6},\frac{3}{28},\ldots$ metre.

14c. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 7m, while the distance between one crest and one trough is 2m. The possible wavelength
(in m) of the wave are :

A) 1,3,5,…. B) $\frac{1}{1},\frac{6}{17},\frac{6}{24},\ldots..$ C) $\frac{2}{3},\frac{2}{9},\frac{2}{15},\ldots$ D) $\frac{1}{2},\frac{1}{4},\frac{1}{6},\ldots..$

Key : c

Sol : Distance between one crest and on tough $=2m$

$$=(2n_1+1)\frac{\lambda}{2}$$

Distance between two crests =7m = $n_2\lambda$

$$\frac{2}{7}=\frac{(2n_1+1)}{2n_2}\Rightarrow 2n_2=14n_1+7$$

Here n_1 and n_2 are integer.

If $n_1=1, n_2=21/2$ $\quad\therefore\ \lambda=\frac{2}{3}$

$n_1=4, n_2=63/2$ $\quad\therefore\ \lambda=2/9$

$n_1=7, n_2=105/2$ $\quad\therefore\ \lambda=2/15$

Hence possible wavelengths $\frac{2}{3},\frac{2}{9},\frac{2}{15},\ldots$ metre.

15. If a resonance tube experiment when the tube is filled with water up to a height 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330m/s, the tuning fork frequency is :

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A) 2200 Hz B) 550 Hz C) 1100 Hz D) 3300 Hz

Key : A

Sol : Here, $l_1=17cm$ and $l_2=24.5$ cm, $V=330$ m/s, $f=?$

$$\lambda=2(l_2-l_1)=2\times(24.5-17)=15cm$$

Now, from $v = f\lambda \Rightarrow 330 = \lambda \times 15 \times 10^{-2}$

$\therefore \lambda = \frac{330}{15} \times 100 = \frac{1100 \times 100}{5} = 2200 Hz$

15a. If a resonance tube experiment when the tube is filled with water up to a height 15cm from bottom it resonates with a given tuning fork. When the water level is raised the next resonace with the same tuning fork occurs at a height of 25cm. If $v = 330 m/\sec$. The tuning fork frequency is

A) 1650 B) 1700 C) 1500 D) 1800

Key : A

Sol : $\ell_1 = 15$ $\ell_2 = 25$ $v = 330 m/\sec$

$\lambda = 2(\ell_2 - \ell_1)$

$\lambda = 2(25 - 15) = 20 cm$

$v = n\lambda$

$330 = n(20 \times 10^{-2})$

$n = \frac{300 \times 100}{20}$

$= 16.5 \times 100 = 1650 Hz$

15b. If a resonance tube experiment when the tube is filled with water up to a height 20cm from bottom it resonates with a given tuning fork. When the water level is raised the next resonace with the same tuning fork occurs at a height of 40cm. If $v = 330 m/\sec$. The tuning fork frequency is

A) 7000Hz B) 8250Hz C) 1400Hz D) 1100 Hz

Key : B

Sol : $l_1 = 20$ $l_2 = 40$ $v = 300 m/\sec$

$\lambda = 2(\ell_2 - \ell_1)$

$= 2(40 - 20) = 2 \times 20 = 40 cm$

$v = n\lambda$

$n = \frac{330}{40 \times 10^{-2}} = \frac{330 \times 100}{40} = 8.25 \times 100$

$n = 8250 Hz$

15c. In a resonance tube the first resonance with a tuning fork occurs at 16cm and second at 49cm. If the velocity of second is 330m/sec. The frequency of tuning fork is

A) 500 B) 300 C) 330 D) 165

Key : A

Sol : Let the resonance column has the end correction to be length of the air column for first resonance to occur $l_1 = 16 cm = 0.16 m$

$l_1 = 49 cm = 0.49$

$l_2 = x = 3\lambda / 4$

$l_2 - x_1 = \lambda / 2$

$0.49 - 0.16 = \lambda / 2$

$\lambda = 0.66m$

$v = n\lambda$

$330 = n \times 0.66$

$n = 500Hz$

16. Speed of a transverse wave on a straight wire (mass 6.0g, length 60 cm and area of cross-section $1.0mm^2$) is $90ms^{-1}$. If the Young's modulus of wire is $16\times10^{11}Nm^{-2}$ the extension of wire over its natural length is : **Jee Main 2020**

A) 0.03 mm B) 0.02 mm C) 0.04 mm D) 0.01 mm

Key : A

Sol : Given, $l = 60cm, m = 6g, A = 1mm^2, v = 90m/s$ and $Y = 16\times10^{11}Nm^{-2}$

Using, $v = \sqrt{\frac{T}{m}\times l} \Rightarrow T = \frac{mv^2}{I}$

Again from, $Y = \frac{T}{A}\Delta L / L_0$

$$\Delta L = \frac{Tl}{YA} = \frac{mv^2 \times l}{l(YA)}$$

$$= \frac{6\times10^3\times90^2}{16\times10^{11}\times10^{-6}} = 3\times10^{-4}m$$

$= 0.03mm$

16a. Speed of a transverse wave on a straight wire (mass 6.0g, length 40 cm and area of cross-section $1.0mm^2$) is $100ms^{-1}$. If the Young's modulus of wire is $16\times10^{11}Nm^{-2}$ the extension of wire over its natural length is :

A) 3.75×10^{-9} B) 4.75×10^{-11} C) 6.35×10^{-11} D) 9.73×10^{-11}

Key :A

Sol: $l = 40cm$ $m = 6g$ $A = 1mm^2$ $V = 100m/\sec$

$y = 16\times10^{11}N/m^2$

$\Delta l = ?$

$$\Delta l = \frac{Tl}{YA} = \frac{mv^2\times l}{\lambda(yA)}$$

$$= \frac{6\times10^{-3}\times(100)^2}{16\times10^{11}\times10^{-6}} = \frac{6\times10}{16\times10^9}$$

$= 3.75\times10^{-9}$

16b. The speed of a transverse wave travelling through a wire having a length 50 cm and man 5g to 80m/sec, $A = 1mm^2$, $y = 16\times10^{11}N/m^2$ find the extension in the wire with respect to its natural length

A) 0.01mm B) 0.3mm C) 0.02mm D) 0.04mm

Key : C

Sol : $v = 80m/\sec$, $A = 1mm^2$. $m = 5g = 5\times10^{-3}kp$

$l = 50cm = 50\times10^{-2}$

$$\mu = \frac{m}{l} = \frac{5\times10^{-3}}{50\times10^{-2}} = 1\times10^{-2}Kg/m$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu v^2 = 10^{-2}\times6400$$

$T = 64N$

$$Y = \frac{F}{A}.\frac{1}{Dl}$$

$$\Delta l = \frac{Fl}{A_4}$$

$$\Delta l = \frac{64\times0.50}{1\times10^{-6}\times16\times10^{11}} = 0.02mm$$

16c. The speed of a transverse wave travelling through a wire having a length 100 cm and man 10g to 160m/sec, $A = 1mm^2$, $y = 32\times10^{11}N/m^2$ find the extension in the wire with respect to its natural length

A) $0.08mm$ B) 0.04mm C) 0.02 mm D) 1mm

Key : B

Sol : $\mu = \frac{10\times10^{-3}}{100\times10^{-2}} = 10^{-2}kg/m$

$$v = \sqrt{\frac{T}{\mu}}$$

$T = v^2\mu = 10^{-2}\times160\times1600 = 256$

$$\Delta l = \frac{Fl}{A_4} = \frac{256\times1}{2\times10^{-6}\times32\times10^{-11}}$$

$\Delta l = 4\times10^{-5}m = 0.04mm$

17. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4N. When the tension is changed to T, the velocity changed to $v/2$. The value of T is closed to : **Jee Main 2020**

A) 2.50×10^4N B) 5.15×10^3N C) 30.5×10^4N D) 10.2×10^2N

Key : B

Sol : The velocity of a transverse wave in a stretched wire is given by

$$v = \sqrt{\frac{T}{\mu}}$$

Where, $T =$ tension in the wire

$\mu =$ linear density of wire

$(\because V\alpha T)$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{v}{v} \times 2 = \sqrt{\frac{2.06 \times 10^4}{T_2}}$$

$$\Rightarrow T_2 = \frac{2.06 \times 10^4}{4} = 0.515 \times 10^4 N$$

$$\Rightarrow T_2 = 5.15 \times 10^3 N$$

17a. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is $3.06 \times 10^4 N$. When the tension is changed to T, the velocity changed to $v/2$. The value of T is closed to :

A) $2.5 \times 10^4 N$ B) $7.65 \times 10^3 N$ C) $3.5 \times 10^{-4} N$ D) $3.76 \times 10^4 N$

Key : B

Sol : $T_1 = 3.06 \times 10^{-4}$

$T_2 = ?$ $v_1 = v$ $v_2 = v/2$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\left(\frac{v}{v}\right)2 = \sqrt{\frac{3.06 \times 10^4}{T_2}}$$

$$4 = \frac{3.06 \times 10^4}{T_2}$$

$T_2 = 3.06 \times 10^4$

$T_2 = 0.765 \times 10^4 N$

$= 7.65 \times 10^3 N$

17b. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is $4.06 \times 10^4 N$. When the tension is changed to T, the velocity changed to $v/4$. The value of T is closed to :

A) $2.53 \times 10^3 N$ B) $4.5 \times 10^4 N$ C) $2.56 \times 10^4 N$ D) $0.56 \times 10^4 N$

Key : A

Sol : $T_1 = 4.06 \times 10^4 N$ $V_1 = v$ $V_2 = v/4$ $T_2 = ?$

$$\frac{v_1}{v_2} = \sqrt{\frac{\mathrm{T}_1}{T_2}}$$

$$\left(\frac{v}{v}\right)2 = \sqrt{\frac{3.06 \times 10^4}{T_2}}$$

$$4 = \frac{3.06 \times 10^4}{T_2}$$

$$16 = \frac{4.06 \times 10^4}{16}$$

$T_2 = 0.25 \times 10^4$

$T_2 = 2.53 \times 10^3$

17c. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is $5.06 \times 10^4 N$. When the tension is changed to T, the velocity changed to $v/8$. The value of T is closed to :

A) $2.53 \times 10^3 N$ B) $5.06 \times 10^4 N$ C) $2.56 \times 10^4 N$ D) $0.56 \times 10^4 N$

Key : B

Sol : $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

$= \sqrt{\frac{5.06 \times 10^4}{T_2}}$

$64 = \frac{25.06 \times 10^4}{T_2}$

$T_2 = 0.6325 \times 10^4 = 6.3 \times 10^4 N$

18. A wire of length L and mass per unit length $6.0 \times 10^{-3} kgm^{-1}$ is put under tension of 540N. Two consecutive frequencies that is resonates at are: 420 Hz and 490 Hz. Then L in meters is

Jee Main 2020

A) 2.1m B) 1.1m C) 8.1m D) 5.1m

Key : A

Sol : Fundamental frequency, $f = 70 Hz$

The fundamental frequency of wire vibrating under tension T is given by

$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

Here, $\mu =$ mass per unit length of the wire

$L =$ length of wire

$70 = \frac{1}{2L}\sqrt{\frac{540}{6 \times 10^{-3}}}$

$\Rightarrow L \approx 2.14m$

18a. A wire of length L and mass per unit length $4.0 \times 10^{-3} kgm^{-1}$ is put under tension of 540N. Two consecutive frequencies that is resonates at are: 430 Hz and 540 Hz. Then L in meters is

A) 0.89 m B) 1.1m C) 8.1m D) 5.1m

Key : A

Sol : Fundamental frequency, $f = 60 Hz$

The fundamental frequency of wire vibrating under tension T is given by

$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

Here, $\mu =$ mass per unit length of the wire

$L =$ length of wire

$$60 = \frac{1}{2L}\sqrt{\frac{540}{4\times10^{-3}}}$$

$\Rightarrow L = 0.89m$

18b. A wire of length L and mass per unit length $4.0\times10^{-3}\,kgm^{-1}$ is put under tension of 440N. Two consecutive frequencies that is resonates at are: 420 Hz and 490 Hz. Then L in meters is

A) 3.36m B) 2.36m C) 8.1m D) 5.1m

Key : B

Sol : Fundamental frequency, $f = 70Hz$

The fundamental frequency of wire vibrating under tension T is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Here, $\mu =$ mass per unit length of the wire

$L =$ length of wire

$$70 = \frac{1}{2L}\sqrt{\frac{440}{4\times10^{-3}}}$$

$\Rightarrow L = 2.36m$

18c. A wire of length L and mass per unit length $8.0\times10^{-3}\,kgm^{-1}$ is put under tension of 640N. Two consecutive frequencies that is resonates at are: 420 Hz and 490 Hz. Then L in meters is

A) 3.36m B) 1.36m C) $2.02m$ D) 5.1m

Key : C

Sol : Fundamental frequency, $f = 70Hz$

The fundamental frequency of wire vibrating under tension T is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Here, $\mu =$ mass per unit length of the wire

$L =$ length of wire

$$70 = \frac{1}{2L}\sqrt{\frac{640}{8\times10^{-3}}}$$

$\Rightarrow L = 2.02m$

19. A student is performing the experiment of resonance column. The diameter of the column tube is 6cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is : **Jee Mains -2021**

A) 13 cm B) 14.8 cm C) 16.6 cm D) 18.4 cm

Key: B

Sol: $\lambda = \frac{V}{f} = \frac{336}{504} = 66.66cm$

$\frac{\lambda}{4} = l + e = l + 0.3D$

$= l + 1.8cm$

$16.66 = l + 1.8cm$

$l = 14.86cm$

19a. A student is performing the experiment of resonance column. The diameter of the column tube is 6cm. The frequency of the tuning fork is 500 Hz. Speed of the sound at the given temperature is 340 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is :

A) 10 cm B) 15 cm C) 17 cm D) 20 cm

Key: C

Sol: $\lambda = \frac{V}{f} = \frac{340}{500} = 0.17m = 17cm$

19b. A student is performing the experiment of resonance column. The diameter of the column tube is 6.25 cm. The frequency of the tuning fork is 400 Hz. Speed of the sound at the given temperature is 350 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is :

A) 15 cm B) 22 cm C) 25 cm D) 30 cm

Key: B

Sol: $\lambda = \frac{V}{f} = \frac{350}{400} = 87.5cm$

$\frac{\lambda}{4} = l = 21.875$

19c. A resonating tube is resonated at 256 Hz. If length of first and second resonating air columns are 32 cm and 100 cm, end correction is ___

A) 1cm B) 2cm C) 0.5 cm D) 3cm

Key: B

Sol: $e = \frac{l_2 - 3l_1}{2} = \frac{100 - 3(32)}{2} = 2cm$

20. A wire of length 30cm, stretched between rigid supports has it's n^{th} and $(n+1)^{th}$ harmonics at 400Hz and 450 Hz respectively. If tension in the string is 2700 N. It's linear mass density is ….. Kg/m. **Jee Main 2022**

Key :3

Sol : Difference in successive harmonic frequencies is equal to fundamental frequency.

$$f_{n+1} - f_n = \frac{V}{21} = \frac{1}{21}\sqrt{\frac{T}{\mu}}$$

$$450 - 400 = \frac{1}{2\times30\times10^{-2}}\sqrt{\frac{2700}{\mu}}$$

$$\mu = \frac{2700}{900} = 3kg/m$$

20a. A wire of length 50cm, stretched between rigid supports has it's p^{th} and $(p+1)^{th}$ harmonics at 500Hz and 550 Hz and respectively. If tension in the string is 4900 N. It's linear mass density is ….. Kg/m Nearly.

Key :2

Sol : Difference in successive harmonic frequencies is equal to fundamental frequency.

$$f_{p+1} - f_p = \frac{V}{21} = \frac{1}{21}\sqrt{\frac{T}{\mu}}$$

$$550 - 500 = \frac{1}{2\times1/2}\sqrt{\frac{4900}{\mu}} \Rightarrow 50 = \frac{70}{\sqrt{\mu}}$$

$$\mu = \frac{4900}{2500} \approx 2kg/m$$

20b. A wire of length 100cm, stretched between rigid supports has it's p^{th} and $(p+2)^{th}$ harmonics at 500 Hz and 600 Hz and respectively. If tension in the string is 5kg.wt. It's linear mass density is ….. $\times10^{-4}$ Kg/m.

Key : 49

Sol : Difference in successive harmonic frequencies is equal to fundamental frequency.

$$f_{p+2} - f_p = \frac{2V}{2\ell} = \frac{2}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$600 - 500 = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{5\times9.8}{\mu}} \Rightarrow \frac{7}{\sqrt{\mu}} = 100$$

$$\mu = 49\times10^{-4}kg/m$$

20c. A wire of length 20cm, stretched between rigid supports has it's p^{th} and $(p+1)^{th}$ harmonics at

100 Hz and 250 Hz and respectively. If tension in the string is 100N. It's linear mass density is ….. $\times10^{-3}$ Kg/m.

Key : 28

Sol : Difference in successive harmonic frequencies is equal to fundamental frequency.

$$f_{p+1} - f_p = \frac{2V}{2\ell} = \frac{2}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$150 = \frac{1}{2(0.2)}\sqrt{\frac{100}{\mu}} \Rightarrow \mu = \frac{1}{36} = 28\times10^{-3}$$

21. Two wires of different linear mass densities are joined, consider junction to be $x=0$. An incident wave $y_i = A_i \sin(\omega t - k_1 x)$ is travelling to the right from the region $x \le 0$. At the boundary the wave is partly reflected and partly transmitted. Find the reflected amplitudes in terms of the incident amplitude. k_2 is wave number of transmitted wave

A) $\left(\dfrac{k_1-k_2}{k_1+k_2}\right)A_i$ B) $\left(\dfrac{k_1+k_2}{k_1-k_2}\right)A_i$ C) $\left(\dfrac{k_1k_2}{k_1-k_2}\right)A_i$ D) $\left(\dfrac{k_1k_2}{k_1+k_2}\right)A_i$

Key : A

Sol : Given

$$y_i = A_i \sin(\omega t - k_1 x)$$

Medium-1 Medium-2

The equation of reflected and transmitted wave are

$$y_r = A_r \sin(\omega t - k_1 x),$$

We have $A_r + A_t = A_i$ ......(i)

And $y_r + y_t = y_i$ .....(ii)

Differentiating equation(ii) partially

$$\frac{\partial y_r}{\partial x} + \frac{\partial y_t}{\partial x} = \frac{\partial y_i}{\partial x}$$

Or $\dfrac{\partial}{\partial x}\left[A_r \sin(\omega t - k_1 x)\right] + \dfrac{\partial}{\partial x}\left[A_t \sin(\omega t - k_2 x)\right] = \dfrac{\partial}{\partial x}\left[A_i \sin(\omega t - k_1 x)\right]$

Or $A_r k_1 \cos\omega t - A_t k_2 \cos\omega t = -A_i k_1 \cos\omega t$ at $x=0$

Or $A_r k_1 - A_t k_2 = -A_i k_1$ .....(iii)

Solving equations (i) & (iii), we get

$$A_r = \left(\frac{k_1-k_2}{k_1+k_2}\right)A_i$$

21. Two wires of different linear mass densities are joined, consider junction to be $x=0$. An incident wave $y_i = A_i \sin(\omega t - k_1 x)$ is travelling to the right from the region $x \le 0$. At the boundary the wave is partly reflected and partly transmitted. Find the transmitted amplitudes in terms of the incident amplitude. k_2 is wave number of transmitted wave

A) $\left(\dfrac{k_1-k_2}{k_1+k_2}\right)A_i$ B) $\left(\dfrac{k_1+k_2}{k_1-k_2}\right)A_i$ C) $\left(\dfrac{2k_2}{k_1-k_2}\right)A_i$ D) $\left(\dfrac{2k_1}{k_1+k_2}\right)A_i$

Key : D

Sol : Given

$$y_i = A_i \sin(\omega t - k_1 x)$$

Medium-1 Medium-2

The equation of reflected and transmitted wave are

$$y_r = A_r \sin(\omega t - k_1 x),$$

We have $A_r + A_t = A_i$ ......(i)

And $y_r + y_t = y_i$ .....(ii)

Differentiating equation(ii) partially

$$\frac{\partial y_r}{\partial x} + \frac{\partial y_t}{\partial x} = \frac{\partial y_i}{\partial x}$$

Or $\frac{\partial}{\partial x}\left[A_r \sin(\omega t - k_1 x)\right] + \frac{\partial}{\partial x}\left[A_t \sin(\omega t - k_2 x)\right] = \frac{\partial}{\partial x}\left[A_i \sin(\omega t - k_1 x)\right]$

Or $A_r k_1 \cos\omega t - A_t k_2 \cos\omega t = -A_i k_1 \cos\omega t$ at $x = 0$

Or $A_r k_1 - A_t k_2 = -A_i k_1$ .....(iii)

Solving equations (i) & (iii), we get

And $A_t = \left(\frac{2k_1}{k_1 + k_2}\right) A_i$

22. A sonometer wire resonates with a given tuning fork forming standing waves with fine antinodes between the two bridges when a mass of 9kg is suspended from the wire. When the mass is replaced by M the wire resonates with same tuning fork forming antinodes for the same position of bridges the value of M is **IIT 2002**

A) 25kg B) 5kg C) 12.5kg D) $\frac{1}{25}kg$

Key : A

Sol : $f_0 = \frac{5}{2l}\sqrt{\frac{9g}{\mu}}$

$= \frac{3}{2l}\sqrt{\frac{Mg}{\mu}}$

$\Rightarrow M = 25kg$

22a. A sonometer wire resonates with a given tuning fork forming standing waves with six nodes between the two bridges when a mass of 16kg is suspended from the wire. When the mass is replaced by M the wire resonates with same tuning fork forming 3 antinodes for double the distance between the bridges the value of M in Kg.

A) 190.2 B) 167.7 C) 13.33 D) 182.8

Key : C

Sol : $n = \frac{p}{2l}\sqrt{\frac{T}{m}} = \frac{p}{2\ell}\sqrt{\frac{mg}{M}}$ $n\alpha \frac{p}{\ell}\sqrt{M}$

$\therefore \frac{P_1}{l_1}\sqrt{M_1} - \frac{P_2}{l_2}\sqrt{M_2}$

$\frac{5}{l}\sqrt{16} = \frac{3}{2l}.\sqrt{M_2}$

$\sqrt{M_2} = \frac{5\times4\times2}{3} = 177.78$

$M_2 = 13.33$

23. A pipe of length l_1, closed at one end, is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if the frequency of first overtone in both the cases is equal **IIT 2004**

A) $\frac{4}{3}l_1\sqrt{\frac{\rho_2}{\rho_1}}$ B) $\frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$ C) $l_1\sqrt{\frac{\rho_2}{\rho_1}}$ D) $l_1\sqrt{\frac{\rho_1}{\rho_2}}$

Key : B

Sol : Frequency of first overtone in closed pipe

$$v = \frac{3}{4l_1}\sqrt{\frac{P}{\sqrt{\rho_1}}}$$

Frequency of first overtone in open pipe

$$v^1 = \frac{1}{l_2} = \sqrt{\frac{P}{\rho_2}}$$

$$\Rightarrow l_2 = \frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$$

23a. A pipe of length l_1, closed at one end, is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of first gas is double that of the other. What is the length of the second pipe if frequency of first overtone in both the cases are equal

A) $\frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$ B) $l_1\sqrt{\frac{\rho_2}{\rho_1}}$ C) $l_1\sqrt{\frac{\rho_1}{\rho_2}}$ D) $4\frac{\sqrt{2}}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$

Key : D

Sol : Compressibility $= \frac{1}{\text{Bulk modulus}}$

For isothermal condition bulk modulus of a gas is equal to pressure

Frequency $\frac{CP_1}{CP_2} = \frac{\rho_2}{\rho_1}$

$$\frac{2C}{C} = \frac{P_2}{P_1}$$

$P_2 = 2P$

$P_1 = P$

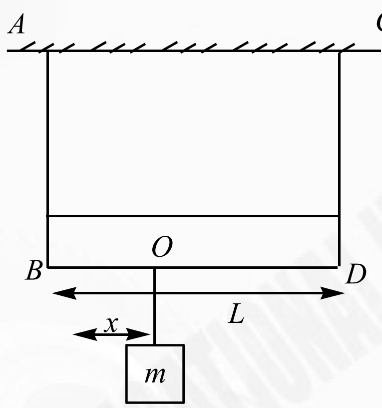
Frequency $\upsilon = \frac{3}{4l_1}\sqrt{\frac{P}{\rho_1}}$

For open pipe $\upsilon = \frac{2}{2l_2}\sqrt{\frac{P_2}{\rho_2}} = \frac{1}{l_2}\sqrt{\frac{2P}{\rho_2}}$

$$\frac{3}{4l_1}\times\sqrt{\frac{P}{\rho_1}}=\frac{1}{l_2}\sqrt{\frac{2P}{\rho_2}}$$

$$\therefore\ l_2=4\frac{\sqrt{2}}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$$

24. A mass less rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal x. Further it is observed that the frequency of first harmonic in AB is equal to second harmonic frequency in CD. Then x is **IIT 2006**



A) $\frac{L}{5}$ B) $\frac{4L}{5}$ C) $\frac{3L}{4}$ D) $\frac{L}{4}$

Key : A

Sol : For rotational equilibrium of mass less rod taking torque about point O

$T_{AB}\times x=T_{CD}\times(l-x)$

For translational equilibrium,

$T_{AB}+T_{CD}=mg$

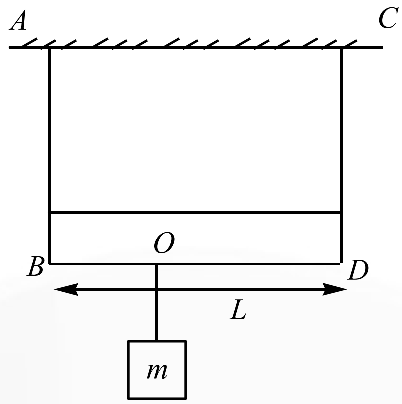
$T_{CD}=\frac{mg}{5}$

$\therefore T_{AB}=\frac{4mg}{5}$

Then $\frac{4mg}{5}\times x=\frac{mg}{5}(L-x)$

$\Rightarrow x=\frac{L}{5}$

24a. A mass less rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal $\frac{l}{5}$. Further it is observed that AB vibrates in fundamental frequency where as CD vibrates in third harmonic then the ratio frequency of AB to that of CD is



Key : B

Sol : For rotational equilibrium $T_{AB}\frac{L}{5}=T_{AB}\frac{4L}{5}$

$\therefore T_A=4T_B$

Fundamental frequency $P_1=1$, 3rd harmonic $P_2=3$

$$n=\frac{P}{2l}\sqrt{\frac{T}{m}}$$

$$n_A=\frac{1}{2l}\sqrt{\frac{T_A}{m}}$$

$$n_B=\frac{3}{2l}\sqrt{\frac{T_B}{m}}$$

$$n_A:n_B=2:3$$

IV) Beats & Echo

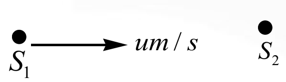
Time period and frequency-Beat frequency, Beat frequency for three sources

1. Two sources of sound S_1 and S_2 produces sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals to **Jee Mains 2019**

A) 2.5 m/s B) 10.0m/s C) 5.5m/s D) 15.0m/s

Key : A

Sol:



$$f_1=\left(\frac{V-u}{V}\right)f \qquad f_2=\left(\frac{V+u}{V}\right)f$$

$$f_2-f_1=\left(\frac{V+u-V+u}{V}\right)f=\frac{2uf}{V}$$

$$10=\frac{2u\times 660}{330}\Rightarrow u=\frac{10}{4}=2.5m/s$$

1a. Two sources of sound S_1 and S_2 produces sound waves of same frequency 990 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 20 beats/s. The velocity of sound is 330 m/s. Then, u equals to

A) 2.8 m/s B) 3.3 m/s C) 4.2 m/s D) 1.8 m/s

Key : B

Sol: For s_1 $f_1 = \left(\frac{v - v_0}{v}\right) f$

For s_2 $f_2 = \left(\frac{v + v_0}{v}\right) f$

$$\Delta f = f_2 - f_1 = \left(\frac{v + v_0 - v + v_0}{v}\right) f$$

$$20 = \frac{2 \times u}{330} 990 \Rightarrow 4 = \frac{20}{6} = 3.3ms^{-1}$$

1b. Two sources of sound S_1 and S_2 produces sound waves of same frequency 1200 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 25 beats/s. The velocity of sound is 330 m/s. Then, u equals to

A) $3.4ms^{-1}$ B) $4.5ms^{-1}$ C) $2.4ms^{-1}$ D) $3ms^{-1}$

Key : A

Sol: For s_1 $f_1 = \left(\frac{v - v_0}{v}\right) f$

For s_2 $f_2 = \left(\frac{v + v_0}{v}\right) f$

$$\Delta f = f_2 - f_1 = \left(\frac{v + v_0 - v + v_0}{v}\right) f = \frac{2v_0}{v} f$$

$$u = v_0 = \frac{\Delta f v}{2f} \Rightarrow u = \frac{25 \times 330}{2 \times 1200} = \frac{82.5}{24} = 3.4ms^{-1}$$

2. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is $f = 1400$ Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to : **Jee Main 2020**

A) $\frac{1}{2} m/s$ B) $1 m/s$ C) $\frac{1}{4} m/s$ D) $\frac{1}{8} m/s$

Key : C

Sol : Number of beats heard $\Delta f = \frac{2v_s}{v}.f$

$$2 = \frac{2 \times v_s}{350} \times 1400$$

$$v_s = \frac{350}{1400} = \frac{5}{20} = \frac{1}{4} ms^{-1}$$

2a. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 4 beats/sec. The oscillation frequency of each tuning fork is $f = 1800$ Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to :

A) $\frac{7}{18} m/s$ B) $\frac{5}{18} m/s$ C) $\frac{4}{15} m/s$ D) $\frac{1}{2} m/s$

Key : A

Sol : Number of beats heard $\Delta f = \frac{2v_s}{v}.f \Rightarrow v_s = \frac{\Delta f v}{2f}$

$$v_s = \frac{4 \times 350}{2 \times 1800} = \frac{700}{1800} = \frac{7}{18} ms^{-1}$$

2b. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 6 beats/sec. The oscillation frequency of each tuning fork is $f = 1200$ Hz and the velocity of sound in air is 330 m/s. The speed of each tuning fork is close to :

A) $0.625 ms^{-1}$ B) $0.4 ms^{-1}$ C) $0.825 ms^{-1}$ D) $0.75 ms^{-1}$

Key : C

Sol : $\Delta f = f_1 - f_2$

$$\Delta f = \frac{2v_s}{v} f$$

$$6 = 2 \times \frac{v_s}{330} 1200 \Rightarrow v_s = \frac{990}{1200} = \frac{33}{40}$$

$$= \frac{8.25}{10} = 0.825 ms^{-1}$$

3. The velocity of sound in a gas, in which two wavelengths 4.08m and 4.16m produce 40 beats in 12s, will be : **Jee Main 2022**

A) $282.8 ms^{-1}$ B) $175.5 ms^{-1}$ C) $353.6 ms^{-1}$ D) $707.2 ms^{-1}$

Key :D

Sol : $\lambda_1 = 4.08m$; $\lambda_2 = 4.16$; $\Delta n = 40/12 \sec$

$$n_1 - n_2 = \Delta n \Rightarrow \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = \frac{40}{12} \Rightarrow v\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

On solving $v = 707.2 ms^{-1}$

3a. The velocity of sound in a gas, in which two wavelengths 5m and 5.2m produce 30 beats in 6s, will be :

A) $520 ms^{-1}$ B) $650 ms^{-1}$ C) $700 ms^{-1}$ D) $740 ms^{-1}$

Key :B

Sol : $\Delta n = \frac{30}{6} = 5$ $\Delta n = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$

$\Rightarrow 5 = \frac{v}{5} - \frac{v}{5.3} \Rightarrow 5 = v\left(\frac{0.2}{26}\right)$

$v = \frac{26 \times 5}{0.1} = 650 ms^{-1}$

4. A man standing between two parallel cliffs fires a gun. If he hears the first echo after 2 sec and the next after 5 sec, the distance between the two cliffs is ____(velocity of sound in air is 350 m/sec)

A) 1225 m B) 1050 m C) 2100 m D) 2450 m

Key : A

Sol : $d = d_1 + d_2 = \frac{vt_1}{2} + \frac{vt_2}{2}$ $\left(t = \frac{2d}{v}\right)$

$d = \frac{v(t_1 + t_2)}{2} = \frac{350 \times 7}{2} = \frac{2450}{2}$

$d = 1225m$

4a. A rifle shot is fired between two parallel mountains. The echo from one mountain is heard after 2 sec later. If the velocity of sound is 360 m/s. The width of the valley is

A) 360 m B) 720 m C) 1080 m D) 1800 m

Key : C

Sol : time of 1st echo $t_1 = 2\sec$

Time of 2nd echo $t_2 = 2 + 2 = 4\sec$

Width of valley $d = \frac{v(t_1 + t_2)}{2} = \frac{360 \times 6}{2}$

$d = 1080m$

5. An engine approaches a wall with constant speed. When it is at a distance of 0.9 km, it blows a whistle, whose echo is heard by the driver after 5 sec. If the speed of sound in air is 330 m/s, the speed of engine is

A) 60 m/s B) 90 m/s C) 30 m/s D) 15 m/s

Key : C

Sol : $d = 0.9km = 900m$

Initial distance $d = \left(\frac{v + v_s}{2}\right)t$

$900 = \left(\frac{330 + v_s}{2}\right)5 \Rightarrow 330 + v_s = \frac{1800}{5} = 360$

$v_s = 360 - 300 = 30m/s$

5a. An engine approaching a hill with constant speed $40ms^{-1}$. When it is at distance d. it blows a whistle, whose echo is heard by the driver after 4sec. If speed of sound in air is $320ms^{-1}$, find value of d

A) 600 m B) 520 m C) 480 m D) 720 m

Key : D

Sol: Initial distance d=$\left(\frac{v+v_s}{2}\right)t$

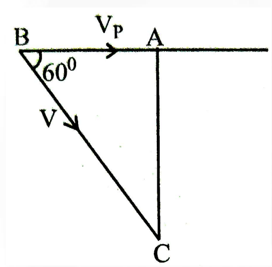
$$\Rightarrow d=\left(\frac{320+40}{2}\right)4=\frac{360}{2}\times 4=180\times 4=720m$$

6. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60^0 with ground level. But he finds the aeroplane right vertically above his position. If V is the speed of sound, speed of the plane is **Jee Mains 2021**

A) $\frac{2V}{\sqrt{3}}$ B) V C) $\frac{V}{2}$ D) $\frac{\sqrt{3}}{2}V$

Key : C

Sol:



$$AB=V_p\times t;BC=Vt;\cos 60=\frac{AB}{BC}$$

$$\frac{1}{2}=\frac{V_p\times t}{Vt};V_p=\frac{V}{2}$$

7. Two vibrating tuning forks produce a progressive wave given by $y_1=2\sin 500\pi t$ and . $y_2=4\sin 508\pi t$. Find time interval between two successive maxima is ___

A) $\frac{1}{4}$sec B) $\frac{1}{2}$sec C) $\frac{3}{4}$sec D) $\frac{1}{8}$sec

Key : A

Sol : $\omega_1=500\pi\Rightarrow 2\pi\ n_1=500\pi\quad n_1=250Hz$

$2\pi n_2=508\pi\quad n_2=254Hz$

Time interval between maximum x $t=\frac{1}{n_2-n_1}=\frac{1}{4}$

V)Doppler's effect

Doppler's effect in source and observer are moving in one dimension, Doppler's effect in source and observer are moving in two dimension, Effect of the source motion on the wavelength, Effect of wind speed.

1. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to **Jee Mains 2019**

A) 753 Hz B) 400 Hz C) 333 Hz D) 666 Hz

Key : D

Sol: In 2nd harmonic $l=\frac{2\lambda}{2} \Rightarrow l=\lambda \Rightarrow f=\frac{v}{\lambda}=\frac{330}{0.5}=660Hz$

$$v_0=\frac{10\times 5}{18}=\frac{50}{18}=\frac{25}{9}$$

$$f=\frac{v+v_0}{v}\times f=\left(\frac{330+\frac{25}{9}}{330}\right)\times 660=666Hz$$

1a. A string of length 60 cm vibrating in 3rd harmonic. A person moving towards string with speed 36 kmph. Speed of sound is 330 ms^{-1} find frequency heard by person

A) 837.5 Hz B) 672.5 Hz C) 808 Hz D) 750 Hz

Key : A

Sol : In 3rd harmonic $l=\frac{3\lambda}{2} \Rightarrow \lambda=\frac{2l}{3}=2\times\frac{60}{3}=40cm$

$$f=\frac{v}{\lambda}=\frac{330}{0.4}=825Hz, v_0=36\times\frac{5}{18}=10ms^{-1}$$

$$f=\frac{v+v_0}{v}\times f=\left(\frac{330+10}{330}\right)\times 825=837.5Hz$$

1b. A string of length 90cm is vibrating in 3rd harmonic. A person moving towards the string at rest with speed 36 km/hr speed of sound is $330ms^{-1}$. Find frequency hear by person

A) 600 Hz B) 566.5Hz C) 552.6 Hz D) 550 Hz

Key : B

Sol : In third harmonic length $l=\frac{3\lambda}{2} \Rightarrow \lambda=\frac{2\ell}{3}=\frac{2\times 90}{3}=60cm$

Frequency $f=\frac{V}{\lambda}=\frac{330}{0.6}=\frac{3300}{6}=550Hz$

Apparent frequency heard by person $f^1=\left(\frac{V+V_0}{V}\right)f$

$$V_0=36\times\frac{5}{18}=10ms^{-1} \Rightarrow f^1=\left(\frac{340}{330}550\right)=566.5Hz$$

1c. An engine standing at the plat form blows a whistle of frequency 305 Hz. If the velocity of sound be 1220 km/hr, the frequency of the whistle as heard by a man running towards the engine with a speed of 20 km/hr is

A) 310 Hz B) 305 Hz C) 300 Hz D) 325 Hz

Key : A

Sol : Apparent frequency heard by man $f^1 = \left(\frac{V+V_0}{V}\right)f = \left(1+\frac{V_0}{V}\right)f$

$$f^1 = \left(1+\frac{20}{1220}\right)305 = \left(1+\frac{1}{61}\right)305 = \frac{62}{61}\times 305 = 310Hz$$

2. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is **Jee Mains 2019**

A) $18/17$ B) $19/18$ C) $20/19$ D) $21/20$

Key : B

Sol: Apparent frequency $f^1 = \left(\frac{v}{v-v_s}\right)f \Rightarrow \frac{f_1}{f_2} = \frac{v}{v-v_{s_1}}\frac{v-v_{s_2}}{v}$

$$= \frac{340-17}{340-34} = \frac{323}{306} = \frac{19}{18}$$

2a. when source of sound is moving towards a stationary observer with velocity $\frac{V}{11}$, frequency registered by observer is f^1. If original frequency of source f. Then find $\frac{f^1}{f}$ (velocity of sound is V)

A) $\frac{10}{11}$ B) $\frac{11}{12}$ C) $\frac{12}{11}$ D) $\frac{11}{10}$

Key : D

Sol : $f^1 = \left(\frac{V}{V-V_s}\right)f = \left(\frac{V}{V-\frac{V}{11}}\right)f \Rightarrow \frac{f^1}{f} = \frac{11}{10}$

2b. A whistling engine is approaching a stationary observer with a velocity of 110m/s. The ratio of frequencies as heard by the observer as the engine approaches and recedes is
$(V = 330ms^{-1})$.

A) 4:3 B) 4:1 C) 3:6 D) 2:1

Key :4

Sol : $f' = f\left(\frac{V}{V-V_s}\right)$ as engine approaches

$f'' = f\left(\frac{V}{V+V_s}\right)$ as engine recedes

$\Rightarrow \frac{f'}{f''} = \frac{V+V_s}{V-V_s} = \frac{440}{220} = 2:1$

3. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ? (Take velocity of sound in air is 350 m/s) **Jee Mains 2019**

A) 1143 Hz B) 857 Hz C) 750 Hz D) 807 Hz

Key : C

Sol: $n_1 = \left(\frac{v}{v-v_s}\right)n$

$n_2 = \left(\frac{v}{v+v_s}\right)n \Rightarrow \frac{n_2}{n_1} = \frac{v-v_s}{v+v_s} \Rightarrow n_2 = \frac{350-50}{350+50} \times 1000 = 750Hz$

3a. A railway engine moving with speed $60ms^{-1}$ passes in front of a stationary listener. The observer measures the frequency of source as 485Hz. What will be apparent frequency of source, when it is moving away from the observer after crossing him ($v = 340m/\sec$)

A) 340Hz B) 324 Hz C) 448 Hz D) 460 Hz

Key : A

Sol : $n_1 = \left(\frac{v}{v-v_s}\right)n$

$n_2 = \left(\frac{v}{v+v_s}\right)n \Rightarrow \frac{n_1}{n_2} = \frac{v+v_s}{v-v_s}$

$\frac{485}{n_2} = \frac{400}{280} \Rightarrow n_2 = \frac{485 \times 28}{40} = 339.5 \approx 340Hz$

3b. A railway engine moving with speed $40ms^{-1}$ passes in front of a stationary listener. The observer measures the frequency of source as 380Hz. What will be apparent frequency of source, when it is moving away from the observer after crossing him ($v = 340m/\sec$)

A) 350Hz B) 400 Hz C) 300 Hz D) 280 Hz

Key : C

Sol : $n_1 = \left(\frac{v}{v-v_s}\right)n$

$n_2 = \left(\frac{v}{v+v_s}\right)n \Rightarrow \frac{n_1}{n_2} = \frac{v+v_s}{v-v_s}$

$\Rightarrow n_2 = \frac{v-v_s}{v+v_s}n_1 = \frac{300}{380} \times 380 = 300Hz$

4. With what speed should a galaxy move outward with respect to earth so that the sodium-D line at wavelength $5890A^0$ …is observed at $5896A^0$? **Jee Mains-2021**

A) 306 km/sec B) 322 km/sec C) 296 km/sec D) 336 km/sec

Key: A

Sol : Apparent wave length

$$\lambda^1 = \left(\frac{v+v_s}{v}\right)\lambda = \left(\frac{c+v_s}{c}\right)\lambda$$

$$5896 = \left(\frac{3\times10^8 + v_s}{3\times10^8}\right)5890$$

$$3\times10^8 + v_s = \frac{5896}{5890}\times3\times10^8$$

$$v_s = (1.00102-1)\times3\times10^8$$

$$= 0.00102\times3\times10^8 \qquad = 102\times3\times10^3\,\text{m/s} \qquad = 306\times km/s$$

4a. A galaxy is moving away from the earth at a speed of 286kms −1 . The shift in the wavelength of a red line at 630nm is_____. [Take the value of speed of light as $3\ \times10^8$]

A) $6\times10^{-10}m$ B) $8\times10^{-10}m$ C) $4\times10^{-10}m$ D) $3\times10^{-10}m$

Key: A

Sol: $\frac{\Delta\lambda}{\lambda}c = v$

$$\Delta\lambda = \frac{v}{c}\times\lambda = \frac{286}{3\times10^5}\times630\times10^{-9} = 6\times10^{-10}m$$

4b. A galaxy is moving out ward with speed 40 km/s w.r.to earth. Find shift in the wave length of D line at $5800A^0$ (in A^0)$\left(c = 3\times10^8 ms^{-1}\right)$

A) 6.24 B) 4.68 C) 7.73 D) 8.2

Key : C

Sol : $\frac{\Delta\lambda}{\lambda}c = v \Rightarrow \Delta\lambda = \frac{v}{c}\lambda$

$$\Delta\lambda = \frac{400\times10^3}{3\times10^8}5800\times10^{-10} \quad = \frac{4\times58}{3}10^{-18+7} = 77.3\times10^{-11} \quad = 7.73\times10^{-10} = 7.73A^0$$

5. An observer moves towards a stationary source of sound with velocity equal to one fifth of the velocity of sound. The percentage change in the frequency will be **Jee Mains-2022**

A) 20% B) 10% C) 5% D) 0%

Key : A

Sol : $n^1 = \left(\frac{v+v_0}{v}\right)n = \left(1+\frac{v_0}{v}\right)n \Rightarrow \frac{\Delta n}{n} = \frac{v_0}{v}$

$$\Rightarrow \Delta n\% = \frac{1}{5}\frac{v}{v}\times100 = 20\%$$

5a. An observer moves towards stationary source of sound with velocity $100ms^{-1}$. The percentage change in frequency will be ____ $\left(v = 340ms^{-1}\right)$

A) 35.2% B) 29.4% C) 21% D) 42%

Key : **B**

Sol : $n^1 = \left(\dfrac{v+v_0}{v}\right)n = n + \dfrac{v_o}{v}n$

$$\Delta n = n^1 - n = \frac{v_0}{v}.n = \frac{100}{340}n$$

$$\Delta n\% = \frac{\Delta n}{n}\times 100 = \frac{100}{340}\times 100 = \frac{1000}{34} = 29.4\%$$

5b. An observer moves towards stationary source of sound with velocity $160ms^{-1}$. The percentage change in frequency will be ____ $\left(v = 350ms^{-1}\right)$

A) 38.6% B) 32.5% C) 40% D) 45.7%

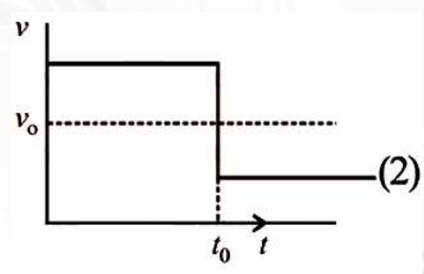
Key : **D**

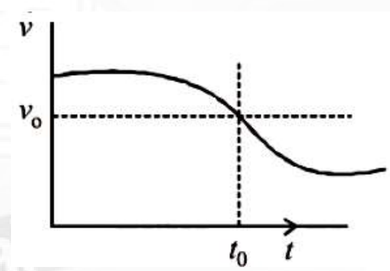
Sol : $n^1 = \left(\dfrac{v+v_0}{v}\right)n = n + \dfrac{v_o}{v}n$

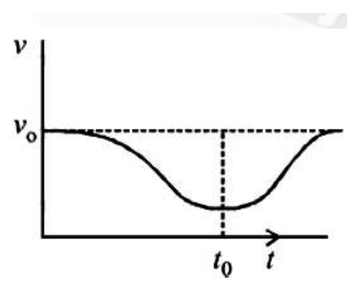
$$\Delta n = n^1 - n = \frac{v_0}{v}n \Rightarrow \frac{\Delta n}{n} = \frac{v_0}{v}$$

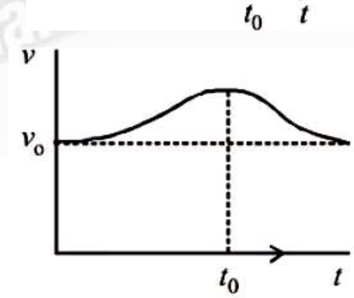
$$\Delta n\% = \frac{160}{350}\times 100 = \frac{320}{7} = 45.7\%$$

6. A sound source S is moving along at straight track with speed v, and is emitting sound of frequency v_0(see fig.) An observer is standing at finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by. (t_0 represent the instant when the distance between the source and observer is minimum) **Jee Main 2020**

A) 

B) 

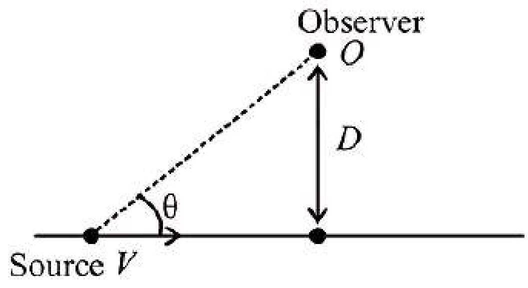
C) 

D) 

Key : B

Sol : Frequency heard by the observer

$$v_{\text{observerd}} = \left(\frac{v_{\text{sound}}}{v_{\text{sound}} - v\cos\theta}\right)v_0$$



Initially θ wiil be less so $\cos\theta$ more.

$\therefore\ v_{\text{observed}}$ more, then it will decrease.

7. A siren placed at a railway platform is emitting sound of frequency 5Khz. A passenger sitting in a moving train 'A' records a frequency of 5.5kHz. While the train approaches the siren. During his return journey in a different train 'B' he records a frequency of 6.0kHz. while approaching the same siren the ratio of velocity of train B to that of 'A' is **IIT 2002**

A) $\frac{242}{252}$ B) 2 C) 14 D) 6

Key : B

Sol : Given $f_A = 5.5kHz$

$f = 5kHz$

$f_B = 6kHz$

$\because P_A = f\left(\frac{v+v_A}{v}\right)$

$\frac{5.5}{5} = 1 + \frac{v_A}{v}$

$\Rightarrow \frac{v_A}{v} = 0.1.....(1)$

$f_B = f\left(\frac{v+v_B}{v}\right)$

$\frac{6}{5} = 1 + \frac{v_B}{v}$ $\quad 0.2 = \frac{v_B}{v}......(2)$

Put 1 and 2

$\frac{v_B}{v_A} = \frac{0.2}{0.1}$ $\quad \frac{v_B}{v_A} = 2$

7a. A stationary body emits sound at frequency 10kHz. A boy sitting in a train 'A' approaching the body observes a frequency as 10.2kHz. The same boy while returing towards the body in other than 'B'. Observes a frequency of 10.3 kHz, then the ratio of velocity of train A to that of B will be

A) 4 B) $\frac{3}{2}$ C) 2 D) $\frac{6}{5}$

Key : B

Sol : For approaching $n^1 = n\left[\frac{v+v_0}{v}\right]$

$\therefore\ 10.2 = n\left[\frac{v+v_A}{v}\right] \qquad 10.3 = 10\left[1+\frac{v_B}{v}\right]$

$1+\frac{v_1}{v} = \frac{10.2}{10} \qquad V_A = \left(\frac{10^2}{10}-1\right).v$

Similar $\qquad V_B = \left(\frac{10^3}{10}-1\right).v$

$\Rightarrow \frac{V_B}{V_A} = \frac{3}{2}$

8. A police car moving at 22 m/s chases a motor cyclist. The police man sounds his horn at 176 Hz, while both of them move toward a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle. If it is given that he does not observe any beats.(330m/s) **IIT 2003**

A) 33 m/s B) 22 m/s C) zero D) 11 m/s

Key : D

Sol : f_1 is the frequency of the police car heard by motorcyclist and f_2 is the frequency of the siren heard by motorcyclist.

$f_1 = \frac{330-v}{330-22}\times 176$

$f_2 = \frac{330+v}{330}\times 165$

$\therefore\ f_1 - f_2 = 0 \Rightarrow v = 22m/s$

8a. A police car is moving at 20m/s chases a motorcyclist. The police man sounds his horn at 180 Hz while both of them moles towards a source of siren of frequency 170Hz coming opposite to both of them with a speed of 10m/s. If motorcyclist hears no beats then the speed of motorcyclist will be (speed of sound 330m/s)

A) 16.2 m/s B) 14.6 m/s C) 20 m/s D) 22 m/s

Key : B

Sol : Frequency observed by the motor cyclist due to police car $f_1 = \frac{330-v_m}{330-20}\times 180$

Frequency observed by motor cyclist due to moving siren $f_2 = \frac{330+v_m}{330-10}\times 170$

For no beats $f_1 = f_2$

$\frac{330-v_m}{330-20}\times 180 = \frac{330+v_m}{330-10}\times 170$

$v_m = 14.6m/s$

Exercise : II

(Numerical / Integer Value based Questions Including PYQ's)

I) Introduction to waves, Travelling wave

Definition of wave, 1-D,2-D,3-D waves, Mechanical and non mechanical waves (Mechanical waves, Non mechanical waves), Longitudinal and transverse waves (Longitudinal wave, Transverse wave), Equation of travelling wave (Equation of plane progressive harmonic wave, Amplitude ;frequency and speed of the wave from the equation), Relation between path difference and phase difference - Equation of plane progressive harmonic wave, Displacement ;velocity and acceleration of the particle, Relation between wave velocity and particle velocity, Speed of the transverse wave in a string, Speed of the transverse wave in solids, Sound wave(Speed of sound wave in solids, Speed of sound wave in liquids and gases, Conversion from displacement variation to pressure variation, Speed of the longitudinal wave), Energy of a progressive wave, Sub-Sonic, Super sonic, Mach number, Audible range, infra sonic and ultra sonic, Intensity of sound wave(Formula for intensity, Variation of intensity with distance, Sound level).

1. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form $y = -0.21\sin(x+30t)$ is produced in it, where x is in meter and is t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2} N$. Value of x is ____. (Round-off to the nearest integer) **Jee Mains 2021**

Key: **1215**

Sol: $T = v^2 \times \mu$

$T = (30)^2 \times 0.135 \times 10^{-1}$

$T = 12.15N$

$T = 1215N \times 10^{-2} N$

$X = 1215$

1a. The mass per unit length of a uniform wire is 0.27 g/cm. A transverse wave of the form $y = -0.21\sin(x+60t)$ is produced in it, where x is in meter and is t is in second. Then, the expected value of tension in the wire is $x \times 10^{-1} N$. Value of x is ____. (Round-off to the nearest integer)

Key: **972**

Sol: $T = v^2 \times \mu$

$T = (60)^2 \times 0.27 \times 10^{-1}$

$T = 972 \times 10^{-1} N$

$X = 972$

1b. The mass per unit length of a uniform wire is 0.54 g/cm. A transverse wave of the form $y = -0.21\sin(x+60t)$ is produced in it, where x is in meter and is t is in second.

Then, the expected value of tension in the wire is $x\times10^{-1}N$. Value of x is ____. (Round-off to the nearest integer)

Sol: $T = v^2\times\mu$

$T=(60)^2\times0.54\times10^{-1}$

$T=1944\times10^{-1}N$

$X=1944$

1c. The mass per unit length of a uniform wire is 0.81 g/cm. A transverse wave of the form

$y = -0.21\sin(x + 20t)$ is produced in it, where x is in meter and is t is in second. Then, the expected value of tension in the wire is $x\times10^{-1}N$. Value of x is ____. (Round-off to the nearest integer)

Key: 324

Sol: $T = v^2\times\mu$

$T=(20)^2\times0.81\times10^{-1}$

$T=324\times10^{-1}N$

$X=324$

2. A wire having linear mass density $9.0\times10^{-4}kg/m$ is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500Hz. The next higher frequency at which the same wire resonates is 550Hz . The length of the wire is…m **Jee Mains 2021**

Key: 10

Sol: $\mu=9.0\times10^{-4}\text{Kg/m}$

$T=900N$

$V=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{900}{90\times10^{-4}}}=1000m/s$

$f_1=500Hz$

$f=550$

$\frac{nV}{2l}=500\ldots$

$\frac{(n+1)V}{2l}=550$

$\ell=\frac{1000}{2\times50}=10m$

2a. A wire having linear mass density $18\times10^{-4}kg/m$ is stretched between two rigid supports with a tension of 1800 N. The wire resonates at a frequency of 500Hz. The next higher frequency at which the same wire resonates is 550Hz . The length of the wire is…m

Key: 10

Sol: $\mu=9.0\times10^{-4}\text{Kg/m}$

$T = 1800N$

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1800}{18\times10^{-4}}} = 1000m/s$

$f_1 = 500Hz$

$f = 550$

$\frac{nV}{2l} = 500\ldots$

$\frac{(n+1)V}{2l} = 550$

$l = \frac{1000}{2\times50} = 10m$

2b. A wire having linear mass density $18\times10^{-4}\,kg/m$ is stretched between two rigid supports with a tension of 1800 N. The wire resonates at a frequency of 500Hz. The next higher frequency at which the same wire resonates is 600Hz . The length of the wire is…m

Key: 5

Sol: $\mu = 18\times10^{-4}\,Kg/m$

$T = 1800N$

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1800}{18\times10^{-4}}} = 1000m/s$

$f_1 = 500Hz$

$f = 600$

$\frac{nV}{2l} = 500\ldots$

$\frac{(n+1)V}{2l} = 600$

$l = \frac{1000}{2\times100} = 5m$

2c. A wire having linear mass density $27\times10^{-4}\,kg/m$ is stretched between two rigid supports with a tension of 2700 N. The wire resonates at a frequency of 500Hz. The next higher frequency at which the same wire resonates is 520Hz . The length of the wire is…m

Key: 25

Sol: $\mu = 27\times10^{-4}\,Kg/m$

$T = 1800N$

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{2700}{27\times10^{-4}}} = 1000m/s$

$f = 500Hz$

$f_1 = 520$

$$\frac{nV}{2l} = 500\ldots$$

$$\frac{(n+1)V}{2l} = 520$$

$$l = \frac{1000}{2\times 20} = 25m$$

3. The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time $t = 0$ and $y = \frac{1}{1+(x-2)^2}$ at $t = 1s$, where x and y are in meters. The shape of wave does not change during the propagation. The velocity of the wave will be __m/s **Jee Mains 2021**

Key: 2

Sol : At $t = 0, y = \frac{1}{1+x^2}$

At time $t = t, y = \frac{1}{1+(x-vt)^2}$

At $t = 1, y = \frac{1}{1+(x-v)^2}$........$(i)$

Given At $t = 1, y = \frac{1}{1+(x-2)^2}$.....$(ii)$

Comparing $(i)\&(ii)$

$v = 2m/s$

3a. The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time $t = 0$ and $y = \frac{1}{1+(x-4)^2}$ at $t = 1s$, where x and y are in meters. The shape of wave does not change during the propagation. The velocity of the wave will be __m/s **Jee Mains 2021**

Key: 4

Sol : At $t = 0, y = \frac{1}{1+x^2}$

At time $t = t, y = \frac{1}{1+(x-vt)^2}$

At $t = 1, y = \frac{1}{1+(x-v)^2}$........$(i)$

Given At $t = 1, y = \frac{1}{1+(x-4)^2}$.....$(ii)$

Comparing $(i)\&(ii)$

$v = 4m/s$

3b. The amplitude of wave disturbance propagating in the positive x-direction is given by $y=\frac{1}{(1+x)^2}$ at time $t=0$ and $y=\frac{1}{1+(x-6)^2}$ at $t=1s$, where x and y are in meters. The shape of wave does not change during the propagation. The velocity of the wave will be __m/s

Key: 6

Sol : At $t=0, y=\frac{1}{1+x^2}$

At time $t=t, y=\frac{1}{1+(x-vt)^2}$

At $t=1, y=\frac{1}{1+(x-v)^2}........(i)$

Given At $t=1, y=\frac{1}{1+(x-6)^2}.....(ii)$

Comparing $(i)\&(ii)$

$v=6m/s$

3c. The amplitude of wave disturbance propagating in the positive x-direction is given by $y=\frac{1}{(1+x)^2}$ at time $t=0$ and $y=\frac{1}{1+(x-8)^2}$ at $t=1s$, where x and y are in meters. The shape of wave does not change during the propagation. The velocity of the wave will be __m/s

Key: 8

Sol : At $t=0, y=\frac{1}{1+x^2}$

At time $t=t, y=\frac{1}{1+(x-vt)^2}$

At $t=1, y=\frac{1}{1+(x-v)^2}........(i)$

Given At $t=1, y=\frac{1}{1+(x-8)^2}.....(ii)$

Comparing $(i)\&(ii)$

$v=8m/s$

4. Two waves executing simple harmonic motion of same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is ____ (degree).

Jee Main 2022

Key :60

Sol : The resultant amplitude

$$A_R=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}$$

$$\sqrt{3}A=\sqrt{A^2+A^2+2A^2\cos\phi}$$

$$3A^2=2A^2+2A^2\cos\phi$$

$$3A^2-2A^2=2A^2\cos\phi$$

$$A^2=2A^2\cos\phi\Rightarrow\cos\phi=\frac{1}{2}$$

$$\therefore\ \phi=60^0$$

4a. Two waves executing simple harmonic motion of same amplitude and frequency are superimposed. The resultant amplitude is equal to the 3 times of amplitude of first wave and the second wave has amplitude twice that of the first motions. The phase difference between the two motions is ____ (degree).

Key : 0

Sol : The resultant amplitude

$$A_R=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}$$

$$3A=\sqrt{A^2+4A^2+2\times A\times 2A\cos\phi}\Rightarrow 9A^2=5A^2+4A^2\cos\phi\Rightarrow\cos\phi=1\Rightarrow\phi=0^0$$

4b. Two waves executing simple harmonic motion of same frequency are superimposed. Find resultant amplitude in terms of amplitude of the first wave and the second wave has amplitude twice that of the first motion. The phase difference between the two motions is 120^0

Key : 2

Sol : The resultant amplitude

$$A_R=\sqrt{A_1^2+A_2^2+2A_1A_2\cos 120}\Rightarrow A_R=\sqrt{3}A=1.732A\approx 2A$$

4c. Two waves executing simple harmonic motion of same frequency are superimposed. Find resultant amplitude in terms of amplitude of the second wave and the second wave has amplitude twice that of the first motion. The phase difference between the two motions is $\frac{\pi}{3}rad$

Key : 1

Sol : The resultant amplitude

$$A_R=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\frac{\pi}{3}}\Rightarrow A_R=\sqrt{\frac{5}{4}A_2^2+A_2^2\cos\frac{\pi}{3}}\Rightarrow A_R=\sqrt{1.25A_2^2+0.5A_2^2}$$

$$A_R=\sqrt{1.75A_2^2}\approx 1.32A_2$$

5. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ____ Hz. **Jee Main 2020**

Key : 106

Sol : $V_{air}=300m/s,\rho_{gas}=2\rho$ air

Using, $V=\sqrt{\frac{B}{\rho}}$

$$\frac{V_{gas}}{V_{air}}=\frac{\sqrt{\frac{B}{2\rho_{air}}}}{\sqrt{\frac{B}{\rho_{air}}}}$$

$$\Rightarrow\ V_{gas}=\frac{V_{air}}{\sqrt{2}}=\frac{300}{\sqrt{2}}=150\sqrt{2}m/s$$

And f f_{nth} harmonic $=\frac{nv}{2L}$ (in open organ pipe)

(L=1 metre given)

$$\therefore\ f_{2nd}\text{ harmonic - }f_{\text{fundamental}}=\frac{2v}{2\times1}-\frac{1}{2\times1}=\frac{v}{2}$$

$$\therefore\ f_{2n}\text{ harmonic - }f_{fundamental}=\frac{150\sqrt{2}}{2}=\frac{150}{\sqrt{2}}\approx106Hz$$

5a. A one metre long (both ends open) organ pipe is kept in a gas that has triple the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ____ Hz.

Key : 86

Sol: $V_{air}=300m/\sec$

$\rho_{gas}=3\rho_{air}$

$V=\sqrt{\frac{B}{\rho}}$

$$\frac{V_{gas}}{V_{air}}=\frac{\sqrt{\frac{B}{3\rho_{air}}}}{\sqrt{\frac{B}{\rho_{air}}}}$$

$$V_{gas}=\frac{v_{air}}{\sqrt{3}}=\frac{300}{\sqrt{3}}$$

f_{nth} harmonic $\frac{xv}{2L}$[open pipe]

$L=1m$

f_{2nd} harmonic - fundamental = $\frac{2v}{2\times1}=\frac{v}{2\times1}$

$$=\frac{2v}{2}-\frac{v}{2}$$

$$=\frac{1}{2}[2-1]\ =v/2\ =\frac{300}{\sqrt{3}}=\frac{150}{\sqrt{3}}\ =86.0$$

5b. A one metre long (both ends open) organ pipe is kept in a gas that has triple the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ____ Hz.

Key : 116.6

Sol : $V_{air} = 330m/\sec$

$f_g = 2f\text{air}$

$v_{gas} = \frac{v_{air}}{\sqrt{2}} = \frac{300}{\sqrt{2}}$

f_{nth} harmonic $= \frac{nv}{2L}$ (Open pipe)

$L = 1m$

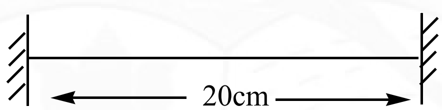
$f_{2harmonic} - f_{fundamental} = \frac{2v}{2\times 1} - \frac{v}{2}$

$= \frac{v}{2}$

$= \frac{330}{\sqrt{2}(2)} = \frac{165}{\sqrt{2}}$

$= 116.6$

6. A 20 cm long string, having a mass of 1.0 g, is fixed at both ends. The tension in the string is 0.5N. the string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation(in centimeters) between the successive nodes on the string.



Key : 5

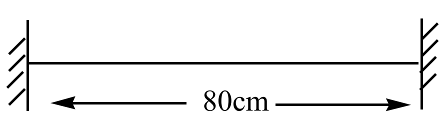
Sol : $v = \sqrt{\frac{T}{\mu}} = 10m/s$

$\lambda = \frac{v}{f} = \frac{10}{100} = 10cm$

Distance between the successive nodes is

$\lambda/2 = 5cm$

6a. A 80 cm long string, having a mass of 2.0 g, is fixed at both ends. The tension in the string is 1N. the string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation(in centimeters) between node and immediate antinode **IIT-2009**



Key : 5

Sol : $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{1}{\frac{1}{400}}}=20m/s$

$\mu=\frac{2\times10^{-3}}{80\times10^{-2}}=\frac{1}{400}kg/m$

$\lambda=\frac{v}{n}=20cm$

$\frac{\lambda}{4}=\frac{20}{4}=5cm$

II) Superposition of waves

Constructive interference - Superposition of waves, Destructive interference - Superposition of waves, Resultant intensity

1. Two waves are simultaneously passing through a string and their equations are : $y_1=A_1\sin k(x-vt), y_2=A_2\sin k(x-vt+x_0)$. Given amplitudes $A_1=12mm A_2=5mm\ x_0=3.5cm$ and wave number $k=6.28cm^{-1}$. The amplitude of resulting wave will be…. mm **Jee Mains 2021**

Key. 7

Sol: $y_1=A_1\sin k(x-vt)$

$y_1=12\sin 6.28(x-vt)$

$y_2=5\sin 6.28(x-vt+3.5)$

$\Delta\phi=\frac{2\pi}{\lambda}(\Delta x)$

$=K(\Delta x)$

$=6.28\times3.5=\frac{7}{2}\times2\pi=7\pi$

$A_{nd}=\sqrt{A_1^2+A_2^2+2A_1\cos\phi}$

$A_{net}=\sqrt{(12)^2+(5)^2+2(12)(5)\cos(7\pi)}$

$=\sqrt{144+25-120}=7mm$

1a. Two waves are simultaneously passing through a string and their equations are : $y_1=A_1\sin k(x-vt), y_2=A_2\sin k(x-vt+x_0)$. Given amplitudes $A_1=24mm A_2=10mm\ x_0=3.5cm$ and wave number $k=6.28cm^{-1}$. The amplitude of resulting wave will be…. mm

Key. 14

Sol: $y_1=A_1\sin k(x-vt)$

$y_1=24\sin 6.28(x-vt)$

$y_2=10\sin 6.28(x-vt+3.5)$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$$= K(\Delta x)$$

$$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$$

$$A_{nd} = \sqrt{A_1^2 + A_2^2 + 2A_1\cos\phi}$$

$$A_{net} = \sqrt{(24)^2 + (10)^2 + 2(24)(10)\cos(7\pi)}$$

$$= \sqrt{144 + 25 - 120} = 14mm$$

1b. Two waves are simultaneously passing through a string and their equations are : $y_1 = A_1 \sin k(x - vt), y_2 = A_2 \sin k(x - vt + x_0)$. Given amplitudes $A_1 = 48mm A_2 = 20mm\ x_0 = 3.5cm$ and wave number $k = 6.28cm^{-1}$.The amplitude of resulting wave will be…. mm

Key. 28

Sol:

$$y_1 = A_1 \sin k(x - vt)$$

$$y_1 = 48\sin 6.28(x - vt)$$

$$y_2 = 20\sin 6.28(x - vt + 3.5)$$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$$= K(\Delta x)$$

$$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$$

$$A_{nd} = \sqrt{A_1^2 + A_2^2 + 2A_1\cos\phi}$$

$$A_{net} = \sqrt{(48)^2 + (20)^2 + 2(48)(20)\cos(7\pi)}$$

$$= \sqrt{144 + 25 - 120} = 28mm$$

1c. Two waves are simultaneously passing through a string and their equations are : $y_1 = A_1 \sin k(x - vt), y_2 = A_2 \sin k(x - vt + x_0)$. Given amplitudes $A_1 = 60mm A_2 = 25mm\ x_0 = 3.5cm$ and wave number $k = 6.28cm^{-1}$.The amplitude of resulting wave will be…. mm

Key. 35

Sol:

$$y_1 = A_1 \sin k(x - vt)$$

$$y_1 = 60\sin 6.28(x - vt)$$

$$y_2 = 25\sin 6.28(x - vt + 3.5)$$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$$= K(\Delta x)$$

$$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$$

$$A_{nd} = \sqrt{A_1^2 + A_2^2 + 2A_1 \cos\phi}$$

$$A_{net} = \sqrt{(60)^2 + (25)^2 + 2(60)(25)\cos(7\pi)}$$

$$= \sqrt{144 + 25 - 120} = 35mm$$

2. Two travelling waves produces a standing wave represented by equation, $y = 1.0\text{mm} \cos\left(1.57\text{cm}^{-1}x\right) \times \sin\left(78.5\text{s}^{-1}\right)t$. The node closest to the in the region $x > 0$ will be at X =…….cm **Jee Mains 2021**

Key: 1

Sol: For node

$$\cos\left(1.57cm^{-1}\right)x = 0$$

$$\left(1.57cm^{-1}\right)x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2(1.57)}cm = 1cm$$

2a. Two travelling waves produces a standing wave represented by equation, $y = 1.0\text{mm} \cos\left(0.785\text{cm}^{-1}x\right) \times \sin\left(78.5\text{s}^{-1}\right)t$. The node closest to the in the region $x > 0$ will be at X =…….cm

Key: 2

Sol: For node

$$\cos\left(0.785cm^{-1}\right)x = 0$$

$$\left(0.785cm^{-1}\right)x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2(0.785)}cm = 2cm$$

2b. Two travelling waves produces a standing wave represented by equation, $y = 1.0\text{mm} \cos\left(0.3925\text{cm}^{-1}x\right) \times \sin\left(78.5\text{s}^{-1}\right)t$. The node closest to the in the region $x > 0$ will be at X =…….cm

Key: 4

Sol: For node

$$\cos\left(0.3925cm^{-1}\right)x = 0$$

$$\left(0.3925cm^{-1}\right)x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2(0.3925)}cm = 4cm$$

2c. Two travelling waves produces a standing wave represented by equation, $y = 1.0\text{mm}\cos\left(0.19625\text{cm}^{-1}x\right)\times\sin\left(78.5\text{s}^{-1}\right)t$. The node closest to the in the region $x > 0$ will be at X =…….cm

Key: 8

Sol: For node

$$\cos\left(0.19625cm^{-1}\right)x = 0$$

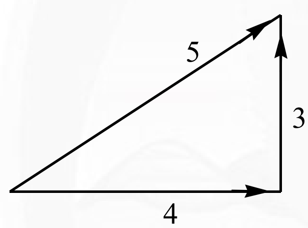
$$\left(0.19625cm^{-1}\right)x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2\left(0.19625\right)}cm = 8cm$$

2. When two progressive waves $y_1 = 4\sin\left(2x-6t\right)$ and $y_2 = 3\sin\left(2x-6t-\pi/2\right)$ are superimposed, the amplitude of the resultant wave is in_____ m. **IIT 2010**

Key : 5

Sol :



Two waves have phase difference of $\pi/2$

2a. When two progressive waves $y_1 = 6\sin\left(4x-8t\right)$ and $y_2 = 8\sin\left(4x-8t-\pi/3\right)$ are superimposed, the amplitude of the resultant wave is in_____ m.

Key : 12

Sol : $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$$A = \sqrt{6^2 + 8^2 + 2\times6\times8\times\cos\frac{\pi}{3}} = \sqrt{148} \simeq 12$$

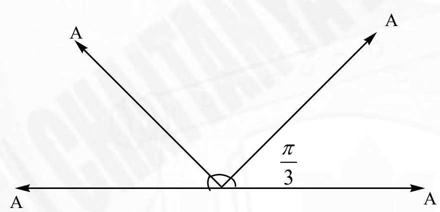
III) Reflection and refraction of waves, Stationary waves

Reflection - Reflection and refraction of waves,(Change in the amplitude and phase of a wave reflected from fixed boundary, Change in the amplitude and phase of a wave reflected from a free boundary) Refraction - Reflection and refraction of waves,(Amplitude of the transmitted wave, Change in the phase of the transmitted wave, Amplitude of the Reflected Wave, Change In the phase of the reflected wave) Equation of stationary wave, Standing waves on string, Standing waves in organ pipes,(Position of nodes for displacement wave, Position of nodes for pressure wave, Position of antinodes displacement wave, Position of antinodes for pressure waves, Stationary waves in stretched string fixed at ends, Stationary longitudinal wave in open organ pipe, Stationary longitudinal wave in closed organ pipe, End correction, Resonance tube).

1. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angle $0, \frac{\pi}{3}, \frac{2\pi}{3}$ & π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is **Adv 2015**

Key: 3

Sol:



$I_0 \propto A^2$

Let amplitude of individual wave be A then amplitude of resulting wave

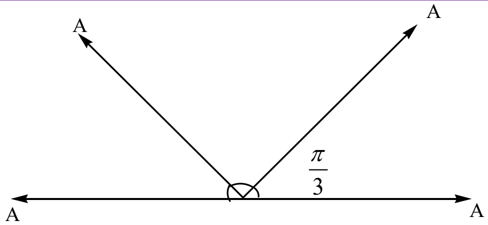
$$2A\sin\left(\frac{\pi}{3}\right) = \sqrt{3}A$$

$$I \propto \left(\sqrt{3}A\right)^2 \qquad \Rightarrow I = 3I_0$$

2. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angle $0, \frac{\pi}{3}, \frac{2\pi}{3}$ & π. When they are superposed, the intensity of the resulting wave is $n \times 0.3I_0$. The value of n is

Key:10

Sol:



$I_0 \propto A^2$

Let amplitude of individual wave be A then amplitude of resulting wave

$2A\sin\left(\frac{\pi}{3}\right) = \sqrt{3}A$

$I \propto \left(\sqrt{3}A\right)^2$ $\Rightarrow I = 3I_0 = n \times 0.3I_0$

$\Rightarrow n = 10$

3. A string of length $1m$ and mass $2\times10^{-5}kg$ is under tension T. When the string vibrates two successive harmonics are found to occur at frequency $750Hz$ and $1000Hz$. The value of tension is---------N. **Adv 2023**

Key: 5

Sol: $\frac{V}{2l} = 1000 - 750 \Rightarrow \frac{V}{2l} = 250$

$V = 2\times(1)\times 250 \Rightarrow V = 500m/s$

$\sqrt{\frac{T}{m}} = 500m \Rightarrow T = (500)^2 m$

$T = 500\times500\times2\times10^{-5} = 25\times2\times10^{-1} = 5N$

3a. A string of length $1m$ and mass $2\times10^{-5}kg$ is under tension T. When the string vibrates two successive harmonics are found to occur at frequency $500Hz$ and $750Hz$. The value of tension is---------N.

Key: 5

Sol: $\frac{V}{2l} = 750 - 500 \Rightarrow \frac{V}{2l} = 250$

$V = 2\times(1)\times 250 \Rightarrow V = 500m/s$

$\sqrt{\frac{T}{m}} = 500m \Rightarrow T = (500)^2 m$

$T = 500\times500\times2\times10^{-5} = 25\times2\times10^{-1} = 5N$

4. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of $2\,ms^{-1}$ in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is $320ms^{-1}$, the smallest value of the percentage change required in the length of the pipe is ______ **IIT -2020**

Key : 0.62 to 0.63

Sol : Let $\ell_1 =$initial length of pipe

$\ell_2 =$new length of pipe

$V_T =$ speed of tuning fork

In close organ pipe, $f = \frac{V}{4\ell_1}$

When tuning fork is moved, $f' = f = \left(\frac{V}{V - V_T}\right) = \frac{V}{4\ell_2}$

$$\Rightarrow \frac{V}{4\ell_1}\left(\frac{V}{V - V_T}\right) = \frac{V}{4\ell_2} \Rightarrow \frac{V - V_T}{V} = \frac{\ell_2}{\ell_1}$$

$$\Rightarrow \frac{\ell_2}{\ell_1} - 1 = \frac{V - V_T}{V} - 1 \Rightarrow \frac{\ell_2 - \ell_1}{\ell_1} = \frac{-V_T}{V}$$

Percentage change required in the length of the pipe

$$\frac{\ell_2 - \ell_1}{\ell_1} \times 100 = \frac{-2}{320} \times 100 = 0.625\%$$

Hence, smallest value of percentage change required in the length of pipe is 0.625%

4a. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 1 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is $320 ms^{-1}$, the smallest value of the percentage change required in the length of the pipe is _____

Key : 0.31

Sol : Let $\ell_1 =$initial length of pipe

$\ell_2 =$new length of pipe

$V_T =$ speed of tuning fork

In close organ pipe, $f = \frac{V}{4\ell_1}$

When tuning fork is moved, $f' = f = \left(\frac{V}{V - V_T}\right) = \frac{V}{4\ell_2}$

$$\Rightarrow \frac{V}{4\ell_1}\left(\frac{V}{V - V_T}\right) = \frac{V}{4\ell_2} \Rightarrow \frac{V - V_T}{V} = \frac{\ell_2}{\ell_1}$$

$$\Rightarrow \frac{\ell_2}{\ell_1} - 1 = \frac{V - V_T}{V} - 1 \Rightarrow \frac{\ell_2 - \ell_1}{\ell_1} = \frac{-V_T}{V}$$

Percentage change required in the length of the pipe

$$\frac{\ell_2 - \ell_1}{\ell_1} \times 100 = \frac{-1}{320} \times 100 = 0.3125$$

Hence, smallest value of percentage change required in the length of pipe is 0.625%

5. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the

pipes are vibrating in their first overtone with same frequency. The length of the open pipe is where is $\frac{X}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ (Round off to the Nearest Integer) **Jee Mains -2021**

Key: 4

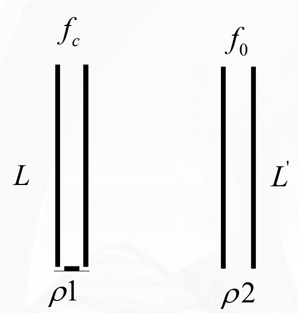
Sol: $f_c = f_0$

$$\frac{3V_c}{4L} = \frac{2V_0}{L'}$$

$$L' = \frac{4L}{3}\frac{V_0}{V_c} = \frac{4L}{3}\sqrt{\frac{B.\rho 1}{\rho 2.B}}\left(B \text{ is bulk modulus}\right)$$

$$= \frac{4L}{3}\sqrt{\frac{\rho 1}{\rho 2}}$$

$$x = 4$$



5a. If 1st overtone of closed pipe has same frequency as 1st overtone of open pipe of same diameter the ratio of the lengths is ___

Key: 1.5

Sol: n_1 open= n_1 closed

$$2\frac{v}{2l_0} = 3\frac{v}{4l_c} \quad \frac{l_c}{l_0} = \frac{3}{2} = 1.5$$

5b. A cylindrical tube open at both ends 'h' has fundamental frequency 'f'. The tube is dipped vertically in water so that half of its length is in water. Find the ratio of new fundamental frequency to old fundamental frequency

Key : 1

Sol : Open pipe $f_0 = \frac{v}{2l}$ closed pipe $f_c = \frac{v}{4\left(\frac{0}{2}\right)}$

$$f_c = f_0 \qquad \frac{f_c}{f_0} = 1$$

5c. For a certain organ pipe three successive resonance frequency are 425, 595 and 765 Hz respectively. The length of the pipe is (speed of sound in air is 340 m/s)____

Key : 100

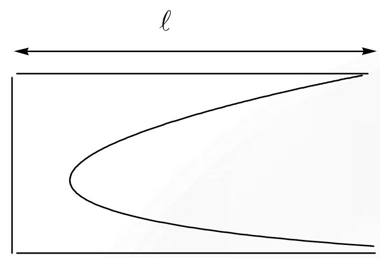
Sol : Frequency are odd multiples of 85 Hz it is closed pipe

$$f = 85Hz \quad l = \frac{v}{4f} = \frac{340\times 100}{4\times 85} = 100cm$$

6. A tuning fork is vibrating at 250Hz . The length of the shortest closed organ pipe that will resonate with the tuning fork will be ...... cm (Take speed of sound in air as $340ms^{-1}$)

Key 34

Sol:



$$\frac{\lambda}{4} = l \Rightarrow \lambda = 4l$$

$$f = \frac{V}{\lambda} = \frac{V}{4l}$$

$$\Rightarrow 250 = \frac{340}{4l}$$

$$\Rightarrow l = \frac{34}{4 \times 25} = 0.34m$$

$$l = 34cm$$

6a. Calculate fundamental frequency of a closed organ pipe of length 66.4cm at 0^0C. If velocity of sound at 0^0C is 332 m/s.

Key : 125

Sol : $n = \frac{v}{4l} = \frac{332 \times 100}{4 \times 66.4} = 125$

6b. Calculate frequency of second harmonic of an open pipe in having length 34cm. If velocity of sound is 340 m/s.

Key : 1000

Sol : $n_1 = 2n = 2\frac{v}{2l} = \frac{340}{0.34} = 1000$

6c. A tuning fork is vibrating at 500Hz . The length of the shortest closed organ pipe that will resonate with the tuning fork will be ...... cm (Take speed of sound in air as $350ms^{-1}$)

Key 35

Sol: $n = \frac{v}{2l}$ $\quad l = \frac{v}{2n} = \frac{350 \times 100}{2 \times 500}$

$= 35cm$

7. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produces a stationary wave whose equation

is given by $y=\left(10\cos\pi x\sin\frac{2\pi t}{T}\right)cm$. The amplitude of the particle at $x=\frac{4}{3}$ cm will be ___cm. **Jee Main 2022**

Key :5

Sol : $A_s=10\cos\pi\left(\frac{4}{3}\right)=5cm$

7a. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produces a stationary wave whose equation is given by $y=\left(10\sin\pi x\cos\frac{2\pi t}{T}\right)cm$. The amplitude of the particle at $x=\frac{1}{2}$ cm will be ___cm.

Key : 10

Sol : $A_s=10\sin\pi x=2A\sin kx=2A\sin\pi(1/2)\Rightarrow A_s=2A=10cm$

7b. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produces a stationary wave whose equation is given by $y=(5\cos\pi x\sin 2\pi nt)cm$. The amplitude of the particle at $x=\frac{2}{3}$ cm will be ___cm.

Key : 2

Sol : $A_s=5\cos\pi 2/3=-5/2=-2.5$

7c. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produces a stationary wave whose equation is given by $y=\left(2\sin 1.57x\cos 6.28\frac{t}{T}\right)cm$. The amplitude of the particle at $x=\frac{1}{3}$ cm will be ___cm.

Key : 100

Sol : $A_s=2\sin\left(\frac{\pi}{2}\times\frac{1}{3}\right)=2\times\frac{1}{2}=1m\Rightarrow 2A=1m=100cm$

8. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe. If the length of the closed organ pipe is 20cm. The length of the open organ pipe is ___ cm. **Jee Main 2022**

Key :80

Sol : $2\left(\frac{v}{2\ell_0}\right)=\frac{v}{4\ell_0}$

$\frac{1}{\ell_0}=\frac{1}{4\times 0.2}=\ell_0=0.8m=80cm$

8a. The first overtone frequency of a closed organ pipe is equal to the fundamental frequency of an open organ pipe. If the length of the closed pipe 30cm. What will be the length of open pipe ?

Key :20

Sol : First overtone of closed organ pipe

$P_1 = \frac{3v}{4l} = \frac{3 \times v}{4 \times 30}$

Fundamental frequency of a open organ pipe

$P_2 = \frac{v}{2l_2}$

$\frac{3v}{2 \times 30} = \frac{v}{2l_2}$

$l_2 = 20cm$

8b. The fundamental frequency of a closed organ pipe equal to the first overtone frequency of an open organ pipe. If the length of open organ pipe is 80cm then the length of the closed organ pipe will be ___cm

Key : 20

Sol : First overtone $(n = 2)$

$n_1 = \frac{2v}{2l} = \frac{2v}{2 \times 0.8} = \frac{5v}{4}$

$n_2 = \frac{v}{4l}$

$n_1 = n_2$

Hence

$\frac{5v}{4} = \frac{v}{4l}$

$l = \frac{1}{5}$

$l = 0.2m$

$l = 20cm$

8c. The first overtone frequency in an open organ pipe is equal to the fundamental frequency of closed organ pipe. If length of the closed organ pipe is 40cm. The length of the open organ pipe is

Key : 160

Sol: $\frac{V}{4l_1} = \frac{V}{l_2}$

$l_2 = 4l_1$

$l_2 = 4 \times 40$

$l_2 = 160cm$

9. A tuning fork of frequency 340 Hz resonates in the fundamental mode with an air column of length 125 cm in a cylindrical tube closed at one end. When water is slowly poured in it, the minimum height of water required for observing resonance once again is ____ cm. (Velocity of sound in air is) **Jee Main 2022**

Key :50

Sol : $\lambda = \frac{V}{\upsilon} = \frac{340 \times 100}{340} = 100cm$

$\frac{\lambda}{4} = 25cm, \frac{3\lambda}{4} = 75cm$

The minimum height of water required for resonance $= 125 - 75 = 50cm$

9a. A tuning fork of frequency 340Hz resonates with an air column of length 120cm in a cylindrical tube in the fundamental made. When water is slowly poured in it, the minimum height of water required for observing resonance are again is $\left(V = 340ms^{-1}\right)$

Key : 45

Sol : $v = n\lambda$

$\lambda = \frac{v}{n} = \frac{340}{340}$

$\lambda = 1m$

First resonating length

$l_1 = \frac{\lambda}{4} = \frac{1}{4}m = 25cm$

Second resonating length

$l_2 = \frac{3\lambda}{4} = 3 \times 25 = 75cm$

Third resonance is not

Possible since the length of tube is 120 cm

Minimum height of water necessary for resonance

$= 120 - 75$

$= 45cm$

9b. A tuning fork of frequency 330Hz resonates with an air column of length 120cm in a cylindrical tube, in the fundamental made. When water is slowly poured in it, the minimum height of water required for observing resonance are again is (velocity of sound 330 m/s)

Key : 45

Sol : $\lambda = \frac{v}{n} = \frac{330}{330} = 1m$

$\lambda = 100cm$

First resonating length

$l_1 = \frac{\lambda}{4} = \frac{100}{4} = 25cm$

$l_2 = \frac{3\lambda}{4} = \frac{3 \times 100}{4} = 75cm$

Minimum height of water necessary for resonance =125-75 cm

=45 cm

9c. A tuning fork of frequency 512 Hz resonates with an air column of length 120cm in cylindrical tube, in the fundamental made. When water is slowly poured in it, the minimum height of water required for observing resonance once again is (velocity of sound 330 m/sec)

Key : 75

Sol : $\lambda = \frac{v}{n} = \frac{512}{330} = 1.5m$

$\lambda = 150cm$

First resonating length

$l_1 = \frac{\lambda}{4} = \frac{150}{4} = 37.5cm$

Second resonating length

$l_2 = \frac{3\lambda}{4} = \frac{3\times 150}{4} = 112.5$

Minimum height of water necessary for resonance $= 187.5 - 112.5$

$= 75cm$

10. In an experiment to determine the velocity of sound in air at room temperature using a resonance tube, the first resonance is observed when the air column has a length of 20.0 cm for a tuning fork of frequency 400 Hz is used. The velocity of the sound at room temperature is. The third resonance is observed when the air column has a length of ____ m. **Jee Main 2022**

Key :104

Sol : $l_1 + e = \lambda/4$ $\quad$ $l_3 + e = 5\lambda/4$

$v = f\left(l_3 - l_1\right)$ $\quad$ $l_3 = 104cm$

10a. In a resonance tube experiment to determine the speed of the sound in air, a pipe of diameter 5cm is used the column in pipe resonates with a tuning fork of frequency 480 Hz. When the minimum length of the air column is 16cm. Find the speed of sound (in m/s) in air column at room temperature

Key : 336

Sol : $f = 480Hz$ $\quad$ $d = 5cm$

$r = 2.cm$

$r = 0.0025m$

Minimum length of column i.e., $l_1 = 16cm \Rightarrow 0.16m$

$e = \frac{l_2 - 3l_1}{2}$ $\quad$ $\left(\because e = 0.6r\right)$

$0.6\times 0.025 = \frac{l_2 - 0.16}{2}$

$l_2 = 0.51m$

There fore speed of sound in air column

$v = 2f\left(l_2 - l_1\right)$

$v = 2\times480(0.51-0.16)$

$= 336 m/\sec$

10b. In a resonance column experiments, a tuning fork of frequency 400Hz is used. The first resonance is observed when the air column has a length of 20 cm and the second resonace is observed when the air column has a length of 62 cm. Find the speed of sound in air

Key : 336

Sol : $v = 2n(l_2 - l_1)$

$= 2\times400\times42\times10^{-2} = 8\times42 = 336 m/s$

10c. In the resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5cm is used. The air column in the pipe resonates with a tuning fork of frequency 500Hz. When the minimum length of the air column is 15.5 cm. find the speed of sound in air at room temperature

Key : 340

Sol : $d = 5cm$

$d = 0.5m$

Minimum length of air column (1min) =15.5 cm

=0.155m

$f = 500 Hz$

Fundamental frequency of closed pipe

$$f = \frac{v}{4(l+e)} = \frac{v}{4(l+0.3d)}$$

$$500 = \frac{v}{4(0.155+0.3\times0.05)}$$

$v = 340 m/\sec$

IV)Beats & Echo

Time period and frequency-Beat frequency, Beat frequency for three sources

1. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be ____ Hz. [Velocity of sound in air is 340 m/s.]**Jee Mains -2021**

Key: 8

Sol:

Speed=7.2km/h =2m/s

Frequency as heard by A $f_A^{'} = f_B\left(\frac{V+V_0}{V-V_s}\right)$

$f_A^{'} = 676\left(\frac{340+2}{340-2}\right)$

$f_A^{'} = 684Hz$

$\therefore f_{Beat} = 684-676$

$= 8Hz$

1a. Two cars are approaching each other at an equal speed of 9 km/hr. When they see each other, both blow horns having frequency of 750 Hz. The beat frequency heard by each driver will be ____ Hz. [Velocity of sound in air is 340 m/s.]

Key: 11

Sol: $v_s = 9\times\frac{5}{18} = 2.5$

When both are approach

Frequency $f^1 = \left(\frac{v+v_s}{v-v_s}\right)f \Rightarrow f^1 = \frac{342.5}{337.5} = 750$

$f^1 = \frac{685}{675}\times 750 = 761Hz$

Beats frequency $\Delta f = f^1 - f = 761-750 = 11Hz$

2. A tuning fork A of unknown frequency produces 5beats/s with a fork of known frequency 340 HZ. When fork A filed, the beat frequency decreases to 2beats/s. What is the frequency of fork A? **Jee Mains -2021**

Key: 335

Sol: Tuning fork A is filed and beat frequency decreases

$n_A < n_B \Rightarrow n_B - n_A = 5 \Rightarrow n_A = 340-5 = 335$

2a. A tuning fork A of unknown frequency produces 8 beats/s with a fork of known frequency 360 HZ. When fork A filed, the beat frequency decreases to 2beats/s. What is the frequency of fork A?

Key: 352

Sol: Tuning fork A is filed and beat frequency decreases

$n_A < n_B \Rightarrow n_B - n_A = 8 \Rightarrow n_A = n_B - 8 = 360-8 = 352$

3. A set of 20 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats with respect to the preceding fork and the frequency of the last fork is twice the frequency of the first, then the frequency of last fork is -Hz. **Jee Main 2022**

Key :152

Sol : $f_n = f_1 + (n-1)d$

$2f = f + 19\times 4 \Rightarrow f = 76$

$\therefore\ f_n = 2f = 152Hz$

3a. A set of 25 tuning forks is arranged in a series of increasing frequencies. If each fork gives 5 beats with respect to the preceding fork and the frequency of the last fork is twice the frequency of the first, then the frequency of first fork is ……Hz.

Key :120

Sol : $f_{last} = f_1 + (n-1)d$ given $f_{last} = 2f_1$

$2f_1 = f_1 + 24\times 5 \qquad \Rightarrow f_1 = 120Hz$

3b. A set of 16 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats with respect to the preceding fork and the frequency of the last fork is twice the frequency of the first, then the frequency of last fork is ……Hz.

Key :120

Sol : $f_n = f_1 + (n-1)d \qquad$ given $f_n = 2f_1$

$2f_1 = f_1(16-1)4 \qquad \Rightarrow f_1 = 15\times 4 = 60$

Frequency of last fork $f_n = 2\times 60 = 120Hz$

4. An observer is riding on a bicycle and moving towards a hill at 18 kmh^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the hill. If the original frequency of the sound as emitted by source is 640Hz and velocity of the sound in air is 320 ms^{-1} the beat frequency between the two sounds, heard by the observer will be ___Hz

Key : 20

Sol : $n_{direct} = \left(\frac{v-v_0}{v}\right)n = \frac{315}{320}\times 640$

$= 630Hz$

$n_{reflected} = \left(\frac{v+v_0}{v}\right)n = \frac{325}{320}\times 640 = 650Hz$

Beat frequency $\Delta n = 650 - 630 = 20Hz$

4a. An observer is riding on a bicycle and moving towards a hill at 36 kmh^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the hill. If the original frequency of the sound as emitted by source is 720Hz and velocity of the sound in air is 320 ms^{-1} the beat frequency between the two sounds, heard by the observer will be ___Hz

Key : 45

Sol : $v_0 = 36 \times \frac{5}{15} = 10ms^{-1}$

$$n_{direct} = \left(\frac{v - v_0}{v}\right)n = \frac{310}{320} \times 720$$

$$= \frac{310 \times 72}{32} = 697.5Hz$$

$$n_{reflected} = \left(\frac{v + v_0}{v}\right)n = \frac{330 \times 720}{320} = 742.5Hz$$

$\therefore$ Beat frequencies $\Delta n = 742.5 - 697.5$

$\Delta n = 45Hz$

5. If the train blowing a whistle of frequency 320 Hz is moving with a velocity of 36km/h. towards a hill from which an echo is heard by the train driver, then frequency of echo will be __(velocity of sound =330 m/s). **Jee Mains -2022**

Key : 340

Sol : $v_s = 36 \times \frac{5}{18} = 10ms^{-1}$

Frequency of reflected sound heard by driver

$$f_2 = \left[\frac{v + v_s}{v - v_s}\right]f = \left(\frac{330 + 10}{330 - 10}\right)320 = 340Hz$$

$$\left(f_1 = \frac{v}{v - v_s}.f \quad f_2 = \frac{v + v_s}{v}.f_1 = \frac{v + v_s}{v - v_s}.f\right)$$

5a. A train moving towards a hill with velocity 54 km/hr blows a whistle of frequency 340 Hz. Find the frequency of echo heard by driver. Velocity of sound is air is 355 m/s.

Key : 370

Sol: $v_s = 54 \times 5/18 = 15ms^{-1}$

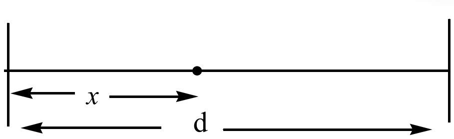
Frequency heard by driver $f^1 = \left(\frac{v + v_s}{v - v_s}\right)f$

$$f^1 = \left(\frac{355 + 15}{355 - 15}\right)340 = 370Hz$$

*6. A scooterist moving with a velocity 36kmph towards a fort wall blows a horn. If he hears echo after 3secons, the distance at which horn is blown from the wall in meters($v = 340ms^{-1}$)

Key : 525

Sol:



$v_s = 36 \times \frac{5}{18} = 10ms^{-1}$

$x = v_s t = 10 \times 3 = 30m$

$$t=\frac{2d-x}{v}\Rightarrow 3=\frac{2d-30}{340}\Rightarrow d=525m$$

*6a. A car moving with velocity 18km/hr towards a wall blows a horn he hears echo after 5 sec. Find the distance at which horn is down form wall $\left(v=345ms^{-1}\right)$

Key : 185

Sol : Horn blown at distanced and echo heard at distance x turn emitted position

$$v=\frac{2d-x}{t}=\frac{2d-v_s t}{t}\Rightarrow 2d-v_s t=vt$$

$$d=\frac{\left(v+v_s\right)t}{2}=\left(\frac{345+5\times 5}{2}\right) \qquad v_s=18\times\frac{5}{18}=5ms^{-1}$$

$$d=\frac{370}{2}=185$$

*7. Two sound waves $y_1=0.3\sin 496\pi t$ and $y_2=0.5\sin 504\pi t$ are superimposed. What will be the ratio of frequency of resultant wave and frequency of amplitude

Key : 125

Sol :

$$2\pi n_1=496\pi\Rightarrow n_1=248Hz$$

$$2\pi n_2=504\pi\Rightarrow n_2=252Hz$$

Frequency of resultant wave $n^1=\frac{n_1+n_2}{2}=\frac{500}{2}=250Hz$

Frequency of amplitude $n''=\frac{n_2-n_1}{2}=\frac{4}{2}=2Hz$

Ratio $\frac{n'}{n''}=\frac{250}{2}=125$

V)Doppler's effect

Doppler's effect in source and observer are moving in one dimension, Doppler's effect in source and observer are moving in two dimension, Effect of the source motion on the wavelength, Effect of wind speed.

1. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of $20ms^{-1}$ with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B in Hz. (speed of sound in air $=340ms^{-1}$)

Jee Mains 2019

Key : 2250

Sol: Observer in A detects 2000Hz

$$f^1 = f\left[\frac{v-v_0}{v-v_s}\right]$$

$$2000 = f\left[\frac{340-20}{340+20}\right] \Rightarrow f = 2000\times\frac{36}{32} = 2250Hz$$

1a. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of $30ms^{-1}$ with respect to the ground. If an observer in car A detects a frequency 2500 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B in Hz. (speed of sound in air $=330ms^{-1}$)

Key : 3000

Sol: Observer in A detects 2000Hz

$$f^1 = f\left[\frac{v-v_0}{v-v_s}\right]$$

$$\Rightarrow f = \frac{360}{300}\times 2500 = 3000Hz$$

1b. A car a is moving at $20ms^{-1}$ with its horn blowing with a frequency of 900 Hz is chasing another car B moving at speed of $9ms^{-1}$ what is the apparent frequency of horn hear by driver of car B. If the wind is blowing in the direction of sound with $10ms^{-1}$ and velocity of sound is $340ms^{-1}$ (in Hz)

Key : 928

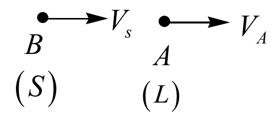
Sol : $V = 340+10 = 350$ $\quad V_0 = 9ms^{-1}$ $\quad V_s = 20ms^{-1}$

$$n^1 = \left[\frac{V-V_0}{V-V_s}\right]n = \frac{341}{330}\times 900 \approx 928Hz$$

1c. Two cars A and B depart simultaneously from the same position and in same direction on a straight road. A starts with initial velocity $2ms^{-1}$ and acceleration $2ms^{-2}$ while B start with initial velocity $2ms^{-1}$. The driver of car A hears a sound of frequency 352Hz emitted by car B after 10sec, after the start, find the actual frequency of the sound as emitted by B. Given velocity of sound $V = 330ms^{-1}$

Key : 375

Sol: $At\ t = 10\sec$, velocity of A is given by



$V_A = u_A + a_A t$
$\Rightarrow V_A = (2)+(2)(10)$
$\Rightarrow V_A = 22ms^{-1}$

Here B is the source and A is the listener. Hence

$$f' = f\left(\frac{v-v_A}{v-v_B}\right)$$

$$\Rightarrow f = \left(\frac{v-v_B}{v-v_A}\right)f'$$

$$\Rightarrow f = \left(\frac{330-2}{330-22}\right)\times 352$$

$$\Rightarrow f = 374.8Hz \approx 375Hz$$

2. A stationary source emits sound waves of frequency 500Hz. Two observes moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530Hz. Their respective speeds are, 2a, 3a in ms^{-1} find value of a (given speed of sound =300 m/s).

Jee Mains 2019

Key : 6

Sol: $480 = 500\left(\frac{300-V_1}{300}\right) \Rightarrow \frac{480}{500} = 1-\frac{V_1}{300}$

$$v_1 = \left(1-\frac{480}{500}\right)300 = 12 = 2\times 6$$

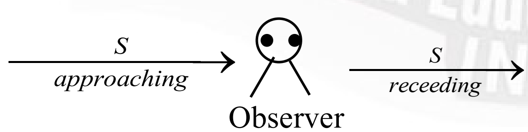
$$530 = 500\left(\frac{300+V_2}{300}\right) \Rightarrow \frac{530}{500} = 1+\frac{V_2}{300} \Rightarrow v_2 = \frac{30}{500}300 = \frac{90}{5} = 18ms^{-1}$$

Hence velocities are 12,18 i.e., $a=6$

2a. The apparent frequency of the whistle of a engine changes in the ratio 9:5 as the engine passes the stationary observer. If the velocity of sound is $350ms^{-1}$, then the speed of the engine is ___ ms^{-1}

Key : 100

Sol :



While approaching apparent frequency $n_1 = \frac{V}{V-V_s}n$

While reducing frequency $n_2 = \frac{V}{V+V_s}.n$

$\frac{n_1}{n_2} = \frac{V_n}{V-V_s}\frac{V+V_s}{V_n}$ given $\frac{n_1}{n_2} = \frac{9}{5}$

$$\Rightarrow \frac{9}{5} = \frac{V+V_s}{V-V_s}$$

$$9V - 9V_s = 5V + 5V_s \Rightarrow 4V = 14V_s$$

$$V_s = \frac{2V}{7} = \frac{2\times 350}{7} = 2\times 50 = 100 ms^{-1}$$

2b. An observe experiences a difference of 4% of actual frequency in the sand from the source when it approaches the observe and when it moves away from him if the speed of sound in air is $350ms^{-1}$, then the speed of the source is ____ ms^{-1}.

Key : 7

Sol : $n_1 = \frac{v}{v-v_s}.n, n_2 = \frac{v}{v+v_s}.n \quad \Delta n = n_1 - n_2$

$$\Delta n \approx \frac{2v_s}{v}.n \Rightarrow \frac{\Delta n}{n} = \frac{2v_s}{v} \Rightarrow \frac{4}{100} = \frac{2v_s}{350}$$

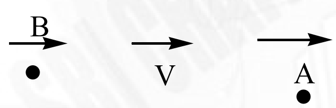
$$v_s = \frac{350\times 4}{2\times 100} = \frac{350}{50} = 7ms^{-1}$$

3. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz, to detect A and receives a reflected sound of frequency 'f'. The value of 'f' is close to. (Speed of sound in water =1500 ms^{-1}) **Jee Mains 2019**

Key : 502

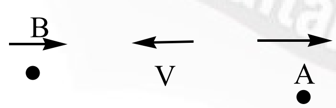
Sol: $V_A = 18\times\frac{5}{18} = 5m/s; V = 1500m/s$

$$V_B = 27\times\frac{5}{18} = 7.5m/s; f = 500$$



$$V_A\left(\frac{V-V_A}{V-V_B}\right)t = \frac{1495}{1492.5}\times 500$$

$$V_A = 500.841 + 2; v_B = v\left(\frac{V+V_B}{V+V_A}\right)f$$



$$v = \frac{1507.5}{1505}\times 500.84 \Rightarrow v = 502Hz$$

3a. A source and an observer move in opposite direction away from each other with speed of $10ms^{-1}$ w.r.to ground. Apparent frequency of the source is 1950 Hz. The natural frequency of the source is ____Hz (velocity of sound is 340 ms^{-1})

Key : 2068

Sol : $n^1 = \left(\frac{v-v_s}{v+v_s}\right)n \Rightarrow 1950 = \frac{330}{350}\times n$

$$n=\frac{1950\times 35}{33}=2068.2\approx 2068$$

3b. A source and an observer move in opposite direction away from each other with speed of $20ms^{-1}$ w.r.to ground. Apparent frequency of the source is 2200 Hz. The natural frequency of the source is ____Hz (velocity of sound is 340 ms^{-1})

Key : 2475

Sol : $n^1=\left(\frac{v-v_s}{v+v_s}\right)n\Rightarrow n=\left(\frac{v+v_s}{v-v_s}\right)n^1=\frac{360}{320}\times 2200=2475Hz$

4. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz. When he hears it after it get reflected from the wall. Find the speed of the bus if speed of the sound is $330ms^{-1}$ (in kmh^{-1})**Jee Main 2020 Sep 4th PM**

Key : 91

Sol : From the Doppler's effect of sound, frequency appeared at wall

$$f_w=\frac{330}{330-v}.f \text{ .... (i)}$$

Here, v=speed of bus,

$f=$actual frequency of source

Frequency heard after reflection from wall (f') is

$$f'=\frac{330+v}{300}.f_w=\frac{330+v}{330-v}.f$$

$$\Rightarrow\ 490=\frac{300+v}{330-v}.420$$

$$\Rightarrow v=\frac{330\times 7}{91}\approx 25.38m/s=91km/hr$$

4a. An observe is riding on a bicycle and moving towards a hill at 18kmph. He hears a sound from a source at some distance behind him directly as well as after its reflection from the hill. If the original frequency of the sound as emitted by source is 640 Hz and velocity of the sound in air is 320m/s. The beat frequency between the two sounds heard by observer will be ______Hz

Key: 20

Sol: $v_0=18kmph$

$v_s=0$

$$V_{ob}=18\times\frac{5}{18}=5m/s$$

$$n_{direct}=n\left(\frac{v-v_0}{v-v_s}\right)=640\left(\frac{320-5}{320-0}\right)$$

$$n_{direct}=630Hz=n_1$$

$$n_{reflect}=n\left(\frac{v+v_0}{v-v_s}\right)=640\left(\frac{v+v_0}{v-v_s}\right)$$

$= 640\left(\frac{320+5}{320}\right) = 650Hz = n_2$

$\Delta n = n_2 \sim n_1 = 650 - 630 = 20Hz$

4b. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 480 Hz to 520 Hz. When he hears it after it get reflected from the wall. Find the speed of the bus if speed of the sound is $330ms^{-1}$ (in kmh^{-1})

Key : 13

Sol : When reflected from wall apparent frequency

$f^1 = \left(\frac{v+v_s}{v-v_s}\right) = f \Rightarrow 520 = \frac{330+v_s}{330-v_s}480$

$\frac{330+v_s}{330-v_s} = \frac{520}{480} \Rightarrow \frac{v}{2\times 330} = \frac{400}{1000}$

$v_s = \frac{40\times 330}{1000} = \frac{132}{10} = 13.2 \approx 13$

5. A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is (in km/hr) :**Jee Main 2020 Sep 5th PM**

Key : 54

Sol : Let f_1 be the frequency heard by wall, $f_1 = \left(\frac{v}{v-v_c}\right)f_0$

Here, $v =$ velocity of sound

$f_0 =$ actual frequency of car horn

Let f_2 be the frequency heard by driver after reflection from wall.

$f_2 = \left(\frac{v+v_c}{v}\right)f_1 = \left(\frac{v+v_c}{v-v_c}\right)f_0$

$\Rightarrow 480 = \left[\frac{345+v_c}{345-v_c}\right]440 \Rightarrow \frac{12}{11} = \frac{345+v_c}{345-v_c}$

$\Rightarrow v_c = 54$ km/hr.

5a. When a car is approaching the observer, the frequency of the Horn is 200Hz. After passing the observer it is 100Hz. If the observer moves with the car, the frequency will be $\frac{x}{3}$ where $x = ?$

Key: 400

Sol: $n_1 = n\left[\frac{v}{v-v_s}\right] = 200........(1)$

$$n_2 = n\left[\frac{v}{v+v_s}\right] = 100........(2)$$

$$\therefore \frac{2}{1} = \frac{v+v_s}{v-v_s} \Rightarrow 2v - 2v_s = v + v_s$$

$$v_s = \frac{v}{3}$$

$$n_1 = 200 = n\left[\frac{v}{v-\frac{v}{3}}\right] = \frac{3v}{2v}\times n = 200$$

$$n = \frac{400}{3} \Rightarrow x = 400$$

5b. When a car is approaching the observer, the frequency of the Horn is 300Hz. After passing the observer it is 180Hz. If the observer moves with the car, the frequency will be $\frac{x}{4}$ where $x = ?$

Key: 900

Sol: $$n_1 = n\left[\frac{v}{v-v_s}\right] = 200........(1)$$

$$n_2 = n\left[\frac{v}{v+v_s}\right] = 100........(2)$$

$$\frac{n_1}{n_2} = \left(\frac{v+v_s}{v-v_s}\right) \Rightarrow \frac{300}{180} = \frac{v+v_s}{v-v_s}$$

$$5v - 5v_s = 3v + 3v_s \Rightarrow 2v = 8v_s$$

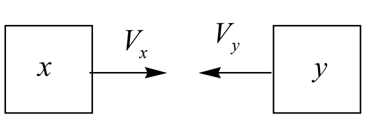
$$v_s = \frac{v}{4} \text{ Now } n_1 = \left(\frac{v}{v-\frac{v}{4}}\right)n \Rightarrow 300 = \frac{v4}{3v}n \Rightarrow \frac{900}{4} = \frac{x}{4}$$

$$x = 900$$

6. Two cars X and Yare approaching each other with velocities 36km/h and 72km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340m/s, the actual frequency of the whistle sound produced is…. Hz **Jee Main 2021**

Key: 1210

Sol:



$$V_x = 36km/hr = 10m/s$$

$$V_y = 72km/hr = 20m/s$$

By doppler's effect

$$f' = f\left(\frac{v+v_0}{v-v_s}\right)$$

$$1320 = f\left(\frac{340+20}{340-10}\right) \Rightarrow f = 1210Hz$$

6a. A man standing on platform observes that the frequency of the sound of a whistle emitted by a train drops by 140 Hz. If the velocity of sound in air is 330 m/s and the speed of the train is 70m/s, the frequency of the whistle is ___Hz

Key : 800

Sol : Velocity of sound in air =330 m/s.

Speed of train = 70m/s

Let initial frequency be f.

$$f^1 = f \times \frac{V}{V+V_s}$$

$$\frac{140}{f^1} = \frac{70}{330} \Rightarrow f^1 = 660Hz$$

$$f = f^1 + 140$$

$$= 800Hz$$

6b. A man standing on platform observes that the frequency of the sound of a whistle emitted by a train drops by 180 Hz. If the velocity of sound in air is 330 m/s and the speed of the train is 80m/s, the frequency of the whistle is ___Hz

Key : 922

Sol : Velocity of sound in air =330 m/s.

Speed of train = 80m/s

Let initial frequency be f.

$$f^1 = f \times \frac{V}{V+V_s} \Rightarrow f = f^1\left(1+\frac{v_s}{v}\right)$$

$$\frac{f}{f^1} - 1 = \frac{v_s}{v} \Rightarrow \frac{f-f^1}{f^1} = \frac{80}{330}$$

$$f^1 = 190 \times \frac{33}{8} = 742.5$$

$$f = f^1 + 180 = 922.5Hz \approx 922Hz$$

7. A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be ....Hz ` **Jee Main 2021**

Key 2025

Sol:

$V_S = 20m/s$ $\quad$ $V_O = 20m/s$

$$f' = f\left(\frac{C-V_0}{C+V_s}\right)$$

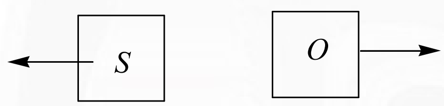
$$1800 = f\left(\frac{340-20}{340+20}\right)$$

$$f = 2025Hz$$

7a. A source and a detector move away from each other in absence of wind with a speed of 40 m/s with respect to the ground. If the detector detects a frequency of 2000Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be ....Hz

Key : 2280

Sol :



$V_S = 40m/s$ $\quad$ $V_O = 20m/s$

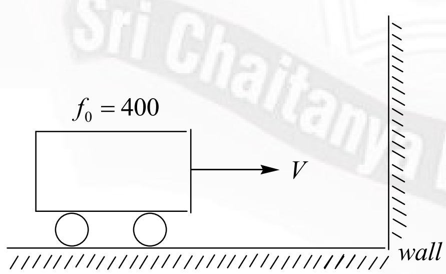
$$f' = f\left(\frac{V-V_0}{V+V_s}\right)$$

$$1800 = \left(\frac{300}{380}\right)f \Rightarrow f = \frac{1800\times 380}{300} = 2280Hz$$

8. The frequency of a car horn encountered a change from 400Hz to 500Hz. When the car approaches a vertical wall. If the speed of sound is 330m/s. Then the speed of car is ___km/h **Jee Mains -2021**

Key: 132

Sol :



Wall as an observer

Frequency received by wall

$$f_1 = f_0\left(\frac{C}{C-V}\right)$$

Again wall as a source

Frequency received by observer on car

$$f_2 = f_1\left(\frac{C+V}{C}\right)$$

$$f_1 = f_0\left(\frac{C+V}{C-V}\right)$$

$$500 = 400\left(\frac{C+V}{C-V}\right)$$

$$\frac{5}{4} = \frac{C+V}{C-V}$$

$$C = 9V$$

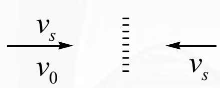
$$V = \frac{C}{9} = \frac{330}{9}m/s$$

$$V = \frac{330}{9}\times\frac{18}{5} = 132km/h$$

8a. A car is moving rapidly towards a wall with velocity 5 m/s sounds horn with frequency 320 Hz. Find frequency of sound heard by driver of car in Hz $\left(v = 330ms^{-1}\right)$

Key : 330 Hz

Sol :



$$v_0 = 0$$

$$v_s = 0$$

$$n_1 = \frac{v}{v-v_s}n \quad n_2 = \frac{v+v_s}{v}.n_1$$

$$\Rightarrow n_2 = \frac{v+v_s}{v}.\frac{v}{v-v_s}n = \left(\frac{v+v_s}{v-v_s}\right)n$$

$$n_2 = \frac{335}{325}.320 = 329.8 \approx 330Hz$$

8b. The frequency of a car horn encountered a change from 500Hz to 800Hz. When the car approaches a vertical wall. If the speed of sound is 330m/s. Then the speed of car is ___m/s

Key: 76

Sol : When reflected from wall apparent frequency

$$f^1 = \left(\frac{v+v_s}{v-v_s}\right) = f \Rightarrow 800 = \frac{330+v_s}{330-v_s}500$$

$$\frac{330+v_s}{330-v_s} = \frac{800}{500} \Rightarrow \frac{v_s}{330} = \frac{300}{1300}$$

$$v_s = \frac{990}{13} = 76.1 \approx 76m/s$$

9. A stationary source emits sound of frequency $f_0 = 492Hz$. The sound is reflected by a large car approaching the source with a speed of $2ms^{-1}$. The reflected signal is

received by the source and superposed with the original.What will be the best frequency of the resulting signal in Hz? (Given that the speed of sound in air is $330ms^{-1}$ and the car reflects the sound at the frequency it has received).**Adv-2017**

Key: 6

Sol: Let f_1 =frequency received by the car then $f_1 = f_0\left[\frac{V+V_c}{V}\right]$

The frequency f_2 received by the source after reflection from the car is given by

$$f_2 = f_1\left[\frac{V}{V-V_c}\right]$$

$$f_2 = f_0\left[\frac{V+V_c}{V-V_c}\right] = 492\left[\frac{330+2}{330-2}\right]$$

$$\frac{492\times 332}{328} = 498Hz$$

Beat frequency = $498-492=6$

9a. A stationary source emits sound of frequency $f_0 = 494Hz$. The sound is reflected by a large car approaching the source with a speed of $2ms^{-1}$. The reflected signal is received by the source and superposed with the original. What will be the best frequency of the resulting signal in Hz? (Given that the speed of sound in air is $330ms^{-1}$ and the car reflects the sound at the frequency it has received).

Key: 6

Sol: Let f_1 =frequency received by the car then $f_1 = f_0\left[\frac{V+V_c}{V}\right]$

The frequency f_2 received by the source after reflection from the car is given by

$$f_2 = f_1\left[\frac{V}{V-V_c}\right]$$

$$f_2 = f_0\left[\frac{V+V_c}{V-V_c}\right] = 494\left[\frac{330+2}{330-2}\right]$$

$$\frac{494\times 332}{328} = 500Hz$$

Beat frequency = $500-494=6$

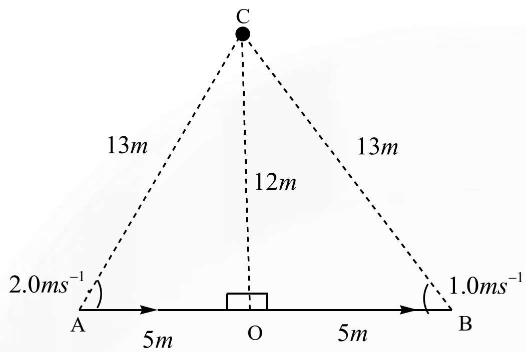
10. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed $1.0ms^{-1}$ and the man behind walks at a speed $2.0ms^{-1}$. A third man is standing at a height $12m$ above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency $1430Hz$. The speed of sound in air is $330ms^{-1}$. At the

instant, when the, moving men are $10m$ apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is------

Adv-2018

Key: 5

Sol:



$$f_A = f\left(\frac{V}{V-2\cos\theta}\right) = 1430\left[\frac{330}{330-2\cos\theta}\right] = 1430\left[\frac{330}{1-\frac{2\cos\theta}{330}}\right] = 1430\left[1+\frac{2\cos\theta}{330}\right]$$

$$f_B = f\left(\frac{V}{V+\cos\theta}\right) = 1430\left[\frac{330}{330+\cos\theta}\right] = 1430\left[1-\frac{\cos\theta}{330}\right]$$

$$\Delta f = f_A - f_B$$

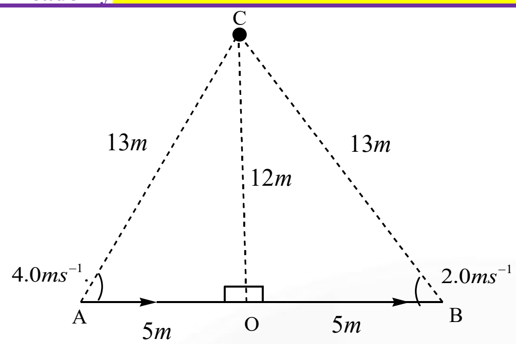
$$\Delta f = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$$

$$\Delta f = 13\left(\frac{5}{13}\right) = 5Hz \qquad \left[\text{From } \angle\text{CAO, } \cos\theta = \frac{5}{13}\right]$$

10a. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed $2.0ms^{-1}$ and the man behind walks at a speed $4.0ms^{-1}$. A third man in standi8ng at a height $12m$ above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency $1430Hz$. The speed of sound in air is $330ms^{-1}$. At the instant, when the, moving men are $10m$ apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is------

Key: 10

Sol:



$$f_A = f\left(\frac{V}{V-4\cos\theta}\right) = 1430\left[\frac{330}{330-4\cos\theta}\right] = 1430\left[\frac{330}{1-\frac{4\cos\theta}{330}}\right] = 1430\left[1+\frac{4\cos\theta}{330}\right]$$

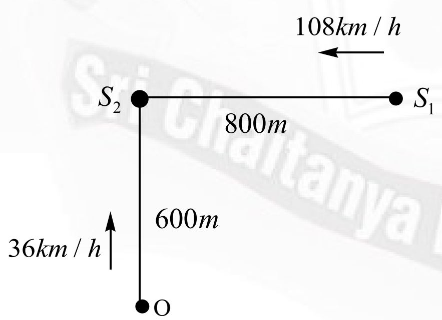
$$f_B = f\left(\frac{V}{V+2\cos\theta}\right) = 1430\left[\frac{330}{330+2\cos\theta}\right] = 1430\left[1-\frac{2\cos\theta}{330}\right]$$

$$\Delta f = f_A - f_B$$

$$\Delta f = 1430\left[\frac{6\cos\theta}{330}\right] = 26\cos\theta$$

$$\Delta f = 26\left(\frac{5}{13}\right) = 10Hz \qquad \left[\text{From } \angle\text{CAO, } \cos\theta = \frac{5}{13}\right]$$

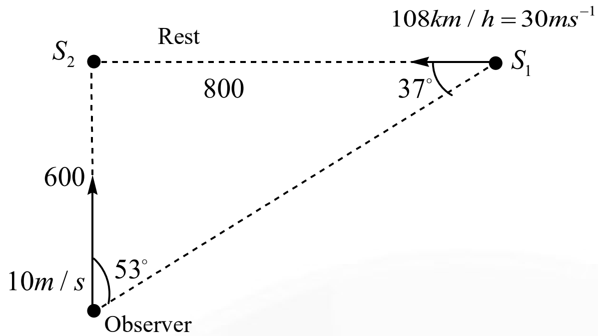
11. A train S_1, moving with a uniform velocity of $108 km/h$, approaches another train S_2 standing on a platform. An observer O moves with a uniform velocity of $36 km/h$ towards S_2, as shown in figure. Both the trains are blowing whistles of same frequency $120 Hz$. When O is $600m$ away from S_2 and distance between S_1 and S_2 $800m$, the number of beats heard by O is____[Speed of the sound=$330 m/s$] **Adv-2019**



Key: 8.12 or 8.13

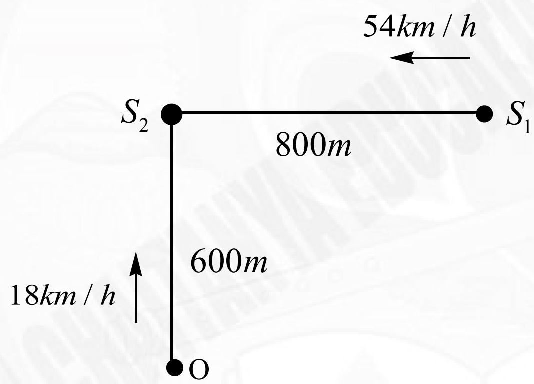
Sol: Speed of sound = $330 m/s$

Calculate beat frequency



$$f_b = 120\left[\left(\frac{330+10\cos 53^\circ}{330-30\cos 37^\circ}\right)-\left(\frac{330+10}{330}\right)\right]=120\left[\frac{336}{306}-\frac{34}{33}\right]=8.12Hz$$

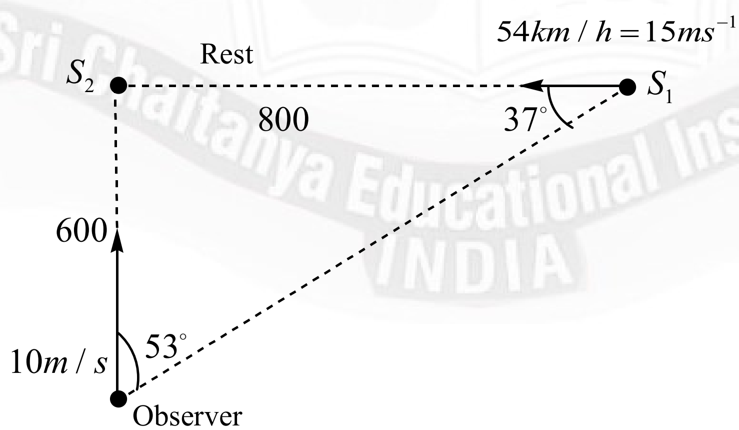
11a. A train S_1, moving with a uniform velocity of $54km/h$, approaches another train S_2 standing on a platform. An observer O moves with a uniform velocity of $18km/h$ towards S_2, as shown in figure. Both the trains are blowing whistles of same frequency $120Hz$. When O is $600m$ away from S_2 and distance between S_1 and S_2 $800m$, the number of beats heard by O is ———— [Speed of the sound=$330m/s$]



Key: 3.84 or 3.85

Sol: Speed of sound = $330m/s$

Calculate beat frequency



$$f_b = 120\left[\left(\frac{330+5\cos 53^\circ}{330-15\cos 37^\circ}\right)-\left(\frac{330+5}{330}\right)\right]=120\left[\frac{333}{318}-\frac{335}{330}\right]=3.84Hz$$

12. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of $2ms^{-1}$ in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is $320ms^{-1}$, the smallest value of the percentage change is required in the length of the pipe is **Adv -2020**

Key: 0.62 to 0.63

Sol: $$f_\infty = \frac{1}{l_1} \Rightarrow f = \frac{k}{l_1}.......(1)$$

($l_1 \Rightarrow$ initial length of pipe)

$$\left(\frac{V}{V-V_r}\right)f = \frac{k}{l_2} \quad \{V_T \text{ Speed of tuning fork, } l_2 \rightarrow \text{new length of pipe}\}........(2)$$

$(1)\div(2)$

$$\frac{V-V_t}{V} = \frac{l_2}{l_1} \Rightarrow \frac{l_2}{l_1} - 1 = \frac{V-V_t}{V} - 1$$

$$\frac{l_2-l_1}{l_1} = \frac{-V_t}{V} \Rightarrow \frac{l_2-l_1}{l_1}\times 100 = \frac{-2}{320}\times 100 = -0.625$$

Therefore smallest value of percentage change required in the length of pipe is 0.625.

12a. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of $4m/s$ in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air $320m/s$, the smallest value of the percentage change is required in the length of the pipe is

Key: 1.25

Sol: $$f_\infty = \frac{1}{l_1} \Rightarrow f = \frac{k}{l_1}.......(1)$$

($l_1 \Rightarrow$ initial length of pipe)

$$\left(\frac{V}{V-V_r}\right)f = \frac{k}{l_2} \quad \{V_T \text{ Speed of tuning fork, } l_2 \rightarrow \text{new length of pipe}\}........(2)$$

$(1)\div(2)$

$$\frac{V-V_t}{V} = \frac{l_2}{l_1} \Rightarrow \frac{l_2}{l_1} - 1 = \frac{V-V_t}{V} - 1$$

$$\frac{l_2-l_1}{l_1} = \frac{-V_t}{V} \Rightarrow \frac{l_2-l_1}{l_1}\times 100 = \frac{-4}{320}\times 100 = 1.25$$

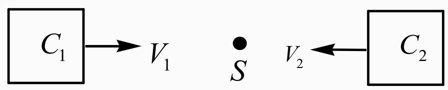
Therefore smallest value of percentage change required in the length of pipe is 1.25.

13. A stationary source is emitting sound at a fixed frequency f_0. Which is reflected by two cars Approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is $330ms^{-1}$. **IIT JEE -2010**

Key : 7

Sol : Firstly, car will be treated as an observer which is approaching the source. Then it will be treated as a source, which is moving in the direction of sound.

Frequency of sound reflected by the car v



$$f_1 = f_0\left(\frac{v+v_1}{v-v_2}\right)$$

And frequency of sound reflected by the car C_2

$$f_2 = f_0\left(\frac{v+v_2}{v-v_2}\right)$$

$$\therefore\ f_1 - f_2 = \left(\frac{1.2}{100}\right)f_0 = f_0\left[\frac{v+v_1}{v-v_1} - \frac{v+v_2}{v-v_2}\right]$$

Or $\left(\frac{1.2}{100}\right)f_0 = \frac{2v(v_1-v_2)}{(v-v_1)(v-v_2)}f_0$

$(v-v_1)=(v-v_2)\approx v$ as v_1 and v_2 are very very less than v.

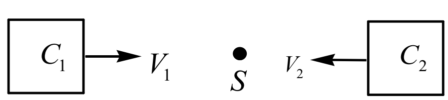
$$\therefore\ \left(\frac{1.2}{100}\right)f_0 = \frac{2(v_1-v_2)}{v}f_0$$

Or $(v_1-v_2) = \frac{v-1.2}{200} = \frac{330\times 1.2}{200} = 1.98ms^{-1} = 7kmh^{-1}$

13a. A stationary source is emitting sound at a fixed frequency f_0. Which is reflected by two cars Approaching the source. The difference between the frequencies of sound reflected from the cars is 2.4% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is $330ms^{-1}$.

Key : 14.25

Sol : Firstly, car will be treated as an observer which is approaching the source. Then it will be treated as a source, which is moving in the direction of sound. Frequency of sound reflected by the car v



$$f_1 = f_0\left(\frac{v+v_1}{v-v_2}\right)$$

And frequency of sound reflected by the car C_2

$$f_2 = f_0\left(\frac{v+v_2}{v-v_2}\right)$$

$$\therefore\ f_1 - f_2 = \left(\frac{2.4}{100}\right)f_0 = f_0\left[\frac{v+v_1}{v-v_1} - \frac{v+v_2}{v-v_2}\right]$$

Or $\left(\frac{2.4}{100}\right)f_0 = \frac{2v(v_1-v_2)}{(v-v_1)(v-v_2)}f_0$

$(v-v_1)=(v-v_2)\approx v$ as v_1 and v_2 are very very less than v.

$$\therefore\ \left(\frac{2.4}{100}\right)f_0 = \frac{2(v_1-v_2)}{v}f_0$$

Or $(v_1 - v_2) = \frac{v-2.4}{200} = \frac{330\times 2.4}{200} = 3.96ms^{-1} = 14.25kmph$

14. A stationary source is emitting sound at a fixed frequency f_0. Which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in kilometers per hours)to the nearest integer ? The cars are moving at constant speed much smaller than the speed of sound which is 330 m/s **IIT -2010**

Key : 7

Sol : $f_{app} = f_0\frac{c+v}{c-v}$

$$df = \frac{2f_0c}{(c-v)^2}dv$$

Where c is the speed of sound and df is equal to 1.2/100 f_0. Hence

$dv \approx 7km/h$

14a. A stationary source is emitting sound at a fixed frequency f_0. Which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 2.35% of f_0. What is the difference in the speeds of the cars (in m/s)to the nearest integer ? The cars are moving at constant speed much smaller than the speed of sound which is 340 m/s

Key : 4

Sol : $f_{app} = f_0\frac{c+v}{c-v}$

$$df = \frac{2f_0c}{(c-v)^2}dv$$

Where c is the speed of sound and df is equal to 2.35/100 f_0. Hence

$dv \approx 4m/s$

Exercise : III

(More than One Answer Type Questions Including PYQ's)

I) Introduction to waves, Travelling wave

Definition of wave, 1-D,2-D,3-D waves, Mechanical and non mechanical waves (Mechanical waves, Non mechanical waves), Longitudinal and transverse waves (Longitudinal wave, Transverse wave), Equation of travelling wave (Equation of plane progressive harmonic wave, Amplitude ;frequency and speed of the wave from the equation), Relation between path difference and phase difference - Equation of plane progressive harmonic wave, Displacement ;velocity and acceleration of the particle, Relation between wave velocity and particle velocity, Speed of the transverse wave in a string, Speed of the transverse wave in solids, Sound wave(Speed of sound wave in solids, Speed of sound wave in liquids and gases, Conversion from displacement variation to pressure variation, Speed of the longitudinal wave), Energy of a progressive wave, Sub-Sonic, Super sonic, Mach number, Audible range, infra sonic and ultra sonic, Intensity of sound wave(Formula for intensity, Variation of intensity with distance, Sound level).

1. As a wave propagates **IIT -1999**

A) The wave intensity remains constant for a plane wave

B) The wave intensity decreases as the inverse of the distance from the source for a spherical wave

C) The wave intensity decreases as the inverse square of the distance from the source for a spherical wave

D) Total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.

Key : A,C,D

Sol : For a plane wave, intensity i.e., energy crossing per unit area per unit time is constant at all points. But for a spherical wave, intensity at a distance r from a point source.

$I\alpha\frac{1}{r^2}$ But the total intensity of the spherical wave over the spherical surface centered at the source remains constant at all time.

For line source $I\alpha\frac{1}{r}$ spherical wave is not produced by the line source.

1a. A harmonic wave is travelling along +ve x-axis, on a stretched string. If wavelength of the wave gets doubled, then

A) Frequency of wave may change

B) Wave speed may change

C) Both frequency and speed of wave may changes

D) Only frequency will change

Key : A,B,C

Sol : Wave length of a wave is a property of source and medium both. So, wavelength can change if either frequency or speed of wave or both change. Here medium property (like tension in string) can change frequency may change which causes the change in the speed of wave, or source frequency may change.

2. In a wave motion $y = a\sin(kx - \omega t), y$ can represent **IIT -1999**

A) Electric field B) Magnetic field C) Displacement D) Pressure

Key : A,B,C,D

Sol : In the wave motion $y = a(kx - \omega t)$, y can represent, electric and magnetic fields in electromagnetic waves and displacement and pressure in sound waves.

2a. In a wave motion longitudinal wave motion $y = y_{max}\sin(kx - \omega t), y$ can represent **IIT -1999**

A) Density B) Flux C) Displacement D) Pressure

Key : A,C,D

Sol : In the wave motion $y = y_{max}\sin(kx - \omega t), y$ can represent displacement and pressure and density in sound waves

3. $y(x,t) = 0.8/\left[(4x+5t)^2 + 5\right]$ represents a moving pulse, where x and y are in meter and t in second. Then **IIT -1999**

A) pulse is moving in +x direction

B) in 2s it will travel a distance of 2.5m

C) Its maximum displacement is 0.16m

D) It is a symmetric pulse

Key : B,C,D

Sol : Comparing the given equation $y(x,t)$

$$= \frac{0.8}{(4x+5t)^2 + 5} = \frac{0.8}{16\left[x + \frac{5}{4}t\right]^2 + 5}$$

With the equation of moving pulse

$y = f(x + vt)$

$v = \frac{5}{4}ms^{-1} = \frac{2.5}{2}ms^{-1}$

So, the wave will travel a distance of 2.5m in 2 sec.

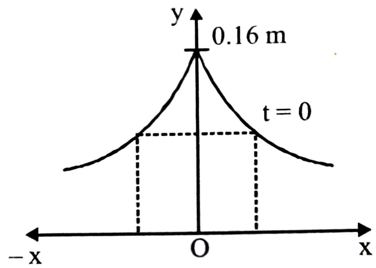
At $x = 0$, $t = 0$, $y = \frac{0.8}{(4x+5t)^2 + 5}$

$= \frac{0.8}{5} = 0.16m$

$\therefore$ maximum displacement is 0.16m

The shape of the pulse at $x = 0$ and $t = 0$ is a shown in figure and it is symmetric.

So, the wave will travel a distance of 2.5 m in 2 sec.



3a. A transverse pulse moving on a long string along x-axis is given as, $y=\frac{2}{(5x-3t)^2+1}$,where x is measured in meter y in centimeter and t is measured in seconds. Then

A) Wave speed $V=\frac{3}{5}m/s$

B) Height of the pulse is $y_{\max}=2cm$

C) The shape of the pulse at $t=0$ is $y=\frac{2}{25x^2+1}$

D) Height of the pulse is $y_{\max}=1cm$

Key : A,B,C

Sol : We can write the equation of the pulse at a time t is given as

$$y=\frac{2}{25\left(x-\frac{3}{5}t\right)^2+1}$$

Hence the wave speed should be, $v=\frac{3}{5}m/s$

The height of the pulse is $y_{\max}=2cm$

The shape of the pulse $t=0$ is $y=\frac{2}{25x^2+1}$

4. A transverse sinusoidal wave of amplitude a, wave length λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a=10^{-3}m$ and $v=10ms^{-1}$, then λ and f are given by **IIT -1998**

A) $\lambda=2\pi\times10^{-2}m$ B) $\lambda=10^{-3}m$ C) $f=10^3Hz/(2\pi)$ D) $f=10^4Hz$

Key : A,C

Sol : For a transverse sinusoidal wave travelling on a string, the maximum velocity $v_{\max}=a\omega$

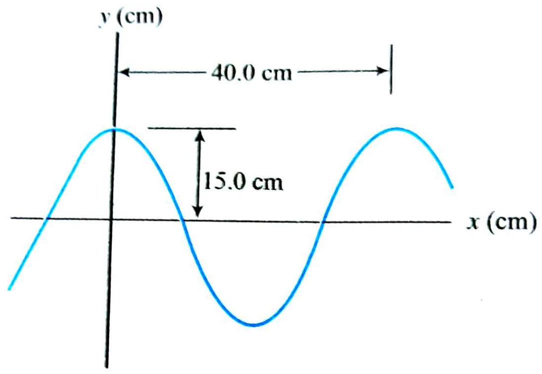
Given maximum velocity $=\frac{V}{10}=\frac{10}{10}=1m/s$

$\therefore\ a\omega=I \Rightarrow 10^{-3}\times2\pi v=1[\because \omega=2\pi v]$

$\Rightarrow v=\frac{1}{2\pi\times10^{-3}}=\frac{10^3}{2\pi}Hz$

And, $\lambda = \frac{v}{\nu} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2} m$

4a. A sinusoidal wave travelling in the positive x-direction has an amplitude of 15cm, wavelength 40 cm and frequency 8 Hz. The vertical displacement of the medium at t=0 and x=0 is also 15cm, as shown in figure. Then



A) Wave number $=\frac{\pi}{20} rad/cm$

B) Time period $=\frac{1}{8}\sec$

C) Angular frequency $\omega = 16\pi rad/s$

D) Speed of the wave $v = 320 cm/s$

Key : A,B,C,D

Sol: $k = \frac{2\pi}{\lambda}$

$T = \frac{1}{f} = \frac{1}{8} s$

$\omega = 2\pi f = 16\pi rad/s$

$v = f\lambda = 320 cm/s$

5. A wave is represented by the equation $y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$ where x is in meters and t is in seconds. The expression represents: **IIT -1990**

A) a wave travelling in the positive x-direction with a velocity 1.5m/s

B) a wave travelling in the negative x-direction with a velocity 1.5m/s

C) a wave travelling in the negative x-direction having a wavelength 0.2m

D) a wave travelling in the positive x-direction having a wavelength 0.2m

Key : B,C

Sol : Given equation, $y = A\sin(10\pi x + 15\pi t + \pi/3)$ comparing this equation with standard equation of a wave travelling in- X direction.

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt + x) + (\phi)\right] \Rightarrow y = A\sin\left[\frac{2\pi v}{\lambda}t + \frac{2\pi}{\lambda}x + \phi\right]$$

$\frac{2\pi v}{\lambda} = 15\pi$ and $\frac{2\pi}{\lambda} = 10\pi$

$\Rightarrow \lambda = \frac{1}{5} = 0.2m$ and $v = \frac{15\pi}{2\pi} \times \frac{1}{5} = 1.5m/s$

5a. A wave equation which gives the displacement along y-direction is given by $y = 10^{-4}\sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave

A) travelling with a velocity of 30 m/s in the negative x-direction

B) of wavelength π metres

C) of frequency $30/\pi$ Hz

D) of amplitude $10^{-4}m$ travelling along the negative x-direction

Key : A,B,C,D

Sol : $y = 10^{-4}\sin(6t + 2x)$

$y = a\sin(\omega t + kx)$

Now, $k = 2, 2\pi/\lambda = 2$ or $\lambda = \pi$ meter

Again $\omega = 60$

Or $2\pi f = 60$ or $f = \frac{60}{2\pi}$ or $f = \frac{30}{\pi}Hz$

Again $v = \frac{\omega}{k} = \frac{60}{2}m/s = 30m/s$

6. A wave disturbance in a medium is described by $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$ where x and y are in meter and t is in second **IIT -1995**

A) A node occurs at $x = 0.15m$

B) An antinode occurs at $x = 0.3m$

C) The speed wave is $5ms^{-1}$

D) The wave length is 0.3m

Key : C

Sol : Comparing the given equation, $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$

$y(x,t) = A\cos(\omega t + \pi/2)\cos kx$

If $kx = \pi/2$, a node occurs;

$\therefore\ 10\pi x = \pi/2 \Rightarrow x = 0.05m$

Also speed of wave

$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5m/s$ and $\lambda = 2\pi/k = 2\pi/10\pi = 0.2m$

6a. A standing wave setup in a medium, the equation of wave is given by $y = 4\cos\frac{\pi x}{3}\sin 40\pi t$ where x and y are in cm and t in sec. Find the velocity of the two component wave

A) $60cm/s$

B) $120cm/s$

C) $180cm/s$

D) $240cm/s$

Key : B

Sol : The given equation is : $y = 4\cos\frac{\pi x}{3}\sin 40\pi t$

Or $y = 2\times 2\cos\frac{2\pi x}{6}\sin\frac{2\pi(120)t}{6}$

The standard equation of a stationary wave given by

We know that $y = 2A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$

Comparing the equation

$A = 2cm, \lambda = 6cm$ and $v = 120cm/s$

II) Superposition of waves, Reflection and refraction of waves, Stationary waves

Constructive interference - Superposition of waves, Destructive interference – Superposition of waves, Resultant intensity, Reflection - Reflection and refraction of waves,(Change in the amplitude and phase of a wave reflected from fixed boundary, Change in the amplitude and phase of a wave reflected from a free boundary) Refraction - Reflection and refraction of waves,(Amplitude of the transmitted wave, Change in the phase of the transmitted wave, Amplitude of the Reflected Wave, Change In the phase of the reflected wave) Equation of stationary wave, Standing waves on string, Standing waves in organ pipes,(Position of nodes for displacement wave, Position of nodes for pressure wave, Position of antinodes displacement wave, Position of antinodes for pressure waves, Stationary waves in stretched string fixed at ends, Stationary longitudinal wave in open organ pipe, Stationary longitudinal wave in closed organ pipe, End correction, Resonance tube).

1. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse of air travels down the pipe. When this pulse reaches the other end of the pipe. **Adv-2012**

A) A high- pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

B) A low- pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

C) A low- pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

D) A high- pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

Key: B,D

Sol: At open end phase of pressure wave charge by π so compression returns as rare-fraction. While at closed end phase of pressure wave does not change so compression return as compression.

1a. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse of air travels down the pipe. When this pulse reaches the other end of the pipe, Which of the following are false.

A) A high- pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

B) A low- pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

C) A low- pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

D) A high- pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

Key: A,C

Sol: At open end phase of pressure wave charge by π so compression returns as rarefaction.

While at closed end phase of pressure wave does not change so compression return as compression.

2. A student is performing the experiment of Resonance Column. The diameter of the column tube is $4cm$. The distance frequency of the tuning for k is $512Hz$. The air temperature is $38^\circ C$ in which the speed of sound is $336m/s$. The zero of the meter scale coincides with the top and of the Resonance column. When first resonance occurs, the reading of the water level in the column is **Adv-2012**

A) 14.0 B) 15.2 C) 16.4 D) 17.6

Key: B

Sol:
$$\frac{V}{4(l+e)}=f$$
$$\Rightarrow l+e=\frac{V}{4f}$$
$$l=\frac{V}{4f}-e$$
Here $e=(0.6)r=(0.6)(2)=1.2cm$

So $l=\dfrac{336\times10^2}{4\times512}-1.2=15.2cm$

2a. A student is performing the experiment of Resonance Column. The diameter of the column tube is $4cm$. The distance frequency of the tuning for k is $486Hz$. The air temperature is $38^\circ C$ in which the speed of sound is $336m/s$. The zero of the meter scale coincides with the top and of the Resonance column. When first resonance occurs, the reading of the water level in the column is

A) 14.0 B) 15.2 C) 16.1 D) 17.6

Key: C

Sol: $\frac{V}{4(l+e)}=f$

$\Rightarrow l+e=\frac{V}{4f}$

$l=\frac{V}{4f}-e$

Here $e=(0.6)r=(0.6)(2)=1.2cm$

So $l=\frac{336\times10^2}{4\times486}-1.2=16.1cm$

3a. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation $y(x,t)=0.01m\sin\left[\left(62.8m^{-1}\right)x\right]\cos\left[\left(628s^{-1}\right)t\right]$. Assuming $\pi=3.14$, the correct statement(s) is (are) **Adv-2013**

A) The number of nodes is 5

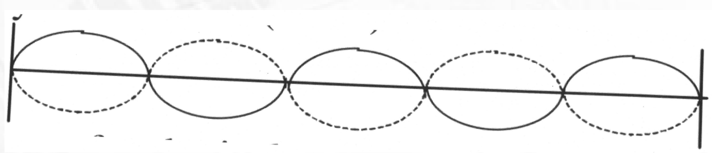
B) The length of the string is $0.25m$

C) The maximum displacement of the mid-point of the string from is equilibrium position is $0.01m$

D) The fundamental frequency is $100Hz$.

Key: B,C

Sol: $y=0.01m\sin(20\pi x)\cos200\pi t$



No. nodes is 6

$20\pi=\frac{2\pi}{\lambda}$

$\therefore\lambda=\frac{1}{10}m=0.1m$

Length of the spring= $0.5\times\frac{1}{2}=0.25$

Mid point is the antinode

Frequency at this mode is $f=\frac{200\pi}{2\pi}=100Hz$

$\therefore$ Fundamental frequency $=\frac{100}{5}=20Hz$

3b. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation $y(x,t)=0.01m\sin\left[\left(62.8m^{-1}\right)x\right]\cos\left[\left(628s^{-1}\right)t\right]$.

Assuming $\pi=3.14$, the correct statement(s) is (are)

A) The number of nodes is 6

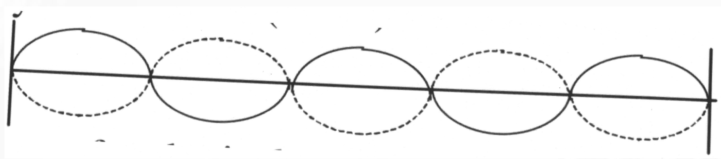
B) The length of the string is $0.25m$

C) The maximum displacement of the mid-point of the string from is equilibrium position is $0.01m$

D) The fundamental frequency is $20Hz$.

Key: A,B,C,D

Sol: $y=0.01m\sin(20\pi x)\cos 200\pi t$



No. nodes is 6

$$20\pi=\frac{2\pi}{\lambda}$$

$$\therefore \lambda=\frac{1}{10}m=0.1m$$

Length of the spring= $0.5\times\frac{1}{2}=0.25$

Mid point is the antinode

Frequency at this mode is $f=\frac{200\pi}{2\pi}=100Hz$

$\therefore$ Fundamental frequency = $\frac{100}{5}=20Hz$

4. One end of a taut string of length $3m$ along the X-axis I fixed at $x=0$. The speed of the waves in the string is $100ms^{-1}$. The other end of the string is vibrating in the y-direction so that stationary waves are setup in the string. The possible waveform(s) of these stationary wave is (are) **Adv-2014**

(a) $y(t)=A\sin\frac{\pi x}{6}\cos\frac{50\pi}{3}$

(b) $y(t)=A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(c) $y(t)=A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(d) $y(t)=A\sin\frac{5\pi x}{2}\cos 250\pi t$

Key: A,C,D

Sol:

$$v = \frac{\varpi}{K}$$

$$v_A = v_B = v_C = v_D = 100ms^{-1}$$

$x = 3$ is an antinode. This eliminates (B).

4a. One end of a taut string of length $3m$ along the X-axis I fixed at $x = 0$. The speed of the waves in the string is $100ms^{-1}$. The other end of the string is vibrating in the y-direction so that stationary waves are setup in the string. Which of the following waveform(s) of these stationary wave is (are) not possible.

(a) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi}{3}$

(b) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(c) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(d) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

Key: B

Sol:

$$v = \frac{\varpi}{K}$$

$$v_A = v_B = v_C = v_D = 100ms^{-1}$$

$x = 3$ is an antinode. This eliminates (B).

5. A student is performing an experiment using a resonance column and a tuning fork of frequency $244s^{-1}$. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is $(0.350 \pm 0.005)m$, the gas in the tube is

(useful information: $\sqrt{167RT} = 640J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590J^{1/2}mole^{-1/2}$. The molar masses M in grams are given in the options. Take the value of $\sqrt{10/M}$ for each gas as given there). **Adv-2014**

A) Neon $\left(M = 20, \sqrt{10/20} = 7/10\right)$

B) Nitrogen $\left(M = 28, \sqrt{10/28} = 3/5\right)$

C) Oxygen $\left(M = 32, \sqrt{10/32} = 9/16\right)$

D) Argon $\left(M = 36, \sqrt{10/36} = 17/32\right)$

Key: D

Sol: $l = \frac{1}{4v}\sqrt{\frac{\gamma RT}{M}}$

Calculations for $\frac{1}{4v}\sqrt{\frac{\gamma RT}{M}}$ for gases mentioned in options A, B, C & D, work out to be $0.459m$, $0.363m$, $0.340m$ & $0.348m$ respectively. As $l = (0.350 \pm 0.005)m$; Hence correct option is D.

5a. A student is performing an experiment using a resonance column and a tuning fork of frequency $244 s^{-1}$. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is $(0.350 \pm 0.005)m$, Which of the following gases are not in the tube.

(useful information: $\sqrt{167RT} = 640 J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590 J^{1/2} mole^{-1/2}$. The molar masses M in grams are given in the options. Take the value of $\sqrt{10/M}$ for each gas as given there).

A) Neon $\left(M = 20, \sqrt{10/20} = 7/10\right)$

B) Nitrogen $\left(M = 28, \sqrt{10/28} = 3/5\right)$

C) Oxygen $\left(M = 32, \sqrt{10/32} = 9/16\right)$

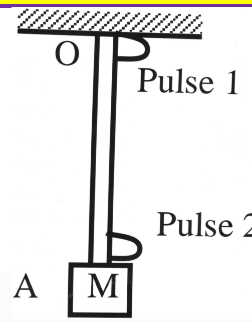
D) Argon $\left(M = 36, \sqrt{10/36} = 17/32\right)$

Key: A,B,C

Sol: $l = \frac{1}{4v}\sqrt{\frac{\gamma RT}{M}}$

Calculations for $\frac{1}{4v}\sqrt{\frac{\gamma RT}{M}}$ for gases mentioned in options A, B, C & D, work out to be $0.459m$, $0.363m$, $0.340m$ & $0.348m$ respectively. As $l = (0.350 \pm 0.005)m$; Hence D is possible and A, B, C are not possible.

6. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of the wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct? **Adv-2017**



A) The time $T_{AO} = T_{OA}$

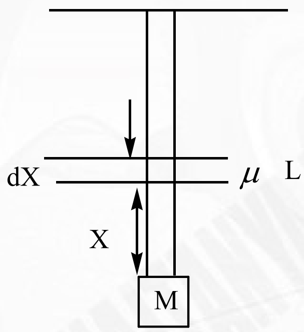
B) The wavelength of pulse 1 becomes longer when it reaches point A.

C) The velocity of any pulse along the rope is independent of its frequency and wavelength.

D) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the mid-point of rope.

Key: A,B,D

Sol



(A) $V = \sqrt{\frac{T}{\mu}}$

Tension at midpoint

$$T = \left(M + \frac{\mu L}{2}\right)g$$

Velocity at midpoint is same for both pulses.

(B) As pulse goes down. Tension decreases, V also decreases hence λ decreases

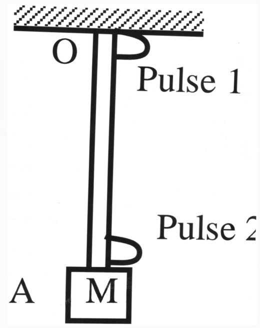
(D) At any height X

$$V = \sqrt{\frac{(M + \mu X)g}{\mu}} = \frac{dX}{dt}$$

$$\int_0^t dt = \sqrt{\frac{\mu}{g}} \int_0^x \frac{dX}{\sqrt{M + \mu X}}$$

$$T_{AO} = T_{OA}$$

6a. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of the wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are incorrect?



A) The time $T_{AO} = T_{OA}$

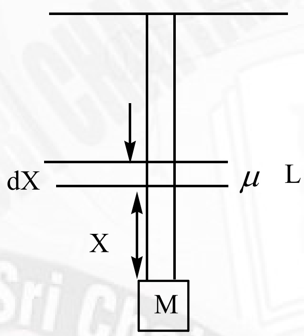
B) The wavelength of pulse 1 becomes longer when it reaches point A.

C) The velocity of any pulse along the rope is independent of its frequency and wavelength.

D) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the mid-point of rope.

Key: D

Sol:



(A) $V = \sqrt{\frac{T}{\mu}}$

Tension at midpoint

$$T = \left(M + \frac{\mu L}{2}\right)g$$

Velocity at midpoint is same for both pulses.

(B) As pulse goes down. Tension decreases, V also decreases hence λ decreases

(D) At any height X

$$V = \sqrt{\frac{(M + \mu X)g}{\mu}} = \frac{dX}{dt}$$

$$\int_0^t dt = \sqrt{\frac{\mu}{g}}\int_0^x \frac{dX}{\sqrt{M+\mu X}} \qquad T_{AO} = T_{OA}$$

7. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency $500Hz$ is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length $50.7cm$ and $83.9cm$. Which of the following statement is (are) true?

Adv-2018

A) The speed of sound determined from this experiment is $332ms^{-1}$

B) The end correction in this experiment is $0.9cm$

C) The wavelength of the sound wave is $66.4cm$

D) The resonance at $50.7cm$ corresponds to the fundamental harmonic

Key: A,B,C

Sol: For 1st resonance, $\frac{3V}{4(L_1+e)} = f_0............(1)$

For 2nd resonance, $\frac{5V}{4(L_2+e)} = f_0............(2)$

$$\frac{3V}{4(L_1+e)} = \frac{5V}{4(L_2+e)}$$

$$3(L_2+e) = 5(L_1+e)$$

$$2e = 3L_2 - 5L_1$$

$$e = \frac{3\times 83.9 - 5\times 50.7}{2} = \frac{251.7-253.5}{2} = \frac{-1.8}{2} = -0.9$$

$$\frac{3V}{4(L_1+e)} = f_0$$

$$V = \frac{500\times 4\times 49.8}{3\times 100} = 20\times 16.6 = 332m/s$$

$$\lambda = \frac{V}{f_0} = \frac{332}{500}\times 100 = 66.4cm$$

7a. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency $500Hz$ is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length $50.7cm$ and $83.9cm$. Which of the following statement is (are) false?

A) The speed of sound determined from this experiment is $332ms^{-1}$

B) The end correction in this experiment is $0.9cm$

C) The wavelength of the sound wave is $66.4cm$

D) The resonance at $50.7cm$ corresponds to the fundamental harmonic

Key: D

Sol: For 1st resonance, $\frac{3V}{4(L_1+e)} = f_0$............(1)

For 2nd resonance, $\frac{5V}{4(L_2+e)} = f_0$............(2)

$$\frac{3V}{4(L_1+e)} = \frac{5V}{4(L_2+e)}$$

$$3(L_2+e) = 5(L_1+e)$$

$$2e = 3L_2 - 5L_1$$

$$e = \frac{3\times 83.9 - 5\times 50.7}{2} = \frac{251.7-253.5}{2} = \frac{-1.8}{2} = -0.9$$

$$\frac{3V}{4(L_1+e)} = f_0$$

$$V = \frac{500\times 4\times 49.8}{3\times 100} = 20\times 16.6 = 332m/s$$

$$\lambda = \frac{V}{f_0} = \frac{332}{500}\times 100 = 66.4cm$$

Fundamental harmonic $\frac{\lambda}{4} = \frac{66.4}{4} = 16.6cm$

8. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is **IIT-1996**

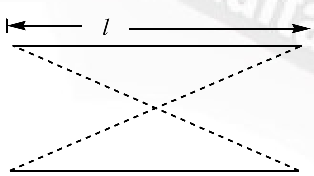
A) 200 Hz B) 300 Hz C) 240 Hz D) 480 Hz

Key : A

Sol : For both end open organ pipe

Fundamental frequency

$V_1 = \frac{c}{2\ell}$ ....(i)



For one end closed organ pipe

For third harmonic frequency

$V_2 = 3\left(\frac{c}{4l}\right)$ .....(ii)

Given $v_2 - v_1 = 100$ .....(iii)

From eq. (i) and (ii)

$\frac{v_2}{v_1} = \frac{3/4}{1/2} = \frac{3}{2} \Rightarrow v_1 = \frac{2}{3}v_2$ ....(iv)

Again, solving eq. (iii) & (iv) we get, $v_1 = 200Hz$.

8a. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 50Hz than the fundamental frequency of the open pipe. The fundamental frequency of the closed pipe is

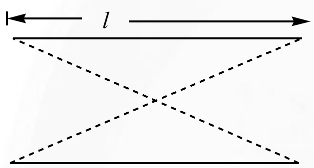
A) 150 Hz B) 100 Hz C) 240 Hz D) 480 Hz

Key : A

Sol : For both end open organ pipe

Fundamental frequency

$V_1 = \frac{c}{2\ell}$ ....(i)



For one end closed organ pipe

For third harmonic frequency

$V_2 = 3\left(\frac{c}{4l}\right)$ .....(ii)

Given $v_2 - v_1 = 50$ .....(iii)

From eq. (i) and (ii)

$\frac{v_2}{v_1} = \frac{3/4}{1/2} = \frac{3}{2} \Rightarrow v_1 = \frac{2}{3}v_2$ ....(iv)

Again, solving eq. (iii) & (iv) we get, $v_1 = 100Hz$.

9. Standing waves can be produced **IIT -1999**

A) on a string clamped at both the ends.

B) on a string clamped at one end free at the other

C) when incident wave gets reflected from a wall

D) when two identical waves with a phase difference of π are moving in the same direction

Key : A,B,C

Sol : Standing waves are produced by two identical waves superposing while travelling in opposite direction.

9a. Mark out the correct statement(s) regarding standing waves.

A) Standing wave appear to be stationary but transfer of energy from one particle to another continues to take place.

B) A standing wave not only appears to be stationary but net transfer of energy from one particle to the other is also equal to zero.

C) A standing wave does not appear to be stationary and net transfer of energy from one particle to the other is also non-zero.

D) A standing waves does not appear to be stationary, but net transfer of energy from one particle to the other is zero.

Key : B

Sol : Standing waves from when two waves of equal amplitude, same frequency, same wavelength travelling in opposite directions superimpose, as a result, the net transfer of energy thorough any cross-section is zero in standing waves.

10. The (x, y) co-ordinates of the corners of a square plate are $(0,0),(L,0),(L,L)$ and $(0,L)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s)for u is (are) (a=positive constant) **IIT -1998**

A) $a\cos(\pi x/2L)\cos(\pi y/2L)$ B) $a\sin(\pi x/L)\sin(\pi y/L)$

C) $a\sin(\pi x/L)\sin(2\pi y/L)$ D) $a\cos(2\pi x/L)\sin(\pi y/L)$

Key : B,C

Sol : The edges of the plate are clamped, so its displacements along the x and y axes will individually be zero at the edges.

Option (a):

$u(x, y)=0$ at $x=L, y=L$

$u(x, y)\neq 0$ at $x=0, y=0$

Option (b)

$u(x, y)=0$ at $x=0, y=0[\because \sin 0=0]$

$u(x, y)=0$ at $x=L, y=L[\because \sin\pi=0]$

Option (c)

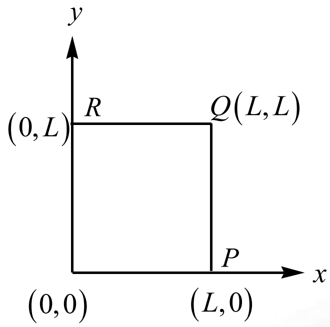
$u(x, y)=0$ at $x=0, y=0[\because \sin 0=0]$

$u(x, y)=0$ at $x=L, y=L[\because \sin\pi=0, \sin 2\pi=0]$

Option (d)

$u(x, y)=0$ at $y=0, y=0, y=L(\because \sin 0=0, \sin\pi=0)$

$u(x, y)\neq 0$ at $x=0, x=L[\because \cos 0=1, \cos 2\pi=1]$



10a. The (x,y) co-ordinates of the corners of a square plate are $(0,0),(L/2,0),(L/2,L/2)$ and $(0,L/2)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x,y)$ denotes the displacement of the plate at the point (x,y) at some instant of time, the possible expression(s)
for u is (are) (a=positive constant)

A) $a\cos(\pi x/L)\cos(\pi y/L)$
B) $a\sin(2\pi x/L)\sin(2\pi y/L)$
C) $a\sin(2\pi x/L)\sin(4\pi y/L)$
D) $a\cos(4\pi x/L)\sin(2\pi y/L)$

Key : B,C

Sol : The edges of the plate are clamped, so its displacements along the x and y axes will individually be zero at the edges.

Option (a):

$u(x,y)=0$ at $x=L/2, y=L/2$

$u(x,y)\neq 0$ at $x=0, y=0$

Option (b)

$u(x,y)=0$ at $x=0, y=0[\because \sin 0=0]$

$u(x,y)=0$ at $x=L/2, y=L/2[\because \sin\pi=0]$

Option (c)

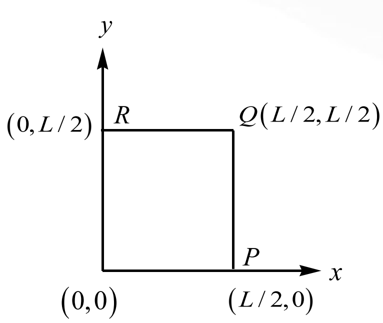
$u(x,y)=0$ at $x=0, y=0[\because \sin 0=0]$

$u(x,y)=0$ at $x=L/2, y=L/2[\because \sin\pi=0, \sin 2\pi=0]$

Option (d)

$u(x,y)=0$ at $y=0, y=0, y=L/2(\because \sin 0=0, \sin\pi=0)$

$u(x,y)\neq 0$ at $x=0, x=L/2[\because \cos 0=1, \cos 2\pi=1]$



11. A 3.6m long vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain height in the pipe. Find the height of water level (from the bottom of the pipe) at which resonance occurs. Neglect end correction. Now, the pipe is filled to a height $H(\approx 3.6m)$. A small hole is drilled very close to its bottom and water is allowed to leak. If the radii of the pipe and the hole are $2\times10^{-2}m$ and $1\times10^{-3}m$ respectively. Then the time interval between the occurrence of first two resonances. Speed of sound in air is 340 m/s and $g=10m/s^2$. **IIT -2000**

A) Expression for the rate of fall of water level in the pipe as a function of H $\left(\frac{-dH}{dt}\right)=\frac{a}{A}\sqrt{2gH}$

B) The time interval between the occurrence of first two resonances 43sec

C) the time interval between the occurrence of first two resonances 23 sec

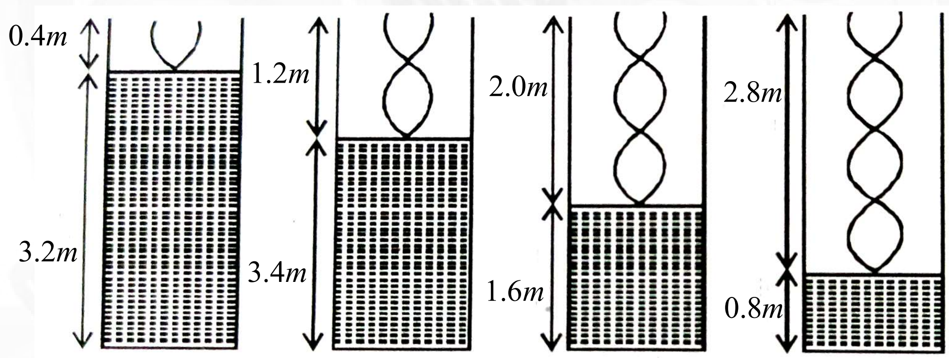
D) Length of air column corresponding to the fundamental frequency is 0.4m

Key : A,B,D

Sol : Speed of sound, $v=340m/s$.

Let ℓ_0 be the length of air column corresponding to the fundamental frequency. Then

$$\frac{v}{4\ell_0}=212.5\Rightarrow \ell_0=\frac{v}{4(212.5)}=\frac{340}{4(212.5)}=0.4m$$



In closed pipe only odd harmonics are obtained. Now, let $\ell_1,\ell_2,\ell_3,\ell_4$, etc. be the lengths corresponding to the 3rd harmonic, 5th harmonic etc. Then

$$3\left(\frac{v}{4\ell_1}\right)=212.5\Rightarrow \ell_1=1.2m;$$

$$5\left(\frac{v}{4\ell_2}\right)=212.5\Rightarrow \ell_2=2.0m$$

$$5\left(\frac{v}{4\ell_3}\right)=212.5\Rightarrow \ell_3=2.8m;$$

$$9\left(\frac{v}{4\ell_4}\right)=212.5\Rightarrow \ell_4=3.6m$$

Or heights of water level are $(3.6-0.4)m,(3.6-1.2)m,(3.6-2.0)m$ and $(3.6-2.8)m$.

Hence heights of water level are 3.2m, 2.4m, 1.6m, and 0.8m.

Let A and a be the area of cross-sections of the pipe and hole respectively. Then

$A = \pi\left(2\times10^{-2}\right) = 1.26\times10^{-3}m^{-2}$

And $a = \pi\left(10^{-3}\right)^2 = 3.14\times10^{-6}m^2$

Velocity of efflux, $v = \sqrt{2gH}$

Continuity equation at 1 and 2 gives.

$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$

So, rate of fall of water level in the pipe,

$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$

Substituting the values, we get

$\frac{-dH}{dt} = \frac{3.14\times10^{-6}}{1.26\times10^{-3}}\sqrt{2\times10\times H}$

$\Rightarrow \frac{-dH}{dt} = \left(1.11\times10^{-2}\right)\sqrt{H}$

Between first two resonance, the water level falls from 3.2 m to 2.4m.

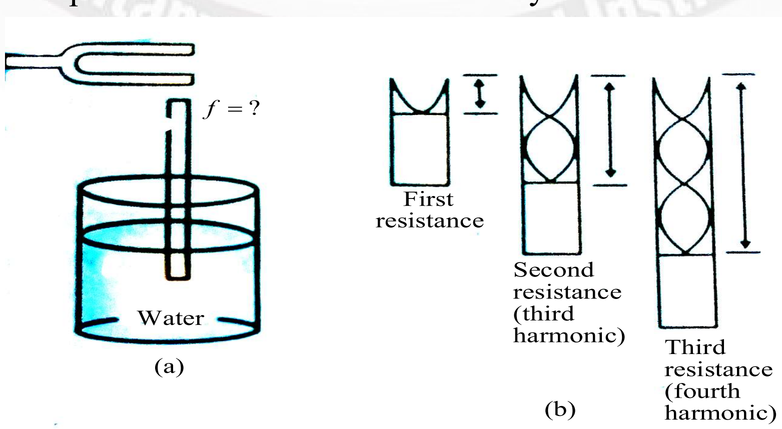
$\therefore\ \frac{dH}{\sqrt{H}} = 1.11\times10^{-2}dt$

$\Rightarrow \int_{3.2}^{2.4}\frac{dH}{\sqrt{H}} = \left(1.11\times10^{-2}\right)\int_0^t dt$

$\Rightarrow\ 2\left[\sqrt{2.4}-\sqrt{3.2}\right] = -\left(1.1\times10^{-2}\right).t$

Or $t \approx 43s$.

11a. A simple apparatus for demonstrating resonate in an air columns is depicted figure. A vertical pipe open at both ends is partially submerge in water, and a tuning fork vibration at an unknown frequency is placed near the top of the pipe. The length L of the air column can be adjusted by moving the pipe vertically. The sound waves generated by the fork are reinforced of the pipe. For a certain pipe, the smallest value of L for which a peak occurs in the sound intensity is 9.00 cm. Then



A) Frequency of the tuning fork is 953 Hz

B) Frequency of the tuning fork is 593 Hz

C) The value of L for the second resonance is 0.270m

D) The value of L for the third resonance is 0.450m

Key : A,C,D

Sol : $f_1 = \frac{v}{4L} = \frac{343m/s}{4(0.090m)} = 953Hz$

$\lambda = \frac{v}{f} = 0.360m$

$L = 3\lambda/4 = 0.270m$

$L = 5\lambda/4 = 0.450m$

12. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. End correction may be neglected. Let P_o denote the mean pressure at any point in the pipe, and ΔP_o the maximum amplitude of pressure variation. Then **IIT -1998**

A) The length L of the air column $\frac{15}{16}m$

B) Amplitude of pressure variation at the middle of the column is $\frac{\Delta P_0}{\sqrt{2}}$

C) Maximum and minimum pressure at the open end of the pipe is P_0

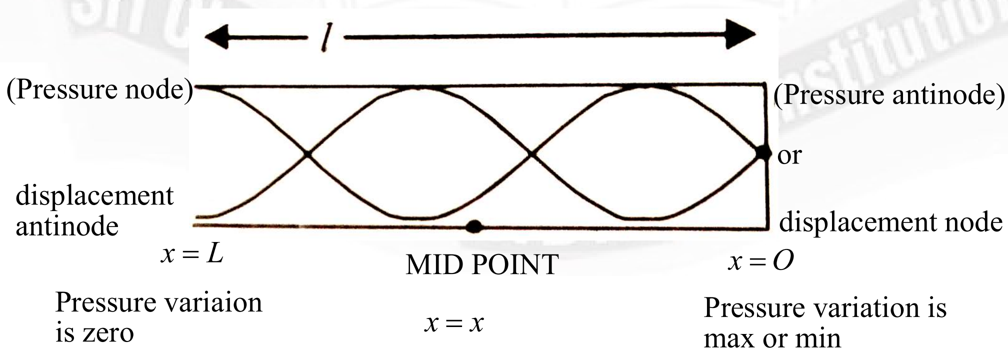
D) Maximum and minimum pressure at the closed end of the pipe is $P_0 + \Delta P_0$ and $P_0 - \Delta P_0$

Key : A,B,C,D

Sol : a) for second overtone of the closed pipe

Frequency, $f = P\left(\frac{v}{4L}\right)$ or $440 = 59\left(\frac{330}{4L}\right)$

$\Rightarrow L = \frac{5\times 330}{4\times 440} \Rightarrow L = \frac{15}{16}m$.



b) At any position x, the pressure

$\Delta P = \Delta P_0 \cos kx \cos \omega t$

Here amplitude $A = \Delta P_o \cos kx = \Delta P_o \cos \frac{2\pi}{\lambda} x$

For $x = \frac{L}{2} = \frac{15}{2 \times 16} = \frac{15}{32} m$ (mid point)

Amplitude $= \Delta P_0 \cos\left[\frac{2\pi}{330/440} \times \frac{15}{32}\right] = \frac{\Delta P_0}{\sqrt{2}}$

c) At open end of pipe, pressure is always same i.e., equal to mean pressure

$\because \Delta P = 0, P_{\max} = P_{\min} = P_0$

d) At the close end of pipe

maximum pressure, $P_{\max} = P_0 + \Delta P_0$

minimum Pressure, $P_{\min} = P_0 - \Delta P_0$

12a. A tube closed at one end has vibrating diaphragm at the other end, which may be assumed to be a displacement node. It is found that when the frequency of the diaphragm is 2000 Hz, a stationary wave pattern is set up in which the distance between adjacent nodes is 8cm. When the frequency is gradually reduced, the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Then

A) The speed of sound in air is 320m/s

B) The distance between adjacent nodes at a frequency of 1600Hz is 10 cm

C) The distance between the diaphragm and the closed end is 40 cm

D) The next lower frequencies at which stationary wave patterns will be obtained is 1200Hz

Key : A,B,C,D

Sol : Since the node-to-node distance is $\lambda/2$

$\lambda/2 = 0.08$ or $\lambda = 0.16m$

a) $c = n\lambda$

$\therefore\ c = 2000 \times 0.16 = 320 m/s$

b) $320 = 1600 \times \lambda$ or $\lambda = 0.2m$

$\therefore$ Distance between nodes=0.2/2=0.1m=10cm

c) Since there are nodes at the ends, the distance between the closed end and the membranes must be exact integrals of $\lambda/2$.

$\therefore\ I = n \times 0.16/2$ and $I = n' \times 0.2/2$

$$\Rightarrow \frac{n}{n'} = \frac{5}{4}$$

When $n = 5$, $n' = 4$

$I = 5 \times 0.16/2 = 0.4m = 40cm$

d) For the next lower frequency n=3

$\therefore\ 0.4 = 3\lambda/2$ or $\lambda = 0.8/3$

Since $c = n\lambda_1$ $n = \frac{320}{0.8/3} = 120 Hz$

13. A string of length 0.4m and mass $10^{-2}kg$ is tightly clamped as its ends. The tension the string is 1.6N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt which allows constructive interference between successive pulses is **IIT -1998**

A) 0.05s B) 0.10s C) 0.20s D) 0.40s

Key : B

Sol : b) For a string frequency $f = \frac{1}{2\ell}\sqrt{\frac{F}{\mu}}$

$\therefore$ Time period, $T = 2\ell\sqrt{\frac{\mu}{F}}$

$$F = 1.6N, \mu = \frac{mass}{length} = \frac{10^{-2}}{0.4} = 2.5\times10^{-2}$$

$$\therefore \ T = 2\times0.4\sqrt{\frac{2.5\times10^{-2}}{1.6}} = 0.1\sec$$

The time required for constructive interference equal to the time period of a wave pulse, hence $\Delta t = 0.1s$

13a. A string of length 0.4m and mass $10^{-2}kg$ is tightly clamped as its ends. The tension the string is 6.4N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt which allows constructive interference between successive pulses is **IIT -1998**

A) 0.05s B) 0.10s C) 0.20s D) 0.40s

Key : A

Sol : b) For a string frequency $f = \frac{1}{2\ell}\sqrt{\frac{F}{\mu}}$

$\therefore$ Time period, $T = 2\ell\sqrt{\frac{\mu}{F}}$

$$F = 6.4N, \mu = \frac{mass}{length} = \frac{10^{-2}}{0.4} = 2.5\times10^{-2}$$

$$\therefore \ T = 2\times0.4\sqrt{\frac{2.5\times10^{-2}}{6.4}} = 0.05\sec$$

The time required for constructive interference equal to the time period of a wave pulse, hence $\Delta t = 0.05s$

III) Doppler's effect, Beats & Echo

Doppler's effect in source and observer are moving in one dimension, Doppler's effect in source and observer are moving in two dimension, Effect of the source motion on the wavelength, Effect of wind speed, Time period and frequency-Beat frequency, Beat frequency for three sources

1. Two vehicles each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity W. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are): **Adv -2013**

A) If the wind blows from the observer to the source, $f_2 > f_1$

B) If the wind blows from the source to the observer, $f_2 > f_1$

C) If the wind blows from the observer to the source, $f_2 < f_1$

D) If the wind blows from the source to the observer, $f_2 < f_1$

Key: A,B

Sol: If wind blows from source to observer

$$f_2 = f_1\left(\frac{V+w+u}{V+w-u}\right)$$

When wind blows from observer towards source

$$f_2 = f_1\left(\frac{V-w+u}{V-w-u}\right)$$

In both cases, $f_2 > f_1$.

1a. Two vehicles each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity W. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. Which of the following statements are false.

(A) If the wind blows from the observer to the source, $f_2 > f_1$

(B) If the wind blows from the source to the observer, $f_2 > f_1$

(C) If the wind blows from the observer to the source, $f_2 < f_1$

(D) If the wind blows from the source to the observer, $f_2 < f_1$

Key: C,D

Sol: If wind blows from source to observer

$$f_2 = f_1\left(\frac{V+w+u}{V+w-u}\right)$$

When wind blows from observer towards source

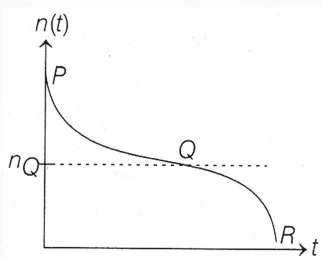
$$f_2 = f_1\left(\frac{V-w+u}{V-w-u}\right)$$

In both cases, $f_2 > f_1$.

2. Two loudspeakers M and N are located $20m$ apart and emit sound at frequencies $118Hz$ & $121Hz$, respectively. A car in initially at a point P, $1800m$ away from the mid-point Q of the line MN and moves towards Q constantly at $60km/h$ along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, $1800m$ away from Q. Let $v(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q & V_R be the beat frequencies measured at locations P, Q & R respectively. The speed of sound in air is $330ms^{-1}$. Which of the following statement(s) is (are) true regarding the sound heard by the person?

Adv-2016

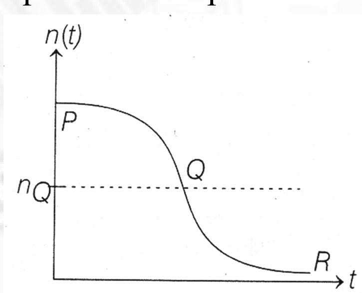
A) The plot below represents schematically the variation of beat frequency with time.



B) The rate of change in beat frequency in maximum when the car passes through Q.

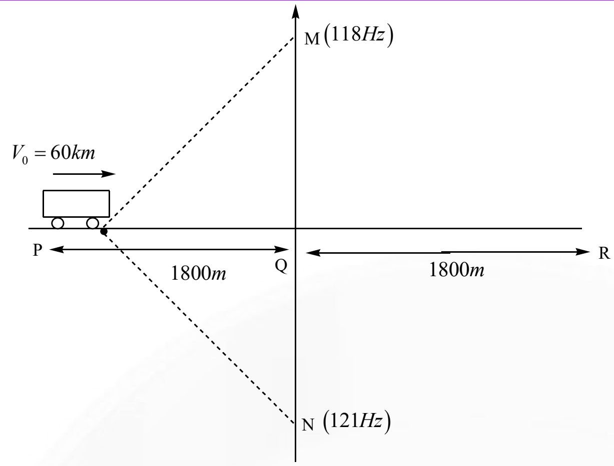
C) $v_P + v_R = 2v_Q$

D) The plot below represents schematically the variations of beat frequency with time.



Key: A,B,C

Sol: Frequency of M received by car



$$f_1 = 118\left(\frac{V + V_0\cos\theta}{V}\right)$$

$$f_2 = 121\left(\frac{V + V_0\cos\theta}{V}\right)$$

No. of beats $n = \Delta f = f_2 - f_1$

$$n = 3\left(\frac{V + V\cos\theta}{V}\right)$$

$$n = 3\left(1 + \frac{V_0}{V}\cos\theta\right)$$

As $\theta \uparrow$, $\cos\theta \downarrow, n \downarrow$

Rate of change of beat frequency

$$\frac{dn}{d\theta} = 3\left[\frac{V_0}{V}(-\sin\theta)\right]$$

$\frac{dn}{d\theta}$ is maximum when $\sin\theta = 1 \quad \theta = 90^\circ$

i.e. car is at point Q.

$$V_P = 3\left(1 + \frac{V_0}{V}\cos\theta\right)$$

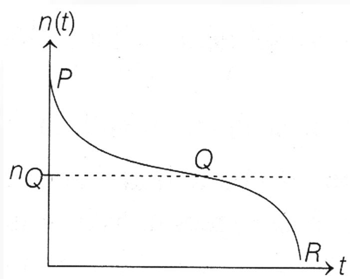
$$V_R = 3\left(1 - \frac{V_0}{V}\cos\theta\right)$$

At Q , no. of beats $V_Q = 121 - 118 = 3$

$$V_Q = \frac{V_P + V_R}{2}$$

2a. Two loudspeakers M and N are located $20m$ apart and emit sound at frequencies $118Hz$ & $121Hz$, respectively. A car in initially at a point P, $1800m$ away from the mid-point Q of the line MN and moves towards Q constantly at $60km/h$ along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, $1800m$ away from Q. Let $v(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q & V_R be the beat frequencies measured at locations P, Q & R respectively. The speed of sound in air is $330ms^{-1}$. Which of the following statement(s) is (are) false regarding the sound heard by the person?

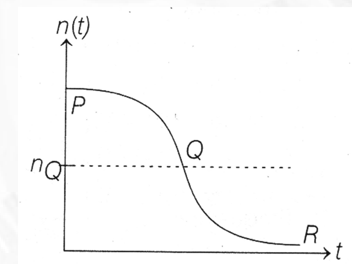
A) The plot below represents schematically the variation of beat frequency with time.



B) The rate of change in beat frequency in maximum when the car passes through Q.

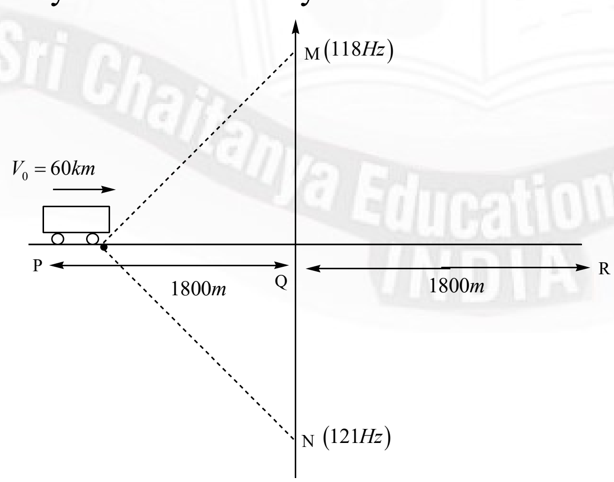
C) $v_P + v_R = 2v_Q$

D) The plot below represents schematically the variations of beat frequency with time.



Key: D

Sol: Frequency of M received by car



$$f_1 = 118\left(\frac{V + V_0\cos\theta}{V}\right)$$

$$f_2 = 121\left(\frac{V + V_0\cos\theta}{V}\right)$$

No. of beats $n = \Delta f = f_2 - f_1$

$$n = 3\left(\frac{V + V\cos\theta}{V}\right)$$

$$n = 3\left(1 + \frac{V_0}{V}\cos\theta\right)$$

As $\theta\uparrow$, $\cos\theta\downarrow, n\downarrow$

Rate of change of beat frequency

$$\frac{dn}{d\theta} = 3\left[\frac{V_0}{V}(-\sin\theta)\right]$$

$\frac{dn}{d\theta}$ is maximum when $\sin\theta = 1 \quad \theta = 90^\circ$

i.e. car is at point Q.

$$V_P = 3\left(1 + \frac{V_0}{V}\cos\theta\right)$$

$$V_R = 3\left(1 - \frac{V_0}{V}\cos\theta\right)$$

At Q , no. of beats $V_Q = 121 - 118 = 3$

$$V_Q = \frac{V_P + V_R}{2}$$

3. A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The frequency end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which the following combination(s) of u and f_s, will the sound reaching the pipe lead to a resonance? **Adv 2021**

A) $u = 0.8v$ & $f_s = f_0$
B) $u = 0.8v$ & $f_s = 2f_0$
C) $u = 0.8v$ & $f_s = 0.5f_0$
D) $u = 0.5v$ & $f_s = 1.5f_0$

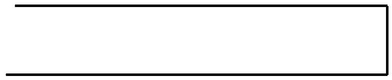
Key: A,D

Sol: $$f = f_s\left(\frac{V}{V-u}\right)$$

A) $f = f_0\left(\frac{V}{V - 0.8V}\right) = 5f_0$
B) $f = 2f_0\left(\frac{V}{V - 0.8V}\right) = 10f_0$

C) $f = 0.5f_0\left(\frac{V}{V - 0.8V}\right) = 2.5f_0$
D) $f = 1.5f_0\left(\frac{V}{V - 0.5V}\right) = 3f_0$

Close



$\therefore$ All odd harmonics are available in closed pipe.

3a. A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The frequency end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which the following combination(s) of u and f_s, will the sound reaching the pipe does not lead to a resonance?

A) $u = 0.8v$ & $f_s = f_0$ B) $u = 0.8v$ & $f_s = 2f_0$

C) $u = 0.8v$ & $f_s = 0.5f_0$ D) $u = 0.5v$ & $f_s = 1.5f_0$

Key: B,C

Sol: $f = f_s\left(\frac{V}{V-u}\right)$

(A) $f = f_0\left(\frac{V}{V-0.8V}\right) = 5f_0$

(B) $f = 2f_0\left(\frac{V}{V-0.8V}\right) = 10f_0$

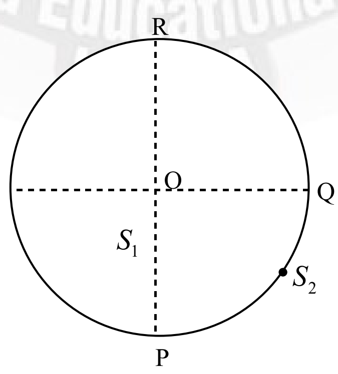
(C) $f = 0.5f_0\left(\frac{V}{V-0.8V}\right) = 2.5f_0$

(D) $f = 1.5f_0\left(\frac{V}{V-0.5V}\right) = 3f_0$

Close

$\therefore$ All odd harmonics are available in closed pipe. Hence correct answer is B, C.

4. s_1 & s_2 are source of sound of frequency $656Hz$. S_1 is moving along OP and S_2 is moving on circle is anticlockwise direction with constant speed of $4\sqrt{2}m/s$, P is a detector which is fixed. Select the correct options $\left(V_{sound} = 300m/s\right)$. **Adv 2023**



A) When S_2 is at Q, then frequency of it detected by detector is approximately $647HZ$.

B) When S_2 is at Q, then frequency detected by detector is $327HZ$.

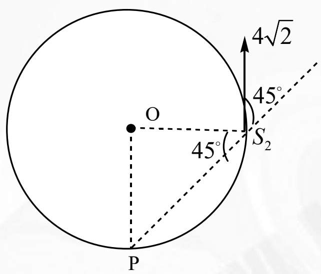
C) When S_2 is at R, and S_1 is coming toward P with velocity $4m/s$, then Beat frequency detected by detector is $8.86HZ$.

D) When S_2 is at R, and S_1 is coming toward P with velocity $4m/s$, then Beat frequency detected by detector is $2.56HZ$.

Key: A,C

Sol: $$f' = 656\left[\frac{300}{300+4\sqrt{2}\times\frac{1}{\sqrt{2}}}\right]$$

$$f' = \frac{656\times 300}{304} \Rightarrow f' = 647.36HZ$$



Option A is correct.

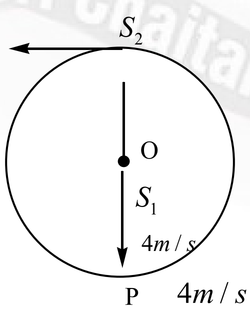
At detector sound from two source are coming.

From S_2 $$f' = 656\left[\frac{300}{300+4\sqrt{2}\cos 90}\right] \Rightarrow f' = 656$$

From S_1 $$f' = 656\left[\frac{300}{300-4}\right] \Rightarrow f' = 664.86$$

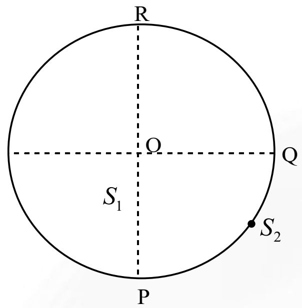
Beat frequency= $f'' - f'$

$$\Rightarrow 664.86 - 656 = 8.86HZ.$$



Option C is correct.

4a. s_1 & s_2 are source of sound of frequency $656 Hz$. S_1 is moving along OP and S_2 is moving on circle is anticlockwise direction with constant speed of $2\sqrt{2} m/s$, P is a detector which is fixed. Select the correct options $\left(V_{sound} = 300 m/s\right)$.



A) When S_2 is at Q, then frequency of it detected by detector is approximately $652 Hz$.

B) When S_2 is at Q, then frequency detected by detector is $327 Hz$

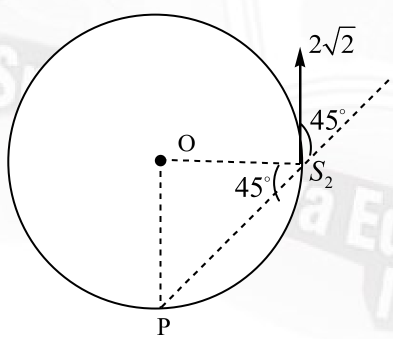
C) When S_2 is at R, and S_1 is coming toward P with velocity $2 m/s$, then Beat frequency
detected by detector is $4.4 Hz$.

D) When S_2 is at R, and S_1 is coming toward P with velocity $2 m/s$, then Beat frequency
detected by detector is $2.56 HZ$.

Key: A,C

Sol:
$$f' = 656\left[\frac{300}{300 + 2\sqrt{2} \times \frac{1}{\sqrt{2}}}\right]$$

$$f' = \frac{656 \times 300}{302} \Rightarrow f' = 651.65 HZ$$



Option A is correct.

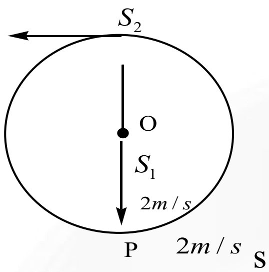
At detector sound from two source are coming.

From S_2 $\quad f' = 656\left[\frac{300}{300 + 2\sqrt{2}\cos 90}\right] \Rightarrow f' = 656$

From S_1 $f' = 656\left[\frac{300}{300-2}\right] \Rightarrow f' = 660.4$

Beat frequency= $f'' - f'$

$\Rightarrow 660.4 - 656 = 4.4 Hz.$



Option C is correct.

5. A band playing music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound obtain an expression for the beat frequency heard by the motorist. Then **IIT -1997**

A) Apparent frequency when heard both direct and reflected by the motorist

$f' = \left(\frac{v+v_m}{v+v_b}\right)f$

B) Apparent frequency when heard both direct and reflected sound by the motorist

$f'' = \left(\frac{v+v_m}{v-v_b}\right)f$

C) Beat frequency heard by motorist $f_b = \frac{2v_b(v+v_m)}{v^2-v_b^2}$

D) None of these

Key : A,B,C

Sol : The motorist hears the beat frequency as he receives two different frequencies one directly from the sound source or band f' and other reflected from the wall.

Apparent frequency when heard both direct and reflected by the motorist.

$f' = \left(\frac{v+v_m}{v+v_b}\right)f$

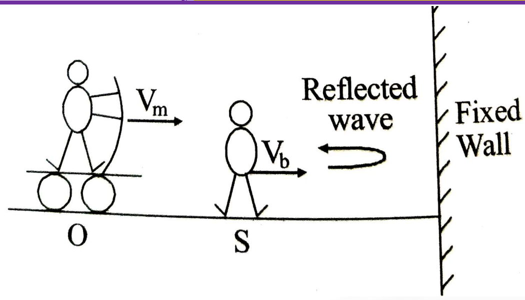
Again apparent frequency when heard both direct and reflected sound by the motorist.

$f'' = \left(\frac{v+v_m}{v-v_b}\right)f$

Hence beat frequency heard by motorist,

$f_b = f'' - f' = \left(\frac{v+v_m}{v+v_b}\right)f - \left(\frac{v+v_m}{v-v_b}\right)f$

Or, $f_b = \frac{2v_b(v+v_m)}{v^2-v_b^2}$



5a. A sound wave of frequency 'f' travels horizontally to the right with speed c. It is reflected from a broad wall moving to the left with speed v. The correct statements is

A) The number of wave striking the surface per second is $f\frac{(c+v)}{c}$

B) The wavelength of the reflected wave is $\frac{c(c-v)}{f(c+v)}$

C) The frequency of reflected wave is $\frac{f(c+v)}{(c-v)}$

D) The number of beats heard by a stationary observer listener to the left to the reflecting surface is $\frac{vf}{(c-v)}$

Key : A,B,C

Sol : Number of waves encountered by the moving plane per unit

Time $=\frac{\text{distance travelled}}{\text{wave length}}=\frac{c+v}{\lambda}=\frac{c}{\lambda}\left(1+\frac{v}{c}\right)=f\left(1+\frac{v}{c}\right)$

f' is the frequency of the incident wave and f'' is the frequency of the reflected wave that a stationary observer meets.

$$f''=\frac{f'}{(1-v/c)}=\frac{f(1+v/c)}{(1-v/c)}=\frac{f(c+v)}{(c-v)}$$

$$\lambda''=\frac{c}{f''}=\frac{c}{f}\left(\frac{c-v}{c+v}\right)$$

Beat frequency $=f''=f=\frac{f(1+v/c)}{(1-v/c)}-f=\frac{2vf}{(c-v)}$

6. A whistle emitting a sound of frequency 440Hz is tied to a string of 1.5m length and rotated with an angular velocity of 20 $rads^{-1}$ in the horizontal plane. Then **IIT -1996**

A) When the source is at the position A, then the frequency heard by the observer will be maximum 484 Hz

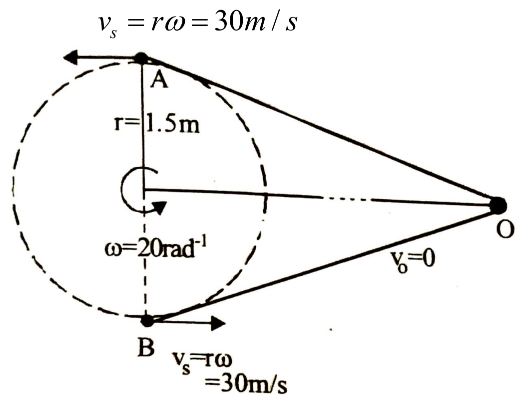
B) When the source is at the position B, then the frequency heard by the observer will be minimum 403 Hz

C) When the source is at the position B, then the frequency heard by the observer will be minimum 203 Hz

D) When the source is at the position A, then the frequency heard by the observer will be maximum 284 Hz

Key : A,B

Sol : The whistle which is emitting sound is being rotated in a circle. The observer is at large distance from the whistle i.e., O



Given : $r = 1.5m$ and $w = 20rads^{-1}$

Speed of source, $v_s = r\omega = 1.5 \times 20 = 30ms^{-1}$

When the source is at the position A, then the frequency heard by the observer will be maximum

$$v' = v\left[\frac{v}{v - v_s}\right] = 440\left[\frac{330}{330 - 30}\right] = 484Hz$$

When the source is at the position B, then the frequency heard by the observer will be minimum

$$v'' = v\left[\frac{v}{v + v_s}\right] = 440\left[\frac{330}{330 + 30}\right] = 403.3Hz$$

Hence the range of frequencies heard by the observer $= 403.3Hz$ to $484Hz$.

6a. A whistle emitting a sound of frequency 440Hz is tied to a string of 1.5m length and rotated with an angular velocity of 40 $rads^{-1}$ in the horizontal plane. Then

A) When the source is at the position A, then the frequency heard by the observer will be maximum 538 Hz

B) When the source is at the position B, then the frequency heard by the observer will be minimum 372 Hz

C) When the source is at the position B, then the frequency heard by the observer will be minimum 273 Hz

D) When the source is at the position A, then the frequency heard by the observer will be maximum 358 Hz

Key : A,B

Sol : The whistle which is emitting sound is being rotated in a circle. The observer is at large distance from the whistle i.e., O

Given : $r = 1.5m$ and $w = 20rads^{-1}$

Speed of source, $v_s = r\omega = 1.5 \times 40 = 60 ms^{-1}$

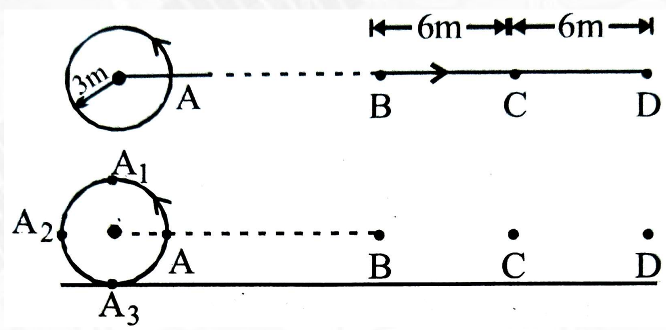
When the source is at the position A, then the frequency heard by the observer will be maximum

$$v' = v\left[\frac{v}{v - v_s}\right] = 440\left[\frac{330}{330-60}\right] = 538 Hz$$

When the source is at the position B, then the frequency heard by the observer will be minimum

$$v'' = v\left[\frac{v}{v + v_s}\right] = 440\left[\frac{330}{330+60}\right] = 372 Hz$$

7. A source of sound is moving along a circular orbit of radius 3 meters with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude BC=CD=6 meter. The frequency of oscillation of the detector is $\frac{5}{\pi}$ per second. The source is at the point A when the detector is at the point B. If the source emits continuous sound wave of frequency 340 Hz, then maximum and the minimum frequencies recorded by the detector. **IIT -1990**

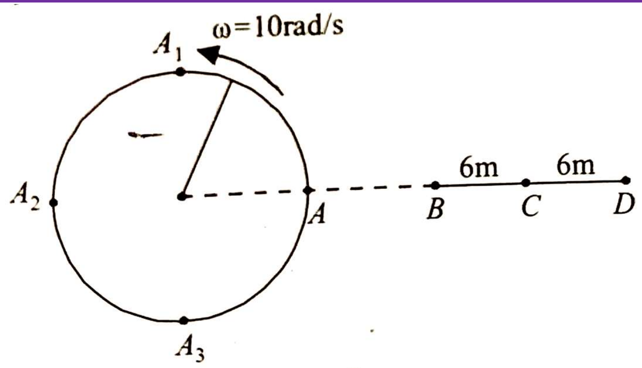


A) maximum frequencies recorded by the detector is 438.7 Hz
B) minimum frequencies recorded by the detector is 257.3 Hz
C) maximum frequencies recorded by the detector is 257 Hz
D) minimum frequencies recorded by the detector is 438 Hz

Key : A,B

Sol : The angular frequency of the detector, $\omega = 2\pi v = 2\pi \times \frac{5}{\pi} = 10 rad/s$

The angular frequency of the detector and the source of sound are equal.



$\Rightarrow$ when the detector is at C moving towards D, the source is at A_1 moving leftwards. It is in this situation that the frequency heard is minimum

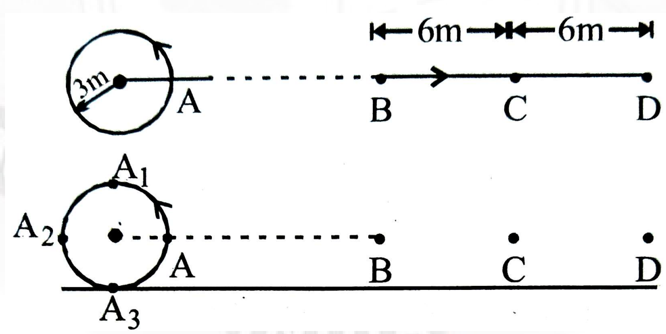
$$\nu_{min} = \nu\left[\frac{v-v_0}{v+v_s}\right] = 340\times\frac{(340-60)}{(340+30)} = 257.3Hz$$

$(\because\ v_0 = A\omega = 6\times10 = 60ms/,\ v_s = R\omega = 3\times10 = 30m/s)$

Again when the detector is at C moving towards B, the source is at A_3 moving rightward. It is in this situation that the frequency heard is maximum.

$$\nu_{max} = \nu\left[\frac{v+v_0}{v-v_s}\right] = 340\times\frac{(340+60)}{(340-30)} = 438.7Hz$$

7a. A source of sound is moving along a circular orbit of radius 3 meters with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude BC=CD=6 meter. The frequency of oscillation of the detector is $\frac{10}{\pi}$ per second. The source is at the point A when the detector is at the point B. If the source emits continuous sound wave of frequency 340 Hz, then maximum and the minimum frequencies recorded by the detector. **IIT JEE -1990**

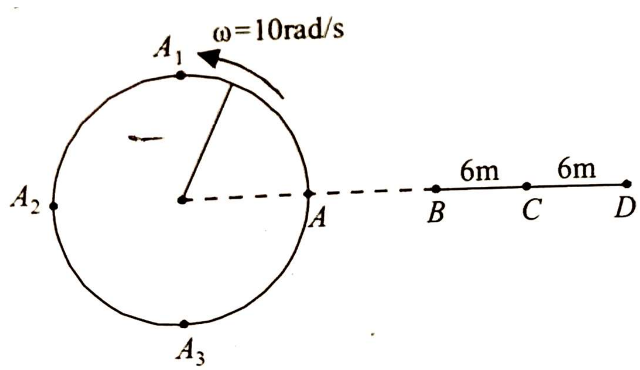


A) maximum frequencies recorded by the detector is 558 Hz
B) minimum frequencies recorded by the detector is 187 Hz
C) maximum frequencies recorded by the detector is 187 Hz
D) minimum frequencies recorded by the detector is 558 Hz

Key : A,B

Sol : The angular frequency of the detector, $\omega = 2\pi\nu = 2\pi\times\frac{10}{\pi} = 20rad/s$

The angular frequency of the detector and the source of sound are equal.



$\Rightarrow$ when the detector is at C moving towards D, the source is at A_1 moving leftwards. It is in this situation that the frequency heard is minimum

$$v_{\min} = v\left[\frac{v - v_0}{v + v_s}\right] = 340 \times \frac{(340 - 120)}{(340 + 60)} = 187Hz$$

($\because$ $v_0 = A\omega = 6 \times 20 = 120m/s$, $v_s = R\omega = 3 \times 20 = 60m/s$)

Again when the detector is at C moving towards B, the source is at A_3 moving rightward. It is in this situation that the frequency heard is maximum.

$$v_{\max} = v\left[\frac{v + v_0}{v - v_s}\right] = 340 \times \frac{(340 + 120)}{(340 - 60)} = 558Hz$$

Exercise : IV

(Matrix Matching/Paragraph Type Questions Including PYQ's)

1. Answer the following by appropriately matching the lists based on the information given in the paragraph. A musical instrument is made using four different metal strings, $1,2,3\ \&\ 4$ with mass per unit length $\mu, 2\mu, 3\mu\ \&\ 4\mu$ respectively. The instrument is played by vibrating the strings by varying the free length in between the range $L_0\ \&\ 2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. List-I gives the above four strings while list-II lists the magnitude of some quantity. **Adv 2019**

List-I	List-II
(I) String- 1 (μ)	(P) 1
(II) String- 2 (2μ)	(Q) $1/2$
(III) String- 3 (3μ)	(R) $1/\sqrt{2}$
(IV) String- 4 (4μ)	(S) $1/\sqrt{3}$
	(T) $3/16$
	(U) $1/16$

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

A) $I \to P, II \to R, III \to S, IV \to Q$

B) $I \to P, II \to Q, III \to T, IV \to S$

C) $I \to Q, II \to S, III \to R, IV \to P$

D) $I \to Q, II \to P, III \to R, IV \to T$

Key: A

Sol: **Case I:** $f_0 = \dfrac{I}{2L_0}\sqrt{\dfrac{T_0}{\mu}}$; **Case II:** $f_1 = \dfrac{I}{2L_0}\sqrt{\dfrac{T_0}{2\mu}} = \dfrac{f_0}{\sqrt{2}}$

Case III: $f_2 = \dfrac{I}{2L_0}\sqrt{\dfrac{T_0}{3\mu}} = \dfrac{f_0}{\sqrt{3}}$; **Case IV:** $f_4 = \dfrac{I}{2L_0}\sqrt{\dfrac{T_0}{4\mu}} = \dfrac{f_0}{2}$

1a. Answer the following by appropriately matching the lists based on the information given in the paragraph.A musical instrument is made using four different metal strings, $1,2,3\ \&\ 4$ with mass per unit length $\mu, 4\mu, 6\mu\ \&\ 8\mu$ respectively. The instrument is played by vibrating the strings by varying the free length in between the range $L_0\ \&\ 2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	List-II
(I) String- 1 (μ)	(P) 1
(II) String- 2 (4μ)	(Q) $1/2$
(III) String- 3 (6μ)	(R) $1/\sqrt{8}$
(IV) String- 4 (8μ)	(S) $1/\sqrt{6}$
	(T) $3/16$
	(U) $1/16$

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

A) $I \to P, II \to R, III \to S, IV \to Q$

B) $I \to P, II \to Q, III \to S, IV \to R$

C) $I \to Q, II \to S, III \to R, IV \to P$

D) $I \to Q, II \to P, III \to R, IV \to T$

Key: B

Sol: **Case I:** $f_0 = \frac{I}{2L_0}\sqrt{\frac{T_0}{\mu}}$; **Case II:** $f_1 = \frac{I}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

Case III: $f_2 = \frac{I}{2L_0}\sqrt{\frac{T_0}{6\mu}} = \frac{f_0}{\sqrt{6}}$; **Case IV:** $f_4 = \frac{I}{2L_0}\sqrt{\frac{T_0}{8\mu}} = \frac{f_0}{\sqrt{8}}$

2. Answer the following by appropriately matching the lists based on the information given in the paragraph. A musical instrument is made using four different metal strings, $1, 2, 3$ & 4 with mass per unit length $\mu, 2\mu, 3\mu$ & 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 & $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. **Adv 2019**

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	List-II
(I) String- 1	(P) 1
(II) String- 2	(Q) $1/2$
(III) String- 3	(R) $1/\sqrt{2}$
(IV) String- 4	(S) $1/\sqrt{3}$
	(T) $3/16$
	(U) $1/16$

The length of the string $1,2,3\,\&\,4$ are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}\,\&\,\frac{7L_0}{4}$, respectively. Strings $1,2,3\,\&\,4$ are vibrated at their $1^{st}, 3^{rd}, 5^{th}\,\&\,14^{th}$ harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be.

A) $I \to P, II \to Q, III \to T, IV \to U$
B) $I \to T, II \to Q, III \to R, IV \to U$
C) $I \to P, II \to Q, III \to R, IV \to T$
D) $I \to P, II \to R, III \to T, IV \to U$

Key: A

Sol: **Case I:** $f_0 = \frac{I}{2L_0}\sqrt{\frac{T_0}{\mu}}$

For string-2

Case II: $\frac{3}{2\times\frac{3L_0}{2}}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}; \quad T_2 = \frac{T_0}{2}$

For string-3

Case III: $\frac{5}{2\times\frac{5L_0}{4}}\sqrt{\frac{T_2}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}; \quad T_3 = \frac{3T_0}{16}$

For string-4

Case IV: $\frac{14}{2\times\frac{7L_0}{4}}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}; \quad T_4 = \frac{T_0}{16}$

2a. Answer the following by appropriately matching the lists based on the information given in the paragraph. A musical instrument is made using four different metal strings, $1,2,3\,\&\,4$ with mass per unit length $\mu, 2\mu, 3\mu\,\&\,4\mu$ respectively. The instrument is played by vibrating the strings by varying the free length in between the range $L_0\,\&\,2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the harmonics.

List-I	**List-II**
(I) String- 1	(P) 1^{st}
(II) String- 2	(Q) 3^{rd}
(III) String- 3	(R) 5^{th}
(IV) String- 4	(S) 7^{th}
	(T) 12^{th}
	(U) 14^{th}

The length of the string 1,2,3 & 4 are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ & $\frac{7L_0}{4}$, respectively.

Strings 1,2,3 & 4 are vibrated at their some harmonics, under the tension $T_0, \frac{T_0}{2}, \frac{3T_0}{16}, \frac{T_0}{16}$ respectively such that all the strings have same frequency. The correct match for the harmonics in the four strings will be.

A) $I \to P, II \to Q, III \to T, IV \to U$

B) $I \to T, II \to Q, III \to R, IV \to U$

C) $I \to P, II \to Q, III \to R, IV \to U$

D) $I \to P, II \to R, III \to T, IV \to U$

Key: C

Sol: **Case I:** $f_0 = \frac{I}{2L_0}\sqrt{\frac{T_0}{\mu}}$

For string-2

Case II: $\frac{3}{2\times\frac{3L_0}{2}}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}; \quad T_2 = \frac{T_0}{2}$

For string-3

Case III: $\frac{5}{2\times\frac{5L_0}{4}}\sqrt{\frac{T_2}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}; \quad T_3 = \frac{3T_0}{16}$

For string-4

Case IV: $\frac{14}{2\times\frac{7L_0}{4}}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}; \quad T_4 = \frac{T_0}{16}$

3. **A loudspeaker diaphragm 0.2m in diameter is vibrating at 1 kHz with an amplitude of $0.01\times10^{-3}m$ assume that the air molecules in the vicinity have the same amplitude of vibration. Density of air is 1.29 kg/m^3. Then match the item given column I to that column II. Take velocity of sound $=340 m/s$.**

Column-I	Column-II
i. Pressure amplitude immediately front of the diaphragm (in W/m^2)	a. 2.7×10^{-2}
ii. Sound intensity in front of the diaphragm (in W/m^2)	b. 2.15×10^{-5}
iii. The acoustic power radiated (in W)	c. 27.55
iv. Intensity at 10 m from the loud speaker (in W/m^2)	d. 0.865

Key : $i-c, ii-d, iii-a, iv-b$

Sol : Pressure amplitude is given by

$P_0 = p\omega\, vA_0 = 1.29\times 2\pi \times 10^3 \times 340 \times \left(0.01\times 10^{-3}\right) = 27.55 N/m^2$

Intensity is given by

$I = \frac{1}{2} p\omega^2 A^2 v = \frac{1}{2}\times 1.29 \times \left(2\pi\times 10^3\right)\times\left(10^{-5}\right)^2 (340) = 0.865 W/m^2$

Power, $P = IA = (0.865)\pi(0.1)^2 = 0.027W = 2.7\times 10^{-2} W$

Intensity at $r = 10m$ is $I = \frac{P_{av}}{A} = \frac{2.7\times 10^{-2}}{4\pi \times 10^2} = 2.15\times 10^{-5} W/m^2$

4. **Match the following columns**

Column-I	Column-II
A) Fundamental frequency of a closed organ pipe is 100Hz	p) Frequency of second harmonic is 300Hz
B) The fundamental frequency of a vibrating string with fixed ends is 150 Hz	q) Frequency of first overtone is 300 Hz
C) The fundamental frequency of an open organ pipe is 150 Hz	r) Frequency of second overtone is 500 Hz
	s) Frequency of third harmonic is 450 Hz
	t) Frequency of third overtone is 600 Hz

Key : $A \to q, r; B \to p, q, s, t; C \to p, q, s, t$

Sol : Since frequency for closed organ pipe $= \frac{(2n+1)}{4L}V$

For fundamental $= \frac{V}{4L} = 100Hz$

Next frequency $= (2n+1)\times 100Hz$

For $n = 1$ frequency =300 Hz

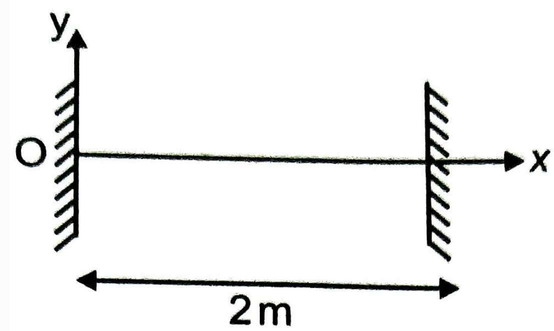
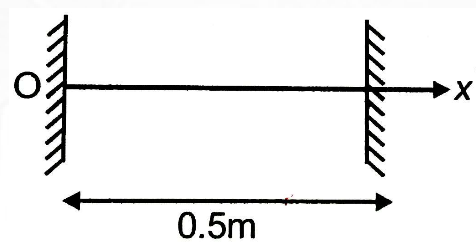
Frequency of a vibrating string $= \frac{n}{2L}V$

Fundamental frequency $= \frac{V}{2L} = 150Hz$

$\therefore$ frequency $= n\times 150Hz$

Similarly, frequency for open pipe $= 150\times nHz$

5. **Column-II lists some equations and arrangements of string. They are either corresponding to stationary wave or progressive wave. Match the entries of column-I with all possible entries in column-II.**

A) Wave length is 1m	p) A string of length 2m vibrating in third overtone (speed =30 m/s)
B) Frequency of wave is 25 Hz	
C) Frequency of wave is 25 Hz	q) $y = A\sin(50\pi t - 2\pi x)$
D) Velocity of transverse wave 25 m/s	r) $y = 2A\cos(2\pi x)\sin 100\pi t$
	s) $y = A\sin(50\pi t - \pi x) + A\sin(50\pi t + \pi x)$
	t) A string fixed at both ends is under tension of 100N. Mass of the string is 0.08kg. [Vibrating in fundamental mode] 

Key : $A \to p,q,r,t; B \to q,s,t; C \to q,t; D \to p,r,s,t$

Sol : A) $\frac{n\lambda}{2} = L$ or, $\lambda = \frac{2\pi}{k}$

$\Rightarrow \lambda = \frac{2L}{n}$

B) $f = \frac{\omega}{2\pi}$

C) Velocity of transformer wave $= \frac{\omega}{k}$ or $\sqrt{\frac{T}{\mu}}$

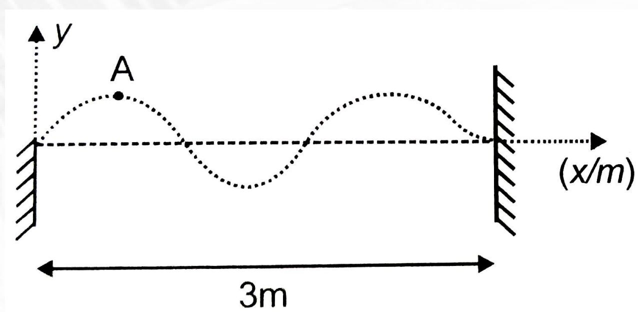
D) Standing waver produce due to same amplitude and frequency.

6. **Figure shows photograph of a string vibrating in certain mode. Length of the string is 3m. At the moment when photograph is taken particle at A was travelling upwards.**

Column-I	**Column-II**
A) Speed of the particle at $x=1/4m$ is equal to speed of particle at	p) $x=3/4m$
B) Velocity of the particle at $x=1/4m$ is parallel to velocity of particle at	q) $x=5/4m$
C) Acceleration of the particle at $x=1/4m$ is parallel to velocity of particle at	r) $x=7/4m$
D) Acceleration of the particle at $x=1/4m$ is equal to acceleration of the particle at	s) $x=9/4m$
	t) $x=1/3m$

Key : $A \to p,q,r,s; B \to p,s,t; C \to q,r; D \to p,s$

Sol : Speed of particle is equal to for energy $\frac{\lambda}{4}$ and velocity and acceleration equal to at every point which is at $\frac{\lambda}{2}$.



Paragraph Type Questions

Passage -1.

Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_1 - \omega_2 = 10^3 Hz$. A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq 2A^2$. **IIT JEE -1993**

I) Find the time interval between successive maxima of the intensity of the signal received by the detector.

A) $2\pi \times 10^{-3} s$ B) $\pi \times 10^{-3} s$ C) $10^{-3} s$ D) None of these

Key : A

II) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

A) $2\pi \times 10^{-3} s$ B) $\frac{\pi}{2}\times 10^{-3}$ C) $10^{-3} s$ D) None of these

Key : B

Sol :I&II

Let the two radio waves be represented by the equations

$y_1 = A\sin 2\pi v_1 t$

$y_2 = A\sin 2\pi v_2 t$

Since, $A_1 = A_2 = A$ and detector is at x=0

The equation of resultant waves according to superposition principle

$y = y_1 + y_2 = A\sin 2\pi v_1 t + A\sin 2\pi v_2 t$

$= A[\sin 2\pi v_1 t + \sin 2\pi v_2 t]$

$= A\times 2\sin\frac{(2\pi v_1 + 2\pi v_2)t}{2}\cos\frac{(2\pi v_1 + 2\pi v_2)t}{2}$

$= 2A\sin\pi(v_1+v_2)t\cos\pi(v_1-v_2)t$

Where the amplitude $A' = 2A\cos\pi(v_1-v_2)t$

Now intensity α $(\text{amplitude})^2$

$\Rightarrow I\alpha A^2$

$1\alpha 4A^2\cos^2\pi(v_1-v_2)t$

Intensity will be maximum when

$\cos^2\pi(v_1-v_2)t = 1$

Or $\cos\pi(v_1-v_2)t = \pm 1$

Or, $\pi(v_1-v_2) = n\pi$

$\Rightarrow \frac{(\omega_1-\omega_2)}{2}t = n\pi$ or $t = \frac{2n\pi}{\omega_1-\omega_2}$

$\therefore$ time interval between two successive maxima

Or, $\therefore \frac{2n\pi}{\omega_1-\omega_2} - \frac{2(n-1)\pi}{\omega_1-\omega_2}$ or $\frac{2\pi}{\omega_1-\omega_2} = \frac{2\pi}{10^3}s$

Time interval between two successive maximas is $2\pi\times 10^{-3}$ sec

ii) The detector can detect if resultant intensity $\geq 2A^2$

$\therefore$ Resultant amplitude $\geq\sqrt{2}A$

Or, $2A\cos\pi(v_1-v_2)t \geq \sqrt{2}A$

Or, $\cos\pi(v_1-v_2)t \geq \frac{1}{\sqrt{2}}$ or, $\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right] \geq \frac{1}{\sqrt{2}}$

The detector lies idle when the values of $\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right]$ is

Between 0 and $\frac{1}{\sqrt{2}}$

$t_1 = \frac{\pi}{\omega_1-\omega_2}$ and $t_2 = \frac{\pi}{2(\omega_1-\omega_2)}$

$\therefore\ t_1 = \frac{\pi}{\omega_1-\omega_2} - \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2(\omega_1-\omega_2)}$

$= \frac{\pi}{\omega_1-\omega_2} - \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2}\times 10^{-3}s$

Passage -2

The displacement of the medium in a second wave is given by the equation $y_1 = A\cos(ax+bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x=0$. The intensity of the reflected wave is 0.64 time that of the incident wave. **IIT JEE -1991**

I) What are the wavelength and frequency of incident wave ?

A) $\frac{2\pi}{a}, \frac{b}{2\pi}$ B) $\frac{b}{2\pi}, \frac{2\pi}{a}$ C) $0, 2\pi$ D) $\frac{1}{2\pi}, 0$

Key : A

II) Write the equation for the reflected wave.

A) $-0.8\cos(ax-bt)$ B) $-0.8A\cos(ax-bt)$

C) $A\cos(ax-bt)$ D) $0.8A\cos(ax+bt)$

Key : B

III) In the resultant wave formed after reflection, find the maximum and minimum values of the particles speeds in the medium.

A) $1.8Ab, 0$ B) $0, 0.9Ab$ C) $0, 0$ D) $3.6Ab, 1.6Ab$

Key : A

Sol : I,II &III

I) Compare the given equation $y_1 = A\cos(ax+bt)$ with the standard equation of a plane progressive wave.

$y = A\cos\left(\frac{2\pi}{\lambda} + 2\pi vt\right) \Rightarrow \frac{2\pi}{\lambda} = a \Rightarrow \lambda = \frac{2\pi}{a}$

Also, $2\pi v = b$

$\therefore$ Frequency of incident wave, $v = \frac{b}{2\pi}$

II) The wave is reflected by an obstacle, it will suffer a phase difference of π. The intensity of the reflected wave is 0.64 times of the incident wave.

Intensity of original wave $I \alpha A^2$

Intensity of reflected wave $I' = 0.64I$

$\Rightarrow I' \alpha A^2 \Rightarrow 0.64 I \alpha A^2 \Rightarrow 0.64A^2 \alpha A'^2 \Rightarrow A' \alpha 0.8A$

So the equation of reflected wave

$y_r = 0.8A\cos(ax+bt+\pi) = -0.8A\cos(ax-bt)$

III) the resultant wave equation

$y = y_i + y_r$

$= A\cos(ax+bt) + \left[-0.8Ab\sin(ax-bt)\right]$

Particle velocity

$v = \frac{dy}{dt} = Ab\sin(ax+bt) - 0.8Ab\sin(ax-bt)$

$= -Ab\left[\sin(ax+bt) + 0/8\sin(ax-bt)\right]$

$= -Ab[\sin ax\cos bt + \cos ax\sin bt + 0.8\sin ax\cos bt - 0.8\cos ax\sin bt]$

$v = -ab\left[1.8\sin ax\cos bt + 0.2\cos ax\sin bt\right]$

The maximum velocity will occur when sin $ax = 1$ and $\cos bt = 1$ under these condition $\cos ax = 0$ and $\sin bt = 0$

$\therefore |v_{\max}| = 1.8Ab$

Also, $|v_{\min}| = 0$

Passage :3

A string 25cm long and having a mass of 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency 8 beats for second heard. It is observed that decreasing the tension in string decreases the beat frequency. The speed of sound in air is $320ms^{-1}$.

1. The frequency of the fundament mode of the closed pipe is

A) 100 Hz B) 200 Hz C) 300 Hz D) 400 Hz

Key : B

Sol : Fundamental frequency of closed pipe

$n_c = \frac{v}{4l} = \frac{320}{4\times 0.4} = 200Hz$

2. The frequency of the string vibrating in its first overtone is

A) 92Hz B) 108 Hz C) 192 Hz D) 208 Hz

Key : D

Sol: $\Delta n = 8$ frequency string in 1st overtone

Is $n_s = n_c \pm 8 = 200 \pm 8$, $n_s = 208Hz$ or $192Hz$ given that Δn decreases if tension decreases $\Rightarrow$ $n_s > n_c$. Hence $n_s = 200 + 8 = 208Hz$

Passage -4

A sinusoidal wave is propagating in negative x-direction in a string stretched along x-axis. A particle of string at x=2cm is found at its mean position and it is moving in positive y-direction at t=1s. The amplitude of the wave, the wavelength and the angular frequency of the wave are 0.1 m, $\pi/4$ m and 4π rad/s, respectively.

1. The equation of the wave is

1) $y = 0.1\sin\left(4\pi(t-1)+8(x-2)\right)$ 2) $y = 0.1\sin\left((t-1)-x(x-2)\right)$

3) $y = 0.1\sin\left(4\pi(t-1)-8(x-2)\right)$ 4) None of these

Key : 1

Sol : The equation of wave moving in negative x-direction, assuming origin of position at x=2 and origin of time (i.e., initial time) at t=1s.

$y = 0.1\sin(4\pi t + 8x)$

Shifting the origin of position to left by 2m, to x=0, Also shifting the origin of time backwards by 1s, that is to t=0s.

$y = 0.1\sin\left[4\pi(t-1)+8x(x-2)\right]$

2. The speed of particle at $x = 2$ m and $t = 1$ s is

1) $0.2\pi m/s$ 2) $0.6\pi m/s$ 3) $0.4\pi m/s$ 4) 0

Key : 3

Sol : As given the particle at x=2 is at mean position at t=1s.

$\therefore$ Its velocity $v = \omega A = 4\pi \times 10.1 = 0.4\pi m/s$

3. The instantaneous power transfer through $x = 2$ m and $t = 1.125$ s is

1) $10 J/s$ 2) $4\pi/3 J/s$ 3) $2\pi/3 J/s$ 4) 0

Key : 4

Sol : Time period of oscillation $T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2}s$

Hence at $t = 1.125s$, that is , at T/4 seconds after $t = 1s$, the particle is at extreme position. Hence, instantaneous power at $x = 2$ at $t = 1.125$ s is zero.

Passage -5

A source of sonic oscillation with frequency $n_0 = 600Hz$ moves away and at right angles to a wall with velocity $u = 30m/s$. A stationary receiver is located on the line of source in succession wall$\rightarrow$ source$\rightarrow$receiver. If velocity of sound propagation is $v = 330m/s$, then

1. The beat frequency recorded by the receiver is

1) 110 Hz 2) 210 Hz 3) 150 Hz 4) 220 Hz

Key : 1

2. The wavelength of direct waves received by the receiver is

1) 50 cm 2) 100 cm 3) 150 cm 4) 90 cm

Key : 1

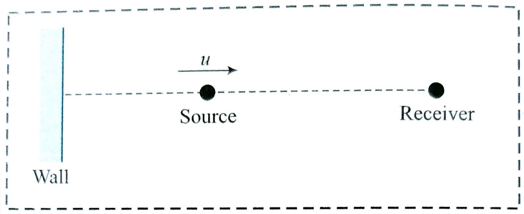
3. The wavelength of reflected waves received by the receiver is

1) 120 cm 2) 50 cm 3) 90 cm 4) 60 cm

Key : 4

Sol : 1-4 :

Since source moves away from the wall, it means that its velocity is towards the receiver as shown in the figure. Hence, frequency of direct sound received by it is greater than natural frequency of the source.



Frequency of direct waves, $n_r = n_0\left(\frac{v}{v-u}\right) = 600Hz$

Frequency of reflected sound is equal to frequency received by the wall. Since source is moving away from the wall, therefore, frequency received by the wall is less than natural frequency of the source which equal to

$n_0\left(\frac{v}{v+u}\right)$

Therefore, the frequency of reflected sound is

$n_r = n_0 = 110Hz$

$\frac{v}{n_d} = 0.5m = 50cm$

And wavelength of reflected waves is

$\frac{v}{n_r} = 0.6m = 60cm$

Passage :6

If two sound waves, $y_1 = 0.3\sin 596\pi\left(t - \frac{x}{330}\right)$ and $y_2 = 0.5\sin 604\pi\left(t - \frac{x}{330}\right)$ are super imposed.

1. What will be the ratio of frequency of resultant wave to the frequency of amplitude

1) 300 2) 200 3) 150 4) 2

Key : 3

Sol : Frequency of resultant wave $n' = \frac{n_1 + n_2}{2}$

$= \frac{298+302}{2} = 300Hz$

Frequency of amplitude $n^{11} = \frac{n_2 - n_1}{2}$

$n^{11} = \frac{302-298}{2} = 2Hz$

$\frac{n^1}{n^{11}} = \frac{300}{2} = 150$

2. Find ratio of maximum to minimum intensities of beats

1) 16 2) 64 3) 4 4) 32

Key : 1

Sol: $\frac{I_{max}}{I_{max}}=\frac{(A_1+A_2)^2}{(A_1-A_2)^2}=\frac{(0.3+0.5)^2}{(0.5-0.3)^2}=\frac{64}{4}=16$

Passage :7

Earthquakes generate sound waves inside the earth. In case of the Earth, both the transverse(S) and longitudinal (P) waves can propagate. Typically, the speed of S waves is about $4.5kms^{-1}$ and that of P waves is $8.0kms^{-1}$. A seismograph records both P and S waves from an earthquake. This difference helps us to find the distance of the point of origin of the earthquake. This point is called the epicentre.

1. If at the location of a seismograph the P waves arrive 2 minute earlier, the distance of the epicentre from the location f the seismograph is

1) $3541.2km$ 2) $1234.3km$ 3) 2468.6 km 4) $3702.9km$

Key : 2

Sol : Let distance of epicentre is d then

$$2\times 60=\frac{d}{4.5}-\frac{d}{8}=\frac{3.5}{8\times 4.5}\times d$$

$$\Rightarrow d=\frac{36\times 2\times 60}{3.5}=\frac{4320}{3.5}=1234.3km$$

2. The reading of the time lag between the arrival of S and P waves gives us the distance of the epicentre from the location of a seismograph. The readings of what minimum number of seismographs would be necessary to pinpoint the location of an epicentre ?

1) 1 2) 2 3) 3 4) 4

Key : 3

Sol : To find the location of an epicentre, a triangle and its circumcentre should be found out so that exact location of epicentre is known.

3. If only 2 seismograph readings are available, how many probable locations of a epicentre could be detected ?

1) 1 2) 2 3) 3 4) 4

Key : 2

Sol : There will be 2 available positions of epicentre, as arc drawn from two different locations of seismograph can intersect at two points.

Passage-8.

Waves $y_1=A\cos(0.5\pi x-100\pi t)$ and $y_2=A\cos(0.46\pi x-92\pi t)$ are travelling along the x-axis. (Here x is meters and t is in seconds) **IIT 2006**

1. Find the number of times intensity is maximum in time interval of 1s.

A) 4 B) 6 C) 8 D) 10

Key : A

Sol : In 1s number of maxima is called the beat frequency. Hence

$f_b=f_1-f_2$

$$= \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4Hz$$

2. The wave velocity of louder sound is

A) 100m/s B) 192m/s C) 200m/s D) 96m/s

Key : C

Sol : Speed of wave

$$v = \frac{\omega}{k}$$

$$v = \frac{100\pi}{0.5\pi} \text{ or } \frac{92\pi}{0.46\pi} = 200m/s$$

3. The number of time $y_1 + y_2 = 0$ at $x = 0$ in 1s is

A) 100 B) 46 C) 192 D) 96

Key : A

Sol : At $x = 0$,

$$y = y_1 + y_2$$

$$= 2A\cos 96\pi t \cos 4\pi t$$

Frequency of $\cos(96\pi t)$ function is 48 Hz and that of $\cos(4\pi t)$ function is 2 Hz.

In 1s $\cos$ function becomes zero at 2f times, where f is the frequency. Therefore, the first function will become zero at 96 times and the second at 4 times. But the second will not overlap with the first. Hence, net y will become zero 100 times in 1s.

Passage-8a.

Two waves of $y_1 = A\cos(0.6\pi x - 102\pi t)$, $y_2 = A\cos[0.553\pi x - 94\pi t]$ are travelling along the x-axis. x is in meters and t in seconds.

1. Find the number of times intensity is maximum in time interval 2 sec

A) 20 B) 4 C) 18 D) 16

Key : C

Sol : $\omega_1 = 2\pi n_1 = 102\pi$

$n_1 = 56$ $\omega_2 = 2\pi n_2 = 94\pi t$

$$n_2 = \frac{94}{2} = 47$$

For 1 second number of beats $\Delta n = n_1 \sim n_2 = 47 \sim 56 = 9$

For 2 seconds $9 \times 2 = 18$

2. The ratio of wave velocity of more louder sound to less louder sound will be

A) 1:1 B) 2:1 C) 1:2 D) 1:3

Key : A

Sol : $v_1 = \frac{\omega}{k_1} = \frac{102\pi}{0.6\pi} = 170$ $\therefore v_1 : v_2 = 1:1$

$$v_2 = \frac{\omega_2}{k_2} = \frac{94\pi}{0.553\pi} = 170$$

3. The number of times $y_1 + y_2 = 0$ at $x = 0$ in 2 seconds

A) 300 B) 400 C) 196 D) 500

Key : C

Sol : $y = y_1 + y_2$

$A\cos(0.6\pi x - 102\pi t) + A\cos[0.553\pi x - 94\pi t] = 0$

$\therefore \cos 102\pi t = -\cos(-94\pi t)$

$\cos 102\pi t = \cos[(2n+1)\pi - 94\pi t]$

$102\pi t = 94\pi t = (2n+1)t$

$t = \dfrac{2n+1}{196}$ at $t = 0$

$n = \dfrac{1}{2}$

At $t = 2$ $2 = \dfrac{2n+1}{196}$

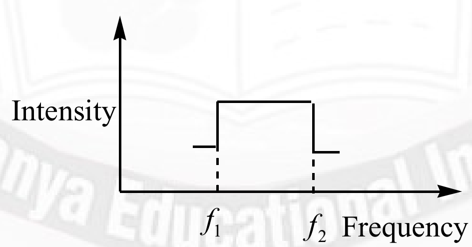
$n = 195.5$

For 0 to 2 seconds

$\dfrac{1}{2} + 195.5 = 196$

Passage-9.

Two trains A and B moving with speeds 20m/s and 30m/s, respectively, in the same direction on the same straight track with B ahead of A. The engine are at the front ends. The engine of train A blows a along whistle. Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800Hz$ to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency to lowest frequency) is thus 320 Hz. The speed of sound in still air is $340m/s$. **IIT 2007**



1. The speed of sound of the whistle is

A) 340 m/s for passengers in A and 310 m/s for passengers in B.

B) 360 m/s for passengers in A and 10 m/s for passengers in B.

C) 310 m/s for passengers in A and 360 m/s for passengers in B.

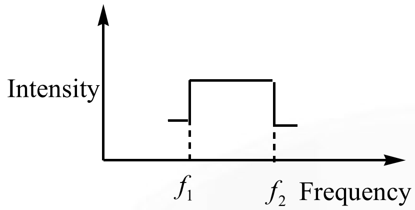
D) 340 m/s for passengers in both the trains.

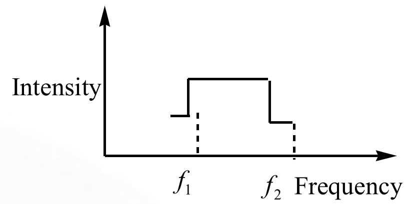
Key : B

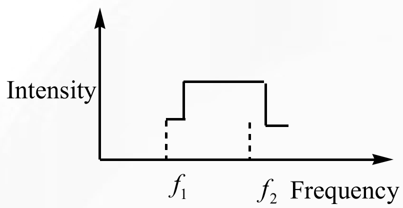
Sol : $v_{SA} = 340 + 20 = 360m/s$

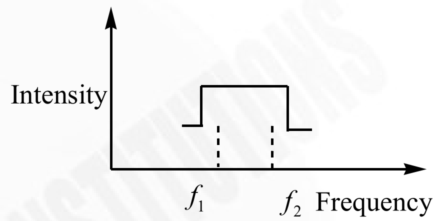
$v_{SB} = 340 - 30 = 310 m/s$

2. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by

(A) 

(B) 

(C) 

(D) 

Key : A

Sol : For the passengers in train A, there is no relative motion between the source and the observer, as both are moving with velocity 20 m/s. Therefore, there is no change in observed frequencies and correspondingly there in no change in their intensities.

3. The spread of frequency as observed by the passengers in train B is

A) 310 Hz B) 330 Hz C) 350 Hz D) 290 Hz

Key : A

Sol : For the passengers in train B the observer is receding with velocity 30m/s and the source is approaching with velocity 20 m/s.

$$f_2' = 800\left(\frac{340-30}{340-20}\right) = 775 Hz$$

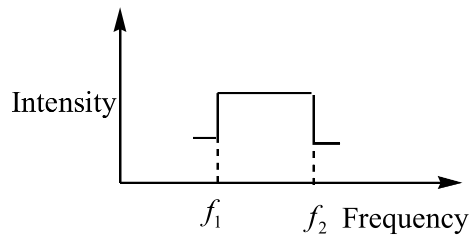
And $f_2' = 1120\left(\frac{340-30}{340-20}\right) = 1085 Hz$

Therefore, spread of frequency

$f_2' - f_1' = 310 Hz$

Passage-9a:

Two trains A and B moving with speeds 30m/s and 20m/s, respectively, in the same direction on the same straight track with B ahead of A. The engine are at the front ends. The engine of train A blows a along whistle. Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800 Hz$ to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency to lowest frequency) is thus 320 Hz. The speed of sound in still air is $340 m/s$.



1. The speed of frequency as observed by the passenger train B is approximately
A) 330Hz B) 371 Hz C) 350 Hz D) 290 Hz

Key : B

Sol : $f_2^{'} = 800\left(\frac{340+20}{340-30}\right) = 927Hz$

And $f_2^{'} = 1120\left(\frac{340+20}{340-30}\right) = 1300.6Hz$

Therefore, spread of frequency

$f_2^{'} - f_1^{'} = 1300 - 921 = 371Hz$

2. The frequency of sound observed by the passengers in train A spreading over___
A) 330 B) 340 C) 320 D) 310

Key : C

Sol : There is no relative motion between passengers A and whistling engine of A.
Therefore frequency spread is 1120Hz to 800 Hz-=320Hz

Passage :10

Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with diatomic gas of molar mass M_B. Both gases are at the same temperature. **IIT 2002**

1. If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A / M_B

Key : $400/189$

Sol : Second harmonic in pipe A is

$2\left[(v_0)A\right]$

Third harmonic of pipe B is

$3\left[(v_0)_B\right] = 2\left[\frac{v}{2l}\right] = 3\left[\frac{v}{4l}\right]$

$= \frac{1}{l}\sqrt{\frac{\gamma_A RT}{M_A}} = \frac{3}{4l}\sqrt{\frac{\gamma_B R}{M_B}}$

Given that second harmonic in pipe A is equal to the third harmonic of pipe B. So

$\frac{1}{l}\sqrt{\frac{\gamma_A RT}{M_A}} = \frac{3}{4l}\sqrt{\frac{\gamma_B RT}{M_B}}$

$\Rightarrow \dfrac{M_A}{M_B} = \dfrac{400}{189}$

2. Now the open end of pipe B is also closed (so that the pipe is close at both ends). Find the raio of the fundamental mode. Calculate the velocity of sound in air

Key : $3/4$

Sol : $[\gamma_A = 1.67 \text{ and } \gamma_B = 1.4]$

$$(V_0)_A = \sqrt{\frac{\gamma_A RT}{M_A}}$$

$$(V_0)_B = \sqrt{\frac{\gamma_A RT}{M_B}}$$

$$\frac{(V_0)_A}{(V_0)_B} = \sqrt{\frac{\gamma_A}{M_A} \times \frac{M_B}{\gamma_B}} = \frac{3}{4}$$

Passage :10a

Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with diatomic gas of molar mass M_B. Temperature of A is four times that of B.

1. If the frequency of first harmonic pipe is octave of the frequency of the third harmonic of fundamental mode in pipe B. The value of $\dfrac{M_A}{M_B}$ is $\left(\gamma_A = \dfrac{5}{3} \quad \gamma_B = \dfrac{7}{5}\right)$

A) $\dfrac{400}{189}$ B) $\dfrac{800}{189}$ C) $\dfrac{1600}{189}$ D) $\dfrac{3200}{189}$

Key : C

Sol : $n_B = 2n_A (octave)$

$$2 \times \frac{v_1}{2l} = \frac{3}{4l} v_2$$

$$T_A = 4T_B$$

$$\sqrt{\frac{\gamma_A RT_A}{M_A}} = \sqrt{\frac{\gamma_B RT_B}{M_B}} \times \frac{3}{4}$$

$$\frac{\frac{5}{3} \times 4T}{M_A} = \frac{9}{16}\left[\frac{7/5 \times T}{M_B}\right]$$

$$\frac{M_A}{M_B} = \frac{1600}{189}$$

2. Now the open end of pipe B is also closed (so that the pipe is close at both ends). Find the fundamental ratio in pipe A to that in pipe B

A) $\dfrac{18}{91}$ B) $\dfrac{9}{80}$ C) $\dfrac{80}{79}$ D) $\dfrac{3}{4}$

Key : D

Sol : $n_A = \frac{1}{2l}\sqrt{\frac{\gamma_A RT_A}{M_A}}$

$$n_B = \frac{1}{2l}\sqrt{\frac{\gamma_B RT_B}{M_B}}$$

$$\frac{N_A}{N_B} = \sqrt{\frac{\frac{5}{3}\times 4T}{M_A} \times \frac{M_B}{7/5T}} = \sqrt{\frac{100\times 189}{3\times 7\times 1600}} = \frac{3}{4}$$

Exercise : V

(Assertion – Reason / Statement – I & II Type Questions)

In the following questions. Statement –I (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

a) **Statement -1** is true, **Statement -2** is true and **Statement -2** is the correct explanation for **Statement -1**

b**) Statement -1** is true, **Statement -2** is true but **Statement -2** is not the correct explanation of **Statement -1**

c) **Statement -1** is true, **Statement -2** is false

d) **Statement -1** is false, **Statement -2** is true.

1. **Statement -1:** Only longitudinal mechanical waves can propagate in gases
Statement -2: Gases have only bulk modulus

Sol : The correct choice is (a). Gases cannot withstand a shearing stress or longitudinal stress.

Hence they do not have shear modulus and Young's modulus; they have only bulk modulus.

2. **Statement -1:** Two sound waves of equal intensity I produced beats. The maximum intensity of sound produced in beats is 4I.
Statement -2: If two waves of amplitudes a_1 and a_2 superpose, the maximum amplitude of the resultant wave $= a_1 + a_2$

Sol : The correct choice is (a). When two waves of amplitudes a_1 and a_2 superpose to produce Beats, the resultant amplitude of the maximum of intensity is $A = a_1 + a_2$. Now, intensity $\alpha\left(\text{amplitude}\right)^2$.Since the two waves have the same intensity, their amplitudes are equal, i.e., $a_1 = a_2 = a$. Thus $A = 2a$. Therefore, $A^2 = 4a^2$ or $I_{\max} = 4I$.

3. **Statement -1:** A medium must possess elasticity in order to support wave motion.
Statement -2: Restoring force does not exit in a medium which does not have elasticity.

Sol : The correct choice is (a)

4. **Statement -1:** Solids can support both longitudinal and transverse mechanical waves but only longitudinal mechanical waves can propagate in gases.

Statement -2: Gases do not have shear modulus.

Sol : The correct choice is (a). Gases cannot withstand a shearing stress. Hence gases do not have any shear modulus: they have only bulk modulus. Solids have Young's modulus, bulk modulus. Therefore, solids can support both transverse and longitudinal waves.

5. **Statement -1:** In standing sound waves, a displacement node is a pressure antinode and vice versa.

Statement -2: In a standing wave, the restoring force is the maximum at a node and minimum at an antinode.

Sol : The correct choice is (c)

6. **Statement -1:** Our ears cannot distinguish two notes, one produced by a violin and other by a sitar, if they have exactly the same intensity and the same frequency.

Statement -2: When musical instrument is played, it produces a fundamental note which a accompanied by a number of overtones called harmonics.

Sol : The correct choice is (d). When a musical instrument is played, it produced a fundamental notetionary observer with a certain velocity and (ii) observer approaching a stationary source of sound with the same velocity.

7. **Statement -1:** Doppler's effect does not occur in case of supersonic source.

Statement -2: A supersonic source produces shock waves.

Sol : The correct choice is (a). If the source of sound is moving at a speed greater than the speed of sound then in a given time the source advances more than the wave. The resultant wave motion is a conical wave called a shock wave which produces a sudden and violent sound.

8. **Statement -1:** If a source of sound moves from a stationary observer, the apparent frequency of sound as heard by the observer is greater than the actual frequency.

Statement -2: The cause of the apparent change in frequency is the change in the wavelength brought about by the motion of the source.

Sol : The correct choice is (d)

9. **Statement -1:** If an observer moves towards a stationary source of sound, the frequency of the sound as heard by him is greater than the actual frequency.

Statement -2: The apparent increase in frequency is due to the fact that the observer intercepts more waves per second when moves towards the source.

Sol : The correct choice is (a)

10. **Statement -1:** If a source of sound is in motion and the observer is stationary, the speed of sound relative to him remains unchanged.

Statement -2: The apparent change in frequency is due to the change in the wave length brought about by the motion of the source.

Sol : The correct choice is (a)

11. **Statement -1:** If the observer is in motion and the source of sound is stationary, the speed of sound relative to him is changed.
Statement -2: The wavelength of sound received by the observer does not change due to his motion .

Sol : The correct choice is (a)

12. **Statement -1:** The apparent frequency is not the same in the following two cases-(i) source approaching a stationary observer with a certain velocity and (ii) observer approaching a stationary source of sound with the same velocity.
Statement -2: The cause of the apparent change in the frequency is different in the two cases.

Sol : The correct choice is (a). In case (i) the speed of sound relative to the observe remains Unchanged; the change in frequency is due to a change in wavelength brought about by the motion of the source in case-(ii) the wavelength of sound remains unchanged; the change in
frequency is due to a change in the speed of sound relative to the observer.

13. **Statement 1:** When pressure of an ideal gas is increased, the speed of sound in gas must increase.
Statement-2: The speed of sound in ideal gas is directly proportional to square root of pressure of the gas.

Sol : Correct choice (D), $v=\sqrt{\frac{\gamma p}{\rho}}$ as p increases ρ also increases keeping $\frac{p}{\rho}$ a constant.

14. **Statement 1:** Two sound waves of same intensity in a particular medium will have displacement amplitude in ratio of $2:1$ if they frequency in the ratio $1:2$.
Statement-2: Two wave of same velocity amplitude in a particular medium have equal intensity.

Sol: Correct choice (A), $I=2\rho a^2\pi^2n^2V$

15. **Statement 1:** An $80 bB$ sound has twice the intensity of a $40 bB$ sound.
Statement-2: Loudness of a sound of a certain intensity I is defined as:
$L(indB)=10\log_{10}\frac{I}{I_0}$

Sol : Correct choice (D)

16. **Statement 1:** When a closed organ pipe vibrates, the pressure of the gas at the closed end remains constant.
Statement-2: In a stationary wave system, displacement nodes are pressure antinodes, and displacement antinodes are pressure nodes.

Sol: Correct choice (D)

17. **Statement 1:** A plane wave of sound travelling in air is incident upon a plane water surface. Angle of incidence is θ. If snell's law is valid for sound waves, it follows that sound will be refracted into water away from normal.
Statement-2: From snell's law, angle of refraction is more than angle of incidence when wave travel from denser to rarer medium.

Sol: Correct choice (A)

18. **Statement 1:** A person is standing near a railway track A train is moving on the track. As train is approaching the person, apparent frequently keeps on increasing and when train has passed the person then apparent frequency keeps on decreasing.
Statement-2: When train is approaching the person then $f = f_0\left[\frac{c}{c-u}\right]$ and when train is moving away from person $f = f_0\left[\frac{c}{c+u}\right]$, c is velocity of sound, u is velocity of train and f_0 is frequency of whistle.

Sol: Correct choice (A), $n_0 = n\times\left(\frac{v-v_0}{v-v_s}\right)$

19. **Statement 1:** A closed organ pipe is vibrating in its first overtone with frequency $340Hz$. Speed of sound in organ pipe is $340m/s$. Length of organ pipe is less than $75cm$.
Statement-2: In case of standing wave in closed pipe, pressure amplitude is maximum at closed end.

Sol: Correct choice (B), $n_c = \frac{v}{4L} \Rightarrow l = \frac{v}{4L}, \frac{340}{4\times340} = \frac{1}{4}m = 25cm$

20. **Statement 1:** Doppler's effect in sound is asymmetric but in light, it is symmetric
Statement-2: In sound, change in frequency depends on the individual velocity of both of the source as well as the observer. In light, change in frequency depends on the relative velocity between source and observer.

Sol: Correct choice (B), Doppler's effect in sound is different when the object is moving towards source and when source move towards observer.
Hence it is asymmetric, Light is symmetric as it just depends on relative motion between the source and the object.

21. **Statement 1:** The propagation of sound in air should be an isothermal process.
Statement-2: As air bad conductor of heat, its temperature does not change by compression or expansion.

Sol : Correct choice (D) The propagation of sound in air is adiabatic process as very little sound energy get dissipated as heat. Air may be a bad conductor of heat but its temperature does change when work is performed on it.

22. **Statement 1:** Velocity of sound in air increases with increase in humidity.
Statement-2: Velocity of sound doesn't depend upon medium.

Sol : Correct choice (C) $v=\sqrt{\frac{\gamma P}{\rho}}$ the ρ or density reduces because of presence of water vapour as both N_2 and O_2 are heavier than H_2O velocity of sound, thus, depends largely on the medium.

23. **Statement 1:** Intensity of sound wave does not change when the listener moves towards or away from the stationary source.
Statement-2: The motion of listener towards a stationary source causes an apparent change in wavelength of sound.

Sol : Correct choice (D), intensity of sound changes when observer moves towards are way from source because of change in frequency. Motion of observes only causes change in frequency of sound and not wavelength.

24. **Statement 1:** A vibrating tuning fork sounds louder when its stem is pressed against desk top.
Statement-2: When a wave reaches another denser medium, part of the wave is reflected.

Sol : Correct choice (B) when a vibrating tuning fork is held in hand only air is set into vibration. When its stem place in contact with the labels the entire labels is set into forced vibrations.

25. **Statement 1:** Longitudinal waves do not exhibit the phenomenon of polarisation.
Statement-2: In longitudinal wave medium particle vibrate in direction normal to the wave propagation.

Sol : Correct choice (C) Polarisation means redirecting a wave to propagate in only one plane. The molecules in a longitudinal wave vibrate along the direction of propagation of wave and hence cannot be redirected by any material.