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A right Choice for the Real Aspirant

Central Office, Bangalore

DIFFERENTIAL EQUATIONS

EXERCISE - IV

MATRIX MATCHING

1. Match the following: -

	Column – I		Column – II
A)	If order and degree of the differential equation formed by differentiating and eliminating the constants from $y = a\sin^2 x + b\cos^2 x + c\sin 2x + d\cos 2x$, where a, b, c, d are arbitrary constants are represented by O and D, then	(P)	$O + 2D = 5$
B)	The order and degree of the differential equation, whose general solution is given by $y = (c_1 + c_2)\sin(x + c_3) - c_4 e^{x+c_5+c_6}$, where $c_1, c_2, c_3, c_4, c_5, c_6$ are arbitrary constants, are O and D, then	(Q	$2O + 3D = 5$
C)	The order and degree of the differential equation satisfying $\sqrt{(1+x^2)} + \sqrt{(1+y^2)} = A(x\sqrt{(1+y^2)} + y\sqrt{(1+x^2)})$ are O and D, then	(R)	$O = D$
D)	If the order and degree of the differential equation of all parabolas whose axis is x-axis are O and D then	(S)	$O^D + D^O = 4$

2. Match the following: -

	Column – I		Column – II
A)	General solution of $x^2ydx=\left(x^3+y^3\right)dy$ is	(P)	$-\frac{1}{xy}+\ln x-\ln y=c$
B)	General solution of $(xy-2y^2)dx=\left(x^2-3xy\right)dy$ is	(Q	$\frac{x^3}{3y^3}=\ln(y)+c$ 5
C)	General solution of $(xy+x^2y^2)ydx+(xy-x^2y^2)xdy=0$ is	(R)	$\frac{x}{y}=2\ln x+3\ln\|y\|=c$
D)	General solution of $(x^2y^2+xy+1)ydx=(x^2y^3-xy+1)x\ dy$ is	(S)	$\ln x-\ln y+\tan^{-1}xy=c$

3. Match the following: -

	Column – I		Column – II
A)	The solution of the D.E. $\left(1+x^2y^2\right)ydx+\left(x^2y^2-1\right)xdy=0$	(P)	$2ye^{2x}=ce^{2x}-1$
B)	The solution of the D.E. $2x^3ydy+\left(1-y^2\right)\left(x^2y^2+y^2-1\right)dx=0$	(Q	$4e^{3x}+3.e^{-4y}=C$
C)	The solution of the D.E. $\dfrac{X+\frac{X^3}{3!}+\frac{X^5}{5!}+\ldots}{1+\frac{X^2}{2!}+\frac{X^4}{4!}+\ldots}=\dfrac{dx-dy}{dx+dy}$ is	(R)	$x^2+y^2=2\ln\frac{y}{x}+c$
D)	The solution of ln $\left(\frac{dy}{dx}\right)=3x+4y$ is	(S)	$x^2y^2=\left(cx-1\right)\left(1-y^2\right)$

PARAGRAPH TYPE QUESTIONS

Formation of D.E :

Passage-1: (4-6)

Rule to solve the equation

$$\alpha_0 \frac{d^n y}{dx^n} + \alpha_1 \frac{d^{n-1} y}{dx^{n-1}} + \alpha_2 \frac{d^{n-2} y}{dx^{n-2}} + ...\alpha_n y = 0 \text{ (Where } \alpha_i \in C \text{ or } \alpha_i \in R \text{)}$$

(i) Write the equation in the symbolic form $(\alpha_0 D^n + \alpha_1 D^{n-1} + \alpha_2 D^{n-2} + ... + \alpha_n)y = 0$

(Where $D \equiv \frac{d}{dx}$)

(ii) Write the auxiliary equation (A.E)

$\alpha_0 D^n + \alpha_1 D^{n-1} + \alpha_2 D^{n-2} + ... + \alpha_n = 0$

Solve it for D as if D were an ordinary algebraic quantity

(iii) From the roots of A.E write down the corresponding part of the complete solution as follows

Root of A.E (Auxiliary equation)	Corresponding part of C.S (Complete Solution)
1. One real root m_1	$c_1 e^{m_1 x}$
2. Two real and different roots m_1, m_2	$c_1 e^{m_1 x} + c_2 e^{m_2 x}$
3. Two real and equal roots m_1, m_2	$(c_1 + c_2 x)e^{m_1 x}$
4. Three real and equal roots m_1, m_1, m_1	$((c_1 + c_2 x + c_3 x^2)e^{m_1 x}$
5. One pair of complex roots $\alpha \pm i\beta$	$e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$
6. Two pairs of complex and equal roots $\alpha \pm i\beta$, $\alpha \pm i\beta$	$e^{\alpha x}\left[(c_1 + c_2 x)\cos \beta x + (c_3 + c_4 x)\sin \beta x\right]$

Now answer the following questions

4. $y = c_1 e^{(2+\sqrt{3})x} + c_2 e^{(2-\sqrt{3})x}$ is solution for (Where c_1 and c_2 are arbitrary constants)

A) $\frac{d^4 y}{dx^4} - 5\frac{d^2 y}{dx^2} + 4y = 0$ B) $\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + y = 0$

C) $\frac{d^4 y}{dx^4} + m^4 y = 0$ D) None of these

5. $y=(c_1+c_2x)\cos 2x+(c_3+c_4x)\sin 2x$ is solution for (Where c_1 and c_2 are arbitrary constants)

A) $\frac{d^4y}{dx^4}+(m^2+n^2)\frac{d^2y}{dx^2}+m^2n^2y=0$ B) $\frac{d^4y}{dx^4}+13\frac{d^2y}{dx^2}+36y=0$

C) $\frac{d^4y}{dx^4}+8\frac{d^2y}{dx^2}+16y=0$ D) $\frac{d^4y}{dx^4}+4y=0$

6. General solution of $\frac{d^4y}{dx^4}-a^4y=0$

A) $y=c_1\cos ax+c_2\sin ax+c_3\cosh ax+c_4\sinh ax$

B) $y=(c_1+c_2)\cos ax+\sin ax$

C) $y=c_1\cos ax+c_2\sin ax+(c_3+c_4x)\cosh ax$

D) None of these

(Where C_1,C_2,C_3,C_4 are arbitrary constants)

PRACTICE QUESTIONS

Paragraph – 2 : (7-9)

The differential equation corresponding to $y=C_1e^{m_1x}+C_2e^{m_2x}+C_3e^{m_3x}$, where C_1,C_2,C_3 are arbitrary constants and m_1,m_2,m_3 are the roots of the equation $m^3-7m+6=0$ is $a\frac{d^3y}{dx^3}+b\frac{d^2y}{dx^2}+c\frac{dy}{dx}+d=0$, where a,b,c,d are constants, where ($m_3<m_1<m_2$).

Answer the following questions:

7. The value of a and b respectively

A) 0, 1 B) 1, 0 C) –1, 0 D) $0,-1$

8. Value of c is

A) 6 B) –7 C) 2 D) -1

9. Value of d is

A) 6 B) –7 C) 2 D) -1

Inspection method of solving D.E :

Passage-3 : (10-12)

If any differential equation in the form $f(f_1(x,y))d(f_1(x,y))+\phi(f_2(x,y))d(f_2(x,y))+...=0$, then each term can be integrated separately

For e.g., $\int \sin xy d(xy)+\int \left(\frac{x}{y}\right)d\left(\frac{x}{y}\right)=-\cos xy+\frac{1}{2}\left(\frac{x}{y}\right)^2+c$

10. The solution of the differential equation $xdy-ydx=\sqrt{x^2-y^2}dx$ is

A) $cx=e^{\sin^{-1}\frac{y}{x}}$ B) $xe^{\sin^{-1}\frac{y}{x}}=c$ C) $x+e^{\sin^{-1}\frac{y}{x}}=c$ D) none of these

11. The solution of the differential equation $(xy^4+y)dx-xdy=0$ is

A) $\frac{x^3}{4}+\frac{1}{2}\left(\frac{x}{y}\right)^2=c$ B) $\frac{x^4}{4}+\frac{1}{3}\left(\frac{x}{y}\right)^3=c$

C) $\frac{x^4}{4}-\frac{1}{2}\left(\frac{x}{y}\right)^2=c$ D) $\frac{x^3}{4}-\frac{1}{2}\left(\frac{x}{y}\right)^2=c$

12. Solution of differential equation $(2x\cos y+y^2\cos x)dx+(2y\sin x-x^2\sin y)dy=0$ is

A) $x^2\cos y+y^2\sin x=c$ B) $x\cos y-y\sin x=c$

C) $x^2\cos^2 y+y^2\sin^2 x=c$ D) none of these

Lagrange's Linear D.E :

Paragraph – 4 : (13-15)

Let $f(x)$ be a differentiable function satisfying $(x-y)f(x+y)-(x+y)f(x-y)$ $=4xy(x^2-y^2)$ for all $x,y\in R$. If $f(1)=1$ then

13. The function f at $x=0$ attains

(A) Local maximum (B) Local minimum

(C) Point of inflexion (D) None of these

14. The value of $\int_{-1}^{2} f(x)dx$ is

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{11}{4}$ (D) $\frac{15}{4}$

15. The area of the region bounded by the curves $y=f(x)$ and $y=x^2$ is

(A) $\frac{1}{4}$sq.units (B) $\frac{1}{12}$sq.units (C) $\frac{7}{12}$sq.units (D) $\frac{11}{12}$sq.units

Passage -5 : (16-18)

Let f be a differentiable function satisfying

$$\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t))dt = 0 \ and \ f(0)=1$$

16. The number of solution(s) of the equation $\begin{vmatrix} f(2x) & f(x) \\ \sin x & 2 \end{vmatrix} = 0 \ in \ (0, 2\pi)$ is :

A) 2 B) 3 C) 4 D) 5

17. The value of $\int_0^{\pi/2} f(x)\,dx$ lies in the interval :

A) $\left(\frac{2}{\pi}, 1\right)$ B) $\left(1, \frac{\pi}{2}\right)$ C) $\left(\frac{3}{2}, \frac{\pi}{2}\right)$ D) $\left(0, \frac{2}{\pi}\right)$

18. The value of $\lim_{x\to 0}\left(\left[\frac{\cos x}{f(x)}\right] + \left[\frac{\cos 2x}{f(2x)}\right] + \left[\frac{\cos 3x}{f(3x)}\right] + ... + \left[\frac{\cos(100x)}{f(100x)}\right]\right)$ is equal to :

Where [k] denotes greatest integer less than or equal to k.

A) 0 B) 4950 C) 5049 D) 5050

Bernoulli's Equation (Reducible to linear equations) :

Passage - 6 : (19-21)

$$\frac{dy}{dx} + Py = Qy^n \quad ....(1)$$

Where P and Q are functions of x along or constants divide each term of equation (1) by

y^n now we get $\frac{dy}{dx} + \frac{y}{x} = y^3 \quad ....(2)$

Let $\frac{1}{y^{n-1}} = v$ so that $\frac{1}{y^n}\frac{dy}{dx} = \frac{1}{1-n}.\frac{dv}{dx}$ substituting in eqn (2) we get

$\frac{dv}{dx} + (1-n)P = Q(1-n)$. This is a Linear D.E.

19. The linear form of $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2 y\,dy = 0$

A) $\dfrac{dt}{dx} - \dfrac{2}{x}t = \dfrac{-2}{x^2}e^{\frac{1}{x^3}}$

B) $\dfrac{dt}{dx} + \dfrac{2}{x}t = \dfrac{-2}{x^2}e^{\frac{1}{x^3}}$

C) $\dfrac{dt}{dx} + \dfrac{2}{x}t = \dfrac{2}{x^2}e^{\frac{1}{x^3}}$

D) $\dfrac{dt}{dx} - \dfrac{2}{x}t = \dfrac{2}{x^2}e^{\frac{1}{x^3}}$

20. The solution of $\dfrac{dy}{dx} + \dfrac{y}{x} = y^3$

A) $2xy^2 + y^2 = 1$

B) $2xy^2 + cx^2y^2 = 1$

C) $2xy^2 + x^2 = 1$

D) $2xy^2 - x^2 = 1$

21. The solution of $\dfrac{dy}{dx} = x^3y^3 - xy$

A) $\dfrac{1}{y^2} = x^2 + 1 - ce^{x^2}$

B) $y^2 = x^2 + 1 + ce^{x^2}$

C) $\dfrac{1}{y^2} = x^2 - 1 - ce^{x^2}$

D) $y^2 = x^2 - 1 + ce^{x^2}$

Geometrical Application of D.E :

Paragraph – 7 : (22–24)

A non-negative differentiable function *f* is defined on the closed interval $[0,1]$ with $f(1) = 0$. For each "*a*", $0 < a < 1$, the line $x = a$ divides the area bounded by $y = f(x)$ and the coordinate axes into two regions having areas A and B, A being the area of the left most region. It is given that $A - B = 2f(a) + 3a + b$ where *b* is a constant independent of *a*.

22. Value of $f(0)$ is

A) $2 - \dfrac{1}{e}$ B) $\dfrac{3}{2}\left(1 - \dfrac{1}{e}\right)$ C) $\dfrac{1}{2}\left(1 + \dfrac{2}{e}\right)$ D) $\dfrac{e}{2}$

23. Slope of normal to $y = f(x)$ at $x = 1/2$ is

A) $2\sqrt{e}$ B) $\sqrt{e}/2$ C) $3\sqrt{e}/2$ D) $2\sqrt{e}/3$

24. Value of *b* is

A) $3e + \dfrac{2}{3}$ B) $\dfrac{3}{2e} - 3$ C) $\dfrac{2}{e} - 1$ D) $\dfrac{3e}{4} + \dfrac{2}{3}$

Paragraph – 8 : (25-27)

A curve passes through origin and its slope at any point (x,y) is $\frac{x-3}{y-4}$

25. The curve is a hyperbola whose eccentricity is

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 4

26. The combined equation of the asymptotes of the given curve is

(A) $x^2-y^2+6x+8y-7=0$ (B) $x^2-y^2-6x+8y-7=0$

(C) $x^2-y^2+6x-8y+7=0$ (D) $x^2-y^2-6x-8y-7=0$

27. The centre of the curve is

(A) $(3,-4)$ (B) $(-3,4)$ (C) $(3,4)$ (D) $(-3,-4)$

PRACTICE QUESTIONS

Paragraph – 9 : (28-30)

Let the differentiable equation of certain curves be given by $\left(1-x^2\right)\frac{dy}{dx}+xy=2x$ then

Answer the following

28. The curves are

A) lines B) parabolas C) lines or circles D) lines or central conics

29. The point through which every chord of the curve represented by the given differential equation is bisected is

a) (2,0) b) (2,1) c)(0,2) d) (4,0)

30. All curves represented by the given differential equation pass through the fixed point

a) (2,1) b) (1,2) c) (2,2) d) (2,0)

Orthogonal Trajectory :
Passage -10 : (31-33)

A curve C with negative slope through the point (0, 1) lies in the first quadrant. The tangent at any point P on it meets the *x*-axis at Q. Such that PQ = 1.

31. The slope of the tangent at $y=\frac{1}{2}$ is

A) -1 B) $-\frac{1}{\sqrt{2}}$ C) $\sqrt{3}$ D) $-\frac{1}{\sqrt{3}}$

32. The area bounded by C and the coordinate axes is

A) 1 B) ln 2 C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

33. The orthogonal trajectories of C are

A) circles B) parabolas C) ellipses D) hyperbolas

Physical Application of D.E :

Paragraph – 11 : (34-36)

A rabbit moving in *xy*-plane starts at origin and runs up y-axis with uniform speed a m/s.At the same time a dog ,running with speed b m/s , starts at the point (1,0) and purses the rabbit. Assuming the path of dog is a curve *y= f(x)* in xy plane (you may use the formula; differential are length of a curve $ds=\sqrt{(dx)^2+(dy)^2}$

Answer the following

34. At any moment of time t, measured from the instant both start $\frac{dy}{dx}=$

a) $y-at$ b) $\frac{y-at}{x}$ c) $(y-at)x$ d) $\frac{x}{y-at}$

35. Differential equation governing the path of the dog is

a) $bxy''=a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ b) $xy'=ab\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

c) $xy''=ab\sqrt{1+y^2}$ d) $xy''=ab\sqrt{1+x^2}$

36. The path of the dog is given by, $\frac{dy}{dx}=$

a) $\frac{x^{a/b}+x^{-a/b}}{2}$ b) $\frac{x^{a/b}-x^{-a/b}}{2}$ c) $x^{a/b}+x^{-a/b}$ d) $x^{a/b}-x^{-a/b}$

PRACTICE QUESTIONS

Passage-12 : (37-41)

A particle moves vertically under gravity in a resisting medium whose resistance per unit mass is kv, where k is a constant and v, the velocity. The equation governing the motion is

$$\frac{du}{dt}=\begin{cases} g-kv, \text{ downward motion} \\ -g-kv, \text{ upward motion}\end{cases}$$

Where *t* is the time and *g*, the acceleration due to gravity.

37. If the particle is projected up with initial velocity *u*, then the time of ascent is

A) $\frac{1}{2k}\ln\left(1+\frac{ku}{g}\right)$ B) $\frac{1}{k}\ln\left(1-\frac{ku}{g}\right)$ C) $\frac{1}{2k}\ln\left(1-\frac{ku}{g}\right)$ D) $\frac{1}{k}\ln\left(1+\frac{ku}{g}\right)$

38. If the particle is projected up with initial velocity *u,* the maximum height reached is

A) $\frac{u}{k}+\frac{g}{k^2}\ln\left(1-\frac{ku}{g}\right)$ B) $\frac{u}{k}+\frac{g}{k^2}\ln\left(1+\frac{ku}{g}\right)$

C) $\frac{u}{k}-\frac{g}{k^2}\ln\left(1-\frac{ku}{g}\right)$ D) $\frac{u}{k}-\frac{g}{k^2}\ln\left(1+\frac{ku}{g}\right)$

39. If the particle is dropped from rest, the velocity *v* at time *t* is

A) $\frac{g}{k}\left(1-e^{-kt}\right)$ B) $\frac{g}{k}\left(1+e^{-kt}\right)$ C) $\frac{g}{2k}\left(1-e^{-2kt}\right)$ D) $\frac{g}{2k}\left(1+e^{-2kt}\right)$

40. If the particle is dropped from rest, the displacement at time t is

A) $\frac{gt}{k}+\frac{g}{k}\left(1-e^{-kt}\right)$ B) $\frac{gt}{k}-\frac{g}{k^2}\left(1-e^{-kt}\right)$

C) $\frac{gt}{k}-\frac{g}{2k^2}\left(1-e^{-2kt}\right)$ D) $\frac{gt}{k}-\frac{g}{k^2}\left(1+e^{-kt}\right)$

41. If the particle is dropped from rest, the displacement s when the velocity is v, is

A) $\frac{v}{k}+\frac{g}{k^2}\ln\left(1-\frac{kv}{g}\right)$ B) $\frac{v}{k}+\frac{g}{k^2}\ln\left(1+\frac{kv}{g}\right)$

C) $-\frac{v}{k}-\frac{g}{k^2}\ln\left(1-\frac{kv}{g}\right)$ D) $\frac{v}{k}-\frac{g}{k^2}\ln\left(1+\frac{kv}{g}\right)$

KEY SHEET

1. **A- P, S; B– P, S; C– Q, R; D – P**	2. **A – Q, ; B – R; C – P; D – S**
3. **A – R; B – S; C – P; D – Q**	

4.	B	**5.**	C	**6.**	A	**7.**	B	**8.**	B
9.	A	**10.**	A	**11.**	B	**12.**	A	**13.**	C
14.	D	**15.**	B	**16.**	B	**17.**	B	**18.**	A
19.	A	**20.**	B	**21.**	A	**22.**	B	**23.**	D
24.	B	**25.**	A	**26.**	B	**27.**	C	**28.**	D
29.	C	**30.**	B	**31.**	D	**32.**	C	**33.**	A
34.	C	**35.**	A	**36.**	B	**37.**	D	**38.**	D
39.	A	**40.**	B	**41.**	C				

HINTS & SOLUTIONS

1. (A) $\because \quad y=\left(\frac{1-\cos 2x}{2}\right)+b\left(\frac{1+\cos 2x}{2}\right)+c\sin 2x+d\cos 2x$

$= A + B \sin 2x + C \cos 2x$

$\therefore \quad \frac{dy}{dx}=2B\cos 2x-2C\sin 2x \Rightarrow \quad \frac{d^2y}{dx^2}=-4B\sin 2x-4C\cos 2x$

$\therefore \quad O = 3, D = 1$

$O + 2D = 5, O^D + D^O = 4, 2^O + 3^D = 8 + 3 = 11$ (P, S)

(B) $\because \quad y = (c_1 + c_2)\sin(x + c_3) - c_4 e^{x+c_5+c_6}.e^x$

Or $y = A\sin(x + B) + Ce^x$ …..(i)

$\therefore \quad \frac{dy}{dx}=A\cos(x+B)+Ce^x$ …..(ii)

Subtracting Eq. (i) from Eq. (ii), then

$$\frac{dy}{dx}-y=A\cos(x+B)-A\sin(x+B)$$

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}=-A\cos(x+B)-A\cos(x+B)$$

$$\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$$

$O = 3, D = 1$

$O + 2D = 5, O^D + D^O = 4, 2^O + 3^D = 11$ (RS)

(C) Put $x = \tan\theta$, $y = \tan\phi$

Then, $(\sec\theta + \sec\phi) = A(\tan\theta\sec\phi + \tan\phi\sec\theta)$

$$\Rightarrow \left(\frac{\cos\theta+\cos\phi}{\cos\theta\cos\phi}\right)=A\left(\frac{\sin\theta+\sin\phi}{\cos\theta\cos\phi}\right)\Rightarrow \cot\left(\frac{\theta+\phi}{2}\right)=A$$

$\Rightarrow \quad \frac{\theta+\phi}{2}=\cot^{-1}A \Rightarrow \theta+\phi = 2\cot^{-1}A$ or $\frac{1}{(1+x^2)}+\frac{1}{(1+y^2)}\frac{dy}{dx}=0$

O = 1, D = 1

Then O = D and 2O + 3D = 5 (QR)

2. A) $x^2ydx=\left(x^3+y^3\right)dy$; $\frac{dy}{dx}=\frac{x^2y}{x^3+y^3}$

Put $y=vx$ and integrating $\log y+c=\frac{x^3}{3y^3}$

B) $\left(xy-2y^2\right)dx=\left(x^2=3xy\right)dy;\ \frac{dy}{dx}=\frac{xy-2y^2}{x^2-3xy}$

Put $Y=vx;\ \frac{x}{y}-2\log x+3\log y=c$

C) Removing the common factor xy

We get $(1+xy)ydx+(1-xy)xdy=0$

Expanding we get $ydx+xdy+x^2y^2\left(\frac{dx}{x}-\frac{dy}{y}\right)=0$

$\frac{-1}{xy}+\ln x-\ln y=c$

D) Rearranging the terms we get $(ydx-xdy)\left(1+x^2y^2\right)+xy(ydx+xdy)=0$

$\frac{dx}{x}-\frac{dy}{y}+\frac{d(xy)}{1+x^2y^2}=0$

$\ln x-\ln y+\tan^{-1}xy=c$

3. A) $(xy)\left(\frac{x}{y}\right)d(xy)=-d\left(\frac{x}{y}\right)\Rightarrow\frac{x^2y^2}{2}=\ln\left(\frac{x}{y}\right)+k$

B) $\frac{2y}{(1-y)^2}\frac{dy}{dx}+\frac{y^2}{1-y^2}\frac{1}{x}=\frac{1}{x^3}$

Put $\frac{y^2}{1-y^2}=u$ then proceed L.D.E.

C) Applying C & D, we get $\frac{dy}{dx}=e^{-2x}$

D) $e^{-4y}.dt=e^{3x}.dx$

4-6. CONCEPTUAL

7– 9 :

$$m^3 - 7m + 6 = 0 \Rightarrow m_1 = 1, m_2 = 2, m_3 = -3$$

$$y = C_1e^x + C_2e^{2x} + C_3e^{-3x} \Rightarrow ye^{-x} = C_1 + C_2e^x + C_3e^{-4x}$$

$$-ye^{-x} + e^{-x}\frac{dy}{dx} = C_2e^x + 4C_3e^{-4x} \Rightarrow -ye^{-2x} + e^{-2x}\frac{dy}{dx} = C_2 - 4C_3e^{-5x}$$

Again diff. w.r.t. to x

$$2ye^{3x} - 3e^{3x}\frac{dy}{dx} + e^{3x}\frac{d^2y}{dx^2} = 20C_3$$

Again diff. w.r.t. to x then

$$(2y)(3e^{3x}) + e^{3x}\left(2\frac{dy}{dx}\right) - 3\left\{e^{3x}\frac{d^2y}{dx^2} + \frac{dy}{dx}3e^{3x}\right\} + e^{3x}\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}(3e^{3x}) = 0$$

dividing $e^{3x} \Rightarrow \frac{d^3y}{dx^3} - 7\frac{dy}{dx} + 6y = 0 \Rightarrow a = 1, b = 0, c = -7, d = 6$

10. $xdy - ydx = \sqrt{x^2 - y^2}\,dx$

$$\frac{xdy - ydx}{x\sqrt{x^2 - y^2}} = \frac{dx}{x}$$

$$\int d\sin^{-1}\left(y/x\right) = \int\frac{dx}{x}$$

$$\sin^{-1} y/x = \ln x + \ln c$$

$$\sin^{-1}\left(y/x\right) = \ln\left(cx\right)$$

$$cx = e^{\sin -1\left(y/x\right)}$$

11. $\left(xy^4 + y\right)dx - xdy = 0$

$$xy^4dx = xdy - ydx$$

$$x^3dx = \frac{x^2\left(xdy - ydx\right)}{y^4}$$

$$x^3dx = \left(\frac{x}{y}\right)^2\frac{xdy - ydx}{y^2}$$

$$x^3dx = -\left(\frac{x}{y}\right)^2 d\left(\frac{x}{y}\right)$$

$$\frac{x^4}{4} = -\frac{1}{3}\left(\frac{x}{y}\right)^3 + C$$

12. $\cos y.2xdx + y^2\cos xdx + \sin x.2ydx - x^2\sin ydy = 0$

$\cos ydx^2 + y^2 d\sin x + \sin xdy^2 + x^2 d\cos y = 0$

$d\left(x^2\cos y\right) + d\left(y^2\sin x\right) = 0$

$x^2\cos y + y^2\sin x = 0$

13-15 :

$$(x-y)f(x+y)-(x+y)f(x-y)=4xy\left(x^2-y^2\right) \Rightarrow \frac{f(x+y)}{x+y}-\frac{f(x-y)}{x-y}=4xy$$

So; $$\frac{f(x+h)}{x+h}-\frac{f(x-h)}{x-h}=4xh \Rightarrow \left[\frac{f(x+h)-f(x)+f(x)}{x+h}\right]-\left[\frac{f(x-h)-f(x)+f(x)}{x-h}\right]=4xh$$

$$\Rightarrow \left[\frac{f(x+h)-f(x)}{x+h}\right]-\left[\frac{f(x-h)-f(x)}{x-h}\right]=4xh+f(x)\left[\frac{1}{x-h}-\frac{1}{x+h}\right]$$

$$\Rightarrow \left[\frac{f(x+h)-f(x)}{x+h}\right]-\left[\frac{f(x-h)-f(x)}{x-h}\right]=4xh+\frac{2hf(x)}{x^2-h^2}$$

$$\Rightarrow \left[\frac{f(x+h)-f(x)}{h}\right].\left(\frac{1}{x+h}\right)+\left[\frac{f(x-h)-f(x)}{-h}\right]\left(\frac{1}{x-h}\right)=4x+\frac{2f(x)}{x^2-h^2}$$

Taking limit of both sides as $h \to 0$;

$$\frac{f'(x)}{x}+\frac{f'(x)}{x}=4x+\frac{2f(x)}{x^2} \quad \Rightarrow \frac{f'(x)}{x}=2x+\frac{f(x)}{x^2}$$

$f'(x)=2x^2+\dfrac{f(x)}{x} \Rightarrow f'(x)+\left[\dfrac{-f(x)}{x}\right]=2x^2$, which is a linear differential equation

Integrating factor $=e^{-\int\frac{1}{x}dx}=\dfrac{1}{x}$

Its solution is

$$f(x).\frac{1}{x}=\int 2x^2\times\frac{1}{x}+c \Rightarrow \frac{f(x)}{x}=x^2+c \Rightarrow f(x)=x^3+cx$$

But $f(1)=1 \ \Rightarrow c=0$

So, $f(x)=x^3$

$f'(x)=3x^2$ and $f''(x)=6x$

Here; $f''(x)=0 \Rightarrow x=0$

So, at $x=0, f(x)$ has point of inflexion

$$\int_{-1}^{2} x^3 dx = \int_{-1}^{1} x^3 dx + \int_{1}^{2} x^3 dx = 0 + \frac{1}{4}\left[x^4\right]_1^2 = \frac{15}{4}$$

The area of the region bounded by the curves $y = f(x)$ and $y = x^2$ is

$$\int_0^1 \left(x^2 - x^3\right) dx = \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}\text{sq.units}$$

16. $f^{-1}\left(f(x)\right).f'(x) - (\cos x - f(x)) = 0$

$xf'(x) + f(x) = \cos x \Rightarrow \ xf(x) = \sin x + C \Rightarrow C = 0$

$\therefore f(x) = \dfrac{\sin x}{x}$ and then proceed we get $\sin x = 0, \tan x = 2$

17. $$\int_0^{\pi/2} f(x)dx = \int_0^{\pi/2} \frac{\sin x}{x} dx$$

$$\frac{2}{\pi} < \frac{\sin x}{x} < 1$$

$$1 < \int_0^{\pi/2} \frac{\sin x}{x} dx < \frac{\pi}{2}$$

18. Conceptual

19. $$\Rightarrow \left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2 y\, dy = 0 \Rightarrow 2y\frac{dy}{dx} - \frac{2}{x}y^2 = \frac{-2e^{\frac{1}{x^3}}}{x^2};\ put\ y^2 = t$$

Proceed like L.D.E.

20. $$\frac{dy}{dx} + \frac{y}{x} = y^3 \ \Rightarrow \ \frac{1}{y^3}\frac{dy}{dx} + \frac{1}{xy^2} = 1;\ put\ \frac{1}{y^2} = t$$

Proceed like L.D.E.

21. $$\frac{1}{y^3}\frac{dy}{dx} + \frac{x}{y^2} = x^3 \ \text{ put } \ -\frac{1}{2y^2} = t$$

Proceed like L.D.E.

22–24 :

$A = \int_0^a f(x)dx$ and $B = \int_a^1 f(x)dx$

$$\int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a) + 3a + b$$

Differentiating w.r.t 'a' on both sides

$$2f(a) = 2f^1(a) + 3 \;\; \forall a \in (0,1)$$

$$\therefore 2\frac{dy}{dx} = 2y - 3 \Rightarrow \frac{2dy}{2y-3} = dx \Rightarrow \log|2y-3| = x + c$$

Using $f(1) = 0$, we get $c = \log 3 - 1 = \ln\left(\frac{3}{e}\right)$

$$\therefore 3 - 2y = 3e^{x-1} \Rightarrow y = \frac{3}{2}\left[1 - e^{x-1}\right]$$

25-27 :

$$\frac{dy}{dx} = \frac{x-3}{y-4} \Rightarrow (y-4)dy = (x-3)dx$$

Integrating both sides we have, $\frac{y^2}{2} - 4y = \frac{x^2}{2} - 3x + C$

But it passes through $(0,0)$ So, $C = 0 \Rightarrow y^2 - 8y = x^2 - 6x$

$$\Rightarrow (x-3)^2 - 9 = (y-4)^2 - 16 \Rightarrow (x-3)^2 - (y-4)^2 = -7$$

$$\Rightarrow (x-3)^2 - (y-4)^2 = -\left(\sqrt{7}\right)^2$$

Which is a rectangular hyperbola whose eccentricity is equal to $\sqrt{2}$ and centre at $(3,4)$

Let $X = x - 3$

$Y = y - 4$

$X^2 - Y^2 = -\left(\sqrt{7}\right)^2$

Its asymptotes are $Y = \pm X$

i.e $y - 4 = \pm(x-3)$

i.e $x - y + 1 = 0$ and $x + y - 7 = 0$

So the combined equation of asymptotes are $x^2 - y^2 - 6x + 8y - 7 = 0$

28. Given DE is $\frac{dy}{dx} + \frac{x}{1-x^2}y = \frac{2x}{1-x^2}$

$$\Rightarrow y = 2 + c\sqrt{1-x^2} \Rightarrow (y-z)^2 = c^2(1-x^2)$$

For c=0 ,we get line,

For $c \neq 0, \frac{x^2}{1} + \frac{(y-z)^2}{c^2} = 1$,a central conic.

29. The point which bisects all chords is the centre of conic (0,2)

30. Also for any $c \neq 0$, curves $\frac{x^2}{1} + \frac{(y-2)^2}{c^2} = 1$ pass through (1,2) or (-1,2)

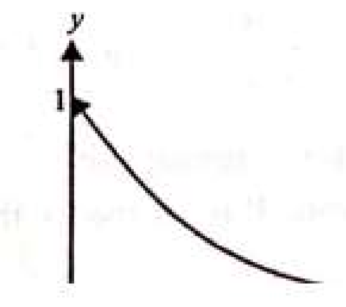
31. $y - y = y'(X - x) \to Q = \left(x - \frac{y}{y'}, 0 \right)$

$PQ^2 = 1 - \left(\frac{y}{y'} \right)^2 + y^2 = 1 \to y' = \frac{-y}{\sqrt{1-y^2}} \to \frac{-\sqrt{1-y^2}}{y} dy = dx$

$y = 1, x = 0 \to x = 0, \theta = \frac{\pi}{2} \to c = 0$

$\therefore$ The curve in parametric form is $x = -\cos\theta - \ln\tan\frac{\theta}{2}, y = \sin\theta$

$x \to \infty, y \to 0 \; as \; \theta \to 0$



$\frac{dy}{dx} = \frac{\cos\theta}{\sin\theta - \cos ec\theta} = -\tan\theta$

$y = \frac{1}{2} \to 0 = \frac{\pi}{6} \to \frac{dy}{dx} = -\frac{1}{\sqrt{3}}$

32. Area = $\int_0^\infty y\,dx, = \sin\theta\; dx = (\sin\theta - \cos ec\theta)d\theta$

$= -\int_0^{\frac{\pi}{2}} \sin\theta(\sin\theta - \cos ec\theta)d\theta = -\int_0^{\frac{\pi}{2}} (\sin^2\theta - 1)d\theta = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

33. The D.E. of c is $y' = -\frac{y}{\sqrt{1-y^2}}$

The D.E. of orthogonal trajectories is $y' = -\frac{\sqrt{1-y^2}}{y} \to dx = \frac{ydy}{\sqrt{1-y^2}}$

Integrating, $x + c = -\sqrt{1-y^2}$

Squaring, $(x+c)^2 = 1 - y^2, (x+c)^2 + y^2 = 1$, family of unit circles with centres on the *x*-axis.

34-36 : At time t ,measured from the instant both start ,rabbit will be at Q(0,at) and dog at P(*x*,*y*)

34. $\therefore \frac{dy}{dt} = \frac{-(at-y)}{x} = \frac{y-at}{x} \Rightarrow xy^1 - y = -at \Rightarrow xy^{11} = -a\frac{dt}{dx} \to 1$

35. Given $\frac{ds}{dt} = b$

$\therefore \ \dfrac{dt}{dx}=\dfrac{dt}{ds}.\dfrac{ds}{dx}=\dfrac{-1}{b}\sqrt{1+\left(\dfrac{dy}{dx}\right)^2} \ =\dfrac{-1}{b}\sqrt{1+\left(y'\right)^2}\rightarrow 2$

.: DE of path of dog is $xy^{11}=\dfrac{a}{b}\sqrt{1+\left(y^1\right)^2}\,(from \ \ 1 \ \ \& \ 2\ )$

36. Putting $y^1=z$ and solving the above , we get $z=\dfrac{dy}{dx}=\dfrac{x^{a/b}-x^{-a/b}}{2}$

37. $\dfrac{dv}{dt}=-\left(g+kv\right)$

$\dfrac{dv}{g+ku}=-dt$

Integrating, $\ln(g+kv)=-kt+A$

$t=0,\ v=u\rightarrow \ln(g+ku)=A$

Subtracting, $\ln\left(\dfrac{g+ku}{g+kv}\right)=kt$

$t=T,\ v=0\rightarrow T=\dfrac{1}{k}\ln\left(1+\dfrac{ku}{g}\right)$

38. $\dfrac{vdv}{dx}=\dfrac{dx}{dt}=-(g+kv)$

$\dfrac{vdv}{g+kv}=-dx$

$\dfrac{kvdv}{g+kv}=-kdx$

$\left(1-\dfrac{g}{g+kv}\right)dv=-kdx$

Integrating, $v-\dfrac{g}{k}\ln\left(g+kv\right)=-kx+A$

$x=0,\ v=u\ \rightarrow\ u-\dfrac{g}{k}\ln\left(g+ku\right)=A$

Eliminating A,

$u-v-\dfrac{g}{k}\ln\left(\dfrac{g+ku}{g+kv}\right)=kx$

$v=0,\ x-H\rightarrow\ H=\dfrac{u}{k}-\dfrac{g}{k^2}\ln\left(1+\dfrac{ku}{g}\right)$

39. $\dfrac{dv}{dt}=g-kv$

$\dfrac{dv}{g-kv}=dt,$ integrating, $\ln(g-kv)=-kt+A$

$v=0,t=0\rightarrow \ln\ g=A$

Eliminating A, $kt = -\ln\left(1 - \frac{kv}{g}\right)$

$1 - \frac{kv}{g} = e^{-kt}$

$v = \frac{g}{k}\left(1 - e^{-kt}\right)$

40. $v = \frac{ds}{dt} = \frac{g}{k}\left(1 - e^{-kt}\right)$

Integrating using s = 0, $t = 0$, $s = \frac{gt}{k} - \frac{g}{k^2}\left(1 - e^{-kt}\right)$

41. $\frac{vdv}{dx} = \frac{dv}{dt} = g - kv$

$\frac{vdv}{g - kv} = dx, \frac{-kvdv}{g - kv} = -kdx \to \left(1 - \frac{g}{g - kv}\right)dv = -kdx$

Integrating using $x = 0$, $v = 0$, $v + \frac{g}{k}\ln\left(1 - \frac{kv}{g}\right) = -kx$

$x = -\frac{v}{k} - \frac{g}{k^2}\ln\left(1 - \frac{kv}{g}\right)$