ANSWER KEYS

1. (2) **2.** (4) **3.** (2) **4.** (1) **5.** (1) **6.** (1) **7.** (4) **8.** (3)
9. (4) **10.** (4)

1. (2)

We have,

$|x-1| \geq |x-3|$

Squaring both sides, we get

$(x-1)^2 \geq (x-3)^2$

$\Rightarrow x^2-2x+1 \geq x^2-6x+9$

$\Rightarrow -2x+1 \geq -6x+9$

$\Rightarrow 4x \geq 8$

$\Rightarrow x \geq 2.$

2. (4)

We have,

$\left|\frac{x^2+6}{5x}\right| \geq 1$

Now, we know that if $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$.

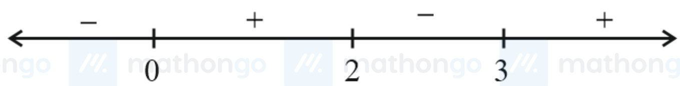
Therefore, first we will solve for

$\frac{x^2+6}{5x} \geq 1$

$\Rightarrow \frac{x^2+6-5x}{5x} \geq 0$

$\Rightarrow \frac{(x-2)(x-3)}{5x} \geq 0$

From the wavy curve, we have



$\therefore x \in (0,2] \cup [3,\infty) \quad \ldots(i)$

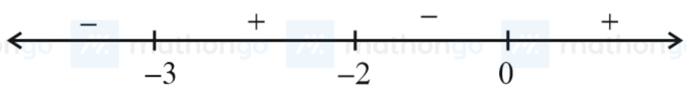
Now,

$\frac{x^2+6}{5x} \leq -1$

$\Rightarrow \frac{x^2+6+5x}{5x} \leq 0$

$\Rightarrow \frac{(x+2)(x+3)}{5x} \leq 0$

From the wavy curve, we have



$\therefore x \in (-\infty,-3] \cup [-2,0) \quad \ldots(ii)$

Combining (i) and (ii), we get

$(-\infty,\ -3] \cup [-2,\ 0) \cup (0,\ 2] \cup [3,\ \infty).$

3. $2-\log_2\left(x^2+3x\right)\geq 0$

(2) $\Rightarrow 2\geq \log_2\left(x^2+3x\right)$

$\Rightarrow x^2+3x-4\leq 0$

$\Rightarrow (x+4)(x-1)\leq 0$

$x\in(-4,1]$ ...(i)

Also $x^2+3x>0$

$\Rightarrow \quad x(x+3)>0$

$\Rightarrow \quad x\in(-\infty,-3)\cup(0,\infty)$ ...(ii)

Combining equations (1) and (2)

$x\in(-4,-3)\cup(0,1]$

Hence, (B) is correct.

4. **(1)** Here, it is given that

$\frac{\log_2\left(x^2-5x+4\right)}{\log_2\left(x^2+1\right)}>1$

$x^2-5x+4>0 \Rightarrow (x-4)(x-1)>0$

$\Rightarrow x\in(-\infty,1)\cup(4,\infty) \qquad \ldots(1)$

$x^2+1>0$ which is true $\forall x\in R \qquad \ldots(2)$

$\log_2\left(x^2-5x+4\right)>\log_2\left(x^2+1\right)$

$x^2-5x+4>x^2+1$

$-5x+3>0$

$x<\frac{3}{5} \quad \ldots(3)$

From equations, (1),(2) and (3) we get

$x\in\left(-\infty,\frac{3}{5}\right)-\{0\}$

5. **(1)** $\log_{0.2}\frac{x+2}{x}\leq 1$

$\Rightarrow \frac{x+2}{x}\geq 0.2$

$\frac{5(x+2)-x}{x}\geq 0$

$\Rightarrow \frac{4x+10}{x}\geq 0$

$x\in\left(-\infty,-\frac{5}{2}\right)\cup(0,\infty)$

Hence, (A) is correct.

6. **(1)** $x - \sqrt{1-|x|} < 0$

Firstly $|x| \leq 1 \Rightarrow x \in [-1, 1]$

If $x \in [-1, 0]$, inequality will always hold true.

If $x \leq [0, 1]$

$x < \sqrt{1-|x|}$

Squaring, we get

$$x^2 \leq 1 - x$$

$$\Rightarrow x^2 + x - 1 < 0$$

$+ \quad - \quad +$

$\frac{-1-\sqrt{5}}{2} \quad \frac{-1+\sqrt{5}}{2}$

$\Rightarrow \quad x \in \left(0, -1 + \frac{\sqrt{5}}{2}\right)$

So, $x \in \left(-1, -1 + \frac{\sqrt{5}}{2}\right)$

Hence, (A) is correct.

7. **(4)** Given $f(x) = [x]$ and $g(x) = |x|$

Now, $f\left(g\left(\frac{8}{5}\right)\right) = f\left(\frac{8}{5}\right) = \left[\frac{8}{5}\right] = 1$

And $g\left(f\left(-\frac{8}{5}\right)\right) = g\left(\left[-\frac{8}{5}\right]\right) = g(-2) = 2$

$\therefore \quad f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(-\frac{8}{5}\right)\right) = 1 - 2 = -1$

8. **(3)**

We have,

$y = 3[x] + 1 = 4[x-1] - 10$

$y = 3[x] + 1 = 4[x] - 14; \ (\because [X + a] = [X] + a, \ a \in Z)$

$\Rightarrow [x] = 15$ and $y = 46$

Then,

$$[x + 2y] = [x] + 2y$$
$$= 15 + 92$$
$$= 107$$

9. **(4)**

Given, $[x]^2 - 5[x] + 6 = 0$

$\Rightarrow [x]^2 - 3[x] - 2[x] + 6 = 0$

(Using middle term factorisation).

$\Rightarrow [x]([x]-3) - 2([x]-3) = 0$

$\Rightarrow ([x]-3)([x]-2) = 0$

$\Rightarrow [x] = 3$ or $[x] = 2$

$\Rightarrow x \in [3, 4)$ or $x \in [2, 3)$

Combining we get, $x \in [2, 4)$.

10. (4)

$y=\{x\}+\{-x\}$
We know that,
$\{x\}+\{-x\}=0; x \in I$
& $\{x\}+\{-x\}=1$; otherwise
$\therefore y=0 \,\forall\, x \in$ I and $y=1$ otherwise
For $x \in[-1,\ 2]$
$y=0$ at $x=-1,\ 0,\ 1,\ 2$
$y=1 \,\forall\, x \in[-1,\ 2]-\{-1,\ 0,\ 1,\ 2\}$