Q1 - 24 June - Shift 2

Let a circle $C : (x-h)^2 + (y-k)^2 = r^2$, $k > 0$, touch the x-axis at (1, 0). If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to _____.

Space for your notes:

Q2 - 25 June - Shift 1

Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 : 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at (–1, 2) and L_2 at (3, – 6), then the equation of the circle C is

(A) $(x-1)^2 + (y-2)^2 = 4$

(B) $(x+1)^2 + (y-2)^2 = 4$

(C) $(x-1)^2 + (y+2)^2 = 16$

(D) $(x-1)^2 + (y-2)^2 = 16$

Space for your notes:

Q3 - 25 June - Shift 1

Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to

Space for your notes:

Q4 - 25 June - Shift 2

A circle touches both the y-axis and the line $x + y = 0$. Then the locus of its center is

(A) $y = \sqrt{2}x$ (B) $x = \sqrt{2}y$

(C) $y^2 - x^2 = 2xy$ (D) $x^2 - y^2 = 2xy$

Space for your notes:

Q5 - 26 June - Shift 1

Let C be a circle passing through the points A(2, –1) and B(3, 4). The line segment AB is not a diameter of C. If r is the radius of C and its centre lies on the circle $(x - 5)^2 + (y - 1)^2 = \frac{13}{2}$, then r^2 is equal to :

(A) 32 (B) $\frac{65}{2}$ (C) $\frac{61}{2}$ (D) 30

Space for your notes:

Q6 - 27 June - Shift 2

The set of values of k for which the circle C : $4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C is :

(A) An empty set (B) $\left(6, \frac{95}{9}\right]$

(C) $\left[\frac{80}{9}, 10\right)$ (D) $\left(9, \frac{92}{9}\right]$

Space for your notes:

Q7 - 27 June - Shift 2

Let a circle C of radius 5 lie below the x-axis. The line $L_1 = 4x + 3y - 2$ passes through the centre P of the circle C and intersects the line $L_2 : 3x - 4y - 11 = 0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x - 12y + 51 = 0$ is

Space for your notes:

Q8 - 28 June - Shift 1

If the tangents drawn at the point O(0, 0) and $P\left(1+\sqrt{5}, 2\right)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to

(A) $\frac{3+\sqrt{5}}{2}$ (B) $\frac{4+2\sqrt{5}}{2}$

(C) $\frac{5+3\sqrt{5}}{2}$ (D) $\frac{7+3\sqrt{5}}{2}$

Space for your notes:

Q9 - 28 June - Shift 1

Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle $C : (x-h)^2 + (y-k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to ____.

Space for your notes:

Q10 - 28 June - Shift 2

If one of the diameters of the circle $x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14=0$ is a chord of the circle $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$, then the value of r^2 is equal to

Space for your notes:

Q11 - 29 June - Shift 1

Let the tangent to the circle $C_1: x^2+y^2=2$ at the point $M(-1,1)$ intersect the circle $C_2: (x-3)^2+(y-2)^2=5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to :

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{1}{6}$ (D) $\frac{5}{3}$

Space for your notes:

Q12 - 29 June - Shift 2

Let a triangle ABC be inscribed in the circle $x^2-\sqrt{2}(x+y)+y^2=0$ such that $\angle BAC=\frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to:

(A) $\left(\sqrt{2}+\sqrt{6}\right)/3$ (B) $\left(\sqrt{6}+\sqrt{3}\right)/2$

(C) $\left(3+\sqrt{3}\right)/4$ (D) $\left(\sqrt{6}+2\sqrt{3}\right)/4$

Space for your notes:

Answer Key

Q1 (7)	**Q2** (C)	**Q3** (7)	**Q4** (D)
Q5 (B)	**Q6** (D)	**Q7** (11)	**Q8** (C)
Q9 (816)	**Q10** (10)	**Q11** (C)	**Q12** (Dropped)

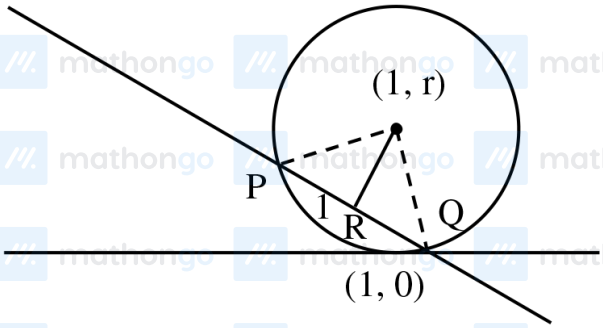
Q1 (7)

$k = r$

$h = 1$

$OP = r,\ PR = 1$

$OR = \left|\frac{r+1}{\sqrt{2}}\right|$



$$r^2 = 1 + \frac{(r+1)^2}{2}$$

$$2r^2 = 2 + r^2 + 1 + 2r$$

$$r^2 - 2r - 3 = 0$$

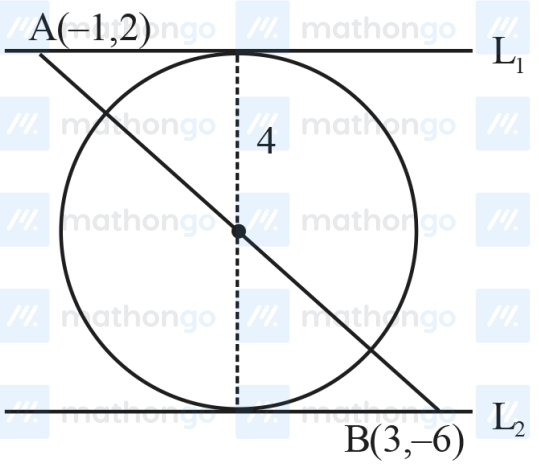
$$(r-3)(r+1) = 0$$

$$\boxed{r = 3}, -1$$

$$h + k + r = 1 + 3 + 3$$

$$= 7$$

Q2 (C)



$L_1 : 4x - 3y + K_1 = 0$

$L_2 : 4x - 3y + K_2 = 0$

now

$-4 - 6 + K_1 = 0 \Rightarrow K_1 = 10$

$12 + 18 + K_2 = 0 \Rightarrow K_2 = -30$

$\Rightarrow$ Tangent to the circle are

$4x - 3y + 10 = 0$

$4x - 3y - 30 = 0$

Length of diameter $2r = \frac{|10+30|}{5} = 8$

$\Rightarrow r = 4$

Now centre is mid point of A & B

$x = 1, y = -2$

Equation of circle

$(x-1)^2 + (y+2)^2 = 16$ **Ans.**

Q3 (7)

$2x^2 - rx + p = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

$y^2 - sy - q = 0 \begin{cases} y_1 \\ y_2 \end{cases}$

Equation of the circle with PQ as diameter is

$2(x^2 + y^2) - rx - 2sy + p - 2q = 0$

on comparing with the given equation

$r = 11,\ s = 7$

$p - 2q = -22$

$\therefore\ 2r + s - 2q + p = 22 + 7 - 22 = 7$

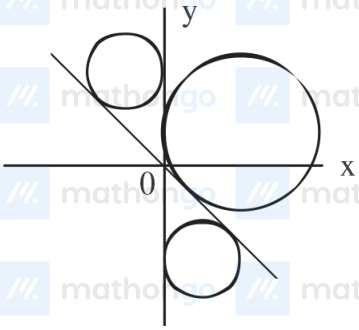
Q4 (D)

Let (h, k) is centre of circle

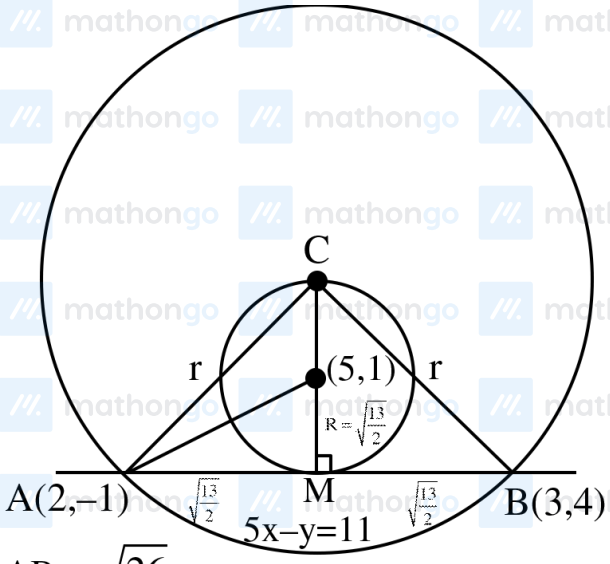
$$\left|\frac{h-k}{\sqrt{2}}\right| = |h|$$

$$k^2 - h^2 + 2hk = 0$$

$\therefore$ Equation of locus is $y^2 - x^2 + 2xy = 0$



Q5 (B)



$AB = \sqrt{26}$

$r^2 = CM^2 + AM^2$

$$= \left(2 \times \sqrt{\frac{13}{2}}\right)^2 + \left(\sqrt{\frac{13}{2}}\right)^2$$

$$r^2 = \frac{65}{2}$$

Q6 (D)

$C : 4x^2 + 4y^2 - 12x + 8y + k = 0$

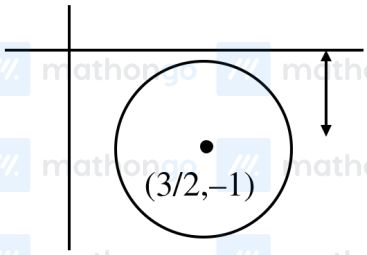
$\Rightarrow x^2 + y^2 - 3x + 2y + \left(\frac{k}{4}\right) = 0$

Centre $\left(\frac{3}{2}, -1\right)$; $r = \sqrt{\frac{13-k}{2}} \Rightarrow k \le 13$ ... (1)

(i) Point $\left(1, \frac{-1}{3}\right)$ lies on or inside circle C

$\Rightarrow S_1 \le 0 \Rightarrow k \le \frac{92}{9}$ ... (2)

(ii) C lies in 4th quadrant



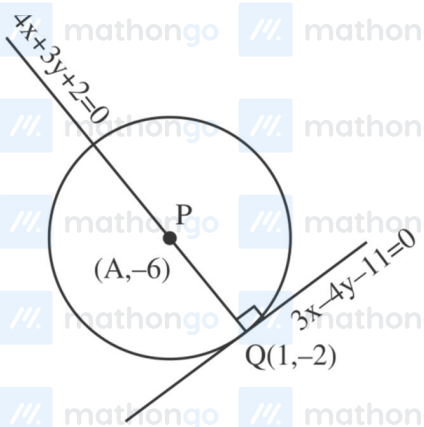
$r < 1$

$\Rightarrow \frac{\sqrt{13-k}}{2} < 1$

$\Rightarrow k < 9$ .... (3)

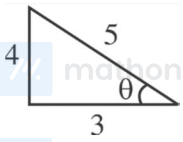
Hence (1) $\cap$ (2) $\cap$ (3) $\Rightarrow$ $k \in \left(9, \frac{92}{9}\right]$

Q7 (11)



$4x + 3y + 2 = 0$

$3x - 4y - 11 = 0$



$$\frac{x}{-25} = \frac{y}{50} = \frac{1}{-25}$$

$$\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \pm 5$$

$$y = -2 + 5\left(-\frac{4}{5}\right) = -6$$

$$x = 1 + 5\left(\frac{3}{5}\right) = 4$$

Req. distance

$$\left|\frac{5(4) - 12(-6) + 51}{13}\right|$$

$$= \left|\frac{20 + 72 + 51}{13}\right|$$

$$= \frac{143}{13} = 11$$

Q8 (C)

Tangent at O

$-(x + 0) - 2(y + 0) = 0$

$\Rightarrow \boxed{x + 2y = 0}$

Tangent at P

$x(1 + \sqrt{5}) + y.2 - (x + 1 + \sqrt{5}) - 2(y + 2 = 0)$

Put $x = -2y$

$-2y(1 + \sqrt{5}) + 2y + 2y - 1 - \sqrt{5} - 2y - 4 = 0$

$-2\sqrt{5}\,y = 5 + \sqrt{5} \Rightarrow y = \left(\frac{\sqrt{5}+1}{2}\right)$

$Q\left(\sqrt{5}+1, -\frac{\sqrt{5}+1}{2}\right)$

Length of tangent $OQ = \frac{5+\sqrt{5}}{2}$

$\text{Area} = \frac{RL^3}{R^2 + L^2}$

$R = \sqrt{5}$

$$= \frac{\sqrt{5} \times \left(\frac{5+\sqrt{5}}{2}\right)^3}{5 + \left(\frac{5+\sqrt{5}}{2}\right)^2}$$

$$= \frac{\sqrt{5}}{2} \times \frac{4 \times \left(125 + 75 + 75\sqrt{5} + 5\sqrt{5}\right)}{\left(20 + 25 + 10\sqrt{5} + 5\right)}$$

$$= \frac{5 + 3\sqrt{5}}{2}$$

Q9 (816)

Normal are

$y + 2x = \sqrt{11} + 7\sqrt{7},$

$2y + x = 2\sqrt{11} + 6\sqrt{7}$

Center of the circle is point of intersection of normals i.e.

$\left(\frac{8\sqrt{7}}{3}, \sqrt{11} + \frac{5\sqrt{7}}{3}\right)$

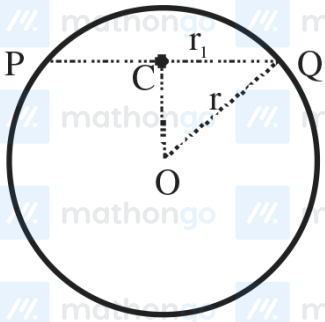
Tangent is $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$

Radius will be $\perp$ distance of tangent from center

i.e. $4\sqrt{\frac{7}{5}}$

Now $(5h - 8k)^2 + 5r^2 = 816$

Q10 (10)



PQ is diameter of circle

$$S: x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14=0$$

$$C\left(\sqrt{2},3\sqrt{2}\right), O\left(2\sqrt{2},2\sqrt{2}\right)$$

$$r_1=\sqrt{6}$$

$$S_1:\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$$

Now in ΔOCQ

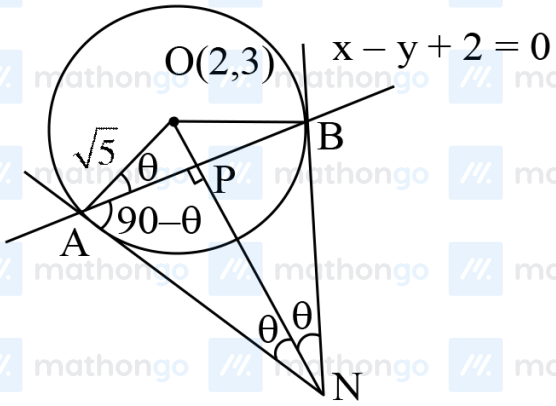
$$|OC|^2+|CQ|^2=|OQ|^2$$

$$4+6=r^2$$

$$r^2=10$$

Q11 (C)

$$OP = \left|\frac{2-3+2}{\sqrt{2}}\right|$$



$$OP = \frac{3}{\sqrt{2}}$$

$$AP = \sqrt{OA^2 - OP^2}$$

$$= \frac{1}{\sqrt{2}}$$

$$\tan\theta = 3$$

$$\therefore \sin\theta = \frac{3}{\sqrt{10}} = \frac{AP}{AN}$$

$$\Rightarrow AN = \frac{\sqrt{5}}{3} = BN$$

$$\text{Area of } \Delta ANB = \frac{1}{2}.\left(AN^2\right)\sin 2\theta = \frac{1}{6}$$

Q12 (Dropped)

Radius of given circle is 1.

$BC = \text{diameter} = 2,\ AB = \sqrt{2}$

$AC = \sqrt{BC^2 - AB^2} = \sqrt{2}$

$\Delta ABC = \frac{1}{2} AB.AC = 1$