system → bothbodies

Sri Chaitanya IIT Academy.,India.

✪ A.P ✪ T.S ✪ KARNATAKA ✪ TAMILNADU ✪ MAHARASTRA ✪ DELHI ✪ RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS & STERLING_BT Paper -1(Adv-2022-P1-Model Date: 10-09-2023

Time: 09.00Am to 12.00Pm RPTA-06 Max. Marks: 180

KEY SHEET

MATHEMATICS

1	**16**	2	**4**	3	**15**	4	**1**	5	**62**	6	**8**
7	**9**	8	**0**	9	**AC**	10	**BCD**	11	**ABD**	12	**ABCD**
13	**ABD**	14	**ABC**	15	**A**	16	**B**	17	**C**	18	**D**

PHYSICS

19	**8**	20	**3**	21	**2**	22	**2.5**	23	**14**	24	**0.375**
25	**20**	26	**0.5**	27	**ABD**	28	**AC**	29	**ABC**	30	**ACD**
31	**ABCD**	32	**AD**	33	**A**	34	**A**	35	**C**	36	**A**

CHEMISTRY

37	**6**	38	**7**	39	**6**	40	**3**	41	**5**	42	**2**
43	**5**	44	**3**	45	**ABCD**	46	**ABC**	47	**AD**	48	**ABC**
49	**ABD**	50	**ACD**	51	**A**	52	**B**	53	**C**	54	**A**

SOLUTIONS
MATHEMATICS

1. Differentiate both sides with respect to x, we get

$$f'(x)=(1-x)\frac{1}{x}+\ln\left(\frac{x}{e}\right)(-1)+f(x)\Rightarrow f'(x)-f(x)=\left(\frac{1}{x}-1-\ln\left(\frac{x}{e}\right)\right)$$

$\therefore$ Multiplying both sides with e^{-x}, we get

$$e^{-x}f'(x)-e^{-x}f(x)=e^{-x}\left(\frac{1}{x}-1-\ln\left(\frac{x}{e}\right)\right)$$

$$\Rightarrow \quad \frac{d}{dx}\left(e^{-x}\cdot f(x)\right)=e^{-x}\left(\frac{1}{x}-1-\ln\left(\frac{x}{e}\right)\right)$$

$\therefore$ on integrating both sides wrt x, we get $e^{-x}f(x)=e^{-x}-\int e^{-x}\left(\ln\left(\frac{x}{e}\right)-\frac{1}{x}\right)dx$

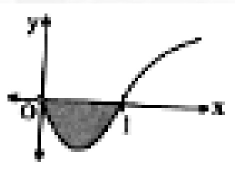
$$=e^{-x}+e^{-x}\cdot\ln\left(\frac{x}{e}\right)+\lambda\Rightarrow f(x)=1+\ln\left(\frac{x}{e}\right)+\lambda e^{x}\quad ...(i)$$

Put x = 1 in original equation we get $f(1)=e$. $...(ii)$

Using (i) and (ii) equation we get $\lambda=1$.

Thus, $f(x)=1+e^{x}+\ln\left(\frac{x}{e}\right) or\ f(x)=e^{x}+\ln x$

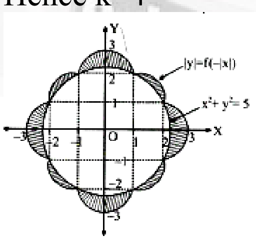
Hence, $g(x)=x\ \ln x$; now $A=\int_0^1 x\ \ln x\, dx=\frac{1}{4}$



2. we have $y=\left|\frac{\pi}{2}-\sin^{-1}(\sin x)\right|-\left|\frac{\pi}{2}-\cos^{-1}(\cos x)\right|=4\pi-2x$

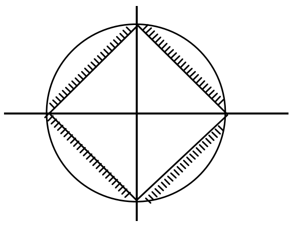
Required area $=\frac{1}{2}\times\left(\frac{\pi}{2}\right)\times\pi=\frac{\pi^2}{4}=\frac{\pi^2}{k}$

Hence k=4



3.

4.



$A=4\left[\frac{\pi}{4}-\frac{1}{2}\right]=\pi-2 \quad [A]=1$

5. Given $y\left(\frac{d^2y}{dx^2}\right)=2\left(\frac{dy}{dx}\right)^2 \Rightarrow \frac{y''}{y'}=\frac{2y'}{y} \Rightarrow \ln y'=2\ln y+\ln a$

$\Rightarrow \frac{y'}{y^2}=a \Rightarrow \int\frac{dy}{y^2}=\int adx \Rightarrow \frac{-1}{y}=ax+b$

Since curve is passing through $(2,2)$ and $\left(8,\frac{1}{2}\right)$

So, $2a+b=\frac{-1}{2}$ ...(i)

$8a+b=-2$ ...(ii)

$\therefore$ On solving (i) and (ii), we get $a=\frac{-1}{4}, b=0$

Hence, $C_1: xy=4$ and curve C_2 is $x^2+y^2=4$.

6. Given $\frac{xdy}{dx}-2y=x^4y^2$.

Now diving by y^2, we get Bernouli's differential equation.

Now after solving we get $\frac{-2}{y}=\frac{x^4}{3}+\frac{C}{x^2}$

Now substitute the values we get $\frac{dy}{dx}=24x^{-5}$

7. Differentiate the given equation wrt x we get, $x.f(x)+\int_0^x f(t)dt+f(x)(1-x)=-e^{-x}$

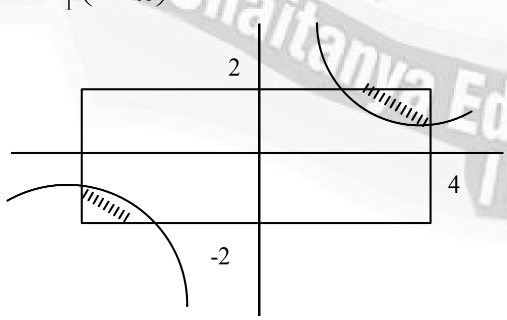
Again Differentiate wrt x we get, $f'(x)+f(x)=e^{-x}$

Now on solving we get $f(x)e^x=x-1$.

8. Differentiate both sides w.r.t x we get $\frac{dy}{dx}-\left(\frac{2x}{1+x^2}\right)y=y^2$

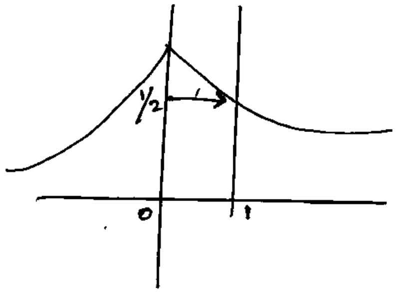
Put $\frac{-1}{y}=t \qquad \frac{dt}{dx}+\left(\frac{2x}{1+x^2}\right)t=1 \qquad f(x)=\frac{-3(1+x^2)}{x^3+3x-3}$

9. $S=2\int_1^4\left(2-\frac{2}{x}\right)dx=4[x-\ln x]_1^4=4(3-\ln^4)$



10. $\int_0^1(2x-x^2-x^n)dx=1-\frac{1}{3}-\frac{1}{n+1}=\frac{1}{2} \Rightarrow n=5$

11. $\frac{1}{e} \le S \le 1 - \frac{1}{e}$; $S \le \int_0^{1/\sqrt{2}} e^{-x^2}\, dx + \int_{1/\sqrt{2}}^{1} e^{-x^2}\, dx$



12. Put $y = vx$

13. On solving the given DE we get, $\frac{-1}{y} = -e^{\frac{1}{x}} + C$

$\because \lim_{x \to 0^-} f(x) = 1 \Rightarrow C = -1$

On solving $\frac{-1}{y} = -e^{\frac{1}{x}} - 1 \Rightarrow y = \frac{1}{1 + e^{\frac{1}{x}}}$

$\frac{dy}{dx} > 0\ \forall x \in R - (0)$

14. $\frac{dy}{dx} - (\cot x) y = \frac{-\sin x}{x^2}$ $\qquad y = \frac{\sin x}{y}$

15. Conceptual based

16. Conceptual based

17. A) $x^2(ydx + xdy) = \frac{xdy - ydx}{y^2} = (xy)\left(\frac{x}{y}\right) d(xy) = d\left(\frac{x}{y}\right)$ $\frac{x^2y^2}{2} = 1h\left(\frac{x}{y}\right) + k$

B) $2x^3ydy + (1 - y^2)(x^2y^2 + y^2 - 1)dx = 0$

$\frac{2y}{(1-y^2)^2} = \frac{dy}{dx} + \frac{y^2}{1-y^2}\frac{1}{x} = \frac{1}{x^3}$ Put $\frac{y^2}{1-y^2} = 4$ $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^3}$

C) Apply C & D, $\frac{dy}{dx} = \frac{e^{-x}}{e^x} = e^{-2x}$

D) *In* $\frac{dy}{dx} = 3x + 4y \Rightarrow \frac{dy}{dx} = e^{3x+4y}$

18. A) $\frac{dy}{dx} = c$, on putting c in equation we get

$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$

B) Different two & elements a & b

C) $2y\frac{dy}{dx} = ya$ substitute a in equation

D) Multiplying equation by e^x, we get

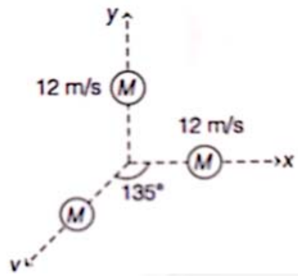
$xy^{ex} - ae^{2x} + b + x^2e^x$ on differentiating, we get

$ye^x + xy,\ ex + xye^x = 2xe^{2x} + xe^x + 2xe^x.$

Divide by e^{2x} & differentiates agree

PHYSICS

19. The momentum of third part will be equal and opposite of the resultant of momentum of rest two equal parts.

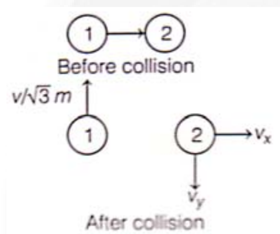


Let u be the velocity of third part. By the conservation of linear momentum,

$3m \times u = m \times 12\sqrt{2} \quad \Rightarrow u = 4\sqrt{2}\text{m/s}$

20. $In\ x-$direction, apply conservation of momentum, we get

$mu_1 + 0 = mu_x \quad \Rightarrow m\upsilon = m\upsilon_x \quad \Rightarrow \upsilon_x = \upsilon$



In y-direction, apply conservation of momentum, we get

$$0 + 0 = m\left(\frac{\upsilon}{\sqrt{3}}\right) - m\upsilon_y \Rightarrow \upsilon_y = \frac{\upsilon}{\sqrt{3}}$$

Velocity of second mass after collision, $\upsilon = \sqrt{\left(\frac{\upsilon}{\sqrt{3}}\right)^2 + \upsilon^2} = \sqrt{\frac{4}{3}\upsilon^2}$ $or\ \upsilon' = \frac{2}{\sqrt{3}}\upsilon$

21. $a_{system} = \frac{5g - 5\mu g}{5+5} = 4\text{m/s}^2 \qquad a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2} = \frac{4(4\hat{i}) + 5(4\hat{j})}{10} = \frac{5\sqrt{4^2+4^2}}{10} = 2\sqrt{2}\text{m/s}^2$

22. First sphere will take a time t_1 to start motion in second sphere on colliding with it, where $t_1 = \frac{L}{u}$. Now speed of second sphere will be $\upsilon_2 = \frac{u}{2}(1+e) = \frac{2}{3}u$

Hence, time taken by second sphere to start motion in third sphere $t_2 = \frac{L}{2/3u} = \frac{3L}{2u}$.

$\therefore \text{Totalotimet} = t_1 + t_2 = \frac{L}{u} + \frac{3L}{2u} = \frac{5L}{2u}$

23. We know that velocity of 2 nd ball after collision is given by

$$\upsilon_2 = \frac{u_1(1+e)m_1}{(m_1+m_2)} + u_2\frac{(m_2-m_1)}{(m_1+m_2)}$$

In present problem $u_2 = 0, m_2 = 2m_1$ and $e = 2/3$, hence $\upsilon_2 = \frac{u\left(1+\frac{2}{3}\right)}{(m_1+2m_1)} = \frac{5}{9}u$ As four exactly similar type of collisions are taking place successively, hence velocity communicated to fifth ball $\upsilon_5 = \left(\frac{5}{9}\right)^4 u$

24. Let ball strikes at a speed u, the$K_1 = \frac{1}{2}mu^2$. Due to collision , tangential component of velocity remains unchanged at $u, \sin 45^0$, but the normal component of velocity change to $u, \sin 45^0 = \frac{1}{2}u \cos 45^0$

$\therefore$ Final velocity of ball after collision $\upsilon = \sqrt{\left(u \sin 45^0\right) + \left(\frac{1}{2}u \cos 45^0\right)^2}$

$$= \sqrt{\left(\frac{u}{\sqrt{2}}\right)^2 + \left(\frac{u}{2\sqrt{2}}\right)^2} = \sqrt{\frac{5}{8}}u$$

Hence, final kinetic energy $K_2 = \frac{1}{2}m\upsilon^2 = \frac{5}{16}mu^2$

$\therefore$ Fractional loss in KE

$$= \frac{K_1 - K_2}{K_1} \qquad \frac{\frac{1}{2}mu^2 - \frac{5}{16}mu^2}{\frac{1}{2}mu^2} = \frac{3}{8}$$

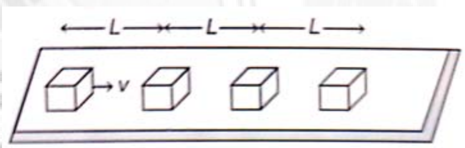
25. Let at the time of explosion velosion velocity of one piece of mass m/2 is $(10\hat{i})$. If velocity of other be v_2 then from conservation law of momentum (since there is no force in horizontal direction), horizontal component of v_2 must be $-10\hat{i}$.

$\therefore$ Relative velocity of two parts in horizontal direction $=20\,\text{ms}^{-1}$ Time taken by ball to fall through 45 m , $= 20\sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} = 3s$ and time taken by ball to fall through first 20 m, $t' = \sqrt{\frac{2h'}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2s$

Hence time taken by ball pieees to fall from 25 m height to ground $= t - t' = -3 - 2 = 1s$.

$\therefore$ Horizontal distance between the two piece at the time of striking on ground $20 \times 1 = 20m$.

26.



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Since, collision is perfectly inelastic, so all the block will stick together one by one and move in a form of combined mass.

Time required to cover distance (d) by first block $= \frac{L}{\upsilon}$. Now first and second block will stick together and move with $\upsilon/2$ velocity (by applying conservation of momentum) and combined system will take $\frac{L}{\upsilon/2} = \frac{2L}{\upsilon}$ to reach upto block third. Now, these three blocks will move with velocity $\upsilon/3$ and combined system will take time $\frac{L}{\upsilon/3} = \frac{3L}{\upsilon}$ to reach up to the fourth block. So, total time $\frac{L}{\upsilon} + \frac{2L}{\upsilon} + \frac{3L}{\upsilon} + ... \frac{(n-1)L}{\upsilon} = \frac{n(n-1)L}{2\upsilon}$ Final velocity of the centre of mass of the system will be υ/n.

27. $f_{\min} = \mu mg \quad a_{boat} = \dfrac{\mu mg}{M}$

a_{man} with respect to boat $= = \mu g + \dfrac{\mu mg}{M} = \dfrac{\mu(M+m)g}{M}$

$\ell = \dfrac{1}{2}\dfrac{\mu(M+m)g}{M}t^2 \quad v = 0(\text{no externalforce})$

28. By symmetry, velocity of each is 5 m/s

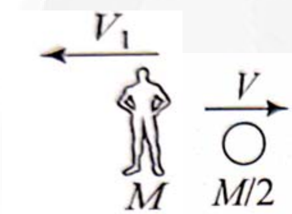
$T = \dfrac{mv^2}{R} = \dfrac{75\times 25}{5} = 375N$

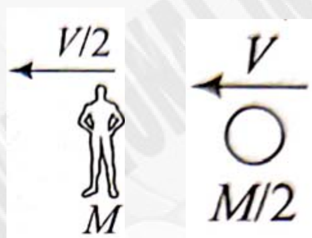
Acceleration $\dfrac{v^2}{R} = \dfrac{25}{5} = 5\ \text{m/s}^2$

29. $X_{CM} = \dfrac{2\times 15}{5} = 6$

$Y_{CM} = \dfrac{3\times 20}{5} = 12$

30. Initially After collision





$MV_1 = MV/2$ $\quad$ MV/2 + MV/2 = 3MV'/2

$V_1 = V/2$ $\quad$ V' = 2V/3

$\therefore \text{Im}\,pulse = M\left(\dfrac{2V}{3} - \dfrac{V}{2}\right) = \dfrac{MV}{6}$

Time $= \dfrac{D}{V} + \dfrac{D + \dfrac{V}{2}\left(\dfrac{D}{V}\right)}{V - (V/2)} = \dfrac{D}{V} + \dfrac{3D}{V} = \dfrac{4D}{V}$

31. Velocity of plank = +4m/s

Velocity of centre of mass, $V_{CM} = 0$

$= \dfrac{40(v_1+4) - 40(v_2-4) + 20(4)}{40+40+20}$

$\Rightarrow v_2 - v_1 = 10$

32. Final momentum in y-axis is zero.

So $MV_1 = \left(\dfrac{5}{12}V_0 \sin\theta\right)$ 3 M

$V_1 = \dfrac{3}{4}V_0$

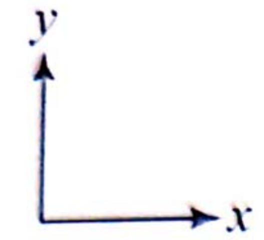
Along $x-axis$ Initial momentum = final momentum $\left(\dfrac{5}{12}V_0 \sin\theta\right)$ $3M = MV_1'$

$V_1' = V_0$

33. $v_{cm} = \dfrac{(3mv) - 2mv}{5m} = \dfrac{v}{5}$ In COM frame

Initial velocity of $A = \left(-v - \frac{v}{5}\right) = -\frac{6v}{5}$ to right

Initial velocity of $B = v - \frac{v}{5} = \frac{4}{5}v = \frac{4}{5}v$ to left Blocks are doing SHM in COM frame with initial position as equilibrium position velocity variation of A in ground frame, considering right as+ v e from $\left(\frac{4v}{5} + \frac{v}{5}\right) = v\, to - \frac{4v}{5} + \frac{v}{5} = -\frac{3v}{5}$ $So \left|v_{A_{max}}\right| = v\, and \left|v_{A_{max}}\right| = 0$

34. 

momentum remains conserved during explosion

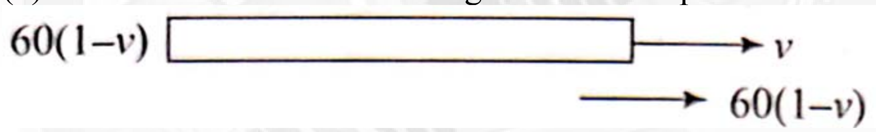
(a) According to shown pieces net momentum of the pieces along x axis is-ve (impossible)

(b) Net momentum of the pieces along y axis is non zero (impossible)

(c) Along x-axis momentum remains positive $m_1v_1 - m_2v_2 > 0$

$\frac{m_1}{m_2} > \frac{v_2}{v_1} > 1 \Rightarrow m_1 > m_2$

(d)Momentum conservation along x and axes is possible for any mass ratio

35. 

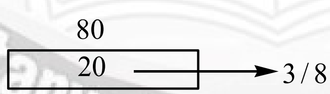
$P_i = P_p \Rightarrow 0 = 60(1+v) + (80+20)v \quad \Rightarrow v = -\frac{3}{8}$ is opposite to 2 m/s velocity of Ram I . e $\frac{3}{8}$ m/s towards rigid .

(b) $0 = 80(1+v) + (60+20)v \Rightarrow v = -\frac{1}{2}$ m/s

(c) $80(1+v) + 60(-1+v) + 20v = 0$

80 - 60 + 800 + 60 v + 20 v = 0- 20 =160 v

$v = -\frac{1}{8}$ m/s

(d)After jump of Ram 

Now $(80+20)\frac{3}{8} = 80(1+v+20v$

$v = -\frac{17}{40}$ m/s

36. Earth is not part of system in P, Q , R, S , T

$\therefore$ GPE is not defined for these systems, and their exists no other form of PE

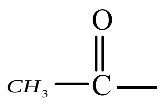
$\therefore$ PE = 0 for them, similarly, Uspring is not defined for option's

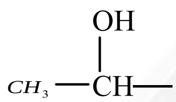
CHEMISTRY

37. $X = \beta$-Hydroxy carbonyl compound

$Y =$ Acids will react with $NaHCo_3$

38.





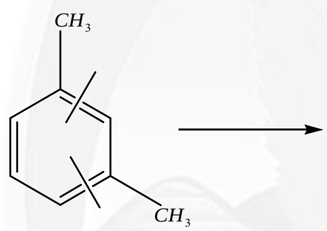
And Hemi Acetals will give positive Halo Form test.

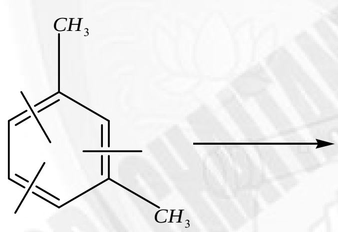
39. 6 moles are consumed

40. Aldehydes are more reactive than ketones to wards nucleophilic Addition reactions

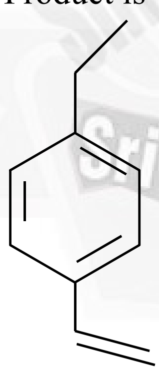
41. Aldehydes without α-Hydrogen will give cannizzaro reaction

42.



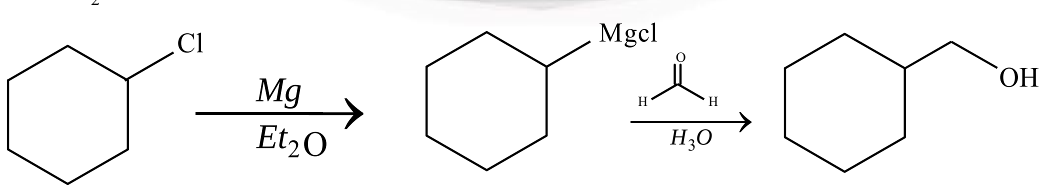


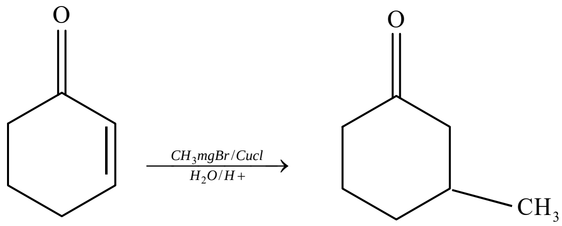
43. Product is



44. Three CO_2 molecules are released

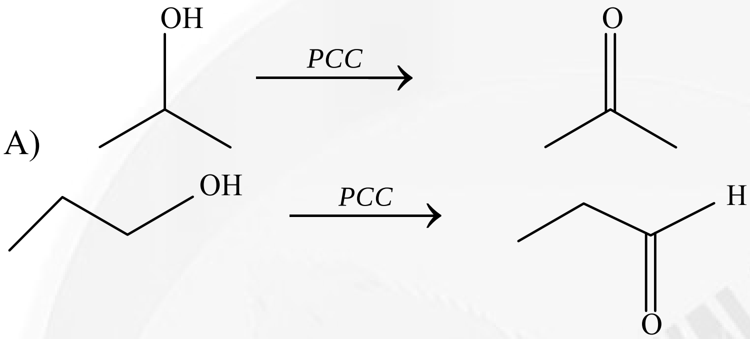
45. A)



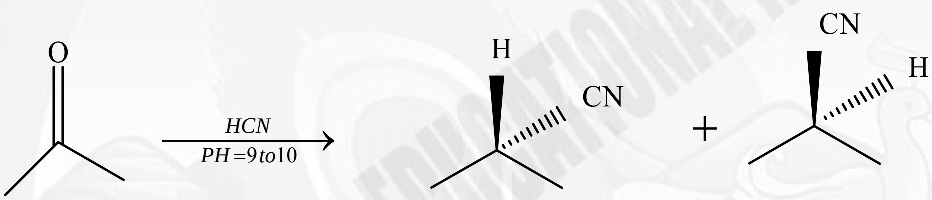


B)
1,4 addition product is major

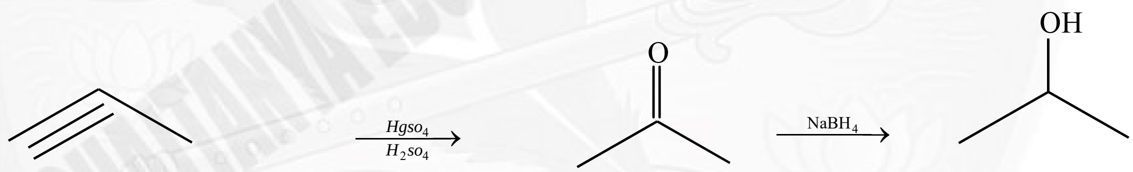
46. A)



B)

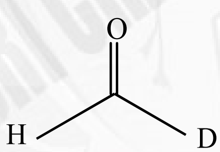


C)



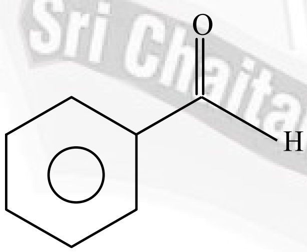
D) Will not form stable hydrate

47. . A)



$(\text{no } \alpha-\text{hydrogen})$

D)



$(\text{no } \alpha-\text{hydrogen})$

48. ABC forms stable hydrates on addition of H_2O
49. Conceptual
50. Compound B doesn't undergoes decarboxylation according to BREDT'S RULE
51. Conceptual
52. Conceptual
53. Conceptual
54. Conceptual