DPP 8.1

Expansion of Determinant

Single Correct Answer Type

1. $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\beta \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$

   if $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$

   (a) 1 (b) 2 (c) 3/2 (d) 1/2

2. If α, β, γ are roots of the equation $x^2(px + q) = r(x + 1)$,

   then the value of determinant $\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix}$ is

   (a) $\alpha\beta\gamma$ (b) $1+\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$

   (c) 0 (d) none of these

3. If $\omega \neq 1$ is cube root of unity and $x + y + z \neq 0$, then

   $\begin{vmatrix} \dfrac{x}{1+\omega} & \dfrac{y}{\omega+\omega^2} & \dfrac{z}{\omega^2+1} \\ \dfrac{y}{\omega+\omega^2} & \dfrac{z}{\omega^2+1} & \dfrac{x}{1+\omega} \\ \dfrac{z}{\omega^2+1} & \dfrac{x}{1+\omega} & \dfrac{y}{\omega+\omega^2} \end{vmatrix} = 0$, if

   (a) $x^2 + y^2 + z^2 = 0$

   (b) $x + y\omega + z\omega^2 = 0$ or $x = y = z$

   (c) $x \neq y \neq z \neq 0$

   (d) $x = 2y = 3z$

4. If $a = \cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3}$, then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}$ is

   (a) purely real (b) purely imaginary

   (c) 0 (d) None of these

5. If 'α' is a root of $x^4 = 1$ with negative principal argument, then the principal argument of $\Delta(\alpha) =$

   $\begin{vmatrix} 1 & 1 & 1 \\ \alpha^n & \alpha^{n+1} & \alpha^{n+3} \\ \dfrac{1}{\alpha^{n+1}} & \dfrac{1}{\alpha^n} & 0 \end{vmatrix}$ is

   (a) $\dfrac{5\pi}{14}$ (b) $-\dfrac{3\pi}{4}$ (c) $\dfrac{\pi}{4}$ (d) $-\dfrac{\pi}{4}$

6. $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants,

   then

   (a) $\Delta_1 = 3(\Delta_2)^2$ (b) $\dfrac{d}{dx}(\Delta_1) = 3\Delta_2$

   (c) $\dfrac{d}{dx}(\Delta_1) = 3(\Delta_2)^2$ (d) $\Delta_1 = 3\Delta_2^{3/2}$

7. If $a^2+b^2+c^2+ab+bc+ca \le 0 \;\forall\, a, b, c \in R$, then value of

the determinant $\begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+a^2 & 1 & (c+a+2)^2 \end{vmatrix}$

equals

(a) 65 (b) $a^2+b^2+c^2+31$

(c) $4(a^2+b^2+c^2)$ (d) 0

8. Product of roots of equation $\begin{vmatrix} 1+2x & 1 & 1-x \\ 2-x & 2+x & 3+x \\ x & 1+x & 1-x^2 \end{vmatrix} = 0$ is

(a) 1/2 (b) 3/4 (c) 4/3 (d) 1/4

Comprehension Type

For Q. 9 to 10

A 3×3 determinant has entries either 1 or -1.

Let S_3 = set of all determinants which contain determinants such that product of elements of any row or any column is -1

For example $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$ is an element of the set S_3.

9. Number of elements of the set S_3 =

(a) 10 (b) 16 (c) 12 (d) 18

10. Number of elements of the set S_n =

(a) 2^n (b) 2^{n-1} (c) 2^{2n} (d) $2^{(n-1)^2}$

Answers Key

Single Correct Answer Type

1. (a) **2.** (c) **3.** (b) **4.** (b) **5.** (b)

6. (b) **7.** (a) **8.** (a)

Comprehension Type

9. (b) **10.** (d)

DPP 8.1

Single Correct Answer Type

1. (a) $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$

$\Rightarrow \quad \sin^2\gamma - \cos\alpha(\cos\alpha - \cos\beta\cos\gamma) + \cos\beta(\cos\alpha\cos\gamma - \cos\beta)$

$= -\cos\alpha(-\cos\beta\cos\gamma) + \cos\beta(\cos\alpha\cos\gamma)$

$\Rightarrow \quad \sin^2\gamma - \cos^2\alpha + 2\cos\alpha\cos\beta\cos\gamma - \cos^2\beta$

$= 2\cos\alpha\cos\beta\cos\gamma$

$\Rightarrow \quad \sin^2\gamma = \cos^2\alpha + \cos^2\beta$

$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

2. (c) The given equation is $px^3 + qx^2 - rx - r = 0$

$$\alpha+\beta+\gamma = \frac{-q}{p},\ \alpha\beta+\beta\gamma+\gamma\alpha = -\frac{r}{p},\ \alpha\beta\gamma = \frac{r}{p}$$

$$D = (1+\alpha)(1+\beta)(1+\gamma) + 1 + 1 - (1+\alpha) - (1+\beta) - (1+\gamma)$$

$$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \beta\gamma$$

$$= \frac{r}{p} - \frac{r}{p} = 0$$

3. (b) As $1+\omega+\omega^2=0$

$$D=\begin{vmatrix} -\frac{x}{\omega^2} & -y & -\frac{z}{\omega} \\ -y & -\frac{z}{\omega} & -\frac{x}{\omega^2} \\ -\frac{z}{\omega} & -\frac{x}{\omega^2} & -y \end{vmatrix} = x^3+y^3+z^3-3xyz$$

$$=\frac{1}{2}(x+y+z)\{(x-y)^2+(y-z)^2+(z-x)^2\}$$

$$=(x+y+z)(x+y\omega+z\omega^2)(x+y\omega^2+z\omega)$$

The determinant varnishes if $x=y=z$ or $x+y\omega+z\omega^2=0$

4. (b) $a=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=\omega^2$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} = 3(\omega-\omega^2) \text{ purely imaginary}$$

5. (b) Clearly $\alpha=-i$ where $i^2=-1$

So $\Delta(\alpha)=0+\alpha^2+1-1-0-\alpha^3$

$$=(-i)^2+1-1-(-i)^3$$

$$=-1+1-1-i=-1-i$$

So, principal argument of $\Delta(\alpha)$ is $-\frac{3\pi}{4}$.

6. (b) $\Delta_1 = x(x^2 - ab) - b(ax - ab) + b(a^2 - ax)$

$= x^3 - 3abx + ab^2 + a^2b$

$\therefore \quad \frac{d}{dx}(\Delta_1) = 3x^2 - 3ab = 3(x^2 - ab) = 3\Delta_2$

7. (a) We have $a^2 + b^2 + c^2 + ab + bc + ca \leq 0$

$\therefore \quad (a+b)^2 + (b+c)^2 + (c+a)^2 \leq 0$

$\therefore \quad a + b = 0, b + c = 0$ and $c + a = 0$

$\therefore \quad a = b = c = 0$

$$\Rightarrow \begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+a^2 & 1 & (c+a+2)^2 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 65$$

8. (a) $f(x) = \begin{vmatrix} 1+2x & 1 & 1-x \\ 2-x & 2+x & 3+x \\ x & 1+x & 1-x^2 \end{vmatrix} = 0$

Constant term $= f(0) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix} = 2 + 2 - 2 - 3 = -1$

Also coefficient of x^4 is '-2'

Hence product of roots is 1/2.

Comprehension Type

9. (b), 10. (d)

For S_n, $a_{11}, a_{12}, a_{13}, \ldots\ldots a_{1(n-1)}$ we have two options '1' or '–1', but for a_{1n} we have only one way depending upon the product $(a_{11} \cdot a_{12} \cdot a_{13} \cdot \ldots \cdot a_{1(n-1)})$

$\therefore$ For R_1 we have 2^{n-1} ways

Similarly for $R_2, R_3, R_4, \ldots\ldots R_{n-1}$ we have 2^{n-1} ways

For R_n we have only one way.

Hence total number of ways $(2^{n-1})^{n-1} = 2^{(n-1)^2}$

For S_3, we have $2^{(3-1)^2} = 16$ elements.

DPP 8.2

Properties of Determinant (Level 1)

Single Correct Answer Type

1. If x, y, z are non-zero real numbers and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$, then $-\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)$ is equal to

(a) 0 (b) 1 (c) 3 (d) 6

2. If $Y = SX$, $Z = tX$ all the variables being differentiable functions of x and lower suffices denote the derivative with respect to x and $\begin{vmatrix} X & Y & Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} \div \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} = X^n$, then $n =$

(a) 1 (b) 2 (c) 3 (d) 4

3. If $w \neq 1$ is a cube root of unity and $\Delta = \begin{vmatrix} x+w^2 & w & 1 \\ w & w^2 & 1+x \\ 1 & x+w & w^2 \end{vmatrix} = 0$, then value of x is

(a) 0 (b) 1 (c) -1 (d) none of these

4. Let $|A| = |a_{ij}|_{3\times 3} \neq 0$. Each element a_{ij} is multiplied by k^{i-j}. Let $|B|$ the resulting determinant, where $k_1|A| + k_2|B| = 0$. Then $k_1 + k_2 =$

(a) 1 (b) -1 (c) 0 (d) 2

5. If α, β, γ are the roots of $x^3 + px^2 + q = 0$, where $q \neq 0$, then $\Delta = \begin{vmatrix} 1/\alpha & 1/\beta & 1/\gamma \\ 1/\beta & 1/\gamma & 1/\alpha \\ 1/\gamma & 1/\alpha & 1/\beta \end{vmatrix}$ equals

(a) $-p/q$ (b) $1/q$
(c) p^2/q (d) None of these

6. If $a - 2b + c = 1$, then the value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is

(a) x (b) $-x$ (c) -1 (d) 1

7. Let $x > 0, y > 0, z > 0$ are respectively the 2nd, 3rd, 4th terms of a G.P and $\Delta = \begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2} \end{vmatrix} = (r-1)^2\left(1-\dfrac{1}{r^2}\right)$ (where r is the common ratio), then

(a) $k=-1$ (b) $k=1$
(c) $k=0$ (d) None of these

8. If $a, b, c, d > 0$; $x \in R$ and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \leq 0$, then $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$

(a) 1 (b) -1
(c) 0 (d) none of these

9. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} - \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

(a) 0 (b) 2^n
(c) ${}^{x+y+z}C_r$ (d) ${}^{x+y+z}C_{r+2}$

10. If $\begin{vmatrix} {}^9C_4 & {}^9C_5 & {}^{10}C_r \\ {}^{10}C_6 & {}^{10}C_7 & {}^{11}C_{r+2} \\ {}^{11}C_8 & {}^{11}C_9 & {}^{12}C_{r+4} \end{vmatrix} = 0$, then the value of r is equal to

(a) 3 (b) 4 (c) 5 (d) 6

11. If either of the two P, Q and R are equal and $P + Q + R = 180°$, then the value of $\begin{vmatrix} 1 & 1+\sin P & \sin P(1+\sin P) \\ 1 & 1+\sin Q & \sin Q(1+\sin Q) \\ 1 & 1+\sin R & \sin R(1+\sin R) \end{vmatrix}$ is

(a) 0 (b) 1
(c) $\sin(P+Q+R)$ (d) $\sin P \sin Q \sin R$

12. In a triangle ABC, if a, b, c are the sides opposite to angles A, B, C respectively, then the value of $\begin{vmatrix} b\cos C & a & c\cos B \\ c\cos A & b & a\cos C \\ a\cos B & c & b\cos A \end{vmatrix}$ is

(a) 1 (b) -1
(c) 0 (d) $a\cos A + b\cos B + c\cos C$

13. If $a = 1 + 2 + 4 + \ldots$ up to n terms
$b = 1 + 3 + 9 + \ldots$ up to n terms
and $c = 1 + 5 + 25 + \ldots$ up to n terms.
then $\Delta = \begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix} =$

(a) $(30)^n$ (b) $(10)^n$ (c) 0 (d) $2^n + 3^n + 5^n$

14. If $a_1, a_2, a_3, 5, 4, a_6, a_7, a_8, a_9$ are in H.P., and the value of the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is D, then the value of $21D/10$ is

(a) 4 (b) 5

(c) 6 (d) 7

15. The value of

$$\begin{vmatrix} 1/c & 1/c & -(a+b)/c^2 \\ -(b+c)/a^2 & 1/a & 1/a \\ -b(b+c)/a^2c & (a+2b+c)/ac & -b(a+b)/ac^2 \end{vmatrix}$$ is

(a) dependent on a, b, c (b) dependent on a

(c) dependent on b (d) independent of a, b and c

Answers Key

Single Correct Answer Type

1. (c) **2.** (c) **3.** (a) **4.** (c) **5.** (d)

6. (c) **7.** (a) **8.** (c) **9.** (a) **10.** (c)

11. (a) **12.** (c) **13.** (c) **14.** (b) **15.** (a)

DPP 8.2

Single Correct Answer Type

1. (c) $$\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$ we get

$$\Delta = \begin{vmatrix} x & 0 & 1 \\ y & 2y & 0 \\ -2z & -2z & 1+3z \end{vmatrix} = 0$$

$$\therefore \quad \Delta=\begin{vmatrix} x & 0 & 0 \\ y & 2y & 1-\frac{y}{x} \\ -2z & -2z & 1+3z+\frac{2z}{x} \end{vmatrix}=0\left(\text{by } C_3\to C_3-\frac{1}{x}C_1\right)$$

$$\therefore \quad x\left[2y\left(1+3z+\frac{2z}{x}\right)+2z\left(1-\frac{y}{x}\right)\right]=0$$

$$\therefore \quad [2xy+6zxy+4yz+2zx-2yz]=0$$

$$\therefore \quad 2(xyz)\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+3\right]=0$$

$$\therefore \quad -\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=3$$

2. (c) $$\Delta=\begin{vmatrix} X & SX & tX \\ X_1 & SX_1+S_1X & tX_1+t_1X \\ X_2 & SX_2+2S_1X_1+S_2X & tX_2+2t_1X_1+t_2X \end{vmatrix}$$

$$\begin{pmatrix} C_2\to C_2-SC_1 \\ C_3\to C_3-tC_1 \end{pmatrix}$$

$$=\begin{vmatrix} X & 0 & 0 \\ X_1 & S_1X & t_1X \\ X_2 & 2S_1X_1+S_2X & 2t_1X_1+t_2X \end{vmatrix}$$

$$=X^2\begin{vmatrix} S_1 & t_1 \\ 2S_1X_1+S_2X & 2t_1X_1+t_2X \end{vmatrix}$$

$$=X^3\begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix}(R_2\to R_2-2X_1R_1)$$

$$\therefore \quad n=3$$

3. (a) Applying $C_1 \to C_1 + C_2 + C_3$,

We get $$\Delta = \begin{vmatrix} x+w^2+w+1 & w & 1 \\ w+w^2+1+x & w^2 & 1+x \\ 1+x+w+w^2 & x+w & w^2 \end{vmatrix}$$

$$= \begin{vmatrix} x & w & 1 \\ x & w^2 & 1+x \\ x & x+w & w^2 \end{vmatrix}$$

[using $1 + w + w^2 = 0$]

Δ is clearly equal to 0 for $x = 0$.

4. (c) $$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$|B| = \begin{vmatrix} a_{11} & k^{-1}a_{12} & k^{-2}a_{13} \\ ka_{21} & a_{22} & k^{-1}a_{23} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix}$$

$$= \frac{1}{k^3}\begin{vmatrix} k^2a_{11} & ka_{12} & a_{13} \\ k^2a_{21} & ka_{22} & a_{33} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix}$$

$$= |A|$$

$$k_1|A| + k_2|B| = 0$$

$$\therefore \quad k_1 + k_2 = 0$$

5. (d) We have $\beta\gamma + \gamma\alpha + \alpha\beta = 0$

$$\Delta = \frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix} \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \\ \alpha\beta & \beta\gamma & \gamma\alpha \end{vmatrix}$$

$$= \frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix} \beta\gamma+\gamma\alpha+\alpha\beta & \gamma\alpha & \alpha\beta \\ \gamma\alpha+\alpha\beta+\beta\gamma & \alpha\beta & \beta\gamma \\ \alpha\beta+\beta\gamma+\gamma\alpha & \beta\gamma & \gamma\alpha \end{vmatrix}$$

[using $C_1 \to C_1 + C_2 + C_3$]

$$= \frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix} 0 & \gamma\alpha & \alpha\beta \\ 0 & \alpha\beta & \beta\gamma \\ 0 & \beta\gamma & \gamma\alpha \end{vmatrix} = 0 \text{ [all zero property].}$$

6. (c) $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Applying $R_1 \to R_1 - 2R_2 + R_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} \qquad (\because a - 2b + c = 1)$$

Applying $R_3 \to R_3 - R_2$, the determinant reduces to

$$\begin{vmatrix} 0 & 0 & 1 \\ x+2 & x+3 & x+b \\ 1 & 1 & c-b \end{vmatrix} = -1$$

7. (a) $\Delta = x^k y^k z^k \begin{vmatrix} 1 & ar & a^2r^2 \\ 1 & ar^2 & a^2r^4 \\ 1 & ar^3 & a^2r^6 \end{vmatrix}$

$$= a^{3k} \cdot r^{6k} \cdot a^3 r^3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & r & r^2 \\ 1 & r^2 & r^4 \end{vmatrix}$$

$$= a^{3(k+1)} \cdot r^{6k+3} \cdot (1-r)(r-r^2)(r^2-1)$$

Clearly, $k = -1$

$\therefore\ \Delta = r^{-2}(1-r)^2(r^2-1)$

$$= (r-1)^2\left(1 - \frac{1}{r^2}\right)$$

8. (c) We have

$(a^2+b^2+c^2)x^2 - 2(ab+bc+cd)x + b^2+c^2+d^2 \le 0$

$\Rightarrow\ (ax-b)^2 + (bx-c)^2 + (cx-d)^2 \le 0$

$\Rightarrow\ (ax-b)^2 + (bx-c)^2 + (cx-d)^2 = 0$

$\Rightarrow\ \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} = x$

$\Rightarrow\ b^2 = ac$ or $2 \log b = \log a + \log c$,

Now, $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$ [Apply $R_1 \to R_1 + R_3 - 2R_2$]

$$= \begin{vmatrix} 0 & 0 & 0 \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$$

9. (a) $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} - \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

In first determinant apply $C_3 \to C_3 + C_2$ and $C_2 \to C_2 + C_1$ and then again $C_3 \to C_3 + C_2$

10. (c) Given $\begin{vmatrix} {}^9C_4 & {}^9C_5 & {}^{10}C_r \\ {}^{10}C_6 & {}^{10}C_7 & {}^{11}C_{r+2} \\ {}^{11}C_8 & {}^{11}C_9 & {}^{12}C_{r+4} \end{vmatrix} = 0$

Apply $C_2 \to C_1 + C_2$

$$\begin{vmatrix} {}^9C_4 & {}^{10}C_5 & {}^{10}C_r \\ {}^{10}C_6 & {}^{11}C_7 & {}^{11}C_{r+2} \\ {}^{11}C_8 & {}^{12}C_9 & {}^{12}C_{r+4} \end{vmatrix} = 0$$

Value of the determinant = 0

$\Rightarrow$ Column 2 is same as column 3

$\Rightarrow \quad r = 5$

11. (a) Applying $C_2 \to C_2 - C_1$ and then $C_3 \to C_3 - C_2$

$$\Delta = \begin{vmatrix} 1 & \sin P & \sin^2 P \\ 1 & \sin Q & \sin^2 Q \\ 1 & \sin R & \sin^2 Q \end{vmatrix} = (\sin P - \sin Q)(\sin Q - \sin P)$$

$(\sin R - \sin P)$

$\Rightarrow \quad \Delta = 0$

12. (c) $\begin{vmatrix} b\cos C & a & c\cos B \\ c\cos A & b & a\cos C \\ a\cos B & c & b\cos A \end{vmatrix}$

Apply $C_1 \to C_1 + C_3$ and using projection rule,

$$D = \begin{vmatrix} b\cos C & a & c\cos B + b\cos C \\ c\cos A & b & a\cos C + c\cos A \\ a\cos B & c & b\cos A + a\cos B \end{vmatrix}$$

$$= \begin{vmatrix} b\cos C & a & a \\ c\cos A & b & b \\ a\cos B & c & c \end{vmatrix} = 0$$

13. (c) $a = 2^n - 1, b = \dfrac{3^n - 1}{2}, c = \dfrac{5^n - 1}{4}$

$$\therefore \quad \Delta = \begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix}$$

$$\Delta = 2 \cdot \begin{vmatrix} 2^n - 1 & 3^n - 1 & 5^n - 1 \\ 1 & 1 & 1 \\ 2^n & 3^n & 5^n \end{vmatrix} = 0$$

14. (b) We have $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

Since $a_n = \dfrac{20}{n}$; $d = \dfrac{1}{20}$

Hence $D = \begin{vmatrix} 20 & \dfrac{20}{2} & \dfrac{20}{3} \\ \dfrac{20}{4} & \dfrac{20}{5} & \dfrac{20}{6} \\ \dfrac{20}{7} & \dfrac{20}{8} & \dfrac{20}{9} \end{vmatrix} = \dfrac{(20)^3}{4 \times 7} \begin{vmatrix} 1 & \dfrac{1}{2} & \dfrac{1}{3} \\ 1 & \dfrac{4}{5} & \dfrac{2}{3} \\ 1 & \dfrac{7}{8} & \dfrac{7}{9} \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \frac{(20)^3}{4 \times 7} \begin{vmatrix} 0 & \dfrac{-3}{10} & \dfrac{-1}{3} \\ 0 & \dfrac{-3}{40} & \dfrac{-1}{9} \\ 1 & \dfrac{7}{8} & \dfrac{7}{9} \end{vmatrix}$$

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15. (a) Multiplying C_1 by a, C_2 by b and C_3 by c, we obtain

$$\Delta = \frac{1}{abc}\begin{vmatrix} \frac{a}{c} & \frac{b}{c} & -\frac{a+b}{c} \\ -\frac{b+c}{a} & \frac{b}{a} & \frac{c}{a} \\ -\frac{b(b+c)}{ac} & \frac{b(a+2b+c)}{ac} & -\frac{b(a+b)}{ac} \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} 0 & \frac{b}{c} & -\frac{a+b}{c} \\ 0 & \frac{b}{a} & \frac{c}{a} \\ 0 & \frac{b(a+2b+c)}{ac} & -\frac{b(a+b)}{ac} \end{vmatrix}$$

This shows that Δ is independent of a, b and c.