Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_STERLING&NUCLEUS_BT Paper -1(Adv-2020-P1-Model Date: 06-08-2023

Time: 09.00Am to 12.00Pm RPTA-01 Max. Marks: 198

KEY SHEET

PHYSICS

1	**C**	2	**B**	3	**B**	4	**D**	5	**C**	6	**C**
7	**BD**	8	**BCD**	9	**ABCD**	10	**ABD**	11	**ABD**	12	**ABCD**
13	**350**	14	**500**	15	**2**	16	**7**	17	**6.5**	18	**15**

CHEMISTRY

19	**B**	20	**C**	21	**C**	22	**C**	23	**D**	24	**D**
25	**ABC**	26	**AD**	27	**AD**	28	**AD**	29	**A**	30	**ABC**
31	**3**	32	**8**	33	**9**	34	**8**	35	**4**	36	**3**

MATHEMATICS

37	**C**	38	**A**	39	**C**	40	**A**	41	**A**	42	**B**
43	**AC**	44	**ABD**	45	**BD**	46	**BC**	47	**BC**	48	**ABCD**
49	**7**	50	**5**	51	**1**	52	**0**	53	**5**	54	**2**

SOLUTIONS
PHYSICS

1. b. At $x=\infty, C=\frac{3}{2}R$

From $PV^x =$ constant

$\Rightarrow P^{1/x}V =$ another constant

So at $x=\infty, V$ = constant

Hence $C=C_v=\frac{5}{2}R$

and then $C_P=C_V+R=\frac{7}{2}R$

At x = 0, P = constant and $C=C'$

At $x=x', C=0$, so the process is adiabatic, hence $x'=\frac{C_P}{C_v}=\frac{7}{5}$

Degree of freedom, f : $C_V=\frac{fR}{2}=3R$

2. c. $PV=\frac{m}{M}RT$ (for ideal gas)

$\therefore MV=\frac{mRT}{P}$

In the position of equilibrium of stopper S,

$P_1=P_2, T_1=T_2, m_1=m_2$

$\Rightarrow A\times 32(360-\alpha)=40\alpha\times A$

$\alpha=160^0$

3. Let 'm' be the mass of ice.

Rate of heat given by the burner is constant. In the first 50 min

$$\frac{dQ}{dt}=\frac{mL}{t_3}=\frac{mkg\times(80\times4.2\times10^3)J/kg}{(50\ min)}...(i)$$

From 50 min to 60 min

$$\frac{dQ}{dt}=\frac{(m+5)S_{H_2O}\Delta\theta}{t_2}$$

$$=\frac{(m+5)kg(4.2\times10^3)J/kg\times2^0C}{10\ min}.....(ii)$$

From eq. (i) and (ii)

$$\frac{80m}{50}=\frac{2(m+5)}{10}$$

$$7m=5\Rightarrow m=\frac{5}{7}kg=0.7\ kg$$

4. $AdP = PA^2x \quad a = \frac{PA^2x}{mV} = w^2x \quad T = 2\pi\sqrt{\frac{mV}{PA^2}}$

5. $ms\Delta\theta = 1600$

6. For gas in A, $P_1 = \left(\frac{RT}{M}\right)\frac{m_A}{V_1}$

$P_2 = \left(\frac{RT}{M}\right)\frac{m_A}{V_2}$

$\therefore \Delta P = P_1 - P_2 = \left(\frac{RT}{M}\right)m_A\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$

putting $V_1 = V$ and $V_2 = 2V$, we get $\Delta P = \frac{RT}{M}\frac{m_A}{2V}$..........(1)

Similarly, for gas in B, $1.5\,\Delta P = \left(\frac{RT}{M}\right)\frac{m_B}{2V}$............(2)

From equ. (1) and (2) we get $2m_B = 3m_A$

7. $P = \frac{dQ}{dt} = m_1 s_1 \frac{d\theta_1}{dt}$

$= 600g \times 4.2\, j/g\, {}^0C \times \frac{40^0 C}{40 \times 60s} = 42W$

Now, $m_1 s_1 \frac{d\theta_1}{dt} = m_2 s_2 \frac{d\theta_2}{dt}$

$\Rightarrow m_2 = 2400\text{ g}$

It takes (60 – 40) = 20 min to reach thermal equilibrium

so, $Pt = m.L \Rightarrow m = \frac{Pt}{L}$

$= \frac{42 \times 1200J}{81 \times 4.2J/gm} = 150g$

$L = \frac{Q}{m} \Rightarrow cal/g,\ C = ms \Rightarrow cal/{}^0C,\ W = \frac{ms}{S_w} \Rightarrow g$

8. $A = 64\text{ mm}^2, T = 2500\text{ K}$ (A = surface area of filament, T = temperature of filament, 'd' is distance of bulb from observer, R_e = radius of pupil of eye)

Point source d = 100 m

A) $P = \sigma AeT^4$

$= 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times 1 \times (2500)^4 (e = 1 \text{ black body}) = 14.175w$

B) Power reaching to eye

$= \frac{P}{4\pi d^2} \times \left(\pi R_e^2\right)$

$= 3.189375 \times 10^{-8} W$

C) $\lambda_m T = b$

$\lambda_m \times 2500 = 2.9 \times 10^{-3}$

$\Rightarrow \lambda_m = 1.16\times10^{-6} = 1160$ nm

D) Power received by one eye of observer = $\left(\frac{hc}{\lambda}\right)\times \dot{N}$

$\dot{N}$ = number of photons entering into eye per second

$= \frac{6.63\times10^{-34}\times3\times10^{8}}{1740\times10^{-9}}\times\dot{N}$

$\Rightarrow \dot{N} = 2.79\times10^{11}$

9. (A) area under the curve is equal to number of molecules of the gas sample area = $\frac{av_0}{2} = N$

(B) $V_{avg} = \frac{\int VdN}{\int dN} = \frac{\int_0^{v_0}\frac{a}{V_0}VdV}{\int_0^{v_0}\frac{a}{V_0}VdV} = \frac{V^{2/3}}{\frac{V^2}{2}}\Big|_0^{V_0} = \frac{2}{3}V_0$

(C) $V_{rms}^2 = \frac{\int V^2dN}{\int dN} = \frac{\int_0^{v_0}V^2\frac{a}{V_0}VdV}{\int_0^{v_0}\frac{a}{V_0}VdV} = \frac{V^{4/4}}{V^{2/2}}\Big|_0^{V_0} = \frac{V_0^2}{2}$

(D) Area under curve from $\frac{V_0}{2}$ to V_0 is $\frac{3}{4}$ of total area

10. Conceptual

11. Conceptual

12. A) $d = \frac{2000\text{ cm}^3}{100} = 20$ cm

B) Separation between piston (final) = compression in spring

C) $P_2 = P_1 + \frac{kn}{A}$

D) $P_1V_1 = P_2V_2$ amu $V_2 = 10^{-2}x$

13. C be the specific heat and L be the latent heat of vapourisation.

From principle of calorimetry,

Heat lost = heat gain

$M_CS_C\Delta T = mC\Delta T + mL$

or $M_CS_C(110-80) = 5C(80-30)+5L.......(i)$

Again, when 100 g liquid is poured and equilibrium temperature is 50^0C

$m_CS_C(80-50) = 100C(50-30).........(ii)$

$\therefore \frac{L}{C} = \frac{1750}{5} = 350^0C$

14. Temp. is increased by $\Delta\theta$ then

$$\Delta l = l\alpha\Delta\theta$$

$$\Rightarrow \Delta\theta = \frac{\Delta l}{l\alpha}$$

$$E_1 = (\rho A l)S\Delta\theta = \rho A l S\frac{\Delta l}{l\alpha}$$

$$E_2 = \frac{1}{2}\left(Y\frac{\Delta l}{l}\right)\left(\frac{\Delta l}{l2}\right)\times A l = \frac{Y(\Delta l)^2 A}{2l}$$

$$\text{So, } \frac{E_1}{E_2} = \frac{\rho A l S\Delta l\times 2l}{l\times Y(\Delta l)^2 A} = \frac{2\rho S l}{\alpha(\Delta l)Y} = 500$$

15. By using Newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta-\theta_s)$$

Solving this differential equation, we have

$$\int_{\theta_0}^{\theta}\frac{d\theta}{\theta-\theta_s} = -k\int_{\theta}^{t}dt$$

This gives, $kt = \ln\frac{\theta_0-\theta_s}{\theta-\theta_s}$

putting $t = t_1$, $\theta_0 = \theta$, $\theta_0 = \theta_2$ we have

$$kt_1 = \ln\frac{\theta_1-\theta_s}{\theta_2-\theta_s}$$

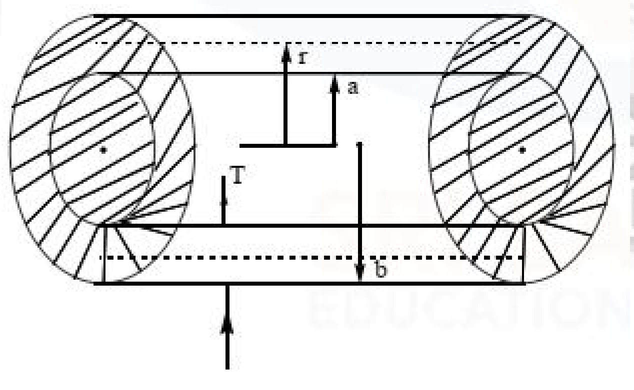
putting $t = t_2$, $\theta_0 = \theta_2$, $\theta = \theta_3$ we have

$$kt_2 = \ln\frac{\theta_2-\theta_s}{\theta_3-\theta_s}$$

By using equation (i) and (ii)

$$\frac{t_1}{t_2} = \frac{\ln\left[(\theta_1-\theta_s)/(\theta_2-\theta_s)\right]}{\ln\left[(\theta_2-\theta_s)/(\theta_3-\theta_s)\right]}$$

16.



At any instant let "P" be the power entered into the cylinder, $P = K2\pi rl\left(\frac{-dt}{-dr}\right)$;

where "r" is the radius of any cylindrical layer between 'a' and 'b'.

$$P = K2\pi l\Delta T \ln\left(\frac{b}{a}\right)$$

ΔT is the instantaneous temperature different of body with surroundings.

$$\text{Then } P = \pi a^2 Sl\frac{dT}{dt} = 2K\pi l\Delta T \ln\left(\frac{b}{a}\right)$$

$$t = \frac{Sa^2}{2K}\ln\left(\frac{b}{a}\right)\ln\frac{(T_0 - T_1)}{(T_0 - T_2)}$$

17. $U^{\beta}\alpha V$

$T^{\beta}\alpha V$

$P^{\beta}\alpha V$

$P^{\beta}V^{\beta}\alpha V$

$P^{\beta}V^{\beta-1} = \text{constant}$

$P(V)^{\frac{(\beta-1)}{\beta}} = \text{constant}$

$$\frac{\Delta U}{\Delta Q} = \frac{{}^{n}C_r\Delta T}{{}^{n}C\Delta T} = \frac{2}{3}$$

$C = {}^{3}C_v$

$$C_V + \frac{R}{1-x} = {}^{3}C_V$$

$$\text{where } x = \frac{\beta - 1}{\beta}$$

$\Rightarrow \beta = -1$

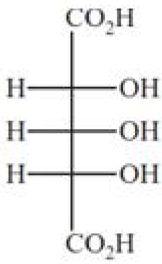
18. $W = \frac{1}{2} \times 3V_0 \times p_0 = \frac{9p_0V_0}{2}$

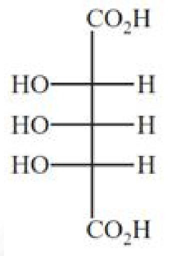
$$Q_{given} \text{ is from 1 to 2} = \frac{1}{2}\times(p_0 + 4p_0)\times 3V_0 + \frac{3}{2}\times(16p_0V_0 - p_0V_0)$$

$$= \frac{15p_0V_0}{2} + \frac{45p_0V_0}{2} = 30p_0v_0 \quad \eta = \frac{9p_0V_0}{30p_0V_0}\times 100 = \frac{3}{20}\times 100 = 15$$

CHEMISTRY

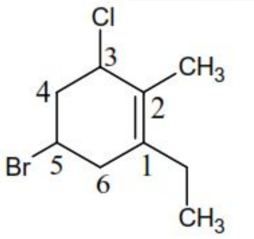
19.

I) 

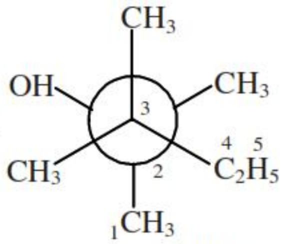
IV) 

I and IV are same

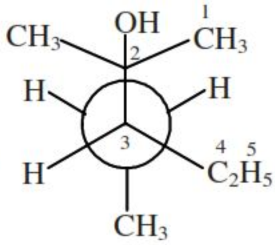
20.



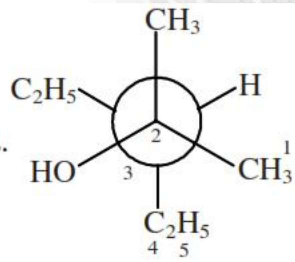
21. Q, & R are same compounds

P. 

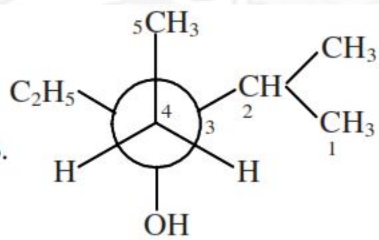
2, 3, 3-trimethyl pentan-2-ol

Q. 

3-ethyl-2-methyl pentan-2-ol

R. 

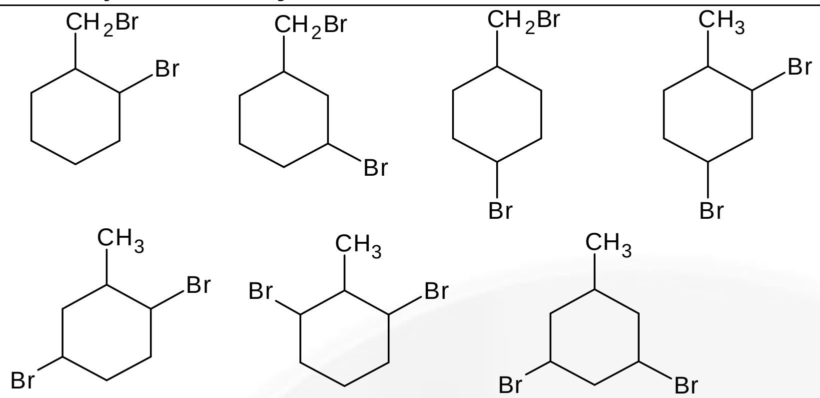
3-ethyl-2-methyl pentan-2-ol

S. 

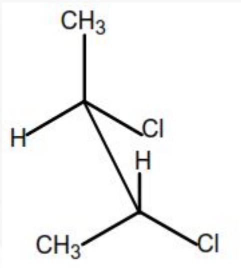
3-ethyl-2-methyl pentan-3-ol

22. I and III can show GI

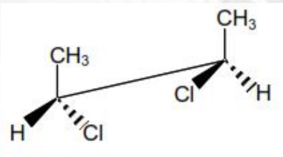
23.



24. C is isomer is more polar than trans,hence it is more soluble in polar solvents
25. A- Enantimer B- Diastereo C- Identical D- No releationship
26. (I) & (III) are identical, (II) & (III) are structural isomers.
27. No place of symmetry so optically active



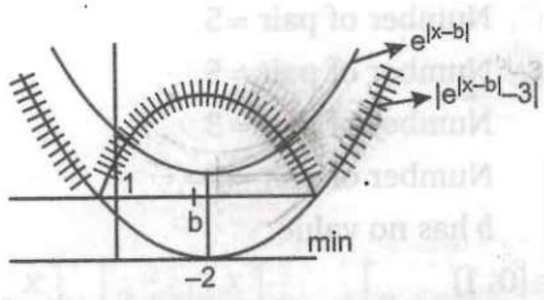
28. In A,D dipolemoments are different
29. Conceptual
30. ABC are meso

D is  chiral (no POS and COS)

31. x = 2, y = 1
22. Number of all stereoisomers $= 2^3 = 8$
33. X = 5, Y = 3, Z = 1
34. pentanal, 2-methylbutanal.(Chiral), 3-methylbutanal 2,2-methylpropanal, 2-pentanone, 3-pentanone 2-methylbutanone
35. Conceptual
36. III, V, VII are optically active

MATHEMATICS

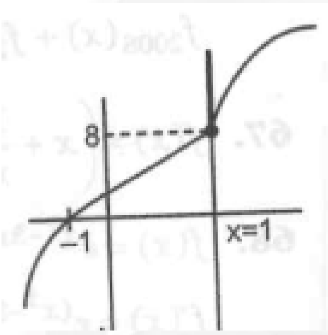
37. For any $b \in R e^{|x-b|}$ is



$|e^{|x-b|} - a|$ has four distinct solutions a > 3 so $a \in (3, \infty)$

38. is one-one when

$2^3 - \ln 1 + b^2 - 3b + 10$



$\Rightarrow b^2 - 3b + 2 = 0$

$\Rightarrow b = 1, 2$

39. $$265\left[\lim_{h\to 0}\frac{h^2+3}{\left(\frac{f(1-h)-f(1)}{-h}\right)\left(\frac{\sin 5h}{h}\right)}\right] = -265 \times \frac{3}{f'(1).5} = \frac{53\times 3}{f'(1)}$$

$$= -\frac{53\times 3}{-53}\ [\because f'(1) = -53] = 3$$

40. $$f(x) = \lim_{n\to\infty}\tan^{-1}\left(4n^2.2\sin^2\frac{x}{2n}\right) = \lim_{n\to\infty}\tan^{-1}\left(8n^2\left(\frac{\sin\frac{x}{2n}}{\frac{x}{2n}}\right).\frac{x^2}{4n^2}\right) = \tan^{-1}(2x^2)$$

$$g(x) = \lim_{n\to\infty}\frac{n^2}{2}\left(\frac{\ln\left(1+\cos^2\frac{2x}{n}-1\right)}{\cos^2\frac{2x}{n}-1}\right)\left(\cos\frac{2x}{n}-1\right) = x^2$$

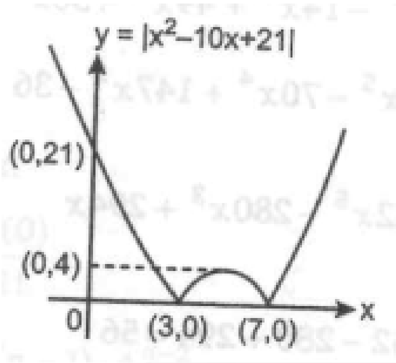
41. $\frac{x}{5}$ is integer at 21 points in [0, 100]

$\frac{x}{2}$ is integer at 51 points in [0, 100]

$\therefore$ but when 'x' is a multiple of 10 then f(x) is continuous, So that respective points should be subtract from both i.e., multiple of 10 are 11 points in [0, 100]

21 + 51 – 11 – 11 = 72 – 22 = 50

42.



43. $h(x)=[\ln x-1]+[1-\ln x]$

$\Rightarrow h(x)=\begin{cases}-1, & \ln x-1\notin I\\ 0, & \ln x-1\in I\end{cases}$

44. $f(-1)=f(0)=0$

So 'f' is not one – one

45. $\lim\limits_{x\to0^+}\dfrac{\cos^{-1}(1-x)\sin^{-1}(1-x)}{\sqrt{2x}(1-x)}=\lim\limits_{x\to0^+}\dfrac{\left(\dfrac{\sin^{-1}\sqrt{2x-x^2}}{\sqrt{2x-x^2}}\right)\sqrt{2x-x^2}.\sin^{-1}(1-x)}{\sqrt{2x}(1-x)}=\dfrac{\pi}{2}$

$\lim\limits_{x\to0^-}\dfrac{\cos^{-1}(-x)\sin^{-1}(-x)}{\sqrt{2(x+1)}(-x)}=\dfrac{\pi}{2\sqrt{2}}$

46. $e^{\lim\limits_{x\to0}\frac{1}{3x^2}\left(p\tan qx^2-3\cos^2x+3\right)}$

$e^{\lim\limits_{x\to0}\frac{pq}{3}\frac{3\left(1-\cos^2x\right)}{3x^2}}$

$\Rightarrow\dfrac{pq}{3}+1=\dfrac{5}{3};\ pq=2$

47. $f(x)=1-(1-x)+(1-x)x^2+(1-x)\left(1-x^2\right)x^3+....+(1-x)\left(1-x^2\right)......\left(1-x^{n-1}\right)x^n$

$=1-(1-x)\left(1-x^2\right)\left(1-x^3\right).....\left(1-x^n\right)=1-\prod\limits_{r=1}^{n}\left(1-x^r\right)$

$(f(x)-1)=-\prod\limits_{r=1}^{n}\left(1-x^r\right)$

48. h(x) = - 1 $\quad x<1$

= |x – 2| + a + 2 – |x| $\quad 1\le x<2$

= |x – 2| + a + 1 – b| $\quad x\ge2$

If h(x) is continuous at x = 1, then a = -3

If h(x) is continuous at x = 2, then b = 1

49. $y=\dfrac{x-\frac{1}{x}}{x^3-\frac{1}{x^3}+2}$ Let $t=x-\dfrac{1}{x}>0$ for $x>1$

$y=\dfrac{t}{t\left(t^2+3\right)+2}$ $x^3-\dfrac{1}{x^3}=t\left(t^2+3\right)$

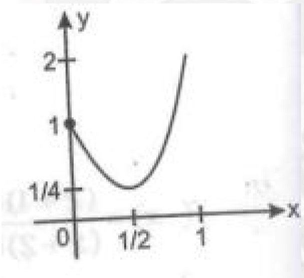
$=\dfrac{t}{t^3+3t+2}$

$=\dfrac{t}{t^2+\frac{2}{t}+3}$ $\left(\begin{array}{l} t^2+\dfrac{2}{t}=t^2+\dfrac{1}{t}+\dfrac{1}{t}\geq 3 \\ \therefore t^2+\dfrac{2}{t}+3\geq 6\ (AM\geq GM) \end{array}\right.$

$y_{max}=\dfrac{1}{\left(t^2+\frac{2}{t}+3\right)_{min}}=\dfrac{1}{6}$

p = 1, q = 6

50. $g(x)=f(x)$ $0\leq x<\dfrac{1}{2}$



$=\dfrac{1}{4}$ $\dfrac{1}{2}\leq x\leq 1$

$=3-x$ $1<x\leq 2$

51. $a\left(x^3-1\right)+(x-1)=0$

$(x-1)\left(ax^2+ax+a+1\right)=0$

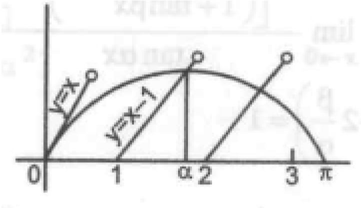
$\alpha,\ \beta\neq 1$ so, $\alpha,\ \beta$ are roots of $ax^2+ax+a+1=0$

$\alpha+\beta=-1,\ \alpha\beta=\dfrac{a+1}{a}$

$=\lim\limits_{x\to\frac{1}{\alpha}}\dfrac{\left[x^2+a\left(x^2+x+1\right)\right]}{\left(e^{1-\alpha x}-1\right)}=\lim\limits_{x\to\frac{1}{\alpha}}\dfrac{(1+a)x^2+ax+a}{\left(\dfrac{e^{1-\alpha x}-1}{1-\alpha x}\right)(1-\alpha x)}$

$=\lim\limits_{x\to\frac{1}{\alpha}}a\dfrac{(1-(\alpha)x)(1-(\beta)x)}{(1-\alpha x)}=\dfrac{a(\alpha-\beta)}{\alpha}$

52. $= \lim_{x \to \alpha^+} \left[\frac{\sin x}{x-1} \right] = 0$



53. $g(f(x)) = x$

$g'(f(x))f'(x) = 1$

$f(1) = \frac{7}{6}$

$\therefore x = 1$

$g'\left(-\frac{7}{6}\right)f'(1) = 1$

$f'(x) = -4.e^{\frac{1-x}{2}}\left(-\frac{1}{2}\right) + x^2 + x + 1$

$\Rightarrow 5a.5^{-3/2} = b$

$\Rightarrow \frac{a}{b} = 5^{\frac{1}{2}}$

$\left(\frac{a}{b}\right)^2 = 5$

$\frac{a^2}{5b^2 g'\left(\frac{-7}{6}\right)} = \frac{5}{5 \times \frac{1}{5}} = 5$

54. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f\left(x\left(1+\frac{h}{x}\right)\right) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{f(x)}{1+h/x} + \frac{f\left(1+\frac{h}{x}\right)}{x} - f(x)}{h} = \lim_{h \to 0} \frac{f(x)\left(-\frac{h}{x}\right)}{h\left(1+\frac{h}{x}\right)} + \frac{f\left(1+\frac{h}{x}\right)}{hx}$

$f'(x) = \frac{-f(x)}{x} + \frac{f'(1)}{x^2}$

$xf'(x) + f(x) = \frac{1}{x}$

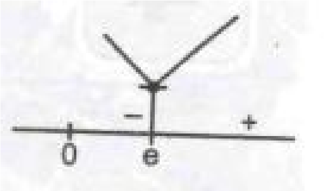
$\frac{d}{dx}(xf(x)) = \frac{1}{x}$

$$xf(x)=\int \frac{1}{x}dx$$

$$xf(x)=\ln x+k$$

$$H'(x)=\frac{\ln x.1-1}{(\ln x)^2}\ H(x)\ge e$$

$$H(e)=e$$



$$\therefore \lim_{x\to e}\left[\frac{1}{f(x)}\right]=2$$