Sri Chaitanya IIT Academy., India.

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

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DIFFERENTIAL EQUATIONS

EXERCISE - II

NUMERICAL/INTEGER ANSWER TYPE QUESTIONS

Order and Degree of D.E :

1. Differential equation of family of curves $y=e^{-x}\left(C_1x+C_2\right)$ is $y_2+\lambda y_1+\mu=0$. Then minimum value of polynomial f in. Whose roots are λ and μ is _____

2. The degree of $x^{\frac{1}{3}}y_1+y_2^{\frac{1}{2}}=y^{\frac{1}{4}}y_2$ is

PRACTICE QUESTIONS

3. The order of the differential Equation $\left[1+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)\right]^{3/4}=\frac{d^2y}{dx^2}$

4. The degree of the D.E. satisfying $y=c(x-c)^2$ is

Formation of D.E. :

5. The D.E. whose solution is $Ax^2+By^2=1$ where A and B are arbitrary constants is of degree.

6. The coinciding solution of two equations
$y^1=y^2+2x-x^4$ *and* $y^1=-y^2-y+2x+x^2+x^4$ is $y=x^n$ then $n+2$ is

7. If $(a+bx)e^{y/x}=x,$ satisfies the D.E. $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^k$ then k is **[Adv.1983]**

PRACTICE QUESTIONS

8. If $y=\sin\left(3\sin^{-1}x\right)$ and a, b, c, d are such that $a\left(1-x^2\right)\frac{d^2y}{dx^2}+bx\frac{dy}{dx}+cy=d$. Then $a+b+c$ value is ______

9. The D.E. of all parabolas each of which has latusrectum 4 and whose axis is parallel to the x-axis is $k(y'')+(y')^3=0$ then k is

Solutions of the D.E. :

(a) Inspection method of solving D.E. :

10. If the curve satisfying $ydx+\left(x+x^2y\right)dy=0$ passes through (1, 1) then $\left(\dfrac{-4\log y(4)+\dfrac{1}{y(4)}}{2}\right)$ is equal to

11. If $xdy=y\left(dx+ydy\right),\ y>0\ and\ y(1)=1$, hen $y(-3)$ is equal to

PRACTICE QUESTIONS

12. IF $y(xy+1)dx+x\left(1+xy+x^2y^2\right)dy=0$ is a differential equation the solution is $Ax^2y^2\log y+Bxy+C=Kx^2y^2$ then $A-B+C=$

(b) Variable separable form :

13. If $y(x)$ is the solutions of the D.E. $(x+2)\dfrac{dt}{dx}=x^2+4x-9,\ x\neq 2\ and\ y(0)=0$. Then $y(-4)=$

14. If $y_1-x\tan(y-x)=1,\ y(0)=\dfrac{\pi}{2}$ then the value of $\sin\left(y(4)-4\right)e^{-8}$ is

PRACTICE QUESTIONS

15. Let $y=f(x)$ be a curve passing through $\left(e,e^e\right)$ which satisfy the differential equation $\left(2ny+xy\log_e x\right)dx-x\log_e xdy=0\ \ x>0,y>0$. If $g(x)=\lim\limits_{n\to\infty}(0.020)f(x)$. Then $\int_{\frac{1}{e}}^{e}g(x)dx$ equal to

16. If $f:R-\{-1\}\to R$ and f is differentiable function satisfies "$f\left(x+f(y)+xf(y)\right)=y+f(x)+yf(x)\forall x,\ y\in R-\{-1\}$ then find the value of $2023\left[1+f(2022)\right]$

(c) Homogeneous Equations :

17. The real value of m for which the substitution $y=u^m$ will transform the differential equation $2x^4y\dfrac{dy}{dx}+y^4=4x^6$ into a homogeneous equation is

18. If the equation of a curve $y=y(x)$ satisfies the differential equation $x\int_0^x y(t)dt=(x+1)\int_0^x ty(t)dt,\ x>0$ and $y(1)=e,\ then\ y\left(\dfrac{1}{2}\right)$ is equal to

(d) Linear D.E. :

19. If $\left(1+x^2\right)y_1 = x(1-y),\ y(0)=\frac{4}{3}$ then $y\left(\sqrt{8}\right)-\frac{1}{9}$ is

20. The solution of $x^3\frac{dy}{dx}+4x^2\tan y = e^x \sec y$ satisfying y(1) = 0 is sin y $= e^x(x-1)x^{-k}$ then k =

21. Differential equation, having $y=\left(\sin^{-1}x\right)^2+A\left(\cos^{-1}x\right)+B$ where A and B are arbitary constants is $\left(p-x^2\right)\frac{d^2y}{dx^2}-\frac{xdy}{dx}=q$ then p+q=__

PRACTICE QUESTIONS

22. If the solution of the differential equation $y=2px+y^2p^3$ is in the form $y^2=2cx+c^k$ then k = __(Where $p=\frac{dy}{dx}$ and 'c' is arbitrary constant)

Bernoulli's Equation (Reducible to linear equations) :

Miscellaneous methods on solving D.E :

23. If the solution of the differential equation $e^{3x}(p-1)+p^3e^{2y}=0$ is in the form $e^y=ce^x+c^k$ then k = __(Where $p=\frac{dy}{dx}$ and 'c' is arbitrary constant)

24. If the solution of the differential equation $y^2(y-xp)=x^4p^2$ is in the form $\frac{1}{y}=\frac{c}{x}+c^k$ then k = __(Where $p=\frac{dy}{dx}$ and 'c' is arbitrary constant)

Application of D.E :

Geometric Application of D.E :

25. A normal is drawn at a point $P(x,y)$ of a curve. It meets the x-axis at Q. If PQ is of constant length k, then show that the differential equation describing such curves is $y\frac{dy}{dx}=\pm\sqrt{k^2-y^2}$ Find the equation of such a curve passing through $(0,k)$ and also passes through (3, 4) then k is ____ **[Adv.1994]**

PRACTICE QUESTIONS

26. A curve passing through the point $(1,1)$ has the property that the perpendicular distance of the origin from normal at any point 'P' of the curve is equal to the distance of P from the x-axis is a circle with radius

KEY SHEET

1.	0.75	**2.**	2	**3.**	1	**4.**	4	**5.**	2
6.	0	**7.**	2	**8.**	0	**9.**	2	**10.**	2
11.	3	**12.**	3	**13.**	0	**14.**	1	**15.**	0
16.	1	**17.**	1.5	**18.**	8	**19.**	1	**20.**	4
21.	3	**22.**	3	**23.**	3	**24.**	2	**25.**	5
26.	1								

HINTS & SOLUTIONS

1. $y=e^{-x}\left(C_1x+C_2\right)$ is $y_2+\lambda y_1+\mu=0 \Rightarrow \lambda=2,\ \mu=1$

$\therefore f(x)=x^2-3x+2$

Then minimum $f(x)=\dfrac{4(1)(2)-9}{4}=0.75$

2. Given equation is $x^{\frac{1}{3}}y_1+y_2{}^{\frac{1}{2}}=y^{\frac{1}{4}}y_2 \Rightarrow x^{\frac{1}{3}}y_1+y_2{}^{\frac{1}{2}}=y^{\frac{1}{4}}y_2$

$\therefore$ Degree = 2

3. Sol : $\left[\left[1+\left(\dfrac{dy}{dx}\right)^2+\sin\left(\dfrac{dy}{dx}\right)\right]^{3/4}\right]^4=\left[\dfrac{d^2y}{dx^2}\right]^4$

So order is 2

4. Given equation $y=c\left(x-c\right)^2$ ….(1) $\Rightarrow y'=2c\left(x-c\right)$ ….(2)

$\Rightarrow \left(y'\right)^2=4c^2\left(x-c\right)$ …..(3)

From (1), (2) and (3), $8y^2=4xy\dfrac{dy}{dx}-\left(\dfrac{dy}{dx}\right)^3$

Degree is 3

5. Given $Ax^2+By^2=1$ ….(i)

$\dfrac{yy}{x}=-\dfrac{A}{B}\Rightarrow \dfrac{x(y'^2+yy'')-yy}{x^2}=0$ or $xyy''+xy^2-yy=0$

Therefore degree = 1

6. $y^2+2x-x^4=-y^2-y+2x+x^2+x^4$

$y=x^2\ (or)\ y=-x^2-\dfrac{1}{2}$

But 2nd function is not satisfied by $y=-x^2-\dfrac{1}{2}$

$\therefore y=x^2 \Rightarrow n=2; n+2=4$

7. $\left(a+bx\right)e^{\frac{y}{x}}=x\Rightarrow e^{\frac{y}{x}}=\dfrac{x}{a+bx}$

Diff. w.r.t. x, we get $e^{\frac{y}{x}}\frac{\left[(x)\frac{dy}{dx}-y\right]}{x^2}=\frac{a+bx-bx}{(a+bx)^2}\Rightarrow\left(x\frac{dy}{dx}-y\right)e^{\frac{y}{x}}=\frac{ax^2}{(a+bx)^2}$

From (i) and (ii) we get $\left(x\frac{dy}{dx}-y\right)=\frac{ax}{a+bx}$

Differentiating (iii) w.r.to x,we get $x\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}=\frac{(a+bx)a-axb}{(a+bx)^2}$

$\Rightarrow x\frac{d^2x}{dx^2}=\frac{a^2}{(a+bx)^2}\Rightarrow x^3\frac{d^2y}{dx^2}=\left(\frac{ax}{a+bx}\right)^2$

Comparing (iii) and (iv), we get $\Rightarrow x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$

8. Given Equation $y=\sin\left(3\sin^{-1}x\right)\Rightarrow\sin^{-1}y=3\sin^{-1}x$

Differentiating on both sides w.r.t. to x, we get

$1\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+9y=0$

$a+b+c=0$

9. $(y-k)^2=4(x-h)$ ….(i)$\Rightarrow k=y-\frac{2}{y'}$ $\Rightarrow 0=y'+\frac{2y''}{(y')^2}$

$\Rightarrow 2(y'')+(y')^3=0$

$\therefore k=2$

10. $\int\frac{1}{(xy)^2}d(xy)=\int\frac{-1}{y}dy-\frac{1}{xy}=-\log y+c$

Passes through (1, 1); -1 = c; $\frac{1}{xy}=\log y+1$

$\frac{1}{4y(4)}=\log y(4)+1;-\log y(4)+\frac{1}{4y(4)}=1\Rightarrow\frac{-4log\left(y(4)\right)+\frac{1}{y(4)}}{2}=2$

11. $-\frac{ydx-xdy}{y^2}=dy\ or\ -d\left(\frac{x}{y}\right)=dy$. Integrating

$-\frac{x}{y}=y+k$ Given when x = 1, y = 1

$\therefore k=-2$ Hence $\frac{x}{y}+y=2$ …(i) involve we get y = -1, 3

$\therefore y=3 \quad \because y>0$

12. $xy^2dx+\left(ydx+xdy\right)+x^2ydy+x^3y^2dy=0$; $(1+xy)d(xy)+x^3y^2dy=0$;

Dividing with x^3y^3 We get $\left(\frac{1}{x^3y^3}+\frac{1}{x^2y^2}\right)d(xy)+\frac{1}{y}dy=0$

Put $xy=t$; Integrate we get $2x^2y^2\log y-2xy-1=Kx^2y^2 \quad \therefore A=2, B=-2, C=-1$

13. $\int dy=\int x+2-\frac{13}{x+2}dx$

$y=\frac{x^2}{2}`+2x-13\log(x+2)+c$

$y(0)=0;\ C=13\log 2$

$y(-4)=0$

14. Put $y-x=t$

$\sin(y-x)=ce^{x^2/2}$

Put $x=0$; $\quad C=1\Rightarrow \sin(y(4)-4)e^{-8}=1$

15. $(2\mathrm{ny}+\mathrm{xy}\log_e \mathrm{x})\mathrm{dx}=\mathrm{x}\log_e \mathrm{xdy}\Rightarrow \frac{\mathrm{dy}}{\mathrm{y}}=\left(\frac{2\mathrm{n}}{\mathrm{x}\log_e \mathrm{x}}+1\right)\mathrm{dx}$

$\Rightarrow \log(\mathrm{y})=2\mathrm{n}\log|\log \mathrm{x}|+\mathrm{x}+\mathrm{c}$ and $\mathrm{c}=0$

$$\text{now}, g(x)=\lim_{n\to\infty} f(x)=\begin{cases}\to\ \infty & \text{if} \quad x<\frac{1}{e}\\ 0 & \text{if} \quad \frac{1}{e}<x \quad <e\\ \to\ \infty & \text{if} \quad x>e\end{cases} \qquad \therefore \int_{\frac{1}{e}}^{e} g(x)dx=0$$

16. " $f\left(x+f(y)+xf(y)\right)=y+f(x)+yf(x)\forall x, y\in R-\{-1\}$ ….(i)

Diff.w.r.t. x as y is constant

We get $f'(x+f(y)+xf(y))(1+f(y))=f'(x)+xf'(x)$ …(ii)

From (i) again diff. w.r.t. y as x is constant

$f'(x+f(y)+xf(y))((1+x)f(y))=1+f'(x)$ …(iii)

From (ii) and (iii)

$\frac{(1+y)f'(y)}{1+f(y)}=\frac{1+f(x)}{(1+x)f'(x)}=\lambda$

$f'(x)=\frac{1+f(x)}{\lambda(1+x)}\Rightarrow f(x)=c(1+x)^{\pm 1}-1$

Put x = y = 0 $f(f(0))=f(0)$

$f(0)=c-1$. Hence $f(c-1)=c-1$

Hence $f(x)=-1\ and\ f(x)=(1+x)-1$

$=x\ and\ c=1 \quad \therefore f(x)=(1+x)^{-1}-1=\frac{-x}{1+x}$

$\Rightarrow 1+f(x)=\frac{1}{1+x} \quad \therefore 1+f(2022)=\frac{1}{2023}=1$

17. $y=u^m \Rightarrow \frac{dy}{dx}=mu^{m-1}$

Hene, $2x^4.u^m.\ mu^{m-1}\frac{dy}{dx}=y^{4m}=4x^6$

$\frac{du}{dx}=\frac{4x^6-u^{4m}}{2mx^4u^{2m-1}} \Rightarrow 4m=6 \Rightarrow m=\frac{3}{2}$

18. Differentiating w.r.t. x, we get $x^2y(x)+\int_0^x ty(t)dt$

Again D.O.B.S. w.r.t. x we get $y(x)=x^2y(x)+y(x)2x+xy(x)$

On integrating we get $\ln y(x)=-\frac{1}{x}-3\ln+\ln c$

So, $y(t)=e \Rightarrow c=e^2 \qquad \therefore\ y\left(\frac{1}{2}\right)=8$

19. Given equation $\frac{dy}{dx}+\frac{x}{1+x^2}y=\frac{x}{1+x^2}$

I.F = $\sqrt{1+x^2}$

$y=1+c\left(1+x^2\right)^{-1/2};\ \frac{4}{3}=y(0)=1+c \Rightarrow c=\frac{1}{3}$

$y\left(\sqrt{8}\right)-\frac{1}{9}=1$

20. Put $\sin y=t \Rightarrow \cos y\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore\ \frac{dt}{dx}+\frac{4}{x}t=\frac{e^x}{x^3}$

$\text{I.F.}=e^{\int 4/x dx}=e^{4 l n x}=x^4$

$\frac{d(t.x^4)}{dx}=xe^x$

$\frac{dy}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}-\frac{A}{\sqrt{1-x^2}} \Rightarrow y^{|}=4y-4B+A^2-2\pi A$

21. $\Rightarrow 2\left(1-x^2\right)y^{|}y^{||}-2x\left(y^{|}\right)^2=4y^{|}$

$\Rightarrow \left(1-x^2\right)y^{||}-xy^{|}=2$

22. The given equation is $y=2px+y^2p^3$ ---(i)

Solving for x, $x=\frac{y}{2p}-\frac{1}{2}y^2p^3$

Differentiating w.r.t y, $\frac{dx}{dy}=\frac{1}{p}=\frac{1}{2p}-\frac{y}{2p^2}\cdot\frac{dp}{dy}-yp^2-y^2p\cdot\frac{dp}{dy}$

or $2p = p - y\dfrac{dp}{dy} - 2yp^3 - 2y^2p^2\dfrac{dp}{dy}$ or $p(1+2yp^2) + y\dfrac{dp}{dy}(1+2yp^2) = 0$

or $(1+2yp^2)\left(p + y\dfrac{dp}{dy}\right) = 0$

Neglecting the first factor which does not involve $\dfrac{dp}{dy}$, we have

$p + y\dfrac{dp}{dy} = 0 \qquad \Rightarrow \dfrac{dp}{p} + \dfrac{dy}{y} = 0$

Integrating $\log p + \log y = \log c$ or $\log py = \log c \Rightarrow py = c$ ---(ii)

Eliminating p between (i) and (ii) $y = 2x\cdot\dfrac{c}{y} + y^2\cdot\dfrac{c^3}{y^3}$ or $y = \dfrac{2cx}{y} + \dfrac{c^3}{y}$

or $y^2 = 2cx + c^3$ which is the required solution

23. (i) put $e^x = X$ and $e^y = Y$

So that $e^x dx = dX$ and $e^y dy = dY \Rightarrow \dfrac{e^y}{e^x}\cdot\dfrac{dy}{dx} = \dfrac{dY}{dX}$ or $\dfrac{Y}{X}p = P$(say) or $p = \dfrac{X}{Y}\cdot P$

$\therefore$ The given equation becomes $X^3\left(\dfrac{X}{Y}P - 1\right) + \dfrac{X^3}{Y^3}.Y^2P^3 = 0$

or $XP - Y + P^3 = 0$ or $Y = PX + P^3$

Which is of Clairaut's form.

$\therefore$ the solution is $Y = cX + c^3$ or $e^y = ce^x + c^3$

24. (i) Put $x = \dfrac{1}{X}$ and $y = \dfrac{1}{Y}$

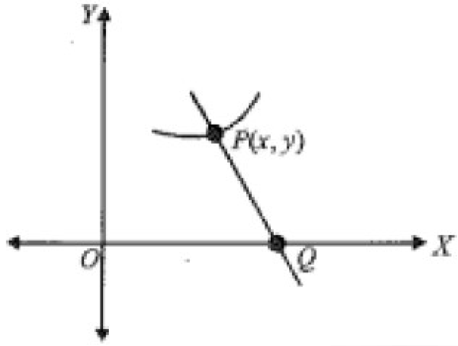
$dx = -\dfrac{1}{X^2}dX$ and $dy = -\dfrac{1}{Y^2}dY \Rightarrow p = \dfrac{dy}{dx} = \dfrac{X^2}{Y^2}\dfrac{dY}{dX} = \dfrac{X^2}{Y^2}P$

the given equation becomes

$\dfrac{1}{Y^2}\left(\dfrac{1}{Y} - \dfrac{1}{X}\cdot\dfrac{X^2}{Y^2}P\right) = \dfrac{1}{X^4}\cdot\dfrac{X^4}{Y^4}P^2 \Rightarrow Y - XP = P^2$ or $Y = PX + P^2$

Which is the Claraut's form

$\therefore$ The solution is $Y = cX + c^2$ or $\dfrac{1}{y} = \dfrac{c}{x} + c^2$

25. 

Given, length of PQ = k

$\therefore\ y\left(\frac{dy}{dx}\right)^2 + y^2 = k^2 \Rightarrow \quad y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

Which is the required differential equation of given curve. On solving this D.E., we get the Eqn. of curve as follows

$\int \frac{ydy}{\sqrt{k^2 - y^2}} = \int \pm dx \Rightarrow -\frac{1}{2}.2\sqrt{k^2 - y^2} = \pm x + c$

$-\sqrt{k^2 - y^2} = \pm x + c$

Since, it passes through (0, k), we get $c = 0$

$\therefore$ Equation of curve is $-\sqrt{k^2 - y^2} = \pm x \Rightarrow x^2 + y^2 = k^2$

The length of normal PQ to any curve $y = f(x)$ is given by $y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

26. Equation of normal at the point $p(x, y)$ is $Y - y = -\frac{dx}{dy}(X - x)\left(\text{let } m = \frac{dx}{dy}\right)$

Let , $m = \frac{dx}{dy} \Rightarrow X + mY - (x + my) = 0$

Distance of perpendicular from the origin to line (i) is $\frac{|x + my|}{\sqrt{1 + m^2}} = |y| \Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

This is homogeneous equation

Let , $y = zx \Rightarrow \frac{dy}{dx} = z + z\frac{dz}{dx} \Rightarrow \frac{2z}{1 + z^2}dz = -\frac{dx}{x}$

Integrating

$\int \frac{2z}{1 + z^2}dz = -\int \frac{dx}{x} \Rightarrow \log\left(1 + z^2\right) = -\log x + c \quad \Rightarrow \left(x^2 + y^2\right) = x.e^c$

This curve passes through $(1,1) \Rightarrow 1 + 1 = 1.e^c \Rightarrow e^c = 2$

The required equation of the curve is $\Rightarrow x^2 + y^2 = 2x$