ANSWER KEYS

1. (3)	**2.** (2)	**3.** (3)	**4.** (1)	**5.** (4)	**6.** (4)	**7.** (4)	**8.** (1)
9. (1)	**10.** (6)						

1. (3)

First find $\alpha^2+\beta^2$ in terms of λ

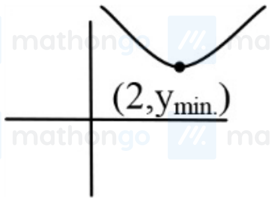
$\alpha+\beta=\lambda-3$ & $\alpha\beta=-\lambda$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(\lambda-3)^2+2\lambda$

$\alpha^2+\beta^2=\lambda^2-6\lambda+2\lambda+9=\lambda^2-4\lambda+9$

Now expression is quadratic in terms of λ.

It will be minimum at $\lambda=\left(-\frac{b}{2a}\right)\Rightarrow\lambda=\left(-\frac{(-4)}{2}\right)\Rightarrow\lambda=2$



$y_{\min}=-\frac{D}{4a}=\frac{-(16-36)}{4}=5$; at $\lambda=2,\ y_{\min}=5$

2. (2)

According to the question,
Let α is the common root of given equation,
Hence,
$\frac{\alpha^2}{b_1c_2-b_2c_1}=\frac{\alpha}{a_2c_1-a_1c_2}=\frac{1}{a_1b_2-a_2b_1}$
Above formula is used to find the common root between,
$a_1x^2+b_1x+c_1=0$
$a_2x^2+b_2x+c_2=0$
Now comparing given equations with above mentioned standard we get,
$a_1=1,\ b_1=b,\ c_1=-1$
and
$a_2=1,\ b_2=1,\ c_2=b$
Now put the above values in the given formula, and we get,
$\frac{\alpha^2}{b^2-(-1)}=\frac{\alpha}{-1-b}=\frac{1}{1-b}$
or $\alpha^2=\frac{b^2+1}{1-b}\quad\ldots.(\text{i})$
and $\alpha=\frac{b+1}{b-1}\quad\ldots.(\text{ii})$

Substitute the value of α in equation (i), we get,
$\left(\frac{b+1}{b-1}\right)^2=\frac{b^2+1}{1-b}$
Solving further we get,
$b^2+2b+1=b^2-b^3+1-b$
$\Rightarrow 3b=-b^3$
$\Rightarrow b^2=-3, b=0$
Hence,
$b=0, i\sqrt{3},\ -i\sqrt{3}$

3. (3)

$2x^2-7x+1=0$
$\Rightarrow x=\frac{7\pm\sqrt{41}}{4}$
$\because$ First equation has irrational roots.
$\therefore$ Both the roots will be common.
$\Rightarrow \frac{a}{2}=\frac{b}{-7}=\frac{2}{1}$
$\Rightarrow a=4,\ b=-14$

4. (1)
Given quation is $x^3 - 12x^2 + 39x - 28 = 0$

$\Rightarrow (x-4)\left(x^2 - 8x + 7\right) = 0$

$\Rightarrow (x-1)(x-4)(x-7) = 0$

$\Rightarrow x = 1,\ 4,\ 7$ or $7,\ 4,\ 1$

so,common difference $= +3or - 3$

5. (4) $e^{\sin x} - e^{-\sin x} - 4 = 0$

$\Rightarrow \left(e^{\sin x}\right)^2 - 4e^{\sin x} - 1 = 0 \Rightarrow t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$

i.e., $e^{\sin x} = 2 + \sqrt{5}$ or $\underbrace{2-\sqrt{5}}_{-\text{ve}}$ (neglected)

$\sin x = \ln\left(2 + \sqrt{5}\right) > 1$ not possible.

$\therefore$ No real roots.

6. (4)
Given equation,
$4\left(x^2 + \frac{1}{x^2}\right) + 16\left(x + \frac{1}{x}\right) - 57 = 0$
Let, $x + \frac{1}{x} = y;\ x^2 + \frac{1}{x^2} = y^2 - 2$
$\Rightarrow 4y^2 + 16y - 65 = 0$
$\Rightarrow y = -\frac{13}{2}$ or $\frac{5}{2}$
When, $y = \frac{5}{2}$
$x + \frac{1}{x} = \frac{5}{2} \Rightarrow x = 2$ or $\frac{1}{2}$
When, $y = -\frac{13}{2}$
$\Rightarrow x + \frac{1}{x} = -13/2$
$\Rightarrow 2x^2 + 13x + 2 = 0$
$\Rightarrow x = \frac{-13 \pm \sqrt{153}}{4}$
Since x is rational, $x = 2$ or $\frac{1}{2}$
Hence, their product is 1.

7. (4) $\alpha + \beta + \gamma = 0,\ \alpha\beta + \beta\gamma + \gamma a = -5,\ \alpha\beta\gamma = -4$

$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = (\alpha + \beta + \gamma)\left(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha\right) = 0$

$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$

$= -12$

8. (1)

Since, $\alpha,\ \beta,\ \gamma$ are the roots of the equation

$2x^3-3x^2+6x+1=0$

$\Rightarrow \alpha+\beta+\gamma=\frac{3}{2}....(1)$

$\Rightarrow \alpha\beta+\beta\gamma+\alpha\gamma=3....(2)$

$\Rightarrow \alpha\beta\gamma=\frac{-1}{2}....(3)$

On squaring Eq. (1), we get

$\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)=\frac{9}{4}$

Using equation (2), we get

$\alpha^2+\beta^2+\gamma^2=\frac{9}{4}-2\times3=\frac{-15}{4}$

9. (1) Let $e^x=t\in(0,\infty)$

Given equation

$t^4+t^3-4t^2+t+1=0$

$t^2+t-4+\frac{1}{t}+\frac{1}{t^2}=0$

$\left(t^2+\frac{1}{t^2}\right)+\left(t+\frac{1}{t}\right)-4=0$

Let $t+\frac{1}{t}=\alpha$

$\left(\alpha^2-2\right)+\alpha-4=0$

$\alpha^2+\alpha-6=0$

$\alpha^2+\alpha-6=0$

$\alpha=-3,2\Rightarrow\alpha=2\Rightarrow e^x+e^{-x}=2$

$x=0$ only solutions

10. (6)

We have,

$\left(7+4\sqrt{3}\right)^{x^2-4x+3}+\left(7-4\sqrt{3}\right)^{x^2-4x+3}=14$

Take

Put

$\left(7+4\sqrt{3}\right)^{x^2-4x+3}=t$

$\Rightarrow\left(\frac{7+4\sqrt{3}}{7-4\sqrt{3}}\times\left(7-4\sqrt{3}\right)\right)^{x^2-4x+3}=t$

$\Rightarrow\left(\frac{1}{7-4\sqrt{3}}\right)^{x^2-4x+3}=t$

$\Rightarrow\left(7-4\sqrt{3}\right)^{x^2-4x+3}=\frac{1}{t}$

Thus, given equation becomes

$t+\frac{1}{t}=14$

$\Rightarrow t^2-14t+1=0$

$\Rightarrow t=\frac{14\pm\sqrt{(14)^2-4(1)(1)}}{2}=7\pm4\sqrt{3}$

Thus,

$x^2-4x+3=1,\ -1$

$\Rightarrow x=2,\ 2\pm\sqrt{2}$