		ulas co 7%		77.	marino go	//	/ <b>//.</b>	GD 7//.	
ANSWER									
1. (3) matho	<b>2.</b> (66) mo	3. (1) ///.	<b>4.</b> (1) mathongo		<b>5.</b> (3) mathongo	6. (1) mathongo	7. (1) mathor	<b>8.</b> (1)	
<b>9.</b> (16)	<b>10.</b> (1)	<b>11.</b> (2)	<b>12.</b> (2)		<b>13.</b> (2)	<b>14.</b> (98)	13. (9)	<b>16.</b> (2)	
17. (1) athor	18. (2)	<b>19.</b> (272)	<b>20.</b> (2)		<b>21.</b> (25)	<b>22.</b> (3)	<b>23.</b> (3)	<b>24.</b> (2)	
<b>25.</b> (25)	<b>26.</b> (13)	<b>27.</b> (2)	<b>28.</b> (2)		<b>29.</b> (9)	<b>30.</b> (1)			
1. (3)									
1									
then $\alpha^2$		$^{2}-2lphaeta=rac{\lambda^{2}}{9}-$	, ,						
	$egin{array}{l}  imes rac{1}{lpha^2eta^2} = 15 = 0 \ 6 = 15 \Rightarrow \lambda = 0 \end{array}$		5 mathongo						
/// Now 6(a			$(eta+eta^2)\big)^2$ go						
$=6\times1$		= 24							
2. (66) We have $\frac{2}{x-1} - \frac{1}{x}$									
	$\frac{-x+1}{(x-2)} = \frac{2}{k}$ mo								
$/\!/\!/ \Rightarrow kx -$	$egin{array}{l} _{3k+2}k \ 3k = 2x^2 - 6x \ - \left( 6 + k  ight) x + 3k \end{array}$								
For no r	real roots $D < 0$ $k^2 - 8(3k + 4)$	ithongo ///.							
$egin{array}{ll} \Rightarrow k^2 + \ \Rightarrow (k - \ ) \ \Rightarrow  k - 6  \end{array}$		z – 32 < 0 atthongo ///.							
$\Rightarrow  \kappa - 0 $ $\Rightarrow 6 - \sqrt{}$	$\sqrt{32} < k < 6 + 1$	$\sqrt{32}$ ngo $/\!/$							
		2, 3, 4, 5, 6, 7, 8, withough							
	uct of roots =(1	-(1-2i+1+ -2i)(1+2i)=	·						
option (1	1)								

#### **Answer Kevs and Solutions**

### **Quadratic Equation** JEE Main Crash Course

Let 
$$x = 3 + \frac{1}{4 + \frac{1}{2 + \frac{1}{2$$

Let 
$$x = 3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{3$$

So, 
$$x = 3 + \frac{1}{4 + \frac{1}{x}} = 3 + \frac{1}{\frac{4x+1}{x}}$$

$$\Rightarrow (4x+1)(x-3)=x$$

$$\Rightarrow 4x^2 + 12x + x - 3 = x \text{ ongo}$$
 /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///

$$\Rightarrow 4x^2 - 12x - 3 = 0$$

$$x = \frac{12\pm\sqrt{(12)^2+12\times4}}{2\times4} = \frac{12\pm\sqrt{12(16)}}{8}$$
 /// mathongo /// mathongo /// mathongo /// mathongo ///

$$=\frac{12\pm4\times2\sqrt{3}}{8}=\frac{3\pm2\sqrt{3}}{2}$$

$$=\frac{127 \times 3 \times 6}{8} = \frac{622 \times 6}{2}$$

$$x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}.$$
 mathongo /// mathongo // mathongo /// mathongo /// mathongo /// mathongo /// mathongo // mat

But only positive value is accepted

So, 
$$x=1.5+\sqrt{3}$$
 mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo

In given equation 
$$x^2+px+q=0,\ p,\ q\in Q$$
, longo w mathongo w mathongo w mathongo w mathongo w







And, we know that the irrational roots of a quadratic equation exist in conjugate pair, if the coefficients are rational.

Thus, if one root of the equation  $x^2 + px + q = 0$  is  $2 - \sqrt{3}$ , then, the other root will be  $2 + \sqrt{3}$ .

Thus, if one root of the equation  $x^2 + px + q = 0$  is  $z - \sqrt{3}$ , then, the other root will be  $z + \sqrt{3}$ . We know that the sum and product of the roots of a quadratic equation  $ax^2 + bx + c = 0$  are respectively  $-\frac{b}{a}$  and  $\frac{c}{a}$ . Therefore, the sum of roots  $2+\sqrt{3}+2-\sqrt{3}=-p$  mathongo /// mathongo // mathongo

Therefore, the sum of roots 
$$2 + \sqrt{3} + 2 - \sqrt{3} = -p$$

$$\Rightarrow p = -4$$

$$\Rightarrow p = -4$$
 And, the product of roots  $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) = q$  mathongo m

$$\Rightarrow q=2^2-\left(\sqrt{3}
ight)^2=1$$

$$(-4)^2 - 4 \times 1 - 12$$

Thus, we have 
$$p^2 - 4q - 12 = (-4)^2 - 4 \times 1 - 12$$
 mathongo math

Thus, the answer is,  $p^2 - 4q - 12 = 0$ . athongo /// mathongo // matho

Given, 
$$x^4 + x^2 + 1 = 0$$

= 16 - 16 = 0.

Using the formula 
$$a^4 + a^2 + 1 = (a^2 + a + 1)(a^2 - a + 1)$$
 mathons with mathons with mathons and mathons and mathons are mathons as  $a^4 + a^2 + 1 = (a^2 + a + 1)(a^2 - a + 1)$ 





We get, 
$$(x^2 + x + 1)(x^2 - x + 1) = 0$$

So  $\alpha^{1011} + \alpha^{2022} - \alpha^{3033} = 1 + 1 - 1 = 1$ 

$$\Rightarrow x = \pm \omega$$
,  $\pm \omega^2$  where  $\omega = 1^{1/3}$  and imaginary. 30 /// mathongo /// mathongo /// mathongo ///



## **Quadratic Equation** JEE Main Crash Course

#### **Answer Kevs and Solutions**

$$x=rac{-\sqrt{6}\pm\sqrt{6-12}}{2}=rac{-\sqrt{6}\pm\sqrt{6}i}{2}=rac{\sqrt{3}\sqrt{2}(-1\pm i)}{\sqrt{2}\sqrt{2}}$$
 mathongo /// mathongo /// mathongo ///

$$=\sqrt{3}\left(\frac{-1}{12}\pm\frac{i}{\sqrt{2}}\right)$$
 athongo /// mathongo /// mathongo /// mathongo /// mathongo ///

$$= \sqrt{3} \left( \cos \frac{3\pi}{4} \pm \left( \sin \frac{3\pi}{4} \right) i \right)$$
 mathongo /// mathongo // mathongo /// mathongo /// mathongo /// mathongo /// mathongo // mathongo //

$$\alpha = \sqrt{3}e^{i\left(\frac{3\pi}{4}\right)}$$
 and  $\beta = \sqrt{3}e^{i\left(\frac{-3\pi}{4}\right)}$  /// mathongo /// mathongo /// mathongo /// mathongo ///

$$\alpha = \sqrt{3}e^{-(4)^2} \text{ and } \beta = \sqrt{3}e^{-(4)^2}$$

$$\text{Now, } (\alpha)^{23} + (\beta)^{23} = (\sqrt{3})^{23} \left[ e^{i\left(\frac{69\pi}{4}\right)} + e^{-i\left(\frac{69\pi}{4}\right)} \right]$$

$$= (\sqrt{3})^{23} \left[ 2\cos\left(\frac{69\pi}{4}\right) \right]$$
mathon
$$= (\sqrt{3})^{23} \left[ 2\cos\left(\frac{69\pi}{4}\right) \right]$$

Similarly 
$$\rightarrow (\alpha)^{14} + (\beta)^{14} = (\sqrt{3})^{14} \left[ 2\cos\left(\frac{45\pi}{4}\right) \right]$$

$$\rightarrow (\alpha)^{15} + (\beta)^{15} = (\sqrt{3})^{15} \left[ 2\cos\left(\frac{45\pi}{4}\right) \right]$$
mathons (mathons)

mathons 
$$o (lpha)^{10} + (eta)^{10} = (\sqrt{3})^{10} \left[ 2\cos\left(rac{30\pi}{4}
ight) 
ight]$$
 mathons  $o$  mathons

Now, 
$$\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$$
 mathongo /// mathongo // mathongo /// mathongo /// mathongo /// mathongo // mathongo // mathongo // mathongo // mathongo

$$= \frac{(\sqrt{3})^{23} \left[ 2\cos\left(16\pi + \frac{5\pi}{4}\right) \right] + (\sqrt{3})^{14} \left[ 2\cos\left(10\pi + \frac{\pi}{2}\right) \right]}{(\sqrt{3})^{15} \left[ 2\cos\left(10\pi + \frac{5\pi}{4}\right) \right] + (\sqrt{3})^{10} \left[ 2\cos\left(16\pi + \frac{3\pi}{2}\right) \right]}$$
 mathongo we mathongo we mathongo we have

$$=\frac{(\sqrt{3})^{23}2\cos\left(\frac{5\pi}{4}\right)}{(\sqrt{3})^{15}2\cos\left(\frac{5\pi}{4}\right)}=(\sqrt{3})^8=81$$
 mathongo /// mathongo // mathongo //

For given quadratic  $375x^2 - 25x - 2 = 0$ , its roots are,

$$\alpha, \beta = \frac{25 \pm \sqrt{25^2 + 2 \times 4 \times 375}}{2 \times 375} \Rightarrow |\alpha| < 1, |\beta| < 1$$
 mathongo /// ma

Now 
$$\lim_{n\to\infty}\sum_{r=1}^n \alpha^r + \lim_{n\to\infty}\sum_{r=1}^n \beta^r$$
 mathongo mathon

Also, 
$$\alpha + \beta - \frac{1}{375}$$
,  $\alpha \beta - \frac{1}{375}$ 

Now  $\lim_{n \to \infty} \sum_{r=1}^{n} \alpha^r + \lim_{n \to \infty} \sum_{r=1}^{n} \beta^r$ 

$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} \begin{bmatrix} a + ar + ar^2 + \dots & \text{infinite terms} \\ & = \frac{a}{1-r} & \text{if } |r| < 1 \end{bmatrix}$$

$$= \frac{(\alpha+\beta) - 2\alpha\beta}{1 - (\alpha+\beta) + \alpha\beta} = \frac{\frac{25}{375} + \frac{4}{375}}{1 - \frac{25}{375} - \frac{2}{375}} = \frac{29}{348} = \frac{1}{12}$$

Mathons

9. (16) Given, mathongo mathongo mathongo	
$P_n = lpha^{ m n} - eta^{ m n}$ and $lpha \ \& \ eta$ are roots of $x^2 - x - 4 = 0$	
Now replacing $n$ with $n-1$ in $P_n=\alpha^{\mathrm{n}}-\beta^{\mathrm{n}}$ we get, $P_{n-1}=(\alpha^{n-1}-\beta^{n-1})$	
Now subtracting $P_{i} - P_{i-1}$ we get	
$\Rightarrow P_n - P_{n-1} = \alpha^{n-2} \left( \alpha^2 - \alpha \right) - \beta^{n-2} \left( \beta^2 - \beta \right)$ Now using the equation $\alpha^2 - \alpha - 4 = 0 \& \beta^2 - \beta - 4 = 0$ we get,	
$P_n-P_{n-1}=4\left(lpha^{n-2}-eta^{n-2} ight)$ $P_n-P_{n-1}=4P_{n-2}$ mathongo /// mathongo	
which implies that $P_{15}-P_{14}=4P_{13}$ and $P_{16}-P_{15}=4P_{14}$	
Now putting the value in given expression $P_{15}P_{16}-P_{14}P_{16}-P_{15}^2+P_{14}P_{15}$	
$=\frac{P_{13}P_{14}}{P_{16}\left(P_{15}-P_{14}\right)-P_{15}\left(P_{15}-P_{14}\right)}$ mathongo /// mathongo	
$ = \frac{\left(P_{15} - P_{14}\right)\left(P_{16} - P_{15}\right)}{P_{13}P_{14}} = \frac{\left(4P_{13}\right)\left(4P_{14}\right)}{P_{13}P_{14}} = 16                                 $	
10. (1)	
Given, $x^2 + 5\sqrt{2}x + 10 = 0$ 30 44 mathongo 44 mathongo	
and $D = a^n - Q^n$	
$\text{Now } \frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} = \frac{P_{17}\left(P_{20} + 5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19} + 5\sqrt{2}P_{18}\right)}$	
$=rac{P_{17}\left(lpha^{20}-eta^{20}+5\sqrt{2}\left(lpha^{19}-eta^{19} ight) ight)}{P_{18}\left(lpha^{19}-eta^{19}+5\sqrt{2}\left(lpha^{18}-eta^{18} ight)} ight)}$ ngo /// mathongo /// mathongo	
$=\frac{P_{17}\left(\alpha^{19}\left(\alpha+5\sqrt{2}\right)-\beta^{19}\left(\beta+5\sqrt{2}\right)\right)}{P_{18}\left(\alpha^{18}\left(\alpha+5\sqrt{2}\right)-\beta^{18}\left(\beta+5\sqrt{2}\right)\right)}$	
Since $\alpha + 5\sqrt{2} = -10/\alpha$ (1)	
$\& \beta + 5\sqrt{2} = -10/\beta$ at $\ensuremath{}(2)$ /// mathongo /// mathongo	
Now put these values in above expression $\frac{P_{17} \left(\alpha^{19} \left(\alpha + 5\sqrt{2}\right) - \beta^{19} \left(\beta + 5\sqrt{2}\right)\right)}{P_{18} \left(\alpha^{18} \left(\alpha + 5\sqrt{2}\right) - \beta^{18} \left(\beta + 5\sqrt{2}\right)\right)} = \frac{-10 P_{17} P_{18}}{-10 P_{18} P_{17}} = 1$	
11. (2) Given equation is- $ (x^2) - 8x + 15  - 2x + 7 = 0$ mathongo	
(x-5)(x-3)  - 2x + 7 = 0	
For $\mathrm{x} \leq 3$ or $x \geq 5$	
$x^2-8x+15-2x+7=0$ ngo /// mathongo /// mathongo $x=5+\sqrt{3}$	
For $3 < x < 5$ , mathongo $x^2 - 8x + 15 + 2x - 7 = 0$ mathongo $x^2 - 8x + 15 + 2x - 7 = 0$	
$x=4$ W Hence sum $=9+\sqrt{3}$ athongo /// mathongo /// mathongo	

## **Quadratic Equation** JEE Main Crash Course

### **Answer Keys and Solutions**

12. (2) athongo											
-----------------	--	--	--	--	--	--	--	--	--	--	--

#### Case-I

/// 
$$x \le 5$$
hongo /// mathongo ///

$$(x+1)^2-(x+1)-rac{3}{4}=0$$
 mathongo mathongo

$$x=rac{1}{2},-rac{3}{2}$$
A mathongo ///, mathongo ///,

$$x=rac{1}{2},-rac{3}{2}$$
/// Case-II mathongo /// mathongo // m

$$(x+1)+(x-5)=\frac{27}{4}$$
 athongo /// mathongo /// mathongo

$$x = \frac{-1 \pm \sqrt{52}}{2}$$
 (rejected as  $x > 5$ ) /// mathongo /// mathong

So sum of roots will be 
$$\alpha + \beta = \sqrt{2}$$
 and product of roots will be  $\alpha\beta = \sqrt{6}$  ...

And also given  $\frac{1}{\alpha^2} + 1$  and  $\frac{1}{\beta^2} + 1$  are roots of  $x^2 + ax + b = 0$ 

So sum of roots will be 
$$-a=\frac{1}{\alpha^2}+1+\frac{1}{\beta^2}+1$$
 mathong /// mathong ///

And similarly product of roots will be, 
$$b = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + 1 + \frac{1}{\alpha^2 \beta^2} \dots (2)$$

Now adding equation (1) & (2) we get, 
$$a+b=\frac{1}{(\alpha\beta)^2}-1=\frac{1}{6}-1=-\frac{5}{6} \ \{ \text{as } \alpha\beta=\sqrt{6} \}$$

Now putting the value of 
$$a+b$$
 in  $x^2-(a+b-2)x+(a+b+2)=0$  
$$\Rightarrow x^2-\left(-\frac{5}{6}-2\right)x+\left(2-\frac{5}{6}\right)=0$$

$$\Rightarrow 6x^2 + 17x + 7 = 0$$
 mathong mathong mathons  $\Rightarrow x = -\frac{7}{3}, x = -\frac{1}{2}$  are the roots, both roots are real and negative.

14. (98) Given 
$$\alpha \& \beta$$
 are roots of  $x^2 - 4\lambda x + 5 = 0$  mathongo mathongo

So, 
$$\alpha+\beta=4\lambda$$
 and  $\alpha\beta=5$   
And  $\alpha \ \& \ \gamma$  are roots of  $x^2-(3\sqrt{2}+2\sqrt{3})x+(7+3\lambda\sqrt{3})=0$ 

And 
$$\alpha & \gamma$$
 are roots of  $x^2 - (5\sqrt{2} + 2\sqrt{3})x + (7 + 3\sqrt{3}) \equiv 0$   
So,  $\alpha + \gamma = 3\sqrt{2} + 2\sqrt{3}$  and  $\alpha \gamma = 7 + 3\lambda\sqrt{3}$   
Given, if  $\beta + \gamma = 3\sqrt{2}$ 

So, from equation (i) and (ii) we get, 
$$\alpha = 2\lambda + \sqrt{3}$$
 and // mathongo // mathongo // mathongo // mathongo //

So, from equation (1) and (11) we get, 
$$\alpha=2\lambda+\sqrt{3}$$
 and  $\beta=2\lambda-\sqrt{3}$ , Now by product of roots we get,  $4\lambda^2-3=5\Rightarrow\lambda=\sqrt{2}$ 

$$\therefore (\alpha + 2\beta + \gamma)^2 = (\alpha + \beta + \beta + \gamma)^2 = (4\lambda + 3\sqrt{2})^2$$

$$= (4\sqrt{2} + 3\sqrt{2})^2 = (7\sqrt{2})^2 = 98$$
mathongo we mathon which is a simple with the second contraction of t

$$= (4\sqrt{2} + 3\sqrt{2}) = (7\sqrt{2}) = 90$$

/// mgthongs // mgthongs

15. (9) athongo ///. mathongo							
$x^2 - 12x + [x] + 31 = 0$							
$\{x\} = x^2 - 11x + 31$ $0 \le x^2 - 11x + 31 < 1$							
$x^2 - 11x + 30 < 0$ $x \in (5, 6)$ mathongo							
so $[x] = 5$ $x^2 - 12x + 5 + 31 = 0$ $x^2 - 12x + 36 = 0$							
$x = 6$ but $x \in (5,6)$ so at $x \in \phi$ mathongo							
$egin{aligned} \mathbf{m} &= 0 \ \mathbf{Now} \ & x^2 - 5 x+2  - 4 = 0 \end{aligned}$							
$x \ge -2$ $x^2 - 5x - 14 = 0$ mathongo							
(x-7)(x+2) = 0 x = 7, -2 x < -2 mathons							
$x^2 + 5x + 6 = 0$							
$x = -3, -2 \ \mathrm{x} = \{7, -2, -3\}$							
$m^2 + mn + n^2 = n^2 = 9$							
16. (2) The Given equation is $x^2 + 9y^2 - $	4x +	3 = 0					
$\Rightarrow 9y^2 + 0y + (x^2 - 4x + 3) = 0$ Make quadratic of $y$ , we have $D \ge 0$							
$\Rightarrow 0-4 imes 9 imes ig(x^2-4x+3ig) {\geq 0}$							
$\Rightarrow x^2 - 3x - x + 3 \le 0 \text{ thongo}$ $\Rightarrow (x - 3)(x - 1) \le 0$							
$x \in [1,3]_{\text{max}}$ mathons							
Now making quadratic in $x$ equation	on is	$\begin{array}{c} \text{mathongo} \\ x^2 - 4x + 3 + \end{array}$	$9y^2$ =	mathongo = 0			
Now making quadratic in $x$ equation $D \geq 0$ $16-4  imes (3+9y^2) \geq 0$	on is	$x^2 - 4x + 3 +$	$9y^2$ =	= 0			
	on is a	$x^2 - 4x + 3 +$ mathongo	9y2 =	= 0 mathongo			
$egin{aligned} D &\geq 0 \ 16 - 4  imes ig(3 + 9y^2ig) &\geq 0 \ \Rightarrow 4 - 3 - 9y^2 &\geq 0 \end{aligned}$	on is :	$x^2 - 4x + 3 +$ mathongo	9 <i>y</i> <sup>2</sup> =	= 0 mathongo mathongo			
$egin{aligned} D &\geq 0 \ 16 - 4  imes (3 + 9y^2) &\geq 0 \ \Rightarrow 4 - 3 - 9y^2 &\geq 0 \ \Rightarrow 9y^2 &\leq 1 \ \Rightarrow y &\in \left[rac{-1}{3}, rac{1}{3} ight] \end{aligned}$	on is a	$x^2 - 4x + 3 +$ mathongo mathongo	9 <i>y</i> <sup>2</sup> =	= 0 mathongo mathongo			
$D\geq 0$ $16-4 imes(3+9y^2)\geq 0$ $\Rightarrow 4-3-9y^2\geq 0$ $\Rightarrow 9y^2\leq 1$ $\Rightarrow y\in\left[rac{-1}{3},rac{1}{3} ight]$	on is : ///. ///.	$x^2 - 4x + 3 +$ mathongo mathongo mathongo	9y <sup>2</sup> = ///.	mathongo mathongo mathongo			

17. (1) athongo /// mathongo /// mathongo /// matho				
Let $\alpha, \beta$ be the roots of the equation				
$x^2 + (3-a)x + 1 - 2a = 0$ mathong /// mathon				
Then, sum of roots $\alpha + \beta = a - 3$				
And product of roots $\alpha\beta=1-2a$ We know that $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$				
$\therefore \alpha^2 + \beta^2 = (a-3)^2 - 2(1-2a)$ $= a^2 - 2a + 7$ mathons				
$= (a-1)^2 + 6$ $\therefore \text{ Minimum value of } \alpha^2 + \beta^2 = 6 \text{ at } a = 1.$				
18. (2)				
We have, ngo /// mathongo /// mathongo /// matho				
$x^{2} + 2(a + 4)x - 5a + 64 > 0$				
If $A>0$ and $D=B^2-4AC<0$ , then $Ax^2+Bx+C>0\ orall x\in R$	ngo ///. mat			
Hence,				
$D < 0 \ \Rightarrow \ 4{\left( {a + 4} \right)^2} - 4{\left( { - 5a + 64} \right)} < 0$				
$\Rightarrow a^2 + 16 + 8a + 5a - 64 < 0$ $\Rightarrow a^2 + 13a - 48 < 0$ mathons with mathons and mathons				
$\Rightarrow (a+16)(a-3) < 0$				
⇒ $a \in (-16,3)$ mathongo mathongo mathongo Possible values for $a$ are $\{-15, -14, \ldots, 2\}$ containing 18 s				
But given range of $a$ is $[-5, 30]$ , hence $a$ would take values $\{-5, -5, -5\}$		1 0 1 23 co	ntaining & integers	
And, $[-5, 30]$ has 36 numbers.	rigo 9,7/2 Anat	1, 0, 1, 2,, 00	ntanning o integers	matho
∴ Required probability				
$= \frac{1}{36}$				
$= \frac{2}{9}$ mathongo /// mathongo /// mathongo /// mathongo /// mathongo				
Given, $(p^2 + q^2)x^2 - 2q(p+r)x + q^2 + r^2 = 0$				
On simplifying we get, $(px-q)^2 + (qx-r)^2 = 0$				
$\Rightarrow px - q = 0 \& qx - r = 0$				
$x \Rightarrow x \pm rac{q}{p} \equiv rac{r}{q}$ /// mathongo /// mathongo /// mathongo				
$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = 4$ [because roots of equation $x^2 - 2x - 8 = 0$ as				
As $p, q, r$ are positive, so $x$ must be 4.				
Now, $q=4p$ and $r=4q=16p$				
So, $\frac{q^2+r^2}{p^2}=\frac{\left(4p\right)^2+\left(4 imes4p\right)^2}{p^2}=16+256=272.$				

### **Quadratic Equation** JEE Main Crash Course

#### **Answer Kevs and Solutions**

20. (2) athongo						
Given $ar^2 - 2hr$	+15 = 0 (i)					

Given 
$$ax^2 - 2bx + 15 = 0$$
 ...(i)

Has repeated roots So 
$$D \equiv 0$$
 ngo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///

$$4b^2-4 imes15 imes a=0$$

$$\Rightarrow b^2 = 15a$$
 ...(ii) mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///.

$$x^2 = 2bx + 21 = 0$$
 Mathongo Root nathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///

Now 
$$\alpha$$
 will satisfy both quadratic mathematic mathematic mathematic mathematics.

$$ax^2-2bx+15=0\ \&\ x^2-2bx+21=0$$
  
W Putting the value we get though  $w$  mathong  $w$ 

$$alpha^2-2blpha+15=0$$

$$(a-1)\alpha^2=6$$
 mathongo mathong

Now in equation (1) product of Root 
$$\alpha^2 = \frac{15}{a}$$
  
So  $\frac{15}{a} = \frac{6}{a-1} \Rightarrow 2a = 5a - 5 \Rightarrow a = \frac{5}{3}$ 

Now 
$$b^2=15a\Rightarrow b^2=15\times \frac{5}{3}\Rightarrow b^2=25$$

So  $b=\pm 5$  when mathong with mathon with mathon mathon mathon with mathon mathon

$$^{\prime\prime\prime}$$
  $_{
m So}$   $_b$   $^{\perp}$   $^{\perp}$   $^{\perp}$   $^{\perp}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$ 

Now in quadratic 
$$x^2 - 2bx + 21 = 0$$
  
Putting the value of  $b$  we get  $100$  /// mothongo // m

Putting the value of 
$$b$$
 we get 
$$x^2 - 10x + 21 = 0 \Rightarrow (x - 7)(x - 3) = 0$$

$$^{\prime\prime\prime}$$
 So  $x=3$  or 7.  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$  mathongo  $^{\prime\prime\prime}$ 

$$x^2+10x+21=0\Rightarrow x=-3 \ or \ x=-7$$
  
So  $lpha=\pm 3 \ \& \ eta=\pm 7$ 

So 
$$\alpha^2+\beta^2=3^2+7^2=9+49=58$$
 mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///

# Quadratic Equation JEE Main Crash Course

#### **Answer Keys and Solutions**

21. (25) thongo /// mathongo ///							
Let $f(x) = (x - \alpha)(x - \beta)$							
It is given that $f(0) = p \Rightarrow \alpha\beta = p$ and $f(1) = \frac{1}{3} \Rightarrow (1 - \alpha)(1 - \beta) = \frac{1}{3}$							
Now let us assume that $\alpha$ is the common factor $f(x) = 0$	on root of $f(x)$ =	= 0 an	d fofofof(x)	= 0			
$fofofof\left( x ight) =0$	mathorigo		rnatriongo'				
$\rightarrow f_0 f_0 f_0 f(\alpha) - 0$							
$\Rightarrow fofof(0) = 0$							
$\Rightarrow fof(p) = 0$							
So, $f(p)$ is either $\alpha$ or $\beta$ .							
Now assuming $(p-\alpha)(p-\beta)=\alpha$							
$\Rightarrow (\alpha\beta - \alpha)(\alpha\beta - \beta) = \alpha \Rightarrow (\beta - 1)(\alpha\beta - \beta) = \frac{\beta}{3} = 1 \left( as (1 - \alpha)(1 - \beta) = \frac{1}{3} \right)$	$(a-1)\beta = 1$						
/// So, $\beta = 3$ go /// mathongo ///							
Now finding $\alpha$ by putting the value of							
$\Rightarrow (1-\alpha)(1-3) = \frac{1}{3}$ $\Rightarrow \alpha = \frac{7}{6}$			0				
So, $f(x) = \left(x - \frac{7}{6}\right)(x - 3)$							
So, $f(-3) = \left(-3 - \frac{7}{6}\right)(-3 - 3) = 25$							
	n O and 1						
22. (3) As there is exactly one root betwee $f(0) \cdot f(1) \le 0$	ii v and 1, 1190						
$\Rightarrow 2(\lambda^2 + 1 - 4\lambda + 2) \le 0 \Rightarrow 2(\lambda^2 - 1)$	$4\lambda + 3$ $< 0$						
$\Rightarrow (\lambda - 1)(\lambda - 3) \le 0 \Rightarrow 2(\lambda + 1)(\lambda - 3) \le 0$	Third Higo						
$\Rightarrow \lambda \in [1, 3]$							
But at $\lambda = 1$ , both roots are 1.							
So, $\lambda  eq 1$ , $\lambda \in (1,3]$							
23. (3) mathongo /// mathongo ///							
$x^3 - 2x^2 + 2x - 1 = 0$							
x = 1 satisfying the equation							
$\therefore x - 1 \text{ is factor of } x^3 - 2x^2 + 2x - $							
$x = (x-1)(x^2 - x + 1) = 0$ $x = 1, \frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}$							
$x=1,-\omega^2,-\omega$							
$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$ $= 1 + (\omega)^{324} + (\omega)^{162}$							
= 1 + 1 + 1 = 3 /// mathongo /// mathongo ///							

## **Answer Keys and Solutions**

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24. (2) athongo /// math Given:			
	$+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10$		
Now, $\sqrt{3}-\sqrt{2}=rac{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{3}+\sqrt{2}\right)}{\sqrt{3}+\sqrt{2}}$	$(\sqrt{2}) = \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}}$ mathongo		
$\Rightarrow \left(\sqrt{3} + \sqrt{2}\right)^{x^2 - 4} + \left(\frac{1}{\sqrt{3}}\right)^{x^2 - 4}$	$\left(\frac{1}{1+\sqrt{2}}\right)^{x^2-4} = 10$ athongo		
$\Rightarrow u^2-10u+1=0 \ \Rightarrow u=rac{10\pm\sqrt{100-4}}{2}$			
$\Rightarrow u = \left(\sqrt{3} \pm \sqrt{2} ight)^2 \ \Rightarrow \left(\sqrt{3} + \sqrt{2} ight)^{x^2-4} = \left(\sqrt{3} + \sqrt{3} +$	ongo (mathongo) $3 \pm \sqrt{2}$		
Therefore, $x^2-4=2\ \&\ x^2-4=-2$ $\Rightarrow x=\pm\sqrt{6}\ \&\ x=\pm\sqrt{2}$			
	$\left(-\sqrt{2}\right)$ mathonag		
Hence, $n(S) = 4$			

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. (25) thongo ///								
Given,								
$\log_2\bigl(9^{2\alpha-4}+13\bigr)-$	$\log_2 \left(3^{2\alpha-4} \cdot \frac{5}{2}\right)$	+1)=2 thongo						
Now let $3^{2\alpha-4} = t$ ,								
$\log_2ig(t^2+13ig)-\log_2ig(t^2+13ig)$	$2(\frac{5t}{2}+1)=2$							
( 2)	,							
$\Rightarrow \log_2 \frac{(t^2+13)}{\left(\frac{5t}{2}+1\right)} = 2$	mathongo							
$\Rightarrow rac{(t^2+13)}{\left(rac{5t}{2}+1 ight)}=2^2$								
$(rac{1}{2}^{+1}) \ \Rightarrow t^2+13=10t+1$	∡mathongo							
$\Rightarrow t^2 - 10t + 9 = 0$								
$\Rightarrow t = 1 \text{ or } 9$								
So,								
$3^{2lpha-4}=1  ext{ or } 9$								
$\Rightarrow 3^{2lpha-4}=3^0  ext{ or } 3^2$	i							
$\Rightarrow 2\alpha - 4 = 0 \text{ or } 2$ $\Rightarrow \alpha = 2, 3$	<u>mathongo</u>							
•								
Now, $\mathrm{x}^2 - 2 \left(\sum_{\alpha \in s} \alpha\right)^2 \mathrm{x}$	$\sum_{i=1}^{n} (a_i + 1)^{i}$	$)^2 \beta = 0$ othonogo						
$x = 2(\sum_{lpha \in s} lpha) x$ $\Rightarrow x^2 - 2((2+3)^2)$	\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \							
$\Rightarrow x^2 - 50x + 25\beta$	$x_j + (3 + 4) $	/// mathongo						
$\Rightarrow x - 50x + 25\beta$ Now for real roots								
$D \ge 0$ ongo ///.								
$\Rightarrow 50^2 - 4  imes 25eta \geq$								
$\Rightarrow 50-2eta \geq 0$								
$\Rightarrow eta \leq 25$								
So, maximum value	of $\beta$ is 25.	///. mathongo						
<b>. (13)</b> We have, $a + b$	o+c=1, ab+	bc + ca = 2 and $ab$	c=3	}				
Now, $a^2 + b^2 + c^2 =$ $\Rightarrow (ab + bc + ca)^2$	$= (a+b+c)^2$	$-2\Sigma ab = -3$						
$\Rightarrow (ab+bc+ca)^2 \ \Rightarrow \varSigma(ab)^2 = -2$	$= \Sigma(ab)^2 + 2a$	$bc\Sigma$ a						
$\Rightarrow \mathcal{Z}(ab) = -2$ So, $a^4 + b^4 + c^4 =$	$(a^2 + b^2 + a^2)^2$	$^{2}/\sqrt{2} \nabla (ab)^{2}$						
=9-2(-2)	(a + b + c)	-22(ab)						
= 13.hongo ///.								

#### **Answer Kevs and Solutions**

### **Quadratic Equation** JEE Main Crash Course

27. (2) Let 
$$e^{2x} = t$$
 /// mathongo // mat

$$\Rightarrow t^2 - t^2 - 3t^2 - t + 1 = 0$$

$$\implies t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) + 3 = 0 \quad \text{mathongo} \quad \text{mat$$

$$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 = 0$$
mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///

Putting 
$$(t + (1/t)) = a$$
, the equation becomes-

$$a^2-a-5=0$$
 mathongo mathongo

So, 
$$a=(t+(1/t))=rac{1+\sqrt{21}}{2}$$
 mathong /// mathong // mathong /// mathong // mathong /

$$e^{(2x)} + \left(\frac{1}{e^{(2x)}}\right) \equiv \frac{1+\sqrt{21}}{2}$$
 mathongo /// mathongo // mat

So, two real solutions and this, graph of the function given cuts 
$$x$$
-axis 2 times

$$e^{4x}+8e^{3x}+13e^{2x}-8e^x+1=0,\ x\in R$$
Now let  $e^x=t$  we get,

$$t^4+8t^3+13t^2-8t+1=0$$
Now divide complete equation by  $t^2$  mathongo /// mathong

$$\Rightarrow t^2 + \frac{1}{t^2} + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

$$\Rightarrow \left(t - \frac{1}{t}\right)^2 + 8\left(t - \frac{1}{t}\right) + 15 = 0$$
/// mathongo // mathon

$$\Rightarrow \left(t - \frac{1}{t}\right)^2 + 8\left(t - \frac{1}{t}\right) + 15 = 0$$

Now let 
$$t-\frac{1}{t}=z$$
 we get, mathongo /// mathongo // mathong

$$\Rightarrow t = \frac{-3\pm\sqrt{13}}{2}, \frac{-5\pm\sqrt{29}}{2} \text{ mono} \qquad \text{mathongo} \qquad \text{mathon$$

So, 
$$e^x = \frac{-3+\sqrt{13}}{2}$$
,  $\frac{-5+\sqrt{29}}{2} = \alpha$ ,  $\beta$  (rejecting negative values as exponential is positive function)

And both 
$$\frac{-3+\sqrt{13}}{2}$$
 and  $\frac{-5+\sqrt{29}}{2} \in (0,1)$  mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///

So, 
$$x = \ln(\alpha)$$
,  $\ln(\beta)$  are both negative,

# **Answer Keys and Solutions**

# Quadratic Equation JEE Main Crash Course

. (9) athongo ///. m						
Given:						
$x^7 + 3x^5 - 13x^3 - 15x$ $\Rightarrow x(x^6 + 3x^4 - 13x^2)$						
So, $x = 0$ is one of the Now,	root					
$(x^6+3x^4-13x^2-15)$ Put $x^2=t$ , then we have	male e e e e e e					
$t^3 + 3t^2 - 13t - 15 = 0$						
$\Rightarrow (t-3)(t^2+6t+5)$						
$\Rightarrow (t-3)(t+1)(t+5)$						
So, $t = 3, t = -1, t = -1$						
Now we are getting						
$x^2=3, x^2=-1, x^2= \ \Rightarrow x=\pm\sqrt{3}, \ \pm i, \pm \sqrt{3}$						
From the given condition	on $ \alpha_1  \ge  \alpha_2  \ge .$	$\ldots \ge  lpha_7 $				
We can clearly say that	$ lpha_7 =0$ and					
and $ lpha_6 =\sqrt{5}= lpha_5 $						
and $ lpha_4 =\sqrt{3}= lpha_3 $ an	ndthongo ///.					
$ \alpha_2 {=1}{=} \alpha_1 $						
So we can have,						
$lpha_1=\sqrt{5}i, \; lpha_2=-\sqrt{5}i$						
$lpha_4=-\sqrt{3},\ lpha_5=i,\ lpha_6$	$_{3}=-i$					
$lpha_7=0$						
Hence, $lpha_1lpha_2-lpha_3lpha_4+lpha_5lpha_6$						
=5-(-3)+1=9						

0. (1) athongo ///. mathongo ///.				
Consider the equation $x^2 + ax + b = 0$				
It has two roots (not necessarily real $\alpha$ : Either $\alpha = \beta$ or $\alpha \neq \beta$	and $\beta$ ) hongo			
, , ,	ot Given than $\alpha^2$	2 is also root		
Case (1) If $\alpha = \beta$ , then it is repeated ro So, $\alpha = \alpha^2 - 2$				
$\Rightarrow (\alpha+1)(\alpha-2)=0$ $\Rightarrow \alpha=-1 \text{ or } \alpha=2$				
When $\alpha = -1$ then $(a,b)$ = $(2,1)$				
$\alpha=2$ then $(a,b)=(-4,4)$				
Case (2) If $\alpha \neq \beta$ , then four possibilities	es are there			
(I) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$				
Here $(\alpha, \beta) = (2, -1)$ or $(-1, 2)$				
Hence $(a,b)$ = $(-(\alpha+\beta),\alpha\beta)$ = $(-1,-2)$				
(II) $lpha=eta^2-2$ and $eta=lpha^2-2$				
Then $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$ Then $\alpha - \beta = \beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \beta)$	$\alpha)$ athongo			
Since $\alpha \neq \beta$ we get $\alpha + \beta = \beta^2 + \alpha^2$				
$lpha + eta = \left(lpha + eta ight)^2 - 2lphaeta - 4$				
Thus $-1=1-2lphaeta-4$ which implies	<b>S</b>			
$\alpha\beta=-1$ . Therefore $(a,b)=(-(\alpha+\beta)$	, lphaeta) thomas $/$			
=(1,-1)				
(III) $\alpha=\alpha^2-2=\beta^2-2$ and $\alpha \neq \beta$ $\Rightarrow \alpha=-\beta$				
Thus $\alpha=2,\beta=-2$ $\alpha=-1,\beta=1$				
Therefore $(a, b) = (0, -4)$ and $(0, -1)$				
(IV) $\beta = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$ i	s same as (III)			
Therefore we get 6 pairs of $(a, b)$				
Which are $(2, 1), (-4, 4), (-1, -2), (1, -2)$	-1), (0, -4), (0, -1)	l) mathongo		