Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 16-09-2023
Time: 09.00Am to 12.00Pm RPTM-067 Max. Marks: 300

KEY SHEET

PHYSICS

1)	4	2)	1	3)	2	4)	2	5)	2
6)	4	7)	4	8)	3	9)	3	10)	2
11)	4	12)	4	13)	1	14)	4	15)	2
16)	1	17)	3	18)	2	19)	1	20)	1
21)	18	22)	48	23)	4	24)	28	25)	30
26)	4	27)	6	28)	4	29)	8	30)	3

CHEMISTRY

31)	2	32)	2	33)	4	34)	2	35)	2
36)	2	37)	2	38)	3	39)	1	40)	1
41)	3	42)	2	43)	2	44)	3	45)	4
46)	1	47)	4	48)	2	49)	2	50)	3
51)	2	52)	6	53)	9	54)	0	55)	9
56)	18	57)	21	58)	6	59)	3	60)	6

MATHEMATICS

61)	2	62)	1	63)	1	64)	1	65)	1
66)	2	67)	3	68)	1	69)	4	70)	4
71)	3	72)	1	73)	1	74)	2	75)	2
76)	3	77)	4	78)	4	79)	1	80)	2
81)	8	82)	4	83)	8	84)	16	85)	4
86)	45	87)	4	88)	2	89)	9	90)	5

SOLUTIONS

PHYSICS

1. $Q \propto A \Rightarrow \frac{Q_1}{Q} = \frac{2(2\pi R^2 + \pi R^2)}{4\pi R^2} = 1.5$

2. $\vec{r}f = \vec{r}i + \Delta\vec{S}_1 + \vec{S}_2 + ...$

   $\vec{r}f = \left(2\hat{i} + 3\hat{j}\right) + 5\hat{i} + 8\hat{j} + \left(-2\hat{i} + 4\hat{j}\right) + \left(-6\hat{j}\right)$

   $\vec{r}f = 5\hat{i} + 9\hat{i}$

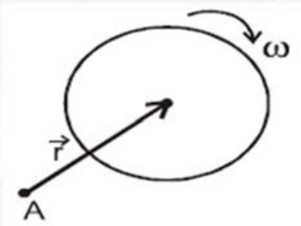
3. $N - mg = 0, \quad f = \mu N$

   $f = ma \quad \Rightarrow a = \mu g = 4m/s^2$

   $u = 0, \vartheta = 2 \quad \vartheta^2 - u^2 = 2as \Rightarrow s = \frac{2^2 - 0^2}{2 \times 4} = 0.5m$

4. Momentum can be zero

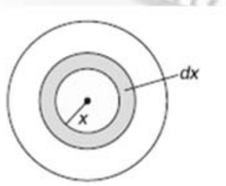
5. The angular momentum of disc about point A is $\vec{L}_A = I_{cm}\vec{\omega} + m\vec{r} \times \vec{v}_{cm}$



   $\vec{v}_{cm} =$ Velocity of centre of mass f disc = 0. $\quad \therefore L = I_{cm}\omega = \frac{1}{2}MR^2\omega$

6. Conceptual
7. Conceptual
8. Initially friction is upwards when the cylinder is to the left of the equilibrium position and downwards when it to the right of equilibrium position.
9. Since angular velocity is constant, acceleration of Com of the disc is zero. Hence the magnitude of acceleration of point S is centripetal acceleration which is constant in magnitude.
10. The cylinder and block will interchange their linear velocity immediately after collision.
11. The force per unit area P is

   $P = \frac{F}{\pi R^2}$



   Torque due to frictional force is

   $\tau = \mu\int(dN)x = \frac{\mu F}{\pi R^2}\int_0^R 2\pi x^2 dx$

   $\Rightarrow \tau = \frac{\mu F \times 2\pi R^3}{3\pi R^2} = \frac{2}{3}\mu FR$

   Hence, the correct answer is (D)

12. Gravity exerts a force mg download on the block, which means that the wedge must exert a force mg upward on the block. Thus, the block exerts a force mg downwards on the wedge, and gravity also exerts a force mg downward on the wedge. Since these forces have no horizontal components, no friction with the ground is necessary to keep the wedge static.

13. Centre of mass of arc is $\frac{2R\sin\alpha}{3\alpha}$

$m_1 = 2m,\ m_2 = m$

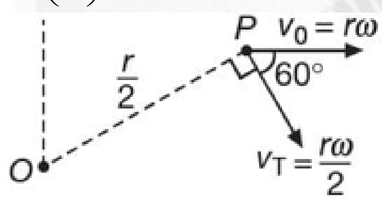
Centre of mass $= \dfrac{2m\dfrac{\left(2R\sin\left(\frac{\pi}{3}\right)\right)}{\left(3\frac{\pi}{3}\right)} + m\dfrac{\left(-2R\sin\left(\frac{\pi}{6}\right)\right)}{3\left(\frac{\pi}{3}\right)}}{3m} = \dfrac{2R\left(\sqrt{3}-1\right)}{3\pi}$

14. Power is additive. If image is now at longer distance, power is reduced.
Hence, must be concave lens but with larger focal distance, so as to form real image.

15. A – (p, q) B - (s) C - (p, q) D – (r)
Conceptual

16. $Fx = \frac{1}{3}ML^2\left(\frac{a_{cm}}{L/2}\right)$

17) The point P will have a velocity v_0 and a tangential velocity v_r and a tangential velocity $v_r\left(\frac{r}{2}\right)\omega = \frac{v_0}{2}$ inclined to each other at $60°$



So $v_P^2 = v_0^2 + \frac{v_0^2}{4} + 2\left(\frac{v_0}{2}\right)\cos 60° \Rightarrow v_P = \frac{v_0\sqrt{7}}{2}$

18. $v_0 = \sqrt{\dfrac{2gh}{1+\frac{k^2}{R^2}}}$ and $\frac{5v_0}{4} = \sqrt{2gh}$

19. Conceptual

20. Both the Statements are true, and Statement-2 is the correct explanation to Statement-1

21. $F = 12t - 3t^2; \tau = 1.5\left(12t - 3t^2\right)$

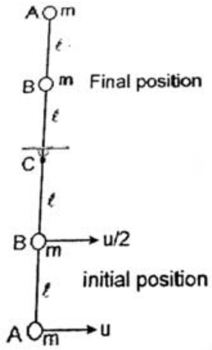
$\alpha = \frac{1.5\left(12t-3t^2\right)}{4.5} = 4t - t^2$ $\qquad \frac{d\omega}{dt} = \left(4t - t^2\right) \Rightarrow \omega = 2t^2 - \frac{t^2}{3}$

To change the direction of motion, pulley need to come to rest momentarily,

$2t^2 - \frac{t^2}{3} = 0 \Rightarrow t = 6\sec$

$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3} \Rightarrow \theta = \frac{2t^3}{3} - \frac{t^4}{12}$ $\qquad \therefore (\theta)_{t=6\sec} = 36rad = \frac{18}{\pi}rev$

22. Let the initial velocity given to the mass at A be u.



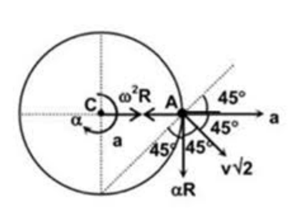
Then the velocity of mass at b is u/2

As the system moves from initial the final position increase in potential energy is $= 4mgl + 2mgl$

Decrease in kinetic energy $= \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{5}{8}mu^2$

From conservation of energy $\frac{5}{8}mu^2 = 6mgl$ or $u = \sqrt{\frac{48}{5}gl}$

23. Velocity of point 'A' $V_A = \sqrt{V^2 + \omega^2 R^2} = v\sqrt{2}$ normal acceleration of point A,



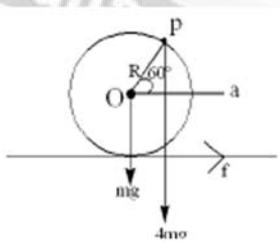
$a_A(n) = \omega^2 R\cos 45^0 + \alpha R\cos 45^0 - a\cos 45^0,$ $a_{A(n)} = \frac{\omega^2 R}{\sqrt{2}} = \frac{V^2}{\sqrt{2}R}$

$\therefore$Radius of curvature of trajectory of point 'A' relative to the ground is

$$r = \frac{(V_A)^2}{{}^aA(n)} = \frac{\left(V\sqrt{2}\right)^2}{\frac{V^2}{\sqrt{2}R}} = 2\sqrt{2}R$$

24. $\pi_0 = I_0\alpha \Rightarrow \frac{2}{5}mR\alpha + f = 2mg$ $\& f = ma = maR$

$\Rightarrow f = \frac{10}{7}mg \Rightarrow \frac{10}{7}mg \le \mu N \Rightarrow \frac{10}{7}mg \le \mu(5mg), \mu \ge \left(\frac{2}{7}\right)$



25. When the extension is maximum, their velocities are equal. From the law of conservation of momentum,

$P_f = P_i \Rightarrow (6)u + (3)u = 6(2) + 3(-1)$ $u = 1ms^{-1}$

This energy is also conserved

$$E_f = E_i \Rightarrow \frac{1}{2}(6)(1)^2 + \frac{1}{2}(3)(1)^2 + \frac{1}{2}Kx_m^2 = \frac{1}{2}(6)(2)^2 + \frac{1}{2}(3)(1)^2$$

$$3 + 1.5 + \frac{1}{2}(200)x_m^2 = 12 + 1.5$$

$$100x_m^2 = 9 \Rightarrow x_m^2 = 0.09 \Rightarrow x_m = 0.3m = 30m$$

26)

$a)\ 4mg\sin\theta - f = 4ma$

$b)\ fR = \left(\frac{8}{3}mR^2\right)\propto$

$c)\ a = R\propto$

$d)\ f = 4\mu mg\cos\theta$

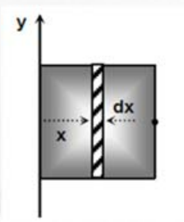
$on\ solving\ \mu = \frac{2}{5} = \frac{4}{10}$

27. $dM = \sigma_0\left(1 - \frac{x}{a}\right)dx$

$$M = \frac{\sigma_0 a^2}{2}$$

$$\Rightarrow d(MOI) = dmx^2$$

$$MOI = \int dMx^2 = \frac{Ma^2}{6}$$



28. Speed of the wall does not change after collision. Hence 2+1 =v – 1 or V =4m/s

29. $P = \beta v^2$; $\frac{mdv}{dt}.v = \beta v^2$; $\int_{v_0}^{2v_0}\frac{dv}{v} = \frac{\beta}{m}\int_0^t dt$

$$2 = \frac{\beta}{m}.t \qquad t = \frac{m\,In2}{\beta}$$

$$t = \frac{4\times 2}{0.693}\times 0.693$$

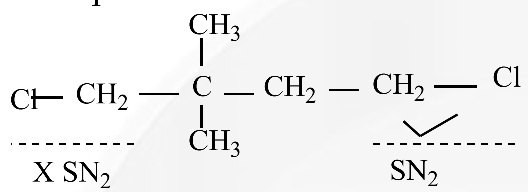
30. Work done equals area under the graph

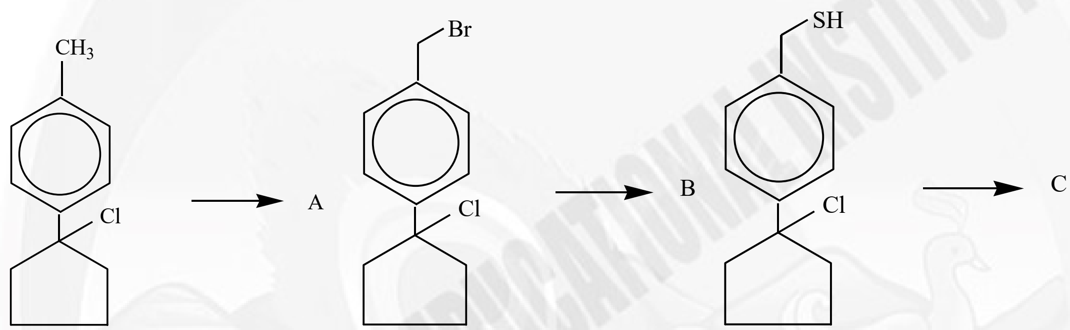
CHEMISTRY

31) a) Localised and getting $\ominus$ Ve change due to +M
b) SIR
c) Delocalised on oxygen
d) Delocalised

32) Conceptual

33) Conceptual

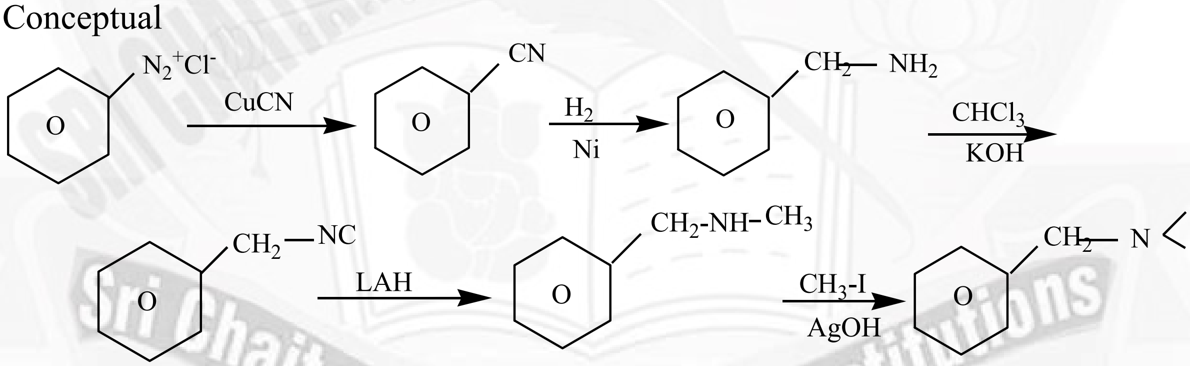
34)


35)


36) a $\longrightarrow$ Hoffmann.
b $\longrightarrow$ No E_2
c $\longrightarrow$ Saytz eff

$\left\{ TS \longrightarrow \text{stability} \right\}$

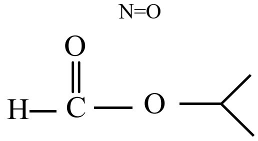
37) Conceptual

38)


39) Victor meyer test

$R-CH_2-OH \xrightarrow[I_2]{P+} R-CH_2-I \xrightarrow{AgNO_2} R-CH_2-NO_2 \xrightarrow[Cl]{NaNO_2+}$

$R-\underset{\underset{N=O}{|}}{CH}-NO_2 \xrightarrow{NaOH} \text{Red.}$

40)


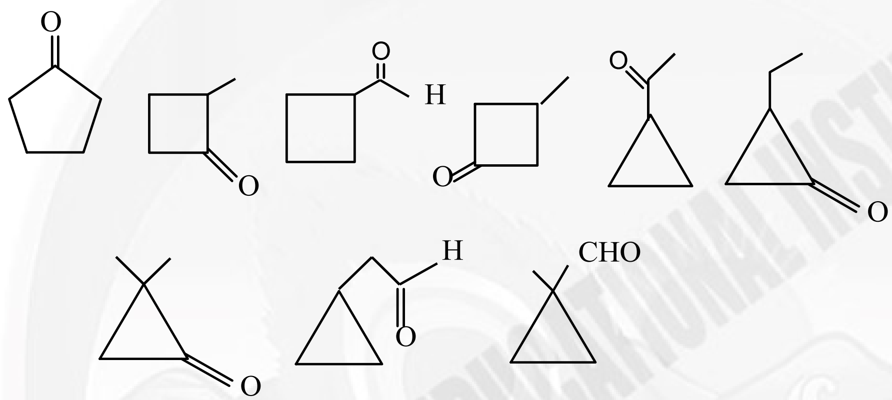
41) Conceptual

42) Conceptual

43) Conceptual

44) Conceptual
45) Conceptual
46) Conceptual
47) Conceptual
48) Conceptual
49) Y has hemiacetal
50) Conceptual
51) 1,3,4,7,10,12,13
52) 1,4,7,8,9,10
53) Explanation

Those isomers which can't show G.I.



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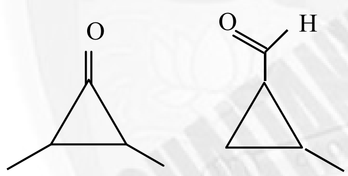


54) Conceptual

55) $x = 2$

$y = 3$

$z = 4$

56) Electrophilic substitution (Intra)

57) $E = \frac{123}{6}$

58) 1 mole of RCOOH will take $\frac{3}{4}$ mole of LAH.

59) $CH_3 - \underset{\displaystyle OH}{\underset{|}{CH}} - \underset{\displaystyle OH}{\underset{|}{CH}} - CH_3$ (3)

60) 1,2,5,6,7,8

MATHEMATICS

61. $y-x=v$

$\frac{dy}{dx}+v.x=-x^3v^2$

$\div v^3$ and put $\frac{1}{v^2}=u$

$\frac{dy}{dx}-2xu=2x^3$

62. $\frac{1+2e^v}{V+2e^v}dv+\frac{dy}{y}=0$ Let $\frac{x}{y}=v$

$x=vy$

$\left(\frac{x}{y}+2e^{\frac{x}{y}}\right)y=c$ $\quad x+2ye^{\frac{x}{y}}=c$

63. F is decreasing function

64. $\div y^2\cos x$

65. Given curves are $y^2-2y+4x+5=0$ *and* $x^2+2x-y+2=0$ or

$(y-1)^2=-4(x+1)$ *and* $(x+1)^2=y-1$.

Shifting origin to (-1, 1), equation of given curves changes to $y^2=-4x$ *and* $X^2=Y$

Hence, statement -1 is true and statement-2 is correct explanation of statement -1.

66. STATEMENT-I In $\left(1+\sqrt{\sin x}\right)<\sqrt{\sin x}<\sqrt{x}$

$$\Rightarrow\int_0^1\left(in\left(1+\sqrt{\sin x}\right)\right)^2.dx<\int_0^1 x.dx=\frac{1}{2}$$

STATEMENT-II

$$f(x)=\sin\left(\frac{2x}{1+x^2}\right)\left(\tan^{-1}x\right)^2\ \forall x\in\left[1,\sqrt{3}\right]$$

$$=\left(\pi-2\tan^{-1}x\right)\left(\tan^{-1}x\right)^2$$

$$\frac{\pi-2\tan^{-1}x+\frac{\tan^{-1}x}{2}+\frac{\tan^{-1}x}{2}}{3}\geq\sqrt[3]{\left(\pi-2\tan^{-1}x\right).\left(\tan^{-1}x\right)^2}$$

$$\Rightarrow\left(\pi-2\tan^{-1}x\right)\left(\tan^{-1}(x)\right)^2\leq\frac{\pi^3}{27}$$

$$\int_1^{\sqrt{3}}f(x).dx\leq\int_1^{\sqrt{3}}\frac{\pi^3}{27}=\frac{\pi^3}{27}\left(\sqrt{3}-1\right)$$

67. $\int\frac{2\left(\cos x+\frac{1}{\cos x}\right)}{\cos^6x+6\cos^2x+4}dx,\ \cos x=t$

68. Conceptual

69. $\int_{0}^{\pi/2} \frac{x^2 I_{n-1}}{I_{n-2}} = n(n-1)$

70. $\left[(x+2)^2 + y(x+2)\right]\frac{dy}{dx} = y^2$

$\Rightarrow y^2 \frac{dx}{dy} - (x+2)y = (x+2)^2$

Let $\frac{1}{(x+2)^2}\frac{dx}{dy} - \frac{1}{(x+2)y} = \frac{1}{y^2}$

$I.F. = e^{\log y} = y$

$G.S = \frac{-y}{x+2} = \log y + c$

As it passes through $(1,3) \Rightarrow C = -1 - \log 3$

$\therefore \frac{-y}{x+2} = \log y - 1 - \log 3$

$\Rightarrow \log\frac{y}{3} = 1 - \frac{y}{x+2}$ ....(1)

Intersection of (1) and $y = x+2$

$\log\frac{y}{3} = 0 \Rightarrow y = 3 \Rightarrow x = 1$

$\therefore$ $(1,3)$ is the only intersection point

Intersection of (1) and $y = (x+2)^2$

$\log\frac{(x+2)^2}{3} = 1 - (x+2)$ or $\log\frac{(x+2)^2}{3} + (x+2) = 1 \because \frac{(x+2)^2}{3} > \frac{4}{3} > 1, \forall x > 0$

$\therefore LHS > 2, \forall x > 0 \Rightarrow$ no solution.

71. $\frac{f^1(x)(1+x^2) - f(x)2x}{(1+x^2)^2} = \frac{2f(x)}{1+x^2}$

$\frac{f^1(x)}{f(x)} = 2 + \frac{2x}{1+x^2}$

$\log(f(x)) = 2x + \log(1+x^2) + C$

$f(0) = 2, C = \log_2$

72. When $\alpha = 1$

$$y = \begin{cases} -x+3, & x<1 \\ x+1, & 1 \le x < 2 \\ 3x-3, & x \ge 2 \end{cases}$$

$\Rightarrow A(1) = \int_0^1 \left(-x+3-2\sqrt{x}\right)dx + \int_1^2 \left(x+1-2\sqrt{x}\right)dx = 5 - \frac{8\sqrt{2}}{3}$

73. $\dfrac{dx}{x}-\dfrac{dy}{y}+\dfrac{\dfrac{dy}{y^2}-\dfrac{dx}{x^2}}{\left(\dfrac{1}{y}-\dfrac{1}{x}\right)^2}=0$

$\log x-\log y-\dfrac{1}{\dfrac{1}{x}-\dfrac{1}{y}}=c \qquad \log\left|\dfrac{x}{y}\right|+\dfrac{xy}{x-y}=c$

74. Conceptual

75. $x=a-h$

$$f\left(a^-\right)=\lim_{h\to0}\frac{\left[\cos\frac{\pi}{2a}(a-h)\right]}{h}=\lim_{h\to0}\frac{\left[\sin\frac{\pi h}{2a}\right]}{h}=0;$$

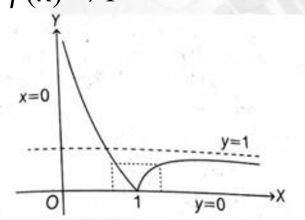
$x=a+h \qquad f\left(a^+\right)=\lim\limits_{h\to0}-\dfrac{\sinh}{2}\tan\left[\dfrac{\pi}{2}+\dfrac{\pi h}{2}\right]=0;$

Hence, $\qquad f\left(a^+\right)=f\left(a^-\right)$

76. We have, $f(x)=\dfrac{|x-1|}{x}=\begin{cases}-\dfrac{(x-1)}{x}, & if\ 0<x\le1\\ \dfrac{x-1}{x}, & if\ x>1\end{cases} \qquad =\begin{cases}\dfrac{1}{x}-1, & if\ 0<x\le1\\ 1-\dfrac{1}{x}, & if\ x>1\end{cases}$

Now, let us draw the graph of $y=f(x)$

Note that when $x\to0$, then $f(x)\to\infty$, when $x=1$, then $f(x)=0$, and when $x\to\infty$, then $f(x)\to1$



Clearly, $f(x)$ is not injective because if $f(x)<1$, then f is many one, as shown in figure.

Also, $f(x)$ is not surjective because range of $f(x)$ is $[0,\infty]$ and but in problem co-domain is $(0,\infty)$, which is wrong. $\therefore f(x)$ is neither injective nor surjective

77. $\dfrac{dy}{dx}+\dfrac{2^{x-y}\left(2^y-1\right)}{2^x-1}=0 \qquad \Rightarrow \dfrac{dy}{dx}+\dfrac{2^x}{2^x-1}\times\dfrac{2^y-1}{2^y}=0$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{-2^x\left(2^y-1\right)}{2^y\left(2^x-1\right)} \qquad \Rightarrow \displaystyle\int\frac{2^y}{2^y-1}dy=-\int\frac{2^x}{2^x-1}dx$

$\Rightarrow \dfrac{1}{1n2}\displaystyle\int\frac{2^y1n}{2^y-1}dy=\frac{-1}{1n2}\int\frac{2^x1n}{2^x-1}dx \qquad \Rightarrow 1n\left|2^y-1\right|=-1n\left|2^x-1\right|+c$

At $x=1,\ y=1,$(given) $\qquad 1n1=-1n1+c$

$0=-0+c\Rightarrow c=0 \quad \Rightarrow \quad 1n\left|2^y-1\right|=-1n\left|2^x-1\right|$

$\Rightarrow 1n\left(\left(2^x-1\right)\left(2^y-1\right)\right)=0\Rightarrow\left(2^x-1\right)\left(2^y-1\right)=1$

At $\quad x=2,\ y=? \qquad \left(2^y-1\right)(4-1)=1$

$2^y = 1+\frac{1}{3} = \frac{4}{3}$ $\quad y = \log_2 4/3 = \log_2 4 - \log_2 3 = 2 - \log_2 3$

78. $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$ $\quad IF = \log x$

79. Use L-H rule $\quad f(x) = \frac{1}{3x} + cx^2$

80. $\int \frac{1+(x+1)\ln x + x(\ln x)^2}{1+(x\ln x)(2+x\ln x)}dx = \int \frac{1+\ln x}{1+x\ln x}dx$

$= \ln(1+x\ln x)+c \quad \therefore e^{f(x)} = 1 + x\ln x \quad e^{f(2)} - 1 = \ln 4$

81. $x^3 = t \quad I_2 = \frac{1}{3}\int_0^1 \frac{dt}{e^t(2-t)}$ $\quad$ as $x \to a+b-x$

$= \frac{1}{3}\int_0^1 \frac{dt}{e^{1-t}(1+t)} \quad = \frac{1}{3}\int_0^1 \frac{et\,dt}{e(1+t)} \quad I_2 = \frac{1}{3e}.I_1$

82. $f(x) = \begin{cases} \frac{x^2}{2} + 2x + 4 & x > 3 \\ ax^2 + bx & x \le 3 \end{cases}$ $\quad \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 189 + 66 = 29$

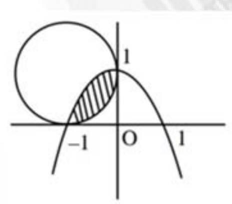
$f^1(x) = f^1(3^+) \Rightarrow 6a + b = 5$

$a = \frac{1}{18}, b = \frac{84}{18}$

83. $f^1(x) = 3x^2 - 6x + c \quad f^1(2) = c = 3 \Rightarrow f^1(x) = 3(x-1)^2$

$f(x) = c(x-1)^3 + D = (2,1)$ lies on it $\Delta = D$ $\quad f(x) = (x-1)^3$

84.



$A = \int_{-1}^{0} (1-x^2) - \left(x - \sqrt{1-(x+1)^2}\right)dx \quad = \int_{-1}^{0} -x^2 + \sqrt{1-(x+1)^2}\,dx$

$= \left(-\frac{x^3}{3} + \frac{x+1}{2} = \sqrt{1-(x+1)^2} + \frac{1}{2}\sin^{-1}\left(\frac{x+1}{1}\right)\right)_{-1}^{0}$

$A = \frac{\pi}{4} - \left(\frac{1}{3}\right) \quad \therefore 12(\pi - 4A) = 12\left(\pi - 4\left(\frac{\pi}{4} - \frac{1}{3}\right)\right) = 16$

85. Replace x with –x and use $f(1), f(2)$ get

$K = 4, \quad f(0) = 0$

$\therefore f(x) = 2x^2$

86. Conceptual

87.

$$F(x)=\int_0^x f(t)dt$$

$$F'(x)=f(x)$$

$$I=\int_0^\pi f'(x)\cos x\,dx+\int_0^\pi F(x)\cos x\,dx=2$$

$$I_1=\int_0^\pi f'(x)\cos x\,dx\,(Let)$$

$$I_1=\cos x\,f(x)]_0^\pi=\int_0^{-\pi}\sin x\,f(x)\,dx$$

$$=(-1)(-6)-f(0)+\int_0^\pi+\sin x\,F'(x)\,dx$$

$$=6-f(0)+I_2 \qquad ....(2)$$

$$I_2=\int_0^\pi \sin x\,F'(x)\,dx \qquad by\,the\ parts\ we get$$

$$=\sin x\,F(x)]_0^\pi-\int_0^\pi \cos x\,F'(x)\,dx$$

$$I_2=-\int_0^\pi \cos F(x)\,dx$$

$$I_1=6-f(0)-\int_0^\pi \cos F(x)\,dx$$

$$I=6-f(0)-\int_0^\pi \cos x\,F\left(x\right)dn+\int_0^\pi F(x)dx=2$$

$$6-2=f(0)$$

$$4=f(0)$$

88. $\frac{z}{1}=\frac{3x-4}{3x+4},\quad \frac{z+1}{z-1}=\frac{-3}{4}x$

$$f(z)=\frac{8}{3(1-z)}+\frac{2}{3} \qquad I=\left\{\frac{8}{3(1-x)}+\frac{2}{3}=\frac{-2}{3}\iota og\left|1-x\right|+\frac{2}{3}x\ EC.\right\}$$

89. Conceptual

90. $\lambda=2\int_0^2 2\sqrt{\frac{2-x}{x}}\qquad =4\pi$