Sri Chaitanya IIT Academy.,India.

A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Jr.Super60_NUCLEUS & STERLING BT **JEE-MAIN** **Date: 23-09-2023**

Time: 09:00AM to 12:00PM **CTM-06/CTM-03_(QMT-01)** **Max. Marks: 300**

KEY SHEET

PHYSICS

1)	3	2)	1	3)	1	4)	1	5)	1
6)	1	7)	4	8)	4	9)	1	10)	1
11)	3	12)	3	13)	3	14)	2	15)	1
16)	4	17)	3	18)	3	19)	2	20)	1
21)	2	22)	63	23)	1250	24)	36	25)	1443
26)	2	27)	45	28)	3	29)	2	30)	18

CHEMISTRY

31)	4	32)	2	33)	1	34)	3	35)	3
36)	1	37)	1	38)	3	39)	2	40)	2
41)	1	42)	1	43)	4	44)	2	45)	1
46)	1	47)	1	48)	2	49)	3	50)	1
51)	25	52)	15	53)	4	54)	1836	55)	8
56)	10	57)	47	58)	6	59)	9	60)	5

MATHEMATICS

61)	3	62)	2	63)	1	64)	4	65)	4
66)	2	67)	1	68)	4	69)	3	70)	2
71)	1	72)	3	73)	1	74)	1	75)	1
76)	4	77)	4	78)	1	79)	2	80)	4
81)	36	82)	8	83)	18	84)	22	85)	18
86)	20	87)	8	88)	9	89)	16	90)	2

SOLUTIONS

PHYSICS

01. $\frac{1}{2}\mu v_r^2 = \frac{1}{2}kx^2$

$x = V_{rer}^2\sqrt{\frac{\mu}{k}}$

$= 2\times\sqrt{\frac{8/6}{10}} = \sqrt{\frac{8}{15}}$

From conservation of momentum

$2\times4+4\times2 = 2v_1+4v_2$

$8 = v_1+2v_2$

From conservation of energy

$\frac{1}{2}\times2\times4^2+\frac{1}{2}\times4\times2^2 = \frac{1}{2}\times2\times v_1^2+\frac{1}{2}\times4\times v_2^2$ …… (ii)

On solving $v_1 = \frac{4}{3}m/s,\ \ v_2 = \frac{10}{3}m/s$

02. $\frac{R_2-r}{h} = \frac{R_2-R_1}{\ell} = \tan\alpha$

$\Rightarrow r = R_1-(R_1-R_2)\frac{h}{l}$

$\Rightarrow r = 0.5\times10^{-3}-(0.5\times10^{-3}-2.5\times10^{-4})\frac{8\times10^{-2}}{10^{-1}}$

$= 0.5\times10^{-3}-(0.25\times10^{-3})\times8\times10^{-1}$

$= 0.5\times10^{-3}-2\times10^{-4}$

$= 0.3\times10^{-3}m$

Surface tension of liquid at 0^0C

$S_0 = \frac{rh\rho g}{2\cos\theta}\ (\theta=0)$

$= \frac{0.3\times10^{-3}\times8\times10^{-2}\times\frac{10^4}{14}\times9.8}{2}$

$= \frac{0.3\times8\times9.8\times10^{-1}}{2\times14} = \frac{0.6\times9.8}{7}\times10^{-1}$

$= 0.084\ N/m$

If $r_0\rightarrow$ radius of cylindrical tube

$S_0 = \frac{r_0h\rho g}{2}(at\ 0^0C)$

$$S_{50} = \frac{r_0 h_{50} \rho g}{2} (at\ 50^0 C)$$

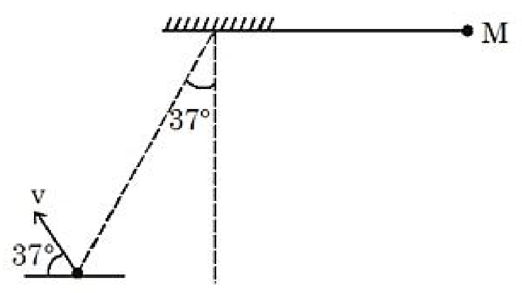
$$\frac{S_{50}}{S_0} = \frac{h_{50}}{h_0} = \frac{5.5\times10^{-2}}{6\times10^{-1}} = \frac{11}{12}$$

$$S_{50} = \frac{11}{12}\times S_0 = \frac{11}{12}\times 0.084 = 0.077 N/m$$

Rate of change of surface tension with temperature

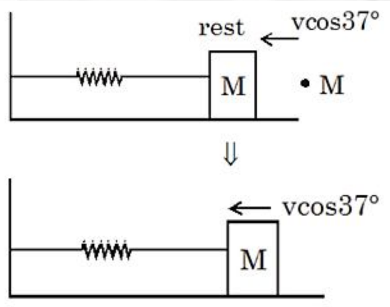
$$\frac{S_{50}-S_0}{50} = \frac{0.077-0.084}{50} = -1.4\times10^{-4}\frac{N}{m^0C}$$

03.



$$mg\ell\cos 37^\circ = \frac{1}{2}mv^2$$

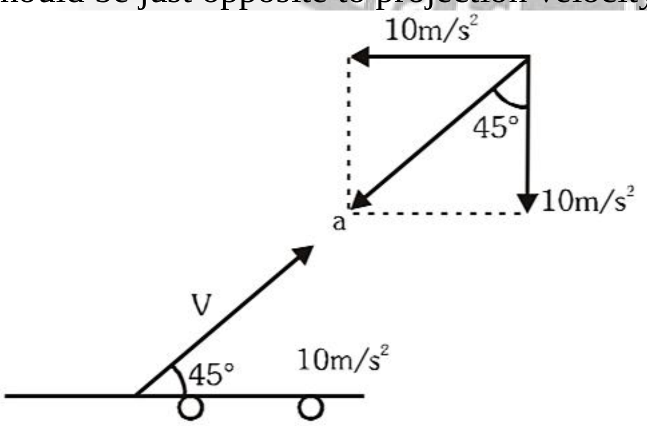
$$\therefore v = \sqrt{20}$$



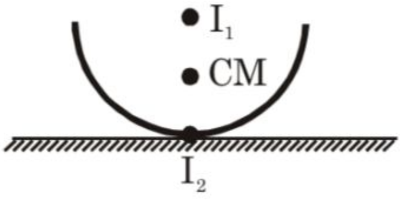
$$\frac{1}{2}m\left(v\cos 37^\circ\right)^2 = \frac{1}{2}kx^2$$

04. Conceptual

05. W.r.t. flat car. Velocity of projection makes angle 45^0 with east. In order to catch the ball without moving trajectory has to be a straight line. So acceleration wrt flat car should be just opposite to projection velocity.



06. $r_{CM} = \frac{R}{2}$



$$I_1 = I_{CM} + m\left(\frac{R}{2}\right)^2 = \frac{2}{3}mR^2$$

$$I_2 = I_{CM} + m\left(\frac{R}{2}\right)^2 = \frac{2}{3}mR^2$$

$$W = \Delta K$$

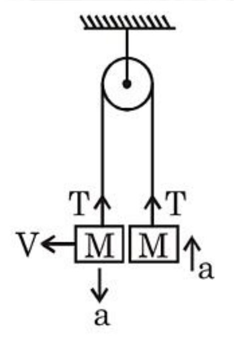
$$mg\frac{R}{2} = \frac{1}{2}\frac{2}{3}mR^2\omega^2 = \frac{3g}{2R}$$

Using $F = ma_{CM}$

$$N - mg = m\frac{\omega^2 R}{2}$$

$$N = mg + \frac{3mg}{4} = \frac{7mg}{4}$$

07.



Before collision with pan

$$V_0 = \sqrt{2g\frac{\ell}{2}} = \sqrt{g\ell}$$

Just after collision

$$\frac{M}{2}V_0 = \left(\frac{M}{2} + M\right)V$$

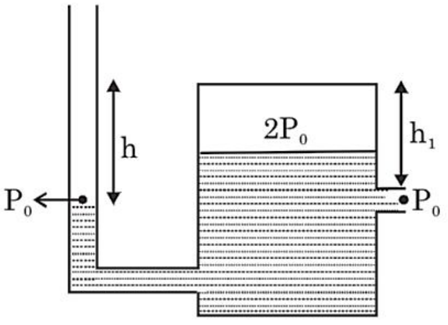
$$V = \frac{V_0}{3}$$

For 'A' $-T + Mg + \frac{MV^2}{\ell} = Ma$ ...(1)

$T - Mg = Ma$ ...(2)

$$1+2 \Rightarrow a=\frac{V^2}{2\ell}=\frac{g}{18}$$

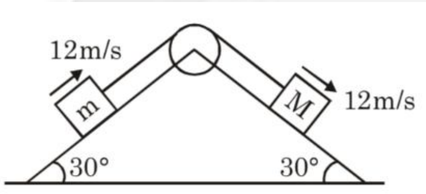
08.



Pressure at same height is same, therefore

$h = h_1$

09.



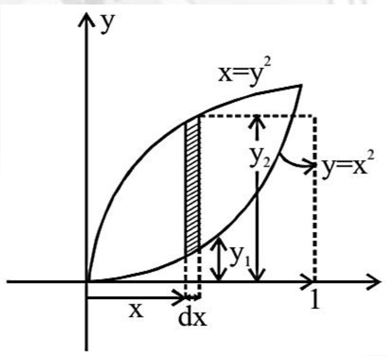
$$a_M = g\sin 30^\circ - \frac{g\cos 30^\circ}{\sqrt{2}} = 5-6 = -1 m/s^2$$

$$a_m = -\left(g\sin 30^\circ + \frac{g\cos 30^\circ}{\sqrt{2}}\right) = -11\, m/s^2$$

As limiting friction is greater than mgsinθ, block 'm' will not move after $t=\frac{12}{11}$sec.

$$v_M = 12-1\times 2 = 10 m/s$$

10.

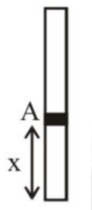


$$x_{cm} = \frac{\int xdm}{\int dm} = \frac{\int_0^1 x\lambda(y_2-y_1)dx}{\int_0^1 \lambda(y_2-y_1)dx}$$

$$=\frac{\int_0^1\left(\sqrt{x}-x^2\right)xdx}{\int_0^1\left(\sqrt{x}-x^2\right)dx}=\frac{9}{20}$$

$$y_{cm}=\frac{\int ydm}{\int dm}=\frac{\int_0^1 y\lambda\left(x_2-x_1\right)dy}{\int_0^1\lambda\left(x_2-x_1\right)dy}=\frac{9}{20}$$

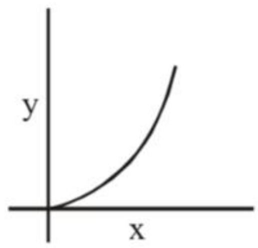
11.



Elastic energy density at

$$A=\frac{1}{2}\frac{(stress)^2}{Y}=\text{constant}$$

$$\frac{1}{2}\frac{\left(\frac{M}{L}\frac{x}{A}g\right)^2}{Y}=\text{constant}$$



$Y\propto x^{2+}$

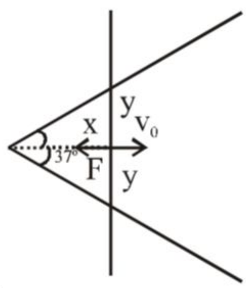
12. Conceptual

13. $\frac{y}{x}=\tan 37^\circ=\frac{3}{4}$

$\Rightarrow y=\frac{3x}{4}$

$F=T.2(2y)$

$=4T\times\frac{3x}{4}$



$F=3Tx(Along-ve\,x)$

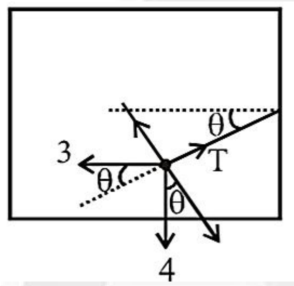
From work energy theorem

$$\int_{x_0}^{2x_0}(-3Tx)\,dx = 0-\frac{1}{2}mv_0^2$$

$$\Rightarrow -3T.\left(\frac{x^2}{2}\right)_{x_0}^{2x_0} = -\frac{mv_0^2}{2}$$

$$\Rightarrow 9x_0^2T = mv_0^2 \Rightarrow v_0 = 3x_0\sqrt{\frac{T}{m}}$$

14.



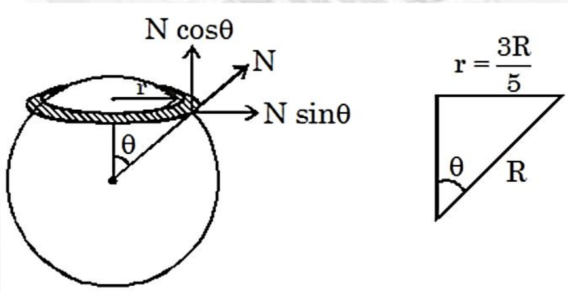
$3\sin\theta = 4\cos\theta$

$\Rightarrow \tan\theta = \frac{4}{3} \Rightarrow \theta = 53^\circ$

15. Conceptual

16. The kinetic energy is minimum in centre of mass reference frame

17.



$\sin\theta = \frac{3R}{5R}$ $\quad\quad$ $\sin\theta = \frac{3}{5}$

$\theta = 37^0$

$\text{N }\sin\theta = T(2\pi) \quad\quad ...(I)$

$\text{N}\cos\theta = W \quad\quad ...(II)$

$Tan\ \theta = \frac{T(2\pi)}{W}$

$\frac{3}{4} = \frac{T(2\pi)}{W}$

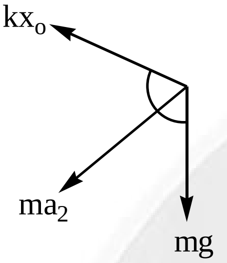
$T = \frac{3W}{8\pi}$

18. $kx_0 = mg$

$$ma_1 = mg\cos 30^\circ \Rightarrow a_1 = g\cos 30^\circ = g\frac{\sqrt{3}}{2}$$

$$ma_2 = mg \Rightarrow a_2 = g$$

$$\therefore \frac{a_1}{a_2} = \frac{\sqrt{3}}{2}$$



19. From FBD of lift.

$T_1 = T_2 + mg$, m=mass of lift

$\Rightarrow T_1 - T_2 = mg$

$\Rightarrow T_{net} = mg$

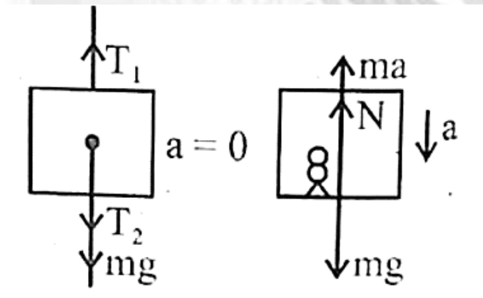
So, (I) is true

From FBD of person,

$N + ma = mg$

$N = mg - ma \Rightarrow N < mg$

So, (II) is false

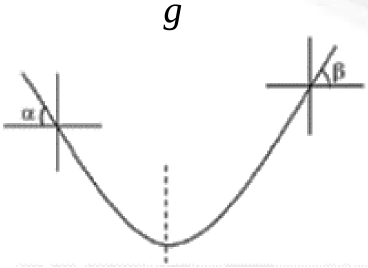


20. $T\cos\alpha = T = T\cos\beta$

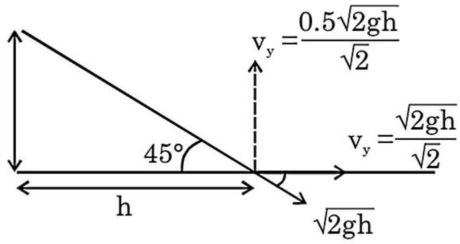
$T_1\sin\alpha + T_2\sin\beta = mg$

$$\Rightarrow mg = T_1\sqrt{1-\left(\frac{T_3}{T_1}\right)^2} + T_2\sqrt{1-\left(\frac{T_3}{T_2}\right)^2}$$

$$\Rightarrow m = \frac{\sqrt{T_1^2 - T_3^2} + \sqrt{T_2^2 - T_3^2}}{g}$$



21.

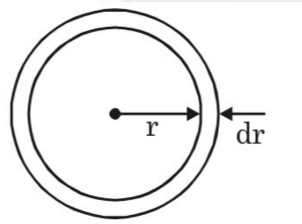


$R = u_x \times T$

$= \sqrt{gh} \times \frac{0.5\sqrt{gh} \times 2}{g} = h$

$\Rightarrow AB = h + h = 2h$

22.

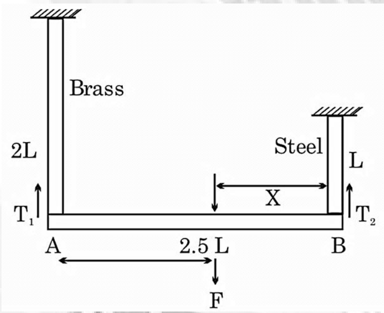


$\tau = \int F.r = \int_0^R \eta(2\pi r\, dr)\left(\frac{rw}{t}\right).r$

$= \frac{2\pi\eta W R^4}{4t} = \frac{2 \times 3.14 \times 1 \times 8 \times 10^{-4}}{4 \times 2 \times 10^3}$

$= 0.625 N - m \approx 0.63 N - m$

23.



$\tau_F = 0$

$T_2 X = T_1(2.5L - X)$

$\Rightarrow \frac{T_2}{T_1} = \left(2.5\frac{L}{X} - 1\right)$

$1 = \left(2.5\frac{L}{X} - 1\right)$

$$\tau = 1.25L$$

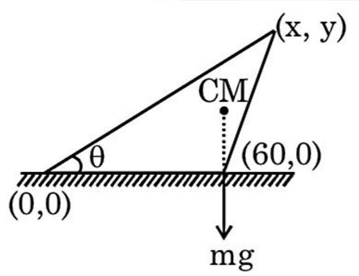
$$Y_E = \sigma$$

$$\Rightarrow T_1 = Y_B \pi 4r^2 \frac{\Delta L}{2L}$$

$$\Rightarrow T_2 = Y_S \pi r^2 \frac{\Delta l}{L}$$

$$= \frac{T_1}{T_2} = \frac{2Y_B}{Y_S} = 1$$

24.



Just to prevent from toppling, mg must pass through toppling point.

$$x_{CM} = 60 = \frac{0+60+x}{3} \Rightarrow x = 120cm$$

$$\text{Area } = \frac{1}{2} \times \frac{60}{100} \times y = 1 \Rightarrow y = \frac{10}{3} m$$

$$\cot\theta = \frac{x}{y} = \frac{120}{100\times 10} \times 3 = 0.36$$

25. $3F_1 + F_2 = 3900$

$$F = \frac{AY}{\ell} x$$

$$3\frac{AY}{\ell} x + \frac{AY}{\ell}\left(x + 0.5\times 10^{-3}\right) = 3900$$

$$4x + 0.5 + 10^{-3} = 3\times 10^{-3}$$

$$4x = 2.5\times 10^{-3}$$

$$x = \frac{2.5}{4}\times 10^{-3}$$

$$F_2 = 1.3\times 10^6 \left(\frac{2.5}{4} + 0.5\right) 10^{-3}$$

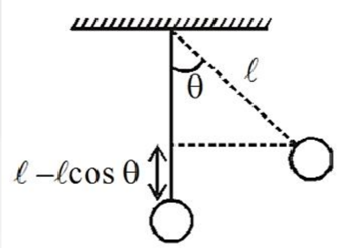
$$= 1.3\times 10^3 \left(\frac{4.5}{4}\right)$$

$$F_2 = 1.46\times 10^3 = 1460N$$

26. $$a = \frac{\rho \frac{V}{2} g + 2\rho \frac{V}{2} g - \rho v g}{\rho V} = \frac{g}{2}$$

$K = 2$

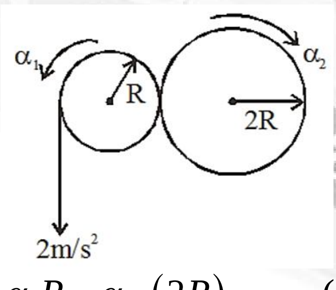
27. At maximum speed, centripetal acceleration will also be maximum.
By W.E.T
$W_g + W_T + W_F = \Delta K$
$-mg\ell(1-\cos\theta) + 0 + mg\ell\sin\theta = \frac{1}{2}mv^2 - 0$



$v^2 = -2g\ell + 2g\ell\cos\theta + 2g\ell\sin\theta$
$v = \sqrt{-2g\ell + 2g\ell\cos\theta + 2g\ell\sin\theta}$
For maximum velocity $\frac{dv}{d\theta} = 0$
$\frac{dv}{d\theta} = \frac{1}{2\sqrt{-2g\ell + 2g\ell\cos\theta + 2g\ell\sin\theta}} \times (-2g\ell\sin\theta + 2\ell\cos\theta)$
$\frac{dv}{d\theta} = 0$
$-2g\ell\sin\theta + 2g\ell\cos\theta = 0$
$2g\ell\sin\theta = 2g\ell\cos\theta \qquad \sin\theta = \cos\theta$
$\theta = \frac{\pi}{4}$

28. $P_1 - P_2 = \frac{1}{2}\rho\omega^2\left[r_1^2 - r_2^2\right]$

29.



$\alpha_1 R = \alpha_2.(2R) \qquad ...(i)$
$\alpha_1 R = 2 \qquad ...(ii)$
Also, $f(2R) = \frac{2\times(2R)^2}{2}\alpha_2$
$\therefore f = 2N.$

30. $v_x = 1 \Rightarrow x = t$ and $v_y = 6t \Rightarrow y = 3t^2 \Rightarrow y = 3x^2$

$$\Rightarrow \frac{dy}{dx} = 6x, \frac{d^2y}{dx^2} = 6 \Rightarrow \left.\frac{dy}{dx}\right|_{x=\frac{\sqrt{3}}{3}} = 2\sqrt{2}$$

As we know that $R = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \dfrac{(1+8)^{3/2}}{6} = 4.5m$

CHEMISTRY

31. Expansion is adiabatic & free

q = 0

w = 0

$\Delta U = q + w$

$\Delta U = 0 \Rightarrow$ No change of internal energy. So change in internal energy will be zero. But for real gas or other substance it is not necessary that $\Delta T = 0$ .Because U = f (P,V,T) for others.

32. Entropy of universe isolated system always increases in a spontaneous process so statement- I is incorrect

33. $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$

$-280 = \Delta_f H(CO_2) + 120 \Rightarrow \Delta_f H(CO_2) = -400$ kJ/mole

$C(s) + O_2(g) \rightarrow CO_2(g)$

$\Delta_1 H(CO_2) = 720 + 500 - 2 \times 710 = -200$ kJ/mole

34. $2C(s) + O_2 \rightarrow 2CO....(1) \qquad \Delta H_1 = -54$ Kcal

$2C(s) + 2O_2 \rightarrow 2CO_2...(2) \qquad \Delta H_2 = -192$ Kcal

$(1)-(2) \Rightarrow 2CO_2 \rightarrow 2CO + O_2$

$\Delta H = -54 + 192 = 138$ Kcal

35.

$$\begin{array}{cccc} 2H_2O & + 2Cl_2 \rightleftharpoons & 4HCl & + O_2 \\ \frac{1}{2} & 2 & 2 & 2 \end{array}$$

$K_p = 12 \times 18$

As $K_p >> 1$ Assume complete forward Reaction

$$\begin{array}{cccc} 2H_2O & + 2Cl_2 \rightleftharpoons & 4HCl & + O_2 \\ \frac{1}{2} & 0 & 6 & 3 \\ \frac{1}{2} & 2x & 6-4x & 3-x \end{array}$$

$$K_p = \frac{(6-4x)^4 (3-x)}{\left(\frac{1}{2}\right)^2 \times (2x)^2} = 12\times 10^8$$

$$\frac{6^4 \times 3}{\left(\frac{1}{2}\right)^2 \times 4x^2} = 12\times 10^8$$

$$P_{O2} = 3.6\times 10^{-3} \text{ atm}$$

36.

$$Sb_2S_3(s) + 3H_2(g) \rightleftharpoons 2Sb(s) + 2H_2S(g)$$
$$\quad 0.01-x \qquad\qquad\qquad x$$

$$nH_2S = \frac{1.198}{239} = 0.005$$

$$\Rightarrow [H_2] = 0.01 - 0.005 = 0.005$$

$$[H_2S] = 0.005 \quad \Rightarrow K_C = 1$$

37. 350K Vessel water evaporates completely & 300 vessesl water vapour condenses until final pressure is 22 mm Hg.

38. At very high pressure

$$Z = 1 + \frac{Pb}{RT}$$

$$\frac{dZ}{dp} = \frac{b}{RT} = \frac{1}{10}\text{atm}^{-1}$$

$$b = \frac{22.4}{273}\times\frac{273}{10} = 2.24 \text{ litre/mole}$$

$$\text{b} = 4\times \text{V} \times \text{N}_A$$

$$V = \frac{2.24\times 10^3}{4\times 6\times 10^{23}} \Rightarrow 9.3\times 10^{-22}\text{cc}$$

39.

$S_1 \to 3s$

$S_2 \to 3d$

$S_3 \to 4s$

$S_4 \to 3p$

For unielectronic species energy depends only on 'n'

$\therefore \quad S_1 = S_2 = S_4 < S_3$

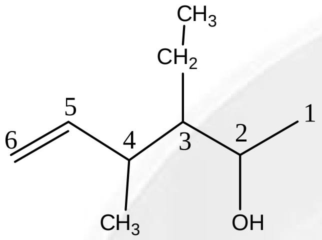
40. The decreasing order of priority of prefix in numbering the carbon chain of an organic compounds is

Bromo > Chloro > Iodo

$$\overset{1}{CH_3}-\underset{\underset{Cl}{|}}{\overset{2}{CH}}-\underset{\underset{Br}{|}}{\overset{3}{CH}}-\underset{\underset{I}{|}}{\overset{4}{CH}}-\overset{5}{CH_2}-\overset{6}{CH_3}$$

3-Bromo-2-chloro-4-idohexane

41.



According to the priority of functional groups, OH is given top priority. Hence, the numbering starts from right side.

42. $\left(Zn^{2l}\right)=\dfrac{Ksp}{\left(OH^{-}\right)^2}$

$\left(Zn\left(OH\right)_4\right)^{2-}=Kc\left[OH^{-}\right]^2$

$\dfrac{ds}{dOH^{-}}=\dfrac{-2Ksp}{\left[OH^{-}\right]^3}+2Kc\left[OH^{-}\right]=0$

$\dfrac{\cancel{2}Ksp}{\left[OH^{-}\right]^3}=\cancel{2}Kc\left[OH^{-}\right]\Rightarrow\left[OH^{-}\right]^4=\dfrac{Ksp}{Kc}$

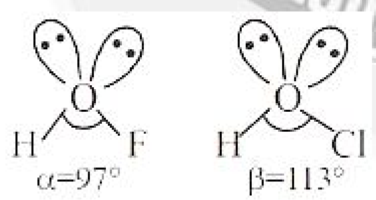
$=\dfrac{10^{-17}}{0.1}=10^{-16}$

$\Rightarrow\left[OH^{-}\right]=10^{-4}\Rightarrow pOH=4$

$pH=10$

43. NOF_3 exists due to dative bond b/w O & N 'N' cannot form 5 bonds

44.



In O-F bond p-character is greater than that of O-Cl bond, also in HOCl, $p\pi-d\pi$ back bonding many also be present, hence $\alpha<\beta$.

45.

$Ca(NH)+2H_2O \rightarrow Ca(OH)_2 + NH_3(g)$
(B)

$2NH_3 + 3CaOCl_2 \rightarrow N_2(g) + 3CaCl_2 + 3H_2O$
(B) (C)

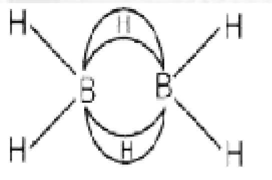
$N_2(g) + 3Mg \rightarrow Mg_3N_2$
(C) (D)

$Mg_3N_2 + 6H_2O \rightarrow 3Mg(OH)_2 + 2NH_3$
(B)

46.
A) $SiC \Rightarrow$ Covalent carbide
B) $Be_2C + 4H_2O \rightarrow 2Be(OH)_2 + CH_4 \uparrow$
C) $CaC_2 + 2H_2O \rightarrow HC \equiv CH \uparrow + Ca(OH)_2$
D) $Mg_2C_3 + 2H_2O \rightarrow 2Mg(OH)_2 + H_3C - C \equiv CH \uparrow$

47. Both correct and S-2 explains 5,

48. Alums are $M_2'(SO_4)M_2''(SO_4)_3.24H_2O$
These are soluble in water. There aqueous solution is acidic both the statements are true but it is not the correct explanation.

49.


Two 3 centre-2-electron bonds
(C) B_2H_6 is e^- deficient species
(E) B_2H_6 is non-Planar molecules
(D) $BF_3 + LiAlH_4 \rightarrow 2B_2H_6 + 3LiF + 3AlF_3$
$NaBH_4 + I_2 \rightarrow B_2H_6 + 2NaI + H_2$

50. Moving down the group stability of lower oxidation state increases Al < Ga < In < Tl

51. $C(s) \rightleftharpoons A + B$
5-x y x
$A + D \rightleftharpoons E(s)$
x 5
y 5-x x

$xy = 5\times10^{-11}$

$y(5-x) = 10^{-10}$

$$\frac{5\times10^{-11}(5-x)}{x} = 10^{-10}$$

$2.5-0.5 \qquad x=x$

$x=\frac{5}{3}$

52. $nHCO_3^- = \frac{183}{61} = 3$

$nSO_4^{2-} = \frac{96}{96} = 1$

$CaO + 2HCO_3^- \rightarrow CaCO_3 + H_2O + CO_3^{2-}$

2 moles of $HCO_3^- \rightarrow 1$ mole of CaO

3 Moles $\rightarrow$?

3/2 = 1.5

$CaO + SO_4^{2-} + H_2O \rightarrow CaSO_4 + 2OH^-$

1 mole of Cao = 1 mole of SO_4^{2-}

$x=1.5, y=0 \qquad 10x+y = 15+0=15$

53. 11. 2L at 0ºC and 1 atm of compound $\Rightarrow \frac{1}{2}$ mole 'X' is always $\geq 15.5g$

$\therefore$ In 1 mole, of compound of 'X' $\geq 31g$

$\therefore$ Atmoic weight of 'X'=31

($\because$ in one mole of compound of any element there should always be 1 mole of atoms of that element)

Also 11.2L of vapour of 'X' weighs $\Rightarrow 62g$

(At 0º C, 1 atm) $\left(\frac{1}{2}\text{mole}\right)$

$\therefore$ Molecular weight of 'X' =124

Atomicity of 'X' $= \frac{124}{31} = 4$

54.

	Chamber I	Chamber II
Initial pressure		
T	$N_2O_4 = 20$ mm	$H_2OV = 20$ mm
1.2 T	$N_2O_4 = 24$ mm	$V = 30$ mm
	$\downarrow$	
	$P_{N_2O_2}$ rem=12 mm	$30+6$
	$P_{NO_2} = 24/4 \Rightarrow 6$	36
	$12+6 \Rightarrow 18$	

55. We have $\Delta E = \frac{3}{4} \times 0.85eV$

as energy $=0.6375$eV the photon will belong to bracket series

(as for bracket series $0.306 \leq E \leq 0.85$)

So $\frac{3}{4}\times 0.85=13.6\left[\frac{1}{4^2}-\frac{1}{n^2}\right]$

$\Rightarrow n=8$; Hence $x=8$

56. $\because$ after adding of NaOH to weak monoprotic acid medium i.e NaOH added is quite lesser

After addition of 50 ml NaoH 0.1M

$HA \quad + \quad NaOH \rightarrow NaA + H_2O$

$(50\text{ml}, \times \text{M})\ (50\text{ml}, 0.1)$

Left moles in solution (50x-5)

$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

$$4.699 = pK_a + \log\left(\frac{5}{50x-5}\right)....(i)$$

After adding of 75 ml NaOH 0.1M

Left millimoles of HA=50x-7.5

Millimoles of NaA formed=7.5

$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

$$5 = pK_a + \log\left(\frac{7.5}{50x-75}\right)...(ii)$$

On subtracting (ii) – (i)

$$0.3010 = \log\left(\frac{7.5}{50x-7.5}\frac{(50x-5)}{5}\right) = \log 2$$

On subtracting x=0.3

$NaA + \qquad HCl \rightarrow NaCl + HA$

15 Millimioles 7.5 Millimoles

$$pH = pK_a + \log\frac{[A^-]}{[HA]} = pK_a + \log\frac{7.5}{7.5}$$

$$pH = pK_a = 5 \Rightarrow cd = 05$$

57. Equivalents of NH_3 evolved

$$=\frac{100\times 0.1\times 2}{1000}-\frac{20\times 0.5}{1000}=\frac{1}{10}$$

Percent of nitrogen in the unknown organic compound

$$=\frac{1}{100}\times\frac{14}{0.3}\times 100 = 46.6$$

58. Planar molecules

$ClF_3, XeF_2, XeF_4, H_2O, H_2S, OCl_2$

59. $NO_3^-, CO_3^{2-}, F_2, Cl_2, Br_2, O_2^{2-}, O_2^-, Li_2^+, He_2^+$

60. $XeF_6\ \ SOF_4, Br\,F_3, I\ F_4^-, PCl_6^-$ have dz^2 orbital in hybridization of central atom

MATHEMATICS

61. $\because\ x = \log_{2a}\left(\frac{bcd}{2}\right)$

$\Rightarrow x + 1 = \log_{2a}\left(\frac{2abcd}{2}\right) = \log_{2a}(abcd)$

62. $Z = 5 - x - y; xy + z(y + x) = 3$

$\Rightarrow xy + (y + x)(5 - (x + y)) = 3$

$\Rightarrow x^2 + y^2 + xy - 5x - 5y + 3 = 0$

As quadratic in y $\quad \Delta \ge 0$

63. $[x^2] + 2 = 3\{x\} < 3$

$\therefore [x^2] = 0 \Rightarrow x \in (-1, 1)$ and $\{x\} = \frac{2}{3}$

$\therefore x = -1 + \frac{2}{3}$ or $\frac{2}{3}$

64. The equation of the plane passing through the point (1, -2, 1) and perpendicular to the planes $2x - 2y + z = 0$ and $x - y + 2z = 4$ is given by

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

Or $x + y + 1 = 0$

Its distance from the point (1, 2, 2) is

$\left|\frac{1 + 2 + 1}{\sqrt{2}}\right| = 2\sqrt{2}$

65. $2\cos\theta + 2\sqrt{2} = 3\sec\theta$

$2\cos^2 + 2\sqrt{2}\cos\theta - 3 = 0$

$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta$ is in Q_1 or Q_4

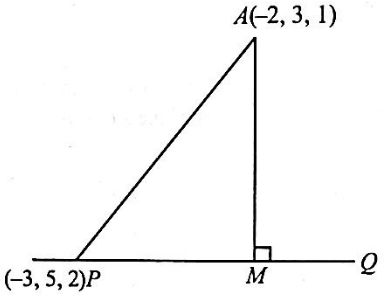
$\Rightarrow \sin\theta = -\frac{1}{\sqrt{2}}, \cot\theta = -1, \tan\theta = 1$

66. Here, $\alpha = \beta = \gamma$

$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore\ \cos\alpha = \frac{1}{\sqrt{3}}$

Direction cosines of PQ are $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{3}\right)$



PM=Projection of AP on PQ

$=\left|(-2+3)\frac{1}{\sqrt{3}}+(3-5)\frac{1}{\sqrt{3}}+(1-2)\frac{1}{\sqrt{3}}\right|$

$=\frac{2}{\sqrt{3}}$

And $Ap=\sqrt{(-2+3)^2+(3-5)^2(1-2)^2}=\sqrt{6}$

$AM=\sqrt{(AP)^2-(PM)^2}=\sqrt{6-\frac{4}{3}}=\sqrt{\frac{14}{3}}$

67. $L=\sin^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$

$\left(\because \sin^2\theta=\frac{1-\cos 2\theta}{2}\right)$

$\Rightarrow L=\left(\frac{1-\cos\left(\frac{\pi}{8}\right)}{2}\right)-\left(\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}\right)$

$\Rightarrow L=\frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{8}\right)\right]$

$\Rightarrow L=\frac{1}{2\sqrt{2}}-\frac{1}{2}\cos\left(\frac{\pi}{8}\right)$

$M=\cos^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$

$$\Rightarrow M = \left(\frac{1+\cos\left(\frac{\pi}{8}\right)}{2}\right) - \left(\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}\right)$$

$$\Rightarrow M = \frac{1}{2}\cos\left(\frac{\pi}{8}\right) + \frac{1}{2\sqrt{2}}$$

68. **Given, vector** $\vec{a}$ is coplanar with vector $\vec{b} + 2\hat{j} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. Also, we have $\vec{a}$ is Perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ and $|\vec{a}| = \sqrt{10}$.

$\therefore \vec{a} = \lambda\vec{b} + \mu\vec{c} = \lambda(2\hat{i} + \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$

$= \hat{i}(2\lambda + \mu) + \hat{j}(\lambda - \mu) + \hat{k}(\lambda + \mu)$ and $\vec{a}.\vec{d} = 0$

$\Rightarrow 3(2\lambda + \mu) + 2(\lambda - \mu) + 6(\lambda + \mu) = 0$

$\Rightarrow 14\lambda + 7\mu = 0 \Rightarrow \mu = -2\lambda$

$\Rightarrow \vec{a} = (0)\hat{i} + 3\lambda\hat{j} + (-\lambda)\hat{k} \Rightarrow |\vec{a}| = \sqrt{10}|\lambda| = \sqrt{10}$

$\Rightarrow |\lambda| = 1 \Rightarrow \lambda = 1$ or -1 Now, as $[\vec{a}\vec{b}\vec{c}] = 0$

$$\therefore [\vec{a}\,\vec{b}\,\vec{c}] + [\vec{a}\vec{b}\vec{d}] + [\vec{a}\vec{c}\vec{d}] = [\vec{a}\vec{b} + \vec{c}\vec{d}] = \begin{vmatrix} 0 & -3\lambda & \lambda \\ 3 & 0 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$\Rightarrow -3\lambda(12) - \lambda(6) = -42\lambda = 42$ or -42. Thus, the possible value is -42

69. $\log_{10}\left(\frac{\sin 2x}{2}\right) = -1 \Rightarrow \sin 2x = \frac{1}{5} 1 + \sin 2x = \frac{n}{10} \Rightarrow n = 12$

70. Any plane through $(1,0,0)$ is

$a(x-1) + by + cz = 0 \qquad (i)$

It passes through (0, 1, 0). Therefore,

$a(0-1) + b(1) + c(0) = 0$ or $-a + b = 0 \qquad (ii)$

(i) Makes an angle of $\frac{\pi}{4}$ with $x + y = 3$, therefore

Or $\frac{1}{\sqrt{2}} = \frac{a+b}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$

Or $a + b = \sqrt{a^2 + b^2 + c^2}$

Squaring, we get

$a^2 + b^2 + 2ab = a^2 + b^2 + c^2$

Or $2ab = c^2 \text{or} 2a^2 = c^2$ [using (ii)]

Or $c = \sqrt{2}a$

Hence, $a:b:c = a:a:\sqrt{2}a$

$=1:1:\sqrt{2}$

71. Conceptual

72. $S = 1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + ...$

$\therefore \frac{S}{6} = \frac{1}{6} + \frac{5}{6^2} + \frac{12}{6^3} + \frac{22}{6^4} + ...$

Subtracting (2) from (1), we get

$\frac{5}{6}S = 1 + \frac{4}{6} + \frac{7}{6^2} + \frac{10}{6^3} + \frac{13}{6^4} + ..$

$\therefore \frac{5}{36}S = \frac{1}{6} + \frac{4}{6^2} + \frac{7}{6^3} + \frac{10}{6^4} + \frac{13}{6^5} + ...$

Sub tracking (4) from (3), we get

$$\frac{25}{36}S = 1 + \frac{3}{6} + \frac{3}{6^2} + \frac{3}{6^3} + ... = 1 + \frac{\frac{3}{6}}{1 - \frac{1}{6}} = \frac{8}{5}$$

$\therefore S = \frac{288}{125}.$

73. $\left(x_1 + \frac{1}{x_1}\right)\left(x_2 + \frac{1}{x_2}\right)\left(x_3 + \frac{1}{x_3}\right)$

$= x_1x_2x_3 + \frac{x_1x_2}{x_3} + \frac{x_2x_3}{x_1} + \frac{x_1x_3}{x_2} + \frac{x_1}{x_2x_3} + \frac{x_2}{x_3x_1} + \frac{x_3}{x_1x_2} + \frac{1}{x_1x_2x_3}$

$= x_1x_2x_3 + \frac{x_1^2x_2^2 + x_2^2x_3^2 + x_1^2x_3^2}{x_1x_2x_3} + \frac{x_1^2 + x_2^2 + x_3^2}{x_1x_2x_3} + \frac{1}{x_1x_2x_3}$

$x_1^2 + x_2^2 + x_3^2 = (x_1 + x_2 + x_3)^2 - 2(x_1x_2 + x_2x_3 + x_3x_1) = -2(3) = -6$

$x_1^2x_2^2 + x_2^2x_3^2 + x_3^2x_1^2 = (x_1x_2 + x_2x_3 + x_3x_1)^2 - 2x_1x_2x_3(x_1 + x_2 + x_3) = 9$

$\Rightarrow \left(x_1 + \frac{1}{x_1}\right)\left(x_2 + \frac{1}{x_2}\right)\left(x_3 + \frac{1}{x_3}\right) = -5 + \frac{9}{-5} + \frac{-6}{-5} + \frac{1}{-5} = -\frac{29}{5}$

74. Conceptual

75. $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\frac{1}{3}=\frac{\pi}{4}-\tan^{-1}\frac{1}{2}=\frac{\pi}{4}-\cot^{-1}2$$

Also $2\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{2/3}{1-\frac{1}{9}}=\tan^{-1}\frac{3}{4}=\sin^{-1}\frac{3}{5}=\frac{\pi}{2}-\cos^{-1}\frac{3}{5}$

76. $|A|=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$

$=(1)(-2)+3(-1)+2(3\alpha)$

$=6\alpha-5$

$|AdjA|=|A|^2$

$\therefore |A|^2=-6\alpha^2+17\alpha-10$

$\therefore (6\alpha-5)^2=-6\alpha^2+17\alpha-10$

$\Rightarrow \alpha=1$ (or) $\Rightarrow \alpha=\frac{5}{6}$

77. $x_1=3+5\cos\theta+5\sin\theta----1$

$4+5\sin\theta-5\cos\theta----2$

(1) + (2) (1) − (2)

$x_1+y_1-7=10\sin\theta$ $x_1-y_1+1=10\cos\theta$

78. $x^2f(x)-2.f\left(\frac{1}{x}\right)=g(x)$ ...(i)

replacing 'x' by $\frac{1}{x}$ then $\frac{1}{x^2}.f\left(\frac{1}{x}\right)-2f(x)=g\left(\frac{1}{x}\right)$ (or)

$2f\left(\frac{1}{x}\right)-4x^2f(x)=2x^2.g\left(\frac{1}{x}\right)$........$(2)$

Solve (i), (ii)

$$\Rightarrow f(x)=-\left(\frac{g(x)+2x^2.g\left(\frac{1}{x}\right)}{3x^2}\right)$$

$$f(-x)=-\left(\frac{g(-x)+2x^2.g\left(-\frac{1}{x}\right)}{3x^2}\right)$$

$f(x)=-f(x)$

($\therefore$ $g(x)$ is an odd and f(x) is an even function)

$f(x)=0\Rightarrow f(5)=0$

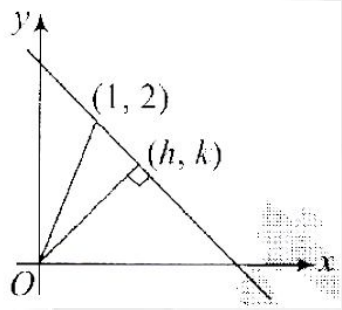
79. By using the properties of determinant

80. Given family of lines is $(4a+3)x-(a+1)y-(2a+1)=0$

Or $(3x-y-1)+a(4x-y-2)=0$

Family of lines passes through the fixed point P which is the intersection of $3x-y=1$ and $4x-y=2$

Solving we get P (1,2)



Now let (h, k) be the foot of perpendicular on each of the family.

$\therefore \frac{k}{h}.\frac{k-2}{h-2}=-1$

$\therefore$ Locus is $x(x-1)+y(y-2)=0$

Or $(2x-1)^2+4(y-1)^2=5$

81. $(\bar{a},\bar{b})=\frac{\pi}{6},\left|[\bar{a}\bar{b}\bar{c}]\right|=\left|(\bar{a}\times\bar{b}).\bar{c}\right|$

$=\left|\bar{a}\times\bar{b}\right|\left|\bar{c}\right|=3.4.\frac{1}{2}.6=36$

82. Give $\left|\vec{b}-\vec{a}\right|=12,\left|\vec{c}\right|=6$ Equation of CD is $\bar{r}=t\bar{c}$ and Equation of AB is

$\vec{r}=\vec{a}+s(\vec{b}-\vec{a})\vec{r}=\vec{a}+s(\vec{b}-\vec{a})\vec{r}=a+s(\vec{b}-\vec{a})$

$S.D=\frac{\left|[\vec{a}\vec{b}-\vec{a}\vec{c}]\right|}{\left|\vec{b}-\vec{a}\right|\times\vec{c}}=8\Rightarrow\left|[\vec{a}\vec{c}\vec{b}]\right|=8\left|(\vec{b}-\vec{a})\times\vec{c}\right|$

$\Rightarrow 6V=8\frac{\left|\vec{c}\right|\left|\vec{b}-\vec{a}\right|}{2}$

$\Rightarrow V=\frac{8\times6\times12}{6\times2}=48\Rightarrow\frac{1}{6}v=8$

83. The characteristic equation for A is $\left|A-\lambda I\right|=0\Rightarrow\begin{vmatrix}4-\lambda & -2\\ \alpha & \beta-\lambda\end{vmatrix}=0$

$\Rightarrow(4-\lambda)(\beta-\lambda)+2\alpha=0$

$\Rightarrow\lambda^2-(\beta+4)\lambda+4\beta+2\alpha=0$

So, the matrix A will satisfy the equation.

$A^2-(\beta+4)A+(4\beta+2\alpha)I=0$

Comparing with the equation $A^2+\gamma A+18I=O$, we get

$-(\beta+4)=\gamma$ and $4\beta+2\alpha=18$

84. Let $a=m+\sqrt{n}\Rightarrow \left.\begin{matrix} f(p)=a \\ f(q)=a \end{matrix}\right\}\Rightarrow x^2+x-a=0$ has root $p\,,q$

$\therefore f\left(\frac{1}{p}\right)+f\left(\frac{1}{q}\right)=\frac{1}{16}\Rightarrow\frac{1}{p^2}+\frac{1}{q^2}+\frac{1}{p}+\frac{1}{q}=\frac{1}{16}\Rightarrow\frac{p^2+q^2}{(pq)^2}+\frac{p+q}{pq}=\frac{1}{16}$

$\Rightarrow\frac{1+2a}{a^2}+\frac{1}{a}=\frac{1}{16}\Rightarrow\frac{1}{a^2}+\frac{3}{a}-\frac{1}{16}=0$

$=16+48a-a^2=0\Rightarrow a^2-48a-16=0$

$\Rightarrow a=\frac{48\pm\sqrt{2304+64}}{2}\Rightarrow a=24\pm\sqrt{592}$

$\Rightarrow a=24+\sqrt{592}$

$\therefore \left.\begin{matrix} m=24 \\ n=592 \end{matrix}\right\}\Rightarrow 100m+n=2992$

85. $AdjA=|A|A^{-1}$

$\left[A(adJA)A^{-1}\right]A=|A|I$

$$\begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

86. For non-trivial solutions $\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix}=0$

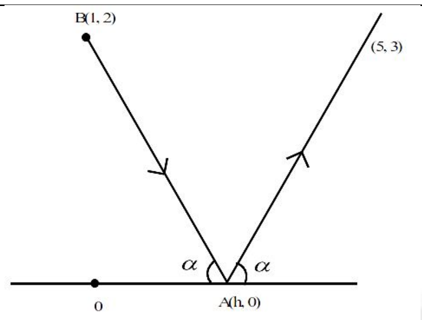
$\therefore \tan^2\theta-\sqrt{3}+\sqrt{7}-\sqrt{3}\tan\theta-\sqrt{7}+\tan\theta=0\Rightarrow\left(\tan\theta-\sqrt{3}\right)\left(\tan\theta+1\right)=0$

$\Rightarrow\tan\theta=\sqrt{3},-1$

$\frac{120}{\pi}\sum\theta=\frac{120}{\pi}\left(\frac{\pi}{3}-\frac{2\pi}{3}-\frac{\pi}{4}+\frac{3\pi}{4}\right)=20$

87. Conceptual

88. Angle of incidence = Angle of reflection $=\alpha$



Slope of the line $= \tan(\pi - \alpha) = -\tan\alpha = -\left(\dfrac{2-0}{1-h}\right)$

$= \dfrac{2}{h-1} = \dfrac{3}{5-h}$ where $A = (h, 0)$

$\Rightarrow 10 - 2h = 3h - 3 \Rightarrow h = \dfrac{13}{5} \therefore OA = \dfrac{13}{5}$

89. $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \Rightarrow |A| = -2$

Now, 2A adj.$(2A) = |2A| I$

$\Rightarrow A adj.(2A) = 4I$ ...(I)

Now, $|A^4| + \left|A^{10} - \left(adj(2A)\right)^{10}\right|$

$= (-2)^4 + \dfrac{\left|A^{20} - A^{10}(adj 2A)^{10}\right|}{|A|^{10}}$

$= 16 + \dfrac{\left|A^{20} - \left(A adj(2A)\right)^{10}\right|}{|A|^{10}} = 16 + \dfrac{\left|A^{20} - 2^{20} I\right|}{2^{10}}$

(from (1))

Now, characteristic roots of A are 2 and -1.

90. $x^2 + y^2 = 3 + 2\left(\cos\dfrac{2\pi}{7} + \cos\dfrac{4\pi}{7} + \cos\dfrac{6\pi}{7}\right) = 2$