Sri Chaitanya IIT Academy.,India.
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A right Choice for the Real Aspirant
ICON Central Office - Madhapur - Hyderabad

Sec:Sr.Super60_STERLING_BT	Paper -2(Adv-2020-P2-Model)	Date: 10-09-2023
Time: 02.00Pm to 05.00Pm	CTA-05	Max. Marks: 180

KEY SHEET

PHYSICS

1	**5**	2	**2**	3	**5**	4	**2**	5	**2**	6	**3**
7	**ABCD**	8	**ABC**	9	**ABC**	10	**ABC**	11	**AC**	12	**ACD**
13	**60**	14	**4.2**	15	**4**	16	**6**	17	**3**	18	**8**

CHEMISTRY

19	**4**	20	**5**	21	**3**	22	**5**	23	**5**	24	**2**
25	**ABCD**	26	**ABCD**	27	**ABCD**	28	**BC**	29	**BC**	30	**ABC**
31	**0.2 - 0.4**	32	**0.33**	33	**3**	34	**322**	35	**3**	36	**9**

MATHEMATICS

37	**2**	38	**4**	39	**8**	40	**0**	41	**1**	42	**6**
43	**ABCD**	44	**ABCD**	45	**AB**	46	**ABC**	47	**ABCD**	48	**ABC**
49	**1**	50	**4**	51	**58**	52	**9**	53	**24**	54	**4.80**

SOLUTIONS

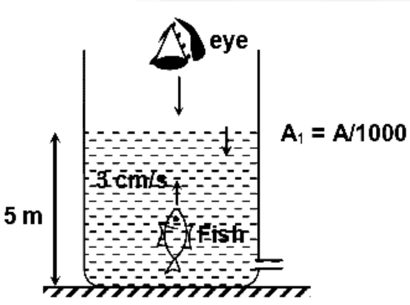
PHYSICS

1. $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$\frac{2.4}{2r}-\frac{1}{-d}=\frac{2.4-1}{r}\ \frac{1}{d}=\frac{1.4-1.2}{r}$

$\frac{1}{d}=\frac{0.2}{r}$ $\Rightarrow d=5r$ $d=5\,cm$

2. Velocity of efflux = $v=\sqrt{2g\times 5}=10\,m/s$



Velocity of surface = $\frac{10\times A/1000}{A}=1\,cm/s$

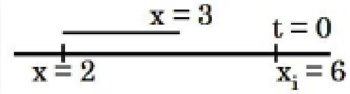
The velocity of fish relative to the observer is $v=\frac{d}{dt}\left(\frac{x}{\mu}+y\right)$

$=\frac{3}{4}\frac{dx}{dt}+\frac{dy}{dt}=\frac{3}{4}\times(-4)+1=-3+1=-2\ cm/s$

3. $x=t^2-4t+6$ at $t=0$ $x_i=6$

at $t=3$ sec $x_f=3$ $v=2t-4$ at $t=2$ $v=0$

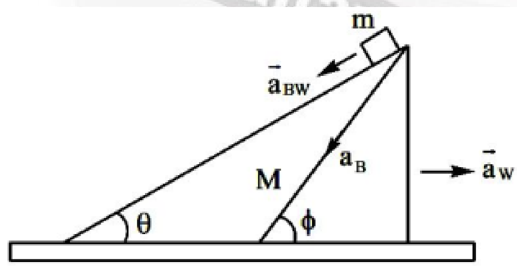
Hence particle changing direction of motion at $t=2$ sec.



At $t=2;\ x=2$

Total distance 5 m

4.



$\frac{a_w}{\sin(\phi-\phi)}=\frac{a_B}{\sin\theta}\Rightarrow\frac{a_w}{a_B}=\frac{\sin(\phi-\theta)}{\sin\theta}$

$ma_B\cos\phi=Ma_w$

$$\Rightarrow \frac{m}{M} = \frac{a_w}{a_B \cos\phi} = \frac{\sin(\phi-\theta)}{\sin\theta.\cos\phi} = \frac{1\times2\times2}{2\times1\times1} = 2$$

5. $\text{Length} \propto G^x c^y h^z$

$$L = \left[M^{-1}L^3T^{-2}\right]^x \left[LT^{-1}\right]\left[ML^2T^{-1}\right]^z$$

By comparing the power of M, L and T in both sides we get

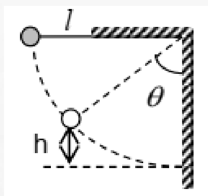
$-x+z=0, 3x+y+2z=1$ and $-2x-y-z=0$

By solving above three equations we get

$$x=\frac{1}{2}, y=-\frac{3}{2}, z=\frac{1}{2}$$

6. Velocity of bob just before collision, $u=\sqrt{2gl}$

The velocity of wall just after collision becomes, $v = e\sqrt{2gl}$



If 'h' is the height attain after first collision, then

$$\frac{1}{2}m\left(e\sqrt{2gl}\right)^2 = mgh \Rightarrow h = e^2 l$$

Height attain after n^{th} collision $h_n = e^{2n} l$

$\Rightarrow l(1-\cos\theta) = e^{2n} l$ $\qquad \Rightarrow 1-\cos\theta = \left(\frac{2}{\sqrt{5}}\right)^{2n}$

for $\theta < 60^0, \left(\frac{4}{5}\right)^n < \frac{1}{2}$ $\qquad \Rightarrow n=3$

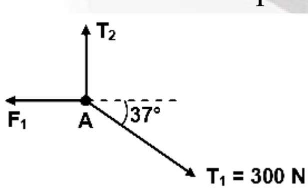
7. $T_2 = T_1 \sin 37^0$ ......(i)

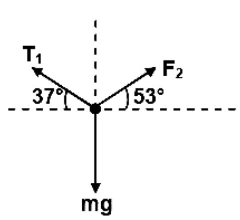
$F_1 = T_1 \cos 37^0$ ......(ii)

$F_2 = T_1 \cos 37^0 / \cos 53^0$ ......(iii)

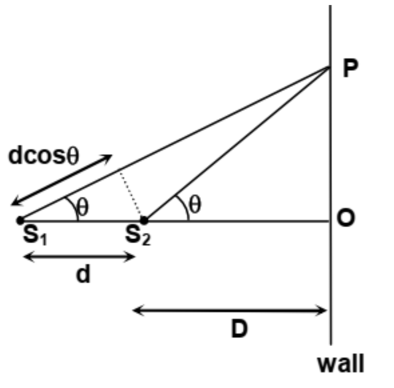
$Mg = T_1 \sin 37^0 + F_2 \sin 53^0$ ......(iv)

From the above equation find F_1, F_2, T_2 and mass of the





8. $\Delta x = S_1P - S_2P$



$d\cos\theta = n\lambda$ .... (i)

$\Delta x_{max} = d = 10\lambda$

$n\lambda = 10\lambda$

n = 10

From equation (i) for the 4th bright ring

$10\lambda\cos\theta = 6\lambda \qquad \theta = 53^0$

$\cos\theta = \frac{3}{5} = \frac{D}{\sqrt{D^2 + y^2}}$

$9D^2 + 9y^2 = 25D^2 \; y = \frac{8}{3}m$

For dark ring path difference is

$\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \frac{9\lambda}{2}, \frac{11\lambda}{2}, \frac{13\lambda}{2}, \frac{15\lambda}{2}, \frac{17\lambda}{2}, \frac{19\lambda}{2}$

9. Since the process in chamber 2 is adiabatic

$\therefore P_0V_0^{\gamma} = P_2V_2^{\gamma} \qquad \therefore P_0V_0^{5/3} = \frac{27}{8}P_0V_2^{5/3}$

$\therefore V_2 = \left(\frac{8}{27}\right)^{3/5} V_0 \quad \therefore$ Volume of chamber

$1 = 2V_0 - V_2 = \left[2 - \left(\frac{8}{27}\right)^{3/5}\right]V_0$

$P_0^{1-\gamma}T_0^{\gamma} = C \qquad \therefore T_2 = \left(\frac{27}{8}\right)^{2/5} T_0$

Work by the gas = $\frac{P_0V_0 - P_2V_2}{\gamma - 1}$

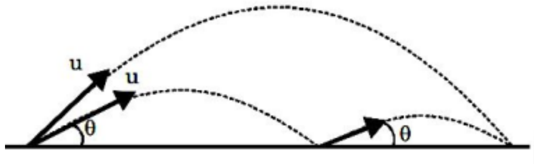
10. $x_{initial} = 0; \; x_{final} = 5$

Displacement = $x_{final} - x_{initial} = 5m$

Distance travelled = 7 + 9 = 9 m

$v = \text{slope of } x - t \text{ graph} = \frac{2}{20} = 0.1\ m/s$

$$< v > = \frac{\text{total distance}}{\text{total time}} = \frac{9}{80} = 0.11\ m/s$$



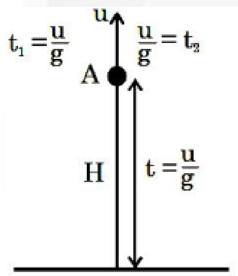
11. $s = ut + \frac{1}{2}at^2$

$12 = 0 + \frac{1}{2}a.(2)^2 \Rightarrow a = 6\ m/s$

$a = g\sin\theta - \mu g\cos\theta$ $\qquad 6 = 8 - 6\mu \Rightarrow 6\mu = 2$

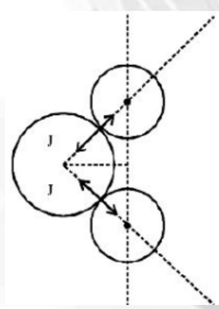
$\mu = \frac{1}{3}$ $\qquad v = u + at = 0 + (2)$

$v = 12\ m/s$ $\qquad s = ut + \frac{1}{2}at^2 = 12(4) + \frac{1}{2}6(4)^2$ $\qquad s = 96m$

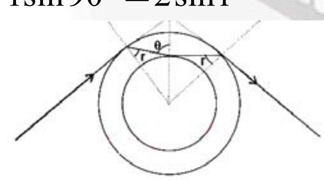


12. 1) $2mv_1\cos\theta = mv$ 2) $\frac{1}{2}m\Omega = 2.\frac{1}{2}mv_1^2$ 3) $\sin\theta = \frac{nd/2}{d}$ 4) $mv = 2m.\ v_1\cos\theta$

$\theta = 45$ and $n = \sqrt{2}$ (for A to stop)



13. $1\sin 90^0 = 2\sin r$



$r = 30^0$

For inner surface $\quad 2\sin C = 1.5\sin 90^0$

$\sin C = 3/4 = 0.75$

Using sine rule $\qquad \frac{\sqrt{3}R}{\sin(\pi-\theta)} = \frac{R}{\sin r}$ $\quad \sin\theta = \frac{\sqrt{3}}{2} = 0.865$

As $\theta > C$ hence total internal reflection occurs. From the geometry angle of deviation is 60^0

14. $LC = \dfrac{\text{pitch}}{\text{no. of circular divisions}} = \dfrac{0.75}{50} = 0.015$ mm

Negative zero error.

20 divisions $\times$ LC $\Rightarrow 20 \times 0.015 = (-0.3)$

Which metal is put

Measured value = 3.75 mm + 10 divisions $\times$ LC

$= 3.75 \text{ mm} + 10 \times 0.015 = 3.90 \text{ mm}$

$TV = MV - (\text{zero error}) \qquad = 3.9 - (-0.3) = 4.20 \text{ mm}$

15. Centre of mass must lie on line L_1

$\dfrac{x}{l} + \dfrac{y}{l} = 1 \qquad 2x + 2y = l \qquad a + b = 4$

16. Ring rises $\Rightarrow$ tension is zero

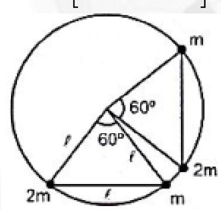
17. The speeds given to 2m will also be possessed by 'm'

$\therefore$ K.E in horizontal position gets converted in P.E in vertical position

$\dfrac{1}{2}2mv^2 + \dfrac{1}{2}mv^2 =$ change in P.E in vertical position

$$\Delta P.E = 2mg\left[l\cos 30^0 - l\cos 60^0\right] + mg\left[l\cos 30^0 + \frac{l}{2}\right]$$

$$2mg\left[\frac{l\sqrt{3}}{2} - \frac{l}{2}\right] + mg\left[\frac{l\sqrt{3}}{2} + \frac{l}{2}\right]$$



$$\therefore mgl\left[\sqrt{3} - 1\right] + mgl\left[\frac{\sqrt{3}+1}{2}\right]$$

$$\therefore mgl\left[\sqrt{3} - 1 + \frac{\sqrt{3}}{2} + \frac{1}{2}\right] = mgl\left[\frac{3\sqrt{3}}{2} - \frac{1}{2}\right]$$

$$K.E = \frac{1}{2}3mv^2 = mgl\left[\frac{3\sqrt{3}-1}{2}\right] \quad \therefore v = \sqrt{\left(\frac{3\sqrt{3}-1}{3}\right)gl}$$

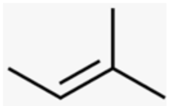
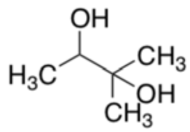
18. 8 m/s

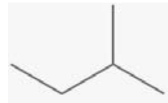
Use LCE between points, $\dfrac{1}{2}mv_1^2 + \dfrac{1}{2}kx_1^2 = \dfrac{1}{2}mv_2^2 + \dfrac{1}{2}kx_2^2 + mgh$

$x_1 = 2m,\ x_2 = 1m$ and $h = 3m$

CHEMISTRY

19. Conceptual

20. P =  Q = 

R = 

i,ii,iii,iv,vi re correct

21. Acetone

22. 5moles CO_2 gas releases

23. a, c, d, e, f are formed optically inactive

24. Conceptual

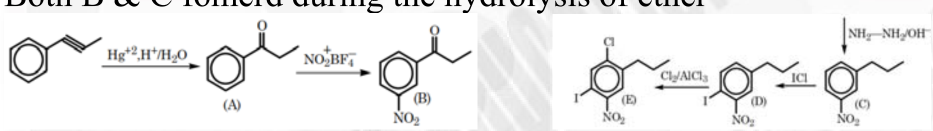
25. Conceptual

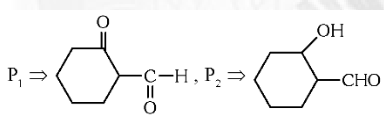
26. A= Intra molecular aldol condensation
B= Stephen's reduction
C is correct
D is correct

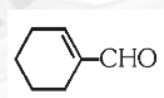
27. ABCD are correct

28. B and C are correct

29. Both B & C fomerd during the hydrolysis of ether

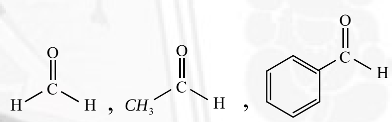
30. 

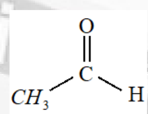
31. 

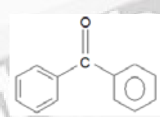
P_3 = 

32. Reaction with NaOI gives =A,B,C,D,J,N,O
lucas test gives faster = I, reaction with Cu/300^0 to give alkene = I

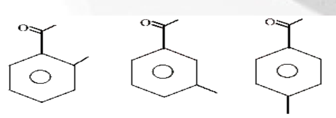
33. i, ii,v,,

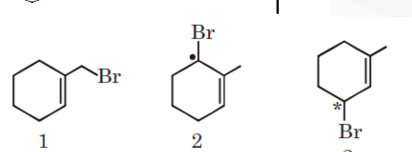
34. x is 3 

y is 2 

, $CH_3-\overset{O}{\overset{||}{C}}-CH_3$ z is 2 , $CH_3-\overset{O}{\overset{||}{C}}-CH_3$, 

35.





1 2 2

2 2

36.

MATHEMATICS

37. $$\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\sum_{n=0}^{\infty}\int_0^1 x^{2^n k}\,dx$$

$$=\int_0^1\sum_{k=1}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{k-1}x^{2^n k}}{k}dx=\int_0^1\sum_{n=0}^{\infty}\ln\left(1+x^{2^n}\right)dx$$

$$=\int_0^1\ln\left((1+x)(1+x^2)(1+x^4).......\right)dx=\int_0^1\ln\left(\frac{1-0}{1-x}\right)dx$$

$$=-\int_0^1\ln x\,dx=-[x\ln x-x]_0^1=1$$

38. $y=\sin(x+y)$

$y'=\cos(x+y)(1+y')$

$$y'=\frac{\cos(x+y)}{1-\cos(x+y)}$$

Given

$$\frac{\cos(x+y)}{1-\cos(x+y)}=\frac{1}{\sqrt{2}-1}\Rightarrow\cos(x+y)=\frac{1}{\sqrt{2}}$$

$$-\left(2\pi+\frac{\pi}{4}\right),-\left(2\pi-\frac{\pi}{4}\right),-\left(4\pi-\frac{\pi}{4}\right)$$

pair of (x, y) $\equiv$

$$\left(\frac{\pi}{4}-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}\right).\left(2\pi+\frac{\pi}{4}-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\left(2\pi-\frac{\pi}{4}+\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)\left(4\pi-\frac{\pi}{4}+\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$$

$$\left(\frac{-\pi}{4}+\frac{1}{\sqrt{2}}.\frac{-1}{\sqrt{2}}\right).\left(-2\pi-\frac{\pi}{4}+\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)\left(-2\pi-\frac{\pi}{4}-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\left(-4\pi+\frac{\pi}{4}-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$$

$$\sum_{k=1}^{n}|\alpha_k|=16\pi.\ \sum_{k=1}^{n}|\beta_k|=\frac{8}{\sqrt{2}}$$

$$\text{so,}\left[\frac{16\pi-\frac{8}{\sqrt{2}}}{11}\right]=\left[\frac{16\pi-2\sqrt{2}}{11}\right]=[4.31]=4$$

39. Let $\theta=\frac{\pi}{4}+x$

$\Rightarrow d\theta=dx$ or $4\theta=\pi+4x\Rightarrow\pi-4\theta=-4x$

$$=\int_{-\pi/2}^{0}\frac{(-4x)\tan\left(\frac{\pi}{4}+x\right)}{1-\tan\left(\frac{\pi}{4}+x\right)}dx$$

$$= -4\int_{-\pi/2}^{0} \frac{\frac{x(1+\tan x)}{1-\tan x}}{1-\frac{1+\tan x}{1-\tan x}}dx$$

$$= -4\int_{-\pi/2}^{0} \frac{x(1+\tan x)}{1-\tan x}.\frac{(1-\tan x)}{(-2)\tan x}dx \;\; = 2\int_{-\frac{\pi}{2}}^{0}\left(\frac{x}{\tan x}+x\right)dx$$

$$I = -\frac{\pi^2}{4} + 2\int_{0}^{\frac{\pi}{2}} \frac{t}{\tan t}dt\ x = I_1$$

$$I_1 = \int_{0}^{\frac{\pi}{2}} t\cot t = \frac{\pi}{2}\ln 2$$

Hence $2.\frac{\pi}{2}\ln 2 - \frac{\pi^2}{4} = \pi\ln 2 - \frac{\pi^2}{4}$

$\Rightarrow k = 2,\ w = 4$

$\Rightarrow kw = 8$

40. $f(x) = \left[\underbrace{\frac{\tan\pi x^3}{}}_{u}\right]\left[\underbrace{\frac{\sqrt{9+\tan^2 x}+\sqrt[3]{27+\tan^3 x}}{}}_{v}\right] \Rightarrow f'(x) = u'v + uv'$

$u = 0$ at $x = -2 \Rightarrow f'(-2) = \sec^2(\pi x^3)3\pi x^2\sqrt{9+\tan^2\pi x}\ \sqrt[3]{27+\tan^3\pi x}$ at $x = -2$

$\therefore f'(-2) = 12\pi\ 3.3 = 108\pi$

$g(x) = u.v \Rightarrow g''(x) = u''v + 2u'v' + uv''$; $u = 1-\cos x$

at $x = 0;\ u = 0,\ u' = 0 \Rightarrow g''(0) = \cos x.\left.\frac{\cos^{-1}x+\tan^{-1}x}{(1+x^2)\cot^{-1}x}\right|_{x=0}$

$$= 1\times\frac{\frac{\pi}{2}+0}{1\times\frac{\pi}{2}} = 1$$

41. $I = \int \frac{\{f'(x)x - f(x)\}dx}{\{f(x)+x\}\sqrt{x(f(x)-x)}}$

$$I = \int \frac{\left\{\frac{f'(x)x - f(x)}{(x)^2}\right\}}{\left(\frac{f(x)}{x}+1\right)\sqrt{\frac{f(x)}{x}-1}}dx$$

Let $\frac{f(x)}{x}-1=t^2 \Rightarrow \frac{f'(x)x-f(x)}{(x)^2}dx=2t\,dt$

$$\therefore I=\int\frac{2t\,dt}{(t^2+2)t}=\frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)+c=\sqrt{2}\tan^{-1}\left\{\sqrt{\frac{f(x)}{2x}-\frac{1}{2}}\right\}+c$$

$$=\sqrt{2}\tan^{-1}\left\{\sqrt{\frac{f(x)-x}{2x}}\right\}+c$$

$m=2, n=2$

42. 1, 2, 2, 3, 3, 3, ……..

$\therefore a_1=1; a_2=a_3=2; a_4=a_5=a_6=3;......$

If $\frac{m(m-1)}{2}+1\le n\le\frac{m(m+1)}{2}$, then $a_n=m$

$$\Rightarrow\frac{m}{\sqrt{\frac{m(m+1)}{2}}}\le\frac{a_n}{\sqrt{n}}\le\frac{m}{\sqrt{1+\frac{m(m-1)}{2}}}$$

$m\to\infty$ $\qquad$ $m\to\infty$

$\sqrt{2}$ $\qquad$ $\sqrt{2}$

43. On integration and using given condition we get g(x) as follows :

$$f(x)=\begin{cases}\tan^{-1}x-\frac{\pi}{6} & \forall x\in[1,\infty)\\ \tan^{-1}x-C & \forall x\in(-\infty,-1]\end{cases}$$

Here $f(\sqrt{3})=\frac{\pi}{6}$ can fix only one branch of f(x), but does not give any information about other branch of f(x). So $f'(-\sqrt{3})=-\frac{\pi}{3}-C$ can be equal to any real number of choosing suitable value of 'C'

44. Here the function inside the integration is even function, so

$$I=2\int_0^\infty\frac{\sin\left(x+\frac{1}{x}\right)\cos\left(x-\frac{1}{x}\right)}{x+\frac{1}{x}}.\,dx=\int_0^\infty\frac{\sin 2x+\sin\frac{2}{x}}{x+\frac{1}{x}}dx......(1)$$

put $x=1/t$, we get $I=\int_\infty^0\frac{\sin\frac{2}{t}+\sin 2t}{\frac{1}{t}+t}.\frac{-dt}{t^2}=\int_0^\infty\frac{\sin 2x+\sin\frac{2}{x}}{x+\frac{1}{x}}\frac{dx}{x^2}......(2)$

Add eq(1) and eq(2). $2I=\int_0^\infty\frac{\sin 2x+\sin\frac{2}{x}}{x}dx......(3)$

$$\int_0^{\infty}\frac{\sin 2x}{x}.dx=\int_0^{\infty}\frac{\sin 2x}{2x}.2dx=\int_0^{\infty}\frac{\sin t}{t}.dt$$

$$\int_0^{\infty}\frac{\sin\frac{2}{x}}{x}dx=\int_0^{\infty}\frac{\sin\frac{2}{x}}{\frac{2}{x}}.\frac{2}{x^2}dx=\int_0^{\infty}\frac{\sin t}{t}dt$$ (using 2/x = t), now using these eq(3) we get

$$I=\int_0^{\infty}\frac{\sin\frac{2}{x}}{x}dx=\int_0^{\infty}\frac{\sin x}{x}dt$$

45. Taking log both sides we get $\ln\beta=\lim_{n\to\infty}\left[\frac{\ln\{(1!)(2!)....(n!)\}}{n^2}-\alpha\ln(n)\right]$

$$=\lim_{n\to\infty}\left[\frac{\ln 1+(\ln 1+\ln 2)+(\ln 1+\ln 2+\ln 3)+....}{n^2}-\alpha\ln(n)\right]$$

$$=\lim_{n\to\infty}\left[\frac{n\ln 1+(n-1)\ln 2+(n-2)\ln 3....}{n^2}-\alpha\ln(n)\right]$$

$$=\lim_{n\to\infty}\left[\frac{1}{n}\sum_{r=1}^{n}\left(\frac{n+1-r}{n}\right)\ln\left(\frac{r}{n}\right)+\left(\frac{1}{2}+\frac{1}{2n}-\alpha\right)\ln n\right]$$

As limit exists

$$\alpha=\frac{1}{2}\text{ and }\ln\beta=\int_0^1(1-x)\ln x\,dx=-\frac{3}{4}$$

$$\Rightarrow\beta=e^{\frac{3}{4}}$$

46. $f(x)=\sec^2 x+2\sec^2 x\tan^2 x$

$$g(x)=\frac{\sin(2nx)}{2\sin x}$$

A) $\int(1+2\tan^2 x)\sec^2 x\,dx$

$$=\tan x+\frac{2}{3}\tan^3 x+C$$

B) $\lim_{n\to 0}\frac{(\sec^2 x+2\sec^2 x\tan^2 x-1).4}{\sin^2 .2nx}=e^{\frac{3}{n^2}}$

C) $I_{2n+2}-I_{2n}=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{\sin(2n+2)x-\sin 2nx}{\sin x}$

$$=\int_0^{\frac{\pi}{2}}\cos(2n+1)x\,dx=\frac{1}{(2n+1)}\sin(2n+1)\frac{\pi}{2}$$

$$=\frac{1}{3}\cdot\frac{1}{5}\cdot-\frac{1}{7}\cdot\frac{1}{9}\cdot\frac{-1}{11}\cdot\frac{1}{13} \text{ for } n\in[1,6]$$

D) $g(x)=0\Rightarrow x=\frac{\pi}{2}$

$\sin 8x=0,\ \sin x\neq 0$

$$\Rightarrow x=\frac{\pi}{8}\cdot\frac{\pi}{4}\cdot\frac{3\pi}{8}\cdot\frac{\pi}{2}\cdot\frac{5\pi}{8}\cdot\frac{3\pi}{4}\cdot\frac{7\pi}{8}$$

47. A) Let $g(x)=e^{-x}\int_0^x f(t)\,dt$

$$g(0)=0=g(1)\Rightarrow g'(c)=0\Rightarrow e^{-c}.f(c)-e^{-c}\int_0^c f(t)dt=0$$

B) apply R.T on $g(x)=(1-x)\,e^x\int_0^x f(x)dx$

C) $g(x)=e^{-x^2/2}\int_0^x f(x)dx$

D) $g(x)=e^{x(x-1)}\int_0^x f(x)dx$

48. $$\int\frac{x^4}{\left(x^4+1\right)^2}dx=\int x.\frac{x^3}{\left(x^4+1\right)^2}\,dx$$

(now use integration by parts using 'x' as first function)

$$=-\frac{x}{4\left(x^4+1\right)}+\frac{1}{4}\int\frac{dx}{x^4+1}=-\frac{x}{4\left(x^4+1\right)}+\frac{1}{8}\int\frac{1-x^2+1+x^2}{x^4+1}\,dx$$

$$=-\frac{x}{4\left(x^4+1\right)}+\frac{1}{8}\int\frac{\left(\frac{1}{x^2}-1\right)}{\left(\frac{1}{x^2}+x^2\right)}dx+\frac{1}{8}\int\frac{\left(\frac{1}{x^2}+1\right)}{\left(\frac{1}{x^2}+x^2\right)}dx$$

$$=\frac{-x}{4\left(x^4+1\right)}-\frac{1}{16\sqrt{2}}\ln\left(\frac{\left(x+\frac{1}{x}\right)-\sqrt{2}}{\left(x+\frac{1}{x}\right)+\sqrt{2}}\right)+\frac{1}{8\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C$$

$$=\frac{-x}{4\left(x^4+1\right)}-\frac{1}{16\sqrt{2}}\ln\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)+\frac{1}{8\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C$$

so $A=4,\ f(x)=\frac{x^2-1}{\sqrt{2}x}$, and $g(x)=x^2+\sqrt{2}x+1$

49. In equation $F^{/3}$ is effectively 3-element Fibonacci sum. So, 1 + 1+ 2 = 4, 3 + 5 + 8 = 16, 13 + 21 + 34 = 68 and so on. So we have the recurrence $F_n^{/3} = F_{n+2} + F_{n+1} + F_n$, so rather clearly we have L_3 as

$$L_3 = \lim_{n\to\infty} \frac{F_{n+2} + F_{n+1} + F_n}{F_{n-1} + F_{n-2} + F_{n-3}}$$

Therefore, we have after multiplying with F_{n-4}

$$L_3 = \frac{\lim_{n\to\infty} \frac{F_{n+2}}{F_{n-4}} + \lim_{n\to\infty} \frac{F_{n+1}}{F_{n-4}} + \lim_{n\to\infty} \frac{F_n}{F_{n-4}}}{\lim_{n\to\infty} \frac{F_{n-1}}{F_{n-4}} + \lim_{n\to\infty} \frac{F_{n-2}}{F_{n-4}} + \lim_{n\to\infty} \frac{F_{n-3}}{F_{n-4}}}$$

Then, we can write L_3 as

$$L_3 = \frac{\phi^6 + \phi^5 + \phi^4}{\phi^3 + \phi^2 + \phi} = \frac{\phi^4.\left(\phi^2 + \phi + 1\right)}{\phi.\left(\phi^2 + \phi + 1\right)} = \phi^3$$

Where $\phi = \frac{1+\sqrt{5}}{2}$

50. $|xy| - r|x| - r|y| + r^2 \le 0$

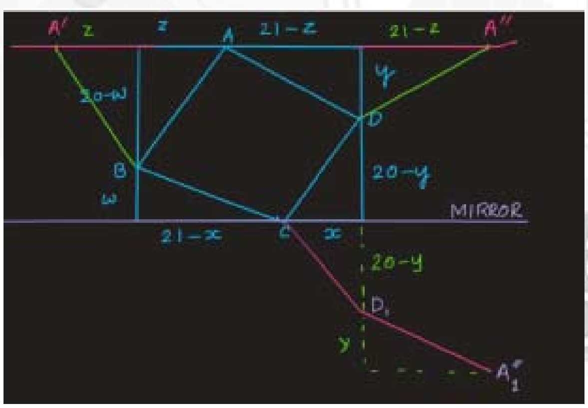
$\Rightarrow \left(|x| - r\right)\left(|y| - r\right) \le 0$

$P(r) = (2r+1)^2 \quad \sum_{r=1}^{r=n} P(r) = \frac{n}{3}\left(4n^2 + 12n + 11\right)$

$$\lim_{n\to\infty} \frac{\frac{n}{3}\left(4n^2 + 12n + 11\right) - \lambda n^3}{an^2 + bn + c} = \frac{1}{2}$$

$\Rightarrow \lambda = \frac{4}{3}$ and $a = 8$

51.



Given expression

$= AB + BC + CD + DA$

$= A'B + BC + CD + DA''$

$= A'A_1''$

$$=\sqrt{\left(A'A''\right)^2+\left(A''A_1''\right)^2}$$

$$=\sqrt{42^2+40^2}=2\times 29$$

52. $\sqrt{x+1}+x=t^3>2x=t^3-t^{-3}$

$$\sqrt{x^2+1}-x=t^{-3}$$

$$\int\left(x-\sqrt{x^2+1}\right)^{1/3}+\left(x+\sqrt{x^2+1}\right)^{1/3}dx=\int\left(t-\frac{1}{t}\right)\left(3t^2+\frac{3}{t^4}\right)\frac{dt}{2}$$

$$=\frac{3}{2}\int\left(t^3+\frac{1}{t^3}-t-\frac{1}{t^5}\right)dt=\frac{3}{2}\left[\frac{t^4}{4}+\frac{t^{-2}}{-2}-\frac{t^2}{2}-\frac{t^{-4}}{-4}\right]$$

$$=\frac{3}{8}\left[t^4+t^{-4}-2\left(t^2+t^{-2}\right)\right]$$

If $x=7$, then $t^3=7+5\sqrt{2}\Rightarrow t^{-3}=5\sqrt{2}-7$

$$\Rightarrow t^3-\frac{1}{t^3}=14\Rightarrow\left(t-\frac{1}{t}\right)^3+3\left(t-\frac{1}{t}\right)=14$$

$$\Rightarrow t-\frac{1}{t}=2\Rightarrow t^2+t^{-2}=6\Rightarrow t^4+t^{-4}=34$$

$$\therefore\int_0^7 f(x)\,dx=\frac{3}{8}\left[(34-12)-(2-4)\right]=\frac{3}{8}[24]=9$$

53. If $q-p$ is maximum then $y=\pm 1$ should be tangents

$$\therefore q-p=4\sqrt{14+10\sqrt{2}}$$

54. $\dfrac{1-a^2+4a}{1+a^2}=\dfrac{3-4b+3b^2}{1-b^2}\Rightarrow\cos x+2\sin x=3\sec y-2\tan y$

$$\Rightarrow(\cos x+2\sin x)\cos y=3-2\sin y$$

$$\Rightarrow\cos x\cos y+2\sin x\cos y+2\sin y=3$$

$$\Rightarrow\frac{\cos x\cos y}{1}=\frac{\sin x\cos y}{2}=\frac{\sin y}{2}\Rightarrow\tan x=2,\ \tan y=\sin x$$