Sri Chaitanya IIT Academy.,India.

✪ A.P ✪ T.S ✪ KARNATAKA ✪ TAMILNADU ✪ MAHARASTRA ✪ DELHI ✪ RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 13-01-2023
Time: 09.00Am to 12.00Pm GTM-06 Max. Marks: 300

KEY SHEET

PHYSICS

1)	**3**	2)	**1**	3)	**3**	4)	**3**	5)	**1**
6)	**4**	7)	**2**	8)	**4**	9)	**4**	10)	**2**
11)	**1**	12)	**1**	13)	**3**	14)	**4**	15)	**3**
16)	**1**	17)	**3**	18)	**2**	19)	**3**	20)	**1**
21)	**20**	22)	**2**	23)	**60**	24)	**175**	25)	**144**
26)	**177**	27)	**31**	28)	**4**	29)	**10**	30)	**6**

CHEMISTRY

31)	**2**	32)	**2**	33)	**1**	34)	**1**	35)	**4**
36)	**4**	37)	**3**	38)	**2**	39)	**4**	40)	**3**
41)	**3**	42)	**3**	43)	**1**	44)	**1**	45)	**2**
46)	**4**	47)	**1**	48)	**3**	49)	**2**	50)	**1**
51)	**2**	52)	**1**	53)	**6**	54)	**1**	55)	**2**
56)	**6**	57)	**5**	58)	**5**	59)	**2**	60)	**3**

MATHEMATICS

61)	**1**	62)	**1**	63)	**1**	64)	**3**	65)	**4**
66)	**1**	67)	**1**	68)	**3**	69)	**1**	70)	**2**
71)	**1**	72)	**4**	73)	**2**	74)	**2**	75)	**2**
76)	**3**	77)	**3**	78)	**2**	79)	**1**	80)	**4**
81)	**1**	82)	**4**	83)	**2**	84)	**6**	85)	**113**
86)	**3**	87)	**16**	88)	**7**	89)	**9**	90)	**5**

SOLUTIONS

PHYSICS

1. Conceptual

2. $\vec{r}_f = \vec{r}_i + \Delta\vec{S}_1 + \vec{S}_2 + ...$

$\vec{r}_f = \left(2\hat{i} + 3\hat{j}\right) + 5\hat{i} + 8\hat{j} + \left(-2\hat{i} + 4\hat{j}\right) + \left(-6\hat{j}\right)$

$\vec{r}_f = 5\hat{i} + 9\hat{i}$

Distance from thrower $= \sqrt{5^2 + 9^2} = \sqrt{106}$

3. Dimension of z is dimension of time and only option C has dimension of time.

4. $speed = \dfrac{coefficent\,of\ t}{coefficent\,of\ x}$ $\qquad 3\times10^8 = \dfrac{6\times10^8}{k} \Rightarrow k = 2$

5. Intensity after passing through 1^{st} polaride $= \dfrac{I_0}{2}$

Let 2^{nd} polaride is at angle θ from first polaride & 3^{rd} polaride is at angle 90-θ from 2^{nd} one then intensity after passing through 2^{nd} polaride = $\dfrac{I_0}{2}\cos^2\theta$

Intensity after passing through 3^{rd} polaride $= \dfrac{I_0}{2}\cos^2\theta\cos^2(90-\theta)$

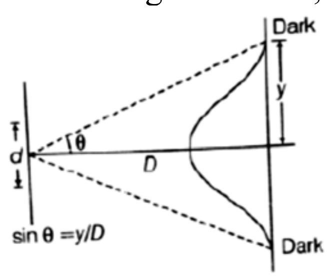
$I_{final} = \dfrac{I_0}{2}\cos^2\theta\sin^2\theta = \dfrac{I_0}{8}\sin^2 2\theta = \dfrac{I_0}{8}\sin^2 2\omega t$

6. For first dark fringe on either side, d $\sin\theta = \lambda$

Or $\dfrac{dy}{D} = \lambda$ $\qquad \therefore y = \dfrac{\lambda D}{d}$

Therefore, distance between two dark fringes on either side $= 2y = \dfrac{2\lambda D}{d}$

Substituting the values, we have



Distance = $= \dfrac{2\left(600\times10^{-6}mm\right)\left(2\times10^3 mm\right)}{(1.0mm)} = 2.4mm$

7. $D = \dfrac{\mu_0 NI}{2\pi R}$

8. $T = 2\pi\sqrt{\dfrac{m_{effective}}{k}}$

If mass increases time period increases. If collision take place of extreme position then no energy loss take place hence amplitude remains same.
If collision take place at mean position then due to inelastic collision energy loss take place & Amplitude decreases.

9. W = area enclose in the cycle

$$W=\frac{1}{2}4\times10^{-4}\times2\times10^{5}=40J$$

10. Zero error is 3 division

$$LC=\frac{0.5}{50}=0.01\text{mm}$$

zero error = 0.03 mm
Reading = 5.0 + 29 $\times$0.01
d = 5.29-0.03 = 5.26 mm

11. For initial condition :

$$\frac{1}{f}=[\mu-1]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\Rightarrow\frac{1}{12}=\left[\frac{3}{2}-1\right]\left[\frac{2}{R}\right]$$

R = 12 cm
When liquid is poured
$u=-12,\quad V=-24,\ f_{eq}=?$

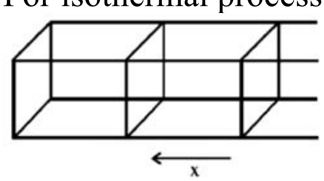
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\text{[Mirror Formula]}\Rightarrow f_{eq}=-8$$

$$P_{eq}=2P_{L_1}+P_{L_2}+P_M$$

$$\frac{1}{8}=2\left(\frac{1}{12}\right)+2P_{L_2}+0\Rightarrow P_{L_2}=\frac{1}{48}=[\mu-1]\left[\frac{1}{R}\right]$$

$$P_{L_2}=\frac{1}{48}=[\mu-1]\left[\frac{1}{R}\right]\Rightarrow\mu=\frac{5}{4}$$

12. For isothermal process



PV = C

PdV + Vdp = 0 $\qquad dp=\frac{-P}{V}.dV$

$$dp=\frac{-p}{V}(Ax)\Rightarrow F_{net}=(dP)A=-\left(\frac{Pa^2}{V}\right)x\Rightarrow \text{F = - kx}$$

13. $$f=\frac{1}{4\ell}\sqrt{\frac{\gamma RT}{M}}$$

Since $\Delta T<<T\Rightarrow\frac{df}{f}=\frac{1}{2}\frac{dT}{T}\Rightarrow df=\frac{1}{2}\times\frac{1}{300}\times10{,}000 \qquad \Rightarrow \qquad df=16.67$

14. As temperature decrease

$E = eA\sigma T^4 \quad \Rightarrow$ Intensity reduce

and $\lambda_{max} T$ = constant

λ_{max} : increase

15. Let resistance per unit length of wire $'AB'$ be k

Case (i) :

x = 0 $\rightarrow$ balance length : ℓ_1

$R_h = 2\Omega$

$i = \dfrac{40}{4+2+2} \Rightarrow \quad i = 5A$

$\therefore$ At balance length

$e = 5k\ell_1$ ....(1)

Case (ii) :

$x = 2 \rightarrow$ Balance length ℓ_2

$R_h = 6\Omega$

$i = \dfrac{40}{4+2+6} \quad \Rightarrow i = 10/3A$

$\therefore$ At balance length,

$e = (10/3)k\ell_2$ ....(2)

From (1) and (2), $5x\ell_1 = (10/3)k\ell_2 \quad \Rightarrow \dfrac{\ell_1}{\ell_2} = \dfrac{2}{3}$

16. $F_y = mg(-\hat{j}) \qquad F_x = \left(\dfrac{\sigma}{2\varepsilon_0}\right)q(\hat{i})$

$a_y = -g(\hat{i}) \quad a_x = \left(\dfrac{\sigma q}{m2\varepsilon_0}\right)\hat{i}$

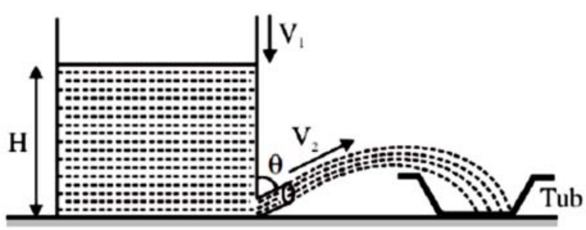
As $S_y = H_{max}, V_y = 0$

Let the time taken to reach max. height be

$t_1 . v_y = u_y + a_y t_1 \quad \Rightarrow 0 = u\sin\theta - gt_1 \Rightarrow \quad t_1 = \dfrac{u\sin\theta}{g}$

At $t = t_1, v_x = u_x + a_x t_1 \Rightarrow v_x = u\cos\theta + \left(\dfrac{\sigma q}{2m\varepsilon_0}\right)\dfrac{u\sin\theta}{g}$

$= u\sin\theta\left[1 + \dfrac{\sigma q \tan\theta}{2m\varepsilon_0 g}\right]$

17. 

Applying bernouli's theorem between point on surface of water and point at orifice taking ground as reference,

$$P_{atm}+\frac{1}{2}\rho V_1^2+\rho gH=P_{atm}+\frac{1}{2}\rho V_2^2 \quad \Rightarrow \quad V_2^2-V_1^2=2gH$$

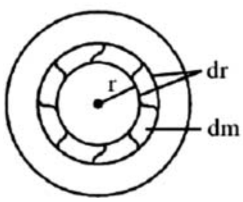
$$\Rightarrow \quad V_2^2-\left(\frac{A_2}{A_1}\right)V_2^2=2gH \qquad [\because A_1V_1=A_2V_2]$$

$$\Rightarrow \quad V_2^2=\frac{2gH}{1-\left(\frac{A_2}{A_1}\right)^2}$$

Substituting $\frac{A_2}{A_1}=\frac{1}{2}, H=0.3m \Rightarrow V_2=2\sqrt{2}$

If $\theta=30^0$

Range $=\frac{V_2^2\sin 2(90-\theta)}{g}=\frac{8\times\frac{\sqrt{3}}{2}}{10}=\frac{2\sqrt{3}}{5}$

18. 

Let 'M' be total mass of earth.
Consider a shell of thickness 'dr' and mass 'dm' at a distance 'r' from centre inside earth,

$$\Rightarrow \quad dm=\rho 4\pi r^2 dr$$

$$M=\int dm \qquad =\int_0^R 4\pi k r^3 dr \qquad =\frac{4\pi kR^4}{4}=\pi kR^4$$

Let field due to earth's gravity at a distance '$2R$' from centre be $T, I\times A=4\pi Gm_{inside}$.

$$\Rightarrow I\times 4\pi(2R)^2=4\pi G\left(\pi kR^4\right)$$

$$\Rightarrow \quad I=\frac{\pi kR^4 G}{4R^2}$$

For a satellite of mass 'm' moving in orbit or '$2R$' radius.

$$mI=\frac{mv^2}{(2R)}$$

$$\Rightarrow I=\frac{V^2}{2R}$$

$$\Rightarrow \frac{\pi k R^2 G}{4} = \frac{V^2}{2R}$$

$$V = \sqrt{\frac{\pi k R^3 G}{2}}$$

19. Conceptual

20. Conceptual

21. $E_x = -\frac{\partial v}{\partial x} = -6\hat{i}$

$$E_y = -\frac{\partial v}{\partial y} = -8\hat{j}$$

$$E_z = -\frac{\partial v}{\partial z} = -8z\hat{k}$$

$\vec{E}_{net}$ at origin $= \sqrt{6^2 + 8^2} = 10$

$$\Rightarrow \quad \left|\vec{F}\right| = \left|q\vec{E}\right| = 20N$$

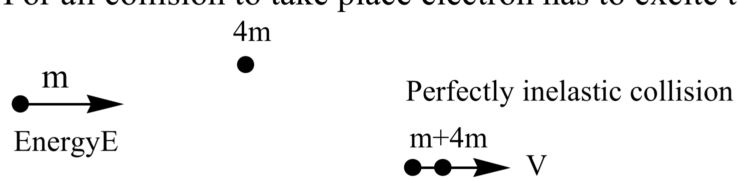
22. Induced field in rod, E = vB

electric field on surface of sphere $= \frac{KQ}{R^2}$

$$\frac{KQ}{R^2} = vB \Rightarrow R^2 = \frac{kQ}{vB} = \frac{9\times10^9\times30}{9\times1}$$

$$R = \sqrt{3}\times10^5 = 1.73\times10^5$$

23. For all collision to take place electron has to excite to n = 3.



mu + 0 = 5m V

Loss $= \frac{1}{2}mu^2 - \frac{1}{2}5m\times\left(\frac{4}{5}\right)^2 = \frac{4E}{5}$

$$\frac{4E}{5} = 12.09\times(2)^2 eV$$

$$\Rightarrow \quad E = 60.45eV$$

24. $\frac{f_{\max}}{\Delta f_{half\ of\ \max\ power}} =$ Quality factor

$\frac{X_C}{R} =$ Quality factor

25. Impulse on block $= \left(\frac{IA}{C}\right)\cos^2 53^\circ \times (\Delta t)$

$$=\frac{(20)\left(10\times10^{-4}\right)}{3\times10^{8}}\times(0.6)^{2}\times6\times10^{-3}$$

$$=\frac{72}{5}\times10h-14kg\ \ m/s$$

Now we have

Impulse = mv

$$\frac{72}{5}\times10^{-14}=1\times10^{-9}v$$

$$v=\frac{72}{5}\times10^{-5}m/s$$

Now we have

$$\frac{1}{2}kx^{2}=\frac{1}{2}mv^{2}$$

$$10^{-5}x^{2}=10^{-9}\times24\times10^{-5}\times24\times10^{-5}$$

$$x=\frac{72}{5}\times10^{-7}m$$

$$N=\frac{7.2}{5}=1.44$$

26. $mv\ell_{\min}=L\Rightarrow\sqrt{2m_{neutron}K_{neutron}}=4\times10^{-34}$

$\Rightarrow\quad \ell_{\min}=1.25\times10^{-14}m=125\times10^{-16}m$

27. $\dfrac{2\sqrt{3}}{\dfrac{0.2}{\sqrt{3}}}$

28. $\dfrac{\Delta V}{V}=\dfrac{2\Delta r}{r}+\dfrac{\Delta l}{l}\Rightarrow\dfrac{\Delta l}{l}=\dfrac{\Delta V}{V}-2\dfrac{\Delta r}{r}=0.204\%$

$\therefore$ Stress $=2\times10^{11}\times\dfrac{0.204}{100}=4.08\times10^{8}N/m^{2}$

29. The diode is forwards biased, so the equivalent resistance of the circuit is $15K\Omega$.

30. $\dfrac{w_1}{w_2}=\dfrac{\lambda_1}{\lambda_2}$

CHEMISTRY

31. Polarizing power of the cation
32. Steam volatile and water insoluble
33. Aromatic aldehydes and ketones do not give positive Fehling's test
34. E_1CB mechanism
35. Presence of unpaired electrons
36. SP mixing
37. Chromyl chloride test
38. $X = B_2H_6; Y = B_3N_3H_6; Z = H_2$
39. Calgon treatment
40. Half filled f orbital configuration
41. SRP vaues
42. Ozone is component of photo chemical smog
43. LiF has high lattice energy
44. Ethene is produced (gas)
45. Adsorbate is concentrated on surface of adsorbent
46. Addition of copper rod does not change direction of current flow.
47. NCERT XI part-1 Page No 181.
48. State-I to state- II is spontaneous, NCERT XI, Equilibrium
49. NCERT lab manual 12 class.
50. One motif for unit cell (NCERT 12 Page No. 6)
51. Two P-H bonds are present
52. Observe chiral carbons in the product
53. There are 6 atoms present in straight line
54. $H_2S_2O_8$
55. Nucelic acids
56. XeO_3
57. $\lambda = $ h/mv

    NCERT (text question)
58. 0.005 moles of Barium chloride in 2L
59. $\pi = CRT = \rho gh$

    Or, $\dfrac{0.2/M}{100/1000} \times 0.0821 \times 300 = \dfrac{1.013 \times 1000 \times 0.2463}{1.013 \times 10^6}$

    $\therefore M = 2 \times 10^5$
60. $\left[H^+\right]_{final} = \dfrac{5 \times 10^{-4}}{500} \times 1000 = 10^{-3} M \Rightarrow p^H = 3.0$

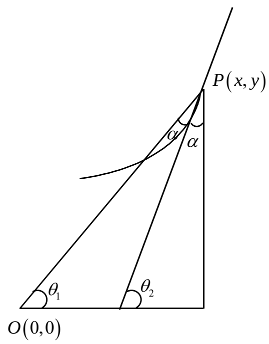
MATHEMATICS

61. $\arg\left(\frac{z-2i}{z+2i}\right)=\frac{\pi}{6} \quad \Rightarrow \arg(z-2i)-\arg(z+2i)=\frac{\pi}{6}$

$\Rightarrow \tan^{-1}\left(\frac{y-2}{x}\right)-\tan^{-1}\left(\frac{y+2}{x}\right)=\frac{\pi}{6} \quad \Rightarrow \frac{xy-2x-xy-2x}{x^2+y^2-4}=\frac{1}{\sqrt{3}}$

$\Rightarrow x^2+y^2-4=-4\sqrt{3}x \quad \Rightarrow x^2+y^2+4\sqrt{3}x-4=0$

$\Rightarrow \left(x+2\sqrt{3}\right)^2+y^2=12+4=4^2 \quad \therefore$ centre $\left(-2\sqrt{3},0\right)$, radius = 4



62.

$\theta_1+\alpha=\theta_2 \qquad \theta_1+2\alpha=\frac{\pi}{2}$

$\theta_1+2(\theta_2-\theta_1)=\frac{\pi}{2} \quad 2\theta_2-\theta_1=\frac{\pi}{2}$

$\frac{2\tan(\theta_2)}{1-(\tan(\theta_2))^2}=\frac{1}{\tan\theta_1} \qquad \frac{2\frac{dy}{dx}}{1-\left(\frac{dy}{dx}\right)^2}=-\frac{x}{y}$

$2y\frac{dy}{dx}=+x\left(\frac{dy}{dx}\right)^2-x \Rightarrow x\left(\frac{dy}{dx}\right)^2-2y\frac{dy}{dx}=x$

63. Three b's and four a's can arrange abababa

Fourth 'b' can take 5 places, $\begin{cases} babababa \\ abbababa \\ ababbaba \\ abababba \\ abababab \end{cases}$

Take 8 place from 12 places and arrange letters in same order, cc dd can take remaining 4 place $\quad \therefore {}^{12}C_8\times 5\times\frac{4!}{2!2!}$

64. Required probability $=1-\frac{{}^6C_1}{2^5}-\frac{{}^6C_1}{2^5}+{}^6C_2.{}^2C_1\frac{1}{2^5}.\frac{1}{2^4}=\frac{175}{216}$

65. $\because \dfrac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} \Rightarrow \tan(3x - 2x) = 1$

Or tan x = 1

Or $x = n\pi + \dfrac{\pi}{4}, n \in I$

But at this value, tan 2x is undefined, hence there is no solution and $\tan x = \dfrac{\sin x}{\cos x}$

$\therefore \tan x$ is not defined when cos x = 0

Or $x = n\pi + \dfrac{\pi}{2}, n \in I$

66. There are two possibilities : either the curves $y = x^2 + u$ and $x = y^2 + u$ intersect in exactly one point, or they intersect in two points but one of the points occurs on the branch $y = -\sqrt{x-u}$.

Case-1 : The two curves are symmetric about y = x, so they must touch that line at exactly one point and not cross it. Therefore, $x = x^2 + u$, so $x^2 - x + u = 0$. This has exactly one solution if the discriminant, $(-1)^2 + 4(1)(u) = 1 + 4u$, equals 0,so $u = \boxed{\dfrac{1}{4}}$.

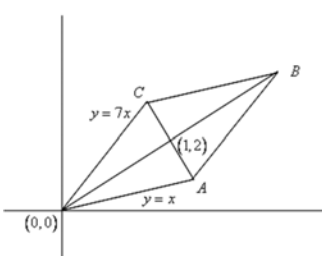
Case-2 : $y = x^2 + u$ intersects the x–axis at $\pm\sqrt{-u}$, while $y = \sqrt{x-u}$ starts x = u and goes up from there. In order for these to intersect in exactly one point, we must have $-\sqrt{-u} < u$, or $-u > u^2$ (note that $-u$ must be positive in order for any intersection points of $y = x^2 + u$ and $x = y^2 + u$ to occur outside the first quadrant). Hence we have $u(u+1) < 0$, or $u \in (-1, 0)$.

67. Equation of diagonal AC is $y - 2 = \dfrac{-1}{2}(x-1) \Rightarrow x + 2y = 5$

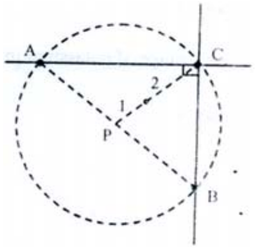
On solving with y = x we get $A = \left(\dfrac{5}{3}, \dfrac{7}{3}\right)$

On solving with y = 7x we get $C = \left(\dfrac{1}{2}, \dfrac{7}{2}\right)$

Clearly B = (24) required area = $\dfrac{1}{2}\begin{vmatrix} 2 & 4 \\ \dfrac{4}{3} & \dfrac{-2}{3} \end{vmatrix} = \left|\dfrac{1}{2} \times \dfrac{-20}{3}\right| = \dfrac{10}{3}$ sq. units



68. As lines are perpendicular to each other 'C' moves on a circle with AB as diameter. Now P, mid–point of AB (which is fixed) when joined with C is median.



$\Rightarrow$ Centroid is moving at a constant distance $\frac{1}{3}(PA)$ from P.

$\Rightarrow$ Locus is a circle

A is point of intersection of x + 4y + 2 = 0 and x – y + 1 = 0 i.e., $\left(-\frac{6}{5},-\frac{1}{5}\right)$

B is point of intersection of 4x – y + 6 = 0 and x + y + 3 = 0 i.e., $\left(\frac{-9}{5},-\frac{6}{5}\right)$

$\Rightarrow P=\left(\frac{-3}{2},\frac{-7}{10}\right)$ therefore locus is $\left(x+\frac{3}{2}\right)^2+\left(y+\frac{7}{10}\right)^2=\frac{17}{50}$

69. $y=\cos\theta\left(\sin\theta+\sqrt{\sin^2\theta+\sin\alpha}\right)$

$\frac{y}{\cos\theta}=\sin\theta+\sqrt{\sin^2\theta+\sin^2\alpha}$

$y\sec\theta-\sin\theta=\sqrt{\sin^2\theta+\sin^2\alpha}$

Squaring on both sides

$y^2\sec^2\theta+\sin^2\theta-2y\tan\theta=\sin^2\theta+\sin^2\alpha$

$y^2+y^2\tan^2\theta-2y\tan\theta-\sin^2\alpha=0$

$\tan\theta\in R\Rightarrow\Delta\ge0$

$4y^2-4.y^2.\left(y^2-\sin^2\alpha\right)\ge0$

$4y^2\left[1-y^2+\sin^2\alpha\right]\ge0$

$\Rightarrow1-y^2+\sin^2\alpha\ge0$

$y^2-1-\sin^2\alpha\le0$

$y^2-\left(1+\sin^2\alpha\right)\le0$

$y\in\left[-\sqrt{1+\sin^2\alpha},\sqrt{1+\sin^2\alpha}\right]$

70. Since, f(x) is differentiable and hence continuous $\forall x\in R$

$\Rightarrow f\left(0^+\right)=f\left(0^-\right)\Rightarrow P(0)=0$ and $\Rightarrow f\left(0^+\right)=f'\left(0^-\right)\Rightarrow P(0)=0$

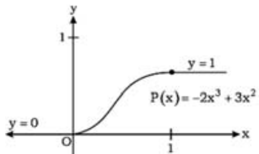
Similarly, continuity at $x=1\Rightarrow P(1)=1$ and differentiability at $x=1\Rightarrow P(1)=0$.

Since, P(x) is a polynomial of least degree and $P'(x)$ vanishes x = 1 and x = 0.
Hence, P(x) must be cubic.

$\therefore P'(x)=kx(x-1)\Rightarrow P(x)=k\left(\frac{x^3}{3}-\frac{x^2}{2}\right)+C$

Since, $P(0)=0\Rightarrow C=0$ and $P(1)=1\Rightarrow k=-6$

Hence, $P(x) = -2x^3 + 3x^2$. $\therefore f(x) = \begin{cases} 0 & if \quad x \le 0 \\ -2x^3 + 3x^2 & if \quad 0 < x < 1 \\ 1 & if \quad x \ge 1 \end{cases}$



Graph of f(x)

71. $S = (n-1)\cos\frac{2\pi}{n} + (n-2)\cos\frac{4\pi}{n} + (n-3)\cos\frac{6\pi}{n} + ... + \cos\frac{2(n-1)\pi}{n}$

$S = 1\cos\frac{2\pi}{n} + 2\cos\frac{4\pi}{n} + ... + (n-1)\cos\frac{2(n-1)\pi}{n}$

$2S = n\left(\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + ... + \cos\frac{2(n-1)\pi}{n}\right)$

$$2S = n\frac{\sin(n-1)\frac{\pi}{n}}{\sin\frac{\pi}{n}}\cos\left(\frac{\frac{2\pi}{n} + \frac{2(n-1)\pi}{n}}{2}\right) = -n$$

72. $a = \tan A; b = \tan B; c = \tan C; 0 < A, B, C < \frac{\pi}{2} 0 < A + C < \frac{\pi}{2}$

$P = 2\cos^2 A + 3\cos^2 C - 2\cos^2(A+C)$

$= 1 + \cos 2A + \frac{3}{2} + \frac{3}{2}\cos 2 + \cos(2A + 2C)$

$= 1 + \cos 2A + \frac{3}{2} + \frac{3}{2}\cos 2C - \cos 2A\cos 2C + \sin 2A\sin 2C$

$= \frac{3}{2} + \frac{3}{2}\cos 2C + (1 - \cos 2C)(\cos 2A + \sin 2C.\sin 2A)$

$< \frac{3}{2} + \frac{3}{2}\cos 2C + \sqrt{(1-\cos 2C)^2 + (\sin 2C)^2}$

$= \frac{3}{2} + \frac{3}{2}\cos 2C + \sqrt{2 - 2\cos 2C} = \frac{5}{2} + \frac{3}{2}\cos 2C + 2\sin C$

$\frac{3}{2} + \frac{3}{2}\left(1 - 2\sin^2 C\right) + 2\sin C$

$\frac{3}{2} + \frac{3}{2} - 3\sin^2 C + 2\sin C$

$\frac{3}{2} + \frac{3}{2} - 3\left[\sin^2 C\frac{2}{3}\sin C + \frac{1}{9} - \frac{1}{9}\right] = +\frac{1}{3} - 3\left[\sec\frac{1}{3}\right]^2$

$= \frac{10}{3}$

73. Using $A.M \ge G.M$

$$x+y=\frac{x}{2}+\frac{x}{2}+y\geq 3\left(\frac{x^2y}{4}\right)^{1/3}$$

Equality holds if and only if $\frac{x}{2}=y\Rightarrow x=2y$.

Also, $2x^2+2xy+3y^2$

$$\frac{2x^2}{8}+.....+\frac{2x^2}{8}+\frac{2zy}{4}+.....+\frac{2xy}{4}+y^2+y^2+y^2$$

$$\geq 15\left\{\left(\frac{2x^2}{8}\right)^8\left(\frac{2xy}{4}\right)^4\left(y^2\right)^3\right\}^{\frac{1}{15}}=15\left(\frac{x^2y}{4}\right)^{\frac{2}{5}}$$

Equality holds if and only if $\frac{2x^2}{8}=\frac{2xy}{4}=y^2$ or x = 2y. Thus

$$k=x+y+\left(2x^2+2xy+y^2\right)^{1/2}\geq\left(3+\sqrt{15}\right)\left(\frac{x^2y}{4}\right)^{1/3}$$

74. Let the numbers be 1, 2, 3, 4, ……, n and the erased number be x then $1\leq x\leq n$

Now, $\dfrac{\frac{n(n+1)}{2}-x}{n-1}=35\frac{7}{17}$

$$\therefore \frac{\frac{n(n+1)}{2}-n}{n-1}\leq 35\frac{7}{17}\leq\frac{\frac{n(n+1)}{2}-1}{n-1}$$

$$\Rightarrow\frac{n}{2}\leq 35\frac{7}{17}\leq\frac{n+2}{2}$$

$$\Rightarrow\frac{n}{2}\leq 35+\frac{7}{17}\leq\frac{n+2}{2}$$

$$\Rightarrow n\leq 70+\frac{14}{17}\leq n+2$$

$$\Rightarrow n=69(or)70$$

at n = 69 ; $\dfrac{\frac{69\times 70}{2}}{68}=35\frac{7}{17}\Rightarrow x=7$

at n = 70 ; $x\notin I$

75. $\sum_{i=1}^{10}\left(x_i-\bar{x}\right)\left(y_i-\bar{y}\right)=80$

$\sum_{i=1}^{10}x_iy_i-y\sum_{i=1}^{10}x_i-x\sum_{i=1}^{10}y_i+\sum_{i=1}^{10}xy=80$ which implies $\sum_{i=1}^{10}x_iy_i-10yx=80$

$$\sigma^2=\frac{\sum_{i=1}^{10}\left(x_i-y_i\right)^2}{10}-\left(\bar{y}-\bar{x}\right)^2=9$$

76. **Statement -1**

x	y	$x \Leftrightarrow y$	$x \vee y$	$x \wedge y$	$\sim(x \wedge y)$	$x \oplus y$	$\sim(x \oplus y)$
T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	F
F	T	F	T	F	T	T	F
F	F	T	F	F	T	F	T

Statement -2 : $\sim(p \to q) \leftrightarrow (\sim p \vee \sim q)$

$\sim(\sim p \vee q) \leftrightarrow \sim(p \wedge q)$

$(p \wedge \sim q) \leftrightarrow \sim(p \wedge q) \equiv p$ (Neither a tautology nor a contradiction)

77. $$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

78. $f^1(0) = \underset{h \to 0}{lt} \dfrac{f(0-h) - f(0)}{-h}$ $= \underset{n \to 0}{lt} \left(\dfrac{-h\left(3e^{-1/h} + 4\right)}{2 - e^{-1/h}} - 0 \right)\left(\dfrac{-1}{h}\right) = 2$

$Rf^1(0) = \underset{h \to 0}{lt} \dfrac{f(0+h) - f(0)}{h}$

$= \underset{h \to 0}{lt} \left(\dfrac{h\left(3e^{1/h} + 4\right)}{2 - e^{1/h}} - 0 \right)\left(\dfrac{1}{h}\right)$ $= \underset{h \to 0}{lt} \left(\dfrac{3 + 4e^{-1/h}}{2e^{-1/h} - 1} \right) = -3$

Since $Lf^1(0) \neq Rf^1(0)$

$\therefore$ f(x) is differentiable at x = 0. But f(x) is continuous at x = 0

79. $\dfrac{dy}{dx} = -\dfrac{x_1^2}{y_1^2}$

Tangent equation is $x_1^2 x + y_1^2 y = x_1^3 + y_1^3$

$\Rightarrow x_1^2 x + y_1^2 y = a^3$

Since, it passes through (x_2, y_2)

$\therefore x_1^2 x_2 + y_1^2 y_2 = a^3$ (1)

and $x_1^3 + y_1^3 = a^3$ (2)

$x_2^3 + y_2^3 = a^3$ (3)

By solving (1), (2), (3) we get result

80. Let $g(x) = x^n f(x) (n > 0)$

$g(0)=0, g(3)=0$

By using rolls theorem, there exist some $\alpha \in (0,3)$ such that

$g^1(\alpha)=0 \Rightarrow \alpha f^1(\alpha)+n f(\alpha)=0, n>0$.

81. $$\int_1^{\sqrt{2}}\left(\left(x^4-2x^2+1\right)^{\frac{1}{3}}-1\right)dx+\int_{-1}^{0}\sqrt{(x+1)^{\frac{3}{2}+1}}\,dx=1$$

$$\Rightarrow \int_1^{\sqrt{2}}\left(x^4-2x^2+1\right)^{\frac{1}{3}}dx+\int_{-1}^{0}\sqrt{(x+1)^{\frac{3}{2}}+1}\,dx=1+\sqrt{2}-1 \Rightarrow I=\sqrt{2}$$

82. Let P and Q be two points t_1 and t_2 respectively whose abscissas are in the ration m : 1.

$\therefore \frac{at_1^2}{at_2^2}=\frac{\mu}{1}$ or $t_1=t_2\sqrt{(\mu)}$

If (h, k) be the point of intersection of tangents at P and Q, then $h=at_1t_2$ $k=a(t_1+t_2)$

Or $h=a\sqrt{(\mu)}t_2^2$ $k=a\left(1+\sqrt{\mu}\right)t_2$

Eliminating t_2 we get the required locus as $y^2=ax\left(\mu^{1/4}+\mu^{-1/4}\right)^2$

83. We first use partial fraction decomposition on this function. Doing so gives us

$$\frac{5x^2-2xy+y^2}{x^2-y^2}=\frac{5x^2-2xy+y^2}{(x+y)(x-y)} \qquad =\frac{3x^2-2xy+3y^2+2\left(x^2-y^2\right)}{(x+y)(x-y)}$$

$$=\frac{x^2+2xy+y^2+2\left(x^2-2xy-y^2\right)}{(x+y)(x-y)}+2=\frac{(x+y)^2+2(x-y)^2}{(x+y)(x-y)}+2$$

$$=\frac{x+y}{x-y}+\frac{2(x-y)}{x+y}+2.$$

We can then apply AM-GM to the first two terms to get

$$\frac{x+y}{x-y}+\frac{2(x-y)}{x+y}\geq 2\sqrt{2}.$$

Thus, the minimum is $\boxed{2+2\sqrt{2}}$, which is achieved when x, y satisfy the equation $(x+y)^2=2(x-y)^2$, which has solutions $y=\left(3\pm 2\sqrt{2}\right)x$. When $y=3\left(3-2\sqrt{2}\right)x$ and $x>0$, then condition $x>y>0$ is satisfied.

84. Since MN is tangent to C_1 at M, $\angle NMQ=\angle MPQ$. Since MN = PN, ΔMNP is isosceles so $\angle MPN=\angle PMN$. It follows that $\angle NPQ=\angle PMQ$. But MN is tangent to C_2 at N, so $\angle NPQ=\angle MNQ$. Hence, $\angle MNQ=\angle PMQ$. Combining this with the fact that

$\angle NMQ = \angle MPQ$, we see that $\Delta PMQ \sim \Delta MNQ$. Then $\frac{PQ}{QM} = \frac{QM}{QN}$, so

$QM^2 = PQ.QN = 3.2 = \boxed{6}$.

85. Since $\{x\} + \{x^2\} = 1$, the value x must satisfy $x + x^2 = n$ for some integer n. The quadratic equation the gives us

$$x = \frac{-1 \pm \sqrt{1+4n}}{2}$$

If we consider when $0 \le x \le 8$, then we must have $\frac{-1+\sqrt{1+4n}}{2} \le 8$. Solving the inequality, we find that x satisfies the equation when $0 \le n \le 72$, giving us 73 possibilities. Likewise, when $-8 \le x < 0$, we must have $\frac{-1-\sqrt{1+4n}}{2} \ge -8$, which has solutions when $0 \le n \le 56$, for a total of 57 possibilities.

Since $\{x\} + \{x^2\} < 2$, we must also eliminate the cases when $\{x\} + \{x^2\} = 0$, which happens only when $-8 \le x \le 8$ is an integer, for a total of 17 possibilities.

Therefore, the total number of solutions is 73 + 57 – 17 = $\boxed{113}$.

86. We know that, $\left|\hat{a}+\hat{b}+\hat{c}\right|^2 = \left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + \left|\hat{c}\right|^2 + 2\left(\left|\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}\right|\right)$

$\left\{2\left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + \left|\hat{c}\right|^2\right\} - 2\left(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}\right)$

$= \left|\hat{a}-\hat{b}\right|^2 + \left|\hat{b}-\hat{c}\right|^2 + \left|\hat{c}-\hat{a}\right|^2 \qquad \Rightarrow \qquad 9 = 3\times 3 - \left|\hat{a}+\hat{b}+\hat{c}\right|^2$

$\Rightarrow \qquad \hat{a}+\hat{b}+\hat{c} = 0 \qquad \Rightarrow \qquad \hat{b}+\hat{c} = -\hat{a}$

$\left|2\hat{a}+5\hat{b}+5\hat{c}\right| = \left|2\hat{a}+5\left(\hat{b}+\hat{c}\right)\right| = \left|2\hat{a}+5\left(-\hat{a}\right)\right| \qquad \left|-3\hat{a}\right| = 3$

87. $\left[\bar{c}\,\bar{d}\,\bar{a}\right]\vec{b} - \left[\bar{c}\,\bar{d}\,\bar{b}\right]\vec{a} + \left[\bar{d}\,\bar{b}\,\bar{a}\right]\vec{c} - \left[\bar{d}\,\bar{b}\,\bar{c}\right]\vec{a} + \left[\bar{b}\,\bar{c}\,\bar{a}\right]\vec{d} - \left[\bar{b}\,\bar{c}\,\bar{d}\right]\vec{a} + k\vec{a} = \bar{0}$

Taking dot with $\vec{b}\times\vec{c}$,

$-3\left[\vec{b}\,\vec{c}\,\vec{d}\right]\left[\vec{a}\,\vec{b}\,\vec{c}\right] + k\left[\vec{a}\,\vec{b}\,\vec{c}\right] + \left[\vec{b}\,\vec{c}\,\vec{a}\right]\left[\vec{b}\,\vec{c}\,\vec{d}\right] = 0$

$\Rightarrow -48\left[\vec{a}\,\vec{b}\,\vec{c}\right] + k\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0 \qquad \Rightarrow k = 48$

88. Let N be $(3\lambda+6, 2\lambda+7, -2\lambda+7)$ such PN is perpendicular to the line.

Then $\lambda = -1 \qquad \therefore N = (3,5,9)$

$\therefore PN = 7$

89. $n(F) = 38, n(B) = 15, n(C) = 20$

$n(F\cup B\cup C) = 58, n(F\cap B\cap C) = 3$

$n(F\cup B\cup C) = n(F + n(B) + n(C) - n(F\cap B) - n(F\cap C) - n(B\cap C) + n(F\cap B\cap C))$

$\Rightarrow n(F\cap B) + n(F\cap C) + n(B\cap C) = 18$

a, denote number of men who got medals in foot ball and basket ball.
b, denote number of men who got medals foot ball and cricket.
c, denote number of men who got medals basket ball and cricket.
d, denote number of men who got medals in all the three sports.
$\therefore a+d+b+d+c+d=18$ (d = 3) or $a+b+c=9$.

90. $175=5^2.7.245=5.7^2.875=5^3 7.1715=5.7^3$

Let $\alpha=\log 5, \quad \beta=\log 7$

$$a=\frac{\log 175}{\log 245}=\frac{2\alpha+\beta}{\alpha+2\beta} \qquad b=\frac{\log 875}{\log 1715}=\frac{3\alpha+\beta}{\alpha+3\beta}$$

$$\frac{1-ab}{a-b}=\frac{(\alpha+2\beta)(\alpha+3\beta)-(2\alpha+\beta)(3\alpha+\beta)}{(2\alpha+\beta)(\alpha+3\beta)-(\alpha+2\beta)(3\alpha+\beta)}$$

$$=\frac{5\left(\beta^2-\alpha^2\right)}{\beta^2-\alpha^2}=5$$