Exercise - 4

Sub Topic: Equation of a straight line in Cartesian and Vector from, angle between two lines, distance between two parallel lines.

MCQ's with ONE or More than ONE Correct Answer Type Questions:

01. There lines $L_1 : \bar{r} = \lambda \bar{i}, \lambda \in R, L_2 : \bar{r} = \bar{k} + \mu \bar{j}, \mu \in R$ and $L_3 : \bar{r} = \bar{i} + \bar{j} + \upsilon \bar{k}, \upsilon \in R,$ are given. For which point (s) Q on L_2 can we find a Point P on L_1 and a point R on L_3 so that P,Q and R are collinear? (ADV-2019)

1. $\bar{k} - \frac{1}{2}\bar{j}$
2. $\bar{k}$
3. $\bar{k} + \bar{j}$
4. $\bar{k} + \frac{1}{2}\bar{j}$

Key : 1,4

Sol : $P(\lambda,0,0)\, on\, L_1, Q(0,\mu,1)\, on\, L_2\, and\, R(1,1,\upsilon)\, on\, L_3.$ Given that P,Q,R are collinear $\therefore \overline{PQ} \| \overline{PR}$

$\Rightarrow \frac{\lambda}{\lambda-1} = \frac{-1}{-\vartheta} \Rightarrow \mu = \frac{\lambda}{\lambda-1}, \vartheta = \frac{\lambda-1}{\lambda} \qquad \frac{\lambda}{\lambda-1} = \frac{-\mu}{-1} = \frac{-1}{-\vartheta}$

$\Rightarrow \mu = \frac{\lambda}{\lambda-1}, \vartheta = \frac{\lambda-1}{\lambda}$

Clearly from the above that $\lambda \neq 0,1$

$\therefore Q\left(0, \frac{\lambda}{\lambda-1}, 1\right)$

(1) For $Q = \bar{K} - \frac{1}{2}\bar{j} \Rightarrow \frac{\lambda}{\lambda-1} = \frac{-1}{2} \Rightarrow 3\lambda = 1 \Rightarrow \lambda = \frac{1}{3}$ which is possible

(2) For $Q = \bar{K} \Rightarrow \frac{\lambda}{\lambda-1} = 0 \Rightarrow \lambda = 0,$ which is not possible

(3) For $Q = \bar{k} + \bar{j} \Rightarrow \frac{\lambda}{\lambda-1} = 1 \Rightarrow \lambda = \lambda - 1$ which is not possible

(4) For $Q = \bar{k} + \frac{1}{2}\bar{j} \Rightarrow \lambda = -1$ which is possible

Hence options (1) & (4) are correct and (2) & (3) are incorrect.

Eg: Let $\bar{r} = \bar{a} + \lambda \bar{l}$ and $\bar{r} = \bar{b} + \mu \bar{m}$ be two lines in the space where $\bar{a} = 5\bar{i} + \bar{j} + 2\bar{k}, \bar{b} = -\bar{i} + 7\bar{j} + 8\bar{k}, \bar{l} = -4\bar{i} + \bar{j} - \bar{k}$ and $\bar{m} = 2\bar{i} - 5\bar{j} - 7\bar{k}$ then the P.V of a point which lies on both of these lines, is

1. $\bar{i} + 2\bar{j} + \bar{k}$
2. $2\bar{i} + \bar{j} + \bar{k}$
3. $\bar{i} + \bar{j} + 2\bar{k}$
4. Non existent as the lines are skew

Key : 1

Sol : $\overline{r_1} = (5\bar{i} + \bar{j} + 2\bar{k}) + \lambda(-4\bar{i} + \bar{j} - \bar{k})$

$\overline{r_2} = (-\bar{i} + 7\bar{j} + 8\bar{k}) + \mu(2\bar{i} - 5\bar{j} - 7\bar{k})$

$P_1 = (5 - 4\lambda, 1 + \lambda, 2 - \lambda), P_2 = (-1 + 2\mu, 7 - 5\mu, 8 - 7\mu)$

$\therefore P_1 = P_2 \Rightarrow 5 - 4\lambda = -1 + 2\mu$

$\Rightarrow 4\lambda + 2\mu = 6 \Rightarrow 2\lambda + 2\mu = 3 \rightarrow (1)$

$1 + \lambda = 7 - 5\mu \Rightarrow \lambda + 5\mu = 6 \rightarrow (2)$

Solve (1) & (2), west $\lambda = 1, \mu = 1$

$\therefore P_1 = (1,2,1), P_2 = (1,2,1)$

$\therefore P_1 = \bar{i} + 2\bar{j} + \bar{k}$

02. Let L_1 *and* L_2 denote the lines $\bar{r} = \bar{i} + \lambda(-\bar{i} + 2\bar{j} + 2\bar{k}), \lambda \in R$ *and* $\bar{r} = \mu(2\bar{i} - \bar{j} + 2\bar{k}), \mu \in R$ respectively. If L_3 is a line which is perpendicular to both L_1 *and* L_2 and cuts both of them, then which of the following options describe $(s)\, L_3$? (ADV-2019)

1. $\bar{r} = \frac{2}{9}(4\bar{i} + \bar{j} + \bar{k}) + t(2\bar{i} + 2\bar{j} - \bar{k}), t \in R$
2. $\bar{r} = \frac{2}{9}(2\bar{i} - \bar{j} + 2\bar{k}) + t(2\bar{i} + 2\bar{j} - \bar{k}), t \in R$
3. $\bar{r} = t(2\bar{i} + 2\bar{j} - \bar{k}), t \in R$
4. $\bar{r} = \frac{1}{3}(2\bar{i} + \bar{k}) + t(2\bar{i} + 2\bar{j} - \bar{k}), t \in R$

Key : 1,2,4

Sol : $L_1 : \bar{r} = \bar{i} + \lambda(-\bar{i} + 2\bar{j} + 2\bar{k}), L_2 : \bar{r} = \mu(2\bar{i} - \bar{j} + 2\bar{k}), \lambda, \mu \in R$ Since L_3 being perpendicular to both L_1 and L_2, is the shortest distance line between L_1 *and* L_2.

$\therefore$ Direction vector of line $L_3 : (-\bar{i} + 2\bar{j} + 2\bar{k}) \times (2\bar{i} - \bar{j} + 2\bar{k}) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = 6\bar{i} + 6\bar{j} - 3\bar{k}$

Since L_1 *and* L_2 are skew lines Let any point on L_1 *and* L_2 be $A(1 - \lambda, 2\lambda,, 2\lambda), B(2\mu, -\mu, 2\mu)$

$\therefore Dr's\ of\ AB = (2\mu + \lambda - 1, -\mu - 2\lambda, 2\mu - 2\lambda)$

$\because AB$ *and* L_3 are representing the same line

$\therefore \frac{2\mu + \lambda - 1}{6} = \frac{-\mu - 2\lambda}{6} = \frac{2\mu - 2\lambda}{-3}$

$\Rightarrow 3\lambda + 3\mu = 1 \rightarrow (1)\ \& 6\lambda - 3\mu = 0 \rightarrow (2)$

Solving (1) & (2) ,we get $\lambda = \frac{1}{9}, \mu = \frac{2}{9}$

$\therefore A\left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right), B\left(\frac{4}{9}, \frac{-2}{9}, \frac{4}{9}\right)$

$\therefore$ Equation of L_3 is given by $\bar{r} = \frac{2}{9}(4\bar{i} + \bar{j} + \bar{k}) + t(2\bar{i} + 2\bar{j} - \bar{k})$

$\therefore (a)$ is correct (or) $\bar{r} = \frac{2}{9}\left(2\bar{i} - \bar{j} + 2\bar{k}\right) + t\left(2\bar{i} + 2\bar{j} - \bar{k}\right) \Rightarrow (b)$ is correct

(OR) $\bar{r} = \frac{2}{9}\left(2\bar{i} - \bar{j} + 2\bar{k}\right) + t\left(2\bar{i} + 2\bar{j} - \bar{k}\right) \Rightarrow (b)$ is correct

Also mid –point of AB is $\left(\frac{2}{3}, O, \frac{1}{3}\right)$

$\therefore L_3$ can also be written as $\bar{r} = \frac{1}{3}\left(2\bar{i} + \bar{k}\right) + t\left(2\bar{i} + 2\bar{j} - \bar{k}\right), t \in R$

$\therefore (d)$ is correct

Clearly (0,0,0) does not lie on $\bar{r} = \frac{2}{9}\left(4\bar{i} + \bar{j} + \bar{k}\right) + t\left(2\bar{i} + 2\bar{j} - \bar{k}\right)$

$\therefore (c)$ is incorrect.

Eg: The line through $\bar{i} + 3\bar{j} + 2\bar{k}$ and perpendicular to the line $\bar{r} = \left(\bar{i} + 2\bar{j} - \bar{k}\right) + \lambda\left(2\bar{i} + \bar{j} + \bar{k}\right) and\, \bar{r} = \left(2\bar{i} + 6\bar{j} + \bar{k}\right) + \mu\left(\bar{i} + 2\bar{j} + 3\bar{k}\right)$ is a

1. $\bar{r} = \left(\bar{i} + 2\bar{j} - \bar{k}\right) + \lambda\left(-\bar{i} + 5\bar{j} - 3\bar{k}\right)$
2. $\bar{r} = \left(\bar{i} + 3\bar{j} + 2\bar{k}\right) + \lambda\left(\bar{i} - 5\bar{j} + 3\bar{k}\right)$
3. $\bar{r} = \left(\bar{i} + 3\bar{j} + 2\bar{k}\right) + \lambda\left(\bar{i} + 5\bar{j} + 3\bar{k}\right)$
4. $\bar{r} = \left(\bar{i} + 3\bar{j} + 2\bar{k}\right) + \lambda\left(-\bar{i} - 5\bar{j} - 3\bar{k}\right)$

Key: 2

Sol : $\bar{r} = \bar{a} + \lambda\bar{b}$ $\quad \bar{a} =$ lines on the line $= \bar{i} + 3\bar{j} + 2\bar{k}$ $\quad \bar{b}$ direction vector of line

Given that $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1} \Rightarrow \overline{r_1} = \left(\bar{i} + 2\bar{j} - \bar{k}\right) + \lambda\left(2\bar{i} + \bar{j} + \bar{k}\right)$

$\overline{r_2} = \overline{a_2} + \mu\overline{b_2} \Rightarrow \overline{r_2} = \left(2\bar{i} + 6\bar{j} + \bar{k}\right) + \mu\left(\bar{i} + 2\bar{j} + 3\bar{k}\right)$

$$\bar{b} = \overline{b_1} \times \overline{b_2} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \bar{i}(3-2) - \bar{j}(6-1) + \bar{k}(4-1) = \bar{i} - 5\bar{j} + 3\bar{k}$$

$\therefore \bar{r} = \left(\bar{i} + 3\bar{j} + 2\bar{k}\right) + \lambda\left(\bar{i} - 5\bar{j} + 3\bar{k}\right)$

03. From a point $P(\lambda, \lambda, \lambda)$, Perpendicular PQ and PR are drawn respectively on the lines $y = x, z = 1,$ and $y = -x\ z = -1.$ If P is such that $\angle$ QPR is a right angle, then the possible values (s) of λ is/are (ADV-2014)

1. $\sqrt{2}$ $\quad$ 2. 1 $\quad$ 3. -1 $\quad$ 4. $-\sqrt{2}$

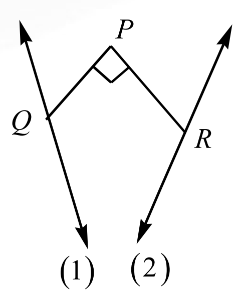
Key : 3

Sol : Given lines are $x = y, z = 1$

$L_1 : \frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{0} = \alpha \rightarrow (1)$

$\therefore Q(\alpha, \alpha, 1)$ and $y = -x, z = -1$

$L_2 : \frac{x-0}{-1} = \frac{y-0}{1} = \frac{z+1}{0} = \beta \rightarrow (2)$



$\therefore R(-\beta,\beta,-1)$ (say)

Direction ratios of PQ are $(\lambda-\alpha,\lambda-\alpha,\lambda-1)$

And Direction ratios of PR are $(\lambda+\beta,\lambda-\beta,\lambda+1)$

$\therefore PQ$ is perpendicular to L_1

$\therefore \lambda-\alpha=0 \Rightarrow \lambda=\alpha$

$\therefore PR$ is perpendicular to L_2

$\therefore -(\lambda+\beta)+\lambda-\beta=0 \Rightarrow \beta=0$

$\therefore$ dr's of PQ are $(0,0,\lambda+1)$

$\therefore$ dr'f of PR are $(\lambda,\lambda,\lambda+1)$

$\therefore \underline{|QPR}=90^0 \Rightarrow (\lambda-1)(\lambda+1)=0 \Rightarrow \lambda=1(or)-1$

But for $\lambda=1,$ we get point Q it self

$\therefore$ we take $\lambda=-1$

Eg : If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}\ z+4=0$ is such that $\sin\theta=\frac{1}{3}$ then the value(s) of λ

1. $\frac{-4}{3}$ 2. $\frac{3}{4}$ 3. $\frac{-3}{5}$ 4. $\frac{5}{3}$

Key : 4

Sol : $\sin\theta=\left|\frac{\bar{n}.\bar{b}}{|\bar{n}||\bar{b}|}\right|$ where $\bar{n}=2\bar{i}-\bar{j}+\sqrt{\lambda}\bar{k}$

$\bar{b}=\bar{i}+2\bar{j}+2\bar{k}$

$\Rightarrow \lambda=5/3$

04. A line L passing through the origin is perpendicular to the lines

$L_1:(3+t)\bar{i}+(-1+2t)\bar{j}+(4+2t)\bar{k}, \quad -\infty<t<\infty$

$L_2:(3+2s)\bar{i}+(3+2s)\bar{j}+(2+s)\bar{k}, \quad -\infty<t<\infty$

Then, the coordinates of the points on L_2 at a distance of $\sqrt{17}$ from the point of intersection of L and L_1 is (are) (ADV-2013)

1. $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ 2. $(-1,-1,0)$ 3. $(1,1,1)$ 4. $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

Key : 2,4

Sol : Given lines are L_1 & L_2

$\therefore$ Direction vector perpendicular to be L_1 *and* L_2

$$\bar{b} = L_1 \times L_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\bar{i} + 3\bar{j} - 2\bar{k}$$

$$\therefore L : \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \lambda$$

Any point on L_1 is $(t+3, 2t-1, 2t+4)$ and any point on L is $(2\lambda, -3\lambda, 2\lambda)$

$\therefore$ Let intersection point of L and L_1 be P.

$t+3 = 2\lambda, 2t-1 = -3\lambda, 2t+4 = 2\lambda$

$\Rightarrow t = -1, \lambda = 1 \therefore P(2, -, 3, 2)$

Any point Q on L_2 is $(2s+3, 2s+3, s+2)$

According to question $PQ = \sqrt{17}$

$\Rightarrow (2s+1)^2 + (2s+6)^2 + s^2 = 17$

$\Rightarrow 9s^2 + 28s + 20 = 0 \Rightarrow s = -2, \frac{-10}{9}$

$\therefore$ Point Q can be $(-1,-1,0)$ and $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

Eg : Let $L_1 : \bar{r} = (\bar{i} - \bar{j}) + t_1(2\bar{i} + 3\bar{j} + \bar{k})$

$L_2 : \bar{r} = (-\bar{i} + 2\bar{j} + 2\bar{k}) + t_2(5\bar{i} + \bar{j})$, then the distance of origin from the plane passing through the point $(1,-1,1)$ and whose normal is perpendicular to both L_1 *and* L_2 is

1. $\frac{19}{\sqrt{195}}$ 2. $\frac{20}{\sqrt{195}}$ 3. $\frac{18}{\sqrt{195}}$ 4. $\frac{17}{\sqrt{195}}$

Key: 1

Sol : GT $L_1 : \bar{r}_1 = \bar{a}_1 + t_1 \bar{b}_1,\ \bar{a}_1 = \bar{i} - \bar{j}, \bar{b}_1 = 2\bar{i} + 3\bar{j} + \bar{k}$

$L_2 : \bar{r}_2 = \bar{a}_2 + t_2\bar{b}_2,\ \bar{a}_2 = -\bar{i} + 2\bar{j} + 2\bar{k},\ \bar{b}_2 = 5\bar{i} + \bar{j}$

Normal $\bar{b} = \bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix} = -\bar{i} + 5\bar{j} - 13\bar{k}$

$\therefore$ Equation of the plane $-1(x-1) + 5(y+1) + (-13)(z-1) = 0$

$\Rightarrow -x + 5y - 13z = -19$

$\therefore$ Perpendicular distance from $(0,0,0)$

$$= \frac{|0+0-0+19|}{\sqrt{1+25+169}} = \frac{19}{\sqrt{195}}$$

<u>Sub topic</u>: Equation of a plane in Different forms, Equations of a plane passing through the intersection of two given planes. Projection of a line on a plane.

05. Let P_1 *and* P_2 be two planes given by $P_1: 10x+15y+12z-60=0, P_2: -2x+5y+4z-20=0$ Which of the following straight lines can be edge of some tetrahedron whose two faces lie on P_1 *and* P_2?

(ADV-2022)

1. $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}$ 2. $\frac{x-6}{-5}=\frac{y}{2}=\frac{z}{3}$ 3. $\frac{x}{-2}=\frac{y-4}{5}=\frac{z}{4}$ 4. $\frac{x}{1}=\frac{y-4}{-1}=\frac{z}{3}$

Key : 1,2

Sol : Thus, equation of pair of planes is $S:(10x+15y+12z-60)(-2x+5y+4z-20)=0$ Now we will obtain a general point of each line and we will solve it with S. If we get more than one value of variable λ, then the line can be the edge of given tetrahedron.

1. From option we have $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}=\lambda$, so, point is $(1,1,5\lambda+1)$

$\therefore (60\lambda-23)(20\lambda-17)=0 \Rightarrow \lambda=\frac{23}{60} \, and \, \frac{17}{20}$

So, it can be the edge of tetrahedron

2. Similarly for option (2) point is $(-5\lambda+6, 2\lambda, 3\lambda)$ So, $(16\lambda)(32\lambda-32)=0 \Rightarrow \lambda=0$ *and* 1
So, it can be the edge of tetrahedron

3. Similarly For option (3), Point is $(-2\lambda, 5\lambda+4, 4\lambda)$ *So*, $(103\lambda)(45\lambda)=0 \Rightarrow \lambda=0$
Only, so it cannot be the edge of tetrahedron

4. Similarly for option (4), Point is $(\lambda, -2\lambda+4, 3\lambda) \Rightarrow (16\lambda)(-2\lambda)=0 \Rightarrow \lambda=0$ only
Hence it cannot be the edge of tetrahedron.

Eg : If the planes $2x+6y+\lambda z+1=0$ and $3x+5y-\lambda z+4=0$ are perpendicular to each other. Then what are the value (s) of λ?

1. 4 2. -4 3. 6 4. -6

Key : 3,4

Sol : Use $a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 6+30-\lambda^2=0$

$\Rightarrow \lambda^2=36$

$\Rightarrow \lambda=\pm 6$

06. Let S be the reflection of a point Q with respect to the plane given by. $\bar{r}=-(t+p)\bar{i}+t\bar{j}+(1+p)\bar{k}$, where $t, p \in R$ and $\bar{i}, \bar{j}, \bar{k}$ are the unit vectors along the three positive coordinate axes. If the position vector of Q and S are $10\bar{i}+15\bar{j}+20\bar{k}$ *and* $\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k}$ respectively, then which of the following is/are true?

(ADV-2022)

1. $3(\alpha+\beta)=-101$ 2. $3(\beta+\lambda)=-71$

3. $3(\gamma+\alpha)=-86$ 4. $3(\alpha+\beta+\gamma)=-121$

Key : 1,2,3

Sol : We are given that equation of plane is $\bar{r} = -(t+p)\bar{i} + tj + (1+p)\bar{k}$

$\Rightarrow \bar{r} = \bar{k} + t\left(-\bar{i}+\bar{j}\right) + p\left(-\bar{i}+\bar{k}\right)$

Now, equation of the plane in standard form is

$\left[\bar{r}-\bar{k} \quad -\bar{i}+\bar{j} \quad -\bar{i}+\bar{k}\right] = 0 \Rightarrow x+y+z=1 \rightarrow (1)$

Given that $Q(10,15,20), S(\alpha,\beta,\gamma)$

From Point of reflection is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

$$\therefore \frac{\alpha-10}{1} = \frac{\beta-15}{1} = \frac{\gamma-20}{1} = \frac{-2(10+15+20-1)}{3}$$

$$\therefore \alpha = \frac{-58}{3}, \beta = \frac{-43}{3}, \gamma = \frac{-28}{3}$$

$\therefore 3(\alpha+\beta) = -101, 3(\beta+\gamma) = -71$

$3(\gamma+\alpha) = -86 \, and \, 3(\alpha+\beta+\gamma) = -129$

So, options (1),(2),(3) are correct and (4) is not correct

Eg : Image of the point P with position vector $7\bar{i}-\bar{j}+2\bar{k}$ is the line whose vector equation is $\bar{r} = \left(9\bar{i}+5\bar{j}+5\bar{k}\right) + \lambda\left(\bar{i}+3\bar{j}+5\bar{k}\right)$ has the position vector Q then Dr's of OQ is (are)

1. $(-9,5,2)$ 2. $(9,5,-2)$ 3. $(9,5,-2)$ 4. $(-9,-5,2)$

Key : 2,4

Sol : $\dfrac{x-9}{1} = \dfrac{y-5}{3} = \dfrac{z-5}{5}$

$\therefore P(7,-1,2), Q(a,b,c)$

D'rs of $PQ = (a-7, b+1, c-2)$

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0 \Rightarrow 1(a-7) + 3(b+1) + 5(c-2) = 0$

$\Rightarrow a + 3b + 5c - 7 + 3 - 10 = 0$

$\Rightarrow a + 3b + 5c = 14 \rightarrow (1)$

Mid point of PQ$\left(\dfrac{7+a}{2}, \dfrac{b-1}{2}, \dfrac{c+2}{2}\right)$

$\therefore x = t+9, y = 3t+5, z = 5t+5$

$$\therefore \frac{a+7}{2} = t+9 \Rightarrow a = 2t+11$$

$$\frac{b-1}{2} = 3t+5 \Rightarrow b = 6t+11$$

$$\frac{c+2}{2} = 5t+1 \Rightarrow c = 10t+8$$

$\therefore 2t + 11 + 18t + 33 + 50t + 40 = 14 \Rightarrow t = 1$

$\therefore a=9, b=5, c=-2$

$\therefore Q(9,5,-2)$

$\therefore Dr's$ of OQ are $(9,5,-2)(or)(-9,-5,2)$

07. Let $L_1\ and\ L_2$ be the following straight lines $L_1:\frac{x-1}{1}=\frac{y}{-1}=\frac{z-1}{3}\ and\ L_2:\frac{x-1}{-3}=\frac{y}{-1}=\frac{z-1}{1}$

Suppose $L:\frac{x-\alpha}{l}=\frac{y-1}{m}=\frac{z-\gamma}{-2}$ lies in the plane containing L_1 and L_2, and passes through the point of intersection of $L_1 \& L_2$. If the lien L bisects the acute angle between the lines $L_1\ and\ L_2$, then which of the following statements is/are True? (ADV – 2020)

1. $\alpha-\gamma=3$ 2. $l+m=2$ 3. $\alpha-\gamma=1$ 4. $l+m=0$

Key : 1,2

Sol : The point of intersection of $L_1\ and\ L_2$ is $(1,0,1)$ $\because$ Line L passes through the point of intersection $(1,0,1)$ of $L_1\ and\ L_2$

$$\therefore \frac{l-\alpha}{l}=\frac{-1}{m}=\frac{1-\gamma}{-2}\rightarrow(1)$$

$\therefore$ Line L bisects the acute angle between the lines $L_1\ and\ L_2$ then

$$\bar{r}=(\bar{i}+\bar{k})+\lambda\left(\frac{\bar{i}-\bar{j}+3\bar{k}-3\bar{i}-\bar{j}+\bar{k}}{\sqrt{11}}\right)$$

$$\bar{r}=(\bar{i}-\bar{k})+t(\bar{i}+\bar{j}-2\bar{k})\rightarrow(2)$$

$\therefore (1)\&(2)$ represents the same line

$$\therefore \frac{l}{1}=\frac{m}{1}=\frac{-2}{-2}\Rightarrow l=m=1$$

$$(1)\Rightarrow\frac{1-\alpha}{1}=-1\Rightarrow\alpha=2$$

And $\frac{1-\gamma}{-2}=-1\Rightarrow\gamma=-1$

$\therefore \alpha-\gamma=2-(-1)=3$

And $l+m=1+1=2$

Eg: If the lines $L_1:\frac{x-1}{3}=\frac{y-\lambda}{1}=\frac{z-3}{2}\ and\ L_3:\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ are coplanar, then the equation of the plane passing through the point of intersection of $L_1\ and\ L_2$ which is at a maximum distance from the origin is

1. $4x+3y+5z-50=0$ 2. $4x-3y+5z-50=0$

3. $4x-3y-5z-50=0$ 4. $4x+3y+5z+50=0$

Key : 1

Sol : $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0(coplanar)$

$\Rightarrow \begin{vmatrix} 2 & 1-\lambda & -1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 2$

$\therefore \frac{x-1}{3} = \frac{y-2}{1} = \frac{z-2}{3} = p \Rightarrow x = 3p+1, y = p+2, z = 2p+3$

& $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3} = q \Rightarrow x = q+3, y = 2q+1, z = 3q+2$

$\therefore 3p+1 = q+3$ & $p+2 = 2q+1$ & $2p+3 = 3q+2$

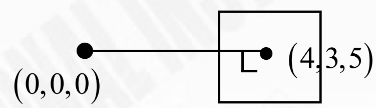
$3p-q = 2 \rightarrow (1)\ p-2q = -1 \rightarrow (2), 2p-2q = -1 \rightarrow (3)$

Solve (1) & (2) we get $P = 1 \,\&\, q = 1$

$x = 3(1)+1 = 4$

$y = 1+2 = 3$

$z = 2(1)+3 = 4$



For maximum distance point (4,3,5)

On the plane form origin

Dr's$(4,3,5)$

$\therefore ax+by+cz+d = 0$

$4x+3y+5z+d = 0$

$4(4)+3(3)+5(5)+d = 0 \Rightarrow d = -50$

$\therefore 4x+3y+5z-50 = 0$

08. Let α, β, γ be real numbers such that $\alpha^2+\beta^2+\gamma^2 \neq 0\, and\, \alpha+\gamma = 1$. Suppose the point $(3,2,-1)$ is the mirror image of the point $(1,0,-1)$ with respect to the plane $\alpha x+\beta y+\gamma z = \delta$. The which of the following statements is/are TRUE? (ADV-2020)

1. $\alpha+\beta = 2$ 2. $\delta-\gamma = 3$ 3. $\delta-\beta = 4$ 4. $\beta+\gamma+\delta = 8$

Key : 1,2,3

Sol : mid-point of $PQ = A(2,1,-1)$

$\therefore Dr's\, of\, PQ = (2,2,0)$

Since PQ perpendicular to the plane and mid-point lies on plane.

$\therefore$ Equation of plane

$2(x-2)+2(y-1)+0(z+1) = 0 \Rightarrow x+y-3 = 0 \Rightarrow x+y = 3$ Compare with $\alpha x+\beta y+\gamma z = \delta$

We get, $\alpha = 1,\ \beta = 1, \gamma = 0, \delta = 3$

$\therefore$ Options (1), (2), (3) are correct

Eg : Let p,q,r be real numbers such that $p^2+q^2+r^2 \neq 0$ and $\alpha+\gamma=0$. Suppose the point (1,2,3) is the mirror image the point $(3,2,1)$ with respect to the plane $px+qy+rz=d$. Then which of t he following statements is/are True?

1. $p+q=1$ 2. $p+q+r=0$ 3. $p+r=0$ 4. $p+q+r+d=0$

Key : 1,2,3,4

Sol : Midpoint of $AB=(2,2,2)$

$Dr's$ of $AB=(3-1,2-2,1-3)=(2,0,-2)$

$\therefore$ Equation of plane $2(x-2)+0(y-2)-2(z-2)=0$

$2x-4+0-2z+4=0 \Rightarrow x-z=0$

Options (1,2,3,4) are correct.

09. Let $P_1: 2x+y-z=3$ and $P_2: x+2y+z=2$ be two planes. Then, which of the following statements is (are) TRUE? (ADV-2018)

1. The line of intersection of P_1 and P_2 has direction rations $(1,2,-1)$
2. The line $\frac{3x-4}{9}=\frac{1-3y}{9}=\frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
3. The acute angle between P_1 and P_2 is 60^0
4. If P_3 is the plane passing through the point $(4,2,-2)$ and perpendicular to the line of intersection of P_1 and P_2 then the distance of the point $(2,1,1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$

Key : 3,4

1. Direction vector of line of intersection of two planes will be given by

$$\overline{n_1}\times\overline{n_2}=\begin{vmatrix} i & \bar{j} & \bar{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}=3\bar{i}-3\bar{j}+3\bar{k}$$

$\therefore$ Dr's of line of intersection of P_1 and P_2 are $1,-,1,1$

$\therefore$ (1) is not correct

2. The standard form of given line as $\frac{x-\frac{4}{3}}{3}=\frac{y-\frac{1}{3}}{-3}=\frac{z}{3}$

$\because 1(3)+(-1)(-3)+1(3)=9\neq 0$

$\therefore$ This line is not perpendicular to line of intersection

$\therefore$ (2) is not correct

3. $\cos\theta=\frac{|2(1)+1(2)+(-1)(1)|}{\sqrt{6}\sqrt{6}}=\frac{1}{2}\Rightarrow\theta=60^0$, Hence (3) is correct

4. Equation of plane $P_3: 1(x-4)-1(y-2)+1(z+2)=0 \Rightarrow x-y+z=0$

$\therefore$ Distance of $(2,1,1)$ from $P_3=\frac{|2-1+1|}{\sqrt{1+1+1}}=\frac{2}{\sqrt{3}}$

Hence (4) is correct.

Eg : Let $P_1 : x+y+2z=3$ and $P_2 : x-2y+z=4$ be two Planes. Let $A(2,4,5), B(4,3,8)$ be two points in space. The equation of plane P_3 through the line of intersection of P_1 *and* P_2 such that the length of the projection upon it of the line segment AB is the least.

1. $2x-y+3z=7$
2. $2x+y+3z=7$
3. The perpendicular distance from origin to the plane P_3 is $\sqrt{7/2}$
4. The Dr's of normal of the plane P_3 is $(2,-,1,3)$

Key : 1,3,4

Sol : Equation of plane P_3 is $P_3 = P_1 + \lambda P_2$

1. $x(1+\lambda)+y(1-2\lambda)+z(2+\lambda)=3+4\lambda$ Dr's of $AB=(2,-,1,3)$

Length of Projection of $\overline{AB}$ on plane $\Rightarrow AB$ must be perpendicular to the pale

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{1+\lambda}{2}=\frac{1-2\lambda}{-1}=\frac{2+\lambda}{3}\Rightarrow \lambda=1$$

$\therefore$ Equation of the plane $2x-y+3z=7$

3. Perpendicular distance from $(0,0,0)$ to plane $=\frac{7}{\sqrt{14}}=\sqrt{\frac{7}{2}}$

4. Dr's of the normal to P_3 is $(2,-1,3)$

10. consider a pyramid OPQRS located in the first octant $(x \ge 0, y \ge 0, z \ge 0)$ with "O" as origin, and OP and OR along the X-axis and Y-axis, respectively. The base OPQR of the pyramid is a square with $OP=3.$ The point S is directly above the mid-point, T of diagonal OQ such that $TS=3$ Then (ADV-2016)

1. The acute angle between OQ and OS is $\pi/3$
2. The equation of the plane containing of the triangle OQS is $x-y=O$
3. The length of the perpendicular from P to the plane containing the triangle OQS is $3/\sqrt{2}$
4. The perpendicular distance from 'O' to the straight line containing RS is $\frac{\sqrt{15}}{2}$

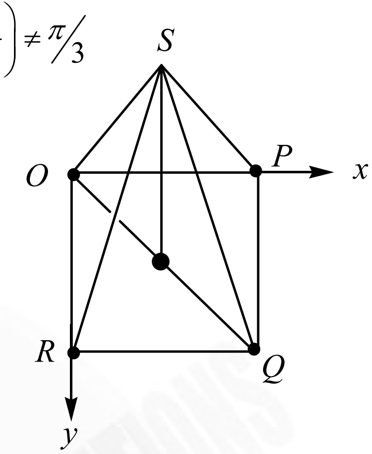
Key : 2,3,4

Sol : According to the given data, the vertices of pyramid OPQRS will be

$O(0,0,0), P(3,0,0), Q(3,3,0)\ R(0,3,0), S\left(\frac{3}{2},\frac{3}{2},3\right)$

Dr's of $OQ=(1,1,0)$, Dr's of OS$=(1,1,2)$

$\therefore$ Acute angle between OQ and OS $= \cos^{-1}\left(\frac{2}{\sqrt{2}\sqrt{6}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \neq \pi/3$



$\therefore$ (1) is not correct. Equation of plane

$$OQS = \begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = O \Rightarrow x - y = 0$$

$\therefore (2)$ is correct

Lengths of perpendicular from $P(3,0,0)$ to plane $x - y = 0$ is

$\frac{|3-0|}{\sqrt{2}} = \frac{3}{\sqrt{2}} \therefore (3)$ is correct

Equation of $RS: \frac{x}{3/2} = \frac{y-3}{-3/2} = \frac{z}{3} (or) \frac{x}{1} = \frac{y-3}{-1} = \frac{z}{2} = \lambda$, Any point on RS is $N(\lambda, -\lambda+3, 2\lambda)$

Since ON is perpendicular to RS

$$\because ON \perp RS \Rightarrow 1(\lambda) - 1(-\lambda+3) + 2(2\lambda) = 0 \Rightarrow \lambda = \frac{1}{2} \Rightarrow N\left(\frac{1}{2}, \frac{5}{2}, 1\right) \Rightarrow ON = \sqrt{\frac{15}{2}}$$

$\Rightarrow (4)$ is correct

Eg : Consider a pyramid PQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with P as the origin, and OP and OR along the x-axis and y-axis respectively. The base OPQR of the pyramid is a square with OP=4. The point S is directly above the midpoint of diagonal OQ such that $TS = 4$. Then which of the following is (are) TRUE?

1. The equation of the plane passing through P,Q and S is $x + y + z = 0$
2. The Dr's of normal to the plane passing through P,Q and S are $(1,1,1)$
3. Perpendicular distance from $(0,0,0)$ to plane $\overleftrightarrow{PQ}$ is $\frac{1}{\sqrt{3}}$
4. The plane $\overleftrightarrow{PQS}$ not passing through origin.

Key : 1,2

Sol : Diagonal $OQ = \sqrt{(OP)^2 + (PQ)^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

$\therefore$ The midpoint of OQ is at $(2,2,0)$ since $TS = 4,$ we can find the coordinates of S by adding 4 to the Z-coordinate of the midpoint of OQ, given us $S = (2,2,4)$

$Dr's\ \overline{PQ} = (4,0,0) - (0,4,0) = (4,-4,0)$

Dr's of $\overline{PS} = (2,2,4) = (2,2,4) - (0,4,0) = (2,-2,4)$

$\therefore \overline{PQ} \times \overline{PS} = (2,2,4) - (0,4,0) = (2,-2,4)$

$$\therefore \overline{PQ} \times \overline{PS} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 4 & -4 & 0 \\ 2 & -2 & 4 \end{vmatrix} = (16,16,16)$$

$\therefore$ Equation of the plane passing through P,Q,S is $16x+16y+16z=0 \Rightarrow x+y+z=0$

So, options (1) & (2) are correct

11. In R^3, Let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie (s) on M? (ADV-2015)

1.$\left(0,\frac{-5}{6}-\frac{-2}{3}\right)$ 2.$\left(\frac{-1}{6},\frac{-1}{3},\frac{1}{6}\right)$ 3.$\left(\frac{-5}{6},O,\frac{1}{6}\right)$ 4.$\left(\frac{-1}{3},O,\frac{2}{3}\right)$

Key : 1,2

Sol : $\therefore$ All the points on L are at a constant distance from P_1 and P_2 that means L is Parallel to

both P_1 and P_2.

$$\therefore \bar{b}=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \bar{i}-3\bar{j}-5\bar{k}$$

$$\therefore L: \frac{x}{1}=\frac{y}{-3}=\frac{z}{-5}=\lambda(say)$$

$\therefore$ Any point on line L is $(\lambda,-3\lambda,-5\lambda)$ Equation of line perpendicular to P_1 drawn from any point on L is $\frac{x-\lambda}{1}=\frac{y+3\lambda}{2}=\frac{z+5\lambda}{-1}=\mu(say)$

$\therefore M(\mu+\lambda, 2\mu-3\lambda, -\mu-5\lambda)$, But M lies on P_1. So, it satisfy the equation of P_1

$$\therefore \mu+\lambda+4\mu-6\lambda+\mu+5\lambda+1=0 \Rightarrow \mu=\frac{-1}{6}$$

$$\therefore M\left(\lambda-\frac{1}{6},-3\lambda-\frac{1}{3},-5\lambda+\frac{1}{6}\right)$$

For locus of M,

$$x=\lambda-\frac{1}{6}, y=-3\lambda-\frac{1}{3}, z=5\lambda+\frac{1}{6}$$

$$\Rightarrow \frac{x+\frac{1}{6}}{1}=\frac{y+\frac{1}{3}}{-3}=\frac{z-\frac{1}{6}}{+5}=\lambda$$

On checking the given point, we find $\left(0,\frac{-5}{6},\frac{-2}{3}\right)$ and $\left(\frac{-1}{6},\frac{-1}{3},\frac{1}{6}\right)$ satisfying the above equation.

Eg : In R^3, Let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and

$P_2: 2x - y + z - 1 = 0$. Let M be the locus of the feet the perpendiculars drawn from the points on L to the plane P_2. Then which of the Points (s) on M?

1. $\left(0, \frac{-5}{6}, \frac{11}{6}\right)$ 2. $\left(0, \frac{5}{6}, \frac{11}{6}\right)$ 3. $\left(\frac{4}{3}, \frac{-19}{6}, \frac{-29}{6}\right)$ 4. $\left(\frac{4}{3}, \frac{19}{6}, \frac{29}{6}\right)$

Key : 2,3

Sol : $\therefore \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \bar{i} - 3\bar{j} - 5\bar{k}$

$\therefore L: \frac{x}{1} = \frac{y}{-3} = \frac{z}{-5} = \lambda(say)$

Any point on the line L is $(\lambda, -3\lambda, -5\lambda)$

$\therefore$ Equation of the line perpendicular to P_2 drawn from any point on L is

$\frac{x-\lambda}{2} = \frac{y+3\lambda}{-1} = \frac{z+5\lambda}{1} = \mu(say)$

$\therefore M(2\mu + \lambda, -\mu - 3\lambda, \mu - 5\lambda)$

But M lies on P_2. So, it satisfy the equation P_2

$\therefore 2(2\mu + \lambda) - 1(-\mu - 3\lambda) + 1(\mu - 5\lambda) - 1 = 0$

$\Rightarrow 6\mu = 1 \Rightarrow \mu = 1/6 \therefore M\left(\frac{1}{3} + \lambda, \frac{-1}{6} - 3\lambda, \frac{1}{6} - 5\lambda\right)$

$\therefore$ For locus of M

$x = \frac{1}{3} + \lambda, y = \frac{-1}{6} - 3\lambda, z = \frac{1}{6} - 5\lambda$

$\therefore \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{6}}{-3} = \frac{z - \frac{1}{6}}{-5} = \lambda$

On checking $\left(O, \frac{5}{6}, \frac{11}{6}\right)$ *and* $\left(\frac{4}{3}, \frac{-19}{6}, \frac{-29}{6}\right)$

Satisfy the above equation

12. In R^3, consider the planes $P_1: y = 0$ *and* $P_2: x + z = 1$. Let P_3 be the plane, different from P_1 *and* P_2, which passes through the intersection of P_1 *and* P_2. If the distance of the point $(0,1,0)$ from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true? (Adv-2015)

1. $2\alpha + \beta + 2\gamma + 2 = 0$ 2. $2\alpha - \beta + 2\gamma + 4 = 0$
3. $2\alpha + \beta - 2\gamma - 10 = 0$ 4. $2\alpha - \beta + 2\gamma - 8 = 0$

Key : 2,4

Sol : $P_3: (x + z - 1) + \lambda y = 0 \Rightarrow x + \lambda y + z - 1 = 0$

Distance of point $(0,1,0)$ *from* P_3

$\frac{|\lambda-1|}{\sqrt{2+\lambda^2}}=1 \Rightarrow \lambda^2-2\lambda+1=\lambda^2+2 \Rightarrow \lambda=-\frac{1}{2}$

Distance of point (α,β,γ) from P_3:

$\frac{|\alpha+\lambda\beta+\gamma-1|}{\sqrt{2+\lambda^2}}=2 \Rightarrow \alpha-\frac{1}{2}\beta+\gamma-1=\pm 3$

$\Rightarrow 2\alpha-\beta+2\gamma-2=\pm 6$

$\Rightarrow 2\alpha-\beta+2\gamma-8=0(OR)\,2\alpha-\beta+2\gamma+4=0$

Hence (2) & (4) are correct.

Eg : Let two planes $P_1: 2x-y+z=2$ and $P_2: x+2y-z=3$ are given, the equation of the plane P_3 through the intersection of P_1 *and* P_2 and the point $(3,2,1)$, and the distance from (α,β,γ) to plane P_3 is $\sqrt{14}$, then which of the following is (are) TRUE?

1. $\alpha-3\beta+2\gamma-13=0$
2. $\alpha-3\beta+2\gamma+15=0$
3. $\alpha-3\beta+2\gamma-14=0$
4. $\alpha-3\beta+2\gamma+14=0$

Key : 1,2

Sol : Plane through the intersection of the plane $2x-y+z-2+k(x+2y-z-3)=0 \to (1)$

Plane passing through the point $(3,2,1) \therefore k=-1$

Put in equation $(1) \Rightarrow x-3y+2z+1=0 \to (2)$

GT distance from (α,β,γ) to plane (2) is $\sqrt{14}$

$\therefore \frac{|\alpha-3\beta+2\gamma+1|}{\sqrt{1+9+4}}=\sqrt{14}$

$\Rightarrow |\alpha-3\beta+2\gamma+1|=14$

$\Rightarrow \alpha-3\beta+2\gamma+1=\pm 14$

$\therefore \alpha-3\beta+2\gamma-13=0, \alpha-3\beta+2\gamma+15=0$

13. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ *and* $L_3: x=5, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take values(s) (Adv-2013)

1. 1
2. 2
3. 3
4. 4

Key : 1,4

Sol : $\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix}=0 \Rightarrow (5-\alpha)(6-5\alpha+\alpha^2-2)=0 \Rightarrow (5-\alpha)(\alpha-1)(\alpha-4)=0 \Rightarrow \alpha=1,4,5$

Hence options (1) & (4) are correct.

Eg : If for some $\alpha \in R$, the lines $L_1: \frac{x+1}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$ and $L_2: \frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, then what is (are) the value (s) of α

1. -4
2. 4
3. 5
4. -5

Key : 1

Sol : $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 2-1 & 1+2 & 1+1 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix}=0 \Rightarrow \alpha=-4$

14. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane (s) containing these two lines is (are) (Adv-2012)

1. $y+2z=-1$ 2. $y+z=-1$ 3. $y-z=-1$ 4. $y-2z=-1$

Key : 2,3

Sol : Given line are coplanar

$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix}=0 \Rightarrow k=\pm 2$

For $k=2,$ equation of the plane is given by $\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix}=0 \Rightarrow y-z+1=0$

For $k=-2,$ Equation of the plane is given by $\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix}=0 \Rightarrow y+z+1=0$

Hence options (2) & (3) are correct

Eg : If the straight lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ are coplanar, what is the plane containing then

1. $7x-14y+7z=0$ 2. $7x-14y+7z=14$

3. $\bar{r}.\left(7\bar{i}-14\bar{j}+7\bar{k}\right)=0$ 4. $\bar{r}.\left(7\bar{i}-14\bar{j}+7\bar{k}\right)=14$

Key : 1,3

Sol : $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{1}$

$A(-1,-3,-5),\ \overline{b_1}=(3,5,7)$

$\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$

$B(2,4,6),\ \overline{b_2}=(1,4,7)$

$\therefore$ Plane containing then $\Rightarrow \left(\bar{r}-\bar{a}\right).\bar{n}=0$ Where $\bar{n}=\overline{b_1}\times\overline{b_2}$

$\Rightarrow \bar{r}.\bar{n}-\bar{a}.\bar{n}=0$ $=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$

$\Rightarrow \bar{r}.\bar{n}=\bar{a}.\bar{n}$ $=7\bar{i}-14\bar{j}+7\bar{k}$

$\Rightarrow \left(r\bar{i}+y\bar{j}+z\bar{k}\right).\left(7\bar{i}+14\bar{j}+7\bar{k}\right)=0$ $\Rightarrow 7x-14y+7z=0$