Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS&STERLING_BT JEE-MAIN Date: 23-09-2023

Time: 09.00Am to 12.00Pm RPTM-08 Max. Marks: 300

KEY SHEET

PHYSICS

1)	2	2)	4	3)	3	4)	1	5)	4
6)	1	7)	4	8)	1	9)	1	10)	1
11)	1	12)	3	13)	4	14)	4	15)	1
16)	2	17)	3	18)	2	19)	1	20)	2
21)	1	22)	2	23)	8	24)	6	25)	8
26)	0	27)	20	28)	50	29)	5	30)	40

CHEMISTRY

31)	3	32)	3	33)	4	34)	4	35)	1
36)	4	37)	1	38)	4	39)	1	40)	1
41)	3	42)	3	43)	3	44)	3	45)	4
46)	1	47)	4	48)	2	49)	4	50)	2
51)	3	52)	3	53)	14	54)	4	55)	36
56)	8	57)	4	58)	12	59)	7	60)	6

MATHEMATICS

61)	1	62)	3	63)	2	64)	4	65)	2
66)	2	67)	4	68)	3	69)	3	70)	2
71)	1	72)	3	73)	2	74)	1	75)	3
76)	1	77)	3	78)	3	79)	1	80)	2
81)	15	82)	36	83)	4	84)	2	85)	2
86)	88	87)	73	88)	5	89)	3	90)	25

SOLUTIONS
PHYSICS

1. Friction force on an inclined plane, for a disc is

$f=\frac{1}{3}mg\sin\alpha$

2. In sphere P, the point of contact has tendency to move towards left w.r.t. surface and hence, friction acts on it towards right.

In sphere Q and R, the point of contact has tendency to move towards right w.r.t. surface and hence, friction acts on both of them towards left.

In sphere S, the point of contact has tendency to move towards left due to rotation and towards right due to translatory motion. It has not been specified in question whether v is less than or greater than ωR. Hence, friction on it may act towards left or right.

3. Initially there is no friction between cylinder and plank.

4. $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

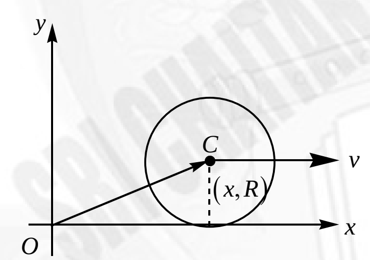
Differentiating w.r.t. t

$\frac{1}{v^2}\frac{dv}{dt}-\left(\frac{1}{u^2}\right)\frac{du}{dt}=0 \quad -\frac{1}{v^2}v_i+\frac{1}{u^2}v_0=0 \qquad v_i=\frac{v^2}{u^2}v_0$

When $f<u<2f, v$ lies beyond 2f $\quad \Rightarrow v>2f$

$\frac{v^2}{u^2}>1 \qquad v_i>V_0$

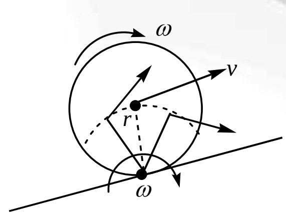
5. The angular momentum of the disc about O is



$\vec{L}_0=m\vec{r}_C\times\vec{v}_C+I_C\vec{\omega} \qquad =m\left(x\hat{i}+R\hat{j}\right)\times vi+\frac{1}{2}mR^2\left(\frac{3v}{2R}\hat{k}\right)$

$=m\left(x\hat{i}+R\hat{j}\right)\times vi+\frac{1}{2}mR^2\left(\frac{3v}{2R}\hat{k}\right)=mvR\left(\hat{j}\times\hat{i}\right)+\frac{3}{4}mvR\hat{k}=-\frac{mvR}{4}\hat{k}$

6.



Instantaneous axis of rotation

7. In all the cases, we observe that the horizontal component of the velocity of the food packet is same as the horizontal component of the velocity of the aeroplane and due to this, at all the instants, both have the same horizontal displacements.

8. External torque is zero; L=constant

9. We have, for an adiabatic process, in relation with temperature and volume

$TV^{\gamma-1} = constant$

Differentiating with respect to T.

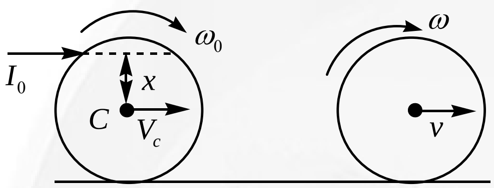
$V^{\gamma-1}.1 + T.(\gamma-1)V^{\gamma-2}\frac{dV}{dT} = 0$ $\qquad \frac{dV}{dT} = -\frac{V}{T(\gamma-1)}$

But from the graph

$\left|\frac{dV}{dT}\right|_{T_0,V_0} = \tan(\pi-\theta) = -\tan\theta$ $\qquad \frac{V_0}{T_0(\gamma-1)} = \tan\theta \Rightarrow \gamma - 1 = \frac{V_0}{T_0\tan\theta}$ $\qquad \gamma = \frac{V_0}{T_0\tan\theta} + 1$

10. Initially ball will gain both linear and angular velocity. Linear impulse: $I_0 = mv_C$

Angular impulse : $I_0 x = I\omega_0 \Rightarrow mv_C x = I\omega_0$



I is moment of inertia about C.

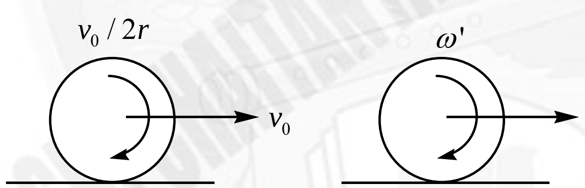
Apply conservation of angular momentum about lowest point.

$I\omega_0 + mv_C r = I\omega + mvr$

Where $\omega = \frac{v}{r}$ at the time of pure rolling

$\Rightarrow \qquad mv_C x + mv_C r = \frac{2}{5}mr^2\frac{v}{r} + mvr \qquad \Rightarrow \qquad v_C(x+r) = \frac{7}{5}rv \Rightarrow \qquad v = \frac{7}{5}v_c\left[\frac{x+r}{r}\right]\left(x = \frac{r}{2}\right)$

11.



$L_i = L_f \qquad I\omega + mv_0 r = I\omega' + mv'r$

12. Let v be the speed of B at lowermost position, the speed of A at lowermost position is 2v.

From conservation of energy $\qquad \frac{1}{2}m(2v)^2 + \frac{1}{2}mv^2 = mg(2a) + mga$

Solving we get $v = \sqrt{\frac{6}{5}ga}$

13. The total KE is $K = K_{plate} + K_{hallow\ sphere} + K_{solid\ sphere}$

Where $K_{plate} = \frac{1}{2}mv^2$

Since the CM of each sphere moves with a velocity $\qquad v_C = \frac{v}{2}$,

$K_{hallow\ sphere} = \frac{1}{2}mv_C^2\left(1 + \frac{K^2}{R^2}\right) \qquad = \frac{1}{2}m\left(\frac{v}{2}\right)^2\left(1+\frac{2}{3}\right) = \frac{5mv^2}{24}$

$K_{hallow\ sphere} = \frac{1}{2}m\left(\frac{v}{2}\right)^2\left(1+\frac{2}{5}\right) = \frac{7}{40}mv^2$

Using the above four equations, $\qquad K = \frac{1}{2}mv^2 + \frac{5}{24}mv^2 + \frac{7}{40}mv^2 = \frac{53}{60}mv^2$

14. Block will stop there if $kx = \mu mg \Rightarrow x = \dfrac{\mu mg}{k}$

$$\frac{mv^2}{2} = \frac{kx^2}{2} + \mu mg(x+1) \Rightarrow v = \sqrt{\frac{26}{5}} m/s$$

15. Let speed of block is v. Then from conservation of linear momentum in horizontal direction velocity of cylinder will by 2v in opposite direction $\left(as\ m = \dfrac{M}{2}\right)$.

Now from conservation of mechanical energy we have

$$mgh = \frac{1}{2}Mv^2 + \frac{1}{2}m(2v)^2$$

Here $h = R - r = 1.0m$

Substituting the values, we get,

$$(1)(10)(1) = \frac{1}{2}(2)(v^2) + \frac{1}{2}(1)(4v^2)$$ Or $3v^2 = 10 \therefore\ v = \sqrt{\dfrac{10}{3}} m/s$

16. Potential energy is defined for conservative force

17. $PV = \dfrac{m}{M}RT$ $\qquad V = \left(\dfrac{mR}{M}\right)\left(\dfrac{T}{P}\right) or V \propto \left(\dfrac{T}{P}\right)$

$\left(\dfrac{T}{P}\right)_A = \dfrac{T_0}{2P_0} and \left(\dfrac{T}{P}\right)_B = \dfrac{T_0}{2P_0}$ $\qquad \left(\dfrac{T}{P}\right)_C = \dfrac{3T_0}{2P_0} and \left(\dfrac{T}{P}\right)_D = T_0 / P_0$

Volume decreases from C to D

Density increases from C to D

18. All the particles on wave front in phase

19. $PV^{-a} = Cons\tan t$

Molar heat capacity in the process

$PV^x = Cons\tan t$, is $C = \dfrac{R}{r-1} + \dfrac{R}{(1-x)} = C_v + \dfrac{R}{1-(-a)} = C_v + \dfrac{R}{1+a}$

At the end of the process V_{rms} is $a^{1/2}$ times. Temperature has become 'a' time $\left(V_{rms} \propto T^{1/2}\right)$

$$\Delta Q = nC\Delta T = nCT(a-1) = nT(a-1)\left[C_v + \frac{R}{1+a}\right]$$

But given $\Delta Q = aPV$ $\qquad$ Or $aPV - \dfrac{(a-1)}{(a+1)}PV = n(a-1)C_v T$

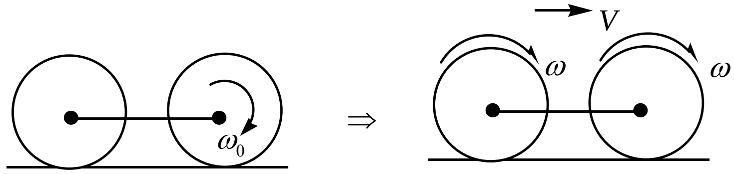
(Since, $PV = nRT$)

Solving $(a^2+1)PV = n(a^2-1)C_V T$ $\qquad$ Or $C_V = \dfrac{R(a^2+1)}{(a^2-1)}$ (insert a = 2)

20. $S = \dfrac{V_0^2}{2a} = \dfrac{V_0^2}{\dfrac{2.g\sin\theta}{1+\frac{2}{5}}} = \dfrac{7V_0^2}{10g\sin\theta}$

21. By law of conservation of angular momentum, we have

$L_i = L_f$ $\qquad$ about bottom most point



$\Rightarrow I\omega_0 = 2(I\omega + mR\upsilon)$ $\Rightarrow \left(\frac{1}{2}mR^2\right)\omega^0 = 2\left(\frac{1}{2}mR^2\omega + mR(\omega R)\right)$

$\Rightarrow \omega = \frac{\omega_0}{6}$ $\Rightarrow \upsilon = \omega R = \frac{\omega_0 R}{6}$

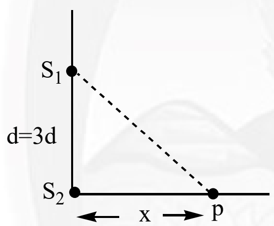
22. From (i) Stefan-Boltzmann law, $P = \sigma AT^4$ and (ii) Wein's displacement law

$= \lambda_m \times T = constant$ $\frac{P_A}{P_B} = \frac{A_A T_A^4}{A_B T_B^4} = \frac{A_A}{A_B} \times \frac{\lambda_B^4}{\lambda_A^4}$

$\therefore \frac{\lambda_A}{\lambda_B} = \left[\frac{A_A}{A_B} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{R_A^2}{R_B^2} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{400 \times 400}{10^4}\right]^{\frac{1}{4}}$ $\therefore \frac{\lambda_A}{\lambda_B} = 2$

23. $\frac{K.E_r}{E_{total}}$

24.



$\frac{5\lambda}{2} = S_1 p - S_2 p$

25. $PV^m = const.$ $\Rightarrow V^m dP + mv^{m-1} Pd\,v = 0$

$\Rightarrow \frac{dP}{dV} = \frac{-mP}{V} = \tan(180 - 53^0)$ $\Rightarrow \frac{4}{3} = m\frac{2x10^5}{4x10^5} \Rightarrow m = 8/3 = 3m = 8$

26. From conservation of energy K.E. + P.E. = E Or $K.E = E - \frac{1}{2}kx^2$

$\therefore K.E.\ at\ x = -\sqrt{\frac{2E}{k}}$ is $E - \frac{1}{2}k\left(\frac{2E}{k}\right) = 0$ $\therefore$ The speed of particle at $x = -\sqrt{\frac{2E}{k}}$ is zero

27. $0 = (50)(1)(2) - 200\omega$ $\omega = \frac{1}{2} rad/s$ $v_{rel} = 1 + 2\left(\frac{1}{2}\right) = 2$ $T = \frac{(2\pi)(2)}{2} = 2\pi s$

28. $(D - y) = -\frac{1}{2}gt^2$ $y - D = \frac{1}{2}gt^2$ $y = D + \frac{1}{2}gt^2$ $\Rightarrow \frac{dy}{dt} = \frac{1}{2}g2t = gt$ $= 50ms^{-1}$

29. $P = \vec{F}.\vec{v}$ $P = (ma)v$

$a = \frac{dv}{dt}$ $v = \frac{20}{1 + \frac{t}{20}}$ $1 + \frac{t}{20} = \frac{20}{v}$ $\frac{1}{20}\frac{dt}{dv} = \frac{20}{v^2}$

$a = \frac{v^2}{400}$ $v = 10m/s$ $a = -\frac{1}{4}m/s^2$ $P = 2 \times \frac{1}{4} \times 10 = 5Watt$

30. $a_c = \frac{v^2}{R} = 2m/s^2$ $a_A = \sqrt{(2a)^2 + \left(\frac{v^2}{R}\right)^2} = \sqrt{36 + 4} = \sqrt{40}m/s^2$ $\frac{a_C}{a_A} = \frac{2}{\sqrt{40}}$

CHEMISTRY

31. Chain propagation steps in methane

$(iii) Cl^{\bullet} + CH_4 \rightarrow CH_3^{\bullet} + HCl$ $(iv) CH_3^{\bullet} + Cl_2 \rightarrow CH_3Cl + Cl^{\bullet}$

32. Ozonolysis

33. $\left.\begin{array}{l} Icl_2^- : SP^3d; linear \\ Icl_4^-; SP^3d^2; square\ planar \end{array}\right\} similar\ structurs$

34. In toluene

(i) Methyl group converts in to acid group by oxidation of $KMnO_4$ in presences of acidic medium followed by nitration

(ii) Methyl group converts in to aldehyde group followed by nitration further oxidized by $KMnO_4$ converts in to acid group

35. Amino acids

36.

Polymers		Monomers
1) Urea-formaldhehyde resine	-	$NH_2-\overset{\overset{O}{\Vert}}{C}-NH_2$ & $H-\overset{\overset{O}{\Vert}}{C}-H$
2) Neoprene	-	$CH_2=\underset{\underset{Cl}{\vert}}{C}-CH=CH_2$
3) pvc	-	$CH_2=CH-Cl$
4) Nylon-6	-	Caprolactum

37. Drugs and their related activity dependence
38. Amines
39. Carboxylic acid group P^{k_a} values.
40. Classification of periodic table
41. Bond angles
42. Bond angles
43. Properties of hydrogen and its isotopes
44. Based on electron affinity
45. Enthalpy of bond dissociation $D_2 > H_2$
46. Chemical bonding
47. Density order :- $H_2O_2 > D_2O > H_2O$
48. $HOCl$ acts as oxidizing agent in presence of H_2O_2 in acidic medium.
49. H_2O_2 stored in plastic bottles and it is explosive nature in presence of sunlight and dust also
50. Chemical reaction can be shown as

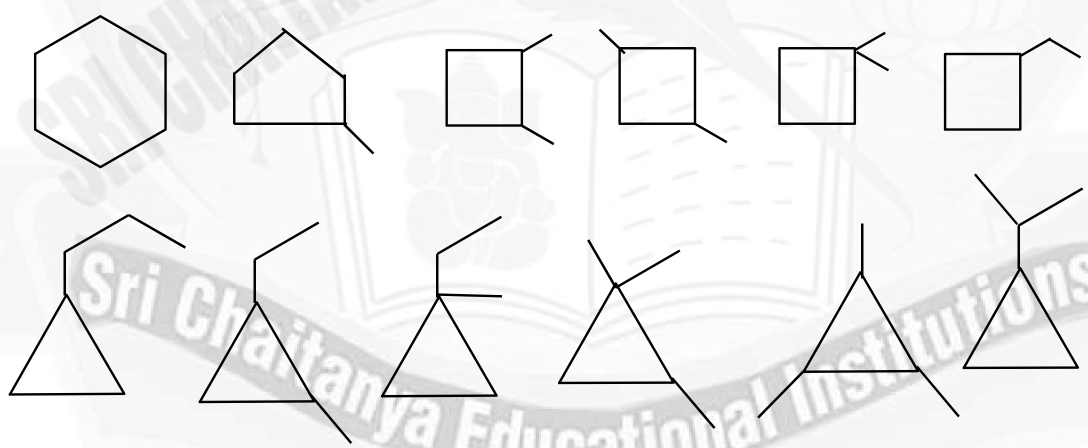
$$\underset{\text{Ethanal}}{CH_3CHO} \xrightarrow[(ii)H_2O]{(i)CH_3MgBr} \underset{\substack{\text{Propan}-2-ol}}{\underset{(A)}{\underset{CH_3}{\underset{|}{CH_3-CH-OH}}}} \xrightarrow[\text{Dehydration}]{H_2SO_4,\Delta} \underset{\text{Propene}}{\underset{(B)}{\underset{H}{\underset{|}{CH_3-C=CH_2}}}}$$

$$\xrightarrow[\text{Hydroboration Oxidation)}]{(i)BH_3;THF \quad (ii)H_2O_2/OH^{\ominus}} \underset{\text{Propan}-1-ol}{\underset{(C)}{CH_3-CH_2-CH_2OH}}$$

Thus, $\overset{\overset{CH_3}{|}}{CH_3CH}-OH$ and $CH_3-CH_2-CH_2OH$ are positional isomers

Hence, option (2) is correct

51. ClO_2, NO, NO_2 are odd electron species
52. Chemical bonding
53. Oxidations and reductions
54. Chemical properties of H_2O_2
55. $K_2S_2O_{8(s)} + 2D_2O_{(l)} \rightarrow 2KDSO_{4(aq)} + D_2O_{2(l)}$
56. $2^n = 2^3 = 8$
57. Nylone-2-Nylone-6; PHBV; Poly glycolic acid; Poly lactic acid
58.



59. $C_4H_{10}; C_6H_{14}; C_5H_{12}; CH_3-CH=CH_2; CH_2=CH_2; C_2H_6; C_3H_8$
60. 1, 2, 3, 4,6, 7

MATHEMATICS

61. $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & 2 & -\mu \\ 3 & -4 & 5 \end{vmatrix} = 0$

$\lambda(10-4\mu)-1(5+3\mu)+1(-4-6)=0$

$\lambda-\mu=2$

$(\mu+2)(10-4\mu)-5-3\mu-10=0$

$\mu=1, \mu=\frac{-5}{4}$

If $\mu=1$ $\quad\lambda=3$

If $\mu=\frac{-5}{4}$, $\quad\lambda=3/4$

$\sum_{(\lambda,\mu)\in S} 80(\lambda^2+\mu^2)=80\left(\frac{34}{16}+10\right)=970$

62. We have

$\vec{a}+\vec{b}=6\vec{p}-\vec{q}$

$\Rightarrow \quad |\vec{a}+\vec{b}|=\sqrt{(6\vec{p}-\vec{q})^2}=\sqrt{36\vec{p}^2+\vec{q}^2-12\vec{p}.\vec{q}}$

$\sqrt{36(8)+9-12(2\sqrt{2})(3)\cos\frac{\pi}{4}}=\sqrt{225}=5$

Similarly,

$|\vec{a}-\vec{b}|=|4\vec{p}+5\vec{q}|=\sqrt{(4\vec{p}+5\vec{q})^2}$

$=\sqrt{16\vec{p}2+25\vec{q}^2+40\vec{p}.\vec{q}}$

$=\sqrt{19(8)+25(9)+40(2\sqrt{2})(3)\cos\frac{\pi}{4}}=\sqrt{617}$

63. Required vector $\bar{c}$ is given by $\lambda\left(\frac{\bar{a}}{|\bar{a}|}+\frac{\bar{b}}{|\bar{b}|}\right)$

Now $\frac{\bar{a}}{|\bar{a}|}=\frac{1}{9}(7\hat{i}-4\hat{j}-4\hat{k})$ and $\frac{\bar{b}}{|\bar{b}|}=\frac{1}{3}(-2\hat{i}-\hat{j}+2\hat{k})$

$\bar{c}=\lambda\left(\frac{1}{9}\hat{i}-\frac{7}{9}\hat{j}+\frac{2}{9}\hat{k}\right)$ $\quad |\bar{c}|=\frac{|\lambda|}{9}\sqrt{54}\Rightarrow 7\sqrt{6}=\frac{|\lambda|}{9}3\sqrt{6}$ $\quad \frac{\lambda}{9}=\pm\frac{7}{3}$

64. Vectors $\bar{a},\bar{b},\bar{c}\ and\ \bar{d}$ are coplanar. Therefore $\sin\alpha+2\sin2\beta+3\sin3\gamma=1$

Now $|\sin\alpha+3\sin2\beta+4\sin3\gamma|$ $\leq\sqrt{1+9+16}\sqrt{\sin^2\alpha+\sin^2 2\beta+\sin^2 3\gamma}$

65. (P) The equation of plane ABC is $y+z-1=0$ Also, equation of line

$L=\frac{x-0}{0}=\frac{y-0}{1}=\frac{z-2}{1}=\lambda$ (say)

So, any point on line L is $(0,\lambda,\lambda+2)$

Put this point is equal of plane ABC, we get $\lambda=\frac{-1}{2}$.

$\therefore (x_0, y_0, z_0) \equiv \left(0, \frac{-1}{2}, \frac{3}{2}\right)$ $\Rightarrow (7x_0 + 2y_0 + 8z_0) = 11$

(Q) $\left[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}\right] = 25 \Rightarrow \left|\left[\vec{a}\vec{b}\vec{c}\right]\right| = 5$

$\therefore \left[\vec{a}+\vec{b}\ \vec{b}+\vec{c}\ \vec{c}+\vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right] = 2(5) = 10$

R) Let equation of plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, which meets axes at $P(a,0,0), Q(0,b,0), R(0,0,c)$.

The centroid of ΔPQR is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\therefore \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = \frac{\lambda}{9}$ … (1)

Also, $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{4}{3} = \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$ … (2)

$\therefore$ From (1) and (2) we get

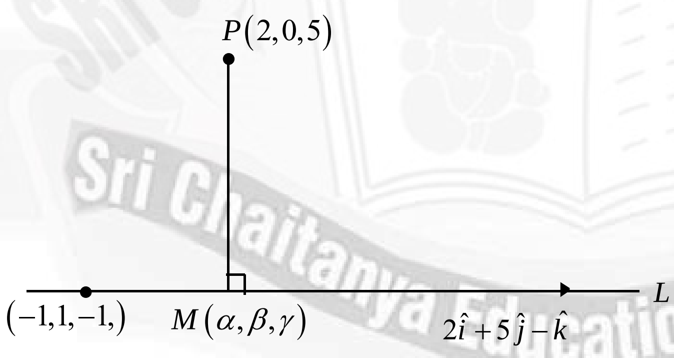
$\frac{4}{3} = \frac{\lambda}{9} \Rightarrow \lambda = 12$

(S) $\vec{\alpha}\times\vec{\beta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ -1 & -2 & -1 \end{vmatrix} = 3\hat{i} - 3\hat{j} + 3k \Rightarrow \left|\vec{\alpha}\times\vec{\beta}\right| = \left(3\sqrt{3}\right)$

$\therefore$ Area of parallelogram $= \frac{1}{2}\left|\sqrt{3}\vec{\alpha}\times 2\vec{\beta}\right|$

$= \sqrt{3}\left|\vec{\alpha}\times\vec{\beta}\right| = \sqrt{3}\left(3\sqrt{3}\right) = 9$

66. $L : \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+2}{-1} = \lambda$ (let)



Let foot of perpendicular be $M(2\lambda-1, 5\lambda+1, -\lambda-1)$.

Now, $\overrightarrow{PM} \perp \left(2\hat{i}+5\hat{j}-\hat{k}\right)$ $\therefore \overrightarrow{PM}.\left(2\hat{i}+5\hat{j}-\hat{k}\right)$

$\Rightarrow 2(3-2\lambda) - 5(5\lambda+1) - (6+\lambda) = 0$ $\Rightarrow \lambda = -\frac{1}{6} \Rightarrow (\alpha,\beta,\gamma) \equiv \left(-\frac{4}{3}, \frac{1}{6}, -\frac{5}{6}\right)$

67. The lines are $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}, \frac{x}{1} = \frac{y-1}{-1} = \frac{z}{0}$

$\vec{a}=0, \vec{c}=\hat{j}, \vec{b}=\hat{i}+\hat{j}+\hat{k}, \vec{d}=\hat{i}-\hat{j}$ $\quad \vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & -1 & 0\end{vmatrix}=\hat{i}+\hat{j}-2\hat{k}. \left|\vec{b}\times\vec{d}\right|=\sqrt{6}$

Shortest Distance $=\dfrac{(\vec{c}-\vec{a}).(\vec{b}\times\vec{d})}{\left|\vec{b}\times\vec{d}\right|}=\hat{j}.\dfrac{(\hat{i}+\hat{j}-2\hat{k})}{\sqrt{6}}=\dfrac{1}{\sqrt{6}}$

68. $I_1=\int_0^{\pi/4} e^t \sec t\,dt \qquad I_2=\int_0^{\pi/4} e^t \sec t \tan t\,dt$

$I+I_2=\int_0^{\pi/4} e^t(\sec t+\sec t\tan t)\,dt \qquad = e^t \sec t\Big|_0^{\pi/4}=\sqrt{2}e^{\pi/4}-1$

Remaining also same

69. Required plane is perpendicular to the planes $2x+4y+5z=8$

And $3x-2y+3z=5$ and passes through the point $(-2,3,5)$.

So, equation of plane is

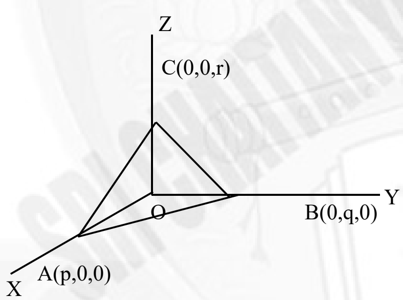
$$\begin{vmatrix}x+2 & y-3 & z-5\\ 2 & 4 & 5\\ 3 & -2 & 3\end{vmatrix}=0$$

Or $22x+9y-16z+97=0$

Comparing with $\alpha x+\beta y+\gamma x+97=0$, we get

$\alpha+\beta+\gamma=22+9-16=15$

70.



$\dfrac{x}{p}+\dfrac{y}{q}+\dfrac{z}{r}=1$

$\dfrac{a}{p}+\dfrac{b}{q}+\dfrac{c}{r}=1$

The equation of the planes through A, B, C planes parallel to the coordinate planes are

$x=p, y=q, z=r \qquad \Rightarrow \dfrac{a}{x}+\dfrac{b}{y}+\dfrac{c}{z}=1$

71. $f(g(h(1)))=f(g(3))=f(-g(-3))=f(-2)=1$

$g(h(f(3)))=g(h(-5))=g(-h(5)) \qquad =g(-1)=-g(1)=-1$

$h(f(g(-1)))=h(f(-g(1)))=h(f(-1)) \qquad =h(f(1))=h(0)$

As h is odd $\Rightarrow h(x)+h(-x)=0$

$h(0)+h(0)=0\Rightarrow h(0)=0$

72. As, $(x+\sin x - x\cos x - \tan x) = x(1-\cos x) + \sin x\left(1-\frac{1}{\cos x}\right)$

$= x(1-\cos x) - \tan x(1-\cos x) = (x-\tan x).(1-\cos x)$

So, $\lim\limits_{x\to 0}\frac{\left(\frac{x-\tan x}{x^3}\right)\left(\frac{1-\cos x}{x^2}\right)}{x^{n-5}} =$ exist and non-zero,

So, $n=5$

73. $g(5)=1 \, and \; f(1)=5$

Now, $g''(5)=$

$-\frac{f''(g(x))}{\left[f'(g(x))\right]^3} \Rightarrow g''(5) = -\frac{f''(g(5))}{\left[f'(g(5))\right]^3} = -\frac{f''(1)}{\left[f'(1)\right]^3}$

$f'(x) = 3x^2+3,\ f'(1)=6$

$f''(x) = 6x,\ f''(1)=6$ $\qquad \therefore g''(5) = \frac{-6}{216} = \frac{-1}{36}$

74. $1.2+2.3+3.4+...+n(n+1)$ $\qquad = \sum n^2 + n = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)(n+2)}{3}$

Let the Given problem be P

$\lim\limits_{n\to\infty}\frac{n(n+1)(n+2)}{3(n^3+2n^2+n)} \le P \le \lim\limits_{n\to\infty}\frac{n(n+1)(n+2)}{3(n^3+2n^2+1)}$

75. $\int (x^{23}+x^{15}+x^7)(2x^{24}+3x^{16}+6x^8)^{1/8}\,dx$

Put $2x^{24}+3x^{16}+6x^8 = t$ $\qquad \therefore \alpha = 54, \beta = 9, \gamma = 8$

76. $I = \int_0^{\pi} l\,n(1+\cos x)\,dx; I = \int_0^{\pi} l\,n(1-\cos x)\,dx$

$2I = \int_0^{\pi} l\,n(1-\cos^2 x)\,dx = \int_0^{\pi} l\,n(\sin^2 x)dx$ $\qquad 2I = 2\int_0^{\pi} l\,n(\sin x)\,dx$

$I = \int_0^{\pi} l\,n(\sin x)dx = 2\int_0^{\pi/2} l\,n(\sin x)\,dx = -2\left(\frac{\pi}{2}\right)l\,n\,2 = -\pi l\,n\,2.$

77. $y^2 = 2x,\ y = x$

By solving $y=0,2, x=0,2$

78. $f'(x) = (x^2-x+2)(x^2-x-2)(x^2-x-6)(x^2-x-12)$

$= (x^2-x+2)(x+1)(x-2)(x+2)(x-3)(x+3)(x-4)$

-3	-2	-1	2	3	4
max	min	max	min	max	min

$f(x)$ has maximum at $x=-3,-1,3$

Sum of values of $x = -3-1+3 = -1$

79. Given equation is, $\frac{dy}{dx}+\frac{y}{x\log_e x}=x, x>1$

This is linear differential equation,

I.F.= $e^{\int\frac{1}{x\log_e x}dx}=\log_e x$

Therefore, the solution is

$y\log_e x=\int x\log_e x dx+C$

Or $y\log_e x=\frac{x^2}{2}\log_e x-\frac{x^2}{4}+C$

Given that $y(2)=2$

$\therefore C=1$

So, $y(e)=\frac{e^2}{2}-\frac{e^2}{4}+1=1+\frac{e^2}{4}$

80. $I=\int_1^2\frac{1+(u-1)^{2009}}{u^{2011}}du\left(where\ 1+x^4=u\right)$ $=\int_1^2 u^{-2011}du+\int_1^2\left(1-\frac{1}{u}\right)^{2009}.\frac{1}{u^2}du$

$=\left.\frac{u^{-2010}}{2010}\right|_1^2+\int_1^{\frac{1}{2}}t^{2009}dt\left(where\ 1-\frac{1}{u}=t\right)$ $=\left.\frac{u^{-2010}}{-2010}\right|_1^2+\left.\frac{t^{2010}}{2010}\right|_1^{1/2}=\frac{-1}{2010}\left[\frac{1}{2^{2010}}-1\right]+\frac{1}{2010}\left[\frac{1}{2^{2010}}-0\right]=\frac{1}{2010}$

81. $\lim_{x\to1^-}f\left(g(x)\right)=5$

$\Rightarrow 15-g=5\Rightarrow g=10$

$\lim_{x\to1^+}f\left(g(x)\right)=5\Rightarrow 2p=5$

82. $x=6\cos^3\theta, y=6\sin^3\theta$

$\frac{dy}{dx}=-\tan\theta$

$T: x\sin\theta+y\cos\theta=6\sin\theta\cos\theta$

$N: x\cos\theta-y\sin\theta=6\cos2\theta$

$P_1=6\sin\theta\cos\theta\Rightarrow 2P_1=6\sin2\theta$

$P_2=6\cos2\theta$

$4P_1^2+P_2^2=36$

83. $f(x)=\int\frac{5x^8+7x^6}{x^{14}\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)^2}dx=\int\frac{\left(\frac{5}{x^6}+\frac{7}{x^8}\right)dx}{\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)2}$ $f(x)=\frac{x^7}{x^2+1+2x^7}+C$

$f(0)=0$ $\Rightarrow C=0$ $f(1)=\frac{1}{4}$ $k=4$

84. Both functions are periodic with period 1

Hence area $=10\int_0^1\left(\sqrt{x}-x^2\right)dx=10\left(\frac{x^{3/2}}{3/2}-\frac{x^3}{3}\right)_0^1=\frac{10}{3}$

85. $P=\frac{x}{1+x^2}$ $Q=\frac{x}{1+x^2}$

I.F= $\sqrt{1+x^2}$

$y\sqrt{1+x^2}=\int\frac{x}{\sqrt{1+x^2}}dx+c$

$y\sqrt{1+x^2}=\sqrt{1+x^2}+c$

$f(0)=\frac{4}{3}$ $\frac{4}{3}=1+c\Rightarrow c=\frac{1}{3}$ $y=1+\frac{1}{3\sqrt{1+x^2}}$

$f\left(\sqrt{8}\right)+\frac{8}{9}=1+\frac{1}{9}+\frac{8}{9}=2$

86. Taking 'O' as the origin, let the p.v's of A, B and C be $\bar{a},\bar{b},\bar{c}$ respectively. Then the position vectors of G_1,G_2,G_3 are $\frac{\bar{b}+\bar{c}}{3},\frac{\bar{c}+\bar{a}}{3},\frac{\bar{a}+\bar{b}}{3}$

$V_1=\frac{1}{6}\left[\bar{a}\bar{b}\bar{c}\right] \quad V_2\left[\overline{OG_1}\,\overline{OG_2}\,\overline{OG_3}\right]$

$V_2=\frac{1}{27}\left[\bar{b}+\bar{c}\ \ \bar{c}+\bar{a}\ \ \bar{a}+\bar{b}\right]=\frac{2}{27}\left[\bar{a}\bar{b}\bar{c}\right]$

$V_2=\frac{2}{27}6V_1\Rightarrow\frac{198V_2}{V_1}=\frac{198\times2\times6}{27}=88$

87. $\left|\vec{b}\times\vec{c}\right|=2\Rightarrow\sin\theta=\frac{1}{2}$ $\qquad\theta=30^o$

Now $2\bar{b}-\bar{c}=\lambda\bar{a}\Rightarrow\left|2\bar{b}-\bar{c}\right|^2=\lambda^2\left|\bar{a}\right|^2$ $\qquad\Rightarrow4\left|\vec{b}\right|^2+\left|\vec{c}\right|^2-4\bar{b}.\bar{c}=\lambda^2$

$\lambda^2=65-8\sqrt{3}\Rightarrow\lambda=\sqrt{65-8\sqrt{3}}$

88. $P=\left(x_1,y_1\right),Q=\left(x_2,y_2\right)$ $\qquad f\left(x\right)=x^7-2x^5+5x^3+8x+5$

$x_1=2,\ \ x_2=-2$

$y_1=f\left(2\right)\qquad y_2=f\left(-2\right)$

$\overrightarrow{OP}+\overrightarrow{OQ}=\left(x_1+x_2\right)\bar{i}+\left(y_1+y_2\right)\bar{j}=\left(f\left(2\right)+f\left(-2\right)\right)\bar{j}$

$=10\bar{j}$

$2M=\left|\overrightarrow{OP}+\overrightarrow{OQ}\right|=10\Rightarrow M=5$

89. Equation of the plane P_1 id

$\begin{vmatrix}x-2 & y-3 & z-4\\2 & 1 & -3\\-1 & 2 & 4\end{vmatrix}=0$ $\qquad\Rightarrow2x-y+z=3$

$P_2:2x-y+z=21$ $\qquad K\sqrt{6}=\frac{18}{\sqrt{6}}=3\sqrt{6}\Rightarrow K=3$

90. Given $\bar{r}=\left(\vec{a}\times\vec{b}\right)\sin x+\left(\vec{b}\times\vec{c}\right)\cos y+2\left(\vec{c}\times\vec{a}\right)$ and $\bar{r}.\left(\bar{a}+\bar{b}+\bar{c}\right)=0$

$\Rightarrow\left[\bar{a}\ \bar{b}\ \bar{c}\right]\left(\sin x+\cos y+2\right)=0$

But $\Rightarrow\left[\bar{a}\ \bar{b}\ \bar{c}\right]\neq0,\ so\ \ \sin x+\cos y=-2$

Which is possible when $\sin x=-1,\cos y=-1$

For $\left(x^2+y^2\right)$ to be minimum $x^2=\frac{\pi^2}{4},y^2=\pi^2$

Hence, the minimum value of $\frac{20}{\pi^2}\left(x^2+y^2\right)=\frac{20}{\pi^2}\left(\frac{5\pi^2}{4}\right)=25$