Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60 | 2020_P2 | Date: 18-09-22

Time: 02.30Pm to 05.30Pm | RPTA-02 | Max. Marks: 198

KEY SHEET

PHYSICS

1	**5**	2	**3**	3	**2**	4	**5**	5	**5**	6	**4**
7	**AD**	8	**AB**	9	**B**	10	**B**	11	**AB**	12	**ABD**
13	**40**	14	**2.5**	15	**7.5**	16	**1**	17	**1.5**	18	**1.4**

CHEMISTRY

19	**4**	20	**2**	21	**8**	22	**4**	23	**2**	24	**8**
25	**ABD**	26	**AB**	27	**ABC**	28	**BCD**	29	**ABC**	30	**ABCD**
31	**68.85**	32	**64**	33	**1.5 – 1.6**	34	**92**	35	**151.1**	36	**84**

MATHEMATICS

37	**4**	38	**3**	39	**9**	40	**5**	41	**3**	42	**3**
43	**BC**	44	**BC**	45	**AC**	46	**BC**	47	**AD**	48	**C**
49	**103**	50	**2620.44**	51	**2019**	52	**0**	53	**2**	54	**0**

SOLUTIONS

PHYSICS

1.
$$\Delta Q_{AB}=nC_p\Delta T=\frac{\gamma}{\gamma-1}nR\Delta T=\frac{\gamma}{\gamma-1}[3P_0V_0-P_0V_0]\qquad =2PV_0\times\frac{\gamma}{\gamma-1}$$
$$\Delta Q_{AC}=\Delta U+\Delta w\quad \frac{nR}{\gamma-1}\Delta T+\frac{1}{2}\times 3V_0\left[P_0+4P_0\right]$$
$$\frac{[16P_0V_0-P_0V_0]}{\gamma-1}+\frac{15P_0V_0}{2}\ 56=2P_0V_0\times\frac{\gamma}{\gamma-1}$$
$$360=15P_0V_0\left[\frac{\gamma-1}{2(\gamma-1)}\right]\ \frac{360}{56}=\frac{15}{4}\frac{(\gamma-1)}{\gamma}\quad 12\gamma=7\gamma+7\ \gamma=\frac{7}{5}=1+\frac{2}{f}\ f=5$$
$$\Delta Q_{AB}=nC_p\Delta T=\frac{\gamma}{\gamma-1}nR\Delta T=\frac{\gamma}{\gamma-1}[3P_0V_0-P_0V_0]\qquad =2PV_0\times\frac{\gamma}{\gamma-1}$$
$$\Delta Q_{AC}=\Delta U+\Delta w\quad \frac{nR}{\gamma-1}\Delta T+\frac{1}{2}\times 3V_0\left[P_0+4P_0\right]$$
$$\frac{[16P_0V_0-P_0V_0]}{\gamma-1}+\frac{15P_0V_0}{2},56=2P_0V_0\times\frac{\gamma}{\gamma-1}$$
$$360=15P_0V_0\left[\frac{\gamma-1}{2(\gamma-1)}\right],\frac{360}{56}=\frac{15}{4}\frac{(\gamma-1)}{\gamma},12\gamma=7\gamma+7,\gamma=\frac{7}{5}=1+\frac{2}{f},f=5$$

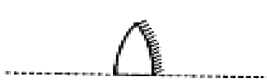
2.
$$8000=\sigma A(2000)^4\ ....(i)$$
$$500=\sigma AT^4.................(ii)$$
$$From\ (i)\ and\ (ii)$$
$$16=\left(\frac{2000}{T}\right)^4$$
$$T=1000K$$
$$\lambda_1T_1=\lambda_2T_2$$
$$\lambda_2=\frac{(1.5\mu m)\times(2000)}{1000}=3\mu m$$

3.

	P	V	T
1	P_0	V_0	T_0
2	αP_0	αV_0	α^2T_0
3	P_0	α^2V_0	α^2T_0
4	$\frac{P_0}{\alpha}$	αV_0	T_0

$$1\to 2;\ PV=nRt,\qquad P=mv\Rightarrow\ mv^2=nRT$$
$$T\alpha V^2\quad \frac{T}{T_0}=\left(\frac{\alpha\omega_0}{v_0}\right)3\to 4;\ T\alpha v^2\Rightarrow\frac{\alpha^2T_0}{T_0}=\frac{v_3^2}{(\alpha v_0)^2}$$

4.



Distance of image of object O from plane mirror= $a+b$. Since, there is no parallax between the images formed by the silvered lens L and plane mirror M, therefore

, two images are formed at the same point .Distance of image $=(a+2b)$ behind lens.

Since , length of the image formed by a plane mirror is always equal to length of the object , therefore, transverse magnification produced by the lens L is equal to 2.Since, distance of object from L is a, therefore, distance of image from L must be equal to 2a.$\therefore (a+2b)=2a \Rightarrow b=\frac{a}{2}$

The silvered lens L may be assumed as a combination of an equi-convex lens and a concave mirror placed in contact with each other co-axially as shown in figure.

Focal length of lens f_1 is given by

$$\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right), f_1=40cm$$

For concave focal length $f_m=\frac{R}{2}=-20cm$

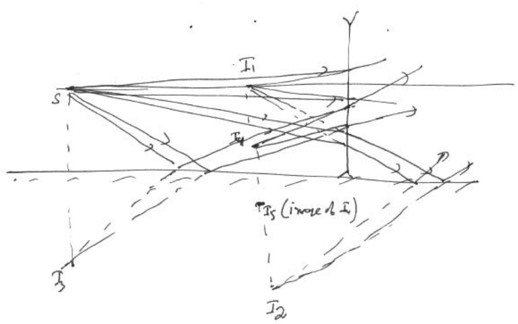
The combination L behaves like a mirror whose equivalent focal length F is given by

$$\frac{1}{F}=\frac{1}{f_m}-\frac{2}{f_1} \Rightarrow F=-10cm$$

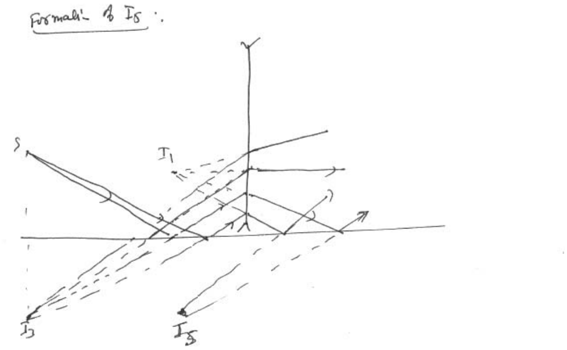
Hence, for the combination $u=-a, v=+2a, F=-10cm$

Using mirror formula , $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \Rightarrow a=5cm$

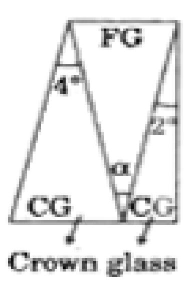
5.



Formation of I_5 :



6.



$\mu_V = 1.51$

$\mu_R = 1.49$

$FG\ \mu_V = 1.77$

$\mu_R = 1.73$

$\theta = \theta_1 - \theta_2 + \theta_3 = 0$

$= (\mu_y - 1)4 - (\mu_y - 1)\alpha + (\mu_y - 1)2 = 0$

$= (1.5 - 1)4 - (1.75 - 1)\alpha + (1.5 - 1)2 = 0$

$\alpha = 4^0$

$\delta_m = \delta_1 - \delta_2 + \delta_3$

$= (\mu_V - \mu_R)4 - (\mu_V - \mu_R)\alpha + (\mu_V - \mu_e)2$

$= 0.02 \times 4 - 0.04 \times 4 + 0.02 \times 2 = .0.04^0$

Magnitude is 0.04^0 but (-)-ve sign indicate spectrum inversed.

$\Rightarrow$Top colour is violet and bottom is red.

7. Point A and C are on the same line passing through origin $\Rightarrow \frac{P_A}{V_A} = \frac{P_C}{V_C}$............$(i)$

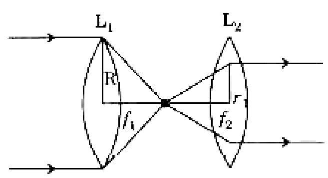
Also $T_A = 200K = \frac{P_A V_A}{nR}$ and Also $T_C = 1800K = \frac{P_C V_C}{nR} \Rightarrow \frac{P_A V_A}{P_C V_C} = \frac{1}{9}$.............$(ii)$

From (i) and (ii) $\frac{V_A}{V_C} = \frac{1}{3}$

8. $P = \sigma a A T_0^4$ When body reaches at T_0 even then it absorbs radiation.

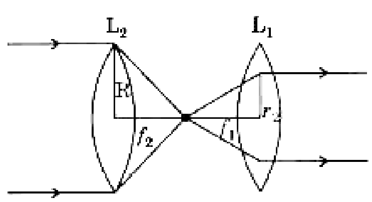
9. Conceptual

10. Case-I $r_1 = \frac{Rf_2}{f_1}$



$I\alpha$ (Aperture) $I\alpha\left(\frac{f_2}{f_1}\right)^2 \Rightarrow I_1 \alpha \frac{1}{k^4}$

Case-II

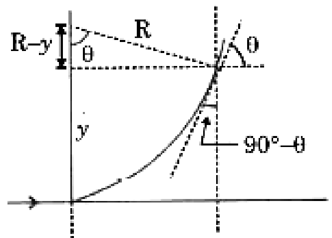


$r_2 = \frac{Rf_1}{f_2},\ I_2 \alpha \left(\frac{f_1}{f_2}\right)^4,\ I_2 \alpha K^4$

11. $\mu_1 \sin r_1 = \sin i_1 \Rightarrow \frac{\mu_1}{\mu_2} \sin i_2 = \sin r_2$

$\frac{\mu_1}{\mu_2} \frac{R_1}{R_2} \frac{\sin i_1}{\mu_1} = \sin r_2,\ r_2 = \sin^{-1}\left(\frac{R_1}{\mu_2 R_2} \sin i_1\right)$

12.



$(a) 1.5+\frac{12.4}{50}\times 0.5=1.624mm \qquad =24\times\frac{0.5}{4.5}=\frac{8}{3}$

$Dis\tan ce=72-\frac{8}{3}=69.33$

$(b)\ NS=24\left(1-\frac{1}{15}\right)+24\left(1-\frac{1}{4/3}\right)$

$=8+6=14$

$Dis\tan ce=72-14=58cm$

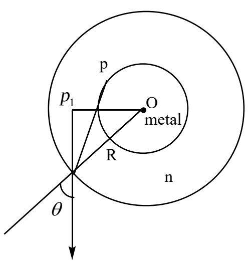
$(c)\frac{d_{pp}}{4/3}=\frac{24}{1}+\frac{24}{5}=\frac{160}{3},\ Dis\tan ce=24+\frac{160}{3}=77.33cm$

13. We can extract maximum work of the engine is reversible or cannot.

$$\frac{Q_A}{Q_B}=\frac{d_B}{T_B}=\frac{-m_A S_A dT_A}{T_A}=+\frac{m_A s_B dT_B}{T_B}\Rightarrow -\int_{T_A}^{T_0}\frac{dT_A}{T_A}=\int_{T_B}^{T_0}\frac{dT_B}{T_B}\Rightarrow T_0=\sqrt{T_A T_B}$$

14. The ray coming from P forms the image at P_1. The OP_1 is the radius of the cylinder an observed from outside. $n\sin i=\sin\theta n\frac{r}{R}=\frac{OP_1}{R}\Rightarrow OP=nr=3.75cm$

The apparent insane in the distance $=2(OP_1-OP)=2(3.75-2.5)cm=2.5$



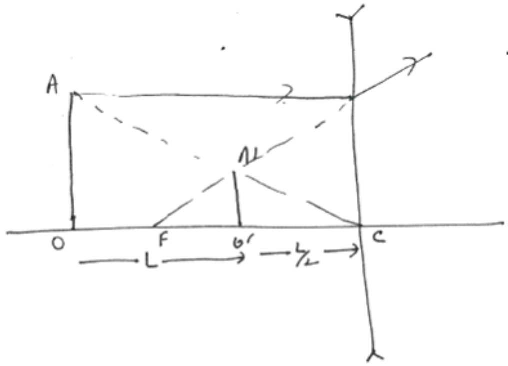
15. $\frac{O^1C}{A^1O^1}=\frac{OC}{OA}$

$\Rightarrow O^1C=OC.\frac{A^1O^1}{OA}=\frac{OC}{3}$

$O^1C=\frac{10+O^1C}{3}\Rightarrow 30^1C=10+O^1C=O^1C=\frac{1}{2}=5cm$

With similar argument $FO^1=\frac{2}{4}$

The $FC=\frac{2}{4}+\frac{2}{2}=\frac{32}{4}=\frac{3\times 10}{4}=7.5cm$



16. $\frac{1}{V}+\frac{1}{mf}=\frac{1}{-f}\quad \frac{1}{V}=-\left(\frac{1}{mf}+1\right)\frac{1}{f}\qquad V=-\frac{mf}{1+m}\ \left|\frac{h_2}{h_1}\right|=\frac{mf}{(1+m)mf}=\frac{1}{m+n}$ n = 1

17. In first case, $\frac{1}{f_{eq}}=\frac{1}{f_m}-\frac{2}{f_L}=\frac{1}{50}-\frac{2}{50}=-\frac{1}{50}$

Thus object must be placed to 100 cm from the system. Now when liquid is used.

$$\frac{1}{f_2}=(\mu-1)\left[-\frac{1}{50}+\frac{1}{100}\right]=(\mu-1)\left[-\frac{1}{100}\right]$$

$$\therefore \frac{1}{f_{eq}}=\frac{1}{f_m}-\frac{2}{f_1}-\frac{2}{f_2}=\frac{1}{50}-\frac{2}{50}+2(\mu-1)\left(\frac{1}{100}\right)$$

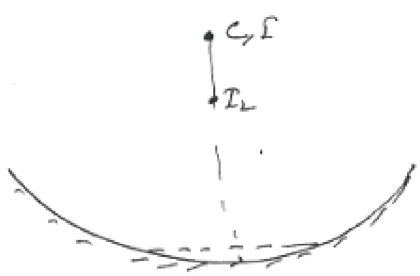
For the image to be formed at ∞, object must be at focus and system must act as concave mirror. $\therefore -\frac{1}{50}+2(\mu-1)\left[\frac{1}{100}\right]=-\frac{1}{100}\Rightarrow 2(\mu-1)\frac{1}{100}=\frac{1}{100}\quad \mu=\frac{3}{2}$

18. The object is at centre of curvature so in first image is formed an itself. This image is contributed by the mirror region in which water is not present. The thick mirror formed due to water in middle has focal length f.

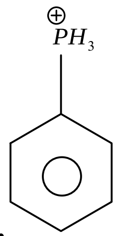
$$\frac{1}{f}=\frac{2}{fw}+\frac{1}{fm}=\frac{2(\mu-1)}{R}+\frac{2}{R}=\frac{2\mu}{R}$$

The image is formed $\frac{1}{v}+\frac{1}{R}=\frac{2\mu}{R}\rightarrow\frac{1}{2}=\frac{2\mu-1}{R}\Rightarrow v=\frac{R}{2\mu-1}$

$$v=\frac{R}{2\mu-1}=R-\ell\qquad \frac{9}{2\mu-1}=9-4=5\Rightarrow 9=10\mu-5\Rightarrow \mu=\frac{14}{10}=1.4$$



CHEMISTRY

19. $\overset{\ominus}{C}Cl_3$,  , $\ddot{P}F_3$, $Cl-\ddot{\underset{\cdot\cdot}{O}}-Cl$

20. $(I,II,V)=S \quad (III,IV)=R$

21. $CaC_2 \xrightarrow{H_2O} CH\equiv CH \xrightarrow{NaNH_2} Na-C\equiv C-Na \xrightarrow{CH_3I} CH_3-C\underset{C}{\equiv}C-CH_3$

$C_4H_6+\dfrac{11}{2}O_2 \rightarrow 4CO_2+3H_2O$ 1 mole of C_4H_6 *requires* $\dfrac{11}{2}$ *moles of* O_2

Mass of $O_2=\dfrac{11}{2}\times 32=x \quad \dfrac{x}{22}=\dfrac{11\times 16}{22}=8$

22. II,III,IV,V

23. $\alpha-$position

24. $FC\equiv C-\underset{*}{C}BrICl(d+l), ClC\equiv C-\underset{*}{C}BrIF,(d+l)$

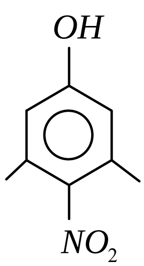
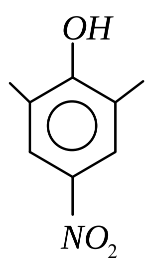
$BrC\equiv C-\underset{*}{C}ClBrI(d+l), IC\equiv C-\underset{*}{C}FClBr(d+l)$

25. Conceptual

26. $\underset{CH}{\overset{CH}{|||}}+H_2SO_4 \rightarrow \underset{CH(HSO_4)_2}{\overset{CH_3}{|}} \xrightarrow{H_2O} \underset{\text{CHO}}{\overset{CH_3}{|}}$

Reducing Compounds can also decolorize Baeyer's reagent.

27. Conceptual

28. Due to SIR  less Acidic than 

29. OMDM does not support rearrangement

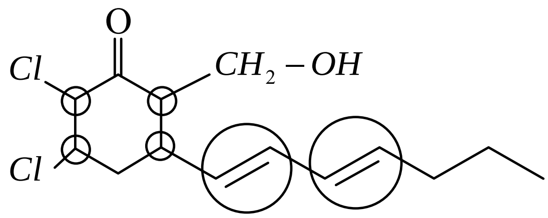
30. All are correct statements

31. Observed rotation = $-23°C$

Consider % of R is "x" and % of S is (100 – x)

$$-23°C=\frac{x(-61)+(100-x)(61)}{100} \Rightarrow x=68.85$$

32.



$$2^6 = 64$$

33. $2CH_3COONa + 2H_2O \rightarrow CH_3 - CH_3 + 2CO_2 + H_2 + 2NaOH$

Gas of cathode is H_2

$2 \times 82\ gm\ of\ CH_3COONa\ produce \rightarrow 2gm\ of\ H_2$

$20gm\ of\ CH_3COONa\ produce \rightarrow ?$

Mass of H_2 produced $= \dfrac{2\times 20}{2\times 82} = 0.2439\,gm$

$CH \equiv CH + 2H_2 \rightarrow CH_3 - CH_3$

$26gm\ of\ C_2H_2\ reduced\ by\ 4gm\ of\ H_2$

$? \quad \rightarrow \quad 0.2439gm$

Weight of $C_2H_2 = \dfrac{26 \times 0.2439}{4} = 1.585$

34. $C_xH_y + \left(x + \dfrac{y}{4}\right)O_2 \rightarrow xCO_2 + \dfrac{y}{2}H_2O$

Moles of $H_2O = \dfrac{72}{18} = 4$ moles of $CO_2 = \dfrac{308}{44} = 7$

$\dfrac{y}{2} = 4[y = 8]$ $[x = 7]$

C_7H_8 $[92]$

35. $CH \equiv CH + Cu_2Cl_2 / NH_4OH \rightarrow Cu - C \equiv C - Cu + 2NH_4Cl$

(Red ppt) $+2H_2O$

Molecular weight = 151.1

36.

$$CH_3 - CH_2 - \underset{\underset{H}{|}}{\overset{\overset{CH_3}{|}}{C}} - HC = CH_2$$

MATHEMATICS

37. $\Delta_1 = 9-4a \quad \Rightarrow \Delta_1+\Delta_2 = a^2-a+5$

$\Delta_2 = a^2-4 \qquad = (a-2)^2+1>0$

38. $\frac{a+b}{2}=\frac{5}{4}\sqrt{ab} \Rightarrow \frac{b}{a}=\frac{1}{4},\left(\frac{H_8-a}{b-H_8}\right)=8\times\frac{1}{4}=2$

39. $\Rightarrow x^2+\frac{1}{x^2}+b+a\left(x+\frac{1}{x}\right)=0 \Rightarrow x+\frac{1}{x}=t$

$t^2-at+b-2=0 \Rightarrow -at+b+t^2-2=0, t^2\in[4,\infty)$

This representation equation of line in a-b plane and a^2+b^2 represents square on this line from O(origin)

$d=\frac{t^2-2}{\sqrt{1+t^2}} \Rightarrow t^2\in[4,\infty), d_{\min}=\frac{2}{\sqrt{5}}$ at $t^2=4 \quad d_{\min}{}^2=\frac{4}{5}=\frac{p}{q}$

40. By the inequality $2ab\le a^2+b^2$, we get

$V_n \le \frac{\sin^2 x_1+\cos^2 x_2}{2}+\frac{\sin^2 x_2+\cos^2 x_3}{2}\ldots\ldots\ldots\frac{\sin^2 x_n+\cos^2 x_1}{2}=\frac{n}{2}$

With inequality for $x_1=x_2=\ldots.x_n=\frac{\pi}{4}$

41. $S=9+16+29+54+103+------+T_n --(i)$

$S= \quad 9+16+29+54+------+T_{n+1}+T_n --(ii)$

$T_n-T_{n-1}=7+6\left(2^{n-2}-1\right)=6.2^{n-2}+1$

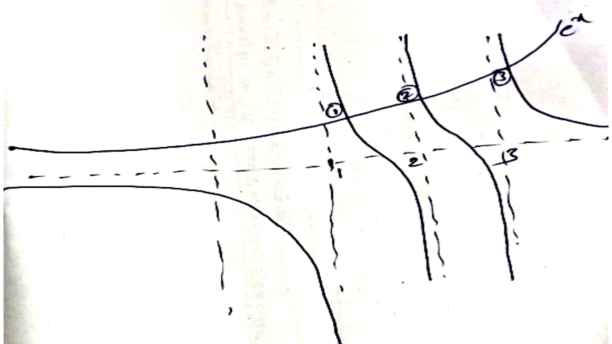
$T_n=6(2)^{n-1}+n+2$

$S=\sum 2^{n-1}+\sum n+\sum 2-6\left(2^n-1\right)+\frac{n(n+5)}{2}$

$n=10, S=6\times(2)^{n-1}+n+2$

42. $f(x)=e^x-\frac{1}{x-1}-\frac{1}{x-2}-\frac{1}{x-3}$

$e^x=\frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3}$



3-Solutions

43. $ax^2+bx+c=0$ has

(i)Both roots at infinite if $a\to 0$ and $b\to 0$

(ii)One root at infinite and other finite if $a\to 0$

(iii)Identity if $a\to 0$ & $b\to 0$ & $c\to 0$

44. $3, A_1, A_2, A_3, 6 \Rightarrow A_1 = 3 + \frac{3}{4} = \frac{15}{4}$

$A_2 = 3 + \frac{6}{4} = \frac{9}{2}; A_3 = 3 + \frac{9}{4} = \frac{21}{4}$ *and* $\frac{1}{3}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{6}$

$\Rightarrow H_1 = \frac{24}{7}, H_2 = 4$ *and* $H_3 = \frac{24}{5}$

$Thus, A_1H_3 - A_2H_2 = \frac{15}{4} \times \frac{24}{5} - \frac{9}{2} \times 4 = 0$

$Now, \sum_{n=1}^{100} \frac{1}{(2n+1)^2 - 1}; T_n = \frac{1}{(2n+2)(2n)} = \frac{1}{4(n)(n+1)} = \frac{(n+1)-(n)}{4(n+1)(n)}$

$\frac{1}{4}\sum_{n=1}^{100}\left(\frac{1}{n} - \frac{1}{n+1}\right) = \frac{1}{4}\left(1 - \frac{1}{101}\right) = \frac{25}{101}$ *and* $\sum_{n=1}^{\infty} 2^{\frac{n}{n+1}}$

$T_n = \frac{n}{2.2^n} = \frac{1}{2}\left[\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + ...\right]$

$T_n = \frac{1}{2}S$

$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^2} + ...\infty; \frac{S}{2} = \frac{1}{2} + \frac{2}{2^3} + ...\infty$

$\Rightarrow \frac{S}{2} = \frac{1}{2} + \frac{1}{2^2} + ...\infty; \frac{S}{2} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} \Rightarrow S = 2$

45. We have

$a_r.a_{r+2} = a_{r+1}^2 - 1 \forall r \geq 0$ *and* $a_0 = 1$

$So,\ a_r + a_{r+2} = a_r + \frac{a_{r+1}^2 - 1}{a_r} = \frac{a_r^2 + a_{r+1}^2 - 1}{a} = \frac{a_r^2 - 1 + a_{r+1}^2}{a}$

$\Rightarrow \frac{a_{r+1}.a_{r+1} + a_{r+1}^2}{a_r} = \frac{(a_{r-1} + a_{r+1}).a_{r+1}}{a_r} .... = \frac{(a_0 + a_2).a_{r+1}}{a_1} = \frac{(1 + a_1^2 - 1) \times a_{r+1}}{a_1} = a_1.a_{r+1}$

$\Rightarrow a_r + a_{r+2} = a_1.a_{r+1}$ *again* $\alpha + \beta = a_1, \alpha\beta = 1$

$So, a_r + a_{r+2} = (\alpha + \beta).a_{r+1}; a_{r+2} - \beta a_{r+1} = \alpha a_{r+1} - a_r = \alpha a_{r+1} - \alpha\beta a_r$

$a_{r+2} - \beta a_{r+1} = a(a_{r+1} - \beta a_r) \Rightarrow a_n - \beta a_{n-1} = \alpha(a_{n-1} - \beta a_{n-2})$

$a_n - \beta a_{n-1} = \alpha(a_{n-1} - \beta a_{n-2}) = \alpha^2(a_{n-2} - \beta a_{m-3}) = \alpha^{n-1}(a_1 - \beta a_0) = \alpha^n$

Add $a_n = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}$

46. $p\left(\frac{Q}{k}\right) = 1 \quad p^{-1} = \left(\frac{Q}{k}\right) \quad (p^{-1})_{23} = \frac{q_{23}}{k} = -\frac{1}{8} \qquad -\frac{(3\alpha + 4)}{20 + 12\alpha} = -\frac{1}{8}\alpha = -1$

$\Rightarrow \det(P) = 20 + 12\alpha = 8 \ (\det P)\left(\det\left(\frac{Q}{k}\right)\right) = 1$

$\frac{8\det(Q)}{k^3} = 1 \Rightarrow |Q| = \frac{k^3}{8} \Rightarrow \frac{k^3}{8} = \frac{k^2}{2} \Rightarrow k = 4 \Rightarrow \det(Q) = 8$

$\det(P.adjQ) = \det P.\det adjQ \qquad = \det P(\det Q)^2 = 8 \times 8^2 = 2^9$

$\det Q.adjP = \det Q\left(\det P\right)^2 = 8\times 8^2 = 2^9$

Alternate $|P|.|Q| = k^3 \Rightarrow |P| = 2k$ $\Rightarrow 6\alpha + 10 = k$ .....(1)

$Also\, PQ = kI \quad |P|Q = k\; adj(P)\; 2kQ = k\, adj(P)$

Comparing q_{23} we get $-\dfrac{k}{4} = -3\alpha - 4$ .....(2)

Solving (1) and (2) we get $\alpha = -1\, and\, k = 4$

47. $AA^T = A^T A = I, BB^T = B^T B = I$

$Now\left(ABA^{-1}\right)^T\left(ABA^{-1}\right) = \left(A^T\right)^{-1} B^T A^T ABA^{-1}$

$= \left(A^T\right)^{-1} A^{-1} = \left(AA^T\right)^{-1} = I$

$\Rightarrow \left(ABA^{-1}\right)^T\left(ABA^{-1}\right) = I$

$As\left(ABA^{-1}\right) is\; symmetric$

$\left(ABA^{-1}\right)^2 = I \Rightarrow ABA^{-1}$ is involutory matrix and $\left(ABA^{-1}\right)^{2017} = ABA^{-1}$

$\because ABA^{-1} \neq I - I$

$ABA^{-1} = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$, where $\alpha^2 + \beta\gamma = 1$

$\Rightarrow tr\left(ABA^{-1}\right) = 0$

$\because \left(ABA^{-1}\right)^2 = I \Rightarrow ABA^{-1}ABA^{-1} = I$

$\Rightarrow B^2 = I \Rightarrow tr\left(B^2\right) = 2$

$B = I, -I$

And $\Rightarrow B = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}, \alpha^2 + \beta\gamma = 1 \Rightarrow |B| = -1$

$\left(adj\dfrac{B}{\sqrt{2}}\right)^2 = \dfrac{1}{2}\left(adjB\right)^2 = \dfrac{1}{2}adjB^2 = \dfrac{I}{2}$

48. $\left(x^5 - x^3 - 4x^2 - 3x - 2\right) + \lambda\left(5x^4 + \alpha x^2 - 8x + \alpha\right)$

$\left(x-2\right)\left(x^2 + x + 1\right)^2 + \lambda\left(5x^4 + \alpha x^2 - 8x + \alpha\right)$

A root is independent λ if its common root is either $x = 2$ or $x = \omega$ $\alpha = -\dfrac{64}{5} or -3$

49. $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 16 & 0 & 0 \end{bmatrix} and\; A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow A^n$ is a null matrix $\forall\, n \geq 3$.

$p^{50} = \left(I + A\right)^{50} = I + 50A + \dfrac{50\times 49}{2}A^2$

$$Q+I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+50\begin{bmatrix}0&0&0\\4&0&0\\16&0&0\end{bmatrix}+25\times 49\begin{bmatrix}0&0&0\\0&0&0\\16&0&0\end{bmatrix}$$

$$\Rightarrow\left(\frac{q_{31}+q_{32}}{q_{21}}\right)=\frac{16(50+25\times 49)+50\times 4}{50\times 4}\qquad=\frac{16\times 51+8}{8}=102+1=103$$

50. We have $\sin^2\theta=\sin^2\theta\left(1-\cos^2\theta\right)=\sin^2\theta-\frac{1}{4}\sin^2 2\theta$

Putting $\theta=12^0,24^0,48^0,96^0,....$ etc, we get

$\sin^4 12^0=\sin^2 12^0-\frac{1}{4}\sin^2 24^0$

$\sin^2 24^0=\sin^2 24^0-\frac{1}{4}\sin^2 48^0$

....

....

Putting the values in S, we get

$S=\sin^2 12^0-\frac{1}{4}\sin^2 3072^0=\frac{4^0-1}{4^0}\sin^2 12^0\Rightarrow P=4^0-1$

51. $T_r=\frac{r+2}{r!+(r+1)!+(r+2)!}=\frac{(r+2)}{r!\{1+(r+1)(r+2)\}}\frac{(r+2)}{r!(r+2)^2}=\frac{1}{r!(r+2)}$

$=\frac{r+1}{(r+2)!}=\frac{(r+2)-1}{(r+2)!},T_r=\frac{1}{2!}-\frac{1}{3!}$

$$\frac{T_n=\frac{1}{(n+1)!}-\frac{1}{(n+2)!}}{Sum=\frac{1}{2!}-\frac{1}{(n+2)!}}\Rightarrow\frac{1}{2!}-\frac{1}{(2021)!}$$

52. $|h(x)+g(x)|>|h(x)|+|g(x)|$ is impossible

So $|h(x)+g(x)|=|h(x)|+|g(x)|$ is true if $h(x)g(x)\geq 0$

Give $h(x)g(x)\leq 0\Rightarrow h(x)g(x)=0$

Give $g(x)\neq 0\Rightarrow h(x)=0\ \forall x\in R$

$\sum_{r=1}^{3}h(x)=h(1)+h(2)+h(3)=0$

53. $B^n=PA^nP\left(P^2=I\right)\Rightarrow PB^nP=P\left(PA^nP\right)P=A^n$

54. Let \`m' is integral solution $(m>0)$

$d=m^4-am^3-bm^2-cm$

$d=m\left(m^3-am^2-bm-c\right)$

$d\geq m....(1)$

$m^3(m-c)=bm^3+cm+d$

$m>a....(2)$

(1) & (2) are contradictions

if $m<0$

$m=-n$

$n^4+cn^3-bn^2+cn-d$

$n^4+(ab-b)n^2+(cn-d)>0$