Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS & STERLING_BT Paper -1(Adv-2020-P1-Model Date: 08-10-2023

Time: 09.00Am to 12.00Pm RPTA-10 Max. Marks: 198

KEY SHEET

PHYSICS

1	**D**	2	**D**	3	**A**	4	**A**	5	**D**	6	**A**
7	**BD**	8	**ACD**	9	**ACD**	10	**BC**	11	**BD**	12	**AC**
13	**0.8**	14	**6**	15	**0.33**	16	**0.75**	17	**4**	18	**5**

CHEMISTRY

19	**C**	20	**C**	21	**A**	22	**C**	23	**C**	24	**A**
25	**ABC**	26	**ABC**	27	**ABCD**	28	**ABCD**	29	**ACD**	30	**ABC**
31	**4.40 - 4.65**	32	**15.53 – 15.54**	33	**228.1 – 229.5**	34	**2.95 – 3.05**	35	**145**	36	**88.24**

MATHEMATICS

37	**B**	38	**B**	39	**D**	40	**B**	41	**C**	42	**C**
43	**ABC**	44	**ABCD**	45	**AC**	46	**AC**	47	**ACD**	48	**ABC**
49	**7**	50	**2**	51	**3**	52	**5**	53	**12**	54	**7**

SOLUTIONS

PHYSICS

1. Maximum restoring force develops at the end where force is applied. This force decreases linearly such that it becomes zero at the other end so stress also decreases linearly.

2. More is the tension in rubber bond more will be deformation hence length of rubber band.

3. At steady state $F_{net} = 0$ $\quad mg - F_b - F_D = 0$

$mg - \rho_1 Vg = F_D \qquad mg - \frac{\rho_s}{3} Vg = F_D \qquad F_D = \frac{2}{3} mg$

Rate of Loss of gravitational P.E of ball $= \frac{d}{dx} mgh = mgv_T$

Rate of Heat Loss = Power of Drag Force $= F_D V = -\frac{2}{3} mgv$

$F_{net} = mg - F_b - 6\pi\eta rv$

$ma = mg - F_b - 6\pi\eta rv \qquad \frac{dv}{dt} = g - \frac{F_b}{m} - \frac{6\pi\eta r}{m} v$

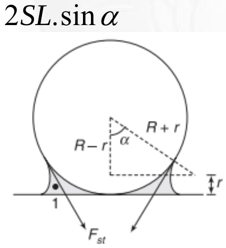
So time constant $\tau = \frac{m}{6\pi\eta r}$ $\qquad$ at $t = \tau, v' = (.63) v_T$

4. From the figure

$\cos\alpha = \frac{R-r}{R+r}$

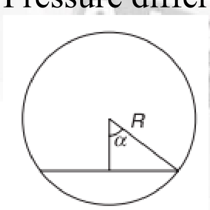
Consider a length L of the cylinder. Force due to surface tension acts tangentially and has a vertical resultant given by

$2SL.\sin\alpha$



Pressure inside the liquid (at 1) is less than atmospheric pressure by $\Delta P = \frac{S}{r}$

Pressure difference causes an additional downward force on the cylinder given by



$\Delta P[\text{Rectangular area } 2R\sin\alpha.L] = \frac{2SRL}{r}\sin\alpha$

$\therefore$ Net downward force due to presence of liquid is $F_{net} = 2SL\sin\alpha + \frac{2SRL}{r}\sin\alpha$

Force per unit length is $\frac{F_{net}}{L} = 2S\left[1 + \frac{R}{r}\right]\sin\alpha$

$= 2S\left[\frac{r+R}{r}\right]\sqrt{1 - \left(\frac{R-r}{R+r}\right)^2}\ [\text{Using (i)}] \qquad = 4S\sqrt{\frac{R}{r}}$

5. Pressure of air in the tube is higher than atmospheric pressure by $\Delta P = \rho gh$

Excess pressure inside bubble is $\Delta P = \frac{4\sigma}{r}$

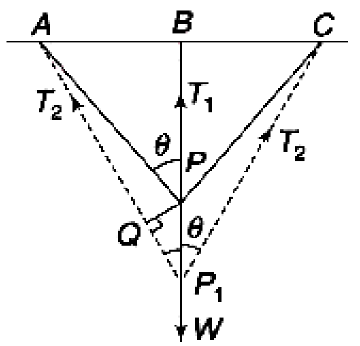
The pressure of air inside the bubble and the entire tube is same.

$\therefore \ \frac{4\sigma}{r} = \rho gh \Rightarrow \sigma = \frac{\rho ghr}{4}$

6. Gauge pressures at B and A are $P_B = \frac{2S}{3R}$

and $P_A = P_B + \frac{2S}{R} = \frac{2S}{3R} + \frac{2S}{R} = \frac{8S}{3R}$ $\therefore \ \frac{P_A}{P_B} = \frac{4}{1}$

7. After the weight is added, the point P moves to P_1 extension in BP is (say)



Drop a perpendicular from P on the line AP_1. Extention of wire AP is

$QP_1 = \Delta l_2\,(say)$

[This is because $AQ \simeq AP$]

Since extension are small,

$\angle APB \simeq \angle AP_1 = B = \theta(say)$ $\ P_1Q = P_1P\cos\theta \Rightarrow \Delta l_2 = \Delta l_1 \cos\theta$ ......(i)

$\therefore \Delta l_1 > \Delta l_2$. It means extension in BP is more.

$Stress(s) = Y\frac{\Delta l}{l}$ $\quad \frac{S_1}{S_2} = \frac{\Delta l_1}{\Delta l_2} \times \frac{l_2}{l_1}$

$= \frac{1}{\cos\theta} \times \frac{1}{\cos\theta}$ $\quad = \frac{1}{\cos^2\theta} > 1$ $\quad \therefore S_{BP} > S_{AP}$

For BP, stress $=$ Y strain $\quad \therefore \ \frac{T_1}{A} = Y.\frac{\Delta l_1}{l_1} \Rightarrow T_1 = YA\frac{\Delta l_1}{l_1}$

For $AP, \frac{T_2}{A} = Y\frac{\Delta l_2}{l_2} \Rightarrow T_2 = YA\frac{\Delta l_2}{l_2}$

$2T_2\cos\theta + T_1 = W$ $\quad 2\left(T_1\cos^2\theta\right)\cos\theta + T_1 = W$

$$T_1 = \frac{W}{1 + 2\cos^3\theta} = \frac{W}{1 + 2\left(\frac{l_1}{l_2}\right)^3}$$

8. When a block attached to a spring is released from its unstretched position, it will go down to stretch the spring by x such that

$\frac{1}{2}kx^2 = mgx \Rightarrow x = \frac{2mg}{k}$

Ultimately, due to air friction, etc., the block will settle down in its equilibrium position, stretching the spring by

$x_0 = \frac{mg}{k} = \left(= \frac{x}{2} \right)$

Thus, energy equal to $\frac{mg}{k}\frac{x}{2}$ is lost forever. This energy is mainly lost an heat in overcoming viscosity of air.

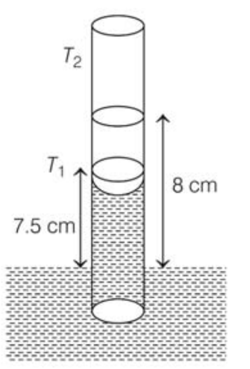
In case of the wire (which is just like a very stiff spring), similar thing happens. We may not be able to observe oscillations as in the case of a spring due to small amplitude. Ultimately, the system settles in equilibrium where half the loss in gravitational Pe of the block is lost as heat.

9. Heights if only single material tubes are used of sufficient length,

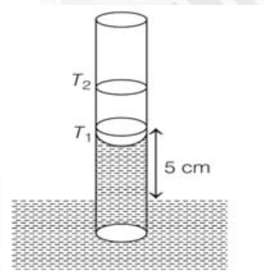
$$h_1 = \frac{2R\cos\theta}{\rho rg} = \frac{2\times 0.075\times\cos 0^\circ}{1000\times 2\times 10^{-4}\times 10} = 7.5\,cm$$

$$h_2 = \frac{2T\cos\theta}{\rho rg} = \frac{2\times 0.075\times\cos 60^\circ}{1000\times 2\times 10^{-4}\times 10} = 3.75\,cm$$

Case - I

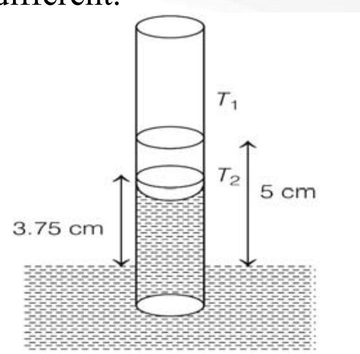


Case – II

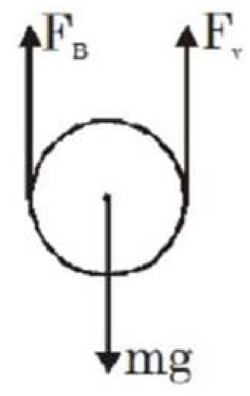


2. Liquid will rise only upto height of 5 cm and meniscus will adjust by changing is radius of curvature. If the liquid goes up in tube 2 then it will not be able to support the weight of the liquid.

3. Weight of water in meniscus will be different in two cases because angle of contact is different.



10. In steady state [equilibrium condition]



$F_B + F_v = mg$ $5 + F_v = 10$ $2.5V = 5$ $V = 2.0\, m/s$

Water will rise upward to fill the space $P = M_w V = (0.5)(2) = 1 kg\, m/s$

Momentum of ball will be downward $P = M_{ball}(V) = 2\, kg\, m/s$

Momentum of system will be $P_{system} = P_w + P_{Ball} = 1 kg\, m/s$ downward

11. $\dfrac{\rho g(h_1 - h_2)\pi r^4}{8\eta L} = \dfrac{Adh_2}{dt} = \dfrac{-Adh_1}{dt} = k$ $dh_1 = \dfrac{-kdt}{A}, dh_2 = \dfrac{+kdt}{A}$

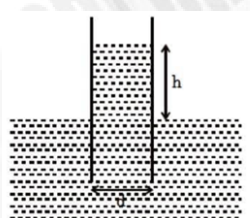
$d(h_1 - h_2) = \dfrac{-2kdt}{A} \Rightarrow k = -\dfrac{A}{2dt} d(h_1 - h_2)$

$\dfrac{\rho g(h_1 - h_2)\pi r^4}{8\eta L} = \dfrac{-Ad(h_1 - h_2)}{2dt}$ $\dfrac{d(\Delta h)}{dt} = \dfrac{-\rho g \pi r^4 2\Delta h}{8\eta LA}$

$\int_{\Delta h}^{\Delta h_0} \dfrac{d(\Delta h)}{\Delta h} = -\dfrac{\rho g \pi r^4}{4\eta LA}\int_0^t dt$ $\Delta h = \Delta h_o e^{-\frac{\rho g \pi r^4}{4\eta LA}t}$

$\Delta h = \dfrac{\Delta h_o}{e}$ in a time $t_0 = \dfrac{4\eta LA}{\rho g \pi r^4} = 2.4 \times 10^4 s$ and $\Delta h \to 0$ as $t \to \alpha$

12.



$2Tl = \rho dlhg$

$h = \dfrac{2T}{\rho g d} = \dfrac{2 \times 7 \times 10^{-2}}{10^3 \times 10 \times 2 \times 10^{-4}} = 7 \times 10^{-2} m$ = 7 cm

Force of mutual attraction between the plats is

$F = \int_0^h \rho g y l dy = \dfrac{\rho g l h^2}{2}$ $= \dfrac{10^3 \times 10 \times 0.10 \times (7 \times 10^{-2})^2}{2} = \dfrac{4.9}{2} = 2.45\, N$

13. $A_1 = \pi r^2; A_2 = \pi(R^2 - r^2)$

$\sigma_1 A_1 + \sigma_2 A_2 = W$ ....(i)

Strain in both sections is same.

$\dfrac{\sigma_1}{Y_1} = \dfrac{\sigma_2}{Y_2}$ ....(ii)

Solving (i) and (ii) gives $\sigma_2 = \dfrac{WY_2}{A_1Y_1 + A_2Y_2}$

Required fraction is: $\dfrac{\sigma_2 A_2}{W} = \dfrac{A_2Y_2}{A_1Y_1 + A_2Y_2} = \dfrac{\left(R^2 - r^2\right)Y_2}{r^2Y_1 + \left(R^2 - r^2\right)Y_2}$

14. $\sigma = -\eta \dfrac{dv}{dy} \quad \Rightarrow \sigma = (1.5)[4 - 2y] \times 2 \qquad \sigma = \dfrac{3}{2}[4 - 2] \times 2 = 6$

15. $\sigma = -\dfrac{\frac{dr}{r}}{\frac{dl}{l}}$

Given $\dfrac{dl}{l} = \dfrac{0.12 \times 10^{-2}}{2} = 0.06 \times 10^{-2}$

Also, since water is incompressible $\Rightarrow$ volume of water remains same.

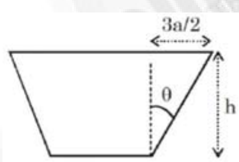
$\therefore \; A_0 \times 2 = A_N \left(2 + 0.08 \times 10^{-2}\right) \qquad \Rightarrow A_N = \dfrac{A_0 \times 2}{2 + 0.08 \times 10^{-2}}$

$\Rightarrow \dfrac{A_N}{A_0} = \dfrac{2}{2\left(1 + 0.04 \times 10^{-2}\right)} = \left(1 + 0.04 \times 10^{-2}\right)^{-1}$

$\Rightarrow \dfrac{\pi\left(r_0 + \Delta r\right)^2}{\pi r_0^2} = 1 - 0.04 \times 10^{-2} \qquad \Rightarrow \left(1 + \dfrac{dr}{r_0}\right)^2 = 1 - 0.04 \times 10^{-2}$

$\Rightarrow \dfrac{dr}{r} = -0.02 \times 10^{-2} \quad \therefore \quad \sigma = \dfrac{-\frac{dr}{r}}{\frac{dl}{l}} = \dfrac{0.02 \times 10^{-2}}{0.06 \times 10^{-2}} = \dfrac{1}{3}$

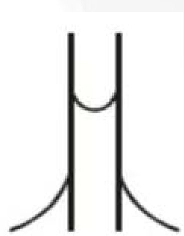
16.



$\tan\theta = \dfrac{3a}{2h} \qquad 2 \times 4aT \times \cos\theta = 4a\lambda g \; \cos\theta = \dfrac{\lambda g}{2T} \qquad \Rightarrow h = \dfrac{3\lambda g a}{2\sqrt{4T^2 - \lambda^2 g^2}}$

17. $m_1a_1 = 6\pi\eta r_1 v_1 \;\; m_2a_2 = 6\pi\eta r_2\left(v_2 - v_1\right) \;\; \dfrac{a_1}{a_2} = \dfrac{r_1v_1m_2}{r_2\left(v_2 - v_1\right)m_1} = \dfrac{4}{3}$

18.



F by capillary on water $= s \times 2\pi r \times 2$

$4\pi s r = mg \qquad 4\pi \times 0.07 \times r = 0.14\,\pi \times 10^{-3} \times 10$

$r = \dfrac{20}{4} = 5\,mm$

CHEMISTRY

19. NCERT

20. Ozone is very unstable in acidic solution and much more stable in basic solution

$$O_3(g) + 2H^+(aq) + 2e^- \rightarrow O_2(g) + H_2O(l)$$
$$E^0 = +2.08V$$
$$O_3(g) + 2H_2O(l) + 2e^- \rightarrow O_2 + 2OH$$
$$E^0 = +1.25V$$

21. With increase in oxidation state, the size of atom decreases. In $S_2O_4^{2-}$, both S are in + 3, in $S_2O_5^{2-}$ one S is in +3 and another S is in + 5 in $S_2O_6^{2-}$ both S are in + 5 oxidation states

22. With increase in the size of P atoms the π bonds become weaker.

23. I_3^- linear the central iodine atom is involved in sp^3 hybridization with three lone pairs occupying the three equatorial positons of trigonal bipyramid structure. During the formation of I_5^- from I_3^-, one on the terminal iodine atom donate a lone pair to I_2 molecule and involves in sp^3 hybridization with two lone pairs and two bond pairs so it is angular.

24. Since iodine atom is large enough it can accommodate 3 chlorine atoms to form ICl_3 but being small, there will be steric hindrance when $3Cl$ atoms surround Bl_2 atoms.

25. FeS_2 contain Fe^{2+} and S_2^{2-} ion. S_2^{2-} structure is similar to peroxide ion O_2^{2-}. Peroxide ions are easily reduced by a two – electron process to O^{2-}. The standard reduction potention for hydrogen peroxide in acid is nearly 1.8 V. Unlike the weak O – O bond in peroxide ion, the S – S bond in disulphide is strong and hence cannot be reduced to S^{2-} by two electron process

26. O – O bond order is 1.5 in ozone which is more than the O – O bond order 1 in H_2O_2. Ozone is endothermic substance while the formation of H_2O_2 is exothermic heat of formation of H_2O_2 liquid is $-187.78\,kJ\,mol^{-1}$ and that of O_3 is $+142.7\,kJ\,mol^{-1}$. So ozone is less stable than H_2O_2. Dipolemoment of H_2O_2 is 2.1 D and that of ozone is 0.53 D.

27. A) Electron affinity of oxygen is least among the oxygen family elements and thus its formation O^{-2} is more endothermic

B) SF_6 is non-polar while in SF_4 is polar

C) Due to steric hindrance SF_6 is chemically inert

D) $SOCl_2$ is the chloro derivatives of H_2SO_3 while SO_2Cl_2 is the chloro derivative of H_2SO_4

28. A) ClF₃ is a strong oxidizing agent and also good fluorinating agent. So it fluorinates the Ag^+ ion and oxidize Cl^- ions when treated with AgCl

$$2AgCl + 2ClF_3 \rightarrow 2AgF_2 + Cl_2 + 2ClF$$

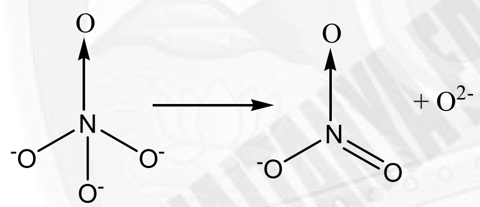
B) Chlorine water oxidized KI_2 brown I_2 solution and then oxidizes to colourless HIO_3

C) PH_3 is a weak base, so the stability of its salt increases with increase in the acidic character of HX. Further the H–X bond strength decreases from HF to HX

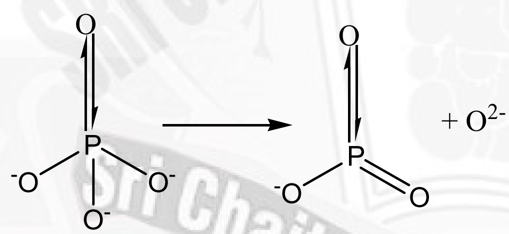
D) ClO_2 is prepared from $NaClO_3$. So use of ClO_2 increase the cost.

29. Acidic character increases down the group from HF to HI, for hydrogen halides. This is mainly due to strong bond between hydrogen and fluorine. But with increase in concentration of HF, increases its acidic strength. At high concentration HF molecules undergo homoassociation to form polyatomic ions (Such as bifluoride HF_2^-) and protons thus greatly increasing the acidity. This leads to protonation of very strong acid like HCl, H_2SO_4 or HNO_3 when using concentrated hydrofluoric acid solutions or liquid HF molecules like BF_3, SbF_5 or NbF_5 etc release the proton easily form HF by accepting the F^- ion converting into BF_4^-, SbF_6^-, NbF_6^- etc. So acidic strength of HF increases in the presence of these compounds

30. Both NO_4^{3-} and PO_4^{3-} have 32 valence electrons, so both have similar Lewis structures. From the Lewis structures for NO_4^{3-} the central N atom has a tetrahedral arrangement of electron pairs. N is small atom. There is probability for steric hindrance by the 4 oxygen atoms around N atom. Further N = O energy (607 kJ/mol) is more than sum of two N-O energies 2×201 = 402 kJ/ mols. So NO_4^{3-} easily converts into stable NO_3^-



Phosphorous atom larger. No steric hindrance Further P = O (544 kJ/mol) is smaller than the sum of two P – O (2×335 = 670 kJ/mol). So PO_4^{3-} is stable than PO_3^-



PO_3^- and NO_3^- each have 24 valence electrons, so both have similar Lewis structures. The planar structure which certain P = O is not much stable as explained above.

31. Number of moles of $F_2 = \dfrac{1\times 2.3}{0.0821\times 300} = 0.095$

Mass of $Te = 1.2\times1.2\times1.2\times6 = 10.368$

Number of moles of $Te + 3F_2 \rightarrow TeF_6$

Number of moles of $Te = \dfrac{10.368}{128} = 0.081$

Limiting agent is F_2

3 moles of F_2 gives 1 moles of TeF_6

0.095 moles of F_2 gives $3.1\times10^{-2} moles\ of\ TeF_6$ $3.1\times10^{-2} moles\ of\ TeF_6\ or\ Te(OH)_6$

$$pH = \frac{p^{k_a} - \log C}{2}$$

32. $2NaN_3 \rightarrow 2Na + 3N_2$

$2KNO_3 + 10Na \rightarrow K_2O + 5Na_2O + N_2$

$10NaN_3 \rightarrow 10Na + 15N_2$

$10NaN_3 + 2KNO_3 \rightarrow K_2O + 5Na_2O + 16N_2$

10×65 2×101

650 202

50 ?

$$\frac{202}{13} = 15.538$$

33. $n = \frac{PV}{RT} = \frac{(760/760\,atm)(138L)}{0.0821\times300} = 5.60$

$6KOH + 3Cl_2 \rightarrow KClO_3 + 5KCl + 3H_2O$

$3\,mols\ of\ Cl_2\ give\ 1\,mol\ of\ KClO_3$

$$\therefore\ Mass\ of\ KClO_3 = 5.60\times\frac{1\,mol\ KClO_3}{3\,mol\ Cl_2}\times122.5 = 228.78$$

34. $H_2O + NaOCl + 2I^- \rightarrow I_2 + NaCl + 2OH^-$

$I_2 + 2Na_2S_2O_3 \rightarrow 2NaI + Na_2S_4O_6$

$$Mass\ of\ NaOCl = \frac{40\times0.1}{1000\times2}\times74.5 = 0.149 \simeq 0.15$$

$\because$ The density of bleach is 1g/cc, then 5cc = 5g

$$Mass\ prsenct\ of\ NaOCl = \frac{0.15}{5}\times100 = 3\%$$

35. $SiF_4 + 2(CH_3)_4NF \rightarrow [(CH_3)_4N]_2^{2+}[SiF_6]^{2-}$

$104\,g\ of\ SiF_4\ gives$ $290\,g$

$52\,g\ of\ SiF_4\ gives$ $145\,g\ of\ product$

36.

$N_2 + 3H_2 \rightarrow 2NH_3$

28 6 34

$280\,kg$ of N_2 by reacting with $60\,kg$ of H_2 should produce $340\,kg$ of NH_3, if there is 100% yield.

Since $300\,kg$ of NH_3 is formed the percent yield is

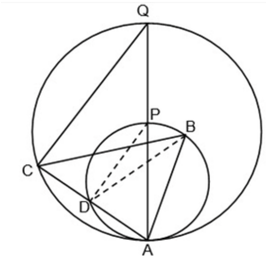
$$\frac{300\times100}{340} = 88.24\%$$

MATHEMATICS

37. $\angle ACQ = \angle ADP = 90°$

Hence ΔACQ and ΔADP are similar triangles. From $\Delta BDA, \frac{BD}{\sin 60°} = 2r$

Therefore $BD = \sqrt{3}r$



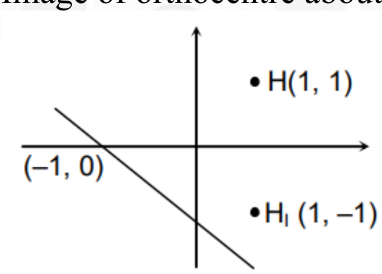
Also, from the similarity of triangles $\frac{AD}{AC} = \frac{AP}{AQ} = \frac{r}{R}$

Therefore $AD = \frac{rx}{R} (\because AC = x)$

Using cosine rule for $\Delta ABD.$

$3r^2 = (BD)^2 = (AB)^2 - 2(AD)(AB)\cos 60° + AD^2 \quad x = \frac{\sqrt{3}Rr}{\sqrt{R^2 - Rr + r^2}}$

38. Image of orthocentre about any side of the triangle lies on its circucircle.



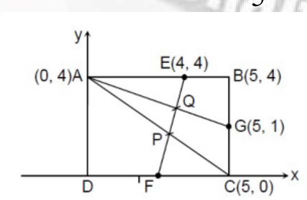
We can observe that $x^2 + y^2 + x + 3y = 0$ is satisfied by $(-1,0)$ and $(1,-1)$ both

So, option (B) is the correct choice.

39. Equation of AG

$y - 4 = \frac{4-1}{0-5}(x-0); y - 4 = -\frac{3}{5}(x)$

$5y - 20 = -3x; y = -\frac{3}{5}x + 4$



Equation AC

$y = -\frac{4}{5}x + 4$

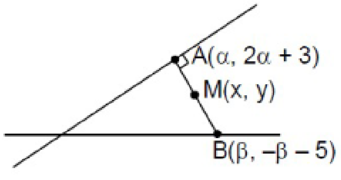
Equation of EF $\quad y = 2x - 7 \Rightarrow \frac{PQ}{EF} = \frac{10}{91}$

40. $\frac{y - (2\alpha + 3)}{x - \alpha} \times 2 = -1$

$2(y-2\alpha-3)=\alpha-x$

$2y-4\alpha-6=\alpha-x$

$2y-6+x=5\alpha \quad ....(1)$



$\dfrac{\beta+\alpha}{2}=x, \dfrac{-\beta-5+2\alpha+3}{2}=y$

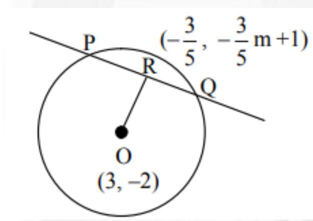
$\beta+\alpha=2x, 2y-2\alpha+2+\alpha=2x$

$2y-2x+2=\alpha$

$-\beta+2\alpha-2=2y, \beta=2y-2\alpha+2$

So, equation $2y-6+x=5(2y-2x+2); 2y-6+x=10y-10x+10; 11x-8y-16=0$

41. $PQ\perp QR \Rightarrow Stope\,OR=-\dfrac{1}{m}=\dfrac{-\frac{3}{5}m+1+2}{-\frac{3}{5}-3}$



$\Rightarrow m^2-5m+6=0 \quad \Rightarrow m=2,3$

42. The line formed is the redical axis of the circles.

43. Equation of tangent at A is $4x-3y-8=0$

Let $y=m(x-2)$ a line thro $(2,0)$ and $m=-7 or \dfrac{1}{7}$

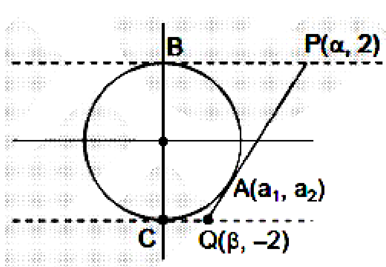
We have $\dfrac{x-2}{-\frac{1}{\sqrt{50}}}=\dfrac{y}{\frac{7}{\sqrt{50}}}=5\sqrt{2} \quad \Rightarrow \quad$ Centre of circle are $(1,7),(3,-7),(9,1)$ and $(-5,-1)$

44. $B(0,2)$ and $C(0,-2)$. Slope of $PQ=\dfrac{4}{\alpha-\beta}$

Equation of $PQ; y-2=\dfrac{4}{\alpha-\beta}(x-\alpha)$

$y(\alpha-\beta)-2(\alpha-\beta)=4x-4\alpha \quad 4x-y(\alpha-\beta)-2(\alpha+\beta)=0$

$\because$ it touches $x^2+y^2=4 \quad \therefore \dfrac{|0-0-2(\alpha+\beta)|}{\sqrt{16+(\alpha-\beta)^2}}=2$



$\Rightarrow a\beta = 4 \Rightarrow$ if $\alpha = 4$ slope of $PQ = \dfrac{4}{3}$

$\therefore$ slope of $OA = -\dfrac{3}{4} \Rightarrow \dfrac{a_2}{a_1} = -\dfrac{3}{4}$

45. Since, $3x + 2y \geq 0$ ....(i)

where $(1,3)(5,0)$ and $(-1,2)$ satisfy Eq.(i). $\therefore$ Option (a) is true.

Again, $2x + y - 13 \geq 0$

is not satisfied by $(1,3)$, $\therefore$ Option (b) is false. $2x - 3y - 12 \leq 0$

is satisfied for all points $\therefore$ Option (c) is true. and $-2x + y \geq 0$

is not satisfied by $(5,0)$, $\therefore$ Option (d) is false.

Thus, (a) and (c) are correct answers.

46. All the points must be collinear.

47. Bisectors are given by $\dfrac{3x-4y}{5} = \pm\left(\dfrac{x+y-4}{\sqrt{2}}\right)$. Bisector containing $(2,1)$ is given by '-' sign which is also obtuse angle bisector

48. Conceptual

49. Orthocentre is $\left(-2, 2\left(\dfrac{1}{m_1} + \dfrac{1}{m_2} + \dfrac{1}{m_3} + \dfrac{1}{m_1 m_2 m_3}\right)\right) = \left(-2, 2\left(\dfrac{26}{24} + \dfrac{1}{24}\right)\right) = \left(-2\dfrac{9}{4}\right)$

So $4\beta + \alpha = 9 - 2 = 7$

50. Let $R(h,k)$ be point of intersection of tangents at P and Q on $x^2 - y^2 = r^2$

$\Rightarrow$ equation of chord of contact PQ is $xh + yk = r^2$

which is also $2x + 4y = 5 \Rightarrow (h,k) \equiv \left(\dfrac{2r^2}{5}, \dfrac{4r^2}{5}\right)$

Mid-point of OR is circum-centre of ΔOPQ

$\Rightarrow \left(\dfrac{r^2}{5}, \dfrac{2r^2}{5}\right)$ lies on $x + 2y = 4 \Rightarrow r = 2$

51. The equation of the chord of contact of the two tangents that can be drawn from $P(4,3)$ to the circle $x^2 + y^2 = 9$ is $(S_1 = 0)$

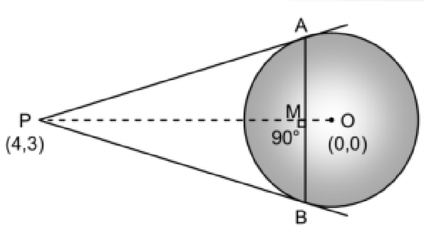
$4x + 3y - 9 = 0$ ....(1)

For area of ΔPAB, we need

$|AB|$ and $|MP|$

Here, $|OP| = \sqrt{(4-0)^2 + (3-0)^2} = 5$

Also, $|OM|$ = distance of $O(0,0)$ from line (1)



$= \dfrac{|0+0-9|}{\sqrt{4^2+3^2}} = \dfrac{9}{5}$ $\therefore |AB| = 2\sqrt{(radius)^2 - OM^2}$

$= 2\sqrt{3^2 - \left(\frac{9}{5}\right)^2} = 2\sqrt{\frac{144}{25}} = \frac{24}{5}$ and $|MP| = |OP| - |OM|$ $= 5 - \frac{9}{5} = \frac{16}{5}$

Here area $\Delta PAB = \frac{1}{2}|AB||MP| = \frac{1}{2}\left(\frac{24}{5}\right)\left(\frac{16}{5}\right) = \frac{192}{25}$ square units $\Rightarrow \frac{64\Delta}{25} = \frac{192}{25} \Rightarrow \Delta = 3.$

52. Centre of C_3 will lie on the radical axis of C_1 and C_2 which is $10x + 6y + 26 = 0.$

Let centre of C_3 is (h,k).

Equation of chord of contact through (h,k) to the C_1 may be given as $hx + ky = 25$ ....(I)

Let the mid point of the chord is (x_1, y_1) the equation of the chord with the help of mid point may be given as $xx_1 + yy_1 = x_1^2 + y_1^2$

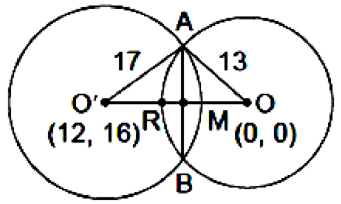
Since (I) and (II) represents same straight line

$\Rightarrow \frac{h}{25} = \frac{x_1}{x_1^2 + y_1^2}, \frac{k}{25} = \frac{y_1}{x_1^2 + y_1^2}$ ....(II)

Since (h,k) lie on the radical axis $10\left(\frac{25x_1}{x_1^2 + y_1^2}\right) + 6\left(\frac{25y_1}{x_1^2 + y_1^2}\right) + 26 = 0$

the locus of (x_1, y_1) is $5x + 3y + \frac{13}{25}\left(x^2 + y^2\right) = 0$

53. Let M be the mid-point of AB



$\Rightarrow AM^2 = 17^2 - x^2 = 13^2 - (20 - x)^2 \Rightarrow x = 13$

$\Rightarrow AB = 4\sqrt{30},$ clearly when C lies at R

Area is maximum $\Rightarrow$Maximum area $= \frac{1}{2} \times 4\sqrt{30} \times 6 = 12\sqrt{30}\ sq.units$

54. Let $A(0,a)$ and $B(b,2b)$

Coordinate of P is $\left(\frac{2a}{5}, \frac{4a}{5}\right)$

Coordinate of Q is $(0, 2b)$

Equation of AB is $y - a - \left(2 - \frac{a}{b}\right)x = 0$

$\because$ It passes through $(1,3)$ hence $a = \frac{b}{b-1}$

Also equation of PQ is $y - 2b = \left(2 - \frac{5b}{a}\right)x$

i.e., $y - 7x + b(x - 2) = 0$

$\therefore$ Fixed point is $\left(\frac{2}{5}, \frac{14}{5}\right)$