DPP 8.3

Properties of Determinant (Level 2)

Single Correct Answer Type

1. The equation $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix}$

$+\begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

(a) has no real solution
(b) has 4 real solutions
(c) has two real and two non-real solutions
(d) has infinite number of solutions, real or non-real

2. Let $\Delta_1 = \begin{vmatrix} ap^2 & 2ap & 1 \\ aq^2 & 2aq & 1 \\ ar^2 & 2ar & 1 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} apq & a(p+q) & 1 \\ aqr & a(q+r) & 1 \\ arp & a(r+p) & 1 \end{vmatrix}$, then

(a) $\Delta_1 = \Delta_2$ (b) $\Delta_2 = 2\Delta_1$
(c) $\Delta_1 = 2\Delta_2$ (d) $\Delta_1 + 2\Delta_2 = 0$

3. Area of triangle whose vertices are $(a, a^2)(b, b^2)(c, c^2)$ is $\frac{1}{2}$, and area of another triangle whose vertices are (p, p^2), (q, q^2) and (r, r^2) is 4, then the value of

$\begin{vmatrix} (1+ap)^2 & (1+bp)^2 & (1+cp)^2 \\ (1+aq)^2 & (1+bq)^2 & (1+cq)^2 \\ (1+ar)^2 & (1+br)^2 & (1+cr)^2 \end{vmatrix}$ is

(a) 2 (b) 4 (c) 8 (d) 16

4. The value of $\begin{vmatrix} \beta\gamma & \beta\gamma' + \beta'\gamma & \beta'\gamma' \\ \gamma\alpha & \gamma\alpha' + \gamma'\alpha & \gamma'\alpha' \\ \alpha\beta & \alpha\beta' + \alpha'\beta & \alpha'\beta' \end{vmatrix}$ is

(a) $(\alpha\beta' - \alpha'\beta)(\beta\gamma' - \beta'\gamma)(\gamma\alpha' - \gamma'\alpha)$
(b) $(\alpha\alpha' - \beta\beta')(\beta\beta' - \gamma\gamma')(\gamma\gamma' - \alpha\alpha')$
(c) $(\alpha\beta' + \alpha'\beta)(\beta\gamma' + \beta'\gamma)(\gamma\alpha' + \gamma'\alpha)$
(d) none of these

5. If $\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix} = 2010$ and if

$\begin{vmatrix} c-a & c-b & ab \\ a-b & a-c & bc \\ b-c & b-a & ca \end{vmatrix} \begin{vmatrix} c-a & c-b & c^2 \\ a-b & a-c & a^2 \\ b-c & b-a & b^2 \end{vmatrix} = p$, then the number of positive divisors of p is

(a) 36 (b) 49 (c) 64 (d) 81

6. If $\begin{vmatrix} a & l & m \\ l & b & n \\ m & n & c \end{vmatrix} \begin{vmatrix} bc-n^2 & mn-lc & ln-bm \\ mn-lc & ac-m^2 & lm-an \\ ln-bm & lm-an & ab-l^2 \end{vmatrix} = 64$, then the

value of $\begin{vmatrix} 2a+3l & 3l+5m & 5m+4a \\ 2l+3b & 3b+5n & 5n+4l \\ 2m+3n & 3n+5c & 5c+4m \end{vmatrix}$ equals

(a) 120 (b) 240 (c) 360 (d) 480

7. The value of $\begin{vmatrix} x^2+y^2 & ax+by & x+y \\ ax+by & a^2+b^2 & a+b \\ x+y & a+b & 2 \end{vmatrix}$ depends on

(a) a (b) b (c) x (d) none of these

8. If $u = ax + by + cz$, $v = ay + bz + cx$, $w = ax + bx + cy$, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$ is

(a) $u^2 + v^2 + w^2 - 2uvw$ (b) $u^3 + v^3 + w^3 - 3uvw$
(c) 0 (d) none of these

9. If the number of positive integral solutions of $u + v + w = n$ be denoted by P_n, then the absolute value of

$\begin{vmatrix} P_n & P_{n+1} & P_{n+2} \\ P_{n+1} & P_{n+2} & P_{n+3} \\ P_{n+2} & P_{n+3} & P_{n+4} \end{vmatrix}$ is

(a) −1 (b) 2 (c) 3 (d) 4

10. If $f(x)$, $h(x)$ are polynomials of degree 4 and $\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix} = mx^4 + nx^3 + rx^2 + sx + t$ be an identity in x, then $\begin{vmatrix} f'''(0) - f''(0) & g'''(0) - g''(0) & h'''(0) - h''(0) \\ a & b & c \\ p & q & r \end{vmatrix}$ is

(a) $2(3n - r)$ (b) $2(2n - 3r)$
(c) $3(n - 2r)$ (d) none of these

11. If $\Delta(x) = \begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$, then coefficient of x in $\Delta(x)$ is

(a) -4 (b) -2 (c) -6 (d) 0

12. If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & x \end{vmatrix}$, then $\lim_{x \to 0} \frac{f(x)}{x^2} =$

(a) 0 (b) 3 (c) 2 (d) 1

Multiple Correct Answers Type

13. If $x \in R, a_i, b_i, c_i \in R$ for $i = 1, 2, 3$ and $\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = 0$, then which of the following may be true?

(a) $x = 1$ (b) $x = -1$
(c) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ (d) none of these

14. If a_i, $i = 1, 2, \ldots, 9$ are perfect odd squares, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is always a multiple of

(a) 4 (b) 7 (c) 16 (d) 64

15. The value of the determinant $\begin{vmatrix} \cos(\theta + \alpha) & -\sin(\theta + \alpha) & \cos 2\alpha \\ \sin \theta & \cos \theta & \sin \alpha \\ -\cos \theta & \sin \theta & \lambda \cos \alpha \end{vmatrix}$ is

(a) independent of θ for all $\lambda \in R$
(b) independent of θ and α when $\lambda = 1$
(c) independent of θ and α when $\lambda = -1$
(d) independent of λ for all θ

Answers Key

Single Correct Answer Type

1. (d) 2. (d) 3. (d) 4. (a) 5. (d)
6. (c) 7. (d) 8. (b) 9. (a) 10. (a)
11. (b) 12. (d)

Multiple Correct Answers Type

13. (a, b, c) 14. (a, c, d) 15. (a, c)

DPP 8.3

Single Correct Answer Type

1. (d) $$\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ -(2+x^2) & 1-5x & 2-3x \end{vmatrix}$$

$$+ \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$$

(taking transpose of 1^{st} determinant)

$$\Rightarrow \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ -(2+x^2)+1-2x & 1-5x+3x-2 & 2-3x+2x-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ -(1+x)^2 & -2x-1 & -1-x \end{vmatrix} = 0$$

Here 1st row and 3rd row are the same

$\Rightarrow$ This is an identity

$\Rightarrow$ Infinite roots

2. (d) $\Delta_1 = \begin{vmatrix} ap^2 & 2ap & 1 \\ aq^2 & 2aq & 1 \\ ar^2 & 2ar & 1 \end{vmatrix}$

$$= 2a^2 \begin{vmatrix} p^2 & p & 1 \\ q^2 & q & 1 \\ r^2 & r & 1 \end{vmatrix}$$

$$= -2a^2(p-q)(q-r)(r-p)$$

$$\Delta_2 = \begin{vmatrix} apq & a(p+q) & 1 \\ aqr & a(q+r) & 1 \\ arp & a(r+p) & 1 \end{vmatrix}$$

$$= a^2 \begin{vmatrix} pq & (p+q) & 1 \\ qr & (q+r) & 1 \\ rp & (r+p) & 1 \end{vmatrix}$$

$$= a^2 \begin{vmatrix} pq-qr & p-r & 0 \\ qr-rp & q-p & 0 \\ rp & (r+p) & 1 \end{vmatrix} \quad (R_1 \to R_1 - R_2, R_2 \to R_2 - R_3)$$

$$= a^2(p-q)(r-p) \begin{vmatrix} -q & -1 & 0 \\ -r & -1 & 0 \\ rp & (r+p) & 1 \end{vmatrix}$$

$$= a^2(p-q)(r-p) \begin{vmatrix} -q+r & 0 & 0 \\ -r & -1 & 0 \\ rp & (r+p) & 1 \end{vmatrix}$$

$$= a^2(p-q)(r-p)(q-r)$$

$\Rightarrow \quad \Delta_1 + 2\Delta_2 = 0$

3. (d) $\begin{vmatrix} (1+ap)^2 & (1+bp)^2 & (1+cp)^2 \\ (1+aq)^2 & (1+bq)^2 & (1+cq)^2 \\ (1+ar)^2 & (1+br)^2 & (1+cr)^2 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 2a & a^2 \\ 1 & 2b & b^2 \\ 1 & 2c & c^2 \end{vmatrix} \times \begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix}$$

$$= 2 \times 2\Delta_1 \cdot 2\Delta_2$$

$$= 8\Delta_1\Delta_2 = 8 \times \frac{1}{2} \times 4 = 16$$

4. (a) $\dfrac{1}{\alpha\beta\gamma} \begin{vmatrix} \alpha\beta\gamma & \alpha\beta\gamma' + \alpha\beta'\gamma & \alpha\beta'\gamma' \\ \alpha\beta\alpha & \beta\gamma\alpha' + \beta\gamma'\alpha & \beta\gamma'\alpha' \\ \alpha\beta\gamma & \alpha\beta'\gamma + \alpha'\beta\gamma & \alpha'\beta'\gamma \end{vmatrix}$

Multiplying R_1 by α, R_2 by β and R_3 by γ and dividing the determinant by $\alpha\beta\gamma$ we have

$$= \frac{1}{\alpha\beta\gamma} \cdot \alpha\beta\gamma \begin{vmatrix} 1 & \alpha\beta\gamma' + \alpha\beta'\gamma & \alpha\beta'\gamma' \\ 1 & \beta\gamma\alpha' + \beta\alpha\gamma' & \beta\gamma'\alpha' \\ 1 & \gamma\alpha\beta' + \gamma\alpha'\beta & \gamma\alpha'\beta' \end{vmatrix}$$

by $R_3 \to R_3 - R_1$

$R_2 \to R_2 - R_1$

$$= \begin{vmatrix} 1 & \alpha\beta\gamma' + \alpha\beta'\gamma & \alpha\beta'\gamma' \\ 0 & \gamma(\alpha'\beta - \alpha\beta') & \gamma'(\alpha'\beta - \alpha\beta') \\ 0 & \beta(\alpha'\gamma - \alpha\gamma') & \beta'(\alpha'\gamma - \alpha\gamma') \end{vmatrix}$$

$$= (\alpha'\beta - \alpha\beta')(\alpha'\gamma - \alpha\gamma') \begin{vmatrix} 1 & \alpha\beta\gamma' + \alpha\beta'\gamma & \alpha\beta'\gamma' \\ 0 & \gamma & \gamma' \\ 0 & \beta & \beta' \end{vmatrix}$$

$$= (\alpha'\beta - \alpha\beta')(\alpha'\gamma - \alpha\gamma')(\gamma\beta' - \beta\gamma')$$

5. (d) $P = \begin{vmatrix} c-a & c-b & ab-c^2 \\ a-b & a-c & bc-a^2 \\ b-c & b-a & ca-b^2 \end{vmatrix} = \begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix}^2 = (2010)^2$

$= (2 \times 3 \times 5 \times 67)^2 = 2^2 3^2 5^2 (67)^2$

No. of divisors of $P = (2+1)(2+1)(2+1)(2+1) = 81$

6. (c) $\Delta\Delta^2 = 64$

$\Rightarrow \quad \Delta^3 = 64 \Rightarrow \Delta = 4$

$$\begin{vmatrix} 2a+3l & 3l+5m & 5m+4a \\ 2l+3b & 3b+5n & 5n+4l \\ 2m+3n & 3n+5c & 5c+4m \end{vmatrix}$$

$= [(2 \times 3 \times 5) + (3 \times 5 \times 4)]\Delta$

$= (30 + 60)\Delta$

$= 90(4)$

$= 360$

7. (d) $\Delta = \begin{vmatrix} x & y & 0 \\ a & b & 0 \\ 1 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} x & y & 0 \\ a & b & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$

8. (b) $\Delta_1\Delta_2 = \begin{vmatrix} ax+by+cz & ay+bz+cx & az+bx+cy \\ bx+cy+az & by+cz+ax & bz+cx+ay \\ cx+ay+bz & cy+az+bx & cz+ax+by \end{vmatrix}$

$$= \begin{vmatrix} u & v & w \\ w & u & v \\ v & w & u \end{vmatrix}$$

$= u(u^2 - vw) - v(wu - v^2) + w(w^2 - uv)$

$= u^3 + v^3 + w^3 - 3uvw$

9. (a) As $u + v + w = n$ and $u, v, w \geq 1$

Now, number of solutions of $u + v + w = n \Rightarrow P_n = {}^{n-1}C_{n-3}$

Similarly $P_{n+1} = {}^{n}C_{n-2}$; $P_{n+2} = {}^{n+1}C_{n-1}$; $P_{n+3} = {}^{n+2}C_n$; $P_{n+4} = {}^{n+3}C_{n+1}$.

$$\text{Now } \Delta = \begin{vmatrix} {}^{n-1}C_{n-3} & {}^{n}C_{n-2} & {}^{n+1}C_{n-1} \\ {}^{n}C_{n-2} & {}^{n+1}C_{n-1} & {}^{n+2}C_{n} \\ {}^{n+1}C_{n-1} & {}^{n+2}C_{n} & {}^{n+3}C_{n+1} \end{vmatrix}$$

$$= \frac{1}{8}\begin{vmatrix} \frac{(n-1)!}{(n-3)!} & \frac{n!}{(n-2)!} & \frac{(n+1)!}{(n-1)!} \\ \frac{n!}{(n-2)!} & \frac{(n+1)!}{(n-1)!} & \frac{(n+2)!}{n!} \\ \frac{(n+1)!}{(n-1)!} & \frac{(n+2)!}{n!} & \frac{(n+3)!}{(n+1)!} \end{vmatrix}$$

$$= \frac{1}{8}\begin{vmatrix} (n-1)(n-2) & n(n-1) & n(n+1) \\ n(n-1) & n(n+1) & (n+1)(n+2) \\ n(n+1) & (n+2)(n+1) & (n+3)(n+2) \end{vmatrix}$$

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & n(n-1) & n \\ 1 & n(n+1) & (n+1) \\ 1 & (n+2)(n+1) & (n+2) \end{vmatrix}$$

(On applying (first) $C_3 \to C_3 - C_2$ and $C_1 \to C_1 - C_2$ (and then) $C_1 \to C_1 + C_3$)

$$= \frac{1}{2}\begin{vmatrix} 1 & n(n-1) & n \\ 0 & 2n & 1 \\ 0 & 2(n+1) & 1 \end{vmatrix} (R_3 \to R_3 - R_2 \text{ and } R_2 \to R_2 - R_1) \Rightarrow \Delta = -1$$

10. (a) Differentiating given equation w.r.t. x, we get

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 4mx^3 + 3nx^2 + 2rx + 5 \qquad (1)$$

Again differentiating w.r.t. x, we get

$$\begin{vmatrix} f''(x) & g''(x) & h''(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 12mx^2 + 6nx + 2r \qquad (2)$$

Again differentiating w.r.t. x, we get

$$\begin{vmatrix} f'''(x) & g'''(x) & h'''(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 24mx + 6n \qquad \ldots(3)$$

Putting $x = 0$ in (2), we get

$$2r = \begin{vmatrix} f''(0) & g''(0) & h''(0) \\ a & b & c \\ p & q & r \end{vmatrix} \qquad (4)$$

Putting $x = 0$ in (3), we get

$$6n = \begin{vmatrix} f'''(0) & g'''(0) & h'''(0) \\ a & b & c \\ p & q & r \end{vmatrix} \qquad (5)$$

From (5) and (4), we get

$$\begin{vmatrix} f'''(0) - f''(0) & g'''(0) - g''(0) & h'''(0) - h''(0) \\ a & b & c \\ p & q & r \end{vmatrix} = 2(3n - r)$$

11. (b) Note that $\Delta(x)$ is a polynomial of degree at most 6 in x.
If $\Delta(x) = a_0 + a_1x + a_2x^2 + \ldots + a_6x^6$, then $\Delta'(x) = a_1 + 2a_2x + \ldots + 6a_6x^5$

$\Rightarrow \quad a_1 = \Delta'(0)$,

Now,

$$\Delta'(x) = \begin{vmatrix} 1 & (x-1)^2 & x^3 \\ 1 & x^2 & (x+1)^3 \\ 1 & (x+1)^2 & (x+2)^3 \end{vmatrix} + \begin{vmatrix} x-2 & 2(x-1) & x^3 \\ x-1 & 2x & (x+1)^3 \\ x & 2(x+1) & (x+2)^3 \end{vmatrix} + \begin{vmatrix} x-2 & (x-1)^2 & 3x^2 \\ x-1 & x^2 & 3(x+1)^2 \\ x & (x+1)^2 & 3(x+2)^2 \end{vmatrix}$$

$$\Rightarrow \quad \Delta'(0) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 8 \end{vmatrix} + \begin{vmatrix} -2 & -2 & 0 \\ -1 & 0 & 1 \\ 0 & 2 & 8 \end{vmatrix} + \begin{vmatrix} -2 & 1 & 0 \\ -1 & 0 & 3 \\ 0 & 1 & 12 \end{vmatrix}$$

$$= -8 - 12 + 18 = -2$$

12. (d) $$f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & x \end{vmatrix}$$

$$\therefore \quad \frac{f(x)}{x^2} = \begin{vmatrix} \frac{\sin x}{x} & \cos x & \frac{\tan x}{x} \\ x^2 & x^2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$\therefore \quad \lim_{x \to 0} \frac{f(x)}{x^2} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 1$$

Multiple Correct Answers Type

13. (a, b, c)

$$\begin{vmatrix} a_1 + b_1x & a_1x + b_1 & c_1 \\ a_2 + b_2x & a_2x + b_2 & c_2 \\ a_3 + b_3x & a_3x + b_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \underbrace{\begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix}}_{D_1} + \underbrace{\begin{vmatrix} b_1x & a_1x + b_1 & c_1 \\ b_2x & a_2x + b_2 & c_2 \\ b_3x & a_3x + b_3 & c_3 \end{vmatrix}}_{D_2} = 0$$

Applying $C_2 \to C_2 - xC_1$ in D_1 and taking x common from C_1 in D_2

$$\Rightarrow \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x\begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and then taking x common from C_2 in D_2

$$\Rightarrow \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x^2 \begin{vmatrix} b_1 & a_1 & c_1 \\ b_2 & a_2 & c_2 \\ b_3 & a_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow (1-x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow x = \pm 1, \text{ or } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

14. (a, c, d)
Let $a_1 = (2m + 1)^2$, $a_2 = (2n + 1)^2$
$\Rightarrow \quad a_1 - a_2 = 4(m(m + 1) - n(n + 1)) = 8k$
so, difference of any two odd square is always a multiple of 8
Now apply $C_1 - C_3$ and $C_2 - C_3$, then C_1 and C_2 both become multiple of 8 so Δ always a multiple of 64.

15. (a, c)

$$\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$$

$$= \frac{1}{\sin\alpha\cos\alpha} \begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Multiplying R_2 and R_3 by $\sin\alpha$ and $\cos\alpha$, respectively]

$$= \frac{1}{\sin\alpha\cos\alpha} \times \begin{vmatrix} 0 & 0 & \cos 2\alpha + \sin^2\alpha + \lambda\cos^2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$= \frac{\cos 2\alpha + \sin^2\alpha + \lambda\cos^2\alpha}{\sin\alpha\cdot\cos\alpha} \begin{vmatrix} \sin\theta\sin\alpha & \cos\theta\sin\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha \end{vmatrix}$$

$$= (\cos^2\alpha + \lambda\cos^2\alpha) \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = (1+\lambda)\cos^2\alpha$$

Therefore, the given determinant is independent of θ for all real values of λ.
Also, $\lambda = -1$, then it is independent of θ and α.

DPP 8.4

System of Equations

Single Correct Answer Type

1. If the system of linear equations

$x + 2ay + az = 0$
$x + 3by + bz = 0$
$x + 4cy + cz = 0$

has a non zero solutions, then a, b, c are in

(a) A.P. (b) G.P.
(c) H.P. (d) satisfies $a + 2b + 3c = 0$

2. The equations $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$, $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$ and $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$ gives non-trivial solution for some values of λ, then the ratio $x : y : z$ when λ has the smallest of these values:

(a) $3:2:1$ (b) $3:3:2$ (c) $1:3:1$ (d) $1:1:1$

3. The system of homogeneous equations $tx + (t + 1)y + (t - 1)z = 0$, $(t + 1)x + ty + (t + 2)z = 0$, $(t - 1)x + (t + 2)y + tz = 0$ has a non-trivial solution for

(a) exactly three real values of t
(b) exactly two real value of t
(c) exactly one real value of t
(d) infinite number of values of t

4. If a, b, c are non-zeros, then the system of equations

$(\alpha + a)x + \alpha y + \alpha z = 0$
$\alpha x + (\alpha + b)y + \alpha z = 0$
$\alpha x + \alpha y + (\alpha + c)z = 0$

has a non-trivial solution if

(a) $2\alpha = a + b + c$ (b) $\alpha^{-1} = a + b + c$
(c) $\alpha + a + b + c = 1$ (d) $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$

5. The values of θ, λ for which the following equations $\sin\theta x - \cos\theta y + (\lambda + 1)z = 0$; $\cos\theta x + \sin\theta y - \lambda z = 0$; $\lambda x + (\lambda + 1)y + \cos\theta z = 0$ have non trivial solution, is

(a) $\theta = n\pi, \lambda \in R - \{0\}$
(b) $\theta = 2n\pi$, λ is any rational number
(c) $\theta = (2n + 1)\pi, \lambda \in R^+, n \in I$
(d) $\theta = (2n + 1)\dfrac{\pi}{2}, \lambda \in R, n \in I$

6. If the system of equations

$x - 2y + z = a$
$2x + y - 2z = b$
and $x + 3y - 3z = c$

have at least one solution, then

(a) $a + b + c = 0$ (b) $a - b + c = 0$
(c) $-a + b + c = 0$ (d) $a + b - c = 0$

7. If A, B, C are the angles of a triangle, the system of equations $(\sin A)x + y + z = \cos A$, $x + (\sin B)y + z = \cos B$, $x + y + (\sin C)z = 1 - \cos C$ has

(a) No solution
(b) Unique solution
(c) Infinitely many solutions
(d) Finitely many solutions

Multiple Correct Answers Type

8. A solution set of the equations $x + 2y + z = 1$, $x + 3y + 4z = k$, $x + 5y + 10z = k^2$ is

(a) $(1 + 5\lambda, -3\lambda, \lambda)$ (b) $(5\lambda - 1, 1 - 3\lambda, \lambda)$
(c) $(1 + 6\lambda, -2\lambda, \lambda)$ (d) $(1 - 6\lambda, \lambda, \lambda)$

9. Consider the system of equations: $x \sin\theta - 2y\cos\theta - az = 0$, $x + 2y + z = 0$, $-x + y + z = 0$, $\theta \in R$

(a) The given system will have infinite solutions for $a = 2$
(b) The number of integer values of a is 3 for the system to have nontrivial solutions.
(c) For $a = 1$ there exists θ for which the system will have infinite solutions
(d) For $a = 3$ there exists θ for which the system will have unique solution

Answers Key

Single Correct Answer Type

1. (c) 2. (d) 3. (c) 4. (d) 5. (d)
6. (b) 7. (b)

Multiple Correct Answers Type

8. (a, b) 9. (b, c, d)

$$\Rightarrow \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x^2 \begin{vmatrix} b_1 & a_1 & c_1 \\ b_2 & a_2 & c_2 \\ b_3 & a_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow (1-x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow x = \pm 1, \text{ or } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

14. (a, c, d)

Let $a_1 = (2m+1)^2$, $a_2 = (2n+1)^2$

$\Rightarrow \quad a_1 - a_2 = 4(m(m+1) - n(n+1)) = 8k$

so, difference of any two odd square is always a multiple of 8

Now apply $C_1 - C_3$ and $C_2 - C_3$, then C_1 and C_2 both become multiple of 8 so Δ always a multiple of 64.

15. (a, c)

$$\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$$

$$= \frac{1}{\sin\alpha\cos\alpha} \begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Multiplying R_2 and R_3 by $\sin\alpha$ and $\cos\alpha$, respectively]

$$= \frac{1}{\sin\alpha\cos\alpha} \times$$

$$\begin{vmatrix} 0 & 0 & \cos 2\alpha + \sin^2\alpha + \lambda\cos^2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$= \frac{\cos 2\alpha + \sin^2\alpha + \lambda\cos^2\alpha}{\sin\alpha\cdot\cos\alpha} \begin{vmatrix} \sin\theta\sin\alpha & \cos\theta\sin\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha \end{vmatrix}$$

$$= (\cos^2\alpha + \lambda\cos^2\alpha) \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = (1+\lambda)\cos^2\alpha$$

Therefore, the given determinant is independent of θ for all real values of λ.

Also, $\lambda = -1$, then it is independent of θ and α.

DPP 8.4

Single Correct Answer Type

1. (c) The system of linear equation has a non zero solution.

$$\Delta = \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$\therefore \quad (3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$

$\therefore \quad bc + ab = 2ac$

$$\therefore \quad \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

$\Rightarrow a, b, c$ are in H.P.

2. (d) $\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$

$\Rightarrow \lambda = 0$ or 3

If $\lambda = 0$, the equations become

$-x + y = 0,$

$-x - 2y + 3z = 0$ and

$2x + y - 3z = 0,$

$$\therefore \quad \frac{x}{6-3} = \frac{y}{6-3} = \frac{z}{-1+4}$$

3. (c) To have a non-trivial solution

$$\begin{vmatrix} t & t+1 & t-1 \\ t+1 & t & t+2 \\ t-1 & t+2 & t \end{vmatrix} = 0$$

$$\Rightarrow \quad 2t + 1 = 0 \Rightarrow t = -\frac{1}{2}$$

4. (d) The given system of equations will have a non-trivial solution if

$$\begin{vmatrix} \alpha+a & \alpha & \alpha \\ \alpha & \alpha+b & \alpha \\ \alpha & \alpha & \alpha+c \end{vmatrix} = 0$$

Operate $R_2 \to R_2 - R_1; R_3 - R_3 - R_1$, then

$$\begin{vmatrix} \alpha+a & \alpha & \alpha \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$

$\Rightarrow \quad \alpha(bc + ca + ab) + abc = 0$

Since, $a, b, c \neq 0$

$$\therefore \quad \frac{1}{\alpha} = -\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

5. (d) For non trivial solution $\begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix} = 0$

$\Rightarrow \sin^2\theta\cos\theta + \lambda^2\cos\theta + (\lambda+1)^2\cos\theta - \sin\theta\lambda(\lambda+1) + \cos^3\theta + \sin\theta\lambda(\lambda+1) = 0$

$\Rightarrow \quad (\sin^2\theta + \cos^2\theta)\cos\theta + \lambda^2\cos\theta + (\lambda+1)^2\cos\theta = 0$

$\Rightarrow \quad \cos\theta + \lambda^2\cos\theta + (\lambda+1)^2\cos\theta = 0$

$\Rightarrow \quad 2\cos\theta(\lambda^2 + \lambda + 1) = 0$

$\Rightarrow \quad \cos\theta = 0$

$$\Rightarrow \quad \theta = (2n+1)\frac{\pi}{2}, \lambda \in R, n \in I$$

6. (b) $\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & -3 \end{vmatrix} = 0$

Hence for atleast one solution

if $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\therefore \quad \Delta_1 = \begin{vmatrix} a & -2 & 1 \\ b & 1 & -2 \\ c & 3 & -3 \end{vmatrix} = 0$$

$\Rightarrow \; a - b + c = 0$

From $\Delta_2 = 0$ and $\Delta_3 = 0$, we get the same condition.

7. (b) Let $\Delta = \begin{vmatrix} \sin A & 1 & 1 \\ 1 & \sin B & 1 \\ 1 & 1 & \sin C \end{vmatrix}$

Apply $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

Then expand along C_1, we get

$\Delta = \sin A(1 - \sin B)(1 - \sin C) + (1 - \sin A)(1 - \sin B) + (1 - \sin A)(1 - \sin C)$

Since A, B, C are angles of a triangle, $0 < \sin A, \sin B, \sin C \leq 1$

$\Rightarrow \; \Delta \neq 0$

Multiple Correct Answers Type

8. (a, b)

The given system of equations is

$x + 2y + z = 1 \quad (1)$

$x + 3y + 4z = k \quad (2)$

$x + 5y + 10z = k^2 \quad (3)$

Subtracting (1) from (2), we get $y + 3z = k - 1$

Subtracting (2) from (3), we get $2y + 6z = k^2 - k$

$\Rightarrow \; 2(k - 1) = k^2 - k$

$\Rightarrow \; k^2 - 3k + 2 = 0$

$\Rightarrow \; k = 1, k = 2$

For $k = 1$;

$y + 3z = 0$ and $y = -3z$

$\Rightarrow \; x - 6z + z = 1$ (from (1))

$\Rightarrow \; x = 1 + 5z$

One set of solution $(1 + 5\lambda, -3\lambda, \lambda)$ where λ is a variable parameter.

For $k = 2$

$y + 3z = 1$ and $y = 1 - 3z$

$x + 2 - 6z + z = 1$ (from (1))

$\Rightarrow \; x = 5z - 1$

$\therefore$ Another set of solution $(5\lambda - 1, 1 - 3\lambda, \lambda)$ where λ is a variable parameter.

9. (b, c, d)

For the system to have nontrivial solution

$$\begin{vmatrix} \sin\theta & -2\cos\theta & -a \\ -1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

i.e. $\sin\theta + 4\cos\theta = 3a$

$\Rightarrow \; -\dfrac{\sqrt{17}}{3} \leq a \leq \dfrac{\sqrt{17}}{3}$

$\Rightarrow$ a has three integer values.