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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60_NUCLEUS & STERLING_BT **Paper -2**(Adv-2020-P2-Model) **Date: 20-08-2023**

Time: 02.00Pm to 05.00Pm **CTA-02** **Max. Marks: 198**

KEY SHEET

PHYSICS

1	**2**	2	**4**	3	**8**	4	**8**	5	**2**	6	**1**
7	**BCD**	8	**ABC**	9	**AC**	10	**AC**	11	**ABCD**	12	**ABCD**
13	**291.6**	14	**37**	15	**24**	16	**127**	17	**1.5**	18	**11**

CHEMISTRY

19	**3**	20	**5**	21	**3**	22	**5**	23	**5**	24	**5**
25	**AB**	26	**ABD**	27	**CD**	28	**BD**	29	**ABCD**	30	**ABCD**
31	**3**	32	**211**	33	**3526**	34	**3421**	35	**630**	36	**0.75**

MATHEMATICS

37	**4**	38	**6**	39	**1**	40	**0**	41	**5**	42	**3**
43	**BD**	44	**ACD**	45	**AB**	46	**BC**	47	**CD**	48	**ACD**
49	**1.5**	50	**6.5**	51	**6.8**	52	**2.66 or 2.67**	53	**1**	54	**3**

SOLUTIONS

PHYSICS

1. Net dispersion $\delta_{net} = (\mu - 1)A[\omega - \omega']$.
Evidently $\delta_{net} = O$ when $\omega' = \omega$.
$\therefore \omega = 0.005$.
Also, comparing $\delta_{net} = [-(\mu-1)A]\omega' + (\mu-1)A\omega$ with $y = mx + c$, the 'y-intercept' comes out to be $(\mu-1)A\omega = 0.010^\circ$.
Now, deviation only due to the first prism is

$$(\mu-1)A = \frac{(\mu-1)A\omega}{\omega}$$

$$= \frac{0.010^\circ}{0.005} = 2^\circ.$$

2. $N = \dfrac{(\mu-1)t}{\lambda}$
i.e., $N = \left(\dfrac{t}{\lambda}\right)\mu - \dfrac{t}{\lambda}$, which is a straight line
$\therefore$ Slope of the line $= \dfrac{t}{\lambda}$
$\therefore \dfrac{t}{600 \times 10^{-9} m} = \dfrac{40}{(2.00 - 1.00)}$
Or, $t = 24\mu m$

3. d sinφ = nλ ………… (1)
d sin(φ+dφ) = (n+1)λ ……….. (2)
(2) –(1) gives
d cos φ . dφ = λ …….. (3)
since Tan φ = y/ D
and differentiating here dφ = β/ 4D and φ = π/3
on substitutions in 3 we get β= 8λD/d

4. Let the radii of the spheres be R, R+a, R+2a and R+3a where a is a constant and the specific heat capacities be
S, Sr, Sr^3 where r is another constant.
$\therefore$ Given, $\left(\dfrac{heat\ capacity\ of\ D}{heat\ capacity\ of\ B}\right):\left(\dfrac{heat\ capacity\ of\ C}{heat\ capacity\ of\ A}\right) = 8:27$

Or $\left[\dfrac{(R+3a)^3 sr^3}{(R+a)^3 sr}\right]:\left[\dfrac{(R+2a)^3 sr^2}{Rs}\right] = 8:27$

Or $\left(1 + \dfrac{2a}{R+a}\right):\left(1 + \dfrac{2a}{R}\right) = 2:3$ Or $\dfrac{2a}{R+a} : \dfrac{2a}{R} = 1:2$

Or $R = a$

$$\frac{m_2}{m_1}=\frac{\frac{4}{3}\pi(R+R)^3\rho}{\frac{4}{3}\pi(R)^3\rho}=\frac{8}{1}.$$

5. The equation of the curve is,

$$T^2=\frac{1}{V}$$

Or $$TV^{V2}=constt.$$

Comparing with $TV^{\gamma-1}=constt,\ \gamma=\frac{3}{2}$ i.e., $C_V=2R$

Using $C_V=\frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2}$, calculate $n_1:n_2$

6. Let the thermal conductivities of the rods AB, BC and BD be K, 2K and 3K respectively. Also, let their lengths be 2L, L and L.
If T be the required temperature of the junction B and assuming $T_1>T>T_2,T_3$, we have

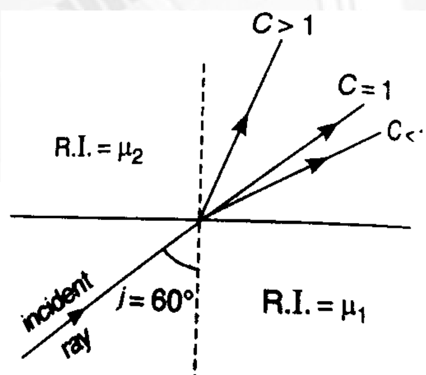
$$\left.\frac{\Delta Q}{\Delta t}\right]_{AB}=\left.\frac{\Delta Q}{\Delta t}\right]_{BC}+\left.\frac{\Delta Q}{\Delta t}\right]_{BD} \qquad \text{(Point rule)}$$

i.e., $$\frac{KA(T_1-T)}{2L}=\frac{2KA(T-T_2)}{L}+\frac{3KA(T-T_3)}{L}$$

$$\frac{(T_1-T)}{2}=2(T-T_2)+3(T-T_3)$$

or $T=\frac{1}{11}(T_1+4T_2+6T_3)$

7. Keeping the first medium fixed (i.e., $\mu_1=const.$) and $i=60^\circ$ (given), let us analyses the situation when μ_2 is varied and hence, $C=\frac{\mu_2}{\mu_1}$ also gets varied.



Initially when $C>1$
i.e., when $\mu_2<\mu_1$
Since $\mu\ge1$
$\therefore$ $\mu_{min}=1$
$\because$ $C=\frac{\mu_2}{\mu_1}$

$\Rightarrow \qquad C_1 = \dfrac{1}{\mu_1}$

From Snell's law,

$\dfrac{\sin 60^\circ}{\sin 90^\circ} = \dfrac{1}{\mu_1} = C_1 = \sqrt{3}/2$

At any instant i.e., when $i = 60^\circ, r = 90^\circ$

$|i - r| = 30^\circ = \pi/6$

When C is increased indefinitely i.e., when $\mu_2 >> \mu_1$

$$\frac{\sin i}{\sin r} >> 1$$

Or, $\sin r \to 0$

Or, $r \to 0$

$\therefore$ In that Situation, $\beta = |i - r|$

$$= |60^\circ - 0|$$

$$= \pi/3$$

When $\mu_2 = \mu_1$, i.e., When C = 1

I = r and hence $|i - r| = 0$

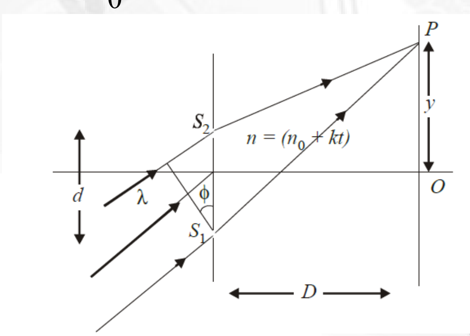
$\therefore$ The point C_2 corresponds to C = 1 at zero deflection.

8. (A) $S_1P - S_2P = \dfrac{dy}{D}$

$\Delta x = (n_0 + kt)\dfrac{dt}{D} - d\sin\phi = 0$

For central maxima.

$\therefore y = \dfrac{D\sin\phi}{n_0 + kt}$ (y-coordinates of central maximum).



(B) $\dfrac{-kD\sin\phi}{(n_0 + kt)^2}$ =velocity of central maximum

(C) For central maxima to be formed at O

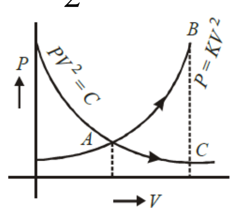
$n'\left(\dfrac{n}{n'} - 1\right)b = d\sin\phi$

Here $n' = n_0 + kt, n$=refractive index of plate.

$$n = n_0 + kt + \frac{d\sin\phi}{b}$$

9. According to first law of thermodynamics,
$dQ = dU + PdV$ (For one mole)
Now molar specific heat is given by
$$C = \frac{dQ}{dT} = \frac{dU + PdV}{dT} = \frac{C_V dT + (RT/V)dV}{dT}$$
$$= C_V + \left[\frac{RT}{V}\right]\left[\frac{dV}{dT}\right] \quad \text{....(i)}$$
Given, $T = T_0 e^{\alpha V}$
or, $dT = \alpha T_0 e^{\alpha V} dV$
$$\therefore \frac{dV}{dT} = \frac{1}{\alpha T_0 e^{\alpha V}} \quad \text{....(ii)}$$
Substituting the value of (dV/dT) from eq. (ii) in eq. (i),
We have
$$C = C_V + \left(\frac{RT}{V}\right)\left[\frac{1}{\alpha T_0 e^{\alpha V}}\right]$$
$$C = C_V + \left(\frac{RT_0 e^{\alpha V}}{\alpha V T_0 e^{\alpha V}}\right) = C_V + \frac{R}{\alpha V} \quad \text{.....(iii)}$$
(C) Given that $P = P_0 e^{\alpha V}$
$$\frac{RT}{V} = P_0 e^{\alpha V} \quad \text{or} \quad T = \frac{P_0}{R} V e^{\alpha V}$$
Now $C = C_V + \left[\frac{RT}{V}\right]\left[\frac{dV}{dT}\right]$
$$= C_V + \left(P_0 e^{\alpha V}\right)\left[\frac{R}{P_0 e^{\alpha V}(1+\alpha V)}\right] = C_V + \frac{R}{(1+\alpha V)}$$

10. From equation $PV = nRT$
$P_C < P_B$, $V_C = V_B$ and $T_B > T_C$
$W_{AB} > W_{AC}$
Also, $(T_B - T_A) > T_C - T_A$ $\therefore$ By 1[st] law, $Q = \Delta U + W$
$$Q = \frac{1}{2} nR\Delta T + W \; ; \; Q_{AB} > Q_{AC}$$



11. $v + u = D$ and $v - u = x$

$\Rightarrow v=\frac{D+x}{2}, u=\frac{D-x}{2}$ and $f=\frac{D^2-x^2}{4D}$

$m_1=\frac{D+x}{D-x}$, $m_2=\frac{D-x}{D+x}$

12. For maxima $d=n\lambda$
For minima $d=(n+1/2)\lambda$
For intensity $\frac{3}{4}$ the of maximum $d=\left(n\pm\frac{1}{3}\right)\frac{\lambda}{2}$

13. Let ρ_0 and V_0 be the density and volume of water displaced by the sphere, at $15^0 C$, when it just happens to get submerged into water.
From principle of floatation,
$V_0\rho_0=150+30=180$ …………………(i)
At a temperature of $35^0 C$, the volume of the sphere increases to $V_0(1+\gamma_a\times 20)$ and the density of water decreases to $\rho_0(1-\gamma_w\times 20)$. If m kg be the additional mass required for submergence, then

$$V_0(1+\gamma_a\times 20)\rho_0(1-\gamma_w\times 20)=(150+m)$$

Or $$V_0\rho_0[1-20(\gamma_w-\gamma_a)]=(150+m) \quad \text{…. (ii)}$$

Dividing Eqn. (ii) by (i), $1-20(\gamma_w-\gamma_a)=(150+m)$
Substituting for γ_w and γ_a and, adopting the required difference in mass $(30\text{-}m)=\Delta m$,

$$1-20(150-3\times 23)\times 10^{-6}=1-\frac{\Delta m}{180}$$

Or $\Delta m=20\times 81\times 180\times 10^{-3}g$ =291.6g.

14. Given, $\rho=\frac{V_3}{V_2}=2$ and γ for a monatomic gas =5/3.
Using, $\eta=1-\left(\frac{1}{\rho}\right)^{\gamma-1}$, we have, the required efficiency as

$$\eta=1-\left(\frac{1}{2}\right)^{\frac{5}{3}-1} \quad =1- 0.63 \quad = 0.37 \text{ or } 37\%$$

15. From geometry $SM^2 = XM \times MN \Rightarrow (XN - MN) \times MN$
$SM^2 \approx XN \times MN \quad \because \quad MN^2$ is negligible

$$XN=\frac{SM^2}{MN}=\frac{(3)^2}{5\times 10^{-1}} = 2R = \frac{9}{0.5} = 18 \Rightarrow R = 9 \text{ cm}$$

Using $\frac{1}{f} = (\mu - 1)\ \frac{1}{R}=\frac{1}{36}$

$$(\mu - 1) = \frac{R}{36}=\frac{9}{36}=\frac{1}{4} \qquad \Rightarrow \mu=1+\frac{1}{4}=\frac{5}{4}$$

$$v=\frac{c}{\mu}=\frac{3\times 10^8\times 4}{5} = 2.4\times 10^8 \text{ m s}^{-1} = 24\times 10^7 \text{ ms}^{-1} \qquad \therefore\ a = 24$$

16. $C = \sin^{-1}\left[\frac{1}{\mu}\right] = 37^0$

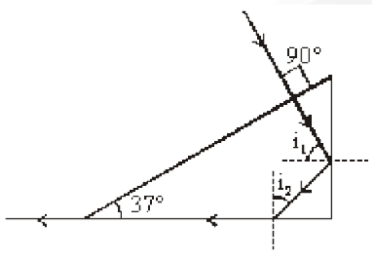
i_1 Can be seen to be > 37

$\therefore$ T.I.R. takes place.

i_2 Can be seen to be = 37

$\therefore$ Grazing emergence takes place.

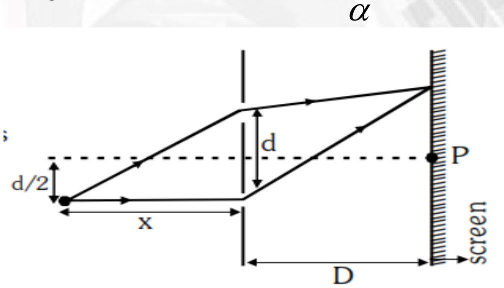
$\therefore$ deviation =127.



17. The central maxima $\frac{dy}{D} = \sqrt{d^2 + x^2} - x = \left[1 + \frac{d^2}{2x^2}\right] - x = \frac{d^2}{2x}$

$$y = \frac{Dd}{2x} \Rightarrow \frac{dy}{dt} = -\frac{Dd}{2x^2}\left(\frac{dx}{dt}\right) = \left(\frac{1\times 0.01}{2\times 0.5\times 0.5}\right)\times(0.001)0.02mm/s$$

$$\Rightarrow y = 2\times 10^{-3}cm/s \Rightarrow \frac{\beta}{\alpha} = 1.5$$



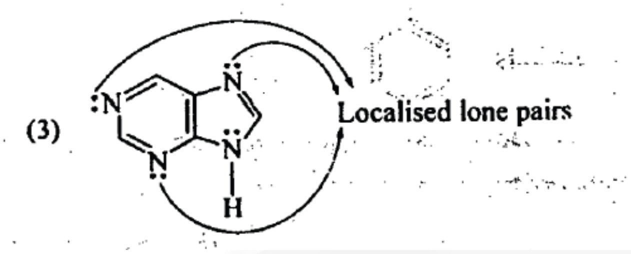
18. To find total energy of a given molecule of a gas we must find its degree of freedom. In molecule of oxygen, it has 2 atoms. So, it has degree of freedom 3T+2R=5, So total internal energy = 5/2RT per mole as gas O_2 is 2 mole so total internal energy of 2 mole oxygen $= \frac{2\times 5}{2}RT = 5RT$. Neon gas is mono atomic so its degree of freedom is only 3 hence total internal energy $= 3/2RT$ per mole

So, total internal energy of 4 mole $Ne = \frac{4\times 3}{2}RT = 6RT$

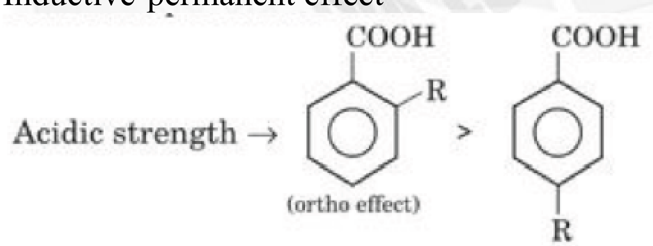
Total internal energy of 2 mole oxygen and 4 moles $Ne = 5RT + 6RT = 11RT$

CHEMISTRY

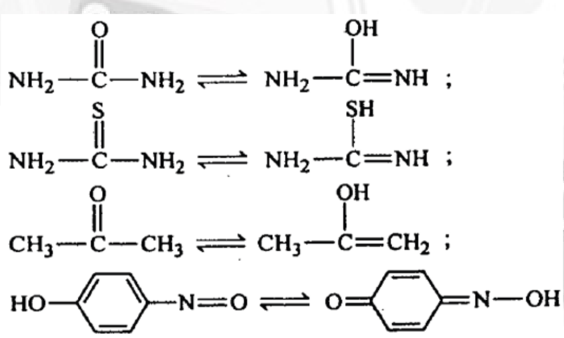
19.



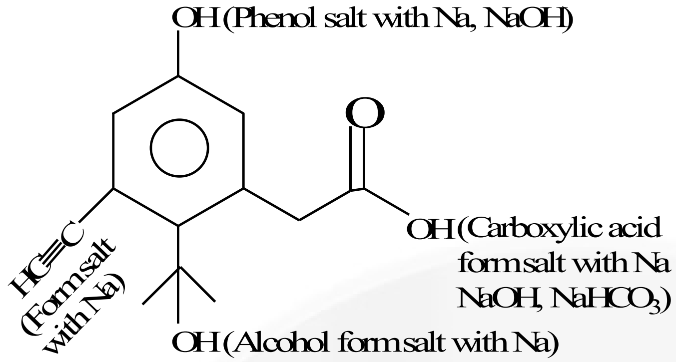
20. a,c,d,e,g
21. These are – phenylacetic acid, *o*-toluic acid, *m*-toluic acid and *p*-toluic acid.
22. Only unsymmetrical alkenes show peroxide effect with HBr. These are – propene, 1-butene, 2-methylpropene, 2-methyl-2-butene and 2-pentene.
23. These are – $CH_3C \equiv CH$, $CH_3CH_2C \equiv CH$, $CH_3C \equiv CCH_3$, $C_6H_5C \equiv CH$, and $CH_3CH_2CH_2C \equiv CH$, please note that $C_6H_5CH(OH)C \equiv CH$ gives $C_6H_5CH = CH\text{-}CHO$ and $C_6H_5C = CCH_3$ gives $C_6H_5COCH_2CH_3$
24. Terminal alkynes on treatment with ammoniacal CuCl give red precipitate of copper alkynide. These are – ethyne, propyne, 3-methyl-l-pentyne, l-butyne, ethynylbenzene.
25. It is homocyclic compound because atoms present in ring are carbons only and according to priority order –COOH is principal functional group and –CN, –OH and $-C_2H_5$ are substitution groups and it is having ethyl at 4th position.
26. Conceptual.
27. Inductive-permanent effect



28. (B) identical
(D) Not even isomer
29.



30. (A,B,C,D): All have finite dipole moments.
31. Conceptual.
32. Conceptual.
33. Conceptual.
34.



COOH COOH COOH COOH

35. Mass =20 gm

Volume = 200 ml

$$c = \frac{mass}{volume} = \frac{20}{200} = 0.1\frac{gm}{ml}$$

Length of tube (l)= 10 dm

Optical rotation $\theta = 30^0$

$$\alpha = \frac{\theta}{CXl}$$

$$\alpha = \frac{30}{0.1X10} = 30$$

(A) If solution is diluted to one litre then new volume

V= 1000ml

$\theta = ?$

$$\alpha = \frac{\theta}{CXl};\ 30 = \frac{\theta}{\frac{20}{1000}X10};\theta = 6°$$

(B) Specific rotation does not change with change in conc. Or volume or length etc. Therefore it is 30^0.

36. $\mu_{net} = 1.5D$

$\mu_{gauche} = 6.0$

$X_{anti} = ?$

$\mu_{net} = \mu_{gauche}X_{gauche} + \mu_{anti}X_{anti}$

$1.5 = 6.0X_{gauche} + 0$

$X_{gauche} = 0.25$

$X_{anti} = 1 - X_{gauche}$

$= 1 - 0.25 = 0.75$

MATHEMATICS

37. Given $f(x+5)\ge f(x)+5$ ....(1)

$f(x+1)\le f(x)+1$ ....(2)

From (1) and (2) we can write

$f(x)+5\le f(x+5)\le f(x+4)+1\le f(x+3)+2$

$\le f(x+2)+3\le f(x+1)+4\le f(x)+5$

$\therefore$ Equality exists everywhere

$\therefore\ f(x+1)=f(x)+1$ $\qquad f(1)=1(given)$

$\therefore\ f(2)=2, f(3)=3,......., f(2013)=2013$

$\therefore\ g(x)=f(x)+5-x\Rightarrow g(2013)=5$

38. $f'(x)\ge -2\forall x\in[-3,3]$

$$f''(x)dx\ge -2\int_{-3}^{3}dx\Rightarrow f'(x)\Big]_{-3}^{3}\ge -12\ \Rightarrow f'(3)-f'(-3)\ge -12$$

$\Rightarrow 0-12\ge -12\Rightarrow$ equality sign holds

$\therefore f''(x)=-2\forall x\in[-3,3]$

$f'(x)=-2x+c\Rightarrow f'(3)=c-6\Rightarrow c-6=0$ $\qquad c=6$

$\therefore f'(x)=-2x+6\ \Rightarrow f(x)=-x^2+6x+k.$

$f(0)=-4\Rightarrow k=-4$ $\qquad \therefore f(x)=6x-x^2-4$

$g(x)=\dfrac{-x^3}{3}+3x^2-4x$ in [-3,3]

$g'(x)=-x^2+6x-4$

$g(-3)=48$ Which is maximum value in [-3,3]

39. $f(x)=2\max\{|x^3-1|,|x^2-1|\}=2|x^3-1|$

40. As $x\to\infty, f(x)\to\infty\Rightarrow g(x)\to\infty$

Put x=f(t)

$$Limit=\lim_{t\to\infty}\frac{g(f(t))\ln f(t)}{f(t)}=\lim_{t\to\infty}\frac{t\ln(t(1+\ln t))}{t(1+\ln t)}=\lim_{t\to\infty}\frac{\ln(t(1+\ln t))}{(1+\ln t)}=1$$

41. $f'(x)=0, f(x)=1, f(x)=-1$ $\qquad \therefore f'(x)=0,$ for atleast 4 points

$f(x)=1,$ for atleast 5 points

$f(x)=-1,$ for atleast 2 points $\qquad$ So $\lambda=11$

42. f(x) is increasing on [-1,3] since $f'(x)>0\ \forall x\in[-1,3]$

and f(x) is decreasing in [-2,-1)

If f(x) has the smallest value at x=-1, then

$\lim_{h\to 0} f(-1-h)\ge f(-1)$

$\Rightarrow K^2-6K\le 0\Rightarrow K\in[0,6]$..................(1)

Also $K^2-6K+8>0 \Rightarrow K<2$ Or $K>4$ ……………..(2)

From (1) and (2) $K \in [0,2) \cup (4,6]$ $\therefore K=0,1,5,6$

Number of positive integers is 3.

43. $f(f(0))=f(2)=8-4+2=6$

$f(f(2))=f(6)=3-2(6)=-9$

$f(f(x))$ is discontinuous at 3 points

44. Conceptual.

45. $\frac{d}{dx}(P(x))+(x-1)^3-((P(x)+1))\geq 0$

$\Rightarrow e^{-x}\left(\frac{dP(x)}{dx}-P(x)+x^3-3x^2+3x-2\right)\geq 0$

$\Rightarrow \frac{d}{dx}\left(P(x)e^{-x}\right)-\frac{d}{dx}e^{-x}x^3-3\frac{d}{dx}xe^{-x}-\frac{d}{dx}e^{-x}\geq o$

$\Rightarrow \frac{d}{dx}\left(P(x)-\left(x^3+3x+1\right)\right)e^{-x}\geq 0$

Let $g(x)=\left(P(x)-(x^3+3x+1)\right)e^{-x}$ is increasing

$g(x)\geq g(0) \quad \Rightarrow \left(P(x)-(x^3+3x+1)\right)e^{-x}\geq 0 \forall x\geq 0$

But $P(x)\leq x^3+3x+1 \forall x\geq 0 \Rightarrow P(x)=x^3+3x+1 \forall x\geq 0$.

46. Let $\phi(x)=e^{-2x}f(x)$ $\phi'(x)=e^{-2x}(f'(x)-2f(x))$

Given $\phi''(x)>0 \forall x\in\left(0,\frac{1}{2}\right)=0$ and $\phi'\left(\frac{1}{4}\right)=0$

$\Rightarrow f'\left(\frac{1}{4}\right)=2f\left(\frac{1}{4}\right);\phi(0)=\phi\left(\frac{1}{2}\right)=0$ $\quad \Rightarrow f(x)<0 \ \forall x\in\left(0,\frac{1}{2}\right)$

So, $f'\left(\frac{3}{8}\right)-2f\left(\frac{3}{8}\right)>0$ $\quad f'\left(\frac{1}{2023}\right)<2f\left(\frac{1}{2023}\right)$

47. $f(x)=-x+1=1-x$

(A) $f(|x|)=1-|x|$, which is continuous $\forall x\in R$

(B) $f(x)=1-x \Rightarrow f^{-1}(x)=1-x \Rightarrow f(x)=f^{-1}(x)$ will have infinite solutions

(C) $(f(0))^2+(f(1))^2+....+(f(10))^2=1+0+1^2+2^2+3^2+....+9^2=1+285=286$

(D) $\tan^{-1}(f(x))=\tan^{-1}(1-x)$ which is derivable $\forall x\in R$

48. $f(x)=\sin^{-1}\left(1-2\sqrt{x}\right)+\cos^{-1}\left(2\sqrt{\sqrt{x}-x}\right)+\tan^{-1}\left(\frac{\sqrt{2}-1-\sqrt{x}}{1+\sqrt{2x}-\sqrt{x}}\right)$

Domain: $x\in(0,1)$

$f(x)=\sin^{-1}\left(1-2\sqrt{x}\right)+\sin^{-1}\left|2\sqrt{x}-1\right|+\tan^{-1}\left(\sqrt{2}-1\right)-\tan^{-1}\left(\sqrt{x}\right)$

$$f(x)=\begin{cases} 2\sin^{-1}\left(1-2\sqrt{x}\right)+\dfrac{\pi}{8}-\tan^{-1}\sqrt{x}, x\in\left(0,\dfrac{1}{4}\right] \\ \dfrac{\pi}{8}-\tan^{-1}\sqrt{x}, x\in\left(\dfrac{1}{4},1\right] \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{2}{\sqrt{1-\left(1-2\sqrt{x}\right)^2}}\left(\frac{-2}{\sqrt{x}}\right)-\dfrac{1}{1+x}.\dfrac{1}{2\sqrt{x}}; & \left(0,\frac{1}{4}\right) \\ -\frac{1}{1+x}.\frac{1}{2\sqrt{x}}; & \left(\frac{1}{4},1\right) \end{cases}$$

$$f'\left(\frac{1^+}{4}\right)=\frac{-4}{5}, f'\left(\frac{1^-}{4}\right)=\frac{-24}{5}, \qquad f'(x)<0$$

$\therefore$ f(x) is a decreasing function in (0,1). Minimum value of f(x) does not exist.

49. For $-1<x<1,\qquad \left\{-x^2\right\}=-x^2+1$

$$\alpha=\lim_{x\to 0}\cos^{-1}\left(\frac{-x^2+1}{x^2+2x+2}\right)=\cos^{-1}\left(\frac{1}{2}\right)=\pi/3$$

$$\beta=\lim_{x\to\infty}\tan^{-1}\left(\frac{e^{-x^2-x}-1}{2e^{-x^2-x}+1}\right)=\tan^{-1}\left(\frac{-1}{1}\right)=-\pi/4$$

$$\frac{1}{2}\left(1-\frac{\tan\beta}{\cos\alpha}\right)=\frac{1}{2}\left(1+\frac{1}{\left(\frac{1}{2}\right)}\right)=\frac{3}{2}=1.5$$

50. $f(x)=K(x-2)^2(x-3)^2+3$

Since $f(1)=7\Rightarrow f(x)=(x-2)^2(x-3)^2+3$

$\therefore f(x)$ has local maximum at $x=\dfrac{5}{2}$,

$$f(5/2)=\frac{49}{16}\Rightarrow p=49, q=16$$

51. $y=f(x)=\ln\left(\dfrac{1-x}{1+x}\right)$

$g=f^{-1}\Rightarrow g(f(x))=x$

$f(x)=\ln 3\Rightarrow x=-\dfrac{1}{2}$

$$g'(y)=\frac{1}{f'(x)}\Rightarrow g'(\ln 3)=-\frac{3}{8}$$

$$g''(y)=\frac{-f''(x)}{\left(f'(x)\right)^3}\Rightarrow g''(\ln 3)=\frac{3}{16}$$

$$g''(\ln 3)-g'(\ln 3)=\frac{9}{16} \qquad p=9, q=16.$$

52. $f(x)=2\tan^{-1}x, g(x)=x+2$

$$f\left(g(x)\right)=2\tan^{-1}(x+2) \qquad \Rightarrow \tan^{-1}(x+2)<\frac{1}{2}$$

$$x+2<\tan\frac{1}{2}\Rightarrow x<\tan\frac{1}{2}-2 \qquad \Rightarrow x\in\left(-10,\tan\frac{1}{2}-2\right)$$

As $\frac{1}{2}<\pi/6\Rightarrow \tan\frac{1}{2}<\frac{1}{\sqrt{3}}\Rightarrow \tan\frac{1}{2}-2<\frac{1}{\sqrt{3}}-2$

Total integers in the range $\{-9,-8,-7,-6,-5,-4,-3,-2\}=8$

$\lambda=8$.

53. $f(x)=\frac{\pi}{2}+\left|\operatorname{sgn}\left(\tan^{-1}\left(\frac{x}{1+x^2}\right)\right)\right|\tan^{-1}x,$

$$=\begin{cases}\frac{\pi}{2}+\tan^{-1}x, x>0\\ \frac{\pi}{2}, x=0\\ \frac{\pi}{2}+\tan^{-1}x, x<0\end{cases} \qquad \therefore\ f(x)=\frac{\pi}{2}+\tan^{-1}x\,\forall x\in R$$

$g(x)=\tan\left(x-\pi/2\right)$

Given equation: $g(x)=k\left(x-\pi/2\right) \quad \Rightarrow \tan\left(x-\pi/2\right)=k\left(x-\pi/2\right)$

$\therefore$Straight line $y=k(x-\pi/2)$ is passing through $(\pi/2,0)$. The line is tangent to the curve $y=g(x)$ at $\left(\pi/2,0\right)$, then slope is equal to 1, K=1

But if the slope is greater than 1, then the line intersect the curve $y=g(x)$ at three distinct points $K\in(1,\infty)$ $\therefore$ a =1.

54. $f'(x)=6\left(x^2-(\lambda+1)x+(2\lambda+1)\right)$

$\therefore$ f(x) has a positive point of local maxima

Therefore, the equation $f'(x)=0$ must have both roots positive and distinct real roots.

$D>0$

$(\lambda+1)^2-4(2\lambda+1)>0$

$\Rightarrow\lambda\in\left(-\infty,3-2\sqrt{3}\right)\cup\left(3+2\sqrt{3},\infty\right)$……………….(1)

So $f'(0)>0\Rightarrow\lambda>-\frac{1}{2}$…………………………(2)

And $\frac{\lambda+1}{2}>0\Rightarrow\lambda>-1$………………………..(3)

Also, $\lambda\in(-10,10)$…………………………….(4)

From (1), (2), (3), & (4) , the integral values of λ is 7, 8, 9.