Mathematics-Straight Lines
Synopsis

Intercept Form:

a) If the portion of the line intercepted between the axes is divided by the point (x_1, y_1) in the ratio m:n , then the equation of the line is $\frac{nx}{x_1}+\frac{my}{y_1}=m+n\ (or)\frac{mx}{x_1}+\frac{ny}{y_1}=m+n$

b) Equation of the line whose intercept between the axes is bisected at the point $(x_1, y_1)\, is\, \frac{x}{x_1}+\frac{y}{y_1}=2$

c) The equation of the line passing through the point (x_1, y_1) and whose intercepts are in the ratio $m:n\, is\ nx+my=nx_1+my_1\ (or)\, mx+ny=mx_1+ny_1$

Normal Form :

The normal form of a line $ax+by+c=0\ is\ \frac{(-a)}{\sqrt{a^2+b^2}}x+\frac{(-b)}{\sqrt{a^2+b^2}}y=\frac{c}{\sqrt{a^2+b^2}}$, if c>0 and $\frac{a}{\sqrt{a^2+b^2}}x+\frac{b}{\sqrt{a^2+b^2}}y=\frac{-c}{\sqrt{a^2+b^2}}, if\ c<0$

Distances

i) The perpendicular distance to the line $ax+by+c=0$

a) from origin is $\frac{|c|}{\sqrt{a^2+b^2}}$ b) from the point $(x_1, y_1)\ is\ \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

ii) The distance of a point (x_1, y_1) from the line $L\equiv ax+by+c=0$ measured along a line making an inclination α with x-axis is $\left|\frac{ax_1+by_1+c}{a\cos\alpha+b\sin\alpha}\right|$

iii) The distance of a point (x_1, y_1) from the line $L\equiv ax+by+c=0$ measured along a line which is parallel to $lx+my+n=0\ is\left|\frac{ax_1+by_1+c}{a\cos\alpha+b\sin\alpha}\right|\ where\ \tan\alpha=\frac{-l}{m}$

iv) The shortest distance between parallel lines $ax+by+c_1=0\ and\ ax+by+c_2=0\ is\ \frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$

v) The equation of a line parallel and lying midway between the above two lines is $ax+by+\frac{c_1+c_2}{2}=0$

Triangles and Quadrilaterals:

i) The area of triangle formed by the line $\frac{x}{a}+\frac{y}{b}=1$ with the co-ordinate axis is $\frac{1}{2}|ab|$

ii) The area of triangle formed by line $ax+by+c=0$ with the co-ordinate axes is $\frac{c^2}{2|ab|}$

iii) Area of rhombus formed by $a|x|+b|y|+c=0$ is $4(area\ of\ \Delta)=\frac{2c^2}{|ab|}$

iv) The area of triangle formed by $a_i x+b_i y+c_i=0, i=1,2,3\ is\ \frac{\Delta^2}{2|\lambda_1\lambda_2\lambda_3|}$ where

$$\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \lambda_1=\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}, \lambda_2=\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix}, \lambda_3=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

v) If p_1, p_2 are distances between parallel sides and θ is angle between adjacent sides of parallelogram then its area is $\left|\frac{p_1 p_2}{\sin\theta}\right|$

vi) Area of parallelogram whose sides are given by the line

$a_1x+b_1y+c_1=0\ ,a_1x+b_1y+c_2=0$

$a_2x+b_2y+d_1=0\ and\ a_2x+b_2y+d_2=0\ is\ \left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right|$

vii) Area of rhombus $=\frac{1}{2}d_1d_2\ where\ d_1, d_2$ are lengths of the diagonals

Angular bisectors of two straight lines

Condition Acute angle bisector obtuse angle bisector. In which angle origin lies

Condition	Acute angle bisector	Obtuse angle bisector	In which angle origin lies
$a_1a_2+b_1b_2>0$	$\frac{a_1x+b_1y+c_1}{\sqrt{a^2+b_1^2}}=-\frac{(a_2x+b_2y+c_2)}{\sqrt{a_2^2+b_2^2}}$	$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\frac{(a_2x+b_2y+c_2)}{\sqrt{a_2^2+b_2^2}}$	Obtuse angle
$a_1a_2+b_1b_2<0$	$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\frac{(a_2x+b_2y+c_2)}{\sqrt{a_2^2+b_2^2}}$	$\frac{a_1x+b_1y+c_1}{\sqrt{a^2+b_1^2}}=-\frac{(a_2x+b_2y+c_2)}{\sqrt{a_2^2+b_2^2}}$	Acute angle

Optimization

Let A and B are two points on same side of line $L\equiv ax+by+c=0$

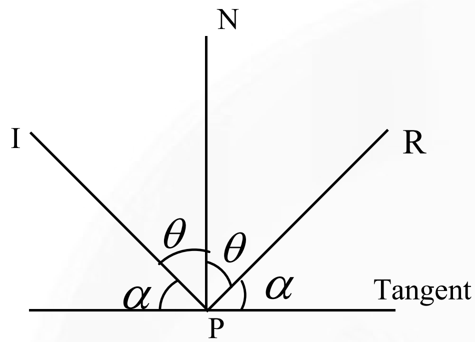
i) The point P such that $PA+PB$ is minimum, is intersection of L=0 and the line joining A to image of B or line joining B to image of A with respect to L=0

ii) The point is P such that $|PA-PB|$ is maximum, is the point of intersection of line L=0 ad line joining A and B.

Reflection in surface

IP= incident ray ; PN= normal to the surface, PR= reflected ray $\angle IPN = \angle NPR$

Angle of incident = Angle of reflection



No. of lines , no. of triangles and no. of circles

Number of lines drawn through the point A which are at a distance d from the point B

a) If AB=d then the no. of lines through A at a distance d from B is 1

b) If AB>d then the no. of lines through A at a distance d from B is 2

c) If AB<D then the no. of lines through A at a distance d from B is 0

Number of right angled triangles with fixed area in a circle depends on height h of the triangle and radius r of the circle

a) If h=r , no. of right angled triangles =2

b) If h< r , no. of right angled triangles =4

c) If h> r , no. of right angled triangles =0

Number of circles touching three lines

a) No circle if the lines are pairwise parallel

b) One circle if the lines are concurrent

c) 2 circles if two lines are parallel and third line cuts parallel line

d) 4 circles if the lines are not concurrent and no two of them are parallel

The line in the family of lines $L_1 + \lambda L_2 = 0$ which is at maximum distance from a point P is perpendicular to $\overline{PA}$, where A is point concurrence of the family of lines

Exercise : I

(Straight Objective Including PYQ's)

Various forms of straight lines

(Point-slope form, Two point form, Slope-intercept form, Intercept form, Normal form, Symmetric form (parametric form), General form of a straight line)

1. If the straight line , $2x-3y+17=0$ is perpendicular to the line passing through the points (7,17) and $(15,\beta)$, then β equals **Jee Mains-2019**

A) $\frac{35}{3}$ B) -5 C) $\frac{-35}{3}$ D) 5

Key: D

Sol:

Equation of st.line $y=\frac{2}{3}x+\frac{17}{3}, Slope=\frac{2}{3}$

Slope of line passing through (7,17) and $(15,\beta)$ is $\frac{\beta-17}{15-7}=\frac{\beta-17}{8}$

Since perpendicular ,

$m_1m_2=-1$

$\Rightarrow\left(\frac{2}{3}\right)\left(\frac{\beta-17}{8}\right)=-1\Rightarrow\beta=5$

1a. The vertices of a triangle are A(10,4), B(-4,9) and C(-2,-1). Find the equation of altitude through A

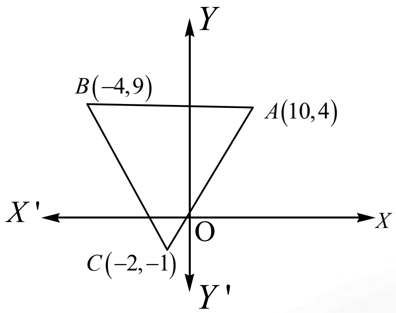
A) $x-5y+10=0$ B) $2x+5y=1$ C) $x-6y+10=0$ D) $x-5y-10=0$

Key: A

Sol:

Slope of $BC=\frac{-1-9}{-2+4}=-5$

Slope of AD altitude $=\frac{-1}{-5}=\frac{1}{5}$



Hence, equation of altitude AD which passes through (10,4) & having slope 1/5 is

$$y-4=\frac{1}{5}(x-10)$$

$$x-5y+10=0$$

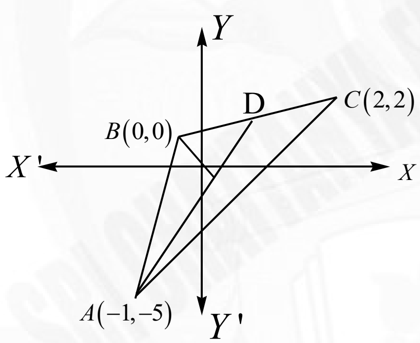
1b. Let ABC be a triangle with A(-1,-5),B(0,0), C(2,2) and let D be the middle point of BC. Find equation of perpendicular drawn from B to AD

A) $x+3y=0$ B) $x+4y=0$ C) $x-3y=0$ D) $x+5y=0$

Key: A

Sol:

D is the middle point of BC. Coordinates of D are $\left(\frac{0+2}{2},\frac{0+2}{2}\right)$



i.e, D(1,1)

Slope of median $AD=\frac{1+5}{1+1}=3$

Slope of BM which is perpendicular to $AD=\frac{-1}{3}$

Equation of line BM is

$$y-0=\frac{-1}{3}(x-0)$$

$$y=\frac{-1}{3}(x)\Rightarrow x+3y=0$$

1c. Find the equation of the straight line bisecting the line segment joining the points (5,3) and (4,4) and making an angle 45^0 with positive direction of x-axis.

A) $x-y-2=0$ B) $x-y-3=0$ C) $x-y-1=0$ D) $x-2y+5=0$

Key: C

Sol: m = slope of line = $\tan 45^0 = 1$

Let A be midpoint of (5,3) & (4,4), then coordinates of A are $\left(\frac{5+4}{2},\frac{3+4}{2}\right), i.e\left(\frac{9}{2},\frac{7}{2}\right)$

Hence, the required equation of the line is $y-\frac{7}{2}=1\left(x-\frac{9}{2}\right)\Rightarrow x-y-1=0$

2. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60^0 with the line $x+y=0$. Then an equation of the line L is **Jee Mains-2019**

A) $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2}$ B) $\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$

C) $\sqrt{3}x+y=8$ D) $x+\sqrt{3}y=8$

Key : A

Sol : If perpendicular makes an angle of 60^0 with line $x+y=0$, then the perpendicular makes an angle of 15^0 or 75^0 with x-axis. The equation of the line

$x\cos 75^0 + y\sin 75^0 = 4$

(or)

$x\cos 15^0 + y\sin 15^0 = 4$

$\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$

(or)

$\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2}$

3. Suppose that the points $(h,k)(1,2)$ and $(-3,4)$ lie on the line L_1. If a line L_2 passing through the points (h,k) and $(4,3)$ is perpendicular to L_1 then $\frac{k}{h}$

A) 3 B) $-\frac{1}{7}$ C) 0 D) $\frac{1}{3}$

Key : D **Jee Mains-2019**

Sol : $(h,k)(1,2)(-3,4)$ are collinear

$$\Rightarrow \begin{vmatrix} h & k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{vmatrix} = 0$$

$\Rightarrow -2h-4k+10=0 \Rightarrow h+2k=5$ ......(1)

$mL_1 = \frac{-1}{2}$; $mL_2 = \frac{K-3}{h-4}$

$mL_1.mL_2 = -1 \Rightarrow 2h-k=5$ ....(2)

From (1) and (2) $\frac{k}{h}=\frac{1}{3}$

4. If the straight line $2x-3y+17=0$ is perpendicular to the line passing through the points $(7,17)$ and $(15,\beta)$ then β equals

A) -5 B) $\frac{-35}{3}$ C) $\frac{35}{3}$ D) 5

Key : D

Sol : $\frac{17-\beta}{-8}\times\frac{2}{3}=-1$

$\beta=5$

5. Let PS be the median of the triangle with vertices $P(2,2), Q(6,-1)$ and $R(7,3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is **IIT-JEE 2000**

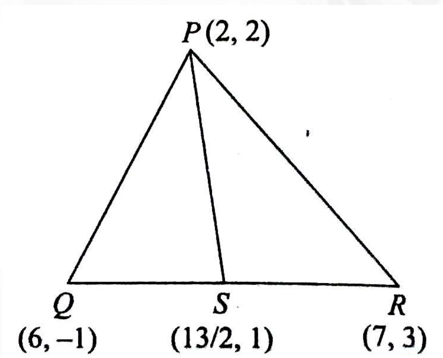
A) $2x-9y-7=0$ B) $2x-9y-11=0$

C) $2x+9y-11=0$ D) $2x+9y+7=0$

Key : D

Sol : S is the midpoint of Q and R. Therefore

$S\equiv\left(\frac{7+6}{2},\frac{3-6}{2}\right)\equiv\left(\frac{13}{2},1\right)$



Now, the slope of PS is $m=\frac{2-1}{2-13/2}=-\frac{2}{9}$

Then the equation of the line passing through $(1,-1)$ and parallel to PS is

$y+1=-\frac{2}{9}(x-1)$

Or $2x+9y+7=0$

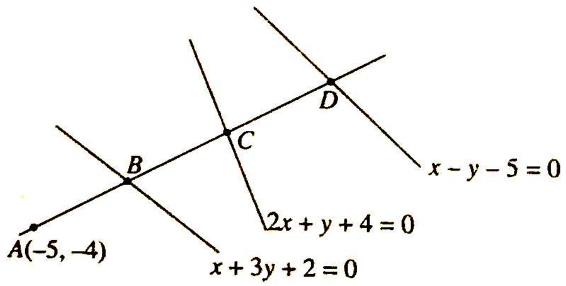
6. A line through $A(-5,-4)$ meets the lines $x+3y+2=0, 2x+y+4=0$, and $x-y-5=0$ at the points B,C, and D, respectively. If $(15/AB)^2+(10/AC)^2=(6/AD)^2$. The equation of the line

IIT-JEE 1993

A) $2x+3y+10=0$ B) $x-y+5=0$ C) $2x+3y+22=0$ D) $x+y+11=0$

Key : C

Sol:



Let θ be the inclination of line through $A(-5,-4)$

Therefore, equation of this line is $\frac{x+5}{\cos\theta} = \frac{y+4}{\sin\theta} = r$ …..(1)

Any point on this line at distance r from point $A(-5,-4)$ is given by

$(-5 + r\cos\theta, -4 + r\sin\theta)$

If $AB = r_1, AC = r_2$ and $AD = r_3$,then

$B(r_1\cos\theta - 5, r_1\sin\theta - 4)$

$C(r_2\cos\theta - 5, r_2\sin\theta - 4)$

$D(r_3\cos\theta - 5, r_3\sin\theta - 4)$

But B lies on $x + 3y + 2 = 0$. Therefore

$r_1\cos\theta - 5 + 3r_1\sin\theta - 12 + 2 = 0$

Or $\frac{12}{\cos\theta + 3\sin\theta} = r_1$

Or $\frac{15}{AB} = \cos\theta + 3\sin\theta$ ….(i)

As C lies on $2x + y + 4 = 0$, we have

$2(r_2\cos\theta - 5) + (r_2\sin\theta - 4) + 4 = 0$

Or $\frac{10}{Ac} = 2\cos\theta + \sin\theta$ …(ii)

Similarly, D lies on $x - y - 5 = 0$. Therefore,

$r_3\cos\theta - 5 - r_3\sin\theta + 4 - 5 = 0$

Or $r_3 = \frac{6}{\cos\theta - \sin\theta} = AD$

Or $\frac{6}{AD} = \cos\theta - \sin\theta$ …(iii)

Now, given that

$\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$

Or $(\cos\theta + 3\sin\theta)^2 + (2\cos\theta + \sin\theta)^2 = (\cos\theta - \sin\theta)^2$

[Using (i), (ii), and (iii)]

Or $4\cos^2 + 9\sin^2\theta + 12\sin\theta\cos\theta = 0$

Or $2\cos\theta + 3\sin\theta = 0$

Or $\tan\theta = -\frac{2}{3}$

Hence, the equation of the required line is

$y + 4 = -\frac{2}{3}(x+5)$

Or $3y + 12 = -2x - 10$

Or $2x + 3y + 22 = 0$

7. A rectangle PQRS, has its side PQ parallel to the line $y = mx$ and vertices P,Q and S on the lines $y = a, x = b$, and $x = -b$, respectively. The the locus of the vertex R is

IIT-JEE 1996

A) $(m^2 - 1)x - my + b(m^2+1) + am = 0$ B) $(m^2 - 1)x + am = 0$

C) $my + b(m^2+1) = 0$ D) $x + y + 1 = 0$

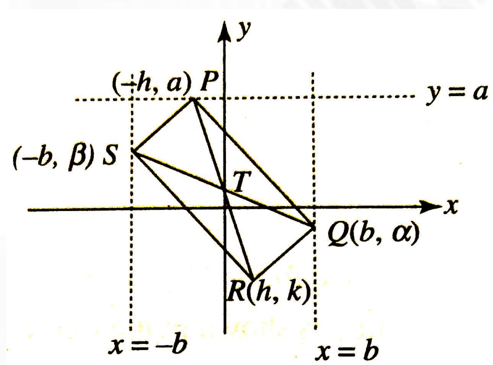
Key : A

Sol: Let the coordinate of Q (b, α) and that of S be $(-b, \beta)$.Let PR and SQ intersect each other at T.

$\because$ T is the mid point of SQ

Since diagonals of a rectangle bisect each other, x co-ordinates of P is –h.

As P lies on y=a, therefore coordinates of P are $(-h, a)$.



Given that PQ is parallel to $y = mx$ and slope of $PQ = m$

$\therefore \frac{\alpha - a}{a + h} = m$

$\Rightarrow \alpha = a + m(b+h)$ …(i)

Also, $RQ \perp PQ \Rightarrow$ slope of $RQ = \frac{-1}{m}$

$\therefore \frac{k-\alpha}{h-b} = \frac{-1}{m} \Rightarrow \alpha = k + \frac{1}{m}(h-b)$ …(ii)

From (i) and (ii), we get

$a + m(b+h) = k + \frac{1}{m}(h-b)$

$\Rightarrow am + m^2(b+h) = km + (h-b)$

$\Rightarrow \left(m^2-1\right)x-my+b\left(m^2+1\right)+am=0$

8. A straight line L through the origin meets the lines $x+y=1$ and $x+y=3$ P and Q, respectively. Through P and Q, two straight lines L_1 and L_2 are drawn, parallel to L_2 intersect at R. Then the locus of R, as L varies, is **IIT-JEE 2002**

A) straight line B) circle C) parabola D) ellipse

Key : A

Sol: The line $y=mx$ meets the given lines at

$P\left(\frac{1}{m+1},\frac{m}{m+1}\right)$ and Q $\left(\frac{3}{m+1},\frac{3m}{m+1}\right)$

Hence, the equation of L_1 is

$$\left(y-\frac{m}{m+1}\right)=2\left(x-\frac{1}{m+1}\right)$$

Or $y-2x-1=-\frac{3}{m+1}$ ...(i)

And that of L_2 is

$$\left(y-\frac{3m}{m+1}\right)=-3\left(x-\frac{3}{m+1}\right)$$

Or $y+3x-3=\frac{6}{m+1}$ ....(ii)

From (i) and (ii), eliminating m, we get

$$\frac{y-2x-1}{y+3x-3}=\frac{1}{2}$$

Or $x-3y+5=0$

Which is a straight line.

9. A straight line L with negative slope passes through the point $(8,2)$ and cuts the positive coordinate axes at point P and Q. Find the absolute minimum value of OP+OQ as L varies, where O is the origin. **IIT-JEE 2002**

A) 7 B) 10 C) 8 D) 18

Key : D

Sol: Let the equation of the line be

$(y-2)=m(x-8)$, where $m<0$

$\therefore\ P\equiv\left(8-\frac{2}{m},0\right)$ and $Q\equiv(0,2-8m)$

Now, $OP+OQ=\left|8-\frac{2}{m}\right|+|2-8m|$

$$=10+\frac{2}{-m}+(-8m)$$

$\geq 10+2\sqrt{\frac{2}{-m}\times(-8m)} \geq 18$

Using AM $\geq GM$

Position of a point w.r.t a line

(Ratio in which the line divides the line joining two points, Position of origin and a point w.r.t given line)

1. Let the point $P(\alpha,\beta)$ be at a unit distance from each of the two lines $L_1: 3x-4y+12=0$ and $L_2: 8x+6y+11=0$. If P lies below L_1 and above L_2, then $100(\alpha+\beta)=$____

A) -14 B) 42 C) -22 D) 14

Key: D

Jee Mains -2022

Sol: $3\alpha-4\beta+12>0$; $\frac{8\alpha+6\beta+11}{10}=1$

$3\alpha-4\beta=-7$ $8\alpha+6\beta=-1$

$\alpha=\frac{-23}{25}$, $\beta=\frac{106}{100}$

$100(\alpha+\beta)-=14$

1a. The range of α for which the points $(\alpha,\alpha+2)$ and $\left(\frac{3\alpha}{2},\alpha^2\right)$ lies on opposite sides of the line $2x+3y-6=0$

A) $(-\infty,-2)$ B) $(0,1)$

C) $(-\infty,-2)\cup(0,1)$ D) $(-\infty,1)\cup(2,\infty)$

Key: C

Sol : Points lie on opposite sides of the line $\Rightarrow L_{11}L_{22}<0$

$5\alpha\left(3\alpha+3\alpha^2-6\right)<0$

$\alpha(\alpha+2)(\alpha-1)<0$

$\alpha\in(-\infty,-2)\cup(0,1)$

1b. If $P\left(1+\frac{t}{\sqrt{2}},2+\frac{t}{\sqrt{2}}\right)$ be any point an a line then the range of values of 't' for which the point P lies between the parallel lines $x+2y=1$ and $2x+4y=15$ is

A) $\frac{-4\sqrt{2}}{5}<t<\frac{5\sqrt{2}}{6}$ B) $\frac{-4\sqrt{2}}{3}<t<\frac{5\sqrt{2}}{6}$ C) $t<\frac{-4\sqrt{2}}{3}$ D) $t<\frac{5\sqrt{2}}{6}$

Key :B

Sol : $f(x,y)=x+2y-1$, $g(x,y)=2x+4y-15$

$f(0,0)=-1<0$, $g(0,0)=-15<0$

$$P=\left(1+\frac{t}{\sqrt{2}},2+\frac{t}{\sqrt{2}}\right)$$

$f(p)>0$: $g(p)<0$

$1+\frac{t}{\sqrt{2}}+2\left(2+\frac{t}{\sqrt{2}}\right)>0$: $t<\frac{-5}{3\sqrt{2}}$

$t>\frac{-4\sqrt{2}}{\sqrt{3}}$

2. Consider the set of all lines $px+qy+r=0$ such that $3p+2q+4r=0$, which one of the following statements is true? **Jee Mains-2019**

A) The lines are concurrent at the point $\left(\frac{3}{4},\frac{1}{2}\right)$

B) Each line passes through the origin

C) The lines are parallel

D) The lines are not concurrent

Key: A

Sol:

Given condition

$3p+2q+4r=0$

$\Rightarrow \frac{3}{4}p+\frac{2}{4}q+r=0$

$Compare\ \ with\ \ px+qy+r=0$

$(x,y)=\left(\frac{3}{4},\frac{1}{2}\right)$

2a. The straight lines $4ax+3by+c=0\ where\ a+b+c=0$ are concurrent at the point

A) (4,3) B) (1/2, 1/3) C) (1/4,1/3) D) none of these

Key: C

Sol:

The set of lines is $4ax+3by+c=0\ where\ a+b+c=0$

Eliminating c will get

$4ax+3by-(a+b)=0$

$(or)\ a(4x-1)+b(3y-1)=0$

This passes through the intersection of the lines

$4x-1=0\ \&\ 3y-1=0, i.e.,\ x=\frac{1}{4}, y=\frac{1}{3}\Rightarrow i.e,\left(\frac{1}{4},\frac{1}{3}\right)$

2b. The straight lines $x+2y-9=0, 3x+5y-5=0\ \&\ ax+by-1=0$ are concurrent, if the straight line $35x-22y+1=0$ passes through the point

A) (a,b) B) (-a,-b) C) (b,a) D) none of these

Key: A

Sol:

The three lines are concurrent if $\begin{vmatrix} 1 & 2 & -9 \\ 3 & 5 & -5 \\ a & b & -1 \end{vmatrix} = 0, \Rightarrow 35a - 22b + 1 = 0$

Which is true if the line $35x - 22y + 1 = 0$ *passes through* (a, b)

2c. If the lines $ax + y + 1 = 0, x + by + 1 = 0 \,\&\, x + y + c = 0$ (a, b, c being distinct & different from *l*) are concurrent , then $\left(\frac{1}{1-a}\right) + \left(\frac{1}{1-b}\right) + \left(\frac{1}{1-c}\right) =$

A) 0 B) $\frac{1}{(a+b+c)}$ C) 1 D) none of these

Key: C

Sol:

As the given lines are concurrent

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0 (\because \quad Apply\ C_2 \to C_2 - C_1 \,\&\, C_3 \Rightarrow C_3 - C_1)$$

$$a(b-1)(c-1) - (c-1)(1-a) - (b-1)(1-a) = 0$$

$$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0 \left[Dividing\ by\ (1-a)(1-b)(1-c) \right]$$

Adding 1 on the both sides , we get $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

3. Let the point $P(\alpha, \beta)$ be at a unit distance from each of two lines $L_1 : 3x - 4y + 12 = 0$ and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2 then $100(\alpha + \beta) =$ **Jee Main-2022**

A) -14 B) 42 C) -22 D) 14

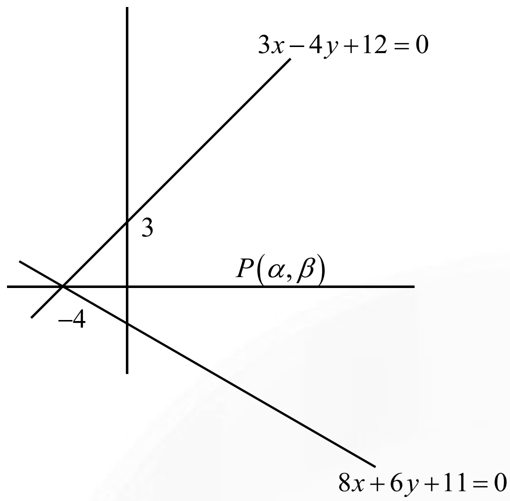
Key : D

Sol : $\frac{3\alpha - 4\beta + 12}{5} = 1$, $\qquad \frac{8\alpha + 6\beta + 11}{10} = 1$

$3\alpha - 4\beta = -7 \qquad 8\alpha + 6\beta = -1$

$\alpha = -\frac{23}{25}$, $\qquad \beta = \frac{106}{100}$

$100(\alpha + \beta) = 14$



3a. Let the point $P(\alpha,\beta)$ be at distance of 2 units from each of two lines $L_1: 6x-8y+48=0$ and $L_2: 4x+3y+12=0$. If P lies below L_1 and above L_2 then $50(\alpha+\beta)=$

A) 25 B) 75 C) 0 D) 50

Key : C

Sol : $\frac{6\alpha-8\beta+48}{-8}<0$, $\frac{4\alpha+3\beta+12}{3}>0$

$6\alpha-8\beta+48>0$, $4\alpha+3\beta+12>0$

$\frac{6\alpha-8\beta+48}{10}=2$, $\frac{4\alpha+3\beta+12}{5}=2$

$3\alpha-4\beta+14=0$, $4\alpha+3\beta+2=0$

$\alpha=-2$, $\beta=2$

$50(\alpha+\beta)=0$

3b. Let the point $P(\alpha,\beta)$ be at a distance of 3 units from each two line $L_1: 3x-4y+12=0$ and $L_2: 12x+5y+37=0$. If P lies below L_1 and above L_2 then $63(\alpha+\beta)=$

A) -7 B) 14 C) -14 D) 7

Key : A

Sol : $\frac{3\alpha-4\beta+12}{-4}<0$, $\frac{12\alpha+5\beta+37}{5}>0$

$3\alpha-4\beta+12>0$, $12\alpha+5\beta+37>0$

$\frac{3\alpha-4\beta+12}{5}=3$, $\frac{12\alpha+5\beta+37}{13}=3$

$3\alpha-4\beta-3=0$, $12\alpha+5\beta-2=0$

$\alpha=\frac{23}{63}$, $\beta=\frac{-30}{63}$

$63(\alpha+\beta)=-7$

3c. Let the point (α,β) be at a distance of 4 units from each of two line $L_1: 12x-5y+60=0$ and $8x+6y+48=0$. If P lies below L_1 and above L_2 then $14(\alpha+\beta)=$

A) 15 B) 14 C) -15 D) 17

Key : C

Sol : $\frac{12\alpha - 5\beta + 60}{13} = 4$

$12\alpha - 5\beta + 8 = 0$

$\frac{8\alpha + 6\beta + 48}{10} = 4$

$8\alpha + 6\beta + 8 = 0$

$4\alpha + 2\beta + 4 = 0$

$\alpha = \frac{-11}{14}$ $\beta = \frac{-4}{14}$

$14[\alpha + \beta] = -15$

4. The rang of α for which the points $(\alpha, \alpha+2)$ and $\left(\frac{3\alpha}{2}, \alpha^2\right)$ lies on opposite sides of the line $2x + 3y - 6 = 0$

A) $(-\infty, -2)$ B) $(0,1)$

C) $(-\infty, -2) \cup (0,1)$ D) $(-\infty, 1) \cup (2,2)$

Key : C

Sol: Points lie on opposite sides of the line

$\Rightarrow L_{11} L_{22} < 0$

$5\alpha\left(3\alpha + 3\alpha^2 - 6\right) < 0$

$\alpha(\alpha+2)(\alpha-1) < 0$

$\alpha \in (-\infty, -2) \cup (0,1)$

5. If the points $(3,4)$ and $(2,-5)$ were to be on the same side of the line $3x - 5y + a = 0$ then

A) $a = -31$ B) $a = 11$ C) $-31 < a < 11$ D) $a < -31$ or $a > 11$

Key : D

Sol: Points lie on same side of line

$\Rightarrow L_{11}.L_{22} > 0$

$(a-11)(a+31) > 0$

$a < -31$ or $a > 11$

6. If each of the points $(a,4), (-2,a)$ lies on the same side of the line joining the points $(2,-1), (5,-3)$ then 'a' is

A) $a \in \left[\frac{-11}{2}, \frac{5}{3}\right]$ B) $a \in \left[\frac{-11}{2}, \frac{4}{3}\right]$ C) $a \in R - \left[\frac{-11}{2}, \frac{5}{3}\right]$ D) $a = \frac{11}{2}, b = \frac{-5}{3}$

Key : C

Sol: Equation of line through $(2,-1)$, $(5,-3)$ is

$$y+1=\frac{-3+1}{5-2}(x-2)$$
$$2x+3y-1=0$$
$(a,4),(-2,a)$ lie same side
$$L_{11}.L_{22}>0$$
$$(2a+12-1)(-4+3a-1)>0$$
$$(2a+11)(3a-5)>0$$
$$a\in\left(-\infty,\frac{-11}{2}\right)\cup\left(\frac{5}{3},\infty\right)$$

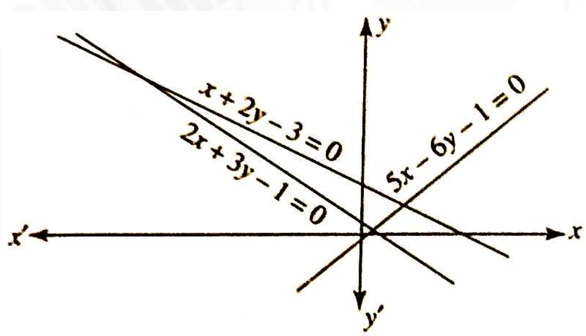
7. All the values of α for which the point (α,α^2) lies inside the triangle formed by the lines
$2x+3y-1=$
$x+2y-3=0$
$5x-6y-1=0$

A) $\left(\frac{-3}{2},-1\right)\cup\left(\frac{1}{2},1\right)$ B) $(-2,-1)$ C) $(2,4)$ D) $\left(-1,\frac{1}{2}\right)$

IIT-JEE 1992

Key : A

Sol: Given lines are $L_1:2x+3y-1=0$
$L_2:5x-6y-1=0$
$L_3:x+2y-3=0$
These lines can be drawn on coordinate axes as shown :



Clearly, from the figure, origin O and point $P(\alpha,\alpha^2)$ must lie on the opposite sides w.r.t $2x+3y-1=0$.
$L_1(0,0):-1<0$
$\Rightarrow L_1(\alpha,\alpha^2):2\alpha+3\alpha^2-1>0$
$\Rightarrow (3\alpha-1)(\alpha+1)>0$
$\Rightarrow \alpha<-1$ or $\alpha>1/3$ ….(i)
O and the point (α,α^2) must lie to the same side w.r.t $x+2y-3=0$. $L_3(0,0):-1<0$

$L_3(0,0): -1<0$

$\Rightarrow L_3(\alpha,\alpha^2): \alpha+2\alpha^2-3<0$

$\Rightarrow (2\alpha+3)(\alpha-1)<0$

$\Rightarrow -3/2<\alpha<1$. ….(ii)

Again O and P (α,α^2) must lie to the same side w.r.t $5x-6y-1=0$

$\Rightarrow L_2(0,0): -1<0$

$\Rightarrow L_2(\alpha,\alpha^2): 5\alpha-6\alpha^2-1<0$

$\Rightarrow 6\alpha^2-5\alpha+1>0$

$\Rightarrow (3\alpha-1)(2\alpha-1)>0$

$\Rightarrow \alpha<1/3$ or $\alpha>1/2$ …(iii)

From (i), (ii) and (iii) common values of α are $(-3/2,-1)\cup(1/2,1)$.

Distances to a line

(Perpendicular distance from a point, Distances of given point from the given line measured along a straight line, Distance between parallel lines)

1. The line, with the slope greater than one, passes through the point $A(4,3)$ and intersects the line $x-y-2=0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line :

A) $2x+y=9$ B) $3x-2y=7$ C) $x+2y=6$ D) $2x-3y=3$

Key : C **Jee Mains -2022**

Sol : Let $B(x_1,x_1-2)$

$$\sqrt{(x_1-4)^2+(x_1-2-3)^2}=\frac{\sqrt{29}}{3}$$

Squaring on both side $18x_1^2-162x_1+340=0$

$x_1=\frac{51}{9}$ or $x_1=\frac{10}{3}$; $y_1=\frac{33}{9}$ or $y_1=\frac{4}{3}$

Option C will satisfy $\left(\frac{10}{3},\frac{4}{3}\right)$

1a. The line with slope greater than one passes through point $A(5,6)$ and intersect the line $x+y-2=0$ at a point B. If the length of the line segment AB is $\sqrt{45}$ then B= ———

A) $(2,0)$ B) $(3,0)$ C) $(4,0)$ D) $(1,2)$

Key:A

Sol:

Let $B(h,2-h)$

AB=$(h-5)^2+(2-h-6)^2=45$

$\Rightarrow h^2-10h+25+h^2+8h+16=45$

$\Rightarrow 2h^2-2h+41=45$

$\Rightarrow 2h^2-2h-4=0$

$\Rightarrow 2h^2-2h-4=0 \Rightarrow (h-2)(h+1)=0 \Rightarrow h=-1$ or $\Rightarrow h=2$

$\therefore B(-1,3)$ or $B(2,0)$

$\therefore$ Slope of the line is >1

$\therefore B(2,0)$

1b. The line with slope less than 1 passes through $A(2,3)$ intersect the line $x-y+3=0$ at B length of the line segment AB is $\sqrt{10}$ then B= ———

A) $(-1,2)$ B) $(2,-2)$ C) $(2,-4)$ D) $(3,-4)$

Key:A

Sol:

Let $B(h,h+3)$

$AB^2=10 \Rightarrow (h-2)^2+h^2=10$

$\Rightarrow 2h^2-4h+4=10$

$\Rightarrow 2h^2-4h-6=10$

$\Rightarrow h^2-2h-3=10$

$\Rightarrow (h-3)(h+1)=0 \Rightarrow h=-1$ or $\Rightarrow h=3$

$B(-1,2)$ OR $B(3,6)$

$\therefore$ Slope of AB <1

$\therefore B(-1,2)$

1c. The equation of the line passing through $(1,2)$ and having distance of 7 units from pt $(8,9)$ is

A) $y=x-1$ B) $x=4$ C) $y=2$ D) $x+y=3$

Key :C

Sol : $y=mx+c \Rightarrow 2=m+c$

$c=2-m$

By verification from options,

$$\frac{8m-9+2-m}{\sqrt{m^2+1}}=7$$

$m-1=\sqrt{m^2+1}$

$2m=0$

$m=0$

$\therefore y=2$

2. If the line, $2x-y+3=0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x-2y+\alpha=0$ and $6x-3y+\beta=0$, respectively, then the sum of all possible values of α and β is

A) 20 B) 28 C) 30 D) 11

Key : C **Jee mains -2020**

Sol : $D_1: 6x-3y+p=0$

Distance between L_1 and L_2

$$\left|\frac{\alpha-6}{2\sqrt{5}}\right|=\frac{1}{\sqrt{5}} \Rightarrow |\alpha-6|=2$$

$\alpha=4,8$

Distance between L_1 and L_3 $\left|\frac{\beta-9}{3\sqrt{5}}\right|=\frac{2}{\sqrt{5}} \Rightarrow |\beta-9|=6$

$\Rightarrow \beta=15,3$

Sum of all values $=4+8+15+3=30$

2a. If the distance between the lines $y-x=2$ and $x-y=\alpha$ is $\sqrt{2}$ and the distance between the lines $4x-3y=5$ and $6y-8x=b$ is 11. Find absolute value of sum of all possible values of $\alpha \,\&\, \beta$

A) 12 B) 24 C) 0 D) 30

Key : B

Sol : a) $y-x=2$, $x-y=\alpha$

$x-y+2=0$, $x-y-\alpha=0$

Distance $=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}=\sqrt{2}$

$|2+\alpha|=2$

$2+\alpha=2$ $\quad$ $2+\alpha=-2$

$\alpha=0$ $\quad$ $\alpha=-4$

b) $4x-3y=5$, $\quad$ $6y-8x=\beta$

$4x-3y-5=0$ $\quad$ $8x-6y+\beta=0$

$4x-3y+\frac{\beta}{2}=0$

Distance $=\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}=11$

$$\frac{\left|\frac{\beta}{2}-(-5)\right|}{\sqrt{4^2+3^2}}=11$$

$$\frac{|\beta+10|}{10}=11$$

$$|\beta+10|=110$$

$\beta+10=110$ $\qquad$ $\beta+10=-110$

$\beta=100$ $\qquad$ $\beta=-120$

Sum of possible values of $\alpha\,\&\,\beta=0-4+100-120=-24$

Absolute value $=|-24|=24$

2b. If the line $5x+3y-7=0$ is at a distance α and β from the lines $15x+9y-20=0$ and $5x+3y+4=0$ respectively then $\alpha+\beta$______

A) $\frac{\sqrt{34}}{3}$ $\qquad$ B) $\frac{\sqrt{34}}{3\sqrt{3}}$ $\qquad$ C) $\frac{1}{3\sqrt{34}}$ $\qquad$ D) $\frac{11}{3\sqrt{34}}$

Key : A

Sol : a) $5x+3y-7=0$

$15x+9y-20=0$

$$5x+3y-\frac{20}{3}=0$$

$$\text{Distance }=\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}$$

$$\alpha=\frac{\left|\frac{-20}{3}+7\right|}{\sqrt{5^2+3^2}}$$

$$\alpha=\frac{-20+21}{3\sqrt{34}}\Rightarrow\alpha=\frac{1}{3\sqrt{34}}$$

b) $5x+3y-7=0$

$5x+3y+4=0$

$$\text{Distance }=\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}$$

$$\beta=\frac{|4+7|}{\sqrt{5^2+3^2}}$$

$$\beta=\frac{11}{\sqrt{34}}$$

$$\Rightarrow\alpha+\beta=\frac{1}{3\sqrt{34}}+\frac{11}{\sqrt{34}}=\frac{1+33}{3\sqrt{34}}=\frac{34}{3\sqrt{34}}=\frac{\sqrt{34}}{3}$$

2c. If the ratio in which the lines $3x+4y+2=0$ divides the distance between $3x+4y+5=0$ and $3x+4y-5=0$ is $\alpha:\beta$ then $\alpha+\beta=$____________

A) 15 $\qquad$ B) 13 $\qquad$ C) 10 $\qquad$ D) 20

Key : C

Sol : Given lines $\qquad$ $3x+4y+2=0\ \ldots\ldots$ (1)

$$3x+4y+5=0 \ldots\ldots.(2)$$
$$3x+4y-5=0 \ldots\ldots.(3)$$

Distance between 1 and 2

$$=\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}=\frac{|5-2|}{\sqrt{3^2+4^2}}=\frac{3}{5}$$

Distance between 1 and 3

$$=\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}=\frac{|-5-2|}{\sqrt{3^2+4^2}}=\frac{7}{5}$$

Ratio $\Rightarrow \alpha:\beta=\frac{3}{5}:\frac{7}{5}$

$\alpha:\beta=3:7$

$\alpha+\beta=3+7=10$

3. If p and q are the lengths of the perpendiculars from the origin on the lines $x\cos ec\alpha-y\sec\alpha=k\cot 2x$ and $x\sin\alpha+y\cos\alpha=k\sin 2\alpha$ then $k^2=$ ______

A) $4p^2+q^2$ B) $2p^2+q^2$ C) p^2+2q^2 D) p^2+4q^2

Key: A **Jee Mains - 2021**

Sol:

$$p=\frac{k\cot 2\alpha}{\sqrt{\cos ec^2\alpha+\sec^2\alpha}}=\frac{k}{2}\cos 2\alpha$$

$\Rightarrow 2P=k\cos 2\alpha$

$$q=\frac{k\sin 2\alpha}{\sqrt{\sin^2\alpha+\cos^2\alpha}}=k\sin 2\alpha$$

$q=k\sin 2\alpha$

$\therefore 4P^2+q^2=k^2$

3a. If p and q are the lengths of the perpendiculars from the origin on the lines $x\sec\alpha+y\operatorname{cosec}\alpha=k\cot 2\alpha$ and $x\cos\alpha+y\sin\alpha=k\sin 2\alpha$ then $k^2=$ ______

A) $2p^2+q^2$ B) $4p^2+q^2$ C) $3p^2+2q^2$ D) p^2+4q^2

Key: B

Sol:

$$p=\frac{k\cot 2\alpha}{\sqrt{\sec^2\alpha+co\sec^2\alpha}}=\frac{k\cot 2\alpha}{\sqrt{\frac{1}{\sin^2\alpha\cos^2\alpha}}}=\frac{k\cot 2\alpha\times\sin 2\alpha}{2}=\frac{k}{2}\cos 2\alpha$$

$\Rightarrow 2P=k\cos 2\alpha$

$$q=\frac{k\sin 2\alpha}{\sqrt{\sin^2\alpha+\cos^2\alpha}}=k\sin 2\alpha$$

$q=k\sin 2\alpha$

$\therefore 4P^2+q^2=k^2$

3b. If p and q are the lengths of the perpendiculars from the origin on the lines $x\,s\,ec\alpha - y\,\text{cosec}\,\alpha = \dfrac{k}{\cos 2\alpha}$ and $x\cos\alpha - y\sin\alpha = k\sec 2\alpha$ then $k^2 =$ ________

A) q^2+4p^2 B) q^2-4p^2 C) $4p^2-q^2$ D) $4p^2+q^2$

Key: B

Sol:

$$p=\frac{\frac{k}{\cos 2\alpha}}{\sqrt{\sec^2\alpha+\cos ec^2\alpha}}=\frac{k}{2\cos 2\alpha}\times\sin 2\alpha=\frac{k}{2}\tan 2\alpha$$

$$\Rightarrow 2P=k\tan 2\alpha$$

$$q=\frac{k\sec\alpha}{\sqrt{\cos^2\alpha+\sin^2\alpha}}=k\sec^2\alpha$$

$$\Rightarrow q=k\sec 2\alpha$$

$$q^2-4p^2=k^2$$

3c. If r and t are the lengths of the perpendiculars from the origin on the lines $x\cos\theta+y\sin\theta=a\cos 2\theta$ and $x\sec\theta+y\cos ec\theta=a$ then $a^2=$ ________

A) r^2+4t^2 B) $4t^2-r^2$ C) $4t^2+r^2$ D) $4r^2+t^2$

Key: A

Sol:

$$r=\frac{|a\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}=|a\cos 2\theta|$$

$$t=\frac{|a|}{\sqrt{\sec^2\theta+\cos ec^2\theta}}=|a|\sin\theta\cos\theta$$

$$\Rightarrow 2t=|a|\sin\theta\cos\theta$$

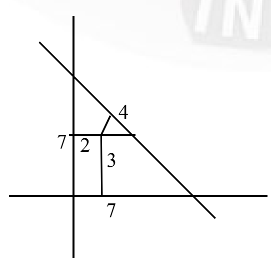
$$r^2+4t^2=a^2$$

4. Slope of a line passing through P(2,3) and intersecting the line $x+y=7$ at a distance of 4 units from P is **Jee Mains -2019**

A) $\dfrac{1-\sqrt{5}}{1+\sqrt{5}}$ B) $\dfrac{1-\sqrt{7}}{1+\sqrt{7}}$ C) $\dfrac{\sqrt{7}-1}{\sqrt{7}+1}$ D) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$

Key: B

Sol:



Since point at 4 units from A(2,3) will be A

$A=(4\cos\theta+2, 4\sin\theta+3)$ and this point will satisfy equation of line $x+y=7$

$\Rightarrow \cos\theta+\sin\theta=\frac{1}{2}$ on squaring

$\sin 2\theta=\frac{-3}{4} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta}=\frac{-3}{4}$

$\Rightarrow 3\tan^2\theta+8\tan\theta+3=0 \Rightarrow \tan\theta=\frac{1-\sqrt{7}}{1+\sqrt{7}}$

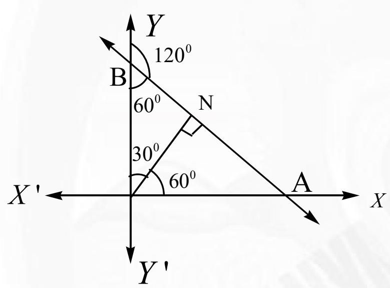
4a. The length of perpendicular from the origin to a line is 9 and the line makes an angle 120^0 with the positive direction of y-axis. Find the equation of the line

A) $x+y\sqrt{3}=18$ B) $\sqrt{3}x+y=18$ C) $3x+\sqrt{3}y=9$ D) $x+4y=16$

Key: A

Sol:

Here $\alpha=60, P=9$



Equation of required line is $x\cos 60^0+y\sin 60^0=9$

$x\left(\frac{1}{2}\right)+y\left(\frac{\sqrt{3}}{2}\right)=9 \Rightarrow x+y\sqrt{3}=18$

4b. Find the equation of the straight line cutting off an intercept of 3 units on negative direction of y-axis and inclined at an angle $\tan^{-1}\left(\frac{3}{5}\right)$ to the axis of x

A) $3x-5y-15=0$ B) $4x-3y-11=0$ C) $5x-3y-15=0$ D) $3x-11y+11=0$

Key: 1

Sol:

$C=-3 \,\&\, \theta=\tan^{-1}\left(\frac{3}{5}\right), \tan\theta=\frac{3}{5}=m$

Now, equation of line is $y=mx+c$

$y=\frac{3}{5}x-3 \Rightarrow 3x-5y-15=0$

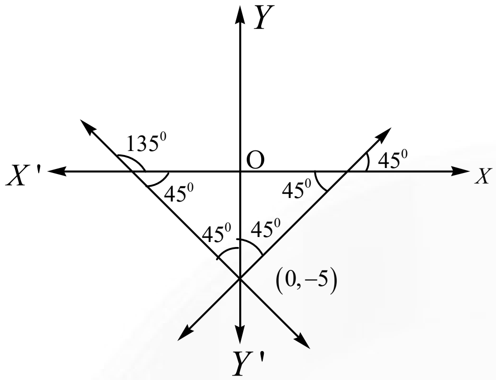
4c. Find the equation of the straight line cutting off an intercept of 5 units on negative direction of y-axis & being equally inclined to the axes

A) $y=\pm x-5$ B) $y=x\pm 6$ C) $y=x\pm 5$ D) $y=\pm x-11$

Key: 1

Sol:

Here, $C=-5, m=\tan 45^0 (or) \tan 135^0$; $m=\pm 1$



Hence, required equation is

$y=(\pm 1)x-5 \;\; \Rightarrow \; y=\pm x-5$

5. A point on the straight line, $3x+5y=15$, which is equidistant from the coordinate axes will be only in **Jee Mains - 2019**

A) 4^{th} quadrant
B) 1^{st} quadrant
C) 1^{st} & 2^{nd} quadrant
D) 1^{st} ,2^{nd} and 4^{th} quadrants

Key: C

Sol:

A point equidistant from both axes means either $y=x(or)y=-x$

Point lies on $3x+5y=15$

$x-y=0$

$3x+5y=15$

$\Rightarrow y=\frac{15}{8}, x=\frac{15}{8}$

$x+y=0$

$\Rightarrow y=\frac{+15}{2}, x=\frac{-15}{2}$

$(x,y)=\left(\frac{-15}{2},\frac{15}{2}\right)\in Q2$

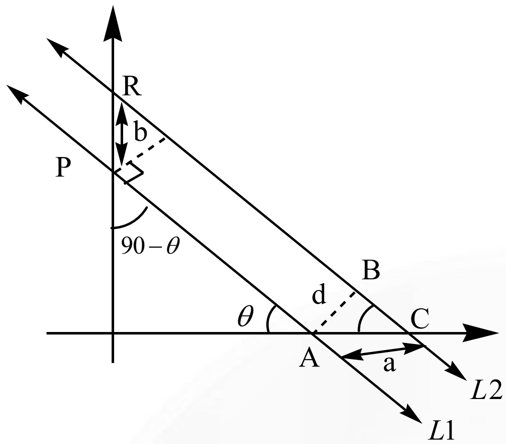
$(x,y)=\left(\frac{15}{8},\frac{15}{8}\right)\in Q1$

5a. Two parallel lines lying in the same quadrant make intercepts a and b on x,y axes respectively between them, then the distance between lines is

A) $\frac{ab}{\sqrt{a^2+b^2}}$
B) $\frac{1}{\sqrt{a^2+b^2}}$
C) $\sqrt{a^2+b^2}$
D) $\frac{1}{a^2}+\frac{1}{b^2}$

Key: A

Sol:



$In \quad \Delta ABC, \sin\theta = \dfrac{d}{a}, In \ \Delta PQR, \cos\theta = \dfrac{d}{b}$

$\sin^2\theta + \cos^2\theta = 1$

$\dfrac{d^2}{a^2} + \dfrac{d^2}{b^2} = 1 \Rightarrow d = \dfrac{ab}{\sqrt{a^2+b^2}}$

5b. All points lying inside the triangle formed by points (1,3),(5,0),(-1,2)satisfy

A) $2x+y-13=0$ B) $3x-4y-12\le 0$ C) $3x+2y\ge 0$ D) $4x+y=0$

Key: C

Sol: By inspection

5c. The equation of the line passing through (1,2) & having distance of 7 units from point (8,9) is

A) $y=3x-1$ B) $x=4$ C) $y=2$ D) $x+y=3$

Key: C

Sol: Distance from (8,9) to line $y-2=m(x-1)$ *is* $7. \Rightarrow m=0$

5d. The equation of locus of a point equidistant from the point (a_1,b_1) and (a_2,b_2) is $(a_1-a_2)x+(b_1-b_2)y+c=0$ then value of 'C' is

A) $\sqrt{a_1^2+b_1^2-a_2^2-b_2^2}$ B) $\dfrac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)$ C) $a_1^2+a_2^2+b_1^2-b_2^2$ D) $\dfrac{1}{2}(a_1^2+a_2^2+b_1^2+b_2^2)$

Key : B

Sol: $(x-a_1)^2+(y-b_1)^2=(x-a_2)+(y-b_2)^2$

$x^2+a_1^2-2a_1x+y^2+b_1^2-2b_1y=x^2+a_2^2-2a_2x+y^2+b_2^2-2b_2y$

$2(a_1-a_2)x+2(b_1-b_2)y+(a_2^2+b_2^2-a_1^2-b_1^2)=0$

Comparing this equation with given equation

$c=\dfrac{1}{2}(a_2^2+b_2^2-a_1^2-\mathbf{b}_1^2)$

6. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q. Then,

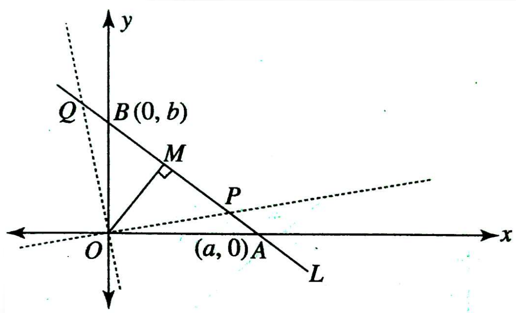
IIT-JEE 1990

A) $a^2+b^2=p^2+q^2$ B) $1/a^2+1/b^2=1/p^2+1/q^2$

C) $a^2+p^2=b^2+q^2$ D) $1/a^2+1/p^2=1/b^2+1/q^2$

Key : B

Sol :



As L has intercepts a and b on the axes, the equation of L is

$\frac{x}{a}+\frac{y}{b}=1$ ……(i)

Let the x-axis and the y-axis be rotated through an angle θ in the anticlockwise direction. In the new system, the intercepts are p and q $(OP=p, OQ=q)$. Therefore, the equation of L w.r.t new coordinates system becomes

$\frac{x}{p}+\frac{y}{q}=1$ ……(ii)

As the origin is fixed in rotation, the distance of line from the origin in both the cases should be same. Hence, we get

$$d=\left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|=\left|\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}\right|$$

Or $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

7. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is

IIT-JEE 1992

A) square B) circle

B) straight line D) two intersecting lines

Key : A

Sol : Let the two perpendicular lines be the coordinate axes. Let (x, y) be the point, sum of whose distances from two axes is 1. Then we must have

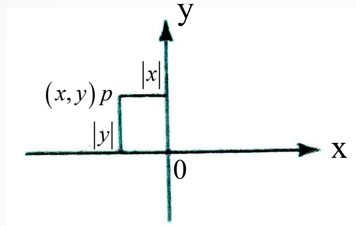
$|x|+|y|=1$

Or $= \pm x \pm y = 1$

The four lines are

$x+y=1, x-y=1, -x+y=1, -x-y=1$

Any two adjacent sides are perpendicular to each other. Also each line is equidistant from the origin. Therefore, the figure formed is a square.



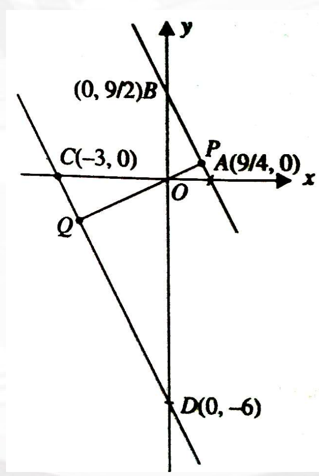
8. A straight line through the origin O meets the parallel lines $4x+2y=9$ and $2x+y+6=0$ at points P and Q. respectively. Then the point O divides the segment PQ in the ratio

IIT-JEE 2002

A) 1:2 B) 3:4 C) 2:1 D) 4:3

Key : B

Sol : Let any line through the origin meets the given lines at P and Q as shown in figure.



Now, from the figure, triangles OAP and OCQ are similar. Therefore,

$$\frac{OP}{OQ}=\frac{OA}{OC}=\frac{9/4}{3}=\frac{3}{4}$$

9. The algebraic sum of the perpendicular distance from the points $(2,0),(0,2)$, and $(1,1)$ to a variable straight line be zero. Then the line passes through a fixed point whose coordinates are__

A) $(1,1)$ B) $(2,2)$ C) $(1,2)$ D) $(2,1)$

IIT-JEE 1991

Key : A

Sol: Let the variable line be

$ax+by+c=0$ ....(1)

Then the perpendicular distance of the line from $(2,0)$ is

$$p_1=\frac{2b+c}{\sqrt{a^2+b^2}}$$

The perpendicular distance of the line from $(0,2)$ is

$$p_2=\frac{2b+c}{\sqrt{a^2+b^2}}$$

The perpendicular distance of the line from $(1,1)$ is

$$p_3=\frac{a+b+c}{\sqrt{a^2+b^2}}$$

According to question

$p_1+p_2+p_3=0$

Or $\dfrac{2a+c+2b+c+a+b+c}{\sqrt{a^2+b^2}}=0$

Or $3a+3b+3c=0$

Or $a+b+c=0$ ..... (ii)

From (i) and (ii), we can say that variable line (i) passes through the fixed point $(1,1)$.

Angle Between Two lines

(Condition for parallel & perpendicular lines)

1. The distance between the two points A and A' which lie on $y=2$ such that both the line segments AB and A'B (where B is the point $(2,3)$) subtend angle $\frac{\pi}{4}$ at the origin, is equal to:

A) 10 B) 48/5 C) 52/5 D)3

Key : C **Jee Mains -2021**

Sol : Let $A=(h,2);B=(2,3),O=(0,0);\angle AOB=\frac{\pi}{4}$

$m_1=$slope of $\overleftrightarrow{OA}=\frac{2}{h}$ and $m_2=$slope of $\overline{OB}=\frac{3}{2}$

$$\Rightarrow \tan\frac{\pi}{4}=\left|\frac{m_1-m_2}{1+m_1m_2}\right|\Rightarrow \pm1=\frac{\frac{2}{h}-\frac{3}{2}}{1+\frac{2}{h}.\frac{3}{2}}$$

$$\Rightarrow \pm1=\frac{4-3h}{2h+6}\Rightarrow h=10 \text{ or } \frac{-2}{5}$$

Hence $A=(10,2), A^1=\left(\frac{-2}{5},2\right)$

$\therefore \quad AA^1=\frac{52}{5}$

1a. The distance between two points A and A' while lie on $y=3$ such that both the line segments AB and $A'B$ (where B is the point $(4,5)$ subtend angle $\frac{\pi}{4}$ at $(0,0)$, is equal to ______

A) $\frac{82}{3}$ B) $\frac{84}{3}$ C) $\frac{84}{4}$ D) $\frac{85}{4}$

Key: A

Sol:

$A(h,3) \qquad 0,(0,0) \qquad B(4,5) \quad \underline{|AOB}=\frac{\pi}{4}$

Slope of OA=$\frac{3}{4}=m_1$

Slope of OB= $\frac{5}{4}=m_2$

$$Tan\frac{\pi}{4}=\left|\frac{\frac{3}{4}-\frac{5}{4}}{1+\frac{3}{4}.\frac{5}{4}}\right|$$

$$1=\left|\frac{12-54}{44+15}\right|$$

$44+15=12-5h$ or $-12+54$

$9h=-3 \qquad h=27$

$h=-\frac{1}{3}$

$A\left(-\frac{1}{3},3\right)$ and $A'(27,3)$

$AA'=27+\frac{1}{3}=\frac{82}{3}$

1b. The distance between two points A and A' while lie on $y=5$ such that both the line segments AB and $A'B$ (where B is the point $(6,7)$ subtend angle $\frac{\pi}{4}$ at equal to ______

A) $\frac{950}{1}$ B) $\frac{850}{13}$ C) $\frac{850}{19}$ D) $\frac{750}{19}$

Key: B

Sol:

$m_1=-\frac{5}{4} \qquad$ m=$\frac{1}{6}$

$$1=\left|\frac{\frac{5}{4}-\frac{7}{6}}{1+\frac{5}{4}\cdot\frac{7}{6}}\right| \Rightarrow 30-7h=\pm(6h+35)$$

$\Rightarrow 30-7h=6h+35$ or $30-7h=-6h-35$ $\quad \therefore A\left(-\frac{5}{13},5\right)$

$\Rightarrow 13h=-5 \Rightarrow h=-\frac{5}{13}$ $\quad h=65$ $\quad \therefore A'(65,5)$

$$AA'=65+\frac{5}{13}=815+\frac{5}{13}=\frac{850}{13}$$

2. The equation of one of the straight line which passes through the point $(1,3)$ and makes an angle $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line $y+1=3\sqrt{2}x$ is

A) $4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$

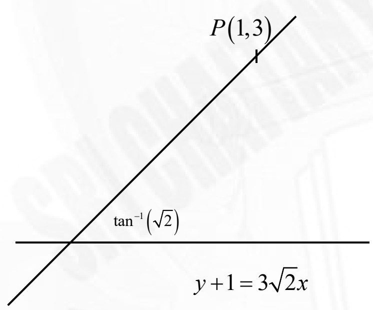
B) $5\sqrt{2}x+4y-\left(15+4\sqrt{2}\right)=0$

C) $4\sqrt{2}x+5y-4\sqrt{2}=0$

D) $4\sqrt{2}x+5y-\left(6+4\sqrt{2}\right)=0$

Key: A **Jee Mains -2021**

Sol:



Let 'm' be slope of line through P

Slope of line $y+1=3\sqrt{2}x$ is $3\sqrt{2}$

Given $\tan\theta=\sqrt{2}$

$$\sqrt{2}=\left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}m}\right|$$

$$35m^2+18\sqrt{2}m-16=0$$

$$m=\frac{-18\sqrt{2}\pm 38\sqrt{2}}{70} \Rightarrow m=\frac{2\sqrt{2}}{7},\frac{-4\sqrt{2}}{5}$$

If $m=\frac{2\sqrt{2}}{7} \Rightarrow y-3=\frac{2\sqrt{2}}{7}(x-1) \Rightarrow 2\sqrt{2x}-7y+21-2\sqrt{2}=0$

$$m=\frac{-4\sqrt{2}}{5} \Rightarrow y-3=\frac{-4\sqrt{2}}{5}(x-1) \Rightarrow 4\sqrt{2x}+5y+21-\left(15+4\sqrt{2}\right)=0$$

1a. The equation of one of the straight line which passes through the point $(1,-1)$ and makes an angle 30° with the straight line $x+y-2=0$ is

A) $\left(2+\sqrt{3}\right)x+y=1+\sqrt{3}$ B) $\left(2+\sqrt{3}\right)x+y+\left(1+\sqrt{3}\right)=0$

C) $\sqrt{3}x+y+1=0$ D) $x-\sqrt{3}y-1=0$

Key: A

Sol: Equation of line $y+1=m(x-1)$

Slope of $x+y-2=0$ is $m=-1$

$$\tan 30^\circ=\left|\frac{-1-m}{1-m}\right|\Rightarrow\frac{1}{\sqrt{3}}=\left|\frac{m+1}{1-m}\right|\Rightarrow\frac{1}{\sqrt{3}}=\pm\left|\frac{m+1}{1-m}\right|$$

$$\frac{1}{\sqrt{3}}=\frac{1+m}{1-m}, \qquad \frac{-1}{\sqrt{3}}=\frac{1+m}{1-m},$$

$$1-m=\sqrt{3}+\sqrt{3}m \qquad -1+m=\sqrt{3}+\sqrt{3}m$$

$$m=\frac{1-\sqrt{3}}{1+\sqrt{3}}=\sqrt{3}-2 \qquad m=\frac{1+\sqrt{3}}{1-\sqrt{3}}=-\left(2+\sqrt{3}\right)$$

$$m=-\left(2+\sqrt{3}\right)$$

Equation is $(y+1)=-\left(2+\sqrt{3}\right)(x-1)$

$$\left(2+\sqrt{3}\right)x+y=1+\sqrt{3}$$

1b. The equation of a one of the straight line which passes through the point $(2,-1)$ and makes an angle 45° with the straight line $2x+3y=1$ is

A) $x+5y-7=0$ B) $5x-y+9=0$

C) $5x+y-9=0$ D) $x-5y+7=0$

Key: C

Sol: Equation of line $(y+1)=m(x-2)$

Slope of $2x+3y-1=0$ is $m=-\frac{2}{3}$

$$\tan 45^\circ=\left|\frac{m+\frac{2}{3}}{1-\frac{2m}{3}}\right|\Rightarrow\left|\frac{3m+2}{3-2m}\right|\Rightarrow\frac{3m+2}{3-2m}=\pm 1$$

$$\frac{3m+2}{3-2m}=1, \qquad \frac{3m+2}{3-2m}=-1$$

$$3m+2=3-2m \qquad 3m+2=-3+2m$$

$$5m=1 \qquad m=-5$$

$$m=\frac{1}{5} \qquad m=-5$$

Equation is $(y+1)=\frac{1}{5}(x-2)$ $\qquad$ $y+1=-5(x-2)$

$$x-5y-7=0 \qquad 5x+y-9=0$$

1c. The equation of a one of the straight line which passes through the point $(2,3)$ and makes an angle $\tan^{-1}(3)$ with the straight line $y=2x+1$ is

A) $x-y-5=0$ B) $x+y-5=0$ C) $x-7y-21=0$ D) $x+7y-21=0$

Key: B

Sol:

Equation of line $y-3=m(x-2)$

Slope of $y=2x+1$ is $m=2$

$$\tan\theta=3\Rightarrow\left|\frac{2-m}{1+2m}\right|=3$$

$$\frac{2-m}{1+2m}=3, \qquad \frac{2-m}{1+2m}=-3$$

$$2-m=3+6m \qquad 2-m=-3-6m$$

$$7m=-1 \qquad -5m=1$$

$$m=-\frac{1}{7} \qquad m=-1$$

Equation is $(y-3)=-\frac{1}{7}(x-2)$ $\qquad y-3=-1(x-2)$

$$7y-21=-x+2 \qquad y-3=-x+2$$

$$x+7y-23=0 \qquad x+y-5=0$$

2. A straight line L through the point $(3,-2)$ is inclined at an angle 60^0 to the line $\sqrt{3}x+y=1$. If L also intersects the x-axis, the equation of L is

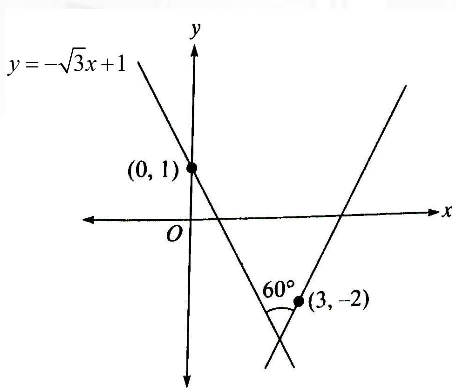
IIT-JEE 2011

A) $y+\sqrt{3}x+2-3\sqrt{3}=0$ B) $y-\sqrt{3}x+2+3\sqrt{3}=0$

C) $\sqrt{3}y-x+3+2\sqrt{3}=0$ D) $\sqrt{3}y+x-3+2\sqrt{3}=0$

Key : B

Sol :



Let the slope of the required line be m. Then

$$\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|=\sqrt{3}$$

$$\therefore\ m+\sqrt{3}=\pm\left(\sqrt{3}-3m\right)$$

$\therefore\ m=0$ or $m=\sqrt{3}$

Therefore, the equation of

$y+2=\sqrt{3}(x-3)$ ($m\neq 0$ as given that line cuts the x-axis)

Or $\sqrt{3}x-y-\left(2+3\sqrt{3}\right)=0$

3. For $a>b>c>0$, the distance between $(1,1)$ and the point of intersection of the liens $ax+by+c=0$ and $bx+ay+c=0$ is less than $2\sqrt{2}$. Then **IIT-JEE 2013**

A) $a+b-c>0$ B) $a-b+c<0$ C) $a-b+c>0$ D) $a+b-c<0$

Key : A

Sol : point of intersection $\left[\frac{-c}{a+b},\frac{-c}{a+b}\right]$

4. All chords of the curve $3x^2-y^2-2x+4y=0$, which subtend a right angle at the origin, pass through a fixed point. Then the coordinates of the point. **IIT-JEE 1991**

A) $(1,2)$ B) $(1,-2)$ C) $(-1,2)$ D) $(1,1)$

Key : B

Sol: The given curve is

$3x^2-y^2-2x+4y=0$

Let $y=mx+c$ be the chord of curve (i) which subtends an angle of 90^0 at the origin.

Then the combined equations of lines joining the point of intersection of curve (i) and chord to the origin can be obtained by making the equation of curve homogeneous with the help of the equation of chord as

$$3x^2-y^2-2x\left(\frac{y-mx}{c}\right)=0$$

Or $3cx^2-cy^2-2xy+2m^2+4uy^2-4mxy=0$

Or $(3c+2m)x^2-2(1+2m)xy+(4-c)y^2=0$

As the lines represented by this pair are perpendicular to each other, we must have

Coefficient of x^2 +coefficient of $y^2=0$

Hence, $3c+2m+4-c=0$

Or $-2=m+c$

Comparing this result with $y=mx+c$, we can see that $y=mx+c$ passes through $(1,-2)$.

Foot of Perpendicular and Image of a point w.r.t a line

1. Image of $P(3,5)$ on the line $y=x+1$ is Q. Then Q lies on

A) $(x-4)^2+(y-2)^2=4$ B) $(x-1)^2+y^2=4$

C) $x^2+y^2=4$ D) $x^2+(y-2)^2=4$

Key : A **Jee Mains-2021**

Sol : Image of $P(3,5)$ on the line $x-y+1=0$ is

$$\frac{x-3}{1}=\frac{y-5}{-1}=\frac{-2(3-5+1)}{2}$$

$x=4, y=4$

$\therefore$ image is $Q(4,4)$

Which lies on $(x-4)^2+(y-2)^2=4$

1a. Image of $P(1,-2)$ w.r.t straight line $2x-3y+5=0$ is Q, then Q lies on

A) $(x-1)^2+(y+2)^2=16$ B) $(x+3)^2+(y-2)^2=4$

C) $(x-3)^2+(y-2)^2=4$ D) $(x+3)^2+y^2=4$

Key : B

Sol: Image of $P(1,-2)$ on the line $2x-3y+5=0$

$$\frac{x-1}{2}=\frac{y+2}{-3}=\frac{-2[2(1)-3(-2)+5]}{2^2+(-3^2)}$$

$x=-3$ $y=4$

Image $Q(-3,4)$ lies on

$(x+3)^2+(y-2)^2=4$

1b. Image of $P(2,4)$ w.r.t straight line $x-y+4=0$ is Q then Q lies on

A) $x^2+(y-3)^2=9$ B) $(x-3)^2+(y-3)^2=9$

C) $(x-3)^2+y^2=9$ D) $x^2+y^2=9$

Key :A

Sol : Image of $P(2,4)$ on the line $x-y+4=0$

$$\frac{x-2}{1}=\frac{y-4}{-1}=\frac{-2(2-4+4)}{1+1}$$

$x=0$ $y=6$

Image $Q(0,6)$ lines on

$x^2+(y-3)^2=9$

1c. Image of $P(-6,8)$ w.r.t straight line $3x+2y-11=0$ is Q then Q lies on

A) $(x-4)^2+(y-4)=100$ B) $(x-4)^2+(y-4)^2=70$

C) $(x-4)^2+(y-4)^2=80$ D) $(x-4)^2+(y-4)^2=90$

Key : C

Sol : Image of $P(-6,8)$ on the line $3x+2y-11=0$

$$\frac{x+6}{3}=\frac{y-8}{2}=\frac{-2[3(-6)+2(8)-11]}{3^2+2^2}$$

$x=0$ $y=12$

Image $Q(0,12)$ lies on

$(x-4)^2+(y-4)^2=80$

2. $A(3,-1),B(1,3),C(2,4)$ are vertices of ΔABC if D is centrod of ΔABC and P is point of intersection of lines $x+3y-1=0$ and $3x-y+1=0$ then which of the following points lies on line joining D and P

A) $8x-11y+6=0$ B) $8x+11y-6=0$ C) $8x-11y-6=0$ D) $8x-11y=0$

Key : A **Jee mains -2020**

Sol : $D(2,2)$

Point of intersection $P\left(-\frac{1}{5},\frac{2}{5}\right)$

Equation of line DP

$8x-11y+6=0$

2a. Let $A(3,8),B(-1,2),C(6-6)$ be three vertices of ΔABC then find out the slope of BC and also the equation of line perpendicular to BC and passing through 'A'

A) $\frac{-7}{6},6x-7y+21=0$ B) $\frac{-3}{4},4x-3y+19=0$

C) $\frac{-8}{7},7x-8y+43=0$ D) $\frac{4}{9},9x-4y+33=0$

Key : C

Sol : $A(3,8),B(-1,2),C(6-6)$

a) Slope of BC

$m \Rightarrow \frac{y_2-y_1}{x_2-x_1}=\frac{-6-2}{6+1}=\frac{-8}{7}$

b) Equation of line $\perp r$ to BC and passing through

'A' = $\left(m_1 \times m_2=-1 \Rightarrow m_2=\frac{-1}{m_1}\right)$

$m_2=\frac{7}{8}$

$y-y_1=m_2(x-x_1)$

$y-8=\frac{7}{8}(x-3)$

$8y-64=7x-21$

$7x-8y+43=0$

2b. Let $A(-3,2)$ and $B(-2,1)$ be the vertices of a triangle ABC. If the centroid of triangle ABC lies on the line $3x+4y+2=0$. Which doesn't pass through any vertex, then the locus of vertex C is ?

A) $4x+3y+5=0$ B) $4x+3y+3=0$ C) $3x+4y+5=0$ D) $3x+4y+3=0$

Key : D

Sol : Let the third vertex be (h,k)

Given $A(-3,2)$ and $B(-2,1)$

Centroid of triangle is $\left(\frac{-3-2+h}{3},\frac{2+1+k}{3}\right)=\left(\frac{h-5}{3},\frac{k+3}{3}\right)$

Given centroid lies on the line $3x+4y+2=0$

$\frac{3(h-5)}{3}+\frac{4(k+3)}{3}+2=0$

$3h+4k+3=0$

The locus of vertex C is $3x+4y+3=0$

2c. Let D be the centroid of the triangle with vertices $(-1,3),(6,-3)$ and $(9,3)$. Let P be the point of intersection of the lines $3x+4y+2=0$ and $2x+3y-1=0$. Then the line passing through the points D and P is $\alpha x+\beta y-\gamma=0$ then $\frac{\gamma-\alpha-\beta}{11}=$

A) 0 B) 2 C) 3 D) -1

Key : C

Sol : D is the centroid of the triangle ABC

$A(-1,3),(6,-3),C(9,3)$

$D=\left(\frac{-1+6+9}{3},\frac{3-3+3}{3}\right)=\left(\frac{14}{3},1\right)$

P is the point of intersection of the lines

$3x+4y+2=0$ ……….(1)

$2x+3y-1=0$ ………..(2)

On solving (1) and (2), we get $P(-10,7)$

Equation of line DP is

$(y-1)=\left(\frac{7-1}{-10-\frac{14}{3}}\right)\left(x-\frac{14}{3}\right)$

$(y-1)=\left(\frac{18}{-44}\right)\left(x-\frac{14}{3}\right)$

$(y-1)=\left(\frac{-9}{22}\right)\left(x-\frac{14}{3}\right)$

$22y-22=-9x+42$

$9x+22y-64=0$

On comparing with $\alpha x+\beta y-\gamma=0$

$\alpha=9,\beta=22,\gamma=64$

$\therefore\ \frac{\gamma-\alpha-\beta}{11}=\frac{64-9-22}{11}$

$=\frac{64-31}{11}=\frac{33}{11}=3$

3. Let A be a fixed point $(0,6)$ and B be a moving point $(2t,0)$. Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of mid-point P of MC ________

A) $3x^2-2y-6=0$ B) $3x^2+2y-6=0$ C) $2x^2+3y-9=0$ D) $2x^2-3y+9=0$

Key: C **Jee Mains-2021**

Sol: M= Midpoint of AB=$(t,3)$

Slope of AB=$\frac{-6}{2t}=\frac{-3}{t}$

Equation of perpendicular bisector of AB is $y-3=\frac{t}{3}(x-t)$

Perpendicular bisector meets y-axis at

$\therefore C=\left(0,3-\frac{t^2}{3}\right)$

Let midpoint of MC=(h,k)

$\therefore (h,k)=\left(\frac{t}{2},3-\frac{t^2}{6}\right)$

$h=\frac{t}{2},\ k=3-\frac{t^2}{6}\ \Rightarrow k=3-\frac{4h^2}{6}$

Locus is $2x^2+3y-9=0$

3a. Let A be a fixed point $(0,3)$ and B be a moving point $(t,0)$. Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of mid-point P of MC ________

A) $8x^2+6y=9$ B) $8x^2-6y=9$ C) $6x^2+8y=-9$ D) $6x^2-8y=9$

Key: A

Sol:

M=$\left(\frac{t}{2},\frac{3}{2}\right)$ Slope of AB=$\frac{0-3}{t-0}=-\frac{3}{t}$

Equation of perpendicular bisector of AB is $y-\frac{3}{2}=\frac{t}{3}\left(x-\frac{t}{2}\right)$

Perpendicular bisector meets y-axis

X=0$\Rightarrow y-\frac{3}{2}=\frac{t}{3}\left(-\frac{t}{2}\right)=\frac{-t^2}{6}$

$\therefore C=\left(0,\frac{3}{2}-\frac{t^2}{6}\right)$

Let P be the midpoint of MC

$$\therefore P=\left(\frac{t}{4},\frac{\frac{3}{2}-\frac{t^2}{6}+\frac{3}{2}}{2}\right)=\left(\frac{t}{4},\frac{3-\frac{t^2}{6}}{2}\right)$$

Let $P=(h,k)\ \Rightarrow (h,k)=\left(\frac{t}{4},\frac{3}{2}-\frac{t^2}{12}\right)$

$\therefore t=4h,\qquad k=\frac{3}{2}-\frac{t^2}{12}$

$$k=\frac{3}{2}-\frac{16h^2}{12}$$

$$k=\frac{3}{2}-\frac{4h^2}{3}\Rightarrow 6k=9-8h^2$$

$\therefore$ Locus is $6y=9-8x^2$

3b. Let A be a fixed point $(3,0)$ and B be a moving point $(0,t)$. Let M be the mid-point of AB and the perpendicular bisector of AB meets the x-axis at C. The locus of mid-point of MC ________

A) $8y^2-6x=9$ B) $8y^2+6x=9$ C) $8y-6x^2=9$ D) $8x^2+6x^2=9$

Key: 2

Sol: M=$\left(\frac{3}{2},\frac{t}{2}\right)$ Slope of AB=$\frac{t-0}{0-3}=-\frac{t}{3}$

Equation of perpendicular bisector of AB is $y-\frac{t}{2}=\frac{3}{t}\left(x-\frac{3}{2}\right)$

Perpendicular bisector meets x-axis

y=0 $\Rightarrow \frac{-t}{2}=\frac{3}{t}\left(x-\frac{3}{2}\right)\Rightarrow x=\frac{3}{2}-\frac{t^2}{6}$

$$\therefore C=\left(\frac{3}{2}-\frac{t^2}{6},0\right)$$

Let P(h,k) be the midpoint of MC

$$\therefore (h,k)=\left(\frac{\frac{3}{2}+\frac{3}{2}-\frac{t^2}{6}}{2},\frac{\frac{t}{2}}{2}\right)$$

h=$\frac{3}{2}-\frac{t^2}{12}$, $\qquad k=\frac{t}{4}$

$$\therefore h=\frac{3}{2}-\frac{16k^2}{12}$$

$$\therefore h=\frac{3}{2}-\frac{4k^2}{3}$$

$\therefore$ Locus is $x=\frac{3}{2}-\frac{4y^2}{3}\ \Rightarrow 6x=9-8y^2$

3c. Let A be a fixed point $(1,0)$ and B be a moving point $(0,k)$. Let P be the mid-point of AB and the perpendicular bisector of AB meets the x-axis at C. The locus of mid-point of PC ________

A) $2x+8y^2=1$ B) $2x-8y^2=-1$ C) $4x-y^2=1$ D) $x+y=2$

Key: A

Sol: P=$\left(\frac{1}{2},\frac{k}{2}\right)$ Slope of AB=$\frac{k-0}{0-1}=-k$

Equation of perpendicular bisector of AB is $y-\frac{k}{2}=\frac{1}{k}\left(x-\frac{1}{2}\right)$

Bisector meets x-axis at C

$\therefore C=\left(\frac{1}{2}-\frac{k^2}{2},0\right)$

Let midpoint of PC=(p,q)

$\therefore (p,q)=\left(\frac{\frac{1}{2}+\frac{1}{2}-\frac{k^2}{2}}{2},\frac{\frac{k}{2}+0}{2}\right)$

$\therefore p=\frac{1}{2}-\frac{k^2}{4},\ q=\frac{k}{4}$

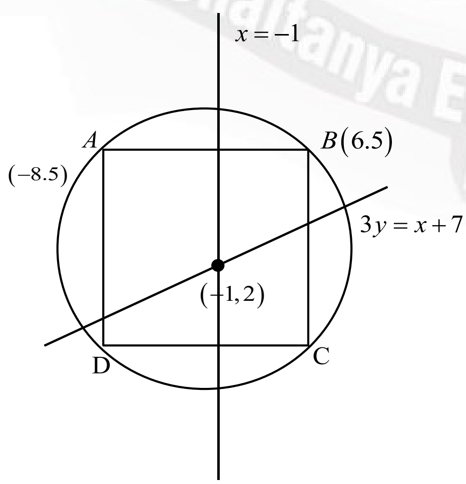
$\therefore p=\frac{1}{2}-\frac{16q^2}{4}\Rightarrow\frac{1}{2}-4q^2$

Locus is $x=\frac{1}{2}-4y^2$

4. A rectangle is inscribed in a circle with a diameter lying along the line $3y=x+7$.. If the two adjacent vertices of the rectangle are $(-8,5)(6,5)$ then the area of the rectangle is

A) 56 B) 84 C) 72 D) 98

Key : B **Jee Mains-2019**



Sol :

Perpendicular bisector of AB will pass from centre

Equation of bisector $x=-1$

Centre $(-1,2)$

Let $D=(\alpha,\beta)$ $\frac{\alpha+6}{2}=-1, \frac{\beta+5}{2}=2$

D= $D=(-8,-1)$, $AD=6, AB=14$

Area $=6\times 14=84$

4a. If the two lines $x+(a-1)y=1, 2x+a^2y=1\left(a\in R-\{0,1\}\right)$ are perpendicular, then the distance of their point of intersection from the origin is

A) $\sqrt{\frac{2}{5}}$ B) $\frac{\sqrt{2}}{5}$ C) $\frac{2}{5}$ D) $\frac{2}{\sqrt{5}}$

Key : A **Jee Mains-2019**

Sol : $m_1m_2=-1$

$\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right)=-1$

$\Rightarrow a^3-a^2+2=0$

$\Rightarrow (a+1)(a^2+2a+2)=0$

$a=-1$

Lines are $x-2y=1, 2x+y=1 \Rightarrow$ Point of intersection = $\left(\frac{3}{5},\frac{-1}{5}\right)$

Distance from origin $=\sqrt{\frac{9}{25}+\frac{1}{25}}=\sqrt{\frac{2}{5}}$

Centers of Triangle

(Centroid, Circumcentre, incentre, Ortho Centre)

1. The distance of the origin from the centroid of the triangle whose sides have the equations $x-2y+1=0$ and $2x-y-1=0$ and whose orthocenter is $\left(\frac{7}{3},\frac{7}{3}\right)$ is

A) $\sqrt{2}$ B) 2 C) $2\sqrt{2}$ D) 4

Key: C **Jee mains -2022**

Sol: $AB\equiv x-2y+1=0$, $AC=2x-y-1=0$

$A=(1,1)$

Attitude from B is $x+2y-7=0, B(3,2)$

Attitude from C is $2x+y-7=0\Rightarrow C(2,3)$

Centroid of $\Delta ABC=E(2,2)$

$OE=2\sqrt{2}$

1a. The line $2x+y=4$ meets x-axis at A and y-axis at B. The perpendicular bisector of AB meets the horizontal line through $(0,-1)$ at C. Let G be the centroid of ΔABC. The $\perp er$ distance from G to AB equal to

A) $\sqrt{5}$ B) $\frac{\sqrt{5}}{3}$ C) $2\sqrt{5}$ D) $3\sqrt{5}$

Key :A

Sol : Slope of CD is $\frac{1}{2}$

$C=(-5,-1)$

Perpendicular distance from

G to AB=$\frac{1}{3}$ (perpendicular distance from C to AB)

1b. If the straight lines $2x+3y-1=0$, $x+2y-1=0$ and $ax+by-1=0$ form a triangle with the origin as orthocentre then $(a,b)=$___

A) $(6,4)$ B) $(-3,3)$ C) $(-8,8)$ D) $(0,7)$

Key :C

Sol : The altitude $\overline{AD}$ is

$2x+3y-1+\lambda(x+2y-1)=0$

If passes $(0,0)$

$\lambda=-1$

$x+y=0$

$m_1m_2=-1$

$b=-a$

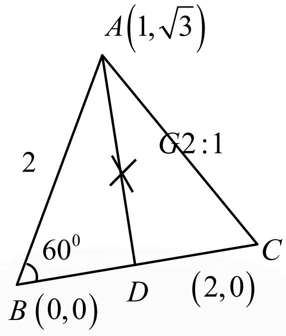
1c. In $\Delta ABC, B=(0,0), AB=2, \angle ABC=\frac{\pi}{3}$ and the middle point of BC has co-ordinates $(2,0)$. Then centroid of triangle is

A) $\left(\frac{5}{3},\frac{1}{3}\right)$ B) $\left(\frac{5}{3},\frac{1}{\sqrt{3}}\right)$ C) $\left(\frac{5}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ D) $\left(\frac{5}{\sqrt{3}},\frac{1}{3}\right)$

Key :B

Sol : Using symmetric form we get

$A=(1,\sqrt{3})$



G divides $\overline{AD}$ in the ratio =2:1

2. The equations of the sides AB,BC and CA of a triangle ABC are $2x+y=0$, $x+Py=39$ and $x-y=3$ respectively and $P(2,3)$ is its circumcentre. The which of the following is Not true ?

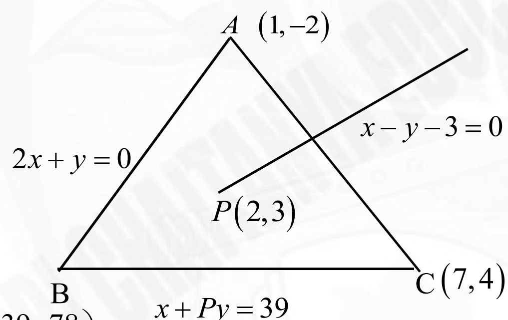
A) $(AC)^2 = 9P$ B) $(AC)^2 + P^2 = 136$

C) $32<\text{area}(\Delta ABC)<36$ D) $34<\text{area}(\Delta ABC)<38$

Key : D **Jee mains -2022**

Sol : Perpendicular bisector of AB

$x+y-5=0$



$\left(\frac{-39}{15},\frac{78}{15}\right)$

$$\frac{x-1}{1}=\frac{y+2}{1}=\frac{-2(-6)}{2}=6$$

$(x,y)=(7,4)$

$7+4P=39$

$4P=32$

$P=8$

$x+8y=39, y=-2x$

Solving $x=\frac{-39}{15}, y=\frac{78}{15}$

$AC^2=72=9P$

$AC^2+P^2=72+64=136$

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & -2 & 1 \\ 7 & 4 & 1 \\ \frac{-39}{15} & \frac{73}{15} & 1 \end{vmatrix}$

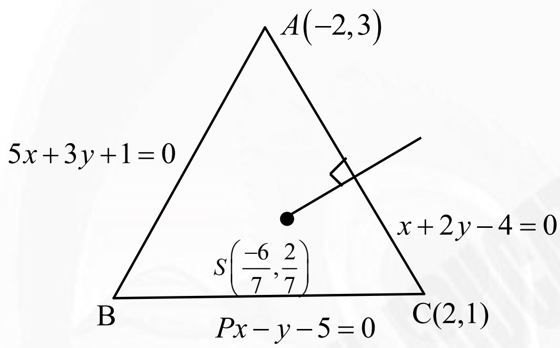
$= 32.4$

2a. The equation of sides AB,BC and CA of a triangle ABC are $5x+3y+1=0$, $Px-y-5=0$ and $x+2y-4=0$ respectively and $S\left(\frac{-6}{7},\frac{2}{7}\right)$ is the circum centre. The P is

A) 3 B) 5 C) -1 D) 2

Key : A

Sol :



Perpendicular bisector of AC

$2x-y+2=0$

Image of a w.r.t $2x-y+2=0$

$\frac{h+2}{2}=\frac{k-3}{-1}=\frac{-2(2(-2)-3+2)}{5}$

$h=2, y=1$

$C=(2,1)$

$2P-1-5=0$

$2P=6$

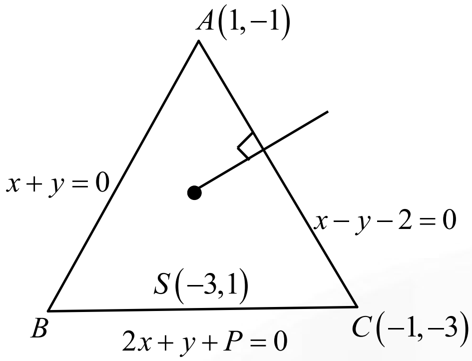
$P=3$

2b. The equation of sides AB, BC and CA of a triangle ABC are $x+y=0$, $2x+y+p$ and $x-y-2=0$ respectively and $S(-3,1)$ is the circumcenters. Then are of triangle ABC is

A) 10 sq.units B) 12 sq.units C) 18 sq.units D) 20 sq.units

Key : B

Sol :



Perpendicular bisector of AC is

$x+y+2=0$

Image of A w.r.t $x+y+2=0$ is

$\frac{h-1}{1}=\frac{k+1}{1}=\frac{-2(1-1+2)}{2}$

$h=-1$, $k=-3$

$(-1,-3)$

$2(-1)-3+P=0$

$BC:2x+y+5=0$

$B(-S,5)$

Area of triangle ABC

$$\Delta=\frac{1}{2}\begin{vmatrix}1 & -1 & 1\\ -5 & 5 & 1\\ -1 & -3 & 1\end{vmatrix}$$

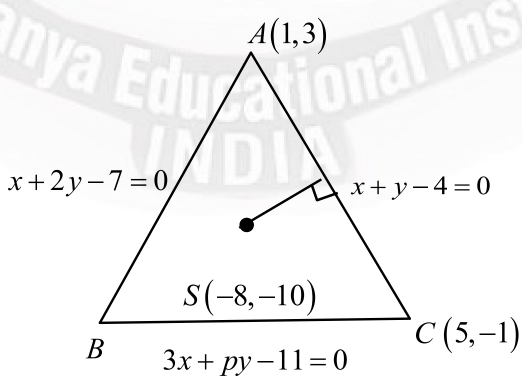
$=12$

2c. The equation of the sides AB, BC and CA of triangle ABC are $x+2y-7=0$, $3x+py-11=0$ and $x+y-4=0$ respectively and $S(-8,-10)$ is the circum centre. Then $AC^2+P^2=$_____

A) 32 B) 64 C) 48 D) 80

Key : C

Sol :



Perpendicular bisector of AC

$x-y-2=0$

Image of A w.r.t $x-y-2=0$

$\frac{h-1}{1}=\frac{k-3}{-1}=\frac{-2(1-3-2)}{2}$

$h=5$, $K=-1$

$C(5,-1)$

$3(5)+P(-1)-11=0$

$P=4$

$AC^2=(5-1)^2+(-1-3)^2$

$=16+16$

=32

$AC^2+P^2=32+16=48$

3. Let $A(\alpha,-2), B(\alpha,6)$ and $C\left(\frac{\alpha}{4},-2\right)$ be the vertices of a ΔABC. If $\left(5,\frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is not correct above ΔABC

A) area is 24　　　B) perimeter is 25

C) circuradius is 5　　　D) inradius is 2

Key: B **Jee mains - 2022**

Sol : $A(\alpha,-2), B(\alpha,6), C\left(\frac{\alpha}{4},-2\right)$

$AC \perp AB$

ΔABC is a right angle

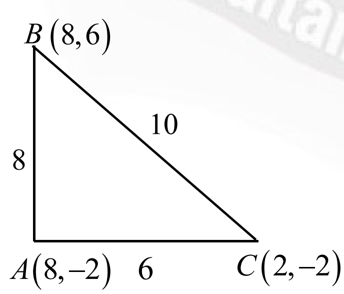
$\angle A=90^0$

Circumcentre = midpoint of BC

$\left(\frac{5\alpha}{8},2\right)=\left(\frac{8+2}{2},\frac{6-2}{2}\right)$

$\frac{5\alpha}{8}=5$, $\frac{\alpha}{4}=2$

$\alpha=8$



3a. In ΔABC, equation to AB is $2x+3y-5=0$, altitude through A is $x-y+4=0$ and altitude through B is $2x-y-1=0$. Then the vertex C is

A) $\left(\frac{-1}{5},\frac{9}{5}\right)$　　B) $\left(\frac{1}{5},\frac{9}{5}\right)$　　C) $\left(\frac{1}{5},\frac{-9}{5}\right)$　　D) $\left(\frac{-1}{5},\frac{-9}{5}\right)$

Key : B

Sol : Ortho centre $=(5,9)$

Altitude through C which is perpendicular to $\overline{AB}$ is $3x-2y+3=0$

3b. In ΔABC, coordinates of A are $(-1,3)$ and equation of the median and altitude through point B are $2x+y=8$ and $2x+3y=8$ respectively then

A) coordinates of C are $(4,0)$
B) coordinates of C are $(3,9)$
C) coordinates of C are $(3,3)$
D) Coordinates of centroid are $(2,2)$

Key: B

Sol :

Equation of $\overline{AC}$ is $3x-2y=K$. It passes $A(-1,3)$

$k=-9$

$3x-2y+9=0$

$M=(1,6),C=(3,9),G(2,4)$

3c. The slopes of sides of a triangle are $-1,-2,3$. If the orthocenter of the triangle is the origin O. Then the locus of its centroid is $\frac{y}{x}=$ ___

A) $\frac{2}{3}$
B) $\frac{2}{5}$
C) $\frac{2}{7}$
D) $\frac{2}{9}$

Key : D

Sol : $OA\perp BC\Rightarrow$ slope $OA=1$, $A(\alpha,\alpha)$

$OB\perp CA\Rightarrow$ slope OB $=\frac{1}{2}$, $B(2\beta,\beta)$

$OC\perp AB\Rightarrow$ slope of $OC=\frac{-1}{3}$, $C(-3\gamma,\gamma)$

$G=\left(-2\gamma,\frac{-4\gamma}{9}\right)$

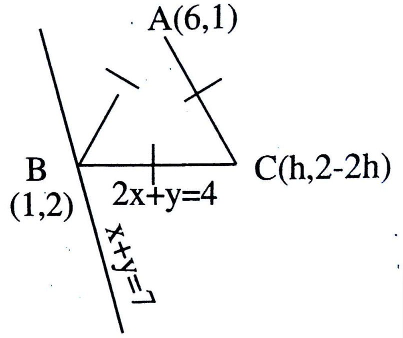
Locus of G is $\frac{y}{x}=\frac{2}{9}$

4. In an isosceles triangle ABC, the vertex A is $(6,1)$ and the equation of the BC is $2x+y=4$. Let the point B lie on the line $x+3y=7$. If (α,β) is the centroid of ΔABC, then 15 $(\alpha+\beta)$ is equal to :

A) 39
B) 41
C) 51
D) 63

Key: C **Jee Mains-2022**

Sol :



Point $B(1,2)$; Now let C be $(h,4-2h)$

(As C lies on $2x+y=4$)

$\because\ \Delta$ is isosceles with base BC

$\therefore\ AB = AC$

$\sqrt{25+1} = \sqrt{(6-h)^2+(2h-3)^2}$

$\sqrt{26} = \sqrt{36+h^2-12h+4h^2+9-12h}$

$26 = 5h^2-24h+45 \Rightarrow 5h^2-24h+19=0$

$h=\frac{19}{5}$ or $h=1$; Thus $c\left(\frac{19}{5},\frac{-18}{5}\right)$

Centroid $\left(\frac{6+1+\frac{19}{5}}{3},\frac{1+2-\frac{18}{5}}{3}\right)$

$\left(\frac{35+19}{15},\frac{15-18}{15}\right);\left(\frac{54}{15},\frac{-3}{15}\right);\alpha=\frac{54}{15};\beta=\frac{-3}{15}$

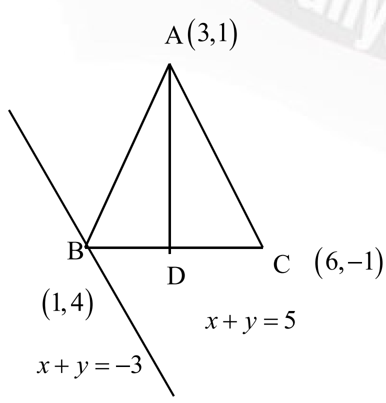
$15(\alpha+\beta)=51$

4a. In an Isosceles triangle PQR, the vertex A is $(3,1)$ and the equation of the BC is $x+y=5$. Let the point B lies on the line $x-y=-3$. If (α,β) is the centroid of ΔABC, then $3(\alpha-\beta)$ is equal to ———

A) 6 B) 10 C) 15 D) 13

Key: A

Sol:



$D \rightarrow (h,k)$ Midpoint of BC

$\rightarrow$Foot of perpendicular of A or BC

$$\frac{h-3}{1}=\frac{k-1}{1}=\frac{-(3+1-5)}{2}=\frac{1}{2}$$

$$h=\frac{7}{2}; \quad h=\frac{3}{2}$$

$$D\left(\frac{7}{2},\frac{3}{2}\right) \rightarrow C=(2D-B)$$

$$=(6,-1)$$

$$\text{G of } \Delta ABC=\left(\frac{10}{3},\frac{4}{3}\right)$$

$$(\alpha,\beta)=\left(\frac{10}{3},\frac{4}{3}\right)$$

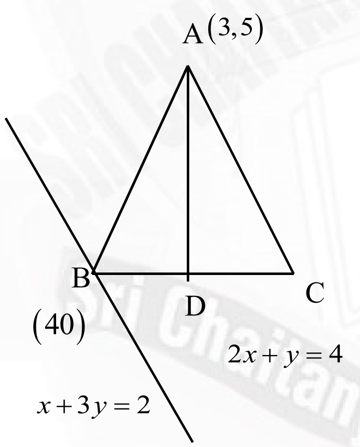
$$3(\alpha,-\beta)=3\left(\frac{10-4}{5}\right)=6$$

4b. In an Isosceles triangle ABC, the vertex A is $(3,5)$ and the equation of the BC is $2x+y=4$. Let the point B lies on the line $x+3y=2$. If (α,β) is the centroid of ΔABC, then $15(\alpha+\beta)$ is equal to ———

A) 10 B) 15 C) 78 D) 56

Key: C

Sol:



$D \rightarrow (h,k)$ Midpoint of BC

$\rightarrow$Foot of perpendicular of A or BC

$$\frac{h-3}{2}=\frac{k-5}{1}=\frac{-(6+5-4)}{5}=\frac{-7}{5}$$

$$\frac{h-3}{2}=\frac{-7}{5} \Rightarrow h=3-\frac{14}{5}=\frac{1}{5}$$

$$k-5=\frac{-7}{5} \Rightarrow h=\frac{7+25}{5}=\frac{15}{5}$$

$D\left(\frac{1}{5}, \frac{18}{5}\right)$

$G \rightarrow AD$ in the ratio 2:1

$$G(\alpha, \beta) = \left(\frac{\frac{2}{5}+3}{3}, \frac{\frac{36}{5}+5}{3}\right)$$

$$= \left(\frac{17}{15}, \frac{61}{15}\right)$$

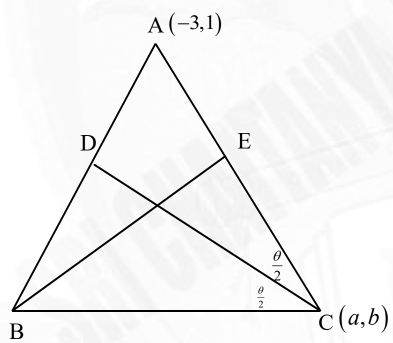
$15(\alpha+\beta) = 17 + 61 = 78$

5. Let ABC be a triangle with $A(-3,-1)$ and $\underline{|ACB} = \theta,\ 0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector is $7x - 4y - 1 = 0$ then $\tan\theta =$ ________

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{4}{3}$ D) 2

Key: C **Jee Mains-2021**

Sol:



Equation of BE $\rightarrow 2x + y - 3 = 0$

Equation of CD $\rightarrow 7x - 4y - 1 = 0$

Let $C(a,b)$

$\therefore E =$ midpoint of AC $= \left(\frac{a-3}{2}, \frac{b+1}{2}\right)$

E lies on BE $\rightarrow 2\left(\frac{a-3}{2}\right) + \left(\frac{b+1}{2}\right) - 3 = 0 \Rightarrow 2a + b = 11$

C lies on CD $\rightarrow 7a - 4b - 1 = 0$

Solving $2a + b - 11 = 0$, $7a - 4b - 1 = 0 \Rightarrow a = 3, b = 5$

$m_1 =$ slope of AC $= \frac{2}{3}$

$m_2 =$ slope of CD $= \frac{7}{4}$

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{1}{2}$

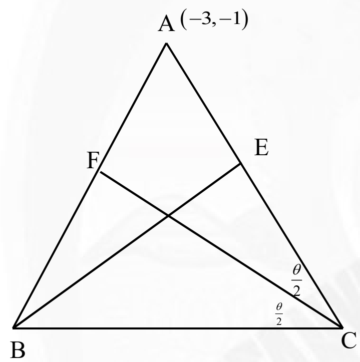
$\therefore \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{4}{3}$

5a. Let in a $\triangle ABC$, $A = (-3,-1)$ and $\underline{|ACB} = \theta$, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is $x + y - 3 = 0$ and the equation of angle bisector is $x - 4y - 1 = 0$ then $\tan\theta =$ ________

A) $\frac{130}{40}$ B) $\frac{13}{84}$ C) $\frac{-13}{8}$ D) $\frac{13}{80}$

Key: B

Sol:



Equation of BE $\rightarrow x + y - 3 = 0$

Equation of CF $\rightarrow x - 4y + 1 = 0$

Let $C(a,b)$

$\therefore E =$ midpoint of AC$=\left(\frac{a+3}{2}, \frac{b-1}{2}\right)$

E lies on BE $\rightarrow \frac{a+3}{2} + \frac{b-1}{2} - 3 = 0 \Rightarrow a + b - 4 = 0$

C lies on CF $\rightarrow a - 4b + 1 = 0$

b=1, a=3

$m_1 =$ slope of AC$=\frac{1+1}{3+3} = \frac{2}{6} = \frac{1}{3}$

$m_2 =$ slope of CF$=\frac{1}{4}$

$\therefore \tan\frac{\theta}{2} = \left|\frac{\frac{1}{3} - \frac{1}{4}}{1 + \frac{1}{3}.\frac{1}{4}}\right| = \left|\frac{1}{13}\right|$

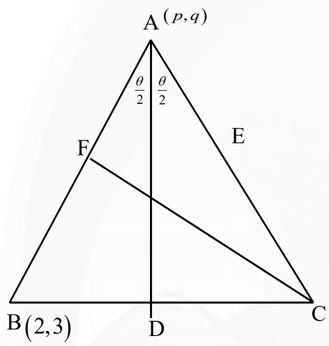
$$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{2.\frac{1}{13}}{1-\frac{1}{169}} = \frac{2}{13}\times\frac{169}{168} = \frac{13}{84}$$

5b. Let in a $\triangle ABC$, $B=(2,3)$ and $\underline{|BAC} = \theta$, $0<\theta<\frac{\pi}{2}$. If the equation of the median through A is $x-y-2=0$ and the equation of angle bisector is $x+4y-3=0$ then $\sin\theta=$ __________

A) $\frac{24}{\sqrt{881}}$ B) $\frac{21}{20}$ C) $\frac{20}{\sqrt{881}}$ D) $\frac{21}{\sqrt{881}}$

Key: D

Sol:



Equation of AD $\rightarrow x-y-2=0$

Equation of CF $\rightarrow x+4y-3=0$

Let $A=(p,q)$

$$F = midpou\text{int}\,ofAB = \left(\frac{p+2}{2},\frac{q+3}{2}\right)$$

F lies on CF $\Rightarrow \frac{p+2}{2}+4\left(\frac{q+3}{2}\right)-3=0$

$$p+4q+8=0$$

$$q=-2,\ \ p=0\ \ \therefore A=(0,-2)$$

$$m_1 = slopeofAD = 1$$

$$m_2 = slopeofAB = \frac{3+2}{2-0} = \frac{5}{2}$$

$$\therefore \tan\frac{\theta}{2} = \left|\frac{1-\frac{5}{2}}{1+1.\frac{5}{2}}\right| = \left|\frac{2-5}{2+5}\right| = \frac{3}{7}$$

$$\therefore \tan\frac{\theta}{2} = \left|\frac{2-\frac{3}{7}}{1-\frac{9}{49}}\right| = \frac{6}{7}\times\frac{49}{40} = \frac{21}{20}$$

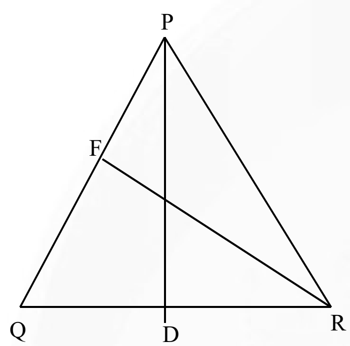
$$\sin\theta = \frac{21}{\sqrt{881}}$$

5c. In a $\triangle PQR$, $Q=(-1,1)$ and the equation of median through R is $x-y+1=0$ and angle bisector of angle P is $2x+y+2=0$ then $\cos\underline{|P} =$

A) $\frac{7}{24}$ B) $\frac{7}{25}$ C) $\frac{24}{25}$ D) $\frac{25}{24}$

Key: C

Sol:



Equation of RF = $x-y+1=0$

Equation of PD= $2x+y+2=0$

Let P= (a,b)

$$\therefore F=\left(\frac{a-1}{2},\frac{b+1}{2}\right)$$

F lies on RF = $\frac{a-1}{2}+\frac{b+1}{2}+1=0$

$= a+b+2=0$

P lies on PD= $2a+b+2=0$

Solving $a+b+2=0, 2a+b+2=0$

$\Rightarrow a=0, b=-2 \Rightarrow p=(0,-2)$

$\therefore m_1 = slopeofPD = -2$

$\therefore m_2 = slopeofPQ = \frac{-2-1}{0+1} = -3$

$$\therefore \tan\frac{\theta}{2} = \left|\frac{-2+3}{1+6}\right| = \frac{1}{7}$$

$$\tan\theta = \frac{2.\frac{1}{7}}{1-\frac{1}{49}} = \frac{2}{7}\times\frac{49}{48} = \frac{7}{24}$$

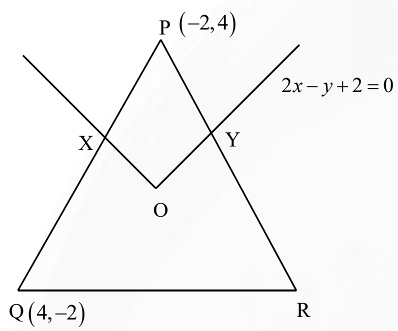
$$\therefore \cos\theta = \frac{24}{25}$$

6. In a $\triangle PQR$, the coordinates of the points P and Q are $(-2,4)$ and $(4,-2)$ respectively of the equation of the perpendicular bisector of PR is $2x-y+2=0$ then the centre of the circum-circle of $\triangle PQR$, is

A) $(-1,0)$ B) $(-2,-2)$ C) $(0,2)$ D) $(1,4)$

Key: B **Jee Mains-2021**

Sol:



X=midpoint of PQ $=(1,1)$

Slope of PQ= $\frac{-2-4}{4+2}=-1$

Equation of OX is $y-1=\pm1(x-1)\Rightarrow y=x$

Solving OX and OY

$2x-y+2=0$, $x-y=0$

$x=-2,\quad y=-2$

$\therefore 0=(-2,-2)$

6a. In $\triangle ABC$ A is $(1,2)$ if the internal angle bisector of B is $2x-y+10=0$ and perpendicular bisector of AC is $y=x$ then the equation of BC is

A) $5x+9y-19=0$ B) $5x-9y-19=0$ C) $5x+9y+19=0$ D) $5x-9y+19=0$

Key: A

Sol:

Image of A w.r.to bisector of B is $(-7,6)$ lies on BC and image of A in the perpendicular bisector of AC is $C(2,1)$.

$\therefore$ Equation of BC is 5x+9y-19=0

6b. In $\triangle ABC$, equation to AB is $2x+3y-5=0$, altitude through A is $x-y+4=0$ and altitude through B is $2x-y-1=0$. Then the vertex of C is

A) $\left(\frac{-1}{5},\frac{9}{5}\right)$ B) $\left(\frac{1}{5},\frac{9}{5}\right)$ C) $\left(\frac{1}{5},\frac{-9}{5}\right)$ D) $\left(\frac{-1}{5},\frac{-9}{5}\right)$

Key: B

Sol:

$x-y+4=0$, $2x-y-1=0$

$H=(5,9)$

Key: 2

Altitude through C is

$3x-2y+3=0$

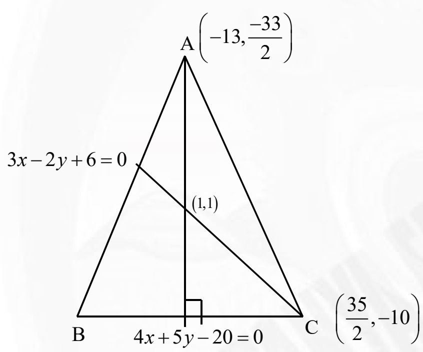
6c. Let the equations of two sides of a triangle be $3x-2y+6=0$ and $4x+5y-20=0$. If the orthocenter of this triangle is at $(1,1)$. Then the equation of its third side is:

A) $26x-122y-1675=0$ B) $26x+61y-1675=0$

C) $122y-26x-1675=0$ D) $122y-26x+1675=0$

Key: A

Sol:



$4x+5y-20=0 \rightarrow 1$

$3x-2y+6=0 \rightarrow 2$

Orthocenter is $(1,1)$

Line perpendicular to $4x+5y-20=0$ passes through $(1,1)$ is $(y-1)=\frac{5}{4}(x-1)$

$\Rightarrow 5x-4y=1 \rightarrow 3$

And line perpendicular to $3x-2y+6=0$ and passes through $(1,1)$

$y-1=-\frac{2}{3}(x-1) \Rightarrow 2x+3y=5 \rightarrow 4$

Solving (1) & (4) we get $C\left(\frac{35}{2},-10\right)$

Solving (2) & (3) we get $A\left(-13,\frac{-33}{2}\right)$

Equations of BC is $26x-122y-1675=0$

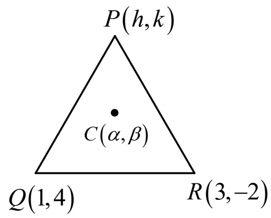
7. A point p moves on the line $2x-3y+4=0$. If Q(1,4) and R(3,-2) are fixed points, then the locus of centroid of ΔPQR is a line

A) with slope 2/3 B) parallel to x- axis

C) with slope 3/2 D) parallel to y-axis

Key: A

Sol:



Let centroid $C(\alpha,\beta)$

$\therefore \alpha = \frac{1+3+h}{3}, \Rightarrow h = 3\alpha - 4$; $\beta = \frac{4-2+k}{3}, \Rightarrow k = 3\beta - 2$

P(h,k) lies on $2x-3y+4=0$

$\therefore 2(3\alpha-4)-3(3\beta-2)+4=0$

$6\alpha - 9\beta + 2 = 0$

Locus is $6x-9y+2=0$

$Slope = \frac{-6}{-9} = \frac{2}{3}$

7a. The algebraic sum of perpendicular distances from the vertices of a triangle to a variable line is 'O' then line through ….. of the triangle

A) In centre B) Ex centre C) Centroid D) Circum centre

Key: C

Sol: Algebraic sum of distances from the three non collinear points to variable line is zero then the line passing through centroid of triangle formed by the points

7b. The centroid of the triangle formed by the lines $x+y-1=0, x-y-1=0, x+3y+3=0$ is

A) $\left(\frac{4}{3},1\right)$ B) $\left(\frac{8}{3},3\right)$ C) $\left(\frac{-4}{3},1\right)$ D) $\left(\frac{-8}{3},3\right)$

Key: A

Sol: Point of intersection of $x+y-1=0 \;\&\; x-y-1=0$ is point of intersection of $x+y-1=0 \;\&\; x-3y+3=0 \text{ is } (3,2)$ point of intersection of $x-3y+3=0, x+y-1=0 \text{ is } (0,1)$

$\therefore Centroid = \left(\frac{4}{3},1\right)$

7c. In $\Delta ABC, B=(0,0), AB=2, \underline{|ABC} = \frac{\pi}{3}$ and the middle point of BC has co-ordinates (2,0), the centroid of triangle is

A) $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ B) $\left(\frac{5}{3},\frac{1}{\sqrt{3}}\right)$ C) $(1,1)$ D) none of these

Key: B

Sol: $B=(0,0), \underline{|ABC} = 60^0, AB = r = 2$

If $A=(x,y)$ *then* $x=x_1+r\cos\theta \Rightarrow 0+2.\frac{1}{2}=1$

$y=y_1+r\sin\theta=0+2.\frac{\sqrt{3}}{2}=\sqrt{3}$,

$A(1,\sqrt{3}),C(4,D)$ Now centroid $=\left(\frac{5}{3},\frac{1}{\sqrt{3}}\right)$

8. Let the equation of two sides of a triangle be $3x-2y+6=0$ and $4x+5y-20=0$. If the orthocenter of this triangle is at $(1,1)$ then the equation of its third sides is

A) $122y-26x-1675=0$ B) $26x+61y+1675=0$

C) $122y+26x+1675=0$ D) $26x-122y-1675=0$

Key : D **Jee Mains-2019**

Sol : Equation of AB is $3x-2y+6=0$ …..(1)

Equation of AC is $4x+5y-20=0$ …..(2)

Equation of BE is $2x+3y-5=0$ ……(3)

Equation of CF is $5x-4y-1=0$ ……(4)

From 1 and 3 is B, 2and 4 is c

Equation of BC $26x-122y-1675=0$

8a. Let the two sides of the triangle be $4x+3y-5=0$ and $x+2y=0$. If the ortho centre of the triangle is at $(-4,-3)$ then equation of the third side is

A) $4x+3y=0$ B) $3x+y=0$ C) $3x-y=0$ D) $3x-4y+2=0$

Key : B

Sol : Equation of AB is $4x+3y-5=0$ ….(1)

Equation of AC is $x+2y=0$ ….(2)

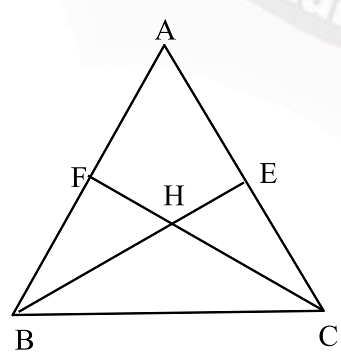
Equation of BE is $2x-y+5=0$ ….(3)

Equation of CF is $3x-4y=0$ ….(4)

From 1 and 3 $B(1-,3)$

From 2 and 4 $C(0,0)$

Equation of BC is $3x+y=0$



8b. Let the two sides of the triangle be $x-y-2=0$ and $2x+y-7=0$. If the orthocenter of the triangle is at $(-4,-6)$ then the equation of third side is

A) $x+y+10=0$ B) $x+y+6=0$ C) $x+y-8=0$ D) $x+y-7=0$

Key : A

Sol : Equation of AB is $x-y-2=0$ ….(1)

Equation of AC is $2x+y-7=0$ ….(2)

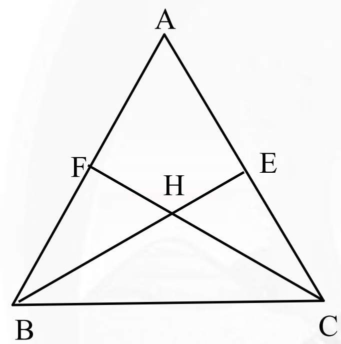
Equation of BE is $x-2y-8=0$ ….(3)

Equation of CF is $x+y+10=0$ ….(4)

From 1 and 3 $B(-4,-6)$

From 2 and 4 $C(17,-27)$

Equation of BC is $x+y+10=0$



8c. Let the two sides of the triangle be $x+y=0$ and $2x+y+5=0$. If the ortho centre of the triangle is at $(1,-1)$ then the equation of third side is

A) $x+y-4=0$ B) $x-y-8=0$ C) $x-y-2=0$ D) $x+4y-2=0$

Key : C

Sol : Equation of AB is $x+y=0$ ….(1)

Equation of AC is $2x+y+5=0$ ….(2)

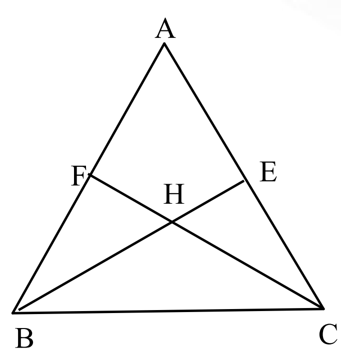
Equation of BE is $x-2y-3=0$ ….(3)

Equation of CF is $x-y-2=0$ ….(4)

From 1 and 3 $B(1,-1)$

From 2 and 4 $C(-1,-3)$

Equation of BC is $x-y-2=0$



9. The orthocenter of the triangle formed by the lines $xy=0$ and $x+y=1$ is

IIT-JEE 1995

A) $(1/2,1/2)$ B) $(1/3,1/3)$ C) $(0,0)$ D) $(1/4,1/4)$

Key : C

Sol : The lines by which triangle is formed are $x=0, y=0$, and $x+y=1$. Clearly, it is a right triangle. We know that in a right-angled triangle, the orthocenter coincides with the vertex at which right angle is formed. Therefore, the orthocenter is $(0,0)$

10. The locus of the orthocenter of the triangle formed by the lines $(1+p)x-py+p(1+p)=0,(1+q)x=qy+q(1+q)=0$ and $y=0$, where $p\neq q$, is

IIT-JEE 2009

A) a hyperbola B) a parabola C) an ellipse D) a straight line

Key : D

Sol : The intersection point $y=0$ with first line is $B(-p,0)$.

The intersection point of $y=0$ with the second line is $A(-q,0)$

The intersection point of the two lines is

$C\left(pq,(p+1)(q+1)\right)$

The altitude from C to AB is $x=pq$

The altitude from B to AC is

$$y=\frac{q}{1+q}(x+p)$$

Solving these two, we get $x=pq$ and $y=-pq$

Therefore, the locus of the orthocenter is $x+y=0$.

Various Triangles and four sided figures

(Area of triangle and parallelogram)

1. Let two points be $A(1,-1)$ and $B(0,2)$. If a point $P(x',y')$ be such that the area of $\Delta PAB=5$ square units and it lies on the line, $3x+y-4\lambda=0$, then a value of λ is :

A) 3 B) -3 C) 4 D) 2

Key : A **Jee mains -2020**

Sol : $D=\frac{1}{2}\begin{vmatrix}0 & 2 & 1\\ 1 & -1 & 1\\ x' & y' & 1\end{vmatrix}$

$-2(1-x')+(y'+x')=\pm 10$

$-2+2x'+y'+x'=\pm 10$

$3x'+y'=12$ or $3x'+y'=-8$

$\lambda=3,-2$

1a. Let two points be $A(6,5)$ and $B(3,2)$. If a point (x,y) be such that the area of $\Delta PAB = 9$ sq. units and lies on the line $x-y-2\lambda=0$. Then the sum of all possible values of λ is ______

A) 1 2) 5 3) 6 D)9

Key : A

Sol : $A(6,5)\quad B(3,2)\quad P(x,y)$

$\Delta PAB = 9$ sq. units

$$\Delta \Rightarrow \frac{1}{2}\begin{vmatrix} 6 & 5 & 1 \\ 3 & 2 & 1 \\ x & y & 1 \end{vmatrix} = 9$$

$$\begin{vmatrix} 6 & 5 & 1 \\ 3 & 2 & 1 \\ x & y & 1 \end{vmatrix} = 18$$

$$|6(2-y)-5(3-x)+1(3y-2x)| = 18$$

$$|12-6y-15+5x+3y-2x| = 18$$

$$|3x-3y-3| = 18$$

$$|x-y-1| = 6$$

a) $x-y-1=6$ b) $x-y-1=-6$

$x-y-7=0 \quad \ldots\ldots(1)$ $x-y+5=0 \ldots\ldots.(2)$

By comparing (1) and (2) with $x-y-2\lambda=0$,

$$\lambda = \frac{7}{2}, -\frac{5}{2}$$

Sum of all possible values of $\lambda = \frac{7}{2}-\frac{5}{2}=\frac{2}{2}=1$

1b. The vertex of a right angled triangle lies on the straight line $2x+y-10=0$ and the two of vertices at points $(2,-3)$ and $(4,1)$, then the area of triangle in sq. units is

A) $\sqrt{10}$ B) 3 C) $\frac{33}{5}$ D) 11

Key : B

Sol : $2x+y-10=0$

$y=10-2x$

$(x,10-2x)$ is the points of 3rd vertex

$(2,-3),(x,10-2x),(4,1)$

$m_1m_2=-1$

$$\left(\frac{10-2x-1}{x-4}\right)\left(\frac{10-2x+3}{x-2}\right)=-1$$

$$(9-2x)(13-2x)=-(x-2)(x-4)$$

$117-18x-26x+4x^2=-x^2+6x-8$

$5x^2-50x+125=0$, solving the above quadratic equations

We get, $x=5$

$y=10-2x \Rightarrow y=0$

$\therefore$ 3rd vertex is $(5,0)$

$A=\frac{1}{2}bh$

$A=\frac{1}{2}\sqrt{(4-5)^2+(1-0)^2}\sqrt{(5-2)^2+(0+3)^2}$

$=\frac{1}{2}\sqrt{2}\times 3\sqrt{2}$

$\therefore$ Area $=3$

1c. The area of the isosceles right angled triangle, which has hypotenuse length as 2 is

A) 1 B) 1.5 C) $\sqrt{2}$ D) $\frac{1}{\sqrt{2}}$

Key : A

Sol : The length of the hypotenuse of an isosceles triangle is 2.

Since it is isosceles, $x^2+x^2=4$, x being the length of the congruent sides

$2x^2=4$

$x^2=2 \Rightarrow x=\sqrt{2}$

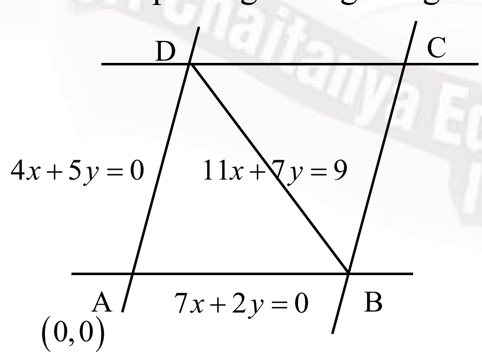
Area thus becomes $\frac{1}{2}\times\sqrt{2}\times\sqrt{2}=1$ Sq. units

2. Two sides of a parallelogram are along the lines $4x+5y=0$ and $7x+2y=0$. If the equation of one of the diagonal of a parallelogram is $11x+7y=9$ then other diagonal passing through the point

A) $(1,2)$ B) $(2,2)$ C) $(2,1)$ D) $(1,3)$

Key :B **Jee Mains -2021**

Sol : Both lines passing through origin



Point D is the point of intersection of $4x+5y=0$ and $11x+7y=9$.

So co-ordinates of point $D\left(\frac{5}{3},\frac{-4}{3}\right)$

Similarly $B\left(\frac{-2}{3},\frac{7}{3}\right)$

Diagonals bisect each other

The middle point of BD

$$\Rightarrow\left(\frac{\frac{5}{3}-\frac{2}{3}}{2},\frac{\frac{-4}{3}+\frac{7}{3}}{2}\right)=\left(\frac{1}{2},\frac{1}{2}\right)$$

Equation of diagonal AC

$$y-0=\frac{\frac{1}{2}-0}{\frac{1}{2}-0}(x-0)$$

$y=x$

Diagonal AC passing through $(2,2)$

2a. Two sides of a parallelogram are along the lines $x+y-3=0$, and $x-y+3=0$. If the equation of one of the diagonals of a parallelogram is $2x-y=0$ then other diagonal passing through the point.

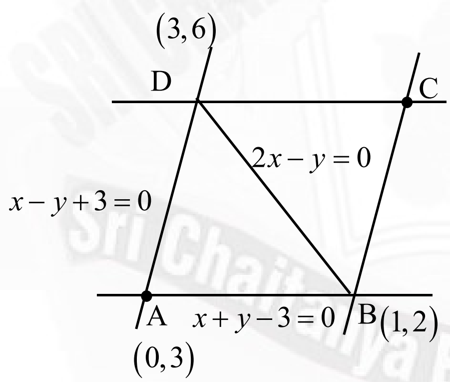
A) $(2,1)$ B) $(2,3)$ C) $(2,4)$ D) $(3,2)$

Key :C

Sol : $A(0,3)$

$B(1,2)$

$D(3,6)$



Mid point of BD $=(2,4)$

Equation of diagonal AC :

$$y-4=\frac{4-3}{2-0}(x-2)$$

$x-2y+6=0$

Passing through $(2,4)$

2b. Two sides of a parallelogram are along the lines $y+2=0$ and $5x-4y+7=0$. If the equation of one of the diagonals of parallelogram is $5x+4y-17=0$ then the other diagonal is

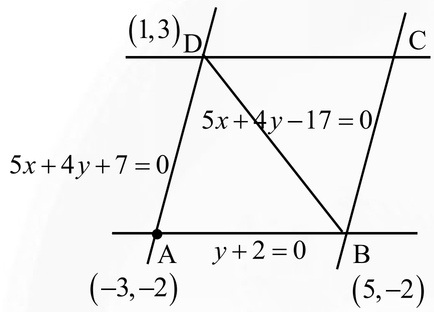
A) $12x-5y+9=0$ B) $9x-5y+12=0$ C) $12x+5y-9=0$ D) $5x-12y-9=0$

Key : D

Sol : $A(-3,-2)$

$B(5,-2)$

$C(1,3)$



Mid point of BD : $\left(3,\frac{1}{2}\right)$

Eq. of diagonal AC : $y+2=\frac{\frac{1}{2}+2}{3+3}(x+3)$

$5x-12y-9=0$

2c. Two sides of a parallelogram are along the lines $x-y-1=0$ and $x-3y+5=0$. If equation of one the diagonal of parallelogram is $x-1=0$, then the other diagonal passing through the point

A) $\left(2,\frac{5}{3}\right)$ B) $\left(3,\frac{7}{2}\right)$ C) $\left(1,\frac{5}{2}\right)$ D) $\left(2,\frac{7}{2}\right)$

Key : A

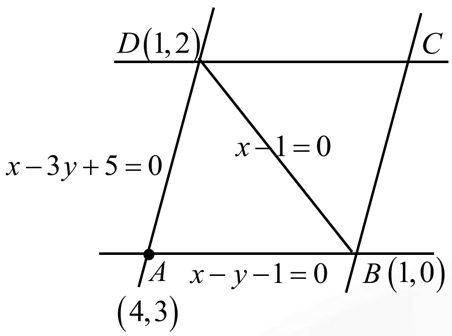
Sol : $A(4,3)$

$B(1,0)$

$D(1,2)$

Mid point of BD : $(1,1)$

Eq. of diagonal AC : $y-1=\frac{3-1}{4-1}(x-1)$



$2x-3y+1=0$

Passing through $\left(2,\frac{5}{3}\right)$

3. Let the area of the triangle with vertices $A(1,\alpha), B(\alpha,0)$, and $C(0,\alpha)$ be 4 sq.units. If the point $(\alpha,-\alpha),(-\alpha,\alpha)$ and (α^2,β) are collinear, then $\beta =$_________

A) 64 B) -8 C) -64 D) 512

Jee Mains 2022

Key : C

Sol. $\frac{1}{2}\begin{vmatrix}\alpha & 0 & 1\\ 1 & \alpha & 1\\ 0 & \alpha & 1\end{vmatrix} = \pm 4$

$\alpha = \pm 8$

$(8,-8),(-8,8),(64,\beta)$ are collinear

Slope $=-1$

$\beta = -64$

3a. If $A(6,3), B(-3,5), C(4,-2)$ and $P(a,b)$ then the ratio of the areas of triangles PBC, ABC is

A) $|a+b|:7$ B) $|a-b|:7$ C) $|a+b+2|:7$ D) $|a+b-2|:7$

Key :D

Sol : Area of ΔPBC : area of ΔABC

$\frac{1}{2}\begin{vmatrix}a+3 & a-4\\ b-5 & b+2\end{vmatrix} : \frac{1}{2}\begin{vmatrix}9 & 2\\ -2 & 5\end{vmatrix}$

$|(a+3)(b+2)-(b-5)(a-4)| : 49$

$|a+b-2| : 7$

3b. Let the area of the triangle with vertices $A(1,\alpha), B(\alpha,0)$ and $C(0,\alpha)$ be 4 sq.units. If the points $(-\alpha,\alpha),(\alpha,-\alpha)$ and $(\alpha^2,-\beta)$ are collinear then $\beta =$_______

A) -8 B) 0 C) -64 D) 5

Key :B

Sol : $\frac{1}{2}\begin{vmatrix} 1 & \alpha & 1 \\ \alpha & 0 & 1 \\ 0 & \alpha & 1 \end{vmatrix} = 4 \Rightarrow \alpha = \pm 8$

$\frac{1}{2}\begin{vmatrix} -\alpha & \alpha & 1 \\ \alpha & -\alpha & 1 \\ \alpha^2 & -\beta & 1 \end{vmatrix} = 0$

$\alpha\beta = 0$

$\beta = 0 \quad (\alpha = \pm 8)$

3c. An equilateral triangle is constructed between two parallel lines $\sqrt{3}x + y - 6 = 0$ and $\sqrt{3}x + y + 9 = 0$ with base on one and vertex on the other. Then the area of triangle is

A) $\frac{200}{\sqrt{3}}$ B) $\frac{225}{4\sqrt{3}}$ C) $\frac{225}{\sqrt{3}}$ D) $\frac{200}{\sqrt{3}}$

Key :B

Sol : Required area $= \frac{p^2}{\sqrt{3}}$

Where P=distance between parallel lines

4. Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is point on $y - 2x = 2$ such that ΔABC is an equilateral triangle. Then the area of the ΔABC is

A) $3\sqrt{3}$ B) $2\sqrt{3}$ C) $\frac{8}{\sqrt{3}}$ D) $\frac{10}{\sqrt{3}}$

Key : C **Jee mains-2023**

Sol : At A $x = y$

$Y - 2x = 2$

$(-2, -2)$

Height from line $x + y = 0$

$h = \frac{4}{\sqrt{2}}$

Area of $\Delta = \frac{\sqrt{3}}{4}\frac{h^2}{\sin^2 60} = \frac{8}{\sqrt{3}}$

4a. The straight lines $7x - 2y + 10 = 0$ and $7x + 2y - 10 = 0$ form an isosceles triangle with the line $y = 2$. The area of this triangle is equal to

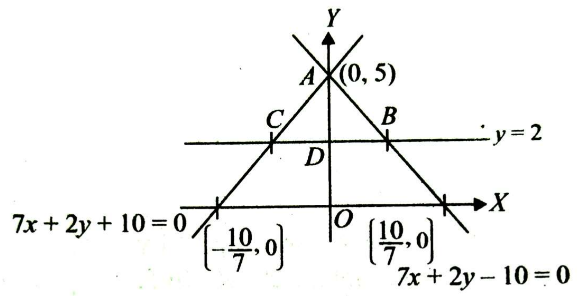
A) 15/7 sq. units B) 10/7 sq. units C) 18/7 sq. units D) None of these

Key :C

Sol : We have

$B=\left(\frac{6}{7},2\right), C=\left(-\frac{6}{7},2\right)$

Or $BC=\frac{12}{7}, AD=3$



$\therefore\ \Delta_{ABC}=\frac{1}{2}\times\frac{12}{7}\times 3=\frac{18}{7}$ sq.units.

4b. If the extremities of the base of an isosceles triangle are the points $(2a,0)$ and $(0,a)$, and the equation of one of the sides is $x=2a$, then the area of the triangle is

A) $5a^2$ sq. units B) $5a^2/2$ sq. units C) $25a^2/2$ sq. units D) None of these

Key :B

Sol : Let the coordinates of the third vertex be $(2a,t)$. Now, AC =BC. Hence,

$t=\sqrt{4a^2+(a-t)^2}$ or $t=\frac{5a}{2}$

So, the coordinates of the third vertex C are $(2a,5a/2)$. Therefore, area of the triangles is

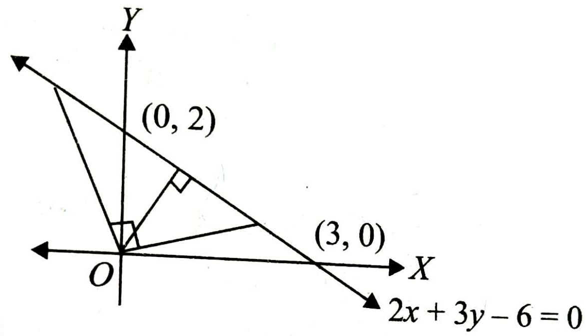
$$\pm\frac{1}{2}\begin{vmatrix}2a & 5a/2 & 1\\ 2a & 0 & 1\\ 0 & a & 1\end{vmatrix}=\begin{vmatrix}a & 5a/3 & 1\\ 0 & -5a/2 & 0\\ 0 & a & 1\end{vmatrix}=\frac{5a^2}{2}\text{sq. units.}$$

4c. If a pair of perpendicular straight lines drawn through the origin forms an isosceles triangle with the line $2x+3y=6$, then area of the triangle so formed is

A) $36/13$ B) $12/17$ C) $13/5$ D) $17/13$

Key : A

Sol : The distance of $(0,0)$ from the line $2x+3y-6=0$ is $6/\sqrt{4+9}=6/\sqrt{13}$. The area of the triangle is $\left(6/\sqrt{13}\right)^2=36/13$.

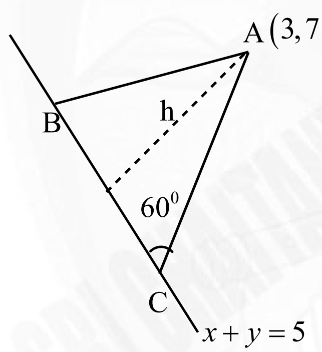


5. Let R be the point $(3,7)$ and let P and Q be two points on the line $x+y=5$ such that PQR is an equilateral triangle. Then the area of ΔPQR is

A) $\frac{25}{4\sqrt{3}}$ B) $\frac{25\sqrt{3}}{2}$ C) $\frac{25}{\sqrt{3}}$ D) $\frac{25}{2\sqrt{3}}$

Key: D **Jee Mains - 2022**

Sol :



$$h=\frac{|3+7-5|}{\sqrt{2}}=\frac{5}{\sqrt{2}}$$

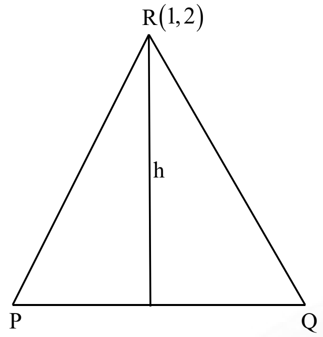
Area of $\Delta ABC=\frac{h^2}{\sqrt{3}}=\frac{25}{2\sqrt{3}}$

5a. Let R be a point $(1,2)$ and let P and Q be two points on the line $2x+y=3$ such that ΔPQR is an equilateral triangle. Then the area of ΔPQR is ______

A) $\frac{1}{5\sqrt{3}}$ B) $\frac{1}{7\sqrt{3}}$ C) $\frac{2}{7\sqrt{3}}$ D) $\frac{3}{7\sqrt{3}}$

Key: A

Sol:



$h = \frac{|2+2-3|}{\sqrt{5}} = \frac{1}{\sqrt{5}}$

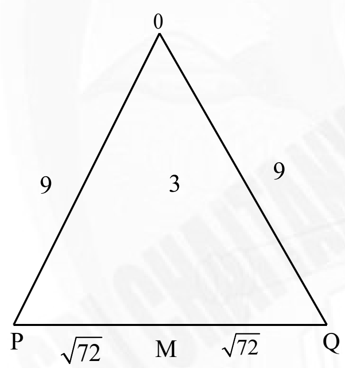
Area of ΔABC = $\frac{h^2}{\sqrt{3}} = \frac{1}{5\sqrt{3}}$ square

5b. Let P and Q are two points on a line $3x+4y-15=0$ such that OP=0R=9. Then the area of triangle OPQ is ———

A) $18\sqrt{2}$ B) $18\sqrt{7}$ C) $15\sqrt{7}$ D) $15\sqrt{3}$

Key: A

Sol:



$PQ = 2PM$

$= 2\sqrt{2}$

Area of triangle=$\frac{1}{2}PQ.OM$

$= \frac{1}{2}.2\sqrt{72}.3 = 18\sqrt{2}$.

6. A point P moves, so that the sum of squares of its distances from the points $(1,2) \& (-2,1)$ is 14. Let $f(x,y)=0$ be the locus of P, which intersects the x-axis at the points A,B and the y-axis at the points C,D. Then the area of the quadrilateral ACBD is equation to :

A) $\frac{9}{2}$ B) $\frac{3\sqrt{17}}{2}$ C) $\frac{3\sqrt{17}}{4}$ D) 9

Key : B **Jee Mains-2022**

Sol : Let $P(x,y)$ be a point

Given point $A(1,2)$ and $B(-2,1)$

$PA^2 + PB^2 = 14$

$(x-1)^2 + (y-2)^2 + (x+2)^2 + (y-1)^2 = 14$

$x^2 + y^2 + x - 3y - 2 = 0$

Intersect x-axis $y = 0$

$x^2 + x - 2 = 0$; $x = -2, 1$

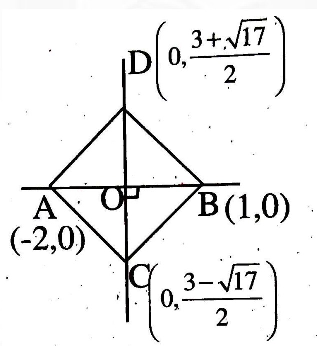
$\therefore\ A(-2,0)B(1,0); y^2 - 3y - 2 = 0$

$y = \dfrac{3 \pm \sqrt{17}}{2}$;

$\therefore C\left(0, \dfrac{3-\sqrt{17}}{2}\right) D\left(0, \dfrac{3+\sqrt{17}}{2}\right)$

Area of quadrilateral

$ACBD = \dfrac{1}{2}\begin{vmatrix} -3 & 0 \\ 0 & -\sqrt{17} \end{vmatrix} = \dfrac{3\sqrt{17}}{2}$



6a. A point P moves, so that the sum of squares of its distances from the points $(1,3)$ & $(-3,1)$ is 26. Let $f(x,y)$ be the locus of P, which intersects the x-axis at A&B and the y-axis at the point C,D then the area of quadrilateral ABCD is ———

A) $\frac{1}{2}(3\sqrt{5})$ B) $\frac{1}{2}(4\sqrt{7})$ C) $\frac{1}{2}(3\sqrt{7})$ D) $\frac{1}{2}(10\sqrt{7})$

Key: B

Sol:

Let P(x,y)be any point

$A(1,3) \quad B(-3,1)$

$PA^2 + PB^2 = 23$

$(x-1)^2 + (y-3)^2 + (x+3)^2 + (y-1)^2 = 23$

$x^2 - 2x + 1 + y^2 - 6y + 9 + x^2 + 6x + y^2 - xy + 1 = 26$

$2x^2 + 2y^2 - 4x - 8y - 6 = 0$

$x^2+y^2-2x-4y-3=0$

$\therefore$ If int x-axis at A and B $\Rightarrow x^2+2x-3=0$

$\Rightarrow x=-3$ or $\Rightarrow x=1$

$A(-3,0) \quad B(1,0)$

$\therefore$ If int y-axis at C and D put x=0 $\Rightarrow y^2-4y-3=0$

$\Rightarrow y=2\pm\sqrt{7}$

$C\left(0,2-\sqrt{7}\right)$ and $D\left(0,2+2\sqrt{7}\right)$

Area of Quadrilateral ABCD= $4\sqrt{7}$

6b. A point P moves, so that the sum of squares of its distances from the points $(2,3)$ & $(-3,2)$ is 32. Let $f(x,y)$ be the locus of P, which intersects the x-axis at A&B and the y-axis at the point C,D then the area of quadrilateral ABCD is ———

A) $\frac{\sqrt{481}}{2}$ B) $\frac{\sqrt{581}}{2}$ C) $\frac{\sqrt{581}}{3}$ D) $\frac{\sqrt{591}}{3}$

Key:A

Sol:

$(x-2)^2+(y-3)^2+(x+3)^2+(y-2)^2=32$

$x^2-4x+4+y^2-6y+9+x^2+6x+4+y^2+4y+4=32$

$2x^2+2y^2+2x+2y+26=32$

$x^2+y^2+x-5y-3=0$

$\therefore$ .If int x-axis at A and B $\Rightarrow x^2+x-2=0$

$$x=\frac{-1\pm\sqrt{1+12}}{2}=\frac{-1\pm\sqrt{13}}{2}$$

$$A\left(\frac{-1-\sqrt{13}}{2},0\right) \quad B\left(\frac{-1+\sqrt{13}}{2},0\right)$$

$\therefore$ If int y-axis at C and D $\Rightarrow y^2-5y-3=0$

$$\Rightarrow y=\frac{5\pm\sqrt{25+12}}{2}=\frac{5\pm\sqrt{37}}{2}$$

$$C\left(0,\frac{5-\sqrt{37}}{2}\right) \quad D\left(0,\frac{5+\sqrt{37}}{2}\right)$$

Area of quadrilateral ABCD $=\frac{1}{2}\begin{vmatrix}\sqrt{13} & 0\\ 0 & \sqrt{37}\end{vmatrix}$

$$=\frac{1}{2}\left(\sqrt{13}.\sqrt{37}\right)$$

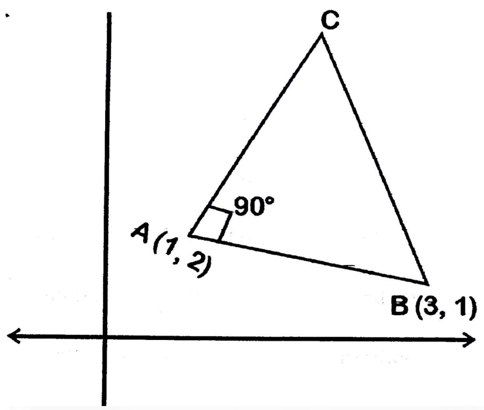
$$=\frac{\sqrt{481}}{2}$$

7. A triangle ABC lying in the first quadrant has two vertices as $A(1,2)$ and $B(3,1)$. If $\angle BAC = 90^0$, and $ar(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

A) $1+\sqrt{5}$ B) $1+2\sqrt{5}$ C) $2\sqrt{5}-1$ D) $2+\sqrt{5}$

Key : B **Jee mains -2020**

Sol :



Slope of line $AB = -\frac{1}{2}$

Slope of line $AC = 2$ & $AB = \sqrt{5}$

$\because \frac{1}{2} AB.AC = 5\sqrt{5}$

$\therefore AC = 10$

$\therefore$ Coordinate of $C = (1+10\cos\theta, 2+10\sin\theta)$

Here $\tan\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}.\sin\theta = \frac{2}{\sqrt{5}}$

Coordinate of $C = (1+2\sqrt{5}, 2+4\sqrt{5})$

Abscissa of vertex C is $1+2\sqrt{5}$

7a. The area of an equilateral triangle whose two vertices are $(1,0);(3,0)$ and third vertex lying in first quadrant is

A) $\sqrt{3}/4$ B) $\sqrt{3}/2$ C) $\sqrt{3}$ D) None of these

Key : C

Sol: CD is perpendicular to AB where D is midpoint of AB $(2,0)$

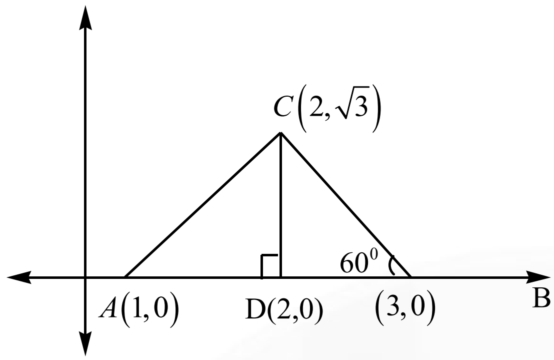
in ΔCDB, $\tan 60^0 = \frac{CD}{DB}$

$\sqrt{3} = \frac{CD}{1}$

$CD = \sqrt{3}$

Area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2 \times \sqrt{3}$

$\Delta = \sqrt{3}$ sq.units



7b. A triangle ABC lying in first quadrant has two vertices as $A(2,3)$ and $B(3,2)$. If $\angle BAC = 90^0$ and $Ar(\Delta ABC) = 3\sqrt{2}$ sq.units then coordinates of point C is

A) $(1+3\sqrt{2}, 2+3\sqrt{2})$ B) $(2-3\sqrt{2}, 3-3\sqrt{2})$ C) $(2+3\sqrt{2}, 3+3\sqrt{2})$ D) $(0, 3+2\sqrt{2})$

Key : C

Sol : Slope of $AB = -1$

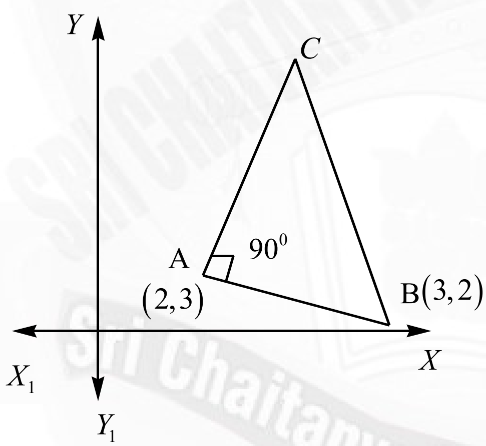
Slope of AC=1

$\therefore$ length of $AB = \sqrt{2}$

$\therefore$ area $= \frac{1}{2}.AB.AC$

$3\sqrt{2} = \frac{1}{2}\sqrt{2}AC$

AC= 6 units



Coordinate of $C = (2+6\cos\theta, 3+6\sin\theta)$

Here $\tan\theta = 1$, $\Rightarrow \sin\theta = \frac{1}{\sqrt{2}} = \cos\theta$

$C = \left(2+\frac{6}{\sqrt{2}}, 3+\frac{6}{\sqrt{2}}\right)$

$C = (2+3\sqrt{2}, 3+3\sqrt{2})$

7c. If, in a $\Delta ABC, b = 12$ units, c=5 units and $\Delta = 30$ sq. units then the distance between vertex A and incentre of the triangle is equal to

A) 2 units B) $2\sqrt{2}$ units C) $\sqrt{2}$ units D) $3\sqrt{2}$ units

Key : C

Sol : $\Delta = \frac{1}{2}bc\sin A$

$30 = \frac{12\times 5}{2}\sin A$

Therefore

$\sin A = 1$

Hence

$A = \frac{\pi}{2}$

Hence it is a right angled triangle

Therefore

$a^2 = b^2 + c^2$

$= 144 + 25$

$= 169$

Hence

$a = 13$

Now

$AI = r\cos ec\frac{A}{2}$

$= \frac{\Delta}{s} = r\cos ec\frac{A}{2}$

$= \frac{30}{12+13+5}.\cos ec45^0 = \sqrt{2}$

8. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a

A) Right angled triangle

B) Equilateral triangle

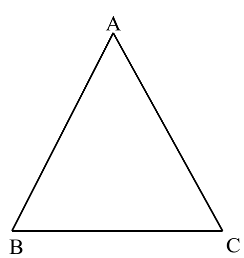
C) Isosceles triangle

D) None of these

Key: C

Jee Mains -2021

Sol:



Solving $x - y = 0,\ 2x + 2y = 3 \Rightarrow B = (1,1)$

Solving $x-y=0,\ 2x+y=6 \Rightarrow A=(2,2)$

Solving $x+2y=3,\ 2x+y=6 \Rightarrow C=(3,0)$

$AB=\sqrt{2},\ \ BC=\sqrt{5}$, $AC=\sqrt{5}$

8a. The triangle with the vertices $(4,3)$, $(-3,2)$, $(1,-6)$ is

A) An obtuse angled triangle

B) An acute angled triangle

C) Right angled

D) Right angled isosceles

Key: B

Sol:

$AB^2=50$, $BC^2=80$, $AC^2=90$

$AB^2+BC^2>AC^2$

8b. The points (a,b), $(-a,-b)$, $(b\sqrt{3},-a\sqrt{3})$ are the vertices of a triangle which is

A) Isosceles B) Equilateral C) Right angled D) Scalene

Key: B

Sol: $AB^2=BC^2=AC^2$

$\therefore$ Triangle is Equilateral

8c. If $(3,2),(-3,2)$, $(0,h)$ are vertices of an equilateral triangle and $h<0$, then h=

A) $2\pm\sqrt{27}$ B) $\sqrt{2}\pm\sqrt{27}$ C) $2-\sqrt{27}$ D) $\sqrt{2}+27$

Key: C

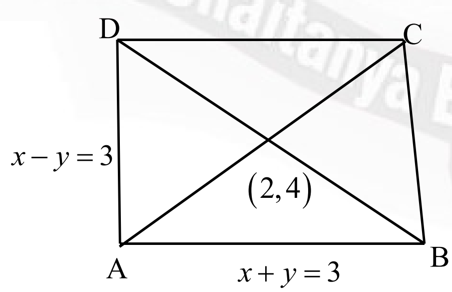
Sol: $AB^2=BC^2=AC^2$

9. Two sides of a parallelogram are along the lines $x+y=3$, $x-y+3=0$. If its diagonals intersect at $(2,4)$ then one of its vertex is

A) $(2,6)$ B) $(2,1)$ C) $(3,5)$ D) $(3,6)$

Key : D **Jee Mains-2019**

Sol:



From

$x+y=3$
$x-y=-3$ $\qquad A(0,3)$

Let $C(x_1y_1)$ then $(2,4)$ is mid point of AC

$C(4,5)$

Equation of CD is $x+y=9$

Equation of BC is $x-y=-1$

Then $D=(3,6)$

9a. If $P(1,2)Q(4,6)R(5,7)S(a,b)$ are the vertices of a parallelogram PQRS then

A) $a=2,b=4$ B) $a=3,b=4$ C) $a=2,b=3$ D) $a=3,b=5$

Key : C

Sol : $\frac{a+4}{2}=\frac{1+5}{2},\frac{b+6}{2}=\frac{2+7}{2}\Rightarrow a=2,b=3$

9b. The diagonals of a parallelogram PQRS are along the lines $x+3y=4,6x-2y=7$ then PQRS must be

A) Rectangle B) Square

C) Cyclic quadrilateral D) Rhombus

Key : D

Sol : Slope of $x+3y=4$ is $\frac{-1}{3}$

Slope of $6x-2y=7$ is 3

$m_1m_2=-1$

$\therefore$ The diagonals are $\perp r$

$\therefore$ PQRS is a rhombus

10. If in a parallelogram ABDC $A(1,2)B(3,4)$ $C(2,5)$ then the equation of the diagonal AD is

A) $5x+3y-11=0$ B) $3x-5y+7=0$ C) $3x+5y-13=0$ D) $5x-3y+1=0$

Key: D **Jee Mains-2019**

Sol : $D=(4,7)$

Equation of AD is $5x-3y+1=0$

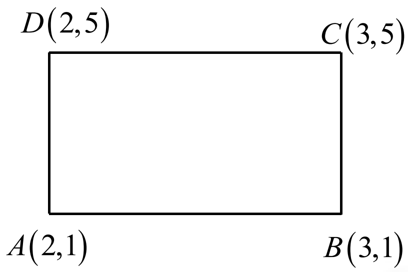
10a. The equations to a pair of opposite sides of parallelogram are $x^2-5x+6=0$ and $y^2-6y+5=0$ then equations to its diagonals are ____

A) $x+4y=13, y=4x-7$ B) $4x+y=13, 4y=x-7$

C) $4x+y=12, y=4x-7$ D) $y-4x=13, y+4x=7$

Key : C

Sol :



The equation of AC is

$y-1=\frac{4}{1}(x-2)\Rightarrow y=4x-7$

Equation of BD is

$x-2=\frac{1}{-4}(y-5)$

$4x+y=13$

10b. Two sides of a rhombus ABCD are parallel to the lines $y=x+2$, $y=7x+3$. If the diagonals of the rhombus intersect at the point $(1,2)$ and the vertex A is on the y-axis. The possible coordinate of A

A) $\left(0,\frac{5}{2}\right)$ B) $(0,5)$ C) $\left(\frac{5}{2},0\right)$ D) $(0,1)$

Key : A

Sol : Let $A(0,a)$

The diagonal AC is parallel to the bisector of the angle between the lines

$y=x+2, y=7x+3$ equation of bisector $\frac{x-y+2}{\sqrt{2}}=\pm\left(\frac{7x-y+3}{\sqrt{50}}\right)$

$2x+4y-7=0, 12x-6y+13=0$

11. The equations to a pair of opposite sides of a parallelogram are $x^2-5x+6=0$ and $y^2-6y+5=0$. The equations to its diagonal are **IIT-JEE 1994**

A) $x+4y=13, y=4x-7$ B) $4x+y=13, 4y=x-7$

C) $4x+y=13, y=4x-7$ D) $y-4x=13, y+4x=7$

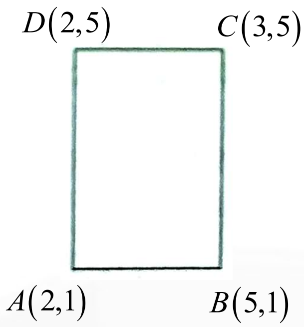
Key : C

Sol : The sides of parallelograms are $x=2, x=3, y=1, y=5$.

$\therefore$ Vertices of parallelogram are as shown in the figure

The equation of diagonal AC is

$\frac{y-1}{5-1}=\frac{x-2}{3-2}$



Or

And the equation of diagonal BD is

$$\frac{x-2}{3-2}=\frac{y-5}{1-5}$$

Or $4x+y=13$

12. Let PQR be a right-angled isosceles triangle, right angled at $P(2,1)$. If the equation of the line QR is $2x+y=3$ then the equation representing the pair of lines PQ and PR is

IIT-JEE 1999

A) $3x^2-3y^2+8xy+20x+10y+25=0$

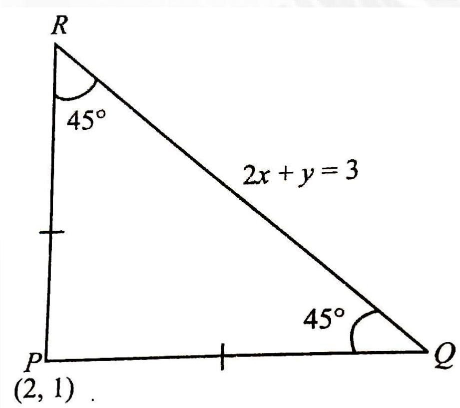
B) $3x^2-3y2+8xy-20x-10y+25=0$

C) $3x^2-3y^2+8xy+10x+15+20=0$

D) $3x^2-3y^2-8xy-15y-20=0$

Key : B

Sol :



Let m be the slope of PQ. Then,

$$\tan 45^0=\left|\frac{m-(-2)}{1+m(-2)}\right|=\left|\frac{m+2}{1-2m}\right|$$

$\therefore\ m+2=1-2m$ or $-1+2m=m+2$

$\therefore\ m=-\frac{1}{3}$ or $m=3$

Hence, the equation of PQ is

$$y-1=-\frac{1}{3}(x-2)$$

Or $x+3y-5=0$

And the equation of PR is

$y-1=3(x-2)$

Or $3x-y-5=0$

Hence, the combined equation of PQ and PR is

$(x+3y-5)(3x-y-5)=0$

13. The area of the parallelogram formed by the lines $y=mx$, $y=nx$, $y=mx+1, y=nx+1$ is equals

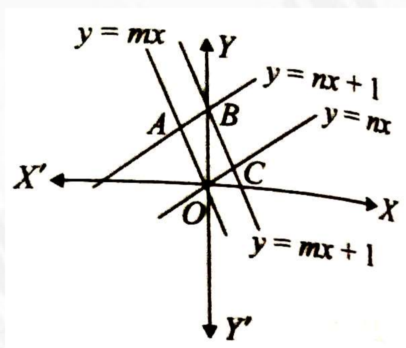
IIT-JEE 2001

A) $\frac{|m+n|}{(m-n)^2}$ B) $\frac{2}{(m+n)^2}$ C) $\frac{1}{(m+n)}$ D) $\frac{1}{(m-n)}$

Key : D

Sol : $y=mx$ and $y=nx$ are line through $(0,0)$, $y=mx+1$ and $y=nx+1$ are lines parallel to $y=mx$ and $y=nx$ respectively with $y-$intercept 1.

The vertices are $O(0,0)$, $A(1/m-n), m/(m-n)$. The area of parallelogram is given by



$$= 2\times\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{1}{m-n} & \frac{m}{m-n} & 1 \end{vmatrix}$$

$$= \frac{1}{|m-n|}$$

14. The diagonals of parallelogram PQRS are along the lines $x+3y=4$ and $6x-2y=7$. Then PQRS must be a **IIT-JEE 1998**

A) rectangle B) square C) cyclic quadrilateral D) rhombus

Key : D

Sol : The slope of $x+3y=4$ is $-1/3$ and the slope of $6x-2y=7$ is 3. Therefore, these two lines are perpendicular, which shows that both diagonals are perpendicular . Hence, PQRS must be a rhomus.

Angular bisector of Two lines

(Acute and Obtuse bisector, the bisector containing or do not containg a given point, internal and external bisector)

1. A light ray emits from the origin making an angle 30^0 with the positive x-axis. After getting reflected by the line $x+y=1$,If this ray intersects x-axis at Q, then the abscissa of Q is

A) $\frac{2}{(\sqrt{3}-1)}$ B) $\frac{2}{3+\sqrt{3}}$ C) $\frac{2}{3-\sqrt{3}}$ D) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$

Key : B **Jee mains - 2023**

Sol : Slope of reflected ray = $\tan 60^0 = \sqrt{3}$

Line $y=\frac{x}{\sqrt{3}}$ intersect $y+x=1$ at $\left(\frac{\sqrt{3}}{\sqrt{3}+1}, \frac{1}{\sqrt{3}+1}\right)$

Equation of reflected ray is

$$y-\frac{1}{\sqrt{3}+1}=\sqrt{3}\left(x-\frac{\sqrt{3}}{\sqrt{3}+1}\right)$$

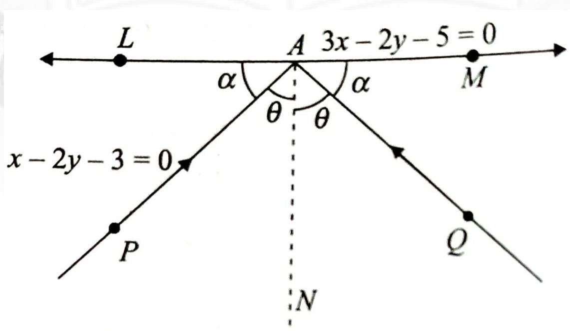
Put $y=0 \Rightarrow x=\frac{2}{3+\sqrt{3}}$

1a. A ray of light is sent along the line $x-2y-3=0$. Upon reaching the line $3x-2y-5=0$, the ray is reflected. Then the equation of the line containing the reflected ray is

A) $29x-2y-31=0$ B) $29x+2y-31=0$ C) $29x+2y+31=0$ D) $29x-2y+31=0$

Key :A

Sol : Solving the equation of LM and PA, the coordinates of A can be obtained. If the slope of AQ is determined, then the equation of AQ can determined. If the slope of AQ is m, then equating the two values of $\tan\theta$ (considering the angles between AL and AP and between AM and AQ), m can be formed.



The equation of line LM is

$3x-2y-5=0$ …(i)

The equation of PA is

$x-2y-5=0$ ....(ii)

Solving (i) and (ii), we got

$x=1, y=-1$

$\therefore\ A=(1,-1)$

Let the slope of AQ be m.

Slope of $LM=\frac{3}{2}$

Slope of $PA=\frac{1}{2}$

i) Let $\angle LAP=\angle QAM=\alpha$

ii) As $\angle LAP=\alpha$

we have $\tan\alpha=\left|\frac{\frac{3}{2}-\frac{1}{2}}{1+\frac{3}{2}\times\frac{1}{2}}\right|=\frac{4}{7}$ ....(iii)

Again, $\angle QAM=\alpha$

$\therefore\ \tan\alpha=\left|\frac{m-\frac{3}{2}}{1+\frac{3}{2}m}\right|=\left|\frac{2m-3}{2+3m}\right|$ .....(iv)

From (ii) and (iv), we have

$\left|\frac{2m-3}{2+3m}\right|=\frac{4}{7}$

Or $\frac{2m-3}{2+3m}=\pm 7$

$\therefore\ m=\frac{1}{2},\frac{29}{2}$

But slope of $AP=\frac{1}{2}$

$\therefore$ slope of $AQ=\frac{29}{2}$

Now, the equation of AQ will be

$y+1=\frac{29}{2}(x-1)$

Or $29x-2y-31=0$

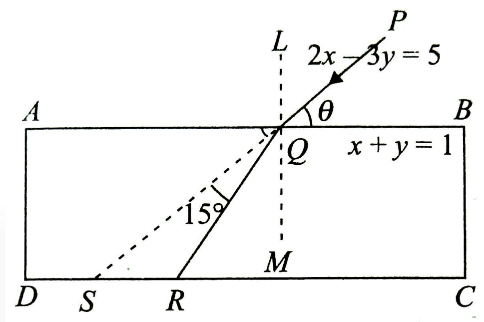
1b. A ray of light is sent along the line $2x-3y=5$. After refracting the line $x+y=1$ it enters the opposite side after turning by 15^0 away from the line $x+y=1$. Then the slope of the line along which the refracted ray travels is

A) $\frac{3\sqrt{3}-8}{1+2\sqrt{3}}$ B) $\frac{3\sqrt{3}-8}{1-2\sqrt{3}}$ C) $\frac{3\sqrt{3}+8}{1+2\sqrt{3}}$ D) $\frac{3\sqrt{3}+8}{1-2\sqrt{3}}$

Key: B

Sol : The equation of line AB is

$x+y=1$ ....(i)



The equation of line QP is

$2x-3y=5$ ....(ii)

QR is the refracted ray. According to the question,

$\angle SQR=15^0$

Solving (i) and (ii), we get

$x=\frac{8}{5}$ and $y=-\frac{3}{5}$

Or $Q=\left(\frac{8}{5},-\frac{3}{5}\right)$

Slope QP$=\frac{2}{3}$

Or Slope of QS $=\frac{2}{3}$

Let Slope of $QR=m$

But $\angle SQR=15^0$

Or $\tan 15^0=\left|\frac{(2/3)-m}{1+(2/3)m}\right|$

Or $2-\sqrt{3}=\left|\frac{2-3m}{1+(2/3)m}\right|$

Or $\pm\left(2-\sqrt{3}\right)=\frac{2-3m}{3+2m}$

Or $2-3m=\pm\left[6-3\sqrt{3}+\left(4-2\sqrt{3}\right)m\right]$

Or $2-3m=\left\{\frac{6-3\sqrt{3}+\left(4-2\sqrt{3}\right)m}{-6+3\sqrt{3}-\left(4-2\sqrt{3}\right)m}\right\}$

Or $m=\frac{3\sqrt{3}-4}{7-2\sqrt{3}},\frac{3\sqrt{3}-08}{1-2\sqrt{3}}$

For the required line,

$m=\frac{3\sqrt{3}-8}{1-2\sqrt{3}}$

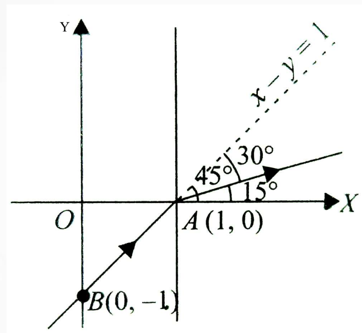
1c. A beam of light is sent along the line $x-y=1$, which after refracting from the x-axis enters the opposite side by turning through 30^0 towards the normal at the point of incidence on the x-axis. Then the equation of the refracted ray is

A) $\left(2-\sqrt{3}\right)x-y=2+\sqrt{3}$ B) $\left(2+\sqrt{3}\right)x-y=2+\sqrt{3}$

C) $\left(2-\sqrt{3}\right)x+y=\left(2+\sqrt{3}\right)$ D) $y=\left(2-\sqrt{3}\right)(x-1)$

Key : D

Sol :



From the figure, refracted ray makes an angle of 15^0 with the positive direction of the x-axis and passes through the point $(1,0)$. Its equation is

$(y-0)=\tan\left(45^0-30^0\right)(x-1)$

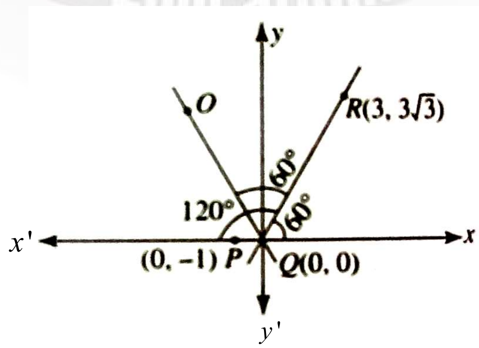
Or $y=\left(2-\sqrt{3}\right)(x-1)$

2. Let $P=(-1,0), Q=(0,0)$, and $R=\left(3,3\sqrt{3}\right)$ be three points. Then the equation of the bisector of $\angle PQR$ is **IIT-JEE 2002**

A) $\left(\sqrt{3}/2\right)x+y=0$ B) $x+\sqrt{3}y=0$ C) $\sqrt{3}x+y=0$ D) $x+\left(\sqrt{3}/2\right)y=0$

Key : C

Sol : The slope of QR is $\left(3\sqrt{3}-0\right)/(3-0)=\sqrt{3}$ i.e., $\theta=60^0$ clearly, $\angle PQR=120^0, OQ$ is the angle bisector of the angle. So line OQ makes 120^0 with the positive direction of the x-axis. Therefore, the equation of the bisector of $\angle PQR$ is $y=x\tan 120^0$ or $y=-\sqrt{3}x$,i.e., $\sqrt{3}x+y=0$

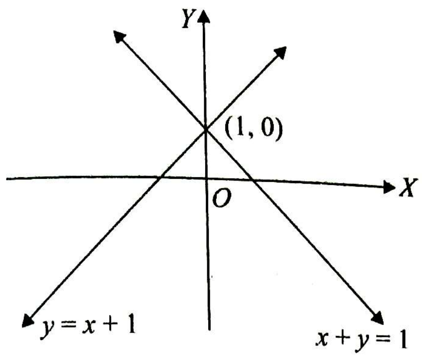


3. Area of the triangle formed by the line $x+y=3$ and the angle bisectors of the pairs of straight lines $x^2-y^2+2y=1$ is

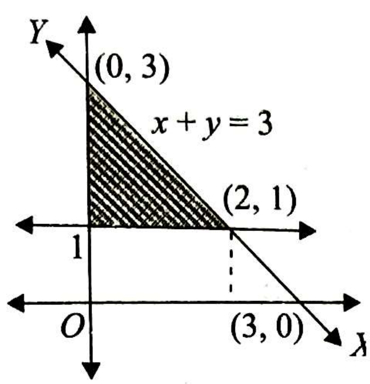
A) 2 sq.units B) 4 sq.units C) 6 sq.units D) 8 sq.units

Key : A **IIT-JEE 2004**

Sol : $x^2-y^2+2y=1$ or $x=\pm(y-1)$



The bisectors of the above lines are $x=0$ and $y=1$.



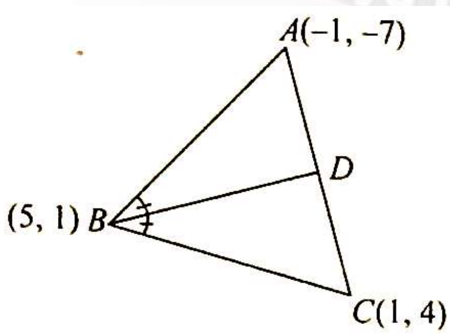
So, the area between $x=0, y=1$, and $x+y=3$ is the shaded region shown in the figure. The area is given by $(1/2)\times 2\times 2=2$ sq.units.

4. The vertices of a triangle are $A(-1,-7), B(5,1)$, and $C(1,4)$. The equation of the bisector of $\angle ABC$ is ____

A) $x-7y+2=0$ B) $x+7y+2=0$ C) $7x-y+2=0$ D) $7x+y+2=0$

Key : A **IIT-JEE 1993**

Sol: Let BD the bisector of $\angle ABC$. Then,



$AD : DC = AB : BC$

And $AB = \sqrt{(5+1)^2 + (1+7)^2} = 10$

$BC = \sqrt{(5-1)^2 + (1-4)^2} = 5$

$\therefore \; AD : DC = 2 : 1$

Therefore, by section formula, $D = (1/3, 1/3)$. Therefore, the equation of BD is

$y - 1 = \dfrac{1/3 - 1}{1/3 - 5}(x - 5)$

Or $y - 1 = \dfrac{-2/3}{-14/3}(x - 5)$

Or $7y - 7 = x - 5$

Or $x - 7y + 2 = 0$

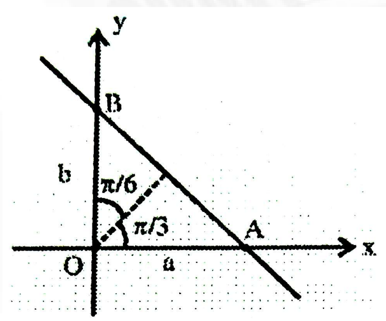
Miscellaneous

1. A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive direction of x-axis and y-axis respectively. If the perpendicular from origin O to this line makes an angle of $\dfrac{\pi}{6}$ with positive direction of y-axis and the area of ΔOAB is $\dfrac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to :

A) $\dfrac{392}{3}$ B) 196 C) $\dfrac{196}{3}$ D) 98

Key : A **Jee Mains -2023**

Sol.



Equation of straight line : $\dfrac{x}{a} + \dfrac{y}{b} = 1$

Or $x\cos\dfrac{\pi}{3} + y\sin\dfrac{\pi}{3} = p$

$\dfrac{x}{2} + \dfrac{y\sqrt{3}}{2} = p$

$\Rightarrow \dfrac{x}{2p} + \dfrac{y}{\dfrac{2p}{\sqrt{3}}} = 1$

Comparing both : $a = 2p, b = \dfrac{2p}{\sqrt{3}}$

Now area of $\Delta OAB = \frac{1}{2}.ab = \frac{98}{3}.\sqrt{3}$

$\frac{1}{2}.2p.\frac{2p}{\sqrt{3}} = \frac{98}{3}.\sqrt{3}$

$p^2 = 49$

$a^2 - b^2 = 4p^2 - \frac{4p^2}{3} = \frac{2}{3}4p^2$

$= \frac{8}{3}.49 = \frac{392}{3}$

1a. Let ABC be a triangle. Let A be the point $(1,2)$, $y = x$ be perpendicular bisector of AB, and $x - 2y + 1 = 0$ be the angle bisector of $\angle C$. I f the equation of BC is given by $ax + by - 5 = 0$, then the value of $a + b$ is

A) 1 B) 2 C) 3 D) 4

Key : D

Sol : The point B is $(2,1)$. The image of $A(1,2)$ on the line $x - 2y + 1 = 0$ is given by

$\frac{x-1}{-1} = \frac{y-2}{-2} = \frac{4}{5}$

Hence, the coordinates of the point are $(9/5, 2/5)$. Since this point lies on BC, the equation of

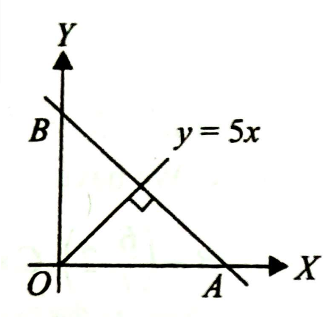
BC is $3x - y - 5 = 0$. Hence, $a + b = 2$.

1b. A line is drawn perpendicular to line $y = 5x$, meeting the coordinate axes at A and B. If the area of triangle OAB is 10 sq. units, where O is the origin, then the equation of drawn line is :

A) $3x - y - 9 = 0$ B) $x + 5y = 10$ C) $x + 4y = 10$ D) $x - 4y = 10$

Key : B

Sol : Let the equation of line be $\frac{x}{a} + \frac{y}{b} = 1$



AB is perpendicular to $y = 5x$. Hence,

$-\frac{b}{a} \times 5 = -1$ or $5b = a$

Area of $\Delta OAB = \frac{1}{2}|ab|$

Or $10=\frac{1}{2}\left|5b^2\right|$

Or $b^2=4$ or $b=\pm 2, a=\pm 10$

The line can be

$\frac{x}{10}+\frac{y}{2}=1$ or $\frac{x}{10}+\frac{y}{2}=-1$

1c. If $x-2y+4=0$ and $2x+y-5=0$ are the sides of an isosceles triangle having area 10 sq. units, the equation of the third side is

A) $3x-y=-9$ B) $3x-y+11=0$ C) $x-3y=19$ D) $3x-y+15=0$

Key :A

Sol : The given lines are mutually perpendicular and intersect at $(6/5,13/5)$.

The equations of angle bisectors of the given lines are $x-2y+4=\pm(2x+y-5)$

i.e., $x+3y=9$ and $3x-y=1$

side BC will be parallel to these bisectors. Let

$AD=a$

Or $AB=a\sqrt{2}$

Area of $\Delta ABC=\frac{1}{2}\times\left(a\sqrt{2}\right)^2=a^2=10$

Or $a=\sqrt{10}$

Let the equation of BC be $x+3y=\lambda$. Then,

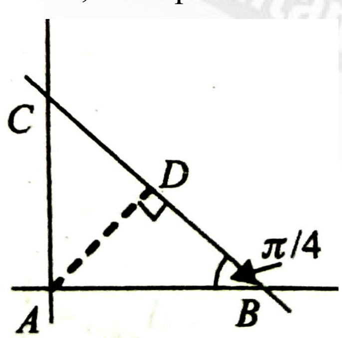
$\sqrt{10}=\frac{(6/5)-(39/5)-\lambda}{\sqrt{10}}$

Or $\lambda=1,19$

Therefore, the equation of BC is $x+3y=-1$ or $x+3y=19$. If the equation of BC is $3x-y=\lambda$, then $\sqrt{10}=\frac{(18/5)-(13/5)-\lambda}{\sqrt{10}}$

Or $\lambda=-9,11$

Hence, the equation of BC is $3x-y=-9$ or $3x-y=11$.



2. Let $A(1,1), B(-4,3), C(-2,-5)$ be vertices of a triangle ABC,P be a point on side BC, and Δ_1 and Δ_2 be the area of triangle APB and ABC, respectively. If $\Delta_1 : \Delta_2 = 4:7$, then the area enclosed by the lines AP, AC and the x-axis is

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) 1

Key : C **Jee Mains-2022**

Sol : $A(1,1)B(-4,3)C(-2,-5)$

Area $(\Delta ABC) = 18$

Let $P(\alpha,\beta)$ lies on BC

Area $\Delta APB = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & 1 \\ -4 & 3 & 1 \end{vmatrix} = \frac{1}{2}|2\alpha + 5\beta - 7|$

Given $\frac{\text{Area } \Delta\text{APB}}{\text{Area } \Delta\text{ABC}} = \frac{4}{7}$

$\Rightarrow |2\alpha + 5\beta - 7| = \frac{144}{7}$

$\Rightarrow 2\alpha + 5\beta - 7 = \pm\frac{144}{7}$ ....(1)

Equation of $\overline{AC}$ is $2x - y - 1 = 0$ ......(2)

It cuts x-axis at $M\left(\frac{1}{2}, 0\right)$

Equation of $\overline{BC}$ $4x + y = 13 = 0$ ....(3)

Solving (1) & (3) we get

$P = \left(\frac{-36}{7}, \frac{53}{7}\right)$ or $\left(\frac{-20}{7}, \frac{-11}{7}\right)$

Since x-coordinates of B,C are -4 and -2 respectively

$\Rightarrow P = \left(\frac{-20}{7}, \frac{-11}{7}\right)$

(-4<x - intercept of P<-2)

Equation $\overline{AP}$ is $2x - 3y + 1 = 0$

$y = 0 \Rightarrow x = \frac{-1}{2}$;

Let $N\left(\frac{-1}{2}, 0\right)$

$$\therefore \text{ Area of } \Delta NAM = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -\frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 1 \end{vmatrix} = \frac{1}{2}$$

2a. Let $A(2,2)$ $B(3,4)$ $C(-1,-4)$ be the vertices of $\triangle ABC$, P be a point or BC, and $\triangle_1$ and $\triangle_2$ be the area of triangle APB and triangle ABL , respectively of $\triangle_1 : \triangle_2 = 3:5$. Then the area of enclosed by AP, AC and x-axis is……

A) $\frac{43}{22}$ B) $\frac{52}{22}$ C) $\frac{62}{23}$ D) $\frac{63}{24}$

Key: A

Sol:

$A(2,2)$ $B(-3,4)$ $C(-1,4)$ Let $P(\alpha,\beta)$ lies on BC

Area of triangle ABC $= \frac{1}{2}\begin{vmatrix} 5 & -2 \\ 3 & 6 \end{vmatrix} = \frac{30+6}{2} = 18$ Square

Area of triangle APB $= \frac{1}{2}\begin{vmatrix} 2-\alpha & 2-\beta \\ 5 & -2 \end{vmatrix} = \frac{1}{2}\left|-4+2\alpha-10+5\beta\right| = \frac{1}{2}\left|2\alpha+5\beta-14\right|$

$$\frac{Areaof\,\triangle APB}{Areaof\,\triangle ABC} = \frac{3}{5}$$

$$\frac{\frac{1}{2}\left|2\alpha+5\beta-14\right|}{18} = \frac{3}{5}$$

$$\left|2\alpha+5\beta-14\right| = \frac{108}{5} \Rightarrow 2\alpha+5\beta-14 = \frac{108}{5} \to 1$$

Equation of AC is $y-2 = \frac{-2}{-3}(x-2)$

$$y-2 = 2x-4 \Rightarrow 2x-y-2 = 0 \to 2$$

It was x-axis at $M(1,0)$

Equation of BL is $\frac{y-4}{-0} = \frac{x+3}{2} \Rightarrow y-4 = -4(x+3) \Rightarrow 4x+y+8 = 0 \to 3$

From 1 & 3

$$2\left(2\alpha+5\beta-14 = \frac{108}{5}\right)$$

$$4\alpha+\beta+8 = 0$$

$$9\beta-36 = \frac{216}{5}$$

$$\beta-4 = \frac{24}{5} \Rightarrow \beta = \frac{44}{5}$$

0r

$$4\alpha+\frac{44}{5}+8 = 0$$

$$4\alpha = -\frac{84}{5} \Rightarrow \alpha = -\frac{21}{5}$$

$P\left(-\frac{21}{5},\frac{44}{5}\right)$

Equation of AP $\Rightarrow y-2=\frac{44}{-21}(x-2)$

Put y=0

$-2=\frac{-44}{21}(x-2) \quad \Rightarrow x=\frac{63}{22} \quad N\left(\frac{63}{22},0\right)$

Area of triangle ANM= $\frac{1}{2}\begin{vmatrix} 1 & 2 \\ \frac{-21}{22} & 2 \end{vmatrix} \Rightarrow \frac{1}{2}\left|2+\frac{21}{11}\right| \Rightarrow \frac{43}{22}$

2b. Let $A(2,3)$ $B(-4,5)$ $C(-3,-4)$ be the vertices of $\triangle ABC$, P be a point or BC, and $\triangle_1$ and $\triangle_2$ be the area of triangle APB and triangle ABL , respectively of $\triangle_1 : \triangle_2 = 3:4$. Then the area of enrolled by AP, AC and x-axis is……

A) $\frac{549}{35}$ B) $\frac{539}{38}$ C) $\frac{559}{37}$ D) $\frac{659}{37}$

Key: A

Sol:

$A(2,3)$ $B(-4,5)$ $C(-3,-4)$ Let $P(\alpha,\beta)$ lies on BC

Area of triangle ABC $=\frac{1}{2}\begin{vmatrix} 6 & -2 \\ 5 & 7 \end{vmatrix}=\frac{40+10}{2}=\frac{52}{2}=26$ Square

Area of triangle APB $=\frac{1}{2}\begin{vmatrix} 2-\alpha & 2-\beta \\ 6 & -2 \end{vmatrix}=\frac{1}{2}|-4+2\alpha-18+6\beta|=\frac{1}{2}|2\alpha+6\beta-22|=|\alpha+3\beta-11|$

$\frac{Areaof\,\triangle APB}{Areaof\,\triangle ABC}=\frac{3}{4}$

$\frac{|\alpha-3\beta-11|}{26}=\frac{3}{4} \Rightarrow \alpha+\beta-11=\pm\frac{39}{2} \rightarrow 1$

Equation of AC is $y-3=\frac{-7}{-5}(x-y) \rightarrow 2$

Put y=0 $\Rightarrow -3=\frac{7}{5}(x-2) \Rightarrow x-2=\frac{-15}{7} \Rightarrow x=2-\frac{15}{7}=-\frac{1}{7}$

$M\left(-\frac{1}{7},0\right)$

Equation of BC is $y-5=\frac{-9}{1}(x+4) \Rightarrow y-5=-9-36 \Rightarrow 9x+y+31=0 \rightarrow 3$

From 1 & 3

$\alpha+\beta-11=39$

$3(9\alpha+\beta+31=0)$

$-2\beta\alpha-104=-\frac{39}{4} \Rightarrow \alpha=-\frac{19}{4} \quad \beta=\frac{47}{4} \quad P\left(-\frac{19}{4},\frac{47}{4}\right)$ OR $P\left(-\frac{13}{4},\frac{7}{4}\right)$

$$-4 < x - 10 \text{ of } P < -3 \qquad \therefore P\left(\frac{-13}{4}, \frac{7}{4}\right)$$

Equation of AP is $y-3=\dfrac{\frac{7}{4}-3}{-\frac{13}{4}-2}(x-2) \Rightarrow -3=\dfrac{-5}{-21}(x-2) \;\Rightarrow x-2=\dfrac{-63}{5}$

$$\Rightarrow x=\frac{-53}{5} \qquad N\left(\frac{-53}{5},10\right)$$

Area of triangle NAM $=\dfrac{1}{2}\begin{vmatrix} 2+\frac{1}{7} & 3 \\ 2+\frac{53}{5} & 3 \end{vmatrix}$

$$=\frac{1}{2}\begin{vmatrix} \frac{15}{7} & 3 \\ \frac{63}{5} & 3 \end{vmatrix}=\frac{1}{2}\left|\frac{45}{7}-\frac{189}{5}\right|$$

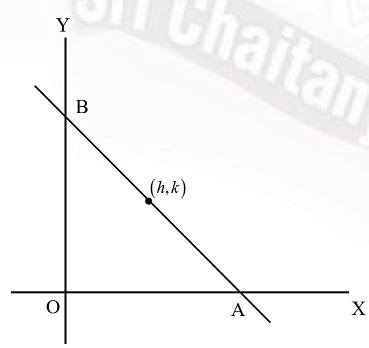
$$=\frac{1}{2}\left|\frac{225-1323}{35}\right|=\frac{1098}{2|35|}=\frac{549}{35}$$

3. A man is walking on a straight line. The A.M of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C. One placed at the points $(1,1)$, $(2,2)$ and $(4,4)$ respectively. Then which of these stones is on the path of the man ________

A) A only B) B only C) C only D) All the above

Key: B **Jee Mains - 2021**

Sol:



Given $A=(1,1)$ $\quad B=(2,2)$ $\quad C=(4,4)$

Let x-intercept be 'a' and
y-intercept be 'b'

Equation of the path is $\frac{x}{a}+\frac{y}{b}=1$

$\therefore (h,k)$ be a point on the path $\Rightarrow \frac{h}{a}+\frac{k}{b}=1$

Also A.M. of reciprocal of a , b is $\frac{1}{4}$ is $\frac{\frac{1}{a}+\frac{1}{b}}{2}=\frac{1}{4}$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{1}{2}$$

$$\Rightarrow \frac{2}{a}+\frac{2}{b}=1$$

$$\therefore (h,k)=(2,2)$$

$\therefore B$ lies on the path

3a. A man is walking on a straight line. The A.M of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{18}$. Three stones A , B and C. One placed at the points $(1,1)$, $(3,3)$ and $(9,9)$ respectively. Then which of these stones is on the path of the man ———

A) A only B) B only C) C only D) All the above

Key: C

Sol:

Given $A=(1,1)$ $B=(3,3)$ $C=(9,9)$

Let x-intercept be ‘a’ and

y-intercept be ‘b’

Equation of the path is $\frac{x}{a}+\frac{y}{b}=1$

$$\Rightarrow \frac{h}{a}+\frac{k}{b}=1$$

$$\frac{\frac{1}{a}+\frac{1}{b}}{2}=\frac{1}{18} \Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{1}{9}$$

$$\Rightarrow \frac{9}{a}+\frac{9}{b}=1$$

$$\therefore (h,k)=(9,9)$$

3b. A man is walking on a straight line. The A.M of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{12}$. Three stones A , B and C. One placed at the points $(2,2)$, $(6,6)$ and $(10,10)$ respectively. Then which of these stones is on the path of the man ———

A) A only B) B only C) C only D) All the above

Key: B

Sol:

Given $A=(2,2)\qquad B=(6,6)\qquad C=(10,10)$

$\dfrac{\frac{1}{a}+\frac{1}{b}}{2}=\dfrac{1}{12}\Rightarrow \dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{6}$

$\therefore (h,k)=(6,6)$

3c. A man is walking on a straight line. The A.M of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{6}$. Three stones A , B and C. One placed at the points $\left(\frac{1}{3},\frac{1}{3}\right)$, $\left(\frac{1}{6},\frac{1}{6}\right)$ and $\left(\frac{1}{12},\frac{1}{12}\right)$ respectively. Then which of these stones is on the path of the man ———

A) A only B) B only C) C only D) All the above

Key: C

Sol:

Given $A=\left(\frac{1}{3},\frac{1}{3}\right)\qquad B=\left(\frac{1}{6},\frac{1}{6}\right)\qquad C=\left(\frac{1}{12},\frac{1}{12}\right)$

$\dfrac{1}{a}+\dfrac{1}{b}\qquad \text{H.M}(a,b)=\dfrac{1}{6}$

$\dfrac{2ab}{a+b}=\dfrac{1}{6}$

$12=\dfrac{1}{a}+\dfrac{1}{b}\qquad\qquad \therefore (h,k)=\left(\frac{1}{12},\frac{1}{12}\right)$

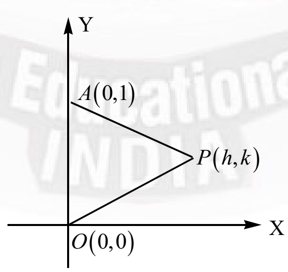
4. Let O(0,0) and A(0,1) be two fixed points then the locus of the point P such that the perimeter of ΔAOP is 4 is ___

A) $8x^2-9y^2+9y=18$ B) $9x^2-8y^2+8y=16$

C) $9x^2+8y^2-8y=16$ D) $8x^2+9y^2-9y=18$

Key: C **Jee Mains-2019**

Sol :



Let P(h,k), OA=1 so OP+AP=3,

$\Rightarrow \sqrt{h^2+k^2}+\sqrt{h^2+(k-1)^2}=3$, S.O.B.S

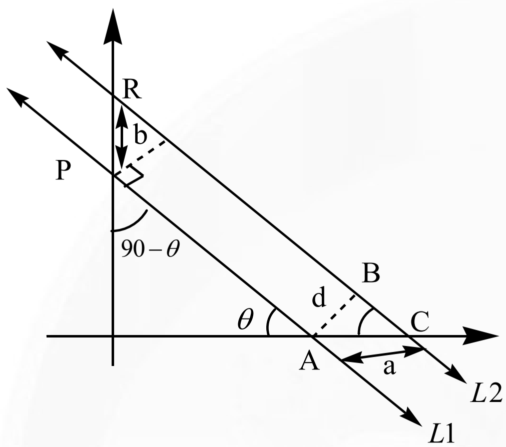
$9h^2+8k^2-8k-16=0$, *Hence locus of p is* $9x^2+8y^2-8y=16$

4a. Two parallel lines lying in the same quadrant make intercepts a and b on x,y axes respectively between them, then the distance between lines is

A) $\frac{ab}{\sqrt{a^2+b^2}}$ B) $\frac{1}{\sqrt{a^2+b^2}}$ C) $\sqrt{a^2+b^2}$ D) $\frac{1}{a^2}+\frac{1}{b^2}$

Key: A

Sol:



$In \quad \Delta ABC, \sin\theta = \frac{d}{a}, In \ \Delta PQR, \cos\theta = \frac{d}{b}$

$\sin^2\theta + \cos^2\theta = 1$

$\frac{d^2}{a^2}+\frac{d^2}{b^2}=1 \Rightarrow d = \frac{ab}{\sqrt{a^2+b^2}}$

4b. All points lying inside the triangle formed by points (1,3),(5,0),(-1,2)satisfy

A) $2x+y-13=0$ B) $3x-4y-12\le 0$ C) $3x+2y\ge 0$ D) $4x+y=0$

Key: C

Sol: By inspection

4c. The equation of the line passing through (1,2) & having distance of 7 units from point (8,9) is

A) $y=3x-1$ B) $x=4$ C) $y=2$ D) $x+y=3$

Key: C

Sol:

Distance from (8,9) to line $y-2=m(x-1)$ *is* $7. \Rightarrow m=0$

Exercise : II

(Numerical / Integer Value based Questions Including PYQ's)

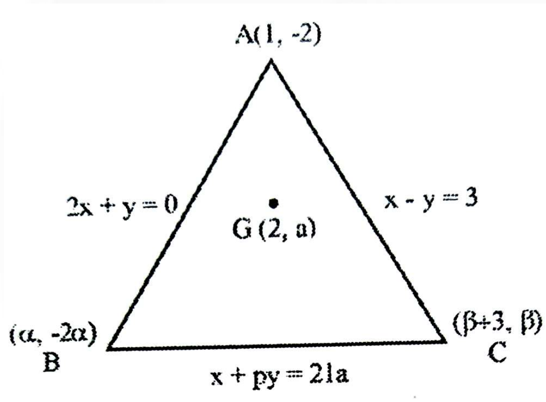
Various forms of straight lines

(Point-slope form, Two point form, Slope-intercept form, Intercept form, Normal form, Symmetric form (parametric form), General form of a straight line)

1. The equations of the sides AB,BC and CA of a triangle ABC are :
$2x+y=0, x+py=21a$
$(a\neq 0)$ and $x-y=3$ respectively. Let $P(2,a)$ be the centroid of ΔABC. Then $(BC)^2$ is equal to

Key : 122

Sol : **Jee Mains - 2023**



Assume $B(\alpha,-2\alpha)$ and $C(\beta+3,\beta)$

$\dfrac{\alpha+\beta+3+1}{3}=2$ also $\dfrac{-2\alpha-2+\beta}{3}=a$

$\Rightarrow \alpha+\beta=2$ $\quad -2\alpha-2+\beta=3a$

$\Rightarrow \beta=2-\alpha$ $\quad -2\alpha-2+2-\alpha=3a \Rightarrow \alpha=-a$

Now both B and C lies as given line

$\alpha-p.2\alpha=21a$

$\alpha(1-2p)=21a$ ....(1)

$-\alpha(1-2p)=21a \Rightarrow p=11$

$\beta+3+p\beta=21a$

$\beta+3+11\beta=21a$

$21\alpha+12\beta+3=0$

Also $\beta=2-\alpha$

$21\alpha+12(2-\alpha)+3=0$

$21\alpha+24-12\alpha+3=0$

$9\alpha+27=0$

$\alpha=-3, \beta=5$

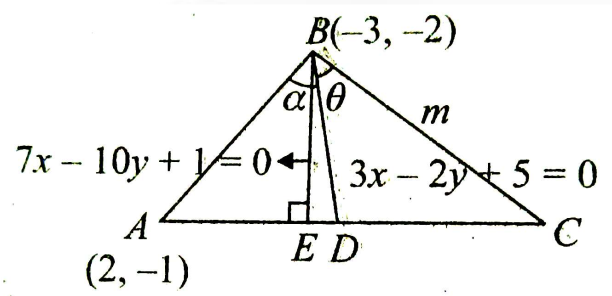
So $BC=\sqrt{122}$ and $(BC)^2=122$

1a. In triangle ABC if, A is $(2,-1)$, and $7x-10y+1=0$ and $3x-2y+5=0$ are the equations of an altitude and an angle bisector, respectively, drawn from B, then the equation of BC is $ax+by+c=0$ then $a+b+c$ is where gcd of a,b,c is 1

Key :23

Sol : BD and BE intersect at B. the coordinates of B are $(-3,-2)$

$m_{AB}=\frac{1}{5}$



$$\left|\frac{\frac{3}{2}-\frac{1}{5}}{1+\frac{3}{10}}\right|=\left|\frac{\frac{3}{2}-m}{1+\frac{3m}{2}}\right|$$

$$\pm1=\frac{3-2m}{2+3m}$$

i.e., $m=\frac{1}{5}$ (rejected) or $m=-5$

The equation of BC is

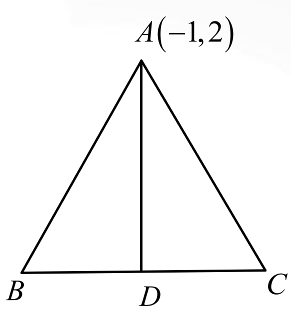
$y+2=-5(x+3)$

Or $5x+y+17=0$

1b. If the equation of the base of an equilateral triangle is $2x-y=1$ and vertex is $(-1,2)$ then the length of the side of the triangle is $\sqrt{\frac{20}{p}}$ then P is

Key : 3

Sol :



$$AD=\frac{|-2-2-1|}{\sqrt{4+1}}=\sqrt{5}$$

$$Tan60 = \frac{\sqrt{5}}{BD}$$

$$\Rightarrow BD = \sqrt{\frac{5}{3}}$$

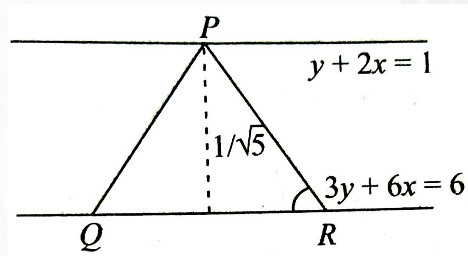
Then BC =2BD

$$= \sqrt{\frac{20}{3}}$$

1c. P is a point on the line $y + 2x = 1$, and Q and R are two points on the line $3y + 6x = 6$ such that triangle PQR is an equilateral triangle. The length of the side of the triangle is $\frac{P}{\sqrt{Q}}$ then $P + Q$ is

Key : 17

Sol :



The given lines are $y + 2x = 1$ and $y + 2x = 2$. The distance between the lines is $(2, -1)/\sqrt{5} = 1/\sqrt{5}$

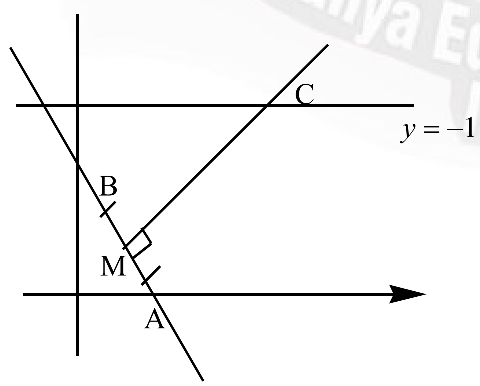
The side length of the triangle is

$$\frac{1}{\sqrt{5}} \cos ec 60^0 = \frac{2}{\sqrt{15}}$$

2. The line $3x + 2y = 24$ meets the y-axis at A and the x-axis at B. the perpendicular bisector of the $\overline{AB}$ meets the line through $(0, -1)$, Parallel to the x-axis at C. If the area of the ΔABC is A, then $\frac{A}{13} =$

Key : 7

Sol :



$A = (8, 0), B = (0, 12)$

$m = (4,6)$

Slope AB $= \frac{12-0}{0-8} = \frac{-3}{2}$

Equation of perpendicular bisector $y-6 = \frac{2}{3}(x-4)$

$\Rightarrow 3y-18 = 2x-8$

$\Rightarrow 2x-3y+10 = 0 \Rightarrow 2x+3+10 = 0 \left(\because\ y=-1\right)$

$\Rightarrow 2x = -13 \Rightarrow x = \frac{-13}{2}$

$C = \left(\frac{-13}{2}, -1\right)$

Area of triangle ABC=91

$\therefore\ \frac{A}{13} = 7$

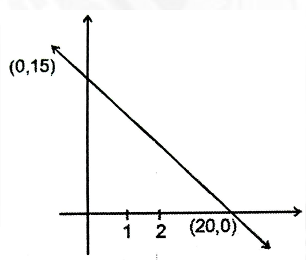
Position of a point w.r.t a line

(Ratio in which the line divides the line joining two points, Position of origin and a point w.r.t given line)

1. A triangle is formed by X-axis, Y-axis and the line $3x+4y=60$. Then the number of points $P(a,b)$ which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is ________ **Jee mains - 2023**

Key : 31

Sol : If $x=1, y=\frac{57}{4}=14.25$



$(1,1)(1,2)-(1,14) \Rightarrow 14$ Pts.,

If $x=2, y=\frac{27}{2}=13.5$

$(2,2)(2,4)......(2,12) \Rightarrow 6$ Pts.,

If $x=3, y=\frac{51}{4}=12.75$

$(3,3)(3,6)-(3,12) \Rightarrow 4$ Pts.,

$(4,4)(4,8) \Rightarrow 2$ Pts.,

$(5,5)(5,10)(6,6)(7,7)(8,8)$ total 31 Pts., 1a. The number of integral points (x, y) (i.e., x and y both are integers) which lie in the first quadrant but not on the coordinate axes and also on the straight line $3x+5y=2007$ is equal to

Key : 133

Sol : We have

$3x+5y=2007$ or $x+\frac{5y}{3}=669$

Clearly, 3 must divide $5y$ and so $y=3k$, for some $k \in N$, thus,

$x+5k=669$

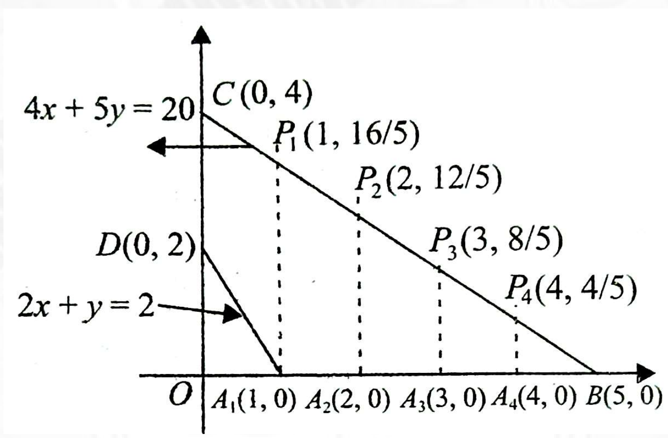
Or $5k \le 668$

Or $k \le \frac{668}{5}$ or $k \le 133$

1b. $P(m,n)$ (where m,n are natural numbers) is any point in the interior of the quadrilateral formed by the pair of lines $xy=0$ and the lines $2x+y-2=0$ and $4x+5y=20$. The possible number of positions of the point P is

Key :6

Sol : There are clearly six points



1c. The number of integral values of m for which the x-coordinate of point of intersection of the lines $3x+4y=9$ and $y=mx+1$ is also an integer is

Key :2

Sol : Solving $3x+4y=9, y=mx+1$, we get

$x=\frac{5}{3+4m}$

Here, x is an integer if $3+4m=1,-1,5,-5$. Hence $m=-2/4$, $-4/4, 2/4, -8/4$. So, m has two integral values.

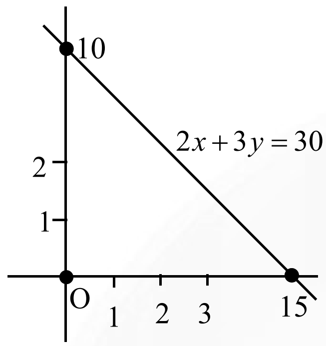
2. A triangle formed by x-axis, y-axis and the line $2x+3y=30$. Then the number of points (a,b)

which lie strictly inside the triangle where a is an integer and b is multiple of a is

______

Key : 17

Sol :



If $x=1$, $y=\frac{140}{15}=9.3$

$(1,1)(1,2).....(1,9)\Rightarrow 9$ points

If $x=2$, $y=\frac{130}{15}=8.66$

$(2,2)(2,4)(2,6)(2,8)\Rightarrow 4$ points

If $x=3$, $y=\frac{40}{5}=8$

$(3,3)(3,6)\Rightarrow 2$ points

If $x=4$, $y=\frac{110}{15}=7.3$

$(4,4)\Rightarrow 1$ point

If $x=5$, $y=\frac{20}{3}=6.6$

$(5,5)\Rightarrow 1$ point

If $x=6$, $y=\frac{30}{5}=6$

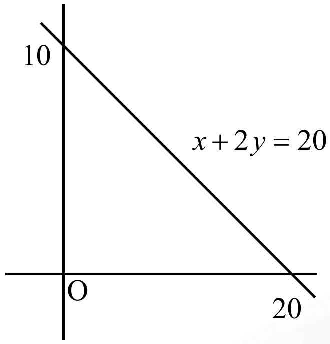
$(6,6)$ but lies on the triangle $\Rightarrow 0$ point

Total =17

2a. A triangle is formed by $x-$ axis, $y-$ axis and the line $x+2y=20$. Then the no. of points $P(a,b)$ which lie strictly inside the triangle, where a is an integer and 6 is multiple of integer is ___

Key : 18

Sol :



$x+2y=20$

If $x=1$, $y=\frac{190}{2}=9.5$

$(1,1)(1,2)(1,3).....(1,9)\Rightarrow 9$ points

If $x=2$, $y=9$

$(2,2)(2,4)(2,6)(2,8)\Rightarrow 4$ points

If $x=3$, $y=\frac{17}{2}=8.5$

$(3,3)(3,6)\Rightarrow 2$ points

If $x=4$, $y=8$

$(4,4),(4,8)\Rightarrow 1$point

If $x=6$, $y=7$

$(6,6)\Rightarrow 1$ points

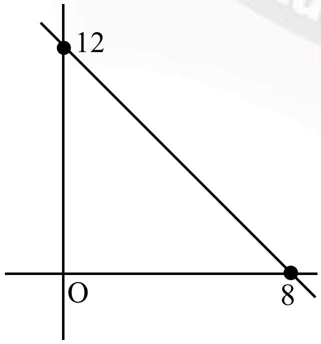
If $x=7$, $y=\frac{13}{2}=6.5\Rightarrow$ no points

Total =18

2b. A triangle formed by x-axis, y-axis and the line $3x+2y=24$. Then the number of points $P(a,b)$ which lie strictly inside the triangle, where a is an integer and b is a multiple of a is ___

Key : 17

Sol:



If $x=1$, $y=\frac{84}{8}=10.5$

$(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$

$(1,7)...(1,10) \Rightarrow 10$ points

If $x=2$, $y=9$

$(2,2)(2,4)(2,6)(2,8) \Rightarrow 4$ points

If $x=3$, $y=\frac{60}{8}=7.5$

$(3,3)(3,6) \Rightarrow 2$ points

If $x=4$, $y=6$

$(4,4) \Rightarrow 1$ point

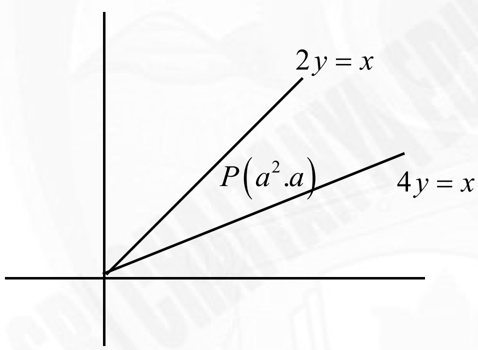
If $x=5, y=\frac{36}{8}=4.5$ no points

Total =17

3. The integral value of 'a' for which the points $P(a^2,a)$ lies in the region corresponding to the acute angle between the lines $2y=x$ and $4y=x$ is _____

Key : 3

Sol :



$P(a^2,a)$ lies below the line

$x-2y=0$

$\frac{a^2-2a}{-2}<0$

$a^2-2a>0$

$a(a-2)>0$

$a \in (-\infty,0) \cup (2,\infty)$

$P(a^2,a)$ lies above the line $x-4y=0$

$\frac{a^2-4a}{-4}>0$

$a^2-4a<0$

$a(a-4)<0$

$a \in (0,4)$

By combining both condition

$2 < a < 4$

a=3

4. The value of K, K$\in$N for which the points $A(1,2)$ and $B(3,k)$ lies on same side of the points line

$4x+3y-16=0$ is

Key :1

Sol : $L = 4x+3y-16=0$

Two points lie on same side

$\Rightarrow L_{11}L_{22} > 0$

$(4+6-16)(12+3k-16) > 0$

$3k-4 < 0$

$3k < 4$

$k < \frac{4}{3}, K \in N$

$k = 1$

Distances to a line

(Perpendicular distance from a point, Distances of given point from the given line measured along a straight line, Distance between parallel lines)

1. Let the point $P(\alpha,\beta)$ be at a unit distance from each of two lines $L_1: 3x-4y+12=0$ and $L_2: 8x+6y+11=0$. If P lies below L_1 and above L_2, then $100(\alpha+\beta)=$__

Key : 14

Sol:

$L_1(0,0) > 0, L_1(\alpha,\beta) > 0$

$\Rightarrow 3\alpha - 4\beta + 12 > 0$

$\perp r$ distance from $P(\alpha,\beta)$ to L_1 is

$1 = \left|\frac{3\alpha - 4\beta + 12}{5}\right|$

$3\alpha - 4\beta + 12 = 5$

$3\alpha - 4\beta + 7 = 0$ ….(1)

$L_2(0,0) > 0$; $L_2(\alpha,\beta) > 0 \Rightarrow 8\alpha + 6\beta + 11 > 0$

Perpendicular from $P(\alpha,\beta)$ to L_2 is

$1 = \left|\frac{8\alpha + 6\beta + 11}{10}\right|$

$8\alpha + 6\beta + 11 = 10$

$8\alpha + 6\beta + 1 = 0$ ….(2)

By solving 1 & 2, $\alpha = \frac{-23}{25}, \beta = \frac{106}{100}$

$$100(\alpha+\beta)=100\left(\frac{-23}{25}+\frac{106}{100}\right)=100\left(\frac{-92}{100}+\frac{106}{100}\right)=14$$

2. The distance between $A(2,3)$ on the line gradient $\frac{3}{4}$ and the point of intersection P of line with $5x+7y+40=0$ is

Key : 8.658

Sol : Distance of a point (x_1, y_1) from the line $L: ax+by+c=0$ making an angle 'α' with x-axis is

$$\left|\frac{ax_1+by_1+c}{a\cos\alpha+b\sin\alpha}\right|$$

Here,

$m=\tan\alpha=\frac{3}{4}\Rightarrow\cos\alpha=\frac{4}{5}$ and $\sin\alpha=\frac{3}{5}$

$a=5, b=7, c=40$ and $(x_1, y_1)=(2,3)$

Distance is $\left|\frac{5(2)+7(3)+40}{5\left(\frac{4}{5}\right)+7\left(\frac{3}{5}\right)}\right|=\frac{355}{41}=8.658$

3. The line parallel to x-axis & passing through the point of intersection of lines $ax+2by+3b=0$ & $-bx+2ay+3a=0$, $(a,b)\neq(0,0)$ is below x-axis at a distance x form it. Then x is

Key : 1.5

Sol : Given lines, $ax+2by+3b=0$, $bx-2ay-3a=0$

Since, required line is parallel to x-axis, x=0

We put x=0 in given equation, we get

$2by=-3b$

$y=\frac{-3}{2}$

$\therefore$ This shows that the required line is below x-axis at a distance $\left(\frac{3}{2}\right)$ from it.

4. The perpendicular distance from the origin to the line passing through $P(1,2)$ such that P bisects the part intercepted between the axes is $\frac{K}{\sqrt{5}}$ then K______

Key : 4

Sol : Equation of the given line $\frac{x}{1}+\frac{y}{2}=2$

$\Rightarrow 2x+y-4=0$

$\therefore$ Required distance $=\frac{4}{\sqrt{5}}$

5. If a line L passes through $(K,2K),(3K,3K)$ and $(3,1),(K\neq 0)$. Then the distance from the origin to the line L is $\frac{h}{\sqrt{K}}$. Then $|h-k|=$____

Key : 4

Sol : $(K,2K),(3K,3K),(3,1)$ are collinear

$\Rightarrow \frac{3K-2K}{3K-K}=\frac{1-3K}{3-3K}\Rightarrow \frac{1}{2}=\frac{1-3K}{3-3K}$

$\Rightarrow 3-3K=2-6K \Rightarrow k=\frac{-1}{3}$

Equation of the line L is

$y-1=\frac{1}{2}(x-3)\Rightarrow x-2y-1=0$

$y-1=\frac{1}{2}(x-3)\Rightarrow x-2y-1=0$

The distance from origin to the line is $\frac{1}{\sqrt{5}}$

6. The distance of the point $(2,3)$ from the line $2x-3y+9=0$ measured along a line $x-y+1=0$ is

$4\sqrt{k}$ then $K=$_______

Key : 2

Sol : Point of intersection of given lines is $(6,7)$ required distance =

$\sqrt{(6-2)^2+(7-3)^2}=\sqrt{32}=4\sqrt{2}$

7. The distance of the line $3x-y=0$ from the point $(4,1)$ measured along a line making an angle 135^0 with the x-axis is $\frac{a\sqrt{b}}{c}$ then $a+b+c=$_____

Key : 17

Sol : Equation of the second line is $y-1=\tan 135^0(x-4)\Rightarrow x+y-5=0$

Point of intersection of $3x-y=0$ and $x+y-5=0$ is $\left(\frac{5}{4},\frac{15}{4}\right)$

$\therefore$ required distance $=\sqrt{\left(4-\frac{5}{4}\right)^2+\left(1-\frac{15}{4}\right)^2}$

$=\frac{11\sqrt{2}}{4}$

8. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point $(13,32)$.

The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is $\frac{h}{\sqrt{K}}$ then $h+K=$_____

Key : 40

Sol : As the line passes through $(13,32)$, we have $\frac{13}{5}+\frac{32}{b}=1 \Rightarrow b=-20$

Thus the line is $\frac{x}{5}-\frac{y}{20}=1 \Rightarrow 4x-y=20$ ...(1)

The equation of the line parallel to $4x-y=20$ has slope is 4. Thus $\frac{-3}{c}=4 \Rightarrow C=\frac{-3}{4}$

Then the equation to line K is $4x-y=-3$

$\therefore$ The distance between $4x-y=20$ and $4x-y=-3$ is $\left|\frac{20+3}{\sqrt{4^2+1^2}}\right|=\frac{23}{\sqrt{17}}$

9. The area of the circle which touches the lines $4x+3y=15$ and $4x+3y=5$ is $k\pi$ then K=____

Key : 1

Sol: Diameter, $2r=\frac{15-5}{\sqrt{16+9}}=\frac{10}{5}=2 \Rightarrow r=1$

$\therefore$ Area $=\pi r^2=\pi$

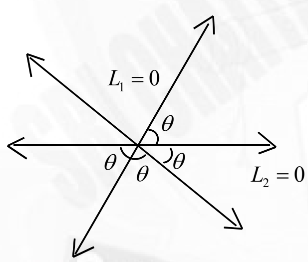
Angle Between Two lines

(Condition for parallel & perpendicular lines)

1. If θ is angle between two lines $L_1=0$, $L_2=0$ and the reflection of L_1 on L_2 the reflection of L_2 on L_1 coincide then $\tan^2\theta=$ ___________

Key : 3

Sol :



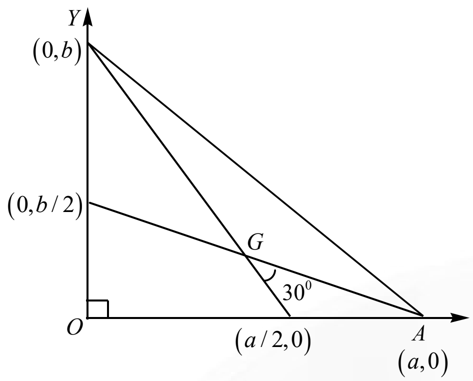
From the figure

$3\theta=180$ or $\theta=60^0$

2. Two medians drawn from the acute angles of a right angled triangle intersect at an angle $\pi/6$. If the length of the hypotenuse of the triangle is 3 units, then the square of the area is _____

Key : 3

Sol : Slope of $AG=-\frac{b}{2a}$



Now, $\tan 30^0 = \dfrac{3b/2a}{1+\left(b^2/a^2\right)}$

Or $\dfrac{1}{2}ab = \left(\dfrac{a^2+b^2}{3\sqrt{3}}\right)$

$= \dfrac{9}{3\sqrt{3}} = \sqrt{3}$ (Putting $a^2+b^2=9$)

3. The line $x+3y-2=0$ bisects the angle between a pair of straight lines of which one has equation $x-7y+5=0$ and the equation of the other line is $ax+by+c=0$ then $|a+b+c| =$ ________

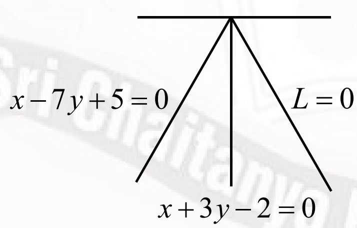
Key : 7

Sol : The family of line through the given lines is

$L \equiv x-7y+5+\lambda(x+3y-2)=0$ .....(i)

For line $L=0$ in the diagram the distance of any point say $(2,0)$ on the line $x+3y-2=0$ from the line $x-7y+5=0$ and the line $L=0$ must be the same. Therefore,

$$\left|\frac{2+5}{\sqrt{50}}\right| = \left|\frac{2+2\lambda+5-2\lambda}{\sqrt{(1+\lambda)^2+(3\lambda-7)^2}}\right|$$



Or $10\lambda^2 - 40\lambda = 0$

i.e., $\lambda = 4$ or 0

Hence, $L=0, \lambda=4$

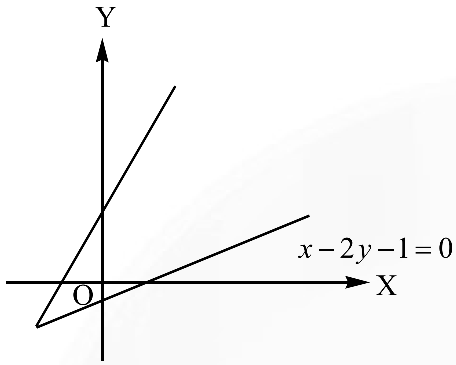
Therefore, the required line is $5x+5y-3=0$.

4. The slope of the bisector of the acute angle between the lines $2x-y+4=0$ and $x-2y=1$ is ________

Key : 1

Sol : Clearly, from the figure, the origin is contained in the acute angle. Writing the equations of the lines

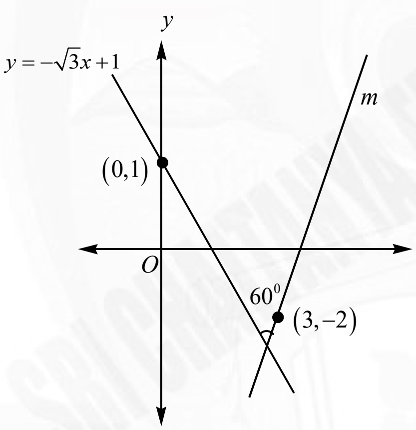
As $2x-y+4=0$ and $-x+2y+1=0$, the required bisector is $\frac{2x-y+4}{\sqrt{5}}=\frac{-x+2y+1}{\sqrt{5}}$



5. A straight line $L=0$ of slope m through the point $(3,-2)$ is inclined at an angle 60^0 to the line $\sqrt{3}x+y=1$. If L also intersects $x-$ axis then $m^2=$

Key : 3

Sol :



Let the slope of the required line be m. Then

Or $m+\sqrt{3}=\pm\left(\sqrt{3}-3m\right)$

Or $4m=0$ or $m=0$

And $2m=2\sqrt{3}$ or $m=\sqrt{3}$

Foot of Perpendicular and Image of a point w.r.t a line

1. The number of integral values of m so that the abscissa of point of intersection of lines $3x+4y=9$ and $y=mx+1$ is also an integer is

Key: 2 **Jee Mains-2023**

Sol: Solving $y=mx+1, 3x+4y=9$

$\Rightarrow 3x+4(mx+1)=9$

$\Rightarrow x=\frac{5}{3+4m}$

Given that ‘x’ is an integer

$\therefore$ possible values of ‘x’ are $\pm 1, \pm 5$

$\frac{5}{3+4m}=1$, $\frac{5}{3+4m}=-1$, $\frac{5}{3+4m}=5$, $\frac{5}{3+4m}=-5$

$m=\frac{1}{2}$, $\quad -2$, $\quad -\frac{1}{2}$, $\quad -2$

But $m \in z \Rightarrow$, $\quad m=-1,-2$

1a. The number of integral values of m so that the abscissa of point of intersection of lines $4x+3y=1$ and $y=mx-1$ is also an integer is

Key: 2

Sol: $4x+3y=1, y=mx-1$

$4x+3mx-3=1$

$x=\frac{4}{4+3m}$

X is an integer $\Rightarrow \frac{4}{4+3m}=1, \Rightarrow \frac{4}{4+3m}=-1, \Rightarrow \frac{4}{4+3m}=4, \Rightarrow \frac{4}{4+3m}=-4,$

$4+3m=4$, $\quad 4+3m=-4 \quad 4+3m=1 \quad 4+3m=-1$

$m=0 \quad m=\frac{-8}{3} \quad m=-1 \quad m=\frac{-5}{3}$

$\therefore m=0,-1$

1b. The number of integral values of λ for which the x- coordinates of the point of intersection of the lines $3x+4y=9$ and $y=\lambda x+1$ is also an integer is m. Then m=------

Key: 2

Sol: x- coordinate of point of intersection of $3x+4y=9$ and $y=\lambda x+1$ is $\frac{5}{3+4\lambda}$

which is an integer if $\lambda=-1$, $\lambda=-2 \therefore m=2$

1c. The number of integral values of m so that the ordinate of point of intersection of lines $my=x+1,$ and $2x+3y=-1$ is also an integer is-------

Key: 2

Sol:

$x=my-1,\ 2x+3y=-1$

$2my-2+3y=-1$

$y=\frac{1}{2m+3}$

Y is an integer $\frac{1}{2m+3}=1, \frac{1}{2m+3}=-1,$

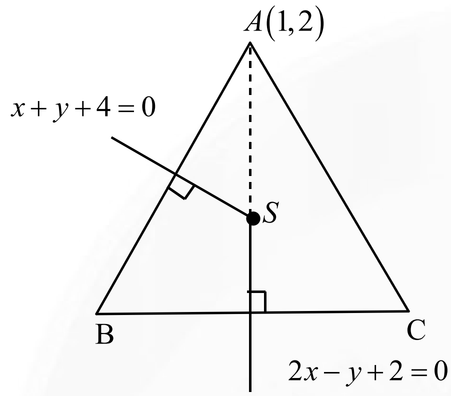
$2m+3=1$, $\quad 2m+3=-1$

$m=-1$, $\quad$ m=-2

2. The point B is the image of A in the line $x+y+4=0$ and C is the image of B in the line $2x-y+2=0$. If $A=(1,2)$ then circum radius of triangle ABC is ___

Key :5

Sol :



$x+y+4=0$ .....(1)

$2x-y+2=0$ .....(2)

Solve equation (1) and (2) we get

Circum centre $=S(-2,-2)$

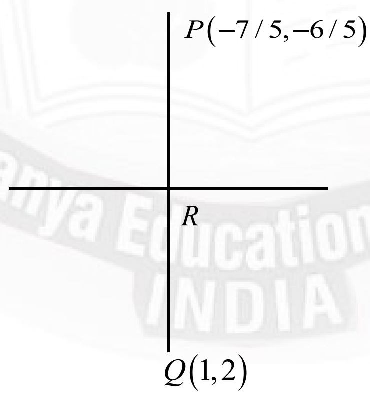
$R=$ circum radius =SA

$=\sqrt{(-2-1)^2+(-2-2)^2}$

$R=5$

3. If the image of $\left(\frac{-7}{5},\frac{-6}{5}\right)$ in a line is $(1,2)$, then the equation of the line is $ax+by+c=0$ then $a+b-c=$_______

Key :8



Sol :

$$R=\left(\frac{1-\frac{7}{5}}{2},\frac{2-\frac{6}{5}}{2}\right)$$

$R=\left(\frac{-1}{5},\frac{2}{5}\right)$

Slope of $\overline{PQ}=\frac{2+\frac{6}{5}}{1+\frac{7}{5}}=\frac{4}{3}$

$y-\frac{2}{5}=\frac{-3}{4}\left(x+\frac{1}{5}\right)$

$3x+4y=1$

4. If $2x+3y=5$ is the $\perp er$ bisector of the line segment joining the points $A(1,1/3)$ and $B(a,b)$ then $39b-26a=$____

Key :7

Sol : $B(a,b)$ is the image of $A\left(1,\frac{1}{3}\right)$

$\frac{a-1}{2}=\frac{b-\frac{1}{3}}{3}=\frac{-2(2+1-5)}{13}$

$\frac{a-1}{2}=\frac{b-\frac{1}{3}}{3}=\frac{4}{13}$

$a=\frac{21}{13},b=\frac{49}{39}$

$39b-26a=7$

5. The image of the line $x+2y=5$ in the line $x-y=2$ is $ax+by-c=0$ then $c-a-b=$____

Key :4

Sol : Image of $ax+by+c=0$ in the line $lx+my+n=0$ is

$(\ell^2+m^2)(ax+by+c)=2(a\ell+bm)(\ell x+my+n)$

$(1+1)(x+2y-5)=2(1-2)(x-y-2)$

$2x+y-7=0$

$a=2,b=1,c=7$

$c-a-b=7-1-2=4$

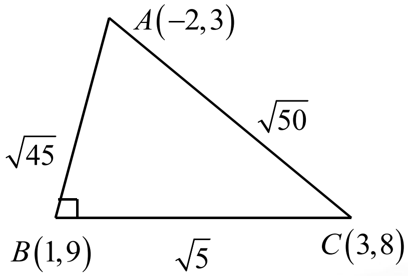
Centers of Triangle

(Centroid, Circumcentre, incentre, Ortho Centre)

1. Consider a triangle having vertices $A(-2,3),B(1,9)$ and $C(3,8)$. If a line passing through the circum centre of triangle ABC, bisects the line BC and intersects y-axis at a point $\left(0,\frac{\alpha}{2}\right)$ then the integral value of real number α is ____

Key : 9 **Jee Mains -2021**

Sol :



$\angle B = 90^0$

Circum centre $\left(\frac{1}{2},\frac{11}{2}\right)$

Mid point of BC= $\left(2,\frac{17}{2}\right)$

Line equation $y-\frac{11}{2}=2\left(x-\frac{1}{2}\right)$

$y=2x+\frac{9}{2}$

Passing through $\left(0,\frac{\alpha}{2}\right)$

$\frac{\alpha}{2}=\frac{9}{2}$

$\alpha=9$

1a. Consider a triangle having vertices $A(2,3)$, $B(1,-5)$ and $C(-1,4)$. If a line L passing through the circum centre of triangle ABC, bisector the line BC and intersects y-axis at $\left(0,\frac{-\alpha}{2}\right)$ then the integral value of real number α is ____

Key : 1

Sol : Circum centre $\left(\frac{-9}{10},\frac{-7}{10}\right)$

Mid point of BC : $\left(0,\frac{-1}{2}\right)$

Equation of L is $4x-6y-3=0$

1b. Consider a triangle with vertices $A(1,3)B(0,-2)C(-3,1)$. If a line L passing through the circum centre of triangle ABC, bisects the line BC and intersect y-axis at $\left(0,\frac{-\alpha}{5}\right)$ then integral value of a real number α is

Key : 7

Sol : Circum centre $(1,-2)$

Mid point of BC : $\left(\frac{-3}{2},\frac{-1}{2}\right)$

Equation of L is $3x+5y+7=0$

1c. Consider a triangle with vertices $A(2,2)B(3,3)C(4,2)$. If a line L passing through the circum centre of triangle ABC, bisect the line BC and intersects at y-axis at $(0,-\alpha)$ then integral value of real number α is

Key : 1

Sol : Circum centre $(3,2)$

Mid point of BC : $\left(\frac{7}{2},\frac{5}{2}\right)$

Equation of L is $x-y-1=0$

2. The equations of the sides AB,BC and CA of a triangle ABC are $2x+y=0, x+py=15a$ and $x-y=3$ respectively. If the orthocenter is $(2,a), -\frac{1}{2}<a<2$ then p is equal to ___

Key : 3 **Jee Mains-2022**

Sol : $2x+y=0......(1), x+py=15a.......(2), x-y=3......(3)$

From (1) and (2) $A=(1,-2)$

From (2) & (3) $c=\left(\frac{15a+3p}{p+1},\frac{15a-3}{p+1}\right)$

B=$(0,0)$, ortho center $=H=(2,a)$

$AH\perp BC \Rightarrow P=a+2\rightarrow(4)$

$CH\perp AB$

$\Rightarrow 13a-2ap-p-4=0.........(5)$

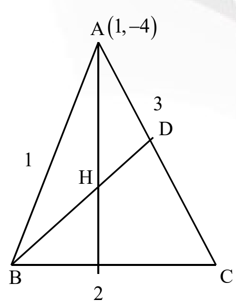
From (4) & (5), P=3

2a. The equations of the sides AB,BC and CA of a triangle ABC are $2x+y=0$ and $x+qy=15a$ and $x-y=3$ respectively. If the orthocenter of triangle is $(2,b), \frac{1}{2}<b<2$ then

______

Key: 3

Sol:



$x+2y=0\rightarrow1$ $\qquad x+9q=15\rightarrow2$ $\qquad x-y=4\rightarrow3$

Id 1&3 $A(1,-2)$

From 2&3 $C\left(\frac{15b+3q}{q-1},\frac{15b-3}{q+1}\right)$

$AH \perp to BC$

$\frac{b+2}{1}.\left(-\frac{1}{q}\right)=-1 \Rightarrow q=b+2$

$CH \perp to AB$

$$\frac{\frac{15b-3}{q+1}-b}{\frac{15b+3q}{q-1}-2}.(-2)=-1$$

$$\Rightarrow \frac{(15b-3-bq-b)^2}{15b+3q-2q+2}=1$$

$30b-6-2bq-2b=15b+q+2$

$13b-2bq-q-8=0$

$13b-2b\left(b+y-(b+y-8)\right)=0$

$13b-2b^2-4b-b-2$

$8b-2b^2-10=0$

$b^2-4b+5=0$

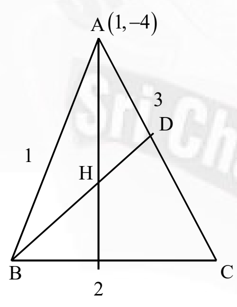
$b=1$ or $b=4$

$b=1$; $q=3$

2(b). The equations of the sides AB,BC and CA of a triangle ABC are $2x+y=0$ and $x+qy=15a$ and $x-y=3$ respectively. If the orthocenter of triangle is $(2,b), b>2$ then ________

Key: 6

Sol:



$x+2y=0 \rightarrow 1$ $\quad x+9q=15 \rightarrow 2$ $\quad x-y=4 \rightarrow 3$

Id 1&3 $A(1,-2)$

From 2&3 $C\left(\frac{15b+3q}{q-1},\frac{15b-3}{q+1}\right)$

$AH \perp to BC$

$\frac{b+2}{1}.\left(-\frac{1}{q}\right)=-1 \Rightarrow q=b+2$

$CH \perp toAB$

$$\frac{\frac{15b-3}{q+1}-b}{\frac{15b+3q}{q-1}-2}.(-2)=-1$$

$$\Rightarrow \frac{(15b-3-bq-b)^2}{15b+3q-2q+2}=1$$

$30b-6-2bq-2b=15b+q+2$

$13b-2bq-q-8=0$

$13b-2b(b+y-(b+y-8))=0$

$13b-2b^2-4b-b-2$

$8b-2b^2-10=0$

$b^2-4b+5=0$

$b=1$ or $b=4$

$b=4$; $q=6$

Various Triangles and four sided figures
(Area of triangle and parallelogram)

1. Let $A(1,0), B(6,2)$ and $C\left(\frac{3}{2},6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB,BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6},-\frac{1}{3}\right)$, is…….

Jee mains -2020

Key : 5

Sol : P will be centroid of ΔABC

$$P\left(\frac{17}{6},\frac{8}{3}\right) \Rightarrow PQ\sqrt{\left(\frac{24}{6}\right)^2+\left(\frac{9}{3}\right)=5}$$

1a. $A(x,y), B(2,4), C(6,8)$ be the vertices of a triangle ABC. If P is a point inside the ΔABC such that $\Delta APC, \Delta APB, \Delta BPC$ have equal areas. If $P=\left(\frac{11}{3},5\right)$, then the length of line segment AS where S is point $\left(\frac{-1}{3},2\right)$

Key :1.67

Sol : P is the centroid of ΔABC

$\frac{x+2+6}{3}=\frac{11}{3}$

$x+8=11$

$x=3$

$\frac{y+4+8}{3}=5$

$y+12=15$

$y=3$

$A(x,y)=(3,3)$

$S\left(\frac{-1}{3},2\right)$

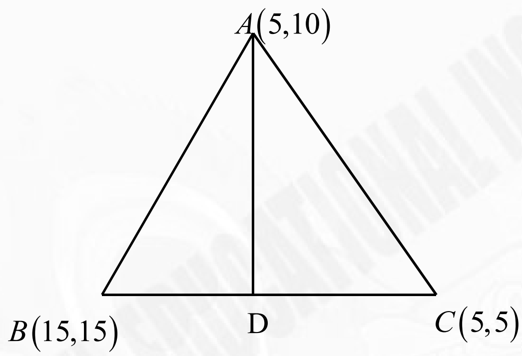
$AS=\sqrt{\left(3+\frac{1}{3}\right)^2+(3-2)^2}=\sqrt{\left(\frac{4}{3}\right)^2+1^2}$

$AS=\sqrt{\frac{16+9}{9}}=\sqrt{\frac{25}{3}}=\frac{5}{3}$ units.

1b. $(5,10),(15,15)$ and $(5,5)$ are the co-ordinates of vertices A,B and C respectively of ΔABC and

P is a point on median AD median AD such that $AP:PD=2:3$. Ratio of the areas of triangle PBC and ABC is P:Q where GCD of P&Q is 1 then P+Q is

Key : 5



Sol :

$Ar(\Delta ABC)=\frac{1}{2}.AB.BC$

$Ar(\Delta PBC)=\frac{1}{2}.PB.BC$

$$\frac{Ar(\Delta ABC)}{AR(\Delta PBC)}=\frac{\frac{1}{2}.AB.BC}{\frac{1}{2}.PB.BC}=\frac{AB}{PB}=\frac{2}{3}=\frac{AP}{PD}$$

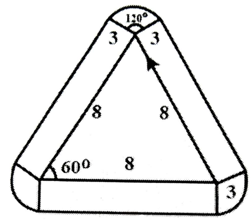
$\therefore$ Areas in ratio 2:3

1c. An equilateral triangle has side length 8. The area of the region containing all points outside the triangle but not more than 3 units from a point on the triangle is

Key : 100.28

Sol : Area $=3.(8.3)+3.\frac{1}{2}r^2\theta$

$=72+\frac{3}{2}.9.\frac{2\pi}{3}=72+9\pi=9(8+\pi)$



2. Let the point of intersections of the lines $x-y+1=0$, $x-2y+3=0$ and $2x-5y+11=0$ are the mid points of sides of triangle ABC. Then area of triangle ABC is

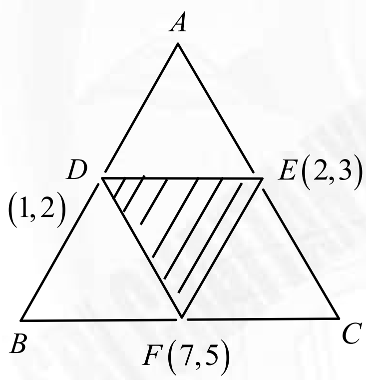
Key : 6 **Jee Mains-2021**

Sol : Intersection points of given lines are

$(1,2)$ $(7,5)$ $(2,3)$

$$\Delta=\frac{1}{2}\begin{vmatrix}1 & 2 & 1\\ 7 & 5 & 1\\ 2 & 3 & 1\end{vmatrix}$$

$$=\frac{3}{2}$$



Area of triangle $DEF=\frac{3}{4}$

Area of triangle ABC= $4\Delta DEF$

$$=4.\left(\frac{3}{2}\right)$$

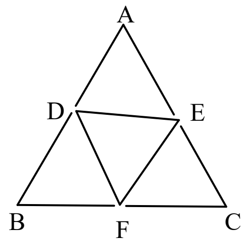
$$=6$$

2a. Let the point of intersection of lines $7x+9y-23=0$, $x+7y+11=0$ and $3x+y+3=0$ are the mid points of sides of triangle ABC. Then area of triangle ABC is ______

Key : 20

Sol: The point of intersection of given lines $(2,1)\left(\frac{-3}{2},\frac{3}{2}\right)(3,-2)$

$$\Delta=\frac{1}{2}\begin{vmatrix}2 & 1 & 1\\ \frac{-3}{2} & \frac{3}{2} & 1\\ 3 & -2 & 1\end{vmatrix}$$



$= 5$

$\Delta DEF = 5$

$\Delta ABC = 4.\Delta DEF$

$= 4(5)$

$= 20$

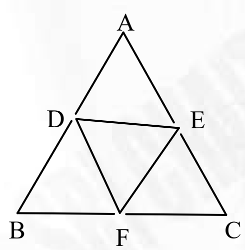
2b. Le the point intersection of lines $x - y = 0$, $x - 2 = 0$ and $y = 0$ are the midpoints of sides of triangle ABC then area of triangle ABC is ___

Key : 2

Sol: The point of intersection of given lines

$(1,1)$ $(1,0)$ $(2,1)$

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$



$= \frac{1}{2}$

$\Delta EF = \frac{1}{2}$

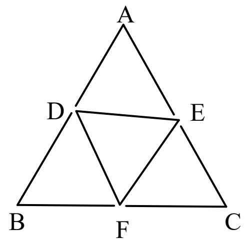
$\Delta ABC = 4\Delta DEF$ $= 4.\frac{1}{2}$ $=2$

2c. Let the point of intersection of lines $x - y = 0$, $x + 2y - 9 = 0$ and $2x - 5y = 0$ are the midpoints of sides of triangle ABC. If area of triangle ABC is P sq. units then [P] is ____

Key : 4

Sol : The point of intersection of lines

$\left(\frac{3}{2}, \frac{3}{2}\right)$ $\left(\frac{5}{2}, 1\right)$ $\left(4, \frac{5}{2}\right)$



$$\Delta = \frac{1}{2}\begin{vmatrix} \frac{3}{2} & \frac{3}{2} & 1 \\ \frac{5}{2} & 1 & 1 \\ 4 & \frac{5}{2} & 1 \end{vmatrix}$$

$$\Delta = \frac{9}{8}$$

$$\Delta DEF = \frac{9}{8}$$

$$\Delta ABC = 4\Delta DEF \qquad = 4.\frac{9}{8} \qquad = \frac{9}{2}$$

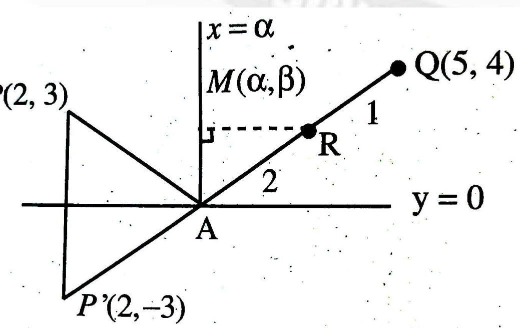
Angular bisector of Two lines

(Acute and Obtuse bisector, the bisector containing or do not containg a given point, internal and external bisector)

1. A ray of light passing through the points $P(2,3)$ reflects on the x-axis at point A and the reflected ray passes through the point $Q(5,4)$. Let R be the point that divides the line segment AQ internally into the ratio 2:1. Let the co-ordinate s of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α,β). Then the value of $7\alpha+3\beta$ is equal to ______

Key : 31 **Jee Mains-2022**

Sol :



By observation we see that $A(\alpha,0)$

And $\beta = y-$coordinate of R

$=\frac{2\times4+1\times0}{2+1}=\frac{8}{3}$……….(1)

Now P. is image of P in $y=0$ which will be $P'(2,-3)$

$\therefore$ Equation of $P.Q$ is $(y+3)=\frac{4+3}{5-2}(x-2)$

i.e., $3y+9=7x-14$

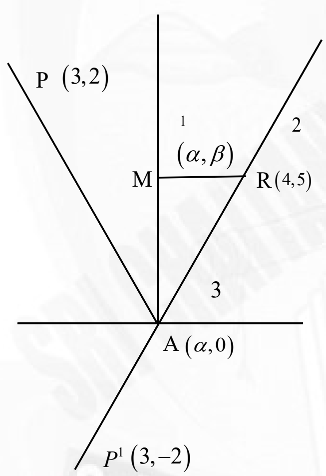
$A\equiv\left(\frac{23}{7},0\right)$ by solving with y=0

$\therefore\ \alpha=\frac{23}{7}$……. (2) by $(1),(2)$

$7\alpha+3\beta=23+8=31$

1a. A ray of light passing through the points $P(3,2)$ reflects on the x-axis at point A and the reflected ray passes through the point $Q(4,5)$. Let R be the point that divides the segment AQ internally in the ratio $3:1$. Let the co-ordinates of the foot of perpendicular M from R on the bisector of the angle PAQ be (α,β). Then the value of $7\alpha+4\beta$ is equal to ______

Key : 31



Sol:

By observation we see that A $(\alpha,0)$

$\beta=y-$coordinate of R= $\frac{15+0}{4}=\frac{15}{4}$

$P^1=$Image of X or x-axis $P^1(3,-2)$

Equation of P^1.Q is $\frac{y+2}{-7}=\frac{x+3}{-1}\Rightarrow y+2=7x-21$

$=7x-y=23$

Put $y=0\Rightarrow A=\left(\frac{23}{7},0\right)$

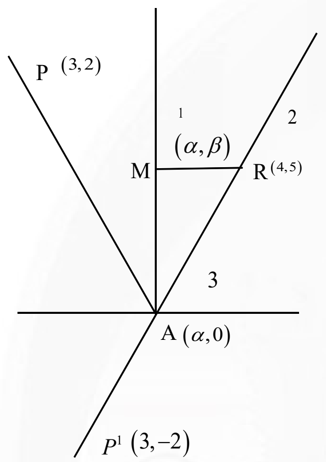
$\alpha=\frac{23}{7}$; $\beta=\frac{15}{4}$

Then $7\alpha+4\beta=-23+15=38$

1b. A ray of light passing through the points $P(3,2)$ reflects on the x-axis at point A and the reflected ray passes through the point $Q(4,5)$. Let R be the point that divides the segment AQ internally in the ratio $3:1$. Let the co-ordinates of the foot of perpendicular M from R on the bisector of the angle PAQ be (α,β). Then the value of $11\alpha-5\beta$ is equal to ______

Key: 38

Sol:



$A(\alpha,0)$ $\quad\quad R\left(\frac{18+2\alpha}{5},\frac{21+0}{5}\right)$

$\beta = y-$co-ordinate of R $=\frac{21}{5}$

$p^1(5,4)$

Equation of P^1Q in $\frac{y+4}{11}=\frac{x-5}{1}$

$y+4=11x-55$

$11x-y=59$

PUT $y=0 \Rightarrow x=\frac{59}{11}$

$A\left(\frac{59}{11},0\right)$

$\alpha=\frac{59}{11};\ \beta=\frac{21}{5}$

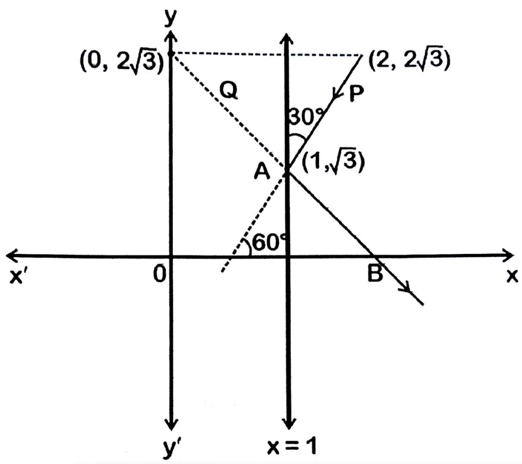
$11\alpha-5\beta \Rightarrow 59-21=38$

2. A ray of light coming from the point $\left(2,2\sqrt{3}\right)$ is incident at an angle 30^0 on the line $x=1$ at the point A. The ray gets reflected on the line $x=1$ and meets x-axis at the point B. Then, then line AB passes through the point.

A) $\left(3,-\frac{1}{\sqrt{3}}\right)$ B) $\left(3,-\sqrt{3}\right)$ C) $\left(4,-\sqrt{3}\right)$ D) $\left(4,-\frac{\sqrt{3}}{3}\right)$

Key : B

Sol :



Equation of incident line AP is

$y-2\sqrt{3}=\sqrt{3}(x-2)$

$\sqrt{3}x-y=0$

Image of P w.r.t line x=1

Equation of reflected Ray AB:

$y-\sqrt{3}=\dfrac{2\sqrt{3}-\sqrt{3}}{0-1}(x-1)$

$\sqrt{3}x+y=2\sqrt{3}$

$\therefore$ point $(3,-\sqrt{3})$ lies on line AB

2a. A ray of light is incident along a line which meets another line $7x-y+1=0$, at the point $(0,1)$. The ray is then reflected from this point along the line, $y+2x=1$. Then the equation of the line of incidence of ray of light is :

a)		b)
A) $41x-25y+25=0$	or	A) $2x+y-1=0$
B) $41x+25y-25=0$	or	B) $x-2y+2=0$
C) $41x+38y-38=0$	or	C) $x+4y+4=0$
D) $41x-38y+38=0$	or	D) $x-4y-4=0$

Key : a-D, b-A

Sol : Let slope of incident ray be m since, angle of incidence = angle of reflection

$\left|\dfrac{m-7}{1+7m}\right|=\left|\dfrac{-2-7}{1-14}\right|=\dfrac{9}{13}$

a)

$\dfrac{m-7}{1+7m}=\dfrac{9}{13}$

$13m-91=9+63m$

b)

$\dfrac{m-7}{1+7m}=\dfrac{-9}{13}$

$13m-91-9-63m$

$50m = -100$

$m = -2$

$(y-1) = -2(x-0)$

$2x + y - 1 = 0$

$76m = 8$

$m = \frac{41}{38}$

$(y-1) = \frac{41}{38}(x-0)$

$38y - 38 - 41x = 0$

$41x - 38y + 38 = 0$

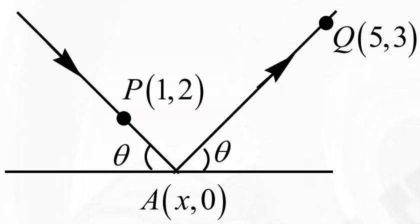
2b. A ray of light coming from the point $(1,2)$ is reflected at a point A on the x-axis and then passes through the point $(5,3)$. The coordinates of the point A are :

A) $\left(\frac{13}{5},0\right)$ B) $\left(\frac{5}{13},0\right)$ C) $(-7,0)$ D) None of these

Key : A

Sol : Let incident ray touch x-axis at $A(x,0)$

From fig.,



Slope of $PA \Rightarrow m_1 = \tan\theta = \frac{2}{1-x}$

Slope of $QA = m_2 = \tan(\pi - \theta) = \frac{3}{5-x}$

$$-\tan\theta = \frac{3}{5-x}$$

$\therefore m_1 = -m_2$

$\frac{2}{1-x} = \frac{-3}{5-x}$

$10 - 2x = -3 + 3x$

$5x = 13$

$x = \frac{13}{5}$

$\therefore$ The co-ordinates of point A are $\left(\frac{13}{5},0\right)$

2c. A ray of light coming from the point $(1,\sqrt{3})$ is incident at an angle 30^0 on the line $x = \frac{1}{2}$ at the point A. The ray gets reflected on the line $x = \frac{1}{2}$ and meets x-axis at point B. Then the line AB passes through the point.

A) $(-2\sqrt{3},3)$ B) $(3,-2\sqrt{3})$ C) $(-3,2\sqrt{3})$ D) $(3\sqrt{3},-2)$

Key : B

Sol : Eq of AP $\Rightarrow$

$\left(y-\sqrt{3}\right)=\sqrt{3}\left(x-1\right)$

$y-\sqrt{3}=\sqrt{3}x-\sqrt{3}$

$\sqrt{3}x-y=0$ ....(1)

Image of P w.r.t line $x=\dfrac{1}{2}$

Is point $Q=\left(0,\sqrt{3}\right)$

By substituting $x=\dfrac{1}{2}$ in (1)

We get $A\left(\dfrac{1}{2},\dfrac{\sqrt{3}}{2}\right)$

Equation of reflected ray AB :

$$\left(y-\frac{\sqrt{3}}{2}\right)=\frac{\frac{\sqrt{3}}{2}-\sqrt{3}}{\frac{1}{2}-0}\left(x-\frac{1}{2}\right)$$

$$\frac{2y-\sqrt{3}}{2}=\frac{\sqrt{3}-2\sqrt{3}}{1}\left(\frac{2x-1}{2}\right)$$

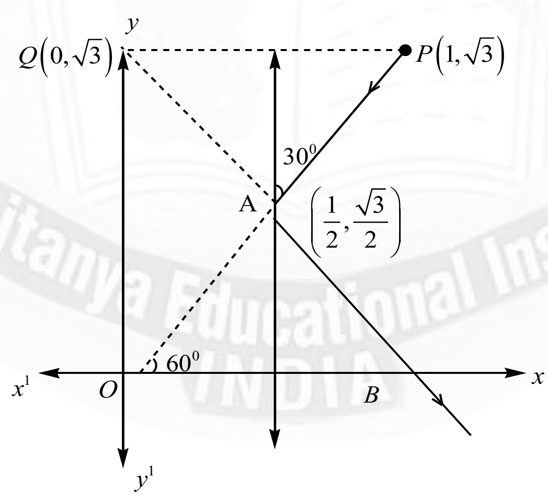
$2y-\sqrt{3}=-\sqrt{3}\left(2x-1\right)$

$2y-\sqrt{3}=-2\sqrt{3}x+\sqrt{3}$

$2\sqrt{3}x+2y-2\sqrt{3}=0$

$\sqrt{3}x+y-\sqrt{3}=0$

$\therefore$ Pont $\left(3,-2\sqrt{3}\right)$ lies on AB



3. The image of the curve $y^2=4x$ with respect to the line $x+y+2=0$ is $\left(x+2\right)^2+k\left(y+2\right)=0$ then K= ___

Key : 4

Sol : $y^2 = 4x \Rightarrow x = t^2, y = 2t$

The image of the point $(t^2, 2t)$ in the line $x + y + 2 = 0$ is $x = -2(t+1), y = -(t^2+2)$

Eliminating t $y = -\left[\left(\frac{x}{2}+1\right)^2 + 2\right]$

$\Rightarrow (x+2)^2 + 4(y+2) = 0$

4. The perpendicular bisector of the line segment joining $P(1,4)$ and $Q(k,3)$ has y-intercept -4 then K is _____

Key : 4

Sol : The perpendicular bisector of PQ is the locus of point which is equidistant from P and Q $(x-1)^2 + (y-4)^2 = (x-k)^2 + (y-3)^2$

$\Rightarrow 2(k-1)x - 2y = k^2 - 8$

y- intercept $\frac{k^2-8}{-2} = -4 \Rightarrow k^2 = 16 \Rightarrow k = 4$

5. Let ABC be a triangle. Let A be the point $(1,2)$, $y = x$ is the $\perp r$ bisector of AB and $x - 2y + 1 = 0$ is the angle bisector of angle C. If the equation of BC is given by $ax + by - 5 = 0$. Then the value of $a + b$ is ____

Key : 2

Sol : Image of A say A^1

w.r.t $x - 2y + 1 = 0$ lies on BC

$\frac{x-1}{1} = \frac{y-2}{-2} = -2\left(\frac{-2}{5}\right) = \frac{4}{5}$ $A^1 = \left(\frac{9}{5}, \frac{2}{5}\right)$

Equation of BC joining $A^1\left(\frac{9}{5}, \frac{2}{5}\right)$ and $B(2,1)$ is $3x - y - 5 = 0 \Rightarrow a + b = 3 - 1 = 2$

Miscellaneous

1. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right), a > 0$, be a fixed point in the $xy-$plane. The image of A in y-axis be B. If the image of B in x-axis is C. If $D(3\cos\theta, a\sin\theta)$ is a point in the fourth quadrant such that the maximum area of ΔACD is 12 square units, then a is equal to _______

Key: 8 **Jee Mains-2022**

Sol: $A = \left(\frac{3}{\sqrt{a}}, \sqrt{a}\right); B = \left(\frac{-3}{\sqrt{a}}, \sqrt{a}\right); C = \left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right)$

Area of ACD

$$= \frac{1}{2}\begin{vmatrix} \frac{6}{\sqrt{a}} & 2\sqrt{a} \\ \frac{3}{\sqrt{a}} - 3\cos\theta & \sqrt{a} - a\cos\theta \end{vmatrix} = 3\sqrt{a}(\cos\theta - \tan\theta)$$

Maximum value $3\sqrt{a}\sqrt{2}=12 \quad a=8$

1a. Let $A\left(\frac{4}{\sqrt{b}},\sqrt{b}\right) b>0$, be a fixed point in $xy-$plane. The image of A in x-axis is B. If the image of B in y-axis is C . If $D(4\cos\theta, b\sin\theta)$ is a point in fourth quadrant such that the maximum area of ΔACD is 24 square units, then b is equal to ———

Key: 18

Sol: $A\left(\frac{4}{\sqrt{b}},\sqrt{b}\right)$ $\quad B\left(\frac{4}{\sqrt{b}},-\sqrt{b}\right)$ $\quad C\left(\frac{-4}{\sqrt{b}},-\sqrt{b}\right)$ $\quad D(4\cos\theta, b\sin\theta)$

Area of ACD$=\frac{1}{2}\begin{vmatrix} \frac{8}{\sqrt{b}} & 2\sqrt{b} \\ \frac{4}{\sqrt{b}}-4\cos\theta & \sqrt{b}-b\cos\theta \end{vmatrix}=4\left(\frac{-b\sin\theta}{\sqrt{b}}+\sqrt{b}\cos\theta\right)$

$=4\sqrt{b}(-\sin\theta+\cos\theta)$

Maximum Area is $=4\sqrt{b}\sqrt{2}=24$

$=2b=36$

$\Rightarrow b=18$

1b. Let $A\left(\frac{5}{b},b\right) b>0$, be a fixed point in $xy-$plane. The image of A in x-axis is B. If the image of B in x-axis is C . If $D(5\cos\theta, b^2\sin\theta)$ is a point in fourth quadrant such that the maximum area of ΔACD is 20 square units, then b is equal to ———

Key: $2\sqrt{2}$

Sol: $A\left(\frac{5}{b},b\right)$ $\quad B\left(\frac{-5}{b},b\right)$ $\quad C\left(\frac{-5}{b},-b\right)$ $\quad D(5\cos\theta, b^2\sin\theta)$

Area of ACD$=\frac{1}{2}\begin{vmatrix} \frac{10}{b} & 2b \\ \frac{5}{b}-5\cos\theta & b-b^2\sin\theta \end{vmatrix}$

$=\frac{1}{2}\left|-10b\sin\theta+10b\cos\theta\right|$

$=5b\left|\cos\theta-\sin\theta\right|$

Maximum Area is $=5b\sqrt{2}=20$

$=b=\frac{4}{\sqrt{2}}$

$\Rightarrow b=2\sqrt{2}$

Exercise : III

(More than One Answer Type Questions Including PYQ's)

Various forms of straight lines

(Point-slope form, Two point form, Slope-intercept form, Intercept form, Normal form, Symmetric form (parametric form), General form of a straight line)

1. A line through $A(-5,-4)$ meets the lines $x+3y+2=0$, $2x+y+4=0$, $x-y-5=0$ at the points B,C&D, respectively such that $\left(\frac{15}{AB}\right)^2+\left(\frac{10}{AC}\right)^2=\left(\frac{6}{AD}\right)^2$ then

A) $\frac{15}{AB}=\cos\theta+3\sin\theta$

B) $\frac{10}{AC}=2\cos\theta+3\sin\theta$

C) $\frac{6}{AD}=\cos\theta-\sin\theta$

D) Slope of the line is $\frac{-2}{3}$

Key : A,B,C,D

Sol : A line through $A(-5,-4)$ with slope $\tan\theta$ is $\frac{x+5}{\cos\theta}=\frac{y+4}{\sin\theta}=r$, any point of the line is $(-5+r\cos\theta,-4+r\sin\theta)$. If this lies on $x+3y+2=0$, we have

$-5+r\cos\theta+3(-4+r\sin\theta)+2=0$.

$r=\frac{15}{AB}=\cos\theta+3\sin\theta$

Similarly we get $\frac{10}{AC}=2\cos\theta+\sin\theta, \frac{6}{AD}=\cos\theta-\sin\theta$

From condition

$$(\cos\theta+3\sin\theta)^2+(2\cos\theta+\sin\theta)^2=(\cos\theta-\sin\theta)^2$$

$$\Rightarrow \tan\theta=\frac{-2}{3}$$

2. A ray of light travelling along the line $x+y=1$ is incident on the x-axis and after refraction is incident it enters the other side of the x-axis by turning $\frac{\pi}{6}$ away from the x-axis. The equation of the line along which the refracted ray travels is

A) $x+(2-\sqrt{3})y=1$

B) $(2-\sqrt{3})x+y=1$

C) $y+(2+\sqrt{3})x=2+\sqrt{3}$

D) $y+(2-\sqrt{3})x=2-\sqrt{3}$

Key : A,C

Sol : The line of the refracted ray passes through the point $(1,0)$ and its slope is $\tan 105^0$

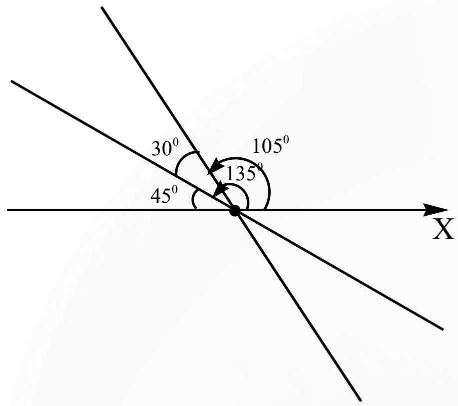
$\therefore$ The equation of the line of the refracted ray is $y-0=\tan 105^0(x-1)$

$y=-(2+\sqrt{3})(x-1)$

$\Rightarrow y+\left(2+\sqrt{3}\right)x=2+\sqrt{3}$

(or)

$\left(2-\sqrt{3}\right)y+x=1$



3. A line is such that its segment between the lines $5x-y+4=0$, and $3x+4y-4=0$ is bisected at the point $(1,5)$. The equation of line is $ax+by+c=0$. Then

A) $b+c=-95$ B) $a+b+c=12$ C) $b+c=3$ D) $a+b+c=0$

Key : A,B

Sol : Given lines are $5x-y+4=0$, $3x+4y-4=0$

$P=(\alpha,5\alpha+4),Q=(x,y)$

$\because$ $R(1,5)$ is mid point of $\overline{PQ}$

$\therefore (1,5)=\left(\dfrac{\alpha+x}{2},\dfrac{5\alpha+4+y}{2}\right)$

$\Rightarrow$ $x=2-\alpha,$ $y=-5\alpha+6$

$\Rightarrow y=6-5\alpha$

$Q(x,y)=(2-\alpha,6-5\alpha)$ lies on $3x+4y-4=0$

$\Rightarrow$ $3(2-\alpha)+4(6-5\alpha)-4=0$

$\Rightarrow$ $\alpha=\dfrac{26}{23}$

Equation of $\overline{PQ}$ in two point form

$$y-5=\left(\frac{\frac{222}{23}-5}{\frac{26}{23}-1}\right)(x-1)$$

$\Rightarrow 107x-3y-92=0$

$ax+by+c=0$

$a=107,b=-3,c=-92$

4. The equation of the lines passing through the point $(1,0)$ and at a distance $\frac{\sqrt{3}}{2}$ from the origin is

A) $\sqrt{3}x+y-\sqrt{3}=0$ B) $x+\sqrt{3}y-\sqrt{3}=0$ C) $\sqrt{3}x-y-\sqrt{3}=0$ D) $x-\sqrt{3}y-\sqrt{3}=0$

Key : A,C

Sol : Let equation of line in slope – point form

$y-0=m(x-1)$

$mx-y-m=0$ ……(1)

$d=\frac{|c|}{\sqrt{a^2+b^2}}\Rightarrow\frac{\sqrt{3}}{2}=\frac{|-m|}{\sqrt{1+m^2}}$

$\Rightarrow 3(1+m^2)=4m^2$

$\Rightarrow m^2=3\Rightarrow m=\pm\sqrt{3}$

$m=\sqrt{3}\Rightarrow\sqrt{3}x-y-\sqrt{3}=0$

$m=-\sqrt{3}\Rightarrow\sqrt{3x}-y-\sqrt{3}=0$

$\Rightarrow\sqrt{3}x+y-\sqrt{3}=0$

5. The straight line $3x+4y-12=0$ meets the coordinates axes A and B. An equilateral triangle ABC is constructed. The possible coordinates of vertex C are

A) $\left[2\left(1-\frac{3\sqrt{3}}{4}\right),\frac{3}{2}\left(1-\frac{4}{\sqrt{3}}\right)\right]$ B) $\left[-2\left(1+\sqrt{3}\right),\frac{3}{2}\left(1-\sqrt{3}\right)\right]$

C) $\left[2\left(1+\sqrt{3}\right),\frac{3}{2}\left(1+\sqrt{3}\right)\right]$ D) $\left[2\left(1+\frac{3\sqrt{3}}{4}\right),\frac{1}{2}\left(1+\frac{4}{\sqrt{3}}\right)\right]$

Key : A,D

Sol : $A=(4,0),B(0,3),M=\left(2,\frac{3}{2}\right)$

$AB=\sqrt{16+9}=5,h=\sqrt{\frac{3}{2}}\times5=\frac{5\sqrt{3}}{2}$

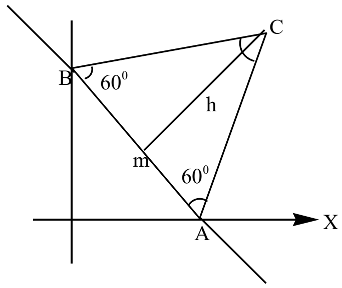
Slope of AB = $\frac{-3}{4}\Rightarrow\tan\theta=\frac{-3}{4}$

Slope $cm=\frac{4}{3}$

Coordinates AC= $(x_1\pm r\cos\theta,y_1\pm r\sin\theta)$

$=\left(2\pm5\frac{\sqrt{3}}{2}\left(\frac{3}{5}\right),\frac{3}{2}\pm5\frac{\sqrt{3}}{2}\left(\frac{4}{5}\right)\right)$

$=\left(2\pm\frac{3\sqrt{3}}{2},\frac{3}{2}\pm2\sqrt{3}\right)$



6. The area of the triangle formed by the intersection of a line parallel to the x-axis and passing through with the lines $y = x$ and $x + y = 2$ is $4h^2$. Then the locus of the point P.

IIT-JEE 2002

A) $y = 2x + 1$ B) $y = -2x + 1$ C) $y = x + 1$ D) $y = -x + 1$

Key : A,B

Sol: A line passing through $P(h, k)$ and parallel to the x-axis is

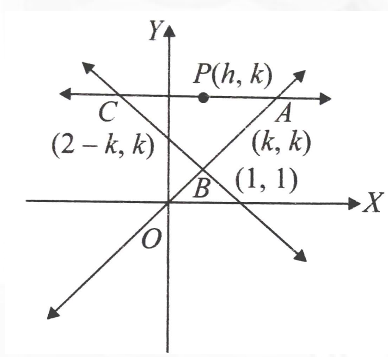
y=k ….(i)

The other two lines given are

$y = x$ ….(ii)

And $x + y = 2$ ….(iii)

Let ABC be the triangle formed by the points of intersection of lines (i), (ii) and (iii), as shown in the figure.



Then $A \equiv (k, k), B \equiv (1, 1)$, and $C \equiv (2 - k, k)$. Therefore,

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2 & -k & k \end{vmatrix} = 4h^2$$

Operating $C_1 \to C_1 \to C_2$, we get

$$\therefore \frac{1}{2}\left|(2 - 2k)(k - 1)\right| = 4h^2$$

$$\therefore (k - 1)^2 = 4h^2$$

$$\therefore k - 1 = 2h \text{ or } k - 1 = -2h$$

$$\therefore k = 2h + 1 \text{ or } k = -2h + 1$$

Hence, the locus of (h, k) is

$y = 2x + 1$ or $y = -2x + 1$

Position of a point w.r.t a line

(Ratio in which the line divides the line joining two points, Position of origin and a point w.r.t given line)

1. The value of θ in $(0,\pi)$ such that the points $(3,5)$ and $(\sin\theta,\cos\theta)$ lie on the same side of the line $x+y-1=0$

A) 0 B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

Key : B,C

Sol : The points line on same side of line L

$L_{11}L_{22}>0$

$(3+5-1)(\sin\theta+\cos\theta-1)>0$

$$\sin\theta+\cos\theta-1>0$$
$$\sin\theta+\cos\theta>1$$
$$\sin\left(\frac{\pi}{4}+\theta\right)>\frac{1}{\sqrt{2}}$$
$$\Rightarrow \frac{\pi}{4}<\frac{\pi}{4}+\theta<\frac{3\pi}{4}$$
$$\Rightarrow \ 0<\theta<\frac{\pi}{2}$$

2. If a point $P(x,y)$ lies on the line $y=x$ such that P and the point $Q(3,4)$ are on the same side of the line $3x-4y=8$ then possible value of x in $(-10,10)$

A) -2 B) 5 C) 7 D) -9

Key : A,B,C

Sol: $P(x,y)$ is a point on $y=x$

$p(x,x)$

$Q(3,4)$

Line : $3x-4y-8=0$

P,Q lie same side of line

$L_{11}L_{22}>0$

$(3x-4x-8)(9-16-18)>0$

$(-x-8)(-15)>0$

$(x+8)15>0$

$x+8>0$

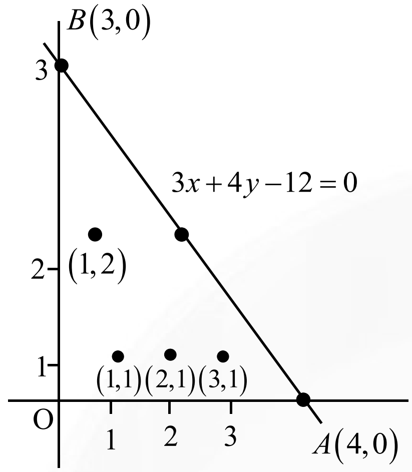
$x>-8$

3. The triangle OAB is formed by x-axis, y-axis and the line $3x+4y-12=0$. Then the interior point (x,y) such that both x and y are y integers

A) $(2,2)$ B) $(1,2)$ C) $(3,1)$ D) $(1,1)$

Key : B,C,D

Sol :



4. If a point $P(\alpha,\beta)$ lie on the line $y=-2x$ such that P and the point $Q(5,4)$ lies opposite sides of the line $4x+5y-20=0$ then

A) $\alpha>\frac{-10}{3}$ B) $\alpha<\frac{-10}{3}$ C) $\beta>\frac{20}{3}$ D) $\beta<\frac{20}{3}$

Key : A,D

Sol : $P(\alpha,\beta)$ is a point on $y=-2x$

$P(\alpha,-2\alpha)$, $Q(5,4)$

$L:4x+5y-20$

P,Q lies opposite sides of line $L_{11}L_{22}<0$

$(4\alpha-10\alpha-20)(20+20-20)<0$

$(-6\alpha-20)<0$

$6\alpha+20>0$

$\alpha>\frac{-20}{6}$

$\alpha>\frac{-10}{3}$

$\beta=-2\alpha$ $\qquad -2\alpha<-2\left(\frac{-10}{3}\right)$

$\beta<\frac{20}{3}$

5. Which of the following is false for the points $(1,2)$ and $(-2,1)$

A) on the same side of the line $4x+2y=1$

B) on the line $4x+2y=1$

C) on opposite side of the line $4x+2y=1$

D) None of these

Key : A,B

Sol: Line $L \equiv 4x+2y-1=0$

$L_{11}L_{22} = (4+4-1)(-8+2-1)$

$=(7)(-7)$

$=-49$

$L_{11}L_{22} < 0$

$(1,2)(-2,1)$ lies on opposite side of line is true

Distances to a line

(Perpendicular distance from a point, Distances of given point from the given line measured along a straight line, Distance between parallel lines)

1. If the lines $L=0, L_1: 2x-y-1=0$ and $L_2: 3x+4y-7=0$ have a common point then

A) If L is situated at a maximum distance from point $(2,3)$, then its equation is $x+2y-3=0$

B) If L is situated at a minimum distance from point $(2,3)$, then its equation is $2x-y-1=0$

C) If L is situated at a maximum distance from $(2,3)$, then its equation is $2x-y-1=0$

D) If L is situated at a minimum distance from $(2,3)$, then its equation is $x+2y-3=0$

Key : A,B

Sol : Point of intersection $L_1: 2x-y-1=0$ and $3x+4y-7=0$ is $(1,1)$

$\therefore$ clearly L passes through $(1,1)$

If it is at maximum distance from $(2,3)$

Then line L will be perpendicular to line joining $(1,1)$ and $(2,3)$

$\therefore$ slope of $L = \frac{-1}{2}$

Equation of L is $y-1 = \frac{-1}{2}(x-1) \Rightarrow x+2y-3=0$

If L is situated at minimum distance from $(2,3)$ then L will line passing through $(1,1)$ and $(2,3)$

$\therefore \; y-1 = 2(x-1) \Rightarrow 2x-y-1=0$

2. The lines $x+2y+3=0$, $x+2y-7=0$ and $2x-y-4=0$ are the sides of a square. The equation of the remaining side of the square can be

A) $2x-y+6=0$ B) $2x-y+8=0$ C) $2x-y-10=0$ D) $2x-y-14=0$

Key : A,D

Key : Distance between $x+2y+3=0$ and $x+2y-7=0$ is $\frac{10}{\sqrt{5}}$. Let the remaining side parallel to $2x-y-4=0$ be $2x-y+\lambda=0$ we have $\frac{|\lambda+4|}{\sqrt{5}} = \frac{10}{\sqrt{5}} \Rightarrow \lambda = 6,-14$

Thus the remaining side is $2x-y+6=0$

Or $2x-y-14=0$

3. The equation of the lines passing through the point $(1,0)$ and at a distance $\frac{\sqrt{3}}{2}$ from the origin is

A) $\sqrt{3}x+y-\sqrt{3}=0$ B) $x+\sqrt{3}y-\sqrt{3}=0$

C) $\sqrt{3}x-y-\sqrt{3}=0$ D) $x-\sqrt{3}y-\sqrt{3}=0$

Key : A,C

Sol : The equations of lines passing through $(1,0)$ are given by $y=m(x-1)$. Its distance from the origin is $\frac{\sqrt{3}}{2}$. Hence $\left|\frac{-m}{\sqrt{1+m^2}}\right|=\frac{\sqrt{3}}{2}\Rightarrow m=\pm\sqrt{3}$

Hence the lines are $\sqrt{3}x+y-\sqrt{3}=0, \sqrt{3}x-y-\sqrt{3}=0$

4. The equation of a straight line passing through the point $(2,3)$ and inclined at an angle of $\tan^{-1}\left(\frac{1}{2}\right)$ with the line $y+2x=5$

A) $y=3$ B) $x=2$ C) $3x+4y-18=0$ D) $4x+3y-17=0$

Key :B,C

Sol : Let the slope of required line be 'm'

Then $\frac{1}{2}=\left|\frac{m-(-2)}{1+(-2)m}\right|$

$\Rightarrow m=\frac{-3}{4}\ \&\ \infty$

Hence the equation of line is

$y-3=\frac{-3}{4}(x-2)$ and $x=2$

5. The equation of the straight line passing through the intersection of $x+2y-19=0$, $x-2y-3=0$ and at a distance of 5 units from $(-2,4)$ is

A) $5x-12y-7=0$ B) $5x+2y+103=0$ C) $5x-12y+7=0$ D) $5x+12y=103$

Key : A,D

Sol : Point of intersection of the lines $x+2y-19=0$, $x-2y-3=0$ is $P(11,4)$

Equation of a line through P is $y-4=m(x-11)\Rightarrow mx-y+(4-11m)=0$

$\therefore \left|\frac{m(-2)-4+4-11m}{\sqrt{m^2+1}}\right|=5\Rightarrow 5\sqrt{m^2+1}=|-13m|$

$\Rightarrow\ 25(m^2+1)=169m^2\Rightarrow m^2=\frac{25}{144}\Rightarrow m=\pm\frac{5}{12}$

$\therefore$ Required lines are

$5x-12y=7$ and $5x+12y=103$

6. The equation of the line passing through the point $(2,3)$ and making an intercept of length 2 units between the lines $y+2x=3$ and $y+2x=5$ **IIT-JEE 1991**

A) $3x+4y-18=0$ B) $y=2$ C) $3x-4y-18=0$ D) $x=2$

Key : AD

Sol : Given lines are

$2x+y-3=0$ …(i)

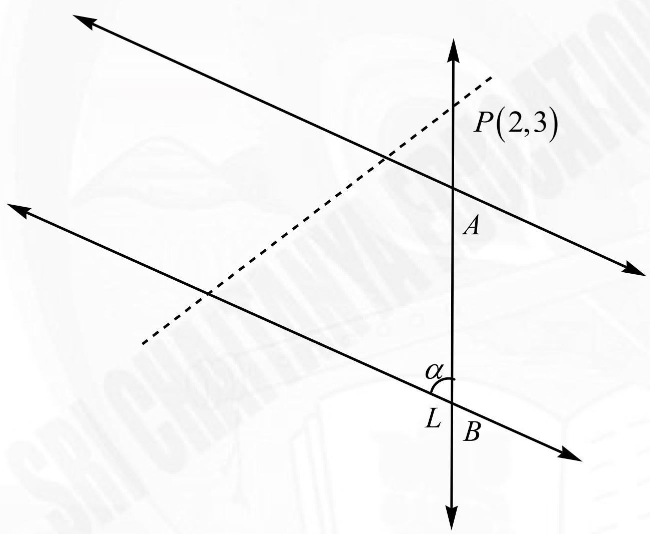
And $2x+y-5=0$ …(ii)

Given AB=2

AL= distance between parallel

Lines (i) and (ii) $=\frac{1-3+5l}{\sqrt{2^2+1^2}}=\frac{2}{\sqrt{5}}$

From ΔALB $\sin\alpha=\frac{AL}{AB}=\frac{\sqrt{5}}{2}=\frac{1}{\sqrt{5}}$



$\therefore \tan\alpha=\frac{1}{2}$

Now slope of given lines is -2.

Let the slope of AB be m. Then

$\tan\alpha=\frac{|1m-(-2)|}{|1+m(-2)|}$

$\Rightarrow \pm\frac{1}{2}=\frac{m+2}{1-2m}$

$\Rightarrow 1-2m=2m+4$ or $2m-1-2m+4$

V $m=-3/4$ or $m=\infty$

Therefore, equation of lines can be $x-2=0$ or $(y-3)=(-3/4)(x-2)$

Or $3x+4y-18=0$

Angle Between Two lines
(Condition for parallel & perpendicular lines)

1. Angle made with the x-axis by a straight line drawn through $(1,2)$ so that it intersects $x+y=4$ at distance $\sqrt{6}/3$ from $(1,2)$ is

A) 105^0 B) 75^0 C) 60^0 D) 15^0

Key : B,D

Sol : Let the angle be θ. Then the equation of the given line is

$$\frac{x-1}{\cos\theta}=\frac{y-2}{\sin\theta}$$

The coordinates of a point on (i) at a distance $\sqrt{6}/3$ from $(1,2)$ are $\left(1+\sqrt{6}/3\cos\theta, 2+\sqrt{6}/3\sin\theta\right)$. This point lies on $x+y=4$. Therefore,

$$1+\frac{\sqrt{6}}{3}\cos\theta+2+\frac{\sqrt{6}}{3}\sin\theta=4$$

Or $\cos\theta+\sin\theta=\sqrt{\frac{3}{2}}$

Or $\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta=\frac{\sqrt{3}}{2}$

Or $\cos\left(\theta-\frac{\pi}{4}\right)=\cos\left(\pm\frac{\pi}{6}\right)$

Or $\theta-\frac{\pi}{4}=\pm\frac{\pi}{6}$

i.e., $\theta=75^0$ or $\theta=15^0$

2. The equation of a straight line passing through the point $(2,3)$ and inclined at an angle of $\tan^{-1}(1/2)$ with the line $y+2x=5$ is

A) $y=3$ B) x=2 C) $3x+4y-18=0$ D) $4x+3y-17=0$

Key : B,C

Sol : Let the slope of line be m. Then

$$\frac{1}{2}=\left|\frac{m-(-2)}{1+(-2)m}\right|$$

Or $m=\frac{3}{4}$ and ∞

Hence, the equation of line is $y-3=-(3/4)(x-2)$ and $x=2$

3. Two straight lines $u=0$ and $v=0$ pass through the origin and the angle between them is $\tan^{-1}(7/9)$. If the ratio of the slope of $v=0$ and $u=0$ is $9/2$, then their equations are

A) $y+3x=0$ and $3y+2x=0$ B) $2y+3x=0$ and $3y+x=0$

C) $2y=3x$ and $3y=x$ D) $y=3x$ and $3y=2x$

Key : A,B,C,D

Sol : Let the slope of $u=0$ be m. Then slope of $v=0$ is $9m/2$. Therefore

$$\frac{7}{9}=\left|\frac{m-\frac{9m}{2}}{1+m\times\frac{9m}{2}}\right|=\left|\frac{-7m}{2+9m^2}\right|$$

Or $9m^2-9m+2=0$ or $9m^2+9m+2=0$

$$m=\frac{9\pm\sqrt{81-72}}{18}=\frac{9\pm 3}{18}=\frac{2}{3},\frac{1}{3}$$

Or $m=\frac{-9\pm 3}{18}=-\frac{2}{3},-\frac{1}{3}$

Therefore, the equation of the line are

i. $3y=x$ and $2y=3x$

ii. $3y=2x$ and $y=3x$

iii. $x+3y=0$ and $3x+2y=0$

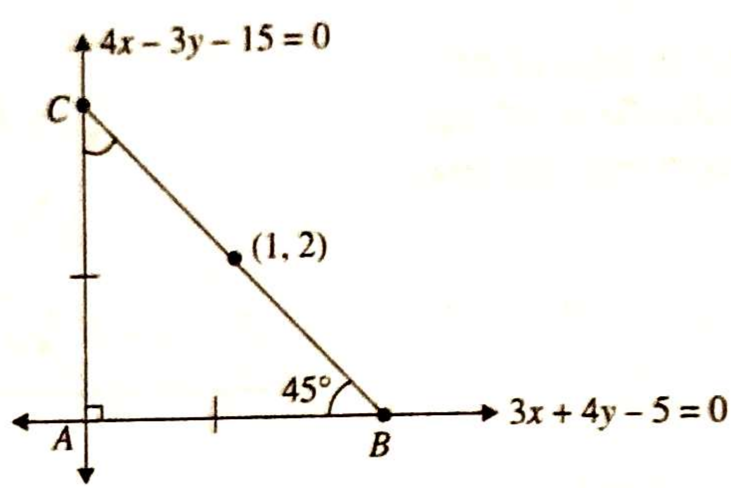
iv. $2x+3y=0$ and $3x+y=0$

4. Straight lines $3x+4y=5$ and $4x-3y=15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB=AC$. The possible equation of the line BC passing through the point $(1,2)$ **IIT-JEE 1990**

A) $x-y+5=0$ B) $x-7y+13=0$ C) $7x+y-9=0$ D) $x+y+5=0$

Key : B,C

Sol: The given straight lines are $3x+4y=5$ and $4x-3y=15$. Clearly these straight lines are perpendicular to each other $(m_1m_2=-1)$, and intersect at A. Now, B and C are the points on these lines such that AB=AC and BC passes through $(1,2)$. From the figure, it is clear that $\angle B=\angle C=45^0$



Let the slope of BC be m. Then,

$$\tan 45^0=\left|\frac{m+3/4}{1-(3/4)m}\right|$$

Or $4m+3=\pm 4(4-3m)$

i.e., $4m+3=4-3m$ or $4m+3=-4+3m$

i.e., $m=\frac{1}{7}$ or $m=-7$

Hence, the equation of BC is

$y-2=\frac{1}{7}(x-1)$ or $y-2=-7(x-1)$

i.e., $7y-14=x-1$ or $y-2=-7x+7$

i.e., $x-7y+13=0$ or $7x+y-9=0$

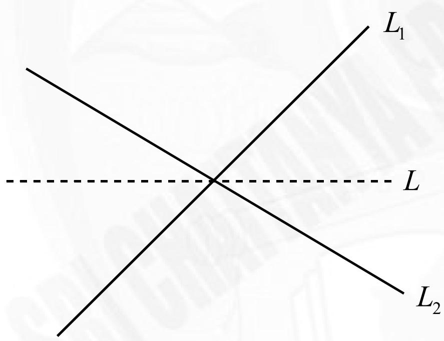
Foot of Perpendicular and Image of a point w.r.t a line

1. Let $L_1: 3x+4y-1=0$ and $L_2: 5x-12y+2=0$ be two given lines. Let image of every point on L_1 with respect a line L lies on L_2 then possible equation of 'L' can be

A) $14x+112y-23=0$ B) $11x-4y=0$ C) $64x-8y-3=0$ D) $52y-45x=7$

Key : A,C

Sol : 'L' must be the angular bisector of L_1 and L_2



$$\frac{3x+4y-1}{5}=\pm\frac{(5x-12y+2)}{13}$$

$39x+527-13=\pm 5(5x+12y+2)$

$14x+112y-23=0$ (or) $64x-8y-3=0$

2. Locus of the image of the point $(2,3)$ in the line $2x-3y+4+K(x-2y+3)=0$, $k\in R$ is a

A) circle with centre $(1,2)$ B) circle of radius 3

C) straight line parallel to x-axis D) circle of radius $\sqrt{2}$

Key : A,D

Sol : $2x-3y+4=0$ ...(1)

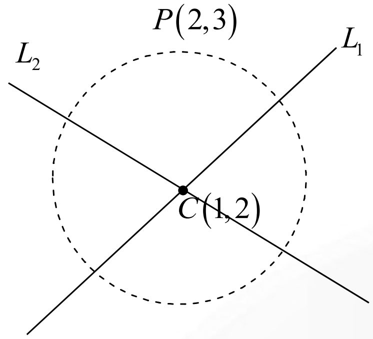
$x-2y+3=0$ ...(2)

Equation (1) and (2) we get

Centre $=C(1,2)$

Radius = CP$=\sqrt{(2-1)^2+(3-2)^2}$

$CP = \sqrt{2}$



3. The locus of feet of perpendiculars drawn from origin to the straight lines passing through $(2,1)$ is

A) a straight line | B) a circle with centre $\left(1, \frac{1}{2}\right)$

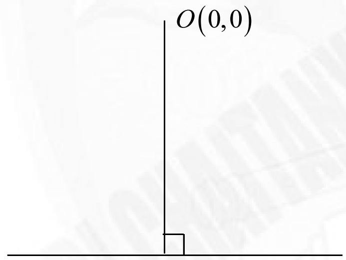
C) radius is $\sqrt{5}$ | D) diameters is $\sqrt{5}$

Key : B,D

Sol : Let $P(x_1, y_1)$ be the feet of

Perpendicular drawn from origin

$\overline{OP} \perp \overline{PA}$



(slope of $\overline{OP}$) x (slope of $\overline{PA}$)$=-1$

$y_1(y_1-1) = -x_1(x_1-2)$

The locus is $x^2 + y^2 + 2x - y = 0$

Centre $=\left(1, \frac{1}{2}\right)$

$r = \sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2}$

4. The equation of line segment AB is $y = x$. If A and B lie on same side of line mirror $2x - y = 1$, then the equation of Image of AB w.r.t to line mirror $2x - y = 1$ is $y = mx + c$ then

A) $m = 70$ B) $C = -6$ C) $m = -7$ D) $C = 6$

Key : A,B

Sol : $A(-1,-1), B(0,0)$ same side of $2x - y - 1 = 0$

Let (h,k) be the image of $A(-1,-1)$

$$\frac{h+1}{2}=\frac{k+1}{-1}=-2\frac{(-2+1-1)}{5}$$

$$h+1=\frac{8}{5},k+1=\frac{-4}{5}$$

$$A^1\left(\frac{3}{5},\frac{-9}{5}\right)$$

Let (h^1,k^1) be the image of $B(0,0)$

$$\frac{h^1}{2}=\frac{k^1}{-1}=\frac{-2(-1)}{5}$$

$$h^1=4/5,k^1=\frac{-2}{5}$$

$$B^1=\left(\frac{4}{5},\frac{-2}{5}\right)$$

Slope of $A^1B^1=7$

Image line is $y+\frac{9}{5}=7\left(x-\frac{3}{5}\right)$

$y=7x-6$

5. For all values of 'θ' the lines represented by the equation $(2\cos\theta+3\sin\theta)x+(3\cos\theta-5\sin\theta)y-(5\cos\theta-2\sin\theta)=0$

A) passes through a fixed point

B) passes through a point $(1,1)$

C) pass through a fixed point whose reflection in the line $x+y=\sqrt{2}$ is $(\sqrt{2}-1,\sqrt{2}-1)$

D) Pass through the origin

Key : A,B,C

Sol : $(2x+3y-5)\cos\theta+(3x-5y+2)\sin\theta=0$

P. I. of lines $2x+3y-5=0$

$3x-5y+2=0$ is $(1,1)$

Let (h,k) be the reflection of $(1,1)$ in the line $x+y=\sqrt{2}$

$$\frac{h-1}{1}=\frac{k-1}{1}=\frac{-2(1+1-\sqrt{2})}{2}$$

$$h=\sqrt{2}-1,K=\sqrt{2}-1$$

$$(h,k)=(\sqrt{2}-1,\sqrt{2}-1)$$

Centers of Triangle

(Centroid, Circumcentre, incentre, Ortho Centre)

1. Two vertices of a triangle are $(-2,3)\&(5,-1)$.If the orthocenter lies at the origin and the centroid on the line $x+y+2=0$, then the third vertex can be

A) $(-5,3)$ B) $(-4,-7)$ C) $(-6,-5)$ D) $(3,-14)$

Key: B

Sol: Let the vertices A(h,k), B(-2,3) and C(5,-1), Orthocentre is O(0,0).

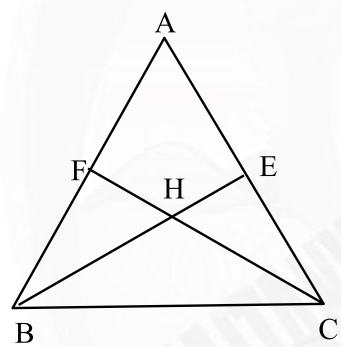
Slope of $AO \times Slope\ of\ BC = -1$

$$\Rightarrow \frac{k}{h} \times \frac{3+1}{-2-5} = -1 \Rightarrow \frac{k}{h} = \frac{7}{4} \longrightarrow (1)$$

Again $Slope\ \ of\ \ BO \times slope\ of\ AC = -1$

$$\Rightarrow \frac{3}{-2} \times \frac{k+1}{h-5} = -1 \Rightarrow \frac{k+1}{h-5} = \frac{2}{3} \longrightarrow (2)$$

Solving equations(1) & (2) , we get $h=-4\ and\ k=-7$.Point $A\,is\,(-4,-7)$

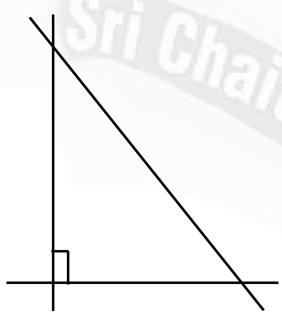


2. If $H(a,b)$ is the ortho centre of triangle formed by the lines $2x+3y-5=0$, $3x-2y-1=0$ and $x+y=5$ then

A) $a+b=2$ B) $a-b=0$ C) $2a+3b=5$ D) $3a-2b=1$

Key : ABCD

Sol :



$H(a,b)$

$2x+3y-5=0$

$3x-2y-1=0$

$x=1 \quad y=1$

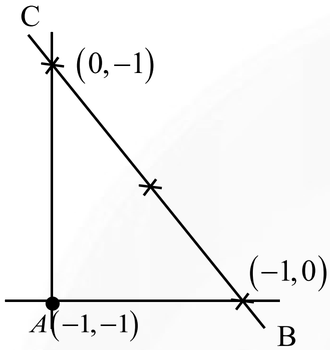
$H(a,b)=(1,1)$

3. If $xy+x+y+1=0$ and $x+y+1=0$ formed a triangle and H,S,G are the ortho centre circum centre and centroid of a triangle then

A) $H(-1,-1)$ B) $S\left(\frac{-1}{2},\frac{-1}{2}\right)$ C) $G\left(\frac{-2}{3},\frac{2}{3}\right)$ D) None

Key :ABC

Sol :



$x+1=0$ ...(1)

$y+1=0$ ...(2)

$x+y+1=0$ ....(3)

Solving 1 and 2 $x=-1$ $y=-1$

$H(-1,-1)$

S is the midpoint of $BC=\left(\frac{-1}{2},\frac{-1}{2}\right)$

$3G=2s+H$

$=\left(\frac{-1-1}{3},\frac{-1-1}{3}\right)=\left(\frac{-2}{3},\frac{-2}{3}\right)$

4. Let $A(1,0)B(6,2),C\left(\frac{3}{2},6\right)$ be the vertices of a triangle ABC if P is any point inside of a triangle such that the triangle APC, triangle APB and triangle BPC have equal area and $Q\left(\frac{-7}{6},\frac{-1}{3}\right)$ then

A) $P\left(\frac{17}{6},\frac{8}{3}\right)$ B) $PQ=5$ C) $P\left(\frac{17}{3},\frac{5}{3}\right)$ D) $PQ=7$

Key : AB

Sol : P is the centroid of triangle ABC

$$P=\left(\frac{1+6+\frac{3}{2}}{3},\frac{0+2+6}{3}\right)=\left(\frac{17}{6},\frac{8}{3}\right)$$

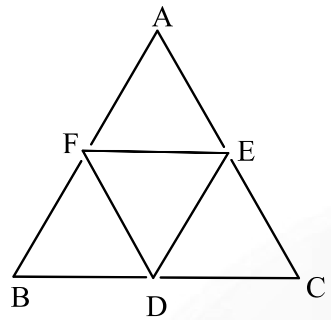
$PQ=5$

5. The coordinates of feet of perpendiculars from the vertices A,B,C of triangle ABC on the opposites are $D(20,25)E(8,16)$ and $F(8,9)$ respectively and triangle ABC is acute angled triangle and ortho centre of the triangle ABC is (a,b) then

A) $a+b=25$ B) $b-a=5$ C) $ab=150$ D) $\frac{a}{b}=\frac{2}{3}$

Key : A,B,C,D

Sol :



If triangle is acute angle triangle then the incentre of ΔDEF is the ortho center of triangle ABC

$H(a,b) =$ use in centre formula for $\Delta DEF = (10,15)$

$\therefore a = 10$ and $b = 15$

Various Triangles and four sided figures

(Area of triangle and parallelogram)

1. If $H = (3,4)$ and $C = (1,2)$ are ortho center and circum centre of ΔPQR and the equation of the side PQ is $x - y + 7 = 0$ then

A) Equation of circum circle is $(x-1)^2 + (y-2)^2 = 80$

B) Equation of circum circle is $(x-1)^2 + (y-2)^2 = 70$

C) Centroid is $\left(\frac{5}{3}, \frac{8}{3}\right)$

D) Circum radius $= \sqrt{70}$

Key : B,C

Sol :

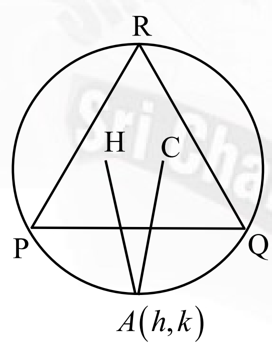


Image of orthocenter

Lies on the circle

Let $A = (h,k)$ be the image

$$\frac{h-3}{1} = \frac{k-4}{-1} = -2\left(\frac{3-4+7}{2}\right)$$

$\therefore h = -3, k = 10$

Radius =CA=$\sqrt{(1+3)^2+(2-10)^2}=\sqrt{80}$

Equation of circle is $(x-1)^2+(y-2)^2=(\sqrt{80})^2$

Centroid divides here H,C in the ratio of 2:1

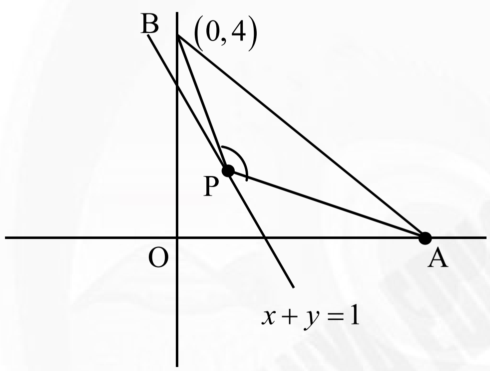
$G=\frac{2C+H}{3}=\left(\frac{5}{3},\frac{8}{3}\right)$

2. If the area of $\Delta OPB=$ area of ΔOPA where O in origin, $A=(6,0),B=(0,4)$ and P lies on the line $x+y=1$ then possible coordinates of P are ___

A) $\left(\frac{3}{5},\frac{2}{5}\right)$ B) $(3,-2)$ C) $(2,-1)$ D) $\left(\frac{1}{2},\frac{1}{2}\right)$

Key : A,B

Sol :



Let $P=(x,1-x)$, be the point

Area of $\Delta OPB=$ area of ΔOPA

$\frac{1}{2}|x.4-(1-x).0|=\frac{1}{2}|x.0-6(1-x)|$

$|4x|=|6(x-1)|\Rightarrow x=3$ or $\frac{3}{5}$

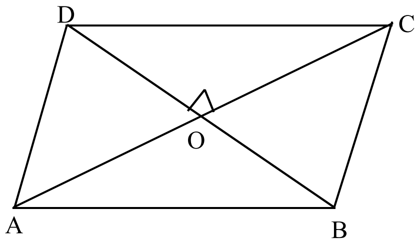
3. A diagonal of rhombus ABCD is member of both the families of lines $(x+y-1)+t(2x+3y-2)=0$ and $(x-y+2)+s(2x-3y+5)=0$ and one of the vertices of the rhombus is $(3,2)$. Of the area of the rhombus is $12\sqrt{5}$ units. Then the other vertices are

A) $\left(\frac{9}{5}-2\sqrt{5},\frac{-2}{5}+\sqrt{5}\right)$ B) $\frac{9}{5}+2\sqrt{5},\frac{-2}{5}+\sqrt{5}$

C) $\left(\frac{9}{5}-2\sqrt{5},\frac{-2}{5}+2\sqrt{5}\right)$ D) $\left(\frac{9}{5}+2\sqrt{5},\frac{-2}{5}-\sqrt{5}\right)$

Key : A,D

Sol :



The point of intersection of $x+y-1=0$, $2x+3y-2=0$ is $(1,0)$

The point of intersection of $x-y+2=0$, $2x-3y+5=0$ is $(-1,1)$

$\therefore$ Let diagonal AC passes through $(1,0),(-1,1)$

$\therefore$ Equation of AC is $x+2y-1=0$

Let $B=(3,2)$

$\therefore$ The vertex $(3,2)$ lies on other diagonal BD. $AC \perp BD \Rightarrow$ equation of BD is $2x-y=4$.

Solving $x+2y-1=0$, $2x-y=4$

$\Rightarrow O=\left(\frac{9}{5},\frac{-2}{5}\right)$

$\therefore$ O is midpoint of BD $\Rightarrow D=\left(\frac{3}{5},\frac{-14}{5}\right)$

Length of $BD=\frac{12\sqrt{5}}{5}$ and Area $=12\sqrt{5}$

$\frac{1}{2}\times AC\times BD=12\sqrt{5}$

$\therefore$ length of AC=10

Using formula of $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$

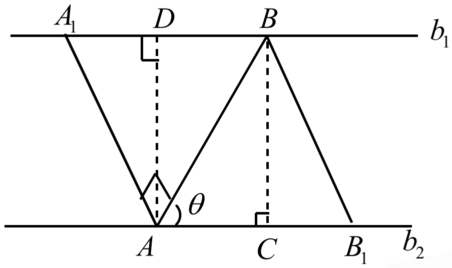
$A=\left(\frac{9}{5}-2\sqrt{5},\frac{-2}{5}+\sqrt{5}\right)$, $C=\left(\frac{9}{5}+2\sqrt{5},\frac{-2}{5}-\sqrt{5}\right)$

4. A line L is drawn from $P(4,3)$ to meet the line $3x+4y+5=0$, $3x+4y+15=0$ at points A,B. From point 'A' a line perpendicular to L is drawn meeting the line L_2 at A_1, similarly from the point 'B' a line perpendicular to L is drawn meeting L_1 at B_1. Thus AA_1BB_1 is parallelogram and find the equation of L such that area of parallelogram AA_1BB_1 is least

A) $7x-y-31=0$ B) $7x+y-3=0$ C) $x+7y+17=0$ D) $x-7y+17=0$

Key : A,D

Sol :



L_1, L_2 are parallel lines

$\Rightarrow$ distance between L_1, L_2

$$= \left|\frac{15-5}{\sqrt{3^2+4^2}}\right| = 2$$

$\therefore \; AD = BC = 2$

Let $\underline{|BAC} = \theta$

$\therefore \; AB = 2\cos ec\theta$

$AA_1 = 2\sec\theta$

Area of parallelogram $= 2 \times \frac{1}{2} \times AB \times AA_1$

$$= 4\sec\theta\cos e\theta = \frac{8}{\sin 2\theta}$$

Area is least of $\theta = \frac{\pi}{4}$

Let slope of AB be m , $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow 1 = \left|\frac{m + \frac{3}{4}}{1 - m.\frac{3}{4}}\right|$$

$\Rightarrow 1 = \left|\frac{4m+3}{4-3m}\right| \Rightarrow m = -7$ or $\frac{1}{7}$

$\therefore$ equation of line L is $y - 3 = -7(x-4) \Rightarrow 9x - y - 31 = 0$

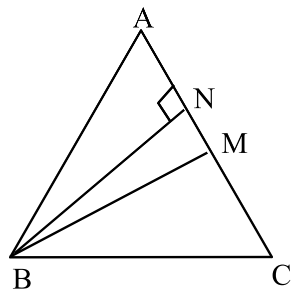
$$y - 3 = \frac{1}{7}(x-4) \Rightarrow x - 7y + 17 = 0$$

5. In a triangle ABC, $A = (2,-1)$ and $7x - 10y + 1 = 0$, $3x - 2y + 5 = 0$ are the equation of altitude and angular bisector of vertex B. The equations of the sides AB, BC are ___ (without finding other vertices)

A) $10x + 7y - 13 = 0$ B) $20x + 4y + 67 = 0$ C) $x - 5y - 7 = 0$ D) $x - 5y + 7 = 0$

Key : A,B,C

Sol :



AC $\perp$ to $7x-10y+1=0$

$\therefore$ slope of $AC=\frac{-10}{7}$

Equation of AC is $10x+7y-13=0$

Equation AB is $(7x-10y+1)+t(3x-2y+5)=0$

AB passes through $A(2,-1)$

$\Rightarrow t=\frac{-25}{13}$

$\therefore$ equation of AB is $x-5y-7=0$

Equation of BC is $(3x-2y+5)+\lambda(7x-10y+1)=0$

Slope of $BC=\frac{3+7\lambda}{2+10\lambda}=m$

Slope of AB=1/5

Slope of angle bisector of AB,BC is $\frac{3}{2}$

$\therefore \left|\frac{m-\frac{3}{2}}{1+3\frac{m}{2}}\right|=\left|\frac{\frac{3}{2}-\frac{1}{5}}{1+\frac{3}{2}.\frac{1}{5}}\right|\Rightarrow m=-5$ or $\frac{1}{5}$

$\frac{3+7\lambda}{2+10\lambda}=-5$

$3+7\lambda=-10-50\lambda$

57+ = −13

$\lambda=\frac{-13}{57}$

$(3x-2y+5)-\frac{13}{57}(7\lambda-10+1)=0$

$20x+4y+67=0$

Angular bisector of Two lines

(Acute and Obtuse bisector, the bisector containing or do not containg a given point, internal and external bisector)

1. In a triangle ABC, $AB = AC$, the equation of AB and AC are $2x+y=1$, $x+2y=2$. If the third side BC passes through the point $(1,2)$ then its equation is ____

A) $x-y+1=0$ B) $x+y-3=0$ C) $2x+y-4=0$ D) $x-2y+3=0$

Key : AB

Sol : The bisector of angle between the lines AB, AC are $\frac{2x+y-1}{\sqrt{5}} = \pm\left(\frac{x+2y-2}{\sqrt{5}}\right)$

$x-y+1=0, x+y-1=0$

Lines parallel to these lines and containing the point

$(1,2)$ are $x-y+1=0$, $x+y-3=0$

2. Two sides of an isosceles triangle are parallel to the coordinate axes. If m_1 and m_2 are the slopes of the bisectors of the acute angles of the triangles then $\frac{m_1}{m_2} =$ ______

A) $\sqrt{2}-1$ B) 1 C) $3-2\sqrt{2}$ D) $3+2\sqrt{2}$

Key : CD

Sol : The angles are invariant under translation we can take the vertices of the triangle as $O(0,0)\,A(1,0)\,B(0,1)$

AD and BE are the bisectors of $\angle A, \angle B$ $\angle OAD = \frac{\pi}{8}, \angle OEB = 3\pi/8$

$m_1 = -\tan\frac{\pi}{8} = -\left(\sqrt{2}-1\right)$

$m_2 = -\tan\frac{3\pi}{8} = -\left(\sqrt{2}+1\right)$

$\frac{m_1}{m_2} = \frac{\sqrt{2}-1}{\sqrt{2}+1} = 3-2\sqrt{2}$ (or) $3+2\sqrt{2}$

According as $|m_1| < |m_2|$ and $|m_1| > |m_2|$.

3. If L_1 and L_2 are two lines belonging to the family of lines $(3+2\lambda)x+(4+3\lambda)y = 7+5\lambda$ such that there are at maximum and minimum distances from the point $(2,3)$ then the equation of the line through the point $(1,2)$ and making equal angles with L_1 and L_2 is ____

A) $x-3y+5=0$ B) $3x+y-5=0$ C) $x+2y-7=0$ D) $2x-y=0$

Key : A,B

Sol : The family of lines

$(3x+4y-7)+\lambda(2x+3y-5)=0$

The lines are concurrent at the point $(1,1)$

Let d be the distance of $B(2,3)$ from the a line through $A(1,1)$, $d = AB\sin\theta$

$d = 0$, minimum when L is along AB

$L_1 \Rightarrow \frac{y-1}{x-1} = 2 \Rightarrow 2x - y - 1 = 0$

$d = AB$, maximum when L is perpendicular to AB

$\therefore \ L_2 \Rightarrow x + 2y - 3 = 0$

The line which is equally inclined to L_1 and L_2 is along a bisector of angle between L_1 and L_2

$$\frac{2x-y-1}{\sqrt{5}} = \pm\left(\frac{x+2y-3}{\sqrt{5}}\right)$$

$\Rightarrow \ x - 3y + 2 = 0$, $3x + y - 4 = 0$

Lines passing through the point $(1,2)$ parallel to above two lines are $x - 3y + 5 = 0$

$3x + y - 5 = 0$

Exercise : IV

(Paragraph / Matrix Matching Type Questions Including PYQ's)

Passage: I.

The coordinates of the feet of the perpendiculars from the vertices A,B and C of a triangle ABC on the opposite sides are D(20,25),E(8,16) and F(8,9) respectively.

01. The maximum number of such triangles ABC is equal to

A) 1 B) 2 C) 4 D) infinitely many

Key: C

02. The coordinates of the orthocenter of the triangle ABC if the triangle is acute angled, are

A) (5,10) B) (50,-5) C) (15,30) D) (10,15)

Key: D

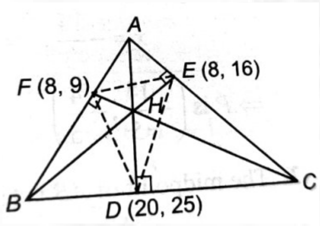
Solution:1&2.

If the triangle ABC be acute angled then the incentre of ΔDEF (the pedal triangle of ΔABC) is the orthocenter of ΔABC.

Now DE=15,EF=7 and FD=20, So the coordinates of incentre of ΔDEF, say H(a,b) are given by

$$a=\frac{15\times 8+7\times 20+20\times 8}{15+7+20}=10$$

$$b=\frac{15\times 9+7\times 25+20\times 16}{15+7+20}=15$$



Thus, the coordinates of incentre of ΔDEF , that is the orthocenter of ΔABC are (10,15)

Further,the triangle ABC as well as triangle HBC satisfy the same conditions. Similarly the triangles HCA and CAB satisfy the same conditions. Thus there are 4 such triangles possible, where ΔABC is acute and other three $\Delta HBC, \Delta HCA$ and ΔHAB are obtuse

Passage : II.

Let A(4,2) and B(2,4) be two given points and L be the straight line $3x+2y+10=0$

03. A point P on L such that $AP+PB$ is minimum is

A) $\left(-\frac{14}{5},-\frac{4}{5}\right)$ B) $\left(\frac{14}{5},-\frac{4}{5}\right)$ C) $\left(-\frac{14}{5},\frac{4}{5}\right)$ D) $\left(\frac{4}{5},\frac{14}{5}\right)$

Key: A

04. A point Q on L such that $|AQ-QB|$ is minimum is

A) (2,2) B) (-2,2) C) (-2,-2) D(2,-2)

Key: C

05. A point R on L such that $|AR-RB|$ is maximum is

A) (-22,28) B) (22,28) C) (-28,22) D) (28,22)

Key: A

Solution:

Solution: 3,4&5

We note that the points lie on the same side of the given line L.

Image of A(4,2) in the line $3x+2y+10=0$

$$A_1(h,k) is \frac{h-4}{3}=\frac{k-2}{2}=\frac{-2(3.4+2.2+10)}{9+4} \longrightarrow (1)$$

$\therefore A_1$ *is* $(-8,-6)$*.The equation of* A_1B *is*

$$y-4=\frac{4+6}{2+8}(x-2) \Rightarrow y=x+2 \longrightarrow (2)$$

Solving $(1)\&(2)$

$$x=-\frac{14}{5}, y=-\frac{4}{5} \Rightarrow P\, is\left(-\frac{14}{5},-\frac{4}{5}\right)$$

The midpoint of AB is (3,3). Slope of AB is $\frac{4-2}{2-4}=-1$.

The equation of the perpendicular bisector of AB is $y-3=1(x-3) \Rightarrow y=x$. It intersects the line (1) at (-2,-2). Thus the point Q (-2,2)

Equation of AB is $x+y=6$.

Solving with eq. (1), we get x=-22,y=28. Thus the point R is (-22,28)

Passage : III.

Family of rays $(\lambda+1)x+(\lambda+2)y+\lambda=0; \lambda \in (-1,0]$ fall on x-axis and gets reflected from it.

06. The equation representing the family of reflected rays is

A) $(1+\mu)x+(\mu-2)y+3=0$ B) $(1-\mu)x+(\mu-2)y+3\mu=0$

C) $(1+\mu)x+(\mu-2)y+3\mu=0$ D) $(1-\mu)x+(5\mu+2)y+3\mu=0$

Key: C

07. The set of allowed values of μ is

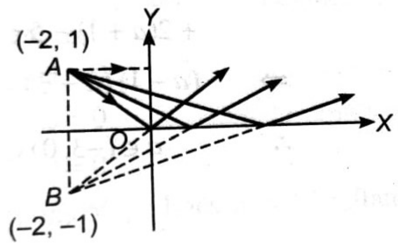
A) $(-1,0]$ B) $R-(0,2)$ C) $\left[\frac{1}{2},\frac{3}{2}\right)$ D) $(-\infty,0)$

Key: A

Solutions: 6 &7

$$(\lambda+1)x+(\lambda+2)y+\lambda=0$$

$$\Rightarrow \lambda(x+y+1)+(x+2y)=0.$$



It represents the family of lines passing through the point A(-2,1). The reflection of A(-2,1) in the X-axis is B(-2,-1). Hence the reflected rays must pass through (-2,-1).
The extreme line of the incident lines are y=1 and $x+2y=0$.
Thus, one of the extreme lines of the reflected rays is $x-2y=0$
Clearly, the otions c passes therough (-2,-1)

The limitng rays of reflected rays are $x-2y=0$ and X-axis. So the slope of the family $(1+\mu)x+(\mu-2)y+3\mu=0$ must satisfy $0<-\left(\frac{1+\mu}{\mu-2}\right)\le\frac{1}{2}$

$\Rightarrow\frac{\mu+1}{\mu-2}<0 \ and \ \frac{3\mu}{2(\mu-2)}\ge 0$

$\Rightarrow -1<\mu<2 \ and \ \mu\le 0 \ or \ \mu>2$

$\therefore \mu\in(-1,0]$

Passage : IV.

Consider the lines $L_1 \ and \ L_2$ **defined by** $L_1 : x\sqrt{2}+y-1=0 \ and \ L_2 : x\sqrt{2}-y+1=0$.
For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from $L_2 \ is \ \lambda^2$. The line $y=2x+1$ meets C at two points R and S. where the distance between R and S is $\sqrt{270}$.
Let the perpendicular bisector of RS meet C at two distinct points $R' \ and \ S'$**. Let D be the square of the distance between** $R' \ and \ S'$

(ADV-2021)

08. The value of λ^2 is____

A) 9 B) 10 C) 8 D) 7

Key: A

09. The value of D is ____

A) 75.14 B) 77.14 C) 79.14 D) 80.14

Key: B

Solution:8 & 9

Locus $C=\left|\frac{(x\sqrt{2}+y-1)(x\sqrt{2}-y+1)}{3}\right|=\lambda^2$

$2x^2-(y-1)^2=\pm 3\lambda^2$ for intersection with $y=2x+1$

$$2x^2-(2x)^2=\pm 3\lambda^2$$

$$2x^2=-3\lambda^2 \ \ (taking\ -ve\ sign)$$

$$x=\pm\sqrt{\frac{3}{2}}\lambda$$

Distance between R and S

$$=2\left|\sqrt{\frac{3}{2}}\lambda\right|\sec\theta \ \ (\tan\theta \text{ is slope of line})$$

$$=\sqrt{6}|\lambda|\sqrt{5},\ So\ \sqrt{30}|\lambda|=\sqrt{270}\,(\lambda=\pm 3)$$

$$\Rightarrow \lambda^2=9$$

Equation of perpendicular bisector $y=-\frac{1}{2}x+1$

For point of intersection $2x^2-\frac{1}{4}x^2=13\lambda^2$

$$x=\pm\sqrt{\frac{12}{7}}\lambda \ (taking\ +ve\,sign)$$

Distance $=\left|2.\sqrt{\frac{2}{7}}.3.\sec\theta\right|=2.\sqrt{\frac{12}{7}}.3.\sqrt{\frac{5}{2}}=3.\sqrt{\frac{60}{7}}$

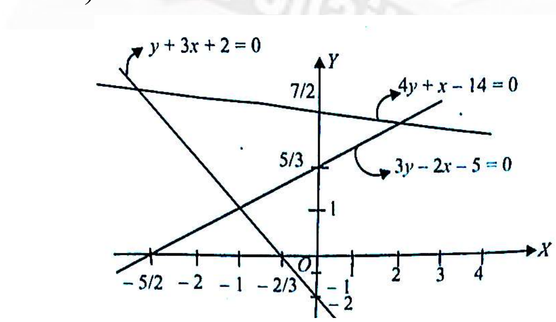
MATRIX MATCH TYPE

1. Consider the triangle formed by the lines $y+3x+2=0, 3y-2x--5=0, 4y+x-14=0$

Column-I	Column-II
a) Value of α if $(0,\alpha)$ lies inside triangle	p) $(-\infty,7/3)\cup(13/4,\infty)$
b) Values of α if $(\alpha,0)$ lies inside triangle	q) $-4/3<\alpha<1/2$
c) Values of α if $(\alpha,2)$ lies inside triangle	r) No value of α
d) Value of α if $(1,\alpha)$ lies outside triangle	s) $5/3<\alpha<7/2$

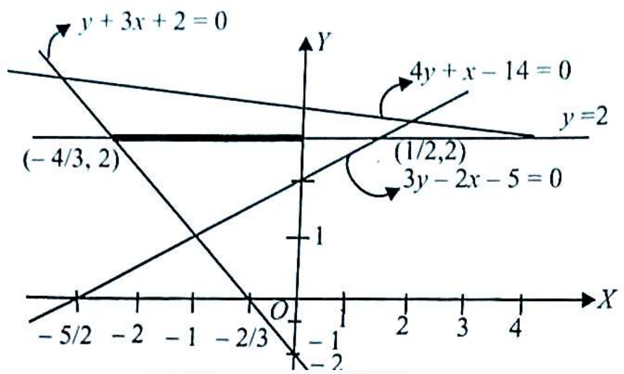
Key : $a\to s; b\to r; c\to q; d\to p$

Sol :a)

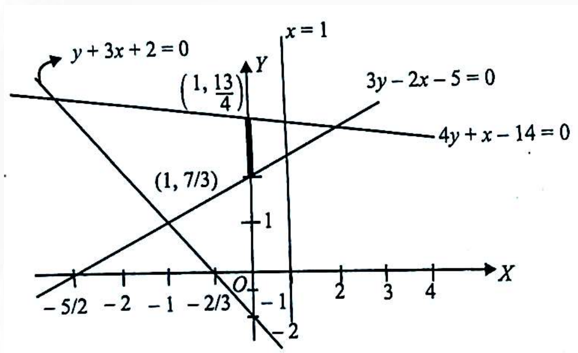


b) Clearly, point $(\alpha,0)$ lies on the x-axis, which is not intersecting any side of triangle, hence no such α exists.

c)



d)

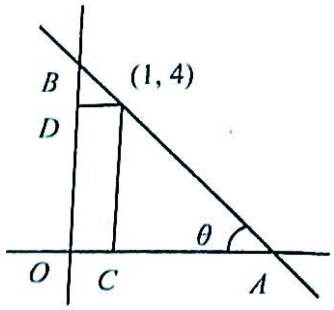


2.

Column-I	Column-II
a) A straight line with negative slope passing through $(1,4)$ meets the coordinate axes at A and B. The minimum length of $OA+OB, O$ being the origin is	p) $5\sqrt{2}$
b) If the point P is symmetric to the point $Q(4,-1)$ with respect to the bisector of the first quadrant, then the length of PQ is	q) $3\sqrt{2}$
c) On the portion of the straight line $x+y=2$ between the axis a square is constructed away from the origin with this portion as one of the its sides. If 'd' de-notes the perpendicular distance of a side of this square from the origin then the maximum value of 'd' is	r) $9/2$
d) If the parametric equation of a line given by $x=4+\lambda/\sqrt{2}$ and $y=-1+\sqrt{2}\lambda$ where λ is a parameter, then the intercept made by the line on the x-axis is	s) 9

Key : $a \to s; b \to p; c \to q; d \to r$

Sol :a)



$OA = 1 + 4\cot\theta$

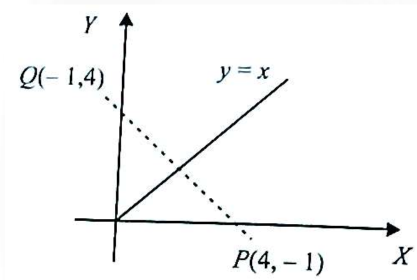
$OB = 4 + \tan\theta$

$OA + OB = 5 + \cot\theta + \tan\theta$

$\geq 5 + 2\sqrt{4\cot\theta\tan\theta}$

$= 5 + (2\times 2) = 9$

b)

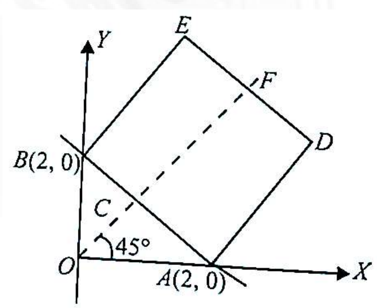


Reflection of $P(4,-1)$ in $y = x$ is $Q(-1,4)$. Hence

$$PQ = \sqrt{(4+1)^2 + (-1-4)^2}$$

$$= \sqrt{50} = 5\sqrt{2}$$

c)



Maximum value of d is

$OF = \sqrt{2} + 2\sqrt{2}$

$= 3\sqrt{2}$

d) The given line is

$x = 4 + \frac{1}{\sqrt{2}}\left(\frac{y+1}{\sqrt{2}}\right) \Rightarrow y = 2x - 9$

Hence, the intercept made, by $x-$axis is 9/2.

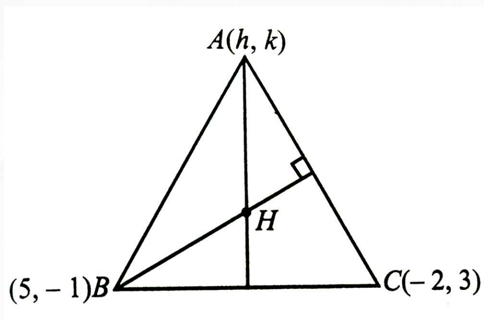
3.

Column-I	Column-II
a) Two vertices of a triangle are $(5,-1)$ and $(-2,3)$. If orthocenter is the origin, then coordinates of the third vertex is	p) $(-4,-7)$
b) A point on the line $x+y=4$ which lines at a unit distance from the line $4x+3y=10$ is	q) $(-7,11)$
c) Orthocentre of the triangle formed by the lines $x+y-1=0, x-y+3=0, 2x+y=7$ is	r) $(2,-2)$
d) If $2a,b,c$ are in A.P., then lines $ax+by=c$ are concurrent at	s) $(-1,2)$

Key : $a \to p; b \to q; c \to s; d \to r$

Sol : a) $AH \perp BC \Rightarrow \left(\frac{k}{h}\right)\left(\frac{3+1}{-2-5}\right) = -1$

$\therefore\ 4k = 7h$ ....(1)



$BH \perp AC \Rightarrow \left(\frac{0+1}{0-5}\right)\left(\frac{k-3}{h+2}\right) = -1$

$\therefore\ k-3 = 5(h+2)$ ...(2)

$\Rightarrow 7h - 12 = 20h + 40$

$\Rightarrow\ 13h = -52$

$\Rightarrow h = -4$

$\therefore\ k = -7$

Hence, point A is $(-4,-7)$

b) $x+y-4=0$ ....(i)

$4x+3y-10=0$ .....(ii)

Let $(h, 4-h)$ be the point on (i). Then,

$$\left|\frac{4h+3(4-h)-10}{5}\right| = 1$$

$\Rightarrow\ h+2 = \pm 5$

$\Rightarrow\ h = 3, h = -7$

Hence, the required point is either $(3,1)$ or $(-7,11)$

c) Since lines $x+y-1=0$ and $x-y+3=0$ are perpendicular. Orthocentre of the triangle is the point of intersection of the lines, i.e., $(-1,2)$.

d) Since $2a,b,c$ are in $A.P.$, so

$b=\dfrac{2a+c}{2}$

$\Rightarrow\ 2a-2b+c=0$

Comparing with line $ax+by+c=0$, we have $x=2$ and $y=-2$. Hence, lines are concurrent at $(2,-2)$

4. Consider the lines given by

$L_1: x+3y-5=0$

$L_2: 3x-ky-1=0$

$L_3: 5x+2y-12=0$

Column-I	Column-II
a) L_1,L_2,L_3 are concurrent if	p) $k=-9$
b) One of L_1,L_2,L_3 is parallel to at least one of the other two if	q) $k=-6/5$
c) L_1,L_2,L_3 form a triangle if	r) $k=5/6$
d) L_1,L_2,L_3 do not form a triangle if	s) $k=5$

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Key : $a\to s; b\to pq; c\to r; d\to pqs$

Sol : Given line are

$L_1: x+3y-5=0$

$L_2: 3x-ky-1=0$

$L_3: 5x+2y-12=0$

L_1 and L_3 are concurrent if

$6-k-1=0$ or $k=5$

For L_1,L_2 to be parallel

$\dfrac{1}{3}=\dfrac{3}{-k}\Rightarrow k=-9$

For L_2,L_3 to be parallel

$\dfrac{3}{5}=\dfrac{-k}{2}\Rightarrow k=\dfrac{-6}{5}$

Thus, for $k=5$, lines are concurrent and for $k=-9,\dfrac{-6}{5}$, at least two lines are parallel.

So, for these value of k, lines will not from triangle.

Obviously, for $k\neq 5,-9,\dfrac{-6}{5}$, lines from triangle.

Exercise : V

(Assertion – Reason / Statement – I & II Type Questions)

In the following questions. Statement –I (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

a) **Statement -1** is true, **Statement -2** is true and **Statement -2** is the correct explanation for **Statement -1**

b) **Statement -1** is true, **Statement -2** is true but **Statement -2** is not the correct explanation of **Statement -1**

c) **Statement -1** is true, **Statement -2** is false

d) **Statement -1** is false, **Statement -2** is true.

1. **Statement 1:** Consider the point $A(0,1)$ and $B(2,0)$ and $'P'$ be a point on the line $4x+3y+9=0$, then coordinates of $'P'$ such that $|PA-PB|$ is maximum is $\left(\frac{-12}{5},\frac{17}{5}\right)$

Statement 2: $|PA-PB| \le |AB|$

Key: D

Sol: Equation of AB is $y-1=\frac{0-1}{2-0}(x-0) \Rightarrow x+2y-2=0$

$|PA-PB| \le |AB|$

Thus $|PA-PB|$ is maximum when A,B and P are collinear.

2. **Statement-1:** A chord y = mx + c of the curve $3x^2 - y^2 - 2x + 4y = 0$, which passes through the point (1, –2), subtend a right angle at the origin.

Statement-2: Lines represented by the equation $(3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$ are perpendicular if c + m + 2 = 0.

Key: A

Sol : Statement-2 is true as the sum of the coefficients of x^2 and $y^2 = 3c + 2m + 4 - c = 0 \Rightarrow$ c + m + 2 = 0 so the lines are perpendicular if c + m + 2 = 0.

In statement-1, let the equation of the chord be y = mx + c, then equation of the pair of lines joining the origin to the points of intersection of the chord and the curve is

$$3x^2 - y^2 - 2x\left(\frac{y-mx}{c}\right) + 4y\left(\frac{y-mx}{c}\right) = 0$$

$\Rightarrow \quad (3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$

which are at right angles if c + m + 2 = 0 (using statement-2) and since the line y = mx + c passes through (1, –2), c + m + 2 =0. So statement-1 is also true.

3. **Statement – 1 :** If algebraic sum of perpendicular distances from (–2, 0), (3, 1) and (4, 2) to the line ax + by + c = 0 is zero then line must pass through $\left(\frac{5}{3},1\right)$ Because

Statement – 2 : If algebraic sum of perpendicular distances from $A_i(x_i, y_i)$ i = 1, 2, 3, ….n to ax + by +c = 0 is zero then line must pass through centroid of polygon having vertices at
(x_i, y_i).

Key.A

Sol : $\sum_{i=1}^{n}\frac{a_0x_i+by_i+c}{\sqrt{a^2+b^2}}=0$

$$a\left(\sum x_i\right)+b\left(\sum y_i\right)+nc=0$$

$$a\frac{\sum_{i=0}^{n}x_i}{n}+b\frac{\sum_{i=0}^{n}y_i}{n}+c=0$$

4. **Statement I :** The point of intersection of the lines joining A(2,3), B(-1,2) and C(-2,1),D(3,4)is an internal point of $\overline{AB}$

Statement II : A(2,3),B(-1,2) are on opposite sides of the line through C(-2,1) and D(3,4)

Key.A

Sol. The line through C,D is $3x-5y+11=0$. $L_A=2>0, L_B=-3<0$.

5. **Statement - 1:** The image of the curve $x^2=4y$ in the line $x+y=2$ is $(y-2)^2+4(x-2)=0$.

Statement - 2: $x^2=4y$ is symmetric with respect to the line $x+y=2$.

Key.C

Sol. $P(2t,t^2)$. Find locus of image of P w.r.t the line $x+y=2$.

6. **Statement-1:** The vertices of a triangle are $A(x_1,x_1\tan\theta_1)$, $B(x_2,x_2\tan\theta_2)$ and $C(x_3,x_3\tan\theta_3)$. If the circumcentre of the triangle ABC coincides with origin and orthocentre $H(x^1,y^1)$ then $\frac{y^{|}}{x^{|}}=\frac{\sin\theta_1+\sin\theta_2+\sin\theta_3}{\cos\theta_1+\cos\theta_2+\cos\theta_3}$

Statement-2: In a triangle circumcentre, centroid and orthocentre are collinear.

Key. A

Sol. since circumcentre is origin and OA = OB = OC = r

OA = $x_1^2+x_1^2\sin^2\theta_1=x_1\sec\theta_1$

$\therefore x_1=r\cos\theta_1$

similarly, $A(r\cos\theta_1,r\sin\theta_1)$, $B(r\cos\theta_2,r\sin\theta_2)$, $C(r\cos\theta_3,r\sin\theta_3)$

cicumcentre (o), centroid (G), and orthocentre (H) are collinear

$\Rightarrow$ slope OH = slope GO

$$\Rightarrow \frac{y^2-0}{x^2-0}=\frac{(y\ coordinate\ of\ G)-0}{(x\ coordinate\ of\ G)-0}$$

$$\Rightarrow \frac{y^1}{x^1}=\frac{\sin\theta_1+\sin\theta_2+\sin\theta_2}{\cos\theta_1+\cos\theta_2+\cos\theta_3}$$

7. Lines L_1, L_2 given by $y-x=0$ and $2x+y=0$ intersect the line L_3 given $y+2=0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

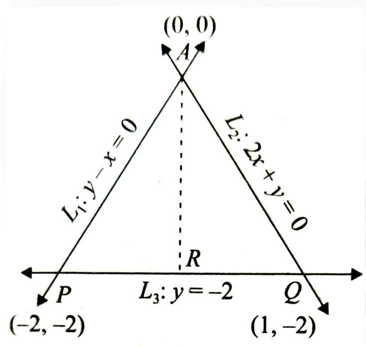
Statement 1: The ratio PR:RQ equals $2\sqrt{2}:\sqrt{5}$

Statement 2: In any triangle, the bisector of an angle divides the triangle into two similar triangles **IIT-JEE 2008**

Key : C

Sol : The point of intersection of L_1 and L_2 is $A(0,0)$.

Also $P\equiv(-2,-2), Q\equiv(1,-2)$



Since AR is the bisector of $\angle PAQ, R$ divides PQ in the same ratio as AP:AQ. Thus $PR:RQ=AP:AQ=2\sqrt{2}:\sqrt{5}$. Hence, statement I is true. Statement 2 is clearly false.