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RPTM-8

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MATHEMATICS

01. Let $\vec{a} = p\hat{i} + \sin\theta\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + p\hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{j} + \hat{k}$. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then all possible values of scalars p and θ respectively are

A) $1, 2n\pi + \frac{\pi}{2} (n \in I)$ B) $2, 2n\pi\ (n \in I)$ C) $3, 3n\pi (n \in I)$ D) $4, 4n\pi (n \in I)$

02. 'I' is the incentre of triangle ABC, whose corresponding sides are a, b, c respectively. $a\overline{IA} + b\overline{IB} + c\overline{IC}$ is always equal to

A) $\vec{0}$ B) $(a+b+c)\overline{BC}$ C) $(\vec{a}+\vec{b}+\vec{c})\overline{AC}$ D) $(a+b+c)\overline{AB}$

03. 4-points whose position Vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar and $(\sin\alpha)\vec{a} + (2\sin 2\beta)\vec{b} + (3\sin 3\gamma)\vec{c} - \vec{d} = \vec{0}$ then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma$ is

A) $\frac{1}{14}$ B) 14 C) 6 D) $\frac{1}{\sqrt{6}}$

04. Let ΔABC be a given triangle. If $|\overline{BA} - t\overline{BC}| \geq |\overline{AC}|$ for any $t \in R$ then ΔABC is

A) Equilateral B) Righangled C) Issosceles D) None of these

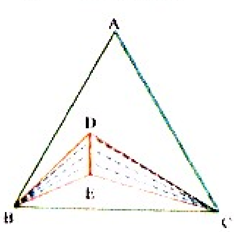
05. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then the unit vector along the angular bisector of $\vec{a}$ and $\vec{b}$ will given by

A) $\frac{\vec{a}-\vec{b}}{2\cos\theta/2}$ B) $\frac{\vec{a}+\vec{b}}{2\cos\theta/2}$ C) $\frac{\vec{a}-\vec{b}}{\cos\theta/2}$ D) $\frac{\vec{a}+\vec{b}}{\cos\theta/2}$

06. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 1 and 2 respectively, and $(1-3\vec{a}.\vec{b})^2 + |2\vec{a} + \vec{b} + 3(\vec{a}\times\vec{b})|^2 = 47$ then the angle between $\vec{a}$ and $\vec{b}$ is

A) $\frac{\pi}{3}$ B) $\pi - \cos^{-1}\left(\frac{1}{4}\right)$ C) $\frac{2\pi}{3}$ D) $\cos^{-1}\left(\frac{1}{4}\right)$

07. In figure, $\overline{AB} = 3\bar{i} - \bar{j}$, $\overline{AC} = 2\bar{i} + 3\bar{j}$ and $\overline{DE} = 4\bar{i} - 2\bar{j}$ then the area of the shaded region in square units is



A) 5 B) 6 C) 7 D) 8

08.	If $\vec{a}' = \hat{i}+\hat{j}$, $\vec{b}' = \hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}' = 2\hat{i}+\hat{j}-\hat{k}$, then the altitude of the parallelepiped formed by the vectors $\vec{a}, \vec{b}$, and $\vec{c}$ having base formed by $\vec{b}$ and $\vec{c}$ is (Where $\vec{a}'$ is reciprocal vector $\vec{a}$, etc.) A) 1 B) $3\sqrt{2}/2$ C) $1/\sqrt{6}$ D) $1/\sqrt{2}$
09.	If the two diagonals of one of its faces are $6\hat{i}+6\hat{k}$ and $4\hat{j}+2\hat{k}$ and of the edges not containing the given diagonals is $\vec{c} = 4\hat{j}-8\hat{k}$, then the valume of a parallelepiped is A) 60 B) 80 C) 100 D) 120
10.	Let $\bar{a}, \bar{b}, \bar{c}$ are three vectors along the adjacent edges of a tetrahedron, if $\lvert\bar{a}\rvert = \lvert\bar{b}\rvert = \lvert\bar{c}\rvert = 2$ and $\bar{a}.\bar{b} = \bar{b}.\bar{c} = \bar{c}.\bar{a} = 1$, then volume of tetrahedron is A) $\frac{1}{\sqrt{2}}$ B) $\frac{2}{\sqrt{3}}$ C) $\sqrt{\frac{3}{2}}$ D) $\frac{2\sqrt{2}}{3}$
11.	$\bar{a}, \bar{b}, \bar{c}$ are three unit vectors equally inclined to each other at an angle $\frac{\pi}{3}$ then the value of $\left[\bar{a}+\bar{b} \;\; \bar{b}+\bar{c} \;\; \bar{c}+\bar{a}\right]^2 + \left[\bar{a}\times\bar{b} \;\; \bar{b}\times\bar{c} \;\; \bar{c}\times\bar{a}\right]$ is A) $\frac{3}{4}$ B) $\frac{5}{2}$ C) $\frac{7}{2}$ D) 4
12.	If $\vec{a}.\vec{b} = \beta$ and $\vec{a}\times\vec{b} = \vec{c}$ then $\vec{b}$ is A) $\frac{(\beta\vec{a}-\vec{a}\times\vec{c})}{\lvert\vec{a}\rvert^2}$ B) $\frac{(\beta\vec{a}+\vec{a}\times\vec{c})}{\lvert\vec{a}\rvert^2}$ C) $\frac{(\beta\vec{c}-\vec{a}\times\vec{c})}{\lvert\vec{a}\rvert^2}$ D) $\frac{(\beta\vec{a}+\vec{a}\times\vec{c})}{\lvert\vec{a}\rvert^2}$
13.	If unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $\lvert\vec{a}-\vec{b}\rvert < 1$ and $0 \le \theta \le \pi$, then θ lies in the interval A) $\left(0, \frac{\pi}{6}\right)$ B) $\left(\frac{5\pi}{6}, \pi\right)$ C) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ D) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$
14.	If the vectors $\vec{a}$ and $\vec{b}$ are linearly independent satisfying $\left(\sqrt{3}\tan\theta+1\right)\vec{a}+\left(\sqrt{3}\sec\theta-2\right)\vec{b} = 0$, then the most general values of θ are A) $n\pi - \frac{\pi}{6}, n \in Z$ B) $2n\pi + \frac{11\pi}{6}, n \in Z$ C) $2n\pi \pm \frac{\pi}{6}, n \in Z$ D) $2n\pi \pm \frac{7\pi}{6}, n \in Z$
15.	The unit vector $\vec{c}$ if $-\hat{i}+\hat{j}-\hat{k}$ bisects the angle between vectors $\vec{c}$ and $3\hat{i}+4\hat{j}$ is

	A) $-\frac{1}{15}\left(11\hat{i}+10\hat{j}-2\hat{k}\right)$ C) $\frac{1}{15}\left(11\hat{i}-10\hat{j}-2\hat{k}\right)$ (B) $\frac{1}{15}\left(-11\hat{i}-10\hat{j}-2\hat{k}\right)$ D) $\frac{1}{15}\left(11\hat{i}-10\hat{j}+2\hat{k}\right)$
16.	**If $\bar{a},\bar{b},\bar{c}$ are unit vectors such that $\lvert\bar{a}+2\bar{b}+3\bar{c}\rvert=\sqrt{3+2\sqrt{2}}$, angle between $\bar{a}$ and $\bar{b}$ is α, angle between $\bar{a}$ and $\bar{c}$ is β and angle between $\bar{b}$ and $\bar{c}$ varies in $\left[\frac{\pi}{2},\frac{2\pi}{3}\right]$ then the greatest value of** $4\cos\alpha+6\cos\beta$ A) $2\sqrt{2}+5$ B) $-2\sqrt{2}+5$ C) $2\sqrt{2}-5$ D) $\sqrt{42}$
17.	**If the vector $\vec{b}=\left(\tan\alpha,-1,2\sqrt{\sin\frac{\alpha}{2}}\right)$ and $\vec{c}=\left(\tan\alpha,\tan\alpha,-\frac{3}{\sqrt{\sin\frac{\alpha}{2}}}\right)$ are orthogonal and a vector $\vec{a}=(1,3,\sin 2\alpha)$ makes an obtuse angle with the z-axis then the value of α is** A) $\alpha=(4n+1)\pi-\tan^{-1}2$ B) $\alpha=(4n+2)\pi+\tan^{-1}3$ C) $\alpha=(4n+1)\pi+\tan^{-1}2$ D) $\alpha=(4n+2)\pi+\tan^{-1}2$
18.	**If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert=1$ and $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=\cos\theta$ then maximum value of θ is** A) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ C) $\frac{2\pi}{3}$ D) $\frac{\pi}{6}$
19.	**If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 2 and 3 respectively such that $\left\lvert 2\left(\vec{a}\times\vec{b}\right)\right\rvert+\left\lvert 3\left(\vec{a}.\vec{b}\right)\right\rvert=k$ then the maximum value of k is** A) $\sqrt{13}$ B) $2\sqrt{13}$ C) $6\sqrt{13}$ D) $10\sqrt{13}$
20.	**Let O be an interior point of ΔABC such that $\overline{OA}+2\overline{OB}+3\overrightarrow{OC}=\vec{0}$, Then the ratio of the area of ΔABC to the area of ΔAOC is** A) 1 B) 2 C) 3 D) 4
21.	**ABCD is a parallelogram A_1 and B_1 are midpoints of sides BC and CD respectively. If $\overline{AA_1}+\overline{AB_1}=\lambda\overline{AC}$ then 2λ**

22.	Let $\vec{A}$ and $\vec{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\vec{A}+\vec{B}\right)$ bisects the internal angle between $\vec{A}$ and $\vec{B}$, then the value of α is
23.	$\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert=\lvert\vec{a}+\vec{b}\rvert=1$, $\vec{a}.\vec{c}=0$, If $\bar{a}=\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}$ and $\vec{a},\hat{k},\vec{b}$ are linearly dependent if maximum, and mimimum value of $\vec{b}.\vec{c}$ is M and m respectively, then $4\left(M^2+m^2\right)$ is equal to
24.	If $\vec{a}=x\hat{i}+(x-1)\hat{j}+\hat{k}$ and $\vec{b}=(x+1)\hat{i}+\hat{j}+a\hat{k}$ always make an acute angle for all $x\in R$, then the least integral value of a is
25.	If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=1$ then the value of $\left[\vec{a}+\vec{b}\;\vec{b}+\vec{c}\;\vec{c}+\vec{a}\right]+\left[\vec{a}\times\vec{b}\;\vec{b}\times\vec{c}\;\vec{c}\times\vec{a}\right]+\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right),\vec{b}\times\left(\vec{c}\times\vec{a}\right),\vec{c}\times\left(\vec{a}\times\vec{b}\right)\right]$ is
26.	Let $\lvert\vec{p}\rvert=\frac{2}{3}\sqrt{2},\lvert\vec{q}\rvert=1$ and the angle between $\vec{p}$ and $\vec{q}$ be $\frac{\pi}{4}$. If a parallelogram is formed with adjacent sides $\vec{a}=\vec{p}-3\vec{q}$ and $\vec{b}=5\vec{p}+2\vec{q}$, then the length of the shorter diagonal is
27.	If $\bar{a},\bar{b},\bar{c}$ are non - coplanar, $\left[\bar{b}\quad\bar{c}\quad\bar{d}\right]=24$ and $\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)+\left(\bar{a}\times\bar{c}\right)\times\left(\bar{d}\times\bar{b}\right)+\left(\bar{a}\times\bar{d}\right)\times\left(\bar{b}\times\bar{c}\right)+k\bar{a}=\bar{0}$ then $\frac{k}{8}=$___.
28.	$\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$, $\vec{c}=\hat{i}+x_1\hat{j}+x_2\hat{k}$ are linearly dependent, $\lvert\vec{c}\rvert=2$ then (i) $x_1^2=$ (ii) $x_2=$
29.	Let $\vec{c}$ be a unit vector coplanar with $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ such that $\vec{c}$ is perpendicular to $\vec{a}$. If P be the projection of $\vec{c}$ along $\vec{b}$ then the value of $\frac{\sqrt{11}}{p}$ is.
30.	The resultant of vectors $\vec{a}$ and $\vec{b}$ is $\vec{c}$. If $\vec{c}$ trisects the angle between $\vec{a}$ and $\vec{b}$, $\lvert\vec{a}\rvert=6,\lvert\vec{b}\rvert=4, then\lvert\vec{c}\rvert$ is

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1) A	2) A	3) A	4) B	5) B	6) C
7) C	8) D	9) D	10) C	11) B	12) A
13) A,B	14) B	15) B	16) C	17) A	18)
19) C	20) C	21) 3	22) 1	23) 6	24) 3
25) 3	26) 5	27) 6	28) 1,2	29) 6	30) 5