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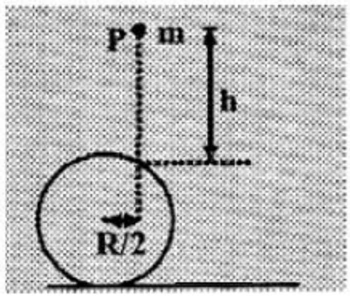
Subject: MPC **Sr.Super60 Papers Speed Test Mains** **Date: 07-09-23**

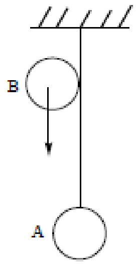
PHYSICS

01. A tiny ball of mass m is released from the state of rest over a large smooth sphere of mass M and radius R, which is at rest on a smooth horizontal surface. If the ball strikes the sphere perfectly inelastically, then the velocity of the sphere after the collision is



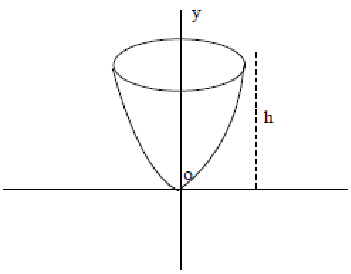
1) $\frac{m\sqrt{6gh}}{2(2M+m)}$ 2) $\frac{m\sqrt{6gh}}{4M+m}$ 3) $\frac{m\sqrt{6gh}}{(2M+m)}$ 4) $\frac{m\sqrt{3gh}}{2(4M+m)}$

02. A small steel ball A is suspended by an inextensible thread of length $l = 1.5m$ from the ceiling. Another identical ball B is thrown vertically downwards as shown in figure, such that its surface remains just in contact with thread during downward motion and collides perfectly elastically with the suspended ball. The suspended ball just completes vertical circle after collision. The velocity of the falling ball just before collision is $\frac{10^2}{n} m/s$. Find the value of n (Take $g = 9.8\ m/s^2$, $\sqrt{2} = 1.4$)



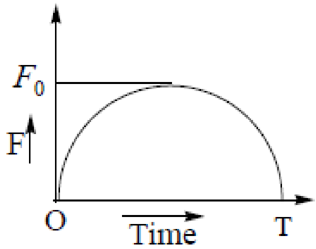
1) 2 2) 4 3) 6 4) 8

03. A paraboloid shaped solid object is formed by rotating a parabola $y = 2x^2$ about y-axis as shown in figure. If the height of the body is 'h', then the distance of centre of mass from origin. (Assume density to be uniform throughout).



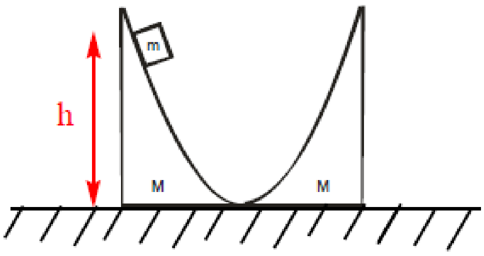
1) $\frac{h}{3}$ 2) $\frac{h}{4}$ 3) $\frac{2h}{3}$ 4) $\frac{2h}{4}$

04. A particle of mass m initially at rest, is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. The graph looks like a semicircle. Then



1) $u = \frac{\pi F_0^2}{2m}$ 2) $u = \frac{\pi T^2}{8m}$ 3) $u = \frac{\pi F_0 T}{4m}$ 4) $u = \frac{\pi F_0 T}{2m}$

05. The inclined surfaces of two movable wedges of same mass M are smoothly conjugated with the horizontal plane as shown in figure. A washer of mass m slides down the left wedge from a height h. If M=m, then h is "n" times the maximum height of washer rise along right wedge. Find "n" (Neglect friction everywhere)

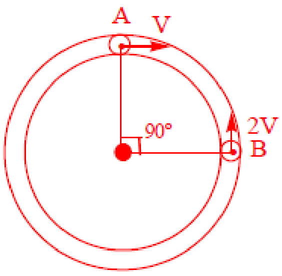


1) 4 2) 2 3) 1 4) 3

06. A bullet is fired from a gun the force on the bullet is given by $F = 800 - 2\times10^6 t^2$ where F is in newton and t in second. The force on the bullet becomes zero as soon as it leaves the barrel. The average impulse imparted to the bullet is $\frac{4x}{3}$ Ns. Find x.

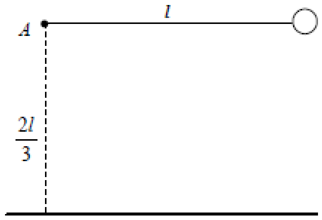
1) 2 2) 8 3) 6 4) 9

07. A small body A of mass m and B of mass 3m and same size as A move towards each other with speeds V and 2V respectively from the positions as shown in figure at t=0, along a smooth horizontal circular track of radius r. After the first elastic collision, they will collide at time t = n t_0 (Neglect the size of the balls when compared to circular track radius r) where t_0 is the time taken before first collision from the instant shown. Find n.



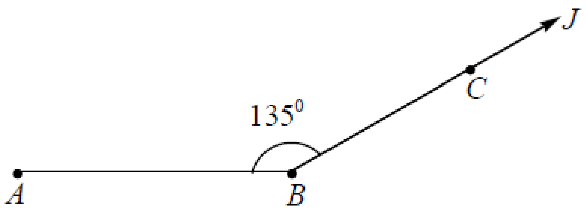
1) 1 2) 2 3) 3 4) 4

08. A is a fixed point at a height $\frac{2l}{3}$ above a perfectly inelastic smooth horizontal plane. A light inextensible string of length l has one end attached to A and other to a heavy ball. The ball is held at the level of A with string just taut and released from rest. The speed of ball just after striking the plane is n times $\sqrt{\frac{gl}{27}}$. Find n. (Neglect the size of the heavy ball when compared to length l)



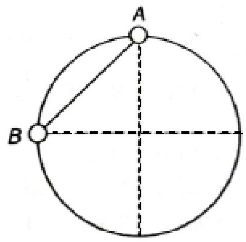
1) 4 2) 1 3) 2 4) 3

09. Three identical particles A, B and C (each of mass m) lie on a smooth horizontal table. Light inextensible strings which are just taut connect AB and BC and $\angle ABC$ is 135^0 as shown in diagram. An impulse J is applied to the particle C in the direction BC for a very short time interval. Then just after applying impulse J, the ratio of speed of B and A is $\sqrt{n}:1$. Find n



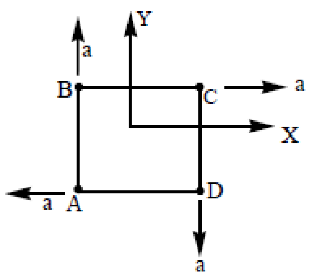
1) 3 2) 5 3) 7 4) 9

10. Two beads A and B of equal mass m are connected by a light inextensible chord. They are constrained to move on a frictionless ring in vertical plane. The beads are released from rest as shown in figure. The tension in the chord just after the release is



1) $\frac{mg}{4}$ 2) $\sqrt{2}\,mg$ 3) $\frac{mg}{2}$ 4) $\frac{mg}{\sqrt{2}}$

11. Four particles A, B, C and D with masses $m_A = m,\ m_B = 2m,\ m_C = 3m\ and\ m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles $(in\ ms^{-2})$ is



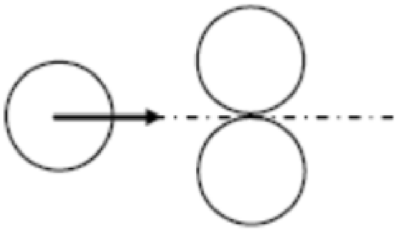
1) $\frac{a}{5}(\hat{i}-\hat{j})$ 2) $a(\hat{i}+\hat{j})$ 3) $zero$ 4) $\frac{a}{5}(\hat{i}+\hat{j})$

12. Consider the following statements
Statement A: A shell moving at a speed of 500 m/s breaks into three identical fragments. Maximum speed that any of the fragments can acquire with respect to centre of mass frame is 500 m/s, if kinetic energy of the system increases by 50%
Statement B: In centre of mass frame linear momentum of system is zero and kinetic energy is minimum
1) Both A,B are correct 2) Both A,B are wrong
3) only A is correct 4) Only B is correct

13. A block of mass 1.9 kg is at rest at the edge of a table of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision, then the kinetic energy just before the combined system strikes the floor, is (Take, $g = 10m/s^2$ and assume there is no rotational motion and loss of energy after the collision is negligible)
1) 20 J 2) 19 J 3) 21 J 4) 23J

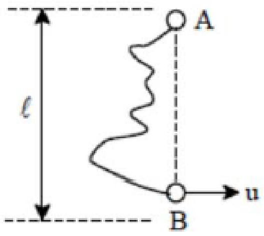
14. An object of mass 40 kg and having velocity 4 m/s collides with another object of mass 60 kg, having velocity 2 m/s in the same direction. The loss of energy when the collision is perfectly inelastic is
1) 392 J **2)** 440 J **3)** 48 J **4)** 110 J

15. Two smooth identical stationary spheres are kept touching each other on a smooth horizontal floor as shown. A third identical sphere moving horizontally with a constant speed hits both stationary spheres symmetrically. If after collision the third sphere moves in same direction with one fourth of its initial speed, the coefficient of restitution will be



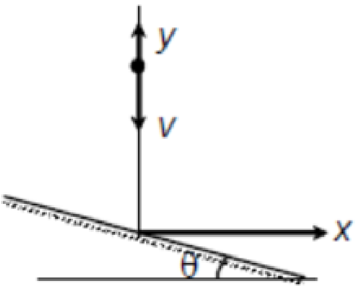
1) 2/3 **2)** 1/3 **3)** 1/4 **4)** 1/6

16. Two balls A & B both of mass m & connected by a light inextensible string of length $2l$. Whole system is on a frictionless horizontal table. Ball B is given a velocity u (as shown) $\perp$r to AB. The velocity of ball A just after the string becomes taut is



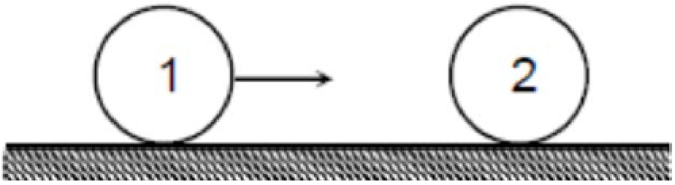
1) $\frac{u\sqrt{3}}{4}$ 2) $u\sqrt{3}$ 3) $\frac{u\sqrt{3}}{2}$ 4) $\frac{u}{2}$

17. An object is moving towards a mirror with a velocity v as shown in figure. If the collision between the mirror and the object is perfectly elastic, then the velocity of the image after collision with mirror in vector form is



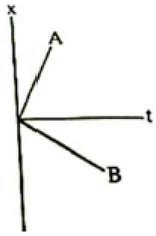
1) $-v\hat{j}$
2) $-v\cos 2\theta\hat{j}+v\cos 2\theta\hat{i}$
3) $-v\hat{i}$
4) $-v\cos 2\theta\hat{j}-v\sin 2\theta\hat{i}$

18. Ball 1 collides with an another identical ball 2 at rest as shown in figure. For what value of coefficient of restitution e, the velocity of second ball becomes two times that of 1 after collision?



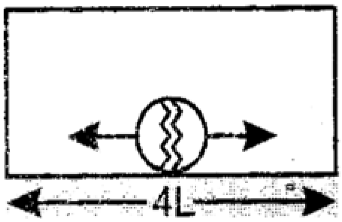
1) 1/3
2) 1/2
3) 1/4
4) 1/6

19. An initially stationary box on a frictionless floor explodes into two pieces; piece A with mass m_A and piece B with mass m_B. These pieces then move across the floor along x axis. Graph of position versus time for the two pieces is given in figure:



1) the graph is not possible
2) $m_A > m_B$
3) $m_A < m_B$
4) $m_A = m_B$

20. A bomb of mass 3m is kept inside a closed box of mass 3m and length 4 L. at its centre. It explodes in two parts of mass m and 2m. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.

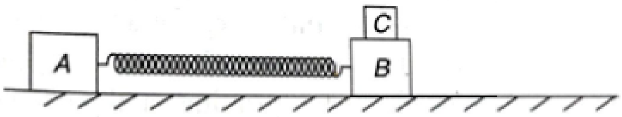


1) 0
2) L/6
3) L/12
4) None of these

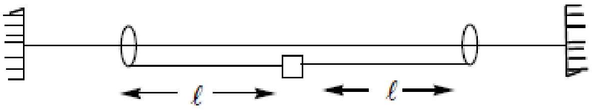
INTEGERS

21. In the arrangement shown in the fig string, springs and the pulley are mass less. Both the springs have a force constant of k and the mass of block B resting on the table is M=1.25 kg. Ball A is released from rest when both the springs are in natural length and just taut. Find the minimum value of mass of A (in kg) so that block B leaves contact with the table at some stage.

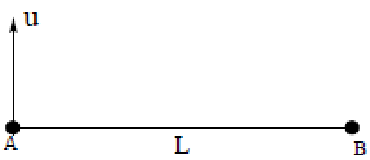
22. Two block A and B are connected to a spring (force constant k = 480 N/m) and placed on a horizontal surface. Another block C is placed on B. The coefficient of friction between the floor and block A is $\mu_1 = 0.5$, whereas there is no friction between B and the floor. Coefficient of friction between C and B is $u_2 = 0.85$. Masses of the blocks are $M_A = 50kg;\ M_B = 28kg$ and $M_C = 2kg$. The system is held at rest with spring compressed by $x_0 = 0.5\ m$, when the system is released, the maximum speed of block B during subsequent motion is (in m/s) $\left(g = 10m/s^2\right)$



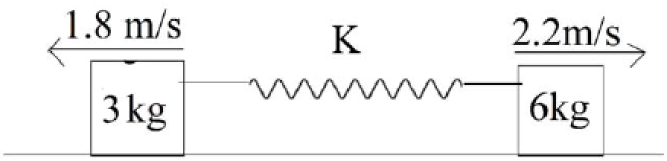
23. Two small rings each of mass 'm' are connected to a block of same mass 'm' through inextensible light strings. Rings are constrained to move along a smooth horizontal rod. Initially system is held at rest (as shown in figure) with the strings just taut. Length of each string is 'ℓ'. The system is released from the position shown speed of the block becomes $v = n\sqrt{\frac{g\ell}{5}}$ (acceleration due to gravity = g) when the strings make an angle of $\theta = 60^\circ$ with vertical. Find n



24. Two particles (A and B) of masses m and 2m are joined by a light rigid rod of length L. The system lies on a smooth horizontal table. The particle (A) of mass m is given a sharp impulse so that it acquires a velocity u =10 m/s perpendicular to the rod in its plane . Maximum speed of particle B during subsequent motion is



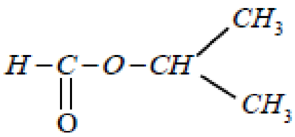
25. Two blocks A and B are connected by a spring of stiffness 512 N/m and placed on a smooth horizontal surface. Initially the spring has its equilibrium length. The indicated velocities are imparted to A and B. The maximum extension of the spring is in cm

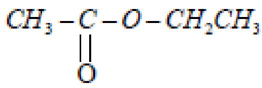


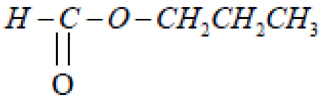
26.	Lucky (55 kg) and Micky (65 kg) are sitting at the two ends of a boat at rest in still water. The boat weighs 100 kg and is 3.0 m long. Lucky walks down to Micky and Shakes hand. The boat gets displaced by ____cm
27.	A batsman hits back a ball of mass 0.4 kg straight in the direction of the bowler without changing its initial speed of 15 ms^{-1}. The impulse imparted to the ball is ______________Ns.
28.	A pendulum of length 2 m consists of a wooden bob of mass 50 g. A bullet of mass 75 g is fired towards the stationary bob with a speed v. The bullet emerges out of the bob with a speed $\frac{v}{3}$ and the bob just completes the vertical circle. The value of v is ________ ms^{-1}. (if $g = 10\ m/s^2$)
29.	Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x}$ m. The value of x is
30.	The distance of centre of mass from end A of a one dimensional rod (AB) having mass density $\rho = \rho_0\left(1-\frac{x^2}{L^2}\right)$ kg/m and length L (in meter) is $\frac{3L}{\alpha}$ m. The value of α is ………. (where x is the distance form end A)

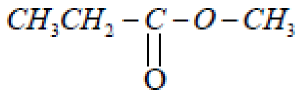
CHEMISTRY

31. An ester $A(C_4H_8O_2)$ on treatment with excess methyl magnesium chloride followed by acidification, gives an alcohol B as the sole organic product. Alcohol B, on oxidation with $NaOCl$ followed by acidification, gives acetic acid, Ester A is:

1) 

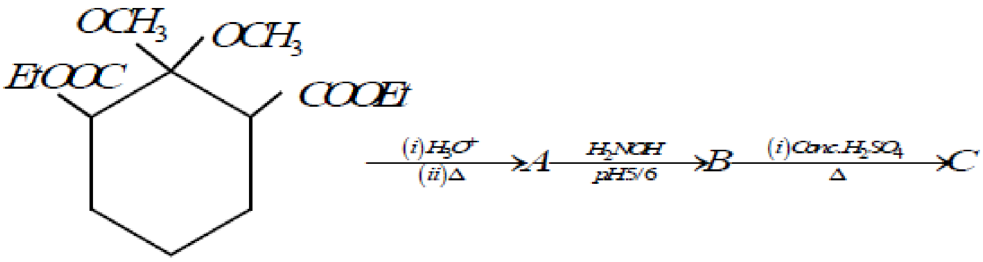
2) 

3) 

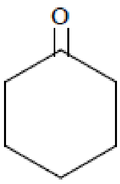
4) 

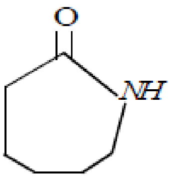
32. Which of the following will not undergo HVZ reaction?

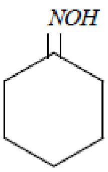
1) 2,2-dimethyl propanoic acid
2) propanoic acid
3) acetic acid
4) 2-methyl propanoic acid

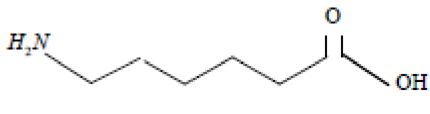
33. 

Compound C in above reaction sequence is

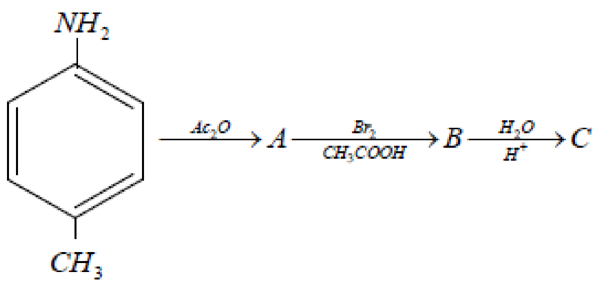
1) 

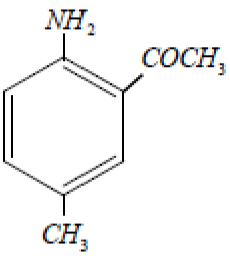
2) 

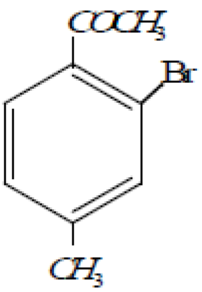
3) 

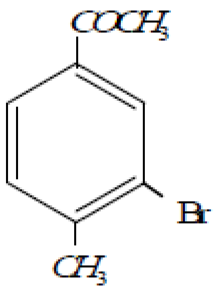
4) 

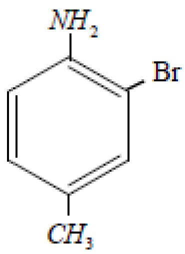
34. The final product C, obtained in this reaction would be

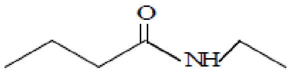
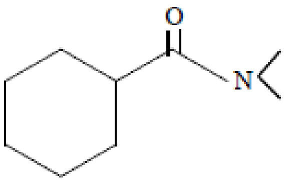
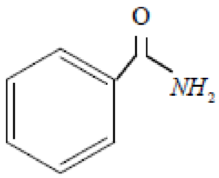
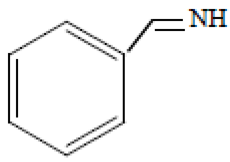


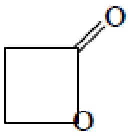
1) 

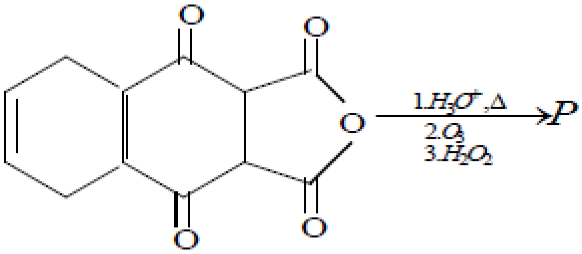
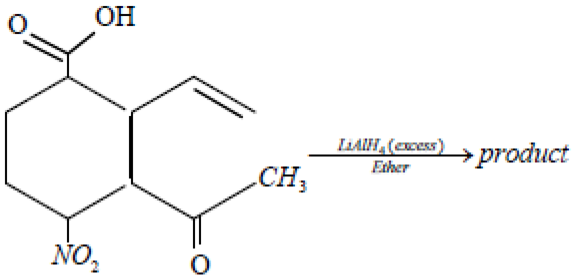
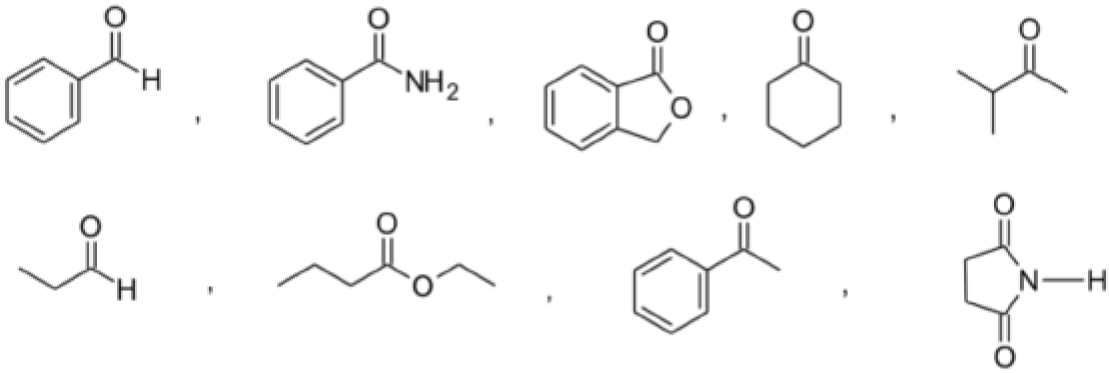
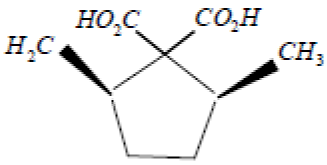
2) 

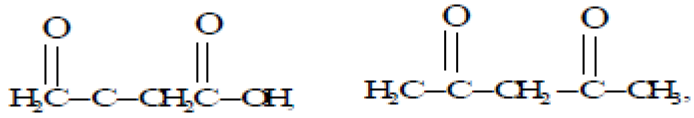
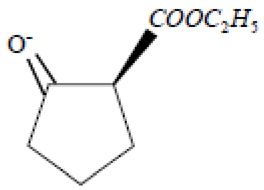
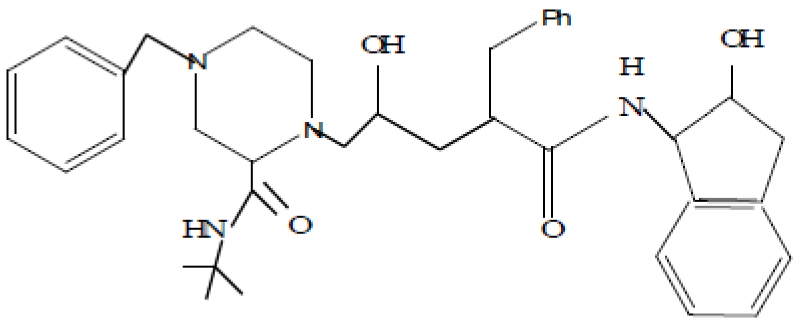
3) 

4) 

35.	The compound which can give amine with KOH and Br_2, is …. 1)  2)  3)  4) 
36.	When sucrose is heated with conc. HNO_3 in presence of V_2O_5, the product formed is: 1) Sucrose nitrate 2) formic acid 3) citric acid 4) oxalic acid
37.	Which is the most volatile? 1) $CH_3CH_2CH_2NH_2$ 2) $(CH_3)_3N$ 3) $\left.\begin{matrix} CH_3CH_2 \\ CH_3 \end{matrix}\right\rangle HN$ 4) CH_3-CH_2-OH
38.	Cyanogen on hydrolysis with $dil.HCl$ gives: 1) formic acid 2) acetic acid 3) glycol 4) oxalic acid
39.	In $C_6H_5COOCH_3 \xrightarrow{LiAlH_4} X$ will be 1) $C_6H_5COOH+CH_3OH$ 2) $C_6H_5CH_2OH+CH_3OH$ 3) $C_6H_5CHO+CH_3COOH$ 4) All of the above
40.	Organic compound $(A)\xrightarrow[rearrangement]{PCl_5} B \xrightarrow[\Delta]{H_2O/H^+} CH_3CH_2NH_2+CH_3COOH$ The compound A is: 1) Syn-ethylmethyl ketoxime 2) Anti-Ethyl methyl ketoxime 3) N-Methyl propionamide 4) N-Ethyl acetamide
41.	Maleic acid on catalytic reduction gives: 1) oxalic acid 2) malonic acid 3) succinic acid 4) tartaric acid
42.	$2CH_3CHO \xrightarrow{Al(OC_2H_5)_3} CH_3COOCH_2CH_3$ This reaction is called: 1) Cannizzaro's reation 2) Aldol condensation 3) Claisen's reaction 4) Tischenko reaction

43.	When ethyl acetate reacts with excess of CH_3MgI, it forms: 1) CH_3COCH_3 2) $CH_3CH_2COCH_3$ 3) $CH_3CHOHCH_2CH_3$ 4) $(CH_3)_3C-OH$
44.	Cyclic ester (A) reacts with methyl alchol to give (B)  $\xrightarrow{CH_3OH}$ (B) What is the compound (B)? 1) $\begin{array}{ccc}CH_2 & - & CH_2\\ \vert & & \vert\\ CH_3O & & COOH\end{array}$ 2) $\begin{array}{ccc}CH_2 & - & CH_2\\ \vert & & \vert\\ HO & & COOCH_3\end{array}$ 3) $\begin{array}{ccc}CH_2 & - & CH_2\\ \vert & & \vert\\ CH_3O & & COOCH_3\end{array}$ 4) $\begin{array}{ccc}CH_2 & - & CH_2\\ \vert & & \vert\\ OH & & CH_2OH\end{array}$
45.	Acetophenone can be converted into amine in a single step by 1) Br_2 / KOH 2) H_2O / OH^- 3) $NH_3 / H_2, Ni / \Delta$ 4) NH_2OH
46.	Succinimide when subjected to Hofmann's bromamide reaction gives a compound having one of the following structures. Select the correct structure: 1) $\begin{array}{l}CH_2COOH\\ \vert\\ CH_2COOH\end{array}$ 2) $\begin{array}{l}CH_2COOH\\ \vert\\ CH_2NH_2\end{array}$ 3) $\begin{array}{l}CH_2NH_2\\ \vert\\ CH_2NH_2\end{array}$ 4) $\begin{array}{l}CH_2CONH_2\\ \vert\\ CH_2CONH_2\end{array}$
47.	Rate of the reaction, $R-\overset{O}{\overset{\Vert}{C}}-Z + Nu^{(-)} \longrightarrow R-\overset{O}{\overset{\Vert}{C}}-Nu + Z^{(-)}$ is fastest when Z is: 1) $OCOCH_3$ 2) NH_2 3) OC_2H_5 4) Cl
48.	Formic acid and formaldehyde can be distinguished by treating with 1)Benedict's solution 2)Tollen's reagent 3)Fehling's solution 4) $NaHCO_3$
49.	(Ester X) $C_4H_8O_2 \xrightarrow[(ii)HOH/HCl]{(i)CH_3MgBr} C_4H_{10}O(Alcohol\,Y)$ Alcohol (Y) gives Lucas test within 5 minutes.Thus(X), and(Y) respectively are: 1) $CH_3COC_2H_5, (CH_3)_3COH$ 2) $HCOOC_3H_7, (CH_3)_2CHOH$ 3) $C_2H_5COOCH_3, (C_2H_5)_3COH$ 4) $HCOOC_3H_7, (CH_3)_3COH$
50.	Secondary amine with Hinsberg reagent forms 1) N-alkyl sulphonamide soluble in KOH solution.

	2) N-alkyl sulphonamide insoluble in KOH solution 3) N-N-dialkyl sulphonamide soluble in KOH solution 4) N,N-dialkyl sulphonamide insoluble in KOH solution
	INTEGERS
51.	The total number of carboxylic acid groups in the product P is 
52.	The number of π bonds in the major product will be____ 
53.	The no. of moles of base (KOH) consumed in the conversion of $RCONH_2$ to RNH_2 using Br_2 is___?
54.	Examine the structural formulas gives below and identify number of compounds which are reduced by $NaBH_4$ 
55.	 When compound is heated, the number of stereoisomers of the product obtained is………

56.	How many of following compound will evolve CO_2 on heating ? $HCCOH, (COOH)_2$, $CH_2(COOH)_2$,   CH_3COOH
57.	 When the above molecules is reduced with excess $LiAlH_4$ followed by acid hydrolysis, how many organic products can be isolated by fractional distillation of product mixture.
58.	A reaction of 0.1 mole of benzylamine with bromomethane gave 23 g of benzyl trimethyl ammonium bromide. The number of moles of bromomethane consumed in this reaction are $n\times10^{-1}$,when n-______(Round off to the nearest integer) (Given: Atomic masses:C:12.0 u, H:1.0 u,N:14.0 u,Br:80.0 u)
59.	The maximum number of moles of CH_3I consumed by one mole of crixivan, a drug used against AIDS is 
60.	How many –OH bonds are present in citric acid?

MATHEMATICS

61.	Area bounded by the curve $f(x)=\frac{x^2-1}{x^2+1}$ and the line y = 1 is: 1) π 2) 2π 3) 3π 4) 4π
62.	The solution of the differential equation $e^{-x}(y+1)dy+(\cos^2 x-\sin 2x)y\,dx=0$ subjected to condition that y=1 when x=0, is: 1) $(y+1)+e^x\cos^2 x=2$ 2) $y+In\ y=e^x\cos^2 x$ 3) $In(y+1)+e^x\cos^2 x=1$ 4) $y+In\ y+e^x\cos^2 x=2$

63.	Let $f(x)=x^2, g(x)=\cos x$ and $\alpha,\beta(\alpha<\beta)$ be the roots of the equation $18x^2-9\pi x+\pi^2=0$. Then the area bounded by the curves $y=fog(x)$, the ordinates $x=\alpha, x=\beta$ and the x-axis in square units is 1) $\frac{1}{2}(\pi-3)$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{12}$
64.	If $f(x)=\max\left\{\sin x,\cos x,\frac{1}{2}\right\}$ then the area of the region bounded by the curves y= f(x), x axis, y axis and the line $x=\frac{5\pi}{3}$ is 1) $\frac{5\pi}{12}+\sqrt{3}$ *square units* 2) $\frac{5\pi}{12}+\frac{\sqrt{3}}{2}$ *square units* 3) $\frac{5\pi}{12}+\sqrt{3}+\sqrt{2}$ *square units* 4) $\frac{5\pi}{12}+\frac{\sqrt{3}}{2}+\sqrt{2}$ *square units*
65.	The area bounded by the curves $y=\sin^{-1}\lvert\sin x\rvert$ and $y=(\sin^{-1}\lvert\sin x\rvert)^2$, $0\le x\le 2\pi$, is 1) $\frac{1}{3}+\frac{\pi^2}{4}$ 2) $\frac{1}{6}+\frac{\pi^3}{8}$ 3) 2 4) $\frac{4}{3}+\pi^2\left(\frac{\pi-3}{6}\right)$
66.	Area bounded between the curves $y=\sqrt{4-x^2}$ and $y^2=3\lvert x\rvert$ in square units is 1) $\frac{2\pi-\sqrt{3}}{3\sqrt{3}}$ 2) $\frac{2\pi-1}{3\sqrt{3}}$ 3) $\frac{2\pi-1}{\sqrt{3}}$ 4) $\frac{2\pi-\sqrt{3}}{3}$
67.	The area enclosed between the curves $y=\ln(x+e), x=\ln\left(\frac{1}{y}\right)$ and x-axis in square units is 1) 1 2) 2 3) 4 4) 6
68.	A point P moves in the xy plane in such a way that $[\lvert x\rvert]+[\lvert y\rvert]=1$, where [.] denotes the greatest integer function. Then the area of the region representing all possible positions of the point P in square units is 1) 2 2) 5 3) 7 4) 8
69.	The general solution of $x\frac{dy}{dx}+(\log x)y=x^{1-\frac{1}{2}\log x}$ is 1) $y=x^{1-\frac{1}{2}\log x}+cx^{-\frac{1}{2}\log x}$ 2) $y.x^{\frac{1}{2}\log x}=x^{\frac{1}{2}\log x}+c$ 3) $y=e^{-\frac{(\log x)^2}{2}}(x+c)$ 4) $y=e^{\frac{1}{2}(\log x)^2}\left(x^{1-\frac{1}{2}(\log x)}-x^{\frac{1}{2}(\log x)}\right)+c$

70. The solution of the differential equation $\frac{x\,dx + y\,dy}{x\,dy - y\,dx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$ is

1) $\sqrt{x^2+y^2} = a\cos\left\{c + \tan^{-1}\frac{y}{x}\right\}$
2) $\sqrt{x^2+y^2} = a\sin\left\{c + \tan^{-1}\frac{y}{x}\right\}$
3) $\sqrt{x^2+y^2} = a\sin\left\{c + \tan^{-1}\frac{x}{y}\right\}$
4) $\sqrt{x^2+y^2} = a\cos\left\{c + \tan^{-1}\frac{x}{y}\right\}$

71. The solution of differential equation $2x^3 y dy + (1-y^2)(x^2y^2 + y^2 - 1)dx = 0$ is

1) $x^2y^2 = (cx+1)(1-y^2)$
2) $x^2y^2 = (cx+1)(1+y^2)$
3) $x^2y^2 = (cx-1)(1-y^2)$
4) None of these

72. The equation of curve passing through (1, 0) and satisfying $\left(y\frac{dy}{dx} + 2x\right)^2 = \left(y^2 + 2x^2\right)\left(1 + \left(\frac{dy}{dx}\right)^2\right)$, is given by

1) $\sqrt{2}x^{\pm\frac{1}{\sqrt{2}}} = \frac{y + \sqrt{y^2 + 2x^2}}{x}$
2) $\sqrt{2}x^{\pm\sqrt{2}} = \frac{y + \sqrt{y^2 + \sqrt{2}x^2}}{x}$
3) $\sqrt{2}y^{\pm\frac{1}{\sqrt{2}}} = \frac{y + \sqrt{x^2 + \sqrt{2}y^2}}{x}$
4) $\sqrt{2}y^{\pm\frac{1}{\sqrt{2}}} = \frac{y + \sqrt{x^2 + 2y^2}}{x}$

73. Solution of $\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$ is

1) $\frac{y}{x} - \frac{1}{x^2 + y^2} = C$
2) $\frac{y}{x} + \frac{1}{x^2 + y^2} = C$
3) $\frac{y}{x} + \frac{1}{\sqrt{x^2 + y^2}} = C$
4) $\frac{y}{x} - \frac{1}{\sqrt{x^2 + y^2}} = C$

74. The family of curves, the sub tangent at any point of which is the arithmetic mean of the coordinates of the point of tangency, is given by

1) $(x-y)^2 = Cy$
2) $(y-x)^2 = Cx$
3) $(x-y)^2 = Cxy$
4) $x^2 = 4ay$

75. Let PQ be the latus rectum of the parabola $y^2 = 4x$ with vertex A. minimum length of the projection of PQ on a tangent drawn in portion PAQ of parabola is

1) 2 **2)** 4 **3)** $2\sqrt{3}$ **4)** $2\sqrt{2}$

76. Let $f: R^+ \to R^+$ is an invertible function such that $f'(x) > 0$ and $f''(x) > 0 \forall x \in [1,5]$. If $f(1) = 1$ and $f(5) = 5$ and area bounded by $y = f(x)$, x-axis, $x = 1$ and $x = 5$ is 8 sq. units. Then area bounded by $y = f^{-1}(x)$, x-axis $x = 1$ and $x = 5$ is

1) 12 **2)** 16 **3)** 18 **4)** 20

77. The area enclosed between the curves $|x| + |y| \geq 2$ and $y^2 = 4\left(1 - \dfrac{x^2}{9}\right)$ is

1) $(6\pi - 4)$ sq. units **2)** $(6\pi - 8)$ sq. units

3) $(3\pi - 4)$ sq. units **4)** $(3\pi - 2)$ sq. units

78. General solution of differential equation of

$f(x)\dfrac{dy}{dx} = f^2(x) + f(x)y + f'(x)y$ (Where c being arbitrary constant)

1) $y = f(x) + ce^x$ **2)** $y = -f(x) + ce^x$

3) $y = -f(x) + ce^x f(x)$ **4)** $y = cf(x) + e^x$

79. If a curve is such that line joining origin to any point $P(x, y)$ on the curve and the line parallel to y-axis through P are equally inclined to tangent to curve at P, then the differential equation of the curve is

1) $x\left(\dfrac{dy}{dx}\right)^2 - 2y\dfrac{dy}{dx} = x$ **2)** $x\left(\dfrac{dy}{dx}\right)^2 + 2y\dfrac{dy}{dx} = x$

3) $y\left(\dfrac{dy}{dx}\right)^2 - 2x\dfrac{dy}{dx} = x$ **4)** $y\left(\dfrac{dy}{dx}\right)^2 - 2y\dfrac{dy}{dx} = x$

80. A function $y = f(x)$ satisfies the differential equation

$(x+1)f'(x) - 2(x^2 + x)f(x) = \dfrac{e^{x^2}}{(x+1)}$; $\forall x > -1$. If $f(0) = 5$, then $f(x)$ is

1) $\left(\dfrac{3x+5}{x+1}\right).e^{x^2}$ **2)** $\left(\dfrac{6x+5}{x+1}\right).e^{x^2}$ **3)** $\left(\dfrac{6x+5}{(x+1)^2}\right).e^{x^2}$ **4)** $\left(\dfrac{5-6x}{x+1}\right).e^{x^2}$

INTEGERS

81. Let a function $f(x)$ be defined in $[-2,2]$ as $f(x) = \begin{cases} \{x\}, & -2 \leq x < -1 \\ |\operatorname{sgn} x|, & -1 \leq x \leq 1 \\ \{-x\}, & 1 < x \leq 2 \end{cases}$, where $\{x\}$ denotes fractional part, then area bounded by graph of $f(x)$ and x–axis is:

82.	Let $y = y(x)$ satisfy the differential equation $\left(2xy + x^2y + \frac{y^3}{3}\right)dx + (x^2 + y^2)dy = 0$. If $y(1) = 1$ and the value of $(y(0))^3 = ke(k \in N)$. Find k.
83.	Let 'f' be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t)dt$, then f(1) + f(2) + f(3) + ... + f(9) =
84.	The area of region represented by $[x+y]+[x-y]=5$, for $x \geq y, x \geq 0, y \geq 0$ in square units is
85.	If $f(x) = \begin{cases} \sqrt{\{x\}}, & x \notin Z \\ 1, & x \in Z \end{cases}$ and $g(x) = \{x\}^2$, (where $\{.\}$ denotes fractional part of x), then area bounded by $f(x)$ and $g(x)$ for $x \in [0,10]$ is
86.	Let $y = (a\sin x + (b+c)\cos x)e^{x+d}$, where a, b, c and d are parameters represent a family of curves, then differential equation for the given family of curves is given by $y'' - \alpha y' + \beta y = 0$, then $\alpha + \beta =$
87.	Let $y = f(x)$ satisfies the differential equation $xy(1+y)dx = dy$. If $f(0) = 1$ and $f(2) = \frac{e^2}{k - e^2}$, then find the value of k
88.	Let f be a differentiable function satisfying the condition $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)} (y \neq 0) \forall x, y \in R$ and $f'(1) = 2$. If the smaller area enclosed by $y = f(x), x^2 + y^2 = 2$ is A, then find $[A]$, where $[\bullet]$ represents the greatest integer function.
89.	The least integer which is greater than or equal to the area of region in $x - y$ plane satisfying $x^6 - x^2 + y^2 \leq 0$ is
90.	The set of points (x, y) in the plane satisfying $x^{2/5} + \lvert y \rvert = 1$ form a curve enclosing a region of area $\frac{p}{q}$ square units, where p and q are relatively prime positive integers. Find $p - q$

KEY SHEET

PHYSICS

1) 2	2) 4	3) 3	4) 3	5) 1	6) 2
7) 4	8) 1	9) 2	10) 4	11) 1	12) 1
13) 3	14) 3	15) 3	16) 1	17) 1	18) 1
19) 3	20) 4	21) 0.62 to 0.63	22) 2	23) 1.73	24) 6.66 to 6.67
25) 25	26) 75	27) 12	28) 10	29) 2	30) 8

CHEMISTRY

31) 1	32) 1	33) 4	34) 4	35) 3	36) 4
37) 2	38) 4	39) 2	40) 1	41) 3	42) 4
43) 4	44) 2	45) 3	46) 2	47) 4	48) 4
49) 1	50) 4	51) 2	52) 1	53) 4	54) 5
55) 2	56) 3	57) 3	58) 3	59) 7	60) 4

MATHEMATICS

61) 2	62) 4	63) 4	64) 4	65) 4	66) 4
67) 1	68) 4	69) 1	70) 2	71) 3	72) 1
73) 1	74) 1	75) 4	76) 2	77) 2	78) 3
79) 1	80) 2	81) 3	82) 4	83) 960	84) 1.5
85) 3.33	86) 4	87) 2	88) 1	89) 2	90) 1

SOLUTIONS

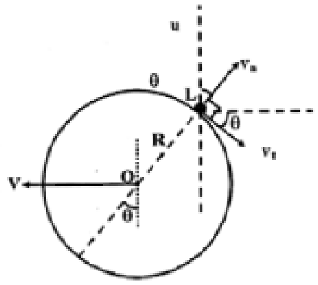
PHYSICS

01. Velocity attained by ball just before it strikes the surface

$u=\sqrt{2gh}$

$\therefore \sin\theta=\frac{R/2}{R} \Rightarrow \theta=30^0$

After collision let the velocity of the sphere be V and the components of velocity of ball along common normal OL and along common tangent be v_n and v_t respectively.



In the absence of external force in the horizontal direction , the linear momentum conserved.

$0+0=-MV+mv_n\sin\theta+mv_t\cos\theta$ (1)

A long the common normal at,

$V\sin\theta-(-v_n)=e[u\cos\theta-0]$ (2)

$e=0$ for inelastic collision.

$\therefore V\sin\theta=-v_n$

Along the common tangent velocity components before and after collision remain same.

$\therefore u\sin\theta=v_t \Rightarrow v_t=\frac{u}{2}$ (4)

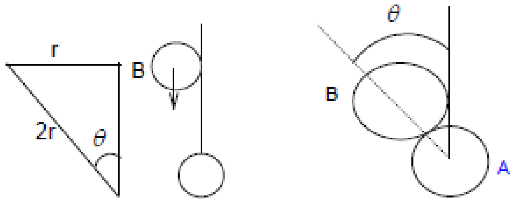
Using (3) and (4) in (1),

$MV=m\left(-\frac{V}{2}\right)\frac{1}{2}+m\left(\frac{u}{2}\right)\frac{\sqrt{3}}{2}$

$\left(M+\frac{m}{2}\right)v=m\frac{u\sqrt{3}}{4}$

Velocity of sphere $V=\frac{m\sqrt{6gh}}{2(2M+m)}$

02.



$\sin\theta=\frac{r}{2r}\Rightarrow\theta=30^0$

let the velocity of B (just before collision) be v_0

From the law of collision

$J-mv_0\cos30^0=mv_1$

$v_1=\frac{J}{m}-v_0\cos30^0$

For ball A

$J\sin30^0=mv_2$

	$v_2 = \frac{J}{2m}$ From the Newton's law of collision $e = \frac{v_2 \sin 30 - (v_1)}{v \cos 30^0 - 0} = 1$ $\therefore J = 1.6 m v_0 \cos 30^0$ So, $u_1 = 0.6 v_0 \cos 30^0$ And $u_2 = 0.8 u_0 \cos 30^0$ Since ball A just completes vertical circle, the refore $u_2 = \sqrt{5gl}$ $\therefore 0.8 u_0 \cos 30^0 = \sqrt{5gl} \Rightarrow u_0 = 12.5 m/s$ $\Rightarrow u_0 = \frac{10^2}{8} m/s$ n = 8
03.	$h_{cm} = \frac{\int y\, dm}{\int dm} = \frac{\int y\, dV}{\int dV}$ $dV = (\pi x^2) dy$
04.	$\Delta P = \text{Area of FT graph} = \frac{\pi r^2}{2} = \frac{\pi (F_0)\left(\frac{T}{2}\right)}{2}$ $mu = \frac{1}{2} \pi F_0 \left(\frac{T}{2}\right)$
05.	$\text{mgh} = \frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2$ $0 = mv_1 - Mv_2$ $v_2 = \frac{mv_1}{M}$ $\text{mgh} = \frac{1}{2} m v_1^2 + \frac{1}{2} \frac{m^2 M}{M^2} v_1^2$ $2\,gh = \frac{(M+m)}{M} v_1^2$ $v_1 = \sqrt{2gh\left(\frac{M}{M+m}\right)}$ $mv_1 = (M + m)\, v_c$ $v_c = \frac{mv_1}{m+M}$ $\frac{1}{2} m v_1^2 = mgh + \frac{1}{2}(M+m) v_c^2$ $h' = h\left(\frac{M}{M+m}\right)^2$

Where v_1 = velocity of washer towards right after following from left wedge

v_2 = velocity of left wedge towards left

v_c = common velocity of washer + right wedge , when there is no relative motion.

06. If $F=0$ find 't' $I=\int_0^t F.dt$

07. Before collision time taken is

$$t_1=\frac{\pi r/2}{3v}=t_0$$

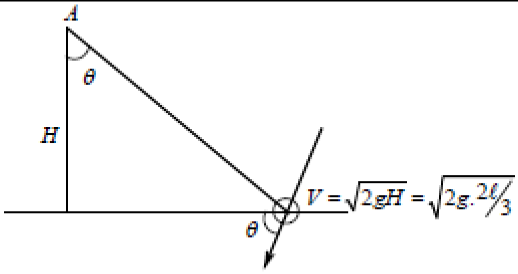
After collision time taken is

$$e=1=\frac{Velocity\ of\ separation}{2V-(-V)}$$

Velocity of separation = 3V

Required time = $\frac{2\pi r}{3V}$

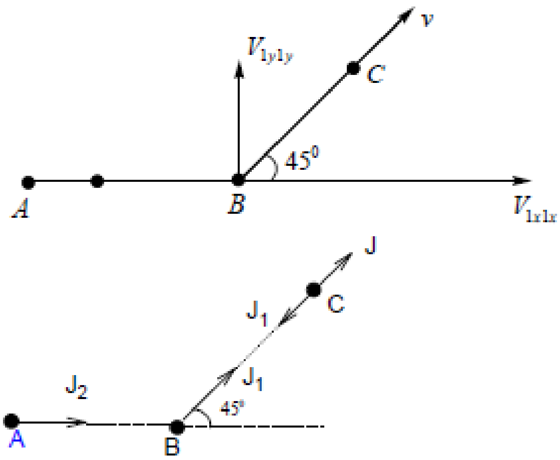
08.



After collision the velocity of the ball $=V\cos\theta$

$$=2\sqrt{g\ell/3}\times 2/3=\frac{4}{3}\sqrt{\frac{g\ell}{3}}$$

09.



The external impulse applied to C causes both strings to jerk exerting internal impulses J_1 and J_2

$V_2 = v_{1x}$----------(1)

$J_1\cos 45^0 - J_2 = mv_{1x}$------------(3)

$J_1\cos 45^0 = mv_{1y}$------------------(4)

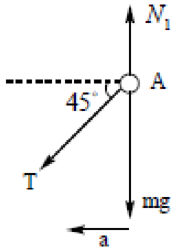
$J - J_1 = mv$-----------------(5)

Also velocities of B&C along BC are are equal i.e., $v_{1y}\cos 45^0 + v_{1x}\cos 45^0 = v$ -----(6)

After solving we get, $v_2 = v_{1x} = \frac{\sqrt{2}J}{7m}$

$v = \frac{3J}{7m}; v_{1y} = \frac{2\sqrt{2}J}{7m}; V_A = \frac{\sqrt{2}J}{7m}, V_B = \frac{\sqrt{10}J}{7m}, J_1 = \frac{3J}{m}$ and $J_2 = \frac{\sqrt{2}J}{7}$

10. Just after the release B moves downwards and A moves horizontally leftwards with the same acceleration say a.

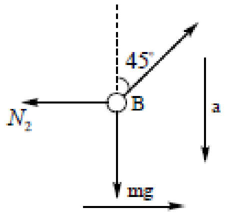


Drawing free body diagram of both A and B, $T\cos 45^\circ = ma$

(or) $T = \sqrt{2}ma$ ……..(i)

$mg - T\cos 45^\circ = ma$ or $mg - ma = ma$

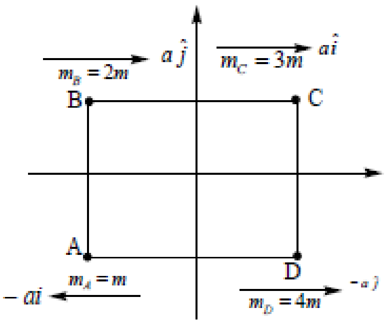
or $a = \frac{g}{2}$ …….(ii)



Substituting this in Eq.(i) , We get

$T = \frac{mg}{\sqrt{2}}$

11. $$a_{CM} = \frac{m_A a_A + m_B a_B + m_C a_C + m_D a_D}{m_A + m_B + m_C + m_D}$$



where , $m_A = m, m_B = 2m, m_C = 3m$ and $m_D = 4m$, $|a_A| = |a_B| = |a_C| = |a_D| = a$ (according to the question)

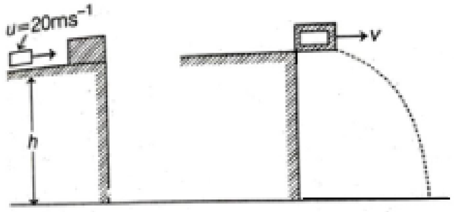
$$a_{CM} = \frac{-ma\hat{i} + 2ma\hat{i} + 3ma\hat{i} - 4ma\hat{j}}{m + 2m + 3m + 4m}$$

$$= \frac{2a\hat{i} - 2a\hat{j}}{10}$$

$$= \frac{a}{5}.(\hat{i} - \hat{j})ms^{-2}$$

12. Conceptual

13. When the bullet undergoes an inelastic collision with block, a part of KE of bullet is lost.



When bullet + block system falls from height h, its total energy (kinetic + potential) becomes kinetic energy, so kinetic energy of bullet + block system at bottom just before collision is equal to total energy just after collision.

Now, by law of conservation of momentum, we have

$$mu = (m+M)v$$

$$\Rightarrow \quad v = \frac{mu}{m+M} = \frac{0.1\times 20}{(0.1+1.9)} = 1ms^{-1}$$

Total energy of bullet and block just after collision

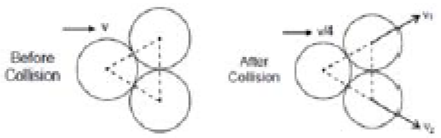
$$= KE + PE = \frac{1}{2}(m+M)v^2 + (m+M)gh$$

$$= \frac{1}{2}\times 2\times 1^2 + 2\times 10\times 1$$

$$= 1 + 20 = 21J$$

14. Loss in KE$=k_i - k_f \quad = \frac{1}{2}(40)\times 4^2 + \frac{1}{2}(60)(2)^2 - \frac{1}{2}(100)\times(2.8)^2 = 48j$

15. $$2mv_1\cos 30^o + m\frac{v}{4} = mv......(i)$$



$$e = \frac{v_1 - \frac{v}{4}\cos 30^o}{v\cos 30^o}..........(ii)$$

16. Applying momentum conservation in the direction of string

$$mu\frac{\sqrt{3}}{2} = 2mv \Rightarrow v = \frac{\sqrt{3}}{4}(u)$$

17. Conceptual

18. mu=3mv V=u/3 $\quad e = -\frac{2u/3 - u/3}{-u} \Rightarrow e = 1/3$

19. $P_{conserved} \Rightarrow v_A$ +ve so v_B -ve

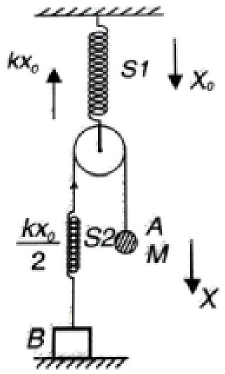
$m_A v_A + m_B v_B = 0,\ v_A > v_B \Rightarrow m_A < m_B$

20. $0 = m\Delta x_{1G} + 2m\Delta x_{2G} + 3m\Delta x_{3G}$

$= (\Delta x_{13} + \Delta x_{3G}) + 2(\Delta x_{23} + \Delta x_{3G}) + 3m\Delta x_{3G}$

$= -2L + 2(2L) + 6\Delta x_{3G} \therefore \Delta x_{3G} = -L/3$

21.



When spring S1 Stretches by x_0, tension in it is kx_0

At this instant tension in S2 shall be $\frac{kx_0}{2}$

It means S2 is stretched by $\frac{x_0}{2}$

When S1 and S2 stretch by x_0 and $\frac{x_0}{2}$, the ball A will fall through a distance

$$x = 2x_0 + \frac{x_0}{2} = \frac{5x_0}{2} \qquad \ldots\ldots\ldots(1)$$

[we are assuming that B does not move]

If A falls through x before coming to rest, $x_0 = \frac{2x}{5}$

$\therefore$ Spring force on $B = \frac{kx_0}{2} = \frac{kx}{5}$

B will just leave the table at this instant if

$$\frac{kx}{5} = Mg \qquad \ldots\ldots\ldots\ldots(2)$$

When ball A falls through x (before coming to rest) principle of conservation of energy says loss in PE of A = Gain in spring PE

$$mg\ x = \frac{1}{2}kx_0^2 + \frac{1}{2}k\left(\frac{x_0}{2}\right)^2$$

$$mg\ x = \frac{1}{2}k\left(\frac{2x}{5}\right)^2 + \frac{1}{2}k\left(\frac{x}{5}\right)^2$$

$\Rightarrow kx = 10mg$

From (2) 5Mg =10 mg

$\therefore m = \frac{M}{2}$

22\. Spring force is maximum when the system is released

$F_{s\max} = kx_0 = 240N$

The limiting friction on A can be

$F_{lA} = \mu_1 M_A g = 250N$

Hence, block A remains fixed and does not move at all .

The limiting friction on C can be

$F_{lc} = \mu_2 M_C g$

Thus maximum acceleration that friction can provide to C is

$a_{c\max} = \mu_2 g = 8.5 \ m/s^2$

Just after the release spring force is maximum and it will cause maximum acceleration in B. Let us assume that there is no slipping between B and C. In that case the maximum acceleration is

$$a_{c\max} = \frac{240}{28+2} = 8.0 \ m/s^2$$

Friction can easily provide this acceleration to block C hence it will not slip over B.

Speed is maximum when the spring acquires its natural length .

$$\frac{1}{2}(M_B + M_C)v_{\max}^2 = \frac{1}{2}kx_0^2$$

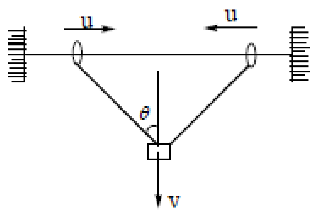
$\Rightarrow v_{\max} = 2 \ m/s$

23\. $v\cos\theta = u\sin\theta$

$v = \sqrt{3}u$ ....(i)

$$mg \ l \cos\theta = mu^2 + \frac{mv^2}{2} \qquad ....(ii)$$

$$u = \sqrt{\frac{g\ell}{5}}, v = \sqrt{\frac{3g\ell}{5}}$$



24\. $$V_{cm} = \frac{mu}{3m} = \frac{u}{3}$$

In COM frame , the system rotates with speed of $A = \dfrac{2u}{3}$ and speed of $B = \dfrac{u}{3}$

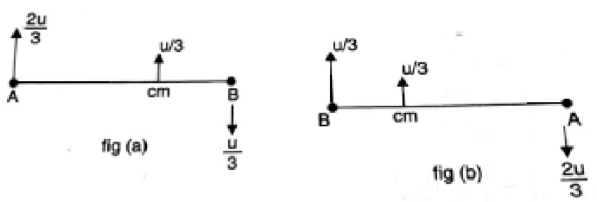
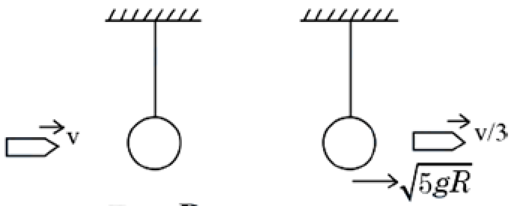
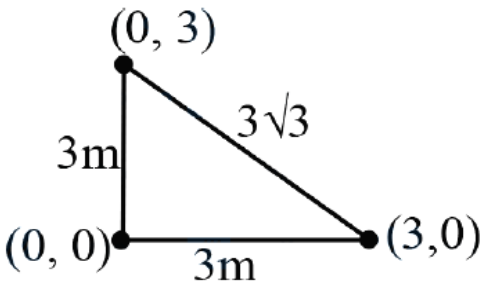


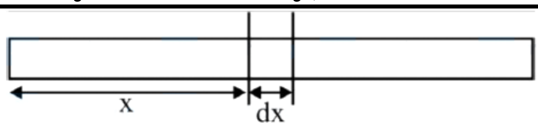
fig (a) fig (b)

The two figures show the velocities of A and B in COM frame.

In ground frame B will have maximum velocity when it is in position indicated in fig (b)

$$\therefore (V_B)_{\max} = \frac{2u}{3}$$

25.	$\frac{1}{2}\mu v_{rel}^2 = 1/2 kx_{max}^2 \Rightarrow 1/2 \times \frac{3\times 6}{3+6}(1.8+2.2)^2 = 1/2\times 512 \times x_{max}^2 \; x_{max} = 1/4m = 25cm$
26.	(B) Kapil and the boat can be considered as one body of mass $m_b = (65+100) = 165kg$ Note that the centre of mass of the system remains unchanged since no external force acts on the system. Let m_s be the mass of Sachin and $\Delta x_s, \Delta x_b$ be the displacements of the combined body of mass m_b and Sachin respectively with reference to the centre of
27.	Impulse = change in momentum $= m[v-(-v)] = 2\ mv$ $= 2 \times 0.4 \times 15 = 12\ Ns$
28.	Considering Only Horizontal direction  $\mathbf{P_i} = \mathbf{P_f}$ $(75v)+0 = 50\left(\sqrt{5gR}\right) + 75\frac{v}{3}$ $75\left(v-\frac{v}{3}\right) = 50\sqrt{100}$ $v = 10m/s$
29.	 $\vec{r}_{com} = \frac{M(0\hat{i}+0\hat{j})+M(3\hat{i})+M(3\hat{j})}{3M}$ $\vec{r}_{com} = \hat{i}+\hat{j}$ $\lvert\vec{r}_{com}\rvert = \sqrt{2} = \sqrt{x}$ $x = 2$

<table>
<tr><td>30.</td><td>


$$dm = \lambda \cdot dx = \lambda_0\left(1-\frac{x^2}{\ell^2}\right)$$

$$X_{cm} = \frac{\int xdm}{\int dm_\ell}$$

$$= \frac{\lambda_0\int_0^\ell\left(x-\frac{x^3}{\ell^2}\right)dx}{\int_0^\ell \lambda_0\left(1-\frac{x^2}{\ell^2}\right)dx} = \frac{\frac{\ell^2}{2}-\frac{\ell^4}{4\ell^2}}{\ell-\frac{\ell^3}{3\ell^2}} = \frac{3\ell}{8}$$
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</table>

CHEMISTRY

<table>
<tr><td>31</td><td>

$$H-\underset{\underset{O}{||}}{C}-O-CH\begin{matrix}CH_3\\CH_3\end{matrix}$$

$$H-\overset{\overset{O}{||}}{C}-O-\overset{CH_3}{CH}-CH_2\xrightarrow[excess]{MeMgBr}\xrightarrow{H_2O}CH_3-\underset{CH_3}{CH}-OH$$

$$\downarrow NaOCl$$

$$CH_3COOH + CHCl_3$$
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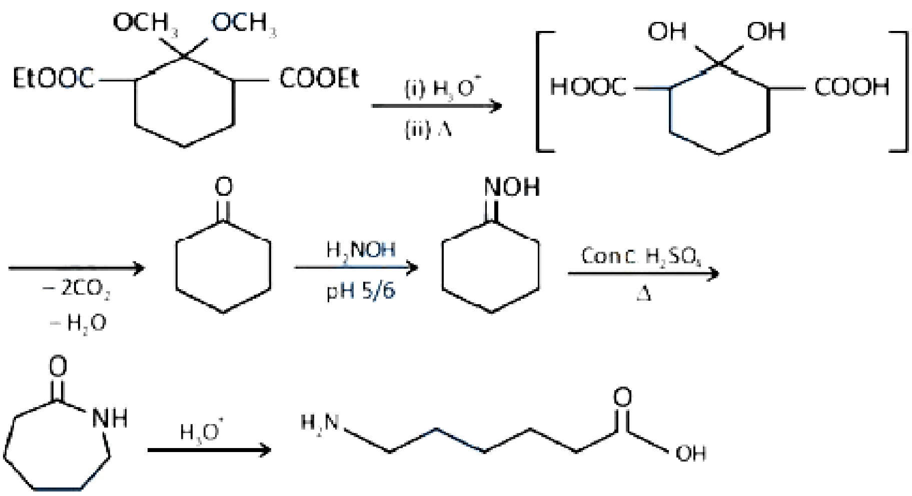
<tr><td>32.</td><td>

$No\ \alpha - hydrogen$
</td></tr>
<tr><td>33.</td><td>


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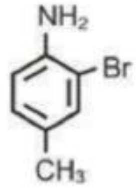
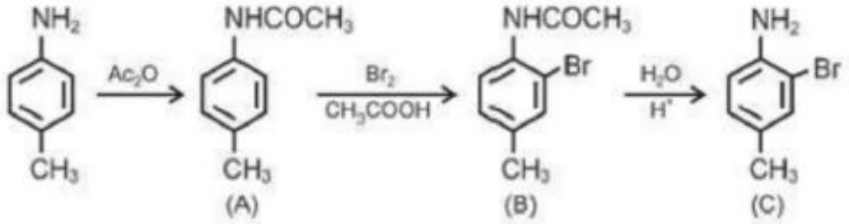
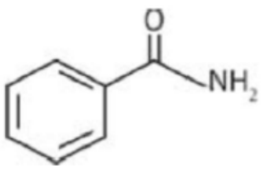
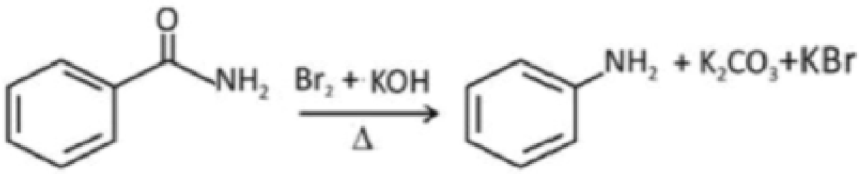
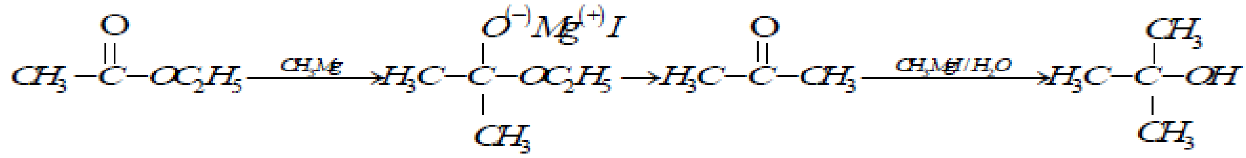
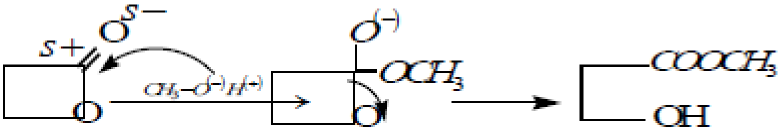
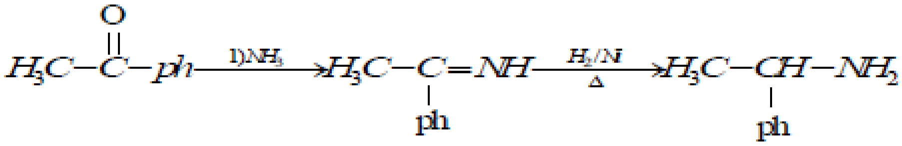
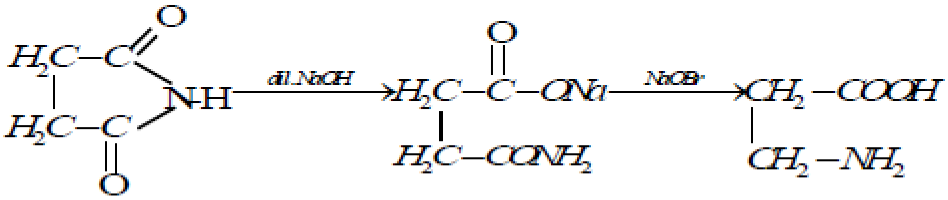
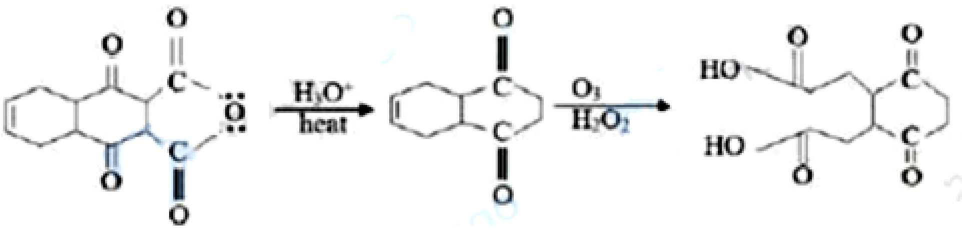
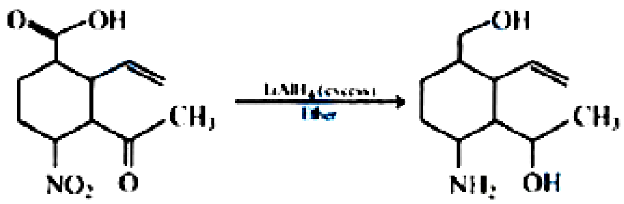
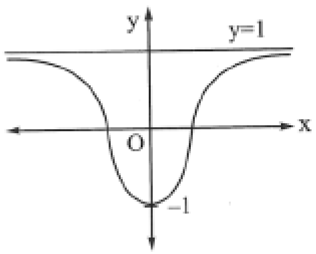
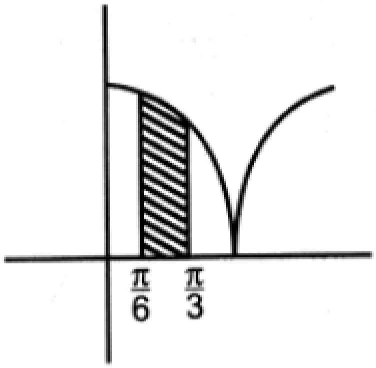
34.	  
35.	
36.	$C_{12}H_{22}O_{11}+36HNO_3 \xrightarrow[\Delta]{V_2O_5} 6H_2C_2O_4+36NO_2+23H_2O$
37.	$(CH_3)_3N$ Volatile nature volatile nature $\propto \frac{1}{\text{Boiling point}}$ $(CH_3)_3N \rightarrow$ No H-bond(less Boiling point) Hence it is most volatile
38.	$(CN)_2 \xrightarrow{H_3O^+} \begin{matrix} COOH \\ \vert \\ COOH \end{matrix}$
39.	2
40.	$\begin{matrix} H_3C \\ H_5C_2 \end{matrix} C=N-OH \xrightarrow[\text{Beckmann's rearrangement}]{PCl_5} H_3C-CO-NH-C_2H_5 \xrightarrow{H_3O^{(+)}} CH_3-COOH+CH_3-CH_2-NH_2$
41.	$\begin{matrix} HC-COOH \\ \Vert \\ HC-COOH \end{matrix} \xrightarrow{H_2/Catalyst} \begin{matrix} CH_2-COOH \\ \vert \\ CH_2-COOH \end{matrix}$
42.	Conceptual

43.	
44.	
45.	
46.	
47.	4
48.	4
49.	Conceptual
50.	4
51.	
52.	$LiAlH_4$ reduces aldehydes, ketones and carboxylic acids to corresponding alcohols and nitro $(-NO_2)$ group to amino $(-NO_2)$ group.But $LiAlH_4$ cannot reduce carbon-carbon double bonds. 
53.	$R-CONH_2+Br_2+4NaOH\xrightarrow{\Delta}R-NH_2+2NaBl+Na_2CO_3+2H_2O$
54.	Sodium borohydride can't reduce amides and esters
55.	2

56.	$CH_2\begin{matrix} COOH \\ COOH \end{matrix} \xrightarrow{\Delta} CH_3COOH + CO_2\uparrow$ $\begin{matrix} COOH \\ \vert \\ COOH \end{matrix} \xrightarrow{\Delta} CO + CO_2\uparrow + H_2O$ $HCOOH \xrightarrow{\Delta} CO_2 + H_2$
57.	3
58.	1 mole of benzyl amine reacts with 3 moles of bromo methane to give 230 gm of benzyl trimethyl ammonium bromide.0.1 mole of benzyl amine reacts with 0.3 moles of bromo methane to give 23 g of of benzyl trimethyl ammonium bromide.
59.	7
60.	$HO-\underset{\substack{\vert \\ CH_2-COOH}}{\overset{\substack{CH_2-COOH \\ \vert}}{C}}-COOH$

MATHEMATICS

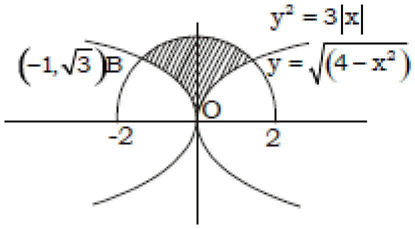
61.	 Area $= 2\int_0^\infty \left(1-\left(\frac{x^2-1}{x^2+1}\right)\right)dx = 4\left(\tan^{-1}x\right)_0^\infty = 2\pi$
62.	$\because \quad \int\left(\frac{y+1}{y}\right)dy = \int e^x\left(\sin 2x - \cos^2 x\right)dx$ $\Rightarrow \quad y + Iny = -e^x\cos^2 x + C$ $\Rightarrow \quad x = 0, y = 1$ $\therefore \quad C = 2$
63.	Here, $y = fog(x) = f\{g(x)\} = (\cos x)^2 = \cos^2 x$ Also, $18x^2 - 9\pi x + \pi^2 = 0$ $\Rightarrow (3x-\pi)(6x-\pi) = 0$ $\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}(\text{as}\,\alpha, \beta)$

	 Required area of curve is $= \int_{\pi/6}^{\pi/3} \cos^2 x dx = \frac{1}{2}\int_{\pi/6}^{\pi/3}(1+\cos 2x)dx$ $= \frac{1}{2}\left\{x+\frac{\sin 2x}{2}\right\}_{\pi/6}^{\pi/3} = \frac{1}{2}\left\{\left(\frac{\pi}{3}-\frac{\pi}{6}\right)+\frac{1}{2}\left(\sin\frac{2\pi}{3}-\sin\frac{2\pi}{6}\right)\right\}$ $= \frac{1}{2}\left\{\frac{\pi}{6}+\frac{1}{2}\left(\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)\right\} = \frac{\pi}{12}$ Hence, (D) is the correct answer.
64.	$f(x) = \begin{cases} \cos x, & 0 \le x \le \frac{\pi}{4} \\ \sin x, & \frac{\pi}{4} \le x \le \frac{5\pi}{6} \\ \frac{1}{2}, & \frac{5\pi}{6} \le x \le \frac{5\pi}{3} \end{cases}$ Required area $= \int_0^{\frac{5\pi}{3}} f(x)dx = \int_0^{\frac{\pi}{4}} \cos x dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{6}} \sin x dx + \int_{5\frac{\pi}{6}}^{5\frac{\pi}{3}} \frac{1}{2}dx$
65.	$\Rightarrow 4\int_0^1 (x-x^2)dx + 4\int_1^{\pi/2}(x^2-x)dx$ $= \frac{4}{3}+4\left[\frac{\pi^3}{24}-\frac{\pi^2}{8}\right]$ $= \frac{4}{3}+\left[\frac{\pi^3}{6}-\frac{\pi^2}{2}\right]$ $= \frac{4}{3}+\pi^2\left(\frac{\pi-3}{6}\right)$

66. Required area $=2\int_0^1\left(\sqrt{4-x^2}-\sqrt{3}x\right)dx$

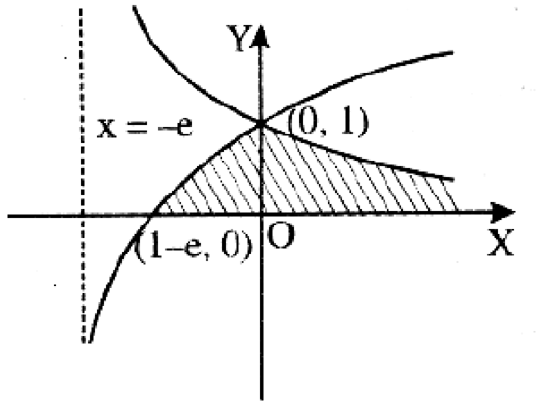
$$=2\left(\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)-\frac{\sqrt{3}.2x^{1/2}}{3}\right)_0$$

$$=\frac{2\pi-\sqrt{3}}{3\sqrt{3}}.$$



67. The given curves are $y=\ln(x+e)$ and $x=\ln\left(\frac{1}{y}\right)\Rightarrow\frac{1}{y}=e^x\Rightarrow y=e^{-x}$

Using transformation of graph we can sketch the curves.



Hence, the required area

$$=\int_{1-e}^{0}\ln(x+e)\,dx+\int_0^\infty e^{-x}dx$$

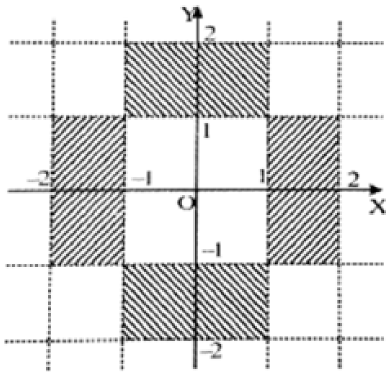
$$=\int_1^e\ln t\,dt+\int_0^\infty e^{-x}dx \text{ (putting } x+e=t\text{)}$$

$$=(t\ln t-t)_1^e-(e^{-x})_0^\infty=1+1=2$$

68. *If* $[|x|]=1$ and $[|y|]=0$ then $1\le|x|<2, 0\le|y|<1$

$\Rightarrow x\in(-2,-1]\cup[1,2), y\in(-1,1)$

If $[|x|]=0,[|y|]=1$, then $x\in(-1,1), y\in(-2,-1]\cup[1,2]$.



Area of the required region $=4(2-1)(1-(-1))=8$

69.	$\frac{dy}{dx}+\frac{\log x}{x}y=x^{-\frac{1}{2}}\log x$ $I.F=e^{\int\frac{\log x}{x}dx}=e^{\frac{(\log x)^2}{2}}=\left(e^{\log x}\right)^{\frac{\log x}{2}}=x^{\frac{1}{2}\log x}$ $G.S is\, x^{\frac{1}{2}\log x}.y=\int dx$ $y\,x^{\frac{1}{2}\log x}=x+c$
70.	Put $x=r\cos\theta,\ y=r\sin\theta$ $\Rightarrow x^2+y^2=r^2,\ \tan\theta=\frac{y}{x}$ $\Rightarrow x\,dx+y\,dy=r\,dr$ $\frac{x\,dy-y\,dx}{x^2}=\sec^2\theta d\theta$ $\therefore$ given equation $\Rightarrow\frac{r\,dr}{r^2d\theta}=\sqrt{\frac{a^2-r^2}{r^2}}$ $\Rightarrow\frac{dr}{\sqrt{a^2-r^2}}=d\theta$ $\Rightarrow\sin^{-1}\left(\frac{r}{a}\right)=\theta+C$ $\Rightarrow r=a\sin(\theta+C)$
71.	$2x^3y\,dy+(1-y^2)(x^2y^2+y^2-1)\,dx=0$ $\frac{2y}{(1-y^2)^2}\frac{dy}{dx}+\frac{y^2}{1-y^2}\frac{1}{x}=\frac{1}{x^3}$ Put $\frac{y^2}{1-y^2}=u\Rightarrow\frac{2y\ dy}{(1-y^2)^2dx}=\frac{du}{dx}\Rightarrow\frac{du}{dx}+\frac{u}{x}=\frac{1}{x^3}$ $u.x=\int\frac{1}{x^2}dx+C\quad\Rightarrow x^2y^2=(Cx-1)(1-y^2)$
72.	The given differential equation can be written as $y^2\left(\frac{dy}{dx}\right)^2+4x^2+4xy.\frac{dy}{dx}=(y^2+2x^2)\left(1+\left(\frac{dy}{dx}\right)^2\right)$ $\Rightarrow\ \frac{dy}{dx}=\frac{y}{x}\pm\sqrt{\frac{1}{2}\left(\frac{y}{x}\right)^2+1}$ ...... (i) Let $y=vx\Rightarrow v+x\frac{dv}{dx}=\frac{dy}{dx}$ $\therefore$ Eq.(i) becomes $v+x\frac{dv}{dx}=v\pm\sqrt{\frac{1}{2}v^2+1}$

or $\displaystyle\int \frac{dv}{\sqrt{\frac{1}{2}v^2+1}} = \int \frac{dx}{x}$

$\Rightarrow \sqrt{2}\log\left|v+\sqrt{v^2+2}\right| = \log|xC|$

$\Rightarrow \sqrt{2}\log\left|\dfrac{y+\sqrt{y^2+2x^2}}{x}\right| = \log|xC|$, putting x = 1 and y = 0

$\Rightarrow C = \left(\sqrt{2}\right)^{\sqrt{2}}$

$\therefore$ Curves are given by $\dfrac{y+\sqrt{y^2+2x^2}}{x} = \sqrt{2}x^{\pm\frac{1}{\sqrt{2}}}$

Hence, (1) is the correct answer.

73. The given equation can be written as

$$\frac{xdx+ydy}{\left(x^2+y^2\right)^2} = \frac{ydx-xdy}{y^2}\cdot\frac{y^2}{x^2}$$

$$\Rightarrow \int \frac{d\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2} = 2\int \frac{1}{x^2/y^2}d\left(\frac{x}{y}\right)$$

Integrating both the sides, we get

$$-\frac{1}{\left(x^2+y^2\right)} = -\frac{1}{(x/y)}+C \Rightarrow \frac{y}{x}-\frac{1}{x^2+y^2} = C$$

74. Let the family of curves be $y = f(x)$

$\tan\theta = \dfrac{l(PP')}{l(TP')}$

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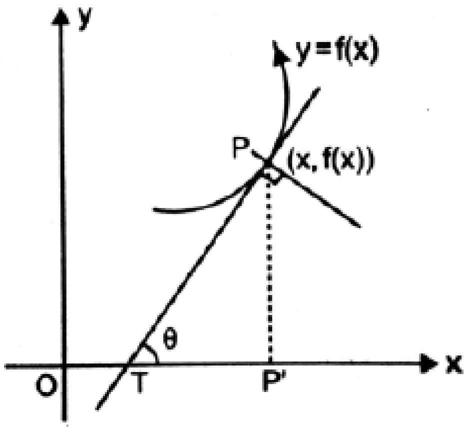


$\therefore$ $l(\text{subtangent}) = \dfrac{f(x)}{f'(x)}$

$\therefore$ $\dfrac{y}{y'} = \dfrac{x+y}{2}$ (given)

$\therefore \quad y' = \dfrac{2y}{x+y}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{2y}{x+y} \qquad \therefore \dfrac{dy}{dx} = \dfrac{x+y}{2xy} \qquad \ldots\ldots \text{(i)}$

It is a homogeneous differential equation.

$\therefore$ Put $x = vy$

Differentiating w.r.t. y, we get

$$\frac{dx}{dy} = v + y\frac{dv}{dy} \qquad \ldots\ldots \text{(ii)}$$

In Eq. (u) replacing $\dfrac{dx}{dy}$ by Eq. (ii), we get

$$v + y\frac{dv}{dy} = \frac{vy+y}{2y} = \frac{1+v}{2}$$

$$\Rightarrow \quad y\frac{dv}{dy} = \frac{1+v}{2} - v = \frac{1+v-2v}{2} = \frac{1-v}{2}$$

$$\Rightarrow \quad \frac{2}{1-v}dv = \frac{dy}{y}$$

Integrating, $\dfrac{2\log|1-v|}{-1} = \log|y| + \log C_1 \left(C_1 > 0\right)$

$\therefore \quad -2\log|y-x| + 2\log|y| = \log|y| + \log C_1$

$\Rightarrow \quad \log|y-x|^2 = \log|y| - \log C_1$

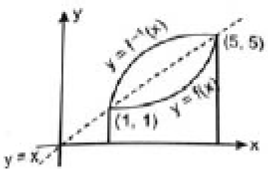
$\Rightarrow \quad \log|y-x|^2 = \log|y| + \log C$, where $\log C = -\log C_1$

$\Rightarrow \quad \log|y-x|^2 = \log|yC|$

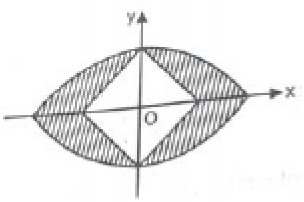
$\Rightarrow \quad (x-y)^2 = Cy$, is the required equation of family of curves.

75. $P'Q' = PQ\cos\left(90^o - \theta\right) = \dfrac{4}{\sqrt{t^2+1}}\left(t^2 < 1\right) \quad \left(P'Q'\right)_{\min} = 2\sqrt{2}$

76.



77. $|x| + |y| \ge 2$ and $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$



Ar. Of ellipse – Ar. Of square $= \pi(2)(3) - 8 = 6\pi - 8$

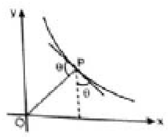
78.

$$\frac{dy}{dx}-\left(1+\frac{f'(x)}{f(x)}\right)y=f(x),\ I.F.=e^{-\int\left(1+\frac{f'(x)}{f(x)}\right)dx}=\frac{e^{-x}}{f(x)}$$

$$\frac{ye^{-x}}{f(x)}=\int e^{-x}dx+C\Rightarrow\frac{ye^{-x}}{f(x)}=-e^{-x}+C$$

79.

$$\tan\theta=\left|\frac{\frac{dy}{dx}-\frac{y}{x}}{1+\frac{y}{x}\cdot\frac{dy}{dx}}\right|=-\frac{dx}{dy}$$

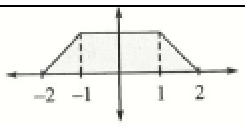


$$\Rightarrow\left(\frac{dy}{dx}\right)^2-\frac{2y}{x}\frac{dy}{dx}=1$$

80.

Let $f(x)=y\frac{dy}{dx}-2xy=\frac{e^{x^2}}{(x+1)^2}$

81.



Area bounded $=\frac{1}{2}\times1\times1+2\times1+\frac{1}{2}\times1\times1=3$

82.

$$\left(2xy\ dx+x^2dy\right)+x^2ydx+\left(\frac{y^3}{3}dx+y^2dy\right)=0$$

$x^2y=t$ and $\frac{y^3}{3}=u$

$(dt+tdx)+(u\,dx+du)=0$

$dt+du=-(t+u)dx$

$$\int\frac{dt+du}{t+u}=-\int dx$$

$In\,(t+u)=-x+C\Rightarrow x^2y+\frac{y^3}{3}=k'e^{-x}$

at $x=1,\ \ y=1$

$\frac{4}{3}=\frac{k'}{e}\Rightarrow k'=\frac{4e}{3}$

$x^2y+\frac{y^3}{3}=\frac{4e}{3}$; put $x=0$

$\frac{y^3}{3}=\frac{4e}{3}$

$\therefore y^3(0)=4e\Rightarrow k=4$

83.

$$f(x) = x^2 + \int_0^x e^{-t} f(x-t)\,dt = x^2 + \int_0^x e^{-(x-t)} f(t)\,dt = x^2 + e^{-x}\int_0^x e^{+t} f(t)\,dt$$

$$f'(x) = 2x + e^{-x}\left(e^x f(x) - 0\right) - e^{-x}\int_0^x e^t f(t)\,dt = 2x + f(x) - (f(x) - x^2) = x^2 + 2x$$

$$\Rightarrow f(x) = \frac{x^3}{3} + x^2 + k = \frac{x^3}{3} + x^2$$

($f(0) = 0$)

$f(1) + f(2) + \ldots\ldots\ + f(9) = 1/3$

$(1^3 + 2^3 + \ldots\ldots\ 9^3) + (1^2 + 1^2 + \ldots\ldots + 9^2)$

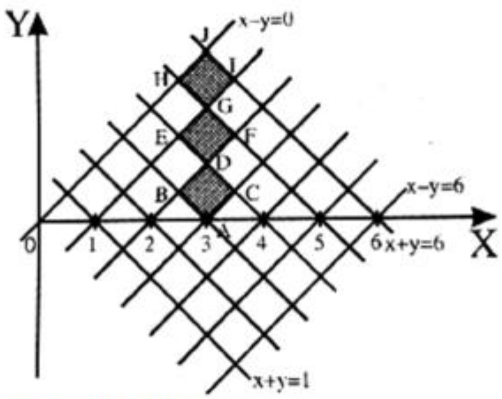
$$\frac{1}{3}\left(\frac{81\times 100}{4}\right) + \frac{9\times 10\times 19}{6} = 960$$

84.

$[x+y]+[x-y]=5$ For $[x-y]=0, [x+y]=5$

$\Rightarrow 0 \le x-y < 1, 5 \le x+y < 6$

Similarly for $1 \le x-y < 2, 4 \le x+y < 5$ and soon.



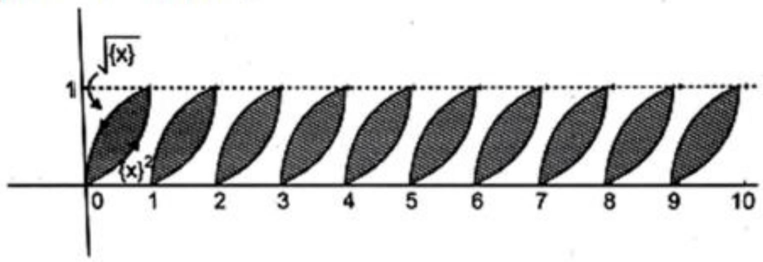
The required area

= area of rectangle (ABCD+DEGF+GHJI)

$= 3\left(\frac{1}{2}.1.1\right) = \frac{3}{2}$ square units

85.

As $f(x) = \begin{cases} \sqrt{\{x\}}, & x \notin z \\ 1, & z \in z \end{cases}$ and $g(x) = \{x\}^2$, where both $f(x)$ and $g(x)$ are [periodic with period '1' shown as;

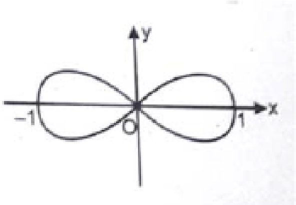


Thus, required area $= 10\int_0^1 \left[\sqrt{\{x\}} - \{x\}^2\right] dx$

$= 10\int_0^1 \left[(x)^{1/2} - x^2\right] dx$

$= 10\left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1$

$= 10\left(\frac{2}{3} - \frac{1}{3}\right) = \frac{10}{3}$ sq units

86.	Conceptual
87.	$xy(1+y)dx = dy \;\; if \;\; (0)=1 \;\; and \;\; f(2)=\dfrac{e^2}{k-e^2}$ $\Rightarrow xdx = \dfrac{1}{y(1+y)}dy \Rightarrow xdx = \dfrac{1+y-y}{y(1+y)}dy$ $\Rightarrow xdx = \left(\dfrac{1}{y}-\dfrac{1}{14y}\right)dy \Rightarrow \int \; xdx = \int \left(\dfrac{1}{y}-\dfrac{1}{1+y}\right)dy$ $\dfrac{x^2}{2} = \log y - \log(1+y) + \log C, f(0)=1 \Rightarrow \dfrac{O^2}{2} = \log_1 - \log_2 + \log C$ $\log C = \log_2 \Rightarrow C = 2$
88.	$f(x)=x^2$ $A = 2\int_0^1 \left(\sqrt{2-x^2}-x^2\right)dx = \dfrac{\pi}{2}+\dfrac{1}{3}$
89.	Required area $= 4\int_0^1 \sqrt{x^2-x^6}\,dx = \int_0^1 4x\sqrt{1-x^4}\,dx = \dfrac{\pi}{2}$  $x^2 = \sin\theta$
90.	$Ar = 4\int_0^1 \left(1-x^{2/5}\right)dx = 4\left(x-\dfrac{5}{7}x^{7/5}\right)_0^1 = \dfrac{8}{7}$