ANSWER KEYS

1. (4.00)	**2.** (1)	**3.** (3)	**4.** (1)	**5.** (2)	**6.** (3)	**7.** (2)	**8.** (2)
9. (3)	**10.** (3)						

1. (4.00)

Given, $\left(\frac{1+i}{1-i}\right)^n = 1$

$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n = 1$

$\Rightarrow \left(\frac{(1+i)^2}{1^2-i^2}\right)^n = 1$

$\Rightarrow \left(\frac{1^2+i^2+2i}{1-i^2}\right)^n = 1$

$\Rightarrow \left(\frac{2i}{2}\right)^n = 1$

$\Rightarrow i^n = 1$

Hence, n is an integer multiple of 4.

So, the smallest positive integer value of n is 4.

2. (1)

We have, $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow z = \frac{3+2i\cos\theta}{1-3i\cos\theta} \times \frac{1+3i\cos\theta}{1+3i\cos\theta}$

$\Rightarrow z = \frac{(3+2i\cos\theta)(1+3i\cos\theta)}{1+9\cos^2\theta}$

$\Rightarrow z = \frac{(3-6\cos^2\theta+8i\cos\theta)}{1+9\cos^2\theta}$

Now, $\text{Re}\left(z\right) = \frac{3-6\cos^2\theta}{1+9\cos^2\theta} = 0$

$\Rightarrow 3 - 6\cos^2\theta = 0$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, $\sin^2 3\theta + \cos^2\theta = \sin^2 3\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{4}\right) = 1.$

3. (3) Let $z = x + iy$

$\therefore \quad \left|\frac{z-25}{z-1}\right| = 5$

$\Rightarrow \left|\frac{(x-25)+iy}{(x-1)+iy}\right| = 5$

$\Rightarrow \quad |(x-25)+iy| = 5|(x-1)+iy|$

$\Rightarrow \quad \sqrt{(x-25)^2+y^2} = 5\sqrt{(x-1)^2+y^2}$

On squaring both sides, we get

$(x-25)^2 + y^2 = 25\left\{(x-1)^2 + y^2\right\}$

$\Rightarrow \quad x^2 - 50x + 625 + y^2 = 25x^2 - 50x + 25 + 25y^2$

$\Rightarrow \quad 24x^2 + 24y^2 = 600$

$\Rightarrow \quad x^2 + y^2 = 25$

$\Rightarrow \quad \sqrt{x^2+y^2} = 5 \qquad \left[\because |z| = \sqrt{(x^2+y^2)}\right]$

$\Rightarrow \quad |z| = 5$

4. **(1)**

If a complex number if purely imaginary, then it must be equal to minus times its conjugate.

$\Rightarrow \frac{z-\alpha}{z+\alpha}=-\left(\frac{\bar{z}-\alpha}{\bar{z}+\alpha}\right)$

$\Rightarrow z\bar{z}+\alpha z-\alpha\bar{z}-\alpha^2=-\left(z\bar{z}-\alpha z+\alpha\bar{z}-\alpha^2\right)$

$\Rightarrow |z|^2=\alpha^2$

$\Rightarrow \alpha^2=4$

$\Rightarrow \alpha=\pm 2$

5. **(2)** $\sqrt{x^2+y^2}-x\leq 1$
$\Rightarrow \sqrt{x^2+y^2}\leq x+1$
$\Rightarrow x^2+y^2\leq x^2+2x+1$
$\Rightarrow y^2\leq 2x+1$

6. **(3)** $\frac{(1+i)^5\left(1+\sqrt{3}i\right)^2}{-2i\left(-\sqrt{3}+i\right)}=\frac{\left(\sqrt{2}\right)^5\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^5.2^2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2}{(2i)\,2\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)}$

$\therefore$ argument $=\frac{5\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{2}+\frac{\pi}{6}=\frac{19\pi}{12}$

$\therefore$ principal argument is $-\frac{5\pi}{12}$

7. **(2)**

We know that, from the property of modulus function if
$|z_1|\leq a,\ a>0\Rightarrow -a\leq z_1\leq a$
So,
$|z+4|\leq 3\Rightarrow -3\leq z+4\leq 3$
$\Rightarrow -6\leq z+1\leq 0$
$\Rightarrow 0\leq -(z+1)\leq 6$
$\Rightarrow 0\leq|z+1|\leq 6$
Hence, the greatest and least values are 6 and 0 respectively.

8. **(2)**
It is given that
$\left|z+\frac{2}{z}\right|=2\ \Rightarrow|z|-\frac{2}{|z|}\leq 2$

$\Rightarrow |z|^2-2|z|-2\leq 0$

This is a quadratic equation in $|z|$

$\therefore \left|z\right|\leq\frac{2\pm\sqrt{4+8}}{2}$
$z\leq 1\pm\sqrt{3}$
$1-\sqrt{3}<z<1+\sqrt{3}$

Hence, maximum value of $|z|$ is $1+\sqrt{3}$

9. **(3)**

We have, $z^2 = \bar{z}$

Let $z = x + iy$

$z^2 = (x+iy)^2 = x^2 - y^2 + i2xy$ (i)

$\bar{z} = x - iy$ ...(ii)

From (i) and (ii)

on equating imaginary parts

$\Rightarrow 2xy = -y$

$\Rightarrow y(2x+1) = 0$

$\Rightarrow y = 0$ or $x = -\frac{1}{2}$

on equating real parts

$\Rightarrow x^2 - y^2 = x$

Case 1: when $y = 0$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$

$\Rightarrow x = 0$ or $x = 1$

Case 2: when $x = -\frac{1}{2}$

$\Rightarrow \frac{1}{4} - y^2 = -\frac{1}{2}$

$\Rightarrow y^2 = \frac{3}{4}$

$\Rightarrow y = \pm\frac{\sqrt{3}}{2}$

hence there exist 4 solution of z

10. **(3)**

$x + iy + \sqrt{2}|(x+1) + iy| + i = 0$

$x + \sqrt{2}\sqrt{(x+1)^2 + y^2} + (y+1)i = 0$

$x + \sqrt{2(x+1)^2 + 2y^2} + (y+1)i = 0$

$x + \sqrt{2(x+1)^2 + 2} = 0$ and $y = -1$

$\sqrt{2(x+1)^2 + 2} = -x$ and $y = -1$

$2(x+1)^2 + 2 = x^2$ and $y = -1$

$x^2 + 4x + 4 = 0$ and $y = -1$

$(x+2)^2 = 0$ and $y = -1$

$x = -2$ and $y = -1$

Hence, $z = x +$ iy $= -2 - i$