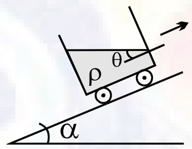
PHYSICS **Max Marks: 60**

Section-I
(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (A)(B),(C) and (D) out of which **ONE** or **MORE THAN ONE** are correct.

1. A fluid container is containing a liquid of density ρ is accelerating upward with acceleration a along the inclined place of inclination α as shown. Then the angle of inclination θ of free surface is :



(A) $\tan^{-1}\left[\frac{a}{g\cos\alpha}\right]$ (B) $\tan^{-1}\left[\frac{a+g\sin\alpha}{g\cos\alpha}\right]$

(C) $\tan^{-1}\left[\frac{a-g\sin\alpha}{g(1+\cos\alpha)}\right]$ (D) $\tan^{-1}\left[\frac{a-g\sin\alpha}{g(1-\cos\alpha)}\right]$

2. A body is imparted a velocity v from the surface of the earth. If v_0 is orbital velocity and v_e be the escape velocity then for

(A) $v = v_0$, the body follows a circular track around the earth.

(B) $v_0 < v < v_e$, the body follows elliptical path and around the earth

(C) $v < v_0$, the body follows elliptical path and returns to surface of earth.

(D) $v > v_e$, the body follows hyperbolic path and escapes the gravitational pull of the earth.

3. Three point masses each of mass m are at the corners of an equilateral triangle of side l. The system rotates about the centre of the triangle with the separation of masses not changing during rotation. If T is the time period of rotation then,

(A) $T \propto l^{\frac{3}{2}}$ (B) $T \propto l^{\frac{1}{2}}$ (C) $T \propto m^{\frac{1}{2}}$ (D) $T \propto m^{-\frac{1}{2}}$

4. Let V and E be the gravitational potential and gravitational field, r is the distance from the centre. Then select the correct alternative(s)

(A) the plot of V vs r is continuous for a spherical shell

(B) the plot of E vs r is discontinuous for a spherical shell

(C)the plot of V vs r is continuous for a solid sphere

(D) the plot of E vs r is discontinuous for a solid sphere

5. For a planet revolving round the sun in an elliptical orbit, the

(A) potential energy is constant

(B) kinetic energy is constant

(C) total mechanical energy is constant

(D) angular momentum about centre of sun is constant

6. A double star consists of two stars having masses m and $2m$ separated by a distance r. Which of the following statement(s) is/are incorrect ?

(A) Radius of circular path of star of mass 2m is $\frac{2r}{3}$

(B) Kinetic energy of 2m mass star is double that of lighter star

(C) Time period of revolution of both are not same

(D) Angular momentum of lighter star is more

7. A solid sphere of radius R and density ρ is attached to one end of a massless spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

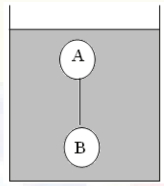
(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the lighter sphere is partially submerged

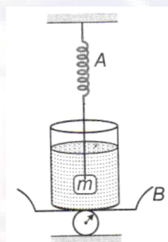
(D) the lighter sphere is completely submerged

8. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



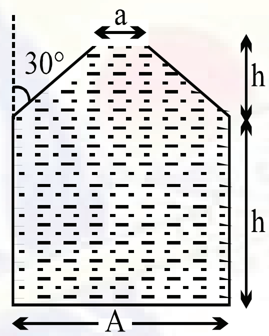
(A) $d_A < d_F$ (B) $d_B > d_F$ (C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

9. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation.



(A) the balance A will read more than 2kg
(B) the balance B will read more than 5 kg
(C) the balance A will read less than 2 kg
(D) the balances A and B will read 2 kg and 5 kg respectively

10. The vessel shown in the figure has two sections. The lower part is a rectangular vessel with area of cross-section A and height h. The upper part is a conical vessel of height h with base area 'A' and top area 'a' and the walls of the vessel are inclined at an angle 30° with the vertical. A liquid of density ρ fills both the sections upto a height 2h. Neglecting atmospheric pressure.



(A) The force F exerted by the liquid on the base of the vessel is $2h\rho g\frac{(A+a)}{2}$

(B) The pressure P at the base of the vessel is $2h\rho g\ \frac{A}{a}$

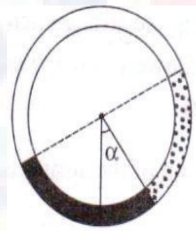
(C) the weight of the liquid W is greater than the force exerted by the liquid on the base

(D) the walls of the vessel exert a downward force (F–W) on the liquid.

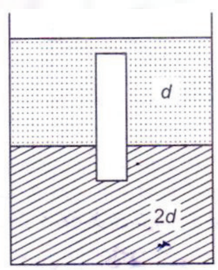
Section-II
(Integer Value Correct Type)

This section contains 10 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

11. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90^0 angle at centre. Radius joining their interface makes an angle $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$ with vertical. Ratio d_1 / d_2 is

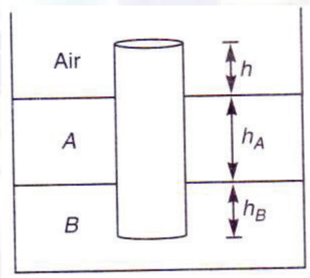


12. A homogeneous solid cylinder of length L and cross-sectional area A/5 is immersed such that it floats with its axis vertical at the liquid-liquid interface with length L/4 in the denser liquid as shown in the figure. The low density liquid is open to atmosphere having pressure p_0. If d = 4kg/ m^3 then, density D of solid is ______ kg/m^3



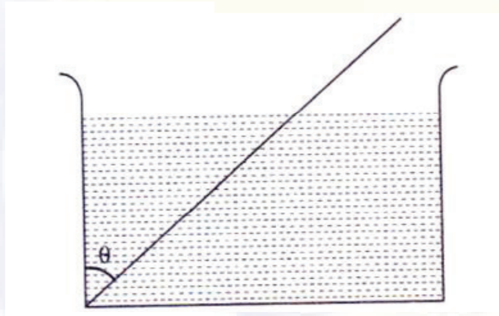
13. A metal ball of density 7800 kg/m^3 is suspected to have a large number of cavities. Its weight 9.8 kg when weighed directly on a balance and 1.5kg less when immersed in water. The percentage by volume of the cavities in the metal ball is approximately 2n%. Find n=?

14. A uniform solid cylinder of density $0.8\ g/cm^3$ floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of the liquids A and B are $0.7\ g/cm^3$ and $1.2\ g/cm^3$, respectively. The height of liquid A is $h_A = 1.2cm$. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8cm$

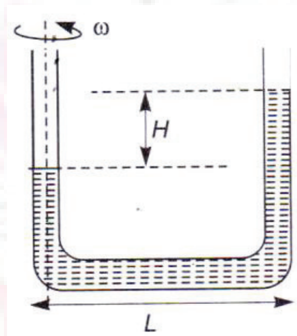


The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. The acceleration of the cylinder immediately after it is released is $\frac{g}{n}$. The value of n is

15. A wooden plank of length 1m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water upto a height 0.5m. The specific gravity of the plank is 0.5. The angle θ that the plank makes with the vertical in the equilibrium position is $\frac{\pi}{n}$. The value of n is (exclude the case $\theta = 0^0$).



16. A U – shaped tube contains a liquid of density ρ and it is rotated about the line as shown in the figure. The difference in the levels of liquid column is $\frac{\omega^2 L^2}{ng}$. The value of n is



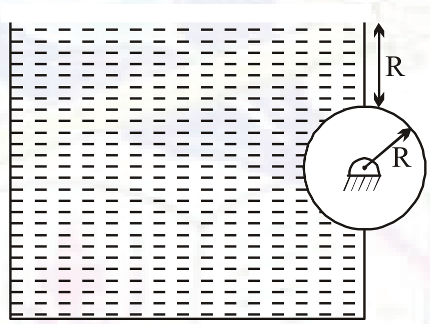
17. A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density d_L. If $d = 2kg/m^3$, $d_L = 4kg/m^3$ and $t_1 = 4\sec$ the time t_2 the ball takes to come back to the position from which it was released is ?

(Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large).

18. Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then T^2 is proportional to $R^{\frac{x}{2}}$ the value of x is

19. A block of ice with an area of cross section A and a height h floats in water of density ρ_0 kept in a beaker of large surface area. The work that should be performed by the external agent to slowly submerge the ice block completely into water if density of ice is ρ_1 is $\frac{Agh^2(\rho_0-\rho_1)^2}{n\rho_0}$ .Then find n=?

20. A cylinder of radius R is kept embedded along the wall of A dam as shown. Take density of water as ρ. Take length as L.(side view shown) (neglect atmospheric pressure)



The vertical force exerted by water pressure on the cylinder is $\frac{\rho\pi R^2 Lg}{n}$. Then find n=?

Sec: Jr.Super60 Dt: 12-09-16

Time: 07:30 AM to 10:30 AM WTA-19 Max.Marks: 180

KEY SHEET

PHYSICS

1	B	2	ABCD	3	AD	4	ABC	5	CD	6	ABC
7	AD	8	ABD	9	BC	10	D	11	3	12	5
13	8	14	6	15	4	16	2	17	8	18	7
19	2	20	2								

CHEMISTRY

21	ABCD	22	ABD	23	ABC	24	D	25	BCD	26	ACD
27	ABCD	28	A	29	AD	30	AC	31	8	32	5
33	6	34	8	35	4	36	3	37	7	38	7
39	3	40	5								

MATHS

41	AC	42	BC	43	ACD	44	BCD	45	AD	46	AB
47	AC	48	ACD	49	ACD	50	ABCD	51	6	52	8
53	1	54	5	55	6	56	5	57	7	58	6
59	2	60	8								

SOLUTIONS

PHYSICS

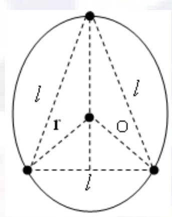
1. Free surface of liquid is perpendicular to effective value of acceleration due to gravity.

2.

$v = v_0$	Circular path around the earth.
$v < v_0$	Elliptical path and body returns to earth
$v > v_0$ but $< v_e$	Elliptical path around the earth will not escape
$v = v_e$	Parabolic path and it escapes from the earth
$v > v_e$	Hyperbolic path and escapes from earth.

3. The force of attraction between any two point masses is responsible for providing the necessary centripetal force to a mass to revolve in a circle of radius r. Using trigonometry, we get

$$\cos 30 = \frac{l/2}{r} = \frac{l}{2r}$$



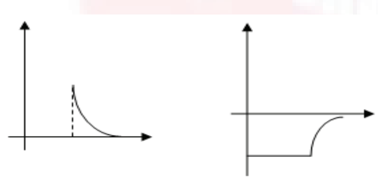
$$\Rightarrow r = \frac{l}{2\cos 30} \qquad \Rightarrow r = \frac{l}{\sqrt{3}}$$

$$\Rightarrow mr\omega^2 = \frac{Gmm}{l^2}\sqrt{3} \qquad \Rightarrow m\frac{l}{\sqrt{3}}\omega^2 = \frac{Gm^2}{l^2}\sqrt{3}$$

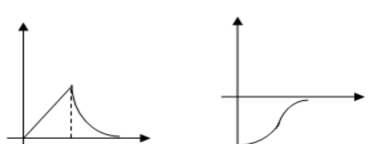
$$\Rightarrow \omega = \sqrt{\frac{3Gm}{l^3}} \qquad T = \frac{2\pi}{\omega}$$

$$\Rightarrow T \propto l^{\frac{3}{2}} \ and \quad T \propto m^{-\frac{1}{2}}$$

4. E – r and V – r graphs for a spherical shell and a solid sphere are shown here.



For a shell



For a solsid sphere

5. Distance from centre of sun and hence the kinetic energy and potential energy keep changing.

6. Both the stars will revolve about their centre of mass. So, if the centre of mass be at a distance x from 2m, then

$$x = \frac{2m(0) + mr}{3m} = \frac{r}{3}$$

So, $r_1 = \frac{2r}{3}$ and $r_2 = \frac{r}{3}$

ω and T will be same for both the stars, so

$K_1 = \frac{1}{2} I_1 \omega^2$ and $K_2 = \frac{1}{2} I_2 \omega^2$

$$\Rightarrow \frac{K_1}{K_2} = \frac{I_1}{I_2} = \frac{m\left(\frac{2r}{3}\right)^2}{2m\left(\frac{r}{3}\right)^2} = 2$$

$L_1 = I_1\omega$ and $L_2 = I_2\omega$ $\quad \Rightarrow \frac{L_1}{L_2} = \frac{I_1}{I_2} = 2$

7. On small sphere

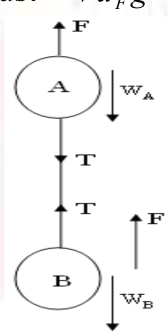
$$\frac{4}{3}\pi R^3(\rho)g + kx = \frac{4}{3}\pi R^3(2\rho)g \quad \ldots\ldots\ldots\ldots\ldots(i)$$

On second sphere (large)

$$\frac{4}{3}\pi R^3(3\rho)g = \frac{4}{3}\pi R^3(2\rho)g + kx \quad \ldots\ldots\ldots\ldots(ii)$$

By Eqs. (I and (ii), we get $x = \frac{4\pi R^3 \rho g}{3k}$

8. F = Upthrust = $Vd_F g$



Equilibrium of A

$$Vd_F g = T + W_A$$
$$= T + Vd_A g \quad \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Equilibrium of B

$$T + Vd_F g = Vd_B g \quad \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Adding Eqs. (i) and (ii), we get

$$2d_f = d_A + d_B$$

$\therefore$ Option (A) is correct.

From Eq.(ii) we can see that, $d_B > d_F$

$\therefore$ Option (B) is correct.

$\therefore$ Correct options are (a), (b) and (d).

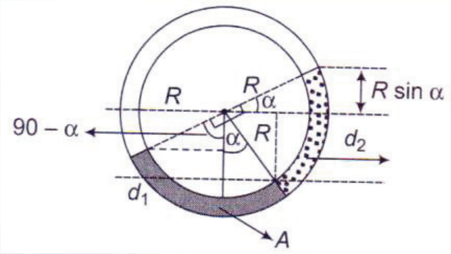
9. Liquid will apply an upthrust on m. An equal force will be exerted (from Newton's third law) on the liquid. Hence, A will read less than 2 kg and B more than 5 kg. Therefore, the correct options are (B) and (C).

10. At bottom $P = \rho g(2h)$

$\therefore$ Force on base$=(2\rho gh)(A)$

11. Equating pressure at A, we get

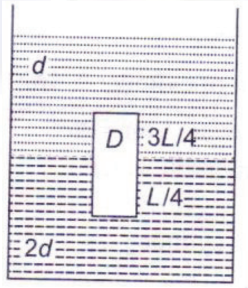
$R\sin\alpha d_2 + R\cos\alpha d_2 + R(1-\cos\alpha)d_1 = R(1-\sin\alpha)d_1$



$(\sin\alpha + \cos\alpha)d_2 = d_1(\cos\alpha - \sin\alpha)$

$\Rightarrow \frac{d_1}{d_2} = \frac{1+\tan\alpha}{1-\tan\alpha}$

12.



Considering vertical equilibrium of cylinder

Weight of cylinder = Upthrust due to upper liquid + upthrust due to lower liquid

$\therefore (A/5)(L)Dg = (A/5)(3L/4)(d)g + (A/5)(L/4)(2d)(g)$

$\therefore \quad D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$

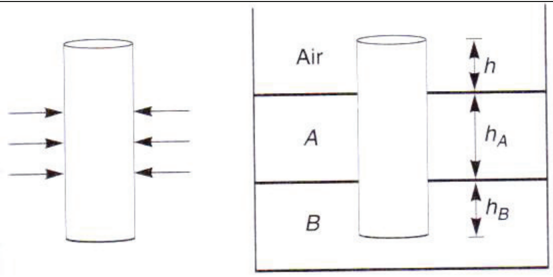
$D = \frac{5}{4}d$

13.

$V_1\rho_1 = 9.8$

$(V_1 + V_2)\rho_2 = 1.5$

14.



Net upward force = extra upthrust = $sh\rho_B g$

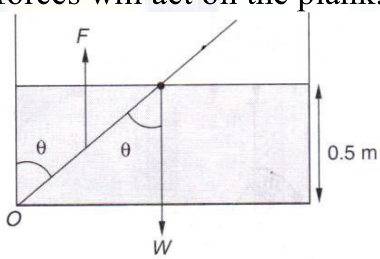
$\therefore$ Net acceleration $a = \frac{\text{force}}{\text{mass of cylinder}}$

Or $a = \frac{sh\rho_B g}{s(h+h_A+h_B)\rho_{\text{cylinder}}}$ or $a = \frac{h\rho_B g}{(h+h_A+h_B)\rho_{\text{cylinder}}}$

Substituting the values of $h, h_A h_B, \rho_B$ and $\rho_{\text{cylinder'}}$

We get, $a = \frac{g}{6}$ (upwards)

15. Submerged length = 0.5 sec θ, F = Upthrust, W = Weight
Three forces will act on the plank.



Force from the hinge at θ.

Taking moments of all three forces about point O. Moment of hinge force will be zero.

$\therefore$ Moment of W (clockwise)

$= \text{Moment of F}\left(\text{anti - clockwise}\right)$

$\therefore (A l g\rho)\frac{l}{2}\sin\theta = A(0.5\sec\theta)(\rho_w)(g)\left(\frac{0.5\sec\theta}{2}\right)\sin\theta$

$\therefore \quad \cos^2\theta = \frac{(0.5)^2(1)}{(l^2)(\rho)}$

$= \frac{(0.5)^2}{(0.5)} = \frac{1}{2}$ (as $l = 1m$

$\therefore \quad \cos\theta = \frac{1}{\sqrt{2}}$ or $\theta = 45^0$

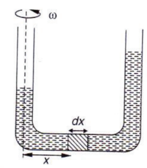
16. For circular motion of small element dx, we have

$dF = (dm)x\,\omega^2$

$\therefore \quad (dp)A = (\rho A dx).x\omega^2$

or $\quad dp = \rho\,\omega^2 x.dx$

$\therefore \int_{p_1}^{p_2} dp = \rho\, \omega^2 \int_0^L x dx$ $\therefore\ p_2 - p_1 = \dfrac{\rho\omega^2 L^2}{2}$



$\therefore\ \rho g H = \dfrac{\rho\omega^2 L^2}{2}$ $\therefore\ H = \dfrac{\omega^2 L^2}{2g}$

17. In elastic collision with the surface, direction of velocity is reversed but its magnitude remains the same. Therefore, time of all = time of rise.

Or time of fall = $\dfrac{t_1}{2}$

Hence, velocity of the ball just before it collides with liquid is

$v = g\dfrac{t_1}{2}$ ....(i)

Retardation inside the liquid,

$a = \dfrac{\text{upthrust-weight}}{\text{mass}}$

$= \dfrac{Vd_{Lg} - V_{dg}}{V_d} = \left(\dfrac{d_L - d}{d}\right)g$ ....(ii)

Time taken to come to rest under this retardation will be

$t = \dfrac{v}{a} = \dfrac{gt_1}{2a} = \dfrac{gt_1}{2\left(\dfrac{d_1 - d}{d}\right)g}$

$= \dfrac{dt_1}{2(d_L - d)}$

Same will be the time to come back on the liquid surface.

Therefore,

t_2 = time the ball takes to came back to the position from where it was released

$= t_1 + 2t = t_1 + \dfrac{dt_1}{d_L - d} = t_1\left[1 + \dfrac{d}{d_L - d}\right]$ or $t_2 = \dfrac{t_1 d_L}{d_L -}d$

18. $\dfrac{mv^2}{R} \propto R^{-5/2}$

$\therefore\ v \propto R^{-3}/4$

Now, $T = \dfrac{2\pi R}{v}$ or $T^2 \propto \left(\dfrac{R}{v}\right)^2$ or $T^2 \propto \left(\dfrac{R}{R^{-3/4}}\right)$ $or\ T^2 \propto R^{7/2}$

19. $F = A\rho_2 g y$

$W = \int F dy$

20. Same as bouncy force on left half of the cylinder if left completely dipped cylinder B