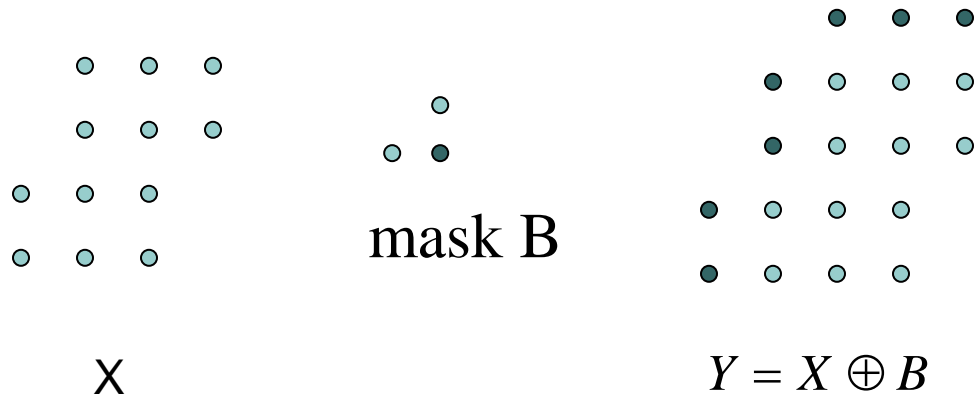

A MATLAB Tour of Morphological Filtering

Dilation

$$Y = X \oplus B = \bigcup_{b \in B} X_b = \bigcup_{x \in X} B_x = B \oplus X$$

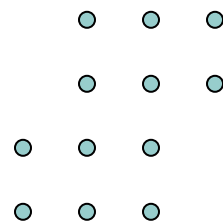


```
>X=zeros(6,7);X(2:3,4:6)=1;X(4:5,3:5)=1;
>se=[0 1;1 1];
>Y=imdilate(X,se);
```

Erosion

$$Y = X \ominus B = \{x : B_x \subset X\}$$

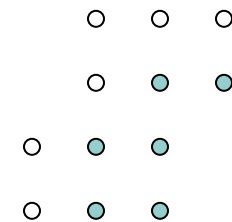
Erosion



X



mask B



$Y = X \ominus B$

```
>X=zeros(6,7);X(2:3,4:6)=1;X(4:5,3:5)=1;
>se=[0 1;1 1];
>Y=imerode(X,se);
```

Relationship Between Dilation and Erosion

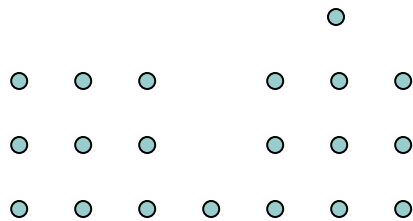
$$(X \ominus B)^c = X^c \oplus \hat{B}$$

```
>se1=[0 1;1 1];  
>se2=fliplr(flipud(se1));  
>X1=imdilate((1-X),se2)  
>X2=1-imerode(X,se1)
```

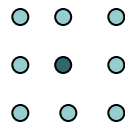
Opening

$$X \circ B = (X \ominus B) \oplus B$$

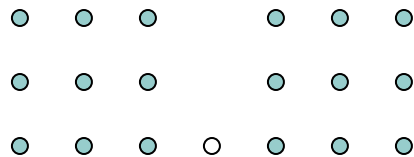
X



mask B



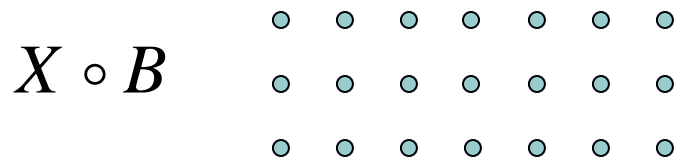
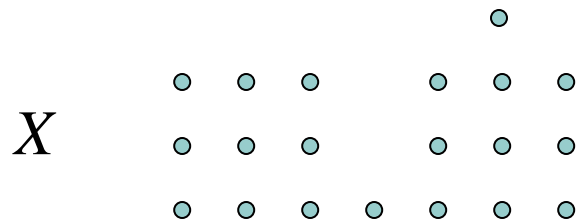
$X \circ B$



```
>X=zeros(6,9);X(3:5,2:4)=1;
>X(3:5,6:8)=1;X(5,5)=1;X(2,7)=1
>se=strel('square',3);
>Y=imdilate(imerode(X,se),se);
%Y=bwmorph(X,'open');
```

Closing

$$X \bullet B = (X \oplus B) \ominus B$$



```
>X=zeros(6,9);X(3:5,2:4)=1;
>X(3:5,6:8)=1;X(5,5)=1;X(2,7)=1
>se=strel('square',3);
>Y=imerode(imdilate(X,se),se);
%Y=bwmorph(X,'close');
```

Relationship Between Opening and Closing

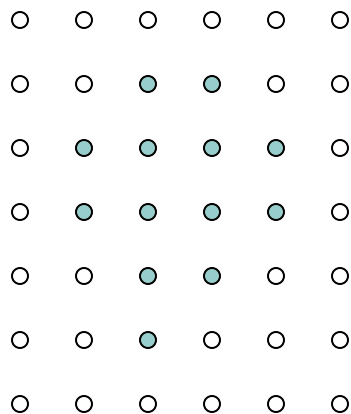
$$(X \bullet B)^C = X^C \circ \hat{B}$$

$$(X \circ B)^C = X^C \bullet \hat{B}$$

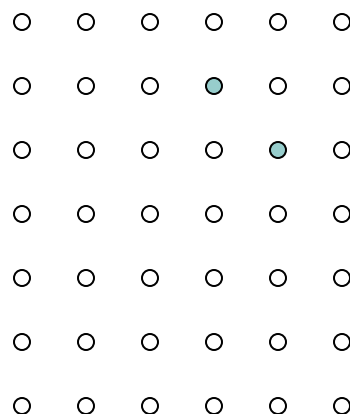
```
>se=strel('line',10,45);  
>se2=fliplr(flipud(se));  
>X1=~imopen(X,se);  
>X2=imclose(~X, se2);  
>isequal(X1,X2)  
>X1=~imclose(X,se);  
>X2=imopen(~X, se2);  
>isequal(X1,X2)
```

Hit-Miss Operator

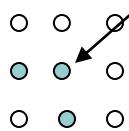
$$X \circledast B = (X \ominus B_1) \cap (X^c \ominus B_2)$$



X

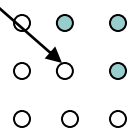


$X \circledast B$



mask B_1

origin



mask B_2

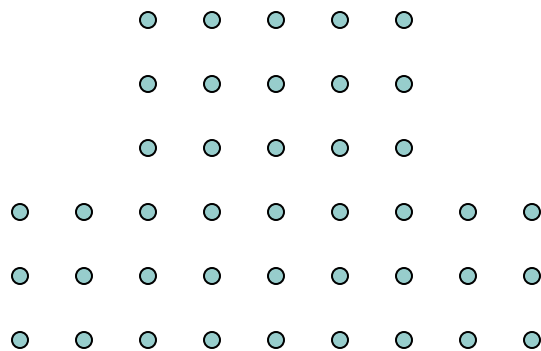
```
>bw = [0 0 0 0 0 0;
        0 0 1 1 0 0;
        0 1 1 1 1 0;
        0 1 1 1 1 0;
        0 1 1 1 1 0;
        0 0 1 1 0 0;
        0 0 1 0 0 0]
```

```
>se = [0 -1 -1;
        1 1 -1; 0 1 0];
```

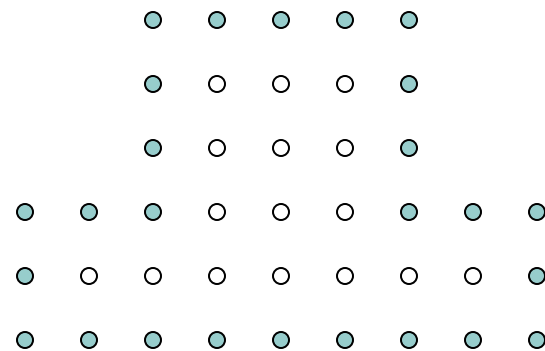
```
>bw2 = bwhitmiss(bw,se)
```


Boundary Extraction

$$\partial X = X - (X \ominus B)$$



X



∂X

```
> X=zeros(8,11);X(2:4,4:8)=1;X(5:7,2:10)=1;
> se=strel('square',3);
➤ Y=X-imerode(X,se);
% Y=imdilate(X,se)-X;
```

Image Example



X

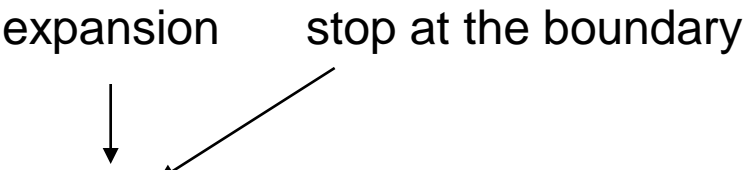
∂X

Region Filling

Pseudo
Codes
of
Region
Filling

Iterations:

expansion stop at the boundary

$$Y_0 = P$$
$$Y_k = (Y_{k-1} \oplus B) \cap X^c, k=1,2,3\dots$$


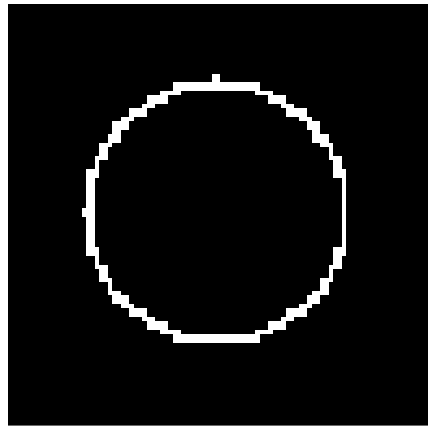
Terminate when $Y_k = Y_{k-1}$, output $Y_k \cup X$

MATLAB
Codes
of
Region
Filling

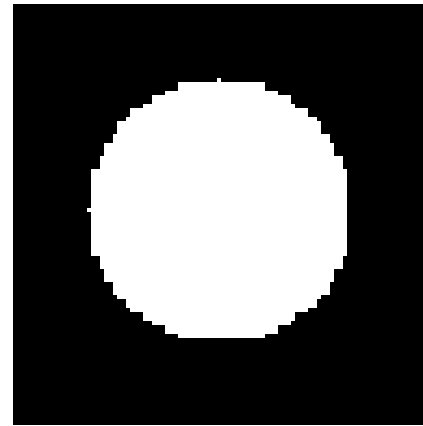
```
function out=region_fill(in,pos,kernel)

current=zeros(size(in));
last=current;
last(pos(1),pos(2))=1;
current=imdilate(last,kernel)&~in;
while any(current(:)~=last(:));
    last=current;
    current=imdilate(last,kernel)&~in;
end
out=current+in;
```

Image Example



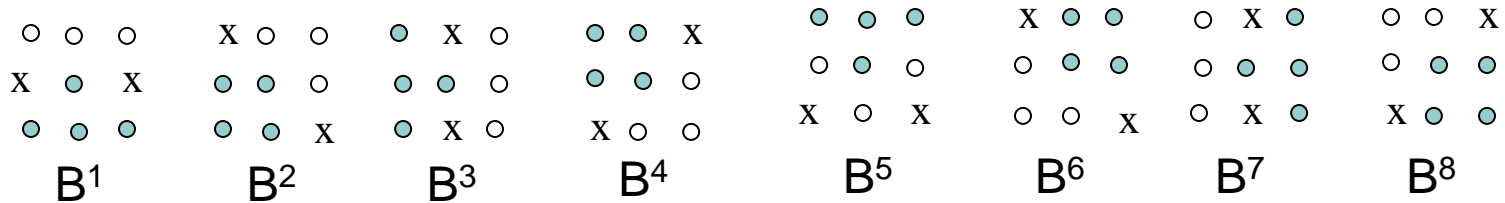
Y



Z

```
>se=strel('square',3);  
>r=round(size(Y,1)/2);  
> c=round(size(Y,2)/2);  
>Z=region_fill(Y,[r,c],se);
```

Thinning



$$X_0 = X$$

$$X_k = (\dots ((X_{k-1} \otimes B^1) \otimes B^2 \dots \otimes B^8)$$

where $X \otimes B = X - X \circledast B$

Stop the iteration when $X_k = X_{k-1}$

Question unanswered on the blackboard:
For B^2, \dots, B^8 , do they operate on $X_{k-1} \otimes B^1$ or X_{k-1} ?

MATLAB Implementation

```
function y=thinning(x,iter)
```

```
se{1}=[-1 -1 -1;0 1 0;1 1 1];  
se{2}=[0 -1 -1;1 1 -1;1 1 0];  
se{3}=fliplr(rot90(se{1}));  
se{4}=flipud(se{2});  
se{5}=flipud(se{1});  
se{6}=fliplr(se{4});  
se{7}=fliplr(se{3});  
se{8}=fliplr(se{2});
```

```
y=x;z=x;  
for i=1:iter  
    for k=1:8  
        %y=y&~bwhitmiss(y,se{k});  
        y=y&~bwhitmiss(z,se{k});  
    end  
    z=y;  
end
```

Scheme
A

Scheme
B

Which One is Right?



Original
image

Scheme
A

Scheme
B

Result by Using BWMORPH



Scheme
A

Scheme
B

```
> y=bwmorph(x,'thin',inf);
```


Summary of Morphological Filtering

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w w = a + z, \text{ for } a \in A\}$	Translates the origin of A to point z .
Reflection	$\hat{B} = \{w w = -b, \text{ for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w w \notin A\}$	Set of points not in A .
Difference	$A - B = \{w w \in A, w \notin B\}$ $= A \cap B^c$	Set of points that belong to A but not to B .
Dilation	$A \oplus B = \{z (\hat{B})_z \cap A \neq \emptyset\}$	"Expands" the boundary of A . (I)
Erosion	$A \ominus B = \{z (B)_z \subseteq A\}$	"Contracts" the boundary of A . (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smooths contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smooths contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

MATLAB codes

`circshift(A,z)`

`fliplr(flipud(B))`

`~A` or `1-A`

`A & ~B`

`imdilate(A,B)`

`imerode(A,B)`

`imopen(A,B)`

`imclose(A,B)`

Summary (Con'd)

Hit-or-miss transform	$A \odot B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c .	<code>bwhitmiss(A,B)</code>
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A . (I)	<code>A & ~ (imerode(A,B))</code>
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p \text{ and } k = 1, 2, 3, \dots$	Fills a region in A , given a point p in the region. (II)	<code>region_fill.m</code>
Thinning	$A \otimes B = A - (A \oplus B)$ $= A \cap (A \oplus B)^c$ $A \otimes \{B\} =$ $((\dots ((A \otimes B^1) \otimes B^2) \dots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$	<p>Thins set A. The first two equations give the basic definition of thinning.</p> <p>The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)</p>	<code>bwmorph(A,'thin');</code>