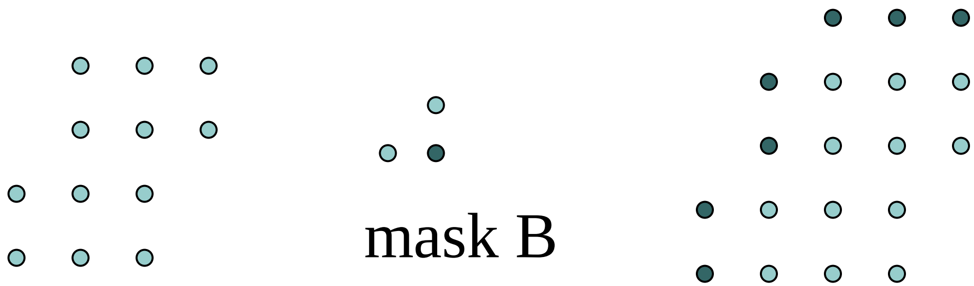
A MATLAB Tour of Morphological Filtering

Dilation

$$Y = X \oplus B = \bigcup_{b \in B} X_b = \bigcup_{x \in X} B_x = B \oplus X$$



X

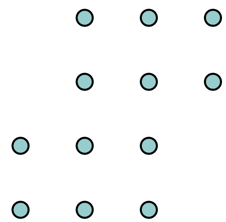
$Y = X \oplus B$

```
>X=zeros(6,7);X(2:3,4:6)=1;X(4:5,3:5)=1;
>se=[0 1;1 1];
>Y=imdilate(X,se);
```

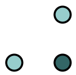
Erosion

$$Y = X \ominus B = \{x : B_x \subset X\}$$

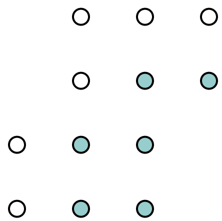
Erosion



X



mask B



$Y = X \ominus B$

```
>X=zeros(6,7);X(2:3,4:6)=1;X(4:5,3:5)=1;
>se=[0 1;1 1];
>Y=imerode(X,se);
```

Relationship Between Dilation and Erosion

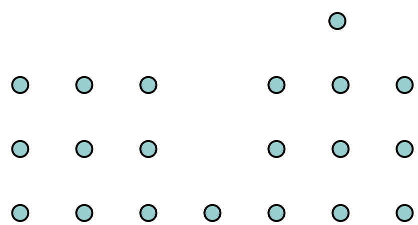
$$(X \ominus B)^c = X^c \oplus \hat{B}$$

```
>se1=[0 1;1 1];
>se2=fliplr(flipud(se1));
>X1=imdilate((1-X),se2)
>X2=1-imerode(X,se1)
```

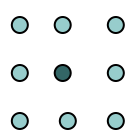
Opening

$$X \circ B = (X \ominus B) \oplus B$$

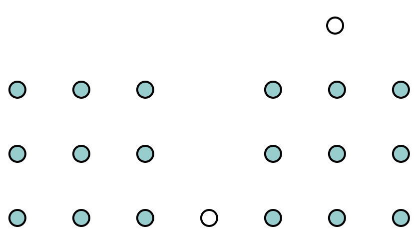
X



mask B



$X \circ B$

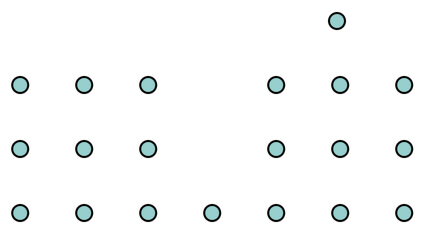


```
>X=zeros(6,9);X(3:5,2:4)=1;
>X(3:5,6:8)=1;X(5,5)=1;X(2,7)=1
>se=strel('square',3);
>Y=imdilate(imerode(X,se),se);
%Y=bwmorph(X,'open');
```

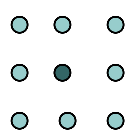
Closing

$$X \bullet B = (X \oplus B) \ominus B$$

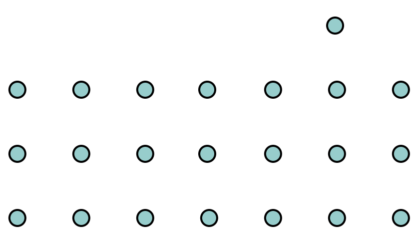
X



mask B



$X \circ B$



```
>X=zeros(6,9);X(3:5,2:4)=1;
>X(3:5,6:8)=1;X(5,5)=1;X(2,7)=1
>se=strel('square',3);
>Y=imerode(imdilate(X,se),se);
%Y=bwmorph(X,'close');
```

Relationship Between Opening and Closing

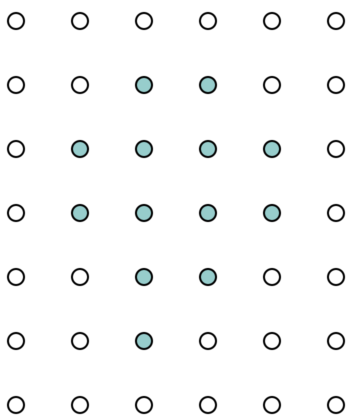
$$(X \bullet B)^C = X^C \circ \hat{B}$$

$$(X \circ B)^C = X^C \bullet \hat{B}$$

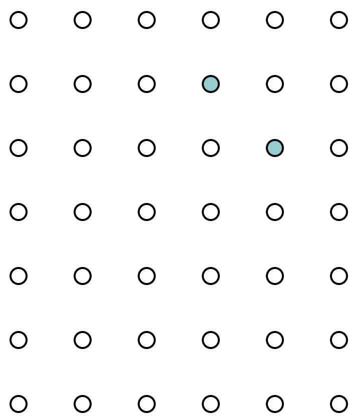
```
>se=strel('line',10,45);
>se2=fliplr(flipud(se));
>X1=~imopen(X,se);
>X2=imclose(~X, se2);
>isequal(X1,X2)
>X1=~imclose(X,se);
>X2=imopen(~X, se2);
>isequal(X1,X2)
```

Hit-Miss Operator

$$X \circledast B = (X \ominus B_1) \cap (X^c \ominus B_2)$$

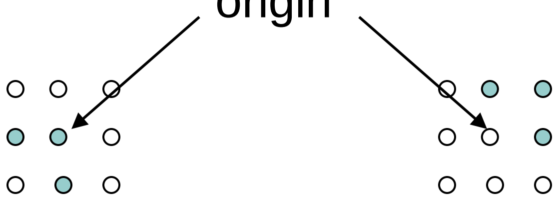


X



$X \circledast B$

origin

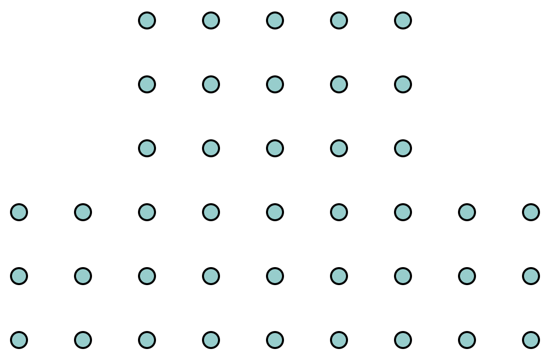


mask B_1 mask B_2

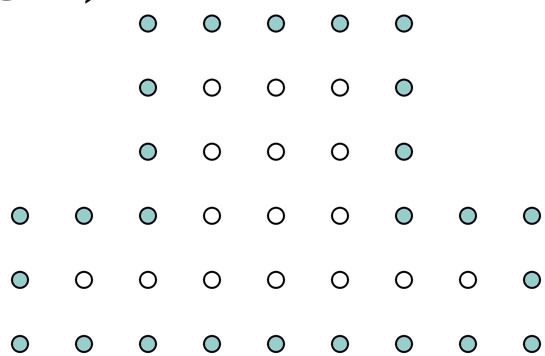
```
>bw = [0 0 0 0 0 0;
         0 0 1 1 0 0;
         0 1 1 1 1 0;
         0 1 1 1 1 0;
         0 0 1 1 0 0;
         0 0 1 0 0 0]
>se = [0 -1 -1;
1  1 -1; 0  1  0];
>bw2 = bwhitmiss(bw,se)
```

Boundary Extraction

$$\partial X = X-(X \ominus B)$$



X



∂X

```
>X=zeros(8,11);X(2:4,4:8)=1;X(5:7,2:10)=1;
> se=strel('square',3);
➢Y=X-imerode(X,se);
%Y=imdilate(X,se)-X;
```

Image Example

X

∂X

Region Filling

Pseudo Codes of Region Filling

Iterations:

expansion stop at the boundary

$$Y_0=P$$

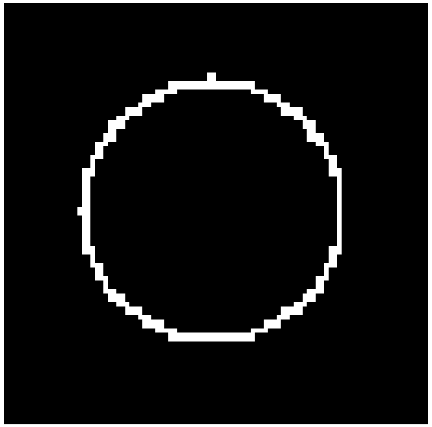
$$Y_k=(Y_{k-1}\oplus B)\cap X^c,\ k=1,2,3\ldots$$

$$\text{Terminate when } Y_k=Y_{k-1}, \text{output } Y_k \cup X$$

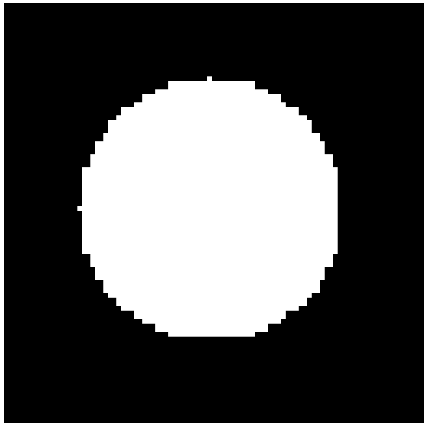
MATLAB Codes of Region Filling

```
function out=region_fill(in,pos,kernel)

current=zeros(size(in));
last=current;
last(pos(1),pos(2))=1;
current=imdilate(last,kernel)&~in;
while any(current(:)~=last(:));
    last=current;
    current=imdilate(last,kernel)&~in;
end
out=current+in;
```

Image Example



Y



Z

```
>se=strel('square',3);
>r=round(size(Y,1)/2);
> c=round(size(Y,2)/2);
>Z=region_fill(Y,[r,c],se);
```

Thinning

○	○	○
x	●	x
●	●	●

B^1

x	○	○
●	●	○
●	●	x

B^2

●	x	○
●	●	○
●	x	○

B^3

●	●	x
●	●	○
x	○	○

B^4

●	●	●
○	●	○
x	○	x

B^5

x	●	●
○	●	●
○	○	x

B^6

○	x	●
○	●	●
○	x	●

B^7

○	○	x
○	●	●
x	●	●

B^8

$$X_0 = X$$

$$X_k = (\ldots((X_{k-1} \otimes B^1) \otimes B^2 \ldots \otimes B^8)$$

where $X \otimes B = X - X \circledast B$

Stop the iteration when $X_k = X_{k-1}$

Question unanswered on the blackboard:
For $B^2_{,}, \ldots, B^8$, do they operate on $X_{k-1} \otimes B^1$ or X_{k-1}?

MATLAB Implementation

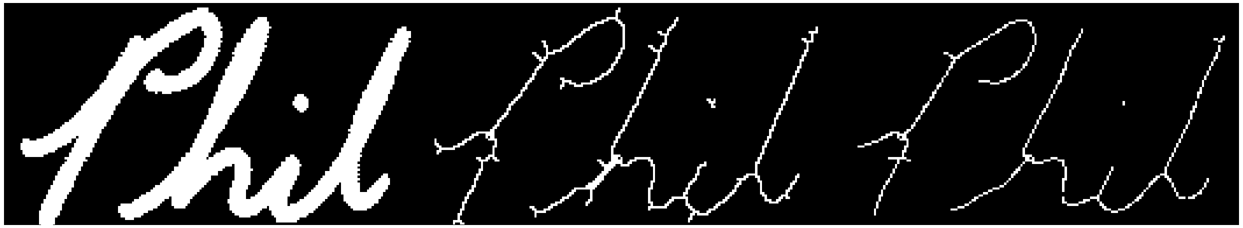
```
function y=thinning(x,iter)

se{1}=[-1 -1 -1;0 1 0;1 1 1];
se{2}=[0 -1 -1;1 1 -1;1 1 0];
se{3}=fliplr(rot90(se{1}));
se{4}=flipud(se{2});
se{5}=flipud(se{1});
se{6}=fliplr(se{4});
se{7}=fliplr(se{3});
se{8}=fliplr(se{2});
```

```
y=x;z=x;
for i=1:iter
for k=1:8
%y=y&~bwhitmiss(y,se{k});
y=y&~bwhitmiss(z,se{k});
end
z=y;
end
```

Scheme A

Scheme B

Which One is Right?



Original image

Scheme A

Scheme B

Result by Using BWMORPH

Scheme A

Scheme B

> y=bwmorph(x,'thin',inf);

Summary of Morphological Filtering

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).	MATLAB codes
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.	circshift(A,z)
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.	fliplr(flipud(B))
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.	~A or 1-A
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.	A &~B
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)	imdilate(A,B)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)	imerode(A,B)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)	imopen(A,B)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)	imclose(A,B)

Summary (Con'd)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.	bwhitmiss(A,B)
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)	A&~(imerode(A,B))
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)	region_fill.m
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)	bwmorph(A,'thin');