

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \dots \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \dots & \dots \end{bmatrix} \quad \boxed{\text{hilbert_cond. m}} \quad \begin{aligned} h_{ij} &= \frac{1}{i+j-1} \\ i, j &= 1, \dots, n \\ (&\gg \text{hilb}(n)) \end{aligned}$$

$$Hx = f \quad x = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \Rightarrow f = H * x$$

$$\tilde{f} = \begin{bmatrix} f_1 + \varepsilon \\ f_2 \\ \vdots \\ f_n + \varepsilon \end{bmatrix} \quad H\tilde{x} = \tilde{f}$$

$$\frac{\|x - \tilde{x}\|}{\|x\|} \quad \begin{array}{l} \text{Errore relativo} \\ \text{sulla soluzione} \end{array} \quad \text{norme } 1, 2, \infty$$

$$\frac{\|f - \tilde{f}\|}{\|f\|} \quad \begin{array}{l} \text{Errore relativo} \\ \text{sul dato (termine noto)} \end{array}$$

$$\underbrace{\frac{\|x - \tilde{x}\|}{\|x\|}}_{\text{err}} \leq K(H) \underbrace{\frac{\|f - \tilde{f}\|}{\|f\|}}_{\text{res}} \Rightarrow K(H) \geq \frac{\text{err}}{\text{res}}$$