

$$1) f(x) = \sqrt{x+2} \quad D = [-2, +\infty)$$

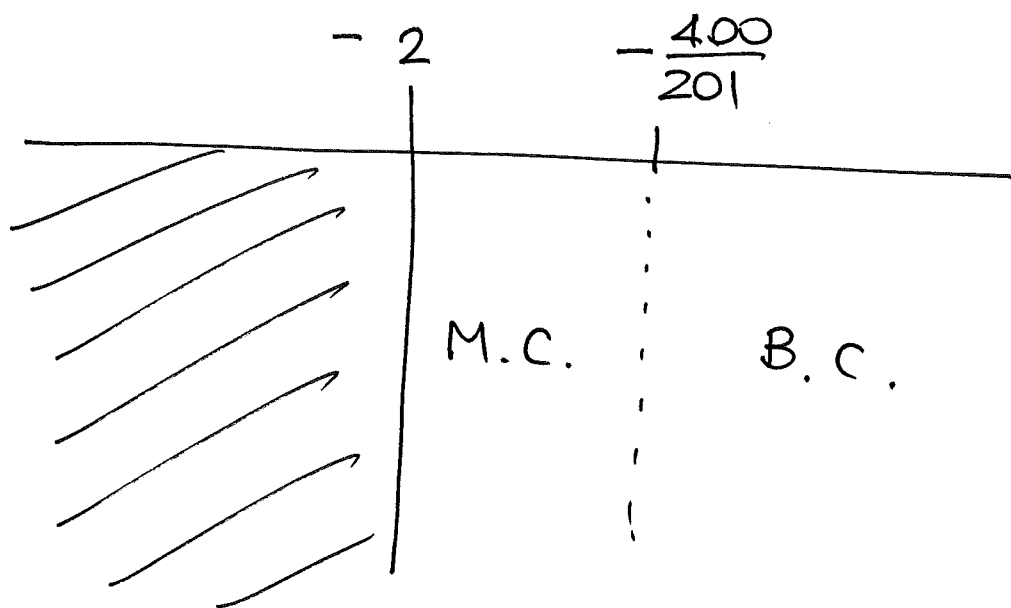
$$K_f(x) = \left| \frac{x f'(x)}{f(x)} \right| = \left| \frac{x \frac{1}{2\sqrt{x+2}}}{\sqrt{x+2}} \right| = \left| \frac{x}{2(x+2)} \right|$$

$$D' = (-2, +\infty)$$

Ben condizionato, es:

$$\left| \frac{x}{2(x+2)} \right| < 100 \dots \Rightarrow \begin{cases} \frac{x}{2(x+2)} < 100 \\ \frac{x}{2(x+2)} > -100 \end{cases}$$

$$\Rightarrow x > -\frac{400}{201} \approx -1.99005$$



$$x_1 = -1.991; \quad x_2 = -1.992;$$

$$f_1 = \text{sqrt}(x_1+2); \quad f_2 = \text{sqrt}(x_2+2);$$

$$A = (f_2 - f_1) / f_1;$$

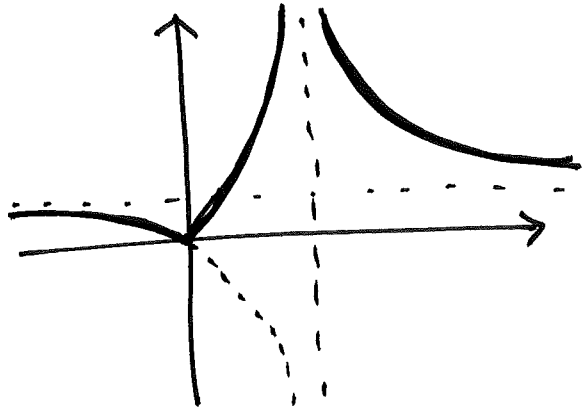
$$B = (x_1 - x_2) / x_1;$$

$$A/B \quad \% \sim \text{condizionamento}$$

2) Cancellazione numerica ~ mal condiz.

$$f(x) = x - 4$$

$$K_f(x) = \left| \frac{x}{x-4} \right| \quad x \neq 4$$



$$x_0 = 4.001 \quad (\text{esatto})$$

$$x = 4.002 \quad (\text{perturbato})$$

$$\Delta x = 10^{-3}$$

$$f(x_0) = 0.001 \quad (\text{ex})$$

$$f(x) = 0.002 \quad (\text{pert.})$$

$$\Delta f = 10^{-3}$$

$$\frac{\Delta f}{f} = \frac{0.001}{0.001} = 1$$

$$\frac{\Delta x}{x} = \frac{0.001}{4.001} \approx 0.00025$$

$$3) f(x) = \sqrt{x^2 + 1} - x = \frac{1}{\sqrt{x^2 + 1} + x}$$

$$\gg x = 4e7;$$

$$y1 = \text{sqrt}(x^2 + 1) - x; \quad y2 = 1 / (\text{sqrt}(x^2 + 1) + x);$$

$$\text{err} = \text{abs}((y1 - y2) / y2);$$

$$(\text{caso } x \rightarrow +\infty)$$

$$4) f(x) = \frac{(1+x)-1}{x} \quad (= 1 \quad \forall x \neq 0)$$

Esempio di cancellazione

$$x = 1e-8, 1e-10, 1e-12, 1e-14$$

$$f = ((1+x)-1)/x$$

$$5) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$n = 10^2 \dots\dots\dots 10^6 \dots\dots\dots 10^{10}$$

$$xn = \left(1 + 1/n\right)^n$$

$$err = \text{abs}(xn - \exp(1)) / \exp(1);$$