IL PROBLEMA DEI MINIMI QUADRATI: CASO DISCRETO, APPROSSIMAZIONE $\mathbb{P}_m$

$(x_i, y_i) \quad i=1,\dots,N$

(i nodi x_i non sono necessariamente distinti)

Sia $m < N$ $\quad (m \ll N)$

Cerco $p_m \in \mathbb{P}_m$ (polinomi algebrici)

t.c.:

$d_i = y_i - p_m(x_i) \quad i=1,\dots,N$

$$\sum_{i=1}^{N} (y_i - p_m^*(x_i))^2 = \min_{p_m \in \mathbb{P}_m} \sum_{i=1}^{N} (y_i - p_m(x_i))^2$$

$$\sum_{i=1}^{N} (d_i^*)^2 = \min \sum_{i=1}^{N} (d_i)^2$$

$$\downarrow$$

$p_m(x) = a_m x^m + a_{m-1} x^{m-1} + \dots + a_1 x + a_0$

$$(\underbrace{a_m, a_{m-1}, \dots, a_2, a_1}_{\in \mathbb{R}^{m+1}}, a_0)$$

Casi particolari

$$m=0 \quad \sum_{i=1}^{N} (y_i - a_0^*)^2 = \min_{a_0 \in \mathbb{R}} \sum_{i=1}^{N} (y_i - a_0)^2$$

$$a_0^* = \frac{1}{N} \sum_{i=1}^{N} y_i = \text{mean}(\underline{y})$$

$m=1$ (retta di regressione)(o dei minimi quadrati)

$$\sum_{i=1}^{N} (y_i - a_1^* x_i - a_0^*)^2 = \min_{(a_1, a_0) \in \mathbb{R}^2} \sum_{i=1}^{N} (y_i - a_1 x_i - a_0)^2$$

MATLAB

input: x, y vettori $[x_1, \dots, x_N]$ $[y_1, \dots, y_N]$ $\downarrow f(x_i)$

m, z (campionamento)

p= polyfit(x, y, m) $\quad p = [p_1 \, p_2 \dots p_{m+1}] \rightarrow \mathbb{P}_m$

pz = polyval(p, z)

Caso particolare: x_i distinti, $m = N-1$:

INTERPOLAZIONE