$\mathcal{S} = \{ s : \mathbb{Z} \to \mathbb{R} \; : \; \exists m \in \mathbb{N} , \; |s_i| = 0 \text{ per } |i| \geq m \}$

spazio vettoriale dei segnali $\quad s, y \in \mathcal{S} \Rightarrow s + y \in \mathcal{S}$

$\alpha \in \mathbb{R}, \; s \in \mathcal{S} \Rightarrow \alpha s \in \mathcal{S}$

$\tau \in \mathbb{Z} \quad s_{(\tau)}$ segnale traslato, $\; s_{(\tau),i} = s_{i-\tau} \quad i \in \mathbb{Z}$

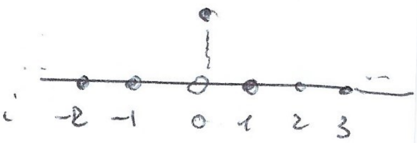
$T : \mathcal{S} \to \mathcal{S}$ 1. operatore lineare $\quad T(\alpha s + \beta y) = \alpha T(s) + \beta T(y)$

$\forall \alpha, \beta \in \mathbb{R}; \; \forall s, y \in \mathcal{S}$

2. invariante per traslazioni $\quad T(s_{(\tau)}) = T(s)_{(\tau)}$

Se valgono 1. e 2. T è detto operatore LTI.

δ "segnale delta" $\quad \delta_i = \begin{cases} 0 & i \neq 0 \\ 1 & i = 0 \end{cases}$



<u>Lemma</u> (di rappresentazione)

Sia $T : \mathcal{S} \to \mathcal{S}$ LTI, posto $h = T(\delta)$ allora per ogni segnale $s \in \mathcal{S}$ $\quad T(s) = s * h \quad$ dove $(s * h)_i = \sum_k s_k h_{i-k}$

<u>Dimostrazione</u> $\quad s \in \mathcal{S} \Rightarrow \quad s = \underbrace{\sum_k s_k \delta_{(k)}}_{\in \mathcal{S}} \quad$ (è una somma finita)

$s_k \in \mathbb{R}, \; \delta_{(k)} \in \mathcal{S}$

$$T(s) = T\Big(\sum_k s_k \delta_{(k)}\Big) \underset{\substack{\text{linearità} \\ \text{di } T}}{=} \sum_k s_k T(\delta_{(k)}) \underset{\substack{\text{inv. per} \\ \text{traslazioni} \\ \text{di } T}}{=} \sum_k s_k T(\delta)_{(k)} =$$

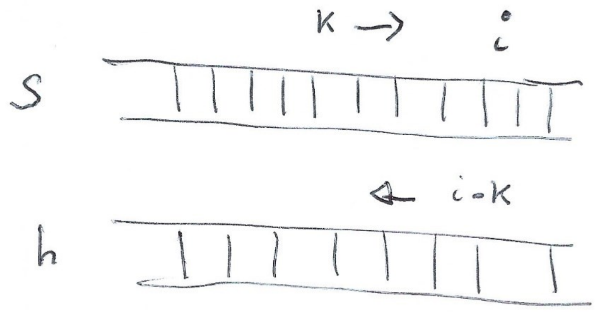
$$\underset{\substack{\text{def. di} \\ h}}{=} \sum_k s_k h_{(k)}$$

Componente i-esima

$$T(s)_i = \sum_k s_k h_{(k),i} = \boxed{\sum_k s_k h_{i-k}}$$

Note l'espressione $\sum_k s_k h_{i-k}$ è una somma finita

percorso, per indici k crescenti, s da "sinistra verso destra" mentre h da "destra verso sinistra"



Esempio

h = 0 ...... α β γ 0 ....
indici = ... -2 -1 0 1 2 ......

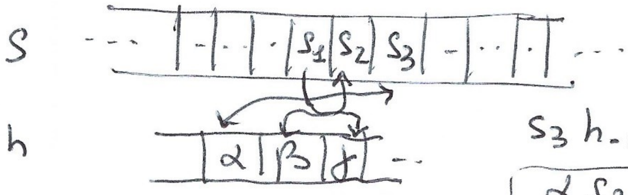
s = ------ s_{-2} s_{-1} s_0 s_1 s_2 s_3 ....

$$(s*h)_2 = \sum_k s_k h_{2-k}$$

$h_{2-k} \neq 0$ solo per

$k=3 \leftarrow 2-k=-1 \rightarrow \alpha$
$k=2 \leftarrow 2-k=0 \rightarrow \beta$
$k=1 \leftarrow 2-k=1 \rightarrow \gamma$

i=2



$$s_3 h_{-1} + s_2 h_0 + s_1 h_1 = \boxed{\alpha s_3 + \beta s_2 + \gamma s_1}$$