

**Formule di quadratura di Newton-Cotes composite.**

Posto  $M \geq 1$ :

$$H = \frac{b-a}{M}, \quad a_i = a + iH, \quad i = 0, \dots, M, \quad a = a_0, \quad b = a_M.$$

( $M = 1 \Rightarrow$  Formule di quadratura semplici).

- Formula del punto medio composita

$$\boxed{\tilde{I}_{PM}^{(c)}} = \boxed{H \sum_{i=1}^M f\left(a_i - \frac{H}{2}\right)}$$

Errore:

$$I(f) - \tilde{I}_{PM}^{(c)} = -\frac{b-a}{24} H^2 f^{(2)}(\eta), \quad \eta \in (a, b) \quad (\text{formula classica})$$

$$I(f) - \tilde{I}_{PM}^{(c)} = \frac{H^2}{24} [f^{(1)}(b) - f^{(1)}(a)] \quad (\text{formula asintotica}).$$

- Formula dei trapezi composita

$$\boxed{\tilde{I}_T^{(c)}} = \frac{H}{2} \sum_{i=1}^M [f(a_{i-1}) + f(a_i)] = \boxed{\frac{H}{2} \left[ f(a) + f(b) + 2 \sum_{i=1}^{M-1} f(a_i) \right]}$$

Errore:

$$I(f) - \tilde{I}_T^{(c)} = -\frac{b-a}{12} H^2 f^{(2)}(\eta), \quad \eta \in (a, b) \quad (\text{formula classica})$$

$$I(f) - \tilde{I}_T^{(c)} = \frac{H^2}{12} [f^{(1)}(a) - f^{(1)}(b)] \quad (\text{formula asintotica}).$$

- Formula di C. Simpson composita

$$\boxed{\tilde{I}_{CS}^{(c)}} = \frac{H}{6} \sum_{i=1}^M \left[ f(a_{i-1}) + 4f\left(a_i - \frac{H}{2}\right) + f(a_i) \right] =$$

$$\boxed{\frac{H}{6} \left[ f(a) + f(b) + 2 \sum_{i=1}^{M-1} f(a_i) + 4 \sum_{i=1}^M f\left(a_i - \frac{H}{2}\right) \right]}$$

Errore:

$$I(f) - \tilde{I}_{CS}^{(c)} = -\frac{b-a}{2880} H^4 f^{(4)}(\eta), \quad \eta \in (a, b) \quad (\text{formula classica})$$

$$I(f) - \tilde{I}_{CS}^{(c)} = \frac{H^4}{2880} [f^{(3)}(a) - f^{(3)}(b)] \quad (\text{formula asintotica}).$$