RISOLUZIONE DI SISTEMI DIAGONALI

$$\begin{cases} a_{11}x_1 = b_1 \\ a_{22}x_2 = b_2 \\ \vdots \\ a_{nn}x_n = b_n \end{cases} \qquad a_{ii}x_i = b_i \quad i = 1, \dots, n$$

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & & & 0 \\ 0 & \cdots & 0 & a_{nn} \end{bmatrix} \qquad \det(A) = a_{11} \cdot a_{22} \cdot \dots \cdot a_{nn}$$

$$= \prod_{i=1}^{n} a_{ii} \neq 0$$

$$\Longleftrightarrow \quad a_{ii} \neq 0 \quad \forall\, i = 1, \dots, n$$

$$x_i = \frac{b_i}{a_{ii}} \qquad i = 1, \dots, n$$

$n = 3$

$$\left[\begin{array}{ccc} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{array}\right] \begin{array}{cc} a_{11} & 0 \\ 0 & a_{22} \\ 0 & 0 \end{array} = a_{11} \cdot a_{22} \cdot a_{33}$$

Autovalori $\quad \det(A - \lambda I) = 0$

$$\det \begin{bmatrix} a_{11}-\lambda & 0 & 0 \\ 0 & a_{22}-\lambda & 0 \\ 0 & 0 & a_{33}-\lambda \end{bmatrix} = (a_{11}-\lambda)(a_{22}-\lambda)(a_{33}-\lambda) = 0$$

$$\lambda_1 = a_{11}, \quad \lambda_2 = a_{22}, \quad \lambda_3 = a_{33}$$

In generale

$$\lambda_i = a_{ii} \quad , i = 1, \dots, n$$

$$\left[\begin{array}{c} \text{Per ogni matrice } n \times n \\ \prod_{i=1}^{n} \lambda_i = \det(A) \end{array}\right]$$

Matrice triangolare inferiore (L)

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & a_{nn} \end{bmatrix}$$

$a_{ij} = 0 \quad i < j$

$a_{ii} \neq 0$

$$\det A = \prod_{i=1}^{n} a_{ii}$$

$\lambda_i = a_{ii} \quad , i = 1, \dots, n$

$$\left[\begin{array}{l} \det(A - \lambda I) = 0 \\ \prod_{i=1}^{n} (a_{ii} - \lambda) = 0 \end{array} \right.$$

Caso 2×2 e 3×3 : $\det(A)$

$$\begin{vmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{vmatrix} \qquad \det A = a_{11} \cdot a_{22}$$

$$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & 0 \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} = a_{11} \cdot a_{22} \cdot a_{33}$$

Autovalori

$$\det \begin{bmatrix} a_{11} - \lambda & 0 \\ a_{21} & a_{22} - \lambda \end{bmatrix} = (a_{11} - \lambda)(a_{22} - \lambda) = 0 \qquad \begin{array}{l} \lambda_1 = a_{11} \\ \lambda_2 = a_{22} \end{array}$$

$$\det \begin{bmatrix} a_{11} - \lambda & 0 & 0 \\ a_{21} & a_{22} - \lambda & 0 \\ a_{31} & a_{32} & a_{33} - \lambda \end{bmatrix} \begin{matrix} a_{11} - \lambda & 0 \\ a_{21} & a_{22} - \lambda \\ a_{31} & a_{32} \end{matrix} = (a_{11} - \lambda)(a_{22} - \lambda)(a_{33} - \lambda) = 0$$

$\lambda_i = a_{ii} \quad i = 1, 2, 3$

Matrice triangolare superiore (U)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ 0 & \cdots & & 0 & a_{nn} \end{bmatrix} \quad a_{ij}=0 \quad i>j$$

$$\det A = \prod_{i=1}^{n} a_{ii}$$

$$\lambda_i = a_{ii}, \quad i=1,\dots,n$$

$$\begin{cases} \det(A-\lambda I)=0 \\ \prod_{i=1}^{n}(a_{ii}-\lambda)=0 \end{cases}$$

Caso 2×2 e 3×3: $\det(A)$

$$\begin{vmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{vmatrix} \quad \det A = a_{11}\cdot a_{22}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ 0 & a_{22} \\ 0 & 0 \end{matrix} = a_{11}\cdot a_{22}\cdot a_{33}$$

Autovalori (per esercizio caso $n=2$)

$$\det \begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ 0 & a_{22}-\lambda & a_{23} \\ 0 & 0 & a_{33}-\lambda \end{bmatrix} \begin{matrix} a_{11}-\lambda & a_{12} \\ 0 & a_{22}-\lambda \\ 0 & 0 \end{matrix} =$$

$$(a_{11}-\lambda)(a_{22}-\lambda)(a_{33}-\lambda)=0$$

$$\lambda_i = a_{ii} \quad i=1,2,3$$

RISOLUZIONE SISTEMI CON MATRICE TRIANGOLARE INFERIORE

$(\det A \neq 0 \iff a_{ii} \neq 0)$ $\quad Ax = b$

$$
\begin{aligned}
a_{11}x_1 &= b_1 \\
a_{21}x_1 + a_{22}x_2 &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \\
&\vdots \\
(a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots + a_{nn}x_n &= 0)
\end{aligned}
$$

1) $x_1 = \dfrac{b_1}{a_{11}}$

2) $a_{21}x_1 + a_{22}x_2 = b_2$ $\qquad x_2 = \dfrac{1}{a_{22}}(b_2 - a_{21}x_1)$

3) $a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

$$x_3 = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1 - a_{32}x_2\right)$$

$A\,(n \times n)$

$$x_1 = \frac{b_1}{a_{11}}$$

$i = 2, \dots, n$ $\qquad x_i = \dfrac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j\right)$

Metodo di sostituzione IN AVANTI

(forward substitution)

RISOLUZIONE SISTEMI CON MATRICE TRIANGOLARE SUPERIORE ($\det A \neq 0 \iff a_{ii} \neq 0$) $Ax = b$

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1$$
$$a_{22} x_2 + a_{23} x_3 = b_2$$
$$a_{33} x_3 = b_3$$

$$\left(a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \dots + a_{1n} x_n = b_1 \right)$$

1) $x_3 = \dfrac{b_3}{a_{33}}$

2) $a_{22} x_2 + a_{23} x_3 = b_2$ $\qquad x_2 = \dfrac{1}{a_{22}} \left(b_2 - a_{23} x_3 \right)$

3) $a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1$

$$x_1 = \frac{1}{a_{11}} \left(b_1 - a_{12} x_2 - a_{13} x_3 \right)$$

$A(n \times n)$

$$x_n = \frac{b_n}{a_{nn}}$$

$i = n-1, \dots, 1$

$$x_i = \frac{1}{a_{ii}} \left(b_i - \sum_{j=i+1}^{n} a_{ij} x_j \right)$$

Metodo di sostituzione ALL' INDIETRO
(backward substitution)