MATLAB – FILL-IN

DIAG
RIGA 1
COLONNA 1

```
>> a=2*eye(6)
>> a(1,2:6)=0.2
>> a(2:6,1)=0.2
>> [l,u,p]=lu(a)
l =
    1.0000         0         0         0         0         0
    0.1000    1.0000         0         0         0         0
    0.1000   -0.0101    1.0000         0         0         0
    0.1000   -0.0101   -0.0102    1.0000         0         0
    0.1000   -0.0101   -0.0102   -0.0103    1.0000         0
    0.1000   -0.0101   -0.0102   -0.0103   -0.0104    1.0000
u =
    2.0000    0.2000    0.2000    0.2000    0.2000    0.2000
         0    1.9800   -0.0200   -0.0200   -0.0200   -0.0200
         0         0    1.9798   -0.0202   -0.0202   -0.0202
         0         0         0    1.9796   -0.0204   -0.0204
         0         0         0         0    1.9794   -0.0206
         0         0         0         0         0    1.9792
```

DIAG
RIGA N (=6)

```
>> a=2*eye(6)
>> a(6,1:5)=0.2
    2.0000         0         0         0         0         0
         0    2.0000         0         0         0         0
         0         0    2.0000         0         0         0
         0         0         0    2.0000         0         0
         0         0         0         0    2.0000         0
    0.2000    0.2000    0.2000    0.2000    0.2000    2.0000
```

COLONNA N

```
>> a(1:5,6)=0.2
>> [l,u,p]=lu(a)
l=
    1.0000         0         0         0         0         0
         0    1.0000         0         0         0         0
         0         0    1.0000         0         0         0
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
    0.1000    0.1000    0.1000    0.1000    0.1000    1.0000
u =
    2.0000         0         0         0         0    0.2000
         0    2.0000         0         0         0    0.2000
         0         0    2.0000         0         0    0.2000
         0         0         0    2.0000         0    0.2000
         0         0         0         0    2.0000    0.2000
         0         0         0         0         0    1.9000
```

Fattorizzazione di matrici tridiagonali (no pivot)

$$A = \begin{bmatrix} a_1 & c_1 & 0 & 0 & 0 \\ b_2 & a_2 & c_2 & 0 & 0 \\ 0 & b_3 & a_3 & c_3 & 0 \\ 0 & 0 & b_4 & a_4 & c_4 \\ 0 & 0 & 0 & b_5 & a_5 \end{bmatrix}$$

Banda $p = 1$
" $q = 1$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \beta_2 & 1 & 0 & 0 & 0 \\ 0 & \beta_3 & 1 & 0 & 0 \\ 0 & 0 & \beta_4 & 1 & 0 \\ 0 & 0 & 0 & \beta_5 & 1 \end{bmatrix} \cdot U \begin{bmatrix} \alpha_1 & \gamma_1 & 0 & 0 & 0 \\ 0 & \alpha_2 & \gamma_2 & 0 & 0 \\ 0 & 0 & \alpha_3 & \gamma_3 & 0 \\ 0 & 0 & 0 & \alpha_4 & \gamma_4 \\ 0 & 0 & 0 & 0 & \alpha_5 \end{bmatrix}$$

$L \cdot U = A$

$R_1 \cdot C_1$: $\alpha_1 = a_1$ $\quad [A]_{1,\bullet}$

$R_1\ C_2$: $\gamma_1 = c_1$

$R_2\ C_1$: $\beta_2 \alpha_1 = b_2$ $\quad \beta_2 = \frac{b_2}{\alpha_1}$

$R_2\ C_2$: $\beta_2 \gamma_1 + \alpha_2 = a_2$ $\quad \alpha_2 = a_2 - \beta_2 \gamma_1$ $\quad [A]_{2,\bullet}$

$R_2\ C_3$: $\gamma_2 = c_2$

$R_3\ C_2$: $\beta_3 \alpha_2 = b_3$ $\quad \beta_3 = \frac{b_3}{\alpha_2}$

$R_3\ C_3$: $\beta_3 \gamma_2 + \alpha_3 = a_3$ $\quad \alpha_3 = a_3 - \beta_3 \gamma_2$ $\quad [A]_{3,\bullet}$

$R_3\ C_4$: $\gamma_3 = c_3$

$R_5\ C_4$: $\beta_5 \alpha_4 = b_5$ $\quad \beta_5 = \frac{b_5}{\alpha_4}$

$R_5\ C_5$: $\beta_5 \gamma_4 + \alpha_5 = a_5$ $\quad \alpha_5 = a_5 - \beta_5 \gamma_4$ $\quad [A]_{5,\bullet}$ (n-esimo)

1ª Riga: $\alpha_1 = a_1$

$\gamma_1 = c_1$

Riga $2, \dots, n$

$$\beta_i = \frac{b_i}{\alpha_{i-1}}$$

$$\alpha_i = a_i - \beta_i \gamma_{i-1}$$

$$\gamma_i = c_i$$

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R_i: $\begin{bmatrix} -1 & 2 & -1 \end{bmatrix}$

$f_1 = f_n = 1 \qquad f_i = 0 \quad i = 2, \dots, n-1$

Risoluzione del sistema $A\underline{x} = \underline{f}$ file Teieis.m

$LU\,\underline{x} = \underline{f}$ $\quad (U\underline{x} = \underline{y})$

- $L\underline{y} = f$

$$\begin{aligned} &y_1 = f_1 \\ &\beta_2 y_1 + y_2 = f_2 \\ &\beta_3 y_2 + y_3 = f_3 \\ &\vdots \\ &\beta_n y_{n-1} + y_n = f_n \end{aligned} \quad \Rightarrow$$

$y_1 = f_1$

$i = 2, \dots, n$

$\quad y_i = f_i - \beta_i * y_{i-1}$

end

- $U\underline{x} = \underline{y}$

$$\begin{aligned} &\alpha_1 x_1 + \gamma_1 x_2 = y_1 \\ &\alpha_2 x_2 + \gamma_2 x_3 = y_2 \\ &\vdots \\ &\alpha_{n-1} x_{n-1} + \gamma_{n-1} x_n = y_{n-1} \\ &\alpha_n x_n = y_n \end{aligned}$$

$$x_n = \frac{y_n}{\alpha_n}$$

$i = n-1, \dots, 1$ (step -1)

$\quad x_i = \frac{1}{\alpha_i}(y_i - \gamma_i x_{i+1})$

end

file ex_Tei.m