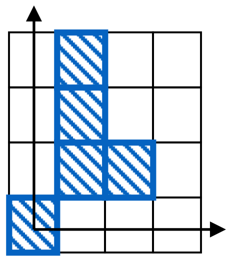
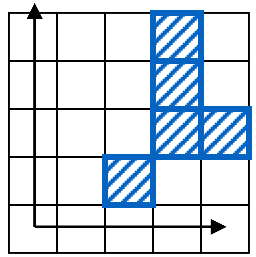
Morfologia matematica (alcune note)

Richiamiamo: traslazione

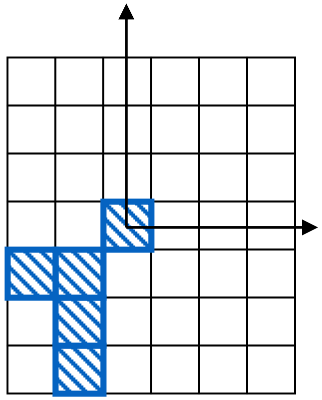
- $A \subseteq Z^n$, $t \in Z^n$
- Traslazione di A rispetto ad un vettore t
  $A_t = \{ c \in Z^n \mid c=a+t, a \in A \}$
- Riflessione di A
  $A^r = \{ c \mid c=-a, a \in A \}$
- Complemento di A
  $A^c = Z^n - A$



A



$A_{(2,1)}$

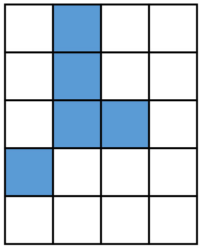


A^r

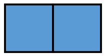
Nota: consideriamo per semplicità
L’intero piano «tassellato», nelle applicazioni regione limitata

Somma di Minkowski (Dilation)

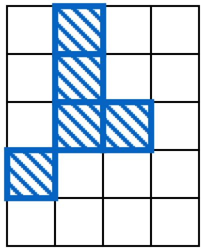
- $A \oplus B = \{ c \in Z^n \mid c = a + b,\ a \in A,\ b \in B \}$
- $A \oplus B = \bigcup A_b\ ,\ b \in B$
  - Si dimostra facilmente: $A \oplus B = B \oplus A$



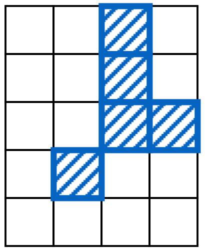
A



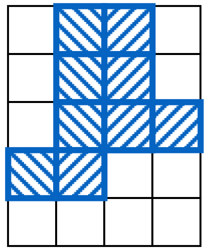
$B = \{ (0,0), (1,0) \}$



$A_{(0,0)}$



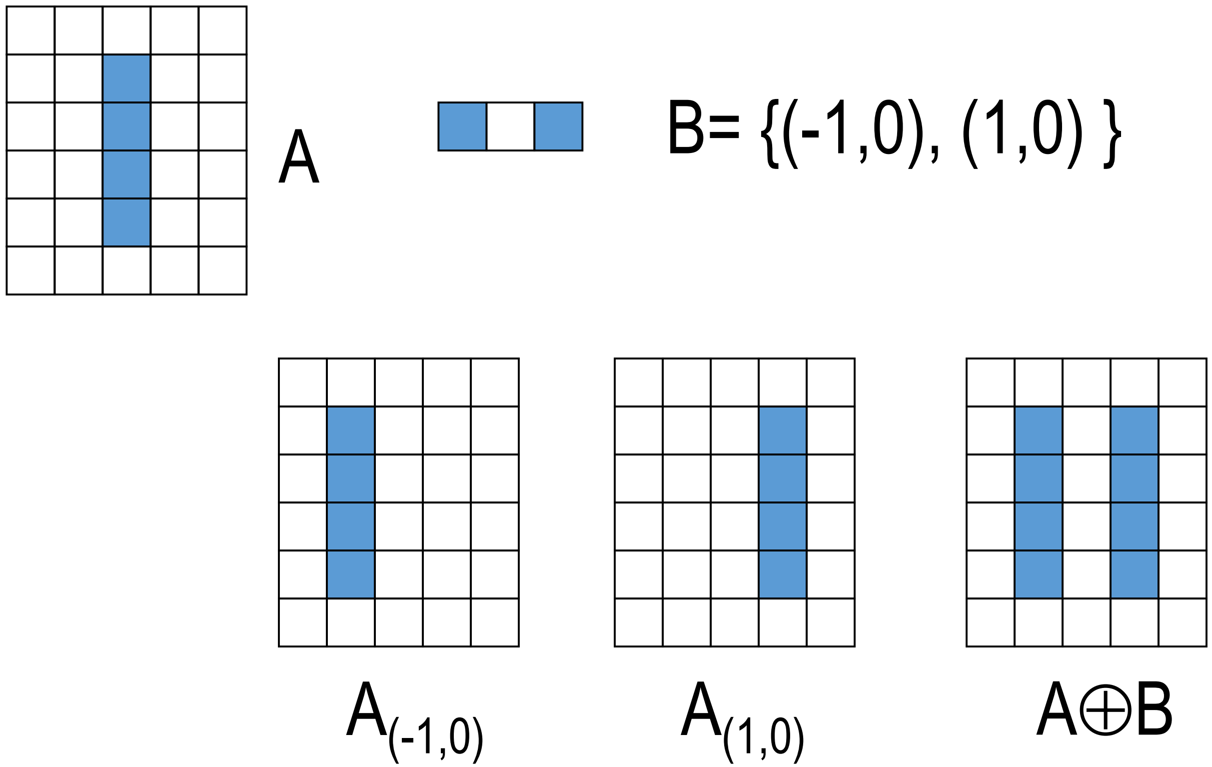
$A_{(1,0)}$



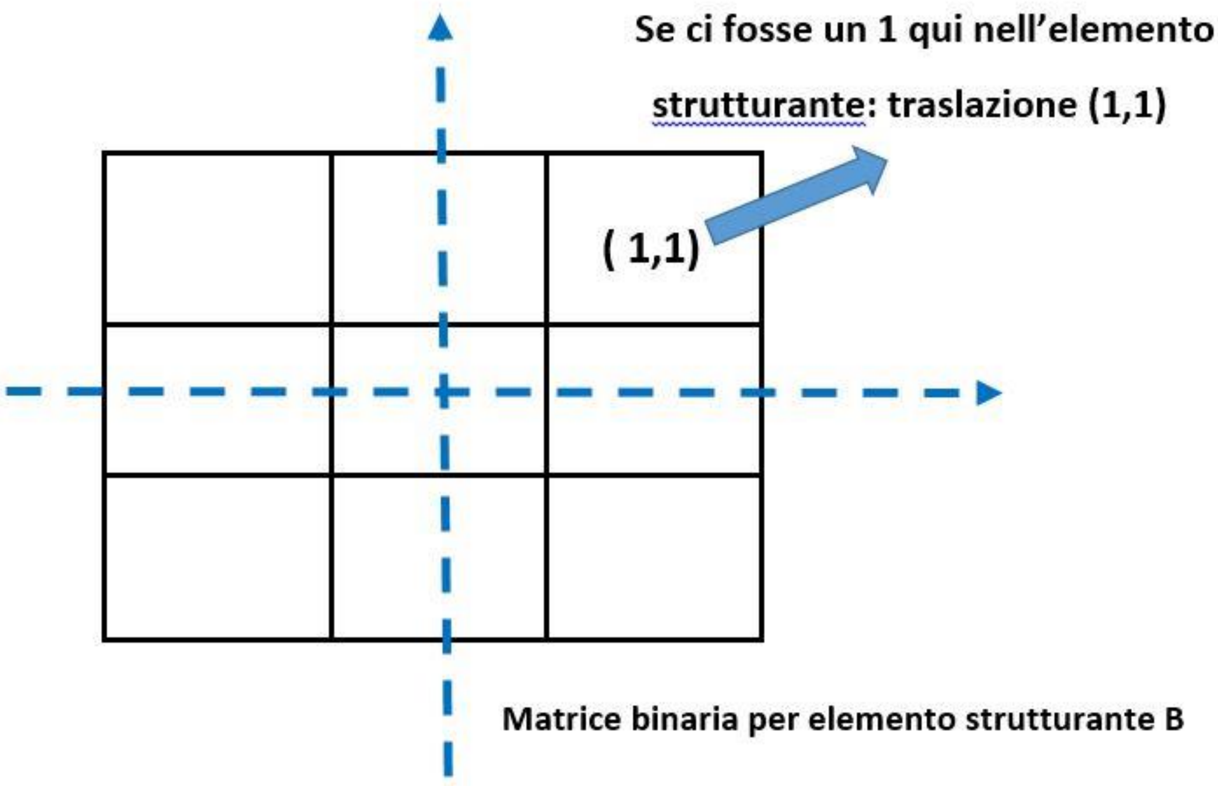
$A \oplus B$

Dilation

L'insieme B viene normalmente definito elemento strutturante



Possibile implementazione



Posso codificare le traslazioni, attenzione se il valore centrale è 0 si perde la trslazione (0,0) ovvero la copia dell'immagine originale nella dilatazione.

Esempio

		1	1	1		
			1	1		
			1	1		

Immagine I
(binaria 0/1)

	1	
1	1	1
	1	

Elemento
Strutturante B

Esempio

Considero solo i valori uguali ad 1 e sommo la matrice B centrata in questi valori in una matrice inizialmente nulla

		1	1	1		
			1	1		
			1	1		

		1				
	1	1	1			
		1				

Esempio

		1	1	1		
			1	1		
			1	1		

		1	1			
	1	2	2	1		
		1	1			

Esempio

		1	1	1		
			1	1		
			1	1		

		1	1	1		
	1	2	3	2	1	
		1	1	1		

Esempio

		1	1	1		
			1	1		
			1	1		

⇨

		1	1	1		
	1	2	4	2	1	
		2	2	2		
			1			

Esempio

		1	1	1		
			1	1		
			1	1		

⇨

		1	1	1		
	1	2	4	3	1	
		2	3	3	1	
			1	1		

Esempio

		1	1	1		
			1	1		
			1	1		

		1	1	1		
	1	2	4	3	1	
		2	4	3	1	
		1	2	2		
			1			

Esempio

		1	1	1		
			1	1		
			1	1		

		1	1	1		
	1	2	4	3	1	
		2	4	4	1	
		1	3	3	1	
			1	1		

Esempio

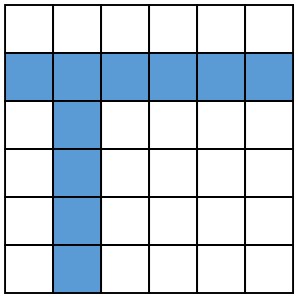
		1	1	1		
			1	1		
			1	1		

		1	1	1		
	1	1	1	1	1	
		1	1	1	1	
		1	1	1	1	
			1	1		

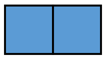
>0

Erosione (Differenza di Minkowski)

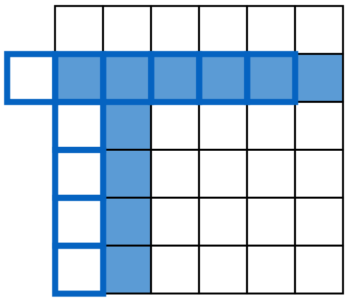
- $A \ominus B = \{ c \in Z^n \mid c+b \in A, \text{ per ogni } b \in B \}$
- $A \ominus B = \cap A_{-b}\ b \in B$



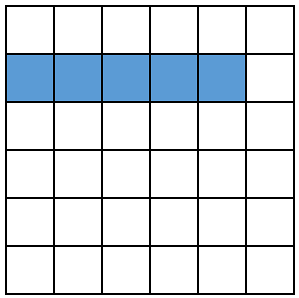
A



B= { (0,0), (1,0) }



$A_{(-1,0)}$



A-B

Posso implementare come dilatazione ma prima faccio riflessione per la matrice B: in questo modo tralso con –B (in Matlab significa riodinare righe e colonne come per la convoluzione).

Al termine considero solo i valori uguali al numero di non zero All’interno di B: ovvero quelli che sono nell’intersezione.

NOTA: nel caso di elemento strutturante simmetrico l’operazione iniziale di riordinamento non cambia B.

Proprietà

$$(A+B)+C=A+(B+C) \qquad (A-B)-C=A-(B+C)$$

$$(A\cup B)+C=(A+C)\cup(B+C) \qquad (A \cap B)-C=(A-C) \cap(B-C)$$

$$A+B= \cup A_b \qquad A-B= \cap A_{-b}$$

$$A\subseteq B\Rightarrow(A+C)\subseteq (B+C) \qquad A\subseteq B\Rightarrow(A-C)\subseteq (B-C)$$

$$(A\cap B)+C\subseteq (A+C)\cap(B+C) \qquad (A\cup B)-C\supseteq (A-C) \cup(B-C)$$

$$A+(B\cup C)=(A+B)\cup(A+C) \qquad A-(B\cup C)=(A-C)\cap(B-C)$$

$$(A+B)^c=A^c -B^r$$

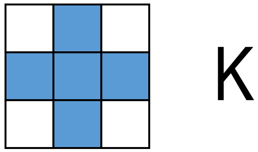
$$A+B_t=(A+B)_t \qquad A-B_t=(A-B)_{-t}$$

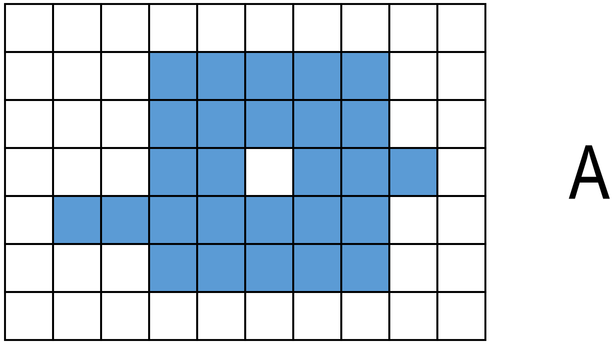
$$A-(B\cap C)\supseteq(A-C)\cup(B-C)$$

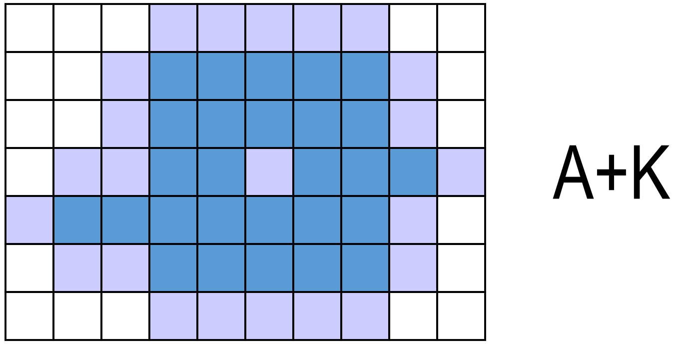
Per semplicità di notazione si è usato +e- per gli operatori erosion e dilation

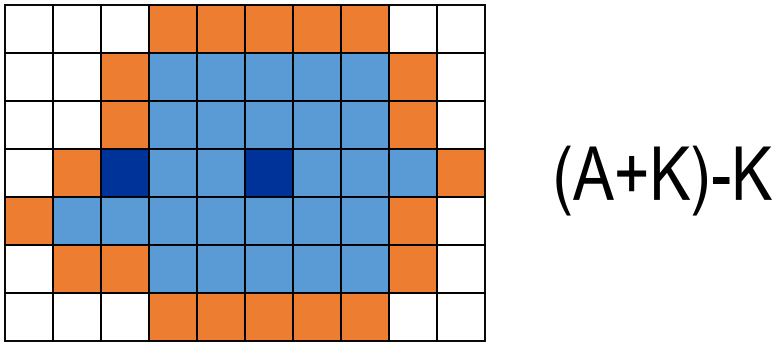
Closing

- C(A, K) = (A+K)-K
  - A⊆C(A,K)=C(C(A,K),K)



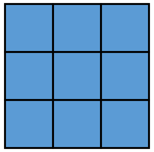




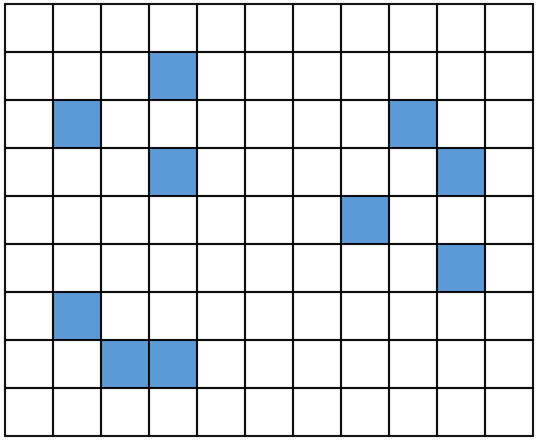


Closing

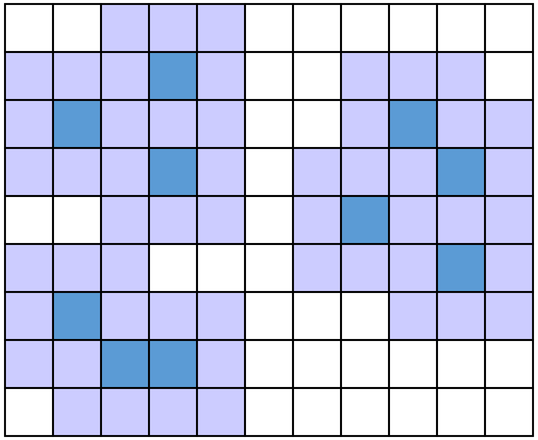
- C(A, K) = (A+K)-K



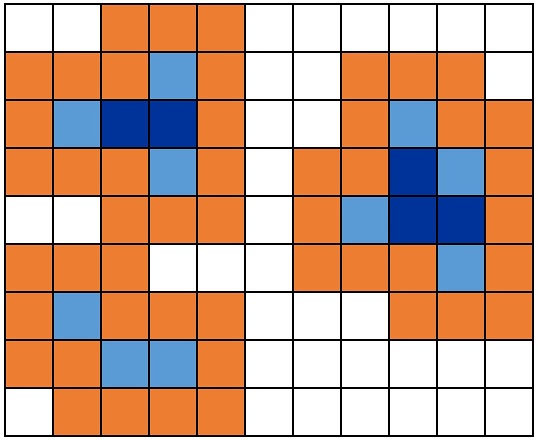
K



A



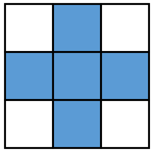
A+K



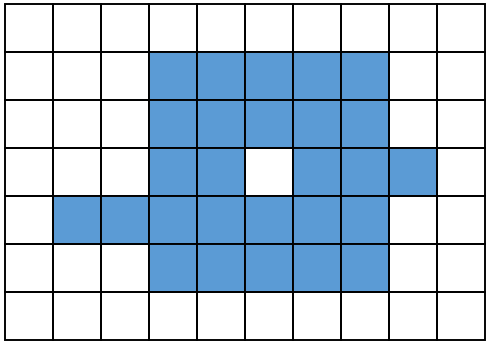
(A+K)-K

Opening

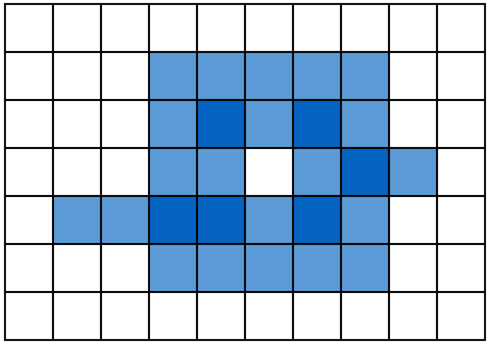
- O(A, K) = (A-K)+K
  - O(O(A,K),K)=O(A,K)⊆A



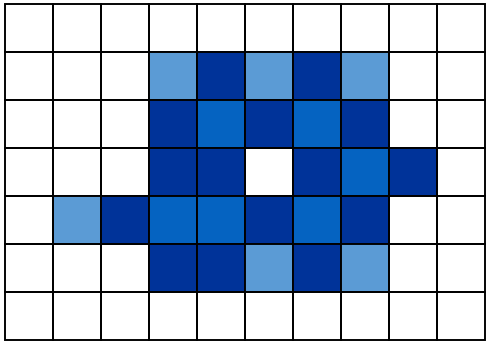
K



A



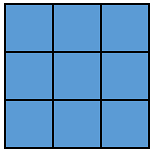
A-K



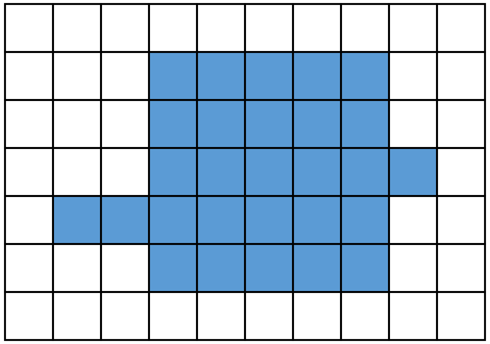
(A-K)+K

Opening

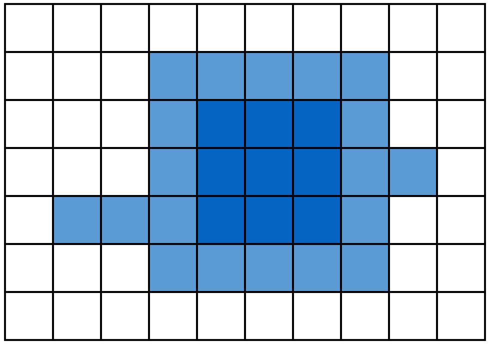
- O(A, K) = (A-K)+K



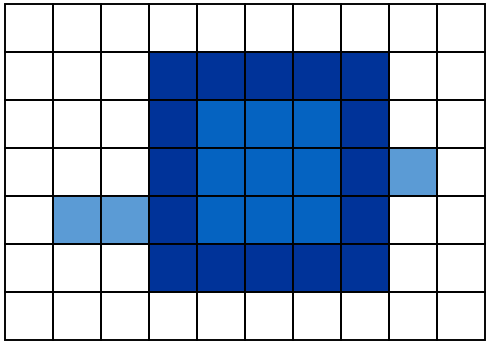
K



A



A-K



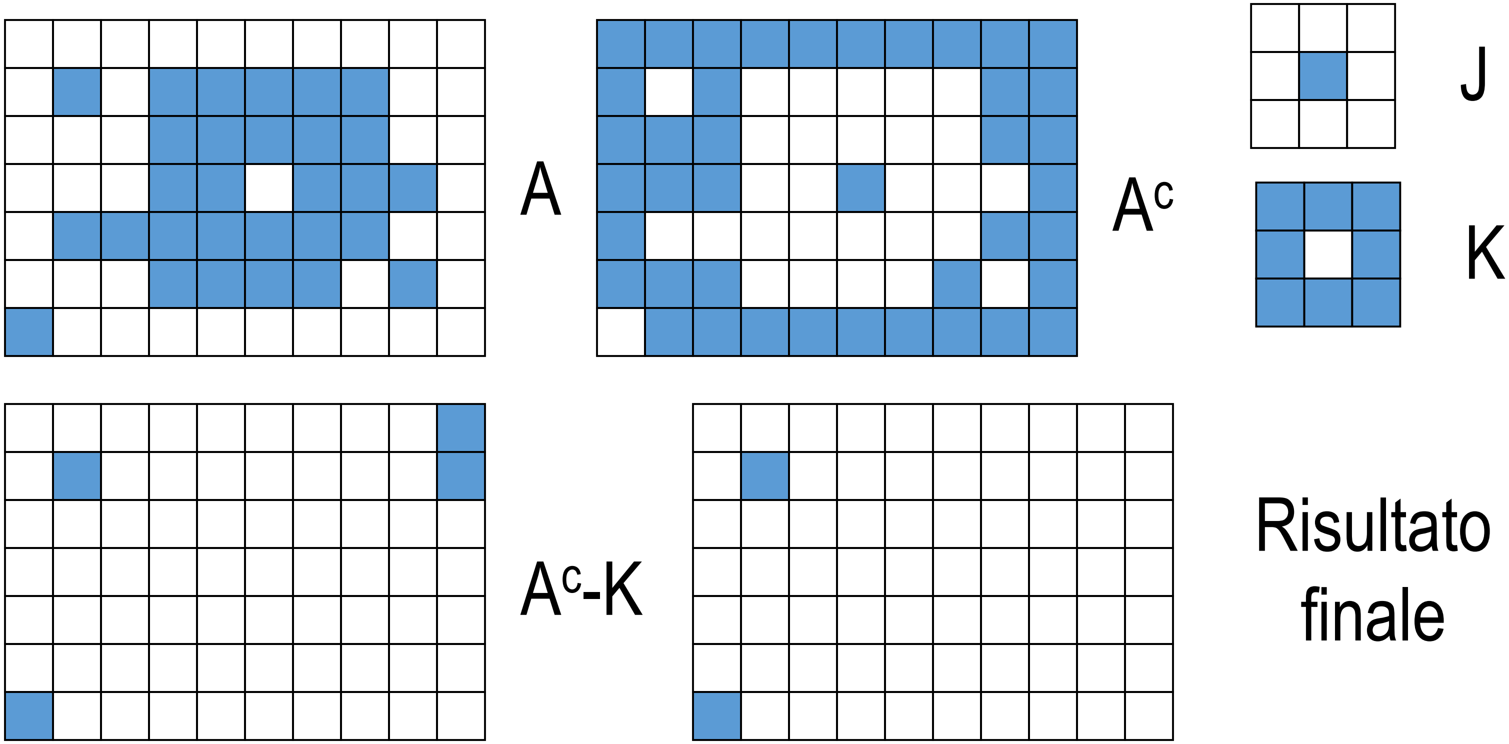
(A-K)+K

Hit or Miss

- $A \otimes (J,K) = (A-J) \cap (A^c-K)$
  - con il vincolo $J \cap K = \emptyset$
- Permette di trovare strutture regolari (template matching)

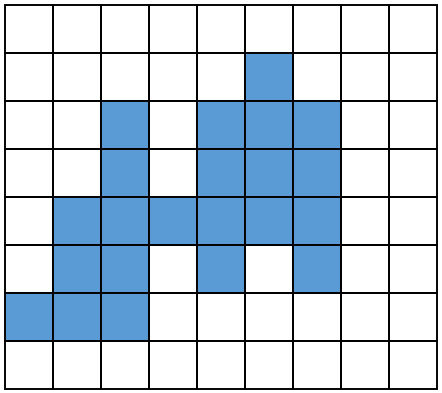
Esempio

- Ricerca di punti isolati (8-connessi)
  - A-J=A

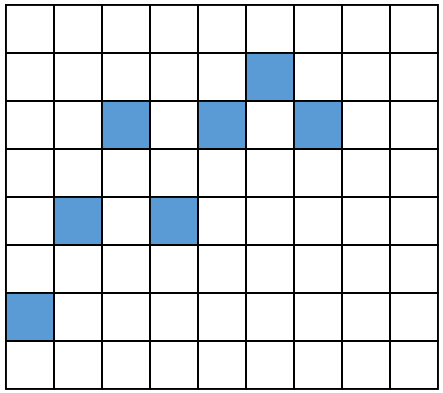


Umbra (estensione alle immagini gray scale)

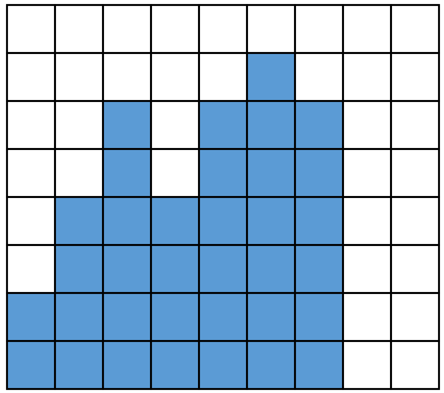
- $A \subseteq E^n$, $F \subseteq E^{n-1}$, $x \in F$, $y \in E$
- Top di un insieme A ($T[A]:F \rightarrow E$):
  $T[A](x) = \max \{ y \mid (x, y) \in A \}$
- Umbra di f ($f:F \rightarrow E$):
  $U[f] = \{ (x, y) \in F \times E \mid y \leq f(x) \}$



Insieme A



Top di A



Umbra di A

Umbra - proprietà

- $T[A] \subseteq A \subseteq U[A] \subseteq Z^n$
- $U[U[A]] \equiv U[A]$

Dilatazione immagini scale di grigio

- Dati: $F, K \subseteq Z^{n-1}$, $f: F \to E$, $k: K \to Z$
- Si definisce dilation di f tramite k
  $f \oplus k = T\{U[f] \oplus U[k]\}$
- **Si dimostra che una definizione equivalente è:**
  $(f \oplus k)(x) = \max\{f(x-z)+k(z) \mid z \in K, x-z \in F\}$
- Dal punto di vista computazionale la complessità è equivalente ad una convoluzione

Erosione immagine a livelli di grigio

- Dati: $F,K \subseteq Z^{n-1}$, $f:F \rightarrow E$, $k:K \rightarrow Z$
- Si definisce erosione di f tramite k
  $f \ominus k = T\{U[f] \ominus U[k]\}$
- **Si dimostra che una definizione equivalente è:**
  $(f \ominus k)(x) = \min\{f(x+z)-k(z) \mid z \in K,\ x+z \in F\}$