



**BTP**  
General calibration: optical & tangential distortions

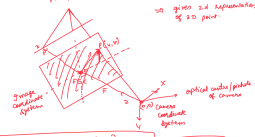
Related Parameters:  $\left\{ \begin{array}{l} \text{compute camera action} \\ \text{parameters, rotation and translation} \\ \text{matrix} \\ \text{(camera distortion)} \end{array} \right\}$

→ Camera model is:  
 ↳ focal length in considered as scaling factor  
 $c_x, c_y$  → location of optical center

3D world coordinate system  $P = (x, y, z)$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

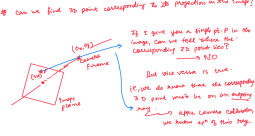
↳ given 2D representation of 3D point.



→ why camera calibration?  
 ① Radial distortion  
 ② Pinhole distortion  
 ↳ caused by lenses of camera.  
 → To remove distortion on our camera lens to take an accurate picture of camera parameters.

→ Zhang's Method 1:-  
 ↳ feature extraction for pinhole camera model

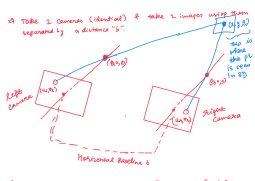
→ Take a bunch of photos of a chessboard grid at different angles and positions.



$$\begin{aligned} 3D \rightarrow 2D \Rightarrow u &= f_x \frac{x}{Z} + c_x \\ v &= f_y \frac{y}{Z} + c_y \end{aligned}$$

$$\begin{aligned} 2D \rightarrow 3D \Rightarrow x &= \frac{u - c_x}{f_x} \cdot Z \\ y &= \frac{v - c_y}{f_y} \cdot Z \end{aligned}$$

↳ 3D point



$$\begin{aligned} u_1 &= f_1 \left( \frac{x}{Z} \right) + c_{x1} & u_2 &= f_2 \left( \frac{x}{Z} \right) + c_{x2} \\ v_1 &= f_1 \left( \frac{y}{Z} \right) + c_{y1} & v_2 &= f_2 \left( \frac{y}{Z} \right) + c_{y2} \end{aligned}$$

↳  $f_1, f_2, c_{x1}, c_{y1}, c_{x2}, c_{y2}$  are known by camera setup.

from eq. ① & ②, we can get  $x, y, z$ .

$$(u_1, v_1) = \left( f_1 \frac{x}{Z} + c_{x1}, f_1 \frac{y}{Z} + c_{y1} \right)$$

$$(u_2, v_2) = \left( f_2 \frac{x}{Z} + c_{x2}, f_2 \frac{y}{Z} + c_{y2} \right)$$

⇒ solving for  $(x, y, z)$ , we get

$$x = \frac{b(u_1 - c_{x1})}{(a_1 - u_1 a_2)}, \quad y = \frac{b(u_2 - c_{x2})}{f_2 (u_1 - u_2)}$$

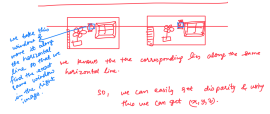
$$z = \frac{b f_1 c_{x1}}{(u_1 - u_2)}$$

Now,  $(u_1 - u_2)$  is called disparity

$$d = \frac{1}{(u_1 - u_2)} \quad \text{if } (u_1 - u_2) \propto \frac{1}{d}$$

↳ if we take images of an object placed very far away from both the cameras (infinity), then doesn't matter the location 's', we will get 2 identical images of the object.

Window based methods → Template matching 1:-



① we need horizontal surfaces.

→ Radial distortion :- cause straight lines to appear curved

function ptc radial distortion invariance

$$x_{\text{distorted}} = x(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

$$y_{\text{distorted}} = y(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

→ Tangential distortion :- occurs when the image-taking lens is not perfectly aligned perfectly parallel to the imaging plane. (lens may shift some from expected)

$$x_{\text{distorted}} = x + [k_4 + k_5] + k_6 (x^2 + y^2)$$

$$y_{\text{distorted}} = y + [k_7 + k_8] + 2k_9 xy$$

clearly, we need  $(c_x, c_y, f_x, f_y, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9)$ .

Radial parameters:  $\left\{ \begin{array}{l} c_x, c_y \text{ camera} \\ f_x, f_y \text{ focal length} \\ k_1, k_2, k_3 \text{ optical center} \end{array} \right\}$

Tangential parameters:  $\left\{ \begin{array}{l} k_4, k_5, k_6, k_7, k_8, k_9 \end{array} \right\}$  corresponds to skewing and translational vectors

$$\text{camera} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Table 1:- Take several images of a chessboard from a camera at different angles & positions. Now, we will compute the camera of the distortion in all images.

Assumption:- when board was kept stationary in xy plane, the two images & camera were moved accordingly.

Now, pass  $(x_1, y_1), (x_2, y_2)$  for  $(x, y)$ .  
 Now, say estimate some  $z$  value.