
MULTI-AGENT AUTOMATED MARKET MAKING

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ABSTRACT

In this white paper we discuss the background and design decisions of our new AMM v2 "Yield Farming" algorithm. We provide a quick review of constant function pricing models (CFPMs). We propose a new CFPM scoring rule designed to promote inventory management, while also providing price protections for retail market orders. We touch on ways AMM v2 may impact pricing, liquidity, and arbitrage on the exchange. We also share the results of running AMM v2 in simulated markets.

Keywords Exchange · Liquidity · Automated Market Making

1 Background

LVL is a US-based financial institution and mobile app that offers free cryptocurrency trading and other consumer financial. LVL operates its own centralized exchange, or spot market, and does not route order flow to institutional market makers or allow them to trade on LVL. To ensure the availability of liquidity on LVL's spot market, we developed and deployed automated market making on LVL in 2019 [1]. Automated market making is available to all LVL Premium members as part of the Autopilot feature of the mobile app. LVL does not participate in automated market making on its own spot market. The automated market making algorithm, which we refer to in this paper as AMM v1, implements the marketing making strategy of Avellaneda and Stoikov [2], the optimal strategy for marketing making on limit order books under the standard brownian motion model of market price. AMM v1 modifies this algorithm in several ways to simplify user configuration and manage asset inventory. Since March 2019, Autopilot accounts running AMM v1 has provided liquidity for millions of dollars of market order flow, while earning a potential income for users. While AMM v1 has demonstrated efficient pricing to market, it has experienced numerous instances where inventory has been either sold or bought out on a pair. When inventory is sold or bought out, buy and sell market orders can no longer be executed, respectively. We refer to this as the inventory management problem, and it is the central motivation for AMM v2.

Since we introduced automated market making on LVL, there has been an explosion of interest in automated market making in decentralized finance, or DeFi. DeFi is a system of censorship-resistant, decentralized financial primitives including lending [3], stable assets [4], and exchanges [5]. DeFi exchanges provide a mechanism of exchange of digital assets between interested parties similar to exchanges and marketplaces found in traditional. These digital assets can include online ad units, prediction market bets, or cryptocurrencies. Because digital assets are an emerging class of assets, DeFi exchanges have historically suffered from low liquidity, which has led to the introduction of automated market makers to "bootstrap" liquidity in nascent digital asset markets [6]. Automated market makers allow market participants to passively allocate their digital assets to the provision of liquidity on an exchange. In return, these participants earn a potential yield on their assets through either trading commissions or a spread on the marginal exchange price [7]. Automated market making algorithms use a *scoring rule* to map liquidity pool reserves and/or a reference market price provided by an oracle, to a marginal asset price. A popular scoring rule for prediction markets is Hanson's *Logarithmic Market Scoring Rule* (LMSR) [8]. Our AMM v1 market making algorithm uses a scoring rule based on stochastic optimal control to price limit orders on a spot market. Bancor introduced a scoring rule based on bond curves [9]. The most well known scoring rule for DeFi exchanges is the *constant product pricing model*, first introduced in the Uniswap Protocol. Constant product pricing is part of a broader family of *constant function pricing*

models (CFPMs) which require any trade to result in reserves in such a way that some function of the reserves is always equal to some constant k . In CPPM exchanges, this function is $xy = k$. CPPMs appear to work exceptionally well at solving the inventory management problem despite their simplicity [10]. CPPMs, and the broader CFPM family have been demonstrated to closely track the reference price under common conditions.

In this white paper we discuss the background and design decisions of our new AMM v2 "Yield Farming" algorithm. We provide a quick review of constant function pricing models (CFPMs). We propose a new CFPM scoring rule designed to promote inventory management, while also providing price protections for retail market orders. We touch on ways AMM v2 may impact pricing, liquidity, and arbitrage on the exchange. We also share the results of running AMM v2 in simulated markets.

2 Constant Function Pricing Models

The section explains the motivation and mathematical formulation of several well known CFPMs. In exploring the mathematical formulations, we deviate from the reference work in formulas and variable names so that the reader may benefit from a more notationally cohesive exploration of multiple different CFPMs. Recall that CFPMs require that the asset reserves, represented in their notional value of a single currency (typically USD), $R_1, R_2, \dots, R_n \in \mathbb{R}^+$ satisfy at all times the relation $\varphi(R_1, R_2, \dots, R_n) = k$. Here, k is the constant and φ is the *constant function*. The constant function is not to be confused with the constant map, which maps the entire domain to a constant. The set of all valid reserve allocations (R_1, R_2, \dots, R_n) , the preimage of k under φ . Not all valid reserve allocations may be reachable based on choice of constant function and market model.

2.1 Constant Product Pricing Model

We begin with Constant Product Pricing Model from Uniswap [5]. This model uses the constant function $xy = k$. We denote the constant function as $\varphi_{\times}(R_1, R_2)$, and valid reserve allocations satisfy:

$$\varphi_{\times}(R_1, R_2) = R_1 R_2 = k \quad (1)$$

An easy way to visualize the set of valid asset allocations (R_1, R_2) is to plot them as a curve on a graph where the x and y axes represent the value of reserves R_1 and R_2 respectively. In this case, the curve is $R_2(R_1) = k/R_1$, the familiar inverse linear function $y = a/x$. Because any trade must result in another point on this curve, it is useful to characterize which trades of a given asset allocation (R_1, R_2) result in a new asset allocation (R'_1, R'_2) satisfying $\varphi_{\times}(R'_1, R'_2)$, and as a result may be considered "valid trades." To analyze this question, we characterize a trade as a pair of fills (Δ_1, Δ_2) . W.l.o.g., we can assume that the market order being executed is trading asset Δ_1 for asset Δ_2 . Execution of the trade (Δ_1, Δ_2) results in a new asset allocation $(R_1, R_2) \rightarrow (R_1 + \Delta_1, R_2 - \Delta_2)$. For this asset allocation to be valid, it must satisfy $\varphi_{\times}(R_1 + \Delta_1, R_2 - \Delta_2) = k$. Through manipulation, we can see that Δ_2 can be expressed as a function of $(\Delta_1, R_1, \text{and } R_2)$:

$$\Delta_2 = \frac{R_2 \Delta_1}{R_1 + \Delta_1} \quad (2)$$

We can define the executed price, $p_{\Delta_2}(\Delta_1)$, on this trade as Δ_1/Δ_2 , which represents the units of Δ_1 required to buy one unit of Δ_2 . The price $p_{\Delta_1}(\Delta_2)$ can be solved using the above result, leading to:

$$p_{\Delta_2}(\Delta_1) = \Delta_1/\Delta_2 = \frac{R_1 \Delta_1 + \Delta_1^2}{R_2 \Delta_1} \quad (3)$$

This reveals two important properties of Constant Product Pricing Models. First, the price to buy Δ_2 scales quadratically in the amount of Δ_1 that is traded, a process called slippage. CPPM's quadratic slippage creates a strong disincentive for traders to buy out R_2 , and in fact R_2 will not ever sell out, as we will see shortly. Secondly, by evaluating the limit of $p_{\Delta_2}(\Delta_1)$ as $(\Delta_1, \Delta_2) \rightarrow (0, 0)$ we can see that the marginal price on the liquidity pool is R_1/R_2 .

We are particularly interested in the case in which R_1 represents US dollars so that $R_1 = R_D$, and R_2 is a cryptocurrency so that $R_2 = R_B = p_0 B$. B is the reserve cryptocurrency holdings, and p_0 is the prevailing market price for the cryptocurrency, measured in dollars. We immediately see that $k = R_D R_B = R_D(p_0 B)$. As in the previous example, we assume a market order buy arrives represented by (Δ_D, Δ_B) . This can also be written as (Δ_D, δ_B) , where δ_B

represents the fill size in the cryptocurrency, and the notional value of the cryptocurrency fill, $p_0\delta_B$ is implicit. Again, δ_B can be closed for offer size Δ_D :

$$\delta_B = \frac{R_B \Delta_D}{p_0(R_D + \Delta_D)} \quad (4)$$

Further, we can calculate the price $p_{\delta_B}(\Delta_D)$ of this trade as:

$$p_{\delta_B}(\Delta_D) = \Delta_D / \delta_B = \frac{p_0(R_D \Delta_D + \Delta_D^2)}{p_0 B \Delta_D} = \frac{R_D \Delta_D + \Delta_D^2}{B \Delta_D} \quad (5)$$

Again, we see that the price to buy δ_B scales quadratically in the amount of Δ_D on offer by the market order. By taking the limit of $p_{\delta_B}(\Delta_D)$ as $(\Delta_D, \delta_B) \rightarrow (0, 0)$, we can the marginal price for the liquidity pool is R_D/B . Furthermore, the market discovery price, which is the marginal price when $R_D = R_B = p_0 B$, is seen to be $R_D/B = R_D/(R_D/P_0) = p_0$!

3 Multi-Agent CFPM

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