1. What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$$\|\vec{w}\| = \sqrt{0.5^2 + 0.5^2} = \boxed{0.707}$$

2. Multiple the following two vectors $(\vec{x} * \vec{w}^T)$, where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$$\begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} = [0.5 \cdot 0.75 + 0.5 \cdot 1.25] = \boxed{[1]}$$

$1 \times 2 \quad 2 \times 1 \quad = \quad 1 \times 1$

3. Multiple the following two vectors ($\vec{x}^T * \vec{w}$) using the vectors from the previous problem.

$$\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.75 & 1.25 \end{bmatrix} = \begin{bmatrix} 0.5 \cdot 0.75 & 0.5 \cdot 1.25 \\ 0.5 \cdot 0.75 & 0.5 \cdot 1.25 \end{bmatrix} = \boxed{\begin{bmatrix} 0.375 & 0.625 \\ 0.375 & 0.625 \end{bmatrix}}$$

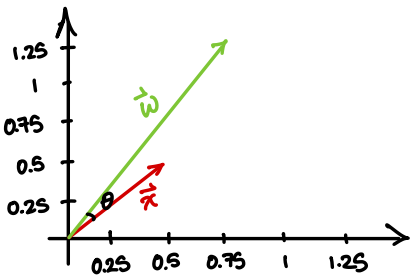
$2 \times 1 \qquad 1 \times 2 \qquad = \qquad 2 \times 2$

4. What is the dot product of $\vec{x}$ and $\vec{w}$ using the values from the previous problem?

$\vec{x} = [0.5,\ 0.5]$ $\quad$ $\vec{w} = [0.75,\ 1.25]$

$\vec{x} \cdot \vec{w} = 0.5 \cdot 0.75 + 0.5 \cdot 1.25 = \boxed{1}$

5. What is the angle between $\vec{x}$ and $\vec{w}$ using the values from the previous problem? Draw the vectors and label the angle that you found.

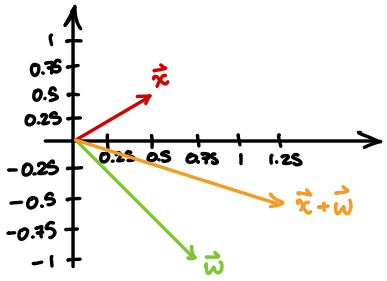


$$\cos\theta = \frac{\vec{x}\cdot\vec{w}}{\|\vec{x}\|\cdot\|\vec{w}\|} = \frac{1}{\sqrt{0.5^2+0.5^2}\sqrt{0.75^2+1.25^2}} \approx 0.970$$

$$\theta = \cos^{-1}(0.970) = \boxed{14.03^\circ}$$

6. Add the following vectors, and draw the resultant and the original vectors.
$\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, -1]$

$\vec{x} + \vec{w} = [1.25, -0.5]$

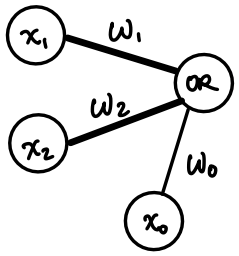


7. What is the difference between prediction and classification?

Predictions are real numbers ($\mathbb{R}$), some continuous value. Classification on the other hand involves using discrete values as output, like classifying between cats, dogs, mice, etc.

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are $w_0 = 0, w_1 = 0.5, w_2 = -0.5$ and the learning parameter $\nu = 0.25$. You may also assume that $x_0 = 1$.

x_1	x_2	OR
0	0	0
0	1	1
1	0	1
1	1	1



Epoch 1:

$w_0 = 0$
$w_1 = 0.5$
$w_2 = -0.5$
$\nu = 0.25$

activation func'n

$y_1 = 1 \cdot w_0 + 0 \cdot w_1 + 0 \cdot w_2 = 0 \Rightarrow 0$

$w_0 \leftarrow 0 - 0.25(0-0) \cdot 1 = 0$
$w_1 \leftarrow 0.5 - 0.25(0-0) \cdot 0 = 0.5$
$w_2 \leftarrow -0.5 - 0.25(0-0) \cdot 0 = -0.5$

$y_2 = 1 \cdot w_0 + 0 \cdot w_1 + 1 \cdot w_2 = -0.5 \Rightarrow 0$

$w_0 \leftarrow 0 - 0.25(0-1) \cdot 1 = 0.25$
$w_1 \leftarrow 0.5 - 0.25(0-1) \cdot 0 = 0.5$
$w_2 \leftarrow -0.5 - 0.25(0-1) \cdot 1 = -0.25$

$y_3 = 1 \cdot w_0 + 1 \cdot w_1 + 0 \cdot w_2 = 0.25 + 0.5 \Rightarrow 1$

$w_0 \leftarrow 0.25 - 0.25(1-1) \cdot 1 = 0.25$
$w_1 \leftarrow 0.5 - 0.25(1-1) \cdot 1 = 0.5$
$w_2 \leftarrow -0.25 - 0.25(1-1) \cdot 0 = -0.25$

$y_4 = 1 \cdot w_0 + 1 \cdot w_1 + 1 \cdot w_2 = 0.25 + 0.5 - 0.25 \Rightarrow 1$

$w_0 \leftarrow 0.25 - 0.25(1-1) \cdot 1 = 0.25$
$w_1 \leftarrow 0.5 - 0.25(1-1) \cdot 1 = 0.5$
$w_2 \leftarrow -0.25 - 0.25(1-1) \cdot 0 = -0.25$

Epoch 2:

$w_0 = 0.25$
$w_1 = 0.5$
$w_2 = -0.25$
$\nu = 0.25$

$y_1 = 1 \cdot w_0 + 0 \cdot w_1 + 0 \cdot w_2 = 0.25 \Rightarrow 1$

$w_0 \leftarrow 0.25 - 0.25(1-0) \cdot 1 = 0$
$w_1 \leftarrow 0.5 - 0.25(1-0) \cdot 0 = 0.5$
$w_2 \leftarrow -0.25 - 0.25(1-0) \cdot 0 = -0.25$

$y_2 = 1 \cdot w_0 + 0 \cdot w_1 + 1 \cdot w_2 = -0.25 \Rightarrow 0$

$w_0 \leftarrow 0 - 0.25(0-1) \cdot 1 = 0.25$
$w_1 \leftarrow 0.5 - 0.25(0-1) \cdot 0 = 0.5$
$w_2 \leftarrow -0.25 - 0.25(0-1) \cdot 1 = 0$

$y_3 = 1 \cdot w_0 + 1 \cdot w_1 + 0 \cdot w_2 = 0.25 + 0.5 \Rightarrow 1$

no update, met target

$y_4 = 1 \cdot w_0 + 1 \cdot w_1 + 1 \cdot w_2 = 0.25 + 0.5 \Rightarrow 1$

no update

Epoch 3:

$w_0 = 0.25$
$w_1 = 0.5$
$w_2 = 0$
$\nu = 0.25$

$y_1 = 1 \cdot w_0 + 0 \cdot w_1 + 0 \cdot w_2 = 0.25 \Rightarrow 1$

$w_0 \leftarrow 0.25 - 0.25(1-0) \cdot 1 = 0$
$w_1 \leftarrow 0.5 - 0.25(1-0) \cdot 0 = 0.5$
$w_2 \leftarrow 0 - 0.25(1-0) \cdot 0 = 0$

$y_2 = 1 \cdot w_0 + 0 \cdot w_1 + 1 \cdot w_2 = 0 \Rightarrow 0$

$w_0 \leftarrow 0 - 0.25(0-1) \cdot 1 = 0.25$
$w_1 \leftarrow 0.5 - 0.25(0-1) \cdot 0 = 0.5$
$w_2 \leftarrow 0 - 0.25(0-1) \cdot 1 = 0.25$

$y_3 = 1 \cdot w_0 + 1 \cdot w_1 + 0 \cdot w_2 = 0.25 + 0.5 \Rightarrow 1$

no update

$y_4 = 1 \cdot w_0 + 1 \cdot w_1 + 1 \cdot w_2 = 0.25 + 0.5 + 0.25 \Rightarrow 1$

no update

Epoch 4:

$w_0 = 0.25$
$w_1 = 0.5$
$w_2 = 0.25$
$\nu = 0.25$

$y_1 = 1 \cdot w_0 + 0 \cdot w_1 + 0 \cdot w_2 = 0.25 \Rightarrow 1$

$w_0 \leftarrow 0.25 - 0.25(1-0) \cdot 1 = 0$
$w_1 \leftarrow 0.5 - 0.25(1-0) \cdot 0 = 0.5$
$w_2 \leftarrow 0.25 - 0.25(1-0) \cdot 0 = 0.25$

$y_2 = 1 \cdot w_0 + 0 \cdot w_1 + 1 \cdot w_2 = 0.25 \Rightarrow 1$

No update

$y_3 = 1 \cdot w_0 + 1 \cdot w_1 + 0 \cdot w_2 = 0.5 \Rightarrow 1$

No update

$y_4 = 1 \cdot w_0 + 1 \cdot w_1 + 1 \cdot w_2 = 0.5 + 0.25 \Rightarrow 1$

No update

Epoch 5:

$w_0 = 0$
$w_1 = 0.5$
$w_2 = 0.25$
$\nu = 0.25$

$y_1 = 1 \cdot w_0 + 0 \cdot w_1 + 0 \cdot w_2 = 0 \Rightarrow 0$

No update

$\therefore$ $\boxed{w_0 = 0,\ w_1 = 0.5,\ w_2 = 0.25}$