1. What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$|\vec{w}| = \sqrt{0.5^2 + 0.5^2} = 0.7071$

2. Multiple the following two vectors $(\vec{x} * \vec{w}^T)$, where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$(\vec{x} * \vec{w}^T) = [0.5, 0.5] \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} = [0.75 \cdot 0.5, 1.25 \cdot 0.5]$
$= [0.375, 0.625]$

3. Multiple the following two vectors ($\vec{x}^T * \vec{w}$) using the vectors from the previous problem.

$$(\vec{x}^T * \vec{w}) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} * [0.75 \ 1.25]$$

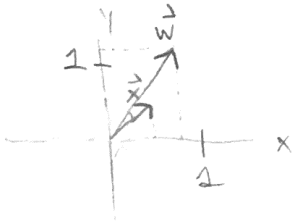
$$= [0.5 \cdot 1.25 + 0.5 \cdot 0.75]$$

$$= [0.625, 0.375]$$

4. What is the dot product of $\vec{x}$ and $\vec{w}$ using the values from the previous problem?

$(0.5, 0.5) \cdot (0.75, 1.25) = 0.5 \cdot 0.75 + 0.5 \cdot 1.25 = 1$

5. What is the angle between $\vec{x}$ and $\vec{w}$ using the values from the previous problem? Draw the vectors and label the angle that you found.



$\vec{x} \cdot \vec{w} = |\vec{x}| \cdot |\vec{w}| \cdot \cos\theta$

$\cos\theta = \frac{\vec{x} \cdot \vec{w}}{|\vec{x}| \cdot |\vec{w}|} = \frac{1}{1.4577 \cdot 0.7071} = 0.9701$

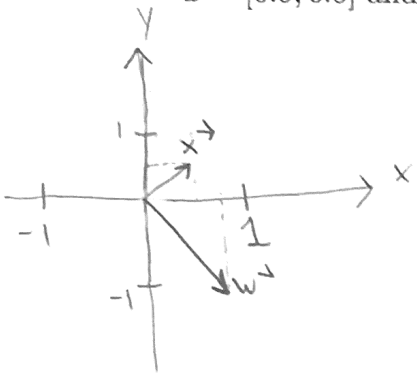
$|\vec{w}| = \sqrt{0.75^2 + 1.25^2} = 1.4577$

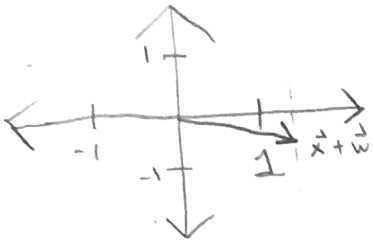
$|\vec{x}| = \sqrt{0.5^2 + 0.5^2} = 0.7071$

$\theta = \cos^{-1}(0.9701) = 14.04^\circ$

6. Add the following vectors, and draw the resultant and the original vectors.
$\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, -1]$

$\vec{x} + \vec{w} = (0.5+0.75, 0.5-1) = (1.25, -0.5)$

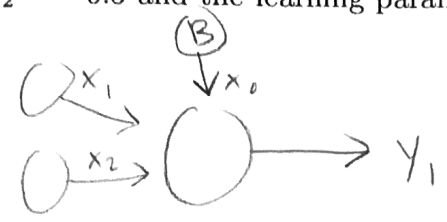




7. What is the difference between prediction and classification?

A prediction is guessing / calculating unknown or future data.
Classification is about categorizing data into sets.

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are $w_0 = 0, w_1 = 0.5, w_2 = -0.5$ and the learning parameter $\nu = 0.25$. You may also assume that $x_0 = 1$.



x_1	x_2	OR	
0	0	0	$g(0 \cdot 0.5 + 0 \cdot -0.5 + 0) = g(0) = 0$ ✓
0	1	1	$g(0 \cdot 0.5 + 1 \cdot -0.5 + 0) = g(-0.5) = 0$ ✗
1	0	1	$g(1 \cdot 0.5 + 0 \cdot -0.5 + 0) = g(0.5) = 1$ ✓
1	1	1	$g(1 \cdot 0.5 + 1 \cdot -0.5 + 0) = g(0) = 0$ ✗

Need to update weights

2- $w_1 = 0.5 - 0.25(0-1) \cdot 0 = 0.5$ same
$w_2 = -0.5 - 0.25(0-1) \cdot 1 = -0.25$ ↑
$w_0 = 0 - 0.25(0-1) \cdot 1 = +0.25$ ↑

4- $w_1 = 0.5 - 0.25(0-1) \cdot 1 = 0.75$ ↑
$w_2 = -0.25 - 0.25(0-1) \cdot 1 = 0$ ↑
$w_0 = 0.25 - 0.25(0-1) \cdot 1 = 0.5$ ↑

2nd iter

x_1	x_2	OR	
0	0	0	$g(0 \cdot 0.75 + 0 \cdot 0 + 0.5) = g(0.5) = 1$ ✗
0	1	1	$g(0 \cdot 0.75 + 1 \cdot 0 + 0.5) = g(0.5) = 1$ ✓
1	0	1	$g(1 \cdot 0.75 + 0 \cdot 0 + 0.5) = g(1.25) = 1$ ✓
1	1	1	$g(1 \cdot 0.75 + 1 \cdot 0 + 0.5) = g(1.25) = 1$ ✓

1- $w_1 = 0.75 - 0.25(1-0) \cdot 0 = 0.75$ same
$w_2 = 0 - 0.25(1-0) \cdot 0 = 0$ same
$w_0 = 0.5 - 0.25(1-0) \cdot 1 = 0.25$ ↓

3rd iter

x_1	x_2	OR	
0	0	0	$g(0 \cdot 0.75 + 0 \cdot 0 + 0.25) = g(0.25) = 1$ ✗
0	1	1	$g(0 \cdot 0.75 + 1 \cdot 0 + 0.25) = g(0.25) = 1$ ✓
1	0	1	$g(1 \cdot 0.75 + 0 \cdot 0 + 0.25) = g(1) = 1$ ✓
1	1	1	$g(1 \cdot 0.75 + 1 \cdot 0 + 0.25) = g(1) = 1$ ✓

1- $w_1 = 1$ same, $w_2 = 0$ same, $w_0 = 0$ ↓

4th iter

x_1	x_2	OR	
0	0	0	$g(0 \cdot 0.75 + 0 \cdot 0 + 0) = g(0) = 0$ ✓
0	1	1	$g(0 \cdot 0.75 + 1 \cdot 0 + 0) = g(0) = 0$ ✗
1	0	1	$g(1 \cdot 0.75 + 0 \cdot 0 + 0) = g(0.75) = 1$ ✓
1	1	1	$g(1 \cdot 0.75 + 1 \cdot 0 + 0) = g(0.75) = 1$ ✓

update $w_1 = 0.75$ same, $w_2 = 0.25$ ↑, $w_0 = 0.25$ ↑

5th

x_1	x_2	OR	
0	0	0	$g(0 \cdot 0.75 + 0 \cdot 0.25 + 0.25) = g(0.25) = 1$ ✗
0	1	1	1 ✓
1	0	1	1 ✓
1	1	1	1 ✓

update $w_1 = 0.75$ same
$w_2 = 0.25$ same
$w_0 = 0$ ↓

6th

x_1	x_2	OR	
0	0	0	0
0	1	1	1 ✓
1	0	1	1
1	1	1	1