

1. What is the magnitude of  $\vec{w} = [0.5, 0.5]$ ?

$$\begin{aligned} & \sqrt{(0.5)(0.5) + (0.5)(0.5)} \\ &= \sqrt{0.25 + 0.25} \\ &= \sqrt{0.5} \end{aligned}$$

2. Multiple the following two vectors ( $\vec{x} * \vec{w}^T$ ), where  $\vec{x} = [0.5, 0.5]$  and  $\vec{w} = [0.75, 1.25]$

$$\begin{aligned} & [0.5, 0.5] \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} \\ &= [0.5 \cdot 0.75 + 0.5 \cdot 1.25] \\ &= [0.375 + 0.625] \\ &= [1] \end{aligned}$$

3. Multiple the following two vectors ( $\vec{x}^T * \vec{w}$ ) using the vectors from the previous problem.

$$\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.75 & 1.25 \end{bmatrix}$$
$$= \begin{bmatrix} 0.5 \cdot 0.75 & 0.5 \cdot 1.25 \\ 0.5 \cdot 0.75 & 0.5 \cdot 1.25 \end{bmatrix}$$
$$= \begin{bmatrix} 0.375 & 0.625 \\ 0.375 & 0.625 \end{bmatrix}$$

4. What is the dot product of  $\vec{x}$  and  $\vec{w}$  using the values from the previous problem?

$$\begin{aligned}
 & [0.5, 0.5] \cdot [0.75, 1.25] \\
 &= 0.5 \cdot 0.75 + 0.5 \cdot 1.25 \\
 &= 0.375 + 0.625 \\
 &= 1
 \end{aligned}$$

5. What is the angle between  $\vec{x}$  and  $\vec{w}$  using the values from the previous problem? Draw the vectors and label the angle that you found.

$$\vec{x} \cdot \vec{w} = 1$$

$$|\vec{x}| = \sqrt{0.5}$$

$$\begin{aligned}
 |\vec{w}| &= \sqrt{(0.75)(0.75) + (1.25)(1.25)} \\
 &= \sqrt{0.5625 + 1.5625} \\
 &= \sqrt{2.125}
 \end{aligned}$$

$$\cos \theta = \frac{\vec{x} \cdot \vec{w}}{|\vec{x}| |\vec{w}|}$$

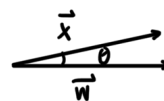
$$\cos \theta = \frac{1}{(\sqrt{0.5})(\sqrt{2.125})}$$

$$\cos \theta = \frac{1}{1.03}$$

$$\cos \theta = 0.97$$

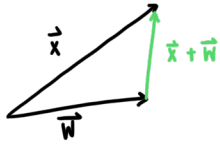
$$\theta \approx \cos^{-1}(0.97)$$

$$\theta \approx 0.25^\circ$$



6. Add the following vectors, and draw the resultant and the original vectors.  
 $\vec{x} = [0.5, 0.5]$  and  $\vec{w} = [0.75, -1]$

$$\begin{aligned}\vec{x} + \vec{w} &= [0.5 + 0.75, 0.5 + -1] \\ &= [1.25, -0.5]\end{aligned}$$



7. What is the difference between prediction and classification?

A prediction is a guess, while a classification is an identifier.

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are  $w_0 = 0, w_1 = 0.5, w_2 = -0.5$  and the learning parameter  $\nu = 0.25$ . You may also assume that  $x_0 = 1$ .

$x_1$	$x_2$	OR	$y_i$
0	0	0	$g(0 \cdot 0 + 0 \cdot 0.5 + 0 \cdot -0.5) = 0 \quad \checkmark$
0	1	1	$g(0 \cdot 0 + 1 \cdot 0.5 + 1 \cdot -0.5) = 0 \quad \times$
1	0	1	$g(1 \cdot 0 + 0 \cdot 0.5 + 1 \cdot -0.5) = -0.5 \Rightarrow 0 \quad \times$
1	1	1	$g(1 \cdot 0 + 1 \cdot 0.5 + 1 \cdot -0.5) = 0 \quad \times$

update:

$$0 - 0.25(0 - 1) \cdot 0 = 0$$

$$0.5 - 0.25(0 - 1) \cdot 1 = 0.25 \quad \uparrow$$

$$-0.5 - 0.25(0 - 1) \cdot -1 = 0.5 \quad \uparrow$$