

1. What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$$|\vec{w}| = \sqrt{(0.5)^2 + (0.5)^2} = \boxed{0.75}$$

2. Multiple the following two vectors ($\vec{x} * \vec{w}^T$), where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$$(\vec{x} * \vec{w}^T) = [0.5, 0.5] * [0.75, 1.25] = [0.375, 0.625]$$

3. Multiple the following two vectors ($\vec{x}^T * \vec{w}$) using the vectors from the previous problem.

$$(\vec{x}^T * \vec{w}) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} * \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$

4. What is the dot product of \vec{x} and \vec{w} using the values from the previous problem?

$$\vec{x} \cdot \vec{w} = [0.5, 0.5] \cdot \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix}$$

$$= 0.5 * 0.75 + 0.5 * 1.25$$

$$= 0.375 + 0.625$$

$$= 1$$

5. What is the angle between \vec{x} and \vec{w} using the values from the previous problem? Draw the vectors and label the angle that you found.

$$\theta = \cos^{-1} \left(\frac{\vec{x} \cdot \vec{w}}{|\vec{x}| |\vec{w}|} \right)$$

$$= \cos^{-1} \left(\frac{1}{(1.457)(0.75)} \right)$$

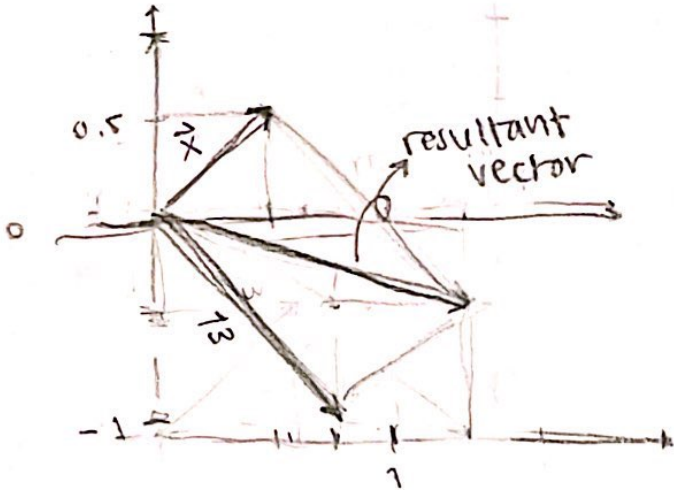
$$= \cos^{-1}(0.914)$$

$$= \boxed{23.84^\circ}$$

$$|\vec{x}| = \sqrt{(0.75)^2 + (1.25)^2} = 1.457$$

6. Add the following vectors, and draw the resultant and the original vectors.
 $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, -1]$

$$\begin{aligned}\vec{x} + \vec{w} &= [0.5, 0.5] + [0.75, -1] \\ &= [1.25, -0.5]\end{aligned}$$



7. What is the difference between prediction and classification?

The difference is that prediction is an attempt to get as close to the answer as possible, whereas classification is the act of dividing information into classes.

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are $w_0 = 0, w_1 = 0.5, w_2 = -0.5$ and the learning parameter $\nu = 0.25$. You may also assume that $x_0 = 1$.

x_1	x_2	OR		correct
0	0	0	$g(0 \cdot 0.05 + 0 \cdot -0.05 + 0) = g(0) = 0$	✓
0	1	1	$g(0 \cdot 0.05 + 1 \cdot -0.05 + 0) = g(-0.05) = 0$	x
1	0	1	$g(1 \cdot 0.05 + 0 \cdot -0.05 + 0) = g(0.05) = 1$	
1	1	1	$g(1 \cdot 0.05 + 1 \cdot -0.05 + 0) = g(0) = 0$	

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ \text{OR} &= 0 \end{aligned} \quad g(0 \cdot 0.05 + 0 \cdot -0.05 + 0) = g(0) = 0 \quad \checkmark$$

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 1 \\ \text{OR} &= 1 \end{aligned} \quad g(0 \cdot 0.05 + 1 \cdot -0.05 + 0) = g(-0.05) = 0 \quad \times$$

Update weights

$$\begin{aligned} w_0 &= 0 - (0.25)(0 - 1) \cdot 1 = 0.25 \\ w_1 &= 0.5 - (0.25)(0 - 1) \cdot 0 = 0.5 \\ w_2 &= -0.5 - (0.25)(0 - 1) \cdot 1 = -0.25 \end{aligned}$$

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 0 \\ \text{OR} &= 1 \end{aligned} \quad g(1 \cdot 0.5 + 0 \cdot -0.25 + 1 \cdot -0.25) = g(0.75) = 1 \quad \checkmark$$

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 1 \\ \text{OR} &= 1 \end{aligned} \quad g(1 \cdot 0.5 + 1 \cdot -0.25 + 1 \cdot -0.25) = g(0.5) = 1 \quad \checkmark$$

Weights: $w_0 = 0.25$
 $w_1 = 0.5$
 $w_2 = -0.25$