

Vectors and Perceptrons

1) $\bar{w} = [0.5, 0.5]$

$$\|\bar{w}\| = \sqrt{0.5^2 + 0.5^2} = \sqrt{0.5}$$

2) $\bar{x} = [0.5, 0.5]$

$$\bar{w} = [0.75, 1.25]$$

$$\bar{x} \times \bar{w}^T = [0.5, 0.5] \times \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix}$$

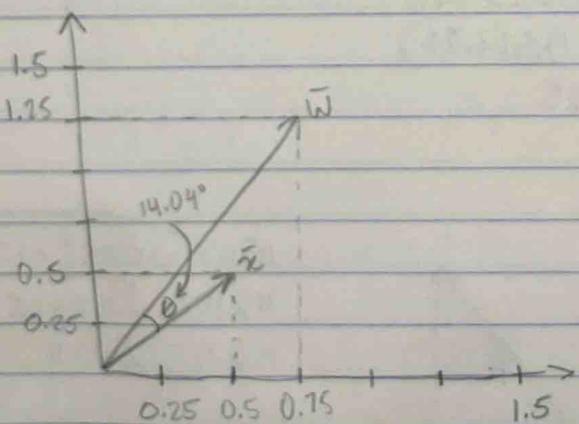
$$\bar{x} \times \bar{w}^T = [0.375, 0.625]$$

3) $\bar{x}^T \times \bar{w} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \times [0.75, 1.25]$

$$\bar{x}^T \times \bar{w} = [0.375, 0.625]$$

4)
$$\begin{aligned} \bar{x} \cdot \bar{w} &= [0.5, 0.5] \cdot [0.75, 1.25] \\ &= 0.5(0.75) + 0.5(1.25) \\ &= 1 \end{aligned}$$

5)

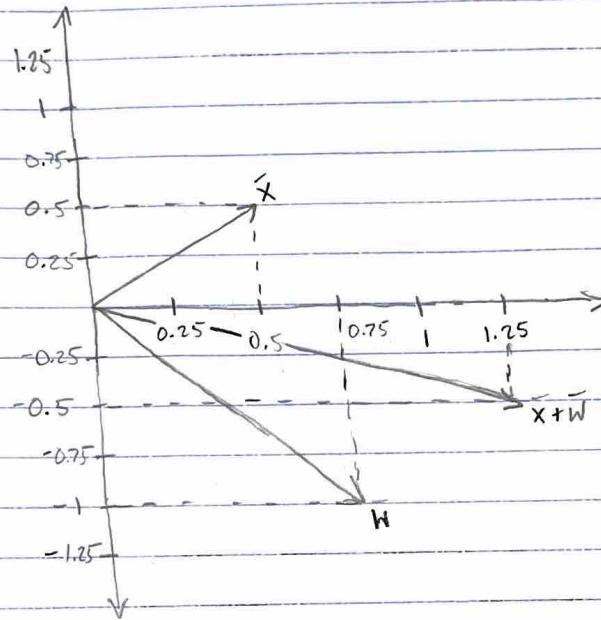


$$\theta = \cos^{-1} \left(\frac{\bar{x} \cdot \bar{w}}{\|\bar{x}\| \|\bar{w}\|} \right) = \cos^{-1} \left(\frac{1}{\sqrt{0.5} \times \sqrt{2.125}} \right) = 0.24498 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = 14.04^\circ$$

$$\|\bar{x}\| = \sqrt{0.5^2 + 0.5^2} = \sqrt{0.5}$$

$$\|\bar{w}\| = \sqrt{0.75^2 + 1.25^2} = \sqrt{2.125}$$

6) $\bar{x} = [0.5, 0.5]$
 $\bar{w} = [0.75, -1]$
 $\bar{x} + \bar{w} = [1.25, -0.5]$



7) Prediction is when a model predicts a continuous value associated with a specific input value/vector. Classification is when a model predicts a specific input value/vector to belong to one of a limited set of categories.

Update After All Inputs

8) $w_0 = 0$

$w_1 = 0.5$

$w_2 = -0.5$

$$w = w - z_2(y-t)x$$

1st Forward

x_0	x_1	x_2	t	$y = x_0(w_0) + x_1(w_1) + x_2(w_2)$
1	0	0	0	$= 1(0) + 0(0.5) + 0(-0.5) = 0 \checkmark$
1	0	1	1	$= 1(0) + 0(0.5) + 1(-0.5) = -0.5 = 0 \times$
1	1	0	1	$= 1(0) + 1(0.5) + 0(-0.5) = 0.5 = 1 \checkmark$
1	1	1	1	$= 1(0) + 1(0.5) + 1(-0.5) = 0 = 0 \times$

1st Backward

$$w_0 = 0 - 0.25(-1)(1) - 0.25(-1)(1)$$

$$w_0 = 0 + 0.25 + 0.25 = \boxed{0.5} \neq w_0$$

$$w_1 = 0.5 - 0.25(-1)(0) - 0.25(-1)(1)$$

$$w_1 = 0.5 + 0.25 = \boxed{0.75} = w_1$$

$$w_2 = -0.5 - 0.25(-1)(1) - 0.25(-1)(1)$$

$$w_2 = -0.5 + 0.25 + 0.25 = \boxed{0} \neq w_2$$

x_0	x_1	x_2	t	$y = x_0(w_0) + x_1(w_1) + x_2(w_2)$
1	0	0	0	$= 1(0.5) + 0(0.75) + 0(0) = 0.5 = 1 \times$
1	0	1	1	$= 1(0.5) + 0(0.75) + 1(0) = 0.5 = 1 \checkmark$
1	1	0	1	$= 1(0.5) + 1(0.75) + 0(0) = 1.25 = 1 \checkmark$
1	1	1	1	$= 1(0.5) + 1(0.75) + 1(0) = 1.25 = 1 \checkmark$

$$w_0 = 0.5 - 0.25(1)(1)$$

$$w_0 = \boxed{0.25}$$

$$w_1 = 0.75 - 0.25(1)(0)$$

$$w_1 = \boxed{0.75}$$

$$w_2 = 0 - 0.25(1)(0)$$

$$w_2 = \boxed{0}$$

x_0	x_1	x_2	t	$y = w_0(x_0) + w_1(x_1) + w_2(x_2)$
1	0	0	0	$= 1(0.25) + 0(0.75) + 0(0) = 0.25 = 1 \times$
1	0	1	1	$= 1(0.25) + 0(0.75) + 1(0) = 0.25 = 1 \checkmark$
1	1	0	1	$= 1(0.25) + 1(0.75) + 0(0) = 1 = 1 \checkmark$
1	1	1	1	$= 1(0.25) + 1(0.75) + 1(0) = 1 = 1 \checkmark$

$$w_0 = 0.25 - 0.25(1)(1)$$

$$w_0 = \boxed{0}$$

$$w_1 = 0.75 - 0.25(1)(0)$$

$$w_1 = \boxed{0.75}$$

$$w_2 = 0 - 0.25(1)(0)$$

$$w_2 = \boxed{0}$$

x_0	x_1	x_2	t	y
1	0	0	0	$= 1(0) + 0(0.75) + 0(0) = 0 \checkmark$
1	0	1	1	$= 1(0) + 0(0.75) + 1(0) = 0 \times$
1	1	0	1	$= 1(0) + 1(0.75) + 0(0) = 0.75 = 1 \checkmark$
1	1	1	1	$= 1(0) + 1(0.75) + 1(0) = 0.75 = 1 \checkmark$

$$w_0 = 0 - 0.25(-1)(1)$$

$$w_0 = \boxed{0.25}$$

$$w_1 = 0.75 - 0.25(-1)(0)$$

$$w_1 = \boxed{0.75}$$

$$w_2 = 0 - 0.25(-1)(1)$$

$$w_2 = \boxed{0.25}$$

$$w_0 = 0.25 \quad w_1 = 0.75 \quad w_2 = 0.25$$

x_0	x_1	x_2	t	y
1	0	0	0	$1(0.25) + 0 + 0 = 0.25 = 1 \times$
1	0	1	1	$0.25 + 0 + 0.25 = 0.5 = 1 \checkmark$
1	1	0	1	$0.25 + 0.75 + 0 = 1 = 1 \checkmark$
1	1	1	1	$0.25 + 0.75 + 0.25 = 1.25 = 1 \checkmark$

$$w_0 = 0.25 - 0.25(1)(1)$$

$$w_0 = \boxed{0}$$

$$w_1 = 0.75 - 0.25(1)(0)$$

$$w_1 = \boxed{0.75}$$

$$w_2 = 0.25 - 0.25(1)(0)$$

$$w_2 = \boxed{0.25}$$

x_0	x_1	x_2	t	y
1	0	0	0	$0 + 0 + 0 = 0 = 0 \checkmark$
1	0	1	1	$0 + 0 + 0.25 = 0.25 = 1 \checkmark$
1	1	0	1	$0 + 0.75 + 0 = 0.75 = 1 \checkmark$
1	1	1	1	$0 + 0.75 + 0.25 = 1 = 1 \checkmark$

Update For Each Vector

8) $w_0 = 0 \quad w_1 = 0.5 \quad w_2 = -0.5 \quad w^{\text{new}} = w^{\text{old}} - \eta (y - t) \cdot x$

in1: $\begin{array}{c|cc|c|c} x_0 & x_1 & x_2 & t & y \\ \hline 1 & 0 & 0 & 0 & 0+0+0=0 \end{array}$

in2: $\begin{array}{c|cc|c|c} x_0 & x_1 & x_2 & t & y \\ \hline 1 & 0 & 1 & 1 & 0+0+-0.5=-0.5=0 \times \end{array}$
* no weight updates

$$w_0 = 0 - 0.25(-1)(1) \quad w_1 = 0.5 - 0.25(-1)(0) \quad w_2 = -0.5 - 0.25(-1)(1)$$

$$w_0 = \boxed{0.25} \quad w_1 = \boxed{0.75} \quad w_2 = \boxed{-0.25}$$

in3: $\begin{array}{c|cc|c|c} x_0 & x_1 & x_2 & t & y \\ \hline 1 & 1 & 0 & 1 & 0.25+0.75+0=1=1 \end{array}$

* no weight updates

in4: $\begin{array}{c|cc|c|c} x_0 & x_1 & x_2 & t & y \\ \hline 1 & 1 & 1 & 1 & 0.25+0.75-0.25=0.75=1 \end{array}$

* no weight update

in1: $\begin{array}{c|cc|c|c} x_0 & x_1 & x_2 & t & y \\ \hline 1 & 0 & 0 & 0 & 0.25+0+0=0.25=1 \times \end{array}$

$$w_0 = 0.25 - 0.25(1)(1) \quad w_1 = 0.75 - 0.25(1)(0) \quad w_2 = -0.25 - 0.25(1)(0)$$

$$w_0 = \boxed{0} \quad w_1 = \boxed{0.75} \quad w_2 = \boxed{-0.25}$$

in2: $\begin{array}{c|cc|c|c} x_0 & x_1 & x_2 & t & y \\ \hline 1 & 0 & 1 & 1 & 0+0+-0.25=0 \times \end{array}$

$$w_0 = 0 - 0.25(1)(1) \quad w_1 = 0.75 - 0.25(-1)(0) \quad w_2 = 0.25 - 0.25(-1)(1)$$

$$w_0 = \boxed{0.25} \quad w_1 = \boxed{0.75} \quad w_2 = \boxed{0}$$

	x_0	x_1	x_2	t	y
in3:	1	1	0	1	$0.25 + 0.75 + 0 = 1 = 1 \checkmark$

* no weight change

	x_0	x_1	x_2	t	y
in4:	1	1	1	1	$0.25 + 0.75 + 0 = 1 = 1 \checkmark$

* no change

	1	0	0	0	$0.25 + 0 + 0 = 0.25 = 1 X$
in1:	1	0	0	0	

$$w_0 = 0.25 - 0.25(1)(1) \quad * \text{no other updates}$$

$$w_0 = \boxed{0} \quad w_1 = \boxed{0.75} \quad w_2 = \boxed{0}$$

	x_0	x_1	x_2	t	y
in2:	1	0	1	1	$0 + 0 + 0 = 0 X$

$$w_0 = 0 - 0.25(-1)(1) \quad w_1 = \boxed{0.75}$$

$$w_0 = \boxed{0.25}$$

$$w_2 = 0 - 0.25(-1)(1)$$

$$w_2 = \boxed{0.25}$$

	x_0	x_1	x_2	t	y
in3:	1	1	0	1	$0.25 + 0.75 + 0 = 1 = 1 \checkmark$

* no change

	x_0	x_1	x_2	t	y
in4:	1	1	1	1	$0.25 + 0.75 + 0.25 = 1.25 = 1 \checkmark$

* no change

	x_0	x_1	x_2	t	y
in1:	1	0	0	0	$0.25 + 0 + 0 = 0.25 = 1 X$

$$w_0 = 0.25 - 0.25(1)(1)$$

$$\boxed{w_0 = 0} \quad \boxed{w_1 = 0.75} \quad \boxed{w_2 = 0.25}$$

	x_0	x_1	x_2	t	y
	1	0	0	0	$0 + 0 + 0 = 0 \checkmark$
	1	0	1	1	$0 + 0 + 0.25 = 0.25 = 1 \checkmark$
	1	1	0	1	$0 + 0.75 + 0 = 0.75 = 1 \checkmark$
	1	1	1	1	$0 + 0.75 + 0.25 = 1 = 1 \checkmark$