

HW 1 - Vectors and Perceptron Worksheet

1) What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$$|\vec{w}| = \sqrt{0.5^2 + 0.5^2} = \sqrt{0.5} = \boxed{0.05\sqrt{2}}$$

2) Multiply the following $(\vec{x}^T * \vec{w}^T)$, where
 $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$$\begin{aligned} \vec{x} * \vec{w}^T &= [0.5 \quad 0.5] * \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} \quad \vec{w}^T = \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} \\ &= 0.5 \cdot 0.75 + 0.5 \cdot 1.25 = 0.375 + 0.625 = \boxed{1.0} \end{aligned}$$

3) Multiply the following two vectors $(\vec{x}^T * \vec{w})$ using the vectors

$$\vec{x}^T = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \quad \vec{x}^T * \vec{w} =$$

$$\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} * \begin{bmatrix} 0.75 & 1.25 \end{bmatrix}$$

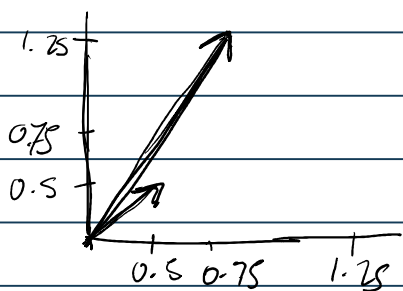
$$= \begin{bmatrix} 0.375 & 0.625 \\ 0.375 & 0.625 \end{bmatrix}$$

Q.) What is the dot product of \vec{x} and \vec{w} ?

$$\vec{x} = [0.5, 0.5] \text{ and } \vec{w} = [0.75, 1.25]$$

$$\begin{aligned}\vec{x} \cdot \vec{w} &= 0.75 \cdot 0.5 + 1.25 \cdot 0.5 \\ &= 0.375 + 0.625 \\ &= 1.0\end{aligned}$$

Q.) Angle between \vec{x} and \vec{w} using the values from the previous problem?



$$|\vec{w}| = \sqrt{0.75^2 + 1.25^2}$$
$$=$$

$$\cos \theta = \frac{\vec{x} \cdot \vec{w}}{|\vec{x}| \cdot |\vec{w}|} \Rightarrow \theta = \cos^{-1} \left(\frac{\vec{x} \cdot \vec{w}}{|\vec{x}| \cdot |\vec{w}|} \right)$$

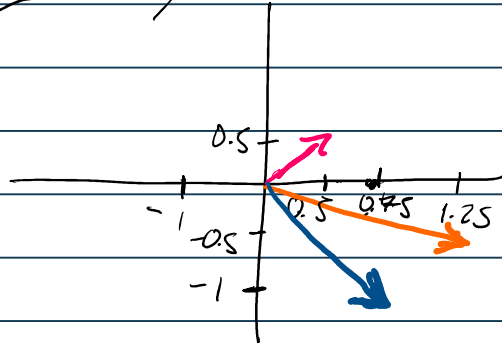
$$\theta = \cos^{-1} \left(\frac{1.0}{0.5\sqrt{2} \cdot \sqrt{2.125}} \right)$$

$$\boxed{\theta = 104.19^\circ}$$

6.) Add the following vectors and draw the resultant and the original vectors.

$$\vec{x} = [0.5, 0.5] \quad \vec{w} = [0.75, -1]$$

$$\vec{x} + \vec{w} = [1.25, -0.5] = \vec{z}$$



7.)

Classification is about categorizing the given data, usually based on similarities & given attributes.

Prediction is about attempting to predict missing or unavailable values in a dataset. Often through models that attempt to fit a shape that gets as close to the data as possible.

Can't sure if
 x_0 represented bias
 weight.

6.) $w_0 = 0, w_1 = 0.5, w_2 = -0.5$
 $V = 0.25, x_0 = 1$

| | x_1 | x_2 | OR |
|----|-------|-------|-----|
| 1. | 0 | 0 | 0 ✓ |
| 2. | 0 | 1 | 1 ✓ |
| 3. | 1 | 0 | 1 ✓ |
| 4. | 1 | 1 | 1 ✓ |

1.) (0,0)

function: $(1)(0) + 0(0.5) + 0(-0.5) = 0$

So, the neuron doesn't fire & is in agreement with the output.

(0,1)

2.) function: $(1)(0) + 0(0.5) + 1(-0.5) = -0.5$

So the value is less than zero, so the neuron doesn't fire, = 0, which is incorrect as the output is 1.

$$w_0 = 0 - 0.25(0 - 1) \times 1 = 0.25$$

$$w_1 = 0.5 - 0.25(0 - 1) \times 0 = 0.5$$

$$w_2 = 0.5 - 0.25(0 - 1) \times 1 = 0.75$$

(1,0) 3.) function: $(1)(0.25) + 1(0.5) + 0(0.75) = 0.75$

So, the value is greater than zero, so the neuron fires = 1. This matches the output. So no changes.

(1,1) 4.) function: $(1)(0.25) + 1(0.5) + 1(0.75) = 1.5$

Since the value is greater than zero, the neuron fires = 1, This matches the output. So no changes are made.

(continued) →

5. (Input 1 again) $(0, 0)$

$$\text{Actual} = (1)(0.25) + 0(0.5) + 0(0.75) = 0.25$$

- Since the value is greater than zero, the neuron fires, so it returns 1. Thus, the output = 0.

$$w_0 \leftarrow 0.25 - 0.25(0.25 - 0) \times 1 = 0.0625$$

$$w_1 \leftarrow 0.5 - 0.25(0.25 - 0) \times 0 = 0.5$$

$$w_2 \leftarrow 0.75 - 0.25(0.25 - 0) \times 0 = 0.75$$

6. $(0, 1)$

$$\text{Actual} = (1)0.0625 + 0(0.5) + 1(0.75) = 0.8125 > 0.$$

So, the neuron fires and returns 1, which matches the output. So no changes are made.

- We can probably conclude at this point that the process will repeat until w_0 is modified to approximate 0, or be less than zero.

The main difficulty here is that $b = 0$, doesn't allow the weight of w_0 to go below zero quickly at all.