Vectors_and_Perceptron_Worksheet

1. What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$|W| = \sqrt{x^2 + y^2}$

$|W| = \sqrt{(0.5)^2 + (0.5)^2}$

$|W| = \sqrt{0.5}$

$\boxed{|W| = 0.7071}$

2. Multiple the following two vectors ($\vec{x} * \vec{w}^T$), where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$\vec{W} = [0.75, 1.25] \Rightarrow \vec{W}^T = \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix}$

$\vec{x} * \vec{w}^T = [0.5, 0.5] * \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} = (0.5 * 0.75) + (0.5 * 1.25) =$

$0.375 + 0.625 = \boxed{1}$

3. Multiple the following two vectors ($\vec{x}^T * \vec{w}$) using the vectors from the previous problem.

$\vec{x} = [0.5, 0.5] \Rightarrow \vec{x}^T = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

$\vec{w} = [0.75, 1.25]$

$\vec{x}^T * \vec{w} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} * [0.75, 1.25] = (0.5 * 0.75) + (0.5 * 1.25) =$

$0.375 + 0.625 = \boxed{1}$

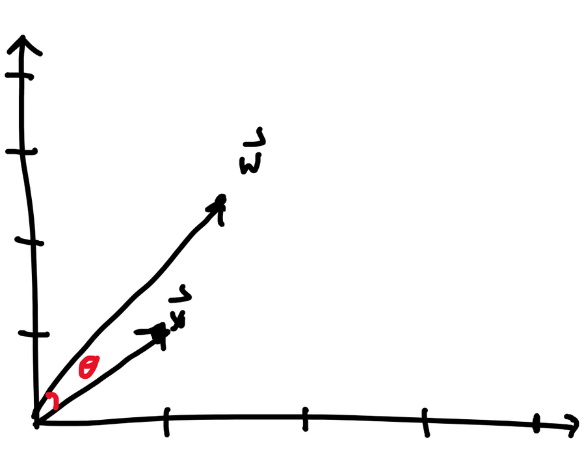
4. What is the dot product of $\vec{x}$ and $\vec{w}$ using the values from the previous problem?

$\vec{x} \cdot \vec{w} \Rightarrow x_1 w_1 + x_2 w_2$

$\vec{x} = [0.5, 0.5]$
$\vec{w} = [0.75, 1.25]$
$x_1 w_1 + x_2 w_2 = (0.5)(0.75) + (0.5)(1.25) = 0.375 + 0.625 = \boxed{1}$

5. What is the angle between $\vec{x}$ and $\vec{w}$ using the values from the previous problem? Draw the vectors and label the angle that you found.

Angle betwen : $\vec{x}, \vec{w} \Rightarrow \theta = \cos^{-1}[(\vec{x} \cdot \vec{w}) / (|x||w|)]$



$\theta = 14.02°$

$\vec{x} = [0.5, 0.5]$
$\vec{w} = [0.75, 1.25]$
$\vec{x} \cdot \vec{w} = 1$
$|x| = \sqrt{(0.5)^2 + (0.5)^2} = 0.7071$
$|w| = \sqrt{(0.75)^2 + (1.25)^2} = 1.4577$

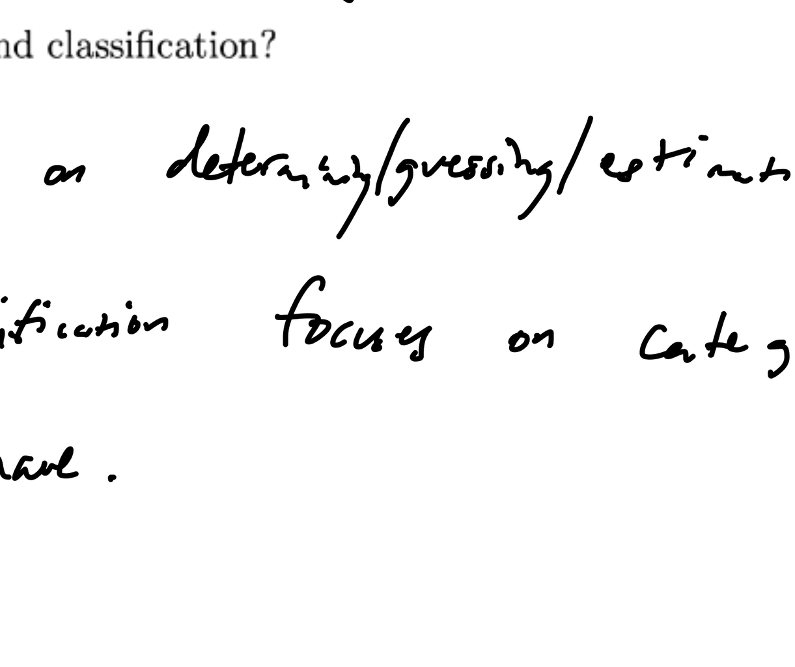
$\theta = \cos^{-1}\left[\frac{1}{(0.7071)(1.4577)}\right]$

$\theta = \cos^{-1}\left[\frac{1}{1.0307}\right]$

$\theta = \cos^{-1}[0.9702] \Rightarrow \theta = 0.24468$ radians ; $14.02°$

6. Add the following vectors, and draw the resultant and the original vectors. $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, -1]$

$\vec{x} = [0.5, 0.5]$
$\vec{w} = [0.75, -1]$
$\vec{x} + \vec{w} = [0.5 + 0.75, 0.5 + -1] = [1.25, -0.5]$



7. What is the difference between prediction and classification?

While prediction focuses on determining/guessing/estimating what a value is, while classification focuses on categorizing the values already have.

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are $w_0 = 0, w_1 = 0.5, w_2 = -0.5$ and the learning parameter $\nu = 0.25$. You may also assume that $x_0 = 1$.

x_1	x_2	OR
0	0	0
0	1	1
1	0	1
1	1	1

V = 0.25
$x_0 = 1 \Rightarrow x_b$

x_1	x_2	t_1	y_1	Correct
0	0	0	$g(0 \times 0.5 + 0 \times -0.5 + 0) = g(0) = 0$	✓
0	1	1	$g(0 \times 0.5 + 1 \times -0.5 + 0) = g(-0.5) = 0$	✗ ⊛
0	1	1	$g(0 \times 0.5 + 1 \times 0.5 + 0) = g(0.5) = 1$	✓
1	0	1	$g(1 \times 0.5 + 0 + 0) = g(0.5) = 1$	✓
1	1	1	$g(1 \times 0.5 + 1 \times 0.5 + 0) = g(1) = 1$	✓

⊛ weight update : $w_{ij} \leftarrow w_{ij} - \nu (y_j - t_j) x_i$

$w_1 = 0.5 - 1(0-1) \cdot 0 = 0.5$
$w_2 = -0.5 - 1(0-1) \cdot 1 = 0.5$ so only 1
$w_b = 0 - 1(0-1) + 0 = 0$