

1. What is the magnitude of  $\vec{w} = [0.5, 0.5]$ ?

$$\begin{aligned} |\vec{w}| &= \sqrt{0.5^2 + 0.5^2} \\ |\vec{w}| &= \sqrt{0.25 + 0.25} \\ |\vec{w}| &= \sqrt{0.5} \\ |\vec{w}| &= 0.7071 \end{aligned}$$

2. Multiple the following two vectors ( $\vec{x} * \vec{w}^T$ ), where  $\vec{x} = [0.5, 0.5]$  and  $\vec{w} = [0.75, 1.25]$

$$\begin{aligned} \vec{x} &= [[0.5, 0.5]] \\ \vec{w} &= [[0.75, 1.25]] \\ \vec{w}^T &= [[0.75], \\ &\quad [1.25]] \\ \vec{x} * \vec{w}^T &= [[0.5*0.75 + 0.5*1.25]] \\ &= [[1]] \end{aligned}$$

3. Multiple the following two vectors ( $\vec{x}^T * \vec{w}$ ) using the vectors from the previous problem.

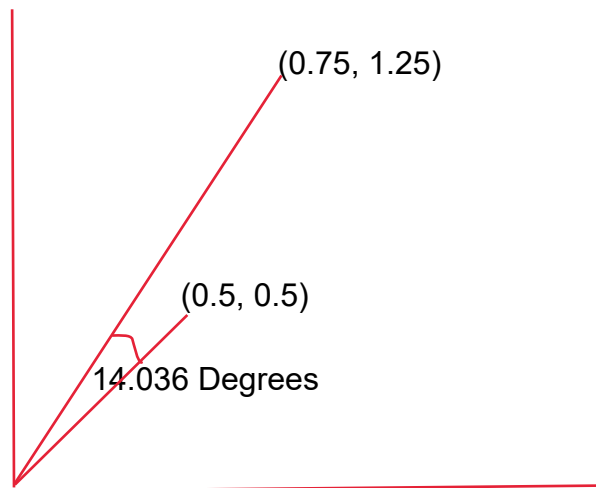
$$\begin{aligned}x &= [[0.5, 0.5]] \\w &= [[0.75, 1.25]] \\x^T &= [[0.5], \\&\quad [0.5]] \\x^T * w &= [[0.5*0.75, 0.5*1.25], \\&\quad [0.5*0.75, 0.5*1.25]] \\&= [[0.375, 0.625], \\&\quad [0.375, 0.625]]\end{aligned}$$

4. What is the dot product of  $\vec{x}$  and  $\vec{w}$  using the values from the previous problem?

$$\begin{aligned}x &= [[0.5, 0.5]] \\w &= [[0.75, 1.25]]\end{aligned}$$

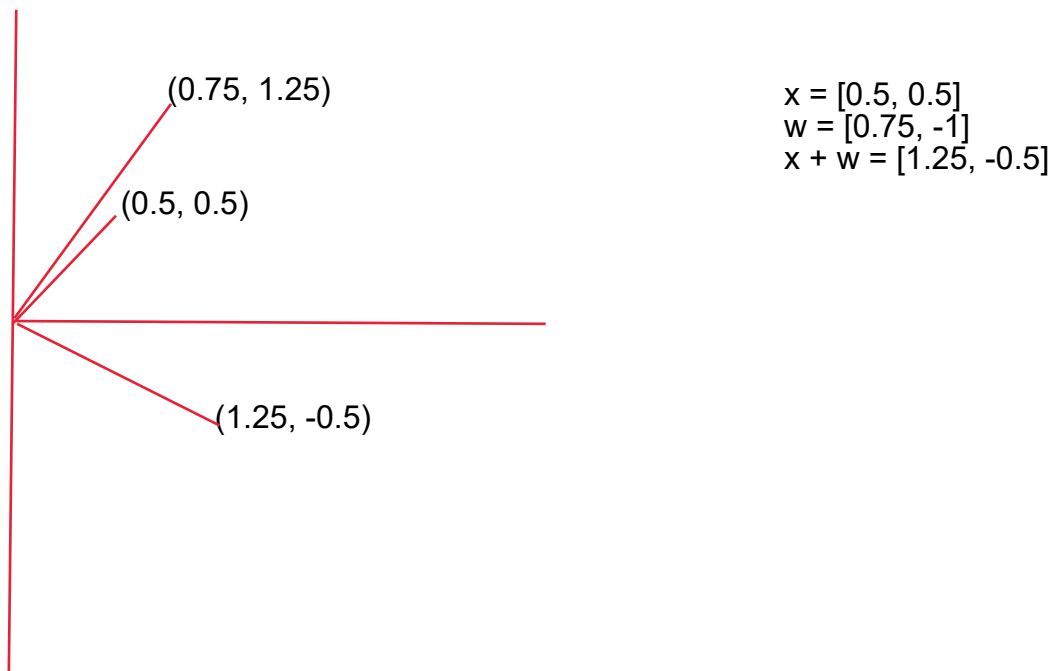
$$\begin{aligned}x \cdot w &= 0.5 \cdot 0.75 + 0.5 \cdot 1.25 \\&= 1\end{aligned}$$

5. What is the angle between  $\vec{x}$  and  $\vec{w}$  using the values from the previous problem? Draw the vectors and label the angle that you found.



$$\begin{aligned}\theta &= \cos^{-1}((x \cdot w) / (|x| \cdot |w|)) \\x \cdot w &= 1 \\|x| &= \sqrt{0.5^2 + 0.5^2} = 0.7071 \\|w| &= \sqrt{0.75^2 + 1.25^2} = 1.458 \\ \theta &= \cos^{-1}(1 / (0.7071 \cdot 1.458)) \\ \theta &= 14.036\end{aligned}$$

6. Add the following vectors, and draw the resultant and the original vectors.  
 $\vec{x} = [0.5, 0.5]$  and  $\vec{w} = [0.75, -1]$

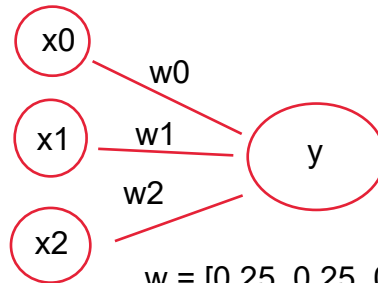


7. What is the difference between prediction and classification?

Prediction tries to predict a missing or unknown element. Classification separates data into categorical groups or classes.

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are  $w_0 = 0, w_1 = 0.5, w_2 = -0.5$  and the learning parameter  $\nu = 0.25$ . You may also assume that  $x_0 = 1$ .

$x_1$	$x_2$	OR
0	0	0
0	1	1
1	0	1
1	1	1



$$\begin{aligned}
 w &= [0, 0.5, -0.5] \\
 x &= [1, 0, 0] \\
 w \cdot x &= 0 \leq 0 \\
 w &= w - 0.25 * (0 - 0) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0, 0.5, -0.5] \\
 x &= [1, 0, 1] \\
 w \cdot x &= -0.5 \leq 0 \\
 w &= w - 0.25 * (0 - 1) * x \\
 w &= w + [0.25, 0, 0.25]
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0, -0.25] \\
 x &= [1, 1, 0] \\
 w \cdot x &= 0.25 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0, -0.25] \\
 x &= [1, 1, 1] \\
 w \cdot x &= 0 \leq 0 \\
 w &= w - 0.25 * (0 - 1) * x \\
 w &= w + [0.25, 0.25, 0.25]
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.5, 0.25, 0] \\
 x &= [1, 0, 0] \\
 w \cdot x &= 0.5 > 0 \\
 w &= w - 0.25 * (1 - 0) * x \\
 w &= w - [0.25, 0, 0]
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0.25, 0] \\
 x &= [1, 0, 1] \\
 w \cdot x &= 0.25 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0.25, 0] \\
 x &= [1, 1, 0] \\
 w \cdot x &= 0.25 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0.25, 0] \\
 x &= [1, 1, 1] \\
 w \cdot x &= 0.5 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0.25, 0] \\
 x &= [1, 0, 0] \\
 w \cdot x &= 0.25 > 0 \\
 w &= w - 0.25 * (1 - 0) * x \\
 w &= w - [0.25, 0, 0]
 \end{aligned}$$

$$\begin{aligned}
 w &= [0, 0.25, 0] \\
 x &= [1, 0, 1] \\
 w \cdot x &= 0 \leq 0 \\
 w &= w - 0.25 * (0 - 1) * x \\
 w &= w + [0.25, 0, 0.25]
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0.25, 0.25] \\
 x &= [1, 1, 0] \\
 w \cdot x &= 0.5 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0.25, 0.25] \\
 x &= [1, 1, 1] \\
 w \cdot x &= 0.75 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0.25, 0.25, 0.25] \\
 x &= [1, 0, 0] \\
 w \cdot x &= 0.25 > 0 \\
 w &= w - 0.25 * (1 - 1) * x \\
 w &= w - [0.25, 0, 0]
 \end{aligned}$$

$$\begin{aligned}
 w &= [0, 0.25, 0.25] \\
 x &= [1, 0, 1] \\
 w \cdot x &= 0.25 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0, 0.25, 0.25] \\
 x &= [1, 1, 0] \\
 w \cdot x &= 0.25 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0, 0.25, 0.25] \\
 x &= [1, 1, 1] \\
 w \cdot x &= 0.5 > 0 \\
 w &= w - 0.25 * (1 - 1) * x
 \end{aligned}$$

$$\begin{aligned}
 w &= [0, 0.25, 0.25] \\
 x &= [1, 0, 0] \\
 w \cdot x &= 0 \leq 0 \\
 w &= w - 0.25 * (0 - 0) * x
 \end{aligned}$$