Vectors and Perception Worksheet

1. magnitude of $\vec{w} = [0.5, 0.5]$

$$|\vec{w}| = \sqrt{0.5^2 + 0.5^2} = \boxed{0.71}$$

2. $\vec{x} = [0.5, 0.5]$ $\vec{w} = [0.75, 1.25]$ $\Rightarrow$ Multiply $\vec{x} * \vec{w}^T$

$$(0.5 \quad 0.5) \times \begin{pmatrix} 0.75 \\ 1.25 \end{pmatrix} = [0.5(0.75) + 0.5(1.25)]$$

$$= \boxed{[\ 1\ ]}$$

3. Multiply $\vec{x}^T * \vec{w}$

$$\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \times (0.75 \quad 1.25) = \begin{bmatrix} 0.5(0.75) & 0.5(1.25) \\ 0.5(0.75) & 0.5(1.25) \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 0.375 & 0.625 \\ 0.375 & 0.625 \end{bmatrix}}$$

4. $x \cdot w = 0.5(0.75) + 0.5(1.25) = 0.375 + 0.625 = \boxed{1}$

5. Angle between $\vec{x}$ and $\vec{w}$

$x \cdot w = 1$

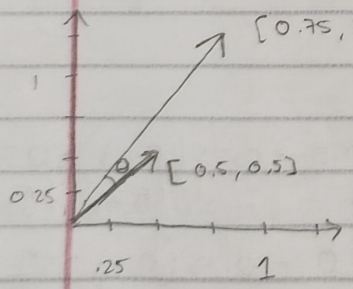
$|x| = 0.71$

$|w| = \sqrt{0.75^2 + 1.25^2} = 1.5$

$\cos\theta = (1)/(0.71 * 1.5) = 0.9428$

$\cos^{-1}(0.9428) = \theta$

$\boxed{\theta = 19.5^\circ}$



Iter 2

X_0	X_1	X_2	OR	Y	
1	0	0	0	$0(0.5) + 0(-0.85) + 1(0.25) \Rightarrow 1$	×
1	0	1	1	$0(0.5) + 1(-0.25) + 1(0) \Rightarrow 0$	×
1	1	0	1	$1(0.5) + 0(0) + 1(0.25) \Rightarrow 1$	✓
1	1	1	1	$1(0.5) + 1(0) + 1(0.25) \Rightarrow 1$	✓

$w_{11} \leftarrow 0.5 - 0.25(1-0)0 = 0.5$ | $w_{12} \leftarrow 0.5 - 0.25(0-1)0 = 0.5$

$w_{21} \leftarrow -0.25 - 0.25(1-0)0 = -0.25$ | $w_{22} \leftarrow -0.25 - 0.25(0-1)1 = 0$

$w_{01} \leftarrow 0.25 - 0.25(1-0)1 = 0$ | $w_{02} \leftarrow 0 - 0.25(0-1)1 = 0.25$

$w_{13} \leftarrow 0.5 - 0.25(1-1)1 = 0.5$ | $w_{14} = 0.5$

$w_{23} \leftarrow 0 - 0.25(1-1)0 = 0$ | $w_{24} = 0$

$w_{03} \leftarrow 0.25 - 0.25(1-1)1 = 0.25$ | $w_{04} = 0.25$

Iter 3

X_0	X_1	X_2	OR	Y	
1	0	0	0	$0(0.5) + 0(0) + 1(0.25) \Rightarrow 1$	×
1	0	1	1	$0(0.5) + 1(0) + 1(0) \Rightarrow 0$	×
1	1	0	1	$0.5 + 0 + 0.25 \Rightarrow 1$	✓
1	1	1	1	$0.5 + 0.25 + 0.25 \Rightarrow 1$	✓

$w_{11} \leftarrow 0.5 - 0.25(1-0)0 = 0.5$ | $w_{12} \leftarrow 0.5 - 0.25(0-1)0 = 0.5$

$w_{21} \leftarrow 0 - 0.25(1-0)0 = 0$ | $w_{22} \leftarrow 0 - 0.25(0-1)1 = +0.25$

$w_{01} \leftarrow 0.25 - 0.25(1-0)1 = 0$ | $w_{02} \leftarrow 0 - 0.25(0-1)1 = 0.25$

$w_{13} \leftarrow 0.5 - 0.25(0)(1) = 0.5$ | $w_{14} = 0.5$ | $w_{14} = 0.5$

$w_{23} \leftarrow 0.25 - 0.25(0)(0) = 0.25$ | $w_{24} = 0.25$ | $w_{24} = 0.25$

$w_{03} \leftarrow 0.25 - 0.25(0)(1) = 0.25$ | $w_{34} = 0.25$ | $w_{04} = 0$

Iter 4

X_0	X_1	X_2	OR	Y	
1	0	0	0	1	X
1	0	1	1	1	✓
1	1	0	1	1	✓
1	1	1	1	1	✓

$w_{11} \leftarrow 0.5 - 0.25(1-0)0 = 0.5$

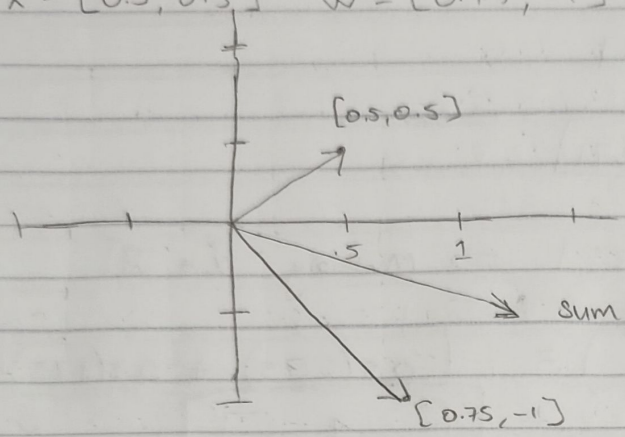
$w_{22} \leftarrow 0.25 - 0.25(1-0)0 = 0.25$

$w_{02} \leftarrow 0.25 - 0.25(1-0)1 = 0$

$w_{12} = 0.5 \quad w_{22} = 0.25 \quad w_{02} = 0$

$w_{13} = 0.5 \quad w_{23} = 0.25 \quad w_{03} = 0$

6. $\vec{x} = [0.5, 0.5]$ $\vec{w} = [0.75, -1]$ $\vec{x} + \vec{w} = [1.25, -0.5]$



7. Prediction vs. Classification

Prediction is estimating a numerical value using prior information, whereas, classification is categorizing or labeling something based on prior given information.

8. $w_0 = 0$, $w_1 = 0.5$ $w_2 = -0.5$, $lr = 0.25$,
$x_0 = 1$

① fixed 1

x_0	x_1	x_2	OR	y
1	0	0	0	$0*0.5 + 0*-0.5 + 0.1 = 0$ ✓
1	0	1	1	$0*0.5 + 1(-0.5) + 0.1 = -0.5 \Rightarrow 0$ ✗
1	1	0	1	$1(0.5) + 0(-0.25) + 1(0.25) = 1$ ✓
1	1	1	1	1 ✓

Activation Function

$w_{ij} \leftarrow w_{ij} - \eta(y_i - t_j) \cdot x_i$

$w_{11} \leftarrow 0.5 - 0.25(0-0) \cdot 0 = 0.5$ $w_{12} \leftarrow 0.5 - 0.25(0-1) \cdot 0 = 0.5$

$w_{21} \leftarrow -0.5 - 0.25(0-0) \cdot 0 = -0.5$ $w_{22} \leftarrow -0.5 - 0.25(0-1) \cdot 1 = -0.25$

$w_{01} \leftarrow 0 - 0.25(0-0) \cdot 1 = 0$ $w_{02} \leftarrow 0 - 0.25(0-1) \cdot 1 = 0.25$

$w_{13} \leftarrow 0.5 - 0.25(1-1) \cdot 1 = 0.5$ | $0.5 = w_1$

$w_{23} \leftarrow -0.25 - 0.25(1-1) \cdot 0 = -0.25$ | $-0.25 = w_2$

$w_{03} \leftarrow 0.25 - 0.25(1-1) \cdot 1 = 0.25$ | $0.25 = w_3$

Iter 5

X_0	X_1	X_2	OR	Y
1	0	0	0	$0.5(0) + 0.25(0) + 0(1) = 0$ ✓
1	0	1	1	$0.5(0) + 0.25(1) + 0(1) = 1$ ✓
1	1	0	1	$0.5(1) + 0.25(0) + 0(1) = 1$ ✓
1	1	1	1	$0.5(1) + 0.25(0) + 0(1) = 1$ ✓

$w_{11} = 0.5$ $w_{12} = 0.5$ $w_{13} = 0.5$

$w_{21} = 0.25$ $w_{22} = 0.25$ $w_{23} = 0.25$

$w_{01} = 0$ $w_{02} = 0$ $w_{03} = 0$

Final Weights

$w_{14} = 0.5$

$w_{24} = 0.25$

$w_{04} = 0$