1. What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$|w| = \sqrt{(0.5)^2 + (0.5)^2} = 0.70710$

2. Multiple the following two vectors $(\vec{x} * \vec{w}^T)$, where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

1x2

1x2

multiplication is not possible as $\vec{x}$ has dimension 1x2 and $\vec{w}$ has dimensions 1x2.

$(m \times n) \times (n \times p)$

$(2 \times 1) \times (1 \times 2)$

3. Multiple the following two vectors ($\vec{x}^T * \vec{w}$) using the vectors from the previous problem.

$$\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \times [0.75, 1.25] = \begin{bmatrix} 0.375 & 0.625 \\ 0.375 & 0.625 \end{bmatrix}$$

4. What is the dot product of $\vec{x}$ and $\vec{w}$ using the values from the previous problem?

$[0.5, 0.5] \cdot [0.75, 1.25] = 0.375 + 0.625 = 1$

5. What is the angle between $\vec{x}$ and $\vec{w}$ using the values from the previous problem? Draw the vectors and label the angle that you found.

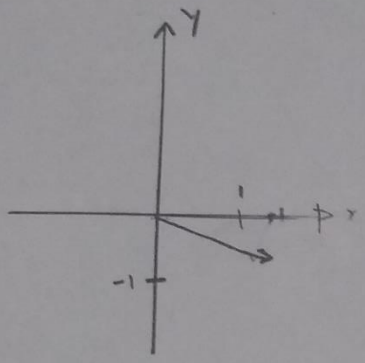
$a \cdot b = |a||b|\cos(\theta)$ $\quad |b| = 1.457$

$$\frac{a \cdot b}{|a||b|} = \cos(\theta)$$

$$\cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right) = \theta = \cos^{-1}\left(\frac{1}{0.7 \cdot 1.5}\right) = 14^{\circ}$$

6. Add the following vectors, and draw the resultant and the original vectors.
$\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, -1]$

$\vec{x} + \vec{w} = [1.25, -0.5]$



7. What is the difference between prediction and classification?

The outputs for classification take on a discrete or categorical set of values while the outputs for prediction may take on a continuous or discrete set of values.

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are $w_0 = 0, w_1 = 0.5, w_2 = -0.5$ and the learning parameter $\nu = 0.25$. You may also assume that $x_0 = 1$.

$$w_{ij} = w_{ij} - \eta(y_{io} - t_{ij})\, x_i$$

x_1	x_2	OR
0	0	0
0	1	1
1	0	1
1	1	1

Weights to predict OR Function

$w_0 = -0.25$

$w_1 = 0.75$

$w_2 = 0.5$

Work attached on back.

Run 1:

$0(1)+0.5(0)-0.5(0)=0$ ✓

$0(1)+0.5(0)-0.5(1)=-0.5$ ×

$0(1)+0.5(1)-0.5(0)=1$ ✓

$0(1)+0.5(1)-0.5(1)=0$ ×

Update w_0

$w_{00}=0-(0-0)\cdot 1=0$

$w_{01}=0-0.25(0-1)\cdot 1=0.25$

$w_{02}=0.25-0.25(1-1)\cdot 1=0$

$w_{03}=0.25-0.25(0-1)=0.5$

Update w_1

$w_{10}=0.5-0.25(0-0)0=0.5$

$w_{11}=0.5-0.25(0-1)0=0.5$

$w_{12}=0.5-0.25(1-1)+=0.5$

$w_{13}=0.5-0.25(0-1)1=0.75$

$w_0=0.5$

$w_1=0.75$

$w_2=0$

Update w_3

$w_{20}=-0.5-0.25(0-0)\cdot 0=-0.5$

$w_{21}=-0.5-0.25(0-1)1=-0.25$

$w_{23}=-0.25-0.25(1-1)=-0.25$

$w_{24}=-0.25-0.25(0-1)=0$

Run 2

$0.5(1)+0.75(0)-0.5(0)=1$ ×

$0.5(1)+0.75(0)-0(1)=1$ ✓

$0.5(1)+0.75(1)+0(0)=1$ ✓

$0.5(1)+0.75(1)+0(1)=1$ ✓

Updated weights

$w_0=0.25$

$w_1=0.75$

$w_2=0$

Update w_0

$w_0=0.5-0.25(1-0)1=0.25$

$w_{01}=0.25-0.25(1-1)=0.25$

⋮

Update w_1

$w_{10}=0.75-0.25(1-0)\cdot 0=0.5$

⋮

Update w_2

$w_{20}=0-0.25(1-0)\cdot 1$

Run 3:

$0.25(1) + .75(0) + 0(1) = 1$

$0.25(1) + 0.75(0) + 0(1) = 1$

$0.25(1) + 0.75(0) + 0(0) = 1$

$0.25(1) + 0.75(1) + 0(1) = 1$

update w_0

$w_{00} = 0.25 + 0.25(1-0)\cdot 1 = 0$

Run 4

$0(1) + 0.75(0) + 0(0) = 0$ ✓

$0(1) + 0.75(0) + 0(1) = 0$ ×

$0(1) + 0.75(1) + 0(1) = 1$ ✓

$0(1) + 0.75(1) + 0(1) = 1$ ✓

update w_0

$w_{0,1} = 0 - 0.25(0-1)\cdot 1 = -0.25 = w_0$

Update w_1

$w_{11} = 0.27 - 0.25(0-1)0 = 0.25$

update w_2

$w_{21} = 0 - 0.25(0-1) = 0.25$

Run 5

$-0.25(1) + 0 + 0 = 0$ ×

$-0.25(1) + 0 + 0.25(1) = 0$ ×

$-0.25(1) + 0.75(1) + 0.25(0) = 1$ ✓

$-0.25(1) + 0.75(1) + 0.25(1) = 1$ ✓

Update

w_0:

$w_0 = -0.25 - 0.25(0-1)(1) = 0$

$w_1 = 0 - 0.25(1-0)(1) = -0.25$

w_2: $w_{21} = 0.25 - 0.25(0-1) = 0.5$

Run 6

$-0.25(1) + 0.75(0) + 0.5(0) = 0$ ✓

$-0.25(1) + 0.75(0) + 0.5(1) = 1$ ✓

$-0.25(1) + 0.75(1) + 0.5(0) = 1$ ✓

$-0.25(1) + 0.75(1) + 0.5(1) = 1$ ✓

Weights final

$w_0 = -0.25$

$w_1 = 0.75$

$w_2 = 0.5$