

1. What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$$\sqrt{0.5^2 + 0.5^2} \approx \boxed{0.707}$$

2. Multiply the following two vectors ($\vec{x} \times \vec{w}^T$) where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$$[0.5, 0.5] \cdot [0.75, 1.25] = 0.5(0.75) + 0.5(1.25) \\ = 0.375 + 0.625 = \boxed{1}$$

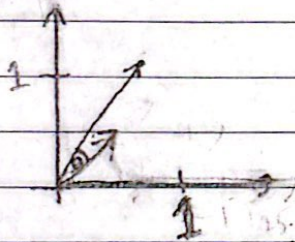
3. ($\vec{x}^T \times \vec{w}$)

$$[0.5, 0.5] \cdot [0.75, 1.25] = \boxed{1}$$

4. $\vec{x} \cdot \vec{y}$

$$[0.5, 0.5] \cdot [0.75, 1.25] = 0.375 + 0.625 = \boxed{1}$$

5.

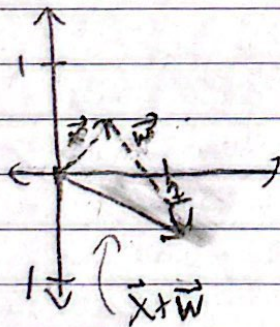


$$|x| = \sqrt{0.5^2 + 0.5^2} \approx 0.707$$

$$|y| = \sqrt{0.75^2 + 1.25^2} \approx 1.461$$

$$\theta = \arccos\left(\frac{1}{|x||y|}\right) = 0.641 \text{ radians} \\ \approx 36.77^\circ$$

6.



$$\vec{x} + \vec{w} = [0.5 + 0.75, 0.5 + (-1)] \\ = [1.25, -0.5]$$

7. Prediction uses its trained model to predict the outcome when given specific values.

Classification uses its trained model to decide whether the input is of a specific group.

8. Find the weights to predict the "OR" function.

Initial weights: $w_0 = 0, w_1 = 0.5, w_2 = -0.5$

Learning parameter: $\eta = 0.25$

$x_0 = 1$

x_1	x_2	OR	y_i	Correct?	new weights
0	0	0	$g(0+0+0) = 0$	✓	
0	1	1	$g(0+0+(-0.5)) = 0$	X	$w_0 = 0.25, w_1 = 0.5, w_2 = -0.25$
1	0	1	$g(0+0.5+0) = 1$	✓	
1	1	1	$g(0+0.5+(-0.5)) = 0$	X	$w_0 = 0.5, w_1 = 0.75, w_2 = 0$

new weights

$$w_0 = 0 - 0.25(0-1)(1) = 0.25$$

$$w_1 = 0.5 - 0.25(0-1)(0) = 0.5$$

$$w_2 = -0.5 - 0.25(0-1)(1) = -0.25$$

↓

$$w_0 = 0.25 - 0.25(0-1)(1) = 0.5$$

$$w_1 = 0.5 - 0.25(0-1)(1) = 0.75$$

$$w_2 = -0.25 - 0.25(0-1)(1) = 0$$

x_1	x_2	t_i	y_i	Correct?	new weights
0	0	0	$g(0.5+0+0) = 1$	X	$w_0 = 0.25, w_1 = 0.75, w_2 = 0$
0	1	1	$g(0.5+0+0) = 1$	✓	
1	0	1	$g(0.5+0.75+0) = 1$	✓	
1	1	1	$g(0.5+0.75+0) = 1$	✓	

8 (cont.)

new weights

$$w_0 = 0.5 - 0.25(1-0)(1) = 0.25$$

$$w_1 = 0.75 - 0.25(1-0)(0) = 0.75$$

$$w_2 = 0 - 0.25(1-0)(0) = 0$$

x_1	x_2	t_1	y_1	correct?	new weights
0	0	0	$g(0.25+0+0)=1$	X	$w_0=0, w_1=0.75, w_2=0$
0	1	1	$g(0.75+0+0)=1$	✓	
1	0	1	$g(0.25+0.75+0)=1$	✓	
1	1	1	$g(0.25+0.75+0)=1$	✓	

Now weights

$$w_0 = 0.25 - 0.25(1-0)(1) = 0$$

$$w_1 = 0.75 - 0.25(1-0)(0) = 0.75$$

$$w_2 = 0 - 0.25(1-0)(0) = 0$$

x_1	x_2	t_1	y_1	correct?	new weights
0	0	0	$g(0+0+0)=0$	✓	
0	1	1	$g(0+0+0)=0$	X	$w_0=0.25, w_1=0.75, w_2=0.25$
1	0	1	$g(0+0.75+0)=1$	✓	
1	1	1	$g(0+0.75+0)=1$	✓	

$$w_0 = 0 - 0.25(0-1)(1) = 0.25$$

$$w_1 = 0.75 - 0.25(0-1)(0) = 0.75$$

$$w_2 = 0 - 0.25(0-1)(1) = 0.25$$

8 (cont.) $w_0 = 0.25, w_1 = 0.75, w_2 = 0.25$

x_1, x_2	t_1	y_1	correct?	new weights
0 0	0	$g(0.25 + 0 + 0) = 1$	X	$w_0 = 0, w_1 = 0.75, w_2 = 0.25$
0 1	1	$g(0.25 + 0 + 0.25) = 1$	✓	
1 0	1	$g(0.25 + 0.75 + 0) = 1$	✓	
1 1	1	$g(0.25 + 0.75 + 0.25) = 1$	✓	

$$w_0 = 0.25 - 0.25(1-0)(1) = 0$$

$$w_1 = 0.75 - 0.25(1-0)(0) = 0.75$$

$$w_2 = 0.25 - 0.25(1-0)(0) = 0.25$$

x_1, x_2	t_1	y_1	correct?
0 0	0	$g(0 + 0 + 0) = 0$	✓
0 1	1	$g(0 + 0 + 0.25) = 1$	✓
1 0	1	$g(0 + 0.75 + 0) = 1$	✓
1 1	1	$g(0 + 0.75 + 0.25) = 1$	✓

$$w_0 = 0$$

$$w_1 = 0.75$$

$$w_2 = 0.25$$