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1. What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$|\vec{w}| = \sqrt{0.5^2 + 0.5^2} = \frac{\sqrt{2}}{2} = 0.707$

2. Multiple the following two vectors ($\vec{x} * \vec{w}^T$), where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$\vec{x} * \vec{w}^T = [0.5 \quad 0.5] * \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} = 1$

3. Multiple the following two vectors ($\vec{x}^T * \vec{w}$) using the vectors from the previous problem.

$$\vec{x}^T * \vec{w} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.75 & 1.25 \end{bmatrix} = \begin{bmatrix} 0.375 & 0.625 \\ 0.375 & 0.625 \end{bmatrix}$$

4. What is the dot product of $\vec{x}$ and $\vec{w}$ using the values from the previous problem?

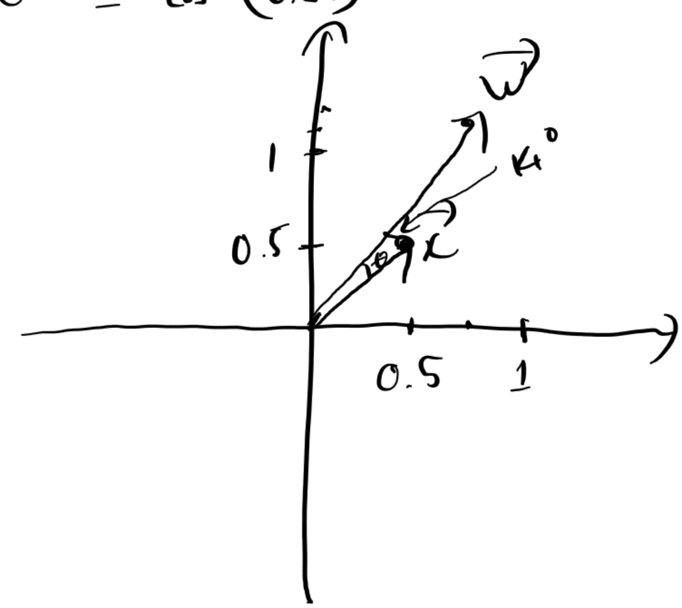
$$\vec{x} \cdot \vec{w} = [0.5 \quad 0.5] \cdot [0.75 \quad 1.25] = 0.5 \cdot 0.75 + 0.5 \cdot 1.25$$

$$= \underline{1}$$

5. What is the angle between $\vec{x}$ and $\vec{w}$ using the values from the previous problem? Draw the vectors and label the angle that you found.

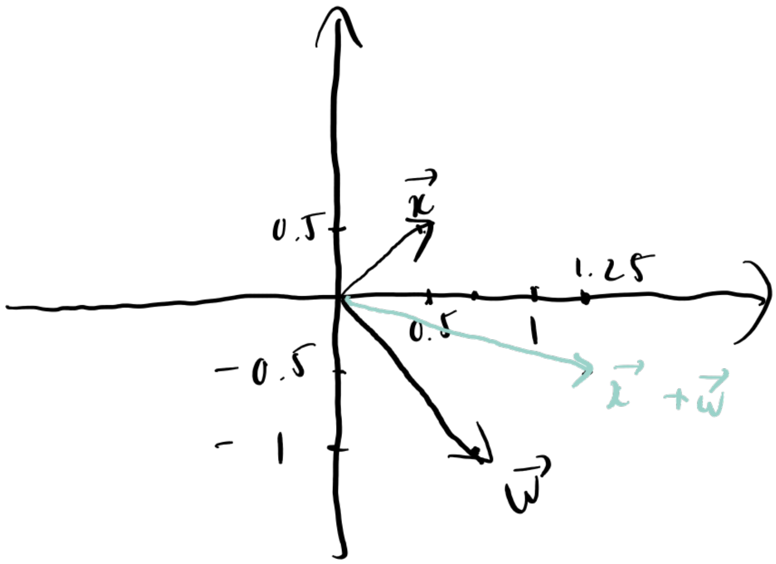
$$\cos(\theta) = \frac{\vec{x} \cdot \vec{w}}{\|\vec{x}\| \cdot \|\vec{w}\|} = \frac{1}{0.707 \cdot 1.46} = 0.97$$

$$\Rightarrow \quad \theta = \cos^{-1}(0.97) = 14.07$$



6. Add the following vectors, and draw the resultant and the original vectors.
   $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, -1]$

$$\vec{x} + \vec{w} = [0.5, 0.5] + [0.75, -1]$$
$$= [1.25 \quad -0.5]$$



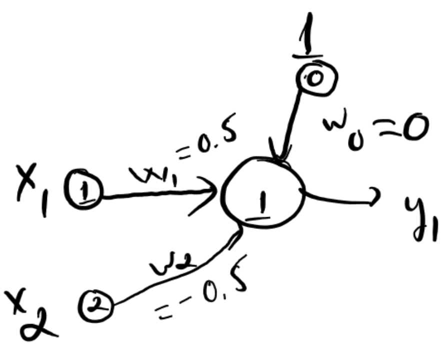
7. What is the difference between prediction and classification?

⊕ Classification : labeling data into different classes based on training data we collect in the past / current

⊕ Prediction : Predicting an unknown element / value in the future based on training data we collect in the past / current (usually a regression problem)

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are $w_0 = 0, w_1 = 0.5, w_2 = -0.5$ and the learning parameter $\nu = 0.25$. You may also assume that $x_0 = 1$.

x_1	x_2	OR
0	0	0
0	1	1
1	0	1
1	1	1



$$y_j = g\left(\sum_{i=0}^{m} w_{ij} x_i\right) = \begin{cases} 1 & \text{if } \sum_{i=1}^{m} w_{ij} x_i > 0 \\ 0 & \text{if } \sum_{i=1}^{m} w_{ij} x_i \leq 0 \end{cases}$$

1st iter

x_1	x_2	t	y
0	0	0	$g(0+0+0) = g(0) = 0$ ✓
0	1	1	$g(0+0-0.5) = g(-0.5) = 0$ ✗
1	0	1	$g(0+0.5-0) = g(0.5) = 1$ ✓
1	1	1	$g(0+0.5-0.5) = g(0) = 0$ ✗

⊗ update weights (using $\mu = 1.0$)

$w_{ij} = w_{ij} - \mu(y_i - t_i) x_i$

- $x_1 = 0$ and $x_2 = 0 \rightarrow$ No update
- $x_1 = 1$ and $x_2 = 1$

$w_0 = \cancel{w_0}^{0} - (0-1) \cdot 1 = 1$

$w_1 = \cancel{w_1}^{0.5} - (0-1) \cdot 0 = 0.5$

$w_2 = \cancel{w_2}^{-0.5} - (0-1) \cdot 1 = 0.5$

- $x_1 = 1$ and $x_2 = 0$

$w_0 = \cancel{w_0}^{1} - (0-1) \cdot 1 = 2$

$w_1 = \cancel{w_1}^{0.5} - (0-1) \cdot 1 = 1.5$

$w_2 = \cancel{w_2}^{0.5} - (0-1) \cdot 0 = 0.5$

- $x_1 = 1$ and $x_2 = 1 \rightarrow$ No update

2nd iter

x_1	x_2	t	y
0	0	0	$g(2+0+0) = g(2) = 1$ ✗
0	1	1	$g(2+0+0.5) = g(2.5) = 1$ ✓
1	0	1	$g(2+1.5+0) = g(2.5) = 1$ ✓
1	1	1	$g(2+1.5+0.5) = g(4) = 1$ ✓

Update weights

$x_1 = 0$ and $x_2 = 0$

$w_0 = \cancel{w_0}^{2} - (1-0) \cdot 1 = 1$

$w_1 = 1.5$

$w_2 = 0.5$

3rd iteration

x_1	x_2	t	y
0	0	0	$g(1+0+0) = g(1) = 1$ ✗
0	1	1	$g(1+0+0.5) = g(1.5) = 1$ ✓
1	0	1	$g(1+1.5+0) = g(1.5) = 1$ ✓
1	1	1	$g(1+1.5+0.5) = g(3) = 1$ ✓

Update weights

$x_1 = 0$ and $x_2 = 0$

$w_0 = w_0' - (1-0).1 = 0$

$w_1 = 1.5$

$w_2 = 0.5$

4th iteration

x_1	x_2	t	y
0	0	0	$g(0+0+0) = g(0) = 0$ ✓
0	1	1	$g(0+0+0.5) = g(0.5) = 1$ ✓
1	0	1	$g(0+1.5+0) = g(1.5) = 1$ ✓
1	1	1	$g(0+1.5+0.5) = g(2) = 1$ ✓