CSC 487 - Vectors and Perceptron Worksheet 4/12/23

1. What is the magnitude of $\vec{w} = [0.5, 0.5]$

$|\vec{w}| = \sqrt{(0.5)^2 + (0.5)^2} = \sqrt{0.5} \approx \boxed{0.7071}$

2. Multiply the following two vectors ($\vec{x} * \vec{w}^t$), where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$$\vec{w}^t = \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix}$$

$$\vec{x} * \vec{w}^t = [0.5, 0.5] * \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} = 0.5(0.75) + 0.5(1.25)$$

$$= \boxed{1}$$

3. Multiply the following vectors ($\vec{x}^t * \vec{w}$)

$$(\vec{x}^t) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$\vec{x}^t * \vec{w} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} * [0.75, 1.25] = 0.5(0.75), 0.5(1.25)$$

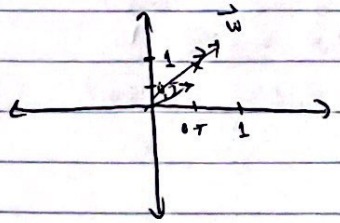
$$0.5(0.75), 0.5(1.25)$$

$$= \boxed{\begin{bmatrix} 0.375, 0.625 \\ 0.375, 0.625 \end{bmatrix}}$$

4. What is the dot product of $\vec{x}$ and $\vec{w}$?

$\vec{x} \cdot \vec{w} = 0.5(0.75) + 0.5(1.25) = \boxed{1}$

5. What is the angle between $\vec{x}$ and $\vec{w}$ using the values from the previous problem?

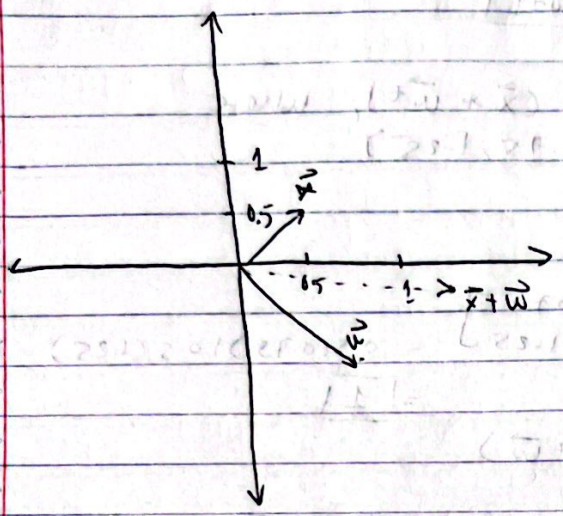


$$\vec{x} \cdot \vec{w} = |\vec{x}||\vec{w}|\cos(\theta)$$

$$1 = (\sqrt{0.5})(\sqrt{0.75^2 + 1.25^2})\cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{0.5}(\sqrt{0.75^2 + 1.25^2})}\right) \approx \boxed{50.19^\circ}$$

6. Add the following vectors, and draw the resultant and the original vectors. $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, -1]$



$\vec{x} + \vec{w} = [1.25, -0.25]$

7. What is the difference between prediction and classification?

Prediction is a value calculated that falls within a continuous range of possible outcomes, whereas classification assigns a value from a discrete set of possible outcomes.

8. "OR" function. $w_0 = \emptyset$, $w_1 = 0.5$, $w_2 = -0.5$, $V = 0.25$; $x_0 = 1$

Epoch 1:

Input 1: $(\emptyset, \emptyset)$

$y = 1(\emptyset) + \emptyset(0.5) + \emptyset(-0.5) = \emptyset \Rightarrow \emptyset$ ✓

Input 2: $(\emptyset, 1)$

$y = 1(\emptyset) + \emptyset(0.5) + 1(-0.5) = -0.5 \Rightarrow \emptyset$ ✗

$w_0 = \emptyset - 0.25(\emptyset - 1) \cdot 1 = 0.25$

$w_1 = 0.5 - 0.25(-1) \cdot (\emptyset) = 0.5$

$w_2 = -0.5 - 0.25(-1) \cdot 1 = -0.25$

Input 3: $(1, \emptyset)$

$y = 1(0.25) + 1(0.5) + \emptyset(-0.25) = 0.75 \Rightarrow 1$ ✓

Input 4: $(1, 1)$

$y = 1(0.25) + 1(0.5) + 1(-0.25) = 0.5 \Rightarrow 1$ ✓

Epoch 2:

Input 1: $(\emptyset, \emptyset)$

$y = 1(0.25) + \emptyset(0.5) + \emptyset(-0.25) = 0.25 \Rightarrow \cancel{\emptyset}$ ✗

$w_0 = 0.25 - 0.25(1) \cdot 1 = \emptyset$

$w_1 = 0.5 - 0.25(1) \cdot \emptyset = 0.5$

$w_2 = -0.25$

Input 2: $(\emptyset, 1)$

$y = 1(\emptyset) + \emptyset(0.5) + 1(-0.25) = -0.25 \Rightarrow \emptyset$ ✗

$w_0 = \emptyset - 0.25(-1) \cdot 1 = +0.25$

$w_1 = 0.5 - \ldots \cdot \emptyset = 0.5$

$w_2 = -0.25 - 0.25(-1) \cdot 1 = +\emptyset$

Input 3: $(1, \emptyset)$

$y = 1(0.25) + 1(0.5) + \emptyset(\emptyset) = 0.75 \Rightarrow 1$ ✓

Input 4: $(1, 1)$

$y = 1(0.25) + 1(0.5) + \emptyset = 0.75 \Rightarrow 1$ ✓

Epoch 3:

Input 1: (0, 0)

$y = 1(0.25) + 0(0.5) + 0(0) = 0.25 \Rightarrow 1$ ✗

$w_0 = 0.25 - 0.25(1) \cdot 1 = 0$

$w_1 = 0.5 - 0.25(1) \cdot 0 = 0.5$

$w_2 = 0$

Input 2: (0, 1)

$y = 1(0) + 0(0.5) + 1(0) = 0 \Rightarrow 0$ ✗

$w_0 = 0 - 0.25(-1) \cdot 1 = 0.25$

$w_1 = 0.5 - 0.25(-1) \cdot 0 = 0.5$

$w_2 = 0 - 0.25(-1) \cdot 1 = 0.25$

Input 3: (1, 0)

$y = 1(0.25) + 1(0.5) + 0 \Rightarrow 1$ ✓

Input 4: (1, 1)

$y = 1(0.25) + 1(0.5) + 1(0.25) = 1 \Rightarrow 1$ ✓

Epoch 4:

Input 1: (0, 0)

$y = 1(0.25) + 0 + 0 - 0.25 \Rightarrow 1$ ✗

$w_0 = 0.25 - 0.25(1 - 0) \cdot 1 = 0$

$w_1 = 0.5 - 0.25(1) \cdot 0 = 0.5$

$w_2 = 0.25$

Input 2: (0, 1)

$y = 1(0) + 0 + 1(0.25) = 0.25 \Rightarrow 1$ ✓

Input 3: (1, 0)

$y = 0 + 0.5 + 0 = 0.5 \Rightarrow 1$ ✓

Input 4: (1, 1)

$y = 0 + 0.5 + 0.25 = 0.75 \Rightarrow 1$ ✓

Epoch 5:

Input 1: (0, 0)

$y = 0 + 0 + 0 = 0 \Rightarrow 0$ ✓

Input 2: (0, 1)

$y = 1(0) + 0(0.5) + 1(0.25) = 0.25 \Rightarrow 1$ ✓

Input 3: (1, 0)

$y = 1(0) + 1(0.5) + 0(0.25) = 0.5 \Rightarrow 1$ ✓

Input 4: (1, 1)

$y = 1(0) + 1(0.5) + 1(0.25) = 0.75 \Rightarrow 1$ ✓

Final weights: $w_0 = 0$, $w_1 = 0.5$, $w_2 = 0.25$