1. What is the magnitude of $\vec{w} = [0.5, 0.5]$?

$$|\vec{w}| = \sqrt{0.5^2 + 0.5^2}$$
$$= \sqrt{0.25 + 0.25}$$
$$= 0.7071$$

2. Multiple the following two vectors $(\vec{x} * \vec{w}^T)$, where $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, 1.25]$

$$[0.5, 0.5] \begin{bmatrix} 0.75 \\ 1.25 \end{bmatrix} = [1]$$

3. Multiple the following two vectors $(\vec{x}^T * \vec{w})$ using the vectors from the previous problem.

$$\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.75 & 1.25 \end{bmatrix} \simeq \begin{bmatrix} 0.375 & 0.625 \\ 0.375 & 0.625 \end{bmatrix}$$

4. What is the dot product of $\vec{x}$ and $\vec{w}$ using the values from the previous problem?

$$\vec{x} \cdot \vec{w} = 0.5 \times 0.75 + 0.5 \times 1.25$$

$$= 0.375 + 0.625$$

$$= 1$$

5. What is the angle between $\vec{x}$ and $\vec{w}$ using the values from the previous problem? Draw the vectors and label the angle that you found.

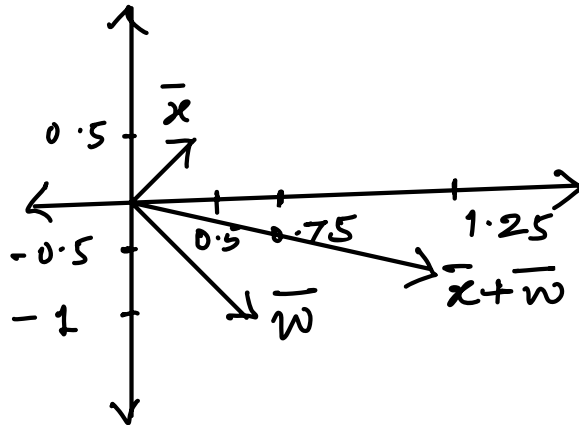$$\vec{x} \cdot \vec{w} = 1$$

$$|\vec{x}| = 0.707$$

$$|\vec{w}| = \sqrt{(0.75)^2 + (1.25)^2}$$

$$= 1.457$$

Now, $\cos\theta = \dfrac{\vec{x} \cdot \vec{w}}{|\vec{x}||\vec{w}|} = \dfrac{1}{0.707 \times 1.457}$

$$= 0.9703$$

$\therefore \quad \theta = \cos^{-1}(0.9703) \approx 14°$

6. Add the following vectors, and draw the resultant and the original vectors.
   $\vec{x} = [0.5, 0.5]$ and $\vec{w} = [0.75, -1]$

$$\overline{x} + \overline{w} = [0.5 + 0.75, \ 0.5 - 1]$$
$$= [1.25, \ -0.5]$$

7. What is the difference between prediction and classification?

Prediction involves giving an estimate value for the output of a given input. Classification is categorizing the input into a class.

8. Using the perceptron learning algorithm and a single neuron, find the weights that correctly predict the "OR" function. Continue updating the weights using the algorithm discussed in class until you converge on a correct solution. Show all of your work. The initial weights are $w_0 = 0, w_1 = 0.5, w_2 = -0.5$ and the learning parameter $\nu = 0.25$. You may also assume that $x_0 = 1$.

x_1	x_2	OR
0	0	0
0	1	1
1	0	1
1	1	1

First pass:

$w_0 = 0$, $w_1 = 0.5$, $w_2 = -0.5$

$0(1) + 0.5(0) + (-0.5)(0) = 0$ ✓

$0(1) + 0.5(0) + (-0.5)(1) = -0.5 \Rightarrow 0$ ✗

$0(1) + 0.5(1) + (-0.5)(0) = 0.5 \Rightarrow 1$ ✓

$0(1) + 0.5(1) + (-0.5)(1) = 0$ ✗

Updating weights:

$w_0 = 0 - 0.25(0-1) \times 1 = 0.25$

$w_1 = 0.5 - 0.25(0-1) \times 0 = 0.5$

$w_2 = -0.5 - 0.25(0-1) \times 1 = -0.25$

Second pass:

$0.25(1) + 0.5(0) - 0.25(0) = 0.25 \approx 1$ ✗

$0.25(1) + 0.5(0) - 0.25(1) = 0$ ✗

$0.25(1) + 0.5(1) - 0.25(0) = 0.75 \approx 1$ ✓

$0.25(1) + 0.5(1) - 0.25(1) = 0.5 \approx 1$ ✓

Updating weights:

$w_0 = 0 - 0.25(0-1) \times 1 = 0.25$

$w_1 = 0.5 - 0.25(0-1) \times 0 = 0.5$

$w_2 = -0.25 - 0.25(0-1) \times 1 = 0$

Third pass:

$$0.25(1) + 0.5(0) + 0 = 0.25 \approx 1 \quad \text{X}$$
$$0.25(1) + 0.5(0) + 0 = 0.25 \approx 1 \quad \checkmark$$
$$0.25(1) + 0.5(1) + 0 = 0.75 \approx 1 \quad \checkmark$$
$$0.25(1) + 0.5(1) + 0 = 0.25 \approx 1 \quad \checkmark$$

Updating weights:

$$w_0 = 0.25 - 0.25(1-0) \times 1 = 0$$
$$w_1 = 0.5 - 0.25(1-0) \times 0 = 0.5$$
$$w_2 = 0 - 0.25(1-0) \times 0 = 0$$

fourth pass:

$$0(1) + 0.5(0) + 0 = 0 \quad \checkmark$$
$$0 + 0.5(1) + 0 = 1 \quad \checkmark$$
$$0 + 0.5(1) + 0 = 0.5 \approx 1 \quad \checkmark$$
$$0 + 0.5(1) + 0 = 0.5 \approx 1 \quad \checkmark$$