

CS492: Probabilistic Programming

# Generative Modelling with Anglican

Hongseok Yang  
KAIST

# CS492: Probabilistic Programming

# Generative Modelling

## with Anglican

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I have limited experience in modelling.  
Better title: Further examples in Anglican.

# What is a model?

1. A probability density or mass function.
2. A simulator or a sampler.

# What is a model?

1. A probability density or mass function.
2. A simulator or a sampler.
  - Generative view.
  - Well-supported by prob. PLs like Anglican.
  - Express how a hypothesis arises and also how it generates observed data.

# Solving a problem via generative modelling

1. Create a simulator (i.e. a generative model).
  - How do candidate solutions arise?
  - How do solutions generate observations?
2. Specify success criteria with conditioning.
3. Solve the problem by posterior inference.

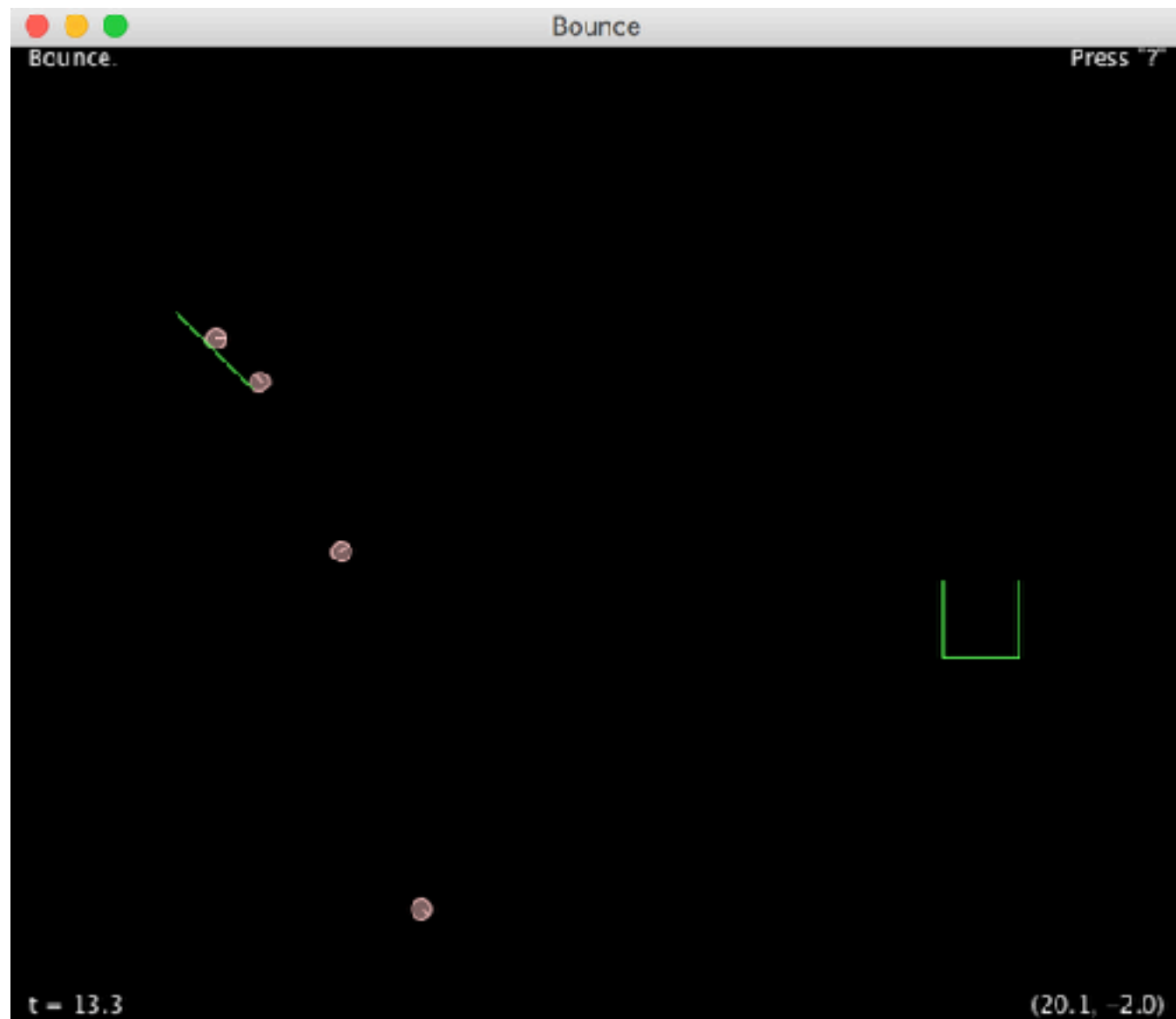
# Learning outcome

1. Can use generative modelling for solving a problem.
2. Can reproduce the solutions of 2D physics and program induction in Anglican.
3. Can write interesting probabilistic programs using the expressive power of Anglican.
4. Can explain Poisson, Gaussian distributions.

# 2D physics

Borrowed from Wood & Paige's practical at MLSS'15

# 2D physics

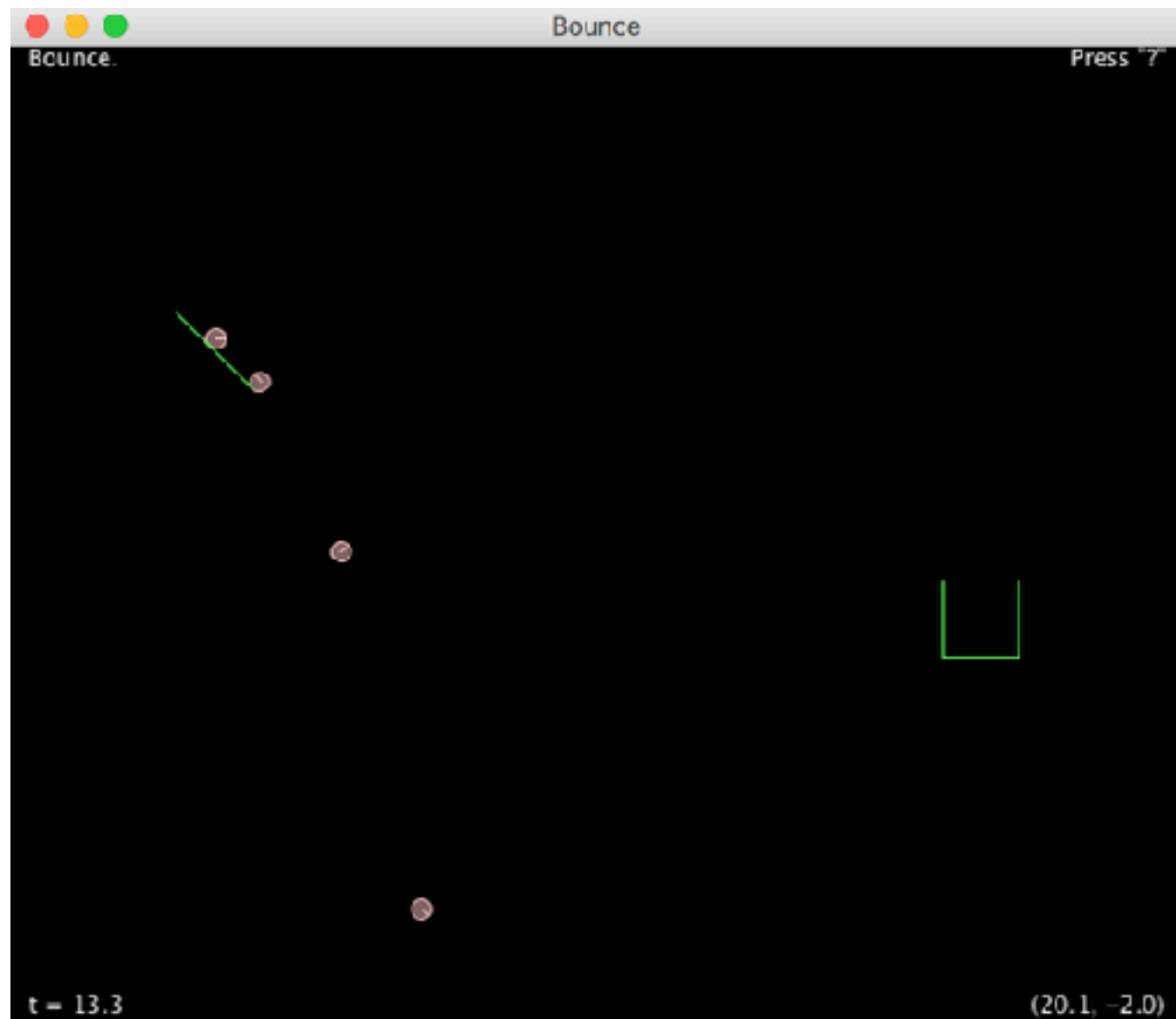


20 balls are falling from top-left.

[Q] How to move them to the bin?



# 2D physics



20 balls are falling from top-left.

[Q] How to move them to the bin?

[A] Put bumpers.

How to solve this in Anglican?

(by means of generative modelling)

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How to solve this in Anglican?

1. Create a simulator.

- Randomly place bumpers. Drop balls.

2. Specify success criteria with conditioning.

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How to solve this in Anglican?

1. Create a simulator.

- Randomly place bumpers. Drop balls.

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(by means of generative modelling)

How to solve this in Anglican?

1. Create a simulator.

- Randomly place bumpers. Drop balls.

2. Specify success criteria with conditioning.

- High likelihood if many balls reach the bin.

3. Solve the problem by posterior inference.



(by means of generative modelling)

How to solve this in Anglican?

1. Create a simulator.

- Randomly place bumpers. Drop balls.

2. Specify success criteria with conditioning.

- High likelihood if many balls reach the bin.

3. Solve the problem by posterior inference.

# Clojure functions for 2D physics in bounce.clj

1. Create a world.

```
(def bumpers (list (list -3 6) (list 2 5)))
```

```
(def start-w (create-world bumpers))
```

2. Simulate 20 balls in the world.

```
(def end-w (simulate-world start-w))
```

3. Count the number of balls in the bin.

```
(def num-balls (balls-in-box end-w))
```

These functions are defined in bounce.clj available at the course webpage.


# Clojure functions in Anglican programs

```
(with-primitive-procedures  
  [create-world  
   simulate-world  
   balls-in-box]
```

```
  (defquery physics [] . . . ))
```

# Clojure functions in Anglican programs

```
(with-primitive-procedures  
  [create-world  
   simulate-world  
   balls-in-box]  
  (defquery physics [] . . . ))
```



Anglican keyword.

Imports primitive Clojure fns.

# Closure functions in Anglican programs

(with-primitive-procedures

`[create-world`

`simulate-world`

`balls-in-box]`



Imported functions.

(defquery physics [] . . . ))

Anglican keyword.

Imports primitive Clojure fns.

1) Simulator. 2) Condition. 3) Inference.

1) Simulator. ~~2) Condition.~~ 3) Inference.

1) Simulator. ~~2) Condition.~~ 3) Inference.

```
(defquery physics0 []
```



```
)
```



1) Simulator. ~~2) Condition.~~ 3) Inference.

```
(with-primitive-procedures  
 [create-world simulate-world balls-in-box]  
 (defquery physics0 []
```



```
)
```

# 1) Simulator. ~~2) Condition.~~ 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics0 []
  (let
   [n-bumpers 8
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)

    . . . . . ]
   . . . . . )))
```

# 1) Simulator. ~~2) Condition.~~ 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics0 []
  (let
   [n-bumpers 8
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]
   . . . . . )))
```

# 1) Simulator. ~~2) Condition.~~ 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics0 []
  (let
   [n-bumpers 8
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]

   (list num-balls bs))))
```

# 1) Simulator. ~~2) Condition.~~ 3) Inference.

(with-primitive-procedures

```
(def lazy-samples0
  (doquery :importance physics0 []))
(def samples0
  (map :result (take-nth 10 (take 2000 (drop 1000 lazy-samples0)))))
(def best-sample0
  (reduce (fn [acc x] (if (> (first x) (first acc)) x acc))
    samples0))
best-sample0
```

#'bounce-worksheet/lazy-samples0

#'bounce-worksheet/samples0

#'bounce-worksheet/best-sample0

```
(2 ((5.039659490241706 2.4748229831103163) (1.6105279484809571
6.095328821668973) (-1.7327828746932634 2.1429922008512325)
(11.46787625011067 3.6077249028398284) (-1.1506281530451017
2.1718715228712937) (-1.752497599843685 5.686404684681266)
(0.3924210062883362 4.924024324154887) (13.669929061656298
3.9039578861003066)))
```

# 1) Simulator. ~~2) Condition.~~ 3) Inference.

(with-primitive-procedures

```
(def lazy-samples0
  (doquery :importance physics0 []))
(def samples0
  (map :result (take-nth 10 (take 2000 (drop 1000 lazy-samples0)))))
(def best-sample0
  (reduce (fn [acc x] (if (> (first x) (first acc)) x acc))
    samples0))
best-sample0
```

#'bounce-worksheet/lazy-samples0

#'bounce-worksheet/samples0

Just two balls in the bin.

```
(2 ((5.039659490241706 2.4748229831103163) (1.6105279484809571
6.095328821668973) (-1.7327828746932634 2.1429922008512325)
(11.46787625011067 3.6077249028398284) (-1.1506281530451017
2.1718715228712937) (-1.752497599843685 5.686404684681266)
(0.3924210062883362 4.924024324154887) (13.669929061656298
3.9039578861003066)))
```

# 1) Simulator. ~~2) Condition.~~ 3) Inference.



(with

```
(def 1  
  (dog  
(def s  
  (map  
(def b  
  (red
```

best-s

#'boun

#'boun

#'boun

```
(2 ((  
6.095  
(11.4  
2.171  
(0.39  
3.903
```

1) Simulator. 2) Condition. 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics1 []
  (let
   [n-bumpers 8
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]

   (list num-balls bs))))
```

[Q] More balls in the bin. Express this goal using observe.

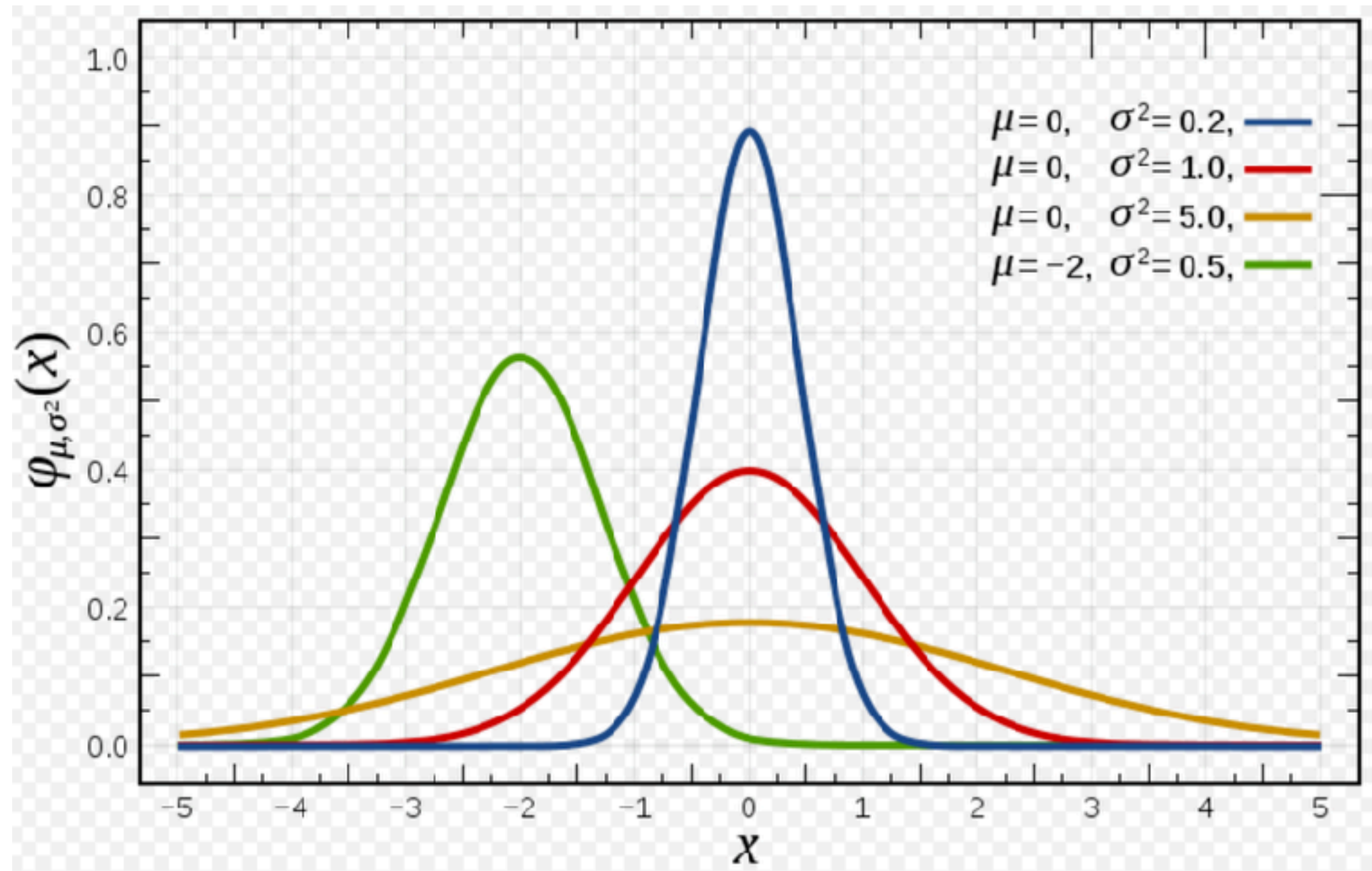


# Normal distribution

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Probability distribution on a real  $x$ .
- Specified by density  $f$ . Also by  $\mu$  and  $\sigma$ .

# Normal distribution

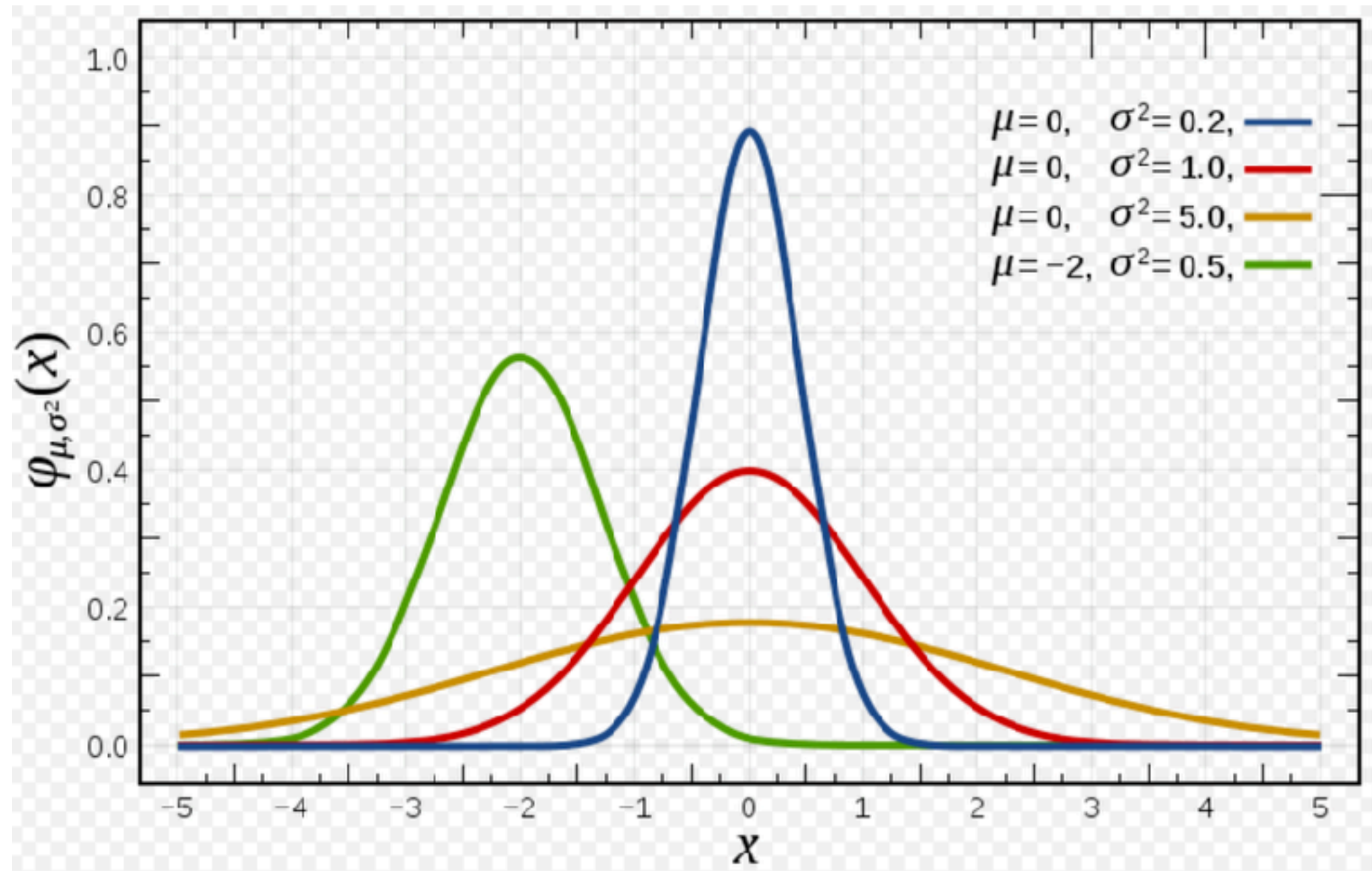


# Normal distribution

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Probability distribution on a real  $x$ .
- Specified by density  $f$ . Also by  $\mu$  and  $\sigma$ .
- [Intuition 1]  $x$  is in  $[\mu-2\sigma, \mu+2\sigma]$  usually.
- [Intuition 2] The chance of  $x$  being further from  $\mu$  decays exponentially.

# Normal distribution



# Normal distribution

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$p(x \mid y) = \frac{p(y \mid x) \times p(x)}{p(y)}$$

- Bayes's rule holds regardless of whether  $p$  is a probability mass or a prob. density.

1) Simulator. 2) Condition. 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics1 []
  (let
   [n-bumpers 8
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]

   (list num-balls bs))))
```

[Q] More balls in the bin. Express this goal using observe.

1) Simulator. 2) Condition. 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics1 []
  (let
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    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]

   (observe (normal num-balls 1) 20)
   (list num-balls bs))))
```

[Q] More balls in the bin. Express this goal using observe.

# 1) Simulator. 2) Condition. 3) Inference.

With primitive procedures

```
(def lazy-samples1
  (doquery :lmh physics1 []))
(def samples1
  (map :result (take-nth 10 (take 2000 (drop 1000 lazy-samples1)))))
(def best-sample1
  (reduce (fn [acc x] (if (> (first x) (first acc)) x acc))
    samples1))
best-sample1
```

```
#'bounce-worksheet/lazy-samples1
```

```
#'bounce-worksheet/samples1
```

```
#'bounce-worksheet/best-sample1
```

```
(18 ((0.42846830943289227 4.955430158101355) (0.11399011311342733
7.047454084589595) (-1.6760284246021682 3.482088149196809)
(2.5215232534350247 2.5347454541889936) (12.119916165254827
7.446364186029621) (9.842132376151833 0.6555153748336084)
(0.383232923290564 0.8118697300596889) (5.731228040496732
5.245105763879869)))
```

[Q] More balls in the bin. Express this goal using observe.



# 1) Simulator. 2) Condition. 3) Inference.

## MCMC algorithm (next lecture)

```
(def lazy-samples1
  (doquery :lmh physics1 []))
(def samples1
  (map :result (take-nth 10 (take 2000 (drop 1000 lazy-samples1)))))
(def best-sample1
  (reduce (fn [acc x] (if (> (first x) (first acc)) x acc))
    samples1))
best-sample1
```

```
#'bounce-worksheet/lazy-samples1
```

```
#'bounce-worksheet/samples1
```

```
#'bounce-worksheet/best-sample1
```

```
(18 ((0.42846830943289227 4.955430158101355) (0.11399011311342733
7.047454084589595) (-1.6760284246021682 3.482088149196809)
(2.5215232534350247 2.5347454541889936) (12.119916165254827
7.446364186029621) (9.842132376151833 0.6555153748336084)
(0.383232923290564 0.8118697300596889) (5.731228040496732
5.245105763879869)))
```

[Q] More balls in the bin. Express this goal using observe.

# 1) Simulator. 2) Condition. 3) Inference.

With primitive procedures

```
(def lazy-samples1
  (doquery :lmh physics1 []))
(def samples1
  (map :result (take-nth 10 (take 2000 (drop 1000 lazy-samples1)))))
(def best-sample1
  (reduce (fn [acc x] (if (> (first x) (first acc)) x acc))
    samples1))
best-sample1
```

```
#'bounce-worksheet/lazy-samples1
```

```
#'bounce-worksheet/samples1
```

18 balls in the bin. sample1

```
(18 ((0.42846830943289227 4.955430158101355) (0.11399011311342733
7.047454084589595) (-1.6760284246021682 3.482088149196809)
(2.5215232534350247 2.5347454541889936) (12.119916165254827
7.446364186029621) (9.842132376151833 0.6555153748336084)
(0.383232923290564 0.8118697300596889) (5.731228040496732
5.245105763879869)))
```

[Q] More balls in the bin. Express this goal using observe.

# 1) Simulator. 2) Condition. 3) Inference.



[Q] More balls in the bin. Express this goal using observe.

# 1) Simulator. 2) Condition. 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics2 []
  (let
   [n-bumpers 8
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]

   (observe (normal num-balls 1) 20)
   (list num-balls bs))))
```

[Q] Reduce the number of bumpers.

## 1) Simulator. 2) Condition. 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics2 []
  (let
   [n-bumpers • • •
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]

   (observe (normal num-balls 1) 20)
   (list num-balls bs))))
```

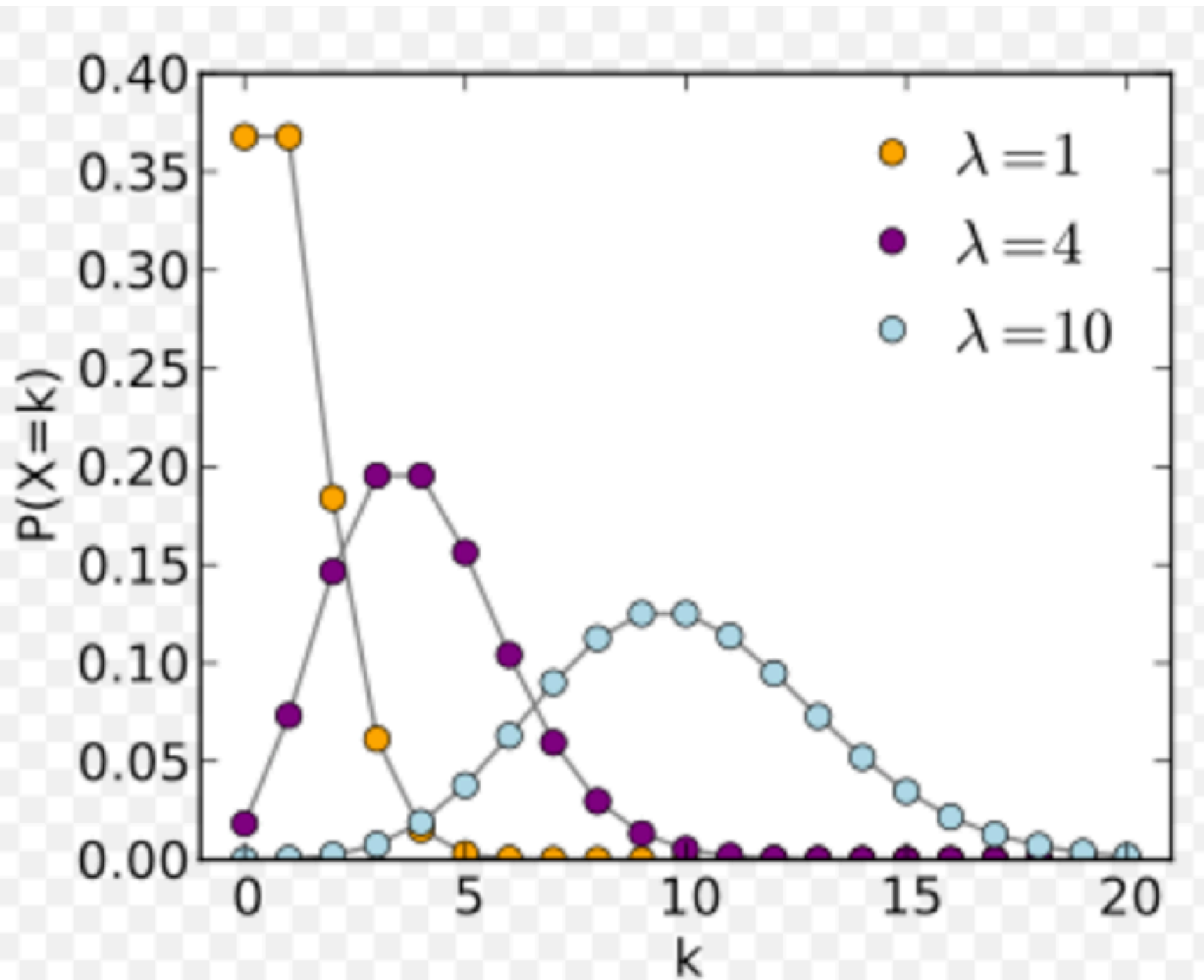
[Q] Reduce the number of bumpers.

# Poisson distribution

$$P(k \text{ events in interval}) = e^{-\lambda} \frac{\lambda^k}{k!}$$

- Distribution on **non-negative integer k**.
- Represents the k independent events happening in a fixed time interval.
- Mean  $\lambda$ . Peak at  $\lambda$  or  $\lambda-1$ .

# Poisson distribution



## 1) Simulator. 2) Condition. 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics2 []
  (let
   [n-bumpers (sample (poisson 6))
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]

   (observe (normal num-balls 1) 20)
   (list num-balls bs))))
```

[Q] Reduce the number of bumpers.



1) Simulator. 2) Condition. 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics2 []
  (let
   [n-bumpers (sample (poisson 6))
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]
   • • • • •
   (observe (normal num-balls 1) 20)
   (list num-balls bs))))
```

[Q] Reduce the number of bumpers.

1) Simulator. 2) Condition. 3) Inference.

```
(with-primitive-procedures
 [create-world simulate-world balls-in-box]
 (defquery physics2 []
  (let
   [n-bumpers (sample (poisson 6))
    f (fn [] (list
              (sample (uniform-continuous -5 14))
              (sample (uniform-continuous 0 10))))
    bs (repeatedly n-bumpers f)
    w0 (create-world bs)
    w1 (simulate-world w0)
    num-balls (balls-in-box w1)]
   (observe (normal n-bumpers 2) 0)
   (observe (normal num-balls 1) 20)
   (list num-balls bs))))
```

[Q] Reduce the number of bumpers.

# 1) Simulator. 2) Condition. 3) Inference.

```
(v
(def lazy-samples2
  (doquery :lmh physics2 []))
(def samples2
  (map :result (take-nth 10 (take 2000 (drop 1000 lazy-samples2)))))
(defn is-better [x y]
  (let [num-bumpers-less (< (count (second x)) (count (second y)))
        num-balls-more (> (first x) (first y))
        num-balls-equal (= (first x) (first y))
        x-above-threshold (> (first x) 15)
        y-above-threshold (> (first y) 15)]
    (or (and x-above-threshold num-bumpers-less)
        (and num-balls-equal num-bumpers-less)
        num-balls-more)))
(def best-sample2
  (reduce (fn [acc x] (if (is-better x acc) x acc))
    samples2))
best-sample2
```

```
#'bounce-worksheet/lazy-samples2
```

```
#'bounce-worksheet/samples2
```

```
#'bounce-worksheet/is-better
```

```
#'bounce-worksheet/best-sample2
```

```
(18 ((3.3372725819978006 8.47981623123972) (6.810994664276837
1.2074164603701054) (11.516669110855627 2.9146326254312993)
(-3.905786355145122 6.597352342859228) (0.5808260753916357
0.6871272205893586)))
```

# 1) Simulator. 2) Condition. 3) Inference.

```
(v
(def lazy-samples2
  (doquery :lmh physics2 []))
(def samples2
  (map :result (take-nth 10 (take 2000 (drop 1000 lazy-samples2)))))
(defn is-better [x y]
  (let [num-bumpers-less (< (count (second x)) (count (second y)))
        num-balls-more (> (first x) (first y))
        num-balls-equal (= (first x) (first y))
        x-above-threshold (> (first x) 15)
        y-above-threshold (> (first y) 15)]
    (or (and x-above-threshold num-bumpers-less)
        (and num-balls-equal num-bumpers-less)
        num-balls-more)))
(def best-sample2
  (reduce (fn [acc x] (if (is-better x acc) x acc))
    samples2))
best-sample2
```

#'bounce-worksheet/lazy-samples2

#'bounce-worksheet/samples2

#'bounce-worksheet/is-better

#'bounce-worksheet/best-sample2

Five bumpers used.

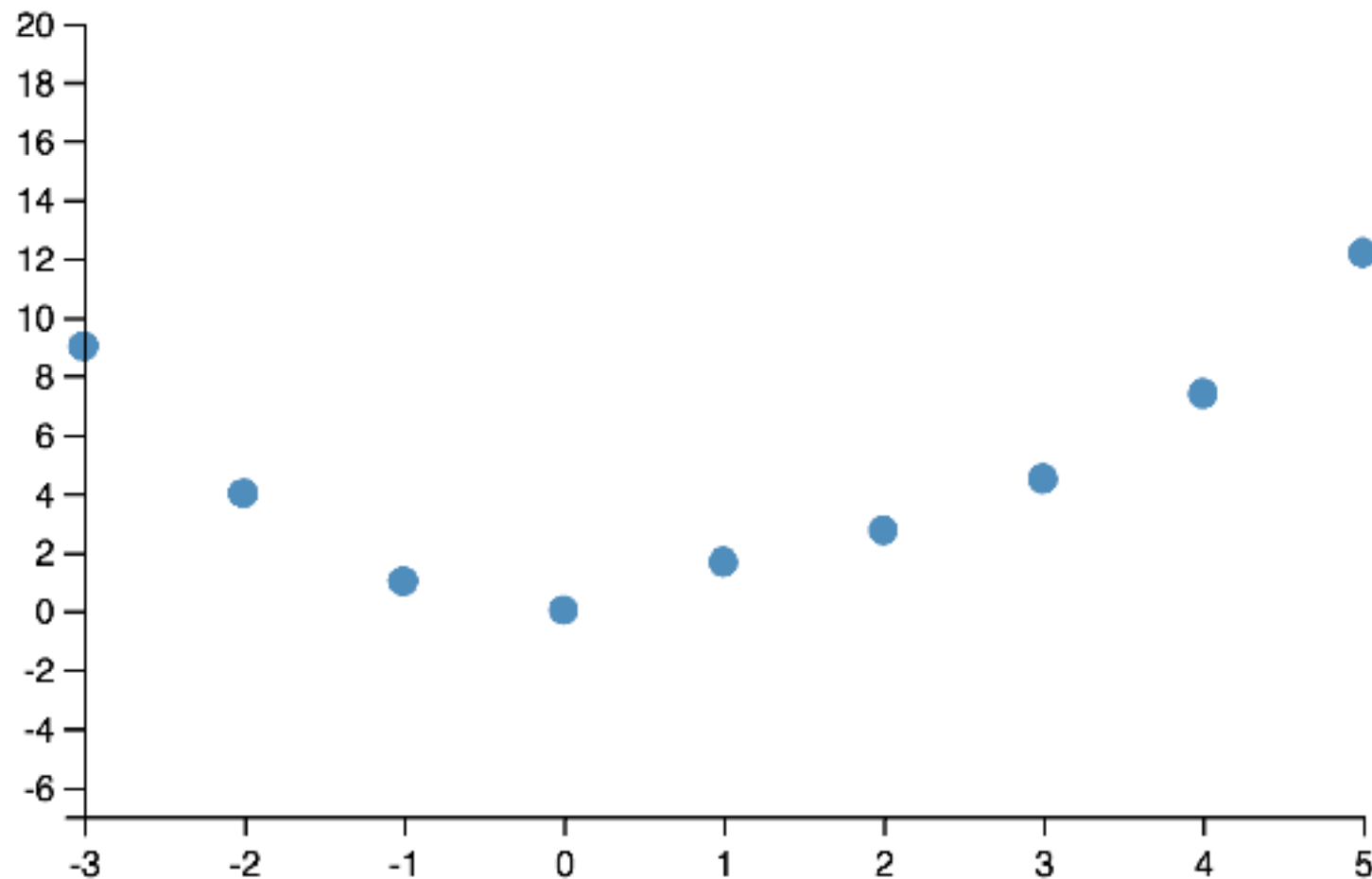
```
(18 ((3.3372725819978006 8.47981623123972) (6.810994664276837
1.2074164603701054) (11.516669110855627 2.9146326254312993)
(-3.905786355145122 6.597352342859228) (0.5808260753916357
0.6871272205893586)))
```

# 1) Simulator. 2) Condition. 3) Inference.



# Baby program induction

# Baby program induction



[Q] Find a Clojure function that interpolates these data points.

# Solve program induction via generative modelling

1. Create a simulator.
2. Specify success criteria with conditioning.
3. Solve the problem by posterior inference.



# Solve program induction via generative modelling

1. Create a simulator.

- Generate expressions using probabilistic grammar.

2. Specify success criteria with conditioning.

- High likelihood if the evaluation of a sampled expression matches data well.

3. Solve the problem by posterior inference.

# Solve program induction via generative modelling

## 1. Create a simulator.

- Generate expressions using probabilistic grammar.

## 2. Specify success criteria with conditioning.

- High likelihood if the evaluation of a sampled expression matches data well.

## 3. Solve the problem by posterior inference.

# Grammar for baby Clojure

$\text{Prog} ::= (\text{fn } [x] \text{ Ex})$

$\text{Ex} ::= \text{Num} \mid x \mid \text{BinEx}$

$\text{Num} ::= -9 \mid -8 \mid \dots \mid 9 \mid 10$

$\text{BEx} ::= (+ \text{ Ex Ex}) \mid (- \text{ Ex Ex}) \mid (* \text{ Ex Ex})$

# Grammar for baby Clojure

**Prog** ::= (fn [x] **Ex**)

**Ex** ::= **Num** | x | **BinEx**

**Num** ::= -9 | -8 | ... | 9 | 10

**BEx** ::= (+ **Ex Ex**) | (- **Ex Ex**) | (\* **Ex Ex**)

4 Nonterminal symbols.

# Grammar for baby Clojure

Prog ::= (fn [x] Ex)

Ex ::= Num | x | BinEx

Num ::= -9 | -8 | ... | 9 | 10

BEx ::= (+ Ex Ex) | (- Ex Ex) | (\* Ex Ex)

4 Nonterminal symbols. Many terminal symbols.

# Grammar for baby Clojure

$\text{Prog} ::= (\text{fn } [x] \text{ Ex})$

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$\text{Num} ::= -9 \mid -8 \mid \dots \mid 9 \mid 10$

$\text{BEx} ::= (+ \text{ Ex Ex}) \mid (- \text{ Ex Ex}) \mid (* \text{ Ex Ex})$

4 Nonterminal symbols. Many terminal symbols.

Production rules tell us how to generate programs.

# Grammar for baby Clojure

## Probabilistic

$\text{Prog} ::= (\text{fn } [x] \text{ Ex})$

$\text{Ex} ::= \text{Num} \mid x \mid \text{BinEx}$

$\text{Num} ::= -9 \mid -8 \mid \dots \mid 9 \mid 10$

$\text{BEx} ::= (+ \text{ Ex Ex}) \mid (- \text{ Ex Ex}) \mid (* \text{ Ex Ex})$

# Grammar for baby Clojure

## Probabilistic

$\text{Prog} ::= (\text{fn } [x] \text{ Ex})$

$\text{Ex} ::= \text{Num} \mid x \mid \text{BinEx}$

$\text{Num} ::= -9 \mid -8 \mid \dots \mid 9 \mid 10$

$\text{BEx} ::= (+ \text{ Ex Ex}) \mid (- \text{ Ex Ex}) \mid (* \text{ Ex Ex})$

For each nonterminal, pick a rule probabilistically.



# Grammar for baby Clojure

## Probabilistic

$\text{Prog} ::= (\text{fn } \overset{1.0}{[x]} \text{ Ex})$

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For each nonterminal, pick a rule probabilistically.

How to represent this prob. grammar in Clojure?

# Quoted expressions

- Data structure for Clojure programs.
- Roughly nested lists (and vectors) of symbols & constants.

```
(def e1 (list '* (list '+ '1 'x) '3))
```

```
(def e2 (list 'fn ['x] e1))
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- Can be converted to a program via eval.

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(eval e2 10),    (eval e1 10)
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(defm gen-e []
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● ● ● ● ● )

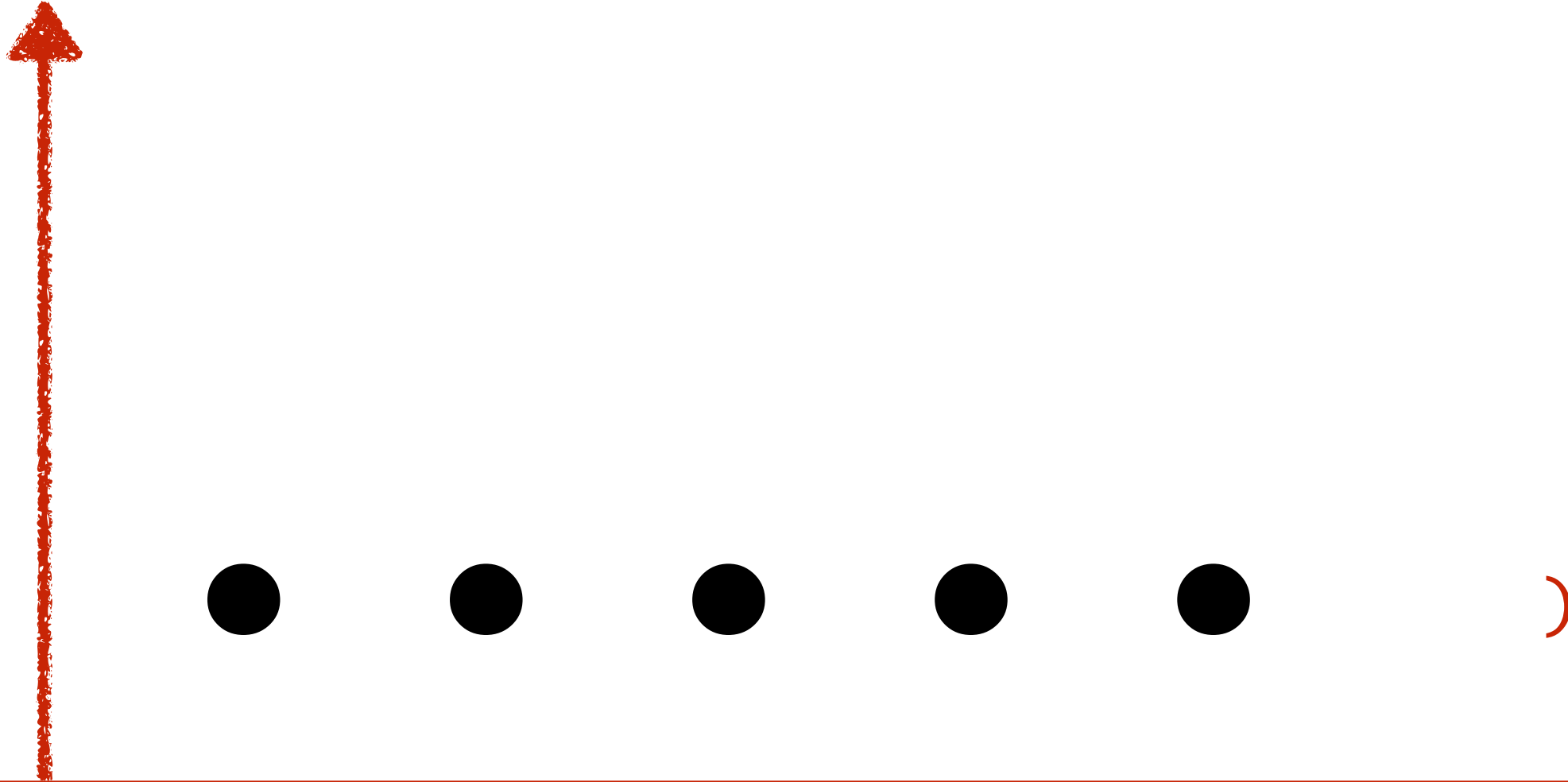
Ex: (Num, x, BinEx) — (.4, .4, .2)

Num: (-9, ..., 10) — (.05, ..., .05)

BEx: (+, -, \*) — (.3, .3, .4)



(defm gen-e []



Anglican's defn.

Anglican functions defined outside of a query.

Ex: (Num, x, BinEx) — (.4, .4, .2)

Num: (-9, ..., 10) — (.05, ..., .05)

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(defm gen-e []  
  (let [t (sample  
          (discrete (list 0.4 0.4 0.2)))]
```

● ● ● ● ● )

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(defm gen-e []
  (let [t (sample
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    (cond
      (= t 0) (sample (uniform-discrete -9 11))
      • • • • • )))

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                • • • • • )
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```

[Q] Complete this Anglican program.

Ex: (Num, x, BinEx) — (.4, .4, .2)  
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      (= t 2) (list (sample (categorical
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```

```

(defquery grammar []
  (let [prog
        prog])

```

Ex: (Num, x, BinEx) — (.4, .4, .2)  
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(defquery grammar []
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    prog))

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Ex: (Num, x, BinEx) — (.4, .4, .2)  
 Num: (-9, ..., 10) — (.05, ..., .05)  
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# Solve program induction via generative modelling

1. Create a simulator.
  - Generate expressions using probabilistic grammar.
2. Specify success criteria with conditioning.
  - High likelihood if the evaluation of a sampled expression matches data well.
3. Solve the problem by posterior inference.

```
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```

```
(defquery grammar []  
  (let [prog (list 'fn ['x] (gen-e))]
```

```
    prog))
```

```
(defm gen-e [] . . . .)
```

```
(defquery grammar [ints outs]  
  (let [prog (list 'fn ['x] (gen-e))]
```

```
    prog))
```

```
(defm gen-e [] . . . .)
```

```
(defn evaluate [f-code v] ((eval f-code) v))
```

```
(with-primitive-procedures [evaluate]  
  (defquery grammar [ints outs]  
    (let [prog (list 'fn ['x] (gen-e))]
```

```
      prog)))
```

```

(defm gen-e [] . . . .)

(defn evaluate [f-code v] ((eval f-code) v))

(with-primitive-procedures [evaluate]
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    (let [prog (list 'fn ['x] (gen-e))
          f (fn [in out]
              (observe
                ● ● ● ● ●
                out)))]
      (map f ins outs)
      prog)))

```

[Q] Complete this.

```

(defm gen-e [] . . . .)

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(with-primitive-procedures [evaluate]
  (defquery grammar [ints outs]
    (let [prog (list 'fn ['x] (gen-e))
          f (fn [in out]
               (observe
                 (normal (evaluate prog in) 1)
                 out)))]
      (map f ins outs)
      prog)))

```

[Q] Complete this.

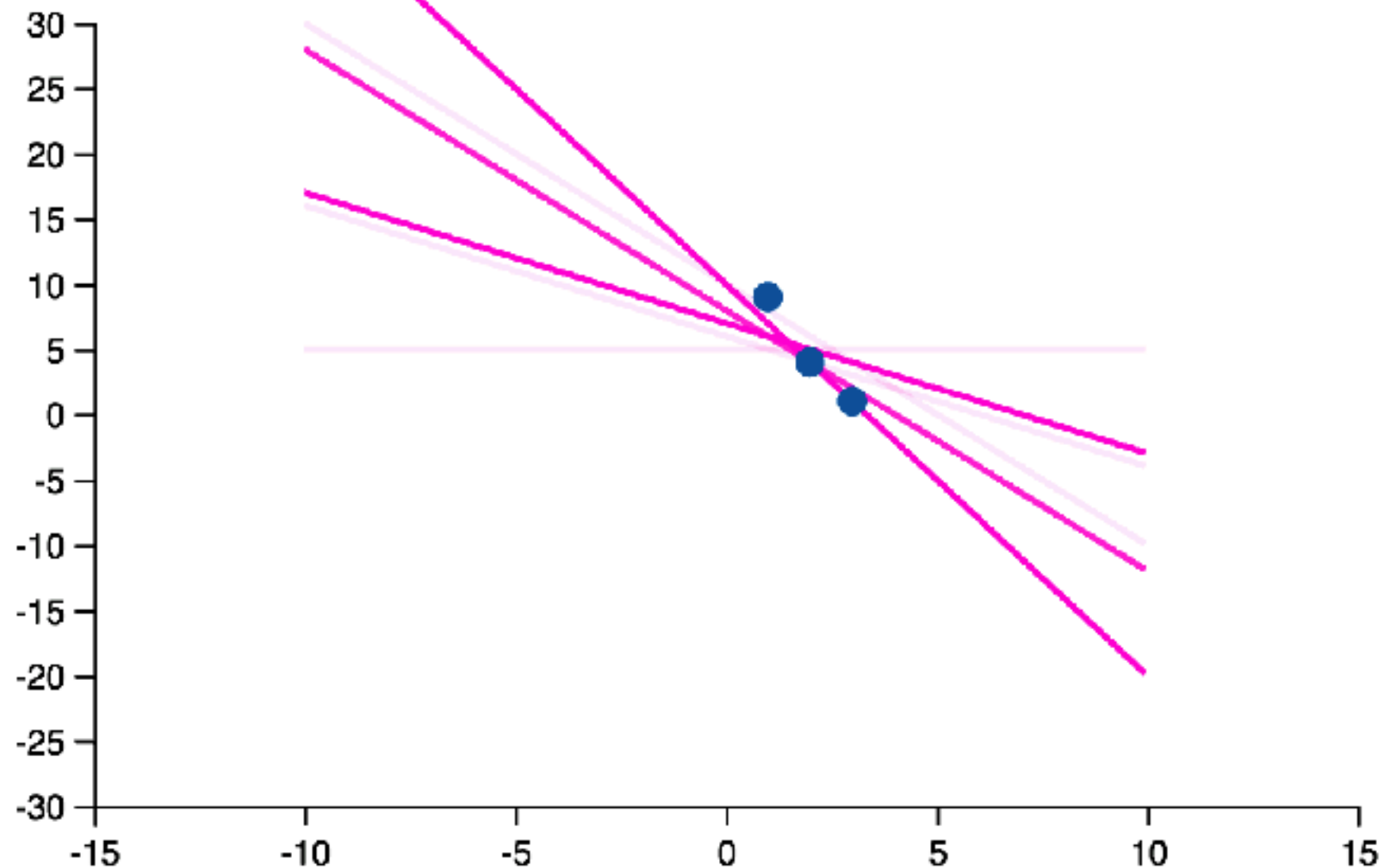
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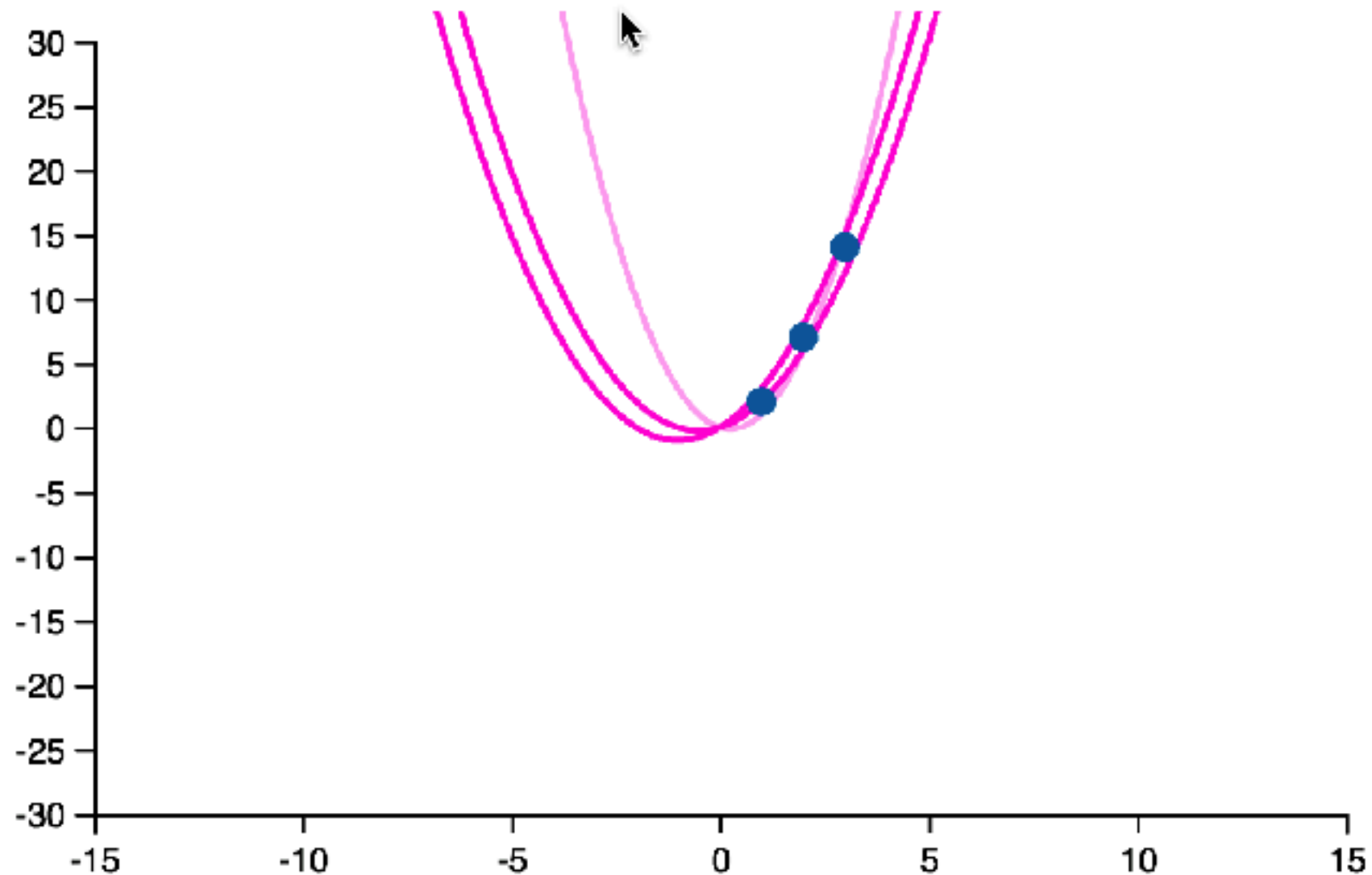
# Result with data set I

```
((fn [x] (+ (* -3 x) 10))  
 (fn [x] (+ (* -2 x) 8))  
 (fn [x] (+ 1 4))  
 (fn [x] (+ (- -1 x) 8))  
 (fn [x] (+ (- 0 x) 7))))
```



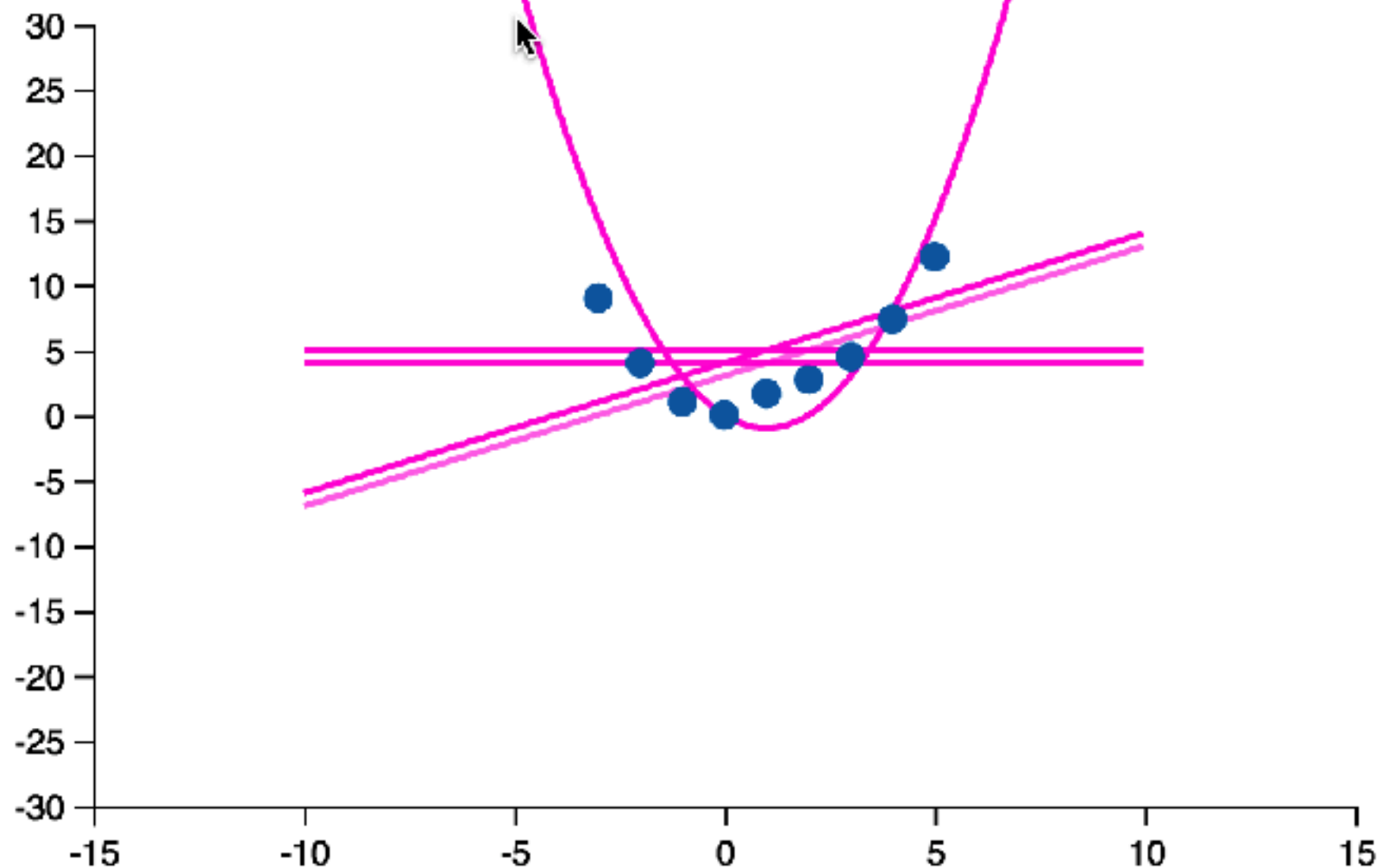
# Result with data set 2

```
((fn [x] (* x (+ (* 2 x) -1))))  
(fn [x] (* x (+ x 1)))  
(fn [x] (* x (+ x (+ 2 0))))  
(fn [x] (* (+ (- x 1) (+ x (- x x))) x))  
(fn [x] (* x (+ 2 (* x 1)))))
```



# Result with data set 3

```
((fn [x] (* x (- (+ 4 x) (* 6 1)))))  
(fn [x] 5)  
(fn [x] (+ 3 x))  
(fn [x] (+ 4 (- x 0)))  
(fn [x] (+ 4 x)))
```



# Summary

- The generative approach suggests to specify a process for generating hypothesis & data.
- This process clarifies hidden assumptions.
- Goes well with probabilistic programming.
- Prob. PLs let us use powerful programming constructs (eval, etc) in modelling.

# Information

- I will put gorilla worksheets for 2d physics and program induction in the webpage.
- To run the 2d physics example, you will have to do two things:
  1. Copy bounce.clj to anglican-user/src.
  2. Copy project.clj to anglican-user.