McMaster University
SFWR ENG 2MD3 Winter 2020 Assignment 4
Due: Sunday March 7, 2020 at 23:55
March 2, 2020

Manuel Lemos, 400177763

Question1 (2 marks each = 8 marks)
Find out the truth values of the following Hoare Triples. (Show that the triple is valid or invalid)

1. {i = j} i = j + i; {i>j}
   FALSE: Consider i = j = 0 precondition is satisfied, but the post condition is not.

2. {i > j} j = i+1; i = j+1; {i > j}
   TRUE: Consider i = 0, j = -3; precondition is satisfied, post condition is also satisfied

3. {x > 0} x = x + 1; {x > 1}
   TRUE: Consider x = 1; precondition is satisfied, post condition is also satisfied

4. {x = 5 > 0} x = x + 1; {x > 1}
   TRUE: providing x is an integer, it will be assigned 1 by the pre-condition, and then incremented to 2, thus satisfying the post condition

Question 2 (2marks each = 8 marks marks)
Find the weakest pre-condition for the following Hoare Triples:

1. {??} x = 5; {x = 5}
   P = wp(x=5, x=5)
   P = {TRUE}

2. {??} x = x + 3; {x = y + 3}
   P = wp(x = x + 3,x = y + 3)
   P = (x + 3 = y + 3)
   P = {x = y}

3. {??} z = x / y; {z < 1}
   P = wp(z = x / y, z < 1)
   P = (x / y < 1)
   P {y > x}

4. {??} x = 3 ∗ y + z; {x ∗ y – z > 0}
   P = wp(x = 3 * y + z, x * y – z > 0)
   P = [(3 * y + z) * y – z > 0]
   P = {3 * y_2 + z * y - z > 0}

Question 3 (2 marks each = 4 marks)
Find the weakest pre-condition for the following sequence of statements:

1. {??} x = z + 1; y = x + y; {y > 5}
   P = wp(x = z + 1, wp(y = x + y, y > 5))
   P = wp(x = z + 1, x + y > 5)
   P = (z + 1 + y > 5)
   P = {y + z > 4}

2. {??} x = x + 1; x = x + 2; {x > 5}
   P = wp(x = x + 1, wp(x = x + 2, x > 5))
   P = wp(x = x + 1, x + 2 > 5)
   P = (x + 3 > 5)
   P = {x > 2}

Question 4 (4 marks)
Find the pre-condition for the following conditional statement corresponding to given post-condition:
{??} if (x != 0){ z = x; } else{ z = x + 1;} {z > 0}
P = wp(if (x != 0) z = x else z = x + 1, z > 0)
P = [(x != 0) ∧ wp(z = x, z > 0) ∨ (x = 0) ∧ wp(z = x + 1, z > 0)]
P = [(x != 0) ∧ (x > 0) ∨ (x = 0) ∧ (x + 1 > 0)]
P = {x > 0} ∨ {x = 0}
P = {x ≥ 0}

Question 5 (4 marks)
What is the loop invariant inside the while loop in the following program?
Show that it is valid before and after the termination of the loop.

```
int Sum(InputArray A, int n) {
        int i = 0, S = 0;
        while(i < n){
                S+= A[i];
                i++;
        }
return S;
}
```

Prior to loop execution; i = 0, S = 0;

Upon termination of the loop; i = n, S = $\sum_{i=0}^{n-1} A[\,i]$

Thus, the loop invariant is: S = $\sum_{k=0}^{i-1} A[\,k]$