

McMaster University
SFWR ENG 2MD3 Winter 2020 Assignment 6
Due: Tuesday March 31, 2020 at 23:55
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Question 1 (8 marks (2+3+3))

1. What is a Complete Binary Tree?

A binary tree with leaves on either a single level or on two adjacent levels such that the leaves on the bottommost level are placed as far left as possible

2. What is the relationship between the number of empty binary trees in an extended binary tree (symbolized by squares) and the number of internal (nonempty) nodes (symbolized by circles)?

Considering a single nonempty node. Will then be paired with 2 empty binary trees.

Adding another node in place of one empty binary tree results in: 3 empty binary trees because this type of binary tree is a full binary tree.

The relationship can then be made that in an extended binary tree, the number of empty binary trees is one greater than the number of internal non-empty nodes.

This makes sense as adding additional non empty nodes does not change the relationship created by the initial one node 2 empty tree case.

3. How many nodes are there in a complete binary tree with levels 0:n, in which level n is completely filled with leaves in all possible positions?

Each level (L) has 2^L nodes, so level 0 has 1 node and level 2 has 4 nodes

This can be represented by; $\sum_{L=0}^n 2^L = 2^0 + 2^1 + \dots + 2^n$

Which can be simplified to: $2^{n+1} - 1$

Question 2 (6 marks (2+2+2))

Answer the following questions with reference to the contiguous sequential representation of a complete binary tree:

1. How do you find the right child of node at $A[i]$?

To find the right child use $A[2 * i + 1]$

2. In an array representing a tree, what nodes have no right child?

If: $2 * i + 1 > n$ no right child exists

Thus, if $i > \frac{n-1}{2}$ no right child exists

In an array of $A[1:7]$, values of $i > 3$ have no right child \rightarrow expression holds

3. Given that there are n nodes in a complete binary tree, how many levels does it have as a function of n ?

In C, can be expressed as: $\text{ceil}(\log_2(n + 1))$

Linguistic notation: (round up) $(\log_2(n + 1))$

Question 3 (10 marks (3+4+3))

Provide answers for:

1. A) How can a heap be used to represent a priority queue?
B) Discuss (no code please) how to perform the operations of item insertion and removal.
C) Particularly describe what happens to the heap when an item is inserted or removed into / from the priority queue and how its properties are restored.

A) A heap is a complete binary tree with values stored in its nodes in an orientation such that no child has a value larger than that of the value of its parent.

Thus, if a heap is used to represent a priority queue, the item of highest priority is located at the root of the tree. (assuming highest priority = largest value)

B) Insertion:
-Add desired value to the leftmost available leaf node
-Restore the heap property among the remaining nodes. Starting at the inserted leaf node and repeatedly exchanging its value with the smaller value of its parents, until no more exchanges are possible.

Removal:
-Highest root node in first array position is removed
-Delete the rightmost leaf on the bottom row
-Place this deleted node's value, into the root node
-Restore the heap property among the remaining nodes. Starting at the root node and repeatedly exchanging its value with the larger of the values of its children, until no more exchanges are possible.

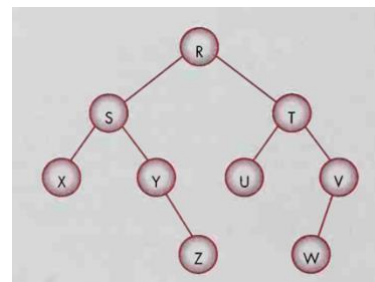
2. Give the PreOrder, InOrder, PostOrder, and LevelOrder traversals of the nodes in the tree of Fig 9.2?

PreOrder Traversal: R S X Y Z T U V W

InOrder Traversal: X S Y Z R U T W V

PostOrder Traversal: X Z Y S U W V T R

LevelOrder Traversal: R S T X Y U V Z W

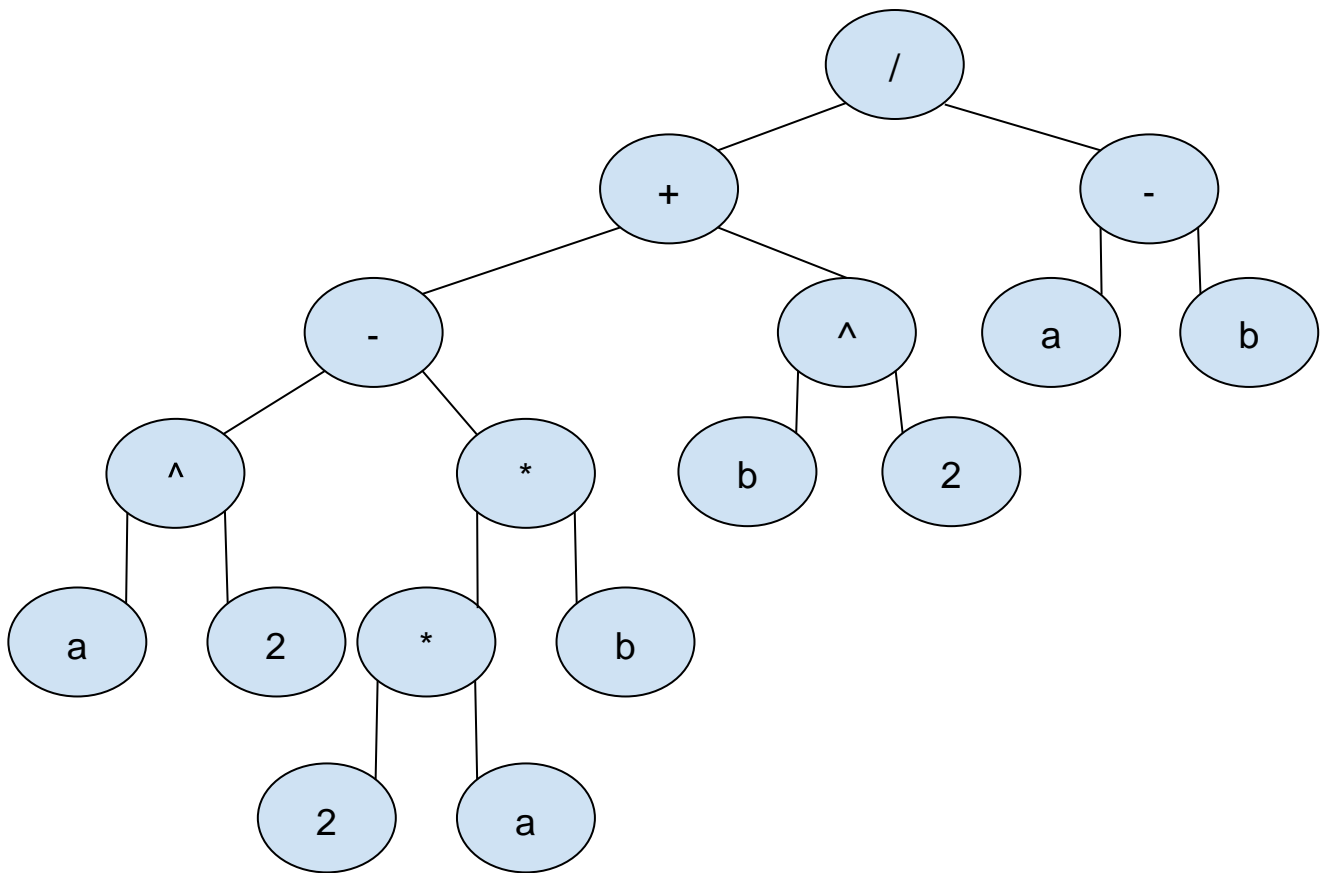


3. Construct the expression tree whose

PostOrder Traversal is: $a^2 2 a^* b^* - b^2 + a b - /$

Grouping: $\langle [\{ a^2 \} \{ (2 a^*) (b)^* \} \{-\}] [b^2] \{ + \} \rangle \langle a b - \rangle \langle / \rangle$

InOrder Traversal is: $a^2 - 2^* a^* b + b^2 / a - b$



Question 4 (6 marks(2+4))

1. Leff Wright wrote the following program to perform search for AirportCodes in binary search trees. Is the program correct or incorrect?

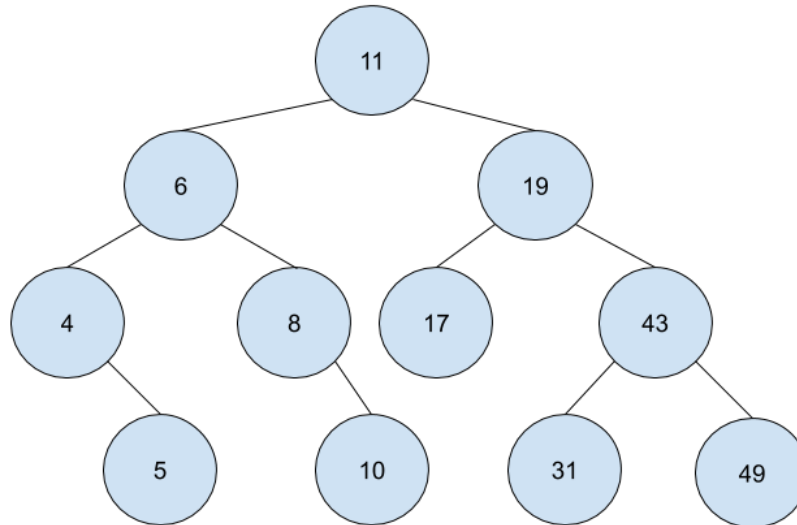
The program is correct -> it functions when implemented

2. Write a recursive version of the above program:

```
TreeNode * BinaryTreeSearch (AirportCode A, TreeNode * T){
    int result;
    if (T == NULL){
        return NULL;
    }
    else if ((result = strcmp(A,T->Airport)) == 0){
        return(T);
    } else if ((result = strcmp(A,T->Airport)) < 0){
        return (BinaryTreeSearch(A,T->LeftLink));
    } else if ((result = strcmp(A,T->Airport)) > 0) {
        return (BinaryTreeSearch(A,T->RightLink));
    } else return NULL;
}
```

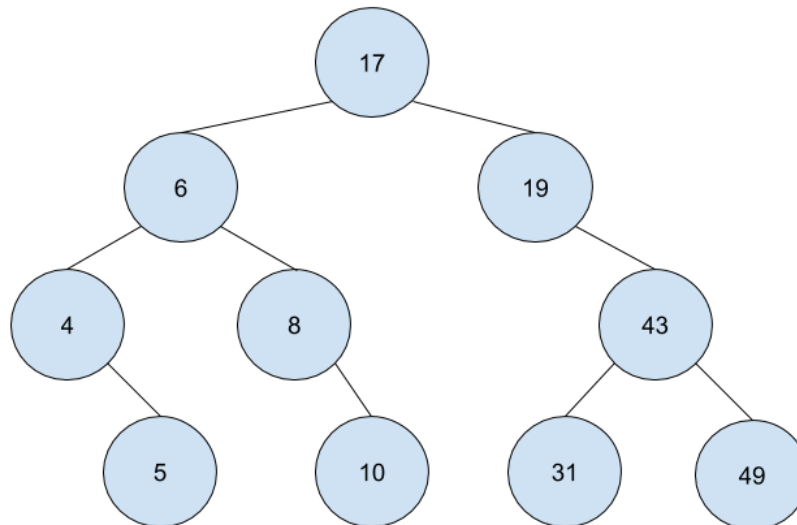
Question 5 (8 marks (4+4))

1. Given the following sequence of key values: 11, 6, 8, 19, 4, 10, 5, 17, 43, 49, 31
Construct a BST starting from left to right.



2. Remove the value 11 from the BST constructed by you above and redraw it making sure it is still a BST.

What BST looks like after 11 is removed and replaced with smallest from right subtree:



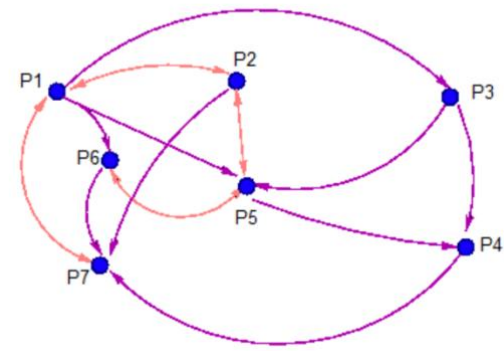
This is still in fact a BST

Question 6 (12 marks (3+3+2+2+2))

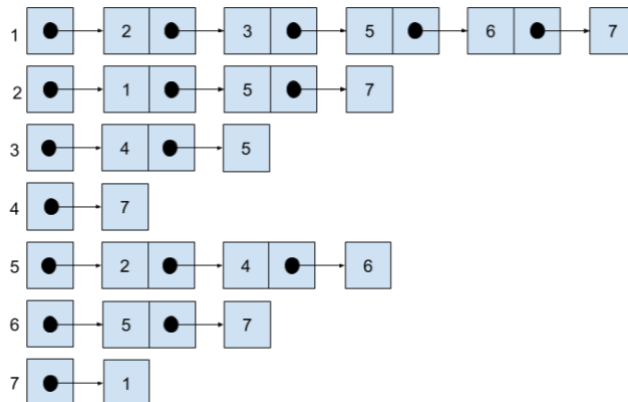
A) Given the Graph:

Aa) Give its adjacency matrix representation

NA	P1	P2	P3	P4	P5	P6	P7
P1	0	1	1	0	1	1	1
P2	1	0	0	0	1	0	1
P3	0	0	0	1	1	0	0
P4	0	0	0	0	0	0	1
P5	0	1	0	1	0	1	0
P6	0	0	0	0	1	0	1
P7	1	0	0	0	0	0	0



Ab) Give its adjacency list representation



Ac) Is it a strongly connected digraph? Justify.

The graph given is a strongly connected digraph. This is because it is possible to reach any node starting from any other node by following the indicated directions on connections

B) Given a universal set of name strings as:

Ali, Alan, Alice, Barb, Bob, Ben, Christie, Cora, Don, Dora, Fred

arranged in alphabetic order. Give a bit string representation of the following sets

Ba) Set of names that begin with C

Names: Christie, Cora

0 0 0 0 0 0 1 1 0 0 0

Bb) Set of names that begin with E

0 0 0 0 0 0 0 0 0 0 0