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Time-dependent CP Measurements : Master Equations and Conventions

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Abstract

The $B^0_{d,s} \to f$ decay rate equations are described from an experimental point of view taking into account untagged and tagged events. All conventions are clearly stated. Concrete examples are shown for the $B_s \to D_s\pi$ and $B_s \to D_sK$ decay modes.

Contents

1 Introduction

The aim of the first part of this note is to derive the decay rate equations for B^0 or B_s^0 mesons to a final state f with a view towards experimental aspects such as flavour tagging. Since the formalism is rather general, we hope that this document can serve as a reference for a wide range of analyses.

There are several conventions in the literature, and not all papers explicitly state the definitions and conventions used. Striving for consistency, we briefly derive all relevant equations from the time evolution of a two-body neutral meson system. Throughout these derivations, we assume that CPT is conserved.

In the second part of the note we apply the presented results to decays $B_s^0 \longrightarrow D_s h$ where h stands for either a K or a π, and derive expressions for the decay rates which are directly suitable for implementation in a time-dependent fit.

Throughout the document we follow the conventions used in [1].

2 Mixing Formalism and Notations

2.1 Mixing Formalism

A B_s^0 meson[1] is produced in a flavour eigenstate as $|B_s^0\rangle$ ($|\bar{B}_s^0\rangle$) at time $t = 0$. Since a B_s^0 ($\bar{B}_s^0$) meson is a superposition of mass eigenstates, and since the mass eigenstates govern the time-evolution of the system, the initially pure B_s^0 ($\bar{B}_s^0$) state will acquire an admixture of $\bar{B}_s^0$ (B_s^0) with time, *i.e.* the fact that the flavour eigenstates do not coincide with the mass eigenstates causes the flavour eigenstates to *mix*.

The relations between the B_s^0-meson mass eigenstates, $|B_{H,L}\rangle$, with masses $m_{H,L}$ and decay widths $\Gamma_{H,L}$, respectively, and their flavour eigenstates, $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$, can be expressed in terms of linear (complex) coefficients p and q:

$$\begin{array}{l} |B_L\rangle \sim p|B_s^0\rangle + q|\bar{B}_s^0\rangle \\ |B_H\rangle \sim p|B_s^0\rangle - q|\bar{B}_s^0\rangle \end{array} \qquad p, q \in \mathbb{C}, \quad |p|^2 + |q|^2 = 1 \quad . \tag{1}$$

The inverse of Eq. 1 is:

$$\begin{array}{l} |B_s^0\rangle \sim q|B_L\rangle + q|B_H\rangle \\ |\bar{B}_s^0\rangle \sim p|B_L\rangle - p|B_H\rangle \end{array} \quad . \tag{2}$$

[1] We here take the example of the B_s^0 meson with no loss of generality. All derivations are equally valid for the B^0 and D^0 meson systems.

The 2-particle system is characterised by 5 physical observables: the mass and decay rate averages, the differences in mass and decay rates, and its "composition fraction" $|q/p|$. The mass and decay rate averages are simply

$$m_s = \frac{m_H + m_L}{2} \quad , \quad \Gamma_s = \frac{\Gamma_H + \Gamma_L}{2} \quad . \tag{3}$$

The differences in mass and decay rates are subject to a sign convention, and all possible combinations are seen in the literature. We follow the convention (more on conventions in Sec. 2.3) that

$$\Delta m_s = m_H - m_L \quad , \quad \Delta\Gamma_s = \Gamma_H - \Gamma_L \quad . \tag{4}$$

While Δm_s is always positive in this definition, the sign of $\Delta\Gamma_s$ depends on which mass eigenstate has the longer lifetime; the Standard Model predicts $\Delta\Gamma_s$ to be negative with the sign convention used [2].

2.2 Time Evolution

The time evolution of a neutral meson system can be described by an effective 2×2 Hamiltonian. This Hamiltonian is not Hermitian, to allow for the decay of the mesons. Using CPT, it can be written in the flavour basis $(|B_s^0\rangle\, |\bar{B}_s^0\rangle)^T$:

$$\boldsymbol{H} = \boldsymbol{M} - \frac{i}{2}\boldsymbol{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{11} - \frac{i}{2}\Gamma_{11} \end{pmatrix} \quad , \tag{5}$$

with the matrices $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$ being hermitian:

$$\boldsymbol{M} = \boldsymbol{M}^\dagger \quad , \quad \boldsymbol{\Gamma} = \boldsymbol{\Gamma}^\dagger \quad . \tag{6}$$

In the basis of the mass eigenstates, $(|B_L\rangle\, |B_H\rangle)^T$, $\boldsymbol{H}$ is diagonal with complex eigenvalues $\omega_{L,H} = m_{L,H} - \frac{i}{2}\Gamma_{L,H}$:

$$\boldsymbol{H} = \begin{pmatrix} \omega_L & 0 \\ 0 & \omega_H \end{pmatrix} \quad , \quad \omega_{L,H} = m_{L,H} - \frac{i}{2}\Gamma_{L,H} \quad . \tag{7}$$

Equations 1 and 2 relate the two bases; with an additional first factor taking care of proper normalisation, the eigenvalue problem becomes:

$$\begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{11} - \frac{i}{2}\Gamma_{11} \end{pmatrix} = \begin{pmatrix} \frac{1}{2|q|^2} & 0 \\ 0 & \frac{1}{2|p|^2} \end{pmatrix} \cdot \begin{pmatrix} q^* & q^* \\ p^* & -p^* \end{pmatrix} \cdot \begin{pmatrix} \omega_L & 0 \\ 0 & \omega_H \end{pmatrix} \cdot \begin{pmatrix} q & p \\ q & -p \end{pmatrix} = \begin{pmatrix} \omega & -p/q\,\Delta\omega/2 \\ -q/p\,\Delta\omega/2 & \omega \end{pmatrix} \tag{8}$$

where $\omega = \frac{1}{2}(\omega_H + \omega_L)$ and $\Delta\omega = \omega_H - \omega_L$. This yields a relation between p, q and the off-diagonal elements of the Hamiltonian:

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \quad . \tag{9}$$

Exploiting the fact that the determinant of the effective Hamiltonian is independent of whether it is calculated with mass or flavour eigenstates leads to

$$\omega_H - \omega_L = 2\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \quad . \tag{10}$$

Comparing Eqs. 8 to 10 allows to resolve the sign ambiguity when taking the square root of Eq. 9:

$$\frac{q}{p} = -\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \quad . \tag{11}$$

The time evolution of the states $|B_s^0(t)\rangle$ and $|\bar{B}_s^0(t)\rangle$ can be expressed in terms of initially pure states $|B_s^0(0)\rangle$ and $|\bar{B}_s^0(0)\rangle$. One possible way to write the time evolution is[2]

$$\begin{aligned} |B_s^0(t)\rangle &= g_+(t)|B_s^0(0)\rangle - \frac{q}{p}g_-(t)|\bar{B}_s^0(0)\rangle \quad , \\ |\bar{B}_s^0(t)\rangle &= g_+(t)|\bar{B}_s^0(0)\rangle - \frac{p}{q}g_-(t)|B_s^0(0)\rangle \quad , \end{aligned} \tag{12}$$

with

[2] Other choices exist, but all these choices must lead to the decay equations presented in Sec. 3.

$$
\begin{aligned}
g_{\pm}(t) &= \frac{1}{2}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t} \pm e^{-im_L t-\frac{1}{2}\Gamma_L t}\right) \\
&= \frac{1}{2}\left(e^{-i\omega_H t} \pm e^{-i\omega_L t}\right) \quad .
\end{aligned} \tag{13}
$$

As such,

$$
\begin{aligned}
|g_{\pm}(t)|^2 &= \frac{1}{4}\left[e^{-\Gamma_H t} + e^{-\Gamma_L t} \pm 2\,\mathcal{R}e\left(e^{-\frac{1}{2}(\Gamma_H+\Gamma_L)t-i(m_H-m_L)}\right)\right] \\
&= \frac{1}{2}e^{-\Gamma_s t}\left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) \pm \cos\left(\Delta m_s t\right)\right]
\end{aligned} \tag{14}
$$

and

$$
\begin{aligned}
g_{+}^{*}(t)g_{-}(t) &= \frac{1}{4}\left[e^{-\Gamma_H t} - e^{-\Gamma_L t} - 2i\,\mathcal{I}m\left(e^{-\frac{1}{2}(\Gamma_H+\Gamma_L)t-i(m_H-m_L)}\right)\right] \\
&= -\frac{1}{2}e^{-\Gamma_s t}\left[\sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + i\sin\left(\Delta m_s t\right)\right] \quad .
\end{aligned} \tag{15}
$$

2.3 Notations and Conventions

Throughout the document, we follow the conventions used in the review on "CP violation in meson decays" [1][3]. The mass and decay width splittings between the heavy $|B_H\rangle$ and light $|B_L\rangle$ states are chosen such that $\Delta m_s = m_H - m_L$ is positive by definition but $\Delta\Gamma_s = \Gamma_H - \Gamma_L$ can be either positive or negative, depending on whether the heavy or the light mass eigenstate lives longer. When neccessary, we will refer to this convention as the "review" convention. For the B_s system, the Standard Model value for $\Delta\Gamma_s$ is predicted to be negative (see *e.g.* [2]).

Since most of our collaborators seem to prefer the convention in which both $\Delta m = m_H - m_L$ and $\Delta\Gamma_s = \Gamma_L - \Gamma_h$ are positive, all decay rate equations given in this document are given again in this other convention in the appendix A which we will also refer to as "table" convention (since the PDG tables with B meson properties use it).

[3]Nota bene: the conventions in this review are different from those used in the PDG tables with B meson properties; for the tables with B meson properties, their choice for the sign of $\Delta\Gamma_s$ is opposite.

3 Decay Rate Equations

The decay rate of a $|B_s^0\rangle$ meson produced at time $t = 0$ to a final state f at time t is given by

$$\frac{d\Gamma_{B_s^0\to f}(t)}{dt} = |\langle f|T|B_s^0(t)\rangle|^2 \quad , \tag{16}$$

where T is the transition matrix element. Similar expressions exist for the CP-conjugates $\bar{B}_s^0$ and $\bar{f}$.

The time-dependent decay rates of the initially produced flavour eigenstates $|B_s^0(t=0)\rangle$ and $|\bar{B}_s^0(t=0)\rangle$ are then given by the four decay equations

$$\frac{d\Gamma_{B_s^0\to f}(t)}{dt\, e^{-\Gamma_s t}} = \tfrac{1}{2}|A_f|^2(1+|\lambda_f|^2) \quad \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t)\right] \tag{17}$$

$$\frac{d\Gamma_{\bar{B}_s^0\to f}(t)}{dt\, e^{-\Gamma_s t}} = \tfrac{1}{2}|A_f|^2\left|\tfrac{p}{q}\right|^2(1+|\lambda_f|^2) \quad \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f \cos(\Delta m_s t) + S_f \sin(\Delta m_s t)\right] \tag{18}$$

$$\frac{d\Gamma_{\bar{B}_s^0\to \bar{f}}(t)}{dt\, e^{-\Gamma_s t}} = \tfrac{1}{2}|\bar{A}_{\bar{f}}|^2(1+|\bar{\lambda}_{\bar{f}}|^2) \quad \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_{\bar{f}} \cos(\Delta m_s t) - S_{\bar{f}} \sin(\Delta m_s t)\right] \tag{19}$$

$$\frac{d\Gamma_{B_s^0\to \bar{f}}(t)}{dt\, e^{-\Gamma_s t}} = \tfrac{1}{2}|\bar{A}_{\bar{f}}|^2\left|\tfrac{q}{p}\right|^2(1+|\bar{\lambda}_{\bar{f}}|^2) \quad \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_{\bar{f}} \cos(\Delta m_s t) + S_{\bar{f}} \sin(\Delta m_s t)\right] \tag{20}$$

where $A_f = \langle f|T|B_s^0\rangle$ and $\bar{A}_{\bar{f}} = \langle \bar{f}|T|\bar{B}_s^0\rangle$ are the decay amplitudes for a $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$ to decay to a final state $|f\rangle$ and $|\bar{f}\rangle$, respectively, and λ_f and $\bar{\lambda}_{\bar{f}}$ are defined as

$$\lambda_f \equiv \frac{1}{\bar{\lambda}_f} = \frac{q}{p}\frac{\bar{A}_f}{A_f} \quad , \quad \bar{\lambda}_{\bar{f}} \equiv \frac{1}{\lambda_{\bar{f}}} = \frac{p}{q}\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} \quad . \tag{21}$$

Similarly, $\bar{A}_f = \langle f|T|\bar{B}_s^0\rangle$ and $A_{\bar{f}} = \langle \bar{f}|T|B_s^0\rangle$. As a consequence of CPT invariance the following identities hold:

$$|\bar{A}_{\bar{f}}| = |A_f| \quad , \quad |\bar{A}_f| = |A_{\bar{f}}| \quad . \tag{22}$$

The CP asymmetry observables $C_f, S_f, D_f, C_{\bar{f}}, S_{\bar{f}}$ and $D_{\bar{f}}$ are given by

$$\begin{aligned} C_f &= \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \quad , \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2} \quad , \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1+|\lambda_f|^2} \quad , \\ C_{\bar{f}} &= \frac{1-|\bar{\lambda}_{\bar{f}}|^2}{1+|\bar{\lambda}_{\bar{f}}|^2} \quad , \quad S_{\bar{f}} = \frac{2\mathcal{I}m(\bar{\lambda}_{\bar{f}})}{1+|\bar{\lambda}_{\bar{f}}|^2} \quad , \quad D_{\bar{f}} = \frac{2\mathcal{R}e(\bar{\lambda}_{\bar{f}})}{1+|\bar{\lambda}_{\bar{f}}|^2} \quad . \end{aligned} \tag{23}$$

Alternatively, one can write the λ variables as a function of the CP asymmetry observables:

$$\begin{aligned} \lambda_f &= \frac{D_f + iS_f}{1+C_f} \quad , \quad |\lambda_f|^2 = \frac{1-C_f}{1+C_f} \quad , \\ \bar{\lambda}_{\bar{f}} &= \frac{D_{\bar{f}} + iS_{\bar{f}}}{1+C_{\bar{f}}} \quad , \quad |\bar{\lambda}_{\bar{f}}|^2 = \frac{1-C_{\bar{f}}}{1+C_{\bar{f}}} \quad . \end{aligned} \tag{24}$$

The observables C, S and D are also called $C \equiv A_{\mathrm{dir}}$, $S \equiv A_{\mathrm{mix}}$ and $D \equiv A_{\Delta\Gamma}$.

There are three classes of CP violating phenomena: CP violation in decay is characterised by $|\bar{A}_{\bar{f}}/A_f| \neq 1$, CP violation in mixing by $|q/p| \neq 1$ and CP violation in the interference between mixing and decay by $\mathcal{I}m(\lambda_f) \neq 0$.

Alternatively, one may classify CP violation as being either direct (where the CP violation has more sources than the phase of the element M_{12} in the effective Hamiltonian; CP violation in decay is an example) or indirect (where the observed CP violating effect is entirely due to the phase of M_{12} in the effective Hamiltonian; CP violation in mixing is an example).

In addition to m_s, Γ_s, Δm_s and $\Delta\Gamma_s$, there are 4 phase-convention independent observables, consisting of three moduli and one phase (difference[4]): $|q/p|$, $|A_f/\bar{A}_{\bar{f}}|$, $|\lambda_f|$, and $\arg(\lambda_f)$.

The decay equations presented in Eq. 20 exhibit an explicit CP structure, *i.e.* they can be written in terms of a CP-even part which does not change sign when CP is applied to either the initial state or the final state, and a CP-odd part which does change sign under a CP operation applied to either the initial or the final state. Neglecting the prefactors, one finds

$$\frac{d\Gamma_{q_i,q_f}(t)}{dt\, e^{-\Gamma_s t}} \sim \left[E_{q_f}(t) + q_i q_f O_{q_f}(t)\right] \quad , \tag{25}$$

[4] $\arg(\lambda_f)$ is the sum of the phase difference between the amplitudes $\bar{A}_f$ and A_f and the phase difference between q and p.

where the initial flavour is denoted by $q_i = \pm 1$ for B_s^0, $\bar{B}_s^0$, the final state by $q_f = \pm 1$ for f, $\bar{f}$, and the E and O terms are given by

$$\begin{aligned} E_{q_f}(t) &= \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{q_f}\sinh\left(\frac{\Delta\Gamma_s t}{2}\right) \quad , \\ O_{q_f}(t) &= C_{q_f}\cos\left(\Delta m_s t\right) - S_{q_f}\sin\left(\Delta m_s t\right) \quad . \end{aligned} \tag{26}$$

The coefficients D_{q_f}, C_{q_f} and S_{q_f} represent D_f, C_f and S_f for $q_f = +1$ and $D_{\bar{f}}$, $C_{\bar{f}}$ and $S_{\bar{f}}$ for $q_f = -1$, respectively.

Splitting the decay rate equations into CP even and CP odd parts has its advantages when dealing with various asymmetries such as a production asymmetry; for details, see *e.g.* [3], p. 136ff.

4 Master Equations for $B_s \to D_s h$

We hereafter apply the equations derived in the previous sections to the decay modes $B_s^0 \to D_s^- \pi^+$ and $B_s^0 \to D_s^\mp K^\pm$ taking into account experimental aspects such as tagging. The aim is to construct expressions for the decay rates which are suitable for implementation in a likelihood fit. To this end, we consider the final state (labelled q_f) and the decision of the flavour tagging algorithms (labelled q_t) to be observables, and derive expressions which give the observed decay rates as a function of the lifetime t, the final state q_f and the tagging decision q_t. Most of the time, we neglect the experimental complication that the resolution of the measurement of the lifetime of a decaying meson is finite.

4.1 Tagging Information

Tagging algorithms determine whether the signal candidate contained a b or a $\bar{b}$ quark at time of production. Each tagged B candidate is thus assigned a value $q_t = +1, -1$ or 0. The value 1 means the B candidate at time of production was a B^0 (or B_s^0) whereas a value of -1 is assigned to candidates which were a $\bar{B}^0$ (or $\bar{B}_s^0$) at time of production. $q_t = 0$ means no tagging information is available for this candidate. In short,

$$q_t = \begin{cases} 0 & \text{untagged candidate} \\ +1 & \text{candidate tagged as } B_{(s)}^0 \text{ at production} \\ -1 & \text{candidate tagged as } \bar{B}_{(s)}^0 \text{ at production} \end{cases} \quad . \tag{27}$$

The experimental effect of tagging the flavour of the initial B meson – when the tagging decision is different from zero – is described by the tagging efficiency ϵ_{tag} (the fraction of candidates to which a tag can be assigned) and the (predicted) mistag rate ω_{tag}. The effective statistical power of a tagged sample (often referred to as *tagging power*) is given by

$$Q_{\mathrm{tag}} = \epsilon_{\mathrm{tag}} \cdot (1 - 2\omega_{\mathrm{tag}})^2 \quad . \tag{28}$$

This factor gives the reduction in effective size of the data sample when compared to perfectly tagged events ($\epsilon_{\mathrm{tag}} = 1$, $\omega_{\mathrm{tag}} = 0$), *i.e.* a sample of N events with a specific Q_{tag} has the same statistical power of a sample of $N\,Q_{\mathrm{tag}}$ events with perfect tagging.

In the sections below it will be handy to express the relative yields of untagged and tagged samples in a combined formula. The effective tagging efficiency is

$$\begin{aligned} \epsilon_{\mathrm{tag}}^{\mathrm{eff}}(q) &= \begin{cases} \epsilon_{\mathrm{tag}} & \text{for tagged events } (q = \pm 1) \\ (1 - \epsilon_{\mathrm{tag}}) & \text{for untagged events } (q = 0) \end{cases} \\ &= |q|\epsilon_{\mathrm{tag}} + (1 - |q|)(1 - \epsilon_{\mathrm{tag}}) \qquad \forall\, q \quad . \end{aligned} \tag{29}$$

4.2 Equations for $B_s^0 \to D_s^- \pi^+$

For the $B_s^0 \to D_s^- \pi^+$ channel only one tree diagram (see Fig. 1) exists, such that a B_s^0 can only decay instantaneously into the $D_s^- \pi^+$ final state, while the decay into $D_s^+ \pi^-$ can only occur after mixing. The decay $B_s^0 \to D_s^- \pi^+$ is therefore flavour specific, leading to $A_{\bar{f}} = \bar{A}_f = 0$, thus $\lambda_f = \bar{\lambda}_{\bar{f}} = 0$. To good approximation, we assume $|q/p| = 1$, *i.e.* there is no CP violation in mixing in the B_s^0 system. This in turn leads to the parameters $D_f = S_f = D_{\bar{f}} = S_{\bar{f}} = 0$ and $C_f = C_{\bar{f}} = 1$.

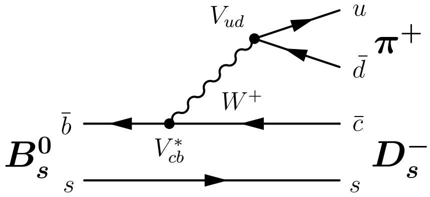

Figure 1: Feynman diagram for $B_s^0 \to D_s^- \pi^+$.

The parameter Δm_s can be obtained from the $B_s^0 \to D_s^- \pi^+$ channel via the flavour asymmetry, defined as

$$A^{\mathrm{flav}}(t) = \frac{\frac{d\Gamma_{\bar{B}_s^0 \to f}(t)}{dt} - \frac{d\Gamma_{B_s^0 \to f}(t)}{dt}}{\frac{d\Gamma_{\bar{B}_s^0 \to f}(t)}{dt} + \frac{d\Gamma_{B_s^0 \to f}(t)}{dt}} = -D \cdot \frac{\cos(\Delta m_s\, t)}{\cosh(\frac{\Delta\Gamma_s t}{2})} \quad . \tag{30}$$

D is a dilution factor with contributions from wrong B_s^0 flavour tagging (for infinitely good resolution in the determination of the lifetime, $D = 1 - 2\omega_{\mathrm{tag}}$, where ω_{tag} is the wrong tag fraction) and experimental decay time resolutions. Using A^{flav}, one can obtain Δm_s, D and optionally $\Delta\Gamma_s$ from the decay rates $d\Gamma_{B_s^0 \to f}/dt$ and $d\Gamma_{\bar{B}_s^0 \to f}/dt$, and similarly from $d\Gamma_{B_s^0 \to \bar{f}}/dt$ and $d\Gamma_{\bar{B}_s^0 \to \bar{f}}/dt$. If the experimental proper time resolution is known, one can disentangle the contributions of the experimental lifetime distribution and the wrong tag fraction ω_{tag}, and hence use D to extract ω_{tag}.

Labelling the final state with q_f (with $q_f = +1$ for the $D_s^- \pi^+$ final state and $q_f = -1$ for the $D_s^+ \pi^-$ final state), the two decay rate equations can be summarised by

$$\begin{aligned} \frac{d\Gamma_{B_s \to D_s \pi}(t, q_f, q_t)}{dt\, e^{-\Gamma_s t}} &= (2 - |q_t|) \cdot \epsilon_{\mathrm{tag}}^{\mathrm{eff}}(q_t) \cdot \cosh\left(\frac{\Delta\Gamma_s\, t}{2}\right) \\ &\quad + q_f q_t \cdot (1 - 2\omega_{\mathrm{tag}}) \cdot \epsilon_{\mathrm{tag}}^{\mathrm{eff}}(q_t) \cdot \cos(\Delta m_s\, t) \quad . \end{aligned} \tag{31}$$

Since the final state identifies the B_s^0 flavour at decay time, the product $q_f q_t$ in the formula above is sometimes replaced by the single factor $q_m = q_f q_t$ which takes the value of $q_m = +1$ for B_s^0 ($\bar{B}_s^0$) mesons which have the same flavour at production and decay time ("unmixed") and the value $q_m = -1$ for those mesons which have different flavours at production and decay time ("mixed"). Untagged candidates are assigned the value $q_m = 0$. The formula then reads:

$$\begin{aligned} \frac{d\Gamma_{B_s \to D_s \pi}(t, q_m)}{dt\, e^{-\Gamma_s t}} &= (2 - |q_m|) \cdot \epsilon_{\mathrm{tag}}^{\mathrm{eff}}(q_m) \cdot \cosh\left(\frac{\Delta\Gamma_s\, t}{2}\right) \\ &\quad + q_m \cdot (1 - 2\omega_{\mathrm{tag}}) \cdot \epsilon_{\mathrm{tag}}^{\mathrm{eff}}(q_m) \cdot \cos(\Delta m_s\, t) \quad . \end{aligned} \tag{32}$$

4.3 Equations for $B_s^0 \to D_s^{\mp} K^{\pm}$

As kaons contain an s quark, two tree diagrams for instantaneous decay exist for the $B_s^0 \to D_s^{\mp} K^{\pm}$ channels (see Fig. 2). Both B_s^0 and $\bar{B}_s^0$ mesons can decay directly (without oscillations) to either $D_s^- K^+$ or $D_s^+ K^-$, therefore this decay channel is not flavour specific and interference occurs between the two contributing amplitudes.

Due to the different coupling constants the two tree diagrams T_1 and T_2 have different magnitudes. The process $B_s^0 \to D_s^+ K^-$ (tree diagram T_2) is suppressed due to flavour change from the third to first quark generation described by V_{ub}, the numerical value of the suppression factor is $|(\rho - i\eta)| \sim 0.36$ (where ρ and η are the parameters from the Wolfenstein parametrisation of the CKM matrix). As can be seen, the suppression is relatively mild, since both amplitudes are of order $A\lambda_{CKM}^3$ in the Wolfenstein parameters A and λ_{CKM} (with $\lambda_{CKM} \sim 0.23$ and $A \sim 0.81$). Thus, one expects a relatively large contribution from the interference between the two amplitudes (compared to decays which are suppressed by one or more powers of λ_{CKM}).

Because of the conservation of CPT, we have $|A_f| = |\bar{A}_{\bar{f}}|$ and $|A_{\bar{f}}| = |\bar{A}_f|$. Using the assumption $|q/p| = 1$ gives $|\lambda_f| = |\bar{\lambda}_{\bar{f}}|$, see Eq. 21. This also implies that $C_f = C_{\bar{f}}$.

The terms λ_f and $\bar{\lambda}_{\bar{f}}$ are then given by

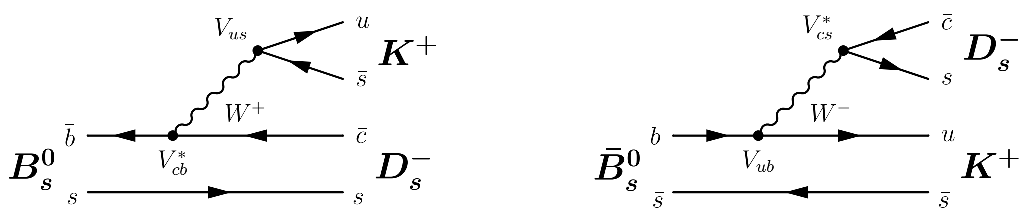

Figure 2: Feynman tree diagrams for $B_s^0 \to D_s^- K^+$ (diagram $T1$, left) and $\bar{B}_s^0 \to D_s^- K^+$ (diagram $T2$, right). The sub-process $T1$ has a larger magnitude than $T2$ due to the different coupling constants.

$$\begin{aligned}
\lambda_{D_s^- K^+} &= \left(\frac{q}{p}\right)\frac{\bar{A}_{D_s^- K^+}}{A_{D_s^- K^+}}\left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right)\left(\frac{V_{ub} V_{cs}^*}{V_{cb}^* V_{us}}\right)\left|\frac{A_2}{A_1}\right| e^{i\Delta_{T1/T2}} \\
&= |\lambda_{D_s^- K^+}| e^{i(\Delta_{T1/T2} - (\gamma+\phi_s))} \quad , \\
\bar{\lambda}_{D_s^+ K^-} &= \left(\frac{p}{q}\right)\frac{A_{D_s^+ K^-}}{\bar{A}_{D_s^+ K^-}} = \left(\frac{V_{tb} V_{ts}^*}{V_{tb}^* V_{ts}}\right)\left(\frac{V_{ub}^* V_{cs}}{V_{cb} V_{us}^*}\right)\left|\frac{A_2}{A_1}\right| e^{i\Delta_{T1/T2}} \\
&= |\lambda_{D_s^- K^+}| e^{i(\Delta_{T1/T2} + (\gamma+\phi_s))} \quad .
\end{aligned} \tag{33}$$

where $|A_2/A_1|$ is the ratio of the hadronic amplitudes between diagrams $T1$ and $T2$, $\Delta_{T1/T2}$ is the strong phase difference between $T1$ and $T2$, and $\gamma + \phi_s$ is the weak phase.

The strong and the weak phases can be extracted using the relations:

$$\gamma + \phi_s = \frac{1}{2}[\arg(\bar{\lambda}_{\bar{f}}) - \arg(\lambda_f)] \quad , \quad \Delta_{T1/T2} = \frac{1}{2}[\arg(\bar{\lambda}_{\bar{f}}) + \arg(\lambda_f)] \quad . \tag{34}$$

The four possible combinations can be differentiated by means of the flavour of the initial B_s-meson and the charge of the bachelor: $q_t = +1, -1, 0$ for a $B_s^0, \bar{B}_s^0$, untagged , respectively, and $q_f = +1, -1$ for a K^+, K^-, respectively.

With the definition of effective D and S observables

$$D^{\text{eff}}(q_f) = \frac{1}{2}\big[(1+q_f)D_f + (1-q_f)D_{\bar{f}}\big] = \begin{cases} D_f & \text{if } q_f = +1 \\ D_{\bar{f}} & \text{if } q_f = -1 \end{cases}$$

and

$$S^{\text{eff}}(q_f) = \frac{1}{2}\big[(1+q_f)S_f + (1-q_f)S_{\bar{f}}\big] = \begin{cases} S_f & \text{if } q_f = +1 \\ S_{\bar{f}} & \text{if } q_f = -1 \end{cases}$$

the four decay rate equations can be summarised by

$$\begin{aligned}
\frac{d\Gamma_{B_s \to D_s K}(t, q_t, q_f)}{dt\, e^{-\Gamma_s t}} &= (2 - |q_t|) \cdot \epsilon_{\text{tag}}^{\text{eff}}(q_t) \cdot \cosh\left(\frac{\Delta\Gamma_s\, t}{2}\right) \\
&\quad + (2 - |q_t|) \cdot \epsilon_{\text{tag}}^{\text{eff}}(q_t) \cdot D^{\text{eff}}(q_f) \cdot \sinh\left(\frac{\Delta\Gamma_s\, t}{2}\right) \\
&\quad + q_t \cdot q_f \cdot (1 - 2\omega_{\text{tag}}) \cdot \epsilon_{\text{tag}}^{\text{eff}}(q_t) \cdot C_f \cdot \cos(\Delta m_s\, t) \\
&\quad - q_t \cdot q_f \cdot (1 - 2\omega_{\text{tag}}) \cdot \epsilon_{\text{tag}}^{\text{eff}}(q_t) \cdot S^{\text{eff}}(q_f) \cdot \sin(\Delta m_s\, t) \quad .
\end{aligned}$$

They depend on the 5 asymmetry observables C_f, S_f, $S_{\bar{f}}$, D_f and $D_{\bar{f}}$.

5 Tagged Samples and Combinatorial Background

In a tagged analysis both the tagging efficiency and the asymmetry of the combinatorial background need to be considered; they are independent of the behaviour observed with signal B mesons.

Denoting $\epsilon_{\mathrm{tag}}^{\mathrm{CombBkg}}$ the tagging efficiency on the combinatorial background, $\omega_{\mathrm{tag}}^{\mathrm{CombBkg}}$ its mistag rate and q_t the tagging decision, the time PDF for the combinatorial background can be expressed as

$$\frac{d\Gamma_{\mathrm{CombBkg}}(t,q_t)}{dt} = \frac{d\Gamma_{\mathrm{CombBkg}}(t)}{dt} \cdot PDF_{\mathrm{CombBkg}}^{\mathrm{tag}}(q_t) \quad , \tag{35}$$

with

$$PDF_{\mathrm{CombBkg}}^{\mathrm{tag}}(q_t) = \begin{cases} (1-\epsilon_{\mathrm{tag}}^{\mathrm{CombBkg}}) & \text{if } q_t = 0 \\ \omega_{\mathrm{tag}}^{\mathrm{CombBkg}} \cdot \epsilon_{\mathrm{tag}}^{\mathrm{CombBkg}} & \text{if } q_t = +1 \\ (1-\omega_{\mathrm{tag}}^{\mathrm{CombBkg}}) \cdot \epsilon_{\mathrm{tag}}^{\mathrm{CombBkg}} & \text{if } q_t = -1 \end{cases} \quad . \tag{36}$$

These last equations can be grouped:

$$\begin{aligned} PDF_{\mathrm{CombBkg}}^{\mathrm{tag}}(q_t) = {} & (2-|q_t|) \cdot \epsilon_{\mathrm{tag}}^{\mathrm{CombBkg,eff}}(q_t) \\ & \cdot \frac{1}{2}\left[(1+q_t)\omega_{\mathrm{tag}}^{\mathrm{CombBkg}} + (1-q_t)(1-\omega_{\mathrm{tag}}^{\mathrm{CombBkg}})\right] \quad . \end{aligned} \tag{37}$$

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A Decay rate equations with $\Delta m = m_H - m_L$ and $\Delta\Gamma_s = \Gamma_L - \Gamma_H$

The aim of this appendix is to quote decay rate equations given in the rest of this document in the convention which has $\Delta m = m_H - m_L$ and $\Delta\Gamma_s = \Gamma_L - \Gamma_H$ ("table" convention).

A.1 Decay Rate Equations

The decay rate equations 20 in the "table" convention read:

$$\frac{d\Gamma_{B_s^0\to f}(t)}{dt\, e^{-\Gamma_s t}} = \frac{1}{2}|A_f|^2(1+|\lambda_f|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t)\right] \tag{38}$$

$$\frac{d\Gamma_{\bar{B}_s^0\to f}(t)}{dt\, e^{-\Gamma_s t}} = \frac{1}{2}|A_f|^2\left|\frac{p}{q}\right|^2(1+|\lambda_f|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f \cos(\Delta m_s t) + S_f \sin(\Delta m_s t)\right] \tag{39}$$

$$\frac{d\Gamma_{\bar{B}_s^0\to \bar{f}}(t)}{dt\, e^{-\Gamma_s t}} = \frac{1}{2}|\bar{A}_{\bar{f}}|^2(1+|\bar{\lambda}_{\bar{f}}|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_{\bar{f}} \cos(\Delta m_s t) - S_{\bar{f}} \sin(\Delta m_s t)\right] \tag{40}$$

$$\frac{d\Gamma_{B_s^0\to \bar{f}}(t)}{dt\, e^{-\Gamma_s t}} = \frac{1}{2}|\bar{A}_{\bar{f}}|^2\left|\frac{q}{p}\right|^2(1+|\bar{\lambda}_{\bar{f}}|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_{\bar{f}} \cos(\Delta m_s t) + S_{\bar{f}} \sin(\Delta m_s t)\right] \tag{41}$$

These equations can be split in even and odd terms just as in equation 25; the even and odd terms now read:

$$\begin{aligned} E_{q_f}(t) &= \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - D_{q_f}\sinh\left(\frac{\Delta\Gamma_s t}{2}\right) \quad , \\ O_{q_f}(t) &= C_{q_f}\cos(\Delta m_s t) - S_{q_f}\sin(\Delta m_s t) \quad . \end{aligned} \tag{42}$$

A.2 Decay rate equations for $B_s^0 \to D_s^- \pi^+$

The decay rate equations for $B_s^0 \to D_s^- \pi^+$ are the same in “review” and “table” convention, since they contain no terms which are odd in $\Delta\Gamma_s$. (This also implies that this decay mode is not sensitive to the sign of $\Delta\Gamma_s$.)

A.3 Decay rate equations for $B_s^0 \to D_s^\mp K^\pm$

The decay rate equations for $B_s^0 \to D_s^\mp K^\pm$ (eq. 35) in the “table” convention become:

$$\begin{aligned}
\frac{d\Gamma_{B_s \to D_s K}(t, q_t, q_f)}{dt\, e^{-\Gamma_s t}} \quad = \quad & (2 - |q_t|) \cdot \epsilon_{\text{tag}}^{\text{eff}}(q_t) \cdot \cosh\left(\frac{\Delta\Gamma_s\, t}{2}\right) \\
& - (2 - |q_t|) \cdot \epsilon_{\text{tag}}^{\text{eff}}(q_t) \cdot D^{\text{eff}}(q_f) \cdot \sinh\left(\frac{\Delta\Gamma_s\, t}{2}\right) \\
& + q_t \cdot q_f \cdot (1 - 2\omega_{\text{tag}}) \cdot \epsilon_{\text{tag}}^{\text{eff}}(q_t) \cdot C_f \cdot \cos(\Delta m_s\, t) \\
& - q_t \cdot q_f \cdot (1 - 2\omega_{\text{tag}}) \cdot \epsilon_{\text{tag}}^{\text{eff}}(q_t) \cdot S^{\text{eff}}(q_f) \cdot \sin(\Delta m_s\, t) \quad .
\end{aligned}$$

References

[1] K. Nakamura *et al.*(Particle Data Group), J. Phys. G 37, 075021 (2010), section 12, "CP violation in meson decays", `http://pdg.lbl.gov/2011/reviews/rpp2011-rev-cp-violation.pdf`

[2] I. Dunietz, R. Fleischer and U. Nierste, "In pursuit of new physics with B_s decays," Phys. Rev. D **63** (2001) 114015, [hep-ph/0012219], `http://arxiv.org/abs/hep-ph/0012219`

[3] R. Aaij, Y. Amhis, A. Bien, T. Brambach, M. Calvi *et al.*, *Tagged time-dependent angular analysis of $B_s \to J/\psi\phi$ decays with 337 pb^{-1} at LHCb*, LHCb-ANA-2011-036, May 2011, `https://cdsweb.cern.ch/record/1349958?ln=en`