

# Chebyshev Functional Link Spline Neural Filter for Nonlinear Dynamic System Identification

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## Abstract

A new approach is suggested to enhance the performance of functional link neural networks (FLNN) for system identification. This approach, called Chebyshev functional link spline neural filter (CFLSNF), employs a spline activation function with flexible interpolation ability to improve the non-linear approximation ability of FLNN. The input signal is extended to higher dimension using chebyshev expansion polynomial which have more computational advantages over other algorithms. To update the weights of activation function an adaptive algorithm is implemented and the Mean Square error with respect to the iterations is studied. The effectiveness of the proposed architecture and algorithm is demonstrated through experiments.

**Keywords:** FLNN, CFLSNF, Chebyshev polynomial, spline activation function, nonlinear system identification, MSE vs Iterations.

## 1 Introduction

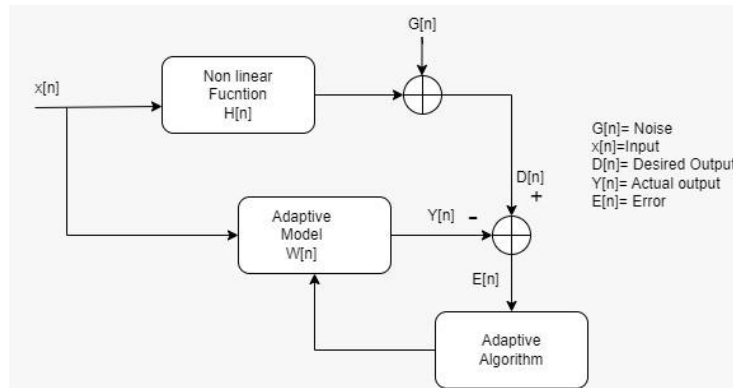
The challenge of identifying complex dynamic objects accurately in control theory arises due to the lack of prior information about the unknown plant. Artificial neural networks (ANNs) have proven to be a powerful learning and simulation tool in approximating complex dynamic nonlinear relationships, especially when the prior information is insufficient. Multilayer perceptron (MLP) and recurrent neural network (RNN) are representative ANNs that have been successfully verified in communication, biomedical, and other fields. However, the computational complexity of these ANNs increases with an increase in iteration times during the training process, which is a common shortcoming. FLNN, a novel single-layer structure, has been developed to address the issue of computational complexity. The primary benefit of FLNN is its ability to reduce the computational burden by expanding the input signal into a higher dimensional space through functional extension. This is accomplished using the Chebyshev expansion function to extend the input signal into a higher dimensional space.

## 2 Literature Survey

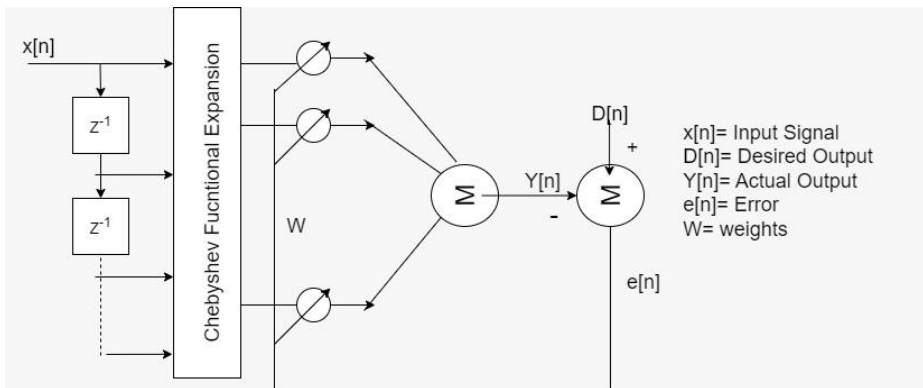
In recent years, there have been significant developments in FLNN structures and algorithms based on various topological structures and combinations. These structures and algorithms have found practical applications in active noise control, nonlinear echo cancellation, and other domains. Typically, FLNNs use the fixed activation function  $\tanh$ , which has limited nonlinear approximation ability due to its almost linear behavior in certain ranges. As a result, researchers have sought to identify activation functions with higher nonlinear approximation capabilities. One promising solution, as proposed in the paper "Chebyshev functional link spline neural filter for non-linear system identification," involves using Chebyshev expansion functions to extend the input signal into higher dimensions, thereby improving the FLNN's nonlinear approximation capabilities. Compared to FLNNs with fixed activation functions, the proposed CFLSNF utilizes an adaptive third-order spline interpolation function to flexibly approximate a wider range of nonlinear curves.

## 3 Proposed Technique:

### 3.1 Block Diagram:



According to Fig1, the input data is sent to both a nonlinear system model and an adaptive model, which calculates the error between the desired and actual output. Adaptive signal processing techniques are used to update the model parameters based on this error. The updated model is used to represent the nonlinear system, and can be used for further analysis or control.



### 3.2 Chebyshev Expansion:

The input signal is expanded to higher dimensions using Chebyshev expansion as shown in Fig2.

The input is extended according to the following expressions:-

$$\begin{aligned}
 T_0(x) &= 1, T_1(x) = x \\
 T_{p+1}(x) &= 2xT_p(x) - T_{p-1}(x) \\
 \phi(k) &= [\phi_1(k), \phi_2(k), \dots, \phi_{N_1}(k)]^T \\
 &= [1, T_1(x(k)), T_1(x(k-1)), \dots, T_1(x(k-m+1)), \\
 &\quad T_1(x(k))T_1(x(k-1)), T_1(x(k))T_1(x(k-2)), \dots, \\
 &\quad T_1(x(k-m))T_1(x(k-m+1)), T_2(x(k)), T_2(x(k-1)), \\
 &\quad \dots, T_2(x(k-m+1))]^T
 \end{aligned}$$

### 3.3 Error function and Weights updation:

Error function is the difference between the desired output and actual output

$$e(i) = D(i) - Y(i);$$

For updation of Weights, we use a method called Stochastic Gradient Descent Method

$$w(k+1) = w(k) + \Delta w(k) = w(k) - \mu_w \frac{\partial J(k)}{\partial w(k)}$$

$$W = W + \text{Lamda\_b} * (Y(i) * Q\_c1 * e(i) / (Y(i) * Q\_c1 * Y(i)' + \text{delta})) - \text{Tau} * \text{sign}(W);$$

Where Lamda\_b = learning rate matrix

## 4 Results and Discussions:

Our CFLSNF model's stability and fast convergence were observed across various noise types (White Gaussian Noise in fig3, Salt and Pepper noise in fig4, speckle noise in fig5) by analyzing the graph of Mean Square Error (MSE) and Iterations. Thus, we conclude that our proposed CFLSNF model is effective in handling all types of noise.

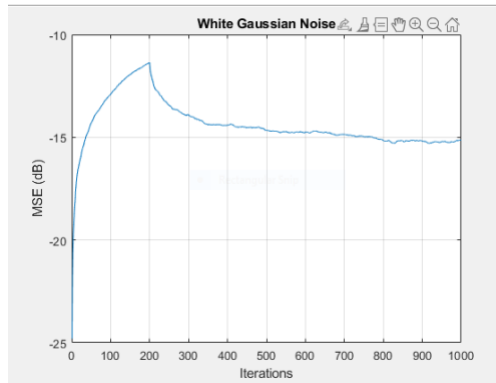


Fig 3

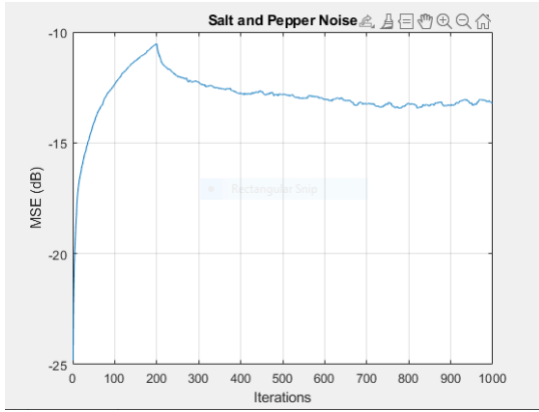


Fig 4

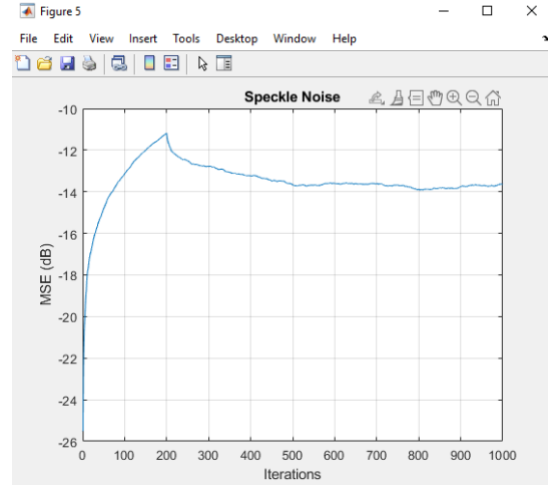


Fig 5

A measure used to evaluate the performance of identifier is the Normalized Mean Square Error (NMSE) and is defined as

$$NMSE(dB) = 10 \times \log_{10} \left[ \frac{1}{\sigma^2 T_D} \sum_{k=1}^{T_D} [y(k) - \hat{y}(k)]^2 \right]$$

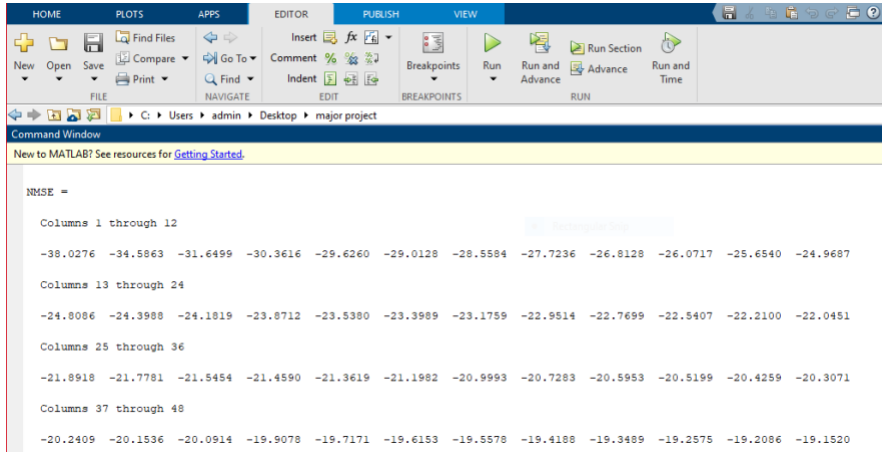


Fig 6

Our proposed model's NMSE was found to be below -18.77db, while the NMSE values for SNN and FLNN were found to be -12.4007db and -18.775db, respectively. These observations indicate that our model is more stable and effective than the SNN and FLNN models.

## 5. Conclusion:

We proposed a new single-layer neural network filter, namely the Chebyshev functional link spline neural filter, for identifying nonlinear dynamic systems. We also discussed its stability conditions and computational complexity. To verify its nonlinear approximation capability, we compared it with the SNN and FLNN models in dynamic system identification scenarios. The resulting NMSE values of CFLSNF were then compared between the SNN and FLNN models. Our findings indicate that the proposed CFLSNF model stronger nonlinear approximation ability and stability compared to the SNN and FLNN models

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