Assignment - I
(Basic Quantum Mechanics)

1. Write down formal definition of expectation value of an observable represented by operator $\hat{A}$.

   Further show that expectation values $\langle \hat{x}\hat{p}_x \rangle$ and $\langle \hat{p}_x \hat{x} \rangle$ are related by

$$\langle \hat{x}\hat{p}_x \rangle - \langle \hat{p}_x \hat{x} \rangle = i\hbar$$

2. Commutator of two operators* $\hat{A}$ and $\hat{B}$ is formally defined

$$[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}.$$

   Using above definition prove following relations.

(a) $[\hat{A},\hat{A}] = 0$ (b) $[\hat{A},\hat{B}] = -[\hat{B},\hat{A}]$

(c) $[\hat{A}+\hat{B},\hat{C}] = [\hat{A},\hat{C}] + [\hat{B},\hat{C}]$

(d) $[\hat{A},\hat{B}C] = [\hat{A},\hat{B}]\hat{C} + \hat{B}[\hat{A},\hat{C}$

(e) $[\hat{A},[\hat{B},\hat{C}]] + [\hat{B},[\hat{C},\hat{A}]] +$

$[\hat{C},[\hat{A},\hat{B}]] = 0$

* Linear operators [Defined in Ex 4]

3. Show that Schrödinger's equation is linear by showing that

$$\Psi(x,t) = a_1 \Psi_1(x,t) + a_2 \Psi_2(x,t)$$

is also a solution, if Ψ_1 and Ψ_2 are themselves solutions of Schrödinger equation.

4. A linear operator is formally defined as

$$\hat{O}[\alpha f(x) + \beta g(x)] = \alpha \hat{O} f(x) + \beta \hat{O} g(x)$$

α, β any arbitrary commuting numbers.

Which of the following operators are linear?

(a) $\hat{O} f(x) = \frac{d}{dx} f(x)$

(b) $\hat{O} f(x) = \sqrt{f(x)}$

(c) $\hat{O} f(x) = \exp[f(x)]$

5. Classically the orbital angular momentum is defined as

$$\vec{L} = \vec{r} \times \vec{p}$$

$\vec{p} \equiv$ linear momentum

$\vec{r} \equiv$ position vector; therefore

$$\left.\begin{aligned} L_x &= y p_z - z p_y \\ L_y &= z p_x - x p_z \\ L_z &= x p_y - y p_x \end{aligned}\right\} \quad \hat{p}_x = -i\hbar \frac{\partial}{\partial x} \text{ and so on.}$$

Replacing these by their corresponding operators, calculate following commutators

- $[\hat{L}_x, \hat{L}_y]$ • $[\hat{L}_y, \hat{L}_z]$

- $[\hat{L}_z, \hat{L}_x]$

- $[\hat{L}^2, \hat{L}_z]$, $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$