Problem

Design for improving productivity?

- A company conducted a survey for the American Society of Interior Designers in which workers were asked which changes in office design would increase productivity.
- Respondents were allowed to answer more than one type of design change.

Reducing noise would increase productivity	70 %
More storage space would increase productivity	67 %
Both the designs would increase productivity	**56%**

- If one of the survey respondents was randomly selected and asked what office design changes would increase worker productivity,
  - what is the probability that this person would select reducing noise *or* more storage space?

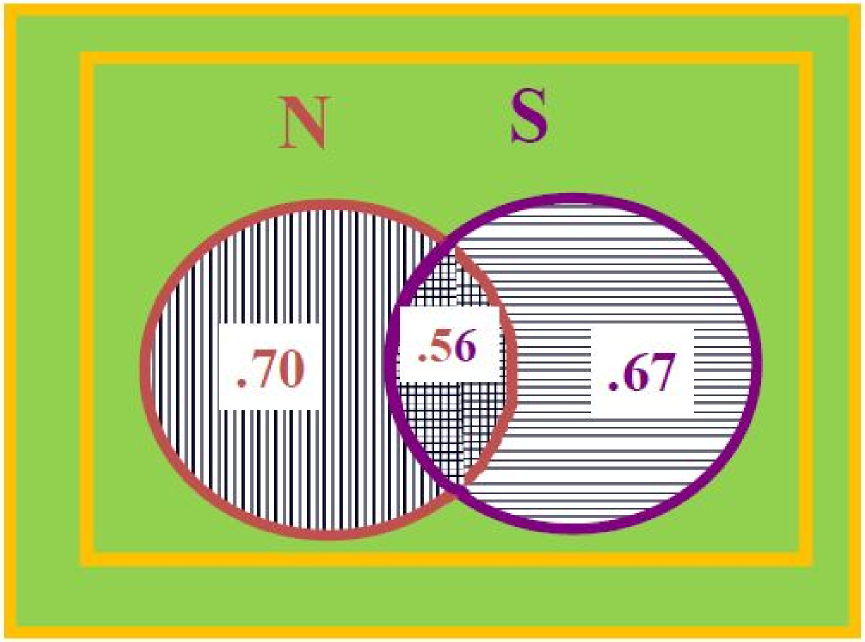
Solution

- Let *N* represent the event "reducing noise."
- Let *S* represent the event "more storage/ filing space."
- The probability of a person responding with *N or S* can be symbolized statistically as a union probability by using the law of addition.

$$P(N \cup S)$$

General Law of Addition -- Example

$$P(N \cup S) = P(N) + P(S) - P(N \cap S)$$



$$P(N) = .70$$

$$P(S) = .67$$

$$P(N \cap S) = .56$$

$$P(N \cup S) = .70 + .67 - .56$$

$$= 0.81$$

Office Design Problem
Probability Matrix

Noise Reduction	Increase Storage Space: Yes	No	Total
Yes	.56	.14	.70
No	.11	.19	.30
Total	.67	.33	1.00

Joint Probability Using a Contingency Table

Event	Event: B_1	B_2	Total
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Joint Probabilities

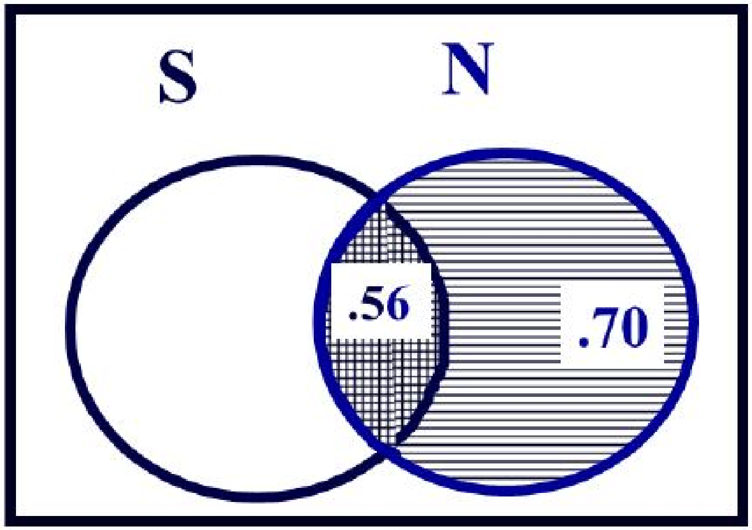
Marginal (Simple) Probabilities

Office Design Problem - Probability Matrix

		Increase Storage Space		
		Yes	No	Total
Noise Reduction	Yes	.56	.14	.70
	No	.11	.19	.30
	Total	.67	.33	1.00

$$
\begin{aligned}
P(N \cup S) &= P(N) + P(S) - P(N \cap S) \\
&= .70 + .67 - .56 \\
&= .81
\end{aligned}
$$

Law of Conditional Probability



$$P(N) = .70$$

$$P(N \cap S) = .56$$

$$P(S|N) = \frac{P(N \cap S)}{P(N)}$$

$$= \frac{.56}{.70}$$

$$= .80$$

Office Design Problem

		Increase Storage Space Yes	No	Total
Noise Reduction	Yes	.56	.14	.70
	No	.11	.19	.30
	Total	.67	.33	1.00

$$P(\overline{N} \mid S) = \frac{P(\overline{N} \cap S)}{P(S)} = \frac{.11}{.67}$$

$$= .164$$

Problem

- A company data reveal that 155 employees worked one of four types of positions.
- Shown here again is the raw values matrix (also called a contingency table) with the frequency counts for each category and for subtotals and totals containing a breakdown of these employees by type of position and by

  Gender

Contingency Table

COMPANY HUMAN RESOURCE DATA

		Gender	
		Male	Female	
Type of Position	Managerial	8	3	11
	Professional	31	13	44
	Technical	52	17	69
	Clerical	9	22	31
		100	55	155

Solution

- If an employee of the company is selected randomly, what is the probability that the employee is female or a professional worker?

$$P(\mathrm{F} \cup \mathrm{P}) = P(\mathrm{F}) + P(\mathrm{P}) - P(\mathrm{F} \cap \mathrm{P})$$

$$P(\mathrm{F} \cup \mathrm{P}) = .355 + .284 - .084 = .555.$$

Problem

- Shown here are the raw values matrix and corresponding probability matrix for the results of a national survey of 200 executives who were asked to identify the geographic locale of their company and their company's industry type.
- The executives were only allowed to select one locale and one industry type.

RAW VALUES MATRIX

		Geographic Location				
		Northeast *D*	*Southeast* *E*	*Midwest* *F*	*West* *G*	
Industry Type	*Finance A*	24	10	8	14	56
	Manufacturing B	30	6	22	12	70
	Communications C	28	18	12	16	74
		82	34	42	42	200

Questions

a. What is the probability that the respondent is from the Midwest (F)?

b. What is the probability that the respondent is from the communications industry (C) or from the Northeast (D)?

c. What is the probability that the respondent is from the Southeast (E) or from the finance industry (A)?

PROBABILITY MATRIX

		Geographic Location				
		Northeast D	Southeast E	Midwest F	West G	
Industry Type	Finance A	.12	.05	.04	.07	.28
	Manufacturing B	.15	.03	.11	.06	.35
	Communications C	.14	.09	.06	.08	.37
		.41	.17	.21	.21	1.00

Mutually Exclusive Events

Type of Position	Gender Male	Female	Total
Managerial	8	3	11
Professional	31	13	44
Technical	52	17	69
Clerical	9	22	31
Total	100	55	155

$$P(T \cup C) = P(T) + P(C)$$
$$= \frac{69}{155} + \frac{31}{155}$$
$$= .645$$

Mutually Exclusive Events

Type of Position	Gender: Male	Female	Total
Managerial	8	3	11
Professional	31	13	44
Technical	52	17	69
Clerical	9	22	31
Total	100	55	155

$$
\begin{aligned}
P(P \cup C) &= P(P) + P(C) \\
&= \frac{44}{155} + \frac{31}{155} \\
&= .484
\end{aligned}
$$

Law of Multiplication

$$\boxed{P(X \cap Y) = P(X) \cdot P(Y|X) = P(Y) \cdot P(X|Y)}$$

Problem

- A company has 140 employees, of which 30 are supervisors.
- Eighty of the employees are married, and 20% of the married employees are supervisors.
- If a company employee is randomly selected, what is the probability that the employee is married and is a supervisor?

		Married		
		Y	N	Sub total
Supervisor	Y	0.1143		30
	N			110
	Sub total	80	60	140

$$P(M) = \frac{80}{140} = 0.5714$$

$$P(S|M) = 0.20$$

$$P(M \cap S) = P(M) \cdot P(S|M)$$

$$= (0.5714)(0.20) = 0.1143$$

Law of Multiplication

Probability Matrix of Employees

Supervisor	Married: Yes	Married: No	Total
Yes	.1143	.1000	.2143
No	.4571	.3286	.7857
Total	.5714	.4286	1.00

$$P(\bar{S}) = 1 - P(S)$$
$$= 1 - 0.2143 = 0.7857$$
$$P(\bar{M} \cap \bar{S}) = P(\bar{S}) - P(M \cap \bar{S})$$
$$= 0.7857 - 0.4571 = 0.3286$$

$$P(M \cap \bar{S}) = P(M) - P(M \cap S)$$
$$= 0.5714 - 0.1143 = 0.4571$$
$$P(\bar{M} \cap S) = P(S) - P(M \cap S)$$
$$= 0.2143 - 0.1143 = 0.1000$$
$$P(\bar{M}) = 1 - P(M)$$
$$= 1 - 0.5714 = 0.4286$$

Special Law of Multiplication for Independent Events

- General Law

$$P(X \cap Y) = P(X) \cdot P(Y \mid X) = P(Y) \cdot P(X \mid Y)$$

- Special Law

If events X and Y are independent,

$P(X) = P(X \mid Y)$, and $P(Y) = P(Y \mid X)$.

Consequently,

$P(X \cap Y) = P(X) \cdot P(Y)$

Law of Conditional Probability

- The conditional probability of X given Y is the joint probability of X and Y divided by the marginal probability of Y.

$$\boxed{P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(Y|X) \cdot P(X)}{P(Y)}}$$

Conditional Probability

- A conditional probability is the probability of one event, given that another event has occurred:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$ → The conditional probability of A given that B has occurred

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$ → The conditional probability of B given that A has occurred

Where P(A and B) = joint probability of A and B

P(A) = marginal probability of A

P(B) = marginal probability of B

Conditional Probability

- If a die is tossed twice we get the sample space

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

- Let A denote the event "sum of two numbers is 9" and B the event "first toss number is 5".
- Conditional probability of event A given event B is denoted by P(A|B) is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Examples

1. A bag contains 8 red balls, 4 green balls and 8 yellow balls. A ball is drawn, it is not a red ball. What is the probability that it is a green ball?
2. A fair coin is tossed twice. Given that first toss resulted in heads, what is the probability that both tosses resulted in heads?

Computing Conditional Probability

- Of the cars on a used car lot, 70% have air conditioning (AC) and 40% have a CD player (CD). 20% of the cars have both.
- What is the probability that a car has a CD player, given that it has AC ?
- We want to find P(CD | AC).

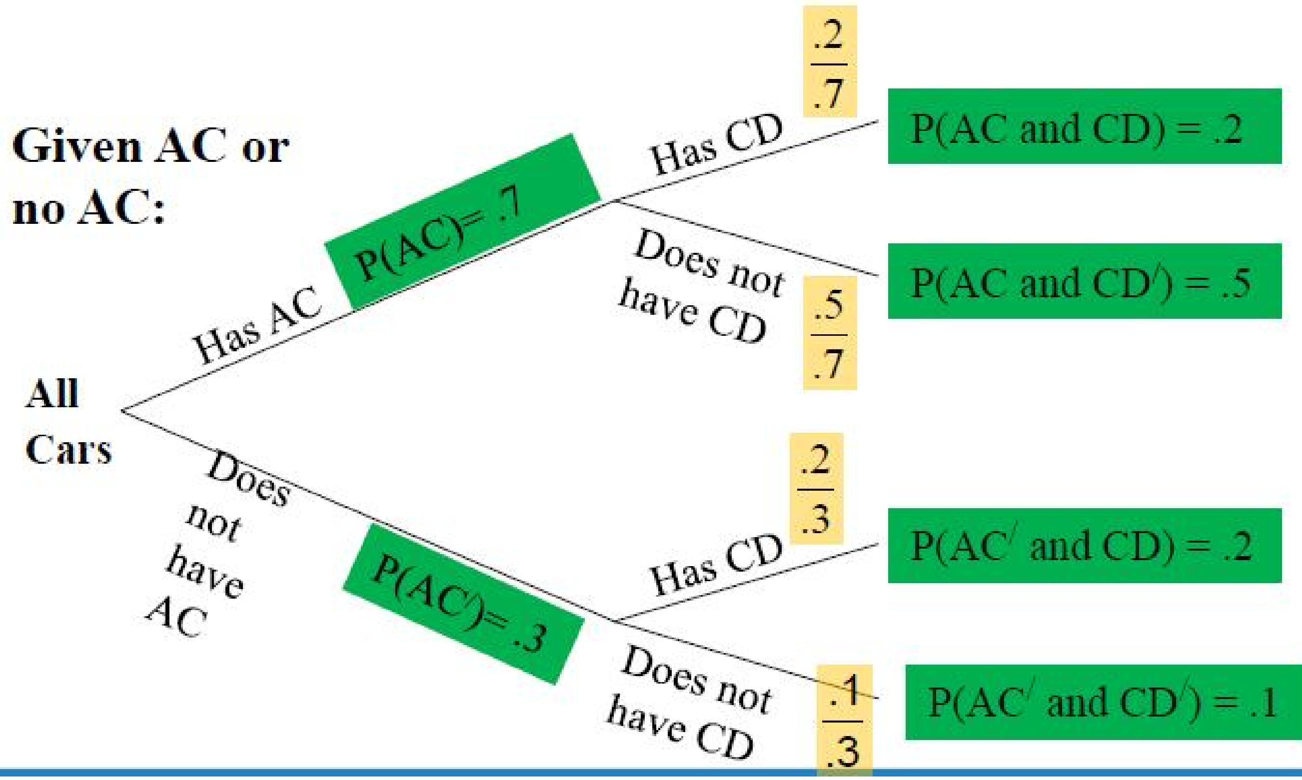
Computing Conditional Probability

	CD	No CD	Total
AC	0.2	0.5	0.7
No AC	0.2	0.1	0.3
Total	0.4	0.6	1.0

$$P(\text{CD} \mid \text{AC}) = \frac{P(\text{CD and AC})}{P(\text{AC})} = \frac{.2}{.7} = .2857$$

Given AC, we only consider the top row (70% of the cars). Of these, 20% have a CD player. 20% of 70% is about 28.57%.

Computing Conditional Probability: Decision Trees



Independent Events

- If X and Y are independent events, the occurrence of Y does not affect the probability of X occurring.
- If X and Y are independent events, the occurrence of X does not affect the probability of Y occurring.

If X and Y are independent events,

$P(X|Y) = P(X)$, and

$P(Y|X) = P(Y)$.

Statistical Independence

- Two events are **independent** if and only if:

$$P(A \mid B) = P(A)$$

- Events A and B are independent when the probability of one event is not affected by the other event

Independent Events Demonstration

	Geographic Location				
	Northeast D	Southeast E	Midwest F	West G	
Finance A	.12	.05	.04	.07	.28
Manufacturing B	.15	.03	.11	.06	.35
Communications C	.14	.09	.06	.08	.37
	.41	.17	.21	.21	1.00

Test the matrix for the 200 executive responses to determine whether industry type is independent of geographic location.

Independent Events Demonstration Contd...

$$P(A|G) = \frac{P(A \cap G)}{P(G)} = \frac{0.07}{0.21} = 0.33 \qquad P(A) = 0.28$$

$$P(A|G) = 0.33 \neq P(A) = 0.28$$

Independent Events

	D	E	
A	8	12	20
B	20	30	50
C	6	9	15
	34	51	85

$$P(A|D) = \frac{8}{34} = .2353$$

$$P(A) = \frac{20}{85} = .2353$$

$$P(A|D) = P(A) = 0.2353$$