Methods of Assigning Probabilities

- Classical method of assigning probability (rules and laws)
- Relative frequency of occurrence (cumulated historical data)
- Subjective Probability (personal intuition or reasoning)

Classical Probability

- Number of outcomes leading to the event divided by the total number of outcomes possible
- Each outcome is equally likely
- Determined *a priori* -- before performing the experiment
- Applicable to games of chance
- Objective -- everyone correctly using the method assigns an identical probability

Classical Probability

$$P(E) = \frac{n_e}{N}$$

Where:

N = total number of outcomes

n_e = number of outcomes in E

Relative Frequency Probability

- Based on historical data
- Computed after performing the experiment
- Number of times an event occurred divided by the number of trials
- Objective -- everyone correctly using the method assigns an identical probability

Relative Frequency Probability

$$P(E) = \frac{n_e}{N}$$

Where:

$N =$ total number of trials

$n_e =$ number of outcomes producing E

Subjective Probability

- Comes from a person's intuition or reasoning
- Subjective -- different individuals may (correctly) assign different numeric probabilities to the same event
- Degree of belief
- Useful for unique (single-trial) experiments
  - New product introduction
  - Initial public offering of common stock
  - Site selection decisions
  - Sporting events

Example

Two fair dice are tossed. Find the probability of each of following:

a) Sum of outcomes of two dice is 5
b) Sum of outcomes of two dice is 7 or 11
c) Outcome of second die is less than first die.
d) Outcomes of both dice are odd

- Solution

Sample space

S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

N= 36

a) Event A "sum of outcomes of two dice is 5"

A = { (1,4), (2,3), (3,2), (4,1 } $N_A = 4$ P(A) = 4/36 = 1/9

b) Event B "sum of outcomes of two dice is 7 or 11

B = { (1,6), (2,5), (3,4), (4,3), (5,2), (5,6), (6,1), (6,5) }

$N_B = 8$ P(B)= 8/36 =2/9

c) Event C "Outcome of second die is less than first die"

$C = \{ (2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4), (6,5)\}$

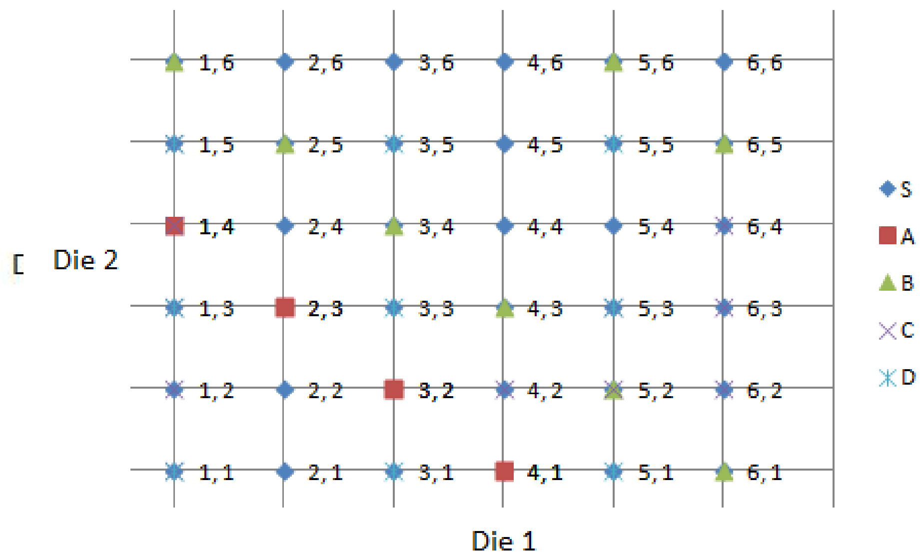
$N_C = 15$ $P(C) = 15/36 = 5/12$

d) Event D "Outcomes of both dice are odd"

$D = \{ (1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5) \}$

$N_D = 9$ $P(D) = 9/36 = 1/4$

This problem can also be solved by considering two dimensional display of sample space.



Example: A die is rolled and a coin is tossed. What is the probability that that die shows ODD number and coin shows HEAD?

Sample space

S = {(1,H), (2,H), (3,H), (4,H), (5,H), (6,H),
(1,T), (2,T), (3,T), (4,T), (5,T), (6,T)}   N = 12

Event

A = "die shows ODD number and coin shows HEAD"

= {(1,H), (3,H), (5,H)} $N_A = 3$

$$P(A) = \frac{N_A}{N} = \frac{3}{12} = \frac{1}{4}$$

Counting the Possibilities

- mn Rule
- Sampling from a Population with Replacement
- Combinations: Sampling from a Population without Replacement

mn Rule

- If an operation can be done m ways and a second operation can be done n ways, then there are mn ways for the two operations to occur in order.
- This rule is easily extend to k stages, with a number of ways equal to

$n_1.n_2.n_3..n_k$

- Example: Toss two coins . The total umber of simple events is 2 x 2 =4

Sampling from a Population with Replacement

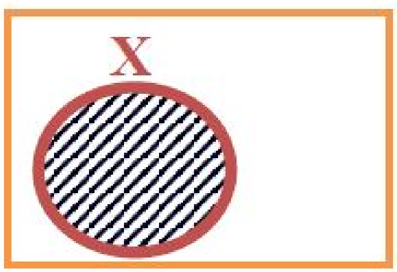
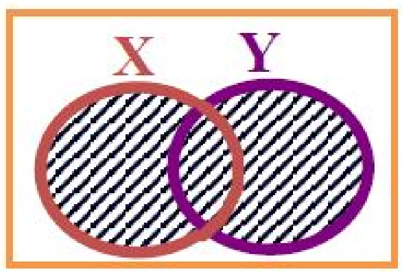
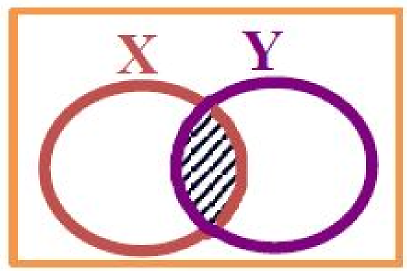
- A tray contains 1,000 individual tax returns. If 3 returns are randomly selected with replacement from the tray, how many possible samples are there?
- $(N)^n = (1{,}000)^3 = 1{,}000{,}000{,}000$

Combinations

- A tray contains 1,000 individual tax returns. If 3 returns are randomly selected without replacement from the tray, how many possible samples are there?

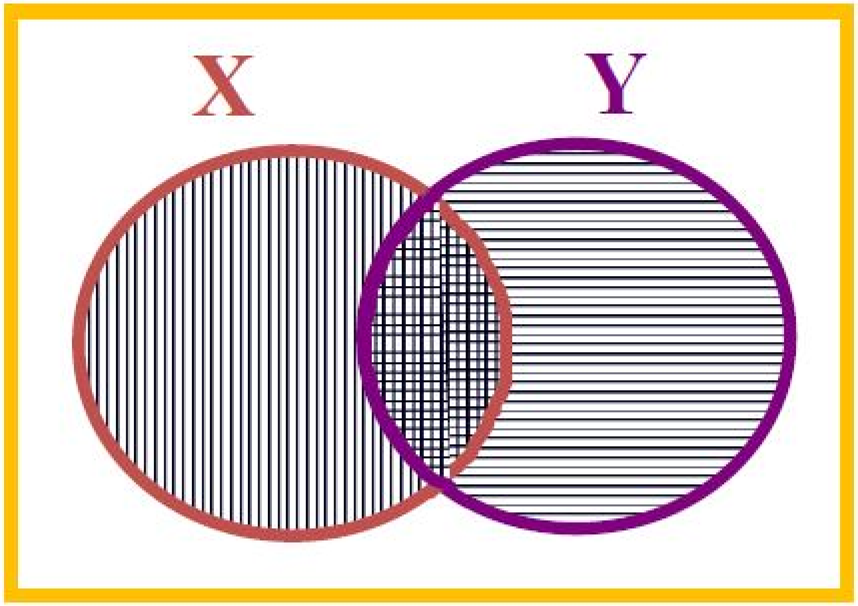
$$\boxed{\binom{N}{n} = \frac{N!}{n!(N-n)!} = \frac{1000!}{3!(1000-3)!} = 166{,}167{,}000}$$

Four Types of Probability

Marginal	Union	Joint	Conditional
$P(X)$	$P(X \cup Y)$	$P(X \cap Y)$	$P(X \mid Y)$
The probability of **X** occurring	The probability of **X or Y** occurring	The probability of **X and Y** occurring	The probability of **X occurring given that Y** has occurred
			

General Law of Addition

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$



Problem

Design for improving productivity?

- A company conducted a survey for the American Society of Interior Designers in which workers were asked which changes in office design would increase productivity.
- Respondents were allowed to answer more than one type of design change.

Reducing noise would increase productivity	70 %
More storage space would increase productivity	67 %
Both the designs would increase productivity	**56%**

- If one of the survey respondents was randomly selected and asked what office design changes would increase worker productivity,
  - what is the probability that this person would select reducing noise *or* more storage space?

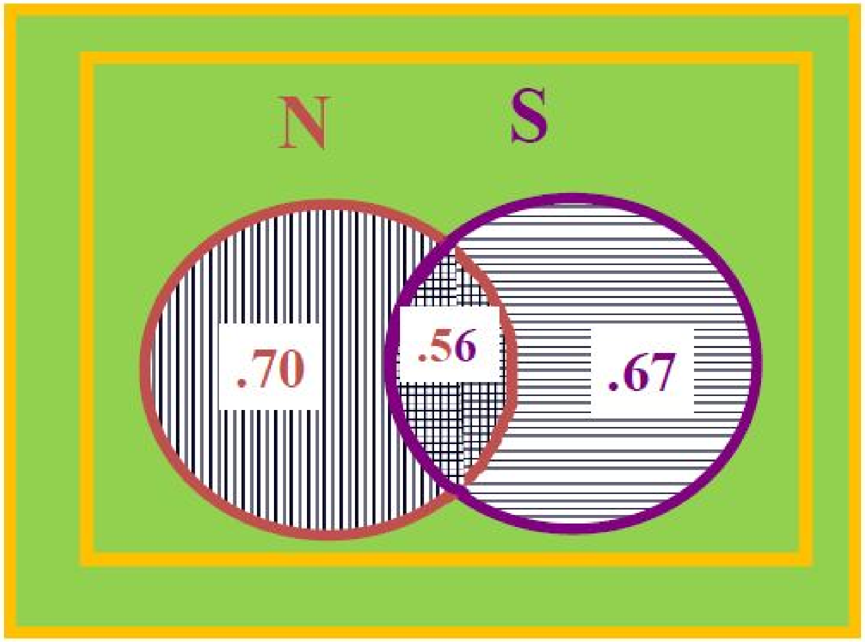
Solution

- Let *N* represent the event "reducing noise."
- Let *S* represent the event "more storage/ filing space."
- The probability of a person responding with *N or S* can be symbolized statistically as a union probability by using the law of addition.

$$P(N \cup S)$$

General Law of Addition -- Example

$$P(N \cup S) = P(N) + P(S) - P(N \cap S)$$



$$P(N) = .70$$

$$P(S) = .67$$

$$P(N \cap S) = .56$$

$$P(N \cup S) = .70 + .67 - .56$$

$$= 0.81$$

Office Design Problem
Probability Matrix

Noise Reduction	Increase Storage Space: Yes	No	Total
Yes	.56	.14	.70
No	.11	.19	.30
Total	.67	.33	1.00

Joint Probability Using a Contingency Table

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Joint Probabilities

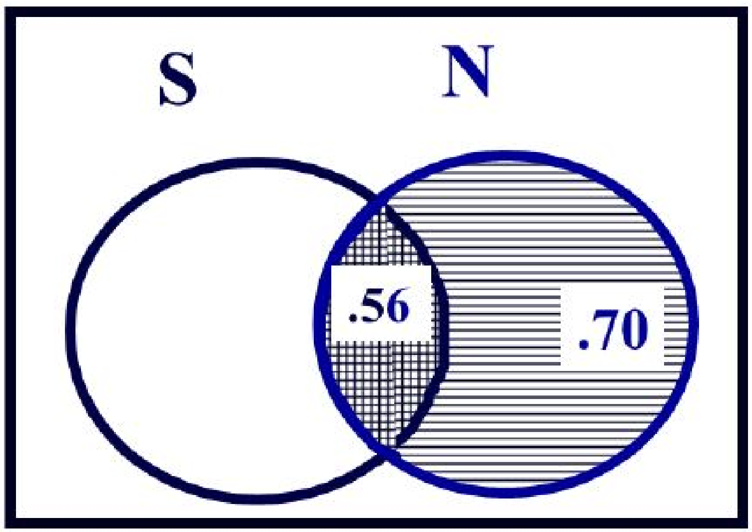
Marginal (Simple) Probabilities

Office Design Problem - Probability Matrix

		Increase Storage Space		
		Yes	No	Total
Noise Reduction	Yes	.56	.14	.70
	No	.11	.19	.30
	Total	.67	.33	1.00

$$
\begin{aligned}
P(N \cup S) &= P(N) + P(S) - P(N \cap S) \\
&= .70 + .67 - .56 \\
&= .81
\end{aligned}
$$

Law of Conditional Probability



$$P(N) = .70$$

$$P(N \cap S) = .56$$

$$P(S|N) = \frac{P(N \cap S)}{P(N)} = \frac{.56}{.70} = .80$$

Office Design Problem

		Increase Storage Space		
		Yes	No	Total
Noise Reduction	Yes	.56	.14	.70
	No	.11	.19	.30
	Total	.67	.33	1.00

$$P(\overline{N} \mid S) = \frac{P(\overline{N} \cap S)}{P(S)} = \frac{.11}{.67}$$

$$= .164$$