

EE23BTECH11208 - Manohar K*

EXERCISE 9.2

14. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P. and obtain the Z-transform of the sequence.

Solution: Given,

symbol	value	description
$x(0)$	8	first term of the series
$x(6)$	26	last term of the series
N	$2 + 5 = 7$	number terms in the series

TABLE I
PARAMETERS

$$x(0) = 8,$$

$$x(6) = 26$$

$$d = \frac{x(6) - x(0)}{N - 1},$$

$$= 3$$

common term in A.P.

$$x(n) = u(n)(x(0) + (n)(d))$$

the A.P. sequence is:

$$8, 11, 14, 17, 20, 23, 26$$

Applying Z Transform:

$$x(n) \xleftrightarrow{Z} X(z) \quad (6)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (7)$$

using eq (5)

$$= \sum_{n=-\infty}^{\infty} (u(n)(x(0) + n(d))) z^{-n} \quad (8)$$

$$= \sum_{n=0}^{\infty} (8(1) + 3n(1)) z^{-n} \quad (9)$$

using eq (??),

$$= 8 \sum_{n=0}^{\infty} z^{-n} + 3 \sum_{n=0}^{\infty} n z^{-n} \quad (10)$$

$$\Rightarrow X(z) = \frac{8}{1 - z^{-1}} + \frac{3z^{-1}}{(1 - z^{-1})^2} \quad (11)$$

$$\{z \in \mathbb{C} : z \neq 1\}$$

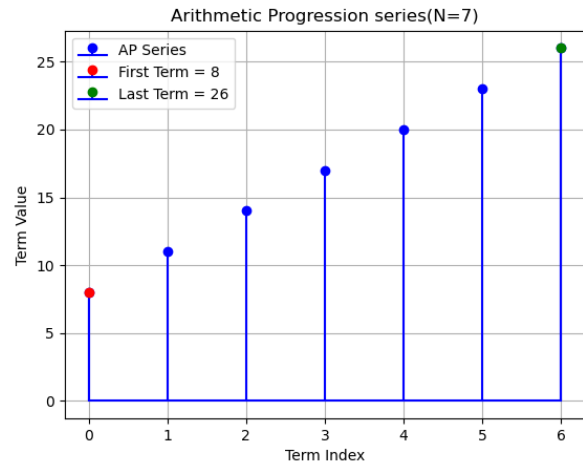


Fig. 1. Plot of $x(n)$ vs n