EE23BTECH11208 - Manohar K*

Exercise 9.2

14. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P. and obtain the Z-transform of the sequence.

Solution: Given,

symbol	value	description
$x(0)$	8	first term of the series
$x(6)$	8	last term of the series
N	$2+5=7$	number terms in the series

TABLE I
Parameters

$$x(0) = 8, \tag{1}$$

$$x(6) = 26 \tag{2}$$

$$d = \frac{x(6) - x(0)}{N-1}, \tag{3}$$

$$d = 3 \tag{4}$$

common term in A.P.

$$x(n) = u(n)\left(x(0) + (n)(d)\right), \tag{5}$$

the A.P. sequence is:

8,11,14,17,20,23,26

Applying Z Transform:

$$x(n) \overset{Z}{\leftrightarrow} X(z) \tag{6}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{7}$$

using eq (5)

$$= \sum_{n=-\infty}^{\infty} \left(u(n)\left(x(0) + n(d)\right)\right) z^{-n} \tag{8}$$

for $n \geq 0$, $u(n) = 1$

$$= \sum_{n=0}^{\infty} (8(1) + 3n(1)) z^{-n} \tag{9}$$

using eq (**??**),

$$= 8\sum_{n=0}^{\infty} z^{-n} + 3\sum_{n}^{\infty} n z^{-n} \tag{10}$$

$$\implies X(z) = \frac{8}{1-z^{-1}} + \frac{3z^{-1}}{(1-z^{-1})^2} \tag{11}$$

$$\{z \in \mathbb{C} : z \neq 1\}$$

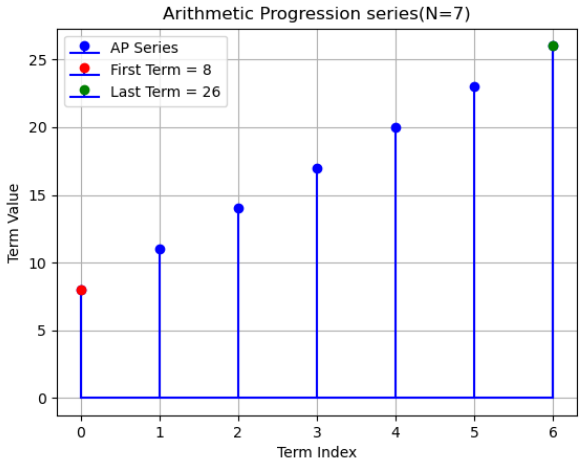

Fig. 1. Plot of x(n) vs n