Types of Distributions - Poisson and Exponential

We'll cover the following

5. Poisson Distribution

This distribution gives us the probability of a given **number of events** happening **in a fixed interval of time**.

Say we have the number of breads sold by a bakery every day. If the average number for seven days is 500, we can predict the probability of a certain day having more sales, e.g., more on Sundays. Another example could be the number of phone calls received by a call center per hour. Poisson distributions can be used to make forecasts about the number of customers or sales on certain days or seasons of the year.

Why are such forecasts important?

Think about it: if more items than necessary are kept in stock, it means a loss for the business. On the other hand, under-stocking would also result in loss because customers need to be turned away due to not having enough stock. Poisson can help businesses estimate when demand is unusually high so that they can plan for the increase in demand in advance while keeping wast of resources to a minimum. However, its applications are not only for sales or specifically business related, some different kinds of examples could be forecasting the number of earthquakes happening, next month or traffic flow and ideal gap distance.

For a distribution to be called a Poisson distribution, the following assumptions need to be in place:

- The number of successes in two disjoint time intervals is independent.

- The probability of a success during a small-time interval is proportional to the entire length of the time interval. The probability of success in an interval approaches zero as the interval becomes smaller.

The probability distribution representing the number of successes occurring in a given time interval is given by this formula:

$$P(X) = \frac{e^{-\mu}\mu^{x}}{x!}$$

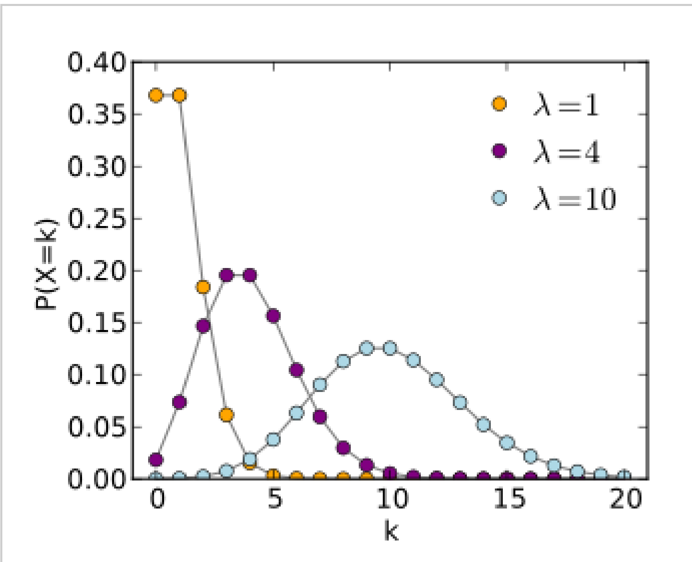
where,
x = 0, 1, 2, 3 …,
e = the natural number *e*,
μ = mean number of successes in the given time interval,
X, Poisson Random Variable, is the number of events in a time interval and *P(X)* is its probability distribution (probability mass function).

The mean, the expected number of occurrences, μ, can be calculated as:

$$\mu = \lambda t$$

where,
λ is the rate at which an event occurs,
t is the length of a time interval

Let's look at a graphical representation of a Poisson distribution, and how it varies with the change in expected number of occurrences:



The horizontal axis is the index k, the number of occurrences. λ is the expected number of occurrences, which need not be an integer. The vertical axis is the probability of k occurrences given λ. The function is defined only at integer values be an integer. The vertical axis is the probability of k occurrences given λ. The function is defined only at integer values of k.

6. Exponential Distribution

Exponential distribution allows us to go a step further from the Poisson distribution. Say we are using Poisson to model the number of accidents in a given time period. *What if we wanted to understand the time interval between the accidents?* This is where exponential distribution comes into play; it allows us to model the time in between each accident.

Some other examples of questions that can be answered by modeling waiting times:

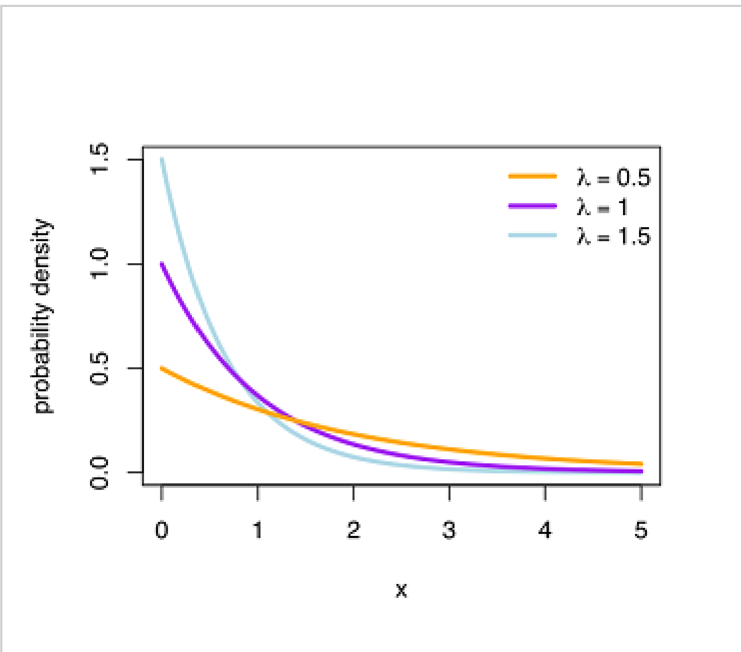
- *How much time will go by before a major earthquake hits a certain area?*
- *How long will a car component last before it needs replacement?*

The probability density function for an exponential distribution is given by:

$$f(x) = \lambda e^{-\lambda x}$$

where,
e = the natural number *e*,
λ = mean time between events,
x = a random variable

A graphical representation of the density function for varying values of the mean time between events looks like this:



We can observe that the greater the rate of events, the faster the curve drops, and the lower the rate, the flatter the curve.

Phew! This was the last one. In the next lesson we are going to put all of this together and recap everything to make sure you firmly grasp these concepts.