Handling Independent Variables

Let's look at just one final example because it shows us how to handle variables which are independent of other variables.

If we have a function

$$f = 2xy + 3x^2z + 4z$$

Where *x*, *y* and *z* are independent of each other. What do we mean by independent? We mean that *x*, *y* and *z* can be any value and don't care what the other variables are — they aren't affected by changes in the other variables. This wasn't the case in the previous example where *y* was $x^3 + x$, so *y* was dependent on *x*.

What is ∂f / ∂x? Let's look at each part of that long expression. The first bit is $2xy$, so the derivative is $2y$. Why is this so simple? It's simple because *y* is not dependent on *x*. What we're asking when we say ∂f / ∂x is how does *f* change when **x** changes. If *y* doesn't depend on *x*, we can treat it like a constant. That *y* might as well be another number like 2 or 3 or 10.

Let's carry on. The next bit is $3x^2$z. We can apply the power reduction rule to get $2 * 3xz$ or $6xz$. We treat *z* as just a boring constant number like 2 or 4 or maybe 100 because *x* and *z* are independent of each other. A change in *z* doesn't affect *x*.

The final bit $4z$ has no *x* in it at all. So it vanishes completely because we treat it like a plain constant number like 2 or 4.

The final answer is

$$\frac{\delta f}{} = 2y + 6xz$$

The important thing in this last example is having the confidence to ignore variables that you know are independent. It makes doing calculus on quite complex expressions drastically simpler, and it is an insight we will need lots when looking at neural networks.

You Can Do Calculus!!!

If you got this far, well done!

You have a genuine insight into what calculus really is, and how it was invented using approximations that get better and better. You can always try these methods on other tough problems that resist normal ways for solving them.

The two techniques we learned, reducing powers and the chain rule, allows us to do quite a lot of calculus, including understanding how neural networks really work and why.

Enjoy your new powers!