Recursive factorial

For positive values of $\mathbf{n}$, let's write $\mathbf{n!}$ as we did before, as a product of numbers starting from n and going down to **1**: $\mathbf{n! = n \cdot (n-1) \cdots 2 \cdot 1}$. But notice that $\mathbf{(n-1) \cdots 2 \cdot 1}$ is another way of writing $\mathbf{(n-1)!}$, and so we can say that $\mathbf{n! = n \cdot (n-1)!}$. Did you see what we just did? We wrote $\mathbf{n!}$ as a product in which one of the factors is $\mathbf{(n-1)!}$. We said that you can compute $\mathbf{n!}$ by computing $\mathbf{(n-1)!}$ and then multiplying the result of computing $\mathbf{(n-1)!}$ by $\mathbf{n}$. You can compute the factorial function on n by first computing the factorial function on $\mathbf{n-1}$. We say that computing $\mathbf{(n-1)!}$ is a subproblem that we solve to compute $\mathbf{n!}$.

Let's look at an example: computing $\mathbf{5!}$.

- You can compute $\mathbf{5!}$ as $\mathbf{5 \cdot 4!}$.
- Now you need to solve the subproblem of computing $\mathbf{4!}$, which you can compute as $\mathbf{4 \cdot 3!}$.
- Now you need to solve the subproblem of computing $\mathbf{3!}$, which is $\mathbf{3 \cdot 2!}$.
- Now $\mathbf{2!}$, which is $\mathbf{2 \cdot 1!}$.
- Now you need to compute $\mathbf{1!}$. You could say that $\mathbf{1!}$ equals $\mathbf{1}$, because it's the product of all the integers from $\mathbf{1}$ through $\mathbf{1}$. Or you can apply the formula that $\mathbf{1! = 1 \cdot 0!}$. Let's do it by applying the formula.
- We defined $\mathbf{0!}$ to equal $\mathbf{1}$.
- Now you can compute $\mathbf{1! = 1 \cdot 0! = 1}$.
- Having computed $\mathbf{1! = 1}$, you can compute $\mathbf{2! = 2 \cdot 1! = 2}$.
- Having computed $\mathbf{2! = 2}$, you can compute $\mathbf{3! = 3 \cdot 2! = 6}$.
- Having computed $\mathbf{3! = 6}$, you can compute $\mathbf{4! = 4 \cdot 3! = 24}$.
- Finally, having computed $\mathbf{4! = 24}$, you can finish up by computing $\mathbf{5! = 5 \cdot 4! = 120}$.

So now we have another way of thinking about how to compute the value of n!, for all nonnegative integers n:

- If **n** = **0**, then declare that **n!** = **1**.
- Otherwise, **n** must be positive. Solve the subproblem of computing **(n−1)!**, multiply this result by **n**, and declare **n!** equal to the result of this product.

When we're computing **n!** in this way, we call the first case, where we immediately know the answer, the **base case**, and we call the second case, where we have to compute the same function but on a different value, the **recursive case**.