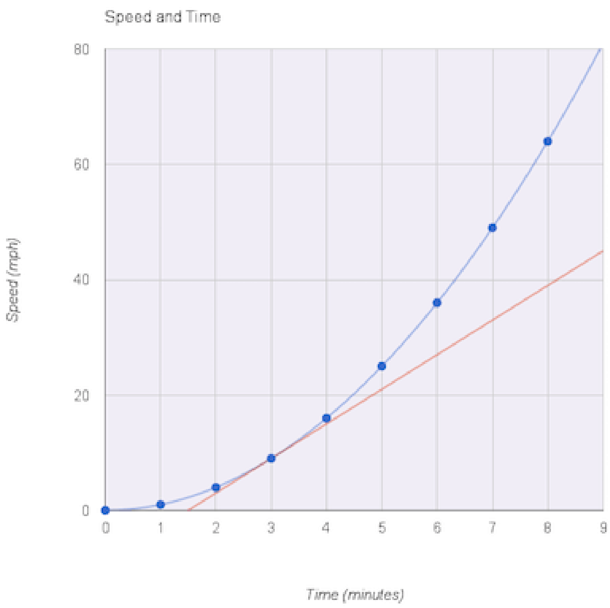
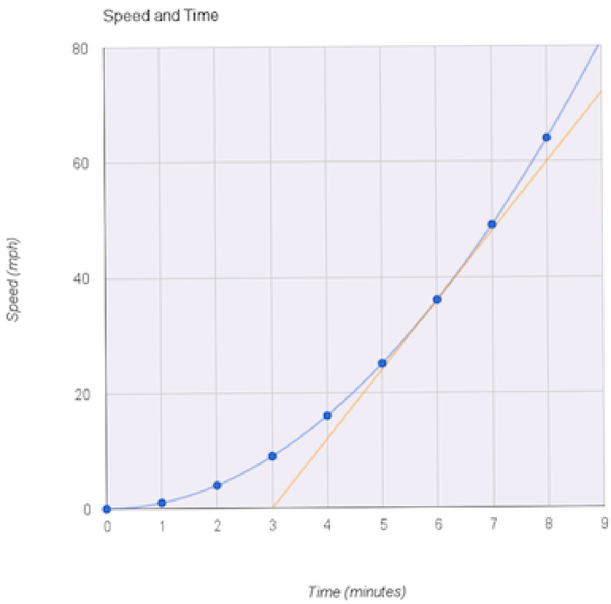
Calculus By Hand

Let’s look more closely at what is happening at time 3 minutes. At 3 minutes, the speed is 9 miles per hour. We know it will be faster after 3 minutes. Let’s compare that with what is happening at 6 minutes. At 6 minutes, the speed is 36 miles per hour. We also know that the speed will be faster after 6 minutes. But we also know that a moment after 6 minutes the speed increase will be greater than an equivalent moment after 3 minutes. There is a real difference between what’s happening at time 3 minutes and time 6 minutes. Let’s visualize this as follows.

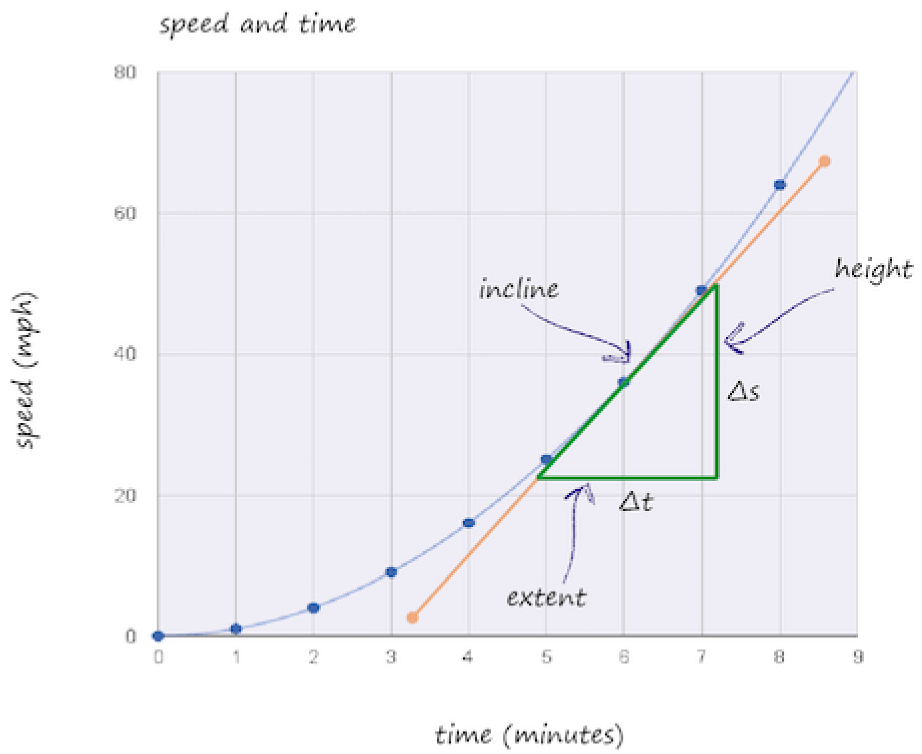




You can see that the slope at time 6 minutes is steeper than at time 3 minutes. These slopes are the rate of change we want. This is an important realization, so let’s say it again. The rate of change of a curve at any point is the slope of the curve at that point.

But how do we measure the slope of a line that is curved? Straight lines were easy, but curvy lines? We could try to *estimate* the slope by drawing a straight line called a *tangent*, which just touches that curved line in a way that tries to be at the same gradient as the curve just at the point. This is in fact how people used to do it before other ways were invented. Let’s try this rough and ready method, just so we have some sympathy with that approach. The ready method, just so we have some sympathy with that approach. The

following diagram shows the speed graph with a tangent touching the speed curve at time 6 minutes.



To work out the slope, or gradient, we know from school maths that we need to divide the height of the incline by the extent. In the diagram, this height (speed) is shown as Δs, and the extent (time) is shown as Δt. That symbol, Δ called "delta", just means a small change. So Δt is a small change in *t*. The slope is Δs / Δt. We could choose any sized triangle for the incline, and use a ruler to measure the height and extent. With my measurements, I just happen to have a triangle with Δs measured as 9.6, and Δt as 0.8. That gives the slope as follows:

$$\text{rate of change at a point} = \text{slope at that point}$$

$$= \frac{\Delta s}{\Delta t}$$

$$= 9.6 / 0.8$$

$$= 12.0$$

We have a key result! The rate of change of speed at time 6 minutes is 12.0 mph per min. You can see that relying on a ruler, and even trying to place a tangent by hand, isn't going to be very accurate. So let's get a tiny bit more sophisticated.