Expected Value of a Random Variable

This chapter discusses the expected values of random variables and how to calculate them.

Expected Value of a random variable

We all understand the concept of average. The average test-score of a class is the sum of each individual student's score divided by the total number of students in the class. The *expected* value of a random variable is somewhat a similar concept. However, note that the outcomes that a random variable can take on don't happen with the same frequency. An outcome that happens more frequently should get a higher weight when computing the "average" for a random variable. The weight here is the probability with which each outcome occurs.

Take the case of the random variable X which we define as the number of heads that occur in 3 flips of a coin. The expected number of heads seen when the experiment of flipping a coin thrice is repeated many many times is shown below:

$$E[X] = P(X = 0) * 0 + P(X = 1) * 1 + P(X = 2) * 2 + P(X = 3) *$$

$$E[X] = \frac{1}{8} * 0 + \frac{3}{8} * 1 + \frac{3}{8} * 2 + \frac{1}{8} * 3$$

$$E[X] = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$E[X] = \frac{12}{8}$$

$$E[X] = 1.5$$

Thus we can make a claim: if we do several experiments of flipping a coin three times and noting down the results from each experiment, we can expect to see heads appearing 1.5 times across all the experiments.