

Big O and Big Omega Notations

Discusses the Big O notation with examples

In the previous section, we defined Θ notation. In this section, we'll discuss big O and big Ω notations. The reader would notice the use of the adjective "big" with these notations and rightly suspect that there exist complementary small o and small ω notations, which we'll discuss in the next lesson.

Big O

Big O is expressed as $O(g(n))$ and is pronounced as "big oh of g of n". We observed that Θ provides both an asymptotically upper and lower tight bound. Big O only provides an asymptotic upper bound which may not necessarily be a tight bound.

Big O is the most commonly talked-about notation when discussing algorithms in industry settings or in interviews. You won't see Θ or other notations being discussed, and for good reason too. Generally, if we are guaranteed that an algorithm will perform no worse than a certain threshold, and that threshold is acceptable for the problem at hand, then knowing its best-case performance or finding a very tight upper bound may not be productive.

Formal Definition

Similar to Θ we define $O(g(n))$ as a set of functions and a function $f(n)$ can be a member of this set if it satisfies the following conditions:

$$0 \leq f(n) \leq cg(n)$$

where c is a positive constant and the inequality holds after n crosses a positive threshold n_0

Explanation

- if $f(n) = \Theta(g(n))$ then it also implies that $f(n) = O(g(n))$

- The notation means that the function $f(n)$ is bounded by above to within a constant factor of $g(n)$.
- Note that the inequality requires $f(n)$ to be greater than 0 after n crosses n_0 .
- The set $O(g(n))$ may also contain other functions also that satisfy the inequality. There could be several values of c that can be used to satisfy the inequality.

Insertion Sort

Previously, we came up with the following $f(n)$ for the insertion sort algorithm

$$f(n) = [2 * (n + 1) + 2n] + [2n + 7[\frac{n(n-1)}{2}]]$$

Now we can attempt to find the $O(g(n))$ for this expression. As a general rule of thumb, when working with big O we can drop the lower order terms and concentrate only on the higher order terms. The term $\frac{n(n-1)}{2}$ is the highest order term, since it involves a square of n . The above expression is thus quadratic, and we can ignore all the constants and linear terms. Thus a tight bound on the expression would be $O(n^2)$. We can prove it below:

$$\frac{n(n-1)}{2} \leq cn^2$$

let $c = 10$ and $n_0 = 10$

You can always pick a different set of constants as long as it satisfies the inequality for all $n > n_0$.

As a general rule of thumb, for an expression consisting of polynomials the expression/function is $O(\text{the highest polynomial degree in the expression})$.

Big Ω

The complementary notation for big O is the *big omega* notation. The big omega notation provides an asymptotic lower bound and is expressed as $\Omega(g(n))$.

Formal Definition

As before, $\Omega(g(n))$ is a set of functions and any function $f(n)$ that satisfies the below constraints belongs to this set and $f(n)$ is said to be big omega of $g(n)$.

$$0 \leq g(n) \leq f(n)$$

where c is a positive constant and the above inequality holds after n crosses a positive threshold n_0

Explanation

- Notice that similar to big O, the definition mandates we only consider positive values for $f(n)$.
- Big Ω is not necessarily a tight lower bound.
- If $f(n) \Theta(g(n))$ then it also implies $f(n) \Omega(g(n))$.

Relation to Θ

One can see that if a function **$f(n)$** is **$\Theta(g(n))$** then it follows that **$f(n)$** must be **$O(g(n))$** and **$\Omega(g(n))$** . In the previous lesson we proved that **$f(n) = 2n^2 - 1$** was $\Theta(n^2)$. We can simply consider the right and left hand sides of the following inequality in isolation to prove that $f(n)$ is also $O(g(n))$ and $\Omega(g(n))$.

$$1(n^2 + 2) < 2n^2 - 1 < 2(n^2 + 2)$$

1

You are told that a function $f(n)$ is $O(n^3)$, does it also imply that the $f(n)$ is $\Theta(n^3)$?

☐ A) No

☐ B) Yes

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You are told that a function $f(n)$ is $O(n)$ and also $\Omega(n)$, does it also imply that the $f(n)$ is $\Theta(n)$?

☐ A) No

☐ B) Yes

CHECK ANSWERS