Coding Example: Minkowski-Bouligand Dimension

This problem is an extension of the previous case study. In this lesson, we'll learn how to find the fractal dimension of the Mandelbrot set.

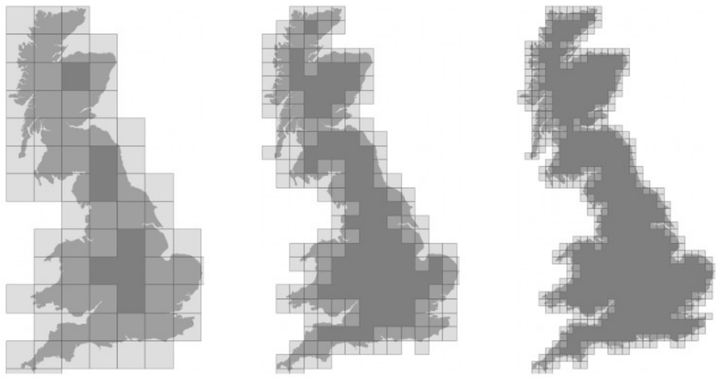
We'll cover the following

> **Note:** You should look at the ufunc.reduceat method that performs a (local) reduce with specified slices over a single axis.

Problem Description

We now want to measure the fractal dimension of the Mandelbrot set using the Minkowski–Bouligand dimension. To do that, we need to do box-counting with a decreasing box size (see figure below). As you can imagine, we cannot use pure Python because it would be way too slow. The goal of the exercise is to write a function using NumPy that takes a two-dimensional float array and returns the fractal dimension. We'll consider values in the array to be normalized (i.e. all values are between 0 and 1).

Given below is the Minkowski-Bouligand dimension:



The Minkowski-Bouligand dimension of the Great Britain coastlines is approximately 1.24.

Complete Solution

Here's the detailed solution to find the fractal dimension:

```
# -----------------------------------------------------------------------------
# From Numpy to Python
# Copyright (2017) Nicolas P. Rougier - BSD license
# More information at https://github.com/rougier/numpy-book
# -----------------------------------------------------------------------------
import numpy as np

#calculate the fractal dimensions
def fractal_dimension(Z, threshold=0.9):
    def boxcount(Z, k):
        S = np.add.reduceat(
            np.add.reduceat(Z, np.arange(0, Z.shape[0], k), axis=0),
                                np.arange(0, Z.shape[1], k), axis=1)
        return len(np.where((S > 0) & (S < k*k))[0])
    Z = (Z < threshold)
    p = min(Z.shape)
    n = 2**np.floor(np.log(p)/np.log(2))
    n = int(np.log(n)/np.log(2))
    sizes = 2**np.arange(n, 1, -1)
    counts = []
    for size in sizes:
        counts.append(boxcount(Z, size))
    coeffs = np.polyfit(np.log(sizes), np.log(counts), 1)
    return -coeffs[0]


if __name__ == '__main__':
    from scipy import misc
```

RUN SAVE RESET

Further Readings

- How To Quickly Compute the Mandelbrot Set in Python, Jean Francois Puget, 2015.
- My Christmas Gift: Mandelbrot Set Computation In Python, Jean Francois Puget, 2015.
- Fast fractals with Python and NumPy, Dan Goodman, 2009.
- Renormalizing the Mandelbrot Escape, Linas Vepstas, 1997.

Now that we have learned Temporal Vectorization, let's look at "Spatial Vectorization" in the next lesson.