20/2/24

UNIT-3 COMBINATORICS

Fundamentals of counting

[1] Rule of sum

If T_1 and T_2 are two different tasks such that T_1 can be performed in 'm' ways and T_2 can be performed in 'n' different ways and the two tasks cannot be performed simultaneously, then performing either of the task T_1 or T_2 will be done in $m + n$ ways.

[2] Rule of product

If a procedure can be broken down into two stages such that the first stage has m possible outcomes and for each of these outcomes there ~~a re~~ are 'n' possible outcomes for second stage. Then the entire procedure can be completed in $m \times n$ ways.

① Consider a person having 8 shirts and 5 ties. Determine the number of ways of choosing shirt and tie.

In the above problem we can observe that there 8 possible ways of selecting a shirt and for every selection, we have 5 ways of selecting a tie. Therefore by rule of product, we have

No. of possible ways of selecting a shirt and tie $= 8 \times 5 = 40$ ways.

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② Suppose a library has 12 books on Math, 10 books on Physics and 16 books on Computers. Determine the number of ways a student can choose one book ~~of~~ from the library.

We can observe that by rule of sum, we have three tasks st selecting one book either from Math, Physics, Computers.

$\therefore$ No. of ways of choosing a book from library

$= 12 + 10 + 16$

$=$ 38 ways

③ A lisense plate has 2 english letters followed by 4 digits. How many license plates will be obtained

i) If repetitions are allowed

ii) If the plates have only ~~vove~~ vowels and even digits (repetitions are allowed)

iii) If no letter ~~are~~ or digit is repeated.

Consider the license plate of the form,

__ __ __ __ __ __

i) Since repetition is allowed, there are 26 alphabets and 10 digits

26	26	~~10~~ 10	10	10	10

Total no. of license plates $= 26 \times 26 \times 10^4$ [By rule of product]

$=$

give explanation

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ii) Vowels: a, e, i, o, u = 5
Even digits : 0, 2, 4, 6, 8 = 5

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$
5 5 5 5 5 5

No. of license plates = 5^6 [Rule of product]

iii) Since repetition is not allowed. (give explanation)

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$
26 25 10 9 8 7

No. of license plates = $26 \times 25 \times 10 \times 9 \times 8 \times 7$ [Rule of product]

4. A bit is made up of either 0 or 1, a byte is a sequence of 8 bits. Find 4
i) the no. of bytes that can be obtained
ii) No of bytes beginning with 11 and ending with 11.
iii) No. of bytes that begin with 11 and don't end with 11.
iv) No. of bytes that begin with 11 or end with 11.

Since a byte is made up of 8 bits and every bit is made up of 0 or 1,
$\therefore$ For each bit we have two ways of choosing/selecting.

i) $\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$
2 2 2 2 2 2 2 2

$\therefore$ No. of bytes $= 2^8$ [Rule of product]
$= 256$

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ii) $\underline{1}\ \underline{1}\ \underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}\ \underline{1}\ \underline{1}$

$\therefore$ No of possible bytes $= 2^4$ [Rule of product]

$= \underline{16}$.

iii) ~~the~~

No. of possible bytes = No. of bytes beginning with 11 − No. of bytes beginning and ending with 11

$= 2^6 - 2^4$

$= 64 - 16$

$= \underline{48}$.

iv) No. of bytes that begin with 11 or end with 11.

note: Take three cases separately

= [No. of bytes that begin with 11] + [No. of bytes that ~~be~~ end with 11] − 2 [No. of bytes that begin and end with 11]

[Rule of sum]

$= 2^6 + 2^6 - 2 \cdot 2^4$ [Rule of product]

$= 2^7 - 2^5$

$= 128 - 32 = \underline{96}$

~~= 112 96~~

⑤ Find the total no. of integers formed using the digits {1, 2, 3, 4} if

i) No digits are repeated

No. of digits = 4

Let S_1, S_2, S_3, S_4 be the integers of one digit, two digit, three digit and 4 digits.

case i) No. of ways of forming $S_1 = 4$

ii) " " " " $S_2 = 4 \times 3 = 12$

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(case iii) No. of ways of forming $S_3 = 4 \times 3 \times 2 = 24$

iv) No. of ways of forming $S_4 = 4 \times 3 \times 2 \times 1 = 24$

[Rule of product]

$\therefore$ By rule of sum, the no. of integers formed

$= 4 + 12 + 24 + 24$

$= \underline{\underline{64}}$

$\frac{48}{12}$ 6

⑥ Find the number of three digit even number formed with no repetitions.

(case i) Ends with 0 $\_\ \_\ 0$

$\therefore$ No. of ways $= 9 \times 8 \times 1$

$= \underline{\underline{72}}$

case ii) Ends with 2 $\_\ \_\ 2$

But 0 cannot be in first

$\therefore$ No. of ways $= 8 \times 8 \times 1 = \underline{\underline{64}}$

(case iii) Ends with 4 $\_\ \_\ 4$

$\therefore$ No. of ways $= 8 \times 8 \times 1 = \underline{\underline{64}}$

(case iv) Ends with 6 $\_\ \_\ 6$

$\therefore$ No. of ways $= 8 \times 8 \times 1 = \underline{\underline{64}}$

(case v) Ends with 8 $\_\ \_\ 8$

$\therefore$ No. of ways $= 8 \times 8 \times 1 = \underline{\underline{64}}$

$\therefore$ Total no. of ways $= 72 + 64 \times 4$

$= \underline{\underline{328}}$

Permutations

Given a collection of n distinct objects and the number of arrangements of r distinct objects is denoted by ^nP_r (or) $P(n,r)$

By using the rule of product we can obtain the number of linear arrangements of 'r' objects as ~~$^nP_r = n\times(n-1)\times(n-2)\times\ldots$~~

$$^nP_r = \frac{n!}{(n-r)!} \qquad 1 \le r < n$$

$$^nP_r = \frac{n\times(n-1)\times(n-2)\times\cdots\times 1}{(n-r)(n-r-1)\times\cdots\times 1}$$

note:

1) If $r = n$, arrangement of all n objects

$$= {}^nP_n = \frac{n!}{(n-n)!} = n!$$

2) To find the number of permutations that can be formed from the collection of 'n' objects where

$$n = \underbrace{n_1}_{\text{no. of type 1 objects}} + \underbrace{n_2}_{\text{no. of type 2 objects}} + \ldots + n_k$$

Then $$^nP_r = \frac{n!}{n_1!\times n_2!\times\ldots\times n_k!}$$

3) Circular arrangement of n different objects is $(n-1)!$

Problems:

① 5 men & 4 women can be seated in a row such that women occupy even spaces. Then how many such arrangements are possible?

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No. of arrangements of women = ${}^4P_4 = 4!$
No. of arrangements of men = ${}^5P_5 = 5!$

By rule of product,
Total no. of arrangements will be equal to $5! \times 4! = 120 \times 24 = 2880$ ways

② In how many ways 6 men and 6 women will be seated in a row if
i) there are no restriction
ii) men and women occupy alternate seats.

i) Since there are no restrictions and total people is 12, total arrangements
$= {}^{12}P_{12} = 12! = 479001600$

ii) (case i) When men take odd place and women taken even.

∴ ~~Toto~~ No. of arrangements $= {}^6P_6 \times {}^6P_6 = 6! \times 6! = 518400$

(case ii) When men taken even place and women take odd place.

∴ No. of arrangements $= {}^6P_6 \times {}^6P_6 = 6! \times 6! = 518400$

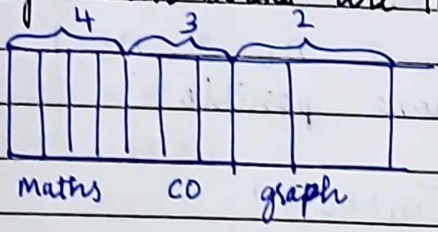
∴ By rule of sum, we can total no. of arrangements men and women can be seated
$= 518400 + 518400$
$= 1036800$

③ 4 Different ~~books~~ math books, 2 diff. graph books and 3 diff. CO books are to be arranged in a shelf. How many different arrangements are possible if

a) books of particular subject are together

b) Only math books are together.

a)



The different math books can be arranged among themselves in 4! ways and CO books in 3! ways and graph books in 2! ways

Since we want all books of particular subject together we now have 3 groups of books which can be arranged in 3! ways.

∴ Total no. of possible arrangements considering the books of particular subjects together

$= 3! \times 4! \times 2! \times 3! \quad = \underline{\underline{1728}}$ ways

b) We take math books as 1 unit as they have to be together. We now have six book arrangements done in 6! ways and have 4 books in math that can be arranged in 4! ways.

∴ Total no. of arrangements where only math books are together $= 6! \times 4!$

$= \underline{\underline{17280}}$ ways.

④ Find the no. of distinguishable permutations of all letters in the word given below.

i) CALCULUS

ii) MATHEMATICS

i) Number of letters, $n = 8$

$n_C = 2$, $n_A = 1$, $n_L = 2$, $n_U = 2$, $n_S = 1$

$\therefore$ No. of permutations possible

$$= \frac{n!}{n_C!\, n_A!\, n_L!\, n_U!\, n_S!}$$

$$= \frac{\cancel{8x}\ 8!}{2!\ 1!\ 2!\ 2!\ 1!}$$

$$= \frac{\overset{4}{\cancel{8}} \times 7 \times \overset{3}{\cancel{6}} \times 5 \times \overset{2}{\cancel{4}} \times 3 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2}}$$

$= 5040$ words

ii) $n = 11$

$n_M = 2$, $n_A = 2$, $n_T = 2$, $n_H = 1$, $n_E = 1$, $n_I = 1$, $n_C = 1$, $n_S = 1$

$\therefore$ No. of permutations

$$= \frac{n!}{n_M!\, n_A!\, n_T!\, n_H!\, n_E!\, n_I!\, n_C!\, n_S!}$$

$$= \frac{11!}{2!\ 2!\ 2!}$$

$= 4989600$ words

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(5) Considering the word MASSASAUGA

i) Find the no. of permutations of the letters of the word.

ii) Find the no. of permutations if all the 4 A's are together.

iii) How many of the permutations begin with S.

No. of letters, $n = 10$

No. of each letter,

$n_M = 1$, $n_A = 4$, $n_S = 3$, $n_U = 1$, $n_G = 1$

i) No. of permutations $= \dfrac{n!}{n_M!\, n_A!\, n_S!\, n_U!\, n_G!}$

$= \dfrac{10!}{1!\, 4!\, 3!\, 1!\, 1!}$

$= \dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{4 \times 3 \times 2 \times 1 \times 3 \times 2}$

$= 25200$ words.

ii) No. of permutations $= \dfrac{7!}{3!\, 1!\, 1!\, 1!}$

$= 840$ words

iii) No. of permutation $= \dfrac{9!}{4!\, 3!}$

$= 7560$ words

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(15) i) How many arrangements are possible for all the letters in the word?

SOCIOLOGICAL. W/O any restrictions

ii) How many arrangements contain AG adjacent to each other?

iii) How many of arrangements have all vowels adjacent?

Soln No. of letters, $n = 12$

No of each letters;

$n_S = 1, n_O = 3, n_C = 2, n_I = 2, n_L = 2, n_G = 1, n_A = 1$

i) No. of arrangements $= \dfrac{12!}{3!\,2!\cdot 2!\,2!}$

$= 9979200$ words

ii) case i) when AG are together

AG SOCIOLOICL

No. of arrangements $= \dfrac{11!}{3!\,2!\,2!\,2!}$

case ii) when GA are together

GA SOCIOLOGICL

No. of arrangements $= \dfrac{11!}{3!\,2!\,2!\,2!}$

Total no. of arrangements $= \dfrac{11!}{3!\,2!\,2!\,2!} + \dfrac{11!}{3!\,2!\,2!\,2!}$

$= \dfrac{11!\ \cancel{2}1}{3!\,2!\,2!\,\cancel{2}1}$

$=$

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iii) All vowels are together
(OIOOIA) SCLGCL

~~Total no. of permutation~~ No. of arrangement $= \frac{7!}{2! \times 2!} = 1260$

Since there are 6 vowels.
$n_A = 1, n_O = 3, n_I = 2$

No. of arrangement of vowels $= \frac{6!}{3!\ 2!\ 1!} = 60$

The total no. of arrangement when all vowels are adjacent $= 1260 \times 60$
$= 75600$ ~~ways~~ words.

7. Consider the word MISSISSIPPI
i) Find the permutations of all letters of word.
ii) How many of them begin with I?
iii) How many of them begin and end with S?
iv) How many of them are possible such that no P are adjacent to each other?

i) No. of letters = 11
No. of each letter,
$n_M = 1, n_I = 4, n_S = 4, n_P = 2$

$\therefore$ No. of arrangements $= \frac{11!}{4!\ 4!\ 2!} = \frac{11 \times 10 \times 9 \times \cancel{8} \times 7 \times \cancel{6} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$
$= 34650$ ways

ii) I MISSISSIPP
~~No~~ $n_I = 3$.

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Since ~~1~~ 1 I is fixed, the total no. of permutations $= \cancel{\frac{11!}{3!4!2!}}$ $= \frac{10!}{3!4!2!}$

$= \cancel{138600}$ $= \cancel{12600}$ ~~20400~~

$= 12600$

iii) S MISSISSIPPI S

$n_S = 2$.

Since S is fixed in beginning and end, total no. of permutations $= \frac{9!}{2!4!2!} = 3780$.

iv) If P is adjacent to each other, take as 1 unit.

Then no. of permutations $= \frac{10!}{4!4!} = 6300$

$\therefore$ No. of permutations where P is not adjacent

= Total possible − when adjacent

= 34650 − 6300

= 28,350

Find beginning with 5, 6, 7

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HW

(8) How many ~~posit~~ positive integers can be formed using the digits 3, 4, 4, 5, 5, 6, 7 if we want the integers to exceed ~~5 followed with 6 zeros~~? ~~cases~~ 5000000?

Case i) Leading digit is 5 $\quad 5\_\_\_\_\_\_$

$\therefore$ No. of permutations = $\frac{6!}{2!}$

~~case~~

Case ii) Leading digit is 6

$6\_\_\_\_\_\_$

$\therefore$ No. of permutations = $\frac{6!}{2!\cdot 2!}$

Case iii) Leading digit is 7

$7\_\_\_\_\_\_$

$\therefore$ No. of permutations = $\frac{6!}{2!\,2!}$

$\therefore$ Total no. of integers $= \frac{6!}{2!} + \frac{6!}{2!2!} + \frac{6!}{2!2!}$

$= \underline{\underline{720}}$.

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(19) Consider 7 people to be arranged on a circular table

i) In how many ways can we arrange them w/o any restrictions ?

ii) How many arrangements are possible if two designated people A and B ~~athy~~ always sit together ?

iii) If two people A and B are not sitting next to each other, then how many arrangements are possible ?

$n = 7$

i) No. of arrangement $= (7-1)! = 6! = 720$

ii) Consider A & B as one unit so $n = 6$.

No. of arrangement $= (n-1)! \times 2! \rightarrow$ A & B

$= (6-1)! \times 2! = 5! \times 2$

$= 240$

iii) No. of arrangement = total possible − when AB are ~~too~~ adjacent

$= 6! - (5! \times 2)$

$= 720 - 240$

$= 480$.

Combinatorics

The process of selecting

The total no. of combinations of r objects being selected from n objects is denoted by

$$\binom{n}{r} = C(n,r) = {}^nC_r = \frac{{}^nP_r}{r!}$$

$$\boxed{{}^nC_r = \frac{n!}{(n-r)!\,r!}}$$

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① How many committees of five members can be formed with one chairperson to be selected from 12 members?

One chairperson out of the 12 members can be selected in ${}^{12}C_1$ ways = 12 ways

The remaining 4 members can be selected out of 11 members in ${}^{11}C_4$ ways.

∴ By rule of product, the total no. of committees formed = $12 \times {}^{11}C_4$

$= \cancel{12 \times 11 \times 10 \times 9 \times 8}$

$= 12 \times \frac{11!}{7! \times 4!} \quad 11 \times 10 \times 9 \times 8$

$= \underline{\underline{3960}}$

② A gym teacher must select nine people from ~~4~~ junior and senior classes for a team. If there are 28 juniors and 25 seniors

i) In ~~wa~~ how many ways selections can be made?

ii) For a team if 4 juniors and 5 seniors are required, in how many ways this selections can be made?

③ A committee of 5 has to be formed from 7 men and 5 women. i) In how many ways selections can be carried out if no restrictions

ii) There must be two men and two women

iii) Even no. of women

iv) There must be atleast one man and atleast one woman.

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(4) A student has to answer 7 out of 10 questions in an exam. How many ways can we make the selection

i) There are no restrictions

ii) If answering 1st two questions are compulsory

iii) He must answer atleast 4 of the first 6 questions.

Ans (2) i) There are 28 juniors and 25 seniors.

To Total girls = 28 + 25 = 53

To No. of ways to make selection = $\binom{53}{9} = {}^{53}C_9$

$= 4431613550$ ways.

ii) ~~If team requires~~ We can select ~~fo~~ four juniors in ${}^{28}C_4$ ways and 5 seniors in ${}^{25}C_5$ ways.

$\therefore$ Total no. of ways of selecting four juniors and five seniors $= {}^{28}C_4 \times {}^{25}C_5$

$= 1087836750$ ways.

Ans (3) i) No. of selections $= {}^{12}C_5 = \frac{12!}{7!\,5!} = \underline{\underline{792}}$

ii) ~~To at~~ We can select two men in ${}^{7}C_2$ ways and 2 women in ${}^{5}C_2$ ways. ~~We~~ ~~can~~ select one person from remaining in ${}^{8}C_1$ ways.

$\therefore$ Total no. of selections $= {}^{7}C_2 \; {}^{5}C_2 \; {}^{8}C_1$

$= \frac{7\times \cancel{6}3\,\cancel{5!}}{2!\,\cancel{5!}} \; \frac{5\times \cancel{4}2\,\cancel{3!}}{2!\,\cancel{3!}} \; \frac{8\,\cancel{7!}}{\cancel{7!}} \; 8$

$= \underline{\underline{1680}}$

iii) Case i) There are 2 women.

$\therefore$ Total selection $= {}^{5}C_2 \times {}^{10}C_3$

$= \frac{5\times \cancel{4}2\,\cancel{3!}}{2!\,\cancel{3!}} \; \frac{10\times \cancel{9}3\times 8\,\cancel{7!}}{\cancel{3!}\,\cancel{7!}} = \underline{\underline{2400}}$

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case ii) There are 4 women

No. of selection $= {}^5C_4 \times {}^8C_1$

$= \frac{5!}{4!\,1!} \times \frac{8!}{1!\,7!}$

$= \underline{40}$.

$\therefore$ Total no. of selections $= 2400 + 40 = \underline{2440}$.

iv) (case i) 1 man + 4 women $= {}^7C_1 \times {}^5C_4 = 7 \times 5 = 35$

(case ii) 2 men + 3 women $= {}^7C_2 \times {}^5C_3 = \frac{7 \times 6}{2} \times \frac{5 \times 4 \times 3}{6}$

$= 210$

(case iii) 3 men + 2 women $= {}^7C_3 \times {}^5C_2 = \frac{7 \times 6 \times 5}{6} \times \frac{5 \times 4}{2} = 350$

(case iv) 4 men + 1 woman $= {}^7C_4 \times {}^5C_1 = \frac{7 \times 6 \times 5}{6} \times 5 = 175$

Total selection $= 35 + 210 + 350 + 175 = \underline{770}$.

Ans (4) i) No. of ways $= {}^{10}C_7 = \frac{10 \times 9 \times 8}{6} = \underline{120 \text{ ways}}$

ii) If first two questions are compulsory, out of 8 we can choose 5 questions.

$\therefore$ Total no. of ways $= {}^8C_5 = \frac{8 \times 7 \times 6}{6} = 42$ ways.

iii) (case i) 4 from first six and 3 from last 4.

This will happen in ${}^6C_4 \times {}^4C_3$ ways.

case ii) 5 from first six and 2 from last 4.

This will happen in ${}^6C_5 \times {}^4C_2$ ways.

(case iii) 6 from first six and 1 from last 4

This will happen in ${}^6C_6 \times {}^4C_1$ ways.

Total selection $= {}^6C_4 \times {}^4C_3 + {}^6C_5 \times {}^4C_2 + {}^6C_6 \times {}^4C_1 = \frac{6 \times 5 \times 4}{2} + 6 \times \frac{4 \times 3}{2} + 4$

$= \underline{100 \text{ ways}}$