Basics of computing

18/11/17
Saturday

Sum Rule - if an event can occur in m ways and another event can occur in n ways and if these events cannot occur simultaneously then one of the 2 events can occur in m+n ways.

Product Rule - If an event H is decomposed into 2 diff. tasks, then if an event A can occur in m ways and event B can occur in n diff ways then the task H can be performed in m×n ways.

Q. A book shelf has 6 different English books, 8 diff French books & 10 diff German books. To find how many ways of selecting:-
(i) 3 books from each language
(ii) 1 book in any one of the lang.

A. (i) $6 \times 8 \times 10 = 480$
(ii) $6 + 8 + 10 = 24$

Q. Suppose a person has 3 shirts and 5 ties, how many diff. ways of choosing a shirt and a tie?

A. $3 \times 5 = 15$

Q. Suppose we wish to construct a sequence of 4 symbol in which first 2 are English letters, next two are single digit nos. If no letter or digit can be repeated. Find the no. of diff sequence we can construct?

A.

26	25	10	9

$26 \times 25 \times 10 \times 9 = 58500$

Q. Suppose a restaurant sells 6 south Indian dishes, 4 North Indian, 3 hot drinks & 2 cold drinks for a breakfast. A student wishes to buy 1 SI dish and 1 hot drink or 1 NI dish & 1 cold drink. In how many ways can he buy his breakfast

A. $6 \times 3 + 4 \times 2$

$= 18 + 8 = 26$

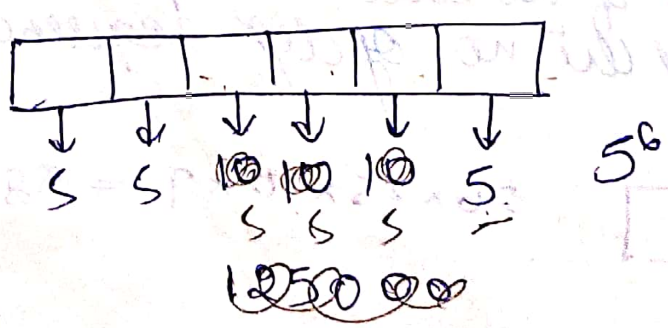
Q. There are 20 married couples in a party. Find the no. of ways of choosing 1 woman & 1 man from the party s.t. the 2 are not married to each other.

A. $20 \times 19 = 380$

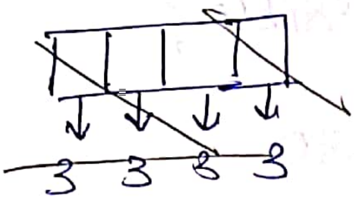
Q. There are 4 bus routes b/w A and B and 3 bus routes b/w B & C. Find the no. of ways a person can make a round trip from A to A through B & C with repeating the ~~[illegible]~~ routes.

A. $4 \times 3 \times 4 \times 3 = 144$ (repetition allowed)

Q. A licence plate consists of 2 English letters which has only a, e, i, o, u followed by 4 digits. If repetitions are allowed then how many licence plates will have a e i o u and even digits.



Q. Find the no. of linear arrangement of 4 letters in BALL.

A. 

81

if there are repeated letters, then:

$$\frac{n!}{n_1!\, n_2! \dots}$$

$n!$ → no. of spaces

$n_1!$ → no. of repeated letters

$$\frac{4!}{2!} = 12$$

Q. No. of linear arrangement of 6 letters in PEPPER.

$$\frac{6!}{3!\,2!} = \frac{6\times\cancel{4}^{2}\times 5}{\cancel{2}} = \underline{\underline{12\times 5 = 60}}$$

Q. ~~Find the no~~ DATABASES

$$\frac{9!}{3!\,2!} = 9\times8\times7\times6\times5\times\cancel{4}\,2 = 30240$$

Q. Find the no. of permutations of the letters of the word. MASSASAUGA. In how many of these all 4 A's are together? How many of them begin with S.

(i) $\frac{10!}{3!\,4!} = 25200.$

(ii) $\frac{7!}{3!} = 840$ – consider the 4 A's as 1 block.

(iii) $\frac{9!}{2!\,4!} = 7560$

Q It is required to seat 5 men & 4 women in a row so that women occupy even places. How many such arrangements are possible?

A.

$\cancel{5\times4\times4\times4}$ $5! \times 4! = 2880$.

$5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1$

Q. In how many ways can 6 men and 6 women be seated in a row
(i) if any person may sit next to any other
(ii) if men and women must occupy alternate seats.

A. (i) $12! = 479{,}001{,}600$

(ii) $6!\,6! \times 2 = 1{,}036{,}800$

Q. 4 diff maths books, 5 diff cs books, 2 diff control theory books are to be arranged in a shelf. How many arrangements are possible
(i) if the books in each particular subject must be together? (ii) only math books must be together

$\to$ to arrange the blocks

A. (i) $\cancel{3!} \times 4! \times 5! \times 3! \times 2!$

(ii) $4! \times 8!$

Q Suppose we are selected r objects from a set of n, the set of r objects being selected is called combination of r ____ which is represented by $^nC_r = \frac{n!}{(n-r)!\,r!}$

Q An airhostess is having a dinner party for some members of her committee. Because of the size of her home, she can invite only 11 out of 20

How many possible ...?

$^{20}C_{11} = \frac{20!}{11!\,9!} = 167,960.$

Q. A gym teacher must select 9 girls from seniors and juniors if there are 25 seniors and 28 juniors.
(i) How many selections can be done?
(ii) If 2 juniors & 1 senior must be in a team
(iii) Team must have 4 juniors and 5 seniors.

A. (i) $^{53}C_9$ =

(ii) $\cancel{^{25}C_1 \times {}^{28}C_2}$ $^{50}C_6$. =

(iii) $^{25}C_5 \times {}^{28}C_4$

Q. A gym teacher must make 4 volley balls team of 9 girls each from 36 girls in how many ways can she select these 4 teams.

A. $^{36}C_9 \times {}^{27}C_9 \times {}^{18}C_9 \times {}^{9}C_9$

=

Q. Find out all ~~combination~~ arrangement of this word. TALLAHASSEE and how many should not have ~~alternate~~ adjacent A's.

A. (i) $\frac{11!}{3!\,2!\,2!\,2!}$

T L L H S S E E

(ii) $\cancel{\frac{9!}{2!\,2!\,2!}}$

$\frac{8!}{2!\,2!\,2!} \times {}^9C_3$

Q A women 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following ways:-

(i) there are no restrictions on choices

(ii) 2 particular persons will not attend separately

(iii) 2 ______ together.

A. (i) $^{11}C_5$

(ii) $^{9}C_{3} + {}^{9}C_5$

(iii) $^{9}C_5 + {}^{10}C_5$ $\quad\quad 2 \times {}^{9}C_4 + {}^{9}C_5$

378 $\quad\quad$ 378.

Combination with repetition

21/11/17
Tuesday

Q. 3 scoops of ice cream from 5 flavours

b	c	d	s	v

$^{r+n-1}C_r$.

c c c $\rightarrow$ 0 0 0 $\rightarrow \rightarrow \rightarrow$

b d v $\quad$ 0 $\rightarrow \rightarrow$ 0 $\rightarrow \rightarrow$ 0

b v v $\quad$ 0 $\rightarrow \rightarrow \rightarrow$ 0 $\rightarrow$ 0 0

Q A donut shop offers 20 kinds of donuts assuming that there are atleast a dozen of each kind. In how many ways we can select a dozen of donuts?

$$C^{20+12-1}_{12} = C^{31}_{12} = 141,120,525$$

Q In how many ways we distribute 7 Bananas & 6 oranges among 4 children so that each child recieves atleast 1 banana.

selecting r items for a set n

$$C^{3+4-1}_{3} \times C^{6+4-1}_{6}$$

$20 \times 84 = 1680$.

6 marks

Q A message is made up of 12 different symbols and is to be transmitted through a comm. channel. In add. to 12 symbols, the transmitter will also send 45 blank spaces b/w the symbols with atleast 3 spaces b/w each pair of consecutive symbols. In how many ways can transmitter send such a mssg?

A. $12! \times C^{11+12-1}_{12} = 12! \times C^{22}_{12} = \underline{3.0974 \times 10^{14}}$

There are 12! ways to arrange 12 diff. symbols. and for each of these arrangements, there are 11 positions b/w the 12 symbols because there must be atleast 3 spaces b/w successive symbols, we use upto 33 of 45 spaces and must now locate the remaining 12 spaces. This is now selection with repetition of size 12 ~~for a~~ from a collection of size 11.

Q Consider the following prog. segment where i, j, k are the integer variables.

A

```
for i=1 to 20 do
for j=1 to i do
for k=1 to j do
print(i*j*k);
```

How many times the print statement is executed?

i | 20

j | i

k | j

$1 \times (2+1) \times ($

selecti of 3 loops 20 within

$^{20+3-1}C_3 = {}^{22}C_3$

$1 \le k \le j \le i \le 20$

$= 1540$

Q. Determine all integer solution of the equation
$x_1 + x_2 + x_3 + x_4 = 7$, $x_i \ge 0$ $\forall\ 1 \le i \le 4$

A. Selecti of 7 items from a set of 4.

$^{7+4-1}C_7$?

Q. In how many ways one can distribute 10 marbles amongst 6 different containers.

A. $^{10+6-1}C_{10} = {}^{15}C_{10} = 3003$

16 marks $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$

Q. Determine the no. of integer solutions of
$x_1 + x_2 + x_3 + x_4 = 32$

a) $x_i \ge 0$ $1 \le i \le 4$

b) $x_i > 0$ $1 \le i \le 4$

c) $x_1, x_2 \ge 5$ $x_3, x_4 \ge 7$

d) $x_i \ge 8$ $1 \le i \le 4$

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A. a) C^{32+4-1}_{32}

b) C^{32+4-1}_{32} - ~~(4+4+3)~~

$C^{28+4-1}_{28} = 4495$

c) $n = 4 \quad r = 8$

x_1	x_2	x_3	x_4
5	5	7	7

$\Rightarrow$ these values already assign the rest is to be distributed

d) 1

Q. In how many ways 10 marbles be distributed among 5 children.

(i) no restriction

(ii) if each child gets atleast 1 marble.

C^{5+5-1}_{5}

A. (i) $C^{10+5-1}_{10} = 1001$

(ii) $C^{5+5-1}_{5} = 126$.

$(x+y)^n = \sum_{r=1}^{n} {}^nC_{n-r}\, x^n \; {}^nC_r\, y^{n-r}$

Binomial theorem

Q Find the binomial coeff of x^5 & y^2 in $(x+y)^7$

A. $(x+y)^7 = \sum_{r=0}^{7} {}^7C_{7-r}\, x^{7-r}\, y^r$

for x^5 $\quad 7-r=5$

$r = 2$

${}^7C_5 = 14$

for y^2 $\quad r=2 \quad {}^7C_5 = 14$

Q Coeff of a^5 & b^2 in the exp. of $(2a-3b)^7$

A. $(2a-3b)^7 = \sum_{r=0}^{n} {}^nC_{n-r} (2a)^{n-r} (-3b)^r$

$7-r = 5$
$r = 2$

${}^7C_5 \times 2^5 (-3)^2 = 14 \times$

23/11/17
Thursday

Q $(x+y)^4$

$\sum_{j=0}^{4} \binom{4}{j} x^{4-j} y^j$

Q $x^{12}y^{13}$ in $(x+y)^{25}$

$\binom{25}{12} = 5200300$

Q. Explain multinomial theorem.

For the integers, n, t the coeff. of $x_1^{n_1}, x_2^{n_2}, x_3^{n_3} \ldots x_t^{n_t}$ in the expansion $(x_1+x_2+\ldots+x_n)^n$ is

$\frac{n!}{n_1! n_2! \ldots n_t!}$ - where each n_i is an integer with $0 \le n_i \le n \ \forall \ 1 \le i \le t$ and $n_1+n_2+\ldots n_t = n$

Q Find the coef. of $x^2y^2z^3$, xyz^5 and x^3z^4 in the expansion of $(x+y+z)^7$

A. $x^2y^2z^3$ $\binom{n}{n_1\, n_2\, n_3} = \binom{7}{2\ 2\ 3} = \frac{7!}{2!2!3!} = 210$

$xy^5 \quad \binom{7}{1\ 1\ 5} \quad \frac{7!}{5!1!1!} = 42$

$x^3z^4 \quad \binom{7}{3\ 0\ 4} = \frac{7!}{3!4!} = 35$

Q. find the term which contain $x^{11}y^4$ in the exp of $(2x^3 - 3xy^2 + z^2)^6$

A general term $\binom{6}{n_1\ n_2\ n_3}(2x^3)^{n_1}(-3xy^2)^{n_2}(z^2)^{n_3}$

$$\binom{6}{n_1\ n_2\ n_3} 2^{n_1} x^{3n_1+n_2} (-3)^{n_2} y^{2n_2} z^{2n_3}$$

$3n_1 + n_2 = 11 \qquad n_1 = 3$

$2n_2 = 4 \qquad n_2 = 2$

$2n_3 = 0 \qquad n_3 = 0$] no need to eliminate z. can be given a power

$$\binom{6}{3\ 2\ 0} 2^3 x^{11}\, 9\, y^4 z^0 = \frac{6!}{3!2!} \times 2^3 \times 9\ x^{11}y^4$$

$$= 4320\, x^{11}y^4$$

Q. xyz^2 in $(2x-y-z)^4$

$$\binom{4}{n_1 n_2\ n_3}(2x)^{n_1}(-y)^{n_2}(-z)^{n_3}$$

$n_1 = 1 \quad n_2 = 1 \quad n_3 = 2$

$$\frac{-4!}{1!1!2!}\, 2\ xyz^2 = -24xyz^2$$

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Q: $a^2b^3c^2d^5$ $(a+2b-3c+2d+5)^{16}$.

$$\binom{16}{n_1\, n_2\, n_3\, n_4\, n_5} a^{n_1}(+2b)^{n_2}(-3c)^{n_3}(2d)^{n_4}5^{n_5}$$

$n_1=2$ $n_2=3$, $n_3=2$ $n_4=5$.

$n_5 = 16-12=4$

$$\frac{16!}{2!\,3!\,2!\,5!\,4!} 2^3(-3)^2\, 2^5\, 5^4\, a^2b^3c^2d^5$$

$= 4.3589\times10^{14} a^2b^3c^2d^5$

Principle of Inclusion and exclusion

$|A\cup B| = |A|+|B| - |A\cap B|$ for inclusion.

Consider a set S with cardinality of $S=n$ and the conditions c_i st. $1\le i\le t$ satisfied by some elements of S, the no. of elements of set S that satisfy none of the conditions c_i is denoted by

$\bar{N} = N(\bar{C_1}\bar{C_2}\bar{C_3}\ldots\bar{C_t})$, given by the formula

$$\bar{N} = N-[N(C_1)+N(C_2)+\ldots+N(C_t)] + [N(C_1C_2)+N(C_1C_3)+\ldots+N(C_1C_t) \ldots + N(C_{t-1}C_t)] - [N(C_1C_2C_3)+N(C_1C_2C_4)+\ldots+N(C_{t-2}C_{t-1}C_t)] + \ldots (-1)^t N(C_1C_2\ldots C_t)$$

OR

$$\overline{N} = N - \sum_{1 \le i \le t} N(c_i) + \sum_{1 \le i \le j \le t} N(c_i c_j) + \ldots$$

$$\ldots + \sum_{1 \le i \le j \le k \le t} N(C_i C_j C_k) + \ldots\ldots (-1)^t N(C_1 C_2 \cdots C_t)$$

$$\boxed{\begin{aligned} &N(C_1) = |A| \\ &N(C_2) = |B| \\ &N(C_1 C_2) = |A \cap B| \\ &|\bar{A} \cap \bar{B}| = N - |A \cup B| \end{aligned}}$$

Q. Among the students in a hostel 12 students study maths, 20 study chem & 8 study biology, 20 physics.

$|A| = 12$ $\quad |A \cap B| = 5$ $\quad |B \cap D| = 4$

$|B| = 20$ $\quad |A \cap C| = 7$ $\quad |C \cap D| = 3$

$|C| = 20$ $\quad |A \cap D| = 4$ $\quad |A \cap B \cap C| = 3$

$|D| = 8$ $\quad |B \cap C| = 16$ $\quad |A \cap B \cap D| = 2$

$|\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}| = 71$

$|B \cap C \cap D| = 2$ $\quad |A \cap B \cap D \cap C| = 2$

$|A \cap C \cap D| = 3$ $\quad$ Find total no. of student.

A. $N = |\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}| + |A \cup B \cup C \cup D|$

$= 71 + 12 + 20 + 20 + 8 - 5 - 7 - 4 - 16 - 4 - 3$
$\quad + 3 + 2 + 2 + 3 - 2$

$= 100$

Q. How many integers between 1 & 300
(i) divisible by atleast one of
(i) 5, 6, 8 (ii) none of 5, 6, 8.

37

$\frac{300}{24}$ $\frac{25}{50}$ 42

A. $|A \cup B \cup C| = S_1 - S_2 + S_3 - \Sigma N(C_i C_j C_k)$

$S_1 = \Sigma N(C_i)$, $S_2 = \Sigma N(C_i C_j)$

$S_1 = N(C_1) + N(C_2) + N(C_3)$
$= 60 + 50 + 37$

40

~~30~~ 8

$S_2 = N(C_1C_2) + N(C_1C_3) + N(C_2C_3)$
$= 10 + 12 + 7$

24×5
300
120

$S_3 = N(C_1C_2C_3) = 2$.

$N = 120$

(ii) $300 - 120 = 180$

27/11/17
Monday

Q. Determine the no. of +ve integers b/w $1 \le i \le 10$
which are not divisible by 2, 3 or 5.

A. 26

$N(C_1) = 50$
$N(C_2) = 33$
$N(C_3) = 20$

$\bar{N} = 100 - (50 + 33 + 20 - 16 - 6 - 10 + 3)$
$= 26$

$N(C_1C_2) = 16$
$N(C_2C_3) = 6$
$N(C_1C_3) = 10$
$N(C_1C_2C_3) = 3$

Q. In how many ways can 26 letters of an alphabet be permuted so that none of the pattern car, dog, pun or byte occurs

A. $N = 26!$

$N(C_1) = (23+1)!$ considering car as 1 letter & removing it from the 26 letters

$N(C_2) = 24!$

$N(C_3) = 24!$

$N(C_4) = 23!$

$N(C_1C_2) = (26-6+2)! = 22!$

$N(C_2C_3) = 22!$

$N(C_3C_4) = 21!$

$N(C_1C_3) = 22!$

$N(C_1C_4) = 21!$

$N(C_2C_4) = 21!$

$N(C_1C_2C_3) = 20!$ $26-9=17+3$

$N(C_2C_3C_4) = 19!$

$N(C_1C_2C_4) = 19!$

$N(C_1C_2C_3C_4) = 17!$

$\overline{N} = N - |A \cup B \cup C \cup D|$

$= N - [24! \times 3 + 23! - (3 \times 22! + 3 \times 21!)$

$+ 19! \times 2 + 20! - 17!]$

$= 4.014 \times 10^{26}$.

Q. Find the no. of permutation from a to z from which none of the patterns spin, game, path or net occurs.

spinel

A. $N = 26!$

$N(C_1) = 23!$
$N(C_2) = 23!$
$N(C_3) = 23!$
$N(C_4) = 24!$

$N(C_1C_2) = 20!$
$N(C_1C_3) = 0 \Rightarrow$ when letters are repeated
$N(C_1C_4) = 0$
$N(C_2C_3) = 0$
$N(C_2C_4) = 0$
$N(C_3C_4) = 0$

$N(C_1C_2C_3) = 0$
$N(C_2C_3C_4) = 0$
$N(C_1C_3C_4) = 0$

$N(C_1C_2C_3C_4) = 0$

$\bar{N} = 26! - (23! \times 3 + 24! - 20!)$

$= 4.0259 \times 10^{26}$

Q. Determine no. of permutation of the letters JNUISGREAT such that none of the words JNU, IS, GREAT occur as consecutive letters such as TSJNUGREAT, UNJGREATSIS, JNUTUISGREA etc. are not allowed

$N = 10!$

$N(C_1) = 8!$ $\quad N(C_1C_2) = 7!$ $\quad N(C_1C_2C_3) = 3!$

$N(C_2) = 9!$ $\quad N(C_2C_3) = 5!$

$N(C_3) = 6!$ $\quad N(C_1C_3) = 4!$

$$\bar{N} = 10! - (8! + 9! + 6! - 7! - 5! - 4! + 3!)$$
$$= 3230058$$

Q Find the no. of non-negative integer soln. of the eq. $x_1 + x_2 + x_3 + x_4 = 18$ s.t. $x_i \le 7$ for $1 \le i \le 4$.

A. $N = {}^{18+4-1}C_{18}$ $\qquad x_i \ge 8$

$N(C_1) = x_i \ge 8$

$= 10 \quad (18-8)$ $\qquad {}^{10+4-1}C_{10}$

$N(C_2) = 10$

$N(C_3) = 10$

$N(C_4) = 10$ $\qquad {}^{13}C_{10}$

$N(C_1C_2) = N(C_2C_3) = N(C_3C_4) = N(C_1C_3) = N(C_2C_4)$
$N(C_1C_4)$
$= {}^5C_2$

$N(C_1C_2C_3) = N(C_2C_3C_4) = N(C_1C_3C_4) = 0$

$N(C_1C_2C_3C_4) = 0$

$$\bar{N} = {}^{21}C_{18} - (4 \times {}^{13}C_{10} - 6 \times {}^5C_2)$$
$$= 246$$

∴ 246 solutions are there for the given condition

Q. Determine how many integer solution are there $x_1+x_2+x_3+x_4=19$

a) $0 \le x_i$ for all $1 \le i \le 4$

b) $0 \le x_i \le 8$ for all $1 \le i \le 4$

A. a) ${}^{19+4-1}C_{19} = 1540$

b) $N = 1540$

$N(C_1) = {}^{13}C_{10} = N(C_2) = N(C_3) = N(C_4)$

$N(C_1C_2) = N(C_2C_3) = N(C_3C_4) = N(C_1C_3) = N(C_1C_4)$

${}^{4}C_2 \times {}^{4}C_1 = N(C_2C_4) = {}^{4}C_1$

$N(C_1C_2C_3) = N(C_2C_3C_4) = N(C_1C_3C_4) = 0$

$N(C_1C_2C_3C_4) = 0$

$\bar{N} = 1540 - [4 \times {}^{13}C_{10} - 6 \times {}^{4}C_1]$

$= 372$

Q.① 6 married couples are to be seated at a circular table. In how many ways can they arrange themselves so that no wife sits next to her husband.

Q.② In certain areas of the countryside there are 5 villages. An engg. is to devise a system of 2 way roads so that after the system is completed no village will be isolated. In how many ways can he do this?

① $N(C_1)$ = fix couple in seated together
$= 10!$

$N(C_2) = 10! = N(C_3) = N(C_4) = N(C_5) = N(C_6)$
$S_1 = 6 \times 10!$

$N = 11!$

$N(C_1C_2) = 9!$ $\qquad S_2 = 15 \times 9!$

$N(C_1C_2C_3) = 8!$ $\qquad S_3 = 20 \times 8!$

$N(C_1C_2C_3C_4) = 7!$ $\qquad S_4 = 15 \times 7!$

$N(C_1C_2C_3C_4C_5) = 6!$ $\qquad S_5 = 6 \times 6!$

$N(C_1C_2C_3C_4C_5C_6) = 5!$ $\qquad S_6 = 5!$

ABC, ACB, BAC, BCA, CAB, CBA

$\overline{N} = 11! - [2 \cdot 6 \times 10! - 2^2 15 \times 9! + 2^3 20 \times 8! - 2^4 15 \times 7! + 2^5 6 \times 6! - 2^6 5!]$

$= \cancel{\ldots}$

$10! \times 2$

$9! \times 2^3$

②

$N(C_1) = \cancel{(4)}\, {}^4C_2 + {}^4C_3 + {}^4C_4$

$N(C_2) = {}^4C_2$ $\qquad S_1 = \cancel{5} \times {}^4C_2$

$N(C_3) = {}^4C_2$

$N(C_4) = {}^4C_2$

$N(C_5) =$

$N(C_1C_2) = {}^3C_2$ $\qquad S_2 = 10 \times ({}^3C_2 + {}^3C_3)$

$N(C_1C_2C_3) = {}^2C_2$ $\qquad S_3 = 10$

$N(C_1C_2C_3C_4) = 0$ $\qquad S_4 = 0$

$N(C_1C_2C_3C_4C_5) = 0$ $\qquad S_5 = 0$

$\cancel{\overline{N} = {}^5C_2 - [5 \times {}^4C_2 - 10 \times {}^3C_2 + 10]}$ $\qquad 25$

$\overline{N} = N - [S_1 - S_2 + S_3] - S_4$

$N = {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$
$= 26$

$N = 26$

$\overline{N} = 26 - 25$
$= 1$

Generalization of principal

keywords: exactly, atleast

28/11/17
Tuesday

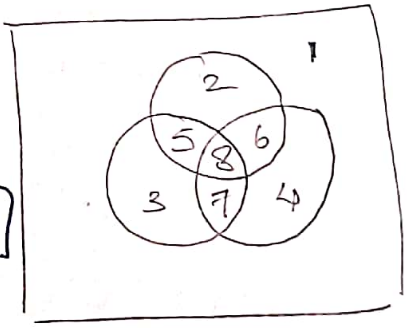
Consider a set S with cardinality of $|S| = N$ with conditions $C_1, C_2 \dots C_t$ satisfied by some of the elements of S. The inclusion & exclusion provides a way to determine $N(\overline{C_1}\,\overline{C_2} \dots \overline{C_t})$ i.e. no. of elements in S that satisfies none of the t conditions

Now we want to determine E_m which denotes no. of elements in S that satisfies exactly m of t conditions

$$E_1 = N(C_1 \overline{C_2}\,\overline{C_3} \dots \overline{C_t}) + N(\overline{C_1} C_2 \overline{C_3} \dots \overline{C_t}) \dots + N(\overline{C_1}\,\overline{C_2} \dots \overline{C_{t-1}} C_t)$$

$$E_2 = N(C_1 C_2 \overline{C_3} \dots \overline{C_t}) + N(\overline{C_1} C_2 C_3 \overline{C_4} \dots \overline{C_t}) + \dots + N(\overline{C_1}\,\overline{C_2} \dots C_{t-1} C_t)$$

$$E_1 = N(C_1) + N(C_2) + N(C_3) - 2[N(C_1 C_2) + N(C_2 C_3) + N(C_1 C_3)] + 3N(C_1 C_2 C_3)$$



$$\Rightarrow \boxed{E_1 = S_1 - 2S_2 + 3S_3}$$

$$\boxed{E_2 = S_2 - 3S_3}$$

for $t = 3$

count elements in region 5, 6, 7 twice & and those in region 8 thrice

six times

$$\boxed{E_m = S_m - \binom{m+1}{1} S_{m+1} + \binom{m+2}{2} S_{m+2}^{t} \ldots \ldots + (-1)^{t-m}\binom{t}{t-m} S_t}$$

for $t=4$,

$$E_1 = E_1 - \binom{2}{1} S_2 + \binom{3}{2} S_3 - \binom{4}{2} S_4$$

$$\boxed{L_m = S_m - \binom{m}{m-1} S_{m+1} + \binom{m+1}{m-1} S_{m+2} \ldots\ldots (-1)^{t-m}\binom{t-1}{m-1} S_t}$$

(least) is atleast 2 conditions

Q. In how many ways can the letters in the word ARRANGEMENT be arrayed so that

(i) there are exactly 2 pairs of consecutive identical letter

(ii) atleas ~~t~~ ______ letters

A. (i) $N = \dfrac{11!}{2!\,2!\,2!\,2!}$ S_1 S

$N(C_1)$ = 2 consecutive A's $= \dfrac{10!}{(2!)^3}$

$N(C_2)$ = R's $= \dfrac{10!}{(2!)^3}$

$N(C_4)$ = N's $= \dfrac{10!}{(2!)^3}$

$N(C_3)$ = E's $= \dfrac{10!}{(2!)^3}$

$S_1 = \dfrac{10!}{(2!)^3} \times 4$

$N(C_1C_2) = \frac{9!}{(2!)^2}$

$N(C_1C_3) =$,, $S_2 = 6 \times \frac{9!}{(2!)^2}$

$N(C_1C_4) =$

$N(C_2C_3) =$

$N(C_2C_4) =$

$N(C_3C_4) =$

$N(C_1C_2C_3) = 8!/2!$

$N(C_1C_2C_4) =$,, $S_3 = 3 \times \frac{8!}{2!}$

$N(C_2C_3C_4) =$,,

$N(C_1C_2C_3C_4) = 7! = S_4$

(i) $E_2 = S_2 - \binom{3}{1} S_3 + \binom{4}{2} S_4$

$= \overset{544320}{\cancel{272160}} - 3 \times 60480 + 6 \times 5040$

$= \cancel{196980} \quad 332640$

(ii) $L_m = S_2 - \binom{2}{1} S_3 + \binom{3}{2} S_4$

$= \overset{544320}{\cancel{272160}} - 2 \times 60480 + 3 \times 5040$

$= \cancel{166320} \quad 398160.$

Q. In how many ways can one arrange the letters in the word CORRESPONDENTS so that—

(i) there is no pair of consecutive identical letters

(ii) there are exactly 2 pair ______

(iii) atleast 3 pair of consecutive ______

A. $N = \dfrac{14!}{2!\,2!\,2!\,2!\,2!}$

$N(C_1)$, $N(C_2)$, $N(C_3)$, $N(C_4)$, $N(C_5)$ $\Bigg\} = \dfrac{13! \times 5}{(2!)^4}$ (S_1)

$N(C_1C_2)$, $N(C_1C_3)$, $N(C_1C_4)$, $N(C_1C_5)$, $N(C_2C_3)$, $N(C_2C_4)$, $N(C_2C_5)$, $N(C_3C_4)$, $N(C_3C_5)$, $N(C_4C_5)$ $\Bigg\} = \dfrac{12! \times 10}{(2!)^3}$

$N(C_2C_4C_5)$, $N(C_1C_4C_5)$, $N(C_1C_2C_3)$, $N(C_1C_2C_4)$, $N(C_1C_2C_5)$, $N(C_2C_3C_4)$, $N(C_2C_3C_5)$, $N(C_3C_4C_5)$ $\Bigg\}$

$^5C_3 = \dfrac{5!}{3!\,2!}$

$S_3 = \dfrac{11! \times 10}{(2!)^2}$

$N(C_1C_2C_3C_4)$, $N(C_1C_2C_3C_5)$, $N(C_2C_3C_4C_5)$, $N(C_1C_3C_4C_5)$, $N(C_1C_2C_4C_5)$ $\Bigg\} = 5 \times \dfrac{10!}{(2!)}$

$N(C_1C_2C_3C_4C_5) = 9!$

(i) $\overline{N} = N - [S_1 - S_2 + S_3 - S_4 + S_5]$

$$= \frac{14!}{(2!)^5} - \left[\frac{5 \times 13!}{(2!)^4} - \frac{10 \times 12!}{(2!)^3} + \frac{10 \times 11!}{(2!)^2} - \frac{5 \times 10!}{(2!)} + 9!\right]$$

$= 1286046720.$

(ii) $E_2 = S_2 - \binom{3}{1} S_3 + \binom{4}{2} S_4 - \binom{5}{3} S_5$

$= \frac{5 \times 13!}{16} - \frac{3 \times 10 \times 12!}{8} + \frac{6 \times 10! \times 11}{4} + 10$

$= 350{,}179{,}200.$

(iii) $L_3 = S_3 - \binom{3}{2} S_4 + \binom{4}{2} S_5$

$= 74753280.$

~~Pigeonhole Principle~~

Q. In how many ways can one arrange all of the letters in the word information so that no pair of consecutive letters occurs more than once.
(Here we want to count the arrangement such as (IINNOOFRMTA, FORTMAIINON) but not (INFORINMOTA (where IN occurs twice)
NORTFNOIAM (... NO ... ...))

A. NOI NO NO

Pigeon hole Principle

If m pigeons occupy n pigeonholes and $m > n$, then atleast 1 pigeonhole must contain 2 or more pigeons in it.

General

If m pigeons occupy n ph then atleast 1 pigeonhole must contain $p+1$ or more pigeons where $p = \frac{m-1}{n}$

Q. A bag contains 12 pairs of socks (each pair in diff. colour). If a person draws the socks 1 by 1 at random, determine atmost how many draws are required to get atleast 1 pair of matching socks.

A. 13

Q. Prove that in any set of 29 persons atleast 5 persons must have been born on the same day of the week

A. no. of days = 7

~~events~~

Rearrangements Dearrangement

30/11/17
Thursday

A permutation of n distinct objects in which none of the objects is in its natural place is called dearrangement.

Q. For eg. permutation of integers of 1 to n where 1 is not in 1st place, 2 is not in 2nd place & so on. Find the no. of dearrangements of 1, 2, 3, 4.

$$d_4 = 4!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\}$$

$$= 24\left(1 - 1 + \tfrac{1}{2} - \tfrac{1}{6} + \tfrac{1}{24}\right)$$

$$= 12 - 4 + 1 = 9$$

$$\boxed{d_n = n!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \cdots \frac{(-1)^n}{n!}\right\} = n! \times \sum_{k=0}^{n} \frac{(-1)^k}{k!}}$$

(cut in)

2413
3421

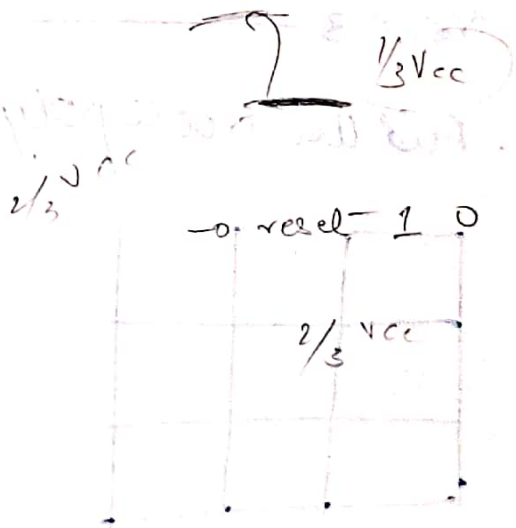
Q. Find d_5, d_6, d_7 & d_8.

A. $d_5 = 5!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right\} = 44$

$d_6 = 265$

$d_7 = 1854$

$d_8 = 14833$.



Rook Polynomial

formula:

$$\boxed{r(C, x) = 1 + r_1 x + r_2 x^2 + r_3 x^3 + \dots}$$

Q.

1	2	3
4		
	5	6

$$r(C, x) = 1 + 6x + 8x^2 + 2x^3$$

r_2

{1, 5} {2, 4} {3, 4} {4, 2} {4, 5}

{1, 6} {2, 6} {3, 5} {4, 5} {4, 6}

r_3

~~{1, 5, 6}~~ {3, 4, 5} {2, 4, 6}

Q.

1	2
3	4

$$r(c, x) = 1 + 4x + 2x^2$$

non taking rooks = not present in same row or column

$r_0 = 1$ (always)

$r_1 = 1$ non taking rooks

$r_2 = 2$ ———

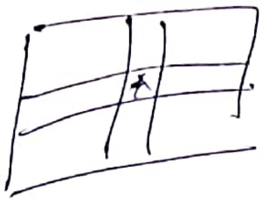
$r_3 = 3$ ———

Q. Find the rook polynomial of 3x3 chessboard.

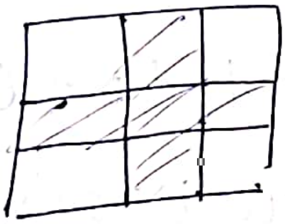
Expansion formula - $r(C,x) = x\,r(D,x) + r(E,x)$.

mark a star as you wish

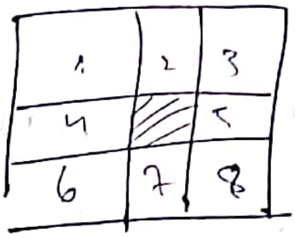
D board is made by removing that row & column.



removing the cell with star



D



E

$r(E,x) = 1+8x+14x^2 + 4x^3$

$r(D,x) = 1+4x+2x^2$

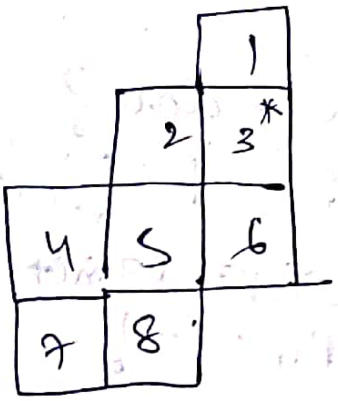
$r(C,x) = 1+8x+14x^2+4x^3+x+4x^2+2x^3$

$= 1+9x+18x^2+6x^3$

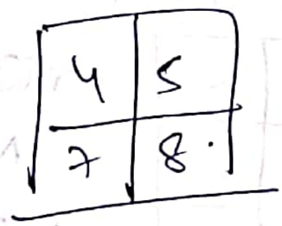
(1,2) (1,4) (1,5) (1,7) (1,8)
(2,4) (2,6) (2,7) (4,8)
(5,7) (6,7) (6,8)

(1,5,7), (2,4,8)
(3,4,7) (6,2,5)

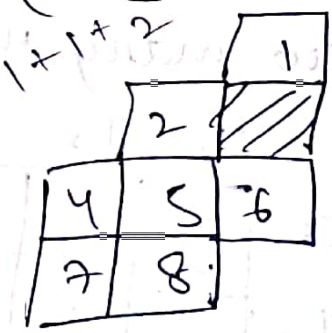
Q.



5+3+1+1+2



D



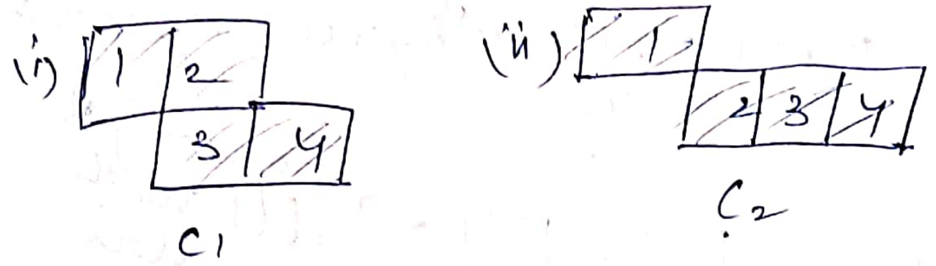
$r(C,x) = 1+4x+2x^2$

$r(C,x) = 1+7x+12x^2 + 3x^3$

(1,2,4) (1,4,8)
(1,2,7) (2,6,7) (1,5,7) (1,6,7) (1,6,8)

$r(C,x) = 1+8x+16x^2+7x^3$

Q. Find the rook polynomial for the shaded chessboards C_1 & C_2.



$r(C,x) = 1+4x+3x^2$ $\qquad$ $r(C,x) = 1+4x+3x^2$

Q. In making seating arrangement for their son's wedding reception Grace & Nick are down to 4 relatives $R_i, 1 \le i \le 4$ who do not get along with one another. There is a single open seat at each of the 5 tables T_j $1 \le j \le 5$. Because of the family differences

- R_1 will not sit at T_1 or T_2 $\qquad {}^4P_3$
- R_2 ———— T_2
- R_3 ———— T_3 or T_4
- R_4 ———— T_4 or T_5

How many no. of arrangement can be done? using principle of inclusion & exclusion

A.

	T_1	T_2	T_3	T_4	T_5
R_1	X	X	1	2	3
R_2	4	X	5	6	7
R_3	8	9	X	X	10
R_4	11	12	13	X	X

~~$r(C,x) = 1 + R_3 ...$~~

$N = {}^5P_4 = 5!$

$N(C_1) = 2 \times 4!$

$N(C_2) = 4!$

$N(C_3) = 2 \times 4!$

$N(C_4) = 2 \times 4!$

$N(C_1C_2) = 3!$

$N(C_1C_3) = 4 \times 3!$

i) R_1 at T_1 R_3 at T_3

(ii) R_1 at T_2 R_3 at T_3

(iii)

(iv)

$N(C_3 C_4) = 3 \times 3!$

$N(C_2 C_4) = 3 \times 3!$

$N(C_2 C_3) = 3 \times 3!$

$N(C_1 C_2 C_3) = 2 \times 2!$

$\underline{A.} = 25.$

$1 + 7x + 16x^2 + 13x^3 + 3x^4$