Date
Page

29/12/23.

UNIT-II - RELATIONS

* Relation

For set A, B any subset of A X B is called binary relation R from a set A to a set B.
Symbolically $R \subseteq A \times B$.

* Binary relation on a set

A binary relation R on ~~set A X A~~ set A is a subset of A X A or a relation from A to A.
Let $A = \{1, 2, 3, 4\}$
The ordered relation $R = \{(a,b) \mid a \text{ divides } b\}$.
$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

* Properties of Relations

$\boxed{1}$ Reflexive Relation

$\forall x \in A, (x,x) \in R$.

eg: $R = \{(a,b) \mid a \leq b\}$.
not reflexive: $R = \{(a,b) \mid a+b \leq 3\}$ [irreflexive]

$\boxed{2}$ Symmetric Relation

if $(x,y) \in R \Rightarrow (y,x) \in R$ for all $x, y \in A$.

eg: ~~R = {(a,b)~~ $A = \{1, 2, 3\}$
$R = \{(1,2), (2,1), (1,3), (3,1)\}$.
$R = \{(1,1), (2,2), (3,3), (2,3)\}$. (reflexive & asymmetric).
$R = \{(1,1), (2,2), (3,3)\}$ (Reflexive & symmetric)

$\boxed{3}$ Transitive Relation

for all $x, y, z \in A$ $(x,y) \in R$ and $(y,z) \in R$ then $(x,z) \in R$
$\forall x \forall y \forall z\ [(x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R]$

$A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,3), (3,4), (2,4)\}$

- Irreflexive, Asymmetric, ~~Ref~~ Transitive

4 Equivalence Relation

↳ Relation that is reflexive, symmetric and transitive

(1) Consider the relation R on set Z where we define aRb when $a \cdot b \geq 0$. Prove that the relation R is a reflexive, symmetric and not transitive.

R is given by.

$R = \{(a,b) \mid ab \geq 0\} \cdot \forall a, b \in Z$.

R is reflexive

$\forall a \in Z \quad (a,a) \in R$ · because $a \cdot a = a^2 \geq 0$.

R is symmetric

$\forall a, b \in Z$ if $[(a,b) \in R \Rightarrow (b,a) \in R]$.

because · $(a,b) \in R$ so $ab \geq 0$.

$(b,a) \in R$ so $ba \geq 0$.

R is not transitive.

Consider $(-1,0) \in R$ since $-1 \times 0 \geq 0$.

$(0,1) \in R$ since $0 \times 1 \geq 0$

but $(-1,1) \notin R$ because $-1 \times 1 \ngeq 0$.

We can infer that R is not a equivalence relation

5 Antisymmetric Relation

Given a ~~set~~ Relation R on set A., R is called anti-symmetric if $\forall a, b \in A$, $(aRb$ and $bRa) \Rightarrow a = b$.

Page

(a) For $A = \{1, 2, 3\}$, consider relation R

(1) $R = \{(1,2), (2,1), (2,3)\}$

Verify different properties on R.

R is irreflexive because $(2,2), (1,1), (3,3) \notin R$.

R is asymmetric because,

$(2,3) \in R$ but $(3,2) \notin R$.

R is not transitive because

$(1,2) \in R$. $(1,2)$

$(2,1) \in R$ $(2,3) \in R$,

but $(1,1) \notin R$. and $(1,3) \notin R$.

R is not antisymmetric because

$(1,2), (2,1) \in R$ but $1 \neq 2$.

(2) $R = \{(1,1), (2,2)\}$

R is ~~not~~ irreflexive because $(3,3) \notin R$.

R is symmetric.

$\forall a, b \in A [(a,b) \in R \Rightarrow (b,a) \in R]$.

R is ~~n~~ transitive

$\forall a, b, c \in A [(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R]$.

R is antisymmetric ~~bco~~

$\forall a, b \in A [(aRb) \text{ and } bRa) \Rightarrow a = b]$.

(b) Consider set A which ~~is~~ are positive divisors of 12.

HW and defined Relation R on A by xRy

if x exactly divides y.

Verify if it is equivalence relation on A.

Date
Page

30/12/23 **Composite Relation**

If A, B and C are the sets with $R \subseteq A \times B$ and $S \subseteq B \times C$, then the composite relation $R \circ S$ is a relation from A to C defined by

$R \circ S = \{(x, z), x \in A, z \in C$, where $\exists y \in B \mid (x, y) \in R$ and $(y, z) \in S\}$

Powers of R

Given a set A and a Relation R on A, we can define the powers of R recursively by

① $R^1 = R$

② For $n \in Z^+$, $R^{n+1} = R \circ R^n$

① For $A = \{1, 2, 3, 4\}$, Let R and S be the relations on A defined by $R = \{(1,2), (1,3), (2,4), (4,4)\}$
$S = \{(1,1), (1,2), (1,3), (2,3), (2,4)\}$

Find $S \circ R$, $R \circ S$, R^2, S^2, R^3

① $R \circ S = \{(1,2), (1,3), (2,4), (4,4)\} \circ \{(1,1), (1,2), (1,3), (2,3), (2,4)\}$

$= \{(1,3), (1,4)\}$

② $S \circ R = \{(1,2), (1,3), (1,4), (2,4)\}$

③ $R^2 = R \circ R = \{(1,4), (2,4), (4,4)\}$

④ $S^2 = S \circ S = \{(1,1), (1,2), (1,3), (1,4)\}$

⑤ $R^3 = R \circ R^2 = \{(1,2), (1,3), (2,4), (4,4)\} \circ \{(1,4), (2,4), (4,4)\}$

$= \{(1,4), (2,4), (4,4)\}$

② If $A = \{1, 2, 3, 4\}$ and $R = \{(1,2), (1,3), (2,4), (3,2)\}$

Find R^2, R^3, R^4

$R^2 = R \circ R = \{(1,4), (1,2), (3,4)\}$

$R^3 = R \circ R^2 = \{(1,4)\}$

$R^4 = R \circ R^3 = \{\}$ ϕ ϕ $\{\}$

note: ① $R \circ S \neq S \circ R$
② We can perform set operations such as ~~and~~ union and intersection on the relations.

Zero-one matrix & Directed graph

[Learn defⁿ from text bk]

① Let us consider $A = \{1,2,3,4\}$ & $B = \{w,x,y,z\}$ and $R = \{(1,x),(2,x),(3,y),(3,z)\}$.
Represent the matrix representation of R.

$$M(R) = \begin{array}{c|c} & \begin{array}{cccc} w & x & y & z \end{array} \leftarrow B \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \uparrow & \\ A & \end{array} \quad 4 \times 4 = |A| \times |B|$$

② Consider $R_2 = \{(w,5),(x,6),\}$ is a relation from B to C where $C = \{5,6,7\}$
$B = \{w,x,y,z\}$
Represent R_2 in matrix

$$M(R_2) = \begin{array}{c|c} & \begin{array}{ccc} 5 & 6 & 7 \end{array} \leftarrow C \\ \begin{array}{c} w \\ x \\ y \\ z \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \uparrow & \\ B & \end{array} \quad 4 \times 3 = |B| \times |C|$$

Compute $M(R_1) \circ M(R_2)$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \Rightarrow \{(1,6), (2,6)\} \Rightarrow R_o R_2$$

$\therefore$ We can observe that $M(R) \cdot M(R_2) = M(R_o R_2)$

<u>note:</u> In the matrix addition and multiplication we assume the stipulation $\boxed{1+1=1}$

(3) Let $A = \{1,2,3,4\}$ and $R = \{(1,1), (1,2), (2,3), (3,2), (3,3), (3,4), (4,2)\}$

i) Express its matrix representation

give description $\hat{M}(R)$ contains 4 rows ...

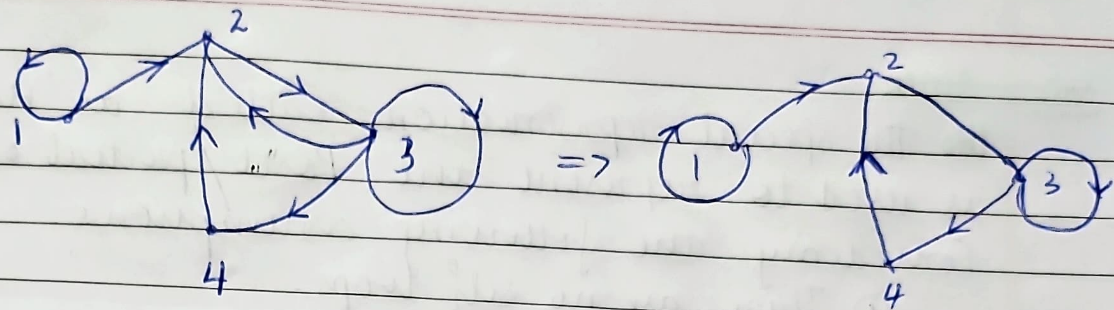
$$M(R) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ \uparrow \\ A \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \leftarrow A \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \\ \\ \\ \end{array} 4 \times 4 = |A| \times |A|$$

ii) Write its directed graph

The directed graph $G = (V, E)$ where ~~V = set of~~

$V = \{1,2,3,4\} = A$

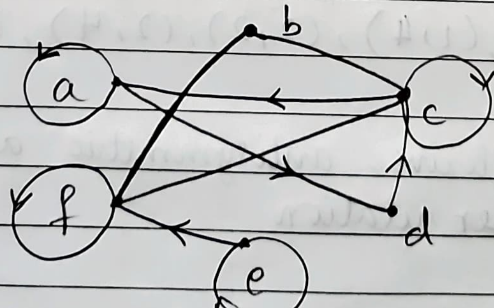
E = edges representing the elements of relation R



④ Let $A = \{a, b, c, d, e, f\}$. Determine the relation $R \subseteq A \times A$ and draw the directed graph associated with R.

$$M(R) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(a,a), (a,d), (b,c), (b,f), (c,a), (c,b), (c,c), (c,f)$
$(d,c), (e,e), (e,f), (f,b), (f,c), (f,f)\}$



Partial Order Relation

R → Reflexive
→ Antisymmetric
→ Transitive

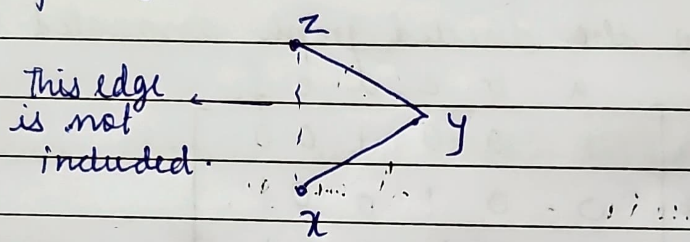
* Since reflexive, we can ignore self loop.
* Transitive, we can remove a to c.
* edges from bottom to up.

The pair (A, R) is called Poset.

~~note~~ Store

The special representation called as 'Hasse Diagram' is used to represent the Poset / partial order relation. Considering the following assumptions:

1. There are no self loops.
2. All the edges move from bottom to top
3. If $(x,y) \in$, $(y,z) \in R \Rightarrow (x,z) \in R$



1. Consider $A = \{1,2,3,4\}$ and R defined as xRy if x divides y.

Write the following:

i) construct relation R.
ii) verify whether relation is partial order
iii) Draw the Hasse diagram.

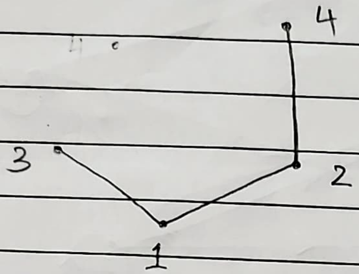
i) $R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

ii) ~~R is~~ R should be reflexive, antisymmetric and transitive to be a partial order relation

- R is reflexive
  $\forall x \in A$, we have the pairs $(1,1), (2,2), (3,3), (4,4)$
- R is antisymmetric
  We observe that we have the pairs where $(x,y) \in R$ and $(y,x) \notin R$ wherein if $(x,y) \in R$ & $(y,x) \in R \Rightarrow x = y$.
- R is transitive
  ~~$(1,1), (1,2) \in R \Rightarrow (1,2) \in R$~~
  ~~$(1,1)(1,3) \in R \Rightarrow (1,3) \in R$~~

We can observe that in the relation $(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$.

iii)

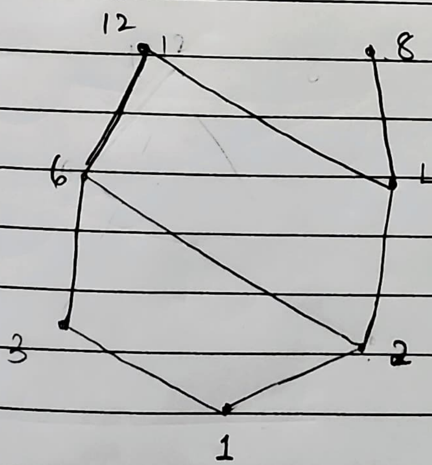


② Let $A = \{1, 2, 3, 4, 6, 8, 12\}$ and R be the partial ordering on A defined by ~~ARB~~
aRb if a divides b.

i) Write the relation
ii) Draw the Hasse Diagram
iii) Write its matrix representation and directed graph.

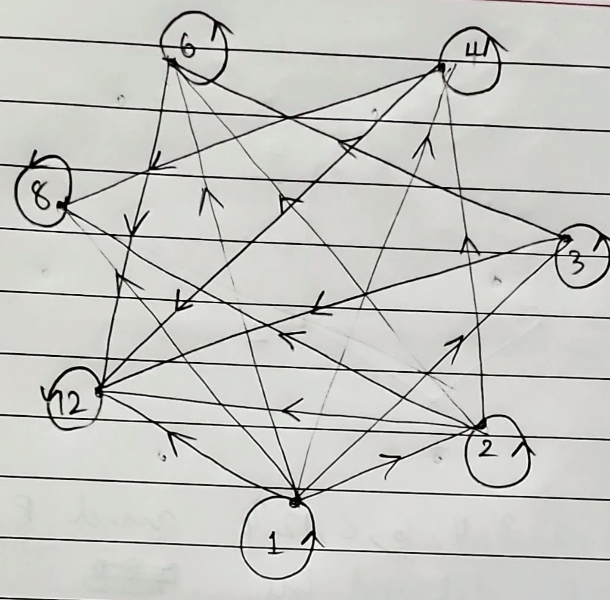
i) $R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (1,12), (2,2), (2,4), (2,6), (2,8), (2,12), (3,3), (3,6), (3,12), (4,4), (4,8), (4,12), (6,6), (6,12), (8,8), (12,12)\}$

ii)



iii)
$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 6 & 8 & 12 \leftarrow A \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 6 \\ 8 \\ 12 \\ \uparrow \\ A \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

ix)



(3) Draw the Hasse Diagram for $A = \{3, 4, 12, 24, 48, 72\}$ for a relation R that exhibits divisibility property.
[xRy if x divides y]

(4) Construct the Hasse diagram for the poset ~~{(D₇₂, |)}~~ $(D_{72}, |)$ → divisibility symbol

A = divisors of 72

{ 1, 2,

classmate

Date

Page

1/1/24.

External Elements of Poset

① If (A, R) is a poset, then we can define the following elements of A

it can be 1 or more, or nill.

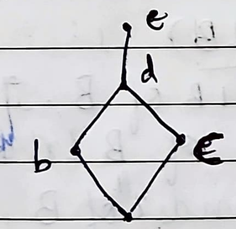
① **Maximal element** - An element $x \in A$ is called max. ele. iff $\forall a \in A, a \neq x \Rightarrow x \not R a$.

② **Minimal element** - An element $y \in A$ is called minimal ele. iff $\forall b \in A, b \neq y \Rightarrow b \not R y$.

③ **Least element** - An element $x \in A$ is called least ele. if $x R a, \forall a \in A$

④ **Greatest element** - An element $x \in A$ is called greatest element if $a R x \ \forall a \in A$.

eg ① Consider the below Hasse diagram.



① Write the relation.

② Identify maximal element, minimal element, greatest & least element wrt A.

where $A = \{a, b, c, d, e\}$.

① $R = \{(a,a), (b,b), (c,c), (d,d), (e,e), (a,b), (a,c), (b,d), (a,d), (c,d), (a,c), (d,e), (b,e), (c,e), (a,e)\}$.

② Max. element is 'e' because $e \not R a, e \not R b, e \not R c, e \not R d$

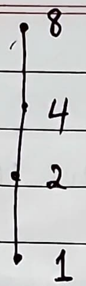
Min element is 'a' because $b \not R a, c \not R a, d \not R a, e \not R a$.

Least element is 'a' because $a R a, a R b, a R c, a R d, a R e$.

Greatest element is 'e' because $a R e, b R e, c R e, d R e, e R e$.

[All elements of A are related to e]

(2)



$R = \{(1,1), (2,2), (4,4), (8,8), (1,2), (2,4), (4,8), (1,4), (2,8), (1,8)\}$

Let (A, R) be a Poset with $B \subseteq A$, then we have the following elements wrt B:

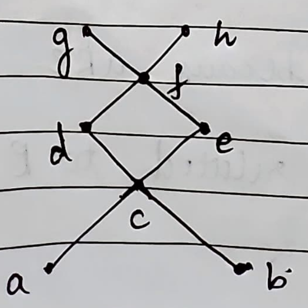
① **Lower bound of B**
An ele. $x \in A$ is called lower bound of B if $x R b$ $\forall b \in B$.

② **Upper bound of B**
Any ele. $y \in A$ is called upper bound of B if $b R y$ $\forall b \in B$.

③ **Greatest lower bound (GLB)**
An ele. $x' \in A$ is called GLB of B if
i) it is a lower bound of B & x' ~~st~~
ii) if $\forall$ other lower bounds x'' of B, we have $x'' R x'$.

④ **Least upper bound (LUB)**
An ele. $y' \in A$ is called LUB of B if
i) it is upper bound of B
ii) if $\forall$ other upper bound y'' of B, we have $y' R y''$.

① Consider the below Hasse Diagram of a Poset (A, R) if $B = \{c, d, e\}$. Find LB, UB, GLB, LUB.



~~R-{(a,a), (b, b~~

i) LB - a, b, c

We can observe that the elements {a, b, c} are related to ~~that~~ all elements of B.

∴ Lower bound of B = {a, b, c}

ii) ~~GLB~~ UB

We can observe that the elements ~~{f, g, h} are~~ of B ~~are~~ {c, d, e} are related to all elements of {f, g, h}

∴ Upper bound of B = {f, g, h}

iii) Least upper bound -

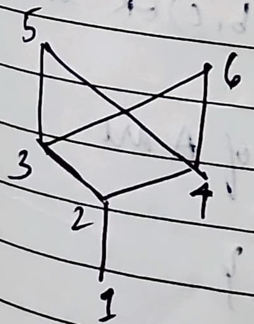
We know that upper bounds of B = {f, g, h} and we observe that f R g & f R h

∴ ~~GLB~~ LUB (B) = f.

iv) GLB.

We know that lower bound of B = {a, b, c} and we observe that a R c & b R c

∴ GLB (B) = c.



B = {3, 4, 5}

i) Maximal - 5, 6.

ii) ~~Minimal~~ - 1.

iii) ~~Least~~ - 1

iv) Max — nil

v) LB - 1, ~~1~~ 2

vi) ~~U~~B - 5

vii) ~~GLB - 2.~~

viii) ~~LUB~~ - 5

x R a

a R x.

~~a~~ R a

a R x

x R b

b R ~~x~~

classmate
Date
Page

17/1/24

Equivalence relation and equivalence class

A relation is said to be equivalence if it satisfies the properties
i) reflexive
ii) symmetric
iii) transitive
If R is an equivalence relation on a set A and $a \in A$, then the equivalence class of 'a' is denoted by $[a]$ or $\bar{a} = \{x \mid x R a\}$

Eg: Consider $A = \{1, 2, 3\}$ and $R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$. verify whether the relation is an equivalence relation. If yes, then determine the equivalence classes of elements of A.

R is reflexive
$\forall a \in A \quad (a,a) \in R$ i.e. $(1,1), (2,2), (3,3) \in R$.
R is symmetric.
$\forall a, b \in A$ if $(a,b) \in R$ then $(b,a) \in R$.
R is transitive clear
$\forall a, b, c \in A$ if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$
$\therefore$ R is an equivalence relation.
The ~~equitance~~ equivalence classes of A are
$[1] = \{1, 2 \mid (1,1), (2,1)\} = \{1, 2\}$
$[2] = \{1, 2 \mid (1,2), (2,2)\} = \{1, 2\}$
$[3] = \{3 \mid (3,3)\} = \{3\}$.

note: i) We observe that ~~equiv~~ $[a]$ contains the element 'a' because ~~as~~ it satisfies reflexive ~~relation~~ ~~pr~~ relation.

ii) If $x R y$ iff $[x] = [y]$ because it satisfies symmetric property / relation.

iii) We can determine the set A by considering union of equivalence classes such that the classes are disjoint.

iv) The equivalence classes will either be $[x] = [y]$ ~~or~~ $[x] \cap [y] = \phi$.

<u>Partition of a set</u>

Consider a set A, there are subsets A_i for some $i \in I = \{1, 2, 3, \ldots\}$ where $\phi \neq A_i \subseteq A$ then Partition of the set A defined as

$P = \{A_1, A_2, \ldots, A_K\}$ such that

i) $A = \bigcup_{i \in I} A_i$

ii) $A_i \cap A_j = \phi$ for every i & j.

① Consider $A = \{1, 2, 3, 4, 5\}$ and $R = \{(1,1), (2,2), (2,3), (3,2), (3,3), (4,4), (4,5), (5,4), (5,5)\}$.
Determine the partition P of A

$[1] = \{1\}$
$[2] = \{2, 3\}$
$[3] = \{2, 3\}$
$[4] = \{4, 5\}$
$[5] = \{4, 5\}$

$P = \{[1], [2], [4]\} = \{\{1\}, \{2,3\}, \{4,5\}\}$.
$P = \{[1], [3], [5]\}$

classmate

Date
Page

24/1/24

② Determine the equivalence relation R on $A = \{1,2,3,4,5,6,7\}$ which has the partition $A = \{1,2\} \cup \{3\} \cup \{4,5,7\} \cup \{6\}$.

Consider the set $\{1,2\}$ of the partition. This subset implies that $[1] = [2] = \{1,2\}$

$\therefore$ We have the pairs $(1,1), (1,2), (2,2), (2,1)$ under the relation R.

Similarly the set $\{4,5,7\}$ implies that under R, we have $[4] = [5] = [7] = \{4,5,7\}$

$\therefore$ The equivalence relation contains $(4,4), (5,4), (7,4), (4,5), (5,5), (7,5), (4,7), (5,7), (7,7)$

Similarly the partition $\{3\}$ & $\{6\}$ implies that under R, we have $(3,3)$ and $(6,6)$

$\therefore R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (1,2), (2,1), (4,5), (5,4), (7,4), (4,7), (7,5), (5,7)\}$

and $|R| = 2^2 + 1^2 + 3^2 + 1^2 = 15$

② **Total Order Relation**

If (A,R) is a poset then R is a total order relation iff $\forall (x,y) \in A$, we have either xRy or yRx.

① Consider $A = \{1,2,4,8\}$. Determine the relation R such that xRy iff x divides y. Determine whether the relation is total order relation. Draw its Hasse Diagram

$R = \{(1,1), (2,2), (4,4), (8,8), (1,2), (1,4), (1,8), (2,4), (2,8), (4,8)\}$

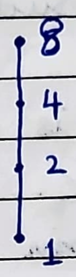
R is reflexive because $\forall x \in A \Rightarrow (x,x) \in R$

R is transitive because $\forall (x,y) \in A$ & $(y,z) \in A \Rightarrow (x,z) \in A$

R is antisymmetric because if (x,y) & $(y,x) \in A \Rightarrow x = y$.

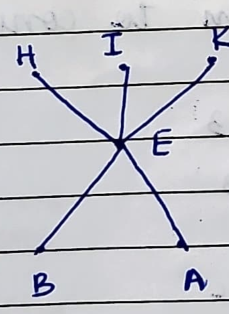
$\therefore$ R is partial order relation.

R is a total order relation because $\forall (x,y) \in A$, we have either xRy or yRx. eg: ~~4R~~ 1R4 but 4 $\not R$ 1



Topological sort Algorithm

Topological sort :
It refers linear ordering of elements in the set such a way that the edges move in the direction left to right.
This is widely used when we need to order the elements / tasks in a linear manner such that the dependency of task is handled appropiately to handle the entire task of execution.



Topological order

A, B, E, H, I, K

Given a hasse diagram of partial order R. We need to find the total order T on this task such that such that $R \subseteq T$.
To obtain this, we apply topological sort algorithm.

25/1/24

~~Topologica~~ **Algorithm :**

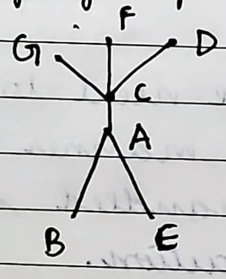
For a partial order R on a set A, $|A| = n$,
STEP i) Set $K=1$. Let A be the hasse diagram of R.

STEP ii) Select a vertex V_K in H_K, such that no edge starts from V_K.

STEP iii) If $K = n$, the process ~~the~~ is completed and we have total order $T : V_n < V_{n-1} < \dots < V_2 < V_1$

STEP iv) If $K < n$, then remove from H_K the vertex V_K and also the edges terminating at V_K. For call the resultant Hasse diagram as H_{K+1}, increment K by 1 and go to STEP ii)

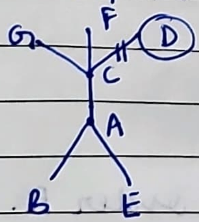
① Obtain the topological sort for the below Hasse diagram by applying topological sort algorithm.



In the above diagram, we have $A = \{A, B, C, D, E, F\}$
$|A| = 7$

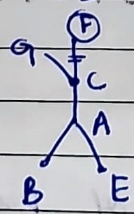
Applying the topological sort algorithm to construct the total order as follows:

$K = 1$
H_1



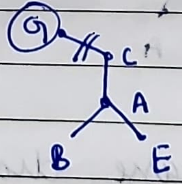
$\boxed{V_1 = D}$

$K = 2$
H_2



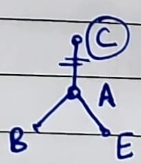
$\boxed{V_2 = F}$

$K = 3$
H_3



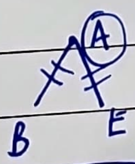
$\boxed{V_3 = G}$

$K = 4$
H_4



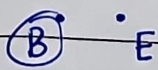
$\boxed{V_4 = C}$

$K = 5$
H_5



$\boxed{V_5 = A}$

$K = 6$



$\boxed{V_6 = B}$

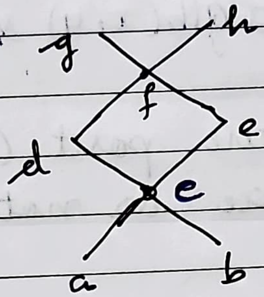
$K = 7$

$\boxed{V_7 = E}$

alphabetical order

T: E < B < A < C < G < F < D

② Obtain the topological sort for the given Hasse diagram.

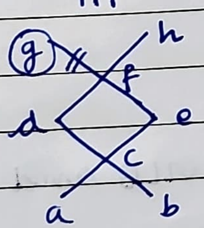


In the above diagram, we have A = {a, b, c, d, e, f, g, h}
|A| = 8.

Applying TSA,

K = 1

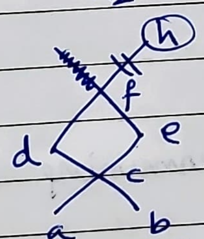
H_1



$\boxed{V_1 = g}$

K = 2

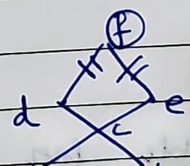
H_2



$\boxed{V_2 = h}$

K = 3

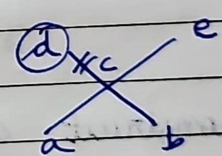
H_3



$\boxed{V_3 = f}$

K = 4

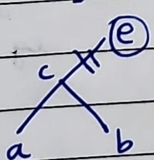
H_4



$\boxed{V_4 = d}$

K = 5

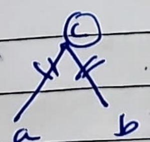
H_5



$\boxed{V_5 = e}$

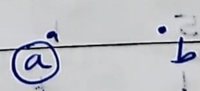
K = 6

H_6



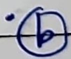
$\boxed{V_6 = c}$

K = 7



$\boxed{V_7 = a}$

K = 8



$\boxed{V_8 = b}$

∴ T: b < a < c < e < d < f < h < g

Lattice

Poset (A, R) is a lattice for all pairs of ~~lattice~~ elements of A have both GLB & LUB i.e.

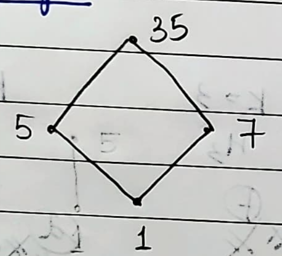
$\forall (x, y) \in A$, there exists $glb(x, y)$ & $lub(x, y)$.

① Draw the Hasse diagram for the poset $(D_{35}, |)$ and check whether it is a lattice or not.

$A = \{1, 5, 7, 35\} = D_{35}$

$R = \{(1,1), (1,5), (1,7), (1,35), (5,5), (5,35), (7,7), (7,35), (35,35)\}$

Hasse diagram:



Construct a table for determining the GLB and LUB of each pair of D_{35}.

glb	1	5	7	35
1	1	1	1	1
5	1	lb = 1,5 5	lb = 1 1	lb = 1,5 5
7	lb = 1 1	lb = 1 1	lb = 7,1 7	lb = 7,1 7
35	lb = 1 1	lb = 1,5 5	lb = 1,7 7	lb = 1,5,7,35 35

GLB
x R b.
GLB
x'' R x'

note: Let B is pair of elements.
eg for (1,1)
B = {1}.
for (1,5)
B = {1,5}

lub	1	5	7	35
1	ub =			
5				
7				
35				

LUB
b R x :
LUB
x' R x''

1,5

1

5 , 1,7

5

lub	1	5	7	35
1	ub = 1, 5, 7, 35 1	ub = 5, 35 5	ub = 7, 35 7	ub = 35 35
5	ub = 5, 35 5	ub = 5, 35 5	ub = 35 35	ub = 35 35
7	ub = 7, 35 7	ub = 35 35	ub = 7, 35 7	ub = 35 35
35	ub = 35 35	ub = 35 35	ub = 35 35	ub = 35 35

We can observe that from the above two tables for every pair of elements of D_{35} there exists GLB and LUB.
Hence the poset $(D_{35}, |)$ is a lattice.