

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Coding Challenge - 1 REACH THE QUEEN PROBLEM REPORT

Submitted by

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Under the guidance of

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Experiential Learning

Design and Analysis of Algorithms – CD343AI

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Question

There are N cities and M directed roads connecting them. The king is at city 1 and his queen is at city N. He needs to find the number of ways to reach the queen. The cities are connected in a way that they don't form directed cycles.

Input Format

The first input line has two integers N and M. Then there are M lines describing the roads. Each line has two integers a and b i.e there is a directed road from city a to city b

Output Format:

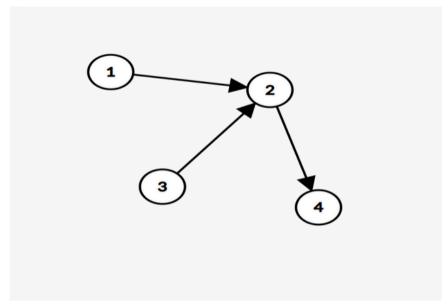
Print the number of ways modulo $10^9 + 7$

Constraints:

$$\begin{split} 1 &\leq N \leq 2*10^5 \\ 1 &\leq M \leq 4*10^5 \\ 1 &\leq a,b \leq N \end{split}$$

Sample Input	Sample Output	%
4 3 1 2 2 4 3 2	1	

Time Limit: 3 Memory Limit: 256 Source Limit: Explanation



There is only 1 way to reach from 1 to 4 i.e 1->2->4

Details on Data Structures and Algorithm

Data Structures:

Graph Representation:

Graph (Adjacency List): The graph is represented using a dictionary where each key is a node, and its value is a list of nodes that it points to. This allows efficient traversal and edge exploration.

In-Degree Array: An array that keeps track of the number of incoming edges for each node. This is essential for Kahn's algorithm to determine the nodes with no incoming edges (in-degree of 0).

2. Topological Sort:

Queue: A deque is used to implement Kahn's algorithm for topological sorting. It helps in efficiently managing the nodes with an in-degree of 0.

3. Dynamic Programming Array:

DP Array: An array where 'dp[i]' represents the number of ways to reach node 'i' from node 1. This is updated as the graph is traversed in topological order.

Algorithm:

1. Graph Construction and In-Degree Calculation:

For each edge (u, v), add v to the adjacency list of u and increment the in-degree of v.

2. Topological Sorting using Kahn's Algorithm:

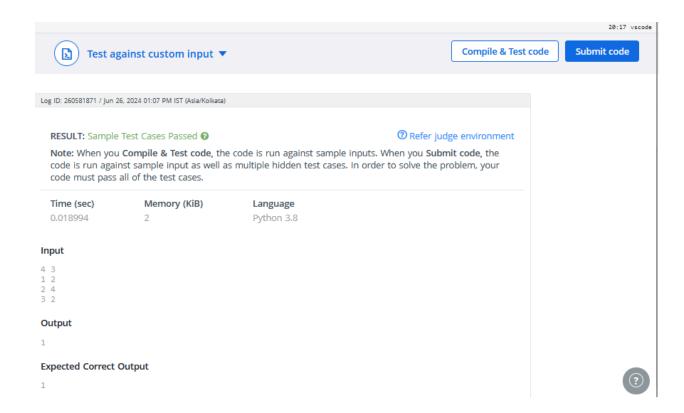
Initialize a queue with all nodes having an in-degree of 0.

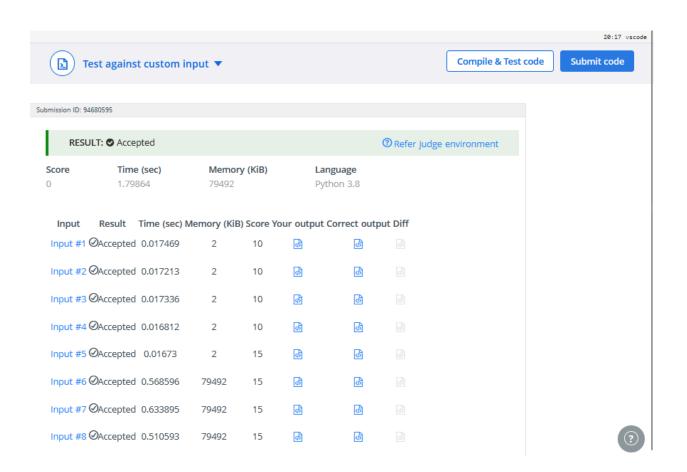
While the queue is not empty, remove a node, add it to the topological order, and reduce the in-degree of its neighbors by 1. If any neighbor's in-degree becomes 0, add it to the queue.

3. Path Counting using Dynamic Programming:

Initialize 'dp[1]' to 1, as there is one way to reach node 1 (starting point). For each node in the topological order, update the dp array for its neighbors. Specifically, for each neighbor, add the current node's dp value to the neighbor's dp value, taking modulo $(10^9 + 7)$.

Code Snippets Screenshots





Time, Space Efficiency

Time Complexity:

Graph Construction: (O(M)) where M is the number of edges. Each edge is processed once.

Topological Sorting (Kahn's Algorithm): (O(N + M)) where N is the number of nodes and M is the number of edges. Each node and edge is processed once. **Dynamic Programming Update:** (O(N + M)). Each node and edge is processed in the order given by the topological sort.

Overall time complexity is (O(N + M)).

Space Complexity:

Graph Storage: (O(N + M)). Adjacency list representation takes space proportional to the number of nodes and edges.

In-Degree Array: (O(N)).

Queue for Topological Sort: (O(N)) in the worst case.

DP Array: (O(N)).

Overall space complexity is (O(N + M)).

Conclusion

The algorithm efficiently calculates the number of distinct paths from node 1 to node N in a DAG using topological sorting and dynamic programming. The time and space complexities are both (O(N+M)), making it suitable for large graphs within the problem constraints.