

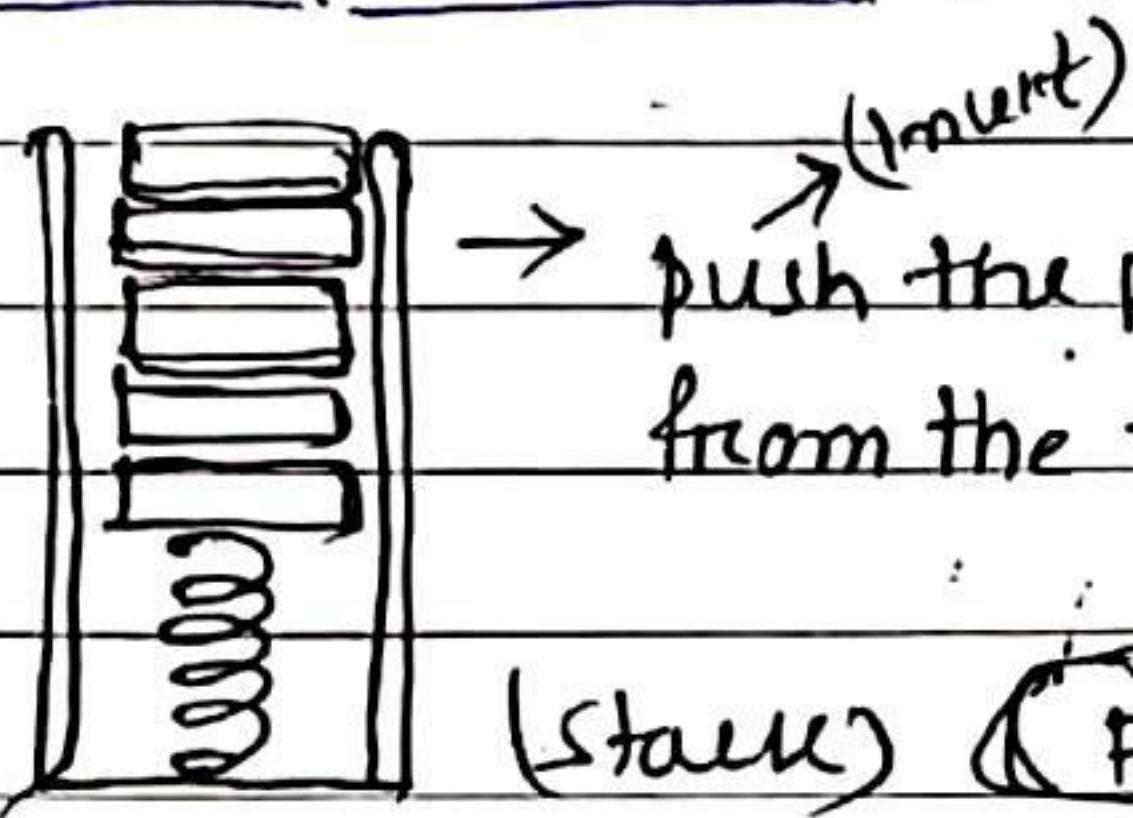
## Stacks and Queues

classmate

Date \_\_\_\_\_

Page \_\_\_\_\_

### • Introduction to Stacks:



→ push the plate from the top and take the plate from the top.

(stack) (FIFO, LIFO)

### • Applications of Stack:

(i) Recursion.

(ii) IF → PF conversion.

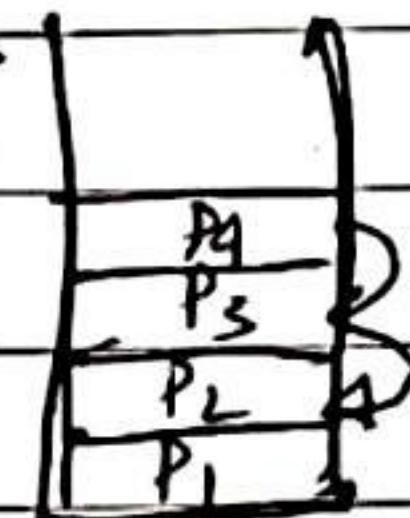
(iii) Parsing

(iv) Browsers.

(v) Editors.

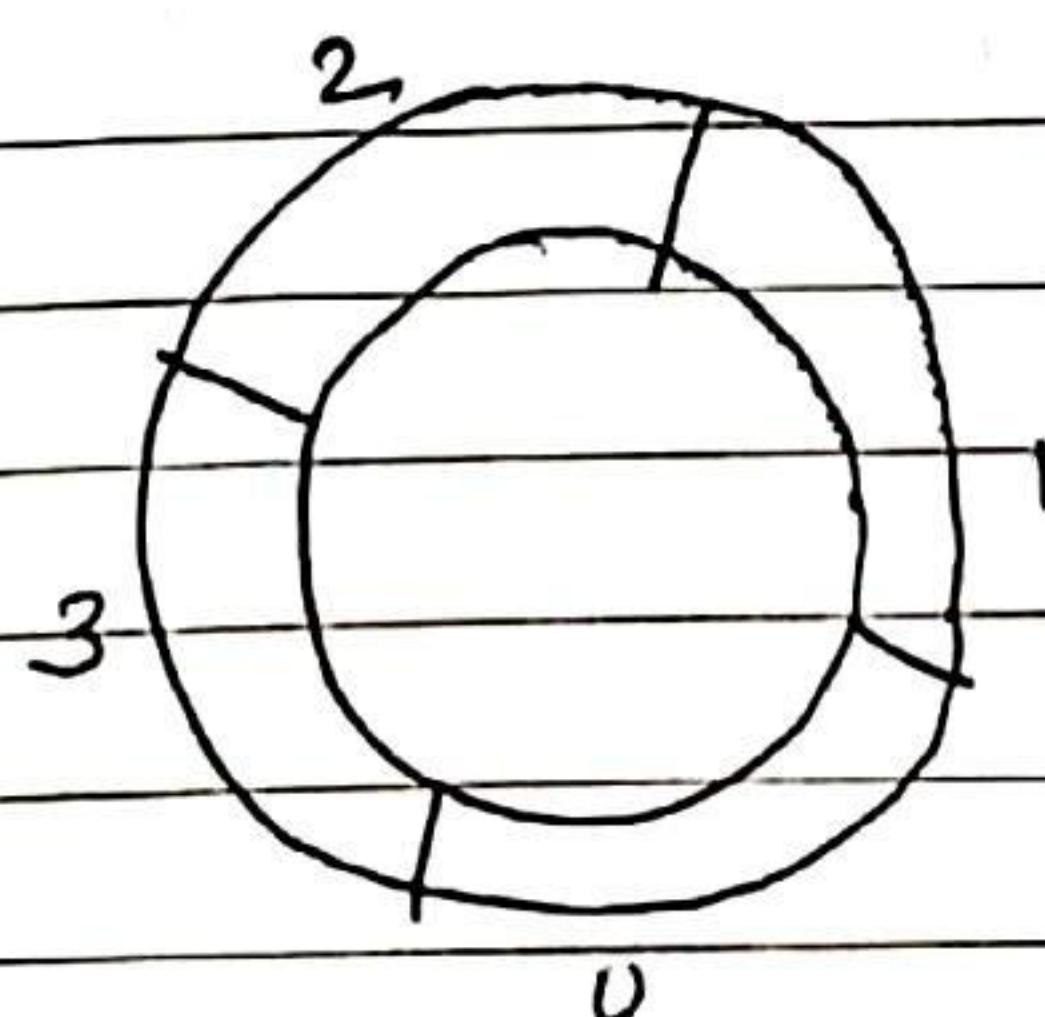
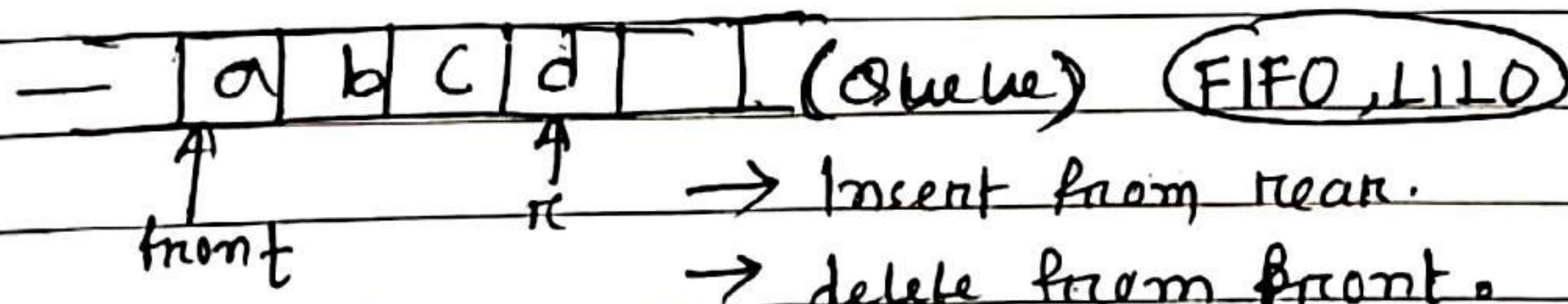
(vi) Tree traversals

and graph traversal.



### • Implementation of a Queue using circular array =

→ Insert the element from one side and delete the element from other side.

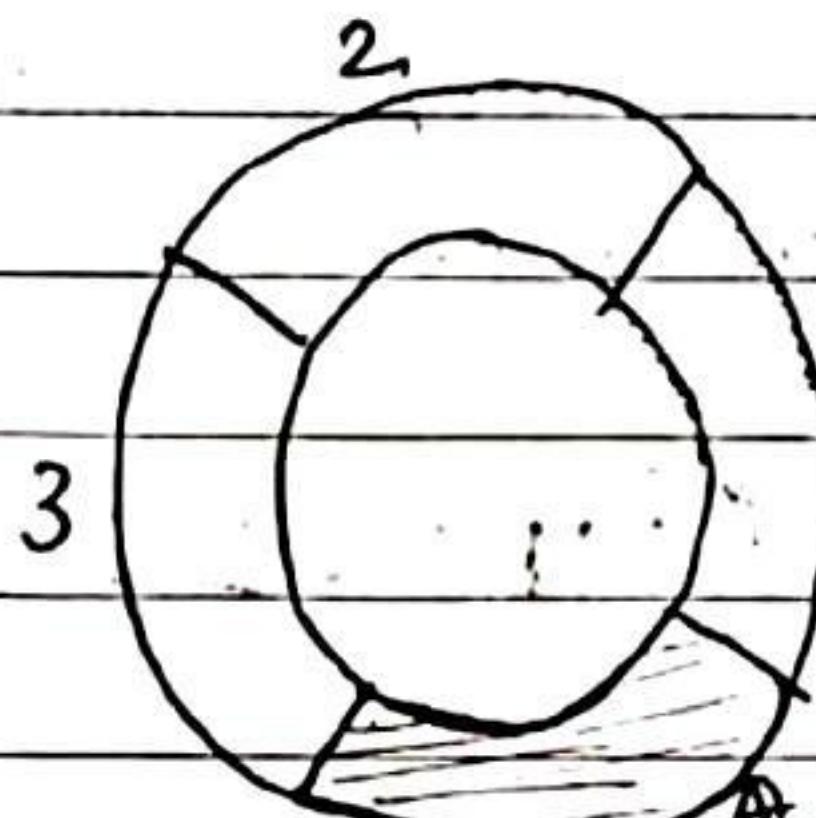


circular queue.

If  $\rightarrow$  in circular queue, if the size of an array ' $n$ ' then we use ' $(n-1)$ '. amount of size.



~~→ one less~~



circular queue.

0 front  $\rightarrow$  should be blank space

### ① Inserting an item in queue =

enqueue(item)

{

rear = (rear+1) mod n;

if (front == rear)

{ pf("Q is full");

if (rear == 0) rear = n - 1;

else

else rear = rear - 1;

return;

}

else {

q[rear] = item;

return;

}

}

② delete an item from queue :-

```

int Dequeue()
{
    if (front == rear)
    {
        pf ("Q is empty");
        return -1;
    }
    else
    {
        front = (front + 1) % n;
        item = q[front];
        return empty item;
    }
}

```

Overflow :-

$$(rear + 1) \bmod n \\ = = front$$

Underflow

front == rear

• Linked List Implementation of Stack :-

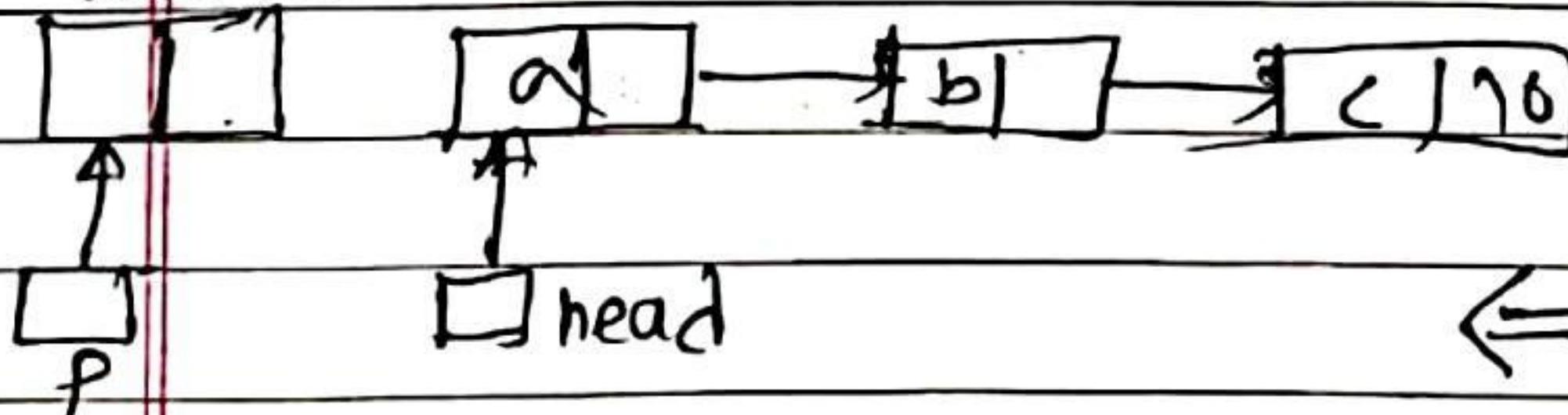
struct node

```

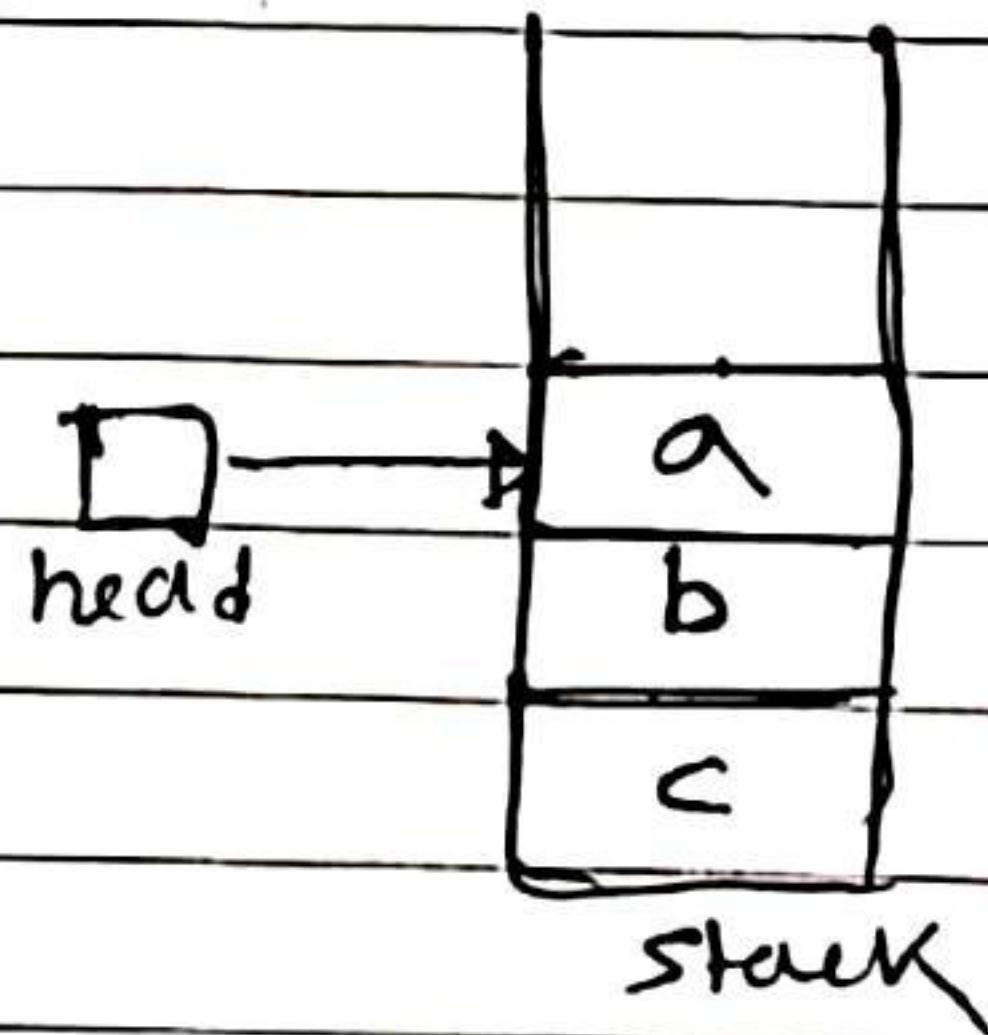
{
    int i;
    struct node *link;
}

```

new node



$\Leftrightarrow$



① Inserting a new item =

push(int item)

{

```
struct node *p = (struct node *) malloc (sizeof(struct node));
if (p == NULL) { pf("error of malloc"); return; }
```

p → data = item;

p → link = head; ~~head~~

p → link = head;

head = p;

}

Time complexity =  $O(1)$  (constant time)

② deletion of item from stack =

③ Deletion code

int pop()

{ struct node \*p;

int item;

```
if (head == NULL) { pf("Underflow"); return -1; }
```

item = head → data;

p = head;

head = head → next;

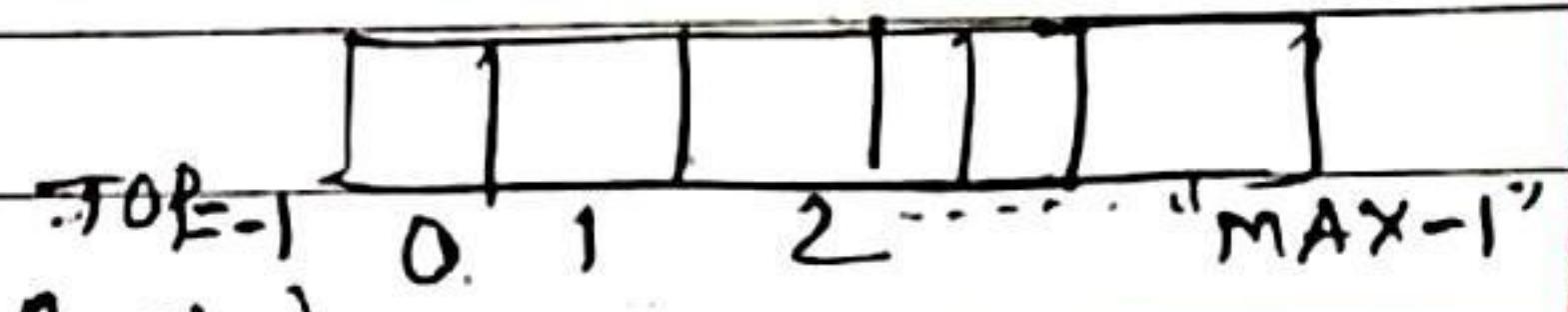
free (p);

Time complexity =  $O(1)$  (constant time)..

- Implementation of stack using array =

```
int stack[MAX];
```

```
int top = -1; // (stack empty)
```



- Pushing a item =

```
void push (int item)
```

```
{
```

```
if (top == MAX-1)
```

```
pf(" overflow");
```

```
else {
```

```
top = top + 1;
```

```
stack[top] = item; }
```

```
}
```

```
return;
```

```
}
```

$\rightarrow \text{stack}[++\text{top}] = \text{item};$

- Poping a item =

```
int pop()
```

```
{ int temp;
```

```
if (top == -1)
```

```
{ pf(" underflow");
```

```
return -1;
```

```
}
```

```
else
```

```
{
```

```
temp = stack[top];
```

```
top = top - 1;
```

```
}
```

```
return temp;
```

```
}
```

$\rightarrow \text{temp} = \text{stack}[\text{top} - 1];$

$\rightarrow$  Time complexity (push, pop) = O(1) Time.

C2-2012

**Question -1**

Suppose a circular queue of capacity  $(n-1)$  elements is implemented with an array of  $n$  elements. Assume that the insertion and deletion operations are carried out using 'REAR' and 'FRONT' as array index variables respectively. Initially,  $\text{REAR} = \text{FRONT} = 0$ .

The conditions to detect queue with full and queue empty are =

→ Queue Full:

$$\text{FRONT} \oplus (\text{REAR} + 1) \bmod n == \text{FRONT}$$

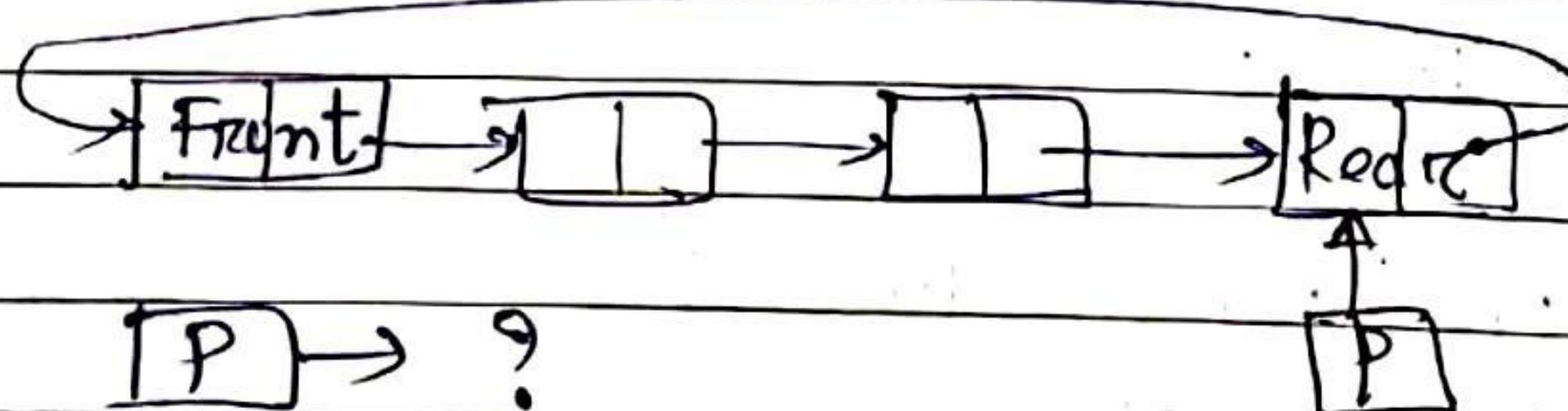
Queue Empty:

$$\text{REAR} = \text{FRONT}$$

C2-2009

**Question -2**

A circularly linked list is used to represent a queue. A single variable 'p' is used to access the queue. To which node should 'p' point to such that both the operations enqueue and dequeue can be performed in constant time?



- a) Read node
- b) FRONT node
- c) not possible.
- d) node next to Front.



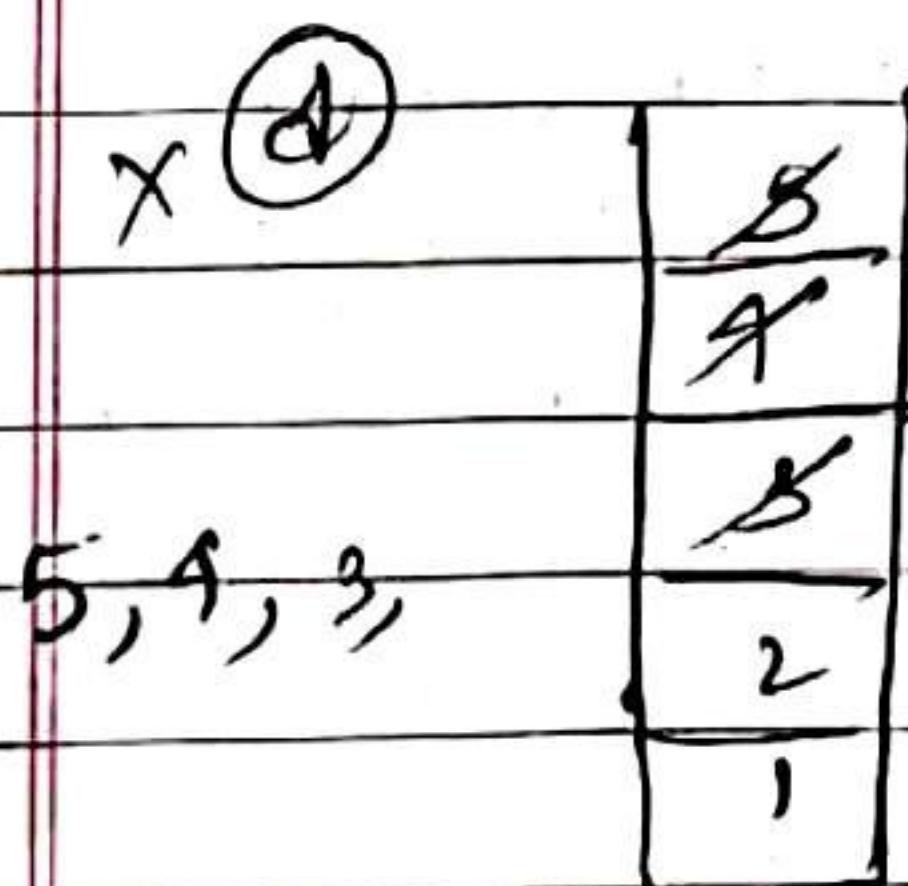
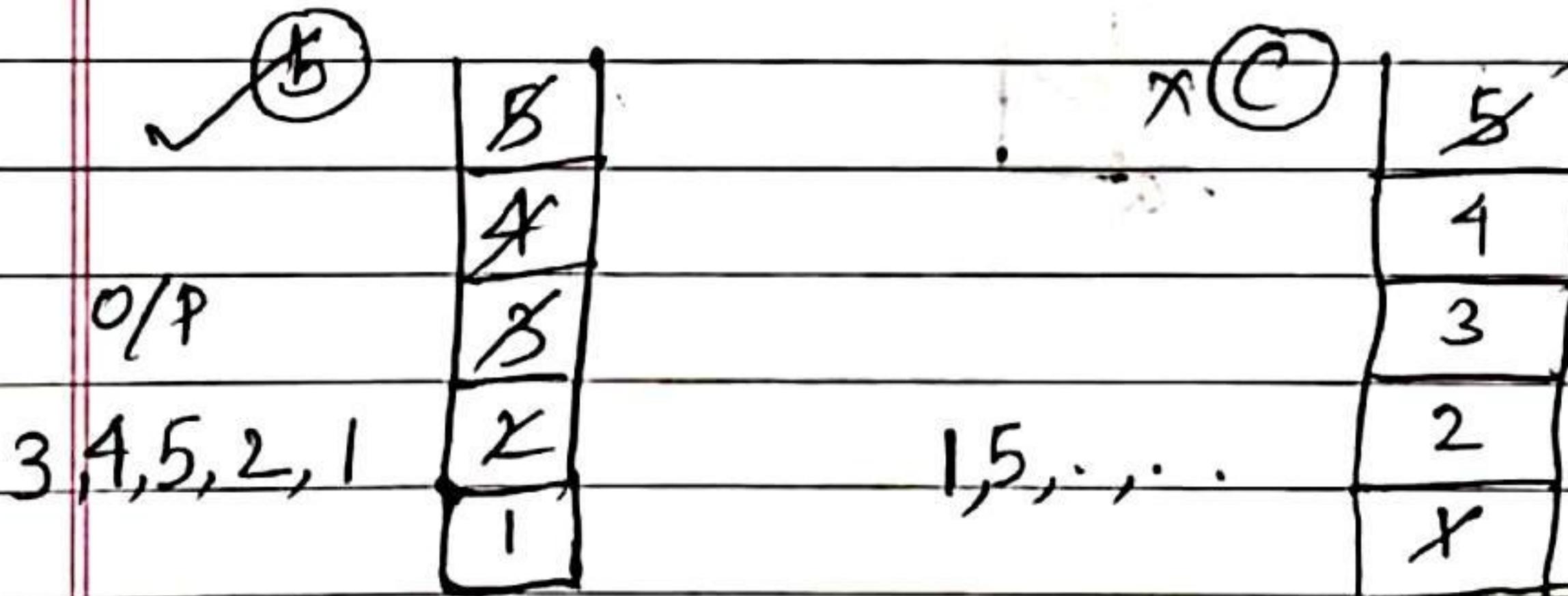
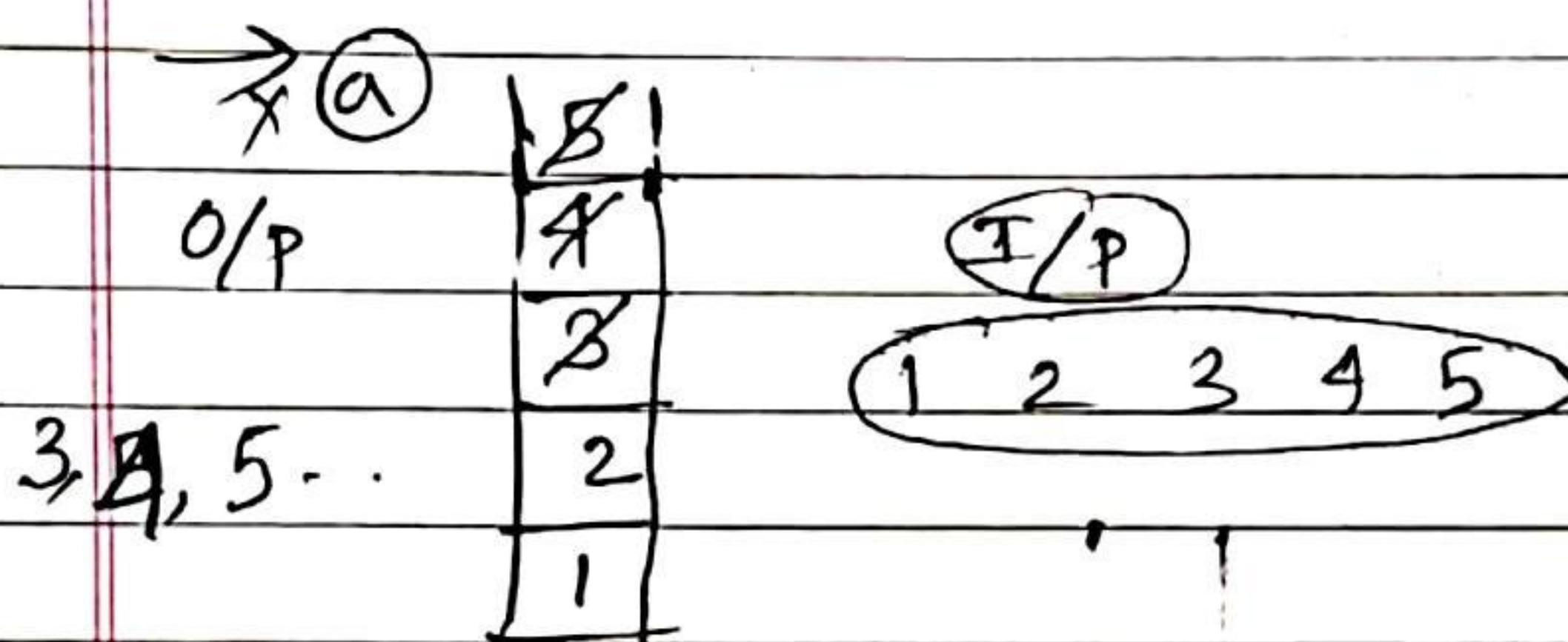
Question-3

Which of the following permutations can be obtained in the O/P (In the same order) using a stack assuming that the I/P is the sequence.

1, 2, 3, 4, 5 in the order?

x a) 3, 4, 5, 1, 2. ✓ b) 3, 4, 5, 2, 1.

x c) 1, 5, 2, 3, 4. x d) 5, 4, 3, 2, 1, 2.

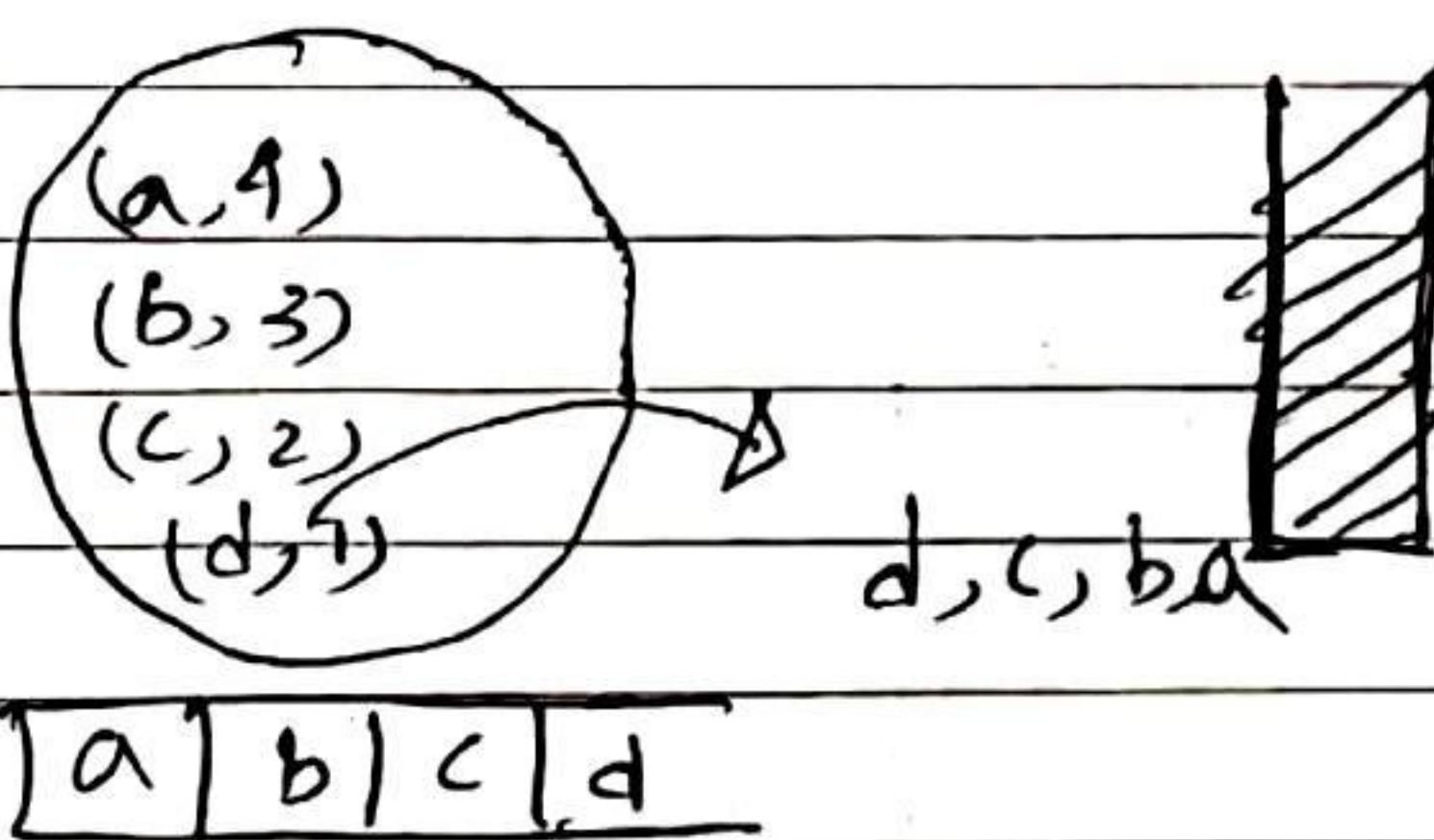


G-199+

Question - 4

A priority queue ' $Q$ ' is used to implement a stack 'S' that stores characters.  $\text{push}(c)$  is implemented as  $\text{Insert}(Q, c, K)$  where  $K$  is an appropriate integer key chosen by the implementation.  $\text{pop}$  is implemented as  $\text{DELETE MIN}(Q)$ . For a sequence of push operations, the keys chosen are in:

- a) Non-decreasing order.
- b) Non-increasing order.
- c) strictly increasing order.
- d) strictly decreasing order.

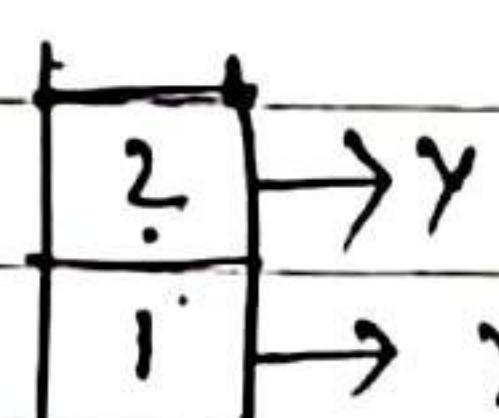
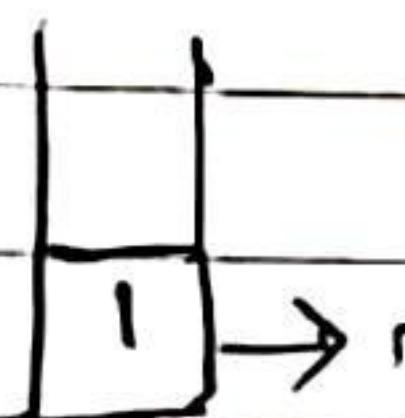


g-2003

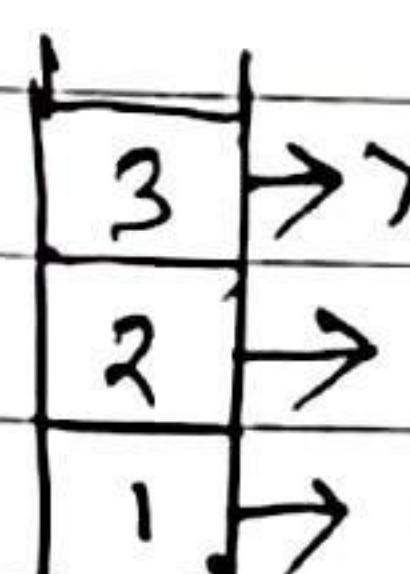
Question - 5

Let ' $S$ ' be a stack of size  $n \geq 1$ . Starting with the empty stack, suppose we push the first ' $n$ ' natural numbers in sequence and then perform ' $n$ ' pop operations. Assume that push and pop operations take ' $x$ ' seconds each, and ' $y$ ' seconds elapse between the end of one such stack operation and the start of the next operation. For  $m \geq 1$ , define the stack-life of ' $m$ ' as the elapsed from end of  $\text{push}(m)$  to the start of the  $\text{pop}$  operation that removes ' $m$ ' from  $S$ . The average stack-life of an element of this stack is:



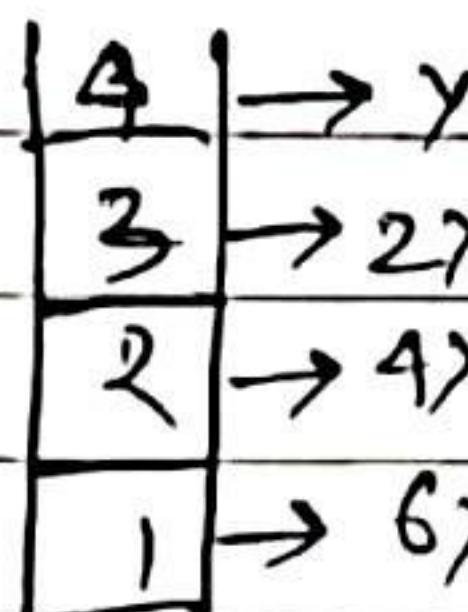


$$= 2x + 2y + y$$



$$2 \rightarrow 2x + 2y + y$$

$$1 \rightarrow 4x + 4y + y$$



$$3 \rightarrow 2x + 2y + y$$

$$2 \rightarrow 4x + 4y + y$$

$$1 \rightarrow 6x + 6y + y$$

for  $n$  elements,

$$\rightarrow 2(n-1)x + 2(n-1)y + ny$$

average life time,

$$2 * \left( \sum_{i=1}^{n-1} i \right) x + 2 \left( \sum_{i=1}^{n-1} i \right) y + ny$$

$$2 * \frac{(n-1)(n-1+1)}{2} * x + 2 * \frac{(n-1)(n-1+1)}{2} * y + ny$$

$\Rightarrow$

$n$

$$\Rightarrow \frac{n(n-1)x + n(n-1)y + ny}{n}$$

$$\Rightarrow (n-1)x + (n-1)y + y$$

$$\Rightarrow (n-1)(x+y) + y$$

$$\Rightarrow n(x+y) - x - y + y$$

$$\Rightarrow n(x+y) - x$$

16-2006

**Question - 6**

Queue is implemented using two stacks  $S_1$  and  $S_2$  as given below:

```
void insert(Q; x) {
    push(S1, x);
}
```

```
void delete(Q) {
    if (stack-empty(S2)) then
        if (stack-empty(S1)) then {
            printf("Q is empty");
            return;
        }
}
```

```
else while (!stack-empty(S1)) {
```

```
    x = pop(S1);

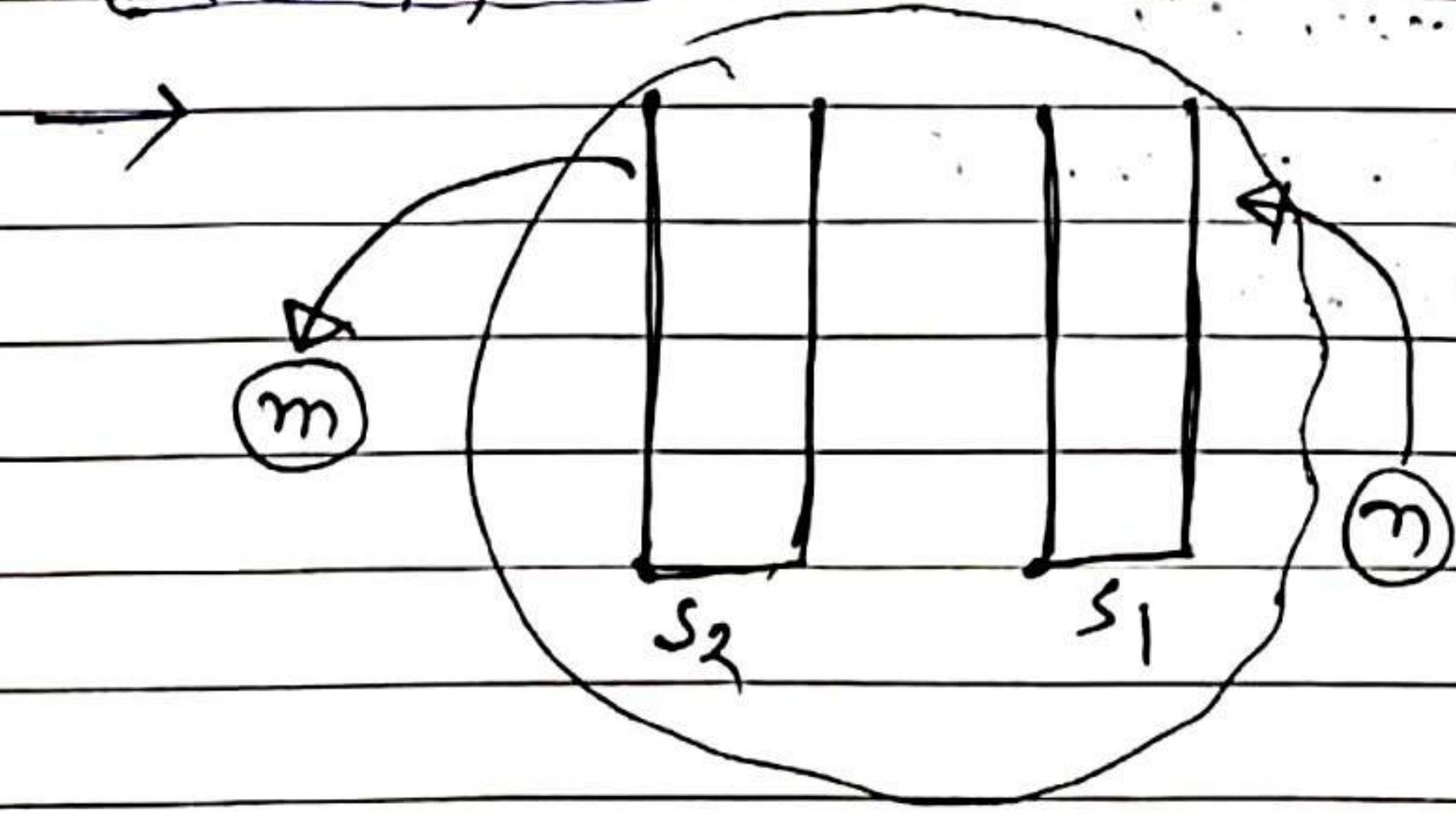
```

```
    push(S2, x);
}
```

```
x = pop(S2);
}
```

$m \rightarrow$  Inserts  
 $m (< m) \rightarrow$  deletes.  
 $x \rightarrow$  Pushes  
 $y \rightarrow$  Pops

What is relation b/w all this?



In Best case :

$$\text{PUSH} : 2m + (n-m) \\ = m+n$$

$$\text{POP} : 2m$$

Worst case :

$$\text{PUSH} : 2n$$

$$\text{POP} : 2(n+m)$$

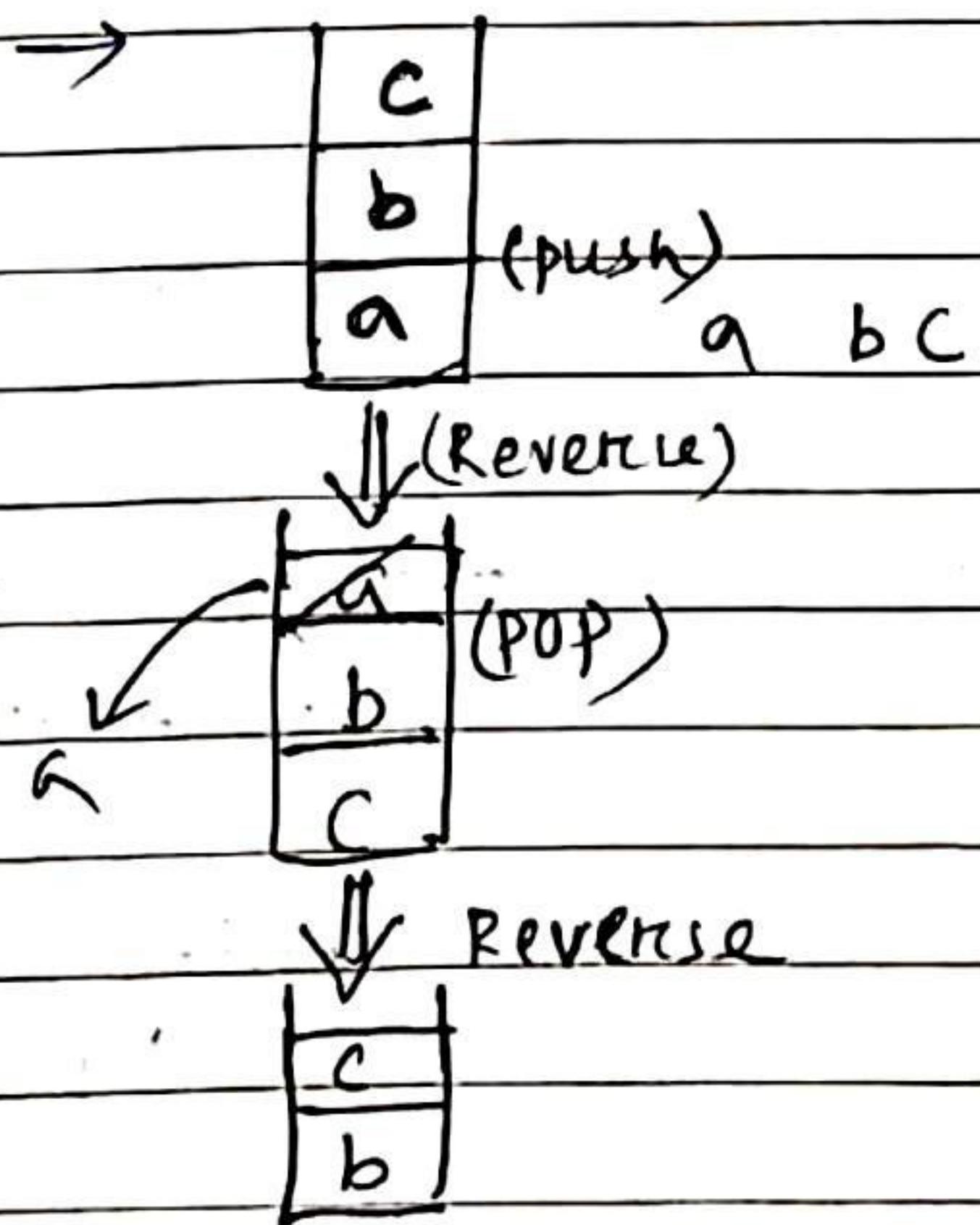
$$m+n \leq x \leq 2n \\ 2m \leq y \leq n+m$$

gate-2000

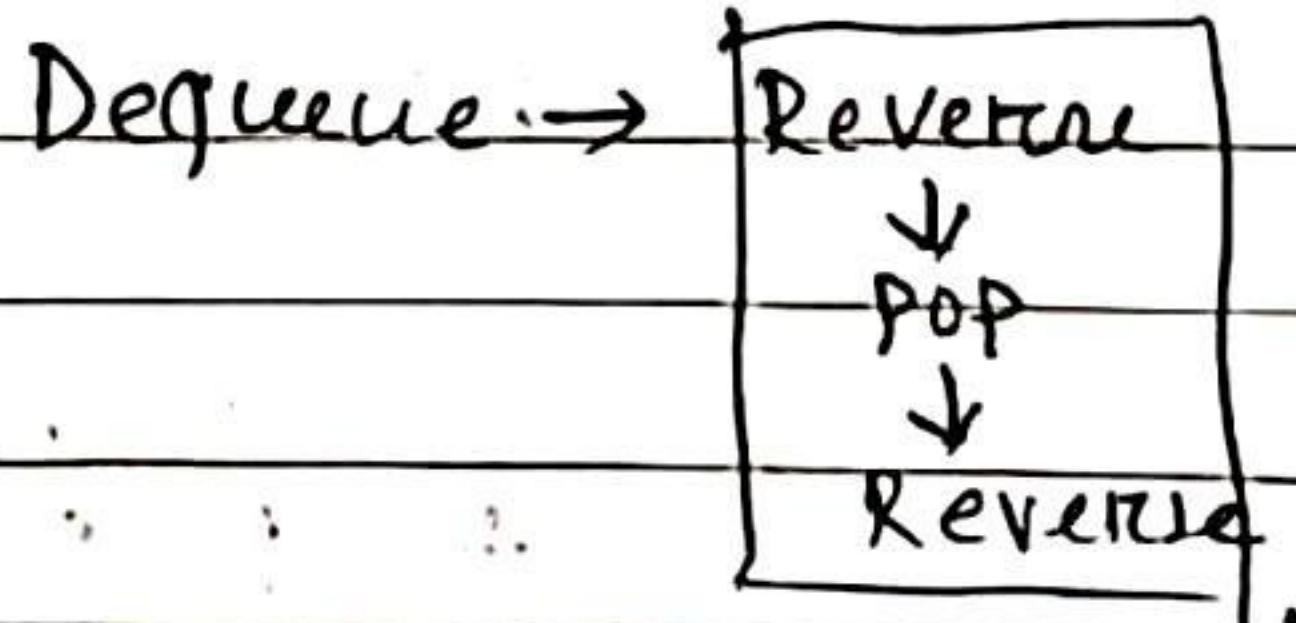
Question - 7)

Suppose a stack implementation supports, in addition to push, and pop, an operation "REVERSE" the order of elements on the stack.

Implement a queue using the above stack implementation, show how to implement "ENQUEUE" using a single operation and "DEQUEUE" using a sequence of 3 operations.



Enqueue → [push]



Infix  $\rightarrow$  postfix

- Infix to postfix conversion algorithm :- (operator stack)

$\rightarrow$  Postfix related anything stack is required.

example:

$$\textcircled{1} \quad a + b \rightarrow \text{Infix expression:}$$

$$ab+ \rightarrow \text{Postfix expression.}$$

$$\textcircled{2} \quad a + b * c \rightarrow \text{Infix exp}$$

$$= \underline{a + b c *}$$

$$= abc * + \rightarrow \text{Postfix expression.}$$

$$\textcircled{3} \quad a + b - c$$

$$= \underline{ab + - c}$$

$$= \underline{\underline{ab + c -}}$$

+ < . (
( < . + )

$$\textcircled{4} \quad a + (b - c) - \text{Infix}$$

$$= \underline{a + b c -}$$

$$= abc - + - \text{Postfix.}$$

$$\textcircled{5} \quad a + b * c - \text{infix}$$

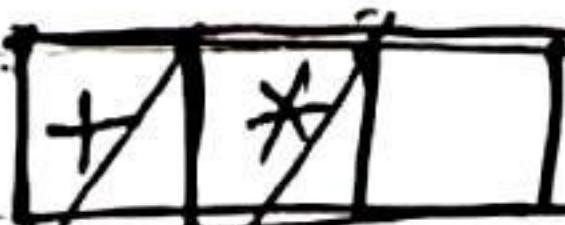
$$= \underline{a + b c *}$$

$$= \underline{\underline{abc * +}} - \text{Postfix}$$

$$\textcircled{6} \quad \begin{matrix} a & + & b & * & c \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \end{matrix} - \text{Infix.}$$

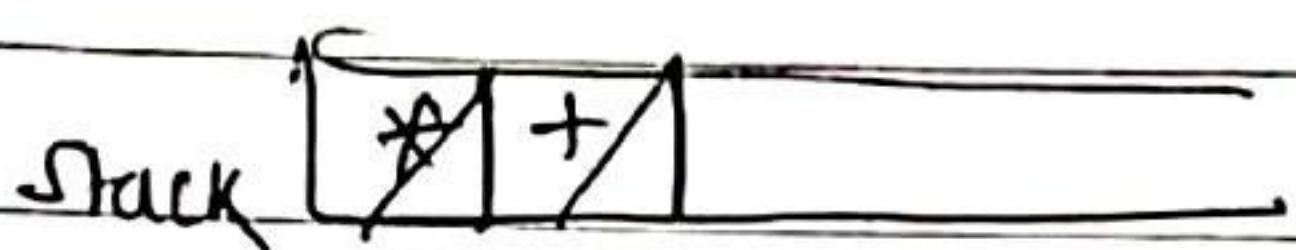
$\rightarrow$  you can push an operator, ~~if~~ only when the inside one is having lower lesser precedence.

Stack



$$= \underline{\underline{abc * +}} - \text{Postfix.}$$

⑦  $a * b + c$  — Infix expression.



$a b * c +$  — postfix expression.

⑧  $a * b + (c - d)$  — Infix



$a b * c d - +$  — postfix expression.

### Algorithm

a) Create a stack.

b) For each character 't' in the i/p.

{  
if ('t' is an operand)  
output 't';

else if ('t' is a right parenthesis)

{

pop and output tokens until a left parenthesis  
( ) is popped (but don't o/p)

else // t is an operator or left parenthesis

{  
pop and o/p tokens until one of lower  
priority than 't' is encountered or a  
left parenthesis is encountered or  
the stack is empty.  
push t

c) pop and o/p all the tokens until the stack is empty.

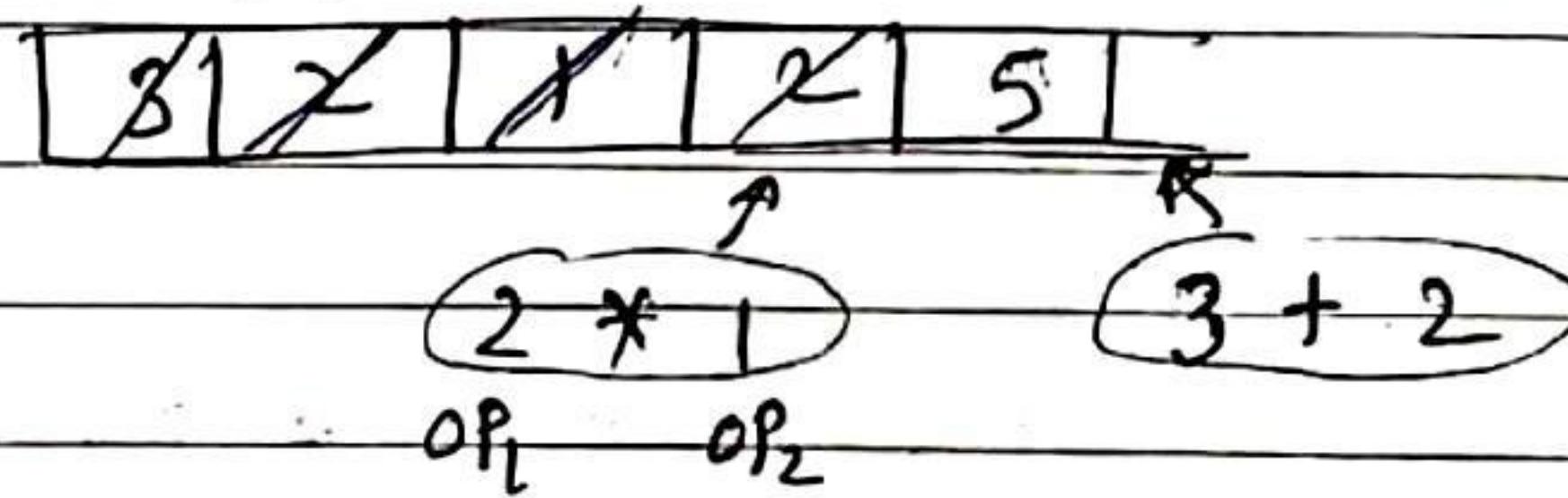
space  
Time complexity —  $O(n)$   
Data structure used — stack  
on push only — operators.

- postfix evaluation algorithm = Operand stack

~~3 + 2 \* 1~~ — Infix

3 + 2 1 \*

~~3 2 1 \* +~~ — postfix  
~~↑ ↑ ↑ ↑ ↑~~



push — only operand.

Algorithm for postfix evaluation =

1. Scan the postfix string from left to right.

2. Initialize an empty stack.

3. Repeat steps 4 and 5 till all the characters are scanned.

4. If the scanned character is an operand, push it onto the stack.

5. If the scanned character is an operator, and if the operator is unary, then pop an element

from the stack. If the operator is binary, then pop two elements from the stack. After popping the elements from the stack apply the operator to those popped elements. push the result on to the stack.

- 6) After all the elements are scanned, the result will be in the stack.