Bayes Theorem – Contingency Tables

Example: weather & temperature

Suppose that the probability of a high temperature is **30%**.
If the temperature is high (low), there is

60% chance for the weather to be sunny (**30%** if low),
20% to be cloudy (**30%** if low) and
20% to be rainy (**40%** if low).

What is P(**T** = high | **W** = sunny)?

temperature (**T**)	sunny	cloudy	rainy	
low	21%	21%	28%	70%
high	18%	6%	6%	30%
	39%	27%	34%	100%

weather (**W**)

P(**T** = high | **W** = sunny) = $\frac{18\%}{39\%} = \sim 46.15\%$

Bayes Theorem – Contingency Tables

Exercise: flu detection

Suppose that the probability to be infected with the flu is **5%**.
If you have the flu, you are tested positive with **90%** probability.
If you are not infected, you are tested positive with **15%** probability.

What is P(**flu** = yes | **test** = positive)?

	positive	negative	
yes			
no			

test

flu

Bayes Theorem – Formula

Example: weather & temperature

Suppose that the probability of a high temperature is **30%**.
If the temperature is high (low), there is
60% chance for the weather to be sunny (**30%** if low),
20% to be cloudy (**30%** if low) and
20% to be rainy (**40%** if low).

	sunny	cloudy	rainy			
	21%	21%	28%	70%	low	temperature (**T**)
	18%	6%	6%	30%	high	
	39%	27%	34%	100%		

weather (**W**)

What is P(**T** = high | **W** = sunny)?

Steps:

P(**T** = high) = 30%

P(**W** = sunny | **T** = high) = 60%

P(**W** = sunny) = P(**T** = high) P(**W** = sunny | **T** = high) + P(**T** = low) P(**W** = sunny | **T** = low)

$= 30\% \cdot 60\% + 70\% \cdot 30\% = 18\% + 21\% = 39\%$

P(**T** = high | **W** = sunny) = $\dfrac{P(T = high)\ P(W = sunny \mid T = high)}{P(W = sunny)} = \dfrac{30\% \cdot 60\%}{39\%} \approx 46.15\%$

Bayes Theorem – Formula

<u>Exercise: flu detection</u>

Suppose that the probability to be infected with the flu is <u>**5%**</u>.
If you have the flu, you are tested positive with <u>**90%**</u> probability.
If you are not infected, you are tested positive with <u>**15%**</u> probability.

What is P(**flu** = yes | **test** = positive)?

<u>Steps:</u>

P(**flu** = yes) =

P(**test** = pos. | **flu** = yes) =

P(**test** = pos.) = P(**flu** = yes) P(**test** = pos. | **flu** = yes) + P(**flu** = no) P(**test** = pos. | **flu** = no)

=

P(**flu** = yes | **test** = pos.) =

Bayes Theorem

<u>Example: weather & temperature</u>

$$P(T = \text{high} \mid W = \text{sunny}) = \frac{P(T = \text{high})\, P(W = \text{sunny} \mid T = \text{high})}{P(W = \text{sunny})}$$

<u>Exercise: flu detection</u>

$$P(\text{flu} = \text{yes} \mid \text{test} = \text{pos}) = \frac{P(\text{flu} = \text{yes})\, P(\text{test} = \text{pos} \mid \text{flu} = \text{yes})}{P(\text{test} = \text{pos})}$$

<u>Bayes Theorem</u>

$$\underbrace{P(A \mid B)}_{\text{posterior}} = \frac{\overbrace{P(A)}^{\text{prior}} \cdot \overbrace{P(B \mid A)}^{\text{likelihood}}}{\underbrace{P(B)}_{\text{evidence}}}$$

Bayes Theorem – Overarching problem

Customer type
{hospital, doctor's office}

↓

Number of products
{1, ≥2}

Type	
Hospital	25%
DO	75%

Type	# products 1 / ≥2
Hospital	90% / 10%
DO	80% / 20%

Exercise: customer cancellation

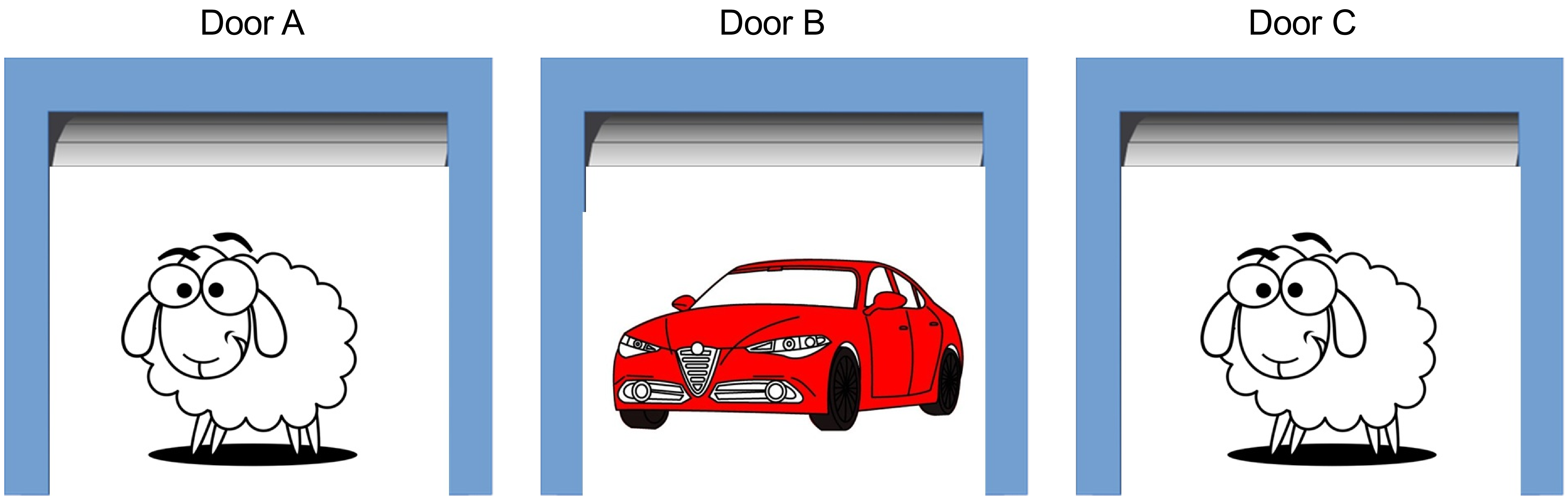
What is P(**type** = hospital | **# prod** = 1)?

Reminder:

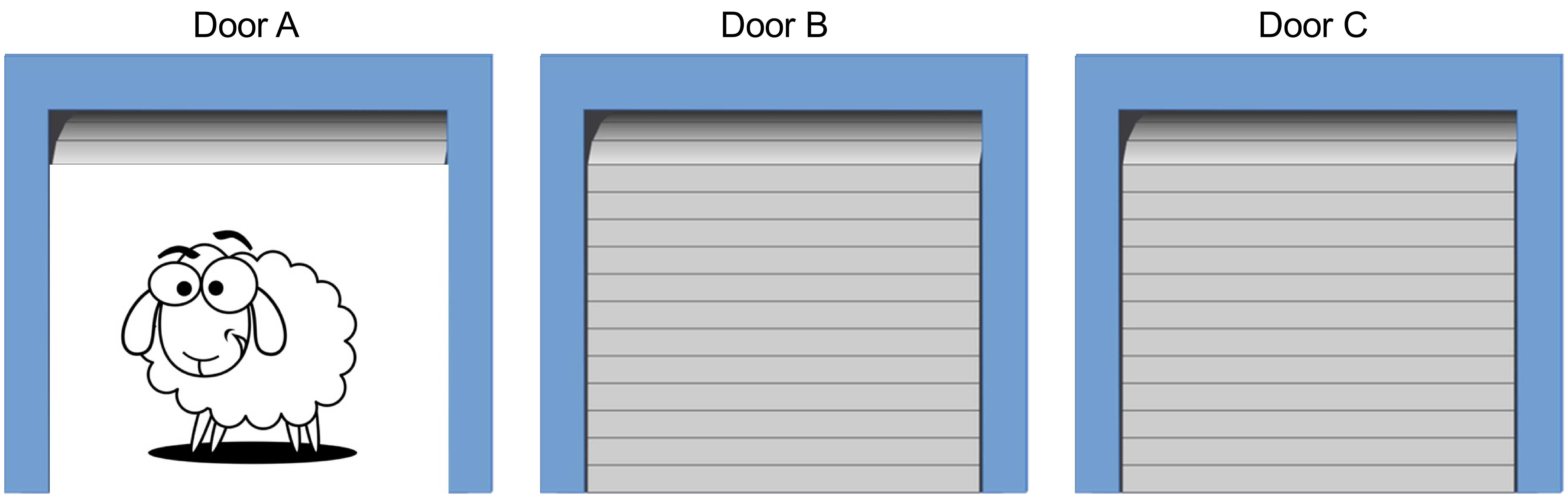
$$P(\textbf{type} = \text{hospital} \mid \textbf{\# prod} = 1) = \frac{P(\textbf{\# prod} = 1 \mid \textbf{type} = \text{hospital})\, P(\textbf{type} = \text{hospital})}{P(\textbf{\# prod} = 1)}$$

Monty Hall Problem

Monty Hall Problem



Monty Hall Problem



You have selected Door C. Monty has opened Door A and revealed a sheep.

Should you switch to Door B?

What is P(**car** = C | **open** = A)?