# Recap on probability

Bayes Theorem

$$P(\mathbf{A} = a \mid \mathbf{B} = b) = \frac{P(\mathbf{B} = b \mid \mathbf{A} = a)\, P(\mathbf{A} = a)}{P(\mathbf{B} = b)} = \frac{P(\mathbf{B} = b, \mathbf{A} = a)}{P(\mathbf{B} = b)}$$

Reformulating probabilities

$$P(\mathbf{A} = a \mid \mathbf{B} = b) = P(\mathbf{A} = a, \mathbf{C} = c \mid \mathbf{B} = b) + P(\mathbf{A} = a, \mathbf{C} = \neg c \mid \mathbf{B} = b)$$

$$= \sum_{c} P(\mathbf{A} = a, \mathbf{C} = c \mid \mathbf{B} = b) = \sum_{c,d} P(\mathbf{A} = a, \mathbf{C} = c, \mathbf{D} = d \mid \mathbf{B} = b)$$

Conditioning

$$P(\mathbf{A} = a \mid \mathbf{B} = b) = \sum_{c} P(\mathbf{A} = a, \mathbf{C} = c \mid \mathbf{B} = b) = \sum_{c} P(\mathbf{A} = a \mid \mathbf{C} = c, \mathbf{B} = b)\, P(\mathbf{C} = c \mid \mathbf{B} = b)$$

$$P(\mathbf{A} = a, \mathbf{B} = b, \mathbf{C} = c) = P(\mathbf{A} = a)\, P(\mathbf{B} = b \mid \mathbf{A} = a)\, P(\mathbf{C} = c \mid \mathbf{A} = a, \mathbf{B} = b)$$

# Calculating posterior in Bayesian network

| T | |
|---|---|
| Low | 40% |
| High | 60% |

Temperature (T)
{low, high}

| T | John running |
|---|---|
| Low | 50% / 50% |
| High | 70% / 30% |

John running
{yes, no}

Kate running
{yes, no}

| T | Kate running |
|---|---|
| Low | 40% / 60% |
| High | 75% / 25% |

| John | Kate | Meet? |
|---|---|---|
| Yes | Yes | 50% / 50% |
| Yes | No | 0% / 100% |
| No | Yes | 0% / 100% |
| No | No | 0% / 100% |

Kate & John meet
{yes, no}

**Questions**

- P(**John** = yes | **meet** = no)
- P(**meet** = yes | **temperature** = high)
- P(**Kate** = yes | **John** = yes)

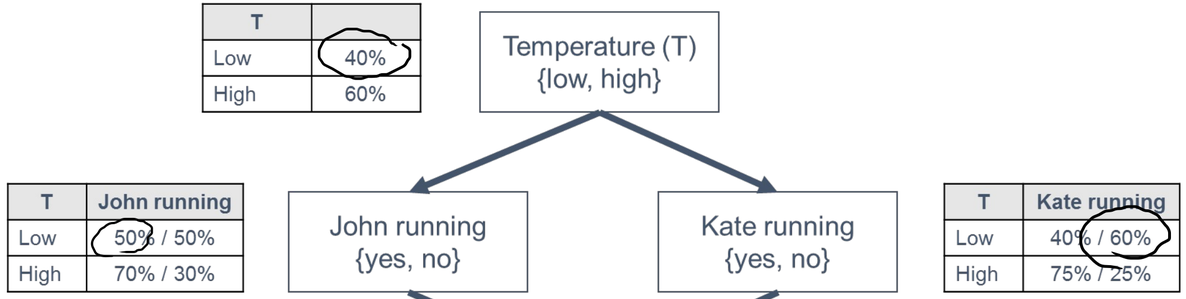
# Approach 1: Enumeration

$$P(\textbf{John} = \text{yes} \mid \textbf{Meet} = \text{no}) = \frac{P(\textbf{J} = \text{yes}, \textbf{M} = \text{no})}{P(\textbf{M} = \text{no})}$$

$$P(\textbf{J} = \text{yes}, \textbf{M} = \text{no}) = \sum_{t,k} P(\textbf{T} = t, \textbf{K} = k, \textbf{J} = \text{yes}, \textbf{M} = \text{no})$$

$$= \sum_{t,k} P(\textbf{T} = t)\, P(\textbf{K} = k \mid \textbf{T} = t)\, P(\textbf{J} = \text{yes} \mid \textbf{K} = k, \textbf{T} = t)\, P(\textbf{M} = \text{no} \mid \textbf{K} = k, \textbf{J} = \text{yes}, \textbf{T} = t)$$

| T | |
|---|---|
| Low | 40% |
| High | 60% |




| T | John running |
|---|---|
| Low | 50% / 50% |
| High | 70% / 30% |

| T | Kate running |
|---|---|
| Low | 40% / 60% |
| High | 75% / 25% |

| John | Kate | Meet? |
|---|---|---|
| Yes | Yes | 50% / 50% |
| Yes | No | 0% / 100% |
| No | Yes | 0% / 100% |
| No | No | 0% / 100% |

| T | K | | | | | |
|---|---|---|---|---|---|---|
| low | yes | 40% | 40% | 50% | 50% | + |
| low | no | 40% | 60% | 50% | 100% | + |
| high | yes | 60% | 75% | 70% | 50% | + |
| high | no | 60% | 75% | 70% | 100% | |

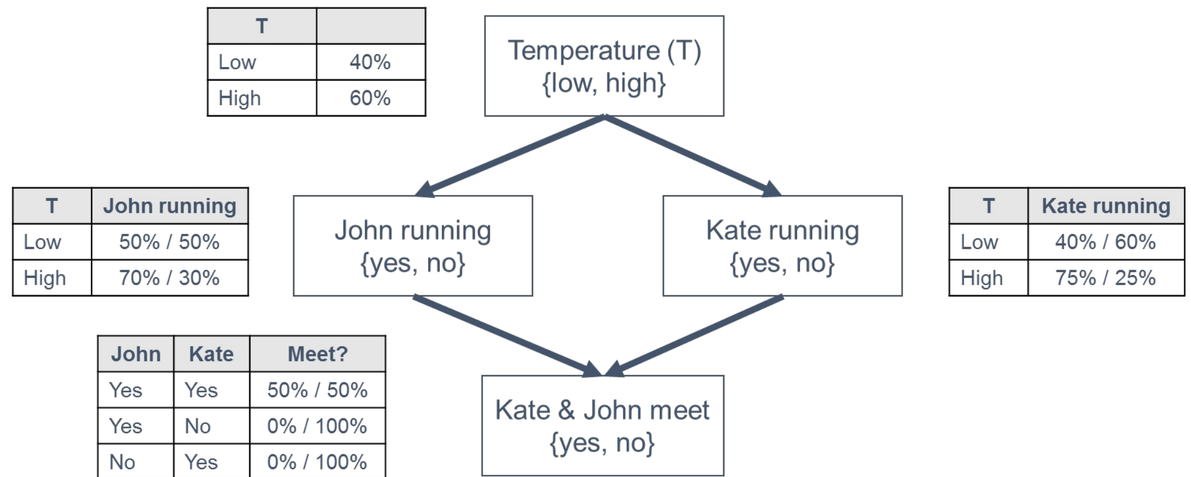
} 42.75%

# Approach 1: Enumeration

$$P(\textbf{John} = \text{yes} \mid \textbf{Meet} = \text{no}) = \frac{P(\textbf{J} = \text{yes}, \textbf{M} = \text{no})}{P(\textbf{M} = \text{no})}$$

$$P(\textbf{M} = \text{no}) = P(\textbf{M} = \text{no}, \textbf{J} = \text{yes}) + P(\textbf{M} = \text{no}, \textbf{J} = \text{no})$$

$$42.25\% + 38\%$$




$$P(\text{John} = \text{yes} \mid \text{Meet} = \text{no}) = \frac{42.25\%}{42.25\% + 38\%} = 52.6\%$$

# Approach 2: Elimination

$$P(\textbf{John} = \text{yes} \mid \textbf{Meet} = \text{no}) = \frac{P(\textbf{J} = \text{yes}, \textbf{M} = \text{no})}{P(\textbf{M} = \text{no})}$$

| T | |
|---|---|
| Low | 40% |
| High | 60% |

Temperature (T)
{low, high}

| T | John running |
|---|---|
| Low | 50% / 50% |
| High | 70% / 30% |

John running
{yes, no}

Kate running
{yes, no}

| T | Kate running |
|---|---|
| Low | 40% / 60% |
| High | 75% / 25% |

| John | Kate | Meet? |
|---|---|---|
| Yes | Yes | 50% / 50% |
| Yes | No | 0% / 100% |
| No | Yes | 0% / 100% |
| No | No | 0% / 100% |

Kate & John meet
{yes, no}

$$P(\textbf{J} = \text{yes}, \textbf{M} = \text{no}) = \sum_{t,k} P(\textbf{T} = t)\, P(\textbf{K} = k \mid \textbf{T} = t)\, P(\textbf{J} = \text{yes} \mid \textbf{T} = t)\, P(\textbf{M} = \text{no} \mid \textbf{K} = k, \textbf{J} = \text{yes})$$

$$= \sum_{k} P(\textbf{M} = \text{no} \mid \textbf{K} = k, \textbf{J} = \text{yes}) \sum_{t} P(\textbf{T} = t)\, P(\textbf{K} = k \mid \textbf{T} = t)\, P(\textbf{J} = \text{yes} \mid \textbf{T} = t)$$

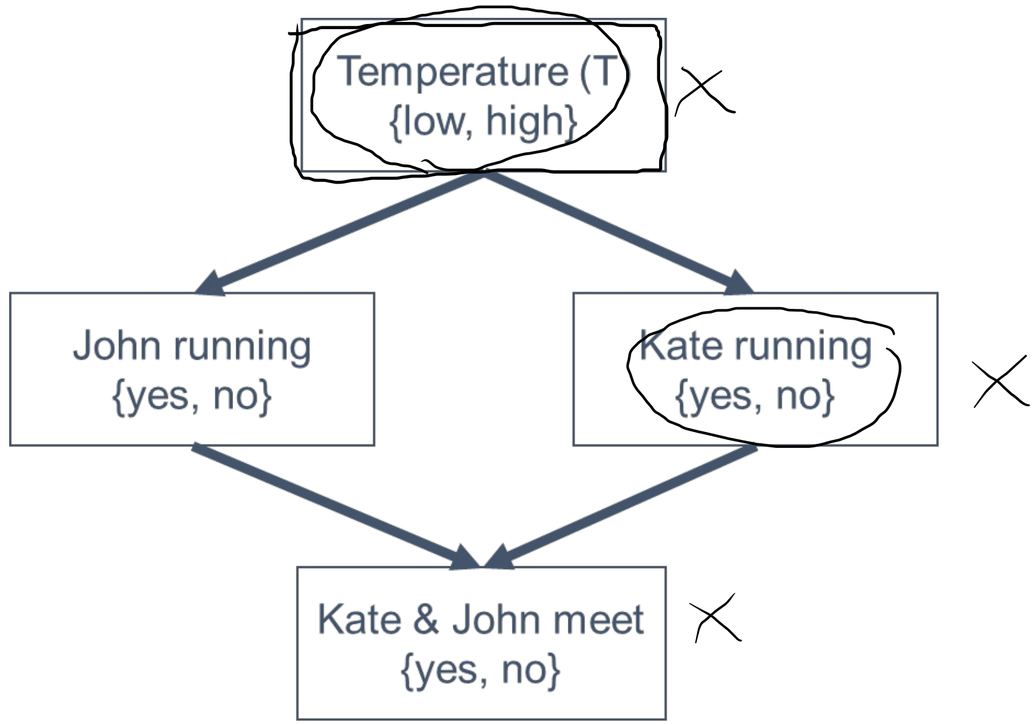
$$= \sum_{t} P(T = t, K = k, J = \text{yes}) = P(K = k, J = \text{yes})$$

yes 35.5% · 50% +
no 22.5% · 100%
= 42.25%

| John | Kate | |
|---|---|---|
| Yes | ~~Yes~~ | 39.5% |
| Yes | No | 22.5% |

low 40% 50% 60%
high 60% 70% 25%

# Enumeration vs Elimination




Enumeration

$$P(\mathbf{J} = \text{yes}, \mathbf{M} = \text{no}) = \sum_{t,k} P(\mathbf{T} = t)\, P(\mathbf{K} = k \mid \mathbf{T} = t)\, P(\mathbf{J} = \text{yes} \mid \mathbf{T} = t)\, P(\mathbf{M} = \text{no} \mid \mathbf{K} = k, \mathbf{J} = \text{yes})$$

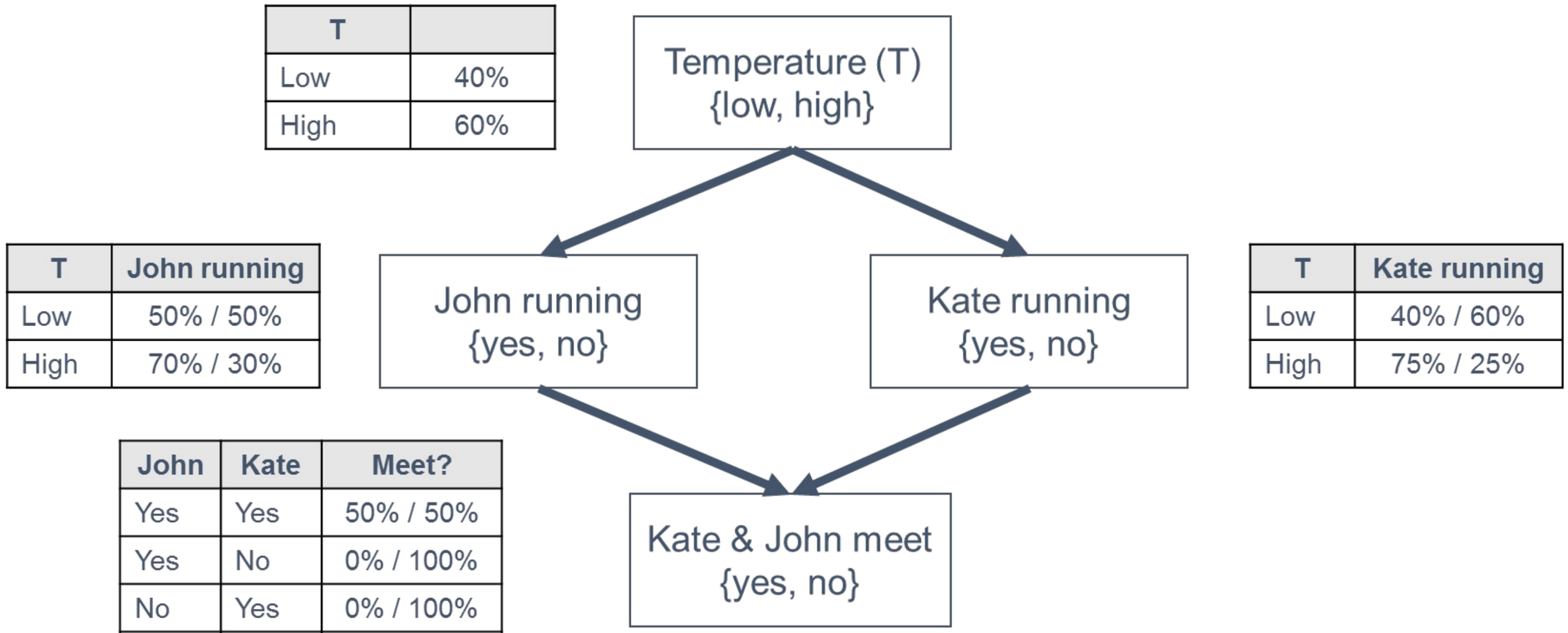
Elimination

$$P(\mathbf{J} = \text{yes}, \mathbf{M} = \text{no}) = \sum_{k} P(\mathbf{M} = \text{no} \mid \mathbf{K} = k, \mathbf{J} = \text{yes}) \sum_{t} P(\mathbf{T} = t)\, P(\mathbf{K} = k \mid \mathbf{T} = t)\, P(\mathbf{J} = \text{yes} \mid \mathbf{T} = t)$$

| John | Kate | |
|---|---|---|
| Yes | Yes | |
| Yes | No | |

# Approach 1: Enumeration

| T | |
|---|---|
| Low | 40% |
| High | 60% |




| T | John running |
|---|---|
| Low | 50% / 50% |
| High | 70% / 30% |

| T | Kate running |
|---|---|
| Low | 40% / 60% |
| High | 75% / 25% |

| John | Kate | Meet? |
|---|---|---|
| Yes | Yes | 50% / 50% |
| Yes | No | 0% / 100% |
| No | Yes | 0% / 100% |
| No | No | 0% / 100% |

Exercise: Solve by **Enumeration**

P(**meet** = yes | **temperature** = high)

# Approach 2: Elimination

| T | |
|---|---|
| Low | 40% |
| High | 60% |

Temperature (T)
{low, high}

| T | John running |
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| Low | 50% / 50% |
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John running
{yes, no}

Kate running
{yes, no}

| T | Kate running |
|---|---|
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| John | Kate | Meet? |
|---|---|---|
| Yes | Yes | 50% / 50% |
| Yes | No | 0% / 100% |
| No | Yes | 0% / 100% |
| No | No | 0% / 100% |

Kate & John meet
{yes, no}

Exercise: Solve by **Elimination**

P(**John** = yes | **Kate** = yes)