Some more probability

Example: weather & temperature

21%	21%	28%	70%	low	temperature (**T**)	P(**W** = sunny | **T** = low) = 0.3
18%	6%	6%	30%	high		P(**W** = sunny | **T** = high) = 0.6
39%	27%	34%	100%			
sunny	cloudy	rainy				
	weather (**W**)					

Some more principles

$$P(\mathbf{W} = \text{sunny} \mid \mathbf{T} = \text{low}) = \frac{P(T = \text{low} \mid W = \text{sunny})\, P(W = \text{sunny})}{P(T = \text{low})} = \frac{\overset{21\%}{P(T = \text{low}, W = \text{sunny})}}{\underset{70\%}{P(T = \text{low})}}$$

$$P(\mathbf{W} = \text{sunny}) = P(\mathbf{W} = \text{sunny}, \mathbf{T} = \text{low}) + P(\mathbf{W} = \text{sunny}, \mathbf{T} = \text{high}) = 21\% + 18\% = 39\%$$

$$P(\mathbf{W} = \text{sunny}, \mathbf{T} = \text{low}) = \underbrace{P(\mathbf{W} = \text{sunny} \mid \mathbf{T} = \text{low})}_{30\%}\, \underbrace{P(\mathbf{T} = \text{low})}_{70\%} = 21\%$$

Some more probability

Example: weather & temperature

21%	21%	28%	70%	low	
18%	6%	6%	30%	high	temperature (**T**)
39%	27%	34%	100%		
sunny	cloudy	rainy			
	weather (**W**)				

$P(\mathbf{W} = \text{sunny} \mid \mathbf{T} = \text{low}) = 0.3$

$P(\mathbf{W} = \text{sunny} \mid \mathbf{T} = \text{high}) = 0.6$

Application

$$P(\mathbf{W} = \text{sunny}) = P(\mathbf{W} = \text{sunny}, \mathbf{T} = \text{low}) + P(\mathbf{W} = \text{sunny}, \mathbf{T} = \text{high})$$

$$= P(\mathbf{W} = \text{sunny} \mid \mathbf{T} = \text{low})\, P(\mathbf{T} = \text{low}) + P(\mathbf{W} = \text{sunny} \mid \mathbf{T} = \text{high})\, P(\mathbf{T} = \text{high})$$

$$= 30\% \quad 70\% + 60\% \quad 30\%$$

$$= 21\% + 18\% = 39\%$$

Joint distribution of three variables

$$P(\mathbf{A} = a, \mathbf{B} = b, \mathbf{C} = c) = P(\mathbf{A} = a \mid \mathbf{B} = b, \mathbf{C} = c)\, P(\mathbf{B} = b \mid \mathbf{C} = c)\, P(\mathbf{C} = c)$$

Amp   low, high

Reformulations assuming B has events {b, b'}

$$P(\mathbf{A} = a \mid \mathbf{C} = c) = P(\mathbf{A} = a, \mathbf{B} = b \mid \mathbf{C} = c) + P(\mathbf{A} = a, \mathbf{B} = b' \mid \mathbf{C} = c)$$

$$P(\mathbf{A} = a, \mathbf{B} = b \mid \mathbf{C} = c) = P(\mathbf{A} = a \mid \mathbf{B} = b, \mathbf{C} = c)\, P(\mathbf{B} = b \mid \mathbf{C} = c)$$

Bayes Theorem with two dependent variables

$$P(\mathbf{A} = a \mid \mathbf{B} = b, \mathbf{C} = c) = \frac{P(\mathbf{B} = b \mid \mathbf{A} = a, \mathbf{C} = c)\, P(\mathbf{A} = a \mid \mathbf{C} = c)}{P(\mathbf{B} = b \mid \mathbf{C} = c)}$$

Absolute & Conditional Independence

Absolute Independence

$$P(A = a \mid B = b) = P(A = a)$$

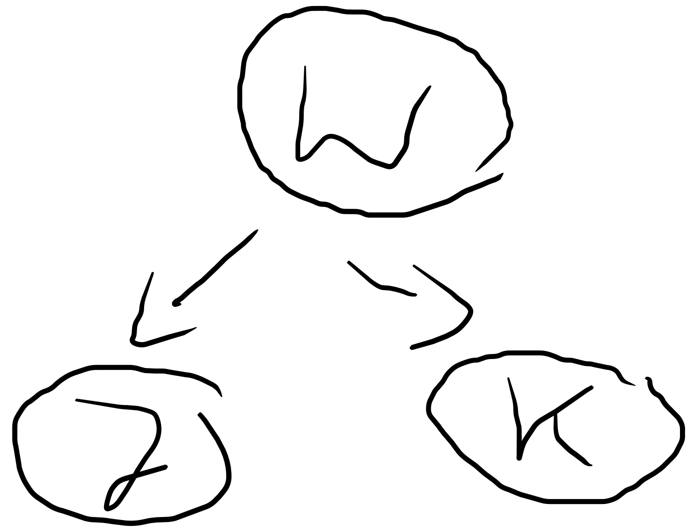
$A \perp B$

"A is indepenent of B" / "Kate running is independent of John running"

Conditional Independence

$$P(A = a \mid B = b, C = c) = P(A = a \mid C = c)$$

$A \perp B \mid C$



"A is independent of B given C" / "Kate running is independent of John running given the weather"

Questions

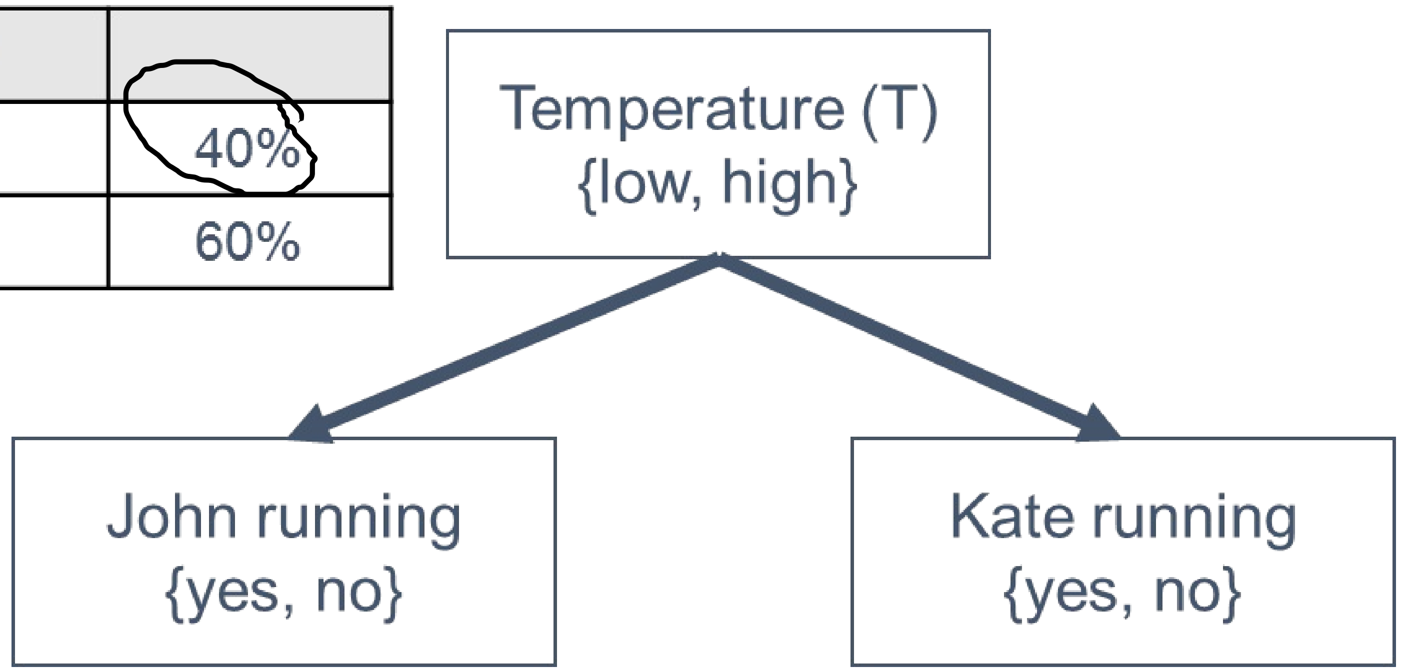
$A \perp B \rightarrow A \perp B \mid C$?

$A \perp B \mid C \rightarrow A \perp B$?

Conditional → Absolute Independence?

Example: John & Kate going for a run

T	
Low	40%
High	60%



T	John running
Low	50% / 50%
High	70% / 30%

T	Kate running
Low	40% / 60%
High	75% / 25%

Conditional independence:

P(**John** = yes | **Kate** = yes, **T** = low) = P(**John** = yes | **T** = low)

P(J = yes | K = yes) = P(J = yes)

Example
P(**John** = yes, **Kate** = yes)

= P(**John** = yes, **Kate** = yes, **T** = low) + P(**John** = yes, Kate = yes, T = high)

= P(**John** = yes | ~~**Kate** =~~ yes, **T** = low) P(**Kate** = yes | **T** = low) P(**T** = low)

30% 40% 40%

= 8% + 31.5%

+ P(**John** = yes | ~~**Kate** =~~ yes, **T** = high) P(**Kate** = yes | **T** = high) P(**T** = high)

70% 75% 60%

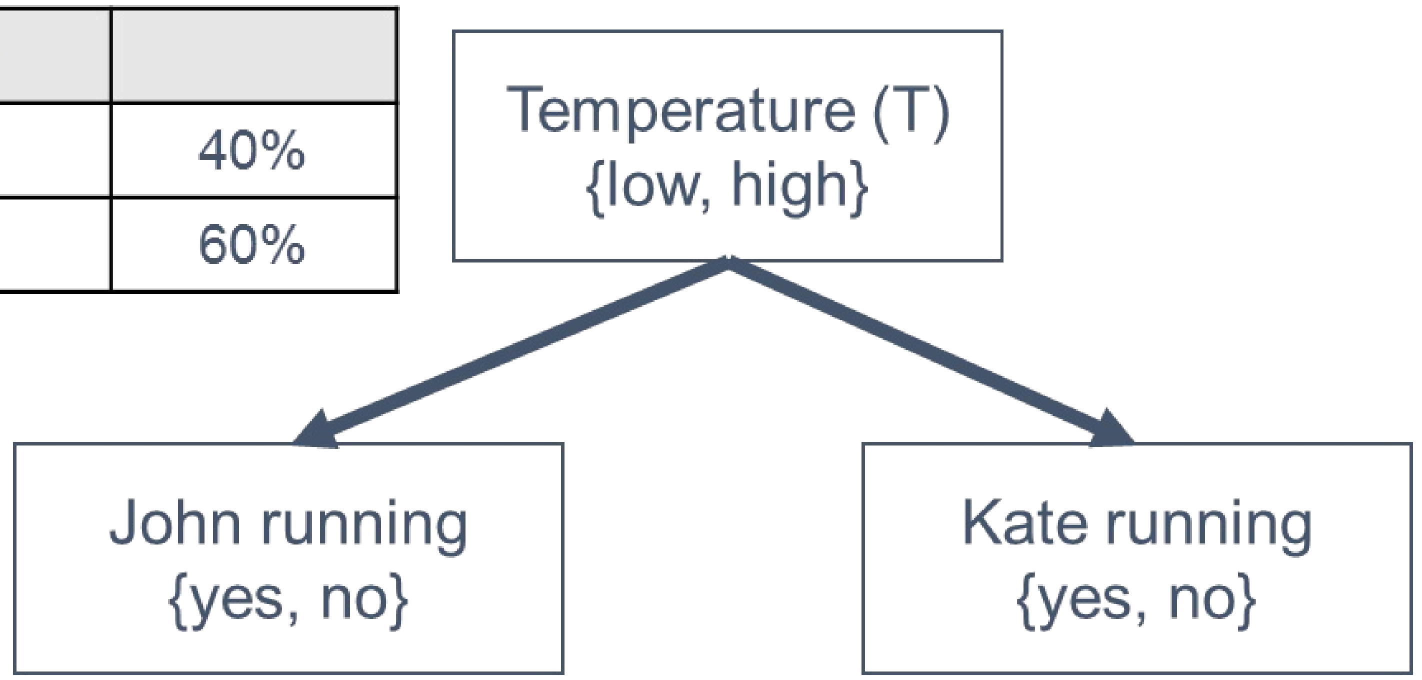
= 39.5%

Conditional → Absolute Independence?

Example: John & Kate going for a run

T	
Low	40%
High	60%

T	**John running**
Low	50% / 50%
High	70% / 30%



T	**Kate running**
Low	40% / 60%
High	75% / 25%

Conditional independence:

P(**John** = yes | **Kate** = yes, **T** = low) = P(**John** = yes | **T** = low)

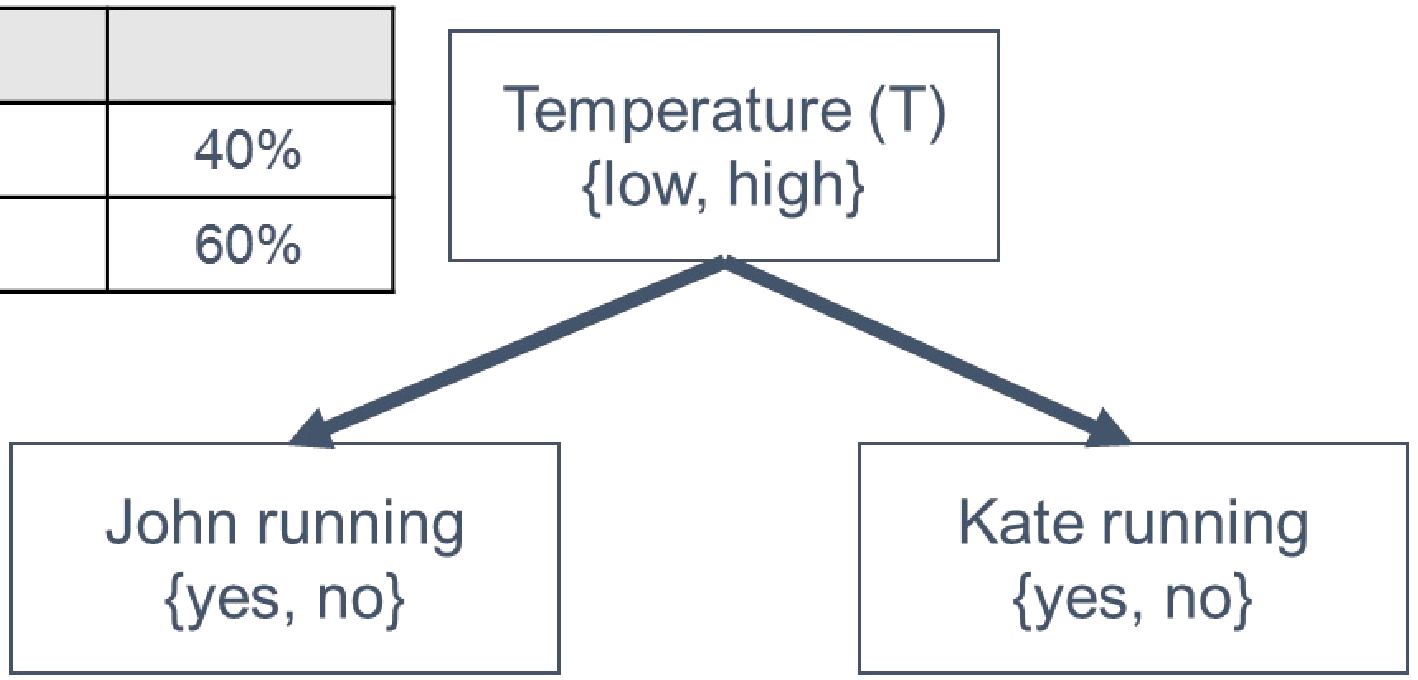
Exercise

P(**John** = no, **Kate** = no) =

Conditional → Absolute Independence?

<u>Example: John & Kate going for a run</u>

T	
Low	40%
High	60%



T	John running
Low	50% / 50%
High	70% / 30%

T	Kate running
Low	40% / 60%
High	75% / 25%

<u>Contigency Table</u>

John \ Kate	yes	no	
yes	39.5%	22.5%	62%
no	21.5%	16.5%	38%
	61%	39%	100%

<u>Absolute Independence?</u>

P(**John** = yes | **Kate** = yes) = $\frac{P(John = yes, Kate = yes)}{P(Kate = yes)} = \frac{39.5\%}{61\%} = 64.75\%$

P(**John** = yes) = 62%

$J \perp K \mid T \not\Rightarrow J \perp K$