HOMEWORK-8: BAYESIAN STATISTICS

Problem 1.1 :

For school-1:

Theta Posterior mean = **9.286568**

Confidence Interval = (**7.777469, 10.826767)**

Sigma Posterior mean = **3.917722**

Sigma Confidence Interval = (**3.018060, 5.164001**)

For School -2:

Theta Posterior mean = **6.963921**

Confidence Interval = (**5.202915, 8.740464)**

Sigma Posterior mean = **7.815385**

Sigma Confidence Interval = (**6.202766, 9.421261**)

For School -3:

Theta Posterior mean = **7.815385**

Confidence Interval = (**6.202766, 9.421261)**

Sigma Posterior mean = **3.737627**

Sigma Confidence Interval = (**2.786237, 5.122137**)

1.2 :

Posterior probabilities for theta_i < theta_j < theta_k:

P(theta_1 < theta_2 < theta_3) = 0.005

P(theta_1 < theta_3 < theta_2) = 0.0034

P(theta_2 < theta_1 < theta_3) = 0.084

P(theta_2 < theta_3 < theta_1) = 0.6733

P(theta_3 < theta_1 < theta_2) = 0.0154

P(theta_3 < theta_2 < theta_1) = 0.2189

1.3 :

Posterior probabilities for Y_i < Y_j < Y_k:

P(Y_1 < Y_2 < Y_3) = 0.0974

P(Y_1 < Y_3 < Y_2) = 0.1095

P(Y_2 < Y_1 < Y_3) = 0.1789

P(Y_2 < Y_3 < Y_1) = 0.2707

P(Y_3 < Y_1 < Y_2) = 0.1443

P(Y_3 < Y_2 < Y_1) = 0.1992

1.4 :

P(theta_1 > theta_2 and theta_3) = 0.8922

P(y_tilde_1 > y_tilde_2 and y_tilde_3) = 0.4699

2:

problem 2(A)

we need to show that,

$$P(\theta|y) \propto \frac{\exp(y\theta)}{(1+\exp(\theta))^n} \exp\left(-\frac{(\theta-\mu)^2}{2\sigma^2}\right)$$

prior distribution for $\theta \sim \text{Normal}(\mu, \sigma^2)$

Binomial likelihood for $(y|\theta)$ with n and $p = \dfrac{\exp(\theta)}{1+\exp(\theta)}$

Likelihood:- For a binomial distribution

$$P(y|\theta) \propto p^y (1-p)^{n-y}$$

$$P(y|\theta) \propto \left(\frac{\exp(\theta)}{1+\exp(\theta)}\right)^y \left(1 - \frac{\exp(\theta)}{1+\exp(\theta)}\right)^{n-y}$$

$$P(y|\theta) \propto \left(\frac{\exp(\theta)}{1+\exp(\theta)}\right)^y \left(\frac{1+\exp(\theta) - \exp(\theta)}{1+\exp(\theta)}\right)^{n-y}$$

$$P(y|\theta) \propto \frac{\exp(y\theta)}{(1+\exp(\theta))^n}$$

prior :- Follows Normal distribution

$$P(\theta) \propto \exp\left(-\frac{(\theta-\mu)^2}{2\sigma^2}\right)$$

Posterior :-

$$P(\theta|y) \propto P(y|\theta)\, P(\theta)$$

$$P(\theta|y) \propto \frac{\exp(y\theta)}{(1+\exp(\theta))^n} \exp\left(-\frac{(\theta-\mu)^2}{2\sigma^2}\right)$$

2.2:

Pr(theta > 0 | y) = 0.5919925

95% CI for theta: [-0.4024024, 0.5105105]