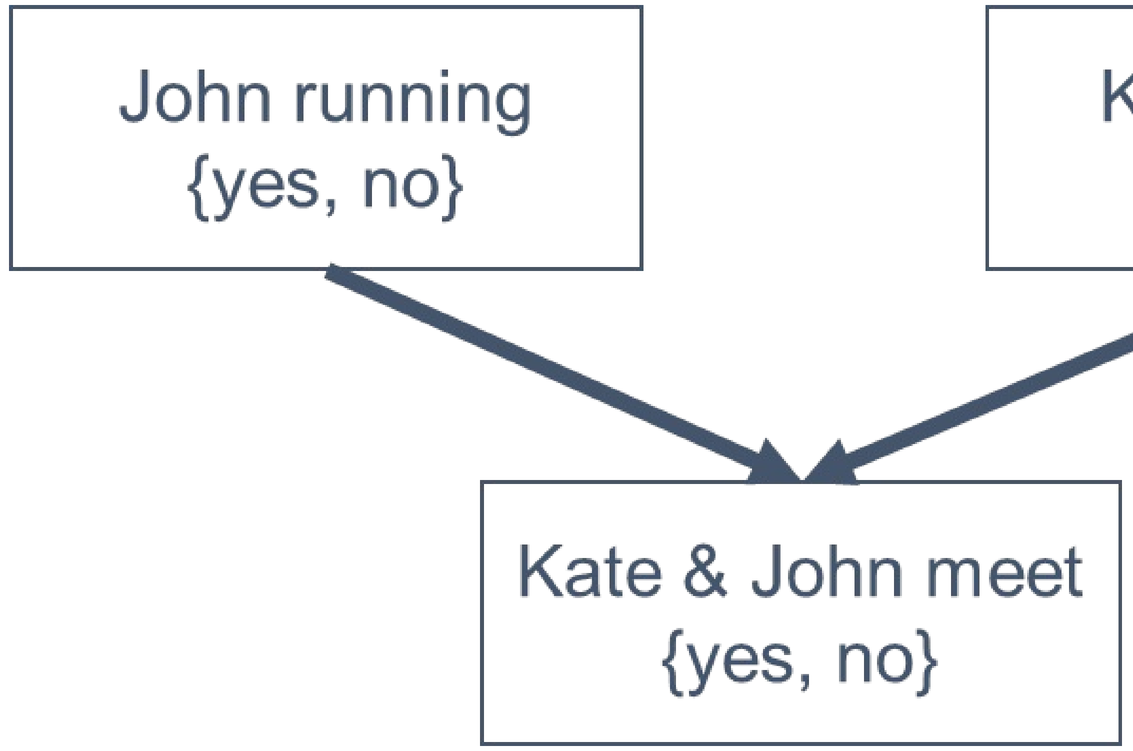
Absolute → Conditional Independence?

Example: John & Kate going for a run

John	
Yes	60%
No	40%



Kate	
Yes	30%
No	70%

John	Kate	Meet?
Yes	Yes	50% / 50%
Yes	No	0% / 100%
No	Yes	0% / 100%
No	No	0% / 100%

Conditional independence:

P(**John** = yes | **Kate** = yes, **Meet** = no) = P(**John** = yes | **Meet** = no)

$\leftarrow P(J = yes \mid K = yes) = P(J = yes)$

Some simplifying notation

P(**John** = yes) = P(J) P(**John** = no) = P(¬J)
P(**Kate** = yes) = P(K) P(**Kate** = no) = P(¬K)
P(**Meet** = yes) = P(M) P(**Meet** = no) = P(¬M)

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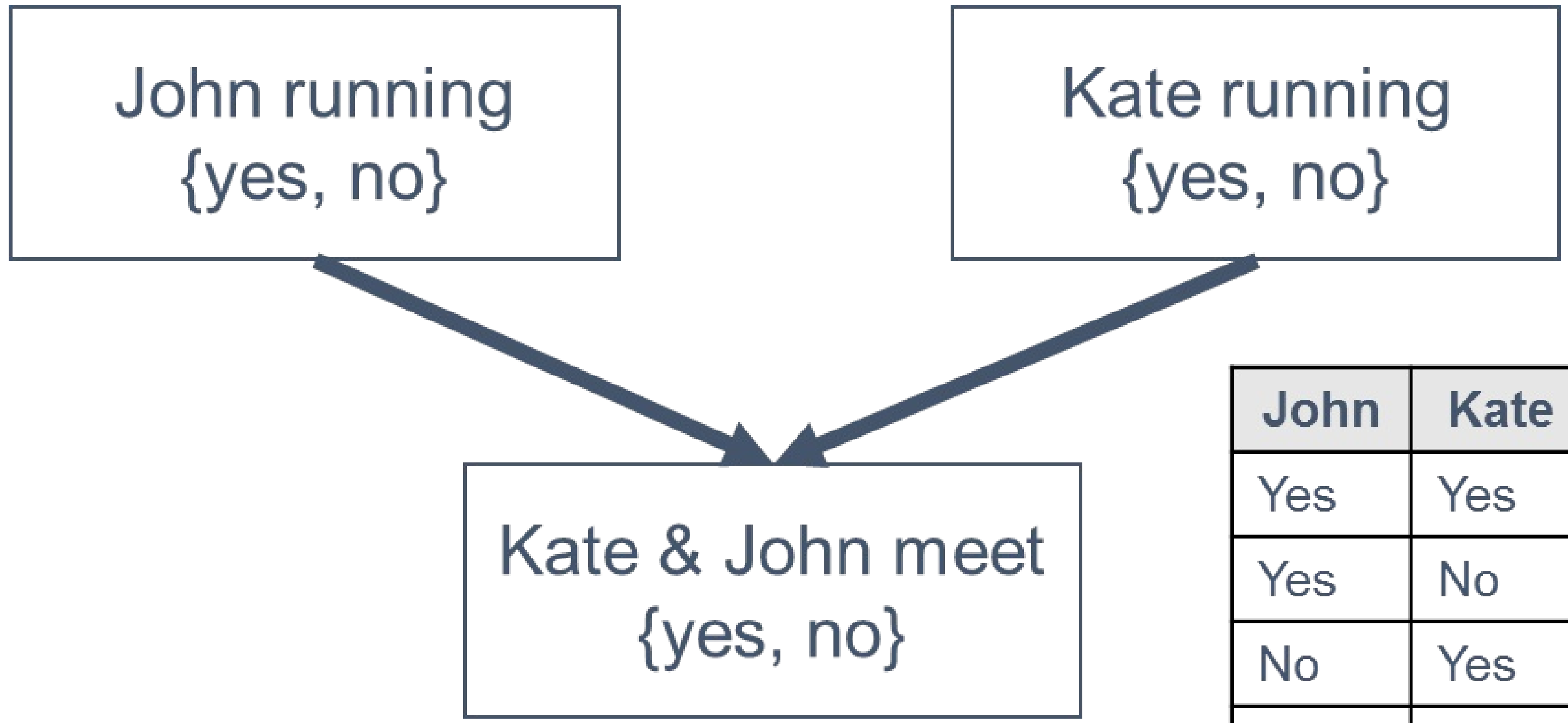
No	40%



Kate	
Yes	30%
No	70%

John	Kate	Meet?
Yes	Yes	50% / 50%
Yes	No	0% / 100%
No	Yes	0% / 100%
No	No	0% / 100%

Conditional independence:

P(**John** = yes | **Kate** = yes, **Meet** = no) = P(**John** = yes | **Meet** = no)

①

$$① \quad P(J \mid K, \neg M) = \frac{P(\neg M \mid J, K)\, \overset{P(J)}{\cancel{P(J \mid K)}}}{P(\neg M \mid K)} = \frac{\overset{50\%}{P(\neg M \mid J, K)}\, \overset{\overset{60\%}{P(J)}}{\cancel{P(J \mid K)}}}{\underset{50\%}{P(\neg M \mid K, J)}\, \underset{\underset{60\%}{P(J)}}{\cancel{P(J \mid K)}} + \underset{100\%}{P(\neg M \mid K, \neg J)}\, \underset{\underset{40\%}{P(\neg J)}}{\cancel{P(\neg J \mid K)}}} = 42.86\%$$

Absolute → Conditional Independence?

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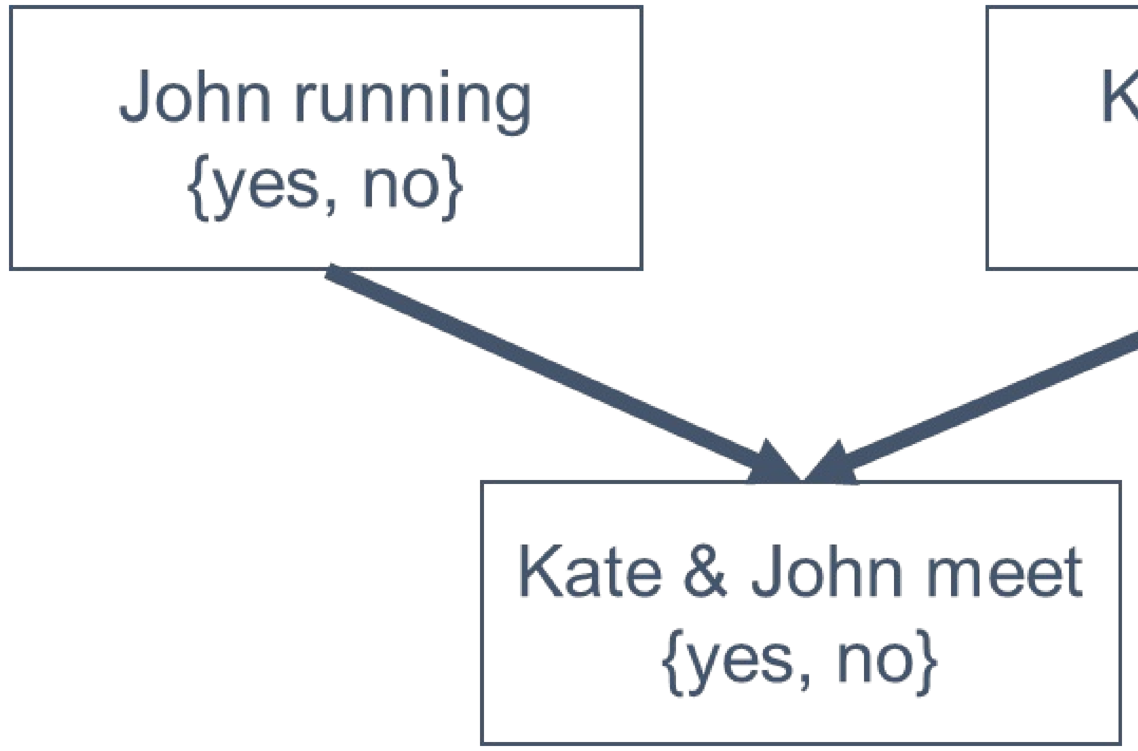

Kate	
Yes	30%
No	70%

John	Kate	Meet?
Yes	Yes	50% / 50%
Yes	No	0% / 100%
No	Yes	0% / 100%
No	No	0% / 100%

Conditional independence:

P(**John** = yes | **Kate** = yes, **Meet** = no) = P(**John** = yes | **Meet** = no)

50% 30%.60% + 100% 70% 60%

$$P(J \mid \neg M) = \frac{\boxed{P(\neg M \mid J)}\, P(J)}{P(\neg M)} = \frac{P(\neg M \mid J, K)\, P(K, J) + P(\neg M \mid J, \neg K)\, \boxed{P(\neg K, J)}}{P(\neg M \mid K, J)\, P(J, K) + P(\neg M \mid \neg K, J)\, P(J, \neg K) + P(\neg M \mid K, \neg J)\, P(\neg J, K) + P(\neg M \mid \neg K, \neg J)\, P(\neg J, \neg K)} = 56.64\%$$

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100% 40% 30% + 100% 40% 70%

Absolute → Conditional Independence?

Example: John & Kate going for a run

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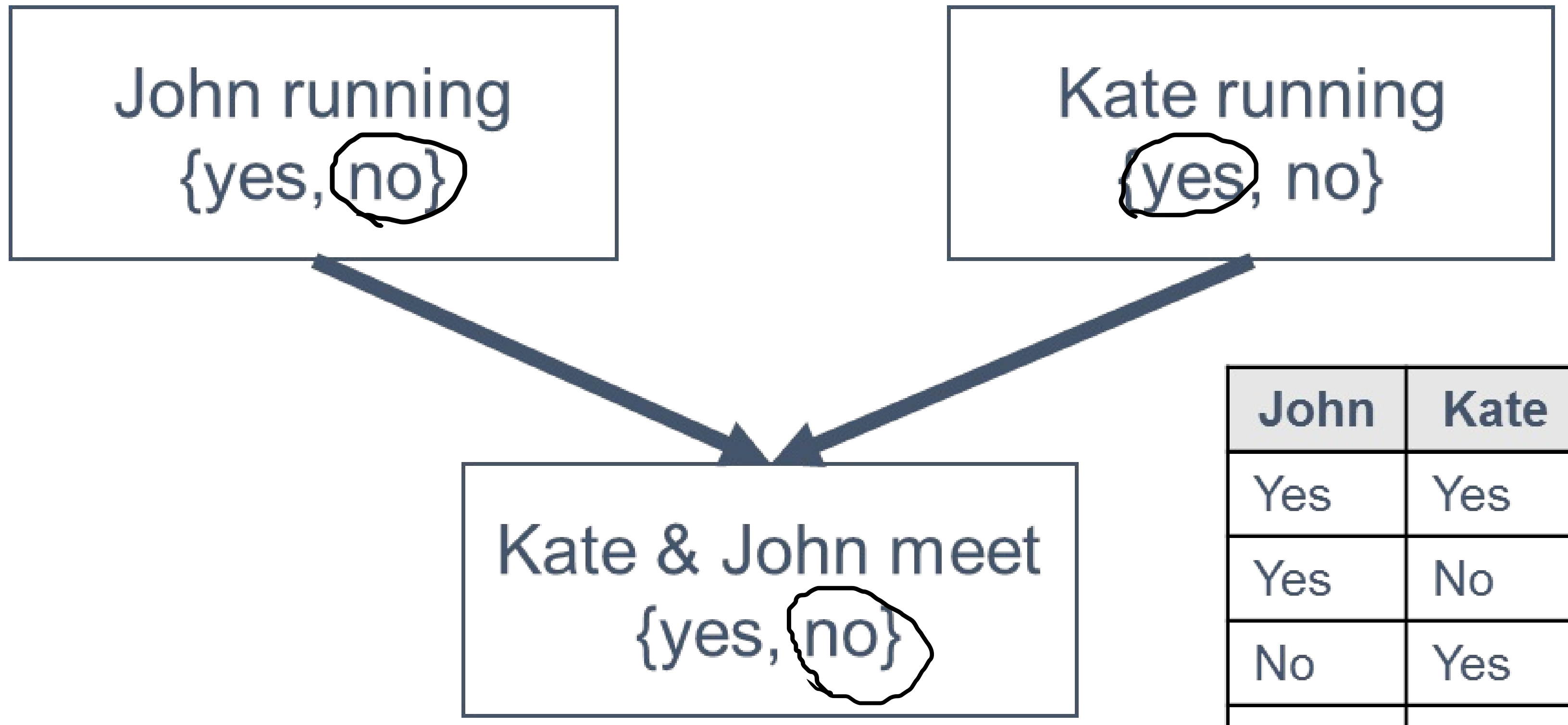
No	40%



Kate	
Yes	30%
No	70%

John	Kate	Meet?
Yes	Yes	50% / 50%
Yes	No	0% / 100%
No	Yes	0% / 100%
No	No	0% / 100%

Conditional independence:

P(**John** = yes | **Kate** = yes, **Meet** = no) = P(**John** = yes | **Meet** = no)

42.86 % $\neq$ 56.04 %

$A \perp B \not\Rightarrow A \perp B \mid C$

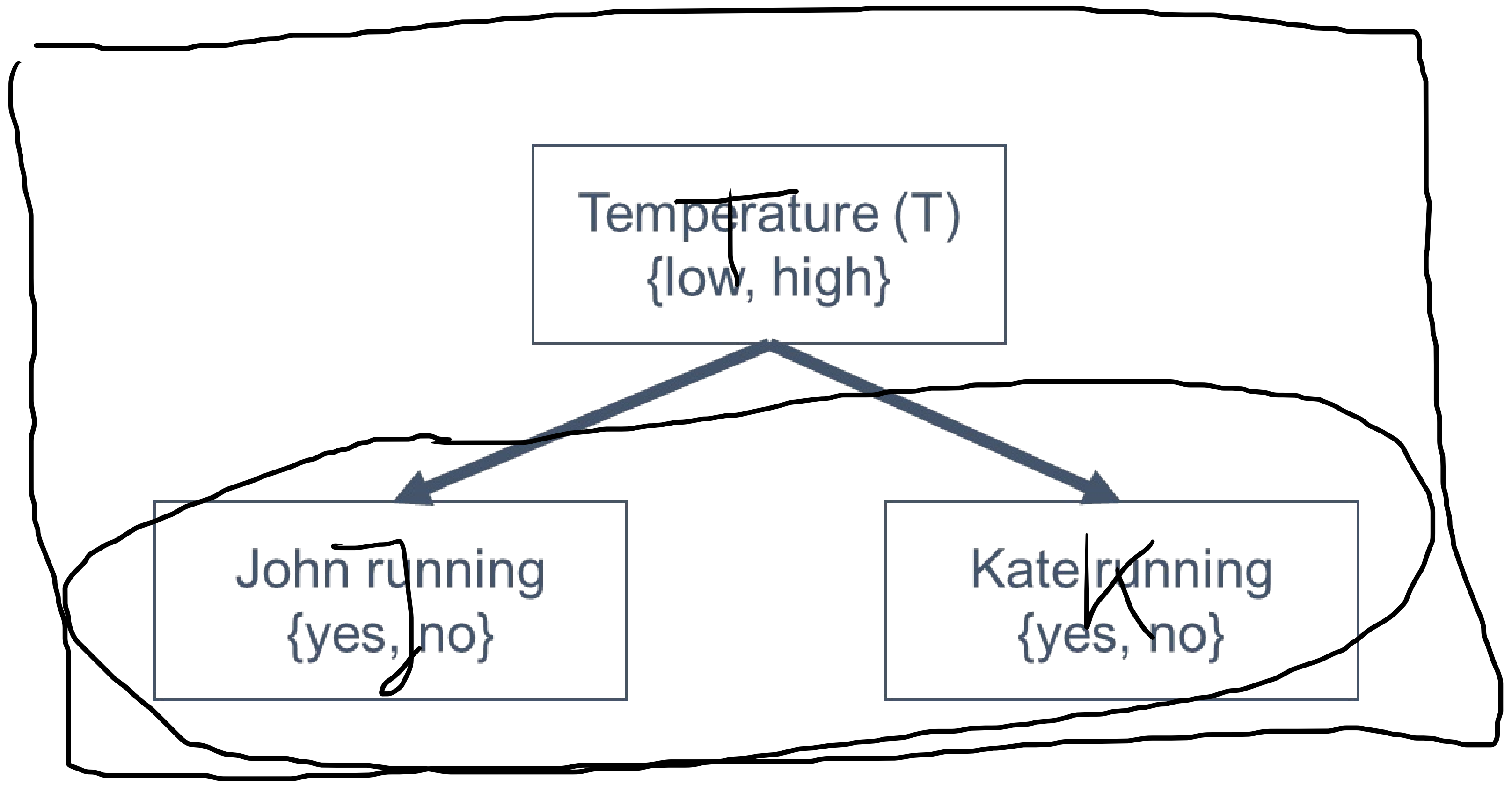
Exercises:

P(**John** = yes | **Kate** = no, **Meet** = no)

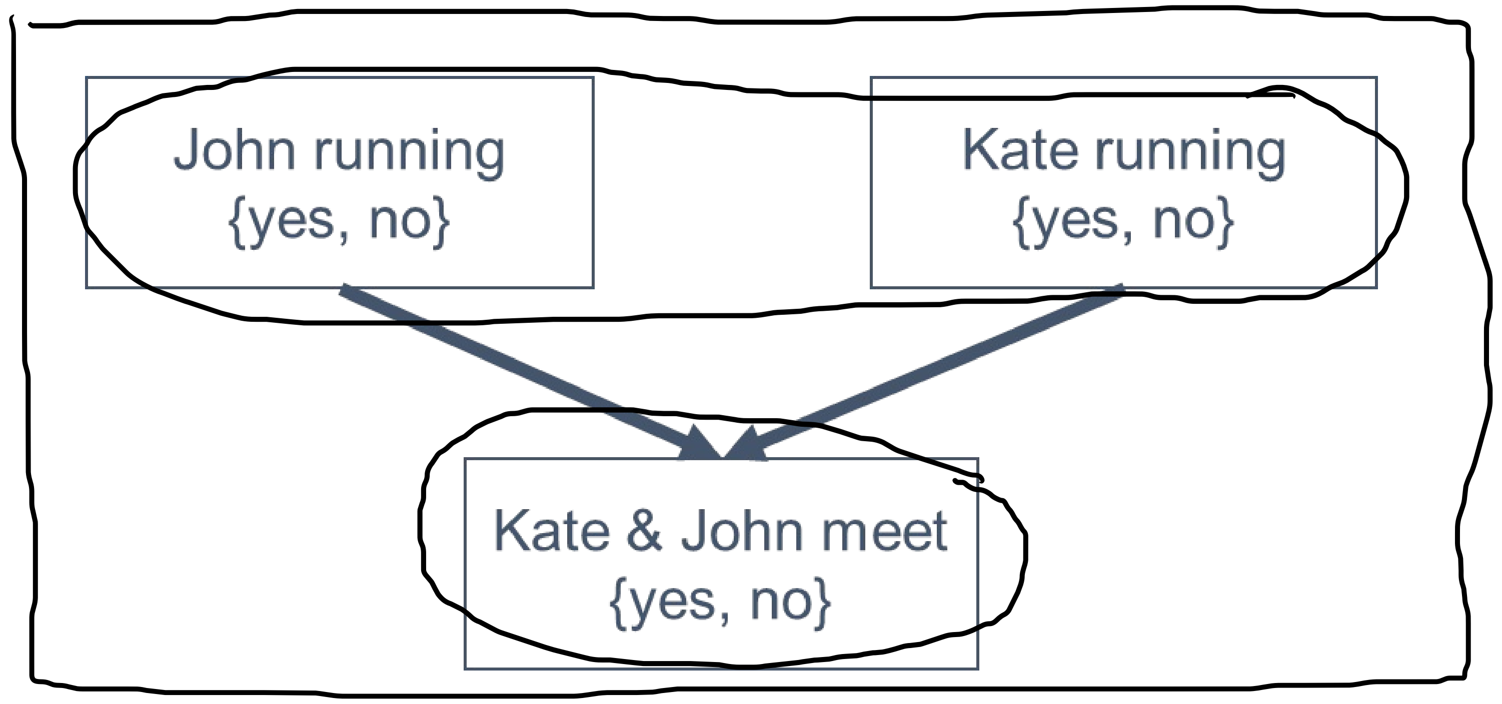
P(**John** = yes | **Meet** = yes)

Summary

Key takeaways



$J \perp K \mid T \not\Rightarrow J \perp K$



$J \perp K \not\Rightarrow J \perp K \mid M$