Multiple Choice Questions

Q1. Suppose $V(x)$ is potential energy function then which one is correct statement?

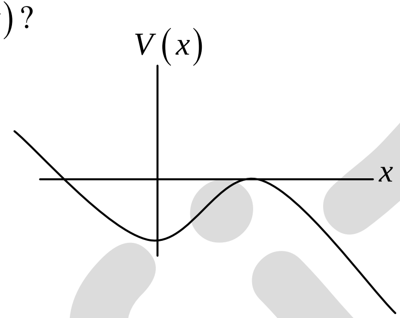
Statement 1: If at any point $x = a_1$, the total energy is equal to potential energy then $x = a_1$ is turning point.

Statement 2: If at any point $x = a_2$ is maxima then a_2 is stable equilibrium point and if any point $x = a_2$ is minima then a_2 is unstable equilibrium point

Statement 3: The phase curve around $x = a_2$ is bounded and execute simple harmonic motion if a_2 is minima of curve

(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) all are correct

Q2. The potential is given by $V(x) = \frac{x^2}{2} - \frac{x^3}{3}$. Which one of the following is a correct plot of $V(x)$?

(a)


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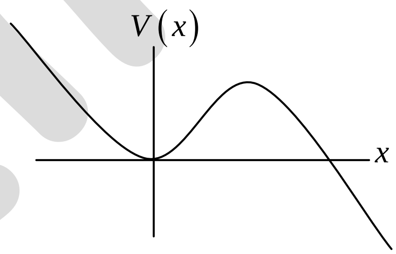

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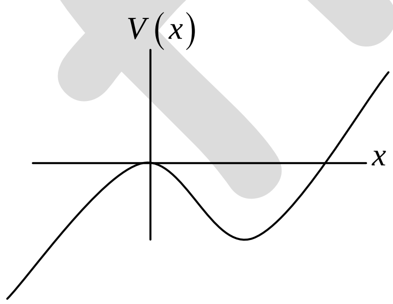
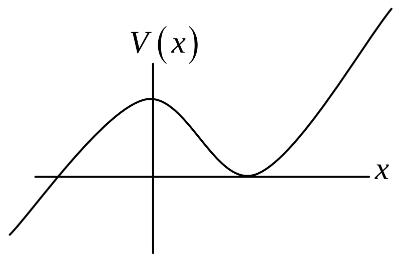


(d)


Q3. A particle of unit mass moves along the x-axis under the influence of a potential, $V(x) = x(x-4)^2$. The particle is found to be in stable equilibrium at the point. The angular frequency of oscillation of the particle is

(a) 2 (b) $\sqrt{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{8}}$

Q4. A particle of unit mass moves in a potential $V(x) = ax + \frac{b}{x}$, where a and b are positive constants. The angular frequency of small oscillations about the minimum of the potential is

(a) $\omega = \sqrt{2}\left(\frac{a^3}{b}\right)^{\frac{1}{4}}$ (b) $\omega = \sqrt{2}\left(\frac{b^3}{a}\right)^{\frac{1}{4}}$

(c) $\omega = 2\left(\frac{a^3}{b}\right)^{\frac{1}{2}}$ (d) $\omega = 2\left(\frac{b^3}{a}\right)^{\frac{1}{2}}$

Q5. We can find state of system in classical mechanics when we can find

(a) Position $x(t)$ at time t

(b) Velocity $\dot{x}(t)$ at time t

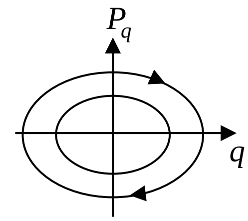
(c) Both position $x(t)$ and velocity $\dot{x}(t)$ at time t

(d) Any of position $x(t)$ and velocity $\dot{x}(t)$ at time t.

Q6. Which of the following system in column A match correctly with column B?

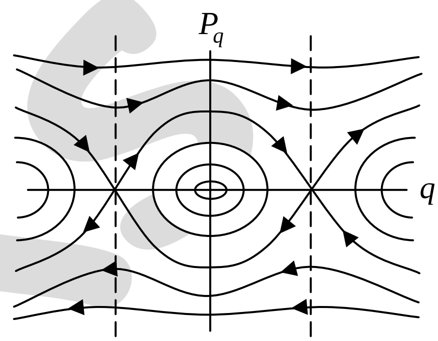
(A) System (B) Phase Curve

(I) One dimensional Harmonic oscillation with potential $V(q)=\frac{1}{2}kq^2$



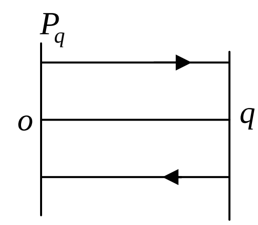
I

(II) Simple Pendulum with potential $V(q)=-k\cos q$



II

(III) Particle trapped in one dimensional rigid box



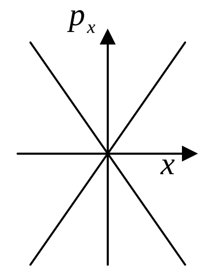
III

(a) (I) and (II) (b) (II) and (III)

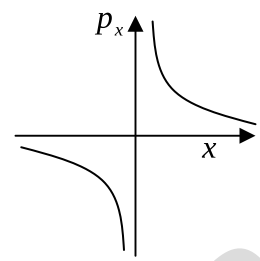
(c) (I) and (III) (d) (I), (II) and (III)

Q7. If potential of the system is given by $V(x) = -kx^2$, which of the following is phase space for $E < 0$?

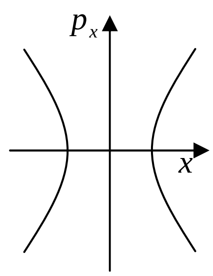
(a)



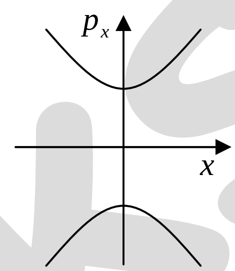
(b)



(c)

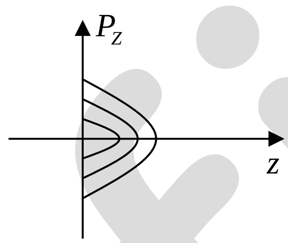


(d)

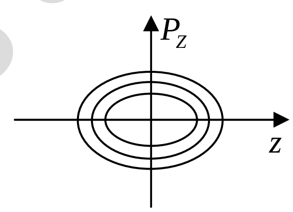


Q8. The trajectory on the $z - p_z$ plane (phase-space trajectory) of a ball confined into a potential $V(z) = z^4$ is approximately given by (neglect friction).

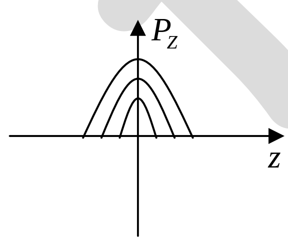
(a)



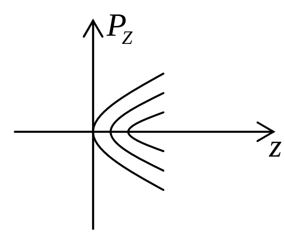
(b)



(c)



(d)



Q9. Consider two systems

System I → Spinless earth is revolving around the sun in the circular orbit

System II → Spinning electron is revolving around the nucleus in the circular orbit.

Choose the correct statement:

(a) System I and II both earth and electron can be treated as body.

(b) In System I earth can be treated as body and in system II electron can be treated as body.

(c) In system I earth can be treated as point particle and in system II electron can be treated as body.

(d) In system I and II, both earth and electron can be treated as point particle.

Q10. Which one is correct statement about generalized co-ordinate.

(a) Generalized co-ordinates are minimum number of co-ordinates required to specify the motion of dynamical system.

(b) Generalized co-ordinates are minimum number of independent co-ordinates required to specify the motion of dynamical system.

(c) Generalized co-ordinates are minimum number of independent co-ordinates required to specify the motion of dynamical system, which may or may not be equal to degree of freedom.

(d) Generalized co-ordinates are minimum number of independent co-ordinates which specify the motion of dynamical system completely and must be equal to degree of freedom of system.

Q11. If the number of degree of freedom is equal to number of constraint equation, then the number of constraint equation can be

(a) $1, 2, 3, ...$ (b) $2, 4, 6, 8, ...$ (c) $3, 6, 9, 12, ...$ (d) $4, 8, 12, ...$

Q12. Which one of the following is not a holonomic constraint?

(a) Simple pendulum

(b) Particle constrained to move on a cone with surface of cone makes an angle α with vertical z - axis.

(c) Movement of gaseous molecule trapped in cubical box of length a.

(d) A bead is constrained to move on a vertical ring, which is confined in a fixed plane.

Q13. Which one is not a non holonomic constraint?

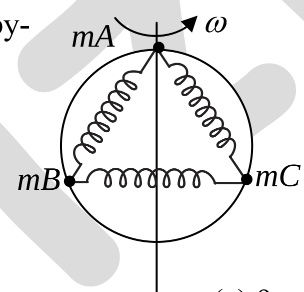
(a) The movement of gaseous molecule trapped in cubical box of length a.

(b) The movement of both points of geometric compass constrained to move on the paper.

(c) Particle constrained to move on inner wall of cone whose surface makes angle α with vertical axis.

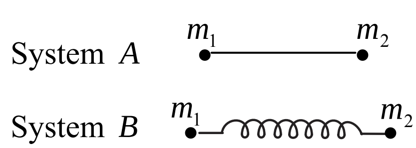
(d) A dog tied with an end of rope of length l and other end is tied with a fixed pole.

Q14. Three Beads of same mass m are connected with spring with constant k. All the Beads are constrained to move on a ring which is rotating about one of its diameter with constant angular velocity ω and a is the radius of ring as shown in figure, then the degree of freedom is given by-



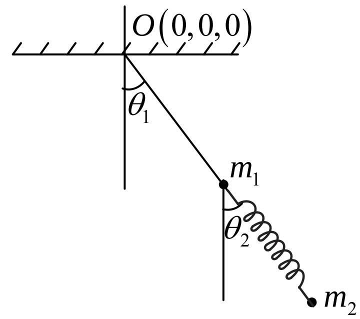
(a) 3 (b) 6 (c) 9 (d) 4

Q15. Consider two systems in which system A consist of two masses attached with rigid rod while system B consist of two masses attached with an ideal spring as shown in figure then degree of freedom of system A and system B are given by (assuming both the systems are in plane)



(a) 3, 3 (b) 4, 4 (c) 3, 4 (d) 4, 3

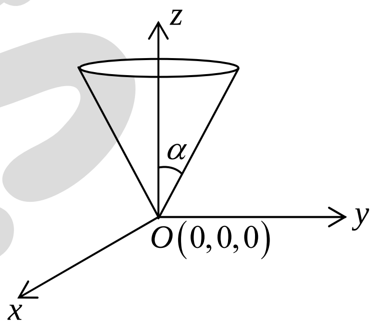
Q16. A double pendulum is constructed, so that the lower bob is attached with ideal spring. If the system is constrained to move in vertical plane as shown in figure then the degrees of freedom of system is given by



(a) 1 (b) 2

(c) 3 (d) 4

Q17. If a particle of mass m is constrained to move on the inner surface of cone with half angle α as shown in figure. Then the equation of constraint in spherical and cylindrical co-ordinate system respectively are



(a) $\theta = \alpha,\ \tan\alpha = \dfrac{r}{z}$ (b) $\tan\alpha = \dfrac{r}{z},\quad \theta = \alpha$

(c) $\theta = \alpha,\ \sin\alpha = \dfrac{r}{z}$ (d) $\sin\alpha = \dfrac{r}{z},\quad \theta = \alpha$

Q18. If $x = a\cos\theta$, then the value of $\dfrac{d^2x}{dt^2}$ is

(a) $\ddot{x} = -a\sin\theta\ddot{\theta}$ (b) $\ddot{x} = -a\sin\theta\ddot{\theta} - a\cos\theta\dot{\theta}$

(c) $\ddot{x} = -a\sin\theta\ddot{\theta} - a\sin\theta\dot{\theta}^2$ (d) $\ddot{x} = -a\sin\theta\ddot{\theta} - a\cos\theta\dot{\theta}^2$

Q19. If $x = x_1 + a\cos\theta$, $y = a\sin\theta$ and $z = 0$, then find the velocity vector $\vec{v}$ as function of x_1 and θ.

(a) $\left(\dot{x}_1 - a\dot{\theta}\sin\theta\right)\hat{i} + a\dot{\theta}\cos\theta\,\hat{j}$ (b) $\left(\dot{x}_1 + a\dot{\theta}\sin\theta\right)\hat{i} - a\dot{\theta}\cos\theta\,\hat{j}$

(c) $\left(\dot{x}_1 - a\dot{\theta}\sin\theta\right)\hat{i} - a\dot{\theta}\cos\theta\,\hat{j}$ (d) $\left(\dot{x}_1 + a\dot{\theta}\sin\theta\right)\hat{i} + a\dot{\theta}\cos\theta\,\hat{j}$

Q20. A particle is constrained to move on vertical elliptical wire under gravity along semi-minor axis. If semi-major axis and semi-minor axis are a and b respectively. And if θ is the angle made by the radial vector with horizontal, then the kinetic energy of the system is given by

(a) $\dfrac{m}{2}\left(a^2 + b^2\right)\dot{\theta}^2$ (b) $\dfrac{m}{2}\left(a^2\cos^2\theta + b^2\sin^2\theta\right)\dot{\theta}^2$

(c) $\dfrac{m}{2}\left(a^2\sin^2\theta - b^2\cos^2\theta\right)\dot{\theta}^2$ (d) $\dfrac{m}{2}\left(a^2\sin^2\theta + b^2\cos^2\theta\right)\dot{\theta}^2$

Q21. A particle moves in two dimensions on the hyperbola $x^2 - 4y^2 = 8$. At a particular instant, it is at the point $(x, y) = (2,1)$ and the x-component of its velocity is 6 (in suitable units). Then the speed at point $(x, y) = (2,1)$ is

(a) $\sqrt{45}$ (b) 7 (c) 3 (d) $\sqrt{27}$

Q22. A particle is constrained to move on a circle $x^2 + y^2 = a^2$. If v_x and v_y are velocity in x and y direction, then $\frac{v_y}{v_x}$ at $y = \frac{a}{2}$ is

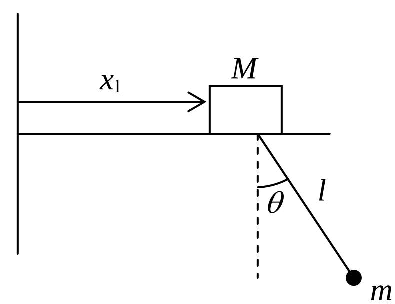
(a) $-\frac{1}{2\sqrt{3}}$ (b) $-2\sqrt{3}$ (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

Q23. If $x = x_1 + a\cos\theta, y = a\sin\theta$ and $z = 0$, then find the kinetic energy $T = \frac{1}{2}m|v|^2$ as a function of x_1 and θ.

(a) $\frac{1}{2}m\left(\dot{x}_1^2 + a^2\dot{\theta}^2\right)$ (b) $\frac{1}{2}m\left(\dot{x}_1^2 + a^2\dot{\theta}^2 + 2\dot{x}_1 a\dot{\theta}\sin\theta\right)$

(c) $\frac{1}{2}m\left(\dot{x}_1^2 + a^2\dot{\theta}^2 + 2\dot{x}_1 a\dot{\theta}\cos\theta\right)$ (d) $\frac{1}{2}m\left(\dot{x}_1^2 + a^2\dot{\theta}^2 - 2\dot{x}_1 a\dot{\theta}\sin\theta\right)$

Q24. A pendulum of length l and mass m is attached to a block of mass M as shown in figure. The block M slides on a horizontal frictional surface while mass m is constrained to move in vertical plane, then kinetic energy $T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\left(\dot{x}_2^2 + \dot{y}_2^2\right)$ in term of x_1 and θ is given by



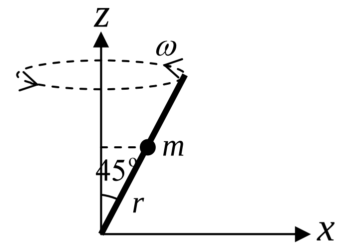
(a) $T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_1^2$

(b) $T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\left(\dot{x}_1^2 + 2l\dot{x}_1\dot{\theta}\cos\theta + l^2\dot{\theta}^2\right)$

(c) $T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\left(\dot{x}_1^2 + l^2\dot{\theta}^2\right)$

(d) $T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\left(\dot{x}_1^2 + l^2\dot{\theta}^2 - 2\dot{x}_1\dot{\theta}\cos\theta\right)$

Q25. A bead of mass m can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant, r is the distance of the bead from the origin. Which of the following is not correct expression for equation of constraint,



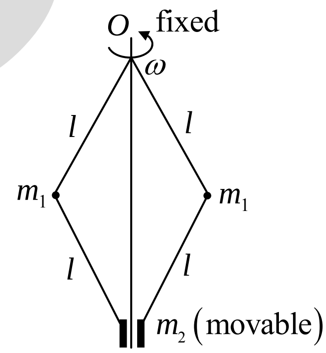
(a) $\theta = \frac{\pi}{4}$ in spherical coordinates (b) $z = \sqrt{x^2 + y^2}$ in Cartesian coordinates.

c) $r = z$ in cylindrical coordinates (d) $z = \frac{r}{\sqrt{2}}$ in spherical coordinates

Q26. A particle of mass m_2 moves on a vertical axis in the system and shown in figure and whole system rotates about this axis with constant angular velocity ω under the action of gravity.

The Degree of freedom of the system is given by.

(a) 1 (b) 2 (c) 3 (d) 4



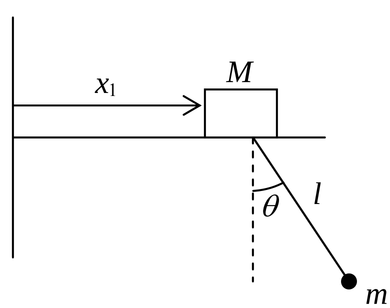
Q27. A pendulum of length l and mass m is attached to a block of mass M as shown in figure. The block M slides on a horizontal frictional surface while mass m is constrained to move in a vertical plane, then degree of freedom is given by

(a) 1

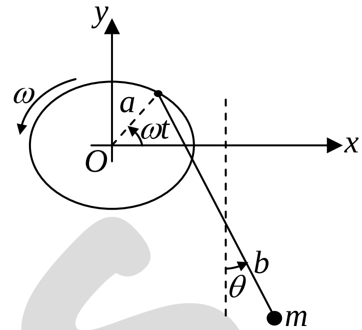
(b) 2

(c) 3

(d) 4



Q28. The point of support of a simple pendulum of length b moves on a massless rim of radius a rotating with constant angular velocity ω as shown is figure. If the origin of the coordinate system is taken at the centre of the rotating rim, then the coordinate of mass m as function of a, b, ω and time t is given by



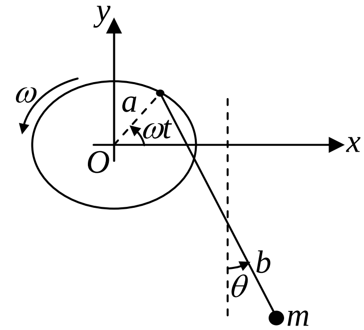
(a) $x = a\sin\omega t + b\sin\theta, y = a\cos\omega t - b\cos\theta$

(b) $x = a\cos\omega t - b\sin\theta, y = a\sin\omega t + b\cos\theta$

(c) $x = a\cos\omega t + b\sin\theta, y = a\sin\omega t - b\cos\theta$

(d) $x = a\cos\omega t - b\sin\theta, y = a\sin\omega t - b\cos\theta$

Q29. The point of support of a simple pendulum of length b moves on a massless rim of radius a rotating with constant angular velocity ω as shown in figure. If the origin of the coordinate system is taken at the centre of the rotating rim, then the velocity of the mass m as function of a, b, ω and time t is given by



(a) $\vec{v} = \left(a\omega\sin\omega t + b\dot{\theta}\cos\theta\right)\hat{i} + \left(a\omega\cos\omega t + b\dot{\theta}\sin\theta\right)\hat{j}$

(b) $\vec{v} = \left(-a\omega\sin\omega t + b\dot{\theta}\cos\theta\right)\hat{i} + \left(a\omega\cos\omega t + b\dot{\theta}\sin\theta\right)\hat{j}$

(c) $\vec{v} = \left(a\omega\sin\omega t + b\dot{\theta}\cos\theta\right)\hat{i} + \left(a\omega\cos\omega t - b\dot{\theta}\sin\theta\right)\hat{j}$

(d) $\vec{v} = \left(-a\omega\sin\omega t + b\dot{\theta}\cos\theta\right)\hat{i} + \left(a\omega\cos\omega t - b\dot{\theta}\sin\theta\right)\hat{j}$

Q30. Shortest distance between two points in a plane is given by

(a) Any curve on plane depending on initial and final condition

(b) May be a straight line

(c) Must be a straight line

(d) Can't say precisely

Q31. A curve passing through the points (x_1, y_1) and (x_2, y_2) when rotated about x-axis gives a minimum surface area given by

(a) $y = c\cos\left(\frac{x+a}{c}\right)$, where a and c are constant

(b) $y = \frac{x+a}{c}$, where a and c are constant

(c) $y = c\cos\ h\left(\frac{x+a}{c}\right)$, where a and c are constant

(d) $y = ce^{i}\left(\frac{x+a}{c}\right)$, where a and c are constant

Q32. If L is Lagrangian of system and $I = \int_{t_1}^{t_2} Ldt$ is defined as the action of system, then consequences of Hamilton's least action principle is

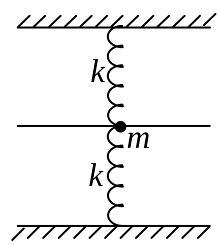
(a) action is zero

(b) variation of action over time t is zero

(c) definition of force

(d) variation of force over time t is zero

Q33. A particle of mass m attached to two identical springs each of length l and spring constant k (see the figure). The equilibrium configuration is the one where the springs are untstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, so Lagrangian of the system is given by



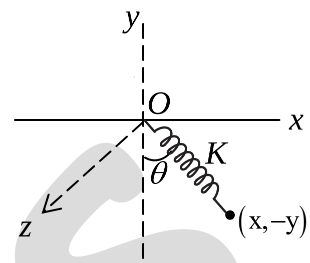
(a) $L = \frac{1}{2}m\dot{x}^2 - kx^2$

(b) $L = \frac{1}{2}m\dot{x}^2 - k\left(x^2 + l^2\right)$

(c) $L = \frac{1}{2}m\dot{x}^2 - k\left[\left(x^2 + l^2\right)^{\frac{1}{2}} - l\right]$

(d) $L = \frac{1}{2}m\dot{x}^2 - k\left[\left(x^2 + l^2\right)^{\frac{1}{2}} - l\right]^2$

Q34. A spring pendulum is defined as a mass m attached to one end of a spring with spring constant k and another end is pivoted at point O as shown in figure. If the particle is constrained to move in xy plane under gravity (in y direction), then the Lagrangian of the system is given by



(a) $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)+mgy+\frac{1}{2}k\left[\sqrt{\left(x^2+y^2\right)}\right]^2$

(b) $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy-\frac{1}{2}k\left[\sqrt{\left(x^2+y^2\right)}\right]^2$

(c) $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy+\frac{1}{2}k\left[\sqrt{\left(x^2+y^2\right)}\right]^2$

(d) $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)+mgy-\frac{1}{2}k\left[\sqrt{\left(x^2+y^2\right)}\right]^2$

Q35. A particle moves under influence of gravity, which acts in z direction and is constrained to move on inner wall of an axially symmetrical vessel, given by $z=\frac{1}{2}b\left(x^2+y^2\right)$, where b is constant. Lagrangian of the particle in cylindrical co-ordinate is given by

(a) $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\dot{z}^2\right)-mgz$

(b) $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\dot{z}^2\right)-\frac{1}{2}mgbr^2$

(c) $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+b^2r^2\dot{\theta}^2\right)-\frac{1}{2}mgbr^2$

(d) $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+b^2r^2\dot{r}^2\right)-\frac{1}{2}mgbr^2$

Q36. Two point masses of mass m_1 and m_2 are connected by a string passing through hole in a smooth table. Assume m_1 constrained to move on table and m_2 constrained to move in vertical direction, then which one is suitable Lagrangian of system?

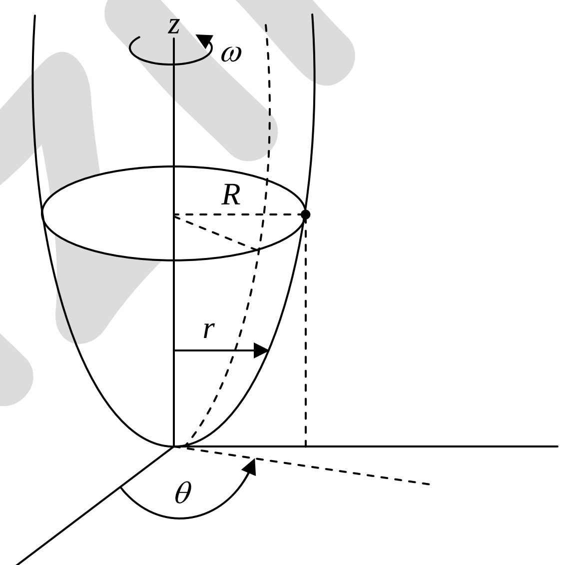
(a) $L = \frac{1}{2}m_1\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{1}{2}m_2\dot{z}^2 - m_2 gz$

(b) $L = \frac{1}{2}m_1\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{1}{2}m_2\dot{z}^2 + m_2 gz$

(c) $L = \frac{1}{2}\left(m_1 + m_2\right)\dot{r}^2 + \frac{1}{2}m_1 r^2\dot{\theta}^2 - m_2 g\left(l - r\right)$

(d) $L = \frac{1}{2}\left(m_1 + m_2\right)\dot{r}^2 + \frac{1}{2}m_1 r^2\dot{\theta}^2 + m_2 g\left(l - r\right)$

Q37. A bead slides along a smooth wire bent in the shape of a parabola $z = cr^2$ (Figure). When the wire is rotating about its vertical symmetry axis with angular velocity ω. Then the lagrangian of the system is:



(a) $L = \frac{m}{2}\left(\dot{r}^2 + r^2\omega^2\right) - mgcr^2$

(b) $L = \frac{m}{2}\left(\dot{r}^2 + r^2\omega^2\right) + mgcr^2$

(c) $L = \frac{m}{2}\left(\dot{r}^2 + 4c^2r^2\dot{r}^2 + r^2\omega^2\right) - mgcr^2$

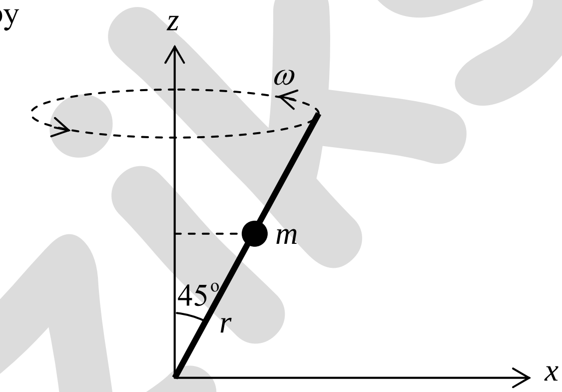
(d) $L = \frac{m}{2}\left(\dot{r}^2 + 4c^2r^2\dot{r}^2 + r^2\omega^2\right) + mgcr^2$

Q38. If Potential $U(r,\dot{r}) = \frac{1}{r}\left(1+\frac{\dot{r}^2}{c^2}\right)$, then generalized force is given by

(a) $-\frac{1}{r^2}\left(1+\frac{\dot{r}^2}{c^2}\right)$ (b) $\frac{1}{rc^2}\left(2\ddot{r}-\frac{\dot{r}^2}{r}\right)+\frac{1}{r^2}$

(c) $-\frac{1}{r^2}$ (d) $\frac{1}{r^2}\left(1+\frac{2\ddot{r}r}{c^2}\right)$

Q39. A bead of mass m can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant, r is the distance of the bead from the origin. The lagrangian of the system is given by



(a) $L=\frac{1}{2}m(\dot{r}^2+r^2\omega^2)-mgr$ (b) $L=\frac{1}{2}m(\dot{r}^2+\frac{1}{\sqrt{2}}r^2\omega^2)-\frac{1}{\sqrt{2}}mgr$

c) $L=\frac{1}{2}m(\dot{r}^2+\frac{1}{2}r^2\omega^2)-mgr$ (d) $L=\frac{1}{2}m(\dot{r}^2+\frac{1}{2}r^2\omega^2)-\frac{1}{\sqrt{2}}mgr$

Q40. A particle of mass m is constrained to move on circumference of mass-less ring of radius 'a' which is also rotating about one of its diameters with angular frequency $\omega = \alpha t + \beta$. Lagrangian of a system is given by

(a) $\frac{1}{2}m\left(a^2\dot{\theta}^2 + a^2\sin^2\theta\alpha^2\right) - mga\cos\theta$

(b) $\frac{1}{2}m\left(a^2\alpha^2 + a^2\sin^2\theta\dot{\phi}^2\right) - mga\cos\theta$

(c) $\frac{1}{2}m\left[a^2(\alpha t + \beta)^2 + a^2\sin^2\theta\dot{\phi}\right] + mga\cos\theta$

(d) $\frac{1}{2}m\left[a^2\dot{\theta}^2 + a^2\sin^2\theta(\alpha t + \beta)^2\right] + mga\cos\theta$

Q41. The Lagrangian of system is given by

$$L = \frac{m}{2}\left(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2\right) - V(x, y).$$

If p_x and p_y are generalized momentum conjugate to x and y respectively, then the value of p_x and p_y are given by

(a) $p_x = m\dot{x}$
$p_y = m\dot{y}$

(b) $p_x = ma\dot{x} + b\dot{y} + c\dot{y}$
$p_y = mc\dot{y} + b\dot{x} + a\dot{x}$

(c) $p_x = ma\dot{x} + mb\dot{x}$
$p_y = mc\dot{y} + mb\dot{y}$

(d) $p_x = ma\dot{x} + mb\dot{y}$
$p_y = mc\dot{y} + mb\dot{x}$

Q42. If Lagrangian of system is given by $L = \frac{1}{2}m\dot{x}^2 e^{vt}$, where p_x is canonical momentum conjugate to x then generalized co-ordinate is given by

(a) $\frac{p_x}{m} \cdot ve^{-vt} + c$, where c is constant

(b) $-\frac{p_x}{m} \cdot ve^{-vt} + c$, where c is constant

(c) $-\frac{p_x}{m} \cdot \frac{e^{-vt}}{v} + c$, where c is constant

(d) $\frac{p_x}{m} \cdot \frac{e^{-vt}}{v} + c$, where c is constant

Q43. The Lagrangian of a system with one degree of freedom q is given by $L = \frac{\alpha \dot{q}^2}{4} + \frac{\beta q^2}{2}$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?

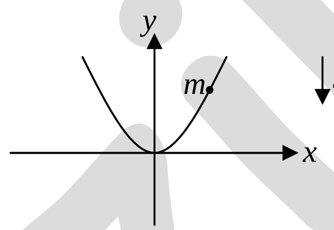
(a) $p_q = \frac{\alpha \dot{q}}{2}$ and it is not a conserved quantity.

(b) $p_q = \alpha \dot{q}$ and it is not a conserved quantity.

(c) $p_q = \alpha \dot{q}$ and it is a conserved quantity.

(d) $p_q = \frac{\alpha \dot{q}}{2}$ and it is conserved quantity.

Q44. Particle of mass m slides under the gravity without friction along the parabolic path $y = ax^2$ axis as shown in the figure. Here a is a constant. Then Lagrangian of the system is given by



(a) $L = \frac{m}{2}\left(1+2a^2x^2\right)\dot{x}^2 - mgax^2$ (b) $L = \frac{m}{2}\left(1+2a^2x^2\right)\dot{x}^2 + mgax^2$

(c) $L = \frac{m}{2}\left(1+4a^2x^2\right)\dot{x}^2 + mgax^2$ (d) $L = \frac{m}{2}\left(1+4a^2x^2\right)\dot{x}^2 - mgax^2$

Q45. The Lagrangian of system in cylindrical co-ordinate is given by

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right) - mgz.$$

If p_r, p_θ and p_z are momentum conjugate to r, θ, and z generalized co-ordinate, then choose **WRONG** statement

(a) $p_r = m\dot{r},\ p_\theta = mr^2\dot{\theta},\ p_z = m\dot{z}$ and external force is acting in z direction

(b) $p_r = m\dot{r},\ p_\theta = mr^2\dot{\theta},\ p_z = m\dot{z}$ and the angular momentum is constant of motion.

(c) $p_r = m\dot{r},\ p_\theta = mr^2\dot{\theta},\ p_z = m\dot{z}$, there are not any external force in $\hat{\theta}$ direction and there is pseudo force.

(d) $p_r = m\dot{r},\ p_\theta = mr^2\dot{\theta},\ p_z = m\dot{z}$, there are not external force in $\hat{\theta}$ direction also there is not any pseudo force in $\hat{r}$ direction.

Q46. If Lagrangian of system is given by $L=\frac{1}{2}\mathrm{I}\dot{\theta}^2+\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)$ where p_r and p_θ are generalized momentum conjugate to r and θ then

(a) $p_\theta=\mathrm{I}\dot{\theta}+m\dot{r}^2\dot{\theta},\ p_r=m\dot{r}$ and both p_θ and p_r are constant of motion

(b) $p_\theta=\mathrm{I}\dot{\theta},\ p_r=m\dot{r}+mr\dot{r}\dot{\theta}$, where p_θ is constant of motion but p_r is not.

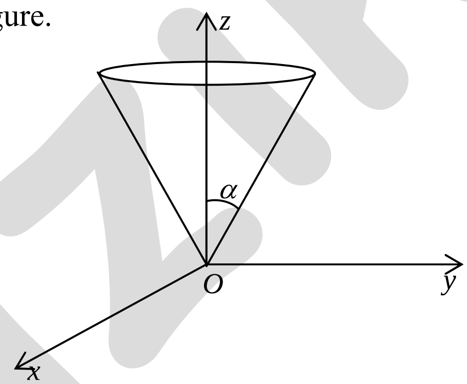
(c) $p_\theta=\mathrm{I}\dot{\theta}+mr^2\dot{\theta},\ p_r=m\dot{r}$, where p_θ is constant of motion but p_r is not.

(d) $p_\theta=\mathrm{I}\dot{\theta}+mr^2\dot{\theta},\ p_r=m\dot{r}+mr\dot{r}\dot{\theta}^2$, where both p_θ and p_r are not constant of motion.

Q47. If Lagrangian is given by $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\dot{z}^2\right)-mgz$, then which quantity is a conserved quantity.

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{z}}$ (c) $\frac{\partial L}{\partial \dot{\theta}}$ (d) $\frac{1}{r}\frac{\partial L}{\partial \dot{\theta}}$

Q48. A particle is constrained to move on inner surface of cone of infinite height and semi-vertical angle α as shown in figure.



The system is under the influence of a gravitational field, which acts along $(-z)$ direction, then which statement is true?

(a) Particle can reach to the vertex O of cone.

(b) Particle can move beyond $z \ge z_{\min}$ or $r > r_{\min}$

(c) Particle can move $z < z_{\max}$ or $r < r_{\max}$

(d) Particle will confined under range at

$$z_{\min} < z < z_{\max}$$
$$r_{\min} < r < r_{\max}$$

Q49. $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\phi}^2\right)-V(r)$, then conserved quantity is

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\theta}}$ (c) $\frac{\partial L}{\partial \dot{\phi}}$ (d) $\frac{\partial L}{\partial \dot{\theta}}+\frac{\partial L}{\partial \dot{\phi}}$

Q50. The Lagrangian of system is given by

$$L=\frac{1}{2}m(\dot{x}^2+a^2\dot{\theta}^2+2a\dot{x}\dot{\theta}\cos\theta)+mga\cos\theta$$

where x and θ are generalized co-ordinates, then which one is **INCORRECT**?

(a) Degree of freedom is two.

(b) Angular momentum of system is conserved

(c) Linear momentum of system is conserved

(d) Force acting in the direction of x is zero.

Q51. If the Lagrangian of system is given by $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-K_1(y-x)+K_2xy$

Then the equation of motion is given by

(a) $m\ddot{x}-K_1-K_2y=0$
$m\ddot{y}-K_1-K_2x=0$

(b) $m\ddot{x}+K_1-K_2y=0$
$m\ddot{y}+K_1-K_2x=0$

(c) $m\ddot{x}-K_1-K_2y=0$
$m\ddot{y}+K_1-K_2x=0$

(d) $m\ddot{x}+K_1-K_2y=0$
$m\ddot{y}-K_1-K_2x=0$

Q52. The equation of motion of a system described by the time-dependent Lagrangian

$$L=e^{-\gamma t}\left[\frac{1}{2}m\dot{x}^2-V(x)\right] \text{ is}$$

(a) $m\ddot{x}+\gamma m\dot{x}+\frac{dV}{dx}=0$ (b) $m\ddot{x}+\gamma m\dot{x}-\frac{dV}{dx}=0$

(c) $m\ddot{x}-\gamma m\dot{x}+\frac{dV}{dx}=0$ (d) $m\ddot{x}+\frac{dV}{dx}=0$

Q53. The Lagrangian of a particle of mass m and coordinate q is given by

$$L = \frac{1}{2}m\dot{q}^2 + \frac{\lambda}{2}q\dot{q}^2,$$

then equation of motion is given by

(a) $\ddot{q}(m+\lambda q) - \frac{\lambda}{2}\dot{q}^2$
(b) $\ddot{q}(m-\lambda q) + \frac{\lambda}{2}\dot{q}^2$
(c) $\ddot{q}(m-\lambda q) - \frac{\lambda}{2}\dot{q}^2$
(d) $\ddot{q}(m+\lambda q) + \frac{\lambda}{2}\dot{q}^2$

Q54. The Lagrangian of spring pendulum is given by $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{1}{2}Kr^2 + mgr\cos\theta$,

then equation of motion is given by

(a) $m\ddot{r} + Kr - mg = 0$
$mr^2\ddot{\theta} + mgr\sin\theta = 0$

(b) $m\ddot{r} - mr\dot{\theta}^2 + Kr - mg\cos\theta = 0$
$mr^2\ddot{\theta} + mgr\sin\theta = 0$

(c) $m\ddot{r} - mr\dot{\theta}^2 + Kr - mg\cos\theta = 0$
$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} + mgr\sin\theta = 0$

(d) $m\ddot{r} - mr\dot{\theta}^2 + Kr + mg\cos\theta = 0$
$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} + mgr\sin\theta = 0$

Q55. Which of the following is correct for Hamiltonian H?

(a) $\int_{t_1}^{t_2} Hdt = 0$
(b) $\delta\int_{t_1}^{t_2} Hdt = 0$
(c) $\int_{t_1}^{t_2} (pq - H)dt = 0$
(d) $\delta\int_{t_1}^{t_2} (p\dot{q} - H)dt = 0$

Q56. Which statement is true?

(a) The concept of conservation of momentum is explained by Lagrangian only and conservation of energy can be explained by both Lagrangian and Hamiltonian.

(b) The concept of conservation of momentum is explained by Hamiltonian only and concept of conservation of Energy can be explained by Lagrangian only.

(c) The concept of conservation of momentum can be explained by both Lagrangian and Hamiltonian and concept of conservation of Energy can be explained by Hamiltonian formulation

(d) Both Lagrangian and Hamiltonian will explain the concept of momentum and energy.

Q57. The kinetic energy is given by $T=\frac{1}{2}m\dot{x}^2$ and potential energy is given by. $V=\frac{1}{2}kx^2$, where x is the generalized coordinate, then the Hamiltonian of the system is given by:

(a) $\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$ (b) $\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2$

(c) $\frac{p_x^2}{2m}-\frac{1}{2}kx^2$ (d) $\frac{p_x^2}{2m}+\frac{1}{2}kx^2$

Q58. If Lagrangian of a system is given by

$$L=\frac{a\dot{x}^2}{4}+\frac{b\dot{y}^2}{4}-kxy$$

then the Hamiltonian of the system is given by.

(a) $\frac{ap_x^2+bp_y^2}{ab}+kxy$ (b) $\frac{ap_x^2+bp_y^2}{ab}-kxy$

(c) $\frac{bp_x^2+ap_y^2}{ab}+kxy$ (d) $\frac{bp_x^2+ap_y^2}{ab}-kxy$

Q59. The time dependent Lagrangian of the system is given by $L=e^{vt}\left(\frac{m\dot{x}^2}{2}-\frac{kx^2}{2}\right)$

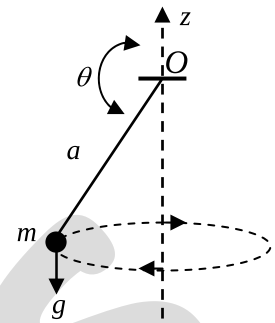
The Hamiltonian of the system is given by

(a) $e^{vt}\left(\frac{p_x^2}{2m}+\frac{kx^2}{2}\right)$ (b) $e^{-vt}\left(\frac{p_x^2}{2m}+\frac{kx^2}{2}\right)$

(c) $e^{vt}\frac{p_x^2}{2m}+e^{-vt}\frac{kx^2}{2}$ (d) $e^{-vt}\frac{p_x^2}{2m}+e^{vt}\frac{kx^2}{2}$

Q60. A particle of mass m is attached to fixed point O by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure.

The Hamiltonian of the particle is



(a) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) - mga\cos\theta$

(b) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + mga\cos\theta$

(c) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) - mga\cos\theta$

(d) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) + mga\cos\theta$

Q61. If Lagrangian of the system is $L = \sqrt{1-\dot{q}^2}$, then Hamiltonian of system is given by

(a) $H = \sqrt{1+p^2}$ (b) $\frac{p^2+1}{\sqrt{1+p^2}}$ (c) $\frac{p^2-1}{\sqrt{1+p^2}}$ (d) $\frac{1-p^2}{\sqrt{1-p^2}}$

Q62. If Hamiltonian of a system is given as

$$H = \frac{P^2 e^{-\nu t}}{2m} + \frac{1}{2}m\omega^2 x^2 e^{\nu t},$$

where x is generalized co-ordinate and p is generalized momentum then the Lagrangian of the system is given by

(a) $L = \frac{1}{2}m(\dot{x}^2 - \omega^2 x^2)e^{-\nu t}$ (b) $L = \frac{1}{2}m\dot{x}^2 e^{-\nu t} - \frac{1}{2}m\omega^2 x^2 e^{\nu t}$

(c) $L = \frac{1}{2}m\dot{x}^2 e^{\nu t} - \frac{1}{2}m\omega^2 x^2 e^{-\nu t}$ (d) $L = \left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2\right)e^{\nu t}$

Q63. If Hamiltonian of the system is given by

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{p_z^2}{2m} - mgz$$

then which one is not an outcome of Hamiltonian equation of motion

(a) $\dot{p}_r = \frac{p_\theta^2}{mr^3}$ and $p_\theta = l$, where l is a constant (b) $\ddot{z} = g$

(c) $P_\theta = mr^2\dot{\theta}$ (d) $mr^2\ddot{\theta} = 0$

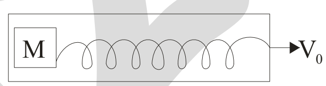
Q64. If T is the kinetic energy of system , V is potential energy of system and Hamiltonian H is explicit function of time then which one of the following will be correct statement?

(a) $\frac{\partial H}{\partial t} = \frac{\partial T}{\partial t} + \frac{\partial V}{\partial t}$ (b) $\frac{\partial H}{\partial t} = -\frac{\partial T}{\partial t} + \frac{\partial V}{\partial t}$

(c) $\frac{\partial H}{\partial t} = \frac{\partial T}{\partial t} - \frac{\partial V}{\partial t}$ (d) $\frac{\partial H}{\partial t} = -\frac{\partial T}{\partial t} - \frac{\partial V}{\partial t}$

Q65. Suppose a point mass m is attached to a spring of force constant k, the other end of which is fixed on a massless cart that is being moved uniformly by an external device with speed V_0, then which one is a correct statement?

Figure to remake



(a) Total momentum is conserve during the motion

(b) Total energy is conserve during the motion

(c) Both momentum and energy is conserve during the motion

(d) Neither momentum nor energy is conserve during the motion

Q66. If $L = \frac{1}{2}m\left[a^2\dot{\theta}^2 + a^2\sin^2\theta(\alpha t + \beta)^2\right] + mga\cos\theta$, then

(a) Angular momentum is conserved.

(b) Total energy E is conserved.

(c) Both the angular momentum and the total energy E is conserved.

(d) Neither angular momentum, nor energy is conserved.

Q67. Consider $[g,f]$ Poisson bracket of g with f, defined as $[g,f]=\sum\frac{\partial g}{\partial q}\frac{\partial f}{\partial p}-\frac{\partial g}{\partial p}\frac{\partial f}{\partial q}$. If H is Hamiltonian of system and f is some function in phase space $f=f(q,p,t)$, then the value of $\frac{df}{dt}$ is given by.

(a) $[f,H]$ (b) $[f,H]-\frac{\partial f}{\partial t}$ (c) $[f,H]+\frac{\partial f}{\partial t}$ (d) None of these

Q68. If J is angular momentum of system and H is Hamiltonian of system, then the value of Poisson Bracket $[J,H]$ gives

(a) Force (b) Torque (c) Power (d) Energy

Q69. Which one of the following is not equivalent to Newton's Law of motion?

(a) $\dot{p}=\{p,H\}$ (b) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=0$

(c) $\frac{\partial H}{\partial q}=\dot{p}$ (d) $F_{ext}=-\frac{\partial V}{\partial q}$

Q70. Consider a free particle of mass m. If F is defined as $F=x-\frac{Pt}{m}$, then which one of the following statements is correct?

(a) $[F,H]=0$, so F is a constant of motion.

(b) $[F,H]\neq 0$, so F is not a constant of motion.

(c) even $[F,H]\neq 0$, but F is a constant of motion

(d) $[F,H]=0$, but F is not a constant of motion.

Q71. The Lagrangian of the particle of mass m is given by

$$L = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{V}{2}\left(x^2 + y^2\right) + W \sin \omega t$$

where V, W and ω are constant of motion, then the conserved quantity is

(a) z component of momentum

(b) z component of angular momentum

(c) z component of linear momentum and z component of angular momentum

(d) Energy, z component of momentum, z component of angular momentum

Q72. If Lagrangian of a simple pendulum of mass m and length l is $L = \frac{p_\theta^2}{2ml^2} - mgl\left(1 - \cos\theta\right)$,

then the value of $\frac{dL}{dt}$ is given by: (Wrong Question)

(a) $-\frac{2g}{l} p_\theta \sin\theta$ (b) $-\frac{g}{l} p_\theta \sin 2\theta$ (c) $\frac{g}{l} p_\theta \cos\theta$ (d) $l p_\theta^2 \cos\theta$

Q73. If $\vec{p}$ is linear momentum defined by $\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$ and $\vec{r}$ is position vector given as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and J_z is z component of angular momentum $J = J_x\hat{i} + J_y\hat{j} + J_z\hat{k}$ and $J_z = xp_y - yp_x$, then which one is not correct?

(a) $\left[p_x, J_z\right] = -p_y$ (b) $\left[y, J_z\right] = x$

(c) $\left[z, J_z\right] = 0$ (d) $\left[y, J_z\right] = -x$

Q74. If J_z is $\hat{z}$ component of angular momentum and P_x is x component of linear momentum, then find the value of poisson bracket $\left[J_x, P_x\right]$

(a) P_x (b) P_y (c) $-P_y$ (d) $-P_x$

Q75. If f, g and h are functions in phase space, it is given that $\left[f, h\right] = 0$ and $\left[g, h\right] = 0$, then the value of $\left[f, g\right]$ is given by

(a) only 0 (b) any constant (c) 1 (d) $|1|$

Q76. If transformation of two canonical co-ordinates $Q = f_1(q, p)$, $P = f_2(q, p)$ is canonical, then choose the correct statement.

Statement I $\rightarrow$ Poisson Bracket $[Q, P] = 1$.

Statement II $\rightarrow$ Jacobian of transformation $\frac{\partial(Q,P)}{\partial(q,p)}$ must be equal to 1.

(a) I (b) II (c) both I and II (d) Neither I nor II

Q77. Which of the following transformation is Canonical?

(1) $Q = -P$ $\quad P = q$

(2) $Q = p\cos q$ $\quad P = p\sin q$

(3) $P = pq^2$ $\quad Q = \frac{-1}{q}$

(4) $Q = \sqrt{2p}\sin q$ $\quad P = \sqrt{2p}\cos q$

(a) 1, 2, 3 are canonical (b) 1, 3, 4 are canonical

(c) 2, 3, 4 are canonical (d) 3,4 are canonical

Q78. If transformations $Q = q + i\alpha p$, $P = q - i\alpha p$ are canonically transformed, then the value of α is given by

(a) $\frac{i}{2}$ (b) $-\frac{i}{2}$ (c) $2i$ (d) $-2i$

Q79. If F_1 and F_2 are generating functions given as $F_1 = qQ, F_2 = qP$, then which one is true?

(a) F_1 is identify transformation and F_2 is inverse transformation

(b) F_1 is inverse transformation and F_2 is identity transformation

(c) Both F_1 and F_2 are inverse transformation

(d) Both F_1 and F_2 are identity transformation

Q80. If canonical transformation $Q=-\frac{1}{q}$ & $P=pq^2$ then the F_3 type generating function is given by

(a) $\frac{-p}{Q}$ (b) $\frac{p}{Q}$ (c) $\frac{Q}{p}$ (d) $\frac{-Q}{P}$

Q81. If Hamiltonian is given by

$H=\frac{p^2}{2}+\frac{q^2}{2}$ is transformed into new Hamiltonian $K(P,Q,t)$ via generating function $F_2=Pq-\frac{1}{2}P^2t$, then new Hamiltonian $K(Q,P,t)$ is given by

(a) $K=\frac{1}{2}(Q+Pt)^2$ (b) $K=\frac{1}{2}Q^2+\frac{P^2t^2}{2}$ (c) $K=\frac{1}{2}(Q-Pt)^2$ (d) $K=\frac{1}{2}Q^2-\frac{P^2t^2}{2}$

Q82. A particle of mass m interacts with potential energy function $V=V(x)$. Particle executes small oscillation about point $x=x_0$, then which one is correct property of $V(x)$

I. At $x=x_0$, the force is zero and momentum is maximum for conservation system.

II. The potential energy can be approximated for small oscillation as $V(x)=V_0+\frac{1}{2}k(x-x_0)^2$ where V_0 is constant and $k=\left.\frac{d^2V}{dx^2}\right|_{x=x_0}$ is force constant.

(a) Only I (b) Only II (c) Both I and II (d) Neither I nor II

Q83. If Lagrangian of particle of mass m is given by $L=\frac{1}{2}ma^2\left(\dot{\theta}^2+\omega^2\sin^2\theta\right)+mga\cos\theta$ then for the small oscillation

(a) Particle will always executes small oscillation

(b) Particle will never execute small oscillation

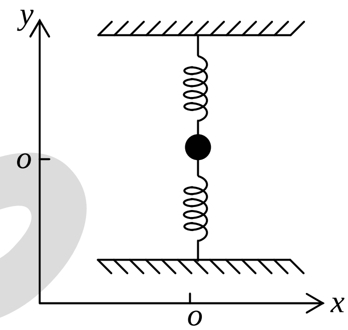
(c) Particle will oscillates if $\omega<\sqrt{\frac{g}{a}}$

(d) Particle will oscillates if $\omega>\sqrt{\frac{g}{a}}$

Q84. The Lagrangian of the system $L = +\frac{1}{2}m\dot{x}^2 + k_1 x\dot{x} - \frac{1}{2}k_2 x^2$ describes the motion of small oscillation then the frequency of the oscillator is given by

(a) $\sqrt{\frac{k_1}{m}}$ (b) $\sqrt{\frac{k_2}{m}}$ (c) $\sqrt{\frac{k_1+k_2}{m}}$ (d) $\sqrt{\left(\frac{k_1 k_2}{k_1+k_2}\right)\frac{1}{m}}$

Q85. A particle of mass m attached to two identical springs each of length l and spring constant k (see the figure). The equilibrium configuration is the one where the springs are untstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, then the Lagrangian of the system for small oscillation is given by



(a) $L = \frac{1}{2}m\dot{x}^2 - kx^2$ (b) $L = \frac{1}{2}m\dot{x}^2 - k\left(\frac{x^4}{4l^2}\right)$

(c) $L = \frac{1}{2}m\dot{x}^2 - k\left[\left(x^2+l^2\right)^{\frac{1}{2}} - l\right]$ (d) $L = \frac{1}{2}m\dot{x}^2 - k\left[\left(x^2+l^2\right)^{\frac{1}{2}} - l\right]^2$

Q86. If the equation of motion is given by

$$b\ddot{\theta}_1 + a\left(2\theta_1 - \theta_2 - \theta_3\right) = 0$$

$$b\ddot{\theta}_2 + a\left(2\theta_2 - \theta_3 - \theta_1\right) = 0$$

$$b\ddot{\theta}_3 + a\left(2\theta_3 - \theta_1 - \theta_2\right) = 0$$

where $\theta_1, \theta_2, \theta_3$ are generalized co-ordinates. Then Normal frequency is given by

(a) $\omega_1 = \sqrt{\frac{a}{b}},\ \omega_2 = \sqrt{\frac{a}{b}},\ \omega_3 = \sqrt{\frac{3a}{b}}$ (b) $\omega_1 = 0,\ \omega_2 = \sqrt{\frac{a}{b}},\ \omega_3 = \sqrt{\frac{3a}{b}}$

(c) $\omega_1 = 0,\ \omega_2 = \sqrt{\frac{a}{a}},\ \omega_3 = \sqrt{\frac{a}{a}}$ (d) $\omega_1 = 0,\ \omega_2 = \sqrt{\frac{3a}{b}},\ \omega_3 = \sqrt{\frac{3a}{b}}$

Q87. If potential energy matrix for small oscillation is given by

$$V=\begin{pmatrix} k & a & 0 & -\frac{k}{2} \\ -\frac{k}{2} & k & b & 0 \\ c & -\frac{k}{2} & k & -\frac{k}{2} \\ -\frac{k}{2} & d & -\frac{k}{2} & k \end{pmatrix}$$

then value of a,b,c and d is given by

(a) $a=-\frac{k}{2}$, $b=-\frac{k}{2}$, $c=-\frac{k}{2}$, $d=0$

(b) $a=0$, $b=0$, $c=-\frac{k}{2}$, $d=-\frac{k}{2}$

(c) $a=-\frac{k}{2}$, $b=-\frac{k}{2}$, $c=0$, $d=0$

(d) $a=-\frac{k}{2}$, $b=0$, $c=-\frac{k}{2}$, $d=0$

Q88. If potential energy is given by $V=k\left[e^{-(\theta_2-\theta_1)}+e^{-(\theta_3-\theta_2)}+e^{-(\theta_1-\theta_3)}\right]$. If $\theta_1,\theta_2,\theta_3$ are generalized co-ordinates and k is constant then potential energy matrix is given by

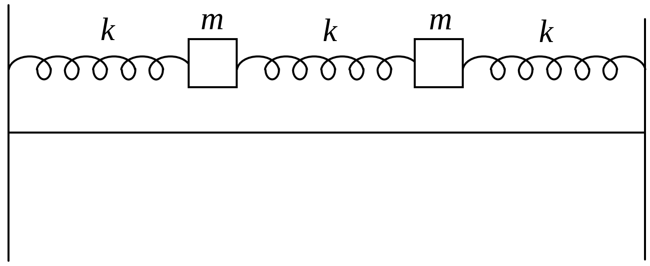
(a) $V=\begin{pmatrix} k & -k & -k \\ -k & k & -k \\ -k & -k & k \end{pmatrix}$

(b) $V=\begin{pmatrix} k & -k & -k \\ -k & 2k & -k \\ -k & -k & k \end{pmatrix}$

(c) $V=\begin{pmatrix} 2k & -k & -k \\ -k & 2k & -k \\ -k & -k & 2k \end{pmatrix}$

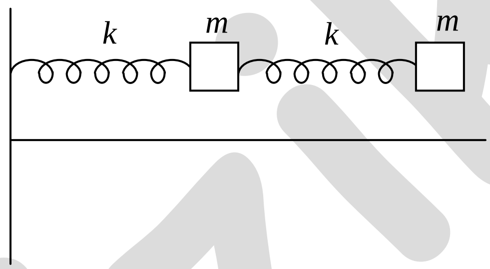
(d) $V=\begin{pmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{pmatrix}$

Q89. If k is spring constant of spring and m is mass of the block and the system executes small oscillation then potential energy matrix of the system is given by



(a) $\begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix}$ (b) $\begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}$ (c) $\begin{bmatrix} k & -2k \\ -2k & k \end{bmatrix}$ (d) $\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$

Q90. If k is spring constant of spring and m is mass of the block and the system executes small oscillation then potential energy matrix of the system is given by



(a) $\begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$ (b) $\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$ (c) $\begin{bmatrix} k & -2k \\ -2k & k \end{bmatrix}$ (d) $\begin{bmatrix} 2k & -2k \\ -2k & k \end{bmatrix}$

Q91. If kinetic energy matrix (T) and potential energy matrix (V) is given by

$$T = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \qquad V = \begin{pmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & -k \end{pmatrix}$$

If Normal frequency is given by $\omega = \sqrt{\frac{k}{m}}$ and corresponding Normal mode is $A\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ then value of A is given by

(a) $A = \frac{1}{\sqrt{2m+M}}$ (b) $A = \frac{1}{\sqrt{2m}}$ (c) $A = \frac{1}{\sqrt{2M}}$ (d) $A = \frac{1}{\sqrt{2m\left(1+\frac{2m}{M}\right)}}$

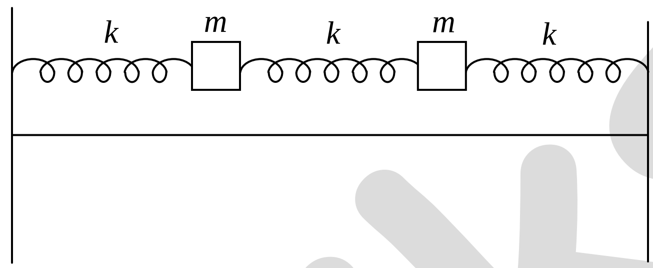
Q92. For the small oscillations about the origin the equation of motion is given by

$$\ddot{x}+2gx-gy=0 \qquad \ddot{y}+2gy-gx=0$$

Then the normal frequency is given by

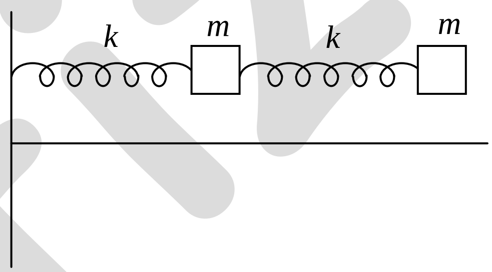
(a) $0,\sqrt{g}$ (b) $0,\sqrt{3g}$ (c) $\sqrt{g},\sqrt{g}$ (d) $\sqrt{g},\sqrt{3g}$

Q93. If k is spring constant of spring and m is mass of the block and system execute small oscillation then natural frequency is given by



(a) $\sqrt{\frac{k}{m}},\sqrt{\frac{2k}{m}}$ (b) $\sqrt{\frac{k}{m}},\sqrt{\frac{3k}{m}}$ (c) $\sqrt{\frac{2k}{m}},\sqrt{\frac{2k}{m}}$ (d) $\sqrt{\frac{2k}{m}},\sqrt{\frac{3k}{m}}$

Q94. If k is spring constant of spring and m is mass of the block and system execute small oscillation then natural frequency is given by



(a) $\sqrt{\frac{k}{m}},\sqrt{\frac{k}{m}}\left(\frac{3+\sqrt{5}}{2}\right)^{1/2}$ (b) $\sqrt{\frac{k}{m}},\sqrt{\frac{k}{m}}\left(\frac{5+\sqrt{3}}{2}\right)^{1/2}$

(c) $\sqrt{\frac{k}{m}}\left(\frac{5-\sqrt{3}}{2}\right)^{1/2}\sqrt{\frac{k}{m}}\left(\frac{5+\sqrt{3}}{2}\right)^{1/2}$ (d) $\omega=\sqrt{\frac{k}{m}}\left(\frac{3-\sqrt{5}}{2}\right)^{1/2},\sqrt{\frac{k}{m}}\left(\frac{3+\sqrt{5}}{2}\right)^{1/2}$

Q95. A planet moves round the sun. At a point P, it is closest to the sun at distance r_1 and has speed v_1 At another point Q, when it is farthest from the sun at a distance r_2, what is its speed ?

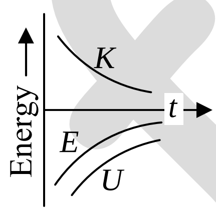
(a) $\frac{r_1^2 v_1}{r_2^2}$ (b) $\frac{r_1 v_1}{r_2}$ (c) $\frac{r_2^2 v_2}{r_1^1}$ (d) $\frac{r_2 v_1}{r_1}$

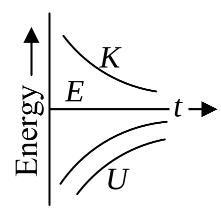
Q96. Two masses m and M are initially at rest at infinite distance apart They approach each other due to gravitational interaction What is their speed of approach at the instant when they are at a distance d apart?

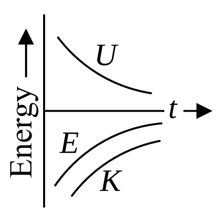
(a) $\left(\frac{2G\left(M^2+m^2\right)^{\frac{1}{2}}}{d}\right)^{\frac{1}{2}}$ (b) $\left(\frac{2GMm}{d\left(M+m\right)}\right)^{\frac{1}{2}}$

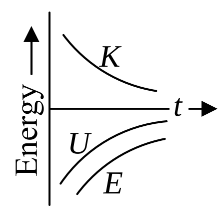
(c) $\left(\frac{2G\left(M+m\right)}{d}\right)^{\frac{1}{2}}$ (d) $\left(\frac{GMm}{d\left(M+m\right)}\right)^{\frac{1}{2}}$

Q97. Which one of the following diagrams correctly depicts the variation of kinetic energy (K), potential energy (U) and total energy (E) of a body in circular planetary motion? (r is the radius of the circle).

(a) 

(b) 

(c) 

(d) 

Q98. If a particle of mass m moves under the influence of a force, then which statement(s) is/are true?

I. If potential is $V(r,\theta) = mgr\cos\theta$ then force is central force.

II. The motion of the particle is influenced under central force, confined into a plane and angular momentum of particle is constant during the motion but direction of angular momentum is parallel to the plane of motion

III. If potential $V(r)$, where r is radial distance of particle from the center of force then force is non conservative force.

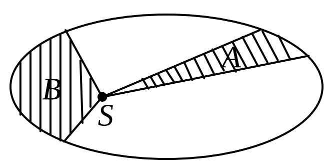
(a) I, II (b) II, III (c) I, II, III (d) None

Q99. A satellite of mass m is orbiting elliptically about the centre O of earth of mass M. The work done on the satellite as it moves from A to B in its orbit is

(a) $\frac{GMm}{r_B}\left(1-\frac{r_B}{r_A}\right)$ (b) $\frac{GMm}{r_A}\left(1-\frac{r_A}{r_B}\right)$

(c) $GMM\left(1-\frac{r_B}{r_A}\right)$ (d) $\frac{GMm}{r_B^2-r_A^2}$

Q100. The figure below shows the motion of a planet around the sun in an elliptical orbit with sun at the focus. The shaded areas A and B shown in the figure can be assumed to be equal. If t_1 and t_2 represent the times for the planet to move from a to c respectively; then



(a) $t_1 < t_2$ (b) $t_1 > t_2$ (c) $t_1 = t_2$ (d) None of these

Q101. If R is the radius of the earth ρ is mean density and G the gravitational constant, then the earth's surface potential will be nearly equal to

(a) $-\frac{\pi\rho G}{R^2}$ (b) $-\frac{4}{3}\pi R^3\rho G$ (c) $-\frac{4}{3}\pi\rho R^2 G$ (d) $-\frac{4}{3}\pi\frac{R}{\rho}G$

Q102. A planet of mass m moves in the gravitational field of the Sun (mass M). If the semi-major and semi-minor axis of the orbit are a and b respectively, the angular momentum of the planet is

(a) $\sqrt{2GMm^2(a+b)}$ (b) $\sqrt{2GMm^2(a-b)}$ (c) $\sqrt{\frac{2GMm^2ab}{a-b}}$ (d) $\sqrt{\frac{2GMm^2ab}{a+b}}$

Q103. Consider a system of two particles of masses $m_1 = m$ and $m_2 = 2m$. The force between the two masses is function of distance between them given as $f(r)\hat{r}$, where $\vec{r}$ is position vector between m_1 and m_2. What is the expression for kinetic energy of the system with respect to centre of mass $R(X,Y,Z)$ reference frame.

(a) $\frac{m}{2}\left(\dot{X}^2+\dot{Y}^2+\dot{Z}^2\right)+\frac{3m}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)$ (b) $\frac{3m}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\frac{m}{3}\left(\dot{X}^2+\dot{Y}^2+\dot{Z}^2\right)$

(c) $\frac{3m}{2}\left(\dot{X}^2+\dot{Y}^2+\dot{Z}^2\right)+\frac{m}{3}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)$ (d) $\frac{m}{3}\left(\dot{X}^2+\dot{Y}^2+\dot{Z}^2\right)+\frac{m}{3}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)$

Q104. A particle of mass m has motion under the influence of a central force. Consider the two statements about the mass m is given below

Statement I: The motion is confined into a plane and angular momentum of system is a constant of motion.

Statement II: The area swept by radius vector is constant during the time.

(a) Statement I is true.

(b) Statement II is true.

(c) Statement I and II are true but both are independent to each other.

(d) Statement I and II are true and both can be dependent to each other.

Q105. Which one is wrong statement about two body central force problem?

(a) Velocity of center of mass is constant during the motion

(b) In centre of mass reference frame the two body problem can be reduced to one body with reduce mass $\mu = \frac{m_1 + m_2}{2}$ where m_1 and m_2 are the masses of the both bodies.

(c) Motion is confined into a plane and angular momentum is perpendicular to the plane of motion.

(d) The total energy of the system is conserved during the motion.

Q106. A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$, where k is a positive constant. The orbit angular momentum of the planet is

(a) $2r_0 km$ (b) $\sqrt{2r_0 km}$ (c) $r_0 km$ (d) $\sqrt{r_0 km}$

Q107. If the gravitational force is assumed to vary inversely as the nth power of distance r, then how does the time period of a, planet revolving around the sun depend upon r?

(a) r^n (b) r^{-n} (c) $r^{(n+1)/2}$ (d) $r^{(n-1)/2}$

Q108. Three objects S_1, S_2 and S_3 having same mass m are in different elliptic orbits of same semi-major axis, about an object of mass M. If the eccentricities of the orbits are e_1, e_2 and e_3 respectively, such that $e_1 < e_2 < e_3$ and if E_1, E_2 and E_3 are their respective mechanical energies, then which one of the following is correct?

(a) $E_1 < E_2 < E_3$ (b) $E_1 > E_2 > E_3$

(c) $E_1 = E_2 = E_3$ (d) E_1, E_2 and E_3 cannot be compared

Q109. A particle is moving in an inverse square field. If the total energy of the particle is positive, then trajectory of particle is

(a) circular (b) elliptical

(c) parabolic (d) hyperbolic

Q110. In a central force field, the trajectory of a particle of mass m and angular momentum J in plane polar coordinates is given by,

$$\frac{1}{r} = \frac{mk}{J^2}(1 + \varepsilon \cos\theta)$$

where, ε is the eccentricity of the particle's motion. Which one of the following choice for ε gives rise to a parabolic trajectory?

(a) $\varepsilon = 0$ (b) $\varepsilon = 1$ (c) $0 < \varepsilon < 1$ (d) $\varepsilon > 1$

Q111. Match the following List I with List II.

List I	List II
A. Eccentricity $e > 0$, energy $(E) > 0$	1. Circular orbit
B. Eccentricity $e = 1$, $E = 0$	2. Elliptical orbit
C. $e < 1, E < 0$	3. Parabolic orbit
D. $e = 0$, $E = -\frac{mk^2}{2J^2}$	4. Hyperbolic orbit

	A	B	C	D
(a)	1	2	3	4
(b)	2	1	3	4
(c)	3	4	1	2
(d)	4	3	2	1

Q112. A satellite in a circular orbit about the earth has a kinetic energy E_K. What is the minimum amount of energy to be added, so that it escape from the earth?

(a) $\frac{E_K}{4}$ (b) $\frac{E_K}{2}$ (c) E_K (d) $2E_K$

Q113. Two particles of identical mass move in circular orbits under a central potential $V(r) = \frac{1}{2}kr^2$. Let l_1 and l_2 be the angular momenta and r_1, r_2 be the radii of the orbits respectively. If $\frac{J_1}{J_2} = 2$, then the value of $\frac{r_1}{r_2}$ is:

(a) $\sqrt{2}$ (b) $1/\sqrt{2}$ (c) 2 (d) $1/2$

Q114. A particle is confined into a plane and it moves under influence of central force given as $f(r) = -\frac{k}{r^2}$. Then the Lagrangian and Hamiltonion is given by

(a) $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\theta^2\right) + \frac{k}{r^2}$

$H = \frac{P_r^2}{2m} + \frac{P_\theta^2}{2mr^2} - \frac{k}{r^2}$

(b) $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\theta^2\right) + \frac{k}{r}$

$H = \frac{1}{2}m\left(\dot{r}^2 + r^2\theta^2\right) - \frac{k}{r^2}$

(c) $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\theta^2\right) - \frac{k}{r}$

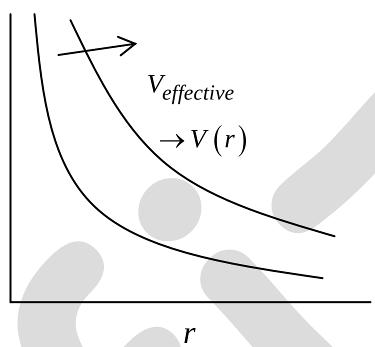
$H = \frac{P_r^2}{2m} + \frac{P_\theta^2}{2mr^2} + \frac{k}{r}$

(d) $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{r}$

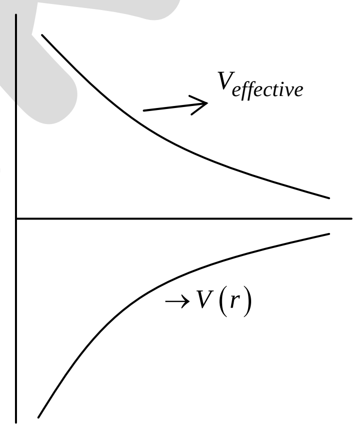
$H = \frac{P_r^2}{2m} + \frac{P_\theta^2}{2mr^2} - \frac{k}{r}$

Q115. For central force problem the potential is given as $V(r) = -\frac{k}{r}$, then which one is the correct plot of potential $V(r)$ and effective potential V_{eff}.

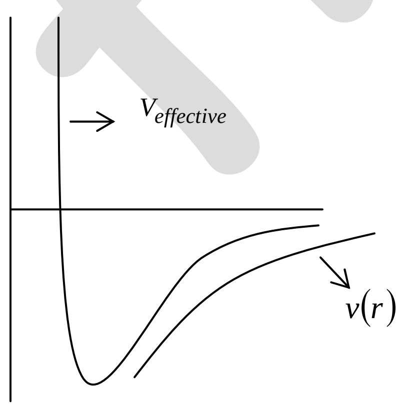
(a)



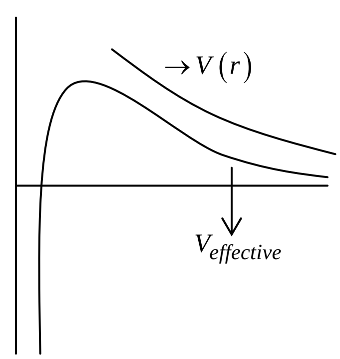
(b)



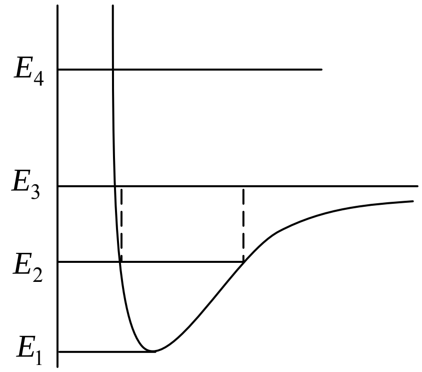
(c)



(d)



Q116. The variation of $V_{effective}$ and distance from centre of force is given as $V(r) = -\frac{k}{r}$



If E_1, E_2, E_3 and E_4 are total energy of different shape of orbit then which one is correctly matched?

(a) E_1 ➔ Ellipse E_2 ➔ circle E_3 ➔ parabolic E_4 ➔ hyperbola

(b) E_1 ➔ hyperbola E_2 ➔ parabolic E_3 ➔ circle E_4 ➔ Ellipse

(c) E_1 ➔ circle E_2 ➔ Ellipse E_3 ➔ parabolic E_4 ➔ hyperbola

(d) E_1 ➔ circle E_2 ➔ parabolic E_3 ➔ Ellipse E_4 ➔ hyperbola

Q117. A particle of mass m moves under influence of central potential given as kr^2. If particle has constant angular momentum l, then the radius of circular orbit is given by

(a) $r_0 = \left(\frac{l^2}{2mk}\right)^{1/4}$ (b) $r_0 = \left(\frac{l^2}{mk}\right)^{1/4}$

(c) $r_0 = \left(\frac{l^2}{2mk}\right)^{1/5}$ (d) $r_0 = \left(\frac{l^2}{mk}\right)^{1/5}$

Q118. A satellite of mass m is in elliptic orbit about earth. At the Perigee (closest approach to the earth) it has altitude of $1100\,km$ and at apogee its altitude is $4100\,km$. If radius of earth is $6400\,km$ then eccentricity of orbit is given by

(a) 0.64 (b) 0.57 (c) 0.27 (d) 0.16

Q119. The circular and parabolic orbits are in an attractive potential given as $V(r) = -\frac{k}{r}$ have the same value of angular momentum. If r_p is perihelion distance of parabola and r_c is radius of circle then ratio of r_c to r_p is given by

(a) 0 (b) ½ (c) 1 (d) 2

Q120. The particle is moving under the influence of central potential given as $v(r) = \frac{1}{2}kr^2$, then bounded motion is possible only for

(a) $-\infty < E < \infty$ (b) $-\infty < E < 0$ (c) $0 < E < \infty$ (d) $E_{\min} < E < \infty$

Q121. Which one is not true about Kepler's law of planetary motion?

(a) The potential between sun and earth is central potential & attractive in nature. The potential is inversely proportional to distance between sun and earth.

(b) Every planet moves in an elliptical orbit around the sun being at centre of ellipse.

(c) The radius vector drawn from the sun to a planet, sweeps out equal area in equal time.

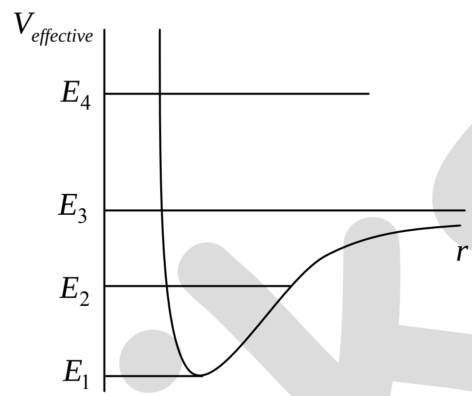
(d) The square of time period of revolution around sun is proportional to cube of semi major axis of ellipse.

Q122. According to Kepler, earth is revolving around the sun. If earth rotates in elliptical orbit given as $\frac{x^2}{16} + \frac{y^2}{25} = 1$, then the co-ordinate of position of sun is given by

(a) $\left(\frac{48}{5}, 0\right)$ $\left(-\frac{48}{5}, 0\right)$ (b) $\left(0, \frac{48}{5}\right)$ $\left(0, -\frac{48}{5}\right)$

(c) $(0, 3)$ $(0, -3)$ (d) $(3, 0)$ $(-3, 0)$

Q123. A satellite revolves around the earth in circular orbit. The shape of V_{eff} for $v(r) = -\frac{k}{r}$ is shown in figure. E_1, E_2, E_3 and E_4 are four values of energies. Suddenly the energy of satellite is increased and satellite settle itself into another closed (bounded) orbit then which one is the energy of old orbit and new orbit respectively:



(a) E_1, E_2 (b) E_2, E_3 (c) E_3, E_4 (d) E_3, E_2

Q124. If a particle of mass m moves under the influence of central force given as, $f(r) = -\frac{k}{r^3}$.

If l is angular momentum of particle, then the shape of orbit is given as

(a) Always unbounded

(b) Always bounded

(c) If $L^2 > mk$ then unbounded and $L^2 < mk$ then bounded

(d) If $L^2 > mk$ then bounded and $L^2 < mk$ then unbounded

Q125. A particle moves under central force field located at $r = 0$ described by $r = e^{-\theta}$, then the magnitude of potential is proportional to

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$

Q126. A satellite in circular orbit about the earth has kinetic energy E_k. What is minimum amount of energy to be added so that it escape from earth's potential

(a) $\frac{3E_k}{2}$ (b) $\frac{E_k}{2}$ (c) E_k (d) $2E_k$

Q127. If potential is defined as $V(r)=\dfrac{K}{2}r^2$, then which one is the correct relationship between average kinetic energy $\langle T\rangle$ and average potential energy $\langle V\rangle$?

(a) $\langle T\rangle=\dfrac{1}{2}\langle V\rangle$ (b) $\langle T\rangle=-\dfrac{1}{2}\langle V\rangle$

(c) $\langle T\rangle=\langle V\rangle$ (d) $\langle T\rangle=\dfrac{3}{2}\langle V\rangle$

Q128. A particle moves under central force field located at $r=0$, described by $r=e^{-\theta}$, then which one is the correct relation between average kinetic energy $\langle T\rangle$ and average potential energy $\langle V\rangle$?

(a) $\langle T\rangle=\langle V\rangle$ (b) $\langle T\rangle=-\langle V\rangle$

(c) $\langle T\rangle=\dfrac{1}{2}\langle V\rangle$ (b) $\langle T\rangle=-\dfrac{1}{2}\langle V\rangle$

Q129. A planet revolves in elliptical orbit about sun, which is centred at one of the foci. Choose the correct statement.

(a) linear momentum is minimum and maximum at perihelion and aphelion respectively, but angular momentum is constant at that point.

(b) linear momentum is constant and angular momentum is minimum and maximum at perihelion and aphelion respectively

(c) linear momentum is zero and angular momentum is constant at the perihelion and aphelion

(d) both linear momentum and angular momentum are minimum and maximum at perihelion and aphelion respectively

Q130. A satellite of mass m revolves around the earth in elliptical orbit. If r_a the apogee and r_p the perigee of the satellite are given as $r_p = 7.5\times10^6\,m$ and $r_a = 10.57\times10^6\,m$.

Consider the potential between earth and satellite is central potential given as $v(r) = -\dfrac{k}{r}$.

Then which statement is correct about the system?

I. The velocity of satellite at apogee and perigee may be nearly 5600 and 7900

II. The velocity of satellite of perigee is purely tangential

III. If total energy of satellite is $-4.5\times10^{10}\,J$ then value of k is $8\times10^{17}\,J/m$

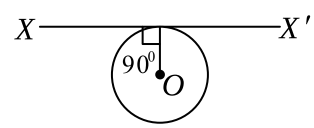
(a) only I, II (b) only II, III (c) only I, III (d) I, II, III

Q131. The magnitude of force between sun and earth is given by $f(r) = -\dfrac{k}{r^2}$. If uniform distribution of dust in the solar system adds to gravitational attraction of the sun on the planet with additional potential = $\dfrac{1}{2}mcr^2$, where m is mass of planet k and c is constant.

Then the period for circular orbit of radius r_o of the planet in combined field is given as

(a) $\dfrac{2\pi}{\sqrt{\dfrac{2k}{mr_o^3}+c}}$ (b) $\dfrac{2\pi}{\sqrt{\dfrac{2k}{mr_o^3}-c}}$ (c) $\dfrac{2\pi}{\sqrt{\dfrac{k}{mr_o^3}+c}}$ (d) $\dfrac{2\pi}{\sqrt{\dfrac{k}{mr_o^3}-c}}$

Q132. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre O as shown. The moment of inertia of the loop about the axis XX' is



(a) $\dfrac{\rho L^3}{8\pi^2}$ (b) $\dfrac{\rho L^3}{16\pi^2}$

(c) $\dfrac{5\rho L^3}{16\pi^2}$ (d) $\dfrac{3\rho L^3}{8\pi^2}$

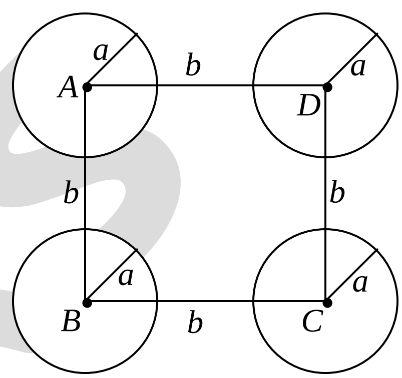
Q133. One quarter sector is cut from a uniform circular disc of radius R. This sector has a mass M. If it is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Then its moment of inertia about the axis of rotation is

(a) $\dfrac{1}{2}MR^2$ (b) $\dfrac{1}{4}MR^2$ (c) $\dfrac{1}{8}MR^2$ (d) $\sqrt{2}MR^2$

Q134. From a circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is

(a) $4MR^2$ (b) $\frac{40}{9}MR^2$ (c) $10MR^2$ (d) $\frac{37}{9}MR^2$

Q135. Four spheres of diameter $2a$ and mass M each are placed with their centres on the four corners of a square of side b. The moment of inertia of the system about one side of the square taken as the axis is



(a) $\frac{8M\left(3a^2+2b^2\right)}{3}$ (b) $\frac{9M\left(6a^2+4b^2\right)}{8}$

(c) $\frac{3M\left(2a^2+4b^2\right)}{7}$ (d) $\frac{2M\left(4a^2+5b^2\right)}{5}$

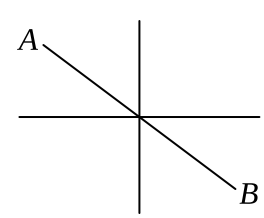
Q136. The ratio of radius of gyration of a circular ring and a disc of the same radius about the axis passing through their centres and perpendicular to their plane is

(a) $\sqrt{5}:1$ (b) $\sqrt{2}:1$ (c) $1:\sqrt{3}$ (d) $1:\sqrt{2}$

Q137. The moment of inertia of a thin rod of length L and mass M about an axis that is perpendicular to the rod and at a distance x from its centre is

(a) $\frac{ML^2}{12}+Mx^2$ (b) $\frac{ML^2}{6}+Mx^2$ (c) $\frac{ML^2}{3}+\frac{Mx^2}{7}$ (d) $\frac{ML^2}{12}+Mx^2$

Q138. Two uniform identical rods each of mass M and length l are joined to form a cross as shown in the figure. The moment of inertia of the cross about the bisector AB is



(a) $\frac{Ml^2}{4}$ (b) $\frac{Ml^2}{5}$

(c) $\frac{Ml^2}{8}$ (d) $\frac{Ml^2}{12}$

Q139. The moment of inertia of a pair of spheres, each having a mass m and radius r, kept in contact about the tangent passing through the point of contact is

(a) $\frac{7mr^2}{5}$ (b) $\frac{9mr^2}{5}$ (c) $\frac{14mr^2}{5}$ (d) $\frac{18mr^2}{5}$

Q140. From a given sample of uniform wire two circular loops P and Q are made, P of radius r and Q of radius nr. If the M.I. of Q about its axis is four times that of P about its axis, the value of n is

(a) $4^{\frac{2}{3}}$ (b) $4^{\frac{1}{3}}$ (c) $4^{\frac{1}{2}}$ (d) $4^{\frac{1}{4}}$

Q141. Two circular discs A and B are of equal masses and thickness but made of metal with densities d_A and $d_B\left(d_A > d_B\right)$. If their moment of inertia about an axis passing through their centres and perpendicular to their faces are I_A and I_B, then

(a) $I_A = I_B$ (b) $I_A > I_B$ (c) $I_A < I_B$ (d) $I_A \geq I_B$

Q142. The angular momentum of a particle rotating under a central force is constant due to

(a) constant force (b) constant linear momentum

(c) constant torque (d) zero torque

Q143. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which of the following will not be affected?

(a) moment of inertia (b) angular momentum

(c) angular velocity (d) rotational kinetic energy

Q144. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its outer end. During the journey of the insect, the angular speed of the disc

(a) remains unchanged (b) continuously decreases

(c) continuously increases (d) first increases and then decreases

Q145. A couple is acting on a two-particle system connected by a massless rod. The resultant motion will be

(a) purely rotational motion (b) purely translational motion

(c) both (a) and (b) (d) Neither (a) nor (b)

Q146. Angular momentum of a system of particles is conserved

(a) when no external force acts on the system

(b) when no external torque acts on the system

(c) when no external impulse acts on the system

(d) when axis of rotation remains the same

Q147. The direction of angular velocity vector is along

(a) the tangent to the circular path (b) the inward radius

(c) the outward radius (d) the axis of rotation

Q148. A body is projected from the ground with some angle to the horizontal. What happens to its angular momentum about the initial position in its motion?

(a) decreases (b) increases

(c) remains the same (d) first increases then decreases

Q149. A disc is rolling, the velocity of its centre of mass is v_{cm}. Which one will be correct?

(a) the velocity of highest point is $2v_{cm}$ and the point of contact is zero.

(b) the velocity of highest point is v_{cm} and the point of contact is v_{cm}

(c) the velocity of highest point is $2v_{cm}$ and point of contact is v_{cm}.

(d) the velocity of highest point is $2v_{cm}$ and point of contact is $2v_{cm}$

Q150. For a hollow cylinder and a solid cylinder rolling without slipping on an incline plane, which of them will reach earlier?

(a) solid cylinder (b) hollow cylinder

(c) both simultaneously (d) can't say anything

Q151. Pick out the correct statement about the torque vector

(a) The torque acting on a particle is given by $\vec{F} \times \vec{r}$, where $\vec{r}$ is the position vector of the particle about the point chosen for calculating torque.

(b) If net torque acting on a particle is zero, its angular momentum is not constant

(c) Zero net torque acting on a system implies zero net force

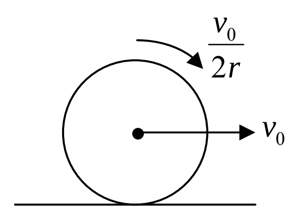
(d) If the angular momentum of a system changes with time, a net torque must act on the system

Q152. A cylinder is released from rest from the top of an inclined plane of inclination θ and length l. If the cylinder rolls without slipping, what will be its speed when it reaches the bottom?

(a) $\sqrt{\frac{4gl\sin\theta}{3}}$ (b) $\sqrt{\frac{2gl\sin\theta}{3}}$

(c) $\sqrt{\frac{5gl\sin\theta}{3}}$ (d) $\sqrt{\frac{6gl\sin\theta}{3}}$

Q153. A sphere of mass M and radius r as shown in the figure slips on a rough horizontal surface. At some time it has translational velocity v_0 and rotational velocity about the centre of mass $\frac{v_0}{2r}$. When the sphere starts pure rolling the translational velocity is



(a) $\frac{2v_0}{7}$ (b) $\frac{3v_0}{9}$ (c) $\frac{4v_0}{9}$ (d) $\frac{6v_0}{7}$

Q154. The centre of a wheel rolling on a plane surface moves with speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed

(a) zero (b) v_0 (c) $\sqrt{2}v_0$ (d) $2v_0$

Q155. Two small kids weighing $10\,kg$ and $15\,kg$ are trying to balance a seesaw of total length $5.0\,m$. If the kid of mass $10\,kg$ sits at an end at what distance from the centre the other kid should sit?

(a) $1.2\,m$ (b) $1.5\,m$ (c) $1.3\,m$ (d) $1.7\,m$

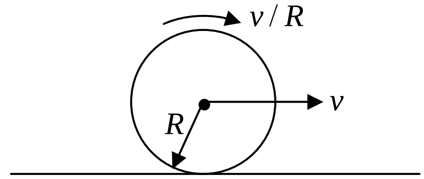
Q156. A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant for the lowermost point of the disc

(a) velocity is v, acceleration is zero

(b) velocity is zero, acceleration is zero

(c) velocity is v, acceleration is $\frac{v^2}{R}$

(d) velocity is zero, acceleration is $\frac{v^2}{R}$



Q157. A ring of mass M and radius a is supported from pivot located at one point of ring, about which it is free to rotate in its own vertical plane. A bead of mass m slides without friction on the ring as shown in figure, then its kinetic energy is given by

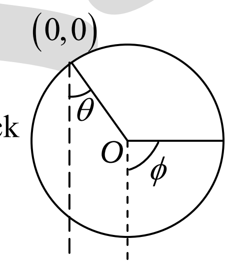
(a) $T = \frac{1}{2}Ma^2\dot{\theta}^2 + \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right)$

(b) $T = Ma^2\dot{\theta}^2 + \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right)$

(c) $T = \frac{1}{2}Ma^2\dot{\theta}^2 + \frac{1}{2}ma^2\left[\dot{\theta}^2 + \dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\cos(\theta-\phi)\right]$

(d) $T = Ma^2\dot{\theta}^2 + \frac{1}{2}ma^2\left[\dot{\theta}^2 + \dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\cos(\theta-\phi)\right]$

figure check



Q158. Moment of Inertia tensor is given by $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ then moment of inertia about the axis $\frac{1}{\sqrt{2}}j + \frac{1}{\sqrt{2}}\hat{k}$ is

(a) 0 (b) 1 (c) 2 (d) 3

Q159. Four masses of equal value m lie in x, y plane at position $(2a,0),(-2a,0),(-a,0)$ and $(a,0)$ respectively. If they are joined by massless rod then moment of inertia tensor is given by

(a) $\begin{pmatrix} 2ma^2 & 0 & 0 \\ 0 & 8ma^2 & 0 \\ 0 & 0 & 10ma^2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 10ma^2 & 0 \\ 0 & 0 & 10ma^2 \end{pmatrix}$

(c) $\begin{pmatrix} 8ma^2 & 0 & 0 \\ 0 & 2ma^2 & 0 \\ 0 & 0 & 10ma^2 \end{pmatrix}$ (d) $\begin{pmatrix} 8ma^2 & 0 & 0 \\ 0 & 2ma^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Q160. If moment of inertia tensor is given by

$$\frac{MR^2}{4}\begin{pmatrix} 10 & -5 & 0 \\ -5 & 6 & 0 \\ 0 & 0 & 16 \end{pmatrix}$$

If system is rotated about y-axis with angular velocity ω, then the angular momentum in x, y, z direction i.e., L_x, L_y and L_z is given by

(a) $L_x = 0,\ L_y = \frac{3}{2}MR^2\omega$ and $L_z = 0$

(b) $L_x = \frac{5}{2}MR^2\omega,\ L_y = \frac{3}{2}MR^2\omega$ and $L_z = 4MR^2\omega$

(c) $L_x = \frac{-5}{2}MR^2\omega,\ L_y = \frac{3}{2}MR^2\omega$ and $L_z = -4MR^2\omega$

(d) $L_x = \frac{-5}{4}MR^2\omega,\ L_y = \frac{3}{2}MR^2\omega$ and $L_z = 0$

Q161. The moment of Inertia tensor of a shaft is given by

$$I = \begin{pmatrix} 6ma^2 & 0 & 2ma^2 \\ 0 & 8ma^2 & 0 \\ 2ma^2 & 0 & 2ma^2 \end{pmatrix}$$

If shaft is rotated about z-axis, then the torque about O (origin) is given by

(a) $2ma^2\omega^2\hat{i}$ (b) $2ma^2\omega^2\hat{j}$

(c) $2ma^2\omega^2\hat{i} + 8ma^2\omega^2\hat{j}$ (d) $2ma^2\omega^2\hat{j} + 8ma^2\omega^2\hat{i}$

Q162. A pendulum consists of a disc of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

(a) $2\pi\sqrt{\frac{l+R}{g}}$ (b) $2\pi\sqrt{\frac{2R^2+2Rl+l^2}{2g(R+l)}}$

(c) $2\pi\sqrt{\frac{2R^2+2Rl+l^2}{g(R+l)}}$ (d) $2\pi\sqrt{\frac{3R^2+4Rl+2l^2}{2g(R+l)}}$

Q163. Consider a cube of volume a^3 and mass M which is situated such that origin O is at one of the corner. Consider the cube has uniform density ρ, then I_{xx} component of moment of inertia tensor is

(a) $\frac{\rho a^5}{3}$ (b) $\frac{2\rho a^5}{3}$ (c) $\frac{3\rho a^5}{2}$ (d) $\frac{2\rho a^5}{5}$

Q164. In a system of unit in which the velocity of light $c=1$, then which of the following is a Lorentz transformation?

(a) $x'=4x, y=y', z'=z, t'=0.25t$

(b) $x'=x-0.5t, y=y', z'=z, t'=t+x$

(c) $x'=1.25x-0.75t, y'=y, z'=z, t'=0.75t-1.25x$

(d) $x'=1.25x-0.75t, y'=y, z'=z, t'=1.25t-0.75x$

Q165. A circle of radius $5m$ lies at rest in x-y plane in the laboratory. For an observer moving with a uniform velocity v along the y direction, the circle appears to be an ellipse with an equation

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

The speed of the observer in terms of the velocity of light c is,

(a) $\frac{9c}{25}$ (b) $\frac{3c}{5}$ (c) $\frac{4c}{5}$ (d) $\frac{16c}{25}$

Q166. An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is

(a) c (b) $-c$ (c) $0.15c$ (d) $-0.15c$

Q167. The area of a disc in its rest frame S is equal to 1 (in some units). The disc will appear distorted to an observer O moving with a speed u with respect to S along the plane of the disc. The area of the disc measured in the rest frame of the observer O is (c is the speed of light in vacuum)

(a) $\left(1-\frac{u^2}{c^2}\right)^{1/2}$ (b) $\left(1-\frac{u^2}{c^2}\right)^{-1/2}$ (c) $\left(1-\frac{u^2}{c^2}\right)$ (d) $\left(1-\frac{u^2}{c^2}\right)^{-1}$

Q168. A light beam is propagating through a block of glass with index of refraction n. If the glass is moving at constant velocity v in the same direction as the beam, the velocity of light in the glass block as measured by an observer in the laboratory is approximately

(a) $u=\frac{c}{n}+v\left(1-\frac{1}{n^2}\right)$ (b) $u=\frac{c}{n}-v\left(1-\frac{1}{n^2}\right)$

(c) $u=\frac{c}{n}+v\left(1+\frac{1}{n^2}\right)$ (d) $u=\frac{c}{n}$

Q169. If fluid is moving with velocity v with respect to stationary narrow tube .If light pulse enter into fluid in the direction of flow. What is speed of light pulse measured by observer who is stationary with respect to tube?

(a) c (b) $\frac{c}{n}$ (c) $\frac{c}{n}\left(\frac{1+\frac{nv}{c}}{1+\frac{v}{nc}}\right)$ (d) $\frac{c}{n}\left(\frac{1+\frac{v}{nc}}{1+\frac{nv}{c}}\right)$

Q170. A light beam is emitted at an angle θ_0 with respect to the x' in S frame which is moving with velocity $u\hat{i}$. Then the angle θ the beam makes with respect to x axis in S frame .

(a) $\theta = \theta_0$

(b) $\dfrac{u\cos\theta_0}{c}$

(c) $\cos\theta = \dfrac{\cos\theta_0 + \frac{u}{c}}{1 + \frac{u}{c}\cos\theta_0}$

(d) $\cos\theta = \dfrac{1 + \frac{u\cos\theta_0}{c}}{\cos\theta_0 + \frac{u}{c}}$

Q171. The relativistic form of Newton's second law of motion is

(a) $F = \dfrac{mc}{\sqrt{c^2 - v^2}}\dfrac{dv}{dt}$

(b) $F = \dfrac{m\sqrt{c^2 - v^2}}{c}\dfrac{dv}{dt}$

(c) $F = \dfrac{mc^3}{\left(c^2 - v^2\right)^{3/2}}\dfrac{dv}{dt}$

(d) $F = m\dfrac{c^2 - v^2}{c^2}\dfrac{dv}{dt}$

Q172. Two particles each of rest mass m collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of the final entity is

(a) $5m/4$ (b) $2m$ (c) $5m/2$ (d) $25m/8$

Q173. According to the special theory of relativity, the speed v of a free particle of mass m and total energy E is:

(a) $v = c\sqrt{1 - \dfrac{mc^2}{E}}$

(b) $v = \sqrt{\dfrac{2E}{m}}\left(1 + \dfrac{mc^2}{E}\right)$

(c) $v = c\sqrt{1 - \left(\dfrac{mc^2}{E}\right)^2}$

(d) $v = c\left(1 + \dfrac{mc^2}{E}\right)$

Q174. The velocity of a particle at which the kinetic energy is equal to its rest energy is (in terms of c, the speed of light in vacuum)

(a) $\sqrt{3}c/2$ (b) $3c/4$ (c) $\sqrt{3/5}c$ (d) $c/\sqrt{2}$

Q175. In the laboratory frame, a particle P of rest mass m_0 is moving in the positive x direction with a speed of $\frac{5}{19}c$. It approaches an identical particle Q, moving in the negative x direction with a speed of $\frac{2}{5}c$. The speed of the particle P in the rest frame of the particle Q is

(a) $\frac{7}{95}c$ (b) $\frac{13}{85}c$ (c) $\frac{3}{5}c$ (d) $\frac{63}{95}c$

Q176. The energy of the particle P in the rest frame of the particle Q is

(a) $\frac{1}{2}m_0c^2$ (b) $\frac{5}{4}m_0c^2$ (c) $\frac{19}{13}m_0c^2$ (d) $\frac{11}{9}m_0c^2$

Q177. If $u(x,y,z,t) = f(x+i\beta y - vt) + g(x - i\beta y - vt)$, where f and g are arbitrary and twice differentiable functions, is a solution of the wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$, then β is

(a) $\left(1-\frac{v}{c}\right)^{1/2}$ (b) $\left(1-\frac{v}{c}\right)$ (c) $\left(1-\frac{v^2}{c^2}\right)^{1/2}$ (d) $\left(1-\frac{v^2}{c^2}\right)$

Q178. A relativistic particle of mass m and velocity $\frac{c}{2}\hat{z}$ is moving towards a wall. The wall is moving with a velocity $\frac{c}{3}\hat{z}$. The velocity of the particle after it suffers an elastic collision is $v\,\hat{z}$ with v equal to

(a) $c/2$ (b) $c/5$ (c) $c/7$ (d) $c/15$

(All the velocities refer to the laboratory frame of reference.)

Q179. The momentum of an electron (mass m) which has the same kinetic energy as its rest mass energy is (c is velocity of light)

(a) $\sqrt{3}mc$ (b) $\sqrt{2}mc$ (c) mc (d) $mc/\sqrt{2}$

Q180. A particle of mass M decays at rest into a massless particle and another particle of mass m. The magnitude of the momentum of each of these relativistic particles is

(a) $\frac{c}{2}\sqrt{M^2-4m^2}$ (b) $\frac{c}{2}\sqrt{M^2+4m^2}$

(c) $\frac{c}{2M}\left(M^2-m^2\right)$ (d) $\frac{c}{2M}\left(M^2+m^2\right)$

Q181. Consider a beam of relativistic particles of kinetic energy K at normal incidence upon a perfectly absorbing surface. The particle flux (number of particles per unit area per unit time) is J and each particle has rest mass m_0. The pressure on the surface is

(a) $\frac{JK}{c}$ (b) $\frac{J\sqrt{K\left(K+m_0c^2\right)}}{c}$

(c) $\frac{J\left(K+m_0c^2\right)}{c}$ (d) $\frac{J\sqrt{K\left(K+2m_0c^2\right)}}{c}$

Solutions

Multiple Choice Questions

Ans. 1: (b)

Solution: At $x = a_1$, $E = V(x)$ i.e. momentum p is zero then $x = a_1$ is turning point.

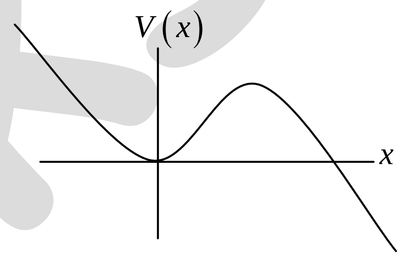
$\frac{\partial V}{\partial x} = 0$ and $\frac{\partial^2 V}{\partial x^2} \geq 0$, then it is minima which is also attractive point so it is stable point.

For small displacement around $x = a_2$ is bounded and execute simple harmonic oscillator.

Ans. 2: (b)

Solution: $\frac{dV}{dx} = 0 \Rightarrow x - x^2 = 0 \Rightarrow x = 0,\ x = 1$ are equilibrium points.

$$\frac{d^2V}{dx^2} = 1 - 2x$$



At $x = 0,\ \frac{d^2V}{dx^2} = 1 > 0$, which is stable equilibrium point.

So, it is minima of the curve.

At $x = 1,\ \frac{d^2V}{dx^2} = -1 < 0$, which is unstable equilibrium point. So it is maxima of the curve.

Ans. 3: (b)

Solution: $V(x) = x(x-4)^2$ for equilibrium point

$$\frac{\partial V}{\partial x} = 0 \Rightarrow (x-4)^2 + 2x(x-4) = 0 \Rightarrow (x-4)(3x-4) = 0$$

$$x = 4,\ x = \frac{4}{3}$$

$$\frac{\partial^2 V}{\partial x^2} = (3x-4) + 3(x-4) = 6x - 16$$

At $x = 4$, $\frac{\partial^2 V}{\partial x^2} = 6x - 16 = 8 > 0$; which is stable equilibrium point

and at $x = \frac{4}{3}$, $\frac{\partial^2 V}{\partial x^2} = 6x - 16 = -4 < 0$, so it is an unstable equilibrium point.

$$\omega = \sqrt{\frac{\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=4}}{m}} = \sqrt{\frac{8}{1}} = \sqrt{8}$$

Ans. 4: (a)

Solution: $V(x) = ax + \frac{b}{x} \Rightarrow \frac{\partial V}{\partial x} = 0 \Rightarrow a - \frac{b}{x^2} = 0 \quad ax^2 - b = 0 \Rightarrow x_0 = \pm\left(\frac{b}{a}\right)^{\frac{1}{2}}$.

Since $\omega = \sqrt{\frac{k}{m}}$, $\mathrm{m} = 1$ and $k = \left.\frac{\partial^2 V}{\partial x^2}\right|_{x=x_0}$ where x_0 is stable equilibrium point.

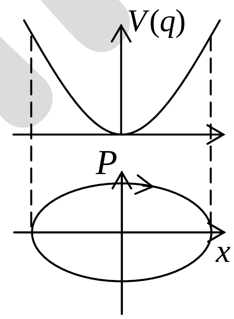
Hence $k = \frac{\partial^2 V}{\partial x^2} = \frac{2b}{x_0^3} = 2\sqrt{\frac{a^3}{b}}$.

Thus $\omega = \sqrt{2}\left(\frac{a^3}{b}\right)^{\frac{1}{4}}$.

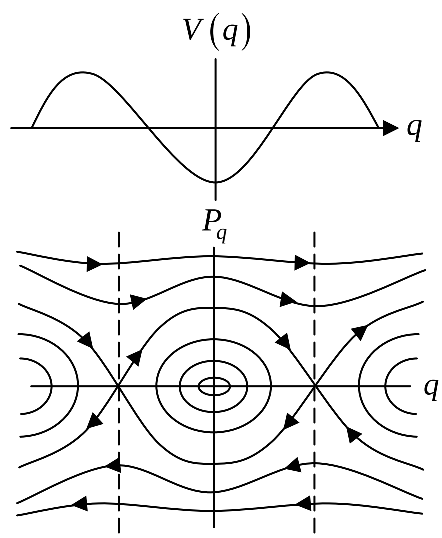
Ans. 5: (c)

Ans. 6: (d)

Solution: For harmonic oscillation, $V(q) = \frac{1}{2}kq^2$



For simple pendulum, $V(q) = -k\cos q$

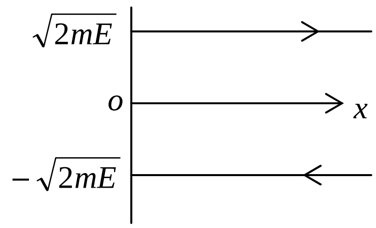


For particle in Box,

$$\frac{P_x^2}{2m} = E$$

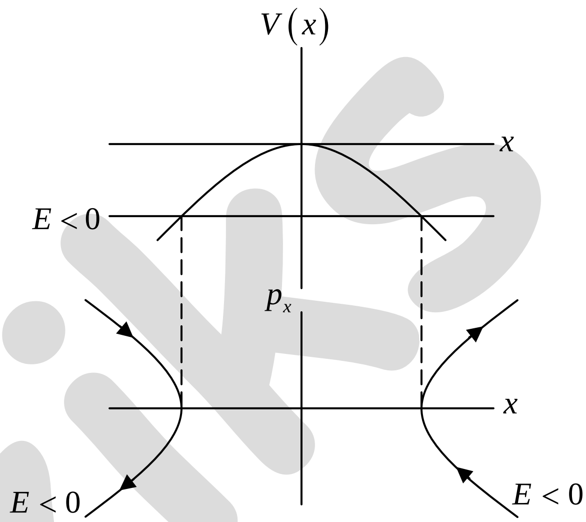
$$\Rightarrow P_x^2 = 2mE \Rightarrow P_x = \pm\sqrt{2mE}$$

II



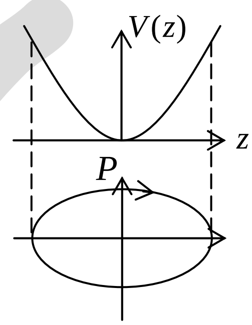
Ans. 7: (c)

Solution:



Ans. 8: (b)

Solution: $V(z) = z^4$



Ans. 9: (c)

Ans. 10: (d)

Ans. 11: (c)

Solution: $D = 3N - k = k \Rightarrow 3N = 2k$

N is integer so k must be divisible by 3.

Ans. 12: (c)

Solution: (a) Simple pendulum $r = a$ in polar co-ordinate

(b) Motion of particle on cone gives $\theta = \alpha$ in spherical co-ordinate

(c) $0 \le x \le a$

$0 \le y \le a$

$0 \le z \le a$

(d) $r = a$ in polar co-ordinate.

Ans. 13: (c)

Solution: (a) $0 \le x \le a$

$0 \le y \le a$

$0 \le z \le a$

(b) $0 \le r \le l$ in polar coordinate, where r is distance between both points, where l is length of compass.

(c) $\theta = \alpha$ in polar co-ordinate

(d) $0 < r \le l$, r is distance from pole.

Ans. 14: (b)

Solution: Equation of constraint in spherical co-ordinate is given by

For A, r_A, θ_A, ϕ_A $\quad r_A = a$

For B, r_B, θ_B, ϕ_B $\quad r_B = a$

For C, r_C, θ_C, ϕ_C $\quad r_C = a$

So, Degree of freedom $3 \times 3 - 3 = 6$

Ans. 15: (c)

Solution: For A, $N = 2,\ z_1 = 0,\ z_2 = 0$ and $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = a$

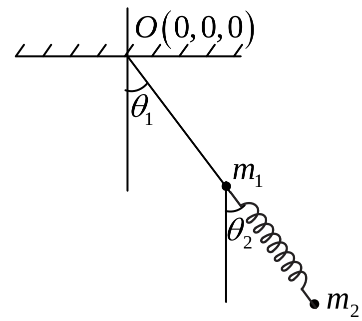
Therefore, number of equation of constraint $K = 3$

And hence, Degree of Freedom $= 3 \times 2 - 3 = 3$

For B, $N = 2,\ z_1 = 0,\ z_2 = 0 \Rightarrow K = 2$

So, Degree of Freedom $= 3 \times 2 - 2 = 4$

Ans. 16: (c)



Solution: Method I: Generalized co-ordinate for m_1 is θ_1

Generalized co-ordinate for m_2 is θ_2 and r.

So, Degree of freedom $=3$.

Method II: Number of particles $N=2$, $\phi_1=c,\ \phi_2=c,\ r=a$

Hence, D.O.F. $=3\times 2-3=3$

Ans. 17: (a)

Solution: In spherical co-ordinate, $\theta=\alpha$

In cylindrical co-ordinate, $\tan\alpha=\dfrac{r}{z}$

Ans. 18: (d)

Solution: $x=a\cos\theta \Rightarrow \dot{x}=-a\sin\theta\dot{\theta} \Rightarrow \ddot{x}=-a\sin\theta\ddot{\theta}-a\cos\theta\dot{\theta}^2$

Ans. 19: (a)

Solution: $d\vec{r}=\dfrac{\partial\vec{r}}{\partial u^1}du^1+\dfrac{\partial\vec{r}}{\partial u^2}du^2+\dfrac{\partial\vec{r}}{\partial u^3}du^3$

$\vec{r}=\left(x_1+a\cos\theta\right)\hat{i}+a\sin\theta\hat{j}+0\hat{k}$ where $u^1=x_1, u^2=\theta,$

$d\vec{r}=\dfrac{\partial\vec{r}}{\partial x_1}dx_1+\dfrac{\partial\vec{r}}{\partial\theta}d\theta \Rightarrow dx_1\hat{i}+\left(-a\sin\theta\hat{i}+a\cos\theta\hat{j}\right)d\theta \Rightarrow (dx_1-a\sin\theta d\theta)\hat{i}+a\cos\theta d\theta\hat{j}$

$\vec{v}=\dfrac{d\vec{r}}{dt}=\left(\dot{x}_1-a\dot{\theta}\sin\theta\right)\hat{i}+a\dot{\theta}\cos\theta\hat{j}$

Ans. 20: (d)

Solution: $T=\dfrac{m}{2}\left(\dot{x}^2+\dot{y}^2\right)$, $x=a\cos\theta$, $y=b\sin\theta$

$T=\dfrac{m}{2}\left(a^2\sin^2\theta+b^2\cos^2\theta\right)\dot{\theta}^2$

Ans. 21: (a)

Solution: $\because x^2-4y^2=8 \Rightarrow 2x\dfrac{dx}{dt}-8y\dfrac{dy}{dt}=0$

$\Rightarrow \quad 2xv_x-8y\ v_y=0 \Rightarrow 2\times 2\times 6-8\times 1\times v_y=0 \Rightarrow v_y=3$

$$v=\sqrt{v_x^2+v_y^2}=\sqrt{6^2+3^2}=\sqrt{36+9}=\sqrt{45}$$

Ans. 22: (c)

Solution: $x^2+y^2=a^2$

$$2x\dot{x}+2y\dot{y}=0\Rightarrow \frac{\dot{y}}{\dot{x}}=-\frac{x}{y}=-\frac{\sqrt{a^2-y^2}}{y}=-\frac{\sqrt{3}/2}{1/2}=-\sqrt{3}$$

Ans. 23: (d)

Solution: $d\vec{r}=\dfrac{\partial \vec{r}}{\partial u^1}du^1+\dfrac{\partial \vec{r}}{\partial u^2}du^2+\dfrac{\partial \vec{r}}{\partial u^3}du^3$

$$\vec{r}=\left(x_1+a\cos\theta\right)\hat{i}+a\sin\theta\,\hat{j}+0\hat{k} \text{ where } u^1=x_1, u^2=\theta,$$

$$d\vec{r}=\frac{\partial \vec{r}}{\partial x_1}dx_1+\frac{\partial \vec{r}}{\partial \theta}d\theta\Rightarrow dx_1\hat{i}+\left(-a\sin\theta\hat{i}+a\cos\theta\,\hat{j}\right)d\theta\Rightarrow (dx_1-a\sin\theta d\theta)\hat{i}+a\cos\theta d\theta\,\hat{j}$$

$$\vec{v}=\frac{d\vec{r}}{dt}=\left(\dot{x}_1-a\dot{\theta}\sin\theta\right)\hat{i}+a\dot{\theta}\cos\theta\,\hat{j}$$

$$T=\frac{1}{2}m|v|^2=\frac{1}{2}m\left(\dot{x}_1^2+a^2\dot{\theta}^2-2\dot{x}_1a\dot{\theta}\sin\theta\right)$$

Ans. 24: (b)

Solution: $T=\dfrac{1}{2}M\dot{x}_1^2+\dfrac{1}{2}m\left(\dot{x}_2^2+\dot{y}_2^2\right)$

$$x_2=x_1+l\cos\theta \qquad y_2=-l\cos\theta$$

$$\dot{x}_2=\dot{x}_1+l\cos\theta\,\dot{\theta} \qquad \dot{y}_2=l\sin\theta\,\dot{\theta}$$

$$T=\frac{1}{2}M\dot{x}_1^2+\frac{1}{2}m\left(\dot{x}_1^2+2l\dot{x}\dot{\theta}\cos\theta+l^2\dot{\theta}^2\right)$$

Ans. 25: (d)

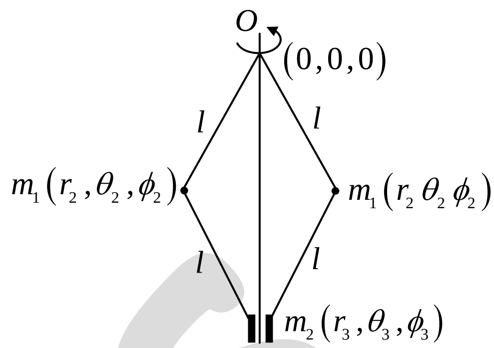
Solution: $\theta=\dfrac{\pi}{4}$ in spherical coordinate

$$\tan\frac{\pi}{4}=\frac{\sqrt{x^2+y^2}}{z} \text{ in Cartesian coordinate}$$

$z = r\cos\frac{\pi}{4} \Rightarrow z = \frac{r}{\sqrt{2}}$ spherical coordinate .

Ans. 26: (b)

Solution: No. of Particle = $N = 3$



Equation of constraint are-

$r_1 = l\,,\; r_2 = l\,,$

$\theta_1 = \theta_2 = \theta \;\; \phi_1 = \phi_2 + c\,,$

$\theta_3 = 0, \phi_3 = 0, r_3 = 2l\cos\theta \;\; 3N - k = 3.3 - 7 = 2$

Ans. 27: (b)

Solution: $y_1 = 0, z_1 = 0 \;\; z_2 = 0\,,\; \sqrt{(x_2 - x_1)^2 + (y_2 - 0)^2} = l$

$3N - k = 3.2 - 4 = 2$

Ans. 28: (c)

Solution: We choose the origin of our coordinate system to be at the center of the rotating rim.

The Cartesian components of mass m become

$x = a\cos\omega t + b\sin\theta, y = a\sin\omega t - b\cos\theta$

Ans. 29: (b)

Solution: $x = a\cos\theta_1 + b\sin\theta, y = a\sin\theta_1 - b\cos\theta$, where $\theta_1 = \omega t$, $u^1 = \theta_1, u^2 = \theta, u^3 = 0$

$$\vec{r} = (a\cos\theta_1 + b\sin\theta)\hat{i} + (a\sin\theta_1 - b\cos\theta)\hat{j}$$

$$d\vec{r} = \frac{\partial\vec{r}}{\partial u^1}du^1 + \frac{\partial\vec{r}}{\partial u^2}du^2 + \frac{\partial\vec{r}}{\partial u^3}du^3$$

$$d\vec{r} = \frac{\partial\vec{r}}{\partial\theta_1}d\theta_1 + \frac{\partial\vec{r}}{\partial\theta}d\theta \Rightarrow \left(a(-\sin\theta_1)\hat{i} + a\cos\theta_1\hat{j}\right)d\theta_1 + \left(b(\cos\theta)\hat{i} + \sin\theta\hat{j}\right)d\theta$$

$$v = \frac{d\vec{r}}{dt} = \left(a(-\sin\theta_1)\hat{i} + a\cos\theta_1\hat{j}\right)\dot{\theta}_1 + \left(b(\cos\theta)\hat{i} + \sin\theta\hat{j}\right)\dot{\theta}$$

$$\vec{v} = \left(-a\dot{\theta}_1\sin\theta_1 + b\dot{\theta}\cos\theta\right)\hat{i} + \left(a\dot{\theta}_1\cos\theta_1 + b\dot{\theta}\sin\theta\right)\hat{j}$$

Put $\theta_1 = \omega t$, $\dot{\theta}_1 = \omega$

$$\vec{v} = \left(-a\omega \sin \omega t + b\dot{\theta}\cos\theta\right)\hat{i} + \left(a\omega\cos\omega t + b\dot{\theta}\sin\theta\right)\hat{j}$$

The velocities are-

$$\left.\begin{aligned}\dot{x} &= -a\omega\sin\omega t + b\dot{\theta}\cos\theta \\ \dot{y} &= a\omega\cos\omega t + b\dot{\theta}\sin\theta\end{aligned}\right\}$$

Ans. 30: (c)

Solution: $S = \int_{x_1}^{x_2} ds$

$$S = \int \sqrt{1+y'^2}\,dx = \int f\left(y, y', x\right)dx$$

$\delta S = 0$ then

$$\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y} = 0 \Rightarrow \frac{y'}{\sqrt{1+y'^2}} = \text{constant} \Rightarrow y = mx + c$$

where m and c are constant.

Ans. 31: (c)

Solution: $I = \int_{x_1}^{x_2} 2\pi y ds$

$$I = \int_{x_1}^{x_2} 2\pi y\sqrt{dx^2 + dy^2} \Rightarrow I = \int_{x_1}^{x_2} 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx$$

$\delta I = 0$

$$f = 2\pi y\sqrt{1+y'^2} \Rightarrow \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y} = 0 \Rightarrow y = c\cos h\left(\frac{x+a}{a}\right)$$

Ans. 32: (c)

Solution: Action is given by $I = \int_{t_1}^{t_2} L\left(q_1, \dot{q}_1, t\right)dt$

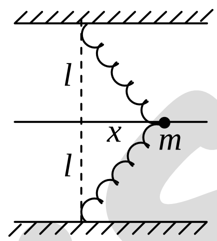
$$\delta I = \delta\int_{t_1}^{t_2} L\left(q_1, \dot{q}_1, t\right)dt = 0$$

Consequence is $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=0$ which is force on the system.

Ans. 33: (d)

Solution: The Lagrangian of system is given by $L=\frac{1}{2}m\dot{x}^2-V(x)$

The potential energy is given by



$$V(x)=\frac{k}{2}\left[\left(x^2+l^2\right)^{\frac{1}{2}}-l\right]^2+\frac{k}{2}\left[\left(x^2+l^2\right)^{\frac{1}{2}}-l\right]^2$$

$$\Rightarrow V(x)=k\left[\left(x^2+l^2\right)^{\frac{1}{2}}-l\right]^2$$

$$L=\frac{1}{2}m\dot{x}^2-k\left[\left(x^2+l^2\right)^{\frac{1}{2}}-l\right]^2$$

Ans. 34: (d)

Solution: The kinetic energy is given by, $T=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)$ potential energy is given by $V=-mgy$ with equation of constraint is $z=0$ then $\dot{z}=0$, so $T=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)$ and $V=-mgy+\frac{1}{2}k\left[\sqrt{x^2+y^2}\right]^2$ (potential energy due to gravity and stored energy due to spring), so Lagrangian in the Cartesian coordinate is

$$L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-\left(-mgy+\frac{1}{2}k\left[\sqrt{\left(x^2+y^2\right)}\right]^2\right),$$

$$L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)+mgy-\frac{1}{2}k\left[\sqrt{\left(x^2+y^2\right)}\right]^2$$

Ans. 35: (d)

Solution: $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\dot{z}^2\right)-mgz$

$$z=\frac{1}{2}br^2,\ \dot{z}=\frac{1}{2}\cdot 2br\dot{r}=br\dot{r}$$

$$L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+b^2r^2\dot{r}^2\right)-\frac{1}{2}mgbr^2$$

Ans. 36: (d)

Solution: $L=\frac{1}{2}m_1\left(\dot{r}^2+r^2\dot{\theta}^2\right)+\frac{1}{2}m_2\dot{z}^2+m_2gz$

Equation of constraint is

$z+r=l\Rightarrow z=\left(l-r\right)\Rightarrow \dot{z}=-\dot{r}$

$\therefore L=\frac{1}{2}m_1\left(\dot{r}^2+r^2\dot{\theta}^2\right)+\frac{1}{2}m_2\dot{z}^2+m_2g\left(l-r\right)$

$=\frac{1}{2}m_1\left(\dot{r}^2+r^2\dot{\theta}^2\right)+\frac{1}{2}m_2\dot{r}^2+m_2g\left(l-r\right)=\frac{1}{2}\left(m_1+m_2\right)\dot{r}^2+\frac{1}{2}m_1r^2\dot{\theta}^2+m_2g\left(l-r\right)$

Ans. 37: (c)

Solution: Because the problem has cylindrical symmetry, we choose r,θ and z as the generalized coordinates. The kinetic energy of the bead is

$$T=\frac{m}{2}\left[\dot{r}^2+\dot{z}^2+\left(r^2\dot{\theta}^2\right)\right]$$

If we choose $U=0$ at $z=0$, the potential energy term is

$$U=mgz$$

But r,z and θ are not independent. The equation of constraint for the parabola is

$$z=cr^2$$

$$\dot{z}=2c\dot{r}r$$

We also have an explicit time dependence of the angular rotation

$$\theta=\omega t\,,\dot{\theta}=\omega$$

We can now construct the Lagrangian as being dependent only on r, because there is no direct θ dependence. $L=T-U=\frac{m}{2}\left(\dot{r}^2+4c^2r^2\dot{r}^2+r^2\omega^2\right)-mgcr^2$

Ans. 38: (b)

Solution: $\frac{\partial U}{\partial \dot{r}}=\frac{2\dot{r}}{rc^2}$ and $\Rightarrow \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{r}}\right)=\frac{2\ddot{r}}{rc^2}-\frac{2\dot{r}^2}{r^2c^2}$

$$\frac{\partial U}{\partial r}=-\frac{1}{r^2}\left(1+\frac{\dot{r}^2}{c^2}\right)$$

therefore the generalized force

$$\frac{d}{dt}\left(\frac{\partial U}{\partial \dot{r}}\right)-\frac{\partial U}{\partial r}=\frac{2\ddot{r}}{rc^2}-\frac{2\dot{r}^2}{r^2c^2}+\frac{1}{r^2}+\frac{\dot{r}^2}{r^2c^2}=\frac{1}{rc^2}\left(2\ddot{r}-\frac{\dot{r}^2}{r}\right)+\frac{1}{r^2}$$

Ans. 39: (d)

Solution: $L=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\phi}^2)-mgr\cos\theta$

equation of constrain is $\theta=\frac{\pi}{4}$ and it is given $\dot{\phi}=\omega$

$$\Rightarrow L=\frac{1}{2}m(\dot{r}^2+\frac{1}{2}r^2\omega^2)-\frac{1}{\sqrt{2}}mgr$$

Ans. 40: (d)

Solution: $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\right)-mgr\cos\theta$

$r=a\Rightarrow\dot{r}=0$

$\dot{\phi}=\omega=\alpha t+\beta$

$$\Rightarrow L=\frac{1}{2}m\left[a^2\dot{\theta}^2+a^2\sin^2\theta\left(\alpha t+\beta\right)^2\right]+mga\cos\theta$$

Ans. 41: (d)

Solution: $\frac{\partial L}{\partial \dot{x}}=p_x=ma\dot{x}+mb\dot{y}$

$$\frac{\partial L}{\partial y}=p_y=mc\dot{y}+mb\dot{x}$$

Ans. 42: (c)

Solution: $\frac{\partial L}{\partial \dot{x}}=p_x$

$$m\dot{x}e^{vt}=p_x\Rightarrow\dot{x}=\frac{p_x}{m}e^{-vt}\Rightarrow x=-\frac{p_x}{m}\frac{e^{-vt}}{v}+c$$

Ans. 43: (a)

Solution: $\frac{\partial L}{\partial \dot{q}} = \frac{\alpha \dot{q}}{2} = p_q$ and $\frac{\partial L}{\partial q} \neq 0$, so $p_q = \frac{\alpha \dot{q}}{2}$ and it is not a conserved quantity.

Ans. 44: (d)

Solution: $y = ax^2 \Rightarrow \dot{y} = 2ax\dot{x}$

$$L = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) - mgy = \frac{m}{2}\left(\dot{x}^2 + 4a^2x^2\dot{x}^2\right) - mgax^2$$

$$L = \frac{m}{2}\left(1 + 4a^2x^2\right)\dot{x}^2 - mgax^2$$

Ans. 45: (d)

Solution: $\frac{\partial L}{\partial r} \neq 0, \frac{\partial L}{\partial \theta} = 0, \frac{\partial L}{\partial r} \neq 0$

So, no external force in θ direction and i.e. angular momentum is constant. Now from Lagrangian equation, $mr\dot{\theta}^2$ is pseudo force, which is mainly centrifugal force.

Ans. 46: (c)

Solution: $\frac{\partial L}{\partial \theta} = 0, \frac{\partial L}{\partial \dot{\theta}} = I\dot{\theta} + mr^2\dot{\theta} = p_\theta$

$\frac{\partial L}{\partial r} \neq 0, \frac{\partial L}{\partial \dot{r}} = m\dot{r}$, since r is not cyclic co-ordinate, so p_r is not a constant of motion.

Ans. 47: (c)

Solution: $\left(\frac{\partial L}{\partial \theta}\right) = 0$ so p_θ is conserved.

Ans. 48: (d)

Solution: $L = \frac{1}{2}m(\dot{r}^2 + r^2\sin^2\alpha\dot{\phi}^2)$

$$P_\phi = mr^2\sin^2\alpha\dot{\phi} = \text{constant}$$

$$\dot{\phi} = \frac{P_\phi}{mr^2\sin^2\alpha}$$

$r \to 0 \Rightarrow \dot{\phi} \to \infty$ which is impossible

$r \to \infty \Rightarrow \dot{\phi} \to 0$ which is also not possible (we want some finite angular velocity)

for $r_{\min} < r < r_{\max}$

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$

$\tan\theta = \dfrac{\sqrt{x^2 + y^2}}{z}$. Here since the particle is constrained to move on the cone, the angle $\theta = \alpha$.

$r = z\tan\alpha$

$z_{\min} < z < z_{\max}$

Ans. 49: (c)

Solution: $\dfrac{\partial L}{\partial \phi} = 0$, so $\dfrac{\partial L}{\partial \dot{\phi}}$ is conserved

Ans. 50: (b)

Solution: Two generalized coordinate so, $DOF = 2$

$$\frac{\partial L}{\partial \theta} \neq 0 \qquad \frac{\partial L}{\partial x} = 0$$

So, angular momentum is not conserved and linear momentum and total energy of system is conserved.

Ans. 51: (c)

Solution: $\dfrac{\partial L}{\partial \dot{x}} = m\dot{x} = p_x$ $\dfrac{\partial L}{\partial x} = +K_1 + K_2 y$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$m\ddot{x} - K_1 - K_2 y = 0$$

$$\frac{\partial L}{\partial \dot{y}} = m\dot{y} = p_y, \ \frac{\partial L}{\partial y} = -K_1 + K_2 x$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = m\ddot{y} + K_1 - K_2 x = 0$$

Ans. 52: (c)

Solution: $\because L = e^{-\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right] \Rightarrow \frac{\partial L}{\partial \dot{x}} = e^{-\gamma t} m\dot{x}$ and $\frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x}e^{-\gamma t}$

$$\because \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \Rightarrow \frac{d}{dt}\left(e^{-\gamma t} m\dot{x}\right) + \frac{\partial V}{\partial x}e^{-\gamma t} = m\ddot{x}e^{-\gamma t} - m\dot{x}\gamma e^{-\gamma t} + \frac{\partial V}{\partial x}e^{-\gamma t} = 0$$

$$\left(m\ddot{x} - m\gamma\dot{x} + \frac{\partial V}{\partial x}\right)e^{-\gamma t} = 0 \Rightarrow m\ddot{x} - \gamma m\dot{x} + \frac{\partial V}{\partial x} = 0$$

Ans. 53: (d)

Solution: $\left(\frac{\partial L}{\partial \dot{q}}\right) = m\dot{q} + \lambda q\dot{q}$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = m\ddot{q} + \lambda q\ddot{q} + \lambda\dot{q}^2$ and

$$\left(\frac{\partial L}{\partial q}\right) = \frac{\lambda}{2}\dot{q}^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0 \Rightarrow m\ddot{q} + \lambda q\ddot{q} + \lambda\dot{q}^2 - \frac{\lambda}{2}\dot{q}^2 = 0$$

$$m\ddot{q} + \lambda q\ddot{q} + \frac{\lambda}{2}\dot{q}^2 = 0 \Rightarrow \ddot{q}(m + \lambda q) + \frac{\lambda}{2}\dot{q}^2$$

Ans. 54: (c)

Solution: $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{1}{2}Kr^2 + mgr\cos\theta$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r}, \frac{\partial L}{\partial r} = mr\dot{\theta}^2 - Kr + mg\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0 \Rightarrow m\ddot{r} - mr\dot{\theta}^2 + Kr - mg\cos\theta = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}, \frac{\partial L}{\partial \theta} = -mgr\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\Rightarrow mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} + mgr\sin\theta = 0$$

Ans. 55: (d)

Solution: Hamilton's principle is given by $\delta I = \delta \int_{t_1}^{t_2} L dt = 0,\ H = p\dot{q} - L$

$$\delta I = \delta \int_{t_1}^{t_1} p\dot{q} - H = 0$$

Ans. 56: (c)

Solution: $\frac{\partial L}{\partial q} = 0$ explains conservation of momentum.

$\frac{\partial H}{\partial q} = 0$ explains conservation of momentum

but conservation of energy can be explained by

$\frac{\partial H}{\partial t} = 0$, so (c) is correct answer.

Ans. 57: (d)

Solution: $L = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} kx^2$ and $H\left(x, p_x, t\right) = \dot{x}p_x - L$

$$p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x} \Rightarrow H = \frac{p_x^2}{2m} + \frac{1}{2} kx^{2.}$$

Ans. 58: (c)

Solution: $L = \frac{a\dot{x}^2}{4} + \frac{b\dot{y}^2}{4} - kxy \Rightarrow \frac{\partial L}{\partial \dot{x}} = \frac{2\dot{x}a}{4} = p_x \Rightarrow \dot{x} = \frac{2p_x}{a}$

$$\frac{\partial L}{\partial \dot{y}} = \frac{2\dot{y}b}{4} = p_y \Rightarrow \dot{y} = \frac{2p_y}{b}$$

$$H = p_x\dot{x} + p_y\dot{y} - L \Rightarrow H = p_x\dot{x} + p_y\dot{y} - \frac{1}{4} a\dot{x}^2 - \frac{1}{4} b\dot{y}^2 + kxy$$

put $\dot{x} = \frac{2p_x}{a}$ and $\dot{y} = \frac{2p_y}{b}$

$$H = \frac{2p_x^2}{a} + \frac{2p_y^2}{b} - \frac{1}{4}\frac{4b_x^2}{a} - \frac{1}{4}\frac{4b_y^2}{b} + kxy = \frac{p_x^2}{a} + \frac{p_y^2}{b} + kxy = \frac{bp_x^2 + ap_y^2}{ab} + kxy$$

Ans. 59: (d)

Solution: $L = e^{vt}\left(\frac{m\dot{x}^2}{2} - \frac{kx^2}{2}\right)$

$$\frac{\partial L}{\partial \dot{x}} = p_x,\ m\dot{x}e^{vt} = p_x \Rightarrow \dot{x} = \frac{p_x}{m}e^{-vt}$$

$$H = \dot{x}p_x - L = \dot{x}p_x - e^{vt}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right)$$

$$\frac{e^{-vt}p_x^2}{m} - e^{vt}e^{-2vt}\frac{p_x^2}{2m} + e^{vt}\frac{1}{2}kx^2 = \frac{e^{-vt}p_x^2}{m} - e^{-vt}\frac{p_x^2}{2m} + e^{vt}\frac{1}{2}kx^2$$

$$H = e^{-vt}\frac{p_x^2}{2m} + e^{vt}\frac{kx^2}{2}$$

Ans. 60: (b)

Solution: The Lagrangian of the particle is $L(\theta,\phi) = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) - mga\cos\theta$

$$\frac{\partial L}{\partial \dot{\theta}} = p_\theta = ma^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{p_\theta}{ma^2},\ \frac{\partial L}{\partial \dot{\phi}} = p_\phi = ma^2\sin^2\dot{\phi} \Rightarrow \dot{\phi} = \frac{p_\theta}{ma^2\sin^2\theta}$$

Put value of $\dot{\theta}$ and $\dot{\phi}$ in

$$H = p_\theta\dot{\theta} + p_\phi\dot{\phi} - L \Rightarrow p_\theta\dot{\theta} + p_\phi\dot{\phi} - \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) + mga\cos\theta$$

So Hamiltonian is given by $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + mga\cos\theta$

Ans. 61: (c)

Solution: $H = p\dot{q} - L \Rightarrow p\dot{q} - \sqrt{1-\dot{q}^2}$

$$\frac{\partial L}{\partial \dot{q}} = p = -\frac{\dot{q}}{\sqrt{1-\dot{q}^2}} \Rightarrow p^2 = \frac{\dot{q}^2}{1-\dot{q}^2} \Rightarrow p^2 - p^2\dot{q}^2 = \dot{q}^2 \Rightarrow \dot{q} = \frac{p}{\sqrt{1+p^2}}$$

$$H = p.\frac{p}{\sqrt{1+p^2}} - \sqrt{1-\frac{p^2}{1+p^2}} \Rightarrow \frac{p^2}{\sqrt{1+p^2}} - \frac{1}{\sqrt{1+p^2}} = \frac{p^2-1}{\sqrt{1+p^2}}$$

Ans. 62: (d)

Solution: $L = p\dot{x} - H$ and $\dfrac{\partial H}{\partial p} = \dot{x}$

$$P = m\dot{x}e^{\nu t} \Rightarrow L = \frac{1}{2}m(\dot{x}^2 - \omega^2 x^2)e^{\nu t}$$

Ans. 63: (d)

Solution: (a) $\dfrac{\partial H}{\partial r} = -\dot{p}_r \Rightarrow \dfrac{-p_\theta^2}{mr^3} = -\dot{p}_r \Rightarrow \dot{p}_r = \dfrac{p_\theta^2}{mr^3}$

$$\frac{\partial H}{\partial \theta} = \dot{p}_\theta \Rightarrow \dot{p}_\theta = 0 \Rightarrow p_\theta = \text{constant}$$

(b) $\dfrac{\partial H}{\partial z} = -\dot{p}_z \Rightarrow -mg = -\dot{p}_z \Rightarrow \dot{p}_z = mg$

$\dfrac{\partial H}{\partial p_z} = \dot{z} \Rightarrow p_z = m\dot{z} \Rightarrow \dot{p}_z = m\ddot{z}$. Using $\dot{p}_z = mg$ gives, $mg = m\ddot{z} \Rightarrow \ddot{z} = g$

(c) $\dfrac{\partial H}{\partial P_\theta} = \dot{\theta} \Rightarrow \dfrac{p_\theta}{mr^2} = \dot{\theta} \Rightarrow p_\theta = mr^2\dot{\theta}$

(d) $\dfrac{\partial H}{\partial \theta} = -\dot{p}_\theta$, since $\dfrac{\partial H}{\partial \theta} = 0$, $\dot{p}_\theta = 0$, this gives

$$p_\theta = \text{constant} = mr^2\dot{\theta} \Rightarrow mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} = 0$$

Ans. 64: (b)

Solution: $\dfrac{\partial H}{\partial t} = -\dfrac{\partial L}{\partial t} = -\dfrac{\partial (T-V)}{\partial t} = -\dfrac{\partial T}{\partial t} + \dfrac{\partial V}{\partial t}$

Ans. 65: (d)

Solution: $L = \dfrac{1}{2}m\dot{x}^2 - \dfrac{k}{2}(x - V_0 t)^2$

$$H = \frac{P_x^2}{2m} + \frac{k}{2}(x - V_0 t)^2$$

$\dfrac{\partial H}{\partial t} \neq 0$, $\dfrac{\partial H}{\partial x} \neq 0$. So neither energy nor momentum is conserved.

Ans. 66: (d)

Solution: $\frac{\partial L}{\partial \theta} \neq 0$ and $\frac{\partial L}{\partial t} \neq 0 \Rightarrow \frac{\partial H}{\partial t} \neq 0$, so energy is not conserved.

Ans. 67: (c)

Solution: $\frac{df}{dt} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial t} + \frac{\partial f}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial f}{\partial t} = \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial H}{\partial q} + \frac{\partial f}{\partial t} \Rightarrow \frac{df}{dt} = [f, H] + \frac{\partial f}{\partial t}$

Ans. 68: (b)

Solution: $\frac{dJ}{dt} = [J, H]$, $\Rightarrow$ Torque, $\tau = [J, H]$

Ans. 69: (c)

Solution: $\frac{\partial H}{\partial q} = -\dot{p}$

Ans. 70: (c)

Solution: $\frac{dF}{dt} = [H, F] + \frac{\partial F}{\partial t} = \frac{P}{m} - \frac{P}{m} = 0 \Rightarrow F =$ constant of motion

Ans. 71: (c)

Solution: $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{V}{2}(x^2 + y^2) - W \sin \omega t$, p_z is z component of momentum

$$\frac{dp_z}{dt} = [p_z, H] + \frac{\partial p_z}{\partial t} \Rightarrow \frac{dp_z}{dt} = 0 \Rightarrow p_z = \text{constant}$$

$J_z = xp_y - yp_x$, where J_z is angular momentum in z direction

$$\left[xp_y - yp_x, \frac{p_x^2}{2m} + \frac{V}{2}x^2\right] + \left[xp_y - yp_x, \frac{p_y^2}{2m} + \frac{Vy^2}{2}\right]$$

$$= [J_z, H_x] + [J_z, H_y]$$

$$= -\frac{p_x p_y}{m} - Vxy + \frac{p_x p_y}{m} + Vxy = 0 \Rightarrow [J_z, H] = 0 \Rightarrow \frac{dJ_z}{dt} = 0 \Rightarrow J_z = \text{constant}$$

$\frac{dH}{dt} = [H, H] + \frac{\partial H}{\partial t} = -W\omega \cos \omega t$, so energy is not constant.

Ans. 72: (a)

Solution: $\frac{dL}{dt}=[L,H]+\frac{\partial L}{\partial t}$, where $L=\frac{p_\theta^2}{2ml^2}-mgl(1-\cos\theta)$. (Wrong Question & Solution)

$$H=\sum_i p_i\dot{q}_i-L=p_\theta\dot{\theta}-L,\ p_\theta=\frac{\partial L}{\partial\dot{\theta}}=ml^2\dot{\theta}\Rightarrow H=\frac{p_\theta^{\ 2}}{2ml^2}+mgl(1-\cos\theta).$$

Hence we have to calculate $[L,H]$ which is only defined into phase space i.e. p_θ and θ.

Then $L=\frac{p_\theta^2}{2ml^2}-mgl(1-\cos\theta)$

Ans. 73: (d)

Solution: We know that $\left[q_i,p_j\right]=\delta_{i,j}$

$$\left[p_x,J_z\right]=\left[p_x,xp_y-yp_x\right]\Rightarrow\left[p_x,xp_y\right]-\left[p_x,yp_x\right]\Rightarrow\left[p_x,x\right]p_y=-p_y$$

$$\left[y,J_z\right]=\left[y,xp_y-yp_x\right]\Rightarrow\left[y,xp_y\right]-\left[y,yp_x\right]\Rightarrow x\left[y,p_y\right]=x$$

$$\left[z,J_z\right]=\left[z,xp_y-yp_x\right]=0$$

Ans. 74: (b)

Solution: $[xP_y-yP_x,P_x]=P_y$

Ans. 75: (b)

Solution: Poisson Bracket follows Jacobi Identity

$$\left[f,[g,h]\right]+\left[g,[h,f]\right]+\left[h,[f,g]\right]=0$$

$$[f,0]+[g,0]+\left[h[f,g]\right]=0$$

$$\left[h,[f,g]\right]=0$$

Ans. 76: (c)

Solution: $[Q,p]=\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}\cdot\frac{\partial p}{\partial q}=1$

$$\frac{\partial(Q,p)}{\partial q,p}=\begin{vmatrix}\frac{\partial Q}{\partial q} & \frac{\partial Q}{p}\\ \frac{\partial p}{\partial q} & \frac{\partial P}{\partial p}\end{vmatrix}=\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial P}\cdot\frac{\partial P}{\partial q}=[Q,P]=1$$

So, both statements are correct.

Ans. 77: (b)

Solution: For (a) $[Q,P]=\frac{\partial Q}{\partial q}.\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}.\frac{\partial P}{\partial q}\Rightarrow 0-(-1.1)=1$, so it is a canonical transformation

(b) $[Q,P]=\frac{\partial Q}{\partial q}.\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}.\frac{\partial P}{\partial q}\Rightarrow -p\sin q.\sin q-p\cos q\cos q=-p$, so it is not a canonical transformation.

(c) $[Q,P]=\frac{\partial Q}{\partial q}.\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}.\frac{\partial P}{\partial q}\Rightarrow \frac{1}{q^2}.q^2-0=1$, so it is a canonical transformation

(d) $[Q,P]=\frac{\partial Q}{\partial q}.\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}.\frac{\partial P}{\partial q}\Rightarrow \sqrt{2p}\cos q.\frac{\sqrt{2}}{\sqrt{p}}\cos q-\frac{\sqrt{2}}{\sqrt{p}}(-\sin q).\sqrt{2p}\sin=1$, so it is a canonical transformation

Ans. 78: (a)

Solution: $\frac{\partial Q}{\partial q}.\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}.\frac{\partial P}{\partial q}=1\Rightarrow -2i\alpha=1\Rightarrow \alpha=-\frac{1}{2i}=\frac{i}{2}$

Canonical Transformation from generating function

Ans. 79: (b)

Solution: $F_1=qQ\Rightarrow\frac{\partial F_1}{\partial Q}=-P\Rightarrow q=-P\ \ \frac{\partial F_1}{\partial q}=p=Q$, so F is inverse transformation

$F_2=qP\Rightarrow\frac{\partial F_2}{\partial q}=p=P,\frac{\partial F_2}{\partial P}=Q=q\rightarrow$ identity transformation

Ans. 80: (b)

Solution: The generating function F, for this transformation is of the 3rd kind, $F_3=F_3(Q,p,t)$

To find F explicitly, use the equation for its derivative (from the table , $P=-\frac{\partial F_3}{\partial Q}$ and substitute the expression for P from equation expressed in terms of p and Q $\Rightarrow \frac{\partial F_3}{\partial Q} = -\frac{p}{Q^2}$. Integrating this with respect to Q results in an equation for the generating function of the transformation given by equation : $F_3 = \frac{p}{Q} + f_1(p)$ Similarly

$$-q = \frac{\partial F_3}{\partial p} \qquad \Rightarrow \frac{\partial F_3}{\partial p} = \frac{1}{Q} \Rightarrow F_3 = \frac{p}{Q} + f_2(Q) \qquad F_3 = \frac{p}{Q} + f_1(p) = \frac{p}{Q} + f_2(Q)$$

$\Rightarrow f_1(p) \Rightarrow f_2(Q) = 0$. Hence $F_3 = \frac{p}{Q}$

Ans. 81: (a)

Solution: $F_2 = Pq - \frac{1}{2}P^2 t \Rightarrow \frac{\partial F_2}{\partial q} = p \Rightarrow p = P$

$$\frac{\partial F_2}{\partial P} = Q \Rightarrow (q - pt) = Q \Rightarrow q = Q + Pt$$

$$\frac{\partial F_2}{\partial t} = -\frac{P^2}{2}$$

$$K = H + \frac{\partial F_2}{\partial t} = \frac{P^2}{2} + \frac{(Q+Pt)^2}{2} - \frac{P^2}{2} = \frac{(Q+Pt)^2}{2}$$

Ans. 82: (c)

Solution: At $x = x_0$ potential energy is minimum. Since total energy is constant therefore, K. E. is maximum. So momentum is maximum.

From Taylor's expansion $V(x)$ can be approximated as

$$V(x) = V(x_0) + (x - x_0)\frac{\partial V}{\partial x}\bigg| + \frac{(x-x_0)^2}{2}\frac{\partial^2 V}{\partial x^2}\bigg|_{x=0} + \ldots\ldots\ldots\ldots$$

$(x - x_0)^3 = (x - x_0)^4 = \ldots\ldots\ldots$ is very small.

So it can be zero.

$$\therefore V(x) = V(x_0) + (x - x_0)\frac{\partial V}{\partial x}\bigg|_{x=x_0} + (x - x_0)^2 \frac{1}{2}\frac{\partial^2 V}{\partial x^2}\bigg|_{x=0}$$

Since, $V(x)$ is minimum i.e., $\left.\frac{\partial V}{\partial x}\right| = 0$

So, force $= 0$

Hence, $V(x) = V(x_0) + (x - x_0)^2 \frac{1}{2}\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=0}$, so $k = \left.\frac{\partial^2 V}{\partial x^2}\right|_{x=x_0}$

Ans. 83: (c)

Solution: $L = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \omega^2 \sin^2\theta\right) + mga\cos\theta$

$$\frac{\partial L}{\partial \dot{\theta}} = ma^2\dot{\theta}, \qquad \frac{\partial L}{\partial \theta} = ma^2\omega^2 \sin\theta\cos\theta - mga\sin\theta$$

From Lagrangian equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$ma^2\ddot{\theta} - ma^2\omega^2\sin\theta\cos\theta + mga\sin\theta = 0$$

For small oscillation $\theta \to 0$

$$\sin\theta \to \theta$$

$$\cos\theta \to 1$$

$$ma^2\ddot{\theta} - ma^2\omega^2\theta + mga\theta = 0$$

$$\ddot{\theta} - \omega^2\theta + \frac{g}{a}\theta = 0$$

$$\ddot{\theta} + \theta\left(\frac{g}{a} - \omega^2\right) = 0$$

If $\frac{g}{a} > \omega^2$ then motion is S.H.M. so (c) is correct.

Ans. 84: (b)

Solution: $L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}k_1 x\dot{x} - \frac{1}{2}k_2 x^2$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} + k_1 x \Rightarrow \frac{\partial L}{\partial x} = k_1\dot{x} - k_2 x$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$$

$$m\ddot{x}+k_1\dot{x}-k_1\dot{x}+k_2x=0 \Rightarrow m\ddot{x}+k_2x=0$$

$$\omega=\sqrt{\frac{k_2}{m}}$$

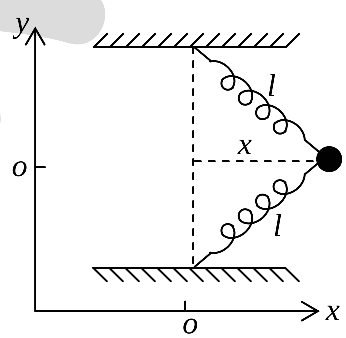
Ans. 85: (b)

Solution: Lagrangian of system is given by $L=\frac{1}{2}m\dot{x}^2-V(x)$

If l is natural length of spring then $V(x)$ is potential energy of system is

$$V(x)=\frac{k}{2}\left[\left(x^2+l^2\right)^{\frac{1}{2}}-l\right]^2+\frac{k}{2}\left[\left(x^2+l^2\right)^{\frac{1}{2}}-l\right]^2$$

$$V(x)=k\left[\left(x^2+l^2\right)^{\frac{1}{2}}-l\right]^2$$



So, Lagrangian is given by $L=\frac{1}{2}m\dot{x}^2-k\left[\left(x^2+l^2\right)^{\frac{1}{2}}-l\right]^2$

(b) For small oscillation one can approximate potential by Taylor expansion

$$V(x)=kl^2\left[\left(1+\frac{x^2}{l^2}\right)^{\frac{1}{2}}-1\right]^2 \Rightarrow V(x)=kl^2\left[\left(1+\frac{1}{2}\frac{x^2}{l^2}-\frac{1}{8}\frac{x^4}{l^4}\right)-1\right]^2$$

$$V(x)=kl^2\left(\frac{x^2}{l^2}\right)^2 \Rightarrow V(x)=k\left(\frac{x^4}{4l^2}\right).$$

So, Lagrangian of system is given by $L=\frac{1}{2}m\dot{x}^2-k\left(\frac{x^4}{4l^2}\right)$

Ans. 86: (d)

Solution: Problem based on T and V matrix:

$\theta_i = c_i e^{i\omega t}$

$$\begin{vmatrix} 2a - b\omega^2 & -a & -a \\ -a & 2a - b\omega^2 & -a \\ -a & -a & 2a - b\omega^2 \end{vmatrix} = 0$$

$\omega_1 = 0,\ \omega_2 = \sqrt{\frac{3a}{b}}$ and $\omega_3 = \sqrt{\frac{3a}{b}}$

Ans. 87: (c)

Solution: V matrix must be symmetric, so $V_{ij} = V_{ji}$. Hence (c) is the correct answer

$$a = V_{12} = V_{21} = -\frac{k}{2},\ b = V_{23} = V_{32} = -\frac{k}{2},\ c = V_{31} = V_{13} = 0,\ d = V_{42} = V_{24} = 0$$

Ans. 88: (c)

Solution: $V = k\left[e^{-(\theta_2-\theta_1)} + e^{-(\theta_3-\theta_2)} + e^{-(\theta_1-\theta_3)}\right]$

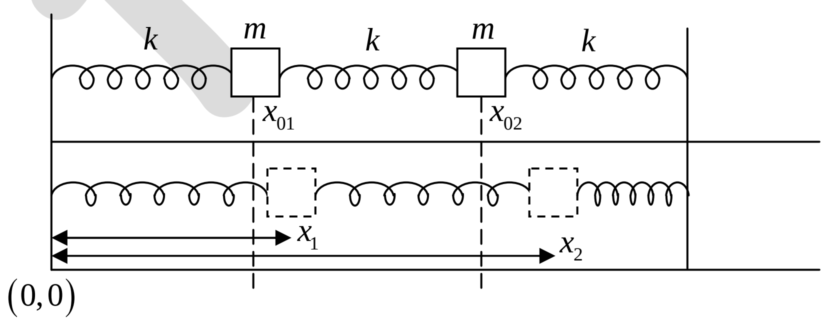
$$= k\left[3 - (\theta_2-\theta_1) - (\theta_3-\theta_2) - (\theta_1-\theta_3) + \frac{1}{2}(\theta_2-\theta_1)^2 + \frac{1}{2}(\theta_3-\theta_2)^2 \frac{1}{2}(\theta_1-\theta_3)^2\right]$$

$$= k\left[3 + \theta_1^2 + \theta_2^2 + \theta_3^2 - \theta_1\theta_2 - \theta_2\theta_3 - \theta_3\theta_1\right]$$

$$V = \begin{pmatrix} 2k & -k & -k \\ -k & 2k & -k \\ -k & -k & 2k \end{pmatrix}$$

Ans. 89: (b)

Solution:



$$V(x) = \frac{1}{2}k(x_1 - x_{01})^2 + \frac{1}{2}k\left((x_2 - x_1) - (x_{02} - x_{01})\right)^2 + \frac{1}{2}k(x_2 - x_{02})^2$$

$$V(x)=\frac{1}{2}k(x_1-x_{01})^2+\frac{1}{2}k\left((x_2-x_{02})-(x_1-x_{01})\right)^2+\frac{1}{2}k(x_2-x_{02})^2$$

It is given $x_1-x_{01}=\eta_1$, $x_2-x_{02}=\eta_2$

$$V(x)=\frac{1}{2}k(\eta_1)^2+\frac{1}{2}k(\eta_2-\eta_1)^2+\frac{1}{2}k(\eta_2)^2$$

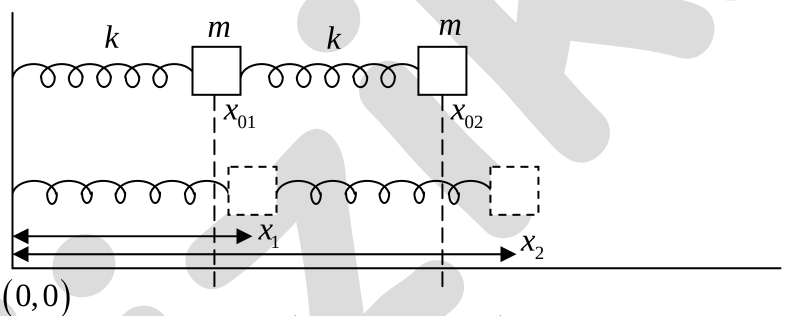
$$V(x)=\frac{1}{2}k\left(2\eta_1^2+2\eta_2^2-2\eta_1\eta_2\right)\Rightarrow\frac{1}{2}k\left(2\eta_1^2+2\eta_2^2-\eta_1\eta_2-\eta_2\eta_1\right)$$

The matrix element $V_{ij}=\dfrac{\partial^2V}{\partial\eta_i\partial\eta_j}$

$$V_{ij}=\begin{bmatrix}2k & -k\\ -k & 2k\end{bmatrix}$$

Ans. 90: (a)

Solution:



The potential energy is given by $V=\frac{1}{2}k(x_1-x_{01})^2+\frac{1}{2}k\left((x_2-x_1)-(x_{02}-x_{01})\right)^2$

It is given $x_1-x_{01}=\eta_1$, $x_2-x_{02}=\eta_2$

$$V=\frac{1}{2}k(\eta_1)^2+\frac{1}{2}k(\eta_2-\eta_1)^2$$

$$V=\frac{1}{2}k(\eta_1)^2+\frac{1}{2}k\left(\eta_1^2+\eta_2^2-2\eta_1\eta_2\right)\Rightarrow V=\frac{1}{2}k\left(2\eta_1^2+\eta_2^2-2\eta_1\eta_2\right)$$

$$\Rightarrow V=\frac{1}{2}k\left(2\eta_1^2+\eta_2^2-\eta_1\eta_2-\eta_2\eta_1\right)$$

$$V_{ij}=\frac{\partial^2V}{\partial\eta_i\partial\eta_j}\Rightarrow V_{ij}=\begin{bmatrix}2k & -k\\ -k & k\end{bmatrix}$$

Ans. 91: (b)

Solution: If $\omega_i = \sqrt{\frac{k}{m}}$ is Normal frequency and u_i is normal mode $A\begin{pmatrix}1\\0\\-1\end{pmatrix}$, then $x_i^T T x_i = 1$

$$A = \frac{1}{\sqrt{2m}} \text{ so } u_i = \frac{1}{\sqrt{2m}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$$

Problem based on Taylor expansion

Ans. 92: (d)

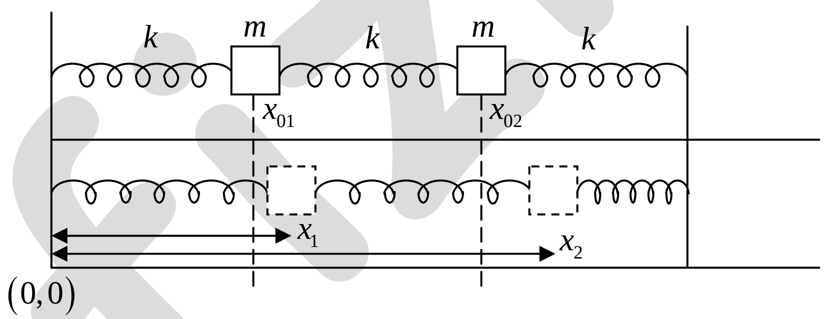
Solution: Put $x = x_0 e^{i\omega t}$ $\quad y = y_0 e^{i\omega t}$

$$\begin{pmatrix} 2g-\omega^2 & -g \\ -g & 2g-\omega^2 \end{pmatrix} = (g-\omega^2)(3g-\omega^2) = 0$$

$$\omega = \sqrt{g} \qquad \omega = \sqrt{3g}$$

Ans. 93: (b)

Solution:



$$V(x) = \frac{1}{2}k(x_1 - x_{01})^2 + \frac{1}{2}k\left((x_2 - x_1) - (x_{02} - x_{01})\right)^2 + \frac{1}{2}k(x_2 - x_{02})^2$$

$$V(x) = \frac{1}{2}k(x_1 - x_{01})^2 + \frac{1}{2}k\left((x_2 - x_{02}) - (x_1 - x_{01})\right)^2 + \frac{1}{2}k(x_2 - x_{02})^2$$

It is given $x_1 - x_{01} = \eta_1$, $x_2 - x_{02} = \eta_2$

$$V(x) = \frac{1}{2}k(\eta_1)^2 + \frac{1}{2}k(\eta_2 - \eta_1)^2 + \frac{1}{2}k(\eta_2)^2$$

$$V(x)=\frac{1}{2}k\left(2\eta_1^2+2\eta_2^2-2\eta_1\eta_2\right)\Rightarrow\frac{1}{2}k\left(2\eta_1^2+2\eta_2^2-\eta_1\eta_2-\eta_2\eta_1\right)$$

The matrix element $V_{ij}=\dfrac{\partial^2 V}{\partial\eta_i\partial\eta_j}\Rightarrow V_{ij}=\begin{bmatrix}2k & -k\\ -k & 2k\end{bmatrix}$

The kinetic energy is given by $T=\dfrac{1}{2}m\dot{x}_1^2+\dfrac{1}{2}m\dot{x}_2^2\Rightarrow\dfrac{1}{2}m\dot{\eta}_1^2+\dfrac{1}{2}m\dot{\eta}_2^2$

The kinetic energy matrix is given by $T=\begin{bmatrix}m & 0\\ 0 & m\end{bmatrix}$

Secular equation is given by $\left|V-\omega^2 T\right|=0$

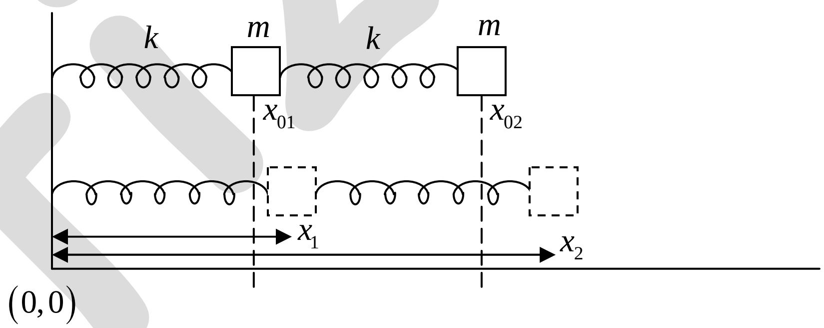
$$\begin{vmatrix}2k-\omega^2 m & -k\\ -k & 2k-\omega^2 m\end{vmatrix}=0$$

$$\left(2k-\omega^2 m\right)^2-k^2=0\Rightarrow\left(2k-\omega^2 m+k\right)\left(2k-\omega^2 m-k\right)=0$$

$$\omega=\sqrt{\frac{k}{m}},\sqrt{\frac{3k}{m}}$$

Ans. 94: (d)

Solution:



The potential energy is given by $V=\dfrac{1}{2}k\left(x_1-x_{01}\right)^2+\dfrac{1}{2}k\left(\left(x_2-x_1\right)-\left(x_{02}-x_{01}\right)\right)^2$

It is given $x_1-x_{01}=\eta_1$, $x_2-x_{02}=\eta_2$

$$V=\frac{1}{2}k\left(\eta_1\right)^2+\frac{1}{2}k\left(\eta_2-\eta_1\right)^2$$

$$V=\frac{1}{2}k(\eta_1)^2+\frac{1}{2}k\left(\eta_1^2+\eta_2^2-2\eta_1\eta_2\right)\Rightarrow V=\frac{1}{2}k\left(2\eta_1^2+\eta_2^2-2\eta_1\eta_2\right)\Rightarrow V=\frac{1}{2}k\left(2\eta_1^2+\eta_2^2-\eta_1\eta_2-\eta_2\eta_1\right)$$

$$V_{ij}=\frac{\partial^2V}{\partial\eta_i\partial\eta_j}\Rightarrow V_{ij}=\begin{bmatrix}2k & -k\\ -k & k\end{bmatrix}$$

The kinetic energy is given by $T=\frac{1}{2}m\dot{x}_1^2+\frac{1}{2}m\dot{x}_2^2\Rightarrow\frac{1}{2}m\dot{\eta}_1^2+\frac{1}{2}m\dot{\eta}_2^2$

The kinetic energy matrix is given by $T=\begin{bmatrix}m & 0\\ 0 & m\end{bmatrix}$

Secular equation is given by $\left|V-\omega^2T\right|=0$

$$\begin{vmatrix}2k-\omega^2m & -k\\ -k & k-\omega^2m\end{vmatrix}=0$$

$$\left(2k-\omega^2m\right)\left(k-\omega^2m\right)-k^2=0\Rightarrow\omega^4m^2-3\omega^2mk+\omega^4m^2=0$$

$$\omega=\sqrt{\frac{k}{m}}\left(\frac{3-\sqrt{5}}{2}\right)^{1/2},\sqrt{\frac{k}{m}}\left(\frac{3+\sqrt{5}}{2}\right)^{1/2}$$

Ans. 95: (b)

Solution: closest from the sun $=r_1$, speed $=v_1$

Farthest from the sun $=r_2$

According to conservation of momentum $mr_1v_1=mr_2v_2$

$$v_2=\frac{r_1v_1}{r_2}$$

Ans. 96: (c)

Solution: If reduce mass $\mu=\frac{mM}{m+M}$, then $\frac{1\mu v^2}{2}=0-\left(-\frac{GMm}{d}\right)\Rightarrow v^2=\frac{2G(M+m)}{d}$

(acceleration due to gravity upward from the earth)

$$v=\sqrt{\frac{2G(M+m)}{d}}$$

Ans. 97: (a)

Solution: For circular motion,

$$\frac{mv^2}{r}=\frac{k}{r^2}\Rightarrow K=\frac{mv^2}{2}=\frac{-k}{r^2},\ E=K+U\text{, where } U=-\frac{k}{r} \text{ and } E=-\frac{k}{2r}$$

Ans. 98: (d)

Solution: I. Is wrong because central force must be function of r i.e., $V(r)$ or $\frac{-\partial V}{\partial r}=F(r)$

II. Direction of angular momentum is perpendicular to plane of motion $\vec{r}\cdot\vec{J}=(\vec{r}\times\vec{b})=0$

III. If potential is $V(r)$, then force is conservative $\left(F=-\frac{\partial V}{\partial r}\hat{r}\right)$

Ans.99: (a)

Solution: By conservation law of energy

$$=\left(-\frac{GM_cm}{r_A}\right)-\left(-\frac{GM_cm}{r_B}\right)=\frac{GM_cm}{r_B}-\frac{GM_cm}{r_A}=\frac{GM_cm}{r_B}\left[1-\frac{r_B}{r_A}\right]$$

Ans. 100: (c)

Solution: Kepler's second law states that the areal velocity swept out by a radius drawn from sun to a planet is constant, i.e.,

$$\frac{dA}{dt}=\text{constant}$$

Ans. 101: (c)

Solution: Potential at earth surface $V=-\frac{GM_e}{R}$ (i)

Mass of earth $M_e=\text{volume}\times\text{density}=\frac{4}{3}\pi R^3\rho$ (ii)

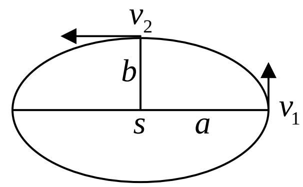
By equation (i) and (ii) becomes

$$V=-\frac{G}{R_e}\left(\frac{4}{3}\pi R^3\rho\right)=-\frac{4}{3}\pi R^2\rho G$$

Ans. 102: (d)

Solution: Assume Sun is at the centre of elliptical orbit.

Conservation of energy $\frac{1}{2}mv_1^2 - \frac{GMm}{a} = \frac{1}{2}mv_2^2 - \frac{GMm}{b}$



Conservation of momentum $L = mv_1 a = mv_2 b$

$$v_2 = v_1\left(\frac{a}{b}\right)$$

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = \frac{GMm}{a} - \frac{GMm}{b} \Rightarrow \frac{1}{2}m\left(v_1^2 - v_1^2\frac{a^2}{b^2}\right) = GMm\left(\frac{b-a}{ab}\right)$$

$$\frac{1}{2}mv_1^2\left(\frac{b^2-a^2}{b^2}\right) = GMm\left(\frac{b-a}{ab}\right) \Rightarrow \frac{1}{2}mv_1^2 = GMm\left(\frac{b}{a}\right)\cdot\frac{1}{(b+a)}$$

$$v_1 = \sqrt{2GM\left(\frac{b}{a}\right)\cdot\frac{1}{(b+a)}}$$

$$L = mv_1 a = m\sqrt{2GM\left(\frac{b}{a}\right)\cdot\left(\frac{1}{b+a}\right)}\cdot a = m\sqrt{\frac{2GMab}{(b+a)}} \Rightarrow L = \sqrt{\frac{2GMm^2ab}{a+b}}$$

Ans. 103: (c)

Solution: Consider $\vec{r} = \vec{r}_2 - \vec{r}_1$, where $\vec{r}_1$ and $\vec{r}_2$ are position vector of m_1 and m_2

$$(m_1+m_2)\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{(m_1+m_2)},\ \vec{r}_1 = \vec{R} - \frac{m_2\vec{r}}{m_1+m_2},\ \vec{r}_2 = \vec{R} + \frac{m_2\vec{r}}{m_1+m_2}$$

$$T = \frac{1}{2}m_1\dot{\vec{r}}_1^{\,2} + \frac{1}{2}m_2\dot{\vec{r}}_2^{\,2} \Rightarrow T = \frac{1}{2}(m_1+m_2)\dot{\vec{R}}^2 + \frac{m_1m_2}{m_1+m_2}\dot{\vec{r}}^{\,2}$$

$$T = \frac{3m}{2}\dot{\vec{R}}^2 + \frac{m}{3}\dot{\vec{r}}^{\,2} \Rightarrow T = \frac{3m}{2}\left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right) + \frac{m}{3}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

$$m_1 + m_2 = 3m,\ \frac{m_1m_2}{m_1+m_2} = \frac{2m^2}{3m} = \frac{2m}{3}$$

Ans. 104: (d)

Solution: $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$

$$\frac{\partial L}{\partial \theta}=0\Rightarrow\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right)=0\Rightarrow mr^2\dot{\theta}=l\quad \text{constant}\Rightarrow\dot{\theta}=\frac{l}{mr^2}$$

$$\frac{dA}{dt}=\frac{1}{2}rr\frac{d\theta}{dt}=\frac{1}{2}r^2\frac{l}{mr^2}\Rightarrow\frac{dA}{dt}=\frac{l}{2m}$$

Ans. 105: (b)

Solution: Reduced mass is $\mu=\dfrac{m_1m_2}{m_1+m_2}$

Ans. 106: (d)

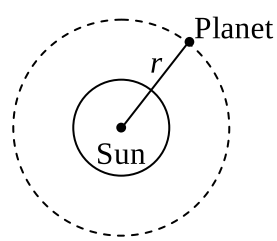
Solution: $V_{effctive}=\dfrac{J^2}{2mr^2}-\dfrac{k}{r}\Rightarrow\dfrac{dV_{effect}}{dr}=-\dfrac{J^2}{mr^3}+\dfrac{k}{r^2}=0$ at $r=r_0$

so $J=\sqrt{r_0km}$

Ans. 107: (c)

Solution: If a planet revolve with velocity v in a circle of radius r, then the centripetal force is given as

$$F=\frac{mv^2}{r}\qquad\text{(i)}$$



The attractive force between sun and planet is given as

$$F=-\frac{k}{r^n}\qquad\text{(ii)}$$

k being a constant

By equations (i) and (ii) $\dfrac{k}{r^n}=\dfrac{mv^2}{r}\Rightarrow v^2=\dfrac{k}{mr^{n-1}}\Rightarrow v=\left(\dfrac{k}{m\,r^{n-1}}\right)^{\frac{1}{2}}$

Hence, time period $=\dfrac{2\pi r}{v}=\dfrac{2\pi r}{\left(\dfrac{k}{m\,r^{n-1}}\right)^{\frac{1}{2}}}=2\pi r\left(\dfrac{mr^{n-1}}{k}\right)^{\frac{1}{2}}=\dfrac{2\pi r.r^{\frac{n-1}{2}}\sqrt{m}}{\sqrt{k}}=2\pi\sqrt{\dfrac{m}{k}}\,r^{\frac{n+1}{2}}$

Hence, time period is proportional to $r^{\frac{n+1}{2}}$

Ans. 108: (b)

Solution: The eccentricity of the ellipse is given as

$$e=\sqrt{1+\frac{2EJ^2}{Mk^2}}$$

where,

$k=$ constant

$M=$ mass of the object

$E=$ mechanical energy of the object is negative

But $e_1<e_2<e_3$ given in question

Then by equation (iii), $E_1>E_2>E_3$

Ans. 109: (d)

Solution: $\varepsilon=\left(1+\frac{2EJ^2}{mk^2}\right)^{\frac{1}{2}}\Rightarrow \varepsilon>1$, since $\left(\frac{J}{k}\right)^2$ is positive

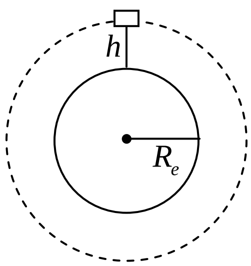
Ans. 110: (b)

Solution: $\frac{1}{r}=\frac{mk}{J^2}(1+\varepsilon\cos\theta)$, for parabolic trajectory $\varepsilon=1$.

Ans. 111: (d)

Ans. 112: (c)

Solution: The potential energy of the satellite at a height h, i.e., at distance (R_e+h) from the centre of the earth is given as



$$PE=-\frac{GM_em}{r}\Rightarrow PE=-\frac{GM_em}{R_e+h}$$

where m is mass of satellite and M_e is mass of earth.

$$\frac{mv^2}{r}=\frac{GM_em}{r^2}\Rightarrow mv^2=\frac{GM_em}{r}\Rightarrow KE=\frac{1}{2}mv^2=\frac{GM_em}{2r}\Rightarrow KE=\frac{GM_em}{2r}$$

But equations, (i) and (ii), we get

$KE = -\frac{1}{2}PE \Rightarrow PE = -2(KE) \Rightarrow PE = -2E_K$ (given $KE = E_K$)

The total energy of satellite $= PE + KE = -2E_K + E_K = -E_K$

The total energy $(KE + PE)$ of satellite is negative and is called the binding energy of the satellite.

Satellite becomes free, if its total energy is non negative. Hence, in order to escape from the earth, the minimum amount of energy to be supplied is E_K.

Ans. 113: (a)

Solution: $V_{eff} = \frac{J^2}{2mr^2} + \frac{1}{2}kr^2$ where J is angular momentum.

Condition for circular orbit $\frac{\partial V_{eff}}{\partial r} = 0 \Rightarrow -\frac{J^2}{mr^3} + kr = 0 \Rightarrow J^2 \propto r^4 \Rightarrow J \propto r^2$.

Thus $\frac{J_1}{J_2} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{J_1}{J_2}} \Rightarrow \frac{r_1}{r_2} = \sqrt{2}$ since $\frac{J_1}{J_2} = 2$.

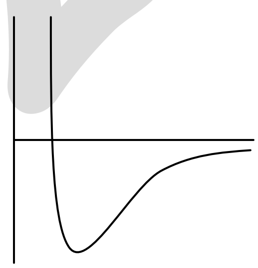
Ans. 114: (d)

Solution: $-\frac{\partial V}{\partial r} = -\frac{k}{r^2} \Rightarrow V = -\frac{k}{r} \Rightarrow L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{r} \Rightarrow H = \frac{P_r^2}{2m} + \frac{P_\theta^2}{2mr^2} - \frac{k}{r}$

Ans. 115: (c)

Solution: $H = \frac{P_r^2}{2m} + \frac{L^2}{2mr^2} - \frac{k}{r}$

$V_{eff} = \frac{J^2}{2mr^2} - \frac{k}{r}$



Ans. 116: (c)

Solution: For E_1 only one value of r is possible and $\frac{\partial V_{eff}}{\partial r} = 0$, so shape of orbit is circle

For E_2, since $E < 0$ and particle is bounded between r_1 and r_2 so $e < 1$

For E_3, since $E = 0$, so shape of orbit is parabolic, so $e = 1$

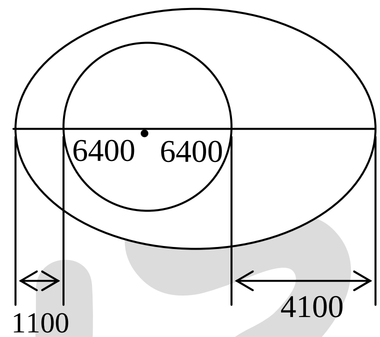
For E_4, since $E > 0$, so shape of orbit is hyperbolic, so $e > 1$, where e = eccentricity

Ans. 117: (a)

Solution: $\frac{l^2}{mr^3}=\frac{\partial V}{\partial r}\Rightarrow\frac{l^2}{mr^3}=2kr$

$$r_0^4=\left(\frac{l^2}{2mk}\right)\Rightarrow r_0=\left(\frac{l^2}{2mk}\right)^{1/4}$$

Ans. 118: (d)



Solution: $r_{\max}=4100+6400$

$$r_{\min}=1100+6400$$

$$e=\frac{r_{\max}-r_{\min}}{r_{\max}+r_{\min}}=\frac{1}{6}=0.16$$

Ans. 119: (d)

Solution: For Keplers potential,

$$V(r)=-\frac{k}{r}$$

$\frac{1}{r}=\frac{mk}{l^2}(1+\in\cos\theta)$, where $\in$ is eccentricity.

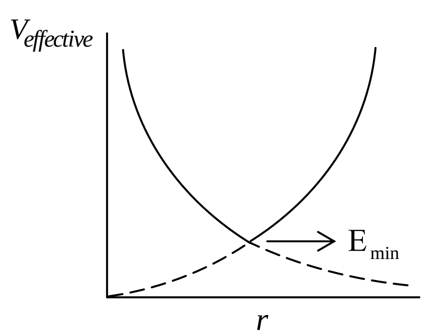
For circle, $\in=0$

So, $\frac{1}{r_c}=\frac{mk}{l^2}\Rightarrow r_c=\frac{l^2}{mk}$

For parabolic shape, $\in=1$ and for closed approach, $\theta=0\Rightarrow\frac{1}{r_p}=\frac{mk}{l^2}(1+1)$

$\Rightarrow r_p=\frac{l^2}{2mk}$, so $\frac{r_c}{r_p}=\frac{l^2/mk}{l^2/2mk}$ and $\frac{r_c}{r_b}=2$

Ans. 120: (d)



Solution: $V_{eff}=\frac{l^2}{2mr^2}+\frac{1}{2}kr^2$

bounded motion is only possible for

$V_{\min}<E<\infty$, $E_{\min}<\infty$

Ans. 121: (b)

Solution: Sun is at one of foci of ellipse not the centre of ellipse.

Ans. 122: (c)

Solution: Semi minor axis = b = 4. According to Kepler, Sun is at any one of foci of the elliptical orbit

Semi major axis $= a = 5$

The co-ordinate of focus of elliptical orbit is $(0, ae)$ and $(0, -ae)$

$$b = a\sqrt{1-e^2} \Rightarrow 4 = 5\sqrt{1-e^2} \Rightarrow e = \frac{3}{5}$$

So, co-ordinate of focus is at $(0, ae)$ and $(0, -ae)$

$$\left(0,\ 5\times\frac{3}{5}\right) \text{and} \left(0,\ -5\times\frac{3}{5}\right)$$

(0, 3) and (0, -3)

Ans. 123: (a)

Solution: Initially satellite is in circle so E_1 is best suited because at circular orbit $\frac{\partial V_{eff}}{\partial r} = 0$.

Now, new orbit is bounded one, so another orbit must be ellipse because Energy increased so E_2 is energy of satellite in new orbit.

Ans. 124: (d)

Solution: Equation of orbit is

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2u^2} f\left(\frac{1}{u}\right) \qquad u = \frac{1}{r} = -\frac{m}{l^2u^2}\left(-ku^3\right)$$

$$\frac{d^2u}{d\theta^2} + \left(1 - \frac{km}{l^2}\right)u = 0 \Rightarrow \frac{d^2u}{d\theta^2} + \left(\frac{l^2 - km}{l^2}\right)u = 0$$

If $l^2 > km$, then bounded

$l^2 < km$, then unbounded.

Ans. 125: (b)

Solution: $r = e^{-\theta}$

$$u = \frac{1}{r} = e^{\theta} \Rightarrow \frac{d^2u}{d\theta^2} = e^{\theta}$$

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2u^2} f\left(\frac{1}{u}\right) \Rightarrow e^{\theta} + e^{\theta} = -\frac{m}{l^2} e^{-2\theta} f\left(\frac{1}{u}\right)$$

$$f\left(\frac{1}{u}\right) \propto e^{3\theta} \Rightarrow f(r) \propto \left(\frac{1}{r}\right)^3 \Rightarrow f(r) \propto \frac{1}{r^3} \Rightarrow -\frac{\partial V}{\partial r} = f(r)$$

$$V = \int f(r) dr \Rightarrow V = \int \frac{1}{r^3} dr \Rightarrow V \propto \frac{1}{r^2}$$

Ans. 126: (c)

Solution: In circle $\frac{mV^2}{r} = \frac{GMm}{r^2} \Rightarrow \frac{1}{2} mV^2 = \frac{GMm}{2r}$

$$P.E. = \frac{GMm}{r} \text{ and } K.E. = \frac{GMm}{2r}$$

$$E = \frac{1}{2} mV^2 + P.E. = \frac{GMm}{2r} - \frac{GMm}{r} \Rightarrow E = -\frac{GMm}{2r} \Rightarrow E = -E_k$$

$$\Delta E = 0 - (-E_k) = E_k$$

Ans. 127: (c)

Solution: From the theorem,

For $V \propto r^{n+1}$,

$$\langle T \rangle = \frac{n+1}{2} \langle V \rangle$$

For potential $\frac{1}{2} Kr^2$, $n = 1 \Rightarrow \langle T \rangle = \frac{1+1}{2} \langle V \rangle \Rightarrow \langle T \rangle = \langle V \rangle$

Ans. 128: (b)

Solution: $r = e^{-\theta}$, $u = \frac{1}{r} = e^{\theta}$

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2u^2} f(1/r)$$

$f(r)\propto\frac{1}{r^3}\Rightarrow V(r)\propto\frac{1}{r^2}$

$V(r)\propto r^{-2}$ or $V(r)\propto r^{-3+1}$

Now, from the virial theorem

$\langle T\rangle=\frac{n+1}{2}\langle V\rangle$, here $n=-3$

So, $\langle T\rangle=-\langle V\rangle$

Ans. 129: (c)

Solution: $E=\frac{1}{2}m\dot{r}^2+\frac{L^2}{2mr^2}-\frac{k}{r}$, put $\dot{r}=0$

$$E-\frac{l^2}{2mr^2}+\frac{k}{r}=0\Rightarrow r^2+\frac{k}{E}r-\frac{l^2}{2mE}=0\Rightarrow\frac{(r_1+r_2)}{2}=a=\frac{-k}{2E}$$

So at perihelion and aphelion linear momentum is zero and angular momentum is always constant during the motion.

Ans. 130: (d)

Solution: For I: $r_p\times v_p=r_a\times v_a\approx 590\times10^8\ m^2/\sec$

For II: at perigee and apogee radial velocity is zero

For III: $E=-\frac{k}{2a}$ where a is semi-major axis

$2a=r_p+r_a\Rightarrow -k=E\cdot 2a$

$k=-\left(4.5\times10^{10}\times18\times10^6\right)=8\times10^{17}\ J/m$

Ans. 131: (c)

Solution: $\frac{l^2}{mr_o^3}=mcr_o+\frac{k}{r_o^2}\Rightarrow l^2=m^2cr_o^4+km_or_o\Rightarrow l=\left(m^2cr_o^4+km_or_o\right)^{1/2}$

$mr^2\dot{\theta}=\left(m^2cr_o^4+km_or_o\right)^{1/2}\Rightarrow\theta=\frac{2\pi}{T}$

$$T = \frac{2\pi}{\sqrt{\frac{k}{mr_o^3} + c}}$$

Ans. 132: (d)

Solution: Mass of the ring $M = \rho L$

If R is the radius of the ring then $L = 2\pi R \Rightarrow R = \dfrac{L}{2\pi}$

M.I of the ring about an axis passing through O and parallel to XX' is

$$I_0 = \frac{MR^2}{2}$$

hence, using parallel axis theorem, M.I about XX' is

$$I_{XX'} = I_0 + MR^2 = \frac{MR^2}{2} + MR^2 \Rightarrow I_{XX'} = \frac{3MR^2}{2} = \frac{3\rho L.\frac{L^2}{4\pi^2}}{2} \text{ or } I_{XX'} = \frac{3\rho L^3}{8\pi^2}$$

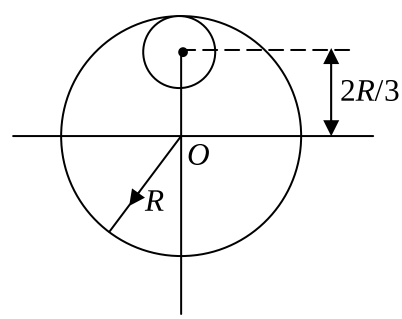
Ans. 133: (a)

Solution: The moment of inertia of the complete disc is $I = \frac{1}{2}(4M)R^2 = 2MR^2$.

From symmetry each quarter of the disc has the same moment of inertia about the axis of rotation, hence required moment of inertia is $\dfrac{2MR^2}{4} = \dfrac{1}{2}MR^2$

Ans. 134: (a)

Solution: Total mass of the disc $= 9\ M$



Radius of disc $= R$

Moment of inertia of the complete disc about a perpendicular axis through O is

$$I_1 = \frac{1}{2} \times 9M \times R^2 = \frac{9}{2}MR^2$$

Mass of the disc removed $= \dfrac{9M}{\pi R^2} \times \left(\dfrac{R}{3}\right)^2 = M$

By the parallel axes theorem, the moment of inertia of small disc about the axis through O,

$$I_2 = \frac{1}{2}M\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{1}{2}MR^2$$

Moment of inertia of the remaining disc about a perpendicular axis through O,

$$I = I_1 - I_2 = \frac{9}{2}MR^2 - \frac{1}{2}MR^2 = 4MR^2$$

Ans. 135: (d)

Solution: The moment of inertia of the system about any side (say CD)

$I = M.I$ of A about $CD +$ moment of inertia of B about $CD + MI$ of C about $CD + MI$ of D about CD

$$= \left(\frac{2}{5}Ma^2 + Mb^2\right) + \left(\frac{2}{5}Ma^2 + Mb^2\right) + \frac{2}{5}Ma^2 + \frac{2}{5}Ma^2 = \frac{2}{5}M\left(4a^2 + 5b^2\right)$$

Ans. 136: (b)

Solution: Let k_1 and k_2 be the radii of gyration of the ring and the disc respectively. Then

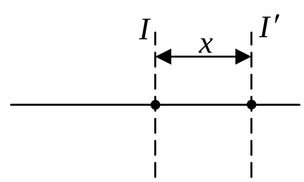
M.I of ring $= MR^2 = Mk_1^2$ or $k_1 = R$

M.I of disc $= \frac{1}{2}MR^2 = Mk_2^2$ or $k_2 = \frac{R}{\sqrt{2}}$

Therefore, $\frac{k_1}{k_2} = \sqrt{2} : 1$

Ans. 137: (d)

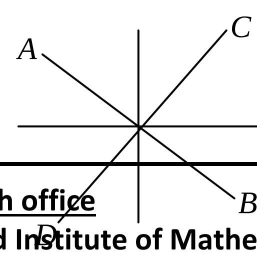
Solution: Using parallel axis theorem



$$I' = I + Mx^2 = \frac{ML^2}{12} + Mx^2$$

Ans. 138: (d)

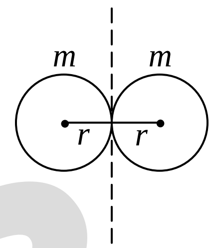
Solution: The M.I of the cross about a line perpendicular to the plane of the figure through the centre of cross is $\frac{Ml^2}{12} \times 2 = \frac{Ml^2}{6}$



The moment of inertia of the cross about the two bisectors are equal by symmetry and according to the theorem of perpendicular axes, the M.I of the cross about the bisector AB is $\frac{Ml^2}{12}$

Ans. 139: (c)

Solution: Using parallel axis theorem the M.I of the system about the tangent



$$=2\left[\frac{2}{5}mr^2+mr^2\right]=\frac{14mr^2}{5}$$

Ans. 140: (c)

Solution: Given that $I_Q=4I_P$. The M.I. of a ring about its axis is, $\text{mass}\times(\text{radius})^2$, let λ be the linear mass density of the wire, then

$$\lambda\times2\pi(nr)\times n^2r^2=4\left(\lambda\times2\pi r\times r^2\right)\Rightarrow n^2=4\Rightarrow n=4^{\frac{1}{3}}$$

Ans. 141: (c)

Solution: Let M be the mass of each disc. Let R_A and R_B be the radii of the discs A and B, respectively. Let t be the thickness of each disc. Then $M=\pi R_A^2td_A=\pi R_B^2td_B$ as $d_A>d_B$ so $R_A^2<R_B^2$

Now $\frac{I_A}{I_B}=\frac{\frac{1}{2}MR_A^2}{\frac{1}{2}MR_B^2}=\frac{R_A^2}{R_B^2}<1$. Therefore, $I_A<I_B$

Ans. 142: (d)

Solution: Torque $\vec{\tau}=\vec{r}\times\vec{F}$. For central force, $\vec{r}$ and $\vec{F}$ lies along the same line. Hence $\vec{\tau}=0$. This implies that angular momentum is constant.

Ans. 143: (b)

Solution: Let M be the mass and r be the radius. Then moment of inertia $=\frac{2}{5}Mr^2$

Hence, $L = \left(\frac{2}{5}Mr^2\right)\omega = \frac{2Mr^2\omega}{5}$

But since, no net torque acts on the sphere, so angular momentum remains conserved. Hence increasing r, decreases ω. Increasing r also increases moment of inertia. Due to changing I and ω rotational kinetic energy also change

Ans. 144: (d)

Solution: Since, no external torque acts on the system (Insect +disc), the angular momentum must remain constant. But, $L = I\omega$ and as the insect moves along the diameter, I increases, hence ω must decrease, from centre to rim. From rim to centre I decreases, hence ω increases.

Ans. 145: (a)

Solution: The net force on the system is zero, hence there will be no translation. But since the net torque is non-zero, there will be rotation

Ans. 146: (b)

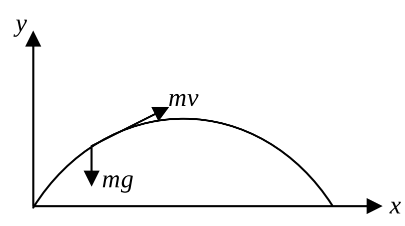
Solution: From the relation

$\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$, where $\vec{L}$ is the total angular momentum and $\vec{\tau}$ the net external torque, we see that, when $\vec{\tau}_{ext} = 0$, $\vec{L}$ is constant

Ans. 147: (d)

Solution: The direction of angular velocity vector is always along the axis of rotation for a rigid body rotating about a fixed axis.

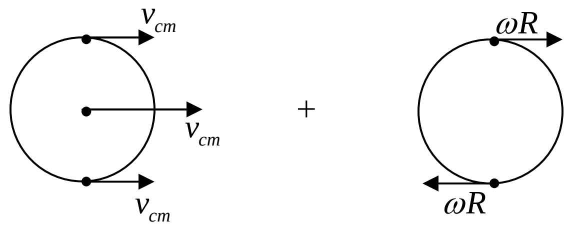
Ans. 148: (b)



Solution: About the initial position, the angular momentum of the particle is in the clockwise sense. Also, the torque due to its weight mg is in the clockwise sense. Hence the angular momentum increases.

Ans. 149: (a)

Solution: The net velocity is the sum of velocity due to translation and velocity due to rotation



For rolling we also have, $v_{cm} = \omega R$. Hence $v_{top} = v_{cm} + v_{cm} = 2v_{cm}$ and $v_{bottom} = v_{cm} - v_{cm} = 0$

Ans. 150: (a)

Solution: The acceleration of a rolling body down an incline plane is given by

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

For hollow cylinder, $\frac{I}{MR^2} = 1$

For solid cylinder, $\frac{I}{MR^2} = \frac{1}{2}$. Hence $a_{\text{hollow}} < a_{\text{solid}}$

Since acceleration of solid cylinder is greater, hence it will reach earlier

Ans. 151: (d)

Solution: The torque acting on the particle is $\vec{\tau} = \vec{r} \times F$ and not $\vec{F} \times \vec{r}$

The relation $\vec{\tau} = \frac{d\vec{L}}{dt}$ says zero net torque means constant angular momentum. It also says changing angular momentum necessarily gives rise to torque. Neither zero net force implies zero net torque nor the zero net torque implies zero net force

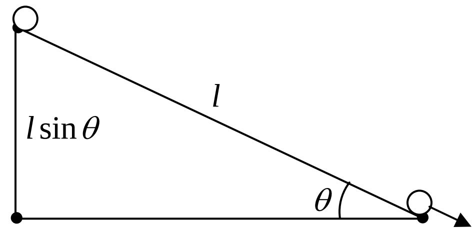
Ans. 152: (a)

Solution: From the conservation of mechanical energy

$$Mgl\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \Rightarrow Mgl\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}.\frac{MR^2}{2}.\frac{v^2}{R^2} \text{ [For rolling } v = \omega R\text{]}$$

$$gl\sin\theta = \frac{v^2}{2} + \frac{v^2}{4}$$

$$\Rightarrow \frac{3v^2}{4} = gl\sin\theta \Rightarrow v = \sqrt{\frac{4gl\sin\theta}{3}}$$



Ans. 153: (d)

Solution: Due to forward slipping $(v_0 > \omega_0 r)$ the frictional force is backward

The translational velocity at time t

$$v(t) = v_0 - \frac{f}{M}t \qquad \text{(i)}$$

The angular velocity at time t

$$\omega(t) = \frac{v_0}{2r} + \alpha t = \frac{v_0}{2r} + \frac{fr}{\frac{2Mr^2}{5}}t \Rightarrow \omega(t) = \frac{v_0}{2r} + \frac{5f}{2Mr}t$$

Rolling will start when $v(t) = r\omega(t)$

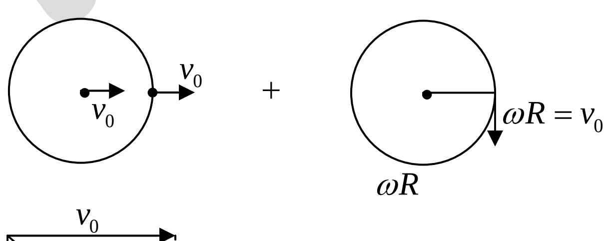
$$\Rightarrow v(t) = \frac{v_0}{2} + \frac{5f}{2M}t \qquad \text{(ii)}$$

Eliminating t, from (i) and (ii)

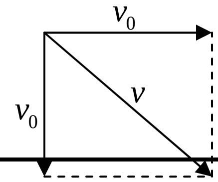
$$\frac{5}{2}v(t) + v(t) = \frac{5}{2}v_0 + \frac{v_0}{2} \text{ or } v(t) = \frac{6v_0}{7}$$

Ans. 154: (c)

Solution: Net velocity of the particle = translational velocity + rotational velocity



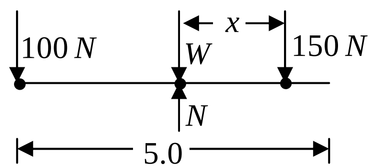
Hence



$\therefore v = \sqrt{v_0^2 + v^2} = \sqrt{2}v_0$

Ans. 155: (d)

Solution: The forces acting on the seesaw are shown in the figure.



Taking torques about the fulcrum

$150 \times x = 100 \times 2.5$ which gives $x = 1.7\ m$

Ans. 156: (d)

Solution: The velocity of the point of contact of rolling body is zero. Since v and ω are constants, the only acceleration of the bottommost point is centripetal acceleration $\frac{v^2}{R}$

Ans. 157: (d)

Solution: co-ordinate of $c.m. = a\sin\theta - a\cos\theta$

Co-ordinate of $m = a\sin\theta + a\sin\phi,\ -a\cos\theta - a\sin\phi$

$T = K.E.$ of ring about $P + K.E.$ of Mass m

$$= \frac{1}{2}\left(2Ma^2\right)\dot{\theta}^2 + \frac{1}{2}ma^2\left[\dot{\theta}^2 + \dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\cos\left(\theta - \phi\right)\right]$$

$$= Ma^2\dot{\theta}^2 + \frac{1}{2}ma^2\left[\dot{\theta}^2 + \dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\cos\left(\theta - \phi\right)\right]$$

Ans. 158: (c)

Solution: $\boldsymbol{a} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

Inertia in the direction of a is given by

$$\boldsymbol{a^T Ta} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = 2 \text{ unit}$$

Ans. 159: (b)

Solution: $I_{xx} = \sum m_i a\left(y_i^2 + z_i^2\right) = 0$ $\qquad$ $I_{xy} = I_{yz} = I_{xz} = 0$

$$I_{yy} = \sum m_i \left(x_i^2 + z_i^2\right) = 10ma^2$$

$$I_{zz} = \sum m_i \left(x_i^2 + y_i^2\right) = 10ma^2$$

Ans. 160: (d)

Solution: $$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \frac{MR^2}{4} \begin{pmatrix} 10 & -5 & 0 \\ -5 & 6 & 0 \\ 0 & 0 & 16 \end{pmatrix} \begin{pmatrix} 0 \\ \omega \\ 0 \end{pmatrix}$$

$$L_x = \frac{-5}{4} MR^2 \omega,\ L_y = \frac{6}{4} MR^2 \omega = \frac{3}{2} MR^2 \omega \text{ and } L_z = 0$$

Ans. 161: (b)

Solution: We know that Torque $\vec{\tau} = \vec{\omega} \times \vec{J}$

$$\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} = \begin{pmatrix} 6ma^2 & 0 & 2ma^2 \\ 0 & 8ma^2 & 0 \\ 2ma^2 & 0 & 2ma^2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}$$

$$\vec{J} = 2m\omega a^2 \left(\hat{i} + \hat{k}\right) \Rightarrow \tau = \vec{\omega} \times \vec{J} = 2ma^2 \omega\ j$$

Ans. 162: (d)

Solution: The moment of inertia about pivotal point is given by

$$I = I_{c.m} + Md^2 = \frac{MR^2}{2} + M(l+R)^2$$

If the ring is displaced by angle θ, then its potential energy is $V = -Mg(l+R)\cos\theta$

The Lagrangian is given by

$$L=\frac{1}{2}I\dot{\theta}^2-V(\theta)=\frac{1}{2}\left[\frac{MR^2}{2}+M(l+R)^2\right]\dot{\theta}^2+Mg(l+R)\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right)-\left(\frac{\partial L}{\partial\theta}\right)=0\ \Rightarrow\left[\frac{MR^2}{2}+M(l+R)^2\right]\ddot{\theta}+Mg(l+R)\sin\theta=0$$

For small oscillation, $\sin\theta=\theta\Rightarrow\left[\frac{MR^2}{2}+M(l+R)^2\right]\ddot{\theta}+Mg(l+R)\theta=0$

Time period is given by $T=2\pi\sqrt{\frac{3R^2+4Rl+2l^2}{2g\left(R+l\right)}}$.

Ans. 163: (b)

Solution: $I_{xx}=\int\rho\left(y^2+z^2\right)dV=\int_0^a\int_0^a\int_0^a\rho\left(y^2+z^2\right)dxdydz=\frac{2\rho a^5}{3}$

Ans. 164: (d)

Ans. 165: (c)

Solution: $3=5\sqrt{1-\frac{v^2}{c^2}}\Rightarrow v=\frac{4c}{5}$

Ans. 166: (b)

Ans. 167: (a)

Solution: Area of disc from S frame is 1 i.e. $\pi a^2=1$ or $\pi a\cdot a=1$

Area of disc from S' frame is $\pi a\cdot b=\pi a\cdot a\sqrt{1-\frac{u^2}{c^2}}\ =1\cdot\sqrt{1-\frac{u^2}{c^2}}=\sqrt{1-\frac{u^2}{c^2}}$

where $b=a\sqrt{1-\frac{u^2}{c^2}}$.

Ans. 168: (a)

Solution: now $u=\dfrac{v+\frac{c}{n}}{1+\frac{v\cdot c}{c^2\cdot n}}=\left(v+\frac{c}{n}\right)\left(1+\frac{v}{cn}\right)^{-1}=\left(v+\frac{c}{n}\right)\left(1-\frac{v}{cn}+\frac{v^2}{c^2n^2}\right)$

$$\Rightarrow v-\frac{v^2}{cn}+\frac{v^3}{c^2n^2}+\frac{c}{n}-\frac{v}{n^2}+\frac{v^2}{cn^3}\Rightarrow u=\frac{c}{n}+v\left(1-\frac{1}{n^2}\right)$$

Ans. 169: (c)

Solution: $u^{'}=\frac{c}{n}$ and $u=\frac{u^{'}+v}{1+\frac{u^{'}v}{c^2}}$

$$u=\frac{c}{n}\left(\frac{1+\frac{nv}{c}}{1+\frac{v}{nc}}\right)$$

Ans. 170: (c)

Solution: $u_x^{'}=c\cos\theta_0\quad v=u$

$$u_x=\frac{u_x^{'}+v}{1+u_x^{'}v/c^2}=\frac{c\cos\theta_0+u}{1+c\cos\theta_0 u/c^2}=\frac{c\cos\theta_0+u}{1+\cos\theta_0 u/c}$$

$$\cos\theta=\frac{u_x}{c}\Rightarrow\cos\theta=\frac{\cos\theta_0+\frac{u}{c}}{1+\frac{u}{c}\cos\theta_0}$$

Ans. 171: (c)

Solution: $P=\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\Rightarrow F=\frac{dP}{dt}=m\frac{dv}{dt}\cdot\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}+mv\left(-\frac{1}{2}\right)\cdot\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\cdot\frac{-2v}{c^2}\frac{dv}{dt}$

$$\Rightarrow F=m\frac{dv}{dt}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(1+\frac{v^2/c^2}{\left(1-\frac{v^2}{c^2}\right)}\right)=\frac{mc^3}{\left(c^2-v^2\right)^{3/2}}\frac{dv}{dt}$$

Ans. 172: (c)

Solution: From conservation of energy

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}+\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=m_1c^2 \Rightarrow \frac{2mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=m_1c^2$$

Since $v=0.6c \Rightarrow m_1=5m/2$

Ans. 173: (c)

Solution: $E=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 1-\frac{v^2}{c^2}=\left(\frac{mc^2}{E}\right)^2 \Rightarrow \frac{v^2}{c^2}=1-\frac{m^2c^4}{E^2} \Rightarrow v=c\sqrt{1-\left(\frac{mc^2}{E}\right)^2}$

Ans. 174: (a)

Solution: $K.E=mc^2-m_0c^2$, rest mass energy $=m_0c^2$

$K.E.=$ rest mass energy

$mc^2-m_0c^2=m_0c^2 \Rightarrow mc^2=2m_0c^2$

$$\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}c^2=2m_0c^2 \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=2 \Rightarrow 4\left(1-\frac{v^2}{c^2}\right)=1 \Rightarrow 4\frac{v^2}{c^2}=3 \Rightarrow v=\frac{\sqrt{3}}{2}c$$

Ans. 175: (c)

Solution: $u_x'=\frac{5}{19}c$, $v=\frac{2}{5}c \Rightarrow u_x=\frac{u_x'+v}{1+\frac{u_x'v}{c^2}}=\frac{3}{5}c$

Ans. 176: (b)

Solution: $E=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{5}{4}m_0c^2$

Ans.177: (c)

Solution: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=\left[f''(x+i\beta y-vt)+g''(x-i\beta y-vt)\right](1-\beta^2)$

$$=\frac{v^2}{c^2}\left[f''(x+i\beta y-vt)+g''(x-i\beta y-vt)\right] \Rightarrow \beta=\left(1-\frac{v^2}{c^2}\right)^{1/2}$$

Ans. 178: (b)

Solution: $v=-\frac{c}{3}\hat{z},\ u_x^{'}=0, u_y^{'}=0, u_z^{'}=\frac{c}{2}$, the speed of particle with respect to wall is

$$u_z=\frac{u_z^{'}+v}{1+\frac{u_z^{'}v}{c^2}}=\frac{c}{5}$$

Ans. 179: (a)

Solution: Kinetic energy $T=mc^2$ and $T=E-mc^2$, $E=2mc^2$

$$E^2=p^2c^2+m^2c^4 \Rightarrow p=\sqrt{3}mc$$

Ans. 180: (c)

Solution: From conservation of momentum mass less particle and particle of mass m have same momentum p and from conservation of energy. $Mc^2=\sqrt{p^2c^2+m^2c^4}+pc$

$$p=\frac{c}{2M}\left(M^2-m^2\right)$$

Ans. 181: (d)

Solution: From conservation of energy

$K+m_0c^2=\sqrt{p^2c^2+m_0c^4}$, so momentum $p=\frac{\sqrt{K\left(K+2m_0c^2\right)}}{c}$

If particle flux (number of particles per unit area per unit time) is J then pressure $P=\frac{F}{A}$

$$P=\frac{J\sqrt{K\left(K+2m_0c^2\right)}}{c}.$$