PHYSICS XII

CHAPTERS 19-23

BOOKLET-4

Contents: **Page No.**

Light Propagation.

Light is a form of energy which generally gives the sensation of sight.

(1) **Different theories**

Newtons corpuscular theory	Huygen's wave theory	Maxwell's EM wave theory	Einstein's quantum theory	de-Broglie's dual theory of light
(i) Based on Rectilinear propagation of light	(i) Light travels in a hypothetical medium ether (high elasticity very low density) as waves	(i) Light travels in the form of EM waves with speed in free space $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	(i) Light is produced, absorbed and propagated as packets of energy called photons	(i) Light propagates both as particles as well as waves
(ii) Light propagates in the form of tiny particles called Corpuscles. Colour of light is due to different size of corpuscles	(ii) He proposed that light waves are of longitudinal nature. Later on it was found that they are transverse	(ii) EM waves consists of electric and magnetic field oscillation and they do not require material medium to travel	(ii) Energy associated with each photon $E = h\nu = \frac{hc}{\lambda}$ h = planks constant $= 6.6 \times 10^{-34}\, J$ - sec ν = frequency λ = wavelength	(ii) Wave nature of light dominates when light interacts with light. The particle nature of light dominates when the light interacts with matter (micro-scopic particles)

(2) **Optical phenomena explained (√) or not explained (×) by the different theories of light**

S. No.	Phenomena	Theory				
		Corpuscular	Wave	E.M. wave	Quantum	Dual
(i)	Rectilinear Propagation	√	√	√	√	√
(ii)	Reflection	√	√	√	√	√
(iii)	Refraction	√	√	√	√	√
(iv)	Dispersion	×	√	√	×	√
(v)	Interference	×	√	√	×	√
(vi)	Diffraction	×	√	√	×	√
(vii)	Polarisation	×	√	√	×	√
(viii)	Double refraction	×	√	√	×	√
(ix)	Doppler's effect	×	√	√	×	√
(x)	Photoelectric effect	×	×	×	√	√

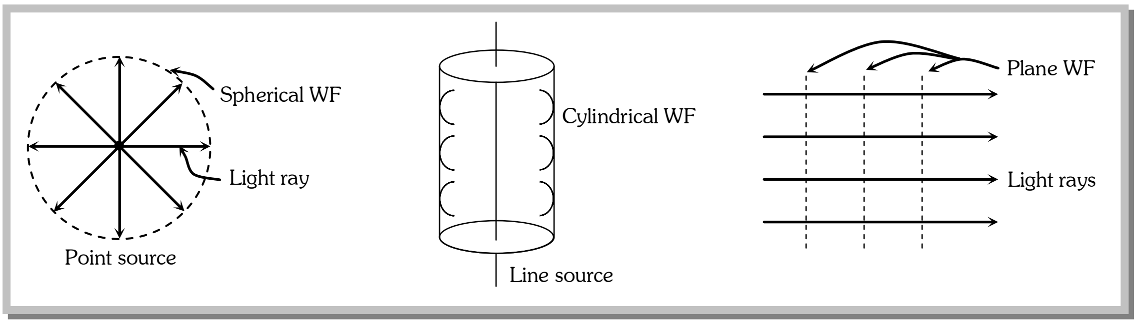
(3) **Wave front**

(i) Suggested by Huygens

(ii) The locus of all particles in a medium, vibrating in the same phase is called Wave Front (WF)

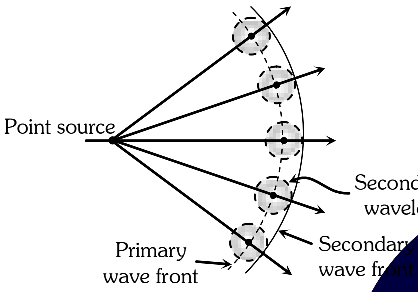
(iii) The direction of propagation of light (ray of light) is perpendicular to the WF.

(iv) **Types of wave front.**



(v) Every point on the given wave front acts as a source of new disturbance called secondary wavelets. Which travel in all directions with the velocity of light in the medium.

A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new wave front at that instant. This is called secondary wave front



Note : ≅Wave front always travels in the forward direction of the medium.

≅ Light rays is always normal to the wave front.

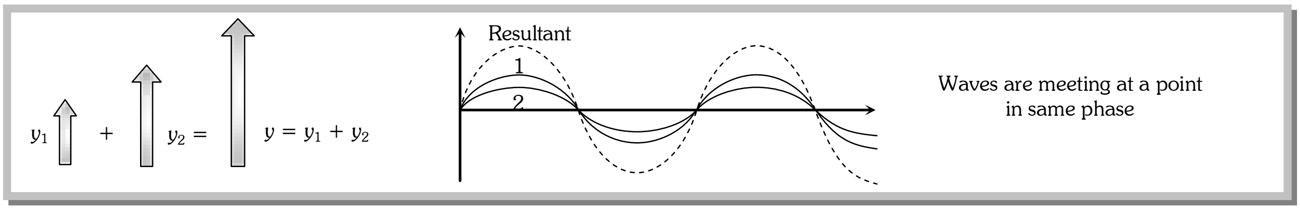
≅ The phase difference between various particles on the wave front is zero.

Principle of Super Position.

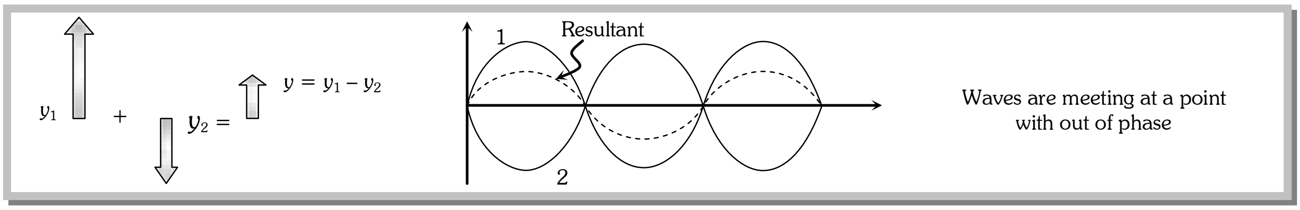
When two or more than two waves superimpose over each other at a common particle of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves. *i.e.* $\vec{y} = \vec{y}_1 + \vec{y}_2$

(1) **Graphical view :**

(i)



(ii)



(2) **Phase / Phase difference / Path difference / Time difference**

(i) Phase : The argument of sine or cosine in the expression for displacement of a wave is defined as the phase. For displacement $y = a \sin \omega t$; term ωt = phase or instantaneous phase

(ii) Phase difference (ϕ) : The difference between the phases of two waves at a point is called phase difference *i.e.* if $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$ so phase difference = ϕ

(iii) Path difference (Δ) : The difference in path length's of two waves meeting at a point is called path difference between the waves at that point. Also $\Delta = \frac{\lambda}{2\pi} \times \phi$

(iv) Time difference ($T.D.$) : Time difference between the waves meeting at a point is $\boldsymbol{T.D.} = \frac{T}{2\pi} \times \phi$

(3) Resultant amplitude and intensity

If suppose we have two waves $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$; where a_1, a_2 = Individual amplitudes, ϕ = Phase difference between the waves at an instant when they are meeting a point. I_1, I_2 = Intensities of individual waves

Resultant amplitude : After superimposition of the given waves resultant amplitude (or the amplitude of resultant wave) is given by $\boldsymbol{A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}}$

For the interfering waves $y_1 = a_1 \sin\omega t$ and $y_2 = a_2 \cos\omega t$, Phase difference between them is 90°. So resultant amplitude $A = \sqrt{a_1^2 + a_2^2}$

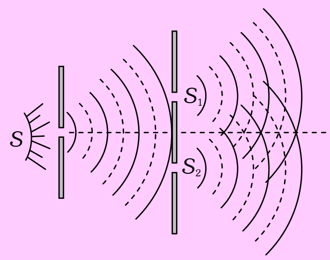
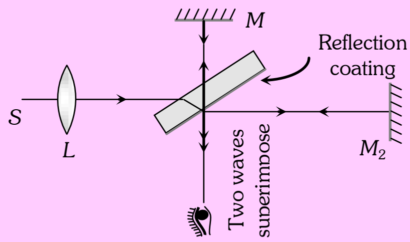
Resultant intensity : As we know intensity ∝ (Amplitude)^2 $\Rightarrow I_1 = ka_1^2, I_2 = ka_2^2$ and $I = kA^2$ (k is a proportionality constant). Hence from the formula of resultant amplitude, we get the following formula of resultant intensity $\boldsymbol{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$

Note : ≅The term $2\sqrt{I_1 I_2}\cos\phi$ is called interference term. For incoherent interference this term is zero so resultant intensity $I = I_1 + I_2$

(4) Coherent sources

The sources of light which emits continuous light waves of the same wavelength, same frequency and in same phase or having a constant phase difference are called coherent sources.

Two coherent sources are produced from a single source of light by adopting any one of the following two methods

Division of wave front	Division of amplitude
The light source is narrow	Light sources is extended. Light wave partly reflected (50%) and partly transmitted (50%)
The wave front emitted by a narrow source is divided in two parts by reflection of refraction.	The amplitude of wave emitted by an extend source of light is divided in two parts by partial reflection and partial refraction.
The coherent sources obtained are imaginary *e.g.* Fresnel's biprism, Llyod's mirror Youngs' double slit *etc.*	The coherent sources obtained are real *e.g.* Newtons rings, Michelson's interferrometer colours in thin films
	

Note : ≅Laser light is highly coherent and monochromatic.

≅ Two sources of light, whose frequencies are not same and phase difference between the waves emitted by them does not remain constant *w.r.t.* time are called non-coherent.

≅ The light emitted by two independent sources (candles, bulbs *etc*.) is non-coherent and interference phenomenon cannot be produced by such two sources.

≅ The average time interval in which a photon or a wave packet is emitted from an atom is defined as the **time of coherence**. It is $\tau_c = \frac{L}{c} = \frac{\text{Distance of coherence}}{\text{Velocity of light}}$, it's value is of the order of 10^{-10} *sec*.

Interference of Light.

When two waves of exactly same frequency (coming from two coherent sources) travels in a medium, in the same direction simultaneously then due to their superposition, at some points intensity of light is maximum while at some other points intensity is minimum. This phenomenon is called Interference of light.

(1) **Types :** It is of following two types

Constructive interference	Destructive interference
(i) When the waves meets a point with same phase, constructive interference is obtained at that point (*i.e.* maximum light)	(i) When the wave meets a point with opposite phase, destructive interference is obtained at that point (*i.e.* minimum light)
(ii) Phase difference between the waves at the point of observation $\phi = 0^o$ or $2n\pi$	(ii) $\phi = 180^o$ or $(2n-1)\pi$; $n = 1, 2, ...$ or $(2n+1)\pi$; $n = 0,1,2.....$
(iii) Path difference between the waves at the point of observation $\Delta = n\lambda$ (*i.e.* even multiple of $\lambda/2$)	(iii) $\Delta = (2n-1)\frac{\lambda}{2}$ (*i.e.* odd multiple of $\lambda/2$)
(iv) Resultant amplitude at the point of observation will be maximum $a_1 = a_2 \Rightarrow A_{min} = 0$ If $a_1 = a_2 = a_0 \Rightarrow A_{max} = 2a_0$	(iv) Resultant amplitude at the point of observation will be minimum $A_{min} = a_1 - a_2$ If $a_1 = a_2 \Rightarrow A_{min} = 0$
(v) Resultant intensity at the point of observation will be maximum $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{max} = 2I_0$	(v) Resultant intensity at the point of observation will be minimum $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{min} = 0$

(2) **Resultant intensity due to two identical waves :**

For two coherent sources the resultant intensity is given by $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

For identical source $I_1 = I_2 = I_0 \Rightarrow I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi = 4I_0\cos^2\frac{\phi}{2}$ $\quad$ $[1 + \cos\theta = 2\cos^2\frac{\theta}{2}]$

Note : ≅ In interference redistribution of energy takes place in the form of maxima and minima.

≅ Average intensity : $I_{av} = \dfrac{I_{max} + I_{min}}{2} = I_1 + I_2 = a_1^2 + a_2^2$

≅ Ratio of maximum and minimum intensities :

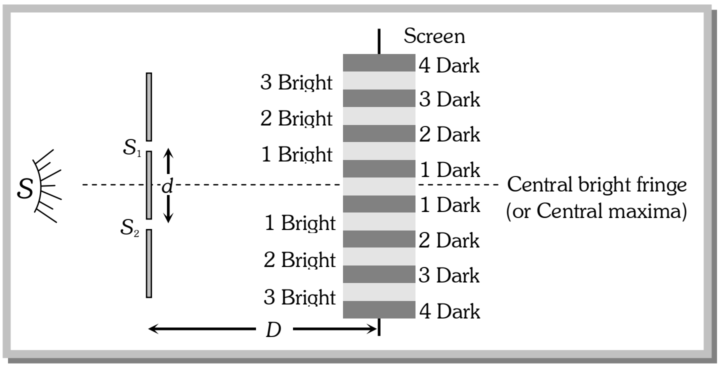
$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1 / I_2} + 1}{\sqrt{I_1 / I_2} - 1}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{a_1 / a_2 + 1}{a_1 / a_2 - 1}\right)^2 \text{ also } \sqrt{\frac{I_1}{I_2}} = \frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{max}}{I_{min}}} + 1}{\sqrt{\frac{I_{max}}{I_{min}}} - 1}\right)$$

≅ If two waves having equal intensity ($I_1 = I_2 = I_0$) meets at two locations P and Q with path difference Δ_1 and Δ_2 respectively then the ratio of resultant intensity at point P and Q will be

$$\frac{I_P}{I_Q} = \frac{\cos^2 \frac{\phi_1}{2}}{\cos^2 \frac{\phi_2}{2}} = \frac{\cos^2\left(\frac{\pi \Delta_1}{\lambda}\right)}{\cos^2\left(\frac{\pi \Delta_2}{\lambda}\right)}$$

Young's Double Slit Experiment (YDSE)

Monochromatic light (single wavelength) falls on two narrow slits S_1 and S_2 which are very close together acts as two coherent sources, when waves coming from two coherent sources (S_1, S_2) superimposes on each other, an interference pattern is obtained on the screen. In YDSE alternate bright and dark bands obtained on the screen. These bands are called Fringes.



d = Distance between slits
D = Distance between slits and screen
λ = Wavelength of monochromatic light emitted from source

(1) Central fringe is always bright, because at central position $\phi = 0^o$ or $\Delta = 0$

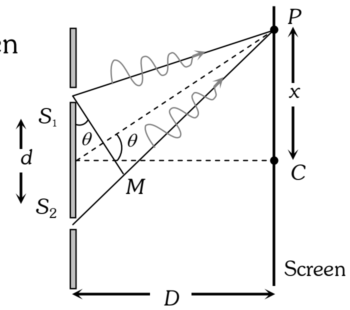
(2) The fringe pattern obtained due to a slit is more bright than that due to a point.

(3) If the slit widths are unequal, the minima will not be complete dark. For very large width uniform illumination occurs.

(4) If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.

(5) If the two coherent sources consist of object and it's reflected image, the central fringe is dark instead of bright one.

(6) **Path difference**



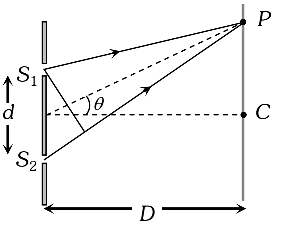
Path difference between the interfering waves meeting at a point P on the screen is given by $\Delta = \frac{xd}{D} = d\sin\theta$

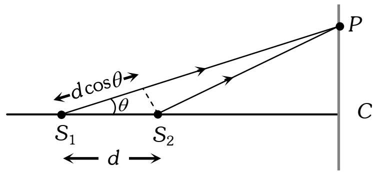
where x is the position of point P from central maxima.

For maxima at P : $\Delta = n\lambda$; where $n = 0, \pm 1, \pm 2, \ldots\ldots$

and For minima at P : $\Delta = \frac{(2n-1)\lambda}{2}$; where $n = \pm 1, \pm 2, \ldots\ldots$

Note : ≅If the slits are vertical, the path difference (Δ) is $d\sin\theta$, so as θ increases, Δ also increases. But if slits are horizontal path difference is $d\cos\theta$, so as θ increases, Δ decreases.





(7) **More about fringe**

(i) All fringes are of equal width. Width of each fringe is $\boldsymbol{\beta = \frac{\lambda D}{d}}$ and angular fringe width $\boldsymbol{\theta = \frac{\lambda}{d} = \frac{\beta}{D}}$

(ii) If the whole YDSE set up is taken in another medium then λ changes so β changes

e.g. in water $\lambda_w = \frac{\lambda_a}{\mu_w} \Rightarrow \beta_w = \frac{\beta_a}{\mu_w} = \frac{3}{4}\beta_a$

(iii) Fringe width $\beta \propto \frac{1}{d}$ *i.e.* with increase in separation between the sources, β decreases.

(iv) Position of n^{th} bright fringe from central maxima $\boldsymbol{x_n = \frac{n\lambda D}{d} = n\beta}$; $n = 0, 1, 2 \ldots.$

(v) Position of n^{th} dark fringe from central maxima $\boldsymbol{x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\beta}{2}}$; $n = 1, 2, 3 \ldots.$

(vi) In YDSE, if n_1 fringes are visible in a field of view with light of wavelength λ_1, while n_2 with light of wavelength λ_2 in the same field, then $\boldsymbol{n_1\lambda_1 = n_2\lambda_2}$.

(vii) Separation (Δx) between fringes

Between n^{th} bright and m^{th} bright fringes $(n > m)$	Between n^{th} bright and m^{th} dark fringe
$\Delta x = (n-m)\beta$	(a) If $n > m$ then $\Delta x = \left(n - m + \frac{1}{2}\right)\beta$ (b) If $n < m$ then $\Delta x = \left(m - n - \frac{1}{2}\right)\beta$

(8) **Identification of central bright fringe**

To identify central bright fringe, monochromatic light is replaced by white light. Due to overlapping central maxima will be white with red edges. On the other side of it we shall get a few coloured band and then uniform illumination.

(9) **Condition for observing sustained interference**

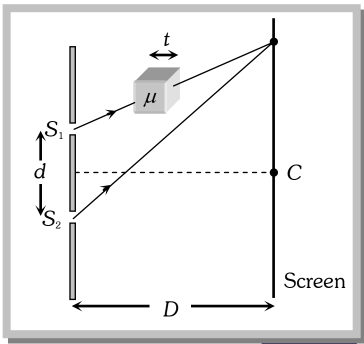
(i) The initial phase difference between the interfering waves must remain constant : Otherwise the interference will not be sustained.

(ii) The frequency and wavelengths of two waves should be equal : If not the phase difference will not remain constant and so the interference will not be sustained.

(iii) The light must be monochromatic : This eliminates overlapping of patterns as each wavelength corresponds to one interference pattern.

(iv) The amplitudes of the waves must be equal : This improves contrast with $I_{max} = 4I_0$ and $I_{min} = 0$.

(v) The sources must be close to each other : Otherwise due to small fringe width $\left(\beta \propto \frac{1}{d}\right)$ the eye can not resolve fringes resulting in uniform illumination.



(10) **Shifting of fringe pattern in YDSE**

If a transparent thin film of mica or glass is put in the path of one of the waves, then the whole fringe pattern gets shifted.

If film is put in the path of upper wave, fringe pattern shifts upward and if film is placed in the path of lower wave, pattern shift downward.

$$\text{Fringe shift} = \frac{D}{d}(\mu-1)t = \frac{\beta}{\lambda}(\mu-1)t$$

$\Rightarrow$ Additional path difference $= (\mu-1)t$

$\Rightarrow$ If shift is equivalent to n fringes then $n = \dfrac{(\mu-1)t}{\lambda}$ or $t = \dfrac{n\lambda}{(\mu-1)}$

$\Rightarrow$ Shift is independent of the order of fringe (*i.e.* shift of zero order maxima = shift of n^{th} order maxima.

$\Rightarrow$ Shift is independent of wavelength.

(11) **Fringe visibility (*V*)**

With the help of visibility, knowledge about coherence, fringe contrast an interference pattern is obtained.

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = 2\frac{\sqrt{I_1 I_2}}{(I_1 + I_2)}$$ If $I_{min} = 0$, $V = 1$ (maximum) *i.e.*, fringe visibility will be best.

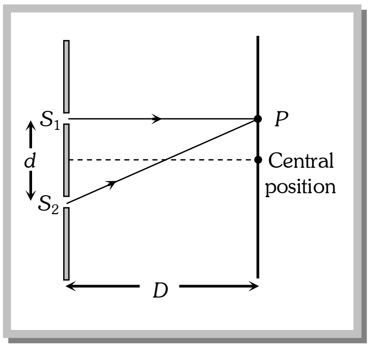
Also if $I_{max} = 0, V = -1$ and If $I_{max} = I_{min}, V = 0$

(12) **Missing wavelength in front of one of the slits in YDSE**

From figure $S_2P = \sqrt{D^2 + d^2}$ and $S_1P = D$

So the path difference between the waves reaching at P

$$\Delta = S_2P - S_1P = \sqrt{D^2 + d^2} - D = D\left(1 + \frac{d^2}{D^2}\right)^{1/2} - D$$



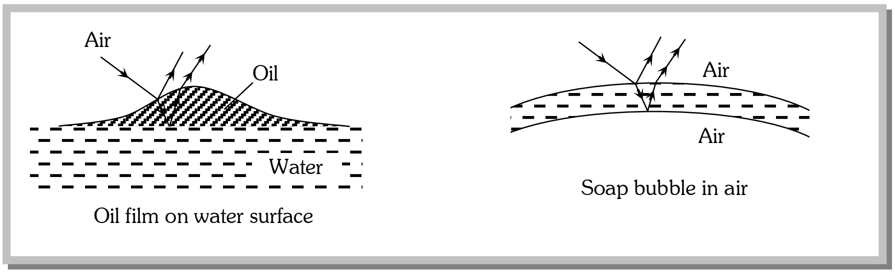
From binomial expansion $\Delta = D\left(1+\frac{1}{2}\frac{d^2}{D^2}\right) - D = \frac{d^2}{2D}$

For Dark at P $\Delta = \frac{d^2}{2D} = \frac{(2n-1)\lambda}{2}$ $\Rightarrow$ Missing wavelength at P $\boldsymbol{\lambda = \frac{d^2}{(2n-1)D}}$

By putting $n = 1, 2, 3 ....$ Missing wavelengths are $\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D} ....$

Illustrations of Interference

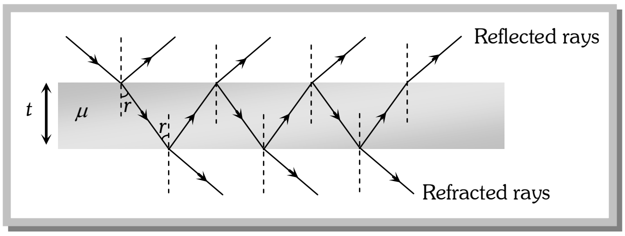
Interference effects are commonly observed in thin films when their thickness is comparable to wavelength of incident light (If it is too thin as compared to wavelength of light it appears dark and if it is too thick, this will result in uniform illumination of film). Thin layer of oil on water surface and soap bubbles shows various colours in white light due to interference of waves reflected from the two surfaces of the film.



Oil film on water surface

Soap bubble in air

(1) **Thin films :** In thin films interference takes place between the waves reflected from it's two surfaces and waves refracted through it.



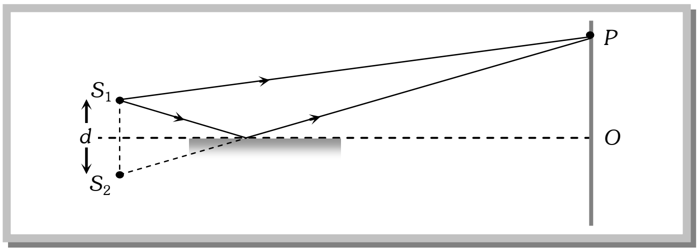
Interference in reflected light	**Interference in refracted light**
Condition of constructive interference (maximum intensity)	Condition of constructive interference (maximum intensity)
$\Delta = 2\mu\, t \cos r = (2n \pm 1)\frac{\lambda}{2}$	$\Delta = 2\mu t \cos r = (2n)\frac{\lambda}{2}$
For normal incidence $r = 0$	For normal incidence
so $2\mu t = (2n \pm 1)\frac{\lambda}{2}$	$2\mu t = n\lambda$
Condition of destructive interference (minimum intensity)	Condition of destructive interference (minimum intensity)
$\Delta = 2\mu t \cos r = (2n)\frac{\lambda}{2}$	$\Delta = 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$
For normal incidence $2\mu t = n\lambda$	For normal incidence $2\mu t = (2n \pm 1)\frac{\lambda}{2}$

Note : ≅The Thickness of the film for interference in visible light is of the order of $10,000\mathring{A}$.

(2) **Lloyd's Mirror**

A plane glass plate (acting as a mirror) is illuminated at almost grazing incidence by a light from a slit S_1. A virtual image S_2 of S_1 is formed closed to S_1 by reflection and these two act as coherent sources. The expression giving the fringe width is the same as for the double slit, but the fringe system differs in one important respect.

In Lloyd's mirror, if the point P, for example, is such that the path difference $S_2P - S_1P$ is a whole number of wavelengths, the fringe at P is dark not bright. This is due to 180° phase change which occurs when light is reflected from a denser medium. This is equivalent to adding an extra half wavelength to the path of the reflected wave. At grazing incidence a fringe is formed at O, where the geometrical path difference between the direct and reflected waves is zero and it follows that it will be dark rather than bright.



Thus, whenever there exists a phase difference of a π between the two interfering beams of light, conditions of maximas and minimas are interchanged, *i.e.*, $\Delta x = n\lambda$ (for minimum intensity)

and $\Delta x = (2n-1)\lambda/2$ (for maximum intensity)

Doppler's Effect in Light

The phenomenon of apparent change in frequency (or wavelength) of the light due to relative motion between the source of light and the observer is called Doppler's effect.

If ν = actual frequency, ν' = Apparent frequency, v = speed of source *w.r.t* stationary observer, c = speed of light

Source of light moves towards the stationary observer ($v << c$)	Source of light moves away from the stationary observer ($v << c$)
(i) Apparent frequency $\nu' = \nu\left(1+\frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1-\frac{v}{c}\right)$	(i) Apparent frequency $\nu' = \nu\left(1-\frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1+\frac{v}{c}\right)$
(ii) Doppler's shift : Apparent wavelength < actual wavelength, So spectrum of the radiation from the source of light shifts towards the red end of spectrum. This is called Red shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$	(ii) Doppler's shift : Apparent wavelength > actual wavelength, So spectrum of the radiation from the source of light shifts towards the violet end of spectrum. This is called Violet shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$

Note : ≅Doppler's shift $(\Delta\lambda)$ and time period of rotation (T) of a star relates as $\Delta\lambda = \frac{\lambda}{c} \times \frac{2\pi r}{T}$; r = radius of star.

Applications of Doppler effect

(i) Determination of speed of moving bodies (aeroplane, submarine etc) in RADAR and SONAR.

(ii) Determination of the velocities of stars and galaxies by spectral shift.

(iii) Determination of rotational motion of sun.

(iv) Explanation of width of spectral lines.

(v) Tracking of satellites. (vi) In medical sciences in echo cardiogram, sonography *etc*.

Concepts

- *The angular thickness of fringe width is defined as* $\delta = \frac{\beta}{D} = \frac{\lambda}{d}$, *which is independent of the screen distance D.*
- *Central maxima means the maxima formed with zero optical path difference. It may be formed anywhere on the screen.*
- *All the wavelengths produce their central maxima at the same position.*
- *The wave with smaller wavelength from its maxima before the wave with longer wavelength.*
- *The first maxima of violet colour is closest and that for the red colour is farthest.*
- *Fringes with blue light are thicker than those for red light.*
- *In an interference pattern, whatever energy disappears at the minimum, appears at the maximum.*
- *In YDSE, the nth maxima always comes before the nth minima.*
- *In YDSE, the ratio* $\frac{I_{\max}}{I_{\min}}$ *is maximum when both the sources have same intensity.*
- *For two interfering waves if initial phase difference between them is* ϕ_0 *and phase difference due to path difference between them is* ϕ'. *Then total phase difference will be* $\phi = \phi_0 + \phi' = \phi_0 + \frac{2\pi}{\lambda}\Delta$.
- *Sometimes maximm number of maximas or minimas are asked in the question which can be obtained on the screen. For this we use the fact that value of* $\sin\theta$ *(or* $\cos\theta$*) can't be greater than 1. For example in the first case when the slits are vertical*

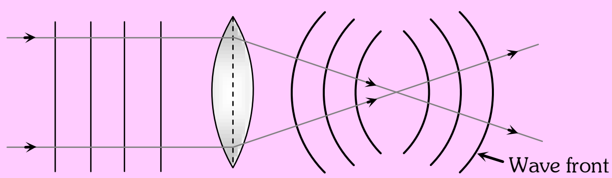
$$\sin\theta = \frac{n\lambda}{d} \qquad \textit{(for maximum intensity)}$$

$$\because \quad \sin\theta \not> 1 \quad \therefore \quad \frac{n\lambda}{d} \not> 1 \quad \text{or} \quad n \not> \frac{d}{\lambda}$$

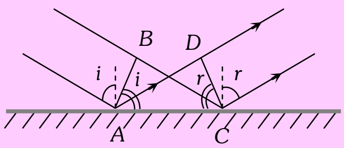
Suppose in some question d/λ *comes out say 4.6, then total number of maximuas on the screen will be 9. Corresponding to* $n = 0, \pm 1, \pm 2, \pm 3$ *and* ± 4.

- ***Shape of wave front***

If rays are parallel, wave front is plane. If rays are converging wave front is spherical of decreasing radius. If rays are diverging wave front is spherical of increasing radius.

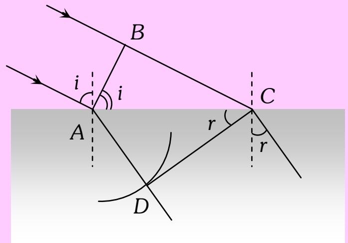


- *Reflection and refraction of wave front*



Reflection

BC = AD and ∠ i = ∠ r



Refraction

$$\frac{BC}{AD} = \frac{v_1}{v_2} = \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

Example

***Example*: 1** If two light waves having same frequency have intensity ratio 4 : 1 and they interfere, the ratio of maximum to minimum intensity in the pattern will be **[BHU 1995; MP PMT 1995; DPMT 1999; CPMT 2003]**

(a) 9 : 1 (b) 3 : 1 (c) 25 : 9 (d) 16 : 25

Solution: (a) By using $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right)^2 = \left(\frac{\sqrt{\frac{4}{1}}+1}{\sqrt{\frac{4}{1}}-1}\right)^2 = \frac{9}{1}$.

***Example*: 2** In Young's double slit experiment using sodium light (λ = 5898Å), 92 fringes are seen. If given colour (λ = 5461Å) is used, how many fringes will be seen **[RPET 1996; JIPMER 2001, 2002]**

(a) 62 (b) 67 (c) 85 (d) 99

Solution: (d) By using $n_1\lambda_1 = n_2\lambda_2 \Rightarrow 92 \times 5898 = n_2 \times 5461 \Rightarrow n_2 = 99$

***Example*: 3** Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point A and π at point B. Then the difference between the resultant intensities at A and B is **[IIT-JEE (Screening) 2001]**

(a) $2I$ (b) $4I$ (c) $5I$ (d) $7I$

Solution: (b) By using $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

At point A : Resultant intensity $I_A = I + 4I + 2\sqrt{I \times 4I}\cos\frac{\pi}{2} = 5I$

At point B : Resultant intensity $I_B = I + 4I + 2\sqrt{I \times 4I}\cos\pi = I$. Hence the difference $= I_A - I_B = 4I$

***Example*: 4** If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, the amplitude of the resulting wave will be about **[MP PMT 2000]**

(a) 7 (b) 6 (c) 5 (d) 3.

Solution: (b) By using $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} \Rightarrow A = \sqrt{(4)^2 + (3)^2 + 2\times 4\times 3\cos\frac{\pi}{3}} = \sqrt{37} \approx 6$.

Example: 5 Two waves being produced by two sources S_1 and S_2. Both sources have zero phase difference and have wavelength λ. The destructive interference of both the waves will occur of point *P* if $(S_1P - S_2P)$ has the value **[MP PET 1987]**

(a) 5λ (b) $\frac{3}{4}\lambda$ (c) 2λ (d) $\frac{11}{2}\lambda$

Solution: (d) For destructive interference, path difference the waves meeting at *P* (*i.e.* $S_1P - S_2P$) must be odd multiple of $\lambda/2$. Hence option (d) is correct.

Example: 6 Two interfering wave (having intensities are 9*I* and 4*I*) path difference between them is 11 λ. The resultant intensity at this point will be

(a) I (b) $9\,I$ (c) $4\,I$ (d) $25\,I$

Solution: (d) Path difference $\Delta = \frac{\lambda}{2\pi}\times\phi \Rightarrow \frac{2\pi}{\lambda}\times 11\lambda = 22\pi$ *i.e.* constructive interference obtained at the same point

So, resultant intensity $I_R = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{9I} + \sqrt{4I})^2 = 25I$.

Example: 7 In interference if $\frac{I_{\max}}{I_{\min}} = \frac{144}{81}$ then what will be the ratio of amplitudes of the interfering wave

(a) $\frac{144}{81}$ (b) $\frac{7}{1}$ (c) $\frac{1}{7}$ (d) $\frac{12}{9}$

Solution: (b) By using $\frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{\max}}{I_{\min}}}+1}{\sqrt{\frac{I_{\max}}{I_{\min}}}-1}\right) = \left(\frac{\sqrt{\frac{144}{81}}+1}{\sqrt{\frac{144}{81}}-1}\right) = \left(\frac{\frac{12}{9}+1}{\frac{12}{5}-1}\right) = \frac{7}{1}$

Example: 8 Two interfering waves having intensities *x* and *y* meets a point with time difference 3*T*/2. What will be the resultant intensity at that point

(a) $(\sqrt{x} + \sqrt{y})$ (b) $(\sqrt{x} + \sqrt{y} + \sqrt{xy})$ (c) $x + y + 2\sqrt{xy}$ (d) $\frac{x+y}{2xy}$

Solution: (c) Time difference T.D. $= \frac{T}{2\pi}\times\phi \Rightarrow \frac{3T}{2} = \frac{T}{2\pi}\times\phi \Rightarrow \phi = 3\pi$; This is the condition of constructive interference.

So resultant intensity $I_R = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$.

Example: 9 In Young's double-slit experiment, an interference pattern is obtained on a screen by a light of wavelength 6000 Å, coming from the coherent sources S_1 and S_2. At certain point *P* on the screen third dark fringe is formed. Then the path difference $S_1P - S_2P$ in microns is **[EAMCET 2003]**

(a) 0.75 (b) 1.5 (c) 3.0 (d) 4.5

Solution: (b) For dark fringe path difference $\Delta = (2n-1)\frac{\lambda}{2}$; here $n = 3$ and $\lambda = 6000\times 10^{-10}\,m$

So $\Delta = (2\times 3 - 1)\times\frac{6\times 10^{-7}}{2} = 15\times 10^{-7}\,m = 1.5\,microns$.

***Example*: 10** In a Young's double slit experiment, the slit separation is 1 *mm* and the screen is 1 *m* from the slit. For a monochromatic light of wavelength 500 *nm*, the distance of 3rd minima from the central maxima is **[Orissa JEE 2003]**

(a) 0.50 *mm* (b) 1.25 *mm* (c) 1.50 *mm* (d) 1.75 *mm*

Solution: (b) Distance of n^{th} minima from central maxima is given as $x = \frac{(2n-1)\lambda D}{2d}$

So here $x = \frac{(2\times 3 - 1)\times 500\times 10^{-9}\times 1}{2\times 10^{-3}} = 1.25\,mm$

***Example*: 11** The two slits at a distance of 1 *mm* are illuminated by the light of wavelength $6.5\times 10^{-7}\,m$. The interference fringes are observed on a screen placed at a distance of 1 *m*. The distance between third dark fringe and fifth bright fringe will be **[NCERT 1982; MP PET 1995; BVP 2003]**

(a) 0.65 *mm* (b) 1.63 *mm* (c) 3.25 *mm* (d) 4.88 *mm*

Solution: (b) Distance between n^{th} bright and m^{th} dark fringe ($n > m$) is given as $x = \left(n - m + \frac{1}{2}\right)\beta = \left(n - m + \frac{1}{2}\right)\frac{\lambda D}{d}$

$\Rightarrow\ x = \left(5 - 3 + \frac{1}{2}\right)\times \frac{6.5\times 10^{-7}\times 1}{1\times 10^{-3}} = 1.63\,mm.$

***Example*: 12** The slits in a Young's double slit experiment have equal widths and the source is placed symmetrically relative to the slits. The intensity at the central fringes is I_0. If one of the slits is closed, the intensity at this point will be **[MP PMT 1999]**

(a) I_0 (b) $I_0/4$ (c) $I_0/2$ (d) $4I_0$

Solution: (b) By using $I_R = 4I\cos^2\frac{\phi}{2}$ {where I = Intensity of each wave}

At central position $\phi = 0°$, hence initially $I_0 = 4I$.

If one slit is closed, no interference takes place so intensity at the same location will be I only *i.e.* intensity become s $\frac{1}{4}th$ or $\frac{I_0}{4}$.

***Example*: 13** In double slit experiment, the angular width of the fringes is 0.20° for the sodium light (λ = 5890 Å). In order to increase the angular width of the fringes by 10%, the necessary change in the wavelength is **[MP PMT 1997]**

(a) Increase of 589 Å (b) Decrease of 589 Å (c) Increase of 6479 Å (d) Zero

Solution: (a) By using $\theta = \frac{\lambda}{d} \Rightarrow \frac{\theta_1}{\theta_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{0.20^o}{(0.20^o + 10\%\,\text{of}\,0.20)} = \frac{5890}{\lambda_2} \Rightarrow \frac{0.20}{0.22} = \frac{5890}{\lambda_2} \Rightarrow \lambda_2 = 6479$

So increase in wavelength = 6479 – 5890 = 589 Å.

***Example*: 14** In Young's experiment, light of wavelength 4000 Å is used, and fringes are formed at 2 *metre* distance and has a fringe width of 0.6 *mm*. If whole of the experiment is performed in a liquid of refractive index 1.5, then width of fringe will be **[MP PMT 1994, 97]**

(a) 0.2 *mm* (b) 0.3 *mm* (c) 0.4 *mm* (d) 1.2 *mm*

Solution: (c) $\beta_{medium} = \frac{\beta_{air}}{\mu} \Rightarrow \beta_{medium} = \frac{0.6}{1.5} = 0.4\,mm.$

***Example*: 15** Two identical sources emitted waves which produces intensity of *k* unit at a point on screen where path difference is λ. What will be intensity at a point on screen at which path difference is $\lambda/4$ **[RPET 1996]**

(a) $\frac{k}{4}$ (b) $\frac{k}{2}$ (c) *k* (d) Zero

Solution: (b) By using phase difference $\phi = \frac{2\pi}{\lambda}(\Delta)$

For path difference λ, phase difference $\phi_1 = 2\pi$ and for path difference $\lambda/4$, phase difference $\phi_2 = \pi/2$.

Also by using $I = 4I_0 \cos^2\frac{\phi}{2} \Rightarrow \frac{I_1}{I_2} = \frac{\cos^2(\phi_1/2)}{\cos^2(\phi_2/2)} \Rightarrow \frac{k}{I_2} = \frac{\cos^2(2\pi/2)}{\cos^2\left(\frac{\pi/2}{2}\right)} = \frac{1}{1/2} \Rightarrow I_2 = \frac{k}{2}$.

***Example*: 16** A thin mica sheet of thickness $2\times10^{-6}\,m$ and refractive index (μ = 1.5) is introduced in the path of the first wave. The wavelength of the wave used is 5000Å. The central bright maximum will shift **[CPMT 1999]**

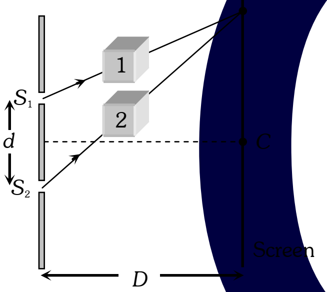
(a) 2 fringes upward (b) 2 fringes downward (c) 10 fringes upward (d) None of these

Solution: (a) By using shift $\Delta x = \frac{\beta}{\lambda}(\mu-1)t \Rightarrow \Delta x = \frac{\beta}{5000\times10^{-10}}(1.5-1)\times2\times10^{-6} = 2\beta$

Since the sheet is placed in the path of the first wave, so shift will be 2 fringes upward.

***Example*: 17** In a *YDSE* fringes are observed by using light of wavelength 4800 Å, if a glass plate (μ = 1.5) is introduced in the path of one of the wave and another plates is introduced in the path of the (μ = 1.8) other wave. The central fringe takes the position of fifth bright fringe. The thickness of plate will be

(a) 8 *micron* (b) 80 *micron* (c) 0.8 *micron* (d) None of these



Solution: (a) Shift due to the first plate $x_1 = \frac{\beta}{\lambda}(\mu_1-1)t$ (Upward)

and shift due to the second $x_2 = \frac{\beta}{\lambda}(\mu_2-1)t$ (Downward)

Hence net shift = $x_2 - x_1 = \frac{\beta}{\lambda}(\mu_2-\mu_1)t$

$\Rightarrow 5\beta = \frac{\beta}{\lambda}(1.8-1.5)t \Rightarrow t = \frac{5\lambda}{0.3} = \frac{5\times4800\times10^{-10}}{0.3} = 8\times10^{-6}\,m = 8\,micron$.

***Example*: 18** In young double slit experiment $\frac{d}{D} = 10^{-4}$ (d = distance between slits, D = distance of screen from the slits). At a point P on the screen resulting intensity is equal to the intensity due to individual slit I_0. Then the distance of point P from the central maxima is (λ = 6000 Å)

(a) 2 *mm* (b) 1 *mm* (c) 0.5 *mm* (d) 4 *mm*

Solution: (a) By using shift $I = 4I_0\cos^2(\phi/2) \Rightarrow I_0 = 4I_0\cos^2(\phi/2) \Rightarrow \cos(\phi/2) = \frac{1}{2}$ or $\frac{\phi}{2} = \frac{\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$

Also path difference $\Delta = \frac{xd}{D} = \frac{\lambda}{2\pi}\times\phi \Rightarrow x\times\left(\frac{d}{D}\right) = \frac{6000\times10^{-10}}{2\pi}\times\frac{2\pi}{3} \Rightarrow x = 2\times10^{-3}\,m = 2mm.$

***Example*: 19** Two identical radiators have a separation of $d = \lambda/4$, where λ is the wavelength of the waves emitted by either source. The initial phase difference between the sources is $\pi/4$. Then the intensity on the screen at a distance point situated at an angle $\theta = 30°$ from the radiators is (here I_0 is the intensity at that point due to one radiator)

(a) I_0 (b) $2I_0$ (c) $3I_0$ (d) $4I_0$

Solution: (a) Initial phase difference $\phi_0 = \frac{\pi}{4}$; Phase difference due to path difference $\phi' = \frac{2\pi}{\lambda}(\Delta)$

where $\Delta = d\sin\theta \Rightarrow \phi' = \frac{2\pi}{\lambda}(d\sin\theta) = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}(\sin 30^o) = \frac{\pi}{4}$

Hence total phase difference $\phi = \phi_0 + \phi' = \frac{\phi}{4}$. By using $I = 4I_0\cos^2(\phi/2) = 4I_0\cos^2\left(\frac{\pi/2}{2}\right) = 2I_0$.

***Example*: 20** In *YDSE* a source of wavelength 6000 Å is used. The screen is placed 1 *m* from the slits. Fringes formed on the screen, are observed by a student sitting close to the slits. The student's eye can distinguish two neighbouring fringes. If they subtend an angle more than 1 minute of arc. What will be the maximum distance between the slits so that the fringes are clearly visible

(a) 2.06 *mm* (b) 2.06 *cm* (c) 2.06×10^{-3} *mm* (d) None of these

Solution: (a) According to given problem angular fringe width $\theta = \frac{\lambda}{d} \geq \frac{\pi}{180 \times 60}$ [As $1' = \frac{\pi}{180 \times 60} rad$]

i.e. $d < \frac{6 \times 10^{-7} \times 180 \times 60}{\pi}$ *i.e.* $d < 2.06 \times 10^{-3} m \Rightarrow d_{max} = 2.06 mm$

***Example*: 21** the maximum intensity in case of interference of *n* identical waves, each of intensity I_0, if the interference is (i) coherent and (ii) incoherent respectively are

(a) n^2I_0, nI_0 (b) nI_0, n^2I_0 (c) nI_0, I_0 (d) $n^2I_0, (n-1)I_0$

Solution: (a) In case of interference of two wave $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$

(i) In case of coherent interference ϕ does not vary with time and so I will be maximum when $\cos\phi = \max = 1$

i.e. $(I_{max})_{co} = I_1 + I_2 + 2\sqrt{I_1I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

So for *n* identical waves each of intensity I_0 $(I_{max})_{co} = (\sqrt{I_0} + \sqrt{I_0} + ...)^2 = (n\sqrt{I_0})^2 = n^2I_0$

(ii)In case of incoherent interference at a given point, ϕ varies randomly with time, so $(\cos\phi)_{av} = 0$ and hence $(I_R)_{Inco} = I_1 + I_2$

So in case of *n* identical waves $(I_R)_{Inco} = I_0 + I_0 + ... = nI_0$

***Example*: 22** The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width. The ratio of the maximum to the minimum intensity in interference pattern will be

(a) $\frac{1}{a}$ (b) $\frac{9}{1}$ (c) $\frac{2}{1}$ (d) $\frac{1}{2}$

Solution: (b) $A_{max} = 2A + A = 3A$ and $A_{min} = 2A - A = A$. Also $\frac{I_{max}}{I_{min}} = \left(\frac{A_{max}}{A_{min}}\right)^2 = \left(\frac{3A}{A}\right)^2 = \frac{9}{1}$

***Example*: 23** A star is moving towards the earth with a speed of $4.5 \times 10^6 m/s$. If the true wavelength of a certain line in the spectrum received from the star is 5890 Å, its apparent wavelength will be about $[c = 3 \times 10^8 m/s]$

[MP PMT 1999]

(a) 5890 Å (b) 5978 Å (c) 5802 Å (d) 5896 Å

Solution: (c) By using $\lambda' = \lambda\left(1 - \frac{v}{c}\right) \Rightarrow \lambda' = 5890\left(1 - \frac{4.5 \times 10^6}{3 \times 10^8}\right) = 5802\,\text{Å}$.

***Example*: 24** Light coming from a star is observed to have a wavelength of 3737 Å, while its real wavelength is 3700 Å. The speed of the star relative to the earth is [Speed of light $= 3 \times 10^8 m/s$] **[MP PET 1997]**

(a) $3 \times 10^5 m/s$ (b) $3 \times 10^6 m/s$ (c) $3.7 \times 10^7 m/s$ (d) $3.7 \times 10^6 m/s$

Solution: (b) By using $\Delta\lambda = \lambda\frac{v}{c} \Rightarrow$ (3737-3700)= $3700 \times \frac{v}{3 \times 10^8} \Rightarrow v = 3 \times 10^6\ m/s$.

***Example*: 25** Light from the constellation Virgo is observed to increase in wavelength by 0.4%. With respect to Earth the constellation is **[MP PMT 1994, 97; MP PET 2003]**

(a) Moving away with velocity $1.2 \times 10^6 m/s$ (b) Coming closer with velocity $1.2 \times 10^6 m/s$

(c) Moving away with velocity $4 \times 10^6 m/s$ (d) Coming closer with velocity $4 \times 10^6 m/s$

Solution: (a) By using $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$; where $\frac{\Delta\lambda}{\lambda} = \frac{0.4}{100}$ and $c = 3 \times 10^8\ m/s \Rightarrow \frac{0.4}{100} = \frac{v}{3 \times 10^8} \Rightarrow v = 1.2 \times 10^6\ m/s$

Since wavelength is increasing *i.e.* it is moving away.

Tricky example: 1

In *YDSE*, distance between the slits is $2 \times 10^{-3}\ m$, slits are illuminated by a light of wavelength 2000Å – 9000 Å. In the field of view at a distance of $10^{-3}\ m$ from the central position which wavelength will be observe. Given distance between slits and screen is 2.5 *m*

(a) 40000 Å (b) 4500 Å (c) 5000 Å (d) 5500 Å

Solution : (b) $x = \frac{n\lambda D}{d} \Rightarrow \lambda = \frac{xd}{nD} = \frac{10^{-3} \times 2 \times 10^{-3}}{n \times 2.5} \Rightarrow \frac{8 \times 10^{-7}}{n} m = \frac{8000}{n}\text{Å}$

For n = 1, 2, 3....... λ = 8000 Å, 4000 Å, $\frac{8000}{3}$ Å.........

Hence only option (a) is correct.

Tricky example: 2

I is the intensity due to a source of light at any point *P* on the screen. If light reaches the point *P* via two different paths (a) direct (b) after reflection from a plane mirror then path difference between two paths is $3\lambda/2$, the intensity at *P* is

(a) *I* (b) Zero (c) 2*I* (d) 4*I*

Solution : (d) Reflection of light from plane mirror gives additional path difference of $\lambda/2$ between two waves

$\therefore$ Total path difference $= \frac{3\lambda}{2} + \frac{\lambda}{2} = 2\lambda$

Which satisfies the condition of maxima. Resultant intensity $= (\sqrt{I} + \sqrt{I})^2 = 4I$.

Tricky example: 3

I *B* *B'*

A ray of light of intensity I is incident on a parallel glass-slab at a point A as shown in figure. It undergoes partial reflection and refraction. At each reflection 25% of incident energy is reflected. The rays AB and $A'B'$ undergo interference. The ratio I_{max}/I_{min} is **[IIT-JEE 1990]**

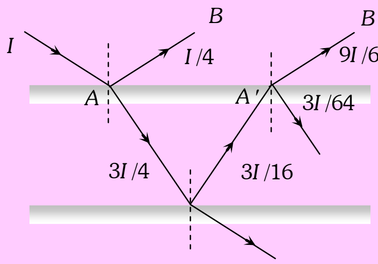
(a) 4 : 1

(b) 8 : 1

(c) 7 : 1

(d) 49 : 1

Solution : (d) From figure $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{64}$ $\Rightarrow \frac{I_2}{I_1} = \frac{9}{16}$



By using $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1}\right) = \left(\frac{\sqrt{\frac{9}{16}}+1}{\sqrt{\frac{9}{16}}-1}\right) = \frac{49}{1}$

Fresnel's Biprism.

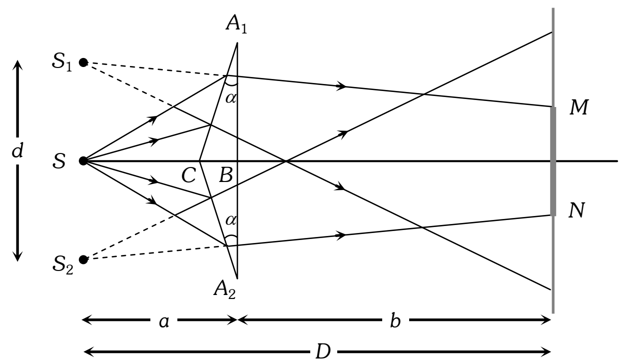
(1) It is an optical device of producing interference of light Fresnel's biprism is made by joining base to base two thin prism (A_1BC and A_2BC as shown in the figure) of very small angle or by grinding a thick glass plate.

(2) Acute angle of prism is about 1/2° and obtuse angle of prism is about 179°.

(3) When a monochromatic light source is kept in front of biprism two coherent virtual source S_1 and S_2 are produced.

(4) Interference fringes are found on the screen (in the MN region) placed behind the biprism interference fringes are formed in the limited region which can be observed with the help eye piece.

(5) Fringe width is measured by a micrometer attached to the eye piece. Fringes are of equal width and its value is $\beta = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{\beta d}{D}$

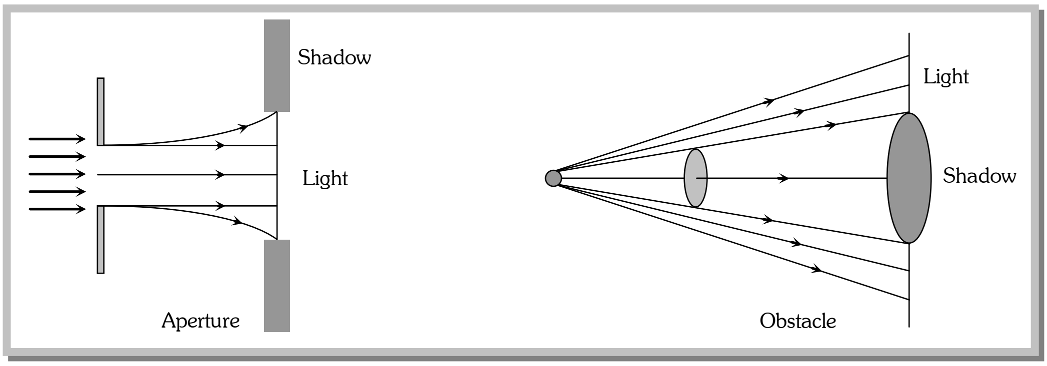


Let the separation between S_1 and S_2 be d and the distance of slits and the screen from the biprism be a and b respectively *i.e.* $D = (a + b)$. If angle of prism is α and refractive index is μ then $d = 2a(\mu - 1)\alpha$

$$\therefore \qquad \lambda = \frac{\beta[2a(\mu-1)\alpha]}{(a+b)} \quad \Rightarrow \quad \beta = \frac{(a+b)\lambda}{2a(\mu-1)\alpha}$$

Diffraction of Light.

It is the phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength of light.



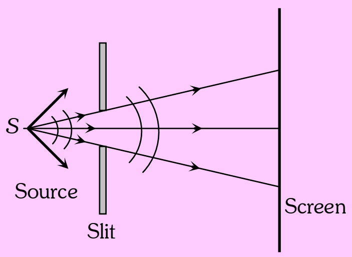
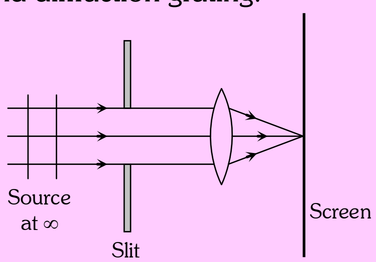
Note : ≅Diffraction is the characteristic of all types of waves.

≅ Greater the wavelength of wave, higher will be it's degree of diffraction.

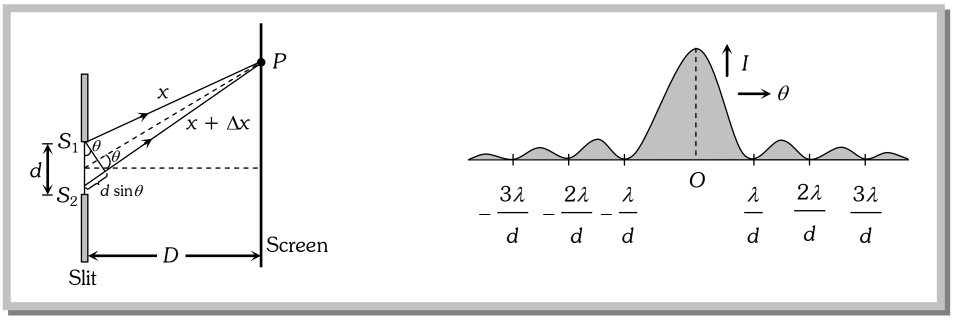
≅ Experimental study of diffraction was extended by Newton as well as Young. Most systematic study carried out by Huygens on the basis of wave theory.

≅The minimum distance at which the observer should be from the obstacle to observe the diffraction of light of wavelength λ around the obstacle of size d is given by $x = \dfrac{d^2}{4\lambda}$.

(1) **Types of diffraction :** The diffraction phenomenon is divided into two types

Fresnel diffraction	Fraunhofer diffraction
(i) If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel type.	(i) In this case both source and screen are effectively at infinite distance from the diffracting device.
(ii) Common examples : Diffraction at a straight edge, narrow wire or small opaque disc etc.	(ii) Common examples : Diffraction at single slit, double slit and diffraction grating.
	

(2) **Diffraction of light at a single slit :** In case of diffraction at a single slit, we get a central bright band with alternate bright (maxima) and dark (minima) bands of decreasing intensity as shown



(i) Width of central maxima $\beta_0 = \frac{2\lambda D}{d}$; and angular width $= \frac{2\lambda}{d}$

(ii) Minima occurs at a point on either side of the central maxima, such that the path difference between the waves from the two ends of the aperture is given by $\Delta = n\lambda$; where $n = 1, 2, 3 ....$

i.e. $d \sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$

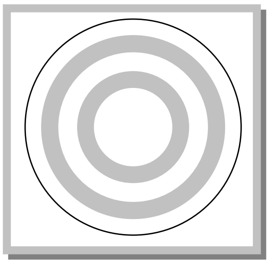
(iii) The secondary maxima occurs, where the path difference between the waves from the two ends of the aperture is given by $\Delta = (2n+1)\frac{\lambda}{2}$; where $n = 1, 2, 3 ....$

i.e. $d \sin\theta = (2n+1)\frac{\lambda}{2} \Rightarrow \sin\theta = \frac{(2n+1)\lambda}{2d}$

(3) **Comparison between interference and diffraction**

Interference	Diffraction
Results due to the superposition of waves from two coherent sources.	Results due to the superposition of wavelets from different parts of same wave front. (single coherent source)
All fringes are of same width $\beta = \frac{\lambda D}{d}$	All secondary fringes are of same width but the central maximum is of double the width $\beta_0 = 2\beta = 2\frac{\lambda D}{d}$
All fringes are of same intensity	Intensity decreases as the order of maximum increases.
Intensity of all minimum may be zero	Intensity of minima is not zero.
Positions of nth maxima and minima $x_{n(\text{Bright})} = \frac{n\lambda D}{d}$, $x_{n(\text{Dark})} = (2n-1)\frac{\lambda D}{d}$	Positions of nth secondary maxima and minima $x_{n(\text{Bright})} = (2n+1)\frac{\lambda D}{d}$, $x_{n(\text{Dark})} = \frac{n\lambda D}{d}$
Path difference for nth maxima $\Delta = n\lambda$	for nth secondary maxima $\Delta = (2n+1)\frac{\lambda}{2}$
Path difference for nth minima $\Delta = (2n-1)\lambda$	Path difference for nth minima $\Delta = n\lambda$

(4) **Diffraction and optical instruments :** The objective lens of optical instrument like telescope or microscope etc. acts like a circular aperture. Due to diffraction of light at a circular aperture, a converging lens cannot form a point image of an object rather it produces a brighter disc known as Airy disc surrounded by alternate dark and bright concentric rings.

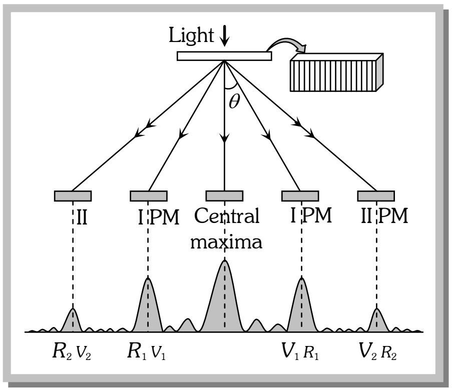


The angular half width of Airy disc $= \theta = \dfrac{1.22\lambda}{D}$ (where D = aperture of lens)

The lateral width of the image $= f\theta$ (where f = focal length of the lens)

Note : ≅Diffraction of light limits the ability of optical instruments to form clear images of objects when they are close to each other.

(5) **Diffraction grating :** Consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the diffraction of principle maxima (PM) is given by $d\sin\theta = n\lambda$; where d = distance between two consecutive slits and is called grating element.

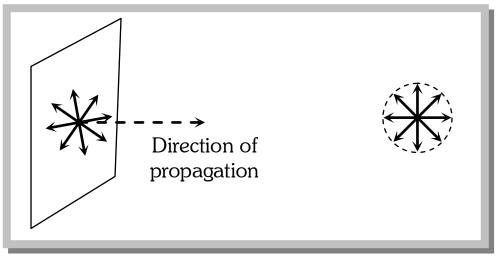


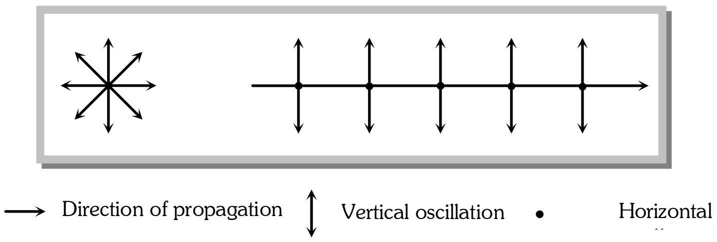
Polarisation of Light

Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. We generally prefer to describe light as electric field oscillations.

(1) **Unpolarised light**

The light having electric field oscillations in all directions in the plane perpendicular to the direction of propagation is called Unpolarised light. The oscillation may be resolved into horizontal and vertical component.





(2) **Polarised light**

The light having oscillations only in one plane is called Polarised or plane polarised light.

(i) The plane in which oscillation occurs in the polarised light is called plane of oscillation.

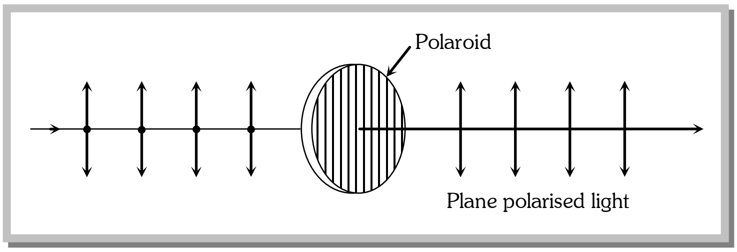
(ii) The plane perpendicular to the plane of oscillation is called plane of polarisation.

(iii) Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.

(3) **Polaroids**

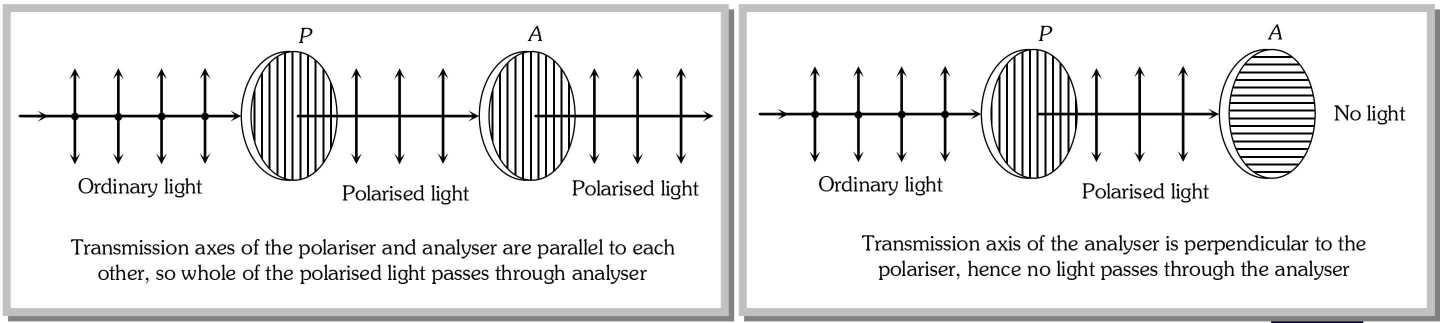
It is a device used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal. **or**

It is a thin film of ultramicroscopic crystals of quinine idosulphate with their optic axis parallel to each other.



(i) Polaroids allow the light oscillations parallel to the transmission axis pass through them.

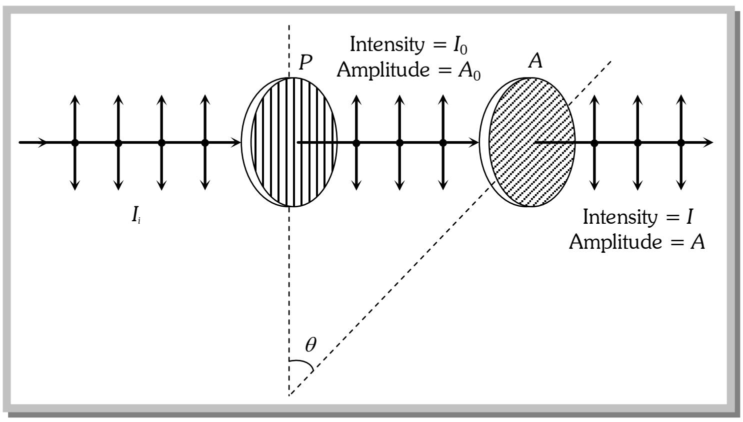
(ii) The crystal or polaroid on which unpolarised light is incident is called polariser. Crystal or polaroid on which polarised light is incident is called analyser.



Note : ≅ When unpolarised light is incident on the polariser, the intensity of the transmitted polarised light is half the intensity of unpolarised light.

(4) Malus law

This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.



(i) $I = I_0 \cos^2\theta$ and $A^2 = A_0^2 \cos^2\theta \Rightarrow A = A_0 \cos\theta$

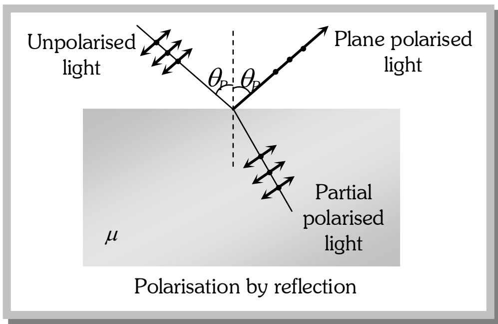
If $\theta = 0^o$, $I = I_0$, $A = A_0$, If $\theta = 45^o$, $I = \frac{I_0}{2}$, $A = \frac{A_0}{\sqrt{2}}$, If $\theta = 90^o$, $I = 0$, $A = 0$

(ii) If $I_i =$ Intensity of unpolarised light.

So $I_0 = \frac{I_i}{2}$ *i.e.* if an unpolarised light is converted into plane polarised light (say by passing it through a plaroid or a Nicol-prism), its intensity becomes half. and $I = \frac{I_i}{2}\cos^2\theta$

Note : ≅Percentage of polarisation $= \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100$

(5) **Brewster's law :** Brewster discovered that when a beam of unpolarised light is reflected from a transparent medium (refractive index $=\mu$), the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation θ_p).



Polarisation by reflection

Also $\mu = \tan\theta_p$ Brewster's law

(i) For $i < \theta_P$ or $i > \theta_P$

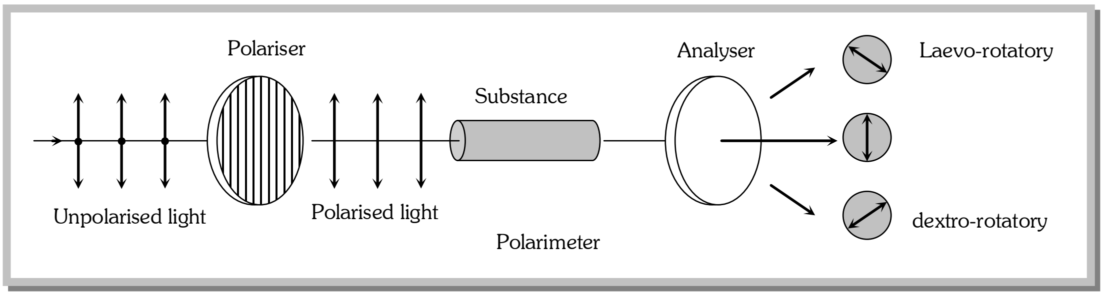
Both reflected and refracted rays becomes partially polarised

(ii) For glass $\theta_P \approx 57^o$, for water $\theta_P \approx 53^o$

(6) **Optical activity and specific rotation**

When plane polarised light passes through certain substances, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle. This phenomenon is called optical activity or optical rotation and the substances optically active.

If the optically active substance rotates the plane of polarisation clockwise (looking against the direction of light), it is said to be *dextro-rotatory* or *right-handed.* However, if the substance rotates the plane of polarisation anti-clockwise, it is called *laevo-rotatory* or *left-handed.*



The optical activity of a substance is related to the asymmetry of the molecule or crystal as a whole, *e.g.*, a solution of cane-sugar is dextro-rotatory due to asymmetrical molecular structure while crystals of quartz are dextro or laevo-rotatory due to structural asymmetry which vanishes when quartz is fused.

Optical activity of a substance is measured with help of polarimeter in terms of 'specific rotation' which is defined as the rotation produced by a solution of length 10 *cm* (1 *dm*) and of unit concentration (*i.e.* 1 *g*/cc) for a given wavelength of light at a given temperature. *i.e.* $[\alpha]_{t^{o}C}^{\lambda} = \frac{\theta}{L \times C}$ where θ is the rotation in length L at concentration C.

(7) **Applications and uses of polarisation**

(i) By determining the polarising angle and using Brewster's law, *i.e.* $\mu = \tan\theta_P$, refractive index of dark transparent substance can be determined.

(ii) It is used to reduce glare.

(iii) In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display **(LCD)**.

(iv) In CD player polarised laser beam acts as needle for producing sound from compact disc which is an encoded digital format.

(v) It has also been used in recording and reproducing three-dimensional pictures.

(vi) Polarisation of scattered sunlight is used for navigation in solar-compass in polar regions.

(vii) Polarised light is used in optical stress analysis known as 'photoelasticity'.

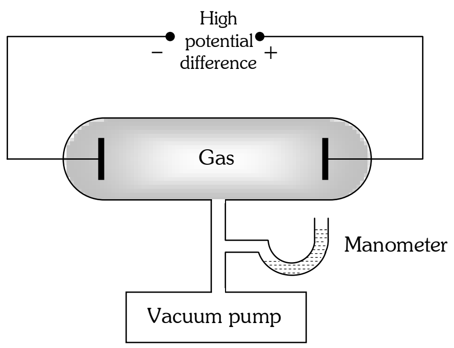
(viii) Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of 'optical activity'.

Electric Discharge Through Gases.

At normal atmospheric pressure, the gases are poor conductor of electricity. If we establish a potential difference (of the order of 30 *kV*) between two electrodes placed in air at a distance of few *cm* from each other, electric conduction starts in the form of sparks.

The passage of electric current through air is called electric discharge through the air.

The discharge of electricity through gases can be systematically studied with the help of discharge tube shown below



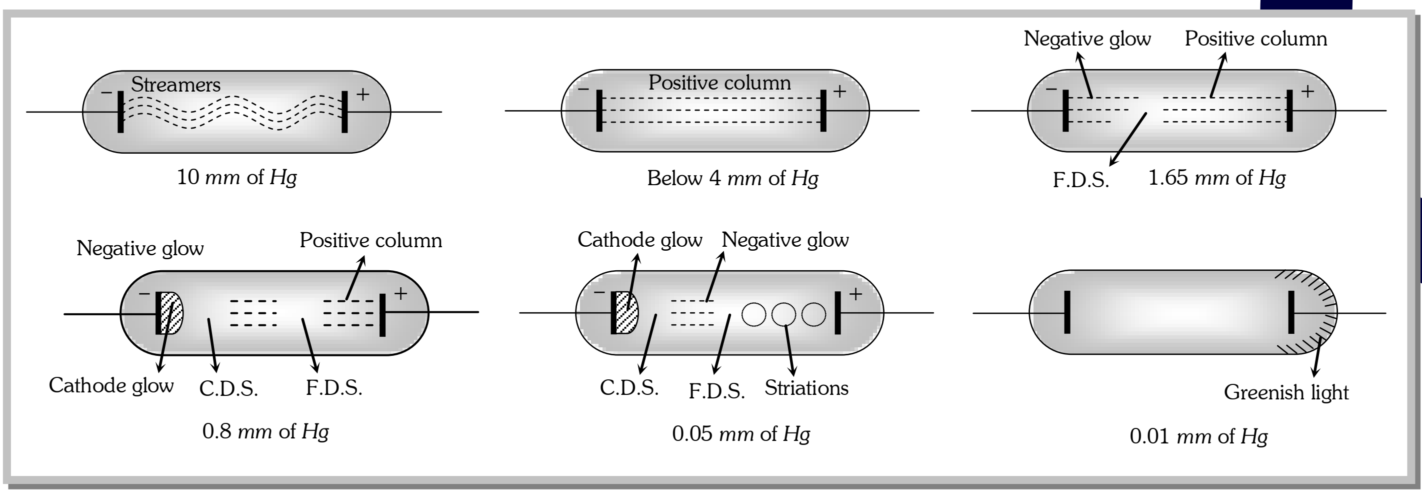
Length of discharge tube ≈ 30 to 40 *cm*

Diameter of the tube ≈ 4*cm*

The discharge tube is filled with the gas through which discharge is to be studied. The pressure of the enclosed gas can be reduced with the help of a vacuum pump and it's value is read by manometer.

Sequence of phenomenon

As the pressure inside the discharge tube is gradually reduced, the following is the sequence of phenomenon that are observed.



(1) At normal pressure no discharge takes place.

(2) At the pressure 10 *mm* of *Hg*, a zig-zag thin red spark runs from one electrode to other and cracking sound is heard.

(3) At the pressure 4 *mm.* of *Hg*, an illumination is observed at the electrodes and the rest of the tube appears dark. This type of discharge is called dark discharge.

(4) When the pressure falls below 4 *mm* of *Hg* then the whole tube is filled with bright light called positive column and colour of light depends upon the nature of gas in the tube as shown in the following table.

Gas	Colour
Air	Purple red
H_2	Blue
N_2	Red
Cl_2	Green
CO_2	Bluish white
Na	Yellow
Neon	Dark red

(5) At a pressure of 1.65 *mm* of *Hg* :

(i) Sky colour light is produced at the cathode it is called as negative glow.

(ii) Positive column shrinks towards the anode and the dark space between positive column and negative glow is called Faradays dark space (FDS)

(6) At a pressure of 0.8 *mm Hg* : At this pressure, negative glow is detached from the cathode and moves towards the anode. The dark space created between cathode and negative glow is called as Crook's dark space length of positive column further reduced. A glow appear at cathode called cathode glow.

(7) At a pressure of 0.05 *mm* of *Hg* : The positive column splits into dark and bright disc of light called striations.

(8) At the pressure of 0.01 or 10^{-2} *mm* of *Hg* some invisible particle move from cathode which on striking with the glass tube of the opposite side of cathode cause the tube to glow. These invisible rays emerging from cathode are called cathode rays.

(9) Finally when pressure drops to nearly 10^{-4} *mm* of *Hg*, there is no discharge in tube.

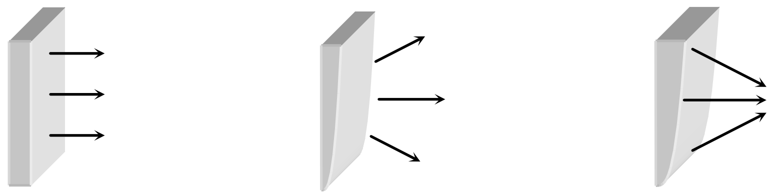
Cathode Rays.

Cathode rays, discovered by sir Willium Crooke are the stream of electrons. They can be produced by using a discharge tube containing gas at a low pressure of the order of 10^{-2} *mm* of *Hg*. At this pressure the gas molecules ionise and the emitted electrons travel towards positive potential of anode. The positive ions hit the cathode to cause emission of electrons from cathode. These electrons also move towards anode. Thus the cathode rays in the discharge tube are the electrons produced due to ionisation of gas and that emitted by cathode due to collision of positive ions.

(1) Properties of cathode rays

(i) Cathode rays travel in straight lines (cast shadows of objects placed in their path)

(ii) Cathode rays emit normally from the cathode surface. Their direction is independent of the position of the anode.



(iii) Cathode rays exert mechanical force on the objects they strike.

(iv) Cathode rays produce heat when they strikes a material surface.

(v) Cathode rays produce fluorescence.

(vi) When cathode rays strike a solid object, specially a metal of high atomic weight and high melting point X-rays are emitted from the objects.

(vii) Cathode rays are deflected by an electric field and also by a magnetic field.

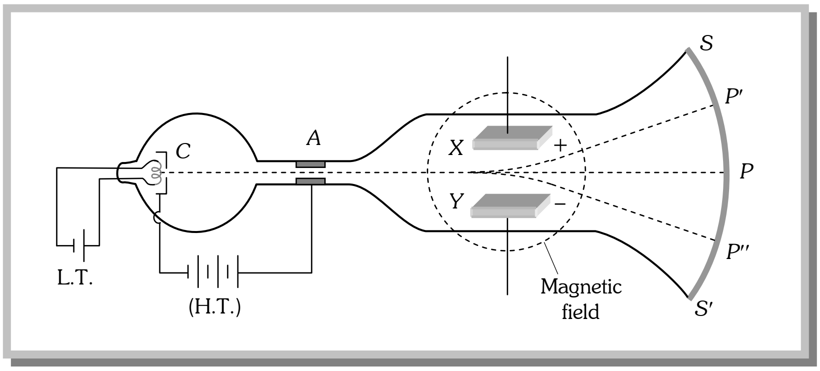
(viii) Cathode rays ionise the gases through which they are passed.

(ix) Cathode rays can penetrate through thin foils of metal.

(x) Cathode rays are found to have velocity ranging $\frac{1}{30}th$ to $\frac{1}{10}th$ of velocity of light.

(2) **J.J. Thomson's method to determine specific charge of electron**

It's working is based on the fact that if a beam of electron is subjected to the crossed electric field $\vec{E}$ and magnetic field $\vec{B}$, it experiences a force due to each field. In case the forces on the electrons in the electron beam due to these fields are equal and opposite, the beam remains undeflected.



C = Cathode, A = Anode, F = Filament, LT = Battery to heat the filament, V = potential difference to accelerate the electrons, SS' = ZnS coated screen, XY = metallic plates (Electric field produced between them)

(i) When no field is applied, the electron beam produces illuminations at point P.

(ii) In the presence of any field (electric and magnetic) electron beam deflected up or down (illumination at P' or P'')

(iii) If both the fields are applied simultaneously and adjusted such that electron beam passes undeflected and produces illumination at point P.

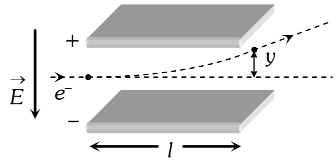
In this case; Electric force = Magnetic force $\Rightarrow eE = evB \Rightarrow v = \frac{E}{B}$; v = velocity of electron

As electron beam accelerated from cathode to anode its potential energy at the cathode appears as gain in the K.E. at the anode. If suppose V is the potential difference between cathode and anode then, potential energy = eV

And gain in kinetic energy at anode will be K.E. $= \frac{1}{2}mv^2$ i.e. $eV = \frac{1}{2}mv^2 \Rightarrow \frac{e}{m} = \frac{v^2}{2V} \Rightarrow \frac{e}{m} = \frac{E^2}{2VB^2}$

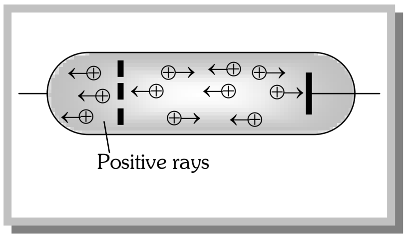
Thomson found, $\frac{e}{m} = 1.77 \times 10^{11} C/kg$.

Note : ≅The deflection of an electron in a purely electric field is given by $y = \frac{1}{2}\left(\frac{eE}{m}\right).\frac{l^2}{v^2}$; where l length of each plate, y = deflection of electron in the field region, v = speed of the electron.



Positive Rays.

Positive rays are sometimes known as the canal rays. These were discovered by Goldstein. If the cathode of a discharge tube has holes in it and the pressure of the gas is around 10^{-3} *mm* of *Hg* then faint luminous glow comes out from each hole on the backside of the cathode. It is said positive rays which are coming out from the holes.



(1) Origin of positive rays

When potential difference is applied across the electrodes, electrons are emitted from the cathode. As they move towards anode, they gain energy. These energetic electrons when collide with the atoms of the gas in the discharge tube, they ionize the atoms. The positive ions so formed at various places between cathode and anode, travel towards the cathode. Since during their motion, the positive ions when reach the cathode, some pass through the holes in the cathode. These streams are the positive rays.

(2) Properties of positive rays

(i) These are positive ions having same mass if the experimental gas does not have isotopes. However if the gas has isotopes then positive rays are group of positive ions having different masses.

(ii) They travels in straight lines and cast shadows of objects placed in their path. But the speed of the positive rays is much smaller than that of cathode rays.

(iii) They are deflected by electric and magnetic fields but the deflections are small as compared to that for cathode rays.

(iv) They show a spectrum of velocities. Different positive ions move with different velocities. Being heavy, their velocity is much less than that of cathode rays.

(v) q/m ratio of these rays depends on the nature of the gas in the tube (while in case of the cathode rays q/m is constant and doesn't depend on the gas in the tube). q/m for hydrogen is maximum.

(vi) They carry energy and momentum. The kinetic energy of positive rays is more than that of cathode rays.

(vii) The value of charge on positive rays is an integral multiple of electronic charge.

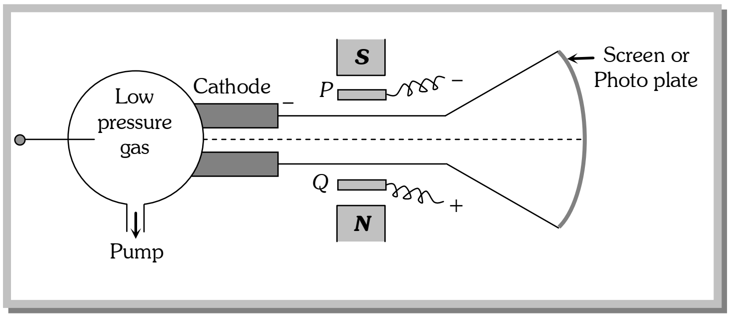
(viii) They cause ionisation (which is much more than that produced by cathode rays).

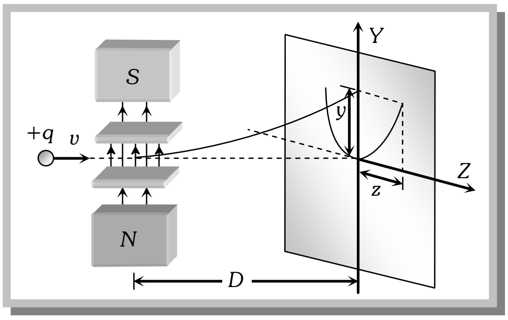
Mass Spectrograph.

It is a device used to determine the mass or (q/m) of positive ions.

(1) Thomson mass spectrograph

It is used to measure atomic masses of various isotopes in gas. This is done by measuring q/m of singly ionised positive ions of the gas.





The positive ions are produced in the bulb at the left hand side. These ions are accelerated towards cathode. Some of the positive ions pass through the fine hole in the cathode. This fine ray of positive ions is subjected to electric field E and magnetic field B and then allowed to strike a fluorescent screen ($\vec{E}\,||\,\vec{B}$ but $\vec{E}$ or $\vec{B} \perp \vec{v}$).

If the initial motion of the ions is in $+x$ direction and electric and magnetic fields are applied along $+y$ axis then force due to electric field is in the direction of y-axis and due to magnetic field it is along z-direction.

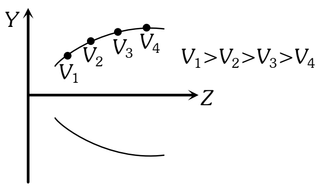
The deflection due to electric field alone $y = \dfrac{qELD}{mv^2}$ ....(i)

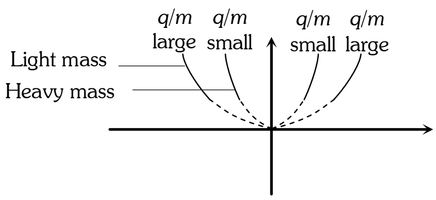
The deflection due to magnetic field alone $z = \dfrac{qBLD}{mv}$ ....(ii)

From equation (i) and (ii)

$z^2 = k\left(\dfrac{q}{m}\right)y$, where $k = \dfrac{B^2LD}{E}$; This is the equation of parabola. It means all the charged particles moving with different velocities but of same q/m value will strike the screen placed in yz plane on a parabolic track as shown in the above figure.

Note : ≅All the positive ions of same. q/m moving with different velocity lie on the same parabola. Higher is the velocity lower is the value of y and z. The ions of different specific charge will lie on different parabola.





≅ The number of parabola tells the number of isotopes present in the given ionic beam.

(2) Bainbridge mass spectrograph

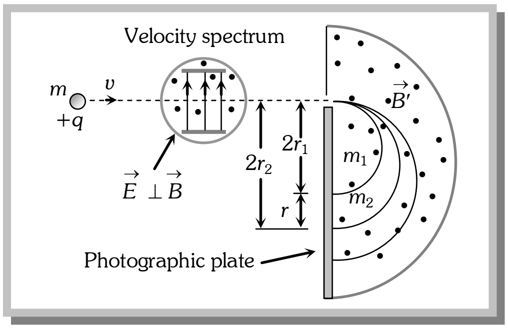
In Bainbridge mass spectrograph, field particles of same velocity are selected by using a velocity selector and then they are subjected to a uniform magnetic field perpendicular to the velocity of the particles. The particles corresponding to different isotopes follow different circular paths as shown in the figure.

(i) **Velocity selector :** The positive ions having a certain velocity v gets isolated from all other velocity particles. In this chamber the electric and magnetic fields are so balanced that the particle moves undeflected. For this the necessary condition is $v = \frac{E}{B}$.

(ii) **Analysing chamber :** In this chamber magnetic field B is applied perpendicular to the direction of motion of the particle. As a result the particles move along a circular path of radius

$$r = \frac{mE}{qBB'} \Rightarrow \frac{q}{m} = \frac{E}{BB'r} \quad \text{also} \quad \frac{r_1}{r_2} = \frac{m_1}{m_2}$$

In this way the particles of different masses gets deflected on circles of different radii and reach on different points on the photo plate.



Note : ≅ Separation between two traces $= d = 2r_2 - 2r_1 \Rightarrow d = \frac{2v(m_2 - m_1)}{qB'}$.

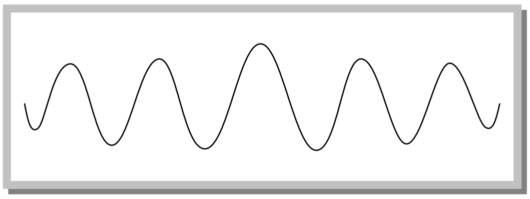
Matter waves (de-Broglie Waves).

According to de-Broglie a moving material particle sometimes acts as a wave and sometimes as a particle.

or

A wave is associated with moving material particle which control the particle in every respect.

The wave associated with moving particle is called matter wave or de-Broglie wave and it propagates in the form of wave packets with group velocity.



(1) de-Broglie wavelength

According to de-Broglie theory, the wavelength of de-Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \qquad \Rightarrow \lambda \propto \frac{1}{p} \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}}$$

Where h = Plank's constant, m = Mass of the particle, v = Speed of the particle, E = Energy of the particle.

The smallest wavelength whose measurement is possible is that of γ-rays.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, α-particle *etc*. is of the order of $10^{-10}\,m$.

(i) **de-Broglie wavelength associated with the charged particles.**

The energy of a charged particle accelerated through potential difference V is $E = \frac{1}{2}mv^2 = qV$

Hence de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$$\lambda_{electron} = \frac{12.27}{\sqrt{V}} \mathring{A}, \quad \lambda_{proton} = \frac{0.286}{\sqrt{V}} \mathring{A}, \quad \lambda_{deutron} = \frac{0.202 \times 10^{-10}}{\sqrt{V}} \mathring{A}, \quad \lambda_{\alpha-particle} = \frac{0.101}{\sqrt{V}} \mathring{A}$$

(ii) **de-Broglie wavelength associated with uncharged particles.**

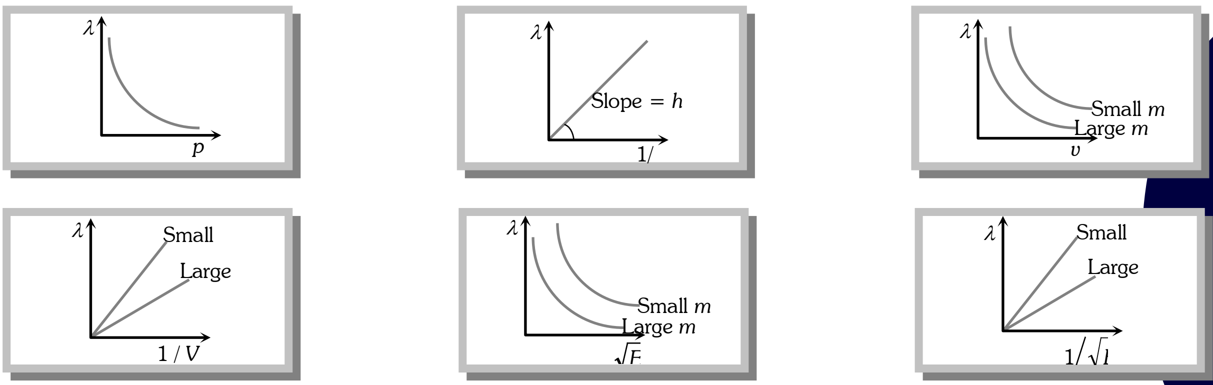
For *Neutron de*-Broglie wavelength is given as $\lambda_{Neutron} = \frac{0.286 \times 10^{-10}}{\sqrt{E(\text{in } eV)}} m = \frac{0.286}{\sqrt{E(\text{in } eV)}} \mathring{A}$

Energy of thermal neutrons at ordinary temperature

$\because \; E = kT \Rightarrow \lambda = \frac{h}{\sqrt{2mkT}}$; where k = Boltzman's constant = 1.38×10^{-23} *Joules/kelvin* , T = Absolute temp.

So $\lambda_{\text{Thermal Neutron}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.07 \times 10^{-17} \times 1.38 \times 10^{-23} T}} = \frac{30.83}{\sqrt{T}} \mathring{A}$

(2) **Some graphs**



Note : ≅ A photon is not a material particle. It is a quanta of energy.

≅ When a particle exhibits wave nature, it is associated with a wave packet, rather then a wave.

(3) **Characteristics of matter waves**

(i) Matter wave represents the probability of finding a particle in space.

(ii) Matter waves are not electromagnetic in nature.

(iii) de-Brogile or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).

(iv) Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles is nature.

(v) Electron microscope works on the basis of de-Broglie waves.

(vi) The electric charge has no effect on the matter waves or their wavelength.

(vii) The phase velocity of the matter waves can be greater than the speed of the light.

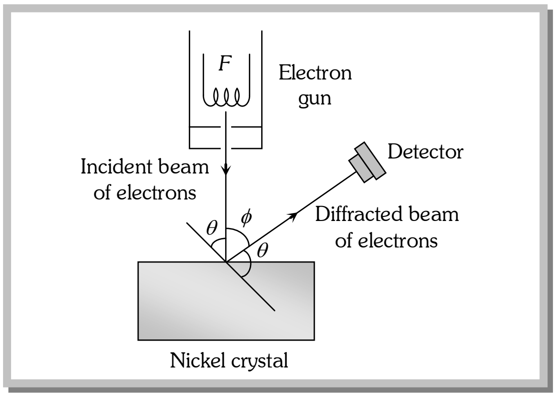
(viii) Matter waves can propagate in vacuum, hence they are not mechanical waves.

(ix) The number of de-Broglie waves associated with n^{th} orbital electron is *n*.

(x) Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

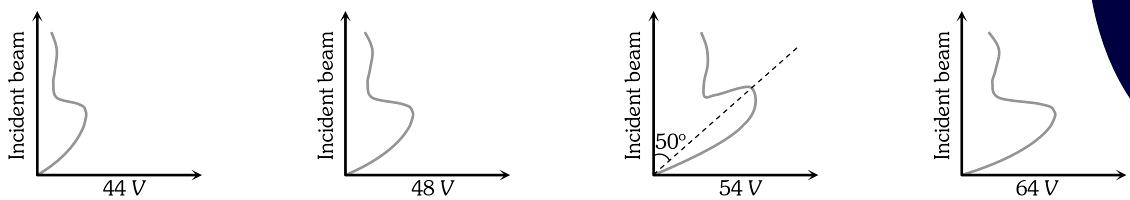
(4) Davision and Germer experiment

It is used to study the scattering of electron from a solid or to verify the wave nature of electron. A beam of electrons emitted by electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. *Ni* crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.



The diffracted beam of electrons is received by the detector which can be positioned at any angle by rotating it about the point of incidence. The energy of the incident beam of electrons can also be varied by changing the applied voltage to the electron gun.

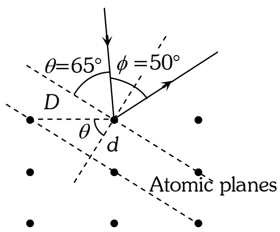
According to classical physics, the intensity of scattered beam of electrons at all scattering angle will be same but Davisson and Germer, found that the intensity of scattered beam of electrons was not the same but different at different angles of scattering.



Intensity is maximum at 54 *V* potential difference and 50° diffraction angle.

If the de-Broglie waves exist for electrons then these should be diffracted as *X*-rays. Using the Bragg's formula $2d\sin\theta = n\lambda$, we can determine the wavelength of these waves.

Where d = distance between diffracting planes, $\theta = \frac{(180-\phi)}{2}$ = glancing angle for incident beam = Bragg's angle.



The distance between diffraction planes in *Ni*-crystal for this experiment is d = 0.91Å and the Bragg's angle = 65°. This gives for $n = 1$, $\lambda = 2 \times 0.91 \times 10^{-10} \sin 65^\circ = 1.65\,Å$

Now the de-Broglie wavelength can also be determined by using the formula $\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.67\,Å$.

Thus the de-Broglie hypothesis is verified.

Heisenberg Uncertainty Principle.

According to Heisenberg's uncertainty principle, it is impossible to measure simultaneously both the position and the momentum of the particle.

Let Δx and Δp be the uncertainty in the simultaneous measurement of the position and momentum of the particle, then $\Delta x \Delta p = \hbar$; where $\hbar = \frac{h}{2\pi}$ and $h = 6.63 \times 10^{-34}$ *J-s* is the Planck's constant.

If $\Delta x = 0$ then $\Delta p = \infty$

and if $\Delta p = 0$ then $\Delta x = \infty$ *i.e.,* if we are able to measure the exact position of the particle (say an electron) then the uncertainty in the measurement of the linear momentum of the particle is infinite. Similarly, if we are able to measure the exact linear momentum of the particle *i.e.*, $\Delta p = 0$, then we can not measure the exact position of the particle at that time.

Photon.

According to Eienstein's quantum theory light propagates in the bundles (packets or quanta) of energy, each bundle being called a photon and possessing energy.

(1) Energy of photon

Energy of each photon is given by $E = h\nu = \frac{hc}{\lambda}$; where c = Speed of light, h = Plank's constant = 6.6×10^{-34} *J-sec*, ν = Frequency in *Hz*, λ = Wavelength of light

Energy of photon in electron volt $E(eV) = \frac{hc}{e\lambda} = \frac{12375}{\lambda(Å)} \approx \frac{12400}{\lambda(Å)}$

(2) Mass of photon

Actually rest mass of the photon is zero. But it's effective mass is given as

$$E = mc^2 = h\nu \Rightarrow m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{c\lambda}$$. This mass is also known as kinetic mass of the photon

(3) Momentum of the photon

$$\text{Momentum } p = m \times c = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

(4) Number of emitted photons

The number of photons emitted per second from a source of monochromatic radiation of wavelength λ and power P is given as $(n) = \frac{P}{E} = \frac{P}{h\nu} = \frac{P\lambda}{hc}$; where E = energy of each photon

(5) Intensity of light (*I*)

Energy crossing per unit area normally per second is called intensity or energy flux i.e. $I = \frac{E}{At} = \frac{P}{A}$ $\left(\frac{E}{t} = P = \text{radiation power}\right)$

At a distance r from a point source of power P intensity is given by $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

Concepts

- *Discovery of positive rays helps in discovering of isotopes.*
- *The de-Broglie wavelength of electrons in first Bohr orbit of an atom is equal to circumference of orbit.*
- *A particle having zero rest mass and non zero energy and momentum must travels with a speed equal to speed of light.*
- ***de-Broglie wave length associates with gas molecules** is given as* $\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3\,mkT}}$ *(Energy of gas molecules at temperature T is* $E = \frac{3}{2}kT$ *)*

Example

Example: 1 The ratio of specific charge of an α-particle to that of a proton is **[BCECE 2003]**

(a) 2 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 3

Solution : (c) Specific charge $= \frac{q}{m}$; Ratio $= \frac{(q/m)_\alpha}{(q/m)_p} = \frac{q_\alpha}{q_p} \times \frac{m_p}{m_\alpha} = \frac{1}{2}$.

Example: 2 The speed of an electron having a wavelength of $10^{-10} m$ is **[AIIMS 2002]**

(a) 7.25×10^6 *m/s* (b) $6.26 \times 10^6 m/s$ (c) $5.25 \times 10^6 m/s$ (d) $4.24 \times 10^6 m/s$

Solution : (a) By using $\lambda_{electron} = \frac{h}{m_e v} \Rightarrow v = \frac{h}{m_e \lambda_e} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}} = 7.25 \times 10^6 m/s$.

Example: 3 In Thomson experiment of finding *e/m* for electrons, beam of electron is replaced by that of muons (particle with same charge as of electrons but mass 208 times that of electrons). No deflection condition in this case satisfied if **[Orissa (Engg.) 2002]**

(a) *B* is increased 208 times (b) *E* is increased 208 times

(c) *B* is increased 14.4 times (d) None of these

Solution : (c) In the condition of no deflection $\frac{e}{m} = \frac{E^2}{2VB^2}$. If m is increased to 208 times then B should be increased by $\sqrt{208} = 14.4$ times.

Example: 4 In a Thomson set-up for the determination of *e/m*, electrons accelerated by 2.5 *kV* enter the region of crossed electric and magnetic fields of strengths $3.6 \times 10^4 Vm^{-1}$ and $1.2 \times 10^{-3} T$ respectively and go through undeflected. The measured value of *e/m* of the electron is equal to **[AMU 2002]**

(a) 1.0×10^{11} *C-kg*$^{-1}$ (b) 1.76×10^{11} *C-kg*$^{-1}$ (c) 1.80×10^{11} *C-kg*$^{-1}$ (d) 1.85×10^{11} *C-kg*$^{-1}$

Solution : (c) By using $\frac{e}{m} = \frac{E^2}{2VB^2} \Rightarrow \frac{e}{m} = \frac{(3.6 \times 10^4)^2}{2 \times 2.5 \times 10^3 \times (1.2 \times 10^{-3})^2} = 1.8 \times 10^{11} C/kg$.

Example: 5 In Bainbridge mass spectrograph a potential difference of 1000 *V* is applied between two plates distant 1 *cm* apart and magnetic field in *B* = 1*T*. The velocity of undeflected positive ions in *m/s* from the velocity selector is **[RPMT 1998]**

(a) $10^7 m/s$ (b) $10^4 m/s$ (c) $10^5 m/s$ (d) $10^2 m/s$

Solution : (c) By using $v = \frac{E}{B}$; where $E = \frac{V}{d} = \frac{1000}{1\times10^{-2}} = 10^5\ V/m \Rightarrow v = \frac{10^5}{1} = 10^5 m/s$.

Example: 6 An electron and a photon have same wavelength. It *p* is the momentum of electron and *E* the energy of photon. The magnitude of *p*/ *E* in S.I. unit is

(a) 3.0×10^8 (b) 3.33×10^{-9} (c) 9.1×10^{-31} (d) 6.64×10^{-34}

Solution : (b) $\lambda = \frac{h}{p}$ (for electron) or $p = \frac{h}{\lambda}$ and $E = \frac{hc}{\lambda}$ (for photon)

$\therefore \quad \frac{p}{E} = \frac{1}{c} = \frac{1}{3\times10^8 m/s} = 3.33\times10^{-9}\ s/m$

Example: 7 The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is *E*. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio λ_1/λ_2 is proportional to **[UPSEAT 2003; IIT-JEE (Screening) 2004]**

(a) E^0 (b) $E^{1/2}$ (c) E^{-1} (d) E^{-2}

Solution : (b) For photon $\lambda_2 = \frac{hc}{E}$ .... (i) and For proton $\lambda_1 = \frac{h}{\sqrt{2mE}}$ ....(ii)

Therefore $\frac{\lambda_1}{\lambda_2} = \frac{E^{1/2}}{\sqrt{2m}\,c} \Rightarrow \frac{\lambda_1}{\lambda_2} \propto E^{1/2}$.

Example: 8 The de-Broglie wavelength of an electron having 80*eV* of energy is nearly ($1eV = 1.6\times10^{-19} J$, Mass of electron $9\times10^{-31}kg$ and Plank's constant 6.6×10^{-34} *J-sec*) **[EAMCET (Engg.) 2001]**

(a) 140 $\mathring{A}$ (b) 0.14 $\mathring{A}$ (c) 14 $\mathring{A}$ (d) 1.4 $\mathring{A}$

Solution : (d) By using $\lambda = \frac{h}{\sqrt{2mE}} = \frac{12.27}{\sqrt{V}}$. If energy is 80 *eV* then accelerating potential difference will be 80 *V*. So $\lambda = \frac{12.27}{\sqrt{80}} = 1.37 \approx 1.4\ \mathring{A}$.

Example: 9 The kinetic energy of electron and proton is $10^{-32}J$. Then the relation between their de-Broglie wavelengths is **[CPMT 1999]**

(a) $\lambda_p < \lambda_e$ (b) $\lambda_p > \lambda_e$ (c) $\lambda_p = \lambda_e$ (d) $\lambda_p = 2\lambda_e$

Solution : (a) By using $\lambda = \frac{h}{\sqrt{2mE}}$ $E = 10^{-32} J$ = Constant for both particles. Hence $\lambda \propto \frac{1}{\sqrt{m}}$

Since $m_p > m_e$ so $\lambda_p < \lambda_e$.

Example: 10 The energy of a proton and an α particle is the same. Then the ratio of the de-Broglie wavelengths of the proton and the α is **[RPET 1991]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

Solution : (b) By using $\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$ $(E - \text{same}) \Rightarrow \frac{\lambda_{proton}}{\lambda_{\alpha-particle}} = \sqrt{\frac{m_\alpha}{m_p}} = \frac{2}{1}$.

Example: 11 The de-Broglie wavelength of a particle accelerated with 150 *volt* potential is 10^{-10} *m*. If it is accelerated by 600 *volts* p.d., its wavelength will be **[RPET 1988]**

(a) 0.25 $\mathring{A}$ (b) 0.5 $\mathring{A}$ (c) 1.5 $\mathring{A}$ (d) 2 $\mathring{A}$

Solution : (b) By using $\lambda \propto \frac{1}{\sqrt{V}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}} \Rightarrow \frac{10^{-10}}{\lambda_2} = \sqrt{\frac{600}{150}} = 2 \Rightarrow \lambda_2 = 0.5\,\mathring{A}.$

***Example*: 12** The de-Broglie wavelength of an electron in an orbit of circumference $2\pi r$ is **[MP PET 1987]**

(a) $2\pi r$ (b) πr (c) $1/2\pi r$ (d) $1/4\pi r$

Solution : (a) According to Bohr's theory $mvr = n\frac{h}{2\pi} \Rightarrow 2\pi r = n\left(\frac{h}{mv}\right) = n\lambda$

For $n = 1$ $\lambda = 2\pi r$

***Example*: 13** The number of photons of wavelength 540 *nm* emitted per second by an electric bulb of power 100*W* is (taking $h = 6\times10^{-34}$ *J-sec*) **[Kerala (Engg.) 2002]**

(a) 100 (b) 1000 (c) 3×10^{20} (d) 3×10^{18}

Solution : (c) By using $n = \frac{P\lambda}{hc} = \frac{100\times540\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}} = 3\times10^{20}$

***Example*: 14** A steel ball of mass 1*kg* is moving with a velocity 1 *m/s*. Then its de-Broglie waves length is equal to

(a) h (b) $h/2$ (c) Zero (d) $1/h$

Solution : (a) By using $\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{\lambda}{1\times1} = h.$

***Example*: 15** The de-Broglie wavelength associated with a hydrogen atom moving with a thermal velocity of 3 *km/s* will be

(a) 1 $\mathring{A}$ (b) 0.66 $\mathring{A}$ (c) 6.6 $\mathring{A}$ (d) 66 $\mathring{A}$

Solution : (b) By using $\lambda = \frac{h}{mv_{rms}} \Rightarrow \lambda = \frac{6.6\times10^{-34}}{2\times1.67\times10^{-27}\times3\times10^{3}} = 0.66\mathring{A}$

***Example*: 16** When the momentum of a proton is changed by an amount P_0, the corresponding change in the de-Broglie wavelength is found to be 0.25%. Then, the original momentum of the proton was **[CPMT 2002]**

(a) p_0 (b) 100 p_0 (c) 400 p_0 (d) 4 p_0

Solution : (c) $\lambda \propto \frac{1}{p} \Rightarrow \frac{\Delta p}{p} = -\frac{\Delta\lambda}{\lambda} \Rightarrow \left|\frac{\Delta p}{p}\right| = \left|\frac{\Delta\lambda}{\lambda}\right| \Rightarrow \frac{p_0}{p} = \frac{0.25}{100} = \frac{1}{400} \Rightarrow p = 400\,p_0.$

***Example*: 17** If the electron has same momentum as that of a photon of wavelength 5200Å, then the velocity of electron in *m /sec* is given by

(a) 10^3 (b) 1.4×10^3 (c) 7×10^{-5} (d) 7.2×10^6

Solution : (b) $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times5200\times10^{-10}} \Rightarrow v = 1.4\times10^3$ *m/s.*

***Example*: 18** The de-Broglie wavelength of a neutron at 27°C is λ. What will be its wavelength at 927°C

(a) $\lambda/2$ (b) $\lambda/3$ (c) $\lambda/4$ (d) $\lambda/9$

Solution : (a) $\lambda_{neutron} \propto \frac{1}{\sqrt{T}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{\lambda}{\lambda_2} = \sqrt{\frac{(273+927)}{(273+27)}} = \sqrt{\frac{1200}{300}} = 2 \Rightarrow \lambda_2 = \frac{\lambda}{2}.$

***Example*: 19** The de-Broglie wavelength of a vehicle is λ. Its load is changed such that its velocity and energy both are doubled. Its new wavelength will be

(a) λ (b) $\frac{\lambda}{2}$ (c) $\frac{\lambda}{4}$ (d) 2λ

Solution : (a) $\lambda = \frac{h}{mv}$ and $E = \frac{1}{2}mv^2 \Rightarrow \lambda = \frac{hv}{2E}$ when v and E both are doubled, λ remains unchanged *i.e.* $\lambda' = \lambda$.

Example: 20 In Thomson mass spectrograph when only electric field of strength 20 *kV/m* is applied, then the displacement of the beam on the screen is 2 *cm*. If length of plates = 5 *cm*, distance from centre of plate to the screen = 20 *cm* and velocity of ions = 10^6 *m/s*, then *q/m* of the ions is

(a) 10^6 *C/kg* (b) 10^7 *C/Kg* (c) 10^8 *C/kg* (d) 10^{11} *C/kg*

Solution : (c) By using $y = \frac{qELD}{mv^2}$; where y = deflection on screen due to electric field only

$$\Rightarrow \frac{q}{m} = \frac{yv^2}{ELD} = \frac{2\times 10^{-2}\times (10^6)^2}{20\times 10^3\times 5\times 10^{-2}\times 0.2} = 10^8\ C/kg.$$

Example: 21 The minimum intensity of light to be detected by human eye is $10^{-10} W/m^2$. The number of photons of wavelength $5.6\times 10^{-7} m$ entering the eye, with pupil area $10^{-6} m^2$, per second for vision will be nearly

(a) 100 (b) 200 (c) 300 (d) 400

Solution : (c) By using $I = \frac{P}{A}$; where P = radiation power

$$\Rightarrow P = I\times A \Rightarrow \frac{nhc}{t\lambda} = IA \quad \Rightarrow \frac{n}{t} = \frac{IA\lambda}{hc}$$

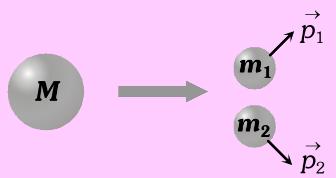
Hence number of photons entering per sec the eye $\left(\frac{n}{t}\right) = \frac{10^{-10}\times 10^{-6}\times 5.6\times 10^{-7}}{6.6\times 10^{-34}\times 3\times 10^8} = 300.$

Tricky example: 1

A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles, λ_1/λ_2 is **[IIT-JEE 1999]**

(a) m_1/m_2 (b) m_2/m_1 (c) 1.0 (d) $\sqrt{m_1}/\sqrt{m_1}$

Solution : (c) According to conservation of momentum *i.e.* $p_1 = p_2$



Hence from $\lambda = \frac{h}{p} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{p_1}{p_2} = \frac{1}{1}$

The curve drawn between velocity and frequency of photon in vacuum will be a **[MP PET 2000]**

(a) Straight line parallel to frequency axis

(b) Straight line parallel to velocity axis

(c) Straight line passing through origin and making an angle of 45° with frequency axis

(d) Hyperbola

Solution : (a) Velocity of photon (*i.e.* light) doesn't depend upon frequency. Hence the graph between velocity of

Velocity of photon (c)

photon and frequency will be as follows.

Photo-electric Effect.

It is the phenomenon of emission of electrons from the surface of metals, when light radiations (Electromagnetic radiations) of suitable frequency fall on them. The emitted electrons are called photoelectrons and the current so produced is called photoelectric current.

This effect is based on the principle of conservation of energy.

(1) **Terms related to photoelectric effect**

(i) **Work function (or threshold energy) (W_0) :** The minimum energy of incident radiation, required to eject the electrons from metallic surface is defined as work function of that surface.

$$W_0 = h\nu_0 = \frac{hc}{\lambda_0} Joules; \quad \nu_0 = \text{Threshold frequency}; \lambda_0 = \text{Threshold wavelength}$$

Work function in electron volt $W_0(eV) = \frac{hc}{e\lambda_0} = \frac{12375}{\lambda_0(\text{Å})}$

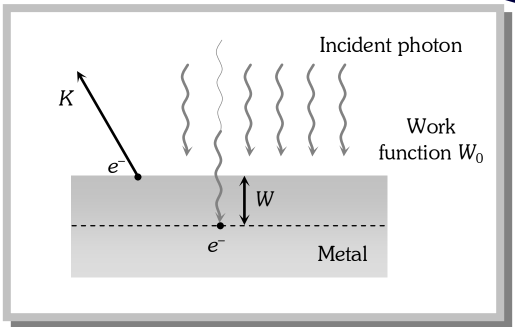
Note : ≅ By coating the metal surface with a layer of barium oxide or strontium oxide it's work function is lowered.

(ii) **Threshold frequency (ν_0) :** The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency.

If incident frequency $\nu < \nu_0 \Rightarrow$ No photoelectron emission

(iii) **Threshold wavelength (λ_0) :** The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as threshold wavelength.

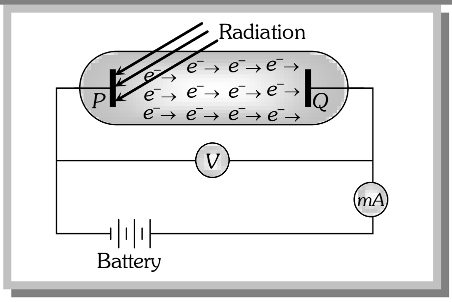
If incident wavelength $\lambda > \lambda_0 \Rightarrow$ No photoelectron emission



(2) **Einstein's photoelectric equation**

According to Einstein, photoelectric effect is the result of one to one inelastic collision between photon and electron in which photon is completely absorbed. So if an electron in a metal absorbs a photon of energy E (= $h\nu$), it uses the energy in three following ways.

(i) Some energy (say W) is used in shifting the electron from interior to the surface of the metal.



(ii) Some energy (say W_0) is used in making the surface electron free from the metal.

(iii) Rest energy will appear as kinetic energy (K) of the emitted photoelectrons.

Hence $E = W + W_0 + K$

For the electrons emitting from surface $W = 0$ so kinetic energy of emitted electron will be max.

Hence $\boldsymbol{E = W_0 + K_{max}}$; This is the Einstein's photoelectric equation

(3) Experimental arrangement to observe photoelectric effect

When light radiations of suitable frequency (or suitable wavelength and suitable energy) falls on plate P, photoelectrons are emitted from P.

(i) If plate Q is at zero potential *w.r.t.* P, very small current flows in the circuit because of some electrons of high kinetic energy are reaching to plate Q, but this current has no practical utility.

(ii) If plate Q is kept at positive potential *w.r.t.* P current starts flowing through the circuit because more electrons are able to reach upto plate Q.

(iii) As the positive potential of plate Q increases, current through the circuit increases but after some time constant current flows through the circuit even positive potential of plate Q is still increasing, because at this condition all the electrons emitted from plate P are already reached up to plate Q. This constant current is called **saturation current.**

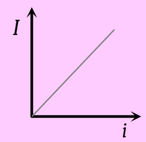
(iv) To increase the photoelectric current further we will have to increase the intensity of incident light.

Photoelectric current (i) depends upon

(a) Potential difference between electrodes (till saturation)

(b) Intensity of incident light (I)

(c) Nature of surface of metal



(v) To decrease the photoelectric current plate Q is maintained at negative potential *w.r.t.* P, as the anode Q is made more and more negative, fewer and fewer electrons will reach the cathode and the photoelectric current decreases.

(vi) At a particular negative potential of plate Q no electron will reach the plate Q and the current will become zero, this negative potential is called **stopping potential** denoted by V_0.

(vii) If we increase further the energy of incident light, kinetic energy of photoelectrons increases and more negative potential should be applied to stop the electrons to reach upto plate Q. Hence $\boldsymbol{eV_0 = K_{max}}$.

Note : ≅ Stopping potential depends only upon frequency or wavelength or energy of incident radiation. It doesn't depend upon intensity of light.

We must remember that intensity of incident light radiation is inversely proportional to the square of distance between source of light and photosensitive plate P *i.e.*, $I \propto \frac{1}{d^2}$ so $I \propto i \propto \frac{1}{d^2}$)

Important formulae

$\Rightarrow h\nu = h\nu_0 + K_{\max}$

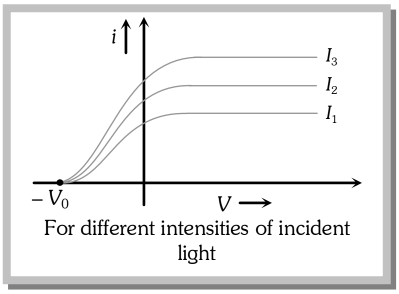
$\Rightarrow K_{\max} = eV_0 = h(\nu - \nu_0)$ $\Rightarrow \frac{1}{2}mv_{\max}^2 = h(\nu - \nu_0)$ $\Rightarrow v_{\max} = \sqrt{\frac{2h(\nu - \nu_0)}{m}}$

$$\Rightarrow K_{\max} = \frac{1}{2}mv_{\max}^2 = eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right) \Rightarrow v_{\max} = \sqrt{\frac{2hc}{m}\frac{(\lambda - \lambda_0)}{\lambda\lambda_0}}$$

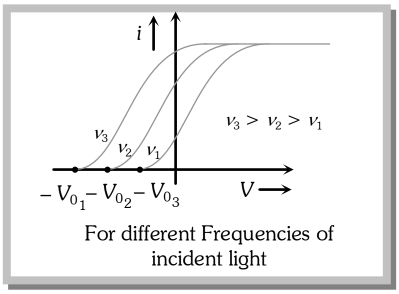
$$\Rightarrow V_0 = \frac{h}{e}(\nu - \nu_0) = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = 12375\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

(4) **Different graphs**

(i) Graph between potential difference between the plates P and Q and photoelectric current

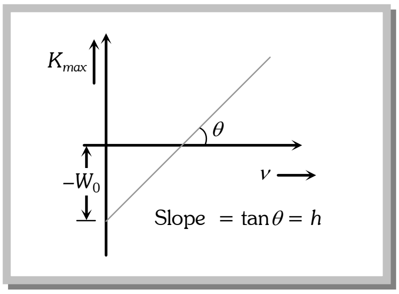


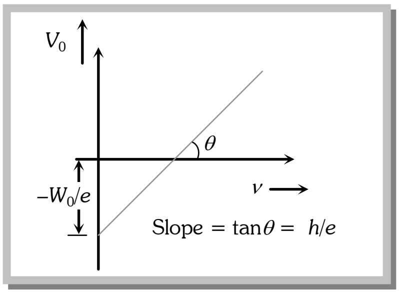
For different intensities of incident light



For different Frequencies of incident light

(ii) Graph between maximum kinetic energy / stopping potential of photoelectrons and frequency of incident light

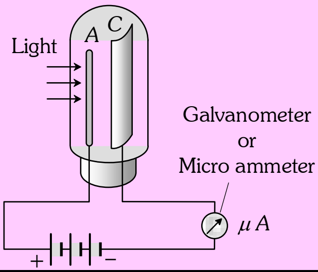
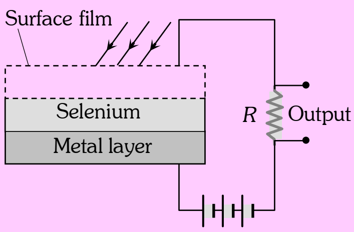
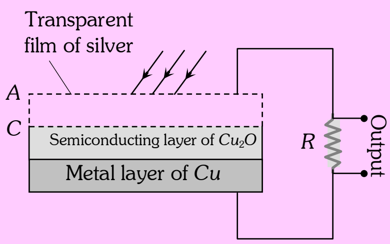




Photoelectric Cell.

A device which converts light energy into electrical energy is called photoelectric cell. It is also known as photocell or electric eye.

Photoelectric cell are mainly of three types

Photo-emissive cell	Photo-conductive cell	Photo-voltaic cell
It consists of an evacuated glass or quartz bulb containing anode A and cathode C. The cathode is semi-cylindrical metal on which a layer of photo-sensitive material is coated.	It is based on the principle that conductivity of a semiconductor increases with increase in the intensity of incident light.	It consists of a Cu plate coated with a thin layer of cuprous oxide (Cu_2O). On this plate is laid a semi transparent thin film of silver.
		

When light incident on the cathode, it emits photo-electrons which are attracted by the anode. The photoelectrons constitute a small current which flows through the external circuit.	In this, a thin layer of some semiconductor (as selenium) is placed below a transparent foil of some metal. This combination is fixed over an iron plate. When light is incident on the transparent foil, the electrical resistance of the semiconductor layer is reduced. Hence a current starts flowing in the battery circuit connected.	When light fall, the electrons emitted from the layer of Cu_2O and move towards the silver film. Then the silver film becomes negatively charged and copper plate becomes positively charged. A potential difference is set up between these two and current is set up in the external resistance.

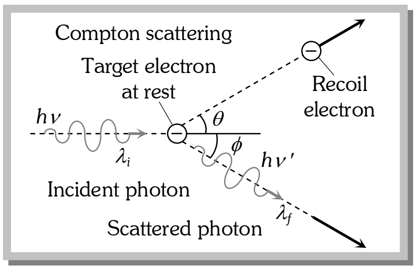
Note : ≅ The photoelectric current can be increased by filling some inert gas like Argon into the bulb. The photoelectrons emitted by cathode ionise the gas by collision and hence the current is increased.

Compton effect

The scattering of a photon by an electron is called Compton effect. The energy and momentum is conserved. Scattered photon will have less energy (more wavelength) as compare to incident photon (less wavelength). The energy lost by the photon is taken by electron as kinetic energy.

The change in wavelength due to Compton effect is called Compton shift. Compton shift

$$\lambda_f - \lambda_i = \frac{h}{m_0 c}(1 - \cos\theta)$$



Note : ≅ Compton effect shows that photon have momentum.

X-rays.

X-rays was discovered by scientist Rontgen that's why they are also called Rontgen rays.

Rontgen discovered that when pressure inside a discharge tube kept 10^{-3} *mm* of *Hg* and potential difference is 25 *kV* then some unknown radiations (X-rays) are emitted by anode.

(1) Production of X-rays

There are three essential requirements for the production of X-rays

(i) A source of electron

(ii) An arrangement to accelerate the electrons

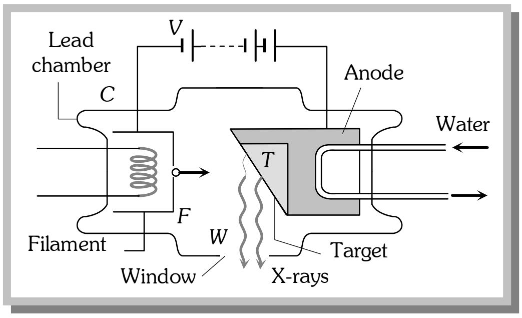
(iii) A target of suitable material of high atomic weight and high melting point on which these high speed electrons strike.

(2) Coolidge X-ray tube

It consists of a highly evacuated glass tube containing cathode and target. The cathode consist of a tungsten filament. The filament is coated with oxides of barium or strontium to have an emission of electrons even at low temperature. The filament is surrounded by a molybdenum cylinder kept at negative potential *w.r.t.* the target.

The target (it's material of high atomic weight, high melting point and high thermal conductivity) made of tungsten or molybdenum is embedded in a copper block.

The face of the target is set at 45° to the incident electron stream.



The filament is heated by passing the current through it. A high potential difference (≈ 10 kV to 80 kV) is applied between the target and cathode to accelerate the electrons which are emitted by filament. The stream of highly energetic electrons are focussed on the target.

Most of the energy of the electrons is converted into heat (above 98%) and only a fraction of the energy of the electrons (about 2%) is used to produce X-rays.

During the operation of the tube, a huge quantity of heat is produced in this target, this heat is conducted through the copper anode to the cooling fins from where it is dissipated by radiation and convection.

(i) **Control of intensity of X-rays** : Intensity implies the number of X-ray photons produced from the target. The intensity of X-rays emitted is directly proportional to the electrons emitted per second from the filament and this can be increased by increasing the filament current. So *intensity of X-rays* $\propto$ *Filament current*

(ii) **Control of quality or penetration power of X-rays :** Quality of X-rays implies the penetrating power of X-rays, which can be controlled by varying the potential difference between the cathode and the target.

For large potential difference, energy of bombarding electrons will be large and hence larger is the penetration power of X-rays.

Depending upon the penetration power, X-rays are of two types

Hard X-rays	Soft X-rays
More penetration power	Less penetration power
More frequency of the order of ≈ 10^{19} Hz	Less frequency of the order of ≈ 10^{16} Hz
Lesser wavelength range (0.1Å – 4Å)	More wavelength range (4Å – 100Å)

Note : ≅ Production of X-ray is the reverse phenomenon of photoelectric effect.

(3) **Properties of X-rays**

(i) X-rays are electromagnetic waves with wavelength range 0.1Å – 100Å.

(ii) The wavelength of X-rays is very small in comparison to the wavelength of light. Hence they carry much more energy (This is the only difference between X-rays and light)

(iii) X-rays are invisible.

(iv) They travel in a straight line with speed of light.

(v) X-rays are measured in Rontgen (measure of ionization power).

(vi) X-rays carry no charge so they are not deflected in magnetic field and electric field.

(vii) $\lambda_{Gama\ rays} < \lambda_{X\text{-}rays} < \lambda_{UV\ rays}$

(viii) They used in the study of crystal structure.

(ix) They ionise the gases

(x) X-rays do not pass through heavy metals and bones.

(xi) They affect photographic plates.

(xii) Long exposure to X-rays is injurious for human body.

(xiii) Lead is the best absorber of X-rays.

(xiv) For X-ray photography of human body parts, $BaSO_4$ is the best absorber.

(xv) They produce photoelectric effect and Compton effect

(xvi) X-rays are not emitted by hydrogen atom.

(xvii) These cannot be used in Radar because they are not reflected by the target.

(xviii) They show all the important properties of light rays like; reflection, refraction, interference, diffraction and polarization *etc*.

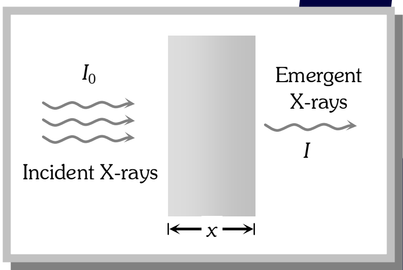
(4) Absorption of X-rays

X-rays are absorbed when they incident on substance.

Intensity of emergent X-rays $I = I_0 e^{-\mu x}$

So intensity of absorbed X-rays $I' = I_0 - I = I_0(1 - e^{-\mu x})$

where x = thickness of absorbing medium, μ = absorption coefficient



Note : ≅ The thickness of medium at which intensity of emergent X-rays becomes half *i.e.* $I' = \frac{I_0}{2}$ is called half value thickness ($x_{1/2}$) and it is given as $x_{1/2} = \frac{0.693}{\mu}$.

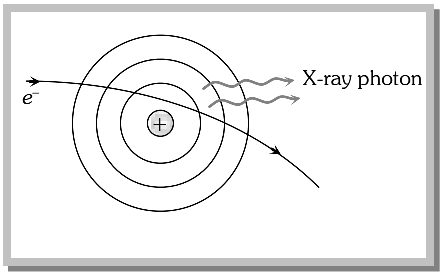
Classification of X-rays.

In X-ray tube, when high speed electrons strikes the target, they penetrate the target. They loses their kinetic energy and comes to rest inside the metal. The electron before finally being stopped makes several collisions with the atoms in the target. At each collision one of the following two types of X-rays may get form.

(1) Continuous X-rays

As an electron passes close to the positive nucleus of atom, the electron is deflected from it's path as shown in figure. This results in deceleration of the electron. The loss in energy of the electron during deceleration is emitted in the form of X-rays.

The X-ray photons emitted so form the continuous X-ray spectrum.



Note : ≅ Continuos X-rays are produced due to the phenomenon called "Bremsstrahlung". It means slowing down or braking radiation.

Minimum wavelength

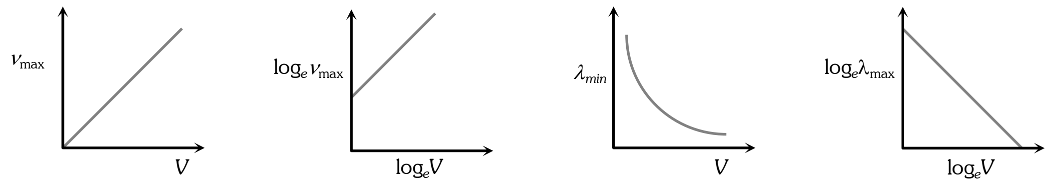
When the electron looses whole of it's energy in a single collision with the atom, an X-ray photon of maximum energy $h\nu_{max}$ is emitted *i.e.* $\frac{1}{2}mv^2 = eV = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$

where v = velocity of electron before collision with target atom, V = potential difference through which electron is accelerated, c = speed of light = 3×10^8 m/s

Maximum frequency of radiations (X-rays) $\qquad \nu_{\max} = \frac{eV}{h}$

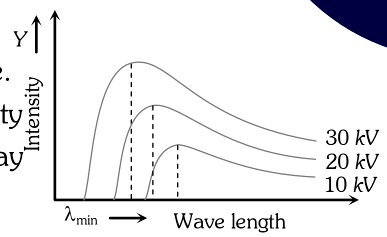
Minimum wave length = cut off wavelength of X-ray $\qquad \lambda_{\min} = \frac{hc}{eV} = \frac{12375}{V}$ Å

Note : ≅ Wavelength of continuous X-ray photon ranges from certain minimum (λ_{min}) to infinity.



Intensity wavelength graph

The continuous X-ray spectra consist of all the wavelengths over a given range. These wavelength are of different intensities. Following figure shows the intensity variation of different wavelengths for various accelerating voltages applied to X-ray tube.

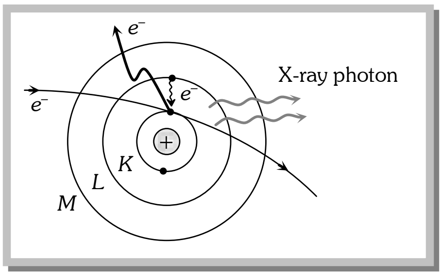


For each voltage, the intensity curve starts at a particular minimum wavelength (λ_{min}). Rises rapidly to a maximum and then drops gradually.

The wavelength at which the intensity is maximum depends on the accelerating voltage, being shorter for higher voltage and vice-versa.

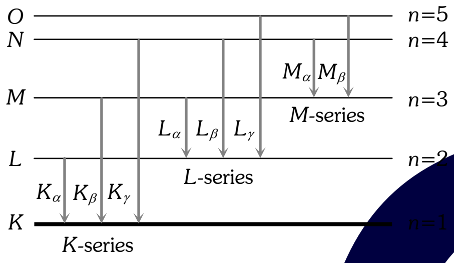
(2) **Characteristic X-rays**

Few of the fast moving electrons having high velocity penetrate the surface atoms of the target material and knock out the tightly bound electrons even from the inner most shells of the atom. Now when the electron is knocked out, a vacancy is created at that place. To fill this vacancy electrons from higher shells jump to fill the created vacancies, we know that when an electron jumps from a higher energy orbit E_1 to lower energy orbit E_2, it radiates energy $(E_1 - E_2)$. Thus this energy difference is radiated in the form of X-rays of very small but definite wavelength which depends upon the target material. The X-ray spectrum consist of sharp lines and is called characteristic X-ray spectrum.



K, L, M, …… series

If the electron striking the target eject an electron from the K-shell of the atom, a vacancy is crated in the K-shell. Immediately an electron from one of the outer shell, say L-shell jumps to the K-shell, emitting an X-ray photon of energy equal to the energy difference between the two shells. Similarly, if an electron from the M-shell jumps to the K-shell, X-ray photon of higher energy is emitted. The X-ray photons emitted due to the jump of electron from the L, M, N shells to the K-shells gives K_α, K_β, K_γ lines of the K-series of the spectrum.



If the electron striking the target ejects an electron from the L-shell of the target atom, an electron from the M, N ….. shells jumps to the L-shell so that X-rays photons of lesser energy are emitted. These photons form the lesser energy emission. These photons form the L-series of the spectrum. In a similar way the formation of M series, N series *etc*. may be explained.

Energy and wavelength of different lines

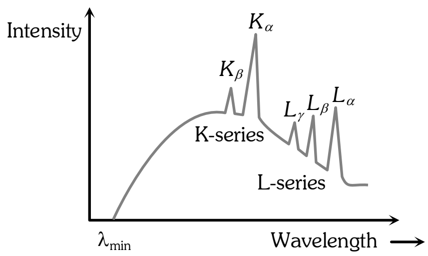
Series	Transition	Energy	Wavelength
K_α	$\underset{(2)}{L} \rightarrow \underset{(1)}{K}$	$E_L - E_K = h\nu_{K\alpha}$	$\lambda_{K\alpha} = \dfrac{hc}{E_L - E_K} = \dfrac{12375}{(E_L - E_K)eV}\text{Å}$
K_β	$\underset{(3)}{M} \rightarrow \underset{(1)}{K}$	$E_M - E_K = h\nu_{K\beta}$	$\lambda_{K\beta} = \dfrac{hc}{E_M - E_K} = \dfrac{12375}{(E_M - E_K)eV}\text{Å}$
L_α	$\underset{(3)}{M} \rightarrow \underset{(2)}{L}$	$E_M - E_L = h\nu_{L\alpha}$	$\lambda_{L\alpha} = \dfrac{hc}{E_M - E_L} = \dfrac{12375}{(E_M - E_L)eV}\text{Å}$

Note : ≅ The wavelength of characteristic X-ray doesn't depend on accelerating voltage. It depends on the atomic number (Z) of the target material.

≅ $\lambda_{K\alpha} < \lambda_{L\alpha} < \lambda_{M\alpha}$ and $\nu_{K\alpha} > \nu_{L\alpha} > \nu_{M\alpha}$

≅ $\lambda_{K\alpha} > \lambda_{L\beta} < \lambda_{K\gamma}$

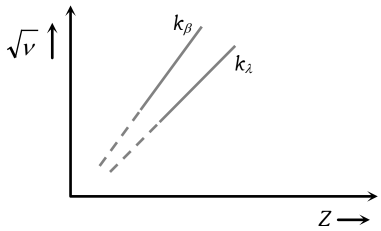
Intensity-wavelength graph



At certain sharply defined wavelengths, the intensity of X-rays is very large as marked K_α, K_β …. As shown in figure. These X-rays are known as characteristic X-rays. At other wavelengths the intensity varies gradually and these X-rays are called continuous X-rays.

Mosley's law

Mosley studied the characteristic X-ray spectrum of a number of a heavy elements and concluded that the spectra of different elements are very similar and with increasing atomic number, the spectral lines merely shift towards higher frequencies.



He also gave the following relation $\sqrt{\nu} = a(Z - b)$

where ν = Frequency of emitted line, Z = Atomic number of target, a = Proportionality constant,

b = Screening constant.

Note : ≅ a and b doesn't depend on the nature of target. Different values of b are as follows

$b = 1$ for K-series

$b = 7.4$ for L-series

$b = 19.2$ for M-series

≅ $(Z - b)$ is called effective atomic number.

More about Mosley's law

(i) It supported Bohr's theory

(ii) It experimentally determined the atomic number (Z) of elements.

(iii) This law established the importance of ordering of elements in periodic table by atomic number and not by atomic weight.

(iv) Gaps in Moseley's data for A = 43, 61, 72, 75 suggested existence of new elements which were later discovered.

(v) The atomic numbers of Cu, Ag and Pt were established to be 29, 47 and 78 respectively.

(vi) When a vacancy occurs in the K-shell, there is still one electron remaining in the K-shell. An electron in the L-shell will feel an effective charge of $(Z - 1)e$ due to $+Ze$ from the nucleus and $-e$ from the remaining K-shell electron, because L-shell orbit is well outside the K-shell orbit.

(vii) Wave length of characteristic spectrum $\frac{1}{\lambda} = R(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and energy of X-ray radiations.

$$\Delta E = h\nu = \frac{hc}{\lambda} = Rhc(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(viii) If transition takes place from $n_2 = 2$ to $n_1 = 1$ (K_α - line)

(a) $a = \sqrt{\frac{3RC}{4}} = 2.47 \times 10^{15}\,Hz$

(b) $\nu_{K\alpha} = RC(Z-1)^2\left(1-\frac{1}{2^2}\right) = \frac{3RC}{4}(Z-1)^2 = 2.47 \times 10^{15}(Z-1)^2\,Hz$

(c) In general the wavelength of all the K-lines are given by $\frac{1}{\lambda_K} = R(Z-1)^2\left(1-\frac{1}{n^2}\right)$ where n = 2, 3, 4, ….

While for K_α line $\lambda_{K\alpha} = \frac{1216}{(Z-1)}\mathring{A}$

(d) $E_{K\alpha} = 10.2(Z-1)^2\,eV$

Uses of X-rays

(i) In study of crystal structure : Structure of DNA was also determined using X-ray diffraction.

(ii) In medical science.

(iii) In radiograph

(iv) In radio therapy

(v) In engineering

(vi) In laboratories

(vii) In detective department

(viii) In art the change occurring in old oil paintings can be examined by X-rays.

Concepts

- *Nearly all metals emits photoelectrons when exposed to UV light. But alkali metals like lithium, sodium, potassium, rubidium and cesium emit photoelectrons even when exposed to visible light.*
- *Oxide coated filament in vacuum tubes is used to emit electrons at relatively lower temperature.*
- *Conduction of electricity in gases at low pressure takes because colliding electrons acquire higher kinetic energy due to increase in mean free path.*
- *Kinetic energy of cathode rays depends on both voltage and work function of cathode.*
- *Photoelectric effect is due to the particle nature of light.*
- *Hydrogen atom does not emit X-rays because it's energy levels are too close to each other.*
- *The essential difference between X-rays and of γ-rays is that, γ-rays emits from nucleus while X-rays from outer part of atom.*
- *There is no time delay between emission of electron and incidence of photon i.e. the electrons are emitted out as soon as the light falls on metal surface.*
- *If light were wave (not photons) it will take about an year take about an year to eject a photoelectron out of the metal surface.*
- *Doze of X-ray are measured in terms of produced ions or free energy via ionisaiton.*
- *Safe doze for human body per week is one Rontgen (One Rontgen is the amount of X-rays which emits 2.5 $\times 10^4$ J free energy through ionization of 1 gm air at NTP*

Example

***Example*: 22** The work function of a substance is 4.0 *eV*. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **[AIEEE 2004]**

(a) 540 *nm* (b) 400 *nm* (c) 310 *nm* (d) 220 *nm*

Solution : (c) By using $\lambda_0 = \dfrac{12375}{W_0(eV)} \Rightarrow \lambda_0 = \dfrac{12375}{4} = 3093.7\,\mathring{A} \simeq 310\,nm$

***Example*: 23** Photo-energy 6 *eV* are incident on a surface of work function 2.1 *eV*. What are the stopping potential **[MP PMT 2004]**

(a) – 5*V* (b) – 1.9 *V* (c) – 3.9 *V* (d) – 8.1 *V*

Solution : (c) By using Einstein's equation $E = W_0 + K_{max} \Rightarrow \quad 6 = 2.1 + K_{\max} \quad \Rightarrow K_{\max} = 3.9\,eV$

Also $V_0 = -\dfrac{K_{\max}}{\rho} = -3.9\,V.$

***Example*: 24** When radiation of wavelength λ is incident on a metallic surface the stopping potential is 4.8 *volts*. If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 *volts*. Then the threshold wavelength for the surface is **[EAMCET (Engg.) 2003]**

(a) 2λ (b) 4λ (c) 6λ (d) 8λ

Solution : (b) By using $V_0 = \dfrac{hc}{e}\left[\dfrac{1}{\lambda} - \dfrac{1}{\lambda_0}\right]$

$$4.8 = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \quad \ldots\ldots \text{(i)} \qquad \text{and} \qquad 1.6 = \frac{hc}{e}\left[\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right] \quad \ldots\ldots \text{(ii)}$$

From equation (i) and (ii) $\lambda_0 = 4\lambda$.

***Example*: 25** When radiation is incident on a photoelectron emitter, the stopping potential is found to be 9 volts. If *e/m* for the electron is $1.8 \times 10^{11}\,Ckg^{-1}$ the maximum velocity of the ejected electrons is **[Kerala (Engg.) 2002]**

(a) $6 \times 10^5 ms^{-1}$ (b) $8 \times 10^5 ms^{-1}$ (c) $1.8 \times 10^6 ms^{-1}$ (d) $1.8 \times 10^5 ms^{-1}$

Solution : (c) $\dfrac{1}{2} m v_{\max}^2 = eV_0 \quad \Rightarrow \quad v_{\max} = \sqrt{2\left(\dfrac{e}{m}\right).V_0} = \sqrt{2 \times 1.8 \times 10^{11} \times 9} = 1.8 \times 10^6\ m/s$.

***Example*: 26** The lowest frequency of light that will cause the emission of photoelectrons from the surface of a metal (for which work function is 1.65 *eV*) will be **[JIPMER 2002]**

(a) $4 \times 10^{10} Hz$ (b) $4 \times 10^{11} Hz$ (c) $4 \times 10^{14} Hz$ (d) $4 \times 10^{-10} Hz$

Solution : (c) Threshold wavelength $\lambda_0 = \dfrac{12375}{W_0(eV)} = \dfrac{12375}{1.65} = 7500\,\mathring{A}$.

$\therefore$ so minimum frequency $\nu_0 = \dfrac{c}{\lambda_0} = \dfrac{3 \times 10^8}{7500 \times 10^{-10}} = 4 \times 10^{14}\,Hz.$

***Example*: 27** Light of two different frequencies whose photons have energies 1 *eV* and 2.5 *eV* respectively, successively illuminates a metal of work function 0.5 *eV*. The ratio of maximum kinetic energy of the emitted electron will be **[AIEEE 2002]**

(a) 1 : 5 (b) 1 : 4 (c) 1 : 2 (d) 1 : 1

Solution : (b) By using $K_{\max} = E - W_0 \Rightarrow \frac{(K_{\max})_1}{(K_{\max})_2} = \frac{1-0.5}{2.5-0.5} = \frac{0.5}{2} = \frac{1}{4}$.

Example: 28 Photoelectric emission is observed from a metallic surface for frequencies ν_1 and ν_2 of the incident light rays $(\nu_1 > \nu_2)$. If the maximum values of kinetic energy of the photoelectrons emitted in the two cases are in the ratio of 1 : k, then the threshold frequency of the metallic surface is **[EAMCET (Engg.) 2001]**

(a) $\frac{\nu_1 - \nu_2}{k-1}$ (b) $\frac{k\nu_1 - \nu_2}{k-1}$ (c) $\frac{k\nu_2 - \nu_1}{k-1}$ (d) $\frac{\nu_2 - \nu_1}{k-1}$

Solution : (b) By using $h\nu - h\nu_0 = k_{\max} \Rightarrow h(\nu_1 - \nu_0) = k_1$ and $h(\nu_1 - \nu_0) = k_2$

Hence $\frac{\nu_1 - \nu_0}{\nu_2 - \nu_0} = \frac{k_1}{k_2} = \frac{1}{k} \Rightarrow \nu_0 = \frac{k\nu_1 - \nu_2}{k-1}$

Example: 29 Light of frequency $8 \times 10^{15} Hz$ is incident on a substance of photoelectric work function 6.125 *eV*. The maximum kinetic energy of the emitted photoelectrons is **[AFMC 2001]**

(a) 17 *eV* (b) 22 *eV* (c) 27 *eV* (d) 37 *eV*

Solution : (c) Energy of incident photon $E = h\nu = 6.6 \times 10^{-34} \times 8 \times 10^{15} = 5.28 \times 10^{-18} J = 33 eV$.

From $E = W_0 + K_{\max} \Rightarrow K_{\max} = E - W_0 = 33 - 6.125 = 26.87 eV \approx 27 eV$.

Example: 30 A photo cell is receiving light from a source placed at a distance of 1 *m*. If the same source is to be placed at a distance of 2 *m*, then the ejected electron **[MNR 1986; UPSEAT 2000, 2001]**

(a) Moves with one-fourth energy as that of the initial energy

(b) Moves with one fourth of momentum as that of the initial momentum

(c) Will be half in number

(d) Will be one-fourth in number

Solution : (d) Number of photons $\propto$ Intensity $\propto \frac{1}{(\text{distance})^2}$

$$\Rightarrow \frac{N_1}{N_2} = \left(\frac{d_2}{d_1}\right)^2 \Rightarrow \frac{N_1}{N_2} - \left(\frac{2}{1}\right)^2 \Rightarrow N_2 = \frac{N_1}{4}.$$

Example: 31 When yellow light incident on a surface no electrons are emitted while green light can emit. If red light is incident on the surface then **[MNR 1998; MH CET 2000; MP PET 2000]**

(a) No electrons are emitted (b) Photons are emitted

(c) Electrons of higher energy are emitted (d) Electrons of lower energy are emitted

Solution : (a) $\lambda_{\text{Green}} < \lambda_{\text{Yellow}} < \lambda_{\text{Red}}$

According to the question λ_{Green} is the maximum wavelength for which photoelectric emission takes place. Hence no emission takes place with red light.

Example: 32 When a metal surface is illuminated by light of wavelengths 400 *nm* and 250 *nm* the maximum velocities of the photoelectrons ejected are v and $2v$ respectively. The work function of the metal is (h = Planck's constant, c = velocity of light in air) **[EMCET (Engg.) 2000]**

(a) $2hc \times 10^6 J$ (b) $1.5hc \times 10^6 J$ (c) $hc \times 10^6 J$ (d) $0.5hc \times 10^6 J$

Solution : (a) By using $E = W_0 + K_{\max} \Rightarrow \frac{hc}{\lambda} = W_0 + \frac{1}{2}mv^2$

$$\frac{hc}{400\times10^{-9}} = W_0 + \frac{1}{2}mv^2 \quad ......(i) \quad \text{and} \quad \frac{hc}{250\times10^{-9}} = W_0 + \frac{1}{2}m(2v)^2 \quad ......(ii)$$

From equation (i) and (ii) $W_0 = 2hc\times10^6 J$.

Example: 33 The work functions of metals A and B are in the ratio 1 : 2. If light of frequencies f and $2f$ are incident on the surfaces of A and B respectively, the ratio of the maximum kinetic energies of photoelectrons emitted is (f is greater than threshold frequency of A, $2f$ is greater than threshold frequency of B) **[EAMCET (Med.) 2000]**

(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

Solution : (b) By using $E = W_0 + K_{max} \Rightarrow E_A = hf = W_A + K_A$ and $E_B = h(2f) = W_B + K_B$

So, $\frac{1}{2} = \frac{W_A + K_A}{W_B + K_B}$ ......(i) also it is given that $\frac{W_A}{W_B} = \frac{1}{2}$ ....(ii)

From equation (i) and (ii) we get $\frac{K_A}{K_B} = \frac{1}{2}$.

Example: 34 When a point source of monochromatic light is at a distance of 0.2m from a photoelectric cell, the cut-off voltage and the saturation current are 0.6 *volt* and 18 *mA* respectively. If the same source is placed 0.6 *m* away from the photoelectric cell, then **[IIT-JEE 1992; MP PMT 1999]**

(a) The stopping potential will be 0.2 *V* (b) The stopping potential will be 0.6 *V*

(c) The saturation current will be 6 *mA* (d) The saturation current will be 18 *mA*

Solution : (b) Photoelectric current $(i) \propto$ Intensity $\propto \frac{1}{(\text{distance})^2}$. If distance becomes 0.6 *m* (*i.e.* three times) so current becomes $\frac{1}{9}$ times *i.e.* 2*mA*.

Also stopping potential is independent of intensity *i.e.* it remains 0.6 *V*.

Example: 35 In a photoemissive cell with exciting wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be **[CBSE 1998]**

(a) $v(3/4)^{1/2}$ (b) $v(4/3)^{1/2}$ (c) Less then $v(4/3)^{1/2}$ (d) Greater then $v(4/3)^{1/2}$

Solution : (d) From $E = W_0 + \frac{1}{2}mv_{max}^2 \Rightarrow v_{max} = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}}$ (where $E = \frac{hc}{\lambda}$)

If wavelength of incident light charges from λ to $\frac{3\lambda}{4}$ (decreases)

Let energy of incident light charges from E to E' and speed of fastest electron changes from v to v' then

$$v = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}} \quad .....(i) \quad \text{and} \quad v' = \sqrt{\frac{2E'}{m} - \frac{2W_0}{m}} \quad ....(ii)$$

As $E \propto \frac{1}{\lambda} \Rightarrow E' = \frac{4}{3}E$ hence $v' = \sqrt{\frac{2\left(\frac{4}{3}E\right)}{m} - \frac{2W_0}{m}} \Rightarrow v' = \left(\frac{4}{3}\right)^{1/2}\sqrt{\frac{2E}{m} - \frac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}}$

$$\Rightarrow \ v' = \left(\frac{4}{3}\right)^{1/2} \qquad X = \sqrt{\frac{2E}{m} - \frac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}} > v \ \text{ so } v' > \left(\frac{4}{3}\right)^{1/2} v.$$

Example: 36 The minimum wavelength of X-rays produced in a coolidge tube operated at potential difference of 40 *kV* is **[BCECE 2003]**

(a) 0.31Å (b) 3.1Å (c) 31Å (d) 311Å

Solution : (a) $\lambda_{\min} = \frac{12375}{40 \times 10^3} = 0.309 Å \approx 0.31\ Å$

Example: 37 The X-ray wavelength of L_α line of platinum (Z = 78) is 1.30Å. The X –ray wavelength of L_a line of Molybdenum (Z = 42) is **[EAMCET (Engg.) 2000]**

(a) 5.41Å (b) 4.20Å (c) 2.70Å (d) 1.35 Å

Solution : (a) The wave length of L_α line is given by $\frac{1}{\lambda} = R(z-7.4)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda \propto \frac{1}{(z-7.4)^2}$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{(z_2-7.4)^2}{(z_1-7.4)^2} \Rightarrow \frac{1.30}{\lambda_2} = \frac{(42-7.4)^2}{(78-7.4)^2} \Rightarrow \lambda_2 = 5.41Å.$$

Example: 38 The cut off wavelength of continuous X-ray from two coolidge tubes operating at 30 *kV* but using different target materials (molybdenum *Z*= 42 and tungsten *Z* = 74) are

(a) 1Å, 3Å (b) 0.3 Å, 0.2 Å (c) 0.414 Å, 0.8 Å (d) 0.414 Å, 0.414 Å

Solution : (d) Cut off wavelength of continuous X-rays depends solely on the voltage applied and does not depend on the material of the target. Hence the two tubes will have the same cut off wavelength.

$$Ve = h\nu = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{Ve} = \frac{6.627 \times 10^{-34} \times 3 \times 10^8}{30 \times 10^3 \times 1.6 \times 10^{-19}} m = 414 \times 10^{-10} m = 0.414\ Å.$$

Tricky example: 3

Two photons, each of energy $2.5 eV$ are simultaneously incident on the metal surface. If the work function of the metal is $4.5\ eV$, then from the surface of metal

(a) Two electrons will be emitted
(b) Not even a single electron will be emitted
(c) One electron will be emitted
(d) More than two electrons will be emitted

Solution : (b) Photoelectric effect is the phenomenon of one to one elastic collision between incident photon and an electron. Here in this question one electron absorbs one photon and gets energy $2.5\ eV$ which is lesser than $4.5\ eV$. Hence no photoelectron emission takes place.

Tricky example: 4

In X-ray tube when the accelerating voltage V is halved, the difference between the wavelength of K_α line and minimum wavelength of continuous X-ray spectrum

(a) Remains constant
(b) Becomes more than two times
(c) Becomes half
(d) Becomes less than two times

Solution : (c) $\Delta\lambda = \lambda_{K_\alpha} - \lambda_{\min}$ when V is halved $\lambda_{\min}$ becomes two times but λ_{K_α} remains the same.

$\therefore \quad \Delta\lambda' = \lambda_{K_\alpha} - 2\lambda_{\min} = 2(\Delta\lambda) - \lambda_{K_\alpha}$

$\therefore \quad \Delta\lambda' < 2(\Delta\lambda)$

Tricky example: 5

Molybdenum emits K_α-photons of energy $18.5\ keV$ and iron emits K_α photons of energy $34.7\ keV$. The times taken by a molybdenum K_α photon and an iron K_α photon to travel $300\ m$ are

(a) ($3\ \mu s$, $15\ \mu s$)
(b) ($15\ \mu s$, $3 \mu s$)
(c) ($1\ \mu s$, $1\ \mu s$)
(d) ($1\ \mu s$, $5 \mu s$)

Solution : (c) Photon have the same speed whatever be their energy, frequency, wavelength, and origin.

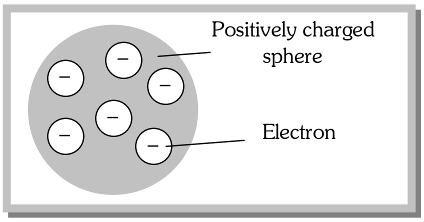
$\therefore$ time of travel of either photon $= \dfrac{300}{3\times 10^8} = 10^{-6}\ s = 1\ \mu s$

Important Atomic Models.

(1) Thomson's model

J.J. Thomson gave the first idea regarding structure of atom. According to this model.

(i) An atom is a solid sphere in which entire and positive charge and it's mass is uniformly distributed and in which negative charge (*i.e.* electron) are embedded like seeds in watermelon.



Success and failure

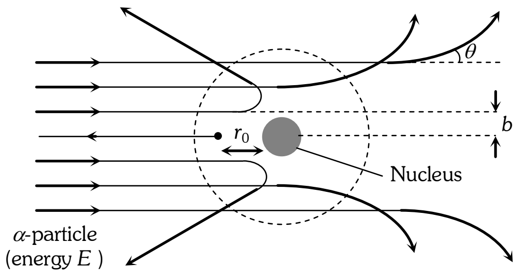
Explained successfully the phenomenon of thermionic emission, photoelectric emission and ionization.

The model fail to explain the scattering of α- particles and it cannot explain the origin of spectral lines observed in the spectrum of hydrogen and other atoms.

(2) Rutherford's model

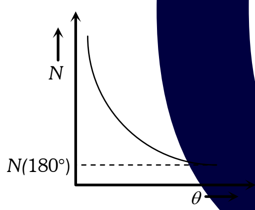
Rutherford's α-particle scattering experiment

Rutherford performed experiments on the scattering of alpha particles by extremely thin gold foils and made the following observations



Number of scattered particles :

$$N \propto \frac{1}{\sin^4(\theta/2)}$$



(i) Most of the α-particles pass through the foil straight away undeflected.

(ii) Some of them are deflected through small angles.

(iii) A few α-particles (1 in 1000) are deflected through the angle more than 90°.

(iv) A few α -particles (very few) returned back *i.e.* deflected by 180°.

(v) Distance of closest approach (Nuclear dimension)

The minimum distance from the nucleus up to which the α-particle approach, is called the distance of closest approach (r_0). From figure $r_0 = \frac{1}{4\pi\varepsilon_0}.\frac{Ze^2}{E}$; $E = \frac{1}{2}mv^2$ = K.E. of α-particle

(vi) Impact parameter (b) : The perpendicular distance of the velocity vector ($\vec{v}$) of the α-particle from the centre of the nucleus when it is far away from the nucleus is known as impact parameter. It is given as

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)} \Rightarrow b \propto \cot(\theta/2)$$

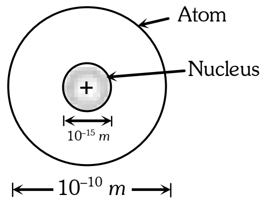
Note : ≅If t is the thickness of the foil and N is the number of α-particles scattered in a particular direction (θ = constant), it was observed that $\frac{N}{t}$ = constant $\Rightarrow \frac{N_1}{N_2} = \frac{t_1}{t_2}$.

After Rutherford's scattering of α-particles experiment, following conclusions were made as regard as atomic structure :

(a) Most of the mass and all of the charge of an atom concentrated in a very small region is called atomic nucleus.

(b) Nucleus is positively charged and it's size is of the order of 10^{-15} $m \approx 1$ *Fermi*.

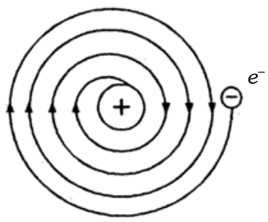
(c) In an atom there is maximum empty space and the electrons revolve around the nucleus in the same way as the planets revolve around the sun.



Size of the nucleus = 1 Fermi = 10^{-15} m
Size of the atom 1 Å = 10^{-10} m

Draw backs

(i) Stability of atom : It could not explain stability of atom because according to classical electrodynamic theory an accelerated charged particle should continuously radiate energy. Thus an electron moving in an circular path around the nucleus should also radiate energy and thus move into smaller and smaller orbits of gradually decreasing radius and it should ultimately fall into nucleus.



Instability of atom

(ii) According to this model the spectrum of atom must be continuous where as practically it is a line spectrum.

(iii) It did not explain the distribution of electrons outside the nucleus.

(3) Bohr's model

Bohr proposed a model for hydrogen atom which is also applicable for some lighter atoms in which a single electron revolves around a stationary nucleus of positive charge Ze (called hydrogen like atom)

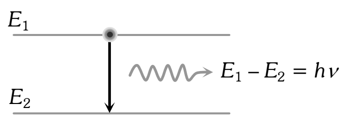
Bohr's model is based on the following postulates.

(i) The electron can revolve only in certain discrete non-radiating orbits, called stationary orbits, for which total angular momentum of the revolving electrons is an integral multiple of $\frac{h}{2\pi}$ $(= \hbar)$

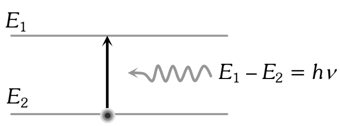
i.e. $L = n\left(\frac{h}{2\pi}\right) = mvr$; where n = 1, 2, 3, ...= Principal quantum number

(ii) The radiation of energy occurs only when an electron jumps from one permitted orbit to another.

When electron jumps from higher energy orbit (E_1) to lower energy orbit (E_2) then difference of energies of these orbits *i.e.* $E_1 - E_2$ emits in the form of photon. But if electron goes from E_2 to E_1 it absorbs the same amount of energy.



Emission



Absorption

Note : ≅According to Bohr theory the momentum of an e^- revolving in second orbit of H_2 atom will be $\frac{h}{\pi}$

≅ For an electron in the n^{th} orbit of hydrogen atom in Bohr model, circumference of orbit $= n\lambda$; where λ = de-Broglie wavelength.

Bohr's Orbits (For Hydrogen and H_2-Like Atoms).

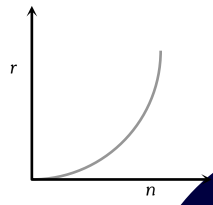
(1) Radius of orbit

For an electron around a stationary nucleus the electrostatics force of attraction provides the necessary centripetal force

i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ .... (i) also $mvr = \frac{nh}{2\pi}$ ....(ii)

From equation (i) and (ii) radius of n^{th} orbit

$$r_n = \frac{n^2h^2}{4\pi^2 kZme^2} = \frac{n^2h^2\varepsilon_0}{\pi mZe^2} = 0.53\frac{n^2}{Z}\text{Å} \qquad \left[\text{where } k = \frac{1}{4\pi\varepsilon_0}\right]$$



$$\Rightarrow \boldsymbol{r_n \propto \frac{n^2}{Z}}$$

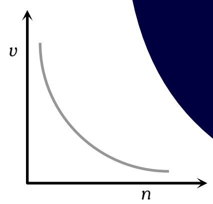
Note : ≅The radius of the innermost orbit (n = 1) hydrogen atom (z = 1) is called Bohr's radius a_0 i.e. $a_0 = 0.53\text{Å}$.

(2) Speed of electron

From the above relations, speed of electron in n^{th} orbit can be calculated as

$$v_n = \frac{2\pi kZe^2}{nh} = \frac{Ze^2}{2\varepsilon_0 nh} = \left(\frac{c}{137}\right).\frac{Z}{n} = 2.2\times 10^6\frac{Z}{n}\, m/sec$$

where (c = speed of light 3×10^8 m/s)



Note : ≅The ratio of speed of an electron in ground state in Bohr's first orbit of hydrogen atom to velocity of light in air is equal to $\frac{e^2}{2\varepsilon_0 ch} = \frac{1}{137}$ (where c = speed of light in air)

(3) Some other quantities

For the revolution of electron in n^{th} orbit, some other quantities are given in the following table

Quantity	Formula	Dependency on n and Z
(1) Angular speed	$\omega_n = \frac{v_n}{r_n} = \frac{\pi mz^2e^4}{2\varepsilon_0^2 n^3h^3}$	$\omega_n \propto \frac{Z^2}{n^3}$
(2) Frequency	$\nu_n = \frac{\omega_n}{2\pi} = \frac{mz^2e^4}{4\varepsilon_0^2 n^3h^3}$	$\nu_n \propto \frac{Z^2}{n^3}$

(3) Time period	$T_n = \dfrac{1}{\nu_n} = \dfrac{4\varepsilon_0^2 n^3 h^3}{m z^2 e^4}$	$T_n \propto \dfrac{n^3}{Z^2}$
(4) Angular momentum	$L_n = m v_n r_n = n\left(\dfrac{h}{2\pi}\right)$	$L_n \propto n$
(5) Corresponding current	$i_n = e\nu_n = \dfrac{m z^2 e^5}{4\varepsilon_0^2 n^3 h^3}$	$i_n \propto \dfrac{Z^2}{n^3}$
(6) Magnetic moment	$M_n = i_n A = i_n\left(\pi r_n^2\right)$ (where $\mu_0 = \dfrac{eh}{4\pi m} =$ Bohr magneton)	$M_n \propto n$
(7) Magnetic field	$B = \dfrac{\mu_0 i_n}{2r_n} = \dfrac{\pi m^2 z^3 e^7 \mu_0}{8\varepsilon_0^3 n^5 h^5}$	$B \propto \dfrac{Z^3}{n^5}$

(4) **Energy**

(i) **Potential energy :** An electron possesses some potential energy because it is found in the field of nucleus potential energy of electron in n^{th} orbit of radius r_n is given by $U = k.\dfrac{(Ze)(-e)}{r_n} = -\dfrac{kZe^2}{r_n}$

(ii) **Kinetic energy :** Electron posses kinetic energy because of it's motion. Closer orbits have greater kinetic energy than outer ones.

As we know $\dfrac{mv^2}{r_n} = \dfrac{k.(Ze)(e)}{r_n^2} \Rightarrow$ Kinetic energy $K = \dfrac{kZe^2}{2r_n} = \dfrac{|U|}{2}$

(iii) **Total energy :** Total energy (E) is the sum of potential energy and kinetic energy *i.e.* $E = K + U$

$\Rightarrow E = -\dfrac{kZe^2}{2r_n}$ also $r_n = \dfrac{n^2 h^2 \varepsilon_0}{\pi m z e^2}$. Hence $E = -\left(\dfrac{me^4}{8\varepsilon_0^2 h^2}\right).\dfrac{z^2}{n^2} = -\left(\dfrac{me^4}{8\varepsilon_0^2 c h^3}\right) ch \dfrac{z^2}{n^2} = -R\,ch\dfrac{Z^2}{n^2} = -13.6\dfrac{Z^2}{n^2} eV$

where $R = \dfrac{me^4}{8\varepsilon_0^2 c h^3} =$ Rydberg's constant $= 1.09 \times 10^7$ *per metre*

Note : ≅Each Bohr orbit has a definite energy

≅ For hydrogen atom ($Z = 1$) $\Rightarrow E_n = -\dfrac{13.6}{n^2} eV$

≅ The state with $n = 1$ has the lowest (most negative) energy. For hydrogen atom it is $E_1 = -13.6\ eV$.

≅ Rch = Rydberg's energy $\simeq 2.17 \times 10^{-18} J \simeq 31.6\, eV$.

≅ $E = -K = \dfrac{U}{2}$.

(iv) **Ionisation energy and potential :** The energy required to ionise an atom is called ionisation energy. It is the energy required to make the electron jump from the present orbit to the infinite orbit.

Hence $E_{ionisation} = E_\infty - E_n = 0 - \left(-13.6\dfrac{Z^2}{n^2}\right) = +\dfrac{13.6 Z^2}{n^2} eV$

For H_2-atom in the ground state $E_{ionisation} = \dfrac{+13.6(1)^2}{n^2} = 13.6\, eV$

The potential through which an electron need to be accelerated so that it acquires energy equal to the ionisation energy is called ionisation potential. $V_{ionisation} = \dfrac{E_{ionisation}}{e}$

(v) **Excitation energy and potential :** When the electron is given energy from external source, it jumps to higher energy level. This phenomenon is called excitation.

The minimum energy required to excite an atom is called excitation energy of the particular excited state and corresponding potential is called exciting potential.

$$E_{Excitation} = E_{Final} - E_{Initial} \text{ and } V_{Excitation} = \frac{E_{excitation}}{e}$$

(vi) **Binding energy (B.E.) :** Binding energy of a system is defined as the energy released when it's constituents are brought from infinity to form the system. It may also be defined as the energy needed to separate it's constituents to large distances. If an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13.6 *eV* energy will be released. The binding energy of a hydrogen atom is therefore 13.6 *eV*.

Note : ≅For hydrogen atom principle quantum number $n = \sqrt{\frac{13.6}{(\text{B.E.})}}$.

(5) **Energy level diagram**

The diagrammatic description of the energy of the electron in different orbits around the nucleus is called energy level diagram.

Energy level diagram of hydrogen/hydrogen like atom

Principle quantum number	Orbit	Excited state	Energy for H_2 – atom	Energy for H_2 – like atom	Ionisation energy from this level (for H_2 – atom)
$n = \infty$	Infinite	Infinite	$E_\infty = 0\ eV$	$0\ eV$	$0\ eV$
$n = 4$	Fourth	Third	$E_4 = -0.85\ eV$	$-0.85\ Z^2$	$+0.85\ eV$
$n = 3$	Third	Second	$E_3 = -1.51\ eV$	$-1.51\ Z^2$	$+1.51\ eV$
$n = 2$	Second	First	$E_2 = -3.4\ eV$	$-3.4\ Z^2$	$+3.4\ eV$
$n = 1$	First	Ground	$E_1 = -13.6\ eV$	$-13.6\ Z^2$	$+13.6\ eV$

Note : ≅In hydrogen atom excitation energy to excite electron from ground state to first excited state will be $-3.4-(-13.6)=10.2\ eV$.

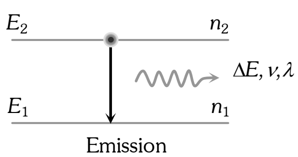
and from ground state to second excited state it is [$-1.51-(-13.6)=12.09\ eV$].

≅ In an H_2 atom when e^- makes a transition from an excited state to the ground state it's kinetic energy increases while potential and total energy decreases.

(6) **Transition of electron**

When an electron makes transition from higher energy level having energy $E_2(n_2)$ to a lower energy level having energy E_1 (n_1) then a photon of frequency ν is emitted

(i) **Energy of emitted radiation**



$$\Delta E = E_2 - E_1 = \frac{-Rc\,hZ^2}{n_2^2} - \left(-\frac{RchZ^2}{n_1^2}\right) = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(ii) **Frequency of emitted radiation**

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h} = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(iii) **Wave number/wavelength**

Wave number is the number of waves in unit length $\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} \Rightarrow \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{13.6Z^2}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

(iv) **Number of spectral lines** : If an electron jumps from higher energy orbit to lower energy orbit it emits raidations with various spectral lines.

If electron falls from orbit n_2 to n_1 then the number of spectral lines emitted is given by

$$N_E = \frac{(n_2 - n_1 + 1)(n_2 - n_1)}{2}$$

If electron falls from n^{th} orbit to ground state (*i.e.* $n_2 = n$ and $n_1 = 1$) then number of spectral lines emitted $\boldsymbol{N_E = \frac{n(n-1)}{2}}$

Note : ≅Absorption spectrum is obtained only for the transition from lowest energy level to higher energy levels. Hence the number of absorption spectral lines will be $(n - 1)$.

(v) **Recoiling of an atom** : Due to the transition of electron, photon is emitted and the atom is recoiled

Recoil momentum of atom = momentum of photon $= \frac{h}{\lambda} = hRZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Also recoil energy of atom $= \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ (where m = mass of recoil atom)

(7) **Draw backs of Bohr's atomic model**

(i) It is valid only for one electron atoms, *e.g.* : $H, He^+, Li^{+2}, Na^{+1}$ etc.

(ii) Orbits were taken as circular but according to Sommerfield these are elliptical.

(iii) Intensity of spectral lines could not be explained.

(iv) Nucleus was taken as stationary but it also rotates on its own axis.

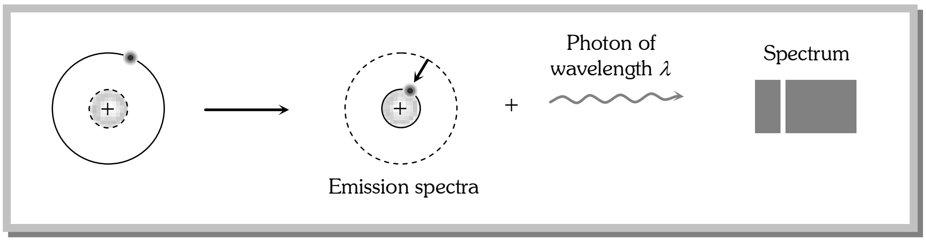
(v) It could not be explained the minute structure in spectrum line.

(vi) This does not explain the Zeeman effect (splitting up of spectral lines in magnetic field) and Stark effect (splitting up in electric field)

(vii) This does not explain the doublets in the spectrum of some of the atoms like sodium (5890Å & 5896Å)

Hydrogen Spectrum and Spectral Series.

When hydrogen atom is excited, it returns to its normal unexcited (or ground state) state by emitting the energy it had absorbed earlier. This energy is given out by the atom in the form of radiations of different wavelengths as the electron jumps down from a higher to a lower orbit. Transition from different orbits cause different wavelengths, these constitute spectral series which are characteristic of the atom emitting them. When observed through a spectroscope, these radiations are imaged as sharp and straight vertical lines of a single colour.



Spectral series

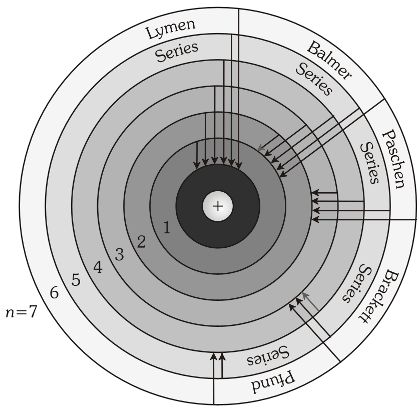
The spectral lines arising from the transition of electron forms a spectra series.

(i) Mainly there are five series and each series is named after it's discover as Lymen series, Balmer series, Paschen series, Bracket series and Pfund series.

(ii) According to the Bohr's theory the wavelength of the radiations emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where n_2 = outer orbit (electron jumps from this orbit), n_1 = inner orbit (electron falls in this orbit)



(iii) First line of the series is called first member, for this line wavelength is maximum (λ_{max})

(iv) Last line of the series ($n_2 = \infty$) is called series limit, for this line wavelength is minimum (λ_{min})

Spectral series	**Transition**	**Wavelength (λ)** $= \frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)R} = \frac{n_1^2}{\left(1 - \frac{n_1^2}{n_2^2}\right)R}$		$\frac{\lambda_{max}}{\lambda_{min}} = \frac{(n+1)^2}{(2n+1)}$	**Region**
		Maximum wavelength $(n_1 = n$ **and** $n_2 = n+1)$	**Minimum wavelength** $(n_2 = \infty, n_1 = n)$		
		$\lambda_{max} = \frac{n^2(n+1)^2}{(2n+1)R}$	$\lambda_{min} = \frac{n^2}{R}$		

1. Lymen series	$n_2 = 2, 3, 4 \ldots\infty$ $n_1 = 1$	$\lambda_{max} = \frac{(1)^2(1+1)^2}{(2\times1+1)R} = \frac{4}{3R}$	$n_1 = n = 1$ $\lambda_{min} = \frac{1}{R}$	$\frac{4}{3}$	Ultraviolet region
2.Balmer series	$n_2 = 3, 4, 5 \ldots\infty$ $n_1 = 2$	$n_1 = n = 2,\ n_2 = 2 + 1 = 3$ $\lambda_{max} = \frac{36}{5R}$	$\lambda_{min} = \frac{4}{R}$	$\frac{9}{5}$	Visible region
3. Paschen series	$n_2 = 4, 5, 6 \ldots\infty$ $n_1 = 3$	$n_1 = n = 3, n_2 = 3 + 1 = 4$ $\lambda_{max} = \frac{144}{7R}$	$n_1 = n = 3$ $\lambda_{min} = \frac{9}{R}$	$\frac{16}{7}$	Infrared region
4. Bracket series	$n_2 = 5, 6, 7 \ldots \infty$ $n_1 = 4$	$n_1 = n = 4, n_2 = 4 + 1 = 5$ $\lambda_{max} = \frac{400}{9R}$	$n_1 = n = 4$ $\lambda_{min} = \frac{16}{R}$	$\frac{25}{9}$	Infrared region
5. Pfund series	$n_2 = 6, 7, 8 \ldots \infty$ $n_1 = 5$	$n_1 = \lambda = 5, n_2 = 5 + 1 = 6$ $\lambda_{max} = \frac{900}{11R}$	$\lambda_{min} = \frac{25}{R}$	$\frac{36}{11}$	Infrared region

Quantum Numbers.

An atom contains large number of shells and subshells. These are distinguished from one another on the basis of their size, shape and orientation (direction) in space. The parameters are expressed in terms of different numbers called quantum number.

Quantum numbers may be defined as a set of four number with the help of which we can get complete information about all the electrons in an atom. It tells us the address of the electron *i.e.* location, energy, the type of orbital occupied and orientation of that orbital.

(1) **Principal Quantum number (n)** : This quantum number determines the main energy level or shell in which the electron is present. The average distance of the electron from the nucleus and the energy of the electron depends on it. $E_n \propto \frac{1}{n^2}$ and $r_n \propto n^2$ (in H-atom)

The principal quantum number takes whole number values, $n = 1, 2, 3, 4, \ldots\ldots \infty$

(2) **Orbital quantum number (l) or azimuthal quantum number (l)**

This represents the number of subshells present in the main shell. These subsidiary orbits within a shell will be denoted as 1, 2, 3, 4 … or *s*, *p*, *d*, *f* … This tells the shape of the subshells.

The orbital angular momentum of the electron is given as $L = \sqrt{l(l+1)}\frac{h}{2\pi}$ (for a particular value of n).

For a given value of n the possible values of l are $l = 0, 1, 2, \ldots\ldots$ upto $(n-1)$

(3) **Magnetic quantum number (m_l)** : An electron due to it's angular motion around the nucleus generates an electric field. This electric field is expected to produce a magnetic field. Under the influence of external magnetic field, the electrons of a subshell can orient themselves in certain preferred regions of space around the nucleus called orbitals.

The magnetic quantum number determines the number of preferred orientations of the electron present in a subshell.

The angular momentum quantum number m can assume all integral value between $-l$ to $+l$ including zero. Thus m_l can be $-1, 0, +1$ for $l = 1$. Total values of m_l associated with a particular value of l is given by $(2l + 1)$.

(4) **Spin (magnetic) quantum number (m_s) :** An electron in atom not only revolves around the nucleus but also spins about its own axis. Since an electron can spin either in clockwise direction or in anticlockwise direction. Therefore for any particular value of magnetic quantum number, spin quantum number can have two values, *i.e.* $m_s = \frac{1}{2}$ (Spin up) or $m_s = -\frac{1}{2}$ (Spin down)

This quantum number helps to explain the magnetic properties of the substance.

Electronic Configurations of Atoms.

The distribution of electrons in different orbitals of an atom is called the electronic configuration of the atom. The filling of electrons in orbitals is governed by the following rules.

(1) **Pauli's exclusion principle**

"It states that no two electrons in an atom can have all the four quantum number (n, l, m_l and m_s) the same."

It means each quantum state of an electron must have a different set of quantum numbers n, l, m_l and m_s. This principle sets an upper limit on the number of electrons that can occupy a shell.

$$N_{\max} \text{ in one shell} = 2n^2; \text{ Thus } N_{\max} \text{ in } K, L, M, N \ldots \text{ shells are } 2, 8, 18, 32,$$

Note : ≅ The maximum number of electrons in a subshell with orbital quantum number l is $2(2l + 1)$.

(2) **Aufbau principle**

Electrons enter the orbitals of lowest energy first.

As a general rule, a new electron enters an empty orbital for which $(n + l)$ is minimum. In case the value $(n + l)$ is equal for two orbitals, the one with lower value of n is filled first.

Thus the electrons are filled in subshells in the following order (memorize)

1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, 4*p*, 5*s*, 4*d*, 5*p*, 6*s*, 4*f*, 5*d*, 6*p*, 7*s*, 5*f*, 6*d*, 7*p*, ……

(3) **Hund's Rule**

When electrons are added to a subshell where more than one orbital of the same energy is available, their spins remain parallel. They occupy different orbitals until each one of them has at least one electron. Pairing starts only when all orbitals are filled up.

Pairing takes place only after filling 3, 5 and 7 electrons in *p*, *d* and *f* orbitals, respectively.

Concepts

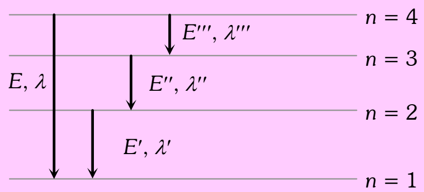
- *With the increase in principal quantum number the energy difference between the two successive energy level decreases, while wavelength of spectral line increases.*

$E' > E'' > E'''$

$\lambda' < \lambda'' < \lambda'''$

$E = E' + E'' + E'''$

$$\frac{1}{\lambda} = \frac{1}{\lambda'} + \frac{1}{\lambda''} + \frac{1}{\lambda'''}$$



- ***Rydberg constant is different for different elements***

$R(=1.09\times10^7\ m^{-1})$ is the value of Rydberg constant when the nucleus is considered to be infinitely massive as compared to the revolving electron. In other words, the nucleus is considered to be stationary.

In case, the nucleus is not infinitely massive or stationary, then the value of Rydberg constant is given as $\boldsymbol{R'}=\dfrac{\boldsymbol{R}}{\boldsymbol{1}+\dfrac{\boldsymbol{m}}{\boldsymbol{M}}}$ where m is the mass of electron and M is the mass of nucleus.

- ***Atomic spectrum is a line spectrum***

  Each atom has it's own characteristic allowed orbits depending upon the electronic configuration. Therefore photons emitted during transition of electrons from one allowed orbit to inner allowed orbit are of some definite energy only. They do not have a continuous graduation of energy. Therefore the spectrum of the emitted light has only some definite lines and therefore atomic spectrum is line spectrum.
- *Just as dots of light of only three colours combine to form almost every conceivable colour on T.V. screen, only about 100 distinct kinds of atoms combine to form all the materials in the universe.*

Example

Example: 1 The ratio of areas within the electron orbits for the first excited state to the ground state for hydrogen atom is **[BCECE 2004]**

(a) 16 : 1 (b) 18 : 1 (c) 4 : 1 (d) 2 : 1

Solution : (a) For a hydrogen atom

Radius $r\propto n^2 \Rightarrow \dfrac{r_1^2}{r_2^2}=\dfrac{n_1^4}{n_2^4} \Rightarrow \dfrac{\pi r_1^2}{\pi r_2^2}=\dfrac{n_1^4}{n_2^4} \Rightarrow \dfrac{A_1}{A_2}=\dfrac{n_1^4}{n_2^4}=\dfrac{2^4}{1^4}=16 \Rightarrow \dfrac{A_1}{A_2}=\dfrac{16}{1}$

Example: 2 The electric potential between a proton and an electron is given by $V=V_0\ln\dfrac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number **[IIT-JEE (Screening) 2003]**

(a) $r_n\propto n$ (b) $r_n\propto 1/n$ (c) $r_n\propto n^2$ (d) $r_n\propto 1/n^2$

Solution : (a) Potential energy $U=eV=eV_0\ln\dfrac{r}{r_0}$

$\therefore$ Force $F=-\left|\dfrac{dU}{dr}\right|=\dfrac{eV_0}{r}$. The force will provide the necessary centripetal force. Hence

$\dfrac{mv^2}{r}=\dfrac{eV_0}{r} \Rightarrow v=\sqrt{\dfrac{eV_0}{m}}$ .....(i) and $mvr=\dfrac{nh}{2\pi}$ .....(ii)

Dividing equation (ii) by (i) we have $mr=\left(\dfrac{nh}{2\pi}\right)\sqrt{\dfrac{m}{eV_0}}$ or $r\propto n$

Example: 3 The innermost orbit of the hydrogen atom has a diameter 1.06 Å. The diameter of tenth orbit is **[UPSEAT 2002]**

(a) 5.3 Å (b) 10.6 Å (c) 53 Å (d) 106 Å

Solution : (d) Using $r\propto n^2 \Rightarrow \dfrac{r_2}{r_1}=\left(\dfrac{n_2}{n_1}\right)^2$ or $\dfrac{d_2}{d_1}=\left(\dfrac{n_2}{n_1}\right)^2 \Rightarrow \dfrac{d_2}{1.06}=\left(\dfrac{10}{1}\right)^2 \Rightarrow d=106$ Å

Example: 4 Energy of the electron in n^{th} orbit of hydrogen atom is given by $E_n=-\dfrac{13.6}{n^2}eV$. The amount of energy needed to transfer electron from first orbit to third orbit is **[MH CET 2002; Kerala PMT 2002]**

(a) 13.6 eV (b) 3.4 eV (c) 12.09 eV (d) 1.51 eV

Solution : (c) Using $E = -\frac{13.6}{n^2} eV$

For $n = 1$, $E_1 = \frac{-13.6}{1^2} = -13.6\,eV$ and for $n = 3$ $E_3 = -\frac{13.6}{3^2} = -1.51\,eV$

So required energy $= E_3 - E_1 = -1.51 - (-13.6) = 12.09\,eV$

Example: 5 If the binding energy of the electron in a hydrogen atom is 13.6 *eV*, the energy required to remove the electron from the first excited state of Li^{++} is **[AIEEE 2003]**

(a) 122.4 *eV* (b) 30.6 *eV* (c) 13.6 *eV* (d) 3.4 *eV*

Solution : (b) Using $E_n = -\frac{13.6 \times Z^2}{n^2} eV$

For first excited state $n = 2$ and for Li^{++}, $Z = 3$

$\therefore\ E = -\frac{13.6}{2^2} \times 3^2 = -\frac{13.6 \times 9}{4} = -30.6\,eV$. Hence, remove the electron from the first excited state of Li^{++} be $30.6\,eV$

Example: 6 The ratio of the wavelengths for $2 \to 1$ transition in Li^{++}, He^{+} and H is **[UPSEAT 2003]**

(a) 1 : 2 : 3 (b) 1 : 4 : 9 (c) 4 : 9 : 36 (d) 3 : 2 : 1

Solution : (c) Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \lambda \propto \frac{1}{Z^2} \Rightarrow \lambda_{Li} : \lambda_{He^+} : \lambda_H = \frac{1}{9} : \frac{1}{4} : \frac{1}{1} = 4 : 9 : 36$

Example: 7 Energy E of a hydrogen atom with principal quantum number n is given by $E = \frac{-13.6}{n^2} eV$. The energy of a photon ejected when the electron jumps $n = 3$ state to $n = 2$ state of hydrogen is approximately **[CBSE PMT/PDT Screening 2004]**

(a) 1.9 *eV* (b) 1.5 *eV* (c) 0.85 *eV* (d) 3.4 *eV*

Solution : (a) $\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36} = 1.9\,eV$

Example: 8 In the Bohr model of the hydrogen atom, let R, υ and E represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantity is proportional to the quantum number n **[KCET 2002]**

(a) *R*/*E* (b) *E*/υ (c) *RE* (d) υR

Solution : (d) Rydberg constant $R = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}$

Velocity $\upsilon = \frac{Ze^2}{2\varepsilon_0 nh}$ and energy $E = -\frac{mZ^2 e^4}{8\varepsilon_0^2 n^2 h^2}$

Now, it is clear from above expressions $R.\upsilon \propto n$

Example: 9 The energy of hydrogen atom in *n*th orbit is E_n, then the energy in *n*th orbit of singly ionised helium atom will be **[CBSE PMT 2001]**

(a) $4E_n$ (b) $E_n/4$ (c) $2E_n$ (d) $E_n/2$

Solution : (a) By using $E = -\frac{13.6 Z^2}{n^2} \Rightarrow \frac{E_H}{E_{He}} = \left(\frac{Z_H}{Z_{He}}\right)^2 = \left(\frac{1}{2}\right)^2 \Rightarrow E_{He} = 4E_n$.

Example: 10 The wavelength of radiation emitted is λ_0 when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be **[SCRA 1998; MP PET 2001]**

(a) $\frac{16}{25}\lambda_0$ (b) $\frac{20}{27}\lambda_0$ (c) $\frac{27}{20}\lambda_0$ (d) $\frac{25}{16}\lambda_0$

Solution : (b) Wavelength of radiation in hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{\lambda_0} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5}{36}R \qquad \text{.....(i)}$$

and $$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = R\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R}{16} \qquad \text{.....(ii)}$$

From equation (i) and (ii) $\frac{\lambda'}{\lambda} = \frac{5R}{36} \times \frac{16}{3R} = \frac{20}{27} \Rightarrow \lambda' = \frac{20}{27}\lambda_0$

Example: 11 If scattering particles are 56 for 90^o angle then this will be at 60^o angle **[RPMT 2000]**

(a) 224 (b) 256 (c) 98 (d) 108

Solution : (a) Using Scattering formula

$$N \propto \frac{1}{\sin^4(\theta/2)} \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_2}{2}\right)}\right]^4 \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin\left(\frac{90^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}\right]^4 = \left[\frac{\sin 45^\circ}{\sin 30^\circ}\right]^4 = 4 \Rightarrow N_2 = 4N_1 = 4 \times 56 = 224$$

Example: 12 When an electron in hydrogen atom is excited, from its 4[th] to 5[th] stationary orbit, the change in angular momentum of electron is (Planck's constant: $h = 6.6 \times 10^{-34}\, J\text{–}s$) **[AFMC 2000]**

(a) $4.16 \times 10^{-34}\, J\text{-}s$ (b) $3.32 \times 10^{-34}\, J\text{-}s$ (c) $1.05 \times 10^{-34}\, J\text{-}s$ (d) $2.08 \times 10^{-34}\, J\text{-}s$

Solution : (c) Change in angular momentum

$$\Delta L = L_2 - L_1 = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi} \Rightarrow \Delta L = \frac{h}{2\pi}(n_2 - n_1) = \frac{6.6 \times 10^{-34}}{2 \times 3.14}(5-4) = 1.05 \times 10^{-34}\, J\text{-}s$$

Example: 13 In hydrogen atom, if the difference in the energy of the electron in n = 2 and n = 3 orbits is E, the ionization energy of hydrogen atom is **[EAMCET (Med.) 2000]**

(a) 13.2 E (b) 7.2 E (c) 5.6 E (d) 3.2 E

Solution : (b) Energy difference between n = 2 and n = 3; $E = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = K\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}K$ .....(i)

Ionization energy of hydrogen atom $n_1 = 1$ and $n_2 = \infty$; $E' = K\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = K$ .....(ii)

From equation (i) and (ii) $E' = \frac{36}{5}E = 7.2E$

Example: 14 In Bohr model of hydrogen atom, the ratio of periods of revolution of an electron in n = 2 and n = 1 orbits is **[EAMCET (Engg.) 2000]**

(a) 2 : 1 (b) 4 : 1 (c) 8 : 1 (d) 16 : 1

Solution : (c) According to Bohr model time period of electron $T \propto n^3 \Rightarrow \frac{T_2}{T_1} = \frac{n_2^3}{n_1^3} = \frac{2^3}{1^3} = \frac{8}{1} \Rightarrow T_2 = 8T_1$.

Example: 15 A double charged lithium atom is equivalent to hydrogen whose atomic number is 3. The wavelength of required radiation for emitting electron from first to third Bohr orbit in Li^{++} will be (Ionisation energy of hydrogen atom is 13.6 eV) **[IIT-JEE 1985; UPSEAT 1999]**

(a) 182.51 $Å$ (b) 177.17 $Å$ (c) 142.25 $Å$ (d) 113.74 $Å$

Solution : (d) Energy of a electron in nth orbit of a hydrogen like atom is given by

$E_n = -13.6\dfrac{Z^2}{n^2}eV$, and $Z = 3$ for Li

Required energy for said transition

$$\Delta E = E_3 - E_1 = 13.6Z^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 13.6 \times 3^2\left[\frac{8}{9}\right] = 108.8\,eV = 108.8 \times 1.6 \times 10^{-19}\,J$$

Now using $\Delta E = \dfrac{hc}{\lambda} \Rightarrow \lambda = \dfrac{hc}{\Delta E} \Rightarrow \lambda = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^8}{108.8 \times 1.6 \times 10^{-19}} = 0.11374 \times 10^{-7}\,m \Rightarrow \lambda = 113.74\,Å$

Example: 16 The absorption transition between two energy states of hydrogen atom are 3. The emission transitions between these states will be **[MP PET 1999]**

(a) 3 (b) 4 (c) 5 (d) 6

Solution : (d) Number of absorption lines = $(n-1) \Rightarrow 3 = (n-1) \Rightarrow n = 4$

Hence number of emitted lines $= \dfrac{n(n-1)}{2} = \dfrac{4(4-1)}{2} = 6$

Example: 17 The energy levels of a certain atom for 1st, 2nd and 3rd levels are E, $4E/3$ and $2E$ respectively. A photon of wavelength λ is emitted for a transition $3 \to 1$. What will be the wavelength of emissions for transition $2 \to 1$ **[CPMT 1996]**

(a) $\lambda/3$ (b) $4\lambda/3$ (c) $3\lambda/4$ (d) 3λ

Solution : (d) For transition $3 \to 1$ $\quad \Delta E = 2E - E = \dfrac{hc}{\lambda} \Rightarrow E = \dfrac{hc}{\lambda}$ .....(i)

For transition $2 \to 1$ $\quad \dfrac{4E}{3} - E = \dfrac{hc}{\lambda'} \Rightarrow E = \dfrac{3hc}{\lambda'}$ .....(ii)

From equation (i) and (ii) $\lambda' = 3\lambda$

Example: 18 Hydrogen atom emits blue light when it changes from $n = 4$ energy level to $n = 2$ level. Which colour of light would the atom emit when it changes from $n = 5$ level to $n = 2$ level **[KCET 1993]**

(a) Red (b) Yellow (c) Green (d) Violet

Solution : (d) In the transition from orbits $5 \to 2$ more energy will be liberated as compared to transition from $4 \to 2$. So emitted photon would be of violet light.

Example: 19 A single electron orbits a stationary nucleus of charge $+Ze$, where Z is a constant. It requires 47.2 eV to excited electron from second Bohr orbit to third Bohr orbit. Find the value of Z **[IIT-JEE 1981]**

(a) 2 (b) 5 (c) 3 (d) 4

Solution : (b) Excitation energy of hydrogen like atom for $n_2 \to n_1$

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)eV \Rightarrow 47.2 = 13.6Z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36}Z^2 \Rightarrow Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 24.98 \simeq 25$$

$\Rightarrow Z = 5$

Example: 20 The first member of the Paschen series in hydrogen spectrum is of wavelength 18,800 Å. The short wavelength limit of Paschen series is **[EAMCET (Med.) 2000]**

(a) 1215 Å (b) 6560 Å (c) 8225 Å (d) 12850 Å

Solution : (c) First member of Paschen series mean it's $\lambda_{max} = \dfrac{144}{7R}$

Short wavelength of Paschen series means $\lambda_{min} = \dfrac{9}{R}$

Hence $\frac{\lambda_{max}}{\lambda_{min}} = \frac{16}{7} \Rightarrow \lambda_{min} = \frac{7}{16} \times \lambda_{max} = \frac{7}{16} \times 18{,}800 = 8225 Å$.

***Example*: 21** Ratio of the wavelengths of first line of Lyman series and first line of Balmer series is **[EAMCET (Engg.) 1995; MP PMT 1997]**

(a) 1 : 3 (b) 27 : 5 (c) 5 : 27 (d) 4 : 9

Solution : (c) For Lyman series $\frac{1}{\lambda_{L_1}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$ .....(i)

For Balmer series $\frac{1}{\lambda_{B_1}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$ .....(ii)

From equation (i) and (ii) $\frac{\lambda_{L_1}}{\lambda_{B_1}} = \frac{5}{27}$.

***Example*: 22** The third line of Balmer series of an ion equivalent to hydrogen atom has wavelength of 108.5 *nm*. The ground state energy of an electron of this ion will be **[RPET 1997]**

(a) 3.4 *eV* (b) 13.6 *eV* (c) 54.4 *eV* (d) 122.4 *eV*

Solution : (c) Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{108.5 \times 10^{-9}} = 1.1 \times 10^7 \times Z^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$

$\Rightarrow \frac{1}{108.5 \times 10^{-9}} = 1.1 \times 10^7 \times Z^2 \times \frac{21}{100} \Rightarrow Z^2 = \frac{100}{108.5 \times 10^{-9} \times 1.1 \times 10^{-7} \times 21} = 4 \Rightarrow Z = 2$

Now Energy in ground state $E = -13.6Z^2\ eV = -13.6 \times 2^2\ eV = -54.4\ eV$

***Example*: 23** Hydrogen (H), deuterium (D), singly ionized helium (He^+) and doubly ionized lithium (Li^{++}) all have one electron around the nucleus. Consider $n = 2$ to $n = 1$ transition. The wavelengths of emitted radiations are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively. Then approximately **[KCET 1994]**

(a) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (b) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (c) $\lambda_1 = 2\lambda_2 = 2\sqrt{2}\lambda_3 = 3\sqrt{2}\lambda_4$ (d) $\lambda_1 = \lambda_2 = 2\lambda_3 = 3\lambda_4$

Solution : (a) Using $\Delta E \propto Z^2$ ($\because$ n_1 and n_2 are same)

$\Rightarrow \frac{hc}{\lambda} \propto Z^2 \Rightarrow \lambda Z^2 = \text{constant} \Rightarrow \lambda_1 Z_1^2 = \lambda_2 Z_2^2 = \lambda_3 Z_3^2 = \lambda_4 Z^4 \Rightarrow \lambda_1 \times 1 = \lambda_2 \times 1^2 = \lambda_3 \times 2^2 = \lambda_4 \times 3^3$

$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$.

***Example*: 24** Hydrogen atom in its ground state is excited by radiation of wavelength 975 Å. How many lines will be there in the emission spectrum **[RPMT 2002]**

(a) 2 (b) 4 (c) 6 (d) 8

Solution : (c) Using $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{975 \times 10^{-10}} = 1.097 \times 10^7\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow n = 4$

Now number of spectral lines $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$.

***Example*: 25** A photon of energy 12.4 *eV* is completely absorbed by a hydrogen atom initially in the ground state so that it is excited. The quantum number of the excited state is **[UPSEAT 2000]**

(a) $n = 1$ (b) $n = 3$ (c) $n = 4$ (d) $n = \infty$

Solution : (c) Let electron absorbing the photon energy reaches to the excited state *n*. Then using energy conservation

$\Rightarrow -\frac{13.6}{n^2} = -13.6 + 12.4 \Rightarrow -\frac{13.6}{n^2} = -1.2 \Rightarrow n^2 = \frac{13.6}{1.2} = 12 \Rightarrow n = 3.46 \simeq 4$

Example: 26 The wave number of the energy emitted when electron comes from fourth orbit to second orbit in hydrogen is 20,397 cm^{-1}. The wave number of the energy for the same transition in He^+ is **[Haryana PMT 2000]**

(a) 5,099 cm^{-1} (b) 20,497 cm^{-1} (c) 40,994 cm^{-1} (d) 81,998 cm^{-1}

Solution : (d) Using $\frac{1}{\lambda} = \bar{\nu} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \bar{\nu} \propto Z^2 \Rightarrow \frac{\bar{\nu}_2}{\bar{\nu}_1} = \left(\frac{Z_2}{Z_1}\right)^2 = \left(\frac{Z}{1}\right)^2 = 4 \Rightarrow \bar{\nu}_2 = \bar{\nu} \times 4 = 81588\, cm^{-1}$.

Example: 27 In an atom, the two electrons move round the nucleus in circular orbits of radii R and $4R$. the ratio of the time taken by them to complete one revolution is

(a) 1/4 (b) 4/1 (c) 8/1 (d) 1/8

Solution : (d) Time period $T \propto \frac{n^3}{Z^2}$

For a given atom (Z = constant) So $T \propto n^3$ .....(i) and radius $R \propto n^2$ .....(ii)

$\therefore$ From equation (i) and (ii) $T \propto R^{3/2} \Rightarrow \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} = \frac{1}{8}$.

Example: 28 Ionisation energy for hydrogen atom in the ground state is E. What is the ionisation energy of Li^{++} atom in the 2nd excited state

(a) E (b) $3E$ (c) $6E$ (d) $9E$

Solution : (a) Ionisation energy of atom in nth state $E_n = \frac{Z^2}{n^2}$

For hydrogen atom in ground state (n = 1) and Z = 1 $\Rightarrow E = E_0$ .....(i)

For Li^{++} atom in 2nd excited state n = 3 and Z = 3, hence $E' = \frac{E_0}{3^2} \times 3^2 = E_0$ .....(ii)

From equation (i) and (ii) $E' = E$.

Example: 29 An electron jumps from n = 4 to n = 1 state in H-atom. The recoil momentum of H-atom (in eV/C) is

(a) 12.75 (b) 6.75 (c) 14.45 (d) 0.85

Solution : (a) The H-atom before the transition was at rest. Therefore from conservation of momentum

Photon momentum = Recoil momentum of H-atom or $P_{recoil} = \frac{h\nu}{c} = \frac{E_4 - E_1}{c} = \frac{-0.85eV - (-13.6\, eV)}{c} = 12.75\frac{eV}{c}$

Example: 30 If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be

[IIT-JEE 1983; CBSE PMT 1991, 93; MP PET 1999; RPET 1993, 2001; RPMT 1999, 2003; J & K CET 2004]

(a) 60 (b) 32 (c) 4 (d) 64

Solution : (a) Maximum value of n = 4

So possible (maximum) no. of elements

$N = 2\times1^2 + 2\times2^2 + 2\times3^2 + 2\times4^2 = 2 + 8 + 18 + 32 = 60$.

Tricky example: 1

If the atom ${}_{100}Fm^{257}$ follows the Bohr model and the radius of ${}_{100}Fm^{257}$ is n times the Bohr radius, then find n

[IIT-JEE (Screening) 2003]

(a) 100 (b) 200 (c) 4 (d) ¼

Solution : (d) $(r_m) = \left(\frac{m^2}{Z}\right)(0.53 Å) = (n \times 0.53 Å) \Rightarrow \frac{m^2}{Z} = n$

$m = 5$ for ${}_{100}Fm^{257}$ (the outermost shell) and $z = 100$

$\therefore \quad n = \frac{(5)^2}{100} = \frac{1}{4}$

Tricky example: 2

An energy of 24.6 *eV* is required to remove one of the electrons from a neutral helium atom. The energy (in *eV*) required to remove both the electrons from a neutral helium atom is **[IIT-JEE 1995]**

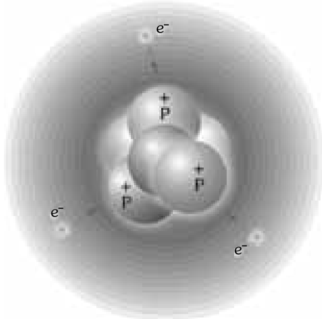
(a) 79.0 (b) 51.8 (c) 49.2 (d) 38.2

Solution : (a) After the removal of first electron remaining atom will be hydrogen like atom.

So energy required to remove second electron from the atom $E = 13.6 \times \frac{2^2}{1} = 54.4\ eV$

$\therefore$ Total energy required = 24.6 + 54.4 = 79 *eV*

Rutherford's α-scattering experiment established that the mass of atom is concentrated with small positively charged region at the centre which is called 'nucleus'.



Nuclei are made up of proton and neutron. The number of protons in a nucleus (called the atomic number or proton number) is represented by the symbol *Z*. The number of neutrons (neutron number) is represented by *N*. The total number of neutrons and protons in a nucleus is called it's mass number *A* so $A = Z + N$.

Neutrons and proton, when described collectively are called ***nucleons.***

Nucleus contains two types of particles : Protons and neutrons

Nuclides are represented as ${}_Z X^A$; where *X* denotes the chemical symbol of the element.

Neutron.

Neutron is a fundamental particle which is essential constituent of all nuclei except that of hydrogen atom. It was discovered by Chadwick.

(1) The charge of neutron : It is neutral

(2) The mass of neutron : $1.6750 \times 10^{-27}\ kg$

(3) It's spin angular momentum : $\frac{1}{2} \times \left(\frac{h}{2\pi}\right) J\text{-}s$

(4) It's magnetic moment : $9.57 \times 10^{-27}\ J/Tesla$

(5) It's half life : 12 minutes

(6) Penetration power : High

(7) Types : Neutrons are of two types slow neutron and fast neutron, both are fully capable of penetrating a nucleus and causing artificial disintegration.

A free neutron outside the nucleus is unstable and decays into proton and electron.

$${}_0n^1 \rightarrow \underset{\text{Proton}}{{}_1H^1} + \underset{\text{Electron}}{{}_{-1}\beta^0} + \underset{\text{Antinutrino}}{\bar{\nu}}$$

Thermal neutrons

Fast neutrons can be converted into slow neutrons by certain materials called moderator's (Paraffin wax, heavy water, graphite) when fast moving neutrons pass through a moderator, they collide with the molecules of the moderator, as a result of this, the energy of moving neutron decreases while that of the molecules of the moderator increases. After sometime they both attains same energy. The neutrons are then in thermal equilibrium with the molecules of the moderator and are called thermal neutrons.

Note : $\cong$Energy of thermal neutron is about 0.025 *eV* and speed is about 2.2 *km/s*.

Nucleus.

(1) Different types of nuclei

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows

(i) **Isotopes :** The atoms of element having same atomic number but different mass number are called isotopes. All isotopes have the same chemical properties. The isotopes of some elements are the following

$$_1H^1,\ _1H^2,\ _1H^3 \quad _8O^{16},\ _8O^{17},\ _8O^{18} \quad _2He^3,\ _2He^4 \quad _{17}Cl^{35},\ _{17}Cl^{37} \quad _{92}U^{235},\ _{92}U^{238}$$

(ii) **Isobars :** The nuclei which have the same mass number (A) but different atomic number (Z) are called isobars. Isobars occupy different positions in periodic table so all isobars have different chemical properties. Some of the examples of isobars are

$$_1H^3 \text{ and } _2He^3, \quad _6C^{14} \text{ and } _7N^{14},\ _8O^{17} \text{ and } _9F^{17}$$

(iii) **Isotones :** The nuclei having equal number of neutrons are called isotones. For them both the atomic number (Z) and mass number (A) are different, but the value of ($A - Z$) is same. Some examples are

$$_4Be^9 \text{ and } _5B^{10},\ _6C^{13} \text{ and } _7N^{14},\ _8O^{18} \text{ and } _9F^{19},\ _3Li^7 \text{ and } _4Be^8,\ _1H^3 \text{ and } _2He^4$$

(iv) **Mirror nuclei :** Nuclei having the same mass number A but with the proton number (Z) and neutron number ($A - Z$) interchanged (or whose atomic number differ by 1) are called mirror nuclei for example.

$$_1H^3 \text{ and } _2He^3,\ _3Li^7 \text{ and } _4Be^7$$

(2) **Size of nucleus**

(i) Nuclear radius : Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where A is the mass number of nucleus *i.e.* $R \propto A^{1/3} \Rightarrow R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}\ m = 1.2\ fm$.

Note : ≅Heavier nuclei are bigger in size than lighter nuclei.

(ii) Nuclear volume : The volume of nucleus is given by $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$

(iii) Nuclear density : Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density}(\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

where m = Average of mass of a nucleon (= mass of proton + mass of neutron = $1.66 \times 10^{-27}\ kg$)

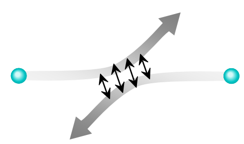
and mA = Mass of nucleus

$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}\ kg/m^3$$

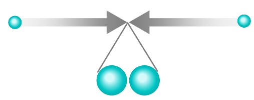
Note : ≅ρ is independent of A, it means ρ is same of all atoms.

≅ Density of a nucleus is maximum at it's centre and decreases as we move outwards from the nucleus.

(3) **Nuclear force**



At low speeds, electromagnetic repulsion prevents the collision of nuclei



At high speeds, nuclei come close enough for the strong force to bind them together.

Forces that keep the nucleons bound in the nucleus are called nuclear forces.

(i) Nuclear forces are short range forces. These do not exist at large distances greater than $10^{-15}\ m$.

(ii) Nuclear forces are the strongest forces in nature.

(iii) These are attractive force and causes stability of the nucleus.

(iv) These forces are charge independent.

(v) Nuclear forces are non-central force.

Nuclear forces are exchange forces

According to scientist Yukawa the nuclear force between the two nucleons is the result of the exchange of particles called mesons between the nucleons.

π - mesons are of three types – Positive π meson (π^+), negative π meson (π^-), neutral π *meson* (π^0)

The force between neutron and proton is due to exchange of charged meson between them *i.e.*

$$p \to \pi^+ + n, \quad n \to p + \pi^-$$

The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^o) between them *i.e.* $p \to p' + \pi^0$ and $n \to n' + \pi^0$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear forces.

Dog-Bone analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

(4) **Atomic mass unit (*amu*)**

The unit in which atomic and nuclear masses are measured is called atomic mass unit (*amu*)

1 *amu* (or 1*u*) $= \frac{1}{12} th$ of mass of ${}_6C^{12}$ atom $= 1.66 \times 10^{-27}$ *kg*

Masses of electron, proton and neutrons

Mass of electron (m_e) = 9.1×10^{-31} *kg* = 0.0005486 *amu*, Mass of proton (m_p) = 1.6726×10^{-27} *kg* = 1.007276 *amu*

Mass of neutron (m_n) = 1.6750×10^{-27} *kg* = 1.00865 *amu*, Mass of hydrogen atom ($m_e + m_p$) = 1.6729×10^{-27} *kg* = 1.0078 *amu*

Mass-energy equivalence

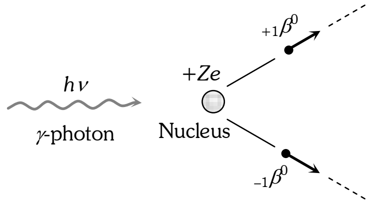
According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by $E = mc^2$

If m = 1 *amu*, $c = 3 \times 10^8$ *m/sec* then $E = 931$ *MeV* *i.e.* 1 *amu* is equivalent to 931 *MeV* or **1 *amu* (or 1 *u*) = 931 *MeV***

(5) **Pair production and pair-annihilation**

When an energetic γ-ray photon falls on a heavy substance. It is absorbed by some nucleus of the substance and an electron and a positron are produced. This phenomenon is called pair production and may be represented by the following equation

$$\underset{(\gamma-\text{photon})}{h\nu} = \underset{(\text{Positron})}{{}_{1}\beta^{0}} + \underset{(\text{Electron})}{{}_{-1}\beta^{0}}$$



The rest-mass energy of each of positron and electron is

$$E_0 = m_0c^2 = (9.1 \times 10^{-31}\ kg) \times (3.0 \times 10^{8}\ m/s)^2$$

$$= 8.2 \times 10^{-14}\ J = \mathbf{0.51\ MeV}$$

Hence, for pair-production it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02\ MeV$. If the energy of γ-photon is less than this, it would cause photo-electric effect or Compton effect on striking the matter.

The converse phenomenon pair-annihilation is also possible. Whenever an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons (energy) are produced. This phenomenon is called pair annihilation and is represented by the following equation.

$$\underset{(\text{Positron})}{{}_{+1}\beta^{0}} + \underset{(\text{Electron})}{{}_{-1}\beta^{0}} = \underset{(\gamma\text{-photon})}{h\nu} + \underset{(\gamma\text{-photon})}{h\nu}$$

(6) Nuclear stability

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ - EM waves. (This process is called radioactivity). The stability of nucleus is determined by many factors. Few such factors are given below :

(i) Neutron-proton ratio $\left(\frac{N}{Z}\text{Ratio}\right)$

The chemical properties of an atom are governed entirely by the number of protons (Z) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal ($N \approx Z$) *i.e.* $\frac{N}{Z} = 1$

Heavy nuclei are stable only when they have more neutrons than protons. Thus heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

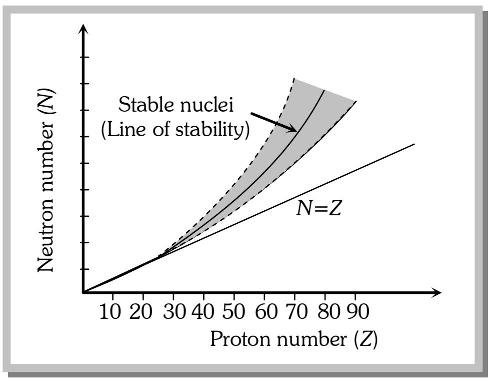
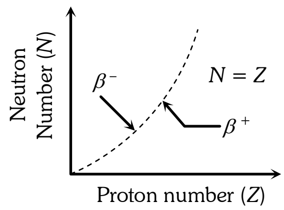


Figure shows a plot of N verses Z for the stable nuclei. For mass number upto about $A = 40$. For larger value of Z the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At Bi ($Z = 83$, $A = 209$), the neutron excess in $N - Z = 43$. There are no stable nuclides with $Z > 83$.

Note : ≅The nuclide ${}_{83}Bi^{209}$ is the heaviest stable nucleus.

≅ A nuclide above the line of stability *i.e.* having excess neutrons, decay through β^- emission (neutron changes into proton). Thus increasing atomic number Z and decreasing neutron number N. In β^- emission, $\dfrac{N}{Z}$ ratio decreases.



A nuclide below the line of stability have excess number of protons. It decays by β^+ emission, results in decreasing Z and increasing N. In β^+ emission, the $\dfrac{N}{Z}$ ratio increases.

(ii) Even or odd numbers of Z or N : The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

It is found that an even-even nucleus (even Z and even N) is more stable (60% of stable nuclide have even Z and even N).

An even-odd nucleus (even Z and odd N) or odd-even nuclide (odd Z and even N) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

Only five stable odd-odd nuclides are known : ${}_1H^2$, ${}_3Li^6$, ${}_5Be^{10}$, ${}_7N^{14}$ and ${}_{75}Ta^{180}$

(iii) Binding energy per nucleon : The stability of a nucleus is determined by value of it's binding energy per nucleon. In general higher the value of binding energy per nucleon, more stable the nucleus is

Mass Defect and Binding Energy.

(1) Mass defect (Δm)

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state. This difference in masses is called mass defect. Hence mass defect

Δm = Sum of masses of nucleons – Mass of nucleus

$$= \{Zm_p + (A - Z)m_n\} - M = \{Zm_p + Zm_e + (A - Z)m_z\} - M'$$

where m_p = Mass of proton, m_n = Mass of each neutron, m_e = Mass of each electron

M = Mass of nucleus, Z = Atomic number, A = Mass number, M' = Mass of atom as a whole.

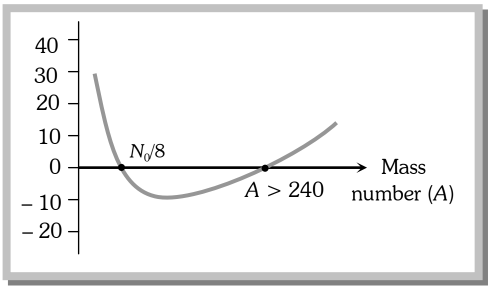
Note : ≅The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

(2) Packing fraction

Mass defect per nucleon is called packing fraction

Packing fraction $(f) = \frac{\Delta m}{A} = \frac{M - A}{A}$ where M = Mass of nucleus, A = Mass number

Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, larger is the stability of the nucleus.



(i) Packing fraction may be of positive, negative or zero value.

(iii) At $A = 16$, $f \to$ Zero

(3) Binding energy (B.E.)

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart (or the same energy is released during the formation of the nucleus). This energy is called the binding energy of the nucleus.

or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect then according to Einstein's mass energy relation

Binding energy $= \Delta m \cdot c^2 = [\{m_p Z + m_n (A - Z)\} - M] \cdot c^2$

(This binding energy is expressed in *joule*, because Δm is measured in *kg*)

If Δm is measured in *amu* then binding energy $= \Delta m\ amu = [\{m_p Z + m_n (A - Z)\} - M]\ amu = \Delta m \times 931\ MeV$

(4) Binding energy per nucleon

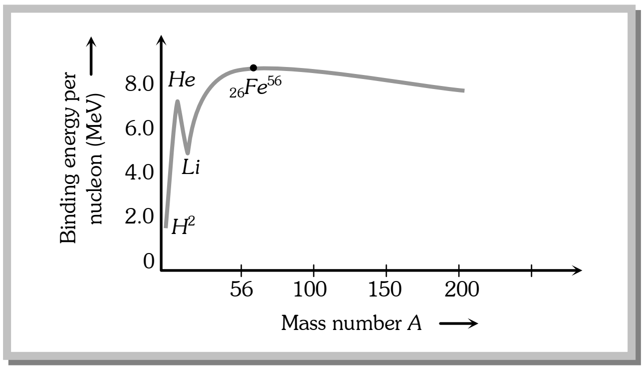
The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

$$\text{Binding energy per nucleon} = \frac{\text{Total binding energy}}{\text{Mass number (i.e. total number of nucleons)}} = \frac{\Delta m \times 931}{A} \frac{MeV}{Nucleon}$$

Binding energy per nucleon $\propto$ Stability of nucleus

Binding Energy Curve.

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.* mass number A)



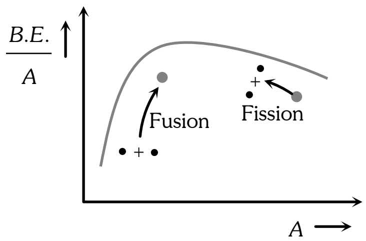
(1) Some nuclei with mass number $A < 20$ have large binding energy per nucleon than their neighbour nuclei. For example ${}_2He^4$, ${}_4Be^8$, ${}_6C^{12}$, ${}_8O^{16}$ and ${}_{10}Ne^{20}$. These nuclei are more stable than their neighbours.

(2) The binding energy per nucleon is maximum for nuclei of mass number $A = 56$ $({}_{26}Fe^{56})$. It's value is 8.8 MeV per nucleon.

(3) For nuclei having $A > 56$, binding energy per nucleon gradually decreases for uranium ($A = 238$), the value of binding energy per nucleon drops to 7.5 MeV.

Note : ≅When a heavy nucleus splits up into lighter nuclei, then binding energy per nucleon of lighter nuclei is more than that of the original heavy nucleus. Thus a large amount of energy is liberated in this process (nuclear fission).

≅ When two very light nuclei combines to form a relatively heavy nucleus, then binding energy per nucleon increases. Thus, energy is released in this process (nuclear fusion).



Nuclear Reactions.

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. The general expression for the nuclear reaction is as follows.

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C} \longrightarrow \underset{\text{(Compound nucleus)}}{Y} + \underset{\text{(Product particles)}}{b} + \underset{\text{(Energy)}}{Q}$$

Here X and a are known as reactants and Y and b are known as products. This reaction is known as (a, b) reaction and can be represented as $X(a, b)\,Y$

(1) Q value or energy of nuclear reaction

The energy absorbed or released during nuclear reaction is known as Q-value of nuclear reaction.

Q-value = (Mass of reactants – mass of products)c^2 *Joules*

= (Mass of reactants – mass of products) *amu*

If $Q < 0$, The nuclear reaction is known as endothermic. (The energy is absorbed in the reaction)

If $Q > 0$, The nuclear reaction is known as exothermic (The energy is released in the reaction)

(2) Law of conservation in nuclear reactions

(i) Conservation of mass number and charge number : In the following nuclear reaction

$${}_2He^4 + {}_7N^{14} \rightarrow {}_8O^{17} + {}_1H^1$$

Mass number (A) →	Before the reaction	After the reaction
	4 +14 = 18	17 + 1 = 18
Charge number (Z) →	2 + 7 = 9	8 + 1 = 9

(ii) Conservation of momentum : Linear momentum/angular momentum of particles before the reaction is equal to the linear/angular momentum of the particles after the reaction. That is $\Sigma p = 0$

(iii) Conservation of energy : Total energy before the reaction is equal to total energy after the reaction. Term Q is added to balance the total energy of the reaction.

(3) Common nuclear reactions

The nuclear reactions lead to artificial transmutation of nuclei. Rutherford was the first to carry out artificial transmutation of nitrogen to oxygen in the year 1919.

$$_2He^4 + {}_7N^{14} \rightarrow {}_9F^{18} \rightarrow {}_8O^{17} + {}_1H^1$$

It is called (α, p) reaction. Some other nuclear reactions are given as follows.

(p, n) reaction $\Rightarrow$ $\quad {}_1H^1 + {}_5B^{11} \rightarrow {}_6C^{12} \rightarrow {}_6C^{11} + {}_0n^1$

(p, α) reaction $\Rightarrow$ $\quad {}_1H^1 + {}_3Li^{11} \rightarrow {}_4Be^8 \rightarrow {}_2He^4 + {}_2He^4$

(p, γ) reaction $\Rightarrow$ $\quad {}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} \rightarrow {}_7N^{13} + \gamma$

(n, p) reaction $\Rightarrow$ $\quad {}_0n^1 + {}_7N^{14} \rightarrow {}_7N^{15} \rightarrow {}_6C^{14} + {}_1H^1$

(γ, n) reaction $\Rightarrow$ $\quad \gamma + {}_1H^2 \rightarrow {}_1H^1 + {}_0n^1$

Nuclear Fission and Fusion.

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

The phenomenon of nuclear fission was discovered by scientist Ottohann and F. Strassman and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(1) Fission reaction of U^{235}

(i) Nuclear reaction :

$$_{92}U^{235} + {}_0n^1 \rightarrow \underset{\text{(unstable nucleus)}}{{}_{92}U^{236}} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q$$

(ii) The energy released in U^{235} fission is about 200 MeV or 0.8 MeV per nucleon.

(iii) By fission of $_{92}U^{235}$, on an average 2.5 neutrons are liberated. These neutrons are called fast neutrons and their energy is about 2 MeV (for each). These fast neutrons can escape from the reaction so as to proceed the chain reaction they are need to slow down.

(iv) Fission of U^{235} occurs by slow neutrons only (of energy about 1eV) or even by thermal neutrons (of energy about 0.025 eV).

(v) 50 kg of U^{235} on fission will release $\approx 4 \times 10^{15}$ J of energy. This is equivalence to 20,000 tones of TNT explosion. The nuclear bomb dropped at Hiroshima had this much explosion power.

(vi) The mass of the compound nucleus must be greater than the sum of masses of fission products.

(vii) The $\frac{\text{Binding energy}}{A}$ of compound nucleus must be less than that of the fission products.

(viii) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments $_{56}Ba$ and $_{36}Kr$ are formed but they may be any stable isotopes of middle weight atoms.

Same other U^{235} fission reactions are

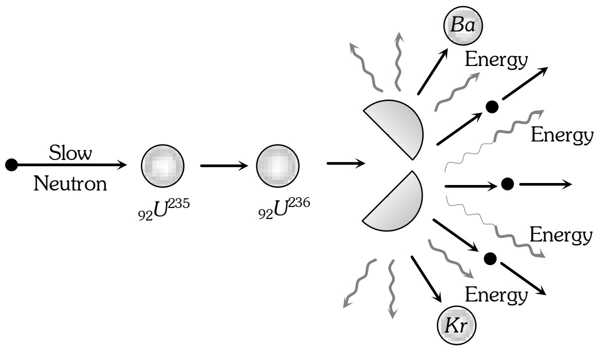
$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1$$

$$\rightarrow {}_{57}La^{148} + {}_{35}Br^{85} + 3\,{}_0n^1$$

$$\rightarrow \text{Many more}$$

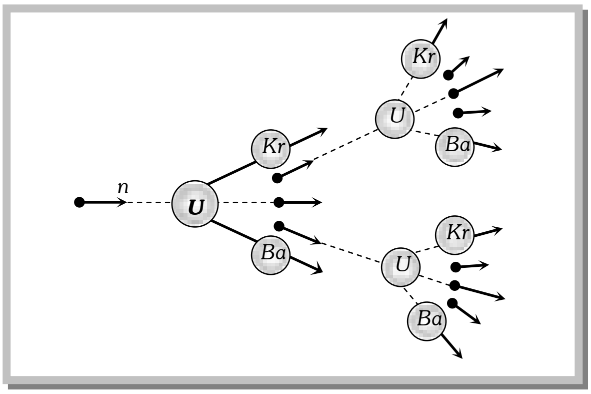
(ix) The neutrons released during the fission process are called prompt neutrons.

(x) Most of energy released appears in the form of kinetic energy of fission fragments.



(2) **Chain reaction**

In nuclear fission, three neutrons are produced along with the release of large energy. Under favourable conditions, these neutrons can cause further fission of other nuclei, producing large number of neutrons. Thus a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.



In the chain reaction, the number of nuclei undergoing fission increases very fast. So, the energy produced takes a tremendous magnitude very soon.

Difficulties in chain reaction

(i) Absorption of neutrons by U^{238}, the major part in natural uranium is the isotope U^{238} (99.3%), the isotope U^{235} is very little (0.7%). It is found that U^{238} is fissionable with fast neutrons, whereas U^{235} is fissionable with slow neutrons. Due to the large percentage of U^{238}, there is more possibility of collision of neutrons with U^{238}. It is found that the neutrons get slowed on coliding with U^{238}, as a result of it further fission of U^{238} is not possible (Because they are slow and they are absorbed by U^{238}). This stops the chain reaction.

Removal : (i) To sustain chain reaction ${}_{92}U^{235}$ is separated from the ordinary uranium. Uranium so obtained $\left({}_{92}U^{235}\right)$ is known as enriched uranium, which is fissionable with the fast and slow neutrons and hence chain reaction can be sustained.

(ii) If neutrons are slowed down by any method to an energy of about 0.3 eV, then the probability of their absorption by U^{238} becomes very low, while the probability of their fissioning U^{235} becomes high. This job is done by moderators. Which reduce the speed of neutron rapidly graphite and heavy water are the example of moderators.

(iii) Critical size : The neutrons emitted during fission are very fast and they travel a large distance before being slowed down. If the size of the fissionable material is small, the neutrons emitted will escape the fissionable material before they are slowed down. Hence chain reaction cannot be sustained.

Removal : The size of the fissionable material should be large than a critical size.

The chain reaction once started will remain steady, accelerate or retard depending upon, a factor called neutron reproduction factor (k). It is defined as follows.

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

$\rightarrow$ If $k = 1$, the chain reaction will be steady. The size of the fissionable material used is said to be the critical size and it's mass, the critical mass.

$\rightarrow$ If $k > 1$, the chain reaction accelerates, resulting in an explosion. The size of the material in this case is super critical. (Atom bomb)

$\rightarrow$ If $k < 1$, the chain reaction gradually comes to a halt. The size of the material used us said to be sub-critical.

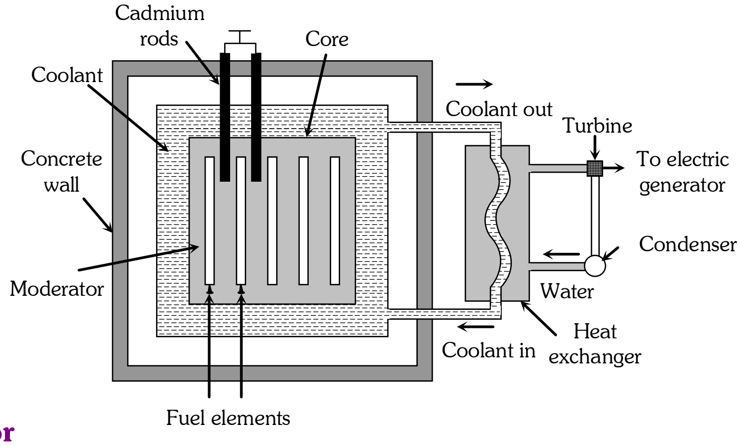
Types of chain reaction : Chain reactions are of following two types

Controlled chain reaction	Uncontrolled chain reaction
Controlled by artificial method	No control over this type of nuclear reaction
All neurons are absorbed except one	More than one neutron takes part into reaction
It's rate is slow	Fast rate
Reproduction factor $k = 1$	Reproduction factor $k > 1$
Energy liberated in this type of reaction is always less than explosive energy	A large amount of energy is liberated in this type of reaction
Chain reaction is the principle of nuclear reactors	Uncontrolled chain reaction is the principle of atom bomb.

Note : ≅The energy released in the explosion of an atom bomb is equal to the energy released by 2000 tonn of TNT and the temperature at the place of explosion is of the order of 10^7 °C.

Nuclear Reactor.

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.



(1) **Parts of nuclear reactor**

(i) **Fissionable material (Fuel) :** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}) Thorium isotope (Th^{232}) and Plutonium isotopes (Pu^{239}, Pu^{240} and Pu^{241}) are the most commonly used fuels in the reactor.

(ii) **Moderator :** Moderator is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O).

(iii) **Control Material :** Control material is used to control the chain reaction and to maintain a stable rate of reaction. This material controls the number of neutrons available for the fission. For example, cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.

(iv) **Coolant :** Coolant is a cooling material which removes the heat generated due to fission in the reactor. Commonly used coolants are water, CO_2 nitrogen *etc.*

(v) **Protective shield :** A protective shield in the form a concrete thick wall surrounds the core of the reactor to save the persons working around the reactor from the hazardous radiations.

Note : ≅It may be noted that Plutonium is the best fuel as compared to other fissionable material. It is because fission in Plutonium can be initiated by both slow and fast neutrons. Moreover it can be obtained from U^{238}.

≅ Nuclear reactor is firstly devised by fermi.

≅ Apsara was the first Indian nuclear reactor.

(2) **Uses of nuclear reactor**

(i) In electric power generation.

(ii) To produce radioactive isotopes for their use in medical science, agriculture and industry.

(iii) In manufacturing of PU^{239} which is used in atom bomb.

(iv) They are used to produce neutron beam of high intensity which is used in the treatment of cancer and nuclear research.

Note : ≅A type of reactor that can produce more fissile fuel than it consumes is the breeder reactor.

Nuclear fusion

In nuclear fusion two or more than two lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy

$$_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4MeV$$

$$_1H^3 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1 + 17.6\,MeV$$

or $$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + 24MeV$$

For fusion high pressure ($\approx 10^6$ *atm*) and high temperature (of the order of 10^7 *K* to 10^8 *K*) is required and so the reaction is called thermonuclear reaction.

Fusion energy is greater then fission energy fission of one uranium atom releases about 200 *MeV* of energy. But the fusion of a deutron $({}_1H^2)$ and triton $({}_1H^3)$ releases about 17.6 *MeV* of energy. However the energy released per nucleon in fission is about 0.85 *MeV* but that in fusion is 4.4 *MeV*. So for the same mass of the fuel, the energy released in fusion is much larger than in fission.

Plasma : The temperature of the order of 10^8 *K* required for thermonuclear reactions leads to the complete ionisation of the atom of light elements. The combination of base nuclei and electron cloud is called plasma. The enormous gravitational field of the sun confines the plasma in the interior of the sun.

The main problem to carryout nuclear fusion in the laboratory is to contain the plasma at a temperature of $10^8 K$. No solid container can tolerate this much temperature. If this problem of containing plasma is solved, then the large quantity of deuterium present in sea water would be able to serve as in-exhaustible source of energy.

Note : ≅To achieve fusion in laboratory a device is used to confine the plasma, called **Tokamak**.

Stellar Energy

Stellar energy is the energy obtained continuously from the sun and the stars. Sun radiates energy at the rate of about 10^{26} *joules* per *second*.

Scientist Hans Bethe suggested that the fusion of hydrogen to form helium (thermo nuclear reaction) is continuously taking place in the sun (or in the other stars) and it is the source of sun's (star's) energy.

The stellar energy is explained by two cycles

Proton-proton cycle	Carbon-nitrogen cycle
$_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + Q_1$	$_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} + Q_1$
$_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + Q_2$	$_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0$
$_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2{}_1H^1 + Q_3$	$_1H^1 + {}_6C^{13} \rightarrow {}_7N^{14} + Q_2$
$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\gamma + 26.7\,MeV$	$_1H^1 + {}_7N^{14} \rightarrow {}_8O^{15} + Q_3$
	$_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + Q_4$

$$_1H^1 + {}_7N^{15} \rightarrow {}_6C^{12} + {}_2He^4$$

$$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_1e^0 + 24.7\,MeV$$

About 90% of the mass of the sun consists of hydrogen and helium.

Nuclear Bomb.

Based on uncontrolled nuclear reactions.

Atom bomb	Hydrogen bomb
Based on fission process it involves the fission of U^{235}	Based on fusion process. Mixture of deutron and tritium is used in it
In this critical size is important	There is no limit to critical size
Explosion is possible at normal temperature and pressure	High temperature and pressure are required
Less energy is released compared to hydrogen bomb	More energy is released as compared to atom bomb so it is more dangerous than atom bomb

Concepts

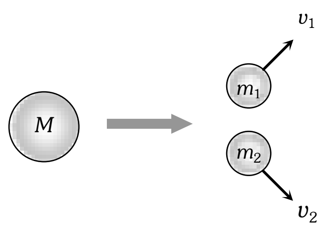
- *A test tube full of base nuclei will weight heavier than the earth.*
- *The nucleus of hydrogen contains only one proton. Therefore we may say that the proton is the nucleus of hydrogen atom.*
- *If the relative abundance of isotopes in an element has a ratio $n_1 : n_2$ whose atomic masses are m_1 and m_2 then atomic mass of the element is* $M = \dfrac{n_1m_1 + n_2m_2}{n_1 + n_2}$

Example

Example: 1 A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio 8 : 1. The ratio of radii of the fragments is **[EAMCET (Engg.) 2001]**

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 2 : 1

Solution : (a)



By conservation of momentum $m_1v_1 = m_2v_2$

$\Rightarrow \dfrac{v_1}{v_2} = \dfrac{8}{1} = \dfrac{m_2}{m_1}$ ...... (i)

Also from $r \propto A^{1/3} \Rightarrow \dfrac{r_1}{r_2} = \left(\dfrac{A_1}{A_2}\right)^{1/3} = \left(\dfrac{1}{8}\right)^{1/3} = \dfrac{1}{2}$

Example: 2 The ratio of radii of nuclei $^{27}_{13}Al$ and $^{125}_{52}Te$ is approximately **[J & K CET 2000]**

(a) 6 : 10 (b) 13 : 52 (c) 40 : 177 (d) 14 : 7

Solution : (a) By using $r \propto A^{1/3} \Rightarrow \dfrac{r_1}{r_2} = \left(\dfrac{A_1}{A_2}\right)^{1/3} = \left(\dfrac{27}{125}\right)^{1/3} = \dfrac{8}{5} = \dfrac{6}{10}$

Example: 3 If Avogadro's number is 6×10^{23} then the number of protons, neutrons and electrons in 14 *g* of $_6C^{14}$ are respectively

(a) 36×10^{23}, 48×10^{23}, 36×10^{23} (b) 36×10^{23}, 36×10^{23}, 36×10^{21}

(c) 48×10^{23}, 36×10^{23}, 48×10^{21} (d) 48×10^{23}, 48×10^{23}, 36×10^{21}

Solution : (a) Since the number of protons, neutrons and electrons in an atom of ${}_6C^{14}$ are 6, 8 and 6 respectively. As 14 *gm* of ${}_6C^{14}$ contains 6×10^{23} atoms, therefore the numbers of protons, neutrons and electrons in 14 *gm* of ${}_6C^{14}$ are $6 \times 6 \times 10^{23} = 36 \times 10^{23}$, $8 \times 6 \times 10^{23} = 48 \times 10^{23}$, $6 \times 6 \times 10^{23} = 36 \times 10^{23}$.

Example: 4 Two Cu^{64} nuclei touch each other. The electrostatics repulsive energy of the system will be

(a) 0.788 *MeV* (b) 7.88 *MeV* (c) 126.15 *MeV* (d) 788 *MeV*

Solution : (c) Radius of each nucleus $R = R_0(A)^{1/3} = 1.2(64)^{1/3} = 4.8\,fm$

Distance between two nuclei (r) = $2R$

So potential energy $U = \dfrac{k \cdot q^2}{r} = \dfrac{9 \times 10^9 \times (1.6 \times 10^{-19} \times 29)^2}{2 \times 4.8 \times 10^{-15} \times 1.6 \times 10^{-19}} = 126.15\,MeV$.

Example: 5 When ${}_{92}U^{235}$ undergoes fission. 0.1% of its original mass is changed into energy. How much energy is released if 1 *kg* of ${}_{92}U^{235}$ undergoes fission **[MP PET 1994; MP PMT/PET 1998; BHU 2001; BVP 2003]**

(a) $9 \times 10^{10}\,J$ (b) $9 \times 10^{11}\,J$ (c) $9 \times 10^{12}\,J$ (d) $9 \times 10^{13}\,J$

Solution : (d) By using $E = \Delta m \cdot c^2 \Rightarrow E = \left(\dfrac{0.1}{100} \times 1\right)(3 \times 10^8)^2 = 9 \times 10^{13}\,J$

Example: 6 1 *g* of hydrogen is converted into 0.993 *g* of helium in a thermonuclear reaction. The energy released is **[EAMCET (Med.) 1995; CPMT 1999]**

(a) $63 \times 10^7\,J$ (b) $63 \times 10^{10}\,J$ (c) $63 \times 10^{14}\,J$ (d) $63 \times 10^{20}\,J$

Solution : (b) $\Delta m = 1 - 0.993 = 0.007\,gm$

$\therefore E = \Delta mc^2 = 0.007 \times 10^{-3} \times (3 \times 10^8)^2 = 63 \times 10^{10}\,J$

Example: 7 The binding energy per nucleon of deuteron $({}_1^2H)$ and helium nucleus $({}_2^4He)$ is 1.1 *MeV* and 7 *MeV* respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is **[MP PMT 1992; Roorkee 1994; IIT-JEE 1996; AIIMS 1997; Haryana PMT 2000; Pb PMT 2001; CPMT 2001; AIEEE 2004]**

(a) 13.9 *MeV* (b) 26.9 *MeV* (c) 23.6 *MeV* (d) 19.2 *MeV*

Solution : (c) ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + Q$

Total binding energy of helium nucleus = 4 × 7 = 28 *MeV*

Total binding energy of each deutron = 2 × 1.1 = 2.2 *MeV*

Hence energy released = 28 – 2 × 2.2 = 23.6 *MeV*

Example: 8 The masses of neutron and proton are 1.0087 *amu* and 1.0073 *amu* respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 *amu*. The binding energy of the helium nucleus will be [1 *amu*= 931 *MeV*] **[CPMT 1986; MP PMT 1995; CBSE 2003]**

(a) 28.4 *MeV* (b) 20.8 *MeV* (c) 27.3 *MeV* (d) 14.2 *MeV*

Solution : (a) Helium nucleus consist of two neutrons and two protons.

So binding energy $E = \Delta m\ amu = \Delta m \times 931\,MeV$

$\Rightarrow E = (2 \times m_p + 2m_n - M) \times 931\,MeV = (2 \times 1.0073 + 2 \times 1.0087 - 4.0015) \times 931 = 28.4\,MeV$

Example: 9 A atomic power reactor furnace can deliver 300 *MW*. The energy released due to fission of each of uranium atom U^{238} is 170 *MeV*. The number of uranium atoms fissioned per hour will be **[UPSEAT 2000]**

(a) 5×10^{15} (b) 10×10^{20} (c) 40×10^{21} (d) 30×10^{25}

Solution : (c) By using $P = \frac{W}{t} = \frac{n \times E}{t}$ where n = Number of uranium atom fissioned and E = Energy released due to each fission so $300 \times 10^6 = \frac{n \times 170 \times 10^6 \times 1.6 \times 10^{-19}}{3600}$ $\Rightarrow n = 40 \times 10^{21}$

Example: 10 The binding energy per nucleon of O^{16} is 7.97 *MeV* and that of O^{17} is 7.75 *MeV*. The energy (in *MeV*) required to remove a neutron from O^{17} is **[IIT-JEE 1995]**

(a) 3.52 (b) 3.64 (c) 4.23 (d) 7.86

Solution : (c) $O^{17} \rightarrow O^{16} + {}_0n^1$

$\therefore$ Energy required = Binding of O^{17} – binding energy of O^{16} = 17 × 7.75 – 16 × 7.97 = 4.23 *MeV*

Example: 11 A gamma ray photon creates an electron-positron pair. If the rest mass energy of an electron is 0.5 *MeV* and the total kinetic energy of the electron-positron pair is 0.78 *MeV*, then the energy of the gamma ray photon must be **[MP PMT 1991]**

(a) 0.78 *MeV* (b) 1.78 *MeV* (c) 1.28 *MeV* (d) 0.28 *MeV*

Solution : (b) Energy of γ-rays photon = 0.5 + 0.5 +0.78 = 1.78 *MeV*

Example: 12 What is the mass of one Curie of U^{234} **[MNR 1985]**

(a) 3.7×10^{10} *gm* (b) 2.348×10^{23} *gm* (c) 1.48×10^{-11} *gm* (d) 6.25×10^{-34} *gm*

Solution : (c) 1 *curie* = 3.71×10^{10} disintegration/*sec* and mass of 6.02×10^{23} atoms of U^{234} = 234 *gm*

$\therefore$ Mass of 3.71×10^{10} atoms $= \frac{234 \times 3.71 \times 10^{10}}{6.02 \times 10^{23}} = 1.48 \times 10^{-11} gm$

Example: 13 In the nuclear fusion reaction ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$, given that the repulsive potential energy between the two nuclei is $-7.7 \times 10^{-14} J$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23} J/K$] **[AIEEE 2003]**

(a) $10^9 K$ (b) $10^7 K$ (c) $10^5 K$ (d) $10^3 K$

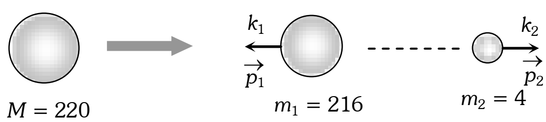
Solution : (a) Kinetic energy of molecules of a gas at a temperature T is 3/2 kT

$\therefore$ To initiate the reaction $\frac{3}{2}kT = 7.7 \times 10^{-14} J$ $\Rightarrow T = 3.7 \times 10^9 K$.

Example: 14 A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 *MeV*. Calculate the kinetic energy of the α-particle **[IIT-JEE (Screening) 2003]**

(a) 4.4 *MeV* (b) 5.4 *MeV* (c) 5.6 *MeV* (d) 6.5 *MeV*

Solution : (b)



Q-value of the reaction is 5.5 *eV* *i.e.* $k_1 + k_2 = 5.5 MeV$ ......(i)

By conservation of linear momentum $p_1 = p_2 \Rightarrow \sqrt{2(216)k_1} = \sqrt{2(4)k_2}$ $\Rightarrow k_2 = 54\ k_1$ ......(ii)

On solving equation (i) and (ii) we get k_2 = 5.4 *MeV*.

Example: 15 Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}_{10}^{20}Ne$ nucleus and M_2 the mass of a ${}_{20}^{40}Ca$ nucleus. Then **[IIT 1998; DPMT 2000]**

(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$ (c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

Solution : (c, d) Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less then the sum of masses of it's constituent particles ${}^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of ${}^{20}_{10}Ne$ nucleus $M_1 < 10(m_p + m_n)$

Also heavier the nucleus, more is he mass defect thus $20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

or $\quad 10(m_p + m_n) > M_2 - M_1$

$\Rightarrow \quad M_2 < M_1 + 10(m_p + m_n) \Rightarrow M_2 < M_1 + M_1 \Rightarrow M_2 < 2M_1$

Tricky example: 1

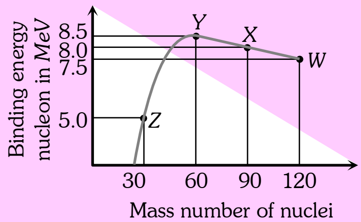
Binding energy per nucleon *vs* mass number curve for nuclei is shown in the figure. *W*, *X*, *Y* and *Z* are four nuclei indicated on the curve. The process that would release energy is **[IIT-JEE 1999]**

(a) $Y \to 2Z$

(b) $W \to X + Z$

(c) $W \to 2Y$

(d) $X \to Y + Z$



Solution : (c) Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleon) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (c) this happens.

Given $\quad W \to 2Y$

Binding energy of reactants = 120 × 7.5 = 900 *MeV*

and binding energy of products = 2 (60 × 8.5) = 1020 *MeV* > 900 *MeV*

Radioactivity.

The phenomenon of spontaneous emission of radiatons by heavy elements is called radioactivity. The elements which shows this phenomenon are called radioactive elements.

(1) Radioactivity was discovered by Henery Becquerel in uranium salt in the year 1896.

(2) After the discovery of radioactivity in uranium, Piere Curie and Madame Curie discovered a new radioactive element called radium (which is 10^6 times more radioactive than uranium)

(3) Some examples of radio active substances are : Uranium, Radium, Thorium, Polonium, Neptunium *etc.*

(4) Radioactivity of a sample cannot be controlled by any physical (pressure, temperature, electric or magnetic field) or chemical changes.

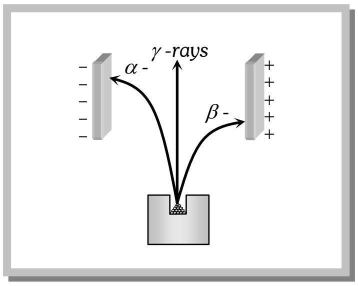
(5) All the elements with atomic number (Z) > 82 are naturally radioactive.

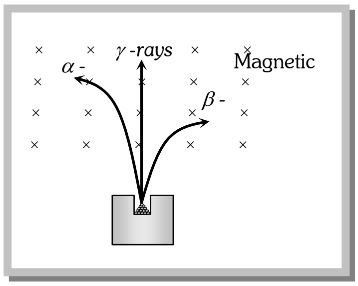
(6) The conversion of lighter elements into radioactive elements by the bombardment of fast moving particles is called artificial or induced radioactivity.

(7) Radioactivity is a nuclear event and not atomic. Hence electronic configuration of atom don't have any relationship with radioactivity.

Nuclear radiatons

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts





(i) Radiations which deflects towards negative plate are called α-rays (stream of positively charged particles)

(ii) Radiations which deflects towards positive plate are called β particles (stream of negatively charged particles)

(iii) Radiations which are undeflected called γ*-rays*. (E.M. waves or photons)

Note : ≅Exactly same results were obtained when these radiations were subjected to magnetic field.

≅ No radioactive substance emits both α and β particles simultaneously. Also γ-rays are emitted after the emission of α or β-particles.

≅ β-particles are not orbital electrons they come from nucleus. The neutron in the nucleus decays into proton and an electron. This electron is emitted out of the nucleus in the form of β-rays.

Properties of α, β and γ-rays

Features	α- particles	β - particles	γ - rays
1. Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electron $(-\beta^0$ or $\beta^-)$	Photons (E.M. waves)
2. Charge	$+ 2e$	$- e$	Zero
3. Mass 4 m_p (m_p = mass of proton = 1.87×10^{-27}	4 m_p	m_e	Massless
4. Speed	$\approx 10^7$ m/s	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 MeV to 9 MeV	All possible values between a minimum certain value to 1.2 MeV	Between a minimum value to 2.23 MeV
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 *cm* of iron (or *Pb*) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete

10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	$_ZX^A \xrightarrow{\alpha-decay} {}_{Z-2}Y^{A-4} + {}_2He^4$ $_ZX^A \xrightarrow{n_\alpha} {}_{Z'}Y^{A'}$ $\Rightarrow \boldsymbol{n_\alpha = \frac{A'-A}{4}}$	$_ZX^A \to {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$ $_ZX^A \xrightarrow{n_\beta} {}_{Z'}X^A$ $\Rightarrow \boldsymbol{n_\beta = (2n_\alpha - Z + Z')}$	$_zX^A \to {}_zX^a + \gamma$

Radioactive Disintegration.

(1) Law of radioactive disintegration

According to Rutherford and Soddy law for radioactive decay is as follows.

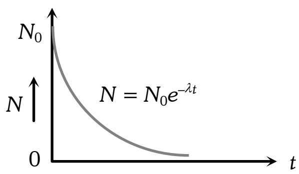
"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant" *i.e.* $-\frac{dN}{dt} \propto N \quad \Rightarrow \frac{dN}{dt} = -\lambda N$. It can be proved that $\boldsymbol{N = N_0 e^{-\lambda t}}$

This equation can also be written in terms of mass *i.e.* $M = M_0 e^{-\lambda t}$

where N = Number of atoms remains undecayed after time t, N_0 = Number of atoms present initially (*i.e.* at $t = 0$), M = Mass of radioactive nuclei at time t, M_0 = Mass of radioactive nuclei at time $t = 0$, $N_0 - N$ = Number of disintegrated nucleus in time t

$\frac{dN}{dt}$ = rate of decay, λ = Decay constant or disintegration constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

Note : $\cong \lambda$ depends only on the nature of substance. It is independent of time and any physical or chemical changes.



(2) Activity

It is defined as the rate of disintegration (or count rate) of the substance (or the number of atoms of any material decaying per second) *i.e.* $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

where A_0 = Activity of $t = 0$, A = Activity after time t

Units of activity (Radioactivity)

It's units are *Becqueral* (*Bq*), Curie (*Ci*) and Rutherford (*Rd*)

1 *Becquerel* = 1 *disintegration/sec*, 1 *Rutherford* = 10^6 *dis/sec*, 1 *Curie* = 3.7×10^{11} *dis/sec*

Note : ≅Activity per *gm* of a substance is known as specific activity. The specific activity of 1 *gm* of radium – 226 is 1 *Curie*.

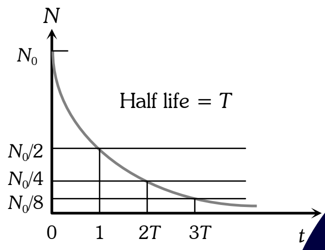
≅ 1 *millicurie* = 37 *Rutherford*

≅ The activity of a radioactive substance decreases as the number of undecayed nuclei decreases with time.

≅ $\text{Activity} \propto \dfrac{1}{\text{Half life}}$

(3) Half life ($T_{1/2}$)

Time interval in which the mass of a radioactive substance or the number of it's atom reduces to half of it's initial value is called the half life of the substance.



i.e. if $N = \dfrac{N_0}{2}$ then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda (T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$

Time (t)	Number of undecayed atoms (N) (N_0 = Number of initial atoms)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed $(N_0 - N)/N_0$ probability of decay
$t = 0$	N_0	1 (100%)	0
$t = T_{1/2}$	$\dfrac{N_0}{2}$	$\dfrac{1}{2}$ (50%)	$\dfrac{1}{2}$ (50%)
$t = 2(T_{1/2})$	$\dfrac{1}{2} \times \dfrac{N_0}{2} = \dfrac{N_0}{(2)^2}$	$\dfrac{1}{4}$ (25%)	$\dfrac{3}{4}$ (75%)
$t = 3(T_{1/2})$	$\dfrac{1}{2} \times \dfrac{N_0}{(2)} = \dfrac{N_0}{(2)^3}$	$\dfrac{1}{8}$ (12.5%)	$\dfrac{7}{8}$ (87.5%)
$t = 10\ (T_{1/2})$	$\dfrac{N_0}{(2)^{10}}$	$\left(\dfrac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = n\ (N_{1/2})$	$\dfrac{N}{(2)^2}$	$\left(\dfrac{1}{2}\right)^n$	$\left\{1 - \left(\dfrac{1}{2}\right)^n\right\}$

Useful relation

After n half-lives, number of undecayed atoms $N = N_0 \left(\dfrac{1}{2}\right)^n = N_0 \left(\dfrac{1}{2}\right)^{t/T_{1/2}}$

(4) Mean (or average) life (τ)

The time for which a radioactive material remains active is defined as mean (average) life of that material.

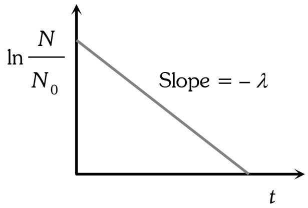
Other definitions

(i) It is defined as the sum of lives of all atoms divided by the total number of atoms

$$i.e.\ \tau = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}} = \frac{1}{\lambda}$$

(ii) From $N = N_0 e^{-\lambda t} \Rightarrow \dfrac{\ln \frac{N}{N_0}}{t} = -\lambda$ slope of the line shown in the graph

i.e. the magnitude of inverse of slope of $\ln \dfrac{N}{N_0}$ *vs* t curve is known as mean life (τ).



(iii) From $N = N_0 e^{-\lambda t}$

If $t = \dfrac{1}{\lambda} = \tau \Rightarrow N = N_0 e^{-1} = N_0\left(\dfrac{1}{e}\right) = 0.37 N_0 = 37\%$ of N_0.

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\dfrac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or

It is the time in which number of decayed atoms ($N_0 - N$) becomes $\left(1 - \dfrac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(iv) From $T_{1/2} = \dfrac{0.693}{\lambda} \Rightarrow \dfrac{1}{\lambda} = \tau = \dfrac{1}{0.693}.(t_{1/2}) = 1.44\,(T_{1/2})$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

Note : ≅ Half life and mean life of a substance doesn't change with time or with pressure, temperature *etc.*

Radioactive Series.

If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Mass number	Series (Nature)	Parent	Stable and product	Integer n	Number of lost particles
$4n$	Thorium (natural)	$_{90}Th^{232}$	$_{82}Pb^{208}$	52	$\alpha = 6, \beta = 4$
$4n + 1$	Neptunium (Artificial)	$_{93}Np^{237}$	$_{83}Bi^{209}$	52	$\alpha = 8, \beta = 5$
$4n + 2$	Uranium (Natural)	$_{92}U^{238}$	$_{82}Pb^{206}$	51	$\alpha = 8, \beta = 6$
$4n + 3$	Actinium (Natural)	$_{89}Ac^{227}$	$_{82}Pb^{207}$	51	$\alpha = 7, \beta = 4$

Note : ≅The $4n + 1$ series starts from $_{94}PU^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.

≅ The $4n + 3$ series actually starts from $_{92}U^{235}$.

Successive Disintegration and Radioactive Equilibrium.

Suppose a radioactive element A disintegrates to form another radioactive element B which intern disintegrates to still another element C; such decays are called successive disintegration.

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$$

Rate of disintegration of $A = \frac{dN_1}{dt} = -\lambda_1 N_1$ (which is also the rate of formation of B)

Rate of disintegration of $B = \frac{dN_2}{dt} = -\lambda_2 N_2$

$\therefore$ Net rate of formation of B = Rate of disintegration of A – Rate of disintegration of B

$= \lambda_1 N_1 - \lambda_2 N_2$

Equilibrium

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to it's rate of production from the previous member.

i.e. $\lambda_1 N_1 = \lambda_2 N_2 \quad \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_2} = \frac{\tau_2}{\tau_1} = \frac{(T_{1/2})}{(T_{1/2})_1}$

Note : $\cong$ In successive disintegration if N_0 is the initial number of nuclei of A at $t = 0$ then number of nuclei of product B at time t is given by $N_2 = \frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$ where $\lambda_1 \lambda_2$ – decay constant of A and B.

Uses of radioactive isotopes

(1) **In medicine**

(i) For testing blood-chromium - 51

(ii) For testing blood circulation - Na - 24

(iii) For detecting brain tumor- Radio mercury - 203

(iv) For detecting fault in thyroid gland - Radio iodine - 131

(v) For cancer - cobalt - 60

(vi) For blood - Gold - 189

(vii) For skin diseases - Phospohorous - 31

(2) **In Archaeology**

(i) For determining age of archaeological sample (carbon dating) C^{14}

(ii) For determining age of meteorites - K^{40}

(iii) For determining age of earth-Lead isotopes

(3) **In agriculture**

(i) For protecting potato crop from earthworm- CO^{60}

(ii) For artificial rains - AgI

(iii) As fertilizers - P^{32}

(4) **As tracers** - (Tracer) : Very small quantity of radioisotopes present in a mixture is known as tracer

(i) Tracer technique is used for studying biochemical reaction in tracer and animals.

(5) **In industries**

(i) For detecting leakage in oil or water pipe lines

(ii) For determining the age of planets.

Concept

1 λ_1, T_1, τ_1

λ

- *If a nuclide can decay simultaneously by two different process which have decay constant λ_1 and λ_2, half life T_1 and T_2 and mean lives τ_1 and τ_2 respectively then*

$$\Rightarrow \lambda = \lambda_1 + \lambda_2$$

$$\Rightarrow T = \frac{T_1 T_2}{T_1 + T_2}$$

$$\Rightarrow \tau = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

Example

Example: 16 When ${}_{90}Th^{228}$ transforms to ${}_{83}Bi^{212}$, then the number of the emitted α–and β–particles is, respectively **[MP PET 2002]**

(a) $8\alpha, 7\beta$ (b) $4\alpha, 7\beta$ (c) $4\alpha, 4\beta$ (d) $4\alpha, 1\beta$

Solution : (d) ${}_{Z=90}Th^{A=228} \to {}_{Z'=83}Bi^{A'=212}$

Number of α-particles emitted $n_\alpha = \frac{A - A'}{4} = \frac{228 - 212}{4} = 4$

Number of β-particles emitted $n_\beta = 2n_\alpha - Z + Z' = 2 \times 4 - 90 + 83 = 1$.

Example: 17 A radioactive substance decays to 1/16th of its initial activity in 40 days. The half-life of the radioactive substance expressed in days is **[AIEEE 2003]**

(a) 2.5 (b) 5 (c) 10 (d) 20

Solution : (c) By using $N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^{40/T_{1/2}} \Rightarrow T_{1/2} = 10$ days.

Example: 18 A sample of radioactive element has a mass of 10 *gm* at an instant *t* = 0. The approximate mass of this element in the sample after two mean lives is **[CBSE PMT 2003]**

(a) 2.50 *gm* (b) 3.70 *gm* (c) 6.30 *gm* (d) 1.35 *gm*

Solution : (d) By using $M = M_0 e^{-\lambda t} \Rightarrow M = 10e^{-\lambda(2\tau)} = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.359\,gm$

Example: 19 The half-life of ${}^{215}At$ is 100 μs. The time taken for the radioactivity of a sample of ${}^{215}At$ to decay to 1/16th of its initial value is **[IIT-JEE (Screening) 2002]**

(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

Solution : (a) By using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{t/100} \Rightarrow t = 400\ \mu\,sec.$

Example: 20 The mean lives of a radioactive substance for α and β emissions are 1620 years and 405 years respectively. After how much time will the activity be reduced to one fourth **[RPET 1999]**

(a) 405 year (b) 1620 year (c) 449 year (d) None of these

Solution : (c) $\lambda_\alpha = \frac{1}{1620}$ *per year* and $\lambda_\beta = \frac{1}{405}$ *per year* and it is given that the fraction of the remained activity $\frac{A}{A_0} = \frac{1}{4}$

Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ *per year*

We know that $A = A_0 e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda}\log_e \frac{A_0}{A} \Rightarrow t = \frac{1}{\lambda}\log_e 4 = \frac{2}{\lambda}\log_e 2 = 324 \times 2 \times 0.693 = 449$ *years*.

***Example*: 21** At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances **[RPMT 1996]**

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

Solution : (a) By using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2} \times \frac{(1/2)^{n_1}}{(1/2)^{n_2}} = \frac{2}{1} \times \frac{\left(\frac{1}{2}\right)^{\frac{2\times 24}{12}}}{\left(\frac{1}{2}\right)^{\frac{2\times 24}{16}}} = \frac{1}{1}$

***Example*: 22** From a newly formed radioactive substance (Half-life 2 *hours*), the intensity of radiation is 64 times the permissible safe level. The minimum time after which work can be done safely from this source is **[IIT 1983; SCRA 1996]**

(a) 6 *hours* (b) 12 *hours* (c) 24 *hours* (d) 128 *hours*

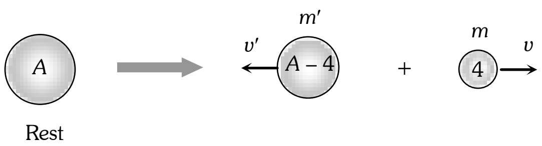
Solution : (b) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{A}{A_0} = \frac{1}{64} = \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^n \Rightarrow n = 6$

$\Rightarrow \frac{t}{T_{1/2}} = 6 \Rightarrow t = 6 \times 2 = 12$ *hours*.

***Example*: 23** nucleus of mass number *A*, originally at rest, emits an α-particle with speed *v*. The daughter nucleus recoils with a speed **[DCE 2000; AIIMS 2004]**

(a) $2v/(A+4)$ (b) $4v/(A+4)$ (c) $4v/(A-4)$ (d) $2v/(A-4)$

Solution : (c)



According to conservation of momentum $4v = (A-4)v' \Rightarrow v' = \frac{4v}{A-4}$.

***Example*: 24** The counting rate observed from a radioactive source at t = 0 second was 1600 counts per second and at *t* = 8 seconds it was 100 counts per second. The counting rate observed as counts per second at *t* = 6 seconds will be **[MP PET 1996; UPSEAT 2000]**

(a) 400 (b) 300 (c) 200 (d) 150

Solution : (c) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow 100 = 1600\left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow T_{1/2} = 2\,sec$

Again by using the same relation the count rate at *t* = 6 *sec* will be $A = 1600\left(\frac{1}{2}\right)^{6/2} = 200$.

***Example*: 25** The kinetic energy of a neutron beam is 0.0837 *eV*. The half-life of neutrons is 693s and the mass of neutrons is 1.675×10^{-27} *kg*. The fraction of decay in travelling a distance of 40*m* will be

(a) 10^{-3} (b) 10^{-4} (c) 10^{-5} (d) 10^{-6}

Solution : (c) $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 0.0837 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}} = 4 \times 10^3\ m/sec$

$\therefore$ Time taken by neutrons to travel a distance of 40 *m* $\Delta t' = \frac{40}{4 \times 10^3} = 10^{-2}\,sec$

$\therefore \frac{dN}{dt} = \lambda N \Rightarrow \frac{dN}{N} = \lambda\, dt$

$\therefore$ Fraction of neutrons decayed in Δt *sec* in $\dfrac{\Delta N}{N} = \lambda\,\Delta t = \dfrac{0.693}{T}\Delta t = \dfrac{0.693}{693}\times 10^{-2} = 10^{-5}$

Example: 26 The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) 77/80 (b) 71/80 (c) 31/32 (d) 15/16

Solution : (c) By using $N = N_0\left(\dfrac{1}{2}\right)^{t/T_{1/2}} \Rightarrow t = \dfrac{T_{1/2}\log_e\left(\dfrac{N_0}{N}\right)}{\log_e(2)} \Rightarrow t \propto \log_e\dfrac{N_0}{N} \Rightarrow \dfrac{t_1}{t_2} = \dfrac{\left(\log_e\dfrac{N_0}{N}\right)_1}{\left(\log_e\dfrac{N_0}{N}\right)_2}$

Hence $\dfrac{6}{10} = \dfrac{\log_e(8/1)}{\log_e(N_0/N)} \Rightarrow \log_e\dfrac{N_0}{N} = \dfrac{10}{6}\log_e(8) = \log_e 32 \Rightarrow \dfrac{N_0}{N} = 32$.

So fraction that decays $= 1 - \dfrac{1}{32} = \dfrac{31}{32}$.

Tricky example: 2

Half-life of a substance is 20 minutes. What is the time between 33% decay and 67% decay **[AIIMS 2000]**

(a) 40 minutes (b) 20 minutes (c) 30 minutes (d) 25 minutes

Solution : (b) Let N_0 be the number of nuclei at beginning

$\therefore$ Number of undecayed nuclei after 33% decay $= 0.67\,N_0$

and number of undecayed nuclei after 67% of decay $= 0.33\,N_0$

$\because$ $0.33\,N_0 \simeq \dfrac{0.67N_0}{2}$ and in the half-life time the number of undecayed nuclei becomes half.