Informed search algorithms & Heuristic Functions

Chapter 3
3.5 and 3.6

Outline

- Best-first search
- Greedy best-first search
- A^* search
- Memory bounded Heuristics search
- Learning to search betters

Best first search

- Best-first search is an instance of the general TREE-SEARCH or GRAPH-SEARCH algorithm in which a node is selected for expansion based on an **evaluation function**, f(n).
- The evaluation function is interpreted as a cost estimate, so the node with the *lowest* evaluation is expanded first.
- Consider Heuristic function to be arbitrary, nonnegative, problem-specific functions, with one constraint: if n is a goal node, then h(n)=0.
- Most best-first algorithms include as a component of f a **heuristic function**, denoted h(n):
  - h(n) = estimated cost of the cheapest path from the state at node *n* to a goal state.

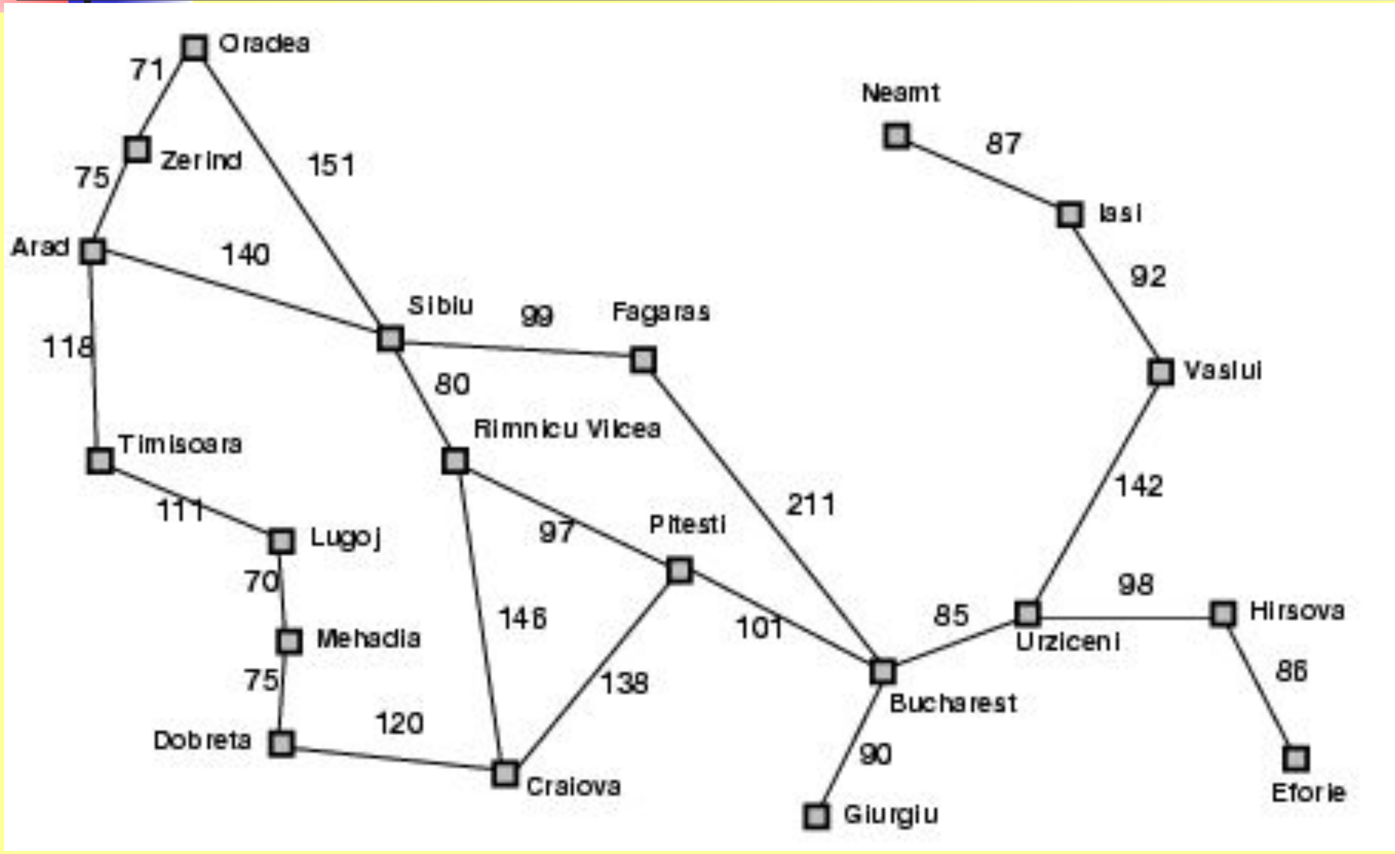
Best-first search

- Idea: use an evaluation function *f(n)* for each node
  - f(n) provides an estimate for the total cost.
  - Expand the node n with smallest f(n).

- Implementation:
  Order the nodes in fringe increasing order of cost.

- Special cases:
  - greedy best-first search
  - A^* search

Romania with straight-line distance



Straight–line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
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Oradea	380
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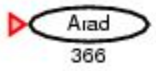
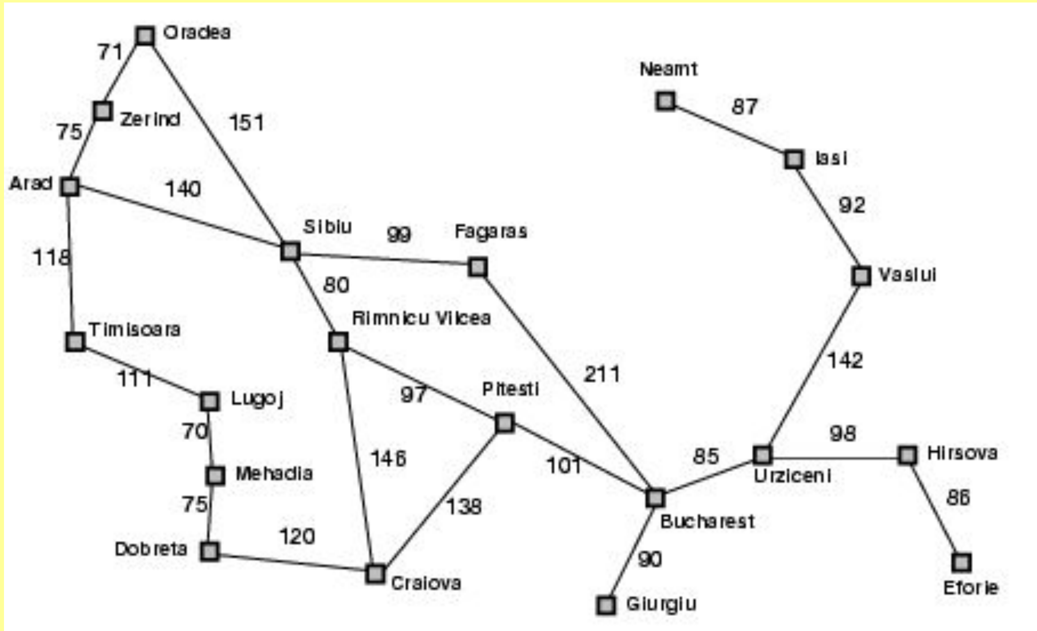
Zerind	374

Greedy best-first search

- f(n) = estimate of cost from *n* to *goal*
- e.g., $f_{SLD}(n)$ = straight-line distance from *n* to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal.

Greedy best-first search example





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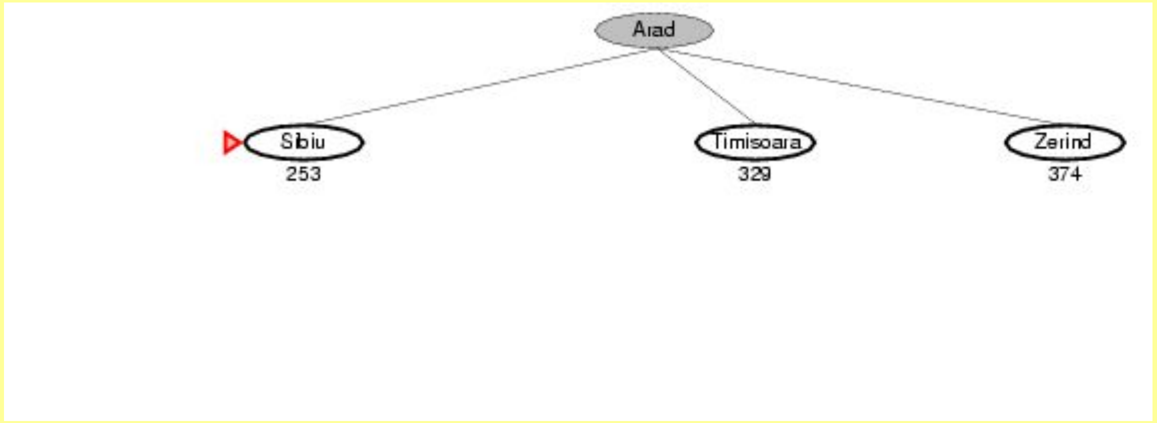

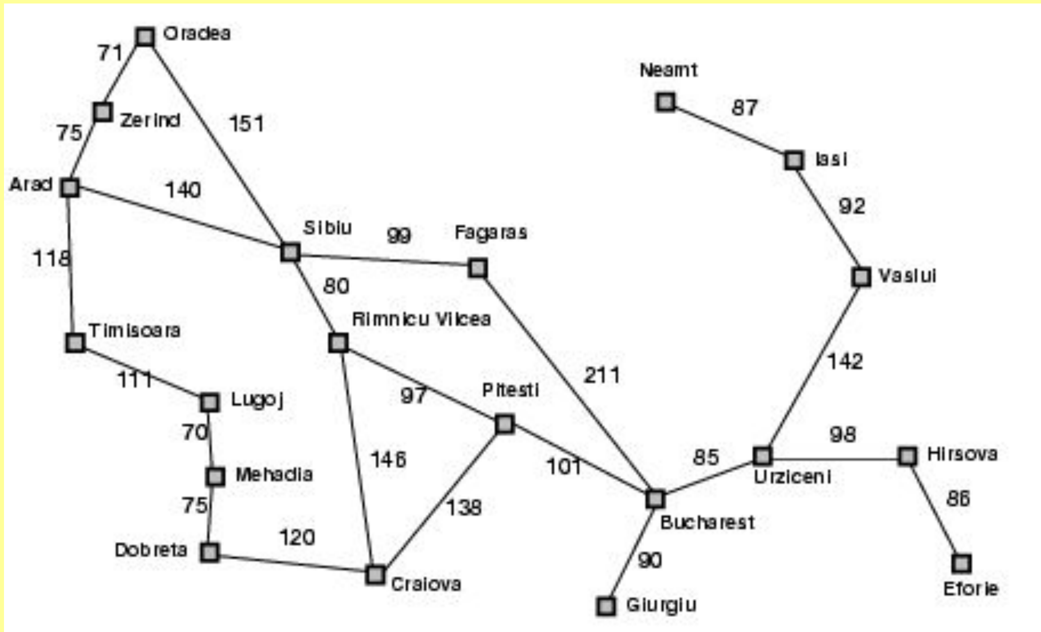

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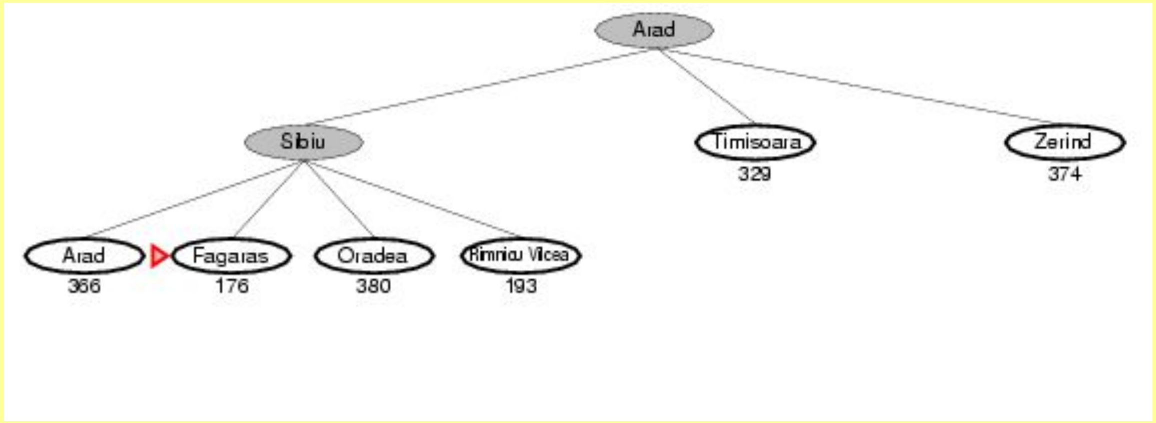

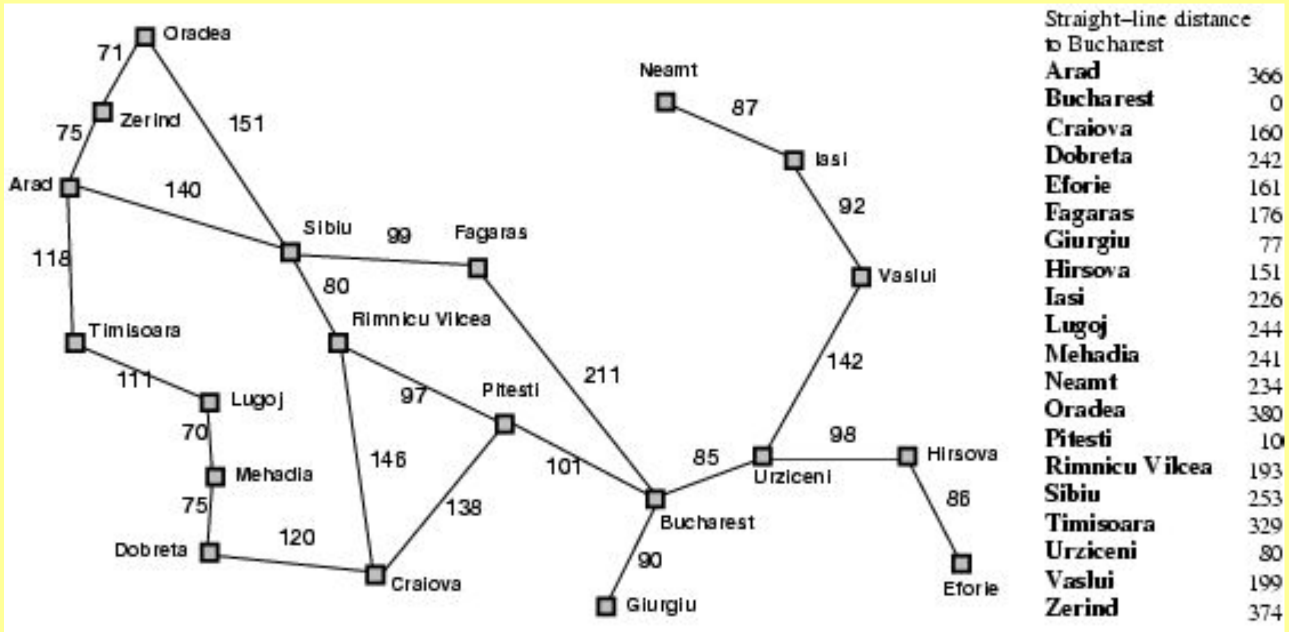

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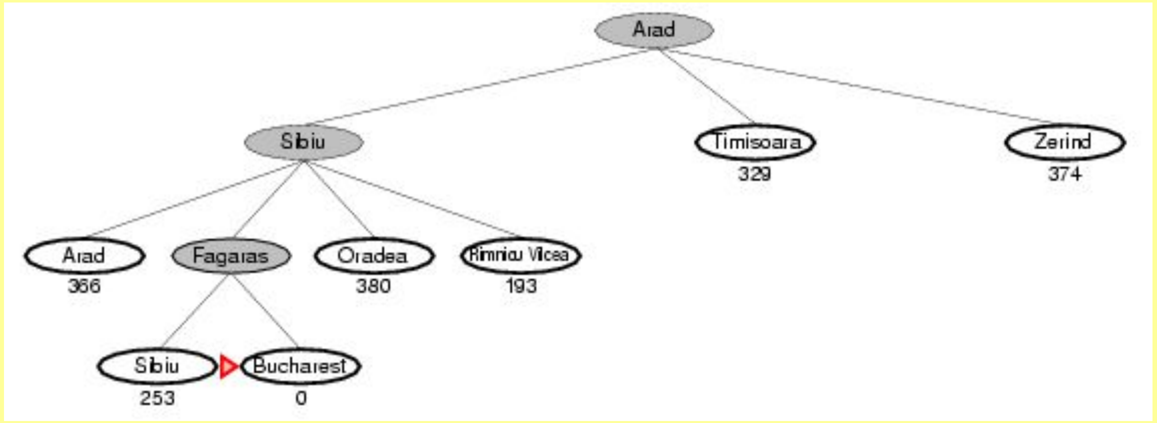

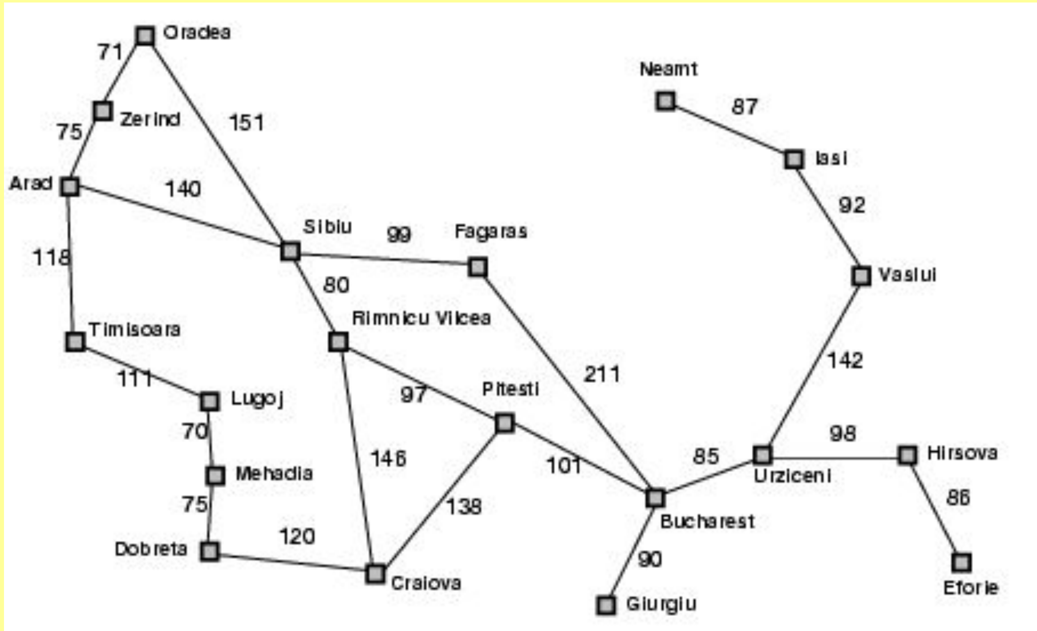

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Properties of greedy best-first search

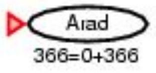
- Complete? No – can get stuck in loops / dead end. (Iasi to Fagarus)
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ - keeps all nodes in memory
- Optimal? No

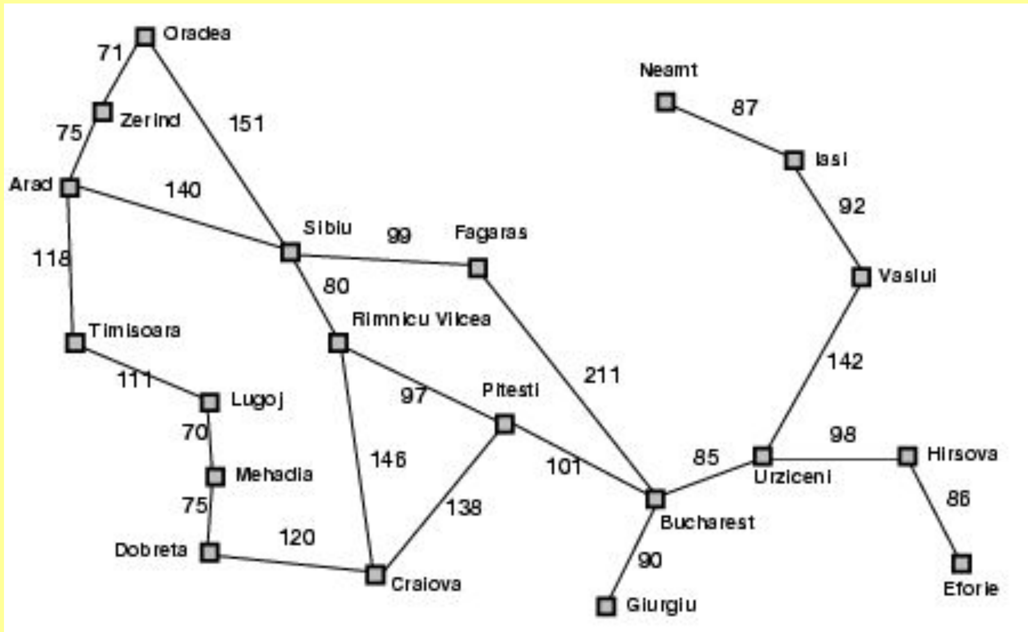
e.g. Arad☐Sibiu☐Rimnicu Virea☐Pitesti☐Bucharest is shorter!

A* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal
- Best First search has $f(n)=h(n)$

A* search example





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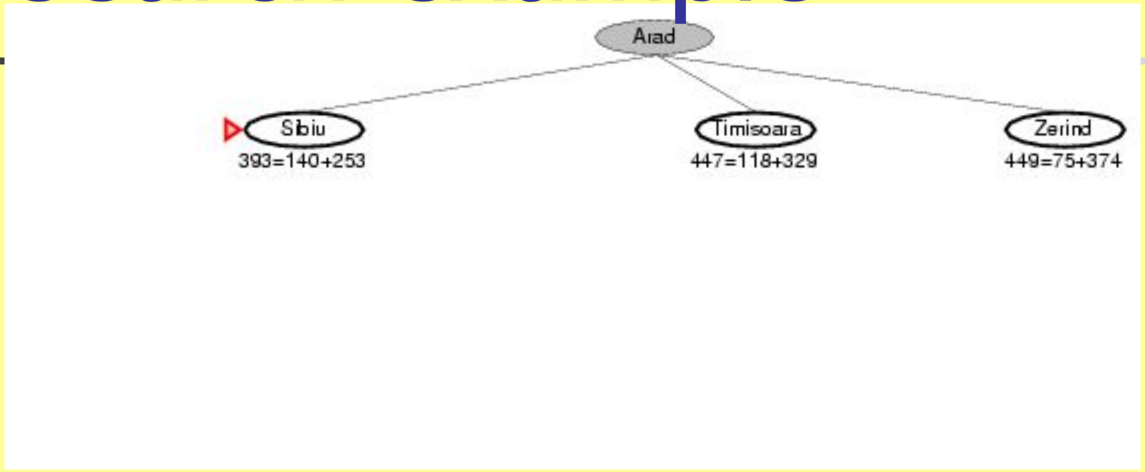

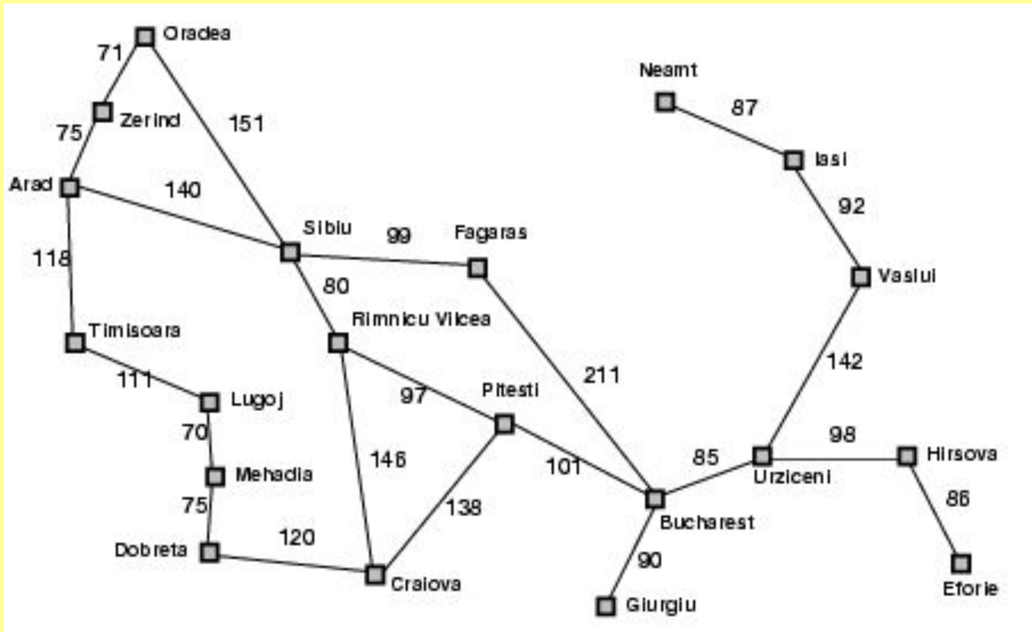

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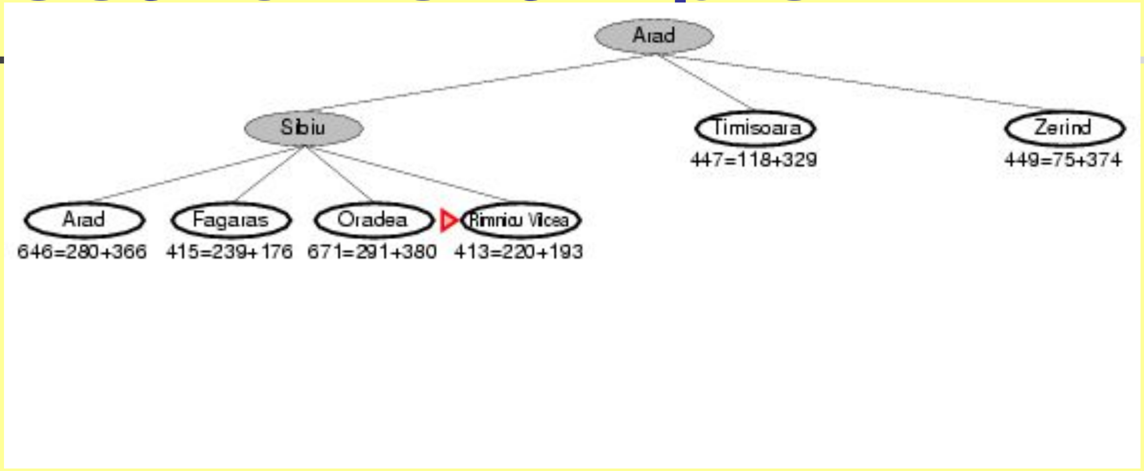

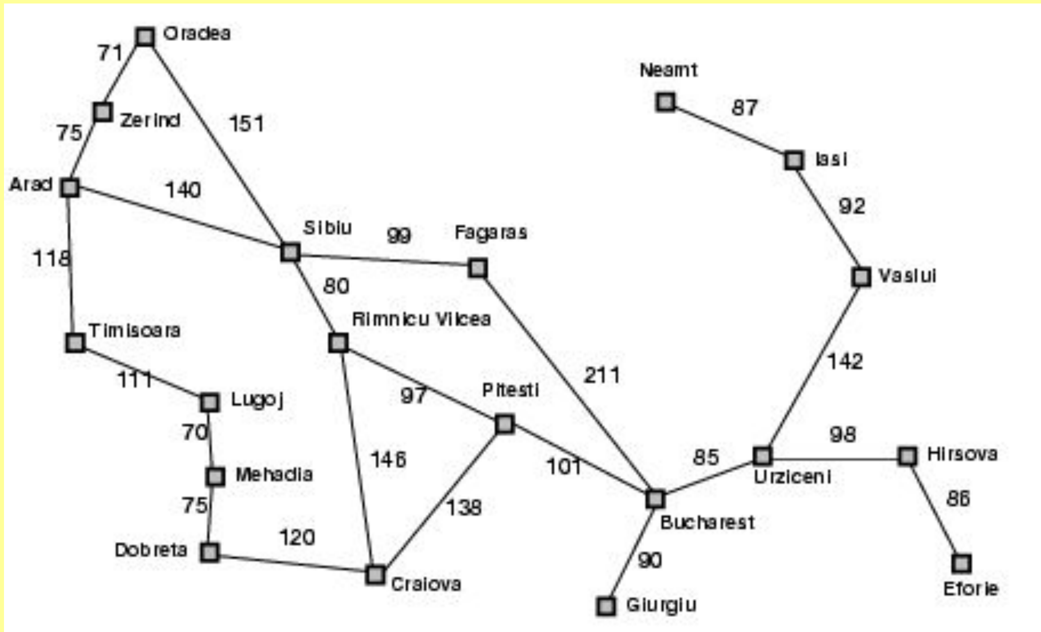

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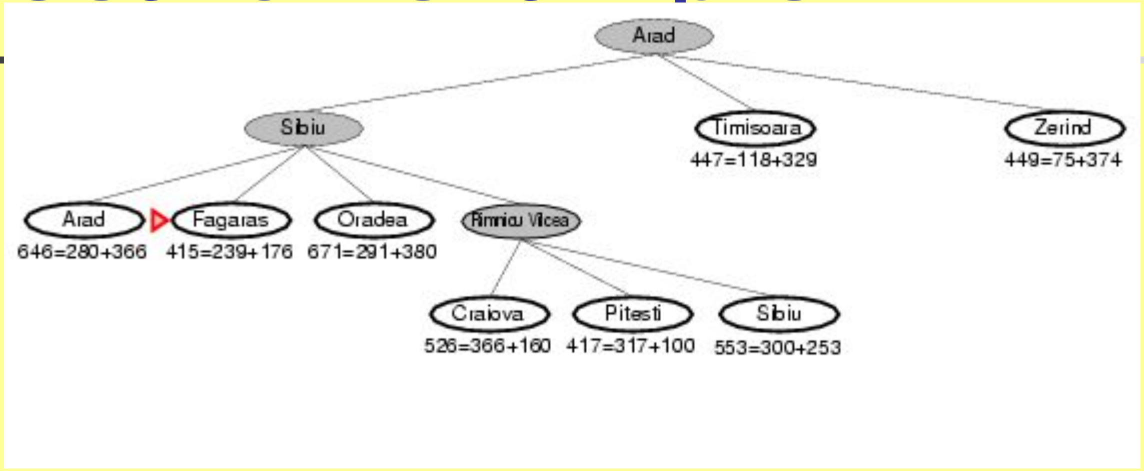
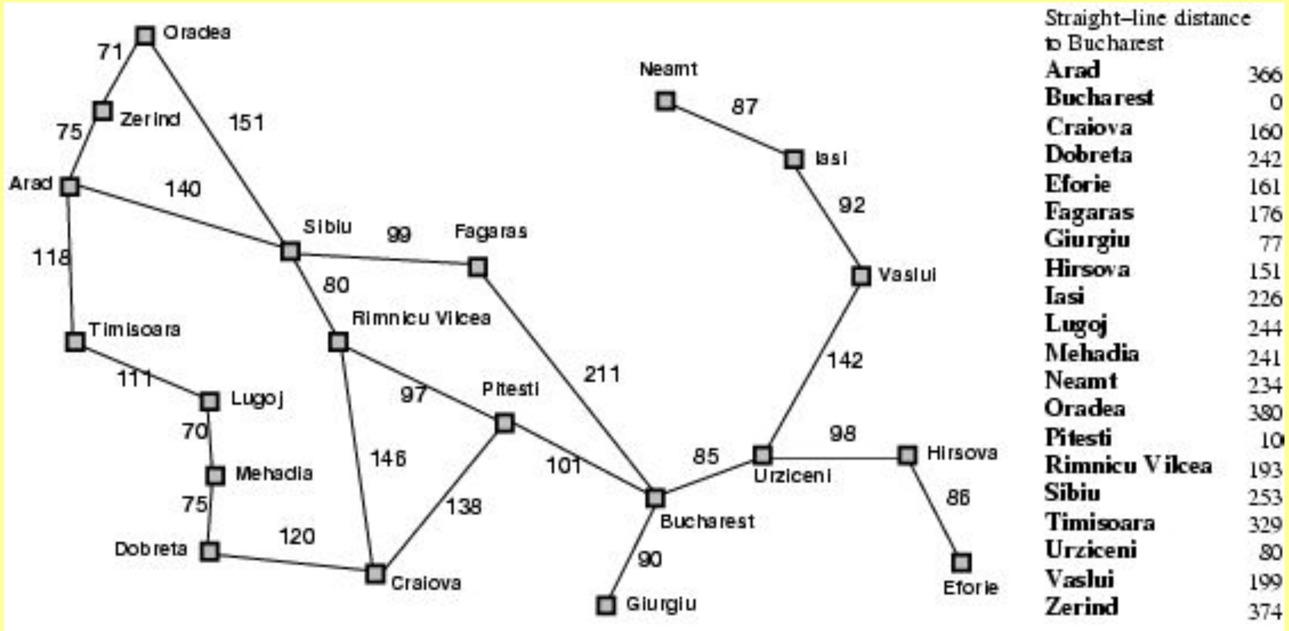
A* search example





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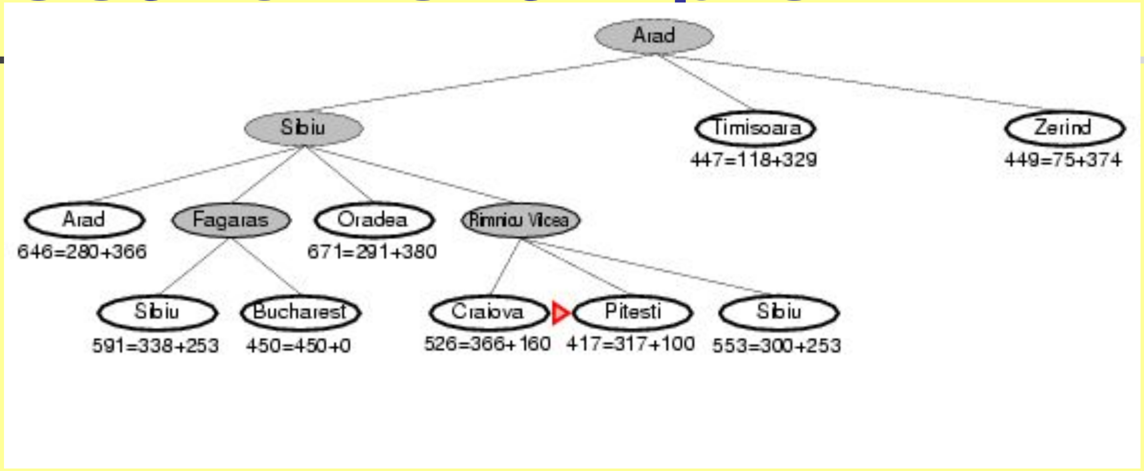
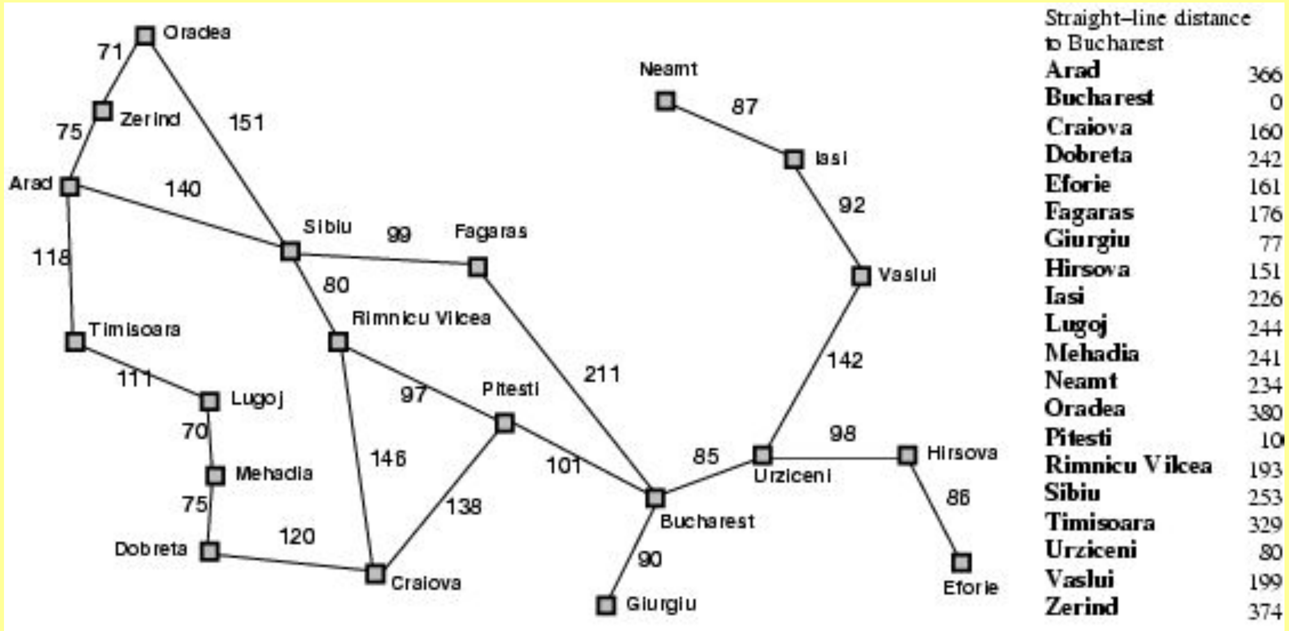
A* search example





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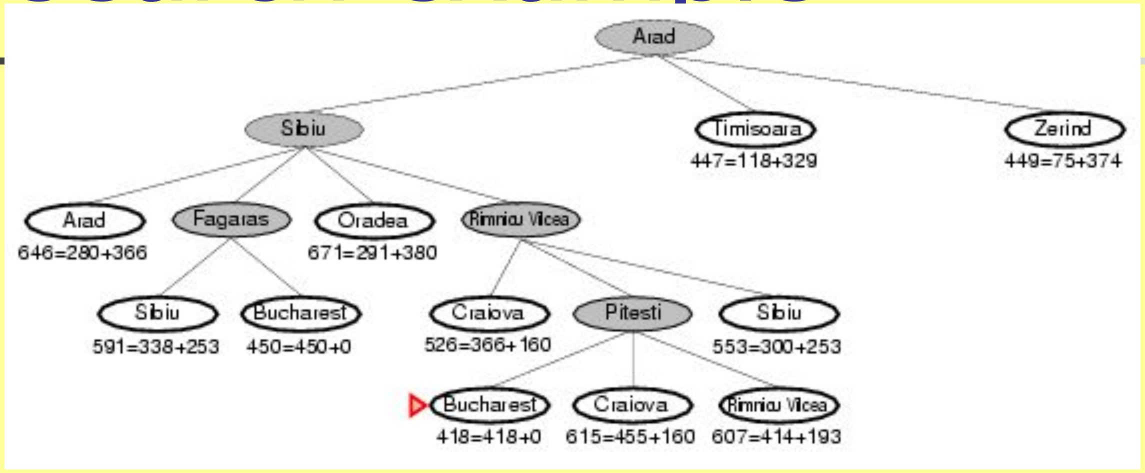
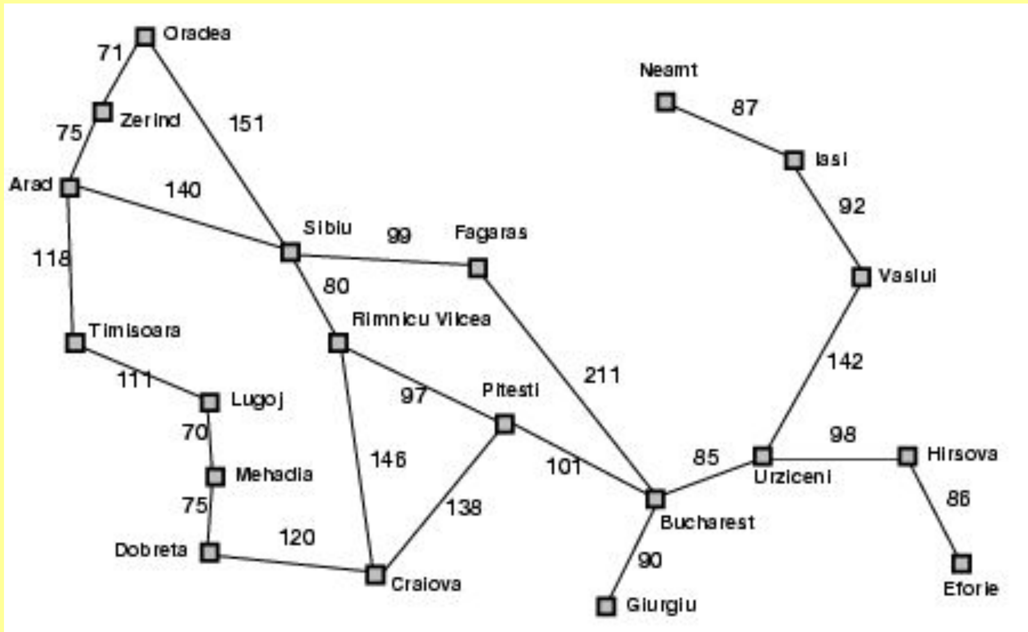
A* search example





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End of class

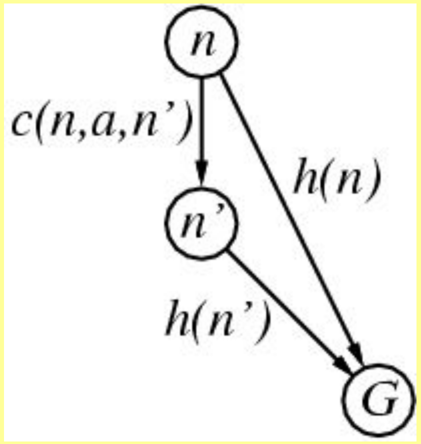
Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A* using `TREE-SEARCH` is optimal *while the graph-search version is optimal if* h(n) *is consistent.*

Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

  $h(n) \le c(n,a,n') + h(n')$



It's the triangle inequality !

- If *h* is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\ge g(n) + h(n) = f(n) \\ f(n') &\ge f(n) \end{aligned}$$

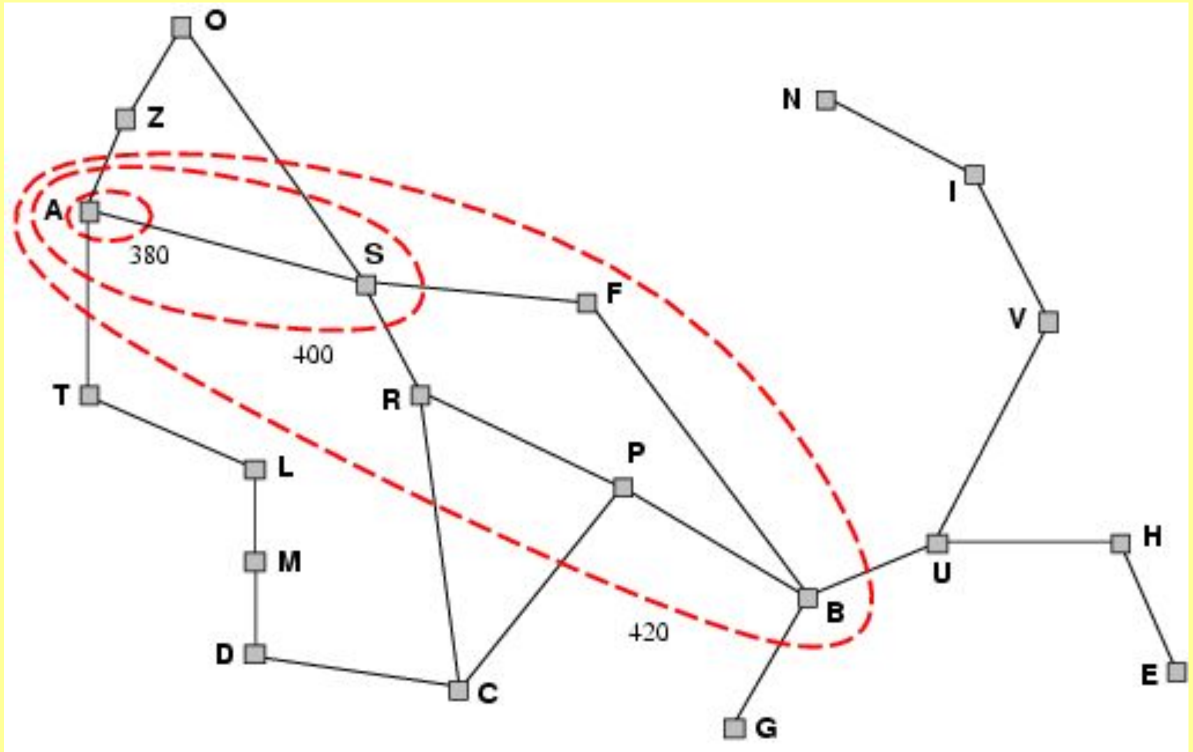
- i.e., *f(n)* is non-decreasing along any path.

- Theorem:
  If *h(n)* is consistent, A* using GRAPH-SEARCH is optimal

keeps all checked nodes in memory to avoid repeated states

Optimality of A*

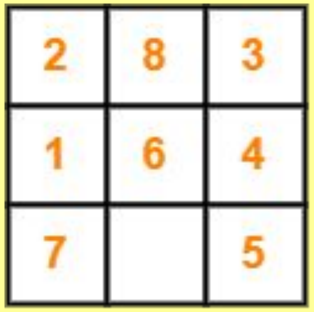
- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* contains all nodes with $f \le f_i$ where $f_i < f_{i+1}$



Problem of A*

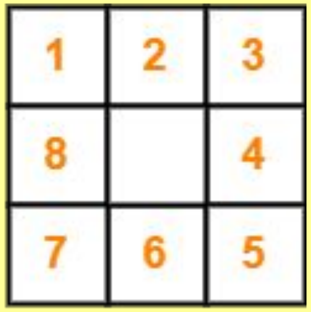
reach the final state from initial state using A* Algorithm.

Consider g(n) = Depth of node and h(n) = Number of misplaced tiles.



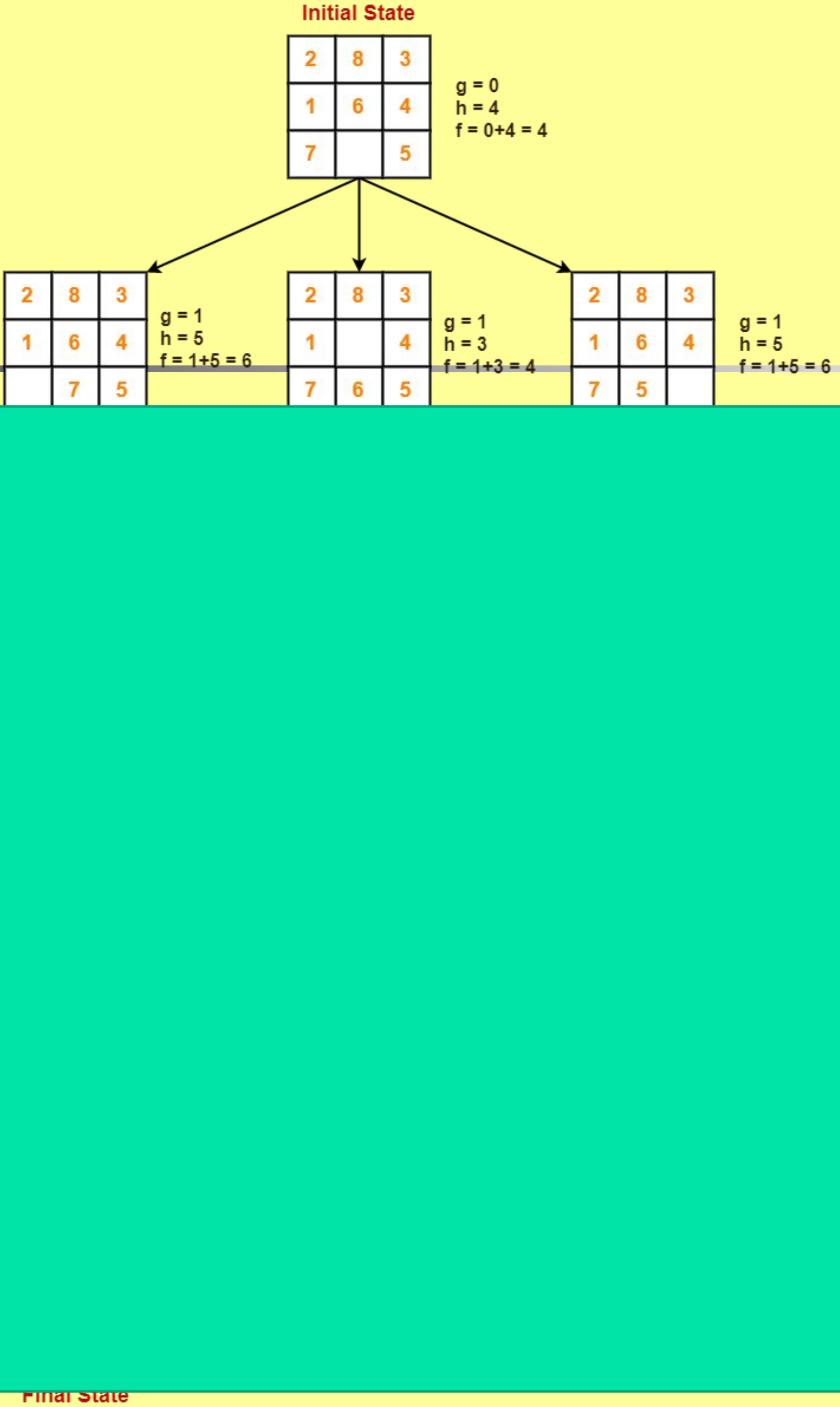
2	8	3
1	6	4
7		5

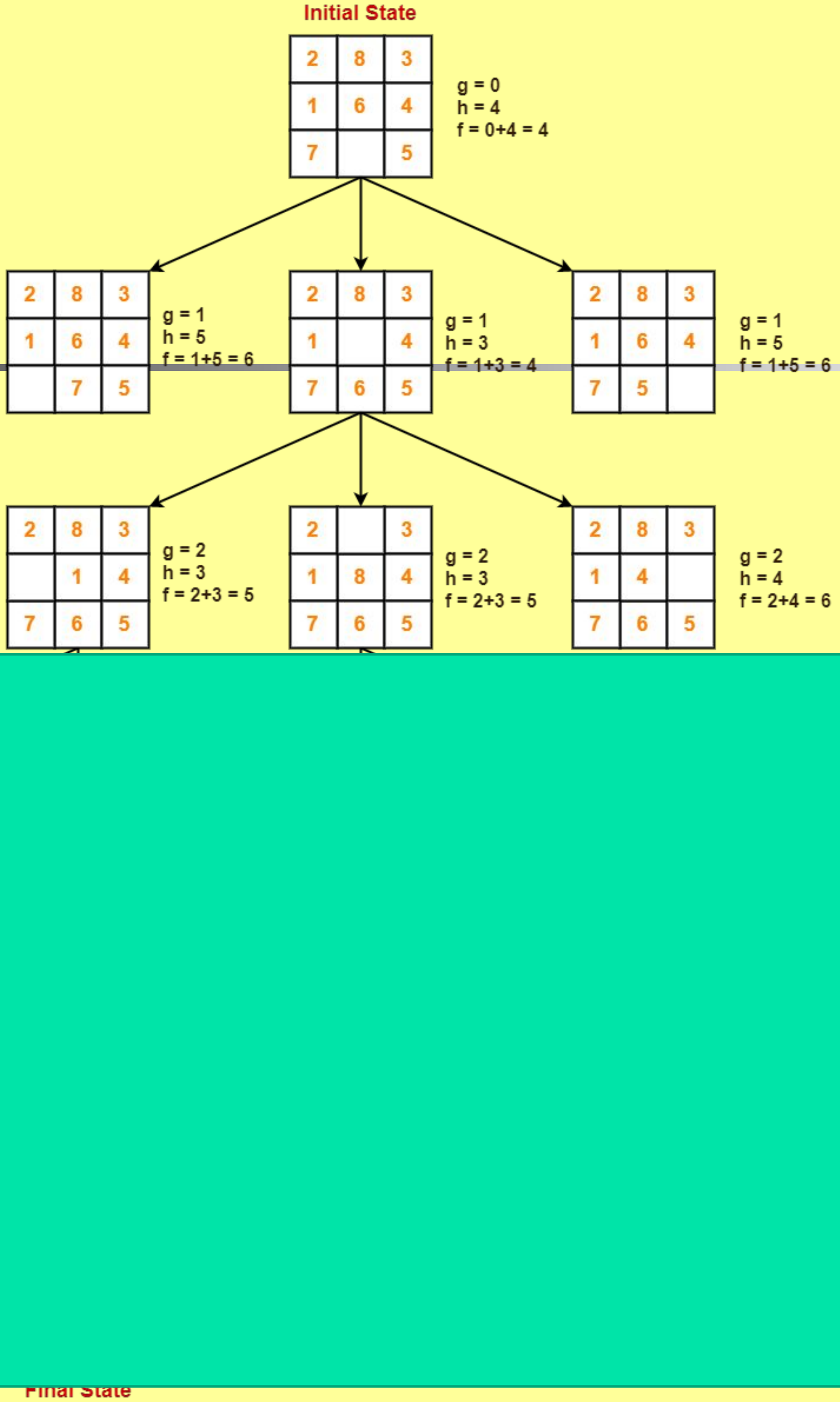
Initial State



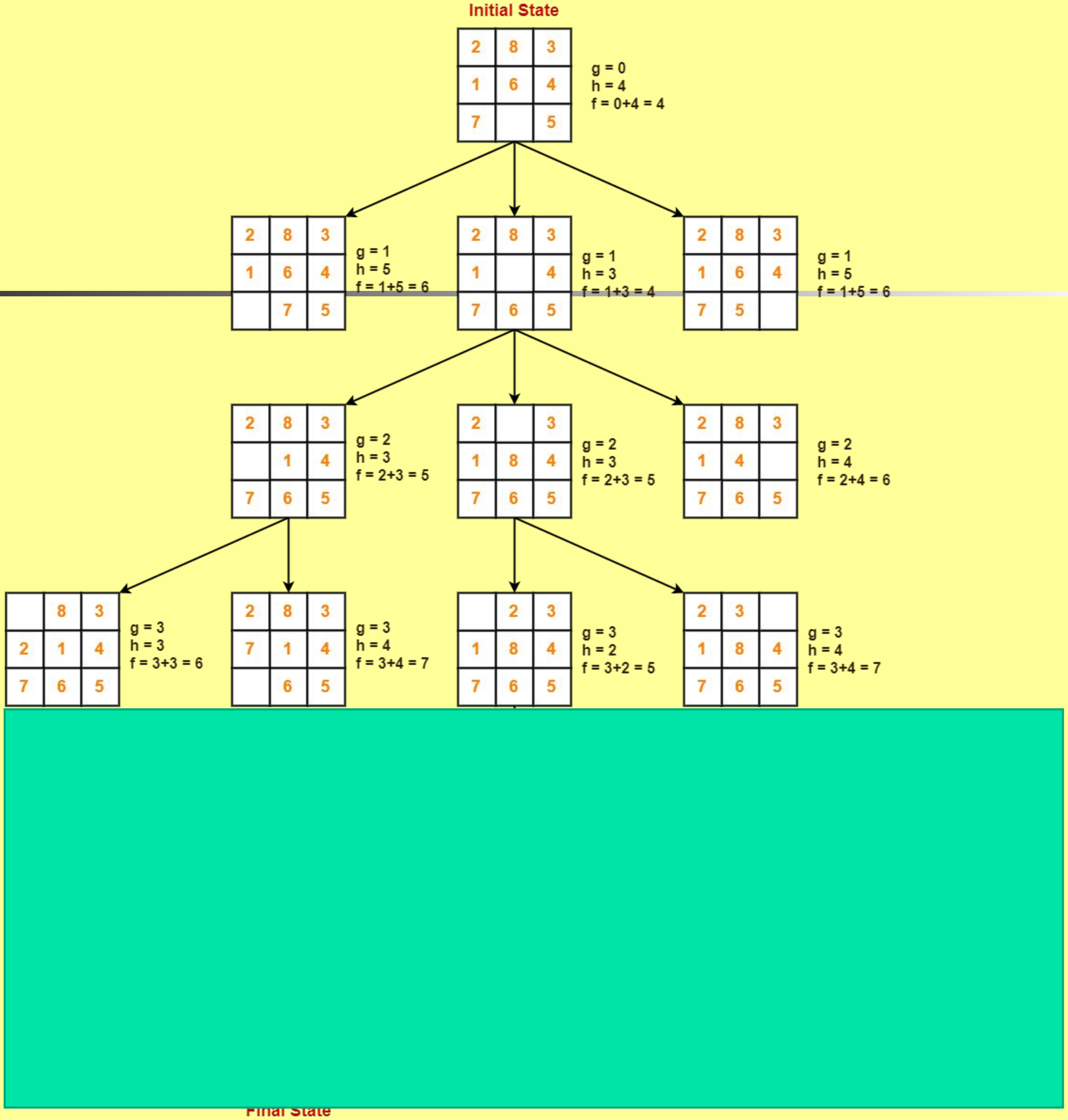
1	2	3
8		4
7	6	5

Final State

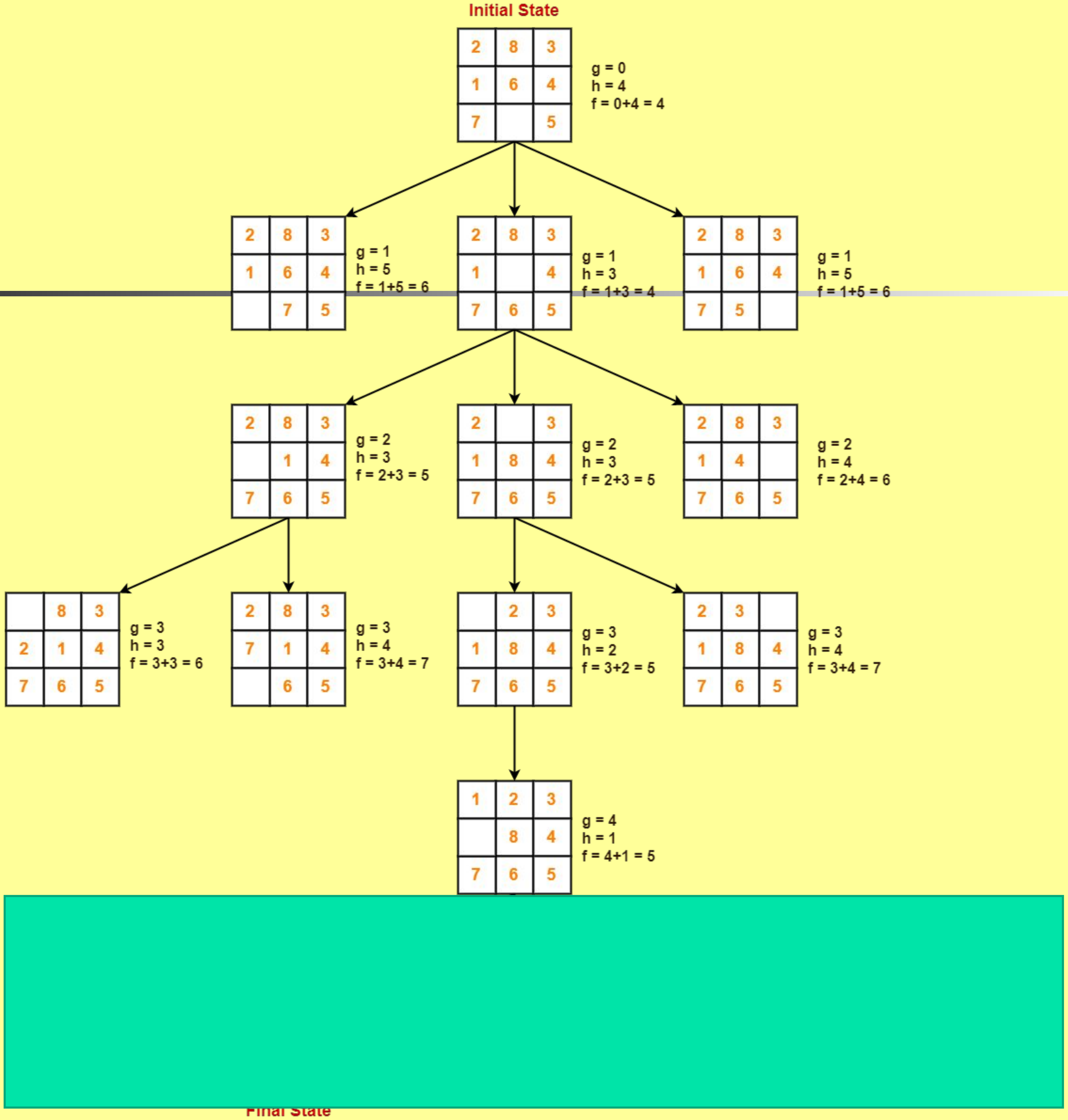

Initial State
2 8 3
1 6 4
7 5
g = 0
h = 4
f = 0+4 = 4
2 8 3
1 6 4
7 5
g = 1
h = 5
f = 1+5 = 6
2 8 3
1 4
7 6 5
g = 1
h = 3
f = 1+3 = 4
2 8 3
1 6 4
7 5
g = 1
h = 5
f = 1+5 = 6
Final State

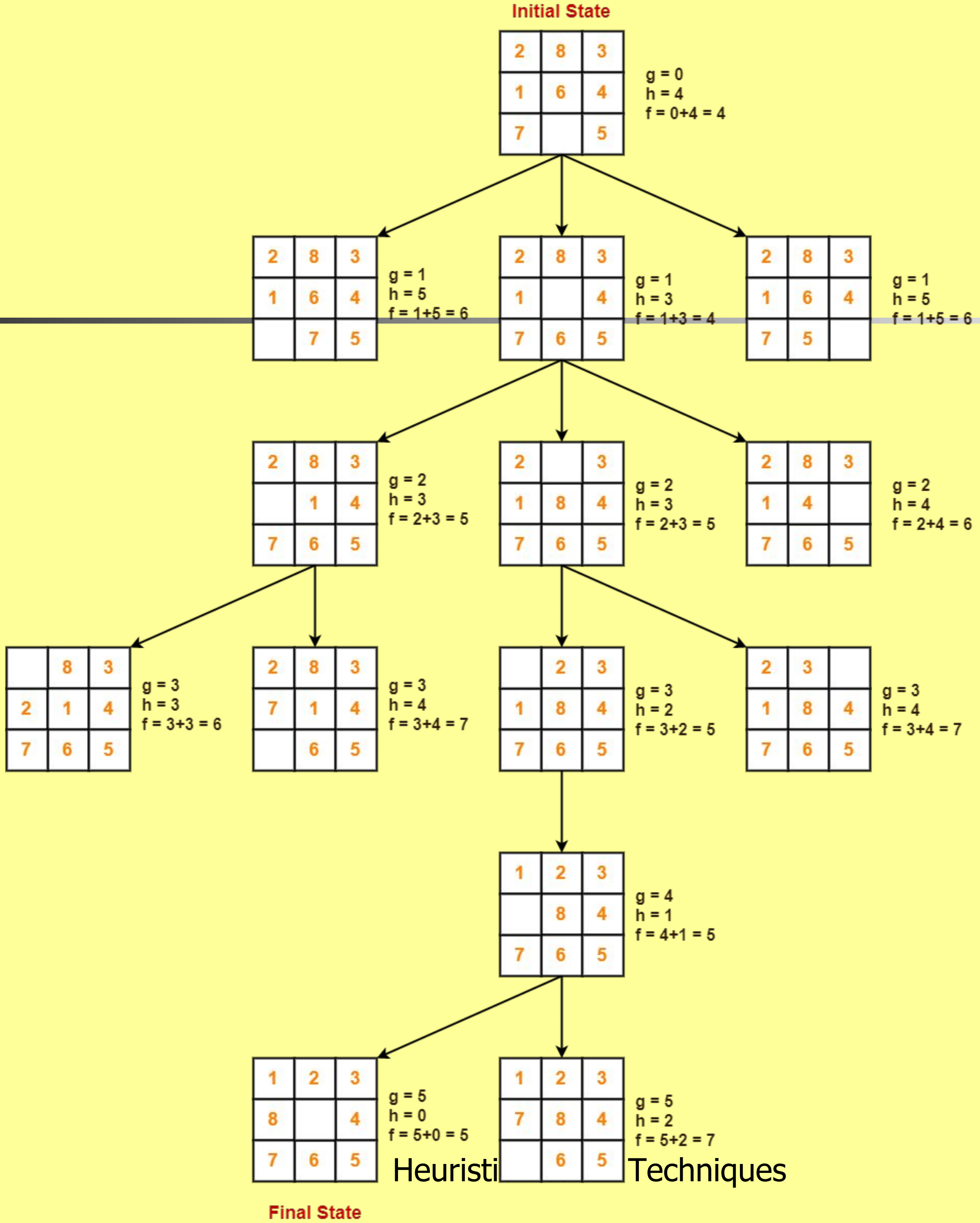


Initial State
g = 0
h = 4
f = 0+4 = 4
g = 1
h = 5
f = 1+5 = 6
g = 1
h = 3
f = 1+3 = 4
g = 1
h = 5
f = 1+5 = 6
g = 2
h = 3
f = 2+3 = 5
g = 2
h = 3
f = 2+3 = 5
g = 2
h = 4
f = 2+4 = 6
Final State



Initial State
g = 0
h = 4
f = 0+4 = 4
g = 1
h = 5
f = 1+5 = 6
g = 1
h = 3
f = 1+3 = 4
g = 1
h = 5
f = 1+5 = 6
g = 2
h = 3
f = 2+3 = 5
g = 2
h = 3
f = 2+3 = 5
g = 2
h = 4
f = 2+4 = 6
g = 3
h = 3
f = 3+3 = 6
g = 3
h = 4
f = 3+4 = 7
g = 3
h = 2
f = 3+2 = 5
g = 3
h = 4
f = 3+4 = 7
Final State


Initial State
2 8 3 / 1 6 4 / 7 _ 5
g = 0
h = 4
f = 0+4 = 4
2 8 3 / 1 6 4 / _ 7 5
g = 1
h = 5
f = 1+5 = 6
2 8 3 / 1 _ 4 / 7 6 5
g = 1
h = 3
f = 1+3 = 4
2 8 3 / 1 6 4 / 7 5 _
g = 1
h = 5
f = 1+5 = 6
2 8 3 / _ 1 4 / 7 6 5
g = 2
h = 3
f = 2+3 = 5
2 _ 3 / 1 8 4 / 7 6 5
g = 2
h = 3
f = 2+3 = 5
2 8 3 / 1 4 _ / 7 6 5
g = 2
h = 4
f = 2+4 = 6
_ 8 3 / 2 1 4 / 7 6 5
g = 3
h = 3
f = 3+3 = 6
2 8 3 / 7 1 4 / _ 6 5
g = 3
h = 4
f = 3+4 = 7
_ 2 3 / 1 8 4 / 7 6 5
g = 3
h = 2
f = 3+2 = 5
2 3 _ / 1 8 4 / 7 6 5
g = 3
h = 4
f = 3+4 = 7
1 2 3 / _ 8 4 / 7 6 5
g = 4
h = 1
f = 4+1 = 5
Final State


Initial State
g = 0
h = 4
f = 0+4 = 4
g = 1
h = 5
f = 1+5 = 6
g = 1
h = 3
f = 1+3 = 4
g = 1
h = 5
f = 1+5 = 6
g = 2
h = 3
f = 2+3 = 5
g = 2
h = 3
f = 2+3 = 5
g = 2
h = 4
f = 2+4 = 6
g = 3
h = 3
f = 3+3 = 6
g = 3
h = 4
f = 3+4 = 7
g = 3
h = 2
f = 3+2 = 5
g = 3
h = 4
f = 3+4 = 7
g = 4
h = 1
f = 4+1 = 5
g = 5
h = 0
f = 5+0 = 5
g = 5
h = 2
f = 5+2 = 7
Final State

Properties of A*

- Complete? Yes (unless there are infinitely many nodes with f $\leq$ *f(G)* , i.e. path-cost > ε)
- Time/Space? Exponential b^d
- Optimal? Yes
- *Optimally Efficient*: Yes (no algorithm with the same heuristic is guaranteed to expand fewer nodes)

Memory Bounded Heuristic Search: Recursive BFS

- How can we solve the memory problem for A* search?
- Idea: Try something like depth first search, but let’s not forget everything about the branches we have partially explored.
- We remember the best f-value we have found so far in the branch we are deleting.

Types of Memory Bounded algorithms

- IDA* - Is practical for many problems with unit step costs and avoids the substantial overhead associated with keeping a sorted queue of nodes. Unfortunately, it suffers from the same difficulties with real valued costs as does the iterative version of uniform-cost search
- RBFS
- MA*

Difference between IDA* and IDDFS

- The main difference between IDA∗ and standard iterative deepening is that the cutoff used is the f-cost (g+h) rather than the depth;
- At each iteration, the cutoff value is the smallest f-cost of any node that exceeded the cutoff on the previous iteration.

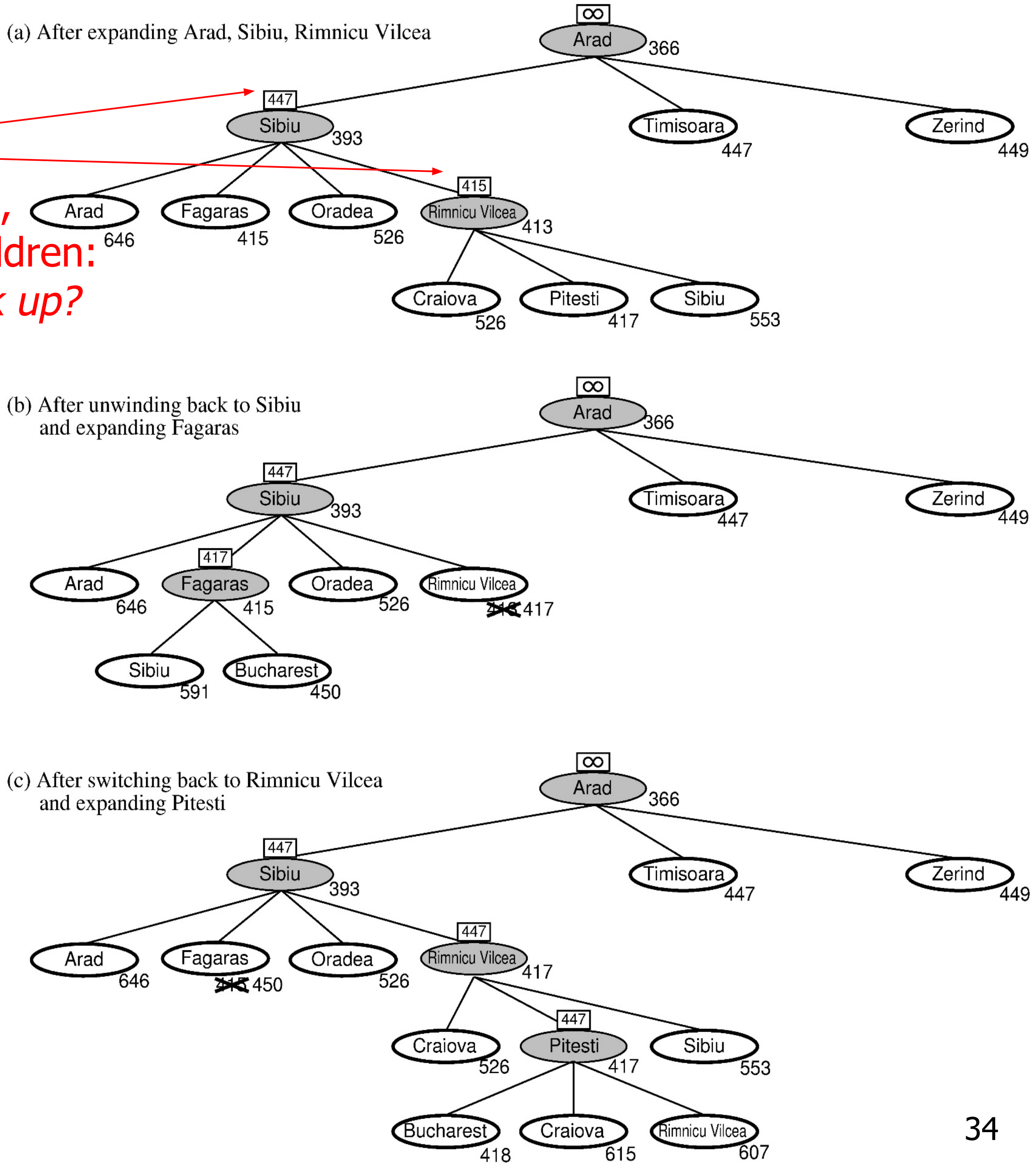
RBFS:

best alternative
over fringe nodes,
which are not children:
do I want to back up?

RBFS changes its mind very often in practice.

This is because the f=g+h become more accurate (less optimistic) as we approach the goal. Hence, higher level nodes have smaller f-values and will be explored first.

Problem: We should keep in memory whatever we can.



End of class

RBFS

- Suffers from excessive node regeneration.
- Like A^* tree search, RBFS is an optimal algorithm if the heuristic function h(n) is admissible.
- Its space complexity is linear in the depth of the deepest optimal solution
- Time complexity is rather difficult to characterize: it depends both on the accuracy of the heuristic function and on how often the best path changes as nodes are expanded.

IDA* and RBFS- Drawback

- IDA* and RBFS suffer from using *too little* memory.
- To utilize available memory
  - **MA*** (memory-bounded A*)
  - **SMA*** (simplified MA*)

Properties of SMA*

- SMA* expands the best leaf and deletes the worst leaf.
- SMA* expands the *newest* best leaf and deletes the *oldest* worst leaf – if F-value same
- SMA* is complete if there is any reachable solution
- It is optimal if any optimal solution is reachable; otherwise, it returns the best reachable solution.
- Time and space complexity is inescapable problem if subset is regenerated.

Learning to search better

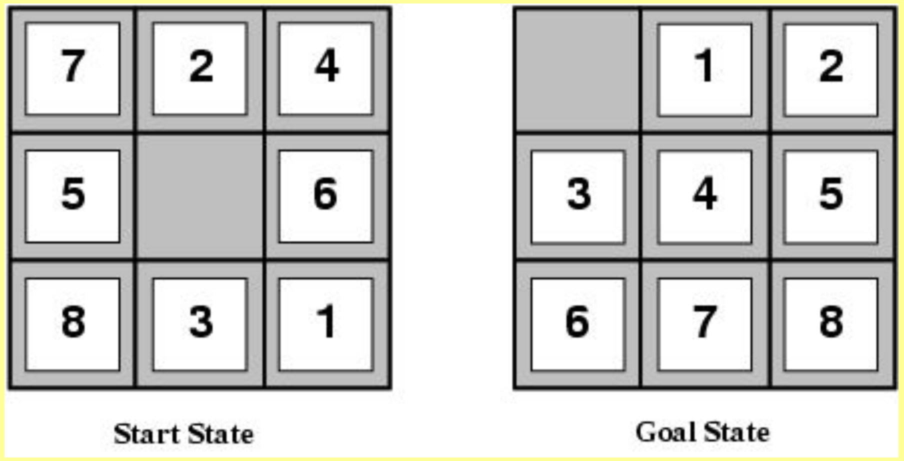
- Could an agent *learn* how to search better?
- **Metalevel state space** captures the internal (computational) state of a program that is searching in an **object-level state space** such as Romania.
- For example, the internal state of the A^* algorithm consists of the current search tree. Each action in the metalevel state space is a computation step that alters the internal state; for example, each computation step in A^* expands a leaf node and adds its successors to the tree.
- **Metalevel learning** algorithm- The goal of learning is to minimize the **total cost** of problem solving, trading off computational expense and path cost.

3.6 Heuristic Functions - Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance/ city block distance

(i.e., no. of squares from desired location of each tile)



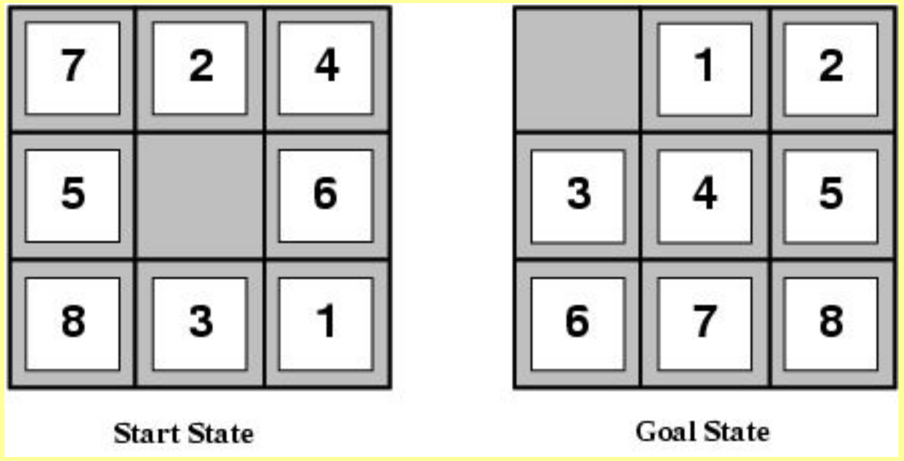
- $h_1(S) = ?$
- $h_2(S) = ?$

Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
  - then h_2 dominates h_1

- h_2 is better for search: it is guaranteed to expand less nodes.

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed problems

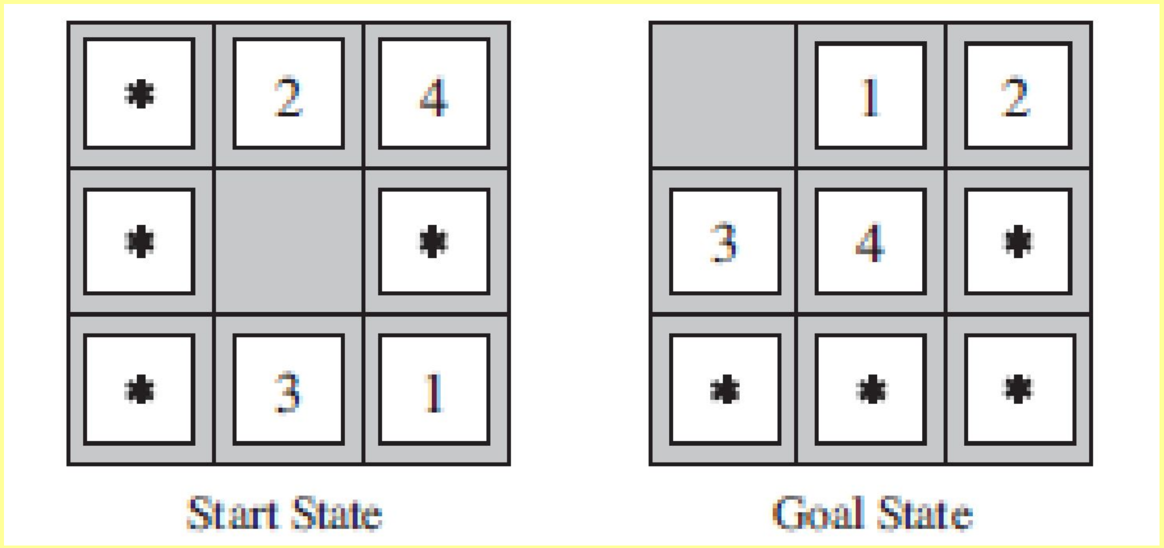
- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Generating admissible heuristics from relaxed problems

- If a collection of admissible heuristics h1 . . .hm is available for a problem and none of them dominates any of the others, which should we choose?
- As it turns out, we need not make a choice.

$$h(n) = \max\{h_1(n), \ldots, h_m(n)\}$$

Generating admissible heuristics from subproblems: Pattern databases



The idea behind **pattern databases** is to store these exact solution costs for every possible subproblem instance—in the example, every possible configuration of the four tiles and the blank.

Learning heuristics from experience

- How could an agent construct to estimate the cost of a solution beginning from the state at node n?
  - to devise relaxed problems
  - to learn from experience
  - Learning algorithm can be used to construct a function h(n) that can (with luck) predict solution costs for other states that arise during search. - neural nets, decision trees, reinforcement learning, inductive learning methods

End of unit 2

Next class – Logical Agents