

DIGITAL DESIGN AND COMPUTER ORGANIZATION

1. Simplify the following Boolean Expressions to minimum literals.

a) $xy + x\bar{y}$
 $\rightarrow x(y + \bar{y})$
 $x(1) [\because y + \bar{y} = 1]$
 x

b) $(\bar{A} \cdot \bar{B})(\bar{A} + \bar{B})$

By applying De-morgan's law

$$(\bar{A} + \bar{B})(\bar{A} \cdot \bar{B})$$

$$(\bar{A} + \bar{B})(A \cdot B)$$

$$A \cdot \bar{A} + A \cdot \bar{B} + \bar{B} \cdot A + \bar{B} \cdot B$$

$$0 + A\bar{B} + \bar{B}A + 0 [\because A \cdot A = 0]$$

$$A\bar{B} + \bar{A}B$$

c) $xyz + \bar{x}y + x\bar{y}\bar{z}$

$$xy(z + \bar{z}) + \bar{x}y$$

$$xy(1) + \bar{x}y [z + \bar{z} = 1]$$

$$y(x + \bar{x}) [x + \bar{x} = 1]$$

$$y$$

d) $xy + x(wz + w\bar{z})$

$$xy + xw(z + \bar{z}) [z + \bar{z} = 1]$$

$$xy + xw$$

$$x(y + w)$$

e) $(\bar{a} + \bar{c})(a + \bar{b} + \bar{c})$

$$a \cdot \bar{a} + \bar{b} \bar{a} + \bar{a} \bar{c} + a \bar{c} + \bar{b} \bar{c} + \bar{c} \cdot \bar{c}$$

$$0 + (a + \bar{a})\bar{c} + \bar{b} \bar{a} + \bar{b} \bar{c} + \bar{c} \cdot \bar{c}$$

$$\bar{c} + \bar{c} + \bar{b} \bar{a} + \bar{b} \bar{c}$$

$$\bar{c} + \bar{b} \bar{c} + \bar{b} \bar{a}$$

$$\bar{c}(b+1) + \bar{b}a$$

$$\bar{c} + \bar{b}a$$

f) $\bar{A}B(\bar{D} + \bar{C}D) + B(A + \bar{A}CD)$
 $\bar{A}B\bar{D} + \bar{A}B\bar{C}D + AB + \bar{A}BCD$

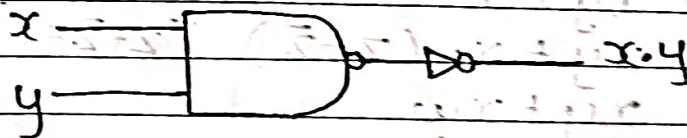
2. What do you mean by Universal gates? Illustrate Universal property of 'NAND' as well as NOR gate by implementing basic gates.

→ A gate is said to be Universal gate because any logic ^{circuit} gate can be implemented with it. We need to show logical operations of AND, OR and complement can be obtained with that gate alone.

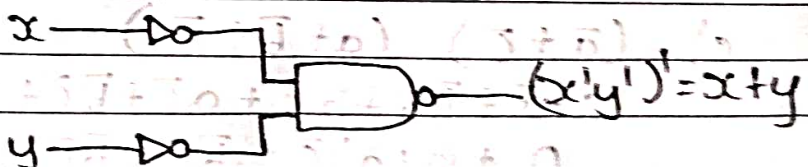
i) NAND as Universal gate

Inverter $x \longrightarrow \text{NAND} \longrightarrow x'$

AND



OR



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ii) NOR as Universal gate.

Inverter $x \rightarrow x'$

OR $x, y \rightarrow x + y$

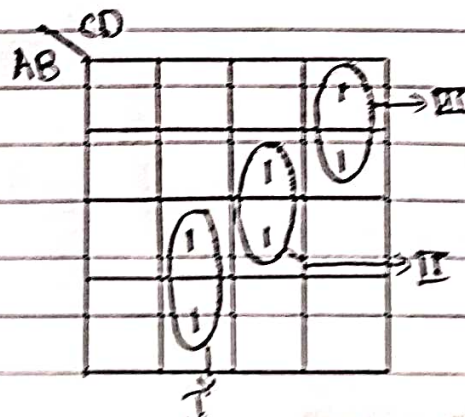
AND $x, y \rightarrow x \cdot y$

3. Define the terms implicant, prime implicant & essential prime implicants.

→ Any minimum OR combination of minterms which makes an SOP function equal to 1, is called implicant. The group must contain either 1, 2, 4, 8, 16... ones.

Prime implicant :- The largest possible group of 1 which can be used to cover given 1 is called prime implicant

Essential prime implicant :- Essential prime implicant involves atleast there is single 1 which cannot be combined in any other way



I → Prime, Implicant, Essential prime implicant

II → Prime implicant

III → P.I, E.P.I

4. Simplify the following using K-Map.

a) $F(x, y, z) = \Sigma(1, 2, 3, 7)$

x \ yz	00	01	11	10
0	0	1	1	1
1			1	

b) $F(x, y, z) = \Sigma(0, 2, 4, 6)$

x \ yz	00	01	11	10
0	1			1
1	1			1

$F = \bar{z}$

Group 1: 0, 2, 4, 6 (minterms where z=0)
Group 2: 1, 3, 5, 7 (minterms where z=1)

Group 1 is a valid group of 4 minterms.
Group 2 is a valid group of 4 minterms.

Group	Minterms	Expression
I	0, 2, 4, 6	\bar{z}
II	1, 3, 5, 7	z
III	0, 1, 4, 5	\bar{x}
IV	2, 3, 6, 7	x

4. Simplify the following using K-Map.

a) $F(x, y, z) = \sum(1, 2, 3, 7)$

$x \backslash yz$	00	01	11	10
0	0	1	1	1
1			1	

b) $F(x, y, z) = \sum(0, 2, 4, 6)$

$x \backslash yz$	00	01	11	10
0	1			1
1	1			1

$F = \bar{z}$

c) $xy + \bar{x}y\bar{z} + \bar{x}y\bar{z}$

$x \backslash yz$	00	01	11	10
0				1
1			1	1

$xyz + \bar{x}y\bar{z} + \bar{x}y\bar{z} + \bar{x}y\bar{z}$

\therefore The simplified version is

$F = y + y\bar{z}$

d) $F(x, y, z) = \bar{x}y + y\bar{z} + \bar{y}\bar{z}$

$x \backslash yz$	00	01	11	10
0	1		1	1
1	1			

$\bar{x}yz + \bar{x}y\bar{z} + \bar{x}y\bar{z} + \bar{x}y\bar{z} + \bar{x}y\bar{z} + \bar{x}y\bar{z}$

$\therefore F = \bar{z} + y$

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c) $F(A, B, C, D) = \Sigma(3, 7, 11, 13, 14, 15)$

CD \ AB	00	01	11	10
00			1	
01			1	
11		1	1	1
10			1	

→ CD

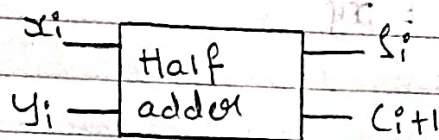
→ CAB

ABD

$$F = ABC + AB'D + CD$$

5. Write truth table of half adder & full adder & explain working of both.

→



Truth table

x_i	y_i	S_i	C_{i+1}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

where, x_i & y_i are the inputs of half adder & S_i is the sum of input bits and C_{i+1} is the carry bit.

By using K-map

y_i	0	1
x_i		
0		1
1	1	

 S_i

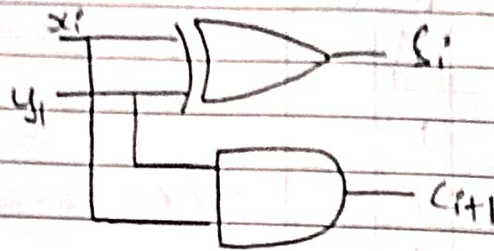
$$F = x_i \bar{y}_i + \bar{x}_i y_i$$

y_i	0	1
x_i		
0		
1		1

 C_{i+1}

$$F = x_i y_i$$

The circuit diagram looks like this

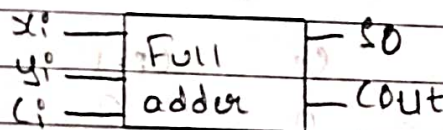


The half adder involves two inputs i.e. addend and augend and produces output S_1 and Carry. The simplified boolean functions can be taken by using k-map i.e.

$$S = xy' + x'y$$

$$C = xy$$

Full adder



Truth table

x_1	y_1	C_1	S_0	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

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By using K-Map

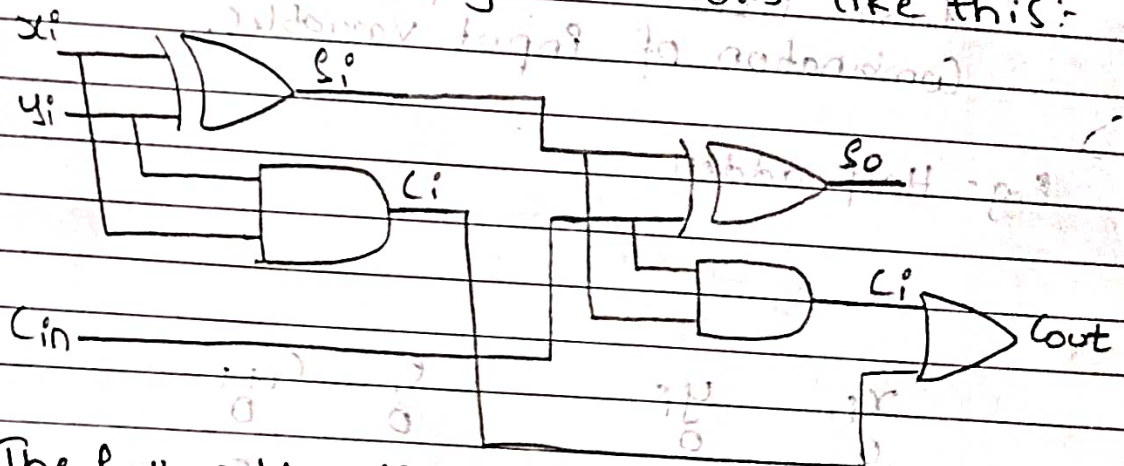
$x_i \backslash y_i z_i$	00	01	11	10
0	1	1	1	1
1	1	1	1	1

$x_i \backslash y_i z_i$	00	01	11	10
0	1	1	1	1
1	1	1	1	1

$$F = x_i \bar{y}_i \bar{z}_i + x_i \bar{y}_i z_i + x_i y_i \bar{z}_i + x_i y_i z_i$$

$$F = x_i c_i + x_i y_i + y_i c_i$$

The Circuit diagram looks like this:



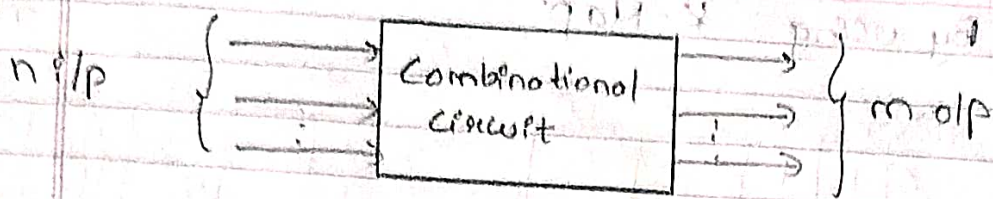
The full adder is a combinational circuit which takes three inputs & gives two outputs i.e. s_o & C_{out} . The simplified boolean function can be taken using K-map i.e.

$$s = x_i \bar{y}_i \bar{c}_i + x_i \bar{y}_i c_i + x_i y_i \bar{c}_i + x_i y_i c_i$$

6. What do you mean by Combinational Circuits? Explain with example.

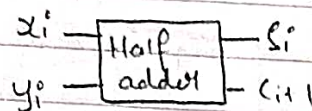
→ A Combinational Circuit consists of logic states whose outputs are determined by the present combination of inputs i.e. no previously stored value or state is taken into consideration here.

Block diagram for Combinational Circuits.



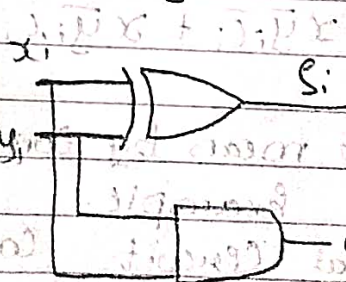
For n , Input Variables, there are 2^n possible combinations of the binary inputs. For each binary input combination there is one possible value for each output. The truth table lists the output values for each combination of input variables.

E.g:- Half adder



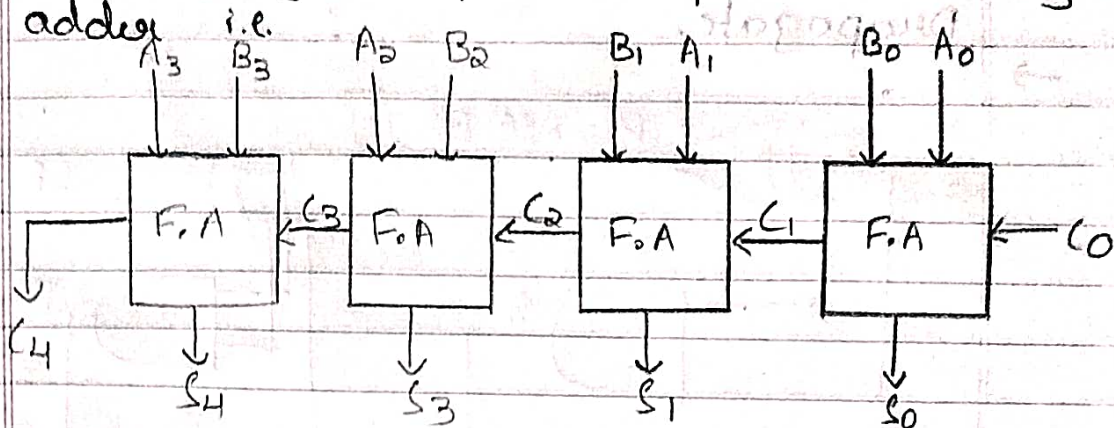
x_i	y_i	S_i	C_{i+1}
0	0	0	0

\therefore The circuit diagram is



7. Explain working of a 4-bit parallel binary adder

→ Considering, a four bit parallel binary adder



where A_0, B_0 are the input to the Full adder & C_0 is the carry bit.

* As shown in fig, firstly the full adder FA1 adds A_0, B_0 along with carry C_0 to generate sum S_0 . The carry C_1 , which is connected to the next adder in chain

* Next, the full adder FA2 uses the carry bit C_1 with the input bits A_1, B_1 to generate sum S_1 & further, carry C_2 is taken to the next adder in chain

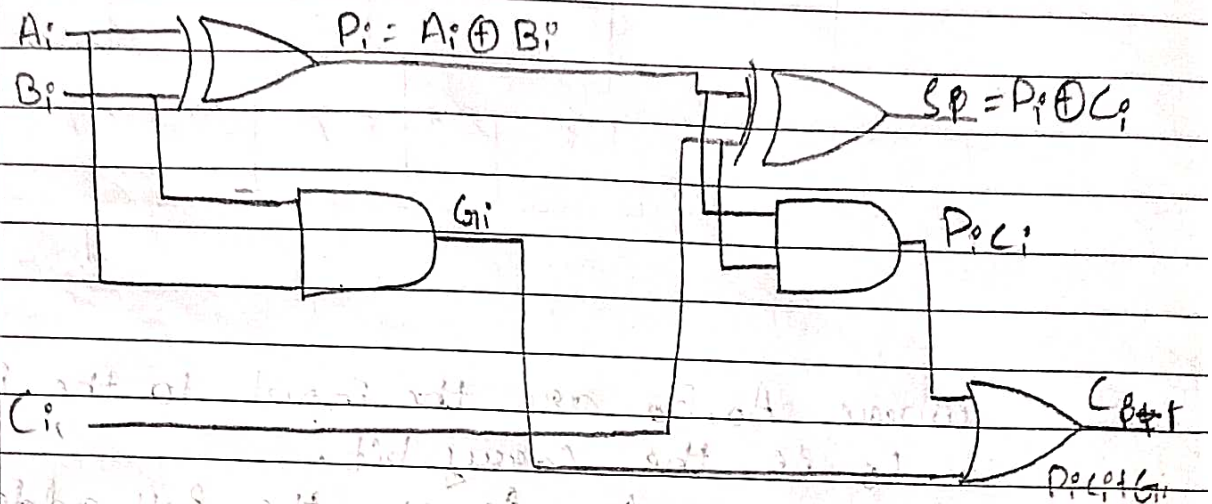
* The process continues till the last full adder FA4 uses the carry bit C_3 to add with its input A_3 & B_3 to generate the last bit of output along last carry bit C_4 .

E.g:

Subscript i:	3	2	1	0	
Input carry	0	1	1	0	C_i
Augend	1	0	1	1	A_i
Addend	0	0	1	1	B_i
Sum	1	1	1	0	S_i
Output carry	0	0	1	1	C_{i+1}

8. With diagram & expression, explain the gate level delays in n-bit adder & how it is minimized using generate & propagate.

→



where, $P_i = \text{Propagate} = A_i \oplus B_i$

$G_i = \text{Generate term} = A_i B_i$

The n-bit adder's output depends on the value of the input carry i.e. the value of P_i at any given stage in the adder is the steady state final value only after the input carry to that stage is propagated.

$$C_{i+1} = P_i C_i + G_i$$

$$C_1 = P_0 C_0 + G_0$$

$$C_2 = P_1 C_1 + G_1$$

$$C_2 = P_1 (P_0 C_0 + G_0) + G_1$$

$$C_2 = P_1 P_0 C_0 + P_1 G_0 + G_1$$

$$C_3 = P_2 C_2 + G_2$$

$$C_3 = G_2 + (P_1 P_0 C_0 + P_1 G_0 + G_1) P_2$$

$$C_3 = G_2 + P_2 G_1 + P_1 P_2 G_0 + P_1 P_2 P_0 C_0$$