1. NUMBERS

IMPORTANT FACTS AND FORMULAE

I..Numeral : In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called ***digits*** to represent any number.

A group of digits, denoting a number is called a ***numeral.***

We represent a number, say 689745132 as shown below :

Ten Crores (10^8)	Crores(10^7)	Ten Lacs (Millions) (10^6)	Lacs(10^5)	Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens(10^1)	Units(10^0)
6	**8**	**9**	**7**	**4**	**5**	**1**	**3**	**2**

We read it as : 'Sixty-eight crores, ninety-seven lacs, forty-five thousand, one hundred and thirty-two'.

II Place Value or Local Value of a Digit in a Numeral :

In the above numeral :

Place value of 2 is (2 x 1) = 2; Place value of 3 is (3 x 10) = 30;

Place value of 1 is (1 x 100) = 100 and so on.

Place value of 6 is $6 \times 10^8 = 600000000$

III.Face Value : The ***face value*** of a digit in a numeral is the value of the digit itself at whatever place it may be. In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

IV.TYPES OF NUMBERS

1.Natural Numbers : Counting numbers 1, 2, 3, 4, 5,..... are called ***natural*** *numbers.*

2.Whole Numbers : All counting numbers together with zero form the set of ***whole numbers.*** Thus,

(i) 0 is the only whole number which is not a natural number.

(ii) Every natural number is a whole number.

3.Integers : All natural numbers, 0 and negatives of counting numbers *i.e.*, {..., -3,-2,-1, 0, 1, 2, 3,.....} together form the set of integers.

(i) **Positive Integers** : {1, 2, 3, 4, ...} is the set of all positive integers.

(ii) **Negative Integers** : {- 1, - 2, - 3,.....} is the set of all negative integers.

(iii) **Non-Positive and Non-Negative Integers** : 0 is neither positive nor negative. So, {0, 1, 2, 3,....} represents the set of non-negative integers, while {0, -1,-2,-3,.....} represents the set of non-positive integers.

4. Even Numbers : A number divisible by 2 is called an even number, e.g., 2, 4, 6, 8, 10, etc.

5. Odd Numbers : A number not divisible by 2 is called an odd number. e.g., 1, 3, 5, 7, 9, 11, etc.

6. Prime Numbers : A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Prime numbers Greater than 100 : Let p be a given number greater than 100. To find out whether it is prime or not, we use the following method :
Find a whole number nearly greater than the square root of p. Let k > *jp. Test whether p is divisible by any prime number less than k. If yes, then p is not prime. Otherwise, p is prime.
e.g,,We have to find whether 191 is a prime number or not. Now, 14 > V191.
Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.
191 is not divisible by any of them. So, 191 is a prime number.

7.Composite Numbers : Numbers greater than 1 which are not prime, are known as composite numbers, e.g., 4, 6, 8, 9, 10, 12.

Note : (i) 1 is neither prime nor composite.
(ii) 2 is the only even number which is prime.
(iii) There are 25 prime numbers between 1 and 100.

8. Co-primes : Two numbers a and b are said to be co-primes, if their H.C.F. is 1. e.g., (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes,

/.TESTS OF DIVISIBILITY

1. Divisibility By 2 : A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.
Ex. 84932 is divisible by 2, while 65935 is not.

2. Divisibility By 3 : A number is divisible by 3, if the sum of its digits is divisible by 3.
Ex.592482 is divisible by 3, since sum of its digits = (5 + 9 + 2 + 4 + 8 + 2) = 30, which is divisible by 3.
But, 864329 is not divisible by 3, since sum of its digits =(8 + 6 + 4 + 3 + 2 + 9) = 32, which is not divisible by 3.

3. Divisibility By 4 : A number is divisible by 4, if the number formed by the last two digits is divisible by 4.
Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.
But, 749282 is not divisible by 4, since the number formed by the last tv/o digits is 82, which is not divisible by 4.

4. Divisibility By 5 : A number is divisible by 5, if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.

5. Divisibility By 6 : A number is divisible by 6, if it is divisible by both 2 and 3. Ex. The number 35256 is clearly divisible by 2.
Sum of its digits = (3 + 5 + 2 + 5 + 6) = 21, which is divisible by 3. Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.

6. Divisibility By 8 : A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.
Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.
But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.

7. Divisibility By 9 : A number is divisible by 9, if the sum of its digits is divisible by 9.
Ex. 60732 is divisible by 9, since sum of digits * (6 + 0 + 7 + 3 + 2) = 18, which is divisible by 9.
But, 68956 is not divisible by 9, since sum of digits = (6 + 8 + 9 + 5 + 6) = 34, which is

not divisible by 9.

8. Divisibility By 10 : A number is divisible by 10, if it ends with 0.

Ex. 96410, 10480 are divisible by 10, while 96375 is not.

9. Divisibility By 11 : A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

Ex. The number 4832718 is divisible by 11, since :

(sum of digits at odd places) - (sum of digits at even places)

$(8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.

10. Divisibility By 12 ; A number is divisible by 12, if it is divisible by both 4 and 3.

Ex. Consider the number 34632.

(i) The number formed by last two digits is 32, which is divisible by 4,

(ii) Sum of digits = $(3 + 4 + 6 + 3 + 2) = 18$, which is divisible by 3. Thus, 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. Divisibility By 14 : A number is divisible by 14, if it is divisible by 2 as well as 7.

12. Divisibility By 15 : A number is divisible by 15, if it is divisible by both 3 and 5.

13. Divisibility By 16 : A number is divisible by 16, if the number formed by the last4 digits is divisible by 16.

Ex.7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. Divisibility By 24 : A given number is divisible by 24, if it is divisible by both3 and 8.

15. Divisibility By 40 : A given number is divisible by 40, if it is divisible by both 5 and 8.

16. Divisibility By 80 : A given number is divisible by 80, if it is divisible by both 5 and 16.

Note : If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.

If p arid q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

Ex. 36 is divisible by both 4 and 6, but it is not divisible by $(4x6) = 24$, since 4 and 6 are not co-primes.

VI MULTIPLICATION BY SHORT CUT METHODS

1. Multiplication By Distributive Law :

(i) $a \times (b + c) = a \times b + a \times c$ (ii) $a x(b-c) = a \times b - a \times c$.

Ex. (i) $567958 \times 99999 = 567958 \times (100000 - 1)$

$= 567958 \times 100000 - 567958 \times 1 = (56795800000 - 567958) = 56795232042$. (ii) $978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000$.

2. Multiplication of a Number By 5^n : Put n zeros to the right of the multiplicand and divide the number so formed by 2^n

Ex. $975436 \times 625 = 975436 \times 5^4 = \frac{9754360000}{16} = 609647600$

VII. BASIC FORMULAE

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$
4. $(a + b)^2 + (a - b)^2 = 2 (a^2 + b^2)$
5. $(a^2 - b^2) = (a + b) (a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$
7. $(a^3 + b^3) = (a + b) (a^2 - ab + b^2)$
8. $(a^3 - b^3) = (a - b) (a^2 + ab + b^2)$
9. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

VIII. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number, then :

Dividend = (Divisor x Quotient) + Remainder

IX. {(i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.

(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

X. PROGRESSION

A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a progression.

1. Arithmetic Progression (A.P.) : If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the *common difference* of the A.P.

An A.P. with first term a and common difference d is given by a, (a + d), (a + 2d),(a + 3d),.....

The **nth term of this A.P. is given by $T_n = a (n - 1) d$.**

The sum of n terms of this A.P.

$S_n = n/2 [2a + (n - 1) d] = n/2$ (first term + last term).

SOME IMPORTANT RESULTS :

(i) $(1 + 2 + 3 + \dots + n) = n(n+1)/2$

(ii) $(1^2 + 2^2 + 3^2 + \dots + n^2) = n (n+1)(2n+1)/6$

(iii) $(1^3 + 2^3 + 3^3 + \dots + n^3) = n^2(n+1)^2$

2. Geometrical Progression (G.P.) : A progression of numbers in which every term bears a constant ratio with its preceding term, is called a geometrical progression.

The constant ratio is called the common ratio of the G.P. A G.P. with first term a and common ratio r is :

$a, ar, ar^2,$

In this G.P. $T_n = ar^{n-1}$

sum of the n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$