IMPORTANT FACTS AND FORMULAE

Factorial Notation: Let n be a positive integer. Then, factorial n, denoted by n! is defined as:

$$n! = n(n-1)(n-2)........3.2.1.$$

Examples: (i) $5! = (5\times 4\times 3\times 2\times 1) = 120$; (ii) $4! = (4\times3\times2\times1) = 24$ etc.
We define, $0! = 1$.

Permutations: The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are: **(ab, ba, ac, bc, cb).**

Ex. 2. All permutations made with the letters a,b,c, taking all at a time are:
(abc, acb, bca, cab, cba).

Number of Permutations: Number of all permutations of n things, taken r at a time, given by:

$$^nP_r = n(n-1)(n-2).....(n-r+1) = n!/(n-r)!$$

Examples: (i) $^6p_2 = (6\times5) = 30$. (ii) $^7p_3 = (7\times6\times5) = 210$.

Cor. Number of all permutations of n things, taken all at a time = n!

An Important Result: If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind, such that $(p_1+p_2+.......p_r) = n$.

Then, number of permutations of these n objects is:

$$n! \;/\; (p_1!).p_2!)......(p_r!)$$

Combinations: Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.
Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c, taking two at a time are **ab, bc, ca.**

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is **abc.**

Ex. 4. Various groups of 2 out of four presons A, B, C, D are:

AB, AC, AD, BC, BD, CD.

Ex. 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of Combinations: The number of all combination of n things, taken r at a time is:

$$^nC_r = n! / (r!)(n-r)! = n(n-1)(n-2).....\text{to } r \text{ factors} / r!$$

Note that: $^nc_r = 1$ and $^nc_0 = 1$.

An Important Result: $^nc_r = {}^nc_{(n-r)}$.

Example: (i) $^{11}c_4 = (11x10x9x8)/(4x3x2x1) = 330$.

(ii) $^{16}c_{13} = {}^{16}c_{(16-13)} = 16x15x14/3! = 16x15x14/3x2x1 = 560$.