Matrix Problems
Circles

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I. Problem Statement

The two circles $x^2+y^2=ax$ and $x^2+y^2=c^2(c>0)$ touch each other if:

1) $2|a|=c$
2) $|a|=c$
3) $a=2c$
4) $|a|=2c$

II. Construction

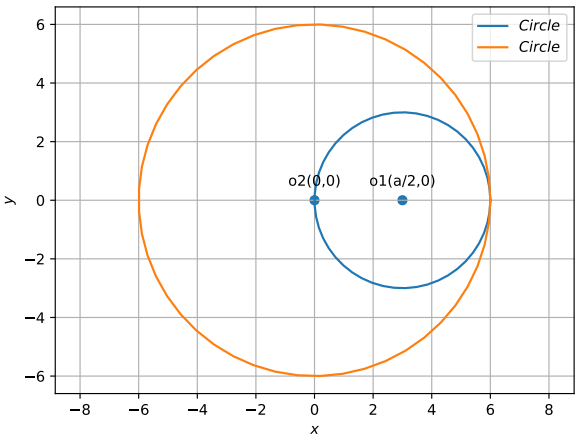

Fig. 1: Figure of construction

III. Solution

Circle equation 1 : $x^2+y^2=ax$
Circle equation 2 : $x^2+y^2=c^2$
The standard equation of the conics is given as :

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{1}$$

The given circle 1 can be expressed as conics with parameters

$$\mathbf{V_1}=\mathbf{I},\mathbf{u_1}=\begin{pmatrix}\frac{a}{2}\\0\end{pmatrix},f_1=0 \tag{2}$$

Radius and Centre are

$$r_1=\sqrt{\mathbf{u}^\top\mathbf{u}-f},\mathbf{a}=-u \tag{3}$$

$$r_1=\sqrt{\frac{a}{2}*\frac{a}{2}}, \tag{4}$$

$$r_1=\pm\frac{a}{2} \tag{5}$$

The given circle 2 can be expressed as conics with parameters

$$\mathbf{V_2}=\mathbf{I},\mathbf{u_2}=\begin{pmatrix}0\\0\end{pmatrix},f_2=-c^2 \tag{6}$$

Radius and Centre are

$$r_2=\sqrt{\mathbf{u}^\top\mathbf{u}-f} \tag{7}$$

$$r_2=\sqrt{0-(-c^2)} \tag{8}$$

$$r_2=\pm c \tag{9}$$

Symbol	Value	Description
a	6	Input Circle Radius
o1	$\begin{pmatrix}\frac{a}{2}\\0\end{pmatrix}$	Centre of circle 1
o2	$\begin{pmatrix}0\\0\end{pmatrix}$	Centre of Circle2

Distance between centres u_1 and u_2 is given by

$$\|\mathbf{u_1}-\mathbf{u_2}\|=\pm\frac{a}{2} \tag{10}$$

The two circles will touch each other iff.

$$r_1\pm r_2=\|\mathbf{u_1}-\mathbf{u_2}\| \tag{11}$$

$$r_1\pm r_2=\pm\frac{a}{2} \tag{12}$$

$$\pm\frac{a}{2}+c=\pm\frac{a}{2} \tag{13}$$

$$c=|a| \tag{14}$$

Hence, option 2 is correct.

Get Python Code for image from

https://github.com/ManojChavva/FWC/blob/main/Matrix/line/code-py/triangle.py

Get LaTex code from

https://github.com/ManojChavva/FWC/blob/main/Matrix/line/line.tex