Matrix Problems
Straight Lines

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I. PROBLEM STATEMENT

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.
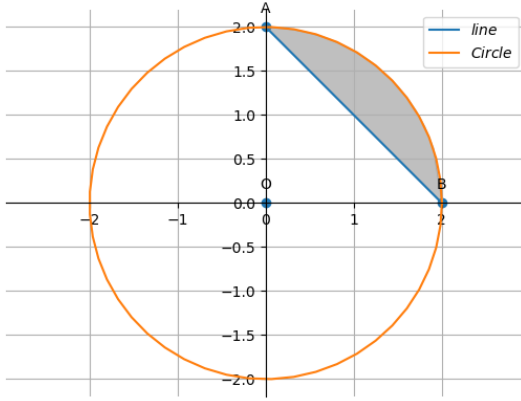1) $2(\pi - 2)$
2) $\pi - 2$
3) $2\pi - 1$
4) $2(\pi + 2)$

Points	coordinates
A	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
B	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Fig. 1: Smaller region between Circle and Line

Given:
Equation of circle is

$$x^2 + y^2 = 4 \tag{1}$$

Equation of line is

$$x + y = 2 \tag{2}$$

To Find:
To find the intersection points and area of shaded region shown in figure

II. CONSTRUCTION

III. SOLUTION

The given circle can be expressed as conics with parameters,

$$\mathbf{V} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \tag{3}$$

$$\mathbf{u} = 0 \tag{4}$$

$$f = -16 \tag{5}$$

The given line equation can be written as

$$\mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + k \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \tag{6}$$

The points of intersection of the line,

$$L: \quad \mathbf{x} = \mathbf{q} + \kappa \mathbf{m} \quad \kappa \in \mathbb{R} \tag{7}$$

with the conic section,

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2 \mathbf{u}^\top \mathbf{x} + f = 0 \tag{8}$$

are given by

$$\mathbf{x}_i = \mathbf{q} + \kappa_i \mathbf{m} \tag{9}$$

where,

$$\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T \left(\mathbf{V} \mathbf{q} + \mathbf{u} \right) \pm \right.$$

$$\left. \sqrt{\left[\mathbf{m}^T \left(\mathbf{V} \mathbf{q} + \mathbf{u} \right) \right]^2 - \left(\mathbf{q}^T \mathbf{V} \mathbf{q} + 2 \mathbf{u}^T \mathbf{q} + f \right) \left(\mathbf{m}^T \mathbf{V} \mathbf{m} \right)} \right) \tag{10}$$

On substituting

$$\mathbf{q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad (11)$$

$$\mathbf{m} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \qquad (12)$$

With the given as in eq(3),(4),(5),
The value of κ ,

$$\kappa = 0, -4 \qquad (13)$$

by substituting eq(13) in eq(6) we get the points of intersection of line with ellipse

$$\mathbf{A} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \qquad (14)$$

$$\mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad (15)$$

From the figure
Total area of portion is given by,
Total Area=(area of circle in first quadrant)-(area of a triangle **AOB**)

Area of triangle

$$\implies A_1 = \int_0^2 (2 - x)\, dx \qquad (16)$$

by solving the above equation we get area of triangle as 2 units

Area of circle

$$\implies A_2 = \int_0^2 \sqrt{4 - x^2}\, dx \qquad (17)$$

by solving the above equation we get area of circle π
the total area is $\implies \mathbf{A} = \pi - 2$

Get Python Code for image from

https://github.com/ManojChavva/FWC/blob/main/Matrix/line/code-py/triangle.py

Get LaTex code from

https://github.com/ManojChavva/FWC/blob/main/Matrix/line/line.tex