Optimisation

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I. Problem Statement

A wire of length 50 m is cut into two pieces. One piece of the wire is bent in the shape of a square and the other in the shape of a circle. What should be the length of each piece so that the combined area of the two is minimum

To Find:
The value of the length of each piece so that the combined area of the two is minimum from the two figures that are square and circle.
Given:
Length of the wire is 50m

II. solution

$$\text{length of the square is} \quad \mathbf{x} \quad \text{m.} \tag{1}$$

Then length of the other piece for the shape of the circle is

$$(50 - x)\text{m} \tag{2}$$

Perimeter of the square with side a is given by:

$$\text{Perimeter of the square} = 4a \tag{3}$$

Now, we know from the above condition that the total length of all four sides of the square is x.

Then, by using equation 3 we get:

$$x = \frac{a}{4} \tag{4}$$

Similarly, we know the formula for the circumference of the circle with radius r is given by:

$$\text{Circumference of a circle}= 2\pi r \tag{5}$$

Now, we know from the above condition that the total length of all circles is (50-x). Then, by using equation 5 we get:

$$r = \frac{50 - x}{2\pi} \tag{6}$$

The standard equation of the line in conics is given as :

$$n^\top \mathbf{x} = c \tag{7}$$

$$\begin{pmatrix} 1 & 2\pi \end{pmatrix} \mathbf{x} = 50 \tag{8}$$

$$\mathbf{x} = \begin{pmatrix} x \\ r \end{pmatrix}, \tag{9}$$

Now by using the formula for the area of the circle and square is:

$$\text{Area of square}= a^2 \tag{10}$$

$$\text{Area of the circle}= \pi r^2 \tag{11}$$

Now, the combined area(A) after substituting the value of a as x^4 and r is

$$A = a^2 + \pi r^2 \tag{12}$$

$$A = \frac{x^2}{16} + \frac{(50 - x)^2}{4\pi} \tag{13}$$

The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{14}$$

$$\mathbf{V} = \begin{bmatrix} 4\pi + 16 & 0 \\ 0 & 0 \end{bmatrix} \tag{15}$$

$$u^\top = \begin{bmatrix} 800 & 0 \end{bmatrix} \tag{16}$$

$$f = 16(50^2) \tag{17}$$

The minimum value is caluculated by using gradient descent method.

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{18}$$

$$\implies x_{n+1} = x_n - \alpha \left(\frac{x}{8} - \frac{50 - x}{6.28} \right) \tag{19}$$

where

1) α = 0.001
2) x_{n+1} is current value
3) x_n is previous value
4) precession = 0.00000001
5) maximum iterations = 100000000

The minimum values obtained from the python code

The given function has minimum value at 28.011 i.e

$$\frac{200}{4+\pi} \tag{20}$$

Then, the length of the circle wire is 50-x and substitute the x value in it, we get:

$$\frac{50\pi}{4+\pi} \tag{21}$$

Hence, the length of square is $\frac{200}{4+\pi}$ m and circle wire is $\frac{50}{4+\pi}m$ to get the minimum area.