Matrix Problems
Straight Lines

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I. Problem Statement

The base of an equilateral triangle with side 2a lies along the y-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

II. Solution

Given ABC is an equilateral triangle i.e

$$AB = BC = CA \tag{1}$$

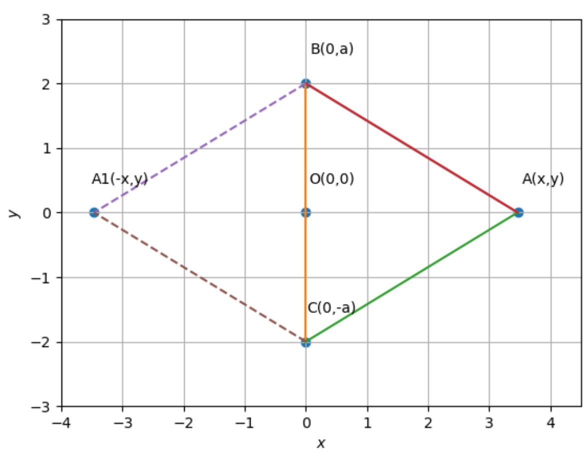

Fig. 1: Equilateral Triangle ABC

Since base with 2a is lies on the y-axis with the mid-point of the base is at origin. The vertices of the two points on y-axis will be

$$\mathbf{B} = \begin{pmatrix} 0 \\ a \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ -a \end{pmatrix} \tag{2}$$

The distance between the two points B and A is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 0 - x \\ a - y \end{pmatrix} \tag{3}$$

Using the definition of the norm,

$$\|\mathbf{B} - \mathbf{A}\| = \left\| \begin{pmatrix} -x \\ a - y \end{pmatrix} \right\| \tag{4}$$

Since, the side of an equilateral triangle is 2a

$$2a = \sqrt{\begin{pmatrix} -x & a - y \end{pmatrix} \begin{pmatrix} -x \\ a - y \end{pmatrix}} \tag{5}$$

$$2a = \sqrt{(x)^2 + (a - y)^2} \tag{6}$$

$$3a^2 = x^2 + y^2 - 2ay \tag{7}$$

Similarly, The distance between the two points C and A is

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 - x \\ -a - y \end{pmatrix} \tag{8}$$

Using the definition of the norm,

$$\|\mathbf{C} - \mathbf{A}\| = \left\| \begin{pmatrix} -x \\ -a - y \end{pmatrix} \right\| \tag{9}$$

Since, the side of an equilateral triangle is 2a

$$2a = \sqrt{\begin{pmatrix} -x & -a - y \end{pmatrix} \begin{pmatrix} -x \\ -a - y \end{pmatrix}} \tag{10}$$

$$2a = \sqrt{(x)^2 + (a + y)^2} \tag{11}$$

$$3a^2 = x^2 + y^2 + 2ay \tag{12}$$

$$\mathbf{AX} = \mathbf{B} \tag{13}$$

Using equation (7) and (12),

$$\begin{pmatrix} 1 & 2a & 1 \\ 1 & -2a & 1 \end{pmatrix} \begin{pmatrix} x^2 \\ y \\ y^2 \end{pmatrix} = \begin{pmatrix} 3a^2 \\ 3a^2 \end{pmatrix} \tag{14}$$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{ccc|c} 1 & 2a & 1 & 3a^2 \\ 1 & -2a & 1 & 3a^2 \end{array}\right) \tag{15}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 2a & 1 & 3a^2 \\ 0 & -4a & 0 & 0 \end{array}\right)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{-4a}} \left(\begin{array}{ccc|c} 1 & 2a & 1 & 3a^2 \\ 0 & 1 & 0 & 0 \end{array}\right)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 2aR_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3a^2 \\ 0 & 1 & 0 & 0 \end{array}\right)$$

$$\implies X = \begin{pmatrix} 3a^2 \\ 0 \end{pmatrix} \tag{16}$$

Using equation (16) we get ,

$$y = 0 \tag{17}$$

$$x^2 = 3a^2 \tag{18}$$

$$x = \pm\sqrt{3}a \tag{19}$$

Hence,the coordinates of the vertices of triangle are

$$\mathbf{A} = (\pm\sqrt{3}a, 0)$$

$$\mathbf{B} = (0, a)$$

$$\mathbf{C} = (0, -a)$$

III. Construction

B and C are the inputs.

Symbol	Value	Description
B	(0, 2)	Vertex B
C	(0, -2)	Vertex C
A	(x,y)	Vertex A
A1	(x1, y1)	Vertex A1

Get Python Code for image from

https://github.com/ManojChavva/FWC/blob/main/Matrix/line/code-py/triangle.py

Get LaTex code from

https://github.com/ManojChavva/FWC/blob/main/Matrix/line/line.tex