

Optimisation

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I. PROBLEM STATEMENT

A wire of length 50 m is cut into two pieces. One piece of the wire is bent in the shape of a square and the other in the shape of a circle. What should be the length of each piece so that the combined area of the two is minimum

To Find:

The value of the length of each piece so that the combined area of the two is minimum from the two figures that are square and circle.

Given:

Length of the wire is 50m

II. SOLUTION

$$\text{length of the square is } x \text{ m.} \quad (1)$$

Then length of the other piece for the shape of the circle is

$$(50 - x)\text{m} \quad (2)$$

Perimeter of the square with side a is given by:

$$\text{Perimeter of the square} = 4a \quad (3)$$

Now, we know from the above condition that the total length of all four sides of the square is x.

Then, by using equation 3 we get:

$$x = \frac{a}{4} \quad (4)$$

Similarly, we know the formula for the circumference of the circle with radius r is given by:

$$\text{Circumference of a circle} = 2\pi r \quad (5)$$

Now, we know from the above condition that the total length of all circles is (50-x). Then, by using equation 5 we get:

$$r = \frac{50 - x}{2\pi} \quad (6)$$

Now by using the formula for the area of the circle and square is:

$$\text{Area of square} = a^2 \quad (7)$$

$$\text{Area of the circle} = \pi r^2 \quad (8)$$

Now, the combined area(A) after substituting the value of a as x^4 and r is

$$A = a^2 + \pi r^2 \quad (9)$$

$$A = \frac{x^2}{16} + \frac{(50 - x)^2}{4\pi} \quad (10)$$

The minimum value is calculated by using gradient descent method.

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \quad (11)$$

$$\Rightarrow x_{n+1} = x_n - \alpha \left(\frac{x}{8} - \frac{50 - x}{6.28} \right) \quad (12)$$

where

- 1) $\alpha = 0.001$
- 2) x_{n+1} is current value
- 3) x_n is previous value
- 4) precession = 0.00000001
- 5) maximum iterations = 100000000

The minimum values obtained from the python code

The given function has minimum value at 28.011 i.e

$$\frac{200}{4 + \pi} \quad (13)$$

Then, the length of the circle wire is 50-x and substitute the x value in it, we get:

$$\frac{50\pi}{4 + \pi} \quad (14)$$

Hence, the length of square is $\frac{200}{4+\pi}$ m and circle wire is $\frac{50\pi}{4+\pi}$ m to get the minimum area.