Matrix Problems
Straight Lines

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Get Python code for the figure from

```
https://github.com/SurabhiSeetha/Fwciith2022/tree/main/Assignment%201/codes/src
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Get LaTex code from

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https://github.com/SurabhiSeetha/Fwciith2022/tree/main/avr%20gcc
```

Symbol	Value	Description
B	(0, 2)	Vertex B
C	(0, -2)	Vertex C
A	(x,y)	Vertex A
A1	(x1, y1)	Vertex A1

I. Problem Statement

The base of an equilateral triangle with side 2a lies along the y-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

II. Solution

Given ABC is an equilateral triangle i.e

$$AB = BC = CA \tag{1}$$

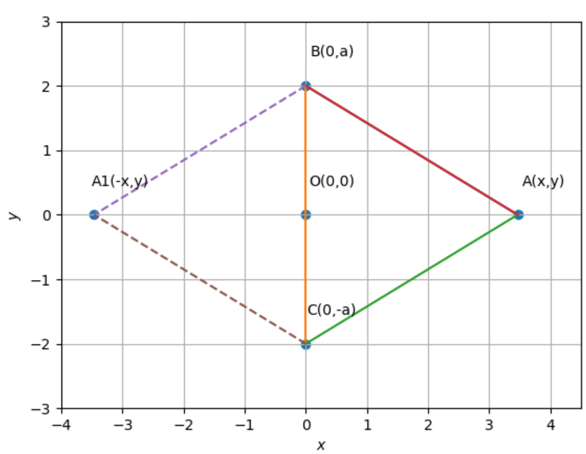

Fig. 1: Equilateral Triangle ABC

III. Construction

B and C are the inputs.
Since base with 2a is lies on the y-axis with the mid-point of the base is at origin. The vertices of the two points on y-axis will be

$$\mathbf{B} = \begin{pmatrix} 0 \\ a \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ -a \end{pmatrix} \tag{2}$$

The distance between the two points B and A is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 0 - x \\ a - y \end{pmatrix} \tag{3}$$

Using the definition of the norm,

$$\|\mathbf{B} - \mathbf{A}\| = \left\| \begin{pmatrix} -x \\ a - y \end{pmatrix} \right\| \tag{4}$$

Since, the side of an equilateral triangle is 2a

$$2a = \sqrt{\begin{pmatrix} -x & a - y \end{pmatrix} \begin{pmatrix} -x \\ a - y \end{pmatrix}} \tag{5}$$

$$2a = \sqrt{(x)^2 + (a - y)^2} \tag{6}$$

Squaring on both sides

$$4a^2 = (x)^2 + (a - y)^2 \tag{7}$$

$$4a^2 = x^2 + a^2 + y^2 - 2ay \tag{8}$$

$$3a^2 = x^2 + y^2 - 2ay \tag{9}$$

Similarly, The distance between the two points C and A is

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 - x \\ -a - y \end{pmatrix} \tag{10}$$

Using the definition of the norm,

$$\|\mathbf{C} - \mathbf{A}\| = \left\| \begin{pmatrix} -x \\ -a - y \end{pmatrix} \right\| \tag{11}$$

Since, the side of an equilateral triangle is 2a

$$2a = \sqrt{\begin{pmatrix} -x & -a-y \end{pmatrix} \begin{pmatrix} -x \\ -a-y \end{pmatrix}} \tag{12}$$

$$2a = \sqrt{(x)^2 + (a+y)^2} \tag{13}$$

Squaring on both sides

$$4a^2 = (x)^2 + (a+y)^2 \tag{14}$$

$$4a^2 = x^2 + a^2 + y^2 + 2ay \tag{15}$$

$$3a^2 = x^2 + y^2 + 2ay \tag{16}$$

Solving equation (9) and (16), we get

$$x = \pm\sqrt{3}a$$

$$y = 0 \tag{17}$$

Hence,the coordinates of the vertices of triangle are
A($\sqrt{3}a, 0), B(0, a) and C(0, a)$
or
$A(\sqrt{3}a, 0), B(0, a) and C(0, a)$.