Matrix Problems
Conics

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I. Problem Statement

Smaller area enclosed by the circle $x^2+y^2=4$ and the line $x+y=2$.

1) $2(\pi-2)$
2) $\pi-2$
3) $2\pi-1$
4) $2(\pi+2)$

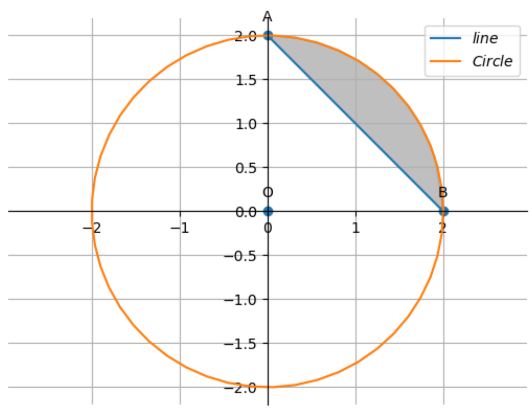

Fig. 1: Smaller region between Circle and Line

Given:
Equation of circle is

$$x^2+y^2=4 \tag{1}$$

Equation of line is

$$x+y=2 \tag{2}$$

To Find:
To find the intersection points and area of shaded region shown in figure

II. Construction

Points	coordinates
A	$\begin{pmatrix}0\\2\end{pmatrix}$
B	$\begin{pmatrix}2\\0\end{pmatrix}$

III. Solution

The given circle can be expressed as conics with parameters,

$$\mathbf{V}=\begin{pmatrix}4&0\\0&4\end{pmatrix} \tag{3}$$

$$\mathbf{u}=0 \tag{4}$$

$$f=-16 \tag{5}$$

The given line equation can be written as

$$\mathbf{x}=\begin{pmatrix}2\\0\end{pmatrix}+k\begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\end{pmatrix} \tag{6}$$

The points of intersection of the line,

$$L: \quad \mathbf{x}=\mathbf{q}+\kappa\mathbf{m} \quad \kappa\in\mathbb{R} \tag{7}$$

with the conic section,

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{8}$$

are given by

$$\mathbf{x}_i=\mathbf{q}+\kappa_i\mathbf{m} \tag{9}$$

where,

$$\kappa_i=\frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\Big(-\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\right]^2-\left(\mathbf{q}^T\mathbf{V}\mathbf{q}+2\mathbf{u}^T\mathbf{q}+f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\Big) \tag{10}$$

On substituting

$$\mathbf{q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{11}$$

$$\mathbf{m} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \tag{12}$$

With the given as in eq(3),(4),(5),
The value of κ ,

$$\kappa = 0, -4 \tag{13}$$

by substituting eq(13) in eq(6) we get the points of intersection of line with ellipse

$$\mathbf{A} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{14}$$

$$\mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{15}$$

From the figure
Total area of portion is given by,
Total Area=(area of circle in first quadrant)-(area of a triangle **AOB**)

Area of triangle

$$\implies A_1 = \int_0^2 (2-x)\,dx \tag{16}$$

by solving the above equation we get area of triangle as 2 units

Area of circle

$$\implies A_2 = \int_0^2 \sqrt{4-x^2}\,dx \tag{17}$$

by solving the above equation we get area of circle π
the total area is $\implies \mathbf{A} = \pi - 2$

Get Python Code for image from

https://github.com/ManojChavva/FWC/blob/main/Matrix/conics/code/conic.py

Get LaTex code from

https://github.com/ManojChavva/FWC/blob/main/Matrix/conics/conic.tex