Optimization

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I. Problem Statement

A wire of length 50 m is cut into two pieces. One piece of the wire is bent in the shape of a square and the other in the shape of a circle. What should be the length of each piece so that the combined area of the two is minimum

To Find:
The value of the length of each piece so that the combined area of the two is minimum from the two figures that are square and circle.
Given:
Length of the wire is 50m

II. solution

$$\text{length of the square is} \quad \mathbf{a} \quad \text{m.} \tag{1}$$

Then length of the other piece for the shape of the circle is

$$(50 - a)\text{m} \tag{2}$$

Perimeter of the square with side a is given by:

$$\text{Perimeter of the square} = 4a \tag{3}$$

Similarly, we know the formula for the circumference of the circle with radius r is given by:

$$\text{Circumference of a circle} = 2\pi r \tag{4}$$

So, the total length is

$$4x + 2\pi r = 50 \tag{5}$$

The standard equation of the line in conics is given as :

$$n^\top \mathbf{x} = c \tag{6}$$

$$\begin{pmatrix} 1 & 2\pi \end{pmatrix} \mathbf{x} = 25 \tag{7}$$

$$\mathbf{x} = \begin{pmatrix} x \\ r \end{pmatrix} \tag{8}$$

Now by using the formula for the area of the circle and square is:

$$\text{Area of square} = a^2 \tag{9}$$

$$\text{Area of the circle} = \pi r^2 \tag{10}$$

Now, the combined area(A)

$$A = a^2 + \pi r^2 \tag{11}$$

The area of two figures is grepresented as :

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{12}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & \pi \end{pmatrix} \tag{13}$$

$$u^\top = \begin{pmatrix} 0 & 0 \end{pmatrix} \tag{14}$$

$$f = 0 \tag{15}$$

The minimum area is

$$\min_x \mathbf{x}^\top \mathbf{V} \mathbf{x} \tag{16}$$

Such that,

$$\begin{pmatrix} 1 & 2\pi \end{pmatrix} \mathbf{x} - 25 == 0 \tag{17}$$

Solving using cvxpy, we get

$$\min_x \mathbf{x}^\top \mathbf{V} \mathbf{x} = 87.53 \quad m^2 \tag{18}$$

The length of each piece is
Square = 4a = 28 m
circle = 2 π r = 21.98 m