

Unit Vector Perpendicular

1 12th Maths - Chapter 10

This is Problem-2 from Exercise 10.4

1. Find a unit vector perpendicular to each of a vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

2 Solution

A unit vector perpendicular

$$(\mathbf{a} + \mathbf{b})^\top \mathbf{x} = 0 \quad (1)$$

$$(\mathbf{a} - \mathbf{b})^\top \mathbf{x} = 0 \quad (2)$$

$$\begin{pmatrix} (\mathbf{a} + \mathbf{b})^\top \\ (\mathbf{a} - \mathbf{b})^\top \end{pmatrix} \mathbf{x} = 0 \quad (3)$$

$$(\mathbf{a} + \mathbf{b} \quad \mathbf{a} - \mathbf{b})^\top \mathbf{x} = 0 \quad (4)$$

Here.

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \quad (5)$$

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \quad (6)$$

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \quad (7)$$

Now using the formula substituing (7) and (8) in (4) and equating,

$$(\mathbf{a} + \mathbf{b} \quad \mathbf{a} - \mathbf{b})^\top \mathbf{x} = 0 \quad (8)$$

$$\begin{pmatrix} 4 & 2 \\ 4 & 0 \\ 0 & 4 \end{pmatrix}^\top \mathbf{x} = 0 \quad (9)$$

$$\begin{pmatrix} 4 & 4 & 0 \\ 2 & 0 & 4 \end{pmatrix} \xleftrightarrow{R_1 = \frac{R_1}{4}} \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 4 \end{pmatrix} \mathbf{x} = 0 \quad (10)$$

$$\xleftrightarrow{R_2 = \frac{R_2}{2}} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix} \mathbf{x} = 0 \quad (11)$$

$$\xleftrightarrow{R_2 = R_1 - R_2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \end{pmatrix} \mathbf{x} = 0 \quad (12)$$

$$\xleftrightarrow{R_2 = \frac{R_2}{-1}} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -2 \end{pmatrix} \mathbf{x} = 0 \quad (13)$$

$$\xleftrightarrow{R_1 = R_1 - R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \end{pmatrix} \mathbf{x} = 0 \quad (14)$$

From (15), we get two equations which is $x_1 + 2x_3 = 0$ and $x_2 - 2x_3 = 0$

$$x_1 + 2x_3 = 0 \quad (15)$$

$$x_2 - 2x_3 = 0 \quad (16)$$

$$x_1 = \begin{pmatrix} -2x_3 \\ 2x_3 \\ x_3 \end{pmatrix} \quad (17)$$

$$= x_3 \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad (18)$$

$$\vec{\mathbf{c}} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad (19)$$

The unit vector

$$\vec{c} = \frac{1}{\|\mathbf{c}\|} \mathbf{c} \quad (20)$$

$$\|\mathbf{c}\| = \sqrt{c^\top c} \quad (21)$$

$$= \sqrt{(-2 \ 2 \ 1) \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}} \quad (22)$$

$$= \sqrt{4 + 4 + 1} \quad (23)$$

$$= 3 \quad (24)$$

$$\vec{c} = \frac{1}{\|\mathbf{c}\|} \mathbf{c} \quad (25)$$

$$= \frac{1}{3} \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad (26)$$

$$\vec{c} = \begin{pmatrix} \frac{-2}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \quad (27)$$