Properties of vectors

1 12^{th} Maths - Exercise 10.4.2

1. Find a unit vector perpendicular to each of a vector $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$ where $\overrightarrow{a} = 3\hat{i}+2\hat{j}+2\hat{k}$ and $\overrightarrow{b} = \hat{i}+2\hat{j}-2\hat{k}$

2 Solution

Now,

$$\text{Let } \mathbf{A} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \tag{1}$$

The cross product or vector product of $\mathbf{A}, \mathbf{B}$ is defined as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \\ \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix} \tag{2}$$

Hence

$$\begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 2 & -2 \end{vmatrix} = \left(-4-4\right) = -8 \tag{3}$$

$$\begin{vmatrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix} = \left(2-(-6)\right) = 8 \tag{4}$$

$$\begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = \left(6-2\right) = 4 \tag{5}$$

which can be represented in matrix form as perpendicular to vector represented by

$$\hat{\mathbf{c}} = \frac{\overrightarrow{\mathbf{c}}}{|\overrightarrow{\mathbf{c}}|} \tag{6}$$

$$\overrightarrow{\mathbf{c}} = \mathbf{A} \times \mathbf{B} = \begin{pmatrix} 8 \\ -8 \\ -4 \end{pmatrix} \tag{7}$$

Hence

$$|\overrightarrow{\mathbf{c}}| = \sqrt{8^2 + (-8^2) + (-4^2)} = 12 \tag{8}$$

Here substiuing the values in (2) so we get

$$\hat{\mathbf{c}} = \begin{pmatrix} 1 \\ 12 \end{pmatrix} \begin{pmatrix} 8 \\ -8 \\ -4 \end{pmatrix} \tag{9}$$

$$\hat{\mathbf{c}} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \tag{10}$$