Properties of Collinear

1 10^{th} Maths - Chapter 7

This is Problem-2 from Exercise 7.3.2

1. Find a unit vector perpendicular to each of a vector $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$ where $\overrightarrow{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\overrightarrow{b}=\hat{i}+2\hat{j}-2\hat{k}$

2 Solution

$$(\mathbf{a}+\mathbf{b})^\top\mathbf{x}=0 \tag{1}$$
$$(\mathbf{a}-\mathbf{b})^\top\mathbf{x}=0 \tag{2}$$
$$\begin{pmatrix}(\mathbf{a}+\mathbf{b})^\top\\(\mathbf{a}-\mathbf{b})^\top\end{pmatrix}\mathbf{x}=0 \tag{3}$$
$$[(\mathbf{a}+\mathbf{b})(\mathbf{a}-\mathbf{b})]^\top\mathbf{x}=0 \tag{4}$$
$$\tag{5}$$

$$\begin{pmatrix}4&2\\4&0\\0&4\end{pmatrix}^\top\mathbf{x}=0 \tag{6}$$
$$\begin{pmatrix}2&1\\2&0\\0&2\end{pmatrix}^\top\mathbf{x}=0 \tag{7}$$
$$\begin{pmatrix}2&2&0\\1&0&2\end{pmatrix}\mathbf{x}=0 \tag{8}$$
$$\xleftrightarrow{R_1=\frac{R_1}{2}}\begin{pmatrix}1&1&0\\1&0&2\end{pmatrix}\mathbf{x}=0 \tag{9}$$
$$\xleftrightarrow{R_2=R_1-R_2}\begin{pmatrix}1&1&0\\0&-1&2\end{pmatrix}\mathbf{x}=0 \tag{10}$$
$$\xleftrightarrow{R_2=\frac{R_2}{-1}}\begin{pmatrix}1&1&0\\0&1&-2\end{pmatrix}\mathbf{x}=0 \tag{11}$$
$$\xleftrightarrow{R_1=R_1-R_2}\begin{pmatrix}1&0&2\\0&1&-2\end{pmatrix}\mathbf{x}=0 \tag{12}$$
$$\tag{13}$$

$$\mathbf{x}_1+2\mathbf{x}_3=0 \tag{14}$$
$$\mathbf{x}_1-2\mathbf{x}_3=0 \tag{15}$$
$$\begin{pmatrix}x1\\x2\\x3\end{pmatrix}=\begin{pmatrix}-2x3\\2x3\\x3\end{pmatrix} \tag{16}$$
$$=x3\begin{pmatrix}-2\\2\\1\end{pmatrix} \tag{17}$$