Unit Vector Perpendicular

1 12^{th} Maths - Chapter 10

This is Problem-2 from Exercise 10.4

1. Find a unit vector perpendicular to each of a vector $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$ where $\overrightarrow{a} = 3\hat{i}+2\hat{j}+2\hat{k}$ and $\overrightarrow{b} = \hat{i}+2\hat{j}-2\hat{k}$

2 Solution

A unit vector perpendicular

$$(\mathbf{a}+\mathbf{b})^\top \mathbf{x} = 0 \tag{1}$$

$$(\mathbf{a}-\mathbf{b})^\top \mathbf{x} = 0 \tag{2}$$

$$\begin{pmatrix} (\mathbf{a}+\mathbf{b})^\top \\ (\mathbf{a}-\mathbf{b})^\top \end{pmatrix} \mathbf{x} = 0 \tag{3}$$

$$\begin{pmatrix} \mathbf{a}+\mathbf{b} & \mathbf{a}-\mathbf{b} \end{pmatrix}^\top \mathbf{x} = 0 \tag{4}$$

Here.

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \tag{5}$$

$$\mathbf{a}+\mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \tag{6}$$

$$\mathbf{a}-\mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \tag{7}$$

Now using the formula substituing (7) and (8) in (4) and equating,

$$\begin{pmatrix}\mathbf{a}+\mathbf{b} & \mathbf{a}-\mathbf{b}\end{pmatrix}^{\top}\mathbf{x} = 0 \tag{8}$$

$$\begin{pmatrix}4 & 2\\4 & 0\\0 & 4\end{pmatrix}^{\top}\mathbf{x} = 0 \tag{9}$$

$$\begin{pmatrix}4 & 4 & 0\\2 & 0 & 4\end{pmatrix}\xleftrightarrow{R_1=\frac{R_1}{4}}\begin{pmatrix}1 & 1 & 0\\2 & 0 & 4\end{pmatrix}\mathbf{x} = 0 \tag{10}$$

$$\xleftrightarrow{R_2=\frac{R_2}{2}}\begin{pmatrix}1 & 1 & 0\\1 & 0 & 2\end{pmatrix}\mathbf{x} = 0 \tag{11}$$

$$\xleftrightarrow{R_2=R_1-R_2}\begin{pmatrix}1 & 1 & 0\\0 & -1 & 2\end{pmatrix}\mathbf{x} = 0 \tag{12}$$

$$\xleftrightarrow{R_2=\frac{R_2}{-1}}\begin{pmatrix}1 & 1 & 0\\0 & 1 & -2\end{pmatrix}\mathbf{x} = 0 \tag{13}$$

$$\xleftrightarrow{R_1=R_1-R_2}\begin{pmatrix}1 & 0 & 2\\0 & 1 & -2\end{pmatrix}\mathbf{x} = 0 \tag{14}$$

From (14), we get two equations which is

$$x_1 + 2x_3 = 0 \tag{15}$$

$$x_2 - 2x_3 = 0 \tag{16}$$

$$\mathbf{x} = \begin{pmatrix}-2x_3\\2x_3\\x_3\end{pmatrix} \tag{17}$$

$$= x_3\begin{pmatrix}-2\\2\\1\end{pmatrix} \tag{18}$$