

# Unit Vector Perpendicular

## 1 12<sup>th</sup> Maths - Chapter 10

This is Problem-2 from Exercise 4.2

- Find a unit vector perpendicular to each of a vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  where  $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

## 2 Solution

A unit vector perpendicular

$$(\mathbf{a} + \mathbf{b})^\top \mathbf{x} = 0 \quad (1)$$

$$(\mathbf{a} - \mathbf{b})^\top \mathbf{x} = 0 \quad (2)$$

$$\begin{pmatrix} (\mathbf{a} + \mathbf{b})^\top \\ (\mathbf{a} - \mathbf{b})^\top \end{pmatrix} \mathbf{x} = 0 \quad (3)$$

$$\begin{pmatrix} \mathbf{a} + \mathbf{b} & \mathbf{a} - \mathbf{b} \end{pmatrix}^\top \mathbf{x} = 0 \quad (4)$$

Here,

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \quad (5)$$

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \quad (6)$$

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \quad (7)$$

Now using the formula substituting (7) and (8) in (4) and equating,

$$\begin{pmatrix} \mathbf{a} + \mathbf{b} & \mathbf{a} - \mathbf{b} \end{pmatrix}^\top \mathbf{x} = 0 \quad (8)$$

$$\begin{pmatrix} 4 & 2 \\ 4 & 0 \\ 0 & 4 \end{pmatrix}^\top \mathbf{x} = 0 \quad (9)$$

$$\begin{pmatrix} 4 & 4 & 0 \\ 2 & 0 & 4 \end{pmatrix} \xrightarrow{R_1 = \frac{R_1}{4}} \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 4 \end{pmatrix} \mathbf{x} = 0 \quad (10)$$

$$\xrightarrow{R_2 = \frac{R_2}{2}} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix} \mathbf{x} = 0 \quad (11)$$

$$\xrightarrow{R_2 = R_1 - R_2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \end{pmatrix} \mathbf{x} = 0 \quad (12)$$

$$\xrightarrow{R_2 = \frac{R_2}{-1}} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -2 \end{pmatrix} \mathbf{x} = 0 \quad (13)$$

$$\xrightarrow{R_1 = R_1 - R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \end{pmatrix} \mathbf{x} = 0 \quad (14)$$

From (15), we get two equations which is  $x_1 + 2x_3 = 0$  and  $x_2 - 2x_3 = 0$

$$\mathbf{x}_1 + 2\mathbf{x}_3 = 0 \quad (15)$$

$$\mathbf{x}_1 - 2\mathbf{x}_3 = 0 \quad (16)$$

$$x_1 = \begin{pmatrix} -2x_3 \\ 2x_3 \\ x_3 \end{pmatrix} \quad (17)$$

$$= x_3 \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad (18)$$

A unit vector perpendicular to

$$\hat{\mathbf{x}}_1 = \frac{1}{3} \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \mathbf{x}_3 \quad (19)$$