

1 12th Maths - Chapter 10

This is Problem-2 from Exercise 10.4

1. Find a unit vector perpendicular to each of a vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

2 Solution

$$(\mathbf{a} + \mathbf{b})^\top \mathbf{x} = 0 \quad (1)$$

$$(\mathbf{a} - \mathbf{b})^\top \mathbf{x} = 0 \quad (2)$$

$$\begin{pmatrix} (\mathbf{a} + \mathbf{b})^\top \\ (\mathbf{a} - \mathbf{b})^\top \end{pmatrix} \mathbf{x} = 0 \quad (3)$$

$$[(\mathbf{a} + \mathbf{b})(\mathbf{a} - \mathbf{b})]^\top \mathbf{x} = 0 \quad (4)$$

$$(5)$$

$$\begin{pmatrix} 4 & 2 \\ 4 & 0 \\ 0 & 4 \end{pmatrix}^\top \mathbf{x} = 0 \quad (6)$$

$$\begin{pmatrix} 2 & 1 \\ 2 & 0 \\ 0 & 2 \end{pmatrix}^\top \mathbf{x} = 0 \quad (7)$$

$$\begin{pmatrix} 2 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \mathbf{x} = 0 \quad (8)$$

$$\xleftrightarrow{R_1 = \frac{R_1}{2}} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix} \mathbf{x} = 0 \quad (9)$$

$$\xleftrightarrow{R_2 = R_1 - R_2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \end{pmatrix} \mathbf{x} = 0 \quad (10)$$

$$\xleftrightarrow{R_2 = \frac{R_2}{-1}} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -2 \end{pmatrix} \mathbf{x} = 0 \quad (11)$$

$$\xleftrightarrow{R_1 = R_1 - R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \end{pmatrix} \mathbf{x} = 0 \quad (12)$$

$$(13)$$

$$\mathbf{x}_1 + 2\mathbf{x}_3 = 0 \quad (14)$$

$$\mathbf{x}_1 - 2\mathbf{x}_3 = 0 \quad (15)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2x_3 \\ 2x_3 \\ x_3 \end{pmatrix} \quad (16)$$

$$= x_3 \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad (17)$$