1. Question

X:	1	2	3	4	5
Y:	60	65	70	75	80

The value of

$$\sum y$$

is.........

a) 320
b) 350
c) 375
b) 379

Answer

Answer: b) 350

2. Question

The regression equation is X=0.1667y-12. Find the value of X if y=20.

a) 8.666
b) -8.666
c) 8.968
d) -5.623

Answer

b) -8.666

3. Question

A person is trading in a share market daily. He is interested to find a relationship, as a mathematical equation, for his returns in a month with respect to the days of that month. The following data shows the profit (y) with respect to the days (x). The amount is given in terms of thousands of Rupees. Identify the mathematical model of the form y = a x + b where

$$\Sigma x = 50, \Sigma y = 80, \Sigma x^2 = 750 \ \& \ \Sigma xy = 1035$$

.

x :	0	5	10	15	20
y :	7	11	16	20	26

Hint:

From the method of least squares, the normal equations are

$$a\sum x + nb = \sum y$$

$$a\sum x^2 + b\sum x = \sum xy$$

Answer

The mathematical modelling is y = a x + b.

From the method of least squares, the normal equations are

$$a\sum x + nb = \sum y$$

$$a\sum x^2 + b\sum x = \sum xy$$

Solving the normal equations, we have,

a = 0.94 and b = 0.66 .... (1)

The mathematical model is y = 0.94 x + 0.66 .... (1)

4. Question

A mechanical engineer, is researching how the carbon content (in percentage) of steel alloys influences their tensile strength (in MPa). He gathered data by testing various steel samples, each with different carbon content, and measuring their corresponding tensile strength. His aims to analyze this relationship to predict the tensile strength of steel alloys based on their carbon content.

Carbon Content (%) (x)	0.1	0.2	0.3	0.4
Tensile Strength (MPa) (y)	400	450	480	500

Determine the regression equation to predict carbon content based on tensile strength of 470 MPa given that b_{xy} =0.0029.
HINT

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

Answer

x-0.25=0.0029(y-457.5)

x-0.25=0.0029y-1.32675 ------------------(2 Mark)

x=0.0029y-1.32675+0.25

x=0.0029y-1.07675 ------------------(1 Mark)

The carbon content based on tensile strength of 470 MPa is x=0.28625 ------(1 Mark)

5. Question

A person is trading in a share market daily. He is interested to find a relationship, as a mathematical equation, for his returns in a month with respect to the days of that month. The following data shows the profit (Y) with respect to the days (X). The amount is given in terms of thousands of Rupees. Form a table to identify

$$\Sigma X, \Sigma Y, \Sigma X^2 \ \& \ \Sigma XY$$

.

X :	0	5	10	15	20
Y:	7	11	16	20	26

Answer

X	Y	X^2	X Y
0	7	0	0
5	11	25	55
10	16	100	160
15	20	225	300
20	26	400	520
$\sum$ = 50	80	750	1035

$$\sum X = 50$$

$$\sum Y = 80$$ .... (1)

$$\sum X^2 = 750$$ .... (1)

$$\sum XY = 1035$$ .... (1)

---

6. Question

Ravi worked for an automobile company and he is interested in predicting the fuel efficiency (miles per gallon, or MPG) of different car models based on their engine size. He collected data on various car models and their engine sizes is given below

Engine size (x)	3	4	5	6	7	8
Fuel efficiency (y)	5	10	15	20	25	30

Compute the regression co-efficient of fuel efficiency on engine size.
HINT:

$$b_{yx} = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$$

Answer

x	y	x^2	y^2	$x \cdot y$
3	5	9	25	15
4	10	16	100	40
5	15	25	225	75
6	20	36	400	120
7	25	49	625	175
8	30	64	900	240
---	---	---	---	---
$\sum x = 33$	$\sum y = 105$	$\sum x^2 = 199$	$\sum y^2 = 2275$	$\sum xy = 665$

(2 MARK)

$$byx = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{6 \cdot 665 - 33 \cdot 105}{6 \cdot 199 - (33)^2}$$

$$= \frac{3990 - 3465}{1194 - 1089}$$

$$= \frac{525}{105}$$

$$= 5$$

(2 MARK)

7. Question

What is the slope of the best fit line y = 0.9 x + 1.3?

a) -0.9

b) 0.9

c) 1.3

d) -1.3

Answer

Answer: b) 0.9

8. Question

Ravi working for an agricultural company. He wants to predict the **crop yield** (in tons per hectare) based on the amount of **water used** (in litres per hectare). He collected the following data from several fields:

Water used (litres/ha) (x)	100	200	300	400
Crop Yield (tons/ha) (y)	2.5	3.5	4.5	5.5

Compute the regression equation and predict the crop yield when 180 kg/ha and 340 kg/ha of water is used given that b_{yx} =0.01.

HINT:

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

Answer

Regression Line y on x

$$y - \bar{y} = byx(x - \bar{x})$$

$$y - 4 = 0.01(x - 250)$$ ----------------------- (2 Mark)

$$y - 4 = 0.01x - 2.5$$

$$y = 0.01x - 2.5 + 4$$

$$y = 0.01x + 1.5$$ ------------------------- (1 Mark)

The crop yield when 180 kg/ha of fertilizer is used is y=3.3. ---------------------- (1 Mark)

The crop yield when 340 kg/ha of fertilizer is used is y=4.9. ---------------------- (1 Mark)

9. Question

Ravi worked for an automobile company and he is interested in predicting the fuel efficiency (miles per gallon, or MPG) of different car models based on their engine size. He collected data on various car models and their engine sizes is given below

Engine size (x)	3	4	5	6	7	8
Fuel efficiency (y)	5	10	15	20	25	30

Identify the average engine size and fuel efficiency of automobile company.

Answer

$$\text{Mean } \bar{x} = \frac{\sum x}{n}$$

$$= \frac{33}{6}$$

$$= 5.5$$ (1 MARK)

$$\text{Mean } \bar{y} = \frac{\sum y}{n}$$

$$= \frac{105}{6}$$

$$= 17.5$$ (1 MARK)

10. Question

The regression equation is X=43*y*-12. Find the value of X if y=10.

a) 418
b) -145
c) -418
d) 145

Answer

a) 418

11. Question

Find the average value of 5,8,10,13.

a) 5
b) 7
c) 10
d) 9

Answer

d) 9

12. Question

X:	1	2	3	4	5
Y:	2	3	5	4	6

The value of

$$\sum x^2$$

is.

a) 40
b) 45
c) 50
d) 55

Answer

Answer: d) 55

13. Question

Find the value of

$$\sum xy$$

in the given data

X	1	3	5	7
Y	3	6	9	12

a) 150
b) 174
c) 184
d) 194

Answer

a) 150

14. Question

Hours Studied (x)	Grade (y)
1	60
2	65
3	70
4	75
5	80

The value of

$$\sum x$$

is........

a) 5
b) 10
c) 15
d) 20

Answer

Answer: c) 15

15. Question

A person is trading in a share market daily. He is interested to find a relationship, as a mathematical equation, for his returns in a month with respect to the days of that month. The following data shows the profit (Y) with respect to the days (X). The amount is given in terms of thousands of Rupees. Find the mathematical model as a straight line of the form Y = a x + b.

X :	0.5	1.0	1.5	2.0	2.5	3.0
Y:	15	17	19	14	10	7

Hint:

From the method of least squares, the normal equations are

$$a\sum x + n\,b = \sum y$$

$$a\sum x^2 + b\sum x = \sum x\,y$$

Answer

The mathematical modelling is y = a x + b.

From the method of least squares, the normal equations are

$$a\sum x + nb = \sum y$$

$$a\sum x^2 + b\sum x = \sum xy$$

And, also $\sum x = 10.5$ .... (1)

$\sum y = 82$ .... (1)

$\sum x\ y = 127$ .... (1)

$\sum x^2 = 22.75$ .... (1)

Solving the normal equations, we have,

a = - 3.7714 and b = 20.2667 .... (1)

The mathematical model is y = - 3.7714 x + 20.2667 .... (1)

16. Question

Ravi working for an agricultural company. He wants to predict the **crop yield** (in tons per hectare) based on the amount of **water used** (in liters per hectare). He collected the following data from several fields:

Water used (liters/ha) (x)	50	60	70	80
Crop Yield (tons/ha) (y)	2	5	7	9

Compute the regression equation (Y on X) for predicting crop yield based on water usage.

Hint:

$$b_{yx} = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

Answer

x	y	x^2	y^2	$x \cdot y$
50	2	2500	4	100
60	5	3600	25	300
70	7	4900	49	490
80	9	6400	81	720
---	---	---	---	---
$\sum x = 260$	$\sum y = 23$	$\sum x^2 = 17400$	$\sum y^2 = 159$	$\sum xy = 1610$

------------------------- (2 Mark)

Mean $\bar{x} = \frac{\sum x}{n}$

$= \frac{260}{4}$

$= 65$

Mean $\bar{y} = \frac{\sum y}{n}$

$= \frac{23}{4}$

$= 5.75$ ------------------------- (1 Mark)

$byx = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$

$= \frac{4 \cdot 1610 - 260 \cdot 23}{4 \cdot 17400 - (260)^2}$

$= \frac{6440 - 5980}{69600 - 67600}$

$= \frac{460}{2000}$

$= 0.23$ ------------------------- (2 Mark)

Regression Line y on x

$y - \bar{y} = byx\left(x - \bar{x}\right)$

$y - 5.75 = 0.23(x - 65)$

$y - 5.75 = 0.23x - 14.95$

$y = 0.23x - 14.95 + 5.75$

$y = 0.23x - 9.2$ ------------------------- (1 Mark)

17. Question

The factorization of quadratic equation

$$x^2 - 7x + 12 = 0$$

is _____

a) (x+3)(x-4)

b) (x-3)(x-4)

c) (x+3)(x+4)

d) (x-3)(x+4)

Answer

b) (x-3)(x-4)

18. Question

If

$$\lambda$$

is an eigen value of an orthogonal matrix A then

$$\frac{1}{\lambda}$$

is an eigen value of

a)

$$AA^T$$

b)

$$A^{-1}$$

c) A

d)

$$A^T$$

Answer

d)

$$A^T$$

19. Question

Let the following quadratic function represents John’s height (h) in meters above the water ‘t’ seconds after he leaves the diving board.

$$h(t) = -4.9\,t^2 + 8t + 5$$

a) Compute the time at which John reach his maximum height.

b) Find the maximum height attained by John.

c) Compute the time at which John hit the water.

Answer

a). The time at which John reaches his maximum height is the x-coordinate of the vertex.

$$t = \frac{-b}{2a} = \frac{-8}{-9.8} = 0.82$$ (1 Mark)

It took John 0.82 seconds to reach his maximum height.

b). The maximum height was reached John at 0.82 seconds.

$$h\,(0.82) = -4.9(0.82)^2 + 8(0.82) + 5 = 8.27$$ (1 Mark)

The maximum height reached by Jeremiah was 8.27 m.

c). When John hits the water, his height will be zero.

$$-4.9t^2 + 8t + 5 = 0 \quad \Rightarrow \quad t = -0.48, \quad t = 2.12$$ (2 Marks)

$$t = 2.12$$

20. Question

The vertex of the quadratic function

$$f\left(x\right) = 3x^2 - 6x + 1$$

is _______

a) (1, 2)
b) (1, -2)
c) (1, 3)
d) (1, -3)

Answer

b) (1, -2)

21. Question

If the quadratic function

$$f\left(x\right) = ax^2 + bx + c$$

has a maximum value, what is the sign of the coefficient ‘a’?

a) It is positive.
b) It is negative.
c) It is zero.
d) It can be any value.

Answer

b) It is negative.

22. Question

The structural stability of a three-level building is analyzed using matrix A, which represents the interaction between the main support beams on each floor: the ground floor beam, the middle floor beam, and the top floor beam. The matrix A is given as

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

a) If the eigen value is 0, the corresponding eigen vector is

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

and the eigen value is 1, the corresponding eigen vector is

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

. Find the eigen vectors corresponding to the eigen value 3.

$$\text{Hint: } \begin{pmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

b) Diagonalize the matrix A which is used to analyze the stability of a three-level building.

Hint: $N^T AN = D$

Answer

When $\lambda = 3$, the corresponding eigen vector is $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$. -------(1m)

The normalized modal matrix $N = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{pmatrix}$ ------(1m)

$$N^T = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix} \text{---------(1m)}$$

$$N^T AN = D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \text{-------(1m)}$$

23. Question

For n data points, the number of normal equations need to solve for fitting a quadratic function is ____

a) 2
b) 3
c) 4
d) 5

Answer

b) 3

24. Question

A group of economists is analysing the financial interactions between three sectors of an economy: agriculture, manufacturing, and services. They use matrix A to represent the flow of resources and influence between these sectors. Each sector is treated as a node in the economy, and their interdependencies are captured by the matrix. The rows and columns of A correspond to Agriculture, Manufacturing, and Services, respectively. The matrix is given by

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

a) If the eigen value is 1, the corresponding eigen vector is

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

and the eigen value is 3, the corresponding eigen vector is

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

. Find the changes in one sector influence the others, providing a deeper understanding of the interconnectedness and resilience of the economy (Eigen vectors) corresponding to the eigen value 4.

$$\text{Hint: } \begin{pmatrix} 3-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

b) Diagonalize the matrix A to identify the most influential sector and the dominant modes of economic influence within the system.

Hint: $N^T AN=D$

Answer

When $\lambda = 4$, the corresponding eigen vector is $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$. --------(1m)

The normalized modal matrix $\underline{\underline{N}} = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}$ -------(1m)

$N^T = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$ ----------(1m)

$N^T AN=D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$--------(1m)

25. Question

The cost, in dollars, of operating a machine per day is given by the quadratic function

$$C(t) = 2t^2 - 84t + 1025$$

where 't' is the time in hours, the machine operates.

a) Compute the number of hours the machine needs to run to reach the minimum cost.

b) Find the minimum cost of running the machine.

c) State true or false with justification.
"The graph of the given quadratic function is a downward parabola."

Answer

a) $h = \frac{-b}{2a} = \frac{84}{2} = 21$ (1 Mark)

Hence 21 hours the machine needs to run to reach the minimum cost.

b) $k = f(21) = 143$ (1 Mark)

The minimum cost of running the machine is 143 dollars.

c) False, since a > 0 the graph will an upward parabola. (1 Mark)

26. Question

The axis of symmetry of the quadratic function

$$f(x) = 2(x-3)^2 + 4$$

is _______

a) x=3
b) x=4
c) x=-3
d) x=-4

Answer

a) x=3

27. Question

A group of students start a small company that produces CDs. The weekly profit (in dollars) is given by the function

$$p(x) = -2x^2 + 80x - 600$$

where x represents the number of CDs produced.

a) Find the break-even points (x-intercepts) and y-intercept of the profit function.
b) Find the vertex of the profit function, and interpret its meaning in the context of this problem.
c) Sketch the profit function.

Answer

a) The x-intercepts are the real solutions of the equation $p(x)=0$.

$$\Rightarrow \quad -2x^2+80x-600=0$$
$$\Rightarrow \quad x=10 \quad or \quad x=30$$

(1 Mark)

The x-intercepts are (10, 0) and (30, 0).

$$p(0)=-600$$

The y-intercept is (0, -600).

(1 Mark)

b) The x-coordinate of the vertex is $h=\dfrac{-b}{2a}=20$

The y-coordinate is

$$k=p(20)=-2(20)^2+80(20)-600=200$$

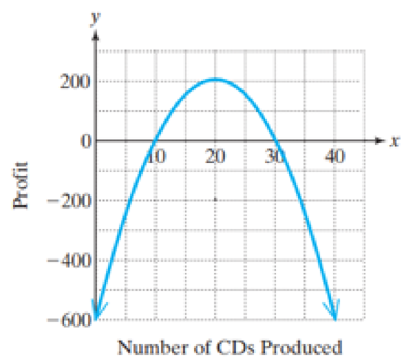
The vertex is (20, 200).

(1 Mark)

A maximum weekly profit of $200 is obtained when 20 CDs are produced.

(1 Mark)

c).



(1 Mark)

28. Question

The table shows the amounts y (in billions of dollars) spent on admission to movie theaters in the United States for the years 1997 to 2003. Let x represent the year, with x=7 corresponding to 1997.

x (year)	7	8	9	10	11	12	13
y (Amount)	6.3	6.9	7.9	8.6	9	9.6	9.9

From the above table we have

$$\Sigma x = 70,\ \Sigma x^2 = 728,\ \Sigma x^3 = 7840,\ \Sigma x^4 = 86996,\ \Sigma y = 58.2,$$

$$\Sigma xy = 599.3,\ \Sigma x^2 y = 6394.7$$

and n=7. Find a quadratic model

$$y = ax^2 + bx + c$$

for the data. Also, find the amount (y) spent on admission to movie theater in the year 2004 (x=14).

Hint: The normal equations are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc;$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x;$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

Answer

Normal equations corresponding to the above data are

728a+70b+7c=58

7840a+728b+70c=599.3

86996 a+ 7840 b+ 728c=6394.7 (1 Mark)

On solving the above linear system of equations

We get the values of a, b and c.

a=-0.0488, b= 1.594, c=-2.55. (1 Mark)

Hence the quadratic model for the data is y = -0.0488 x2+1.594 x +2.55. (1 Mark)

The amount spent on admission to movie theatre y in the year 14 is 15.3 dollars. (1 Mark)

29. Question

A rock is thrown upward from the top of a 112-foot high cliff overlooking the ocean at a speed of 96 feet per second. Compute the following:

a) Form equation for the rock’s height above ground.
b) Compute the time at which the rock reach the maximum height.
c) Compute the maximum height attained by the rock.
d) Compute the time at which the rock hit the ocean.

Hint : The position function of the rock is

$$s(t) = -16^2 + v_0 t + s_0$$

.

Answer

a) $s(t) = -16t^2 + 96t + 122$

(1 Mark)

b) $h = \frac{-b}{2a} = \frac{-96}{-32} = 3$

Maximum height will be reached after 3 seconds.

(1 Mark)

c) $k = f\left(\frac{-b}{2a}\right) = f(0) = -16(3)^2 + 96(2.5) + 112 = 256$

Maximum height is 256 ft.

(1 Mark)

d)

$-16t^2 + 96t + 112 = 0$

$t = -1, \quad t = 7$

t = 7 seconds

(1 Mark)

30. Question

The general equation that gives the height of such an object is called a position equation, and on Earth's surface it has the form $s(t)=-16t^2+80t+40$ where t represent the time(in seconds). Compute the time at which the ball reaches the maximum height and maximum height attained by the ball. Also, find the time at which the ball hit the ground.

Answer

The ball's height above ground can be modeled by the equation $s(t)=-16t^2+80t+40$.
The ball reaches the maximum height at the vertex of the parabola.
h=-b/2a=5/2=2.5 seconds (1 mark)
The ball reaches the maximum height after 2.5 seconds.
To find the maximum height, find the y coordinate of the vertex of the parabola.
$k=f(-b/2a)=f(2.5)=-16(2.5^2)+80(2.5)+40=140$ feet. (2 mark)
The ball reaches a maximum height of 140 feet.
To find when the ball hits the ground, we need to determine when the height is zero
s(t)=0 implies that $-16t^2+80t+40=0$
on solving the above quadratic equation, we get time t=5.458 seconds. (1 mark)
the ball will hit the ground after about 5.458 seconds.

31. Question

A symmetric matrix can always be diagonalized by

a) an orthogonal matrix whose columns are eigenvectors of the given matrix
b) an arbitrary orthogonal matrix
c) a diagonal matrix
d) a triangular matrix

Answer

a) an orthogonal matrix whose columns are eigenvectors of the given matrix

32. Question

The factorization of quadratic function

$$x^2 + 5x + 6 = 0$$

is _____
a) (x-2)(x+3)
b) (x+2)(x-3)

c) (x+2)(x+3)
d) (x-2)(x-3)

Answer
c) (x+2)(x+3)

33. Question
Which statistical method is commonly used to estimate the parameters of a power function?

a) Maximum likelihood estimation

b) Method of moments

c) Least squares method

d) Bayesian estimation

Answer
c) Least squares method

34. Question
Consider the power function

$$y = a x^{b}$$

and the model of the power function is log(y) = log (a) + b log(x) and if the value of log(a) =1.2 and b = 0.75, then find the value of y if x = 10.
a) y = 89.13
b) y = 81.39
c) y = 83.13
d) y = 89.91

Answer
a) y = 89.13

35. Question
A computer scientist is analyzing the relationship between processor speed S (in GHz) and performance P (in millions of instructions per second, MIPS) of a system

$$P = a S^{b}$$

.The data is:

Speed (GHz)	1	2	3	4	5
Performance (MIPS)	100	210	360	590	820
$\log_{10}(S)$	0.000	0.301	0.477	0.602	0.699
$\log_{10}(P)$	2.000	2.322	2.556	2.771	2.914

If the normal equations are 5A + 2.079 b = 12.563 ; 2.079 A + 1.169 b = 5.623
Find the value of a and b and fix the power curve.

Answer

On solving the normal equations, we get

A = 1.967 ------------------ (1)

b = 1.311 ---------------- (1)

A = $\log_{10}(a) \Longrightarrow$ $a=10^A \Longrightarrow$ $a = 10^{(1.967)}$ $\Longrightarrow a = 92.683$ --------------- (1)

The power curve is $P = (92.683)\, S^{(1.311)}$ --------------------- (1)

36. Question

The method of least squares can be generalized to fit which of the following types of models?

a) Only linear models
b) Any model that can be linearized
c) Only power functions
d) Only exponential models

Answer

b) Any model that can be linearized

37. Question

Find the first derivative and the second derivative to determine the rate of change of the given function with respect to x.

$$f(x)=2x^3-9x^2+12$$

Answer

Given the function $f(x)=2x^3-9x^2+12$

The first derivative is $f'(x)=6x^2-18x$ --------- (1 Mark)

The second derivative is $f''(x)=12x-18$ --------- (1 Mark)

38. Question

An electrical engineer records the voltage x and current y in a circuit to investigate their relationship. The data collected is:

voltage x	0	1	2	3	4	6
current y	1	6	17	34	57	0

Compute the two normal equations

Hint

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4 + d\sum x^5$$

$$\sum x^3y = a\sum x^3 + b\sum x^4 + c\sum x^5 + d\sum x^6$$

Answer

From the table values

$$\sum x^2 y$$

=1292,

$$\sum x^2$$

=55,

$$\sum x^3$$

=225,

$$\sum x^4$$

=979,   -------- 1 mark

$$\sum x^5$$

=4425,

$$\sum x^3 y$$

=4708,

$$\sum x^6$$

= 20515   --------- 1 mark

$$55a + 225b + 979c + 4425d = 1292$$

$$225a + 979b + 4425c + 20515d = 4708$$

------------------------2 marks

---

39. Question

An engineer needs to align a satellite dish to maximize signal reception. The dish must be adjusted precisely to align with a satellite positioned in the sky. The signal strength varies with the sine of the angle of rotation, with the maximum signal strength occurring when the dish is aligned at an angle of pi/3 radians (60 degrees) from its starting position. To achieve precise alignment, the engineer wants to understand how small adjustments around this optimal angle affect the signal strength.

Expand f(x) = 4cosx into a Taylor series about the point x=

$$\frac{1}{1^3}+\frac{1}{2^3}+\frac{1}{3^3}+\cdots$$

to approximate the signal strength for small deviations from the optimal angle.
Hint:

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \ldots\ldots$$

Answer

f(x)=4cosx $\quad f\left(\frac{\pi}{3}\right)=2$ (1 Mark)

f ' (x)=-4sinx $\quad f'\left(\frac{\pi}{3}\right)=-2\sqrt{3}$ (1 Mark)

f '' (x)=4cosx $\quad f''\left(\frac{\pi}{3}\right)=-2$ (1 Mark)

f ''' (x)=-4sinx $\quad f'''\left(\frac{\pi}{3}\right)=2\sqrt{3}$ (1 Mark)

Taylor's series

$$f(x)=2-2\sqrt{3}\left(x-\frac{\pi}{3}\right)-\left(x-\frac{\pi}{3}\right)^2+\frac{\sqrt{3}}{3}\left(x-\frac{\pi}{3}\right)^3+\cdots\ldots\ldots$$ (2 Marks)

40. Question

A study examines the relationship between the age (x) of a child and their weight (y). The following data shows the age (in years) and corresponding weight (in kilograms):

Age (years)	1	2	3	4	5
Weight (kg)	9.5	12.0	14.5	16.8	20.1
$\log_{10}(x)$	0.000	0.301	0.477	0.602	0.699
$\log_{10}(y)$	0.978	1.079	1.161	1.225	1.303

If the normal equations are 5A + 2.079 b = 5.746 ; 2.079 A + 1.169 b = 2.527

On solving normal equations, we get A = 0.961 and b = 0.453

Fit a power curve of the form

$$y = a x^{b}$$

where y is the weight and x is the age, using logarithms to base 10.

Answer

$A = \log_{10}(a) \Rightarrow a=10^{A} \Rightarrow a = 10^{(0.961)} \Rightarrow a = 9.141$ -------------------- (2)

The power curve of the form $y = a\,x^{b} = (9.141)\,x^{(0.453)}$ ---------------- (1)

41. Question

Which of the following is one of the normal equation of

$$y = a x^{b}$$

?

a)

$$\Sigma \log(xy) = n \log(a) + b\,\Sigma \log(x)$$

b)

$$\Sigma \log(xy) = n \log(a) + x\,\Sigma \log(b)$$

c)

$$\Sigma \log(x) \log(y) = \log(a)\,\Sigma \log(x) + b\,\Sigma (\log x)^2$$

d)

$$\Sigma \log(y) = n \log(a) + b\,\Sigma (\log x)^2$$

Answer

c) $\Sigma \log(x) \log(y) = \log(a) \Sigma \log(x) + b \Sigma (\log x)^2$

42. Question

A financial analyst wants to analyze the relationship between the age of a car (x) and its resale value (y). Understanding this relationship can help him to estimate the depreciation rate of cars over time and make informed decisions regarding car purchases and sales. To investigate this relationship, collect data on the ages and corresponding resale values of several cars.

x	-2	-1	0	1	2
y	40	50	62	58	60

Frame the normal equations to identify the relationship between the age of a car (x) and its resale value (y)
Hint:
The normal equations are

$$\sum y = an + b\sum x + c\sum x^2 + d\sum x^3$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 + d\sum x^4$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4 + d\sum x^5$$

$$\sum x^3 y = a\sum x^3 + b\sum x^4 + c\sum x^5 + d\sum x^6$$

Answer

x^2	x^3	x^4	x^5	x^6	$x \cdot y$	$x^2 \cdot y$	$x^3 \cdot y$
4	-8	16	-32	64	-80	160	-320
1	-1	1	-1	1	-50	50	-50
0	0	0	0	0	0	0	0
1	1	1	1	1	58	58	58
4	8	16	32	64	120	240	480
---	---	---	---	---	---	---	---
10	0	34	0	130	48	508	168

---------------------2 marks

$$270 = 5a + 0b + 10c + 0d$$

$$48 = 0a + 10b + 0c + 34d$$

----------------- 2 marks

$$508 = 10a + 0b + 34c + 0d$$

$$168 = 0a + 34b + 0c + 130d$$

------------------------1 mark

43. Question

What is the first term of the Maclaurin series of cos(x)?

a) 1
b) 0

c) x
d)

$$-x^2$$

Answer

a) 1

44. Question

Expansion of

$$e^x$$

at x = 0 is ________

a) f(x) = 1+x+….
b) f(x) = 1-x+….
c) f(x) = 1+2x+….
d) f(x) = 2+x+….

Answer

a) f(x) = 1+x+….

45. Question

At a local maximum, the second derivative of the function is typically ---------------

a) Positive
b) Negative
c) Zero
d) Undefined

Answer

b) Negative

46. Question

A critical point occurs when ------------

a)

$$f(x)=0$$

b)

$$f'(x)=0$$

c)

$$f''(x)=0$$

d)

$$f'''(x)=0$$

Answer

b)

$$f'(x)=0$$

47. Question

Imagine a car is moving along a straight road, and at each second, it covers the distance . This process continues indefinitely. We want to estimate whether the car will travel a finite distance or infinite distance.

Answer

Using comparison test

and select a series which is larger than

$$a_n$$

.

is a convergent series by p series test --- (2)

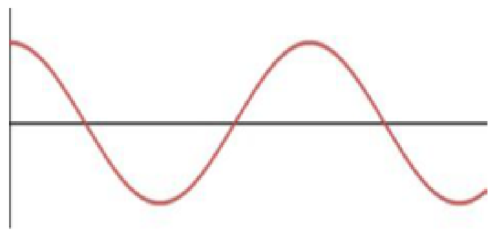
$$a_n < b_n$$

for all n, therefore the given series is converge. ---(1)

The car travels only a finite distance. --- (1)

48. Question

A harp is having a tightly stretched string. While playing it, the vibration of the string is monitored especially around the point where the harpist initially plucks the string and the displacement of the wave pattern is of the form f(x) = log x



Find the Taylor series of the wave pattern f(x) to approximate the displacement of the string at x=1.

Hint: The Taylor's series for the displacement

$$f(x) = f(a) + (x-a)f'(a) + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \ldots\ldots$$

Answer

$$f(x) = \log x \qquad f(1) = 0 \qquad (1\,Mark)$$

$$f'(x) = \frac{1}{x} \qquad f'(1) = 1 \qquad (1\,Mark)$$

$$f''(x) = \frac{-1}{x^2} \qquad f''(1) = -1 \qquad (1\,Mark)$$

$$f'''(x) = \frac{2}{x^3} \qquad f'''(1) = 2$$

Taylor's series

$$f(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \cdots\ldots \qquad (1\,Mark)$$

49. Question

If the value of y is as follows

y	250	300	350	330	370

Find the value of

$$\sum y$$

a) 1600
b)1800
c) 1500
d) 1300

Answer

a)1600

50. Question

A coffee plantation in Brazil has seen a surge in demand for its premium coffee beans in South Korea**.** As the demand for specialty coffee grows among South Korean consumers, the company has been exporting increasing quantities each year. The table below shows the amount of coffee beans (in tons) exported from Brazil to South Korea over the past few years.

Years since exports began (x)	2	3	4	5
Amount of coffee beans exported (y)	27.8	62.1	110	161

a) Construct and solve the normal equations to determine the best fit exponential equation for the quantity of coffee beans exported over the years.
Hint:

$$\sum Y = nA + B\sum x$$

$$\sum xY = A\sum x + B\sum x^2$$

and $\sum Y=7.4854$, $\sum x \cdot Y=27.4671$

b) As the company wants to model the export growth to forecast future demand and optimize the production, fit an exponential model

$$y = a\,e^{bx}$$

for the given data.
Hint:

$$y = a\,e^{bx}$$

b)As the company wants to model the export growth to forecast future demand and optimize production fit an exponential model

$$y = a\,e^{bx}$$

for the given data.
Hint:

$$y = a\,e^{bx}$$

takes the form

$$log\ y = log\ a + bx\ log\ e \Rightarrow Y = A + Bx$$

where,

$$Y = log\ y;\ A = log\ a\ and\ B = b\ log_{10}\ e$$

$$a = Antilog_{10}\ A\ and\ b = \frac{B}{log_{10}\ e}$$

Answer

x	x^2
2	4
3	9
4	16
5	25
$\sum x=14$	$\sum x^2=54$

The Normal equations are

$4A+14B=7.4854$____ (1 Mark)

$14A+54B=27.4671$____ (1 Mark)

A=0.9836 and B=0.25364____ (1 Mark)

b) a=9.629 and b=1.7932

$y = 9.629\ e^{1.7932x}$____ (1 Mark)

51. Question

In the exponential growth model, what does the variable k represent?

a) The initial amount

b) The growth rate

c) The time period

d) The final amount

Answer

b) The growth rate

52. Question

Imagine a small island with a unique species of rabbits. The island is isolated and provides an ideal environment for these rabbits to thrive. The population of rabbits for (t) number of years is P(t).

a) Find the rate of change of population with respect to (t) if the exponential growth of the population is $P(t) = e^{2t}$ and express the rate of change in exponential series.

b) If the exponential growth is $P(t) = e^{t}$, find the number of rabbits between the years t=2 and t=3.

Answer

a) P' (t) =$2e^{2t}$

$$2e^{2t} = 2 + \frac{2(2t)^1}{1!} + \frac{2(2t)^2}{2!} + \frac{2(2t)^3}{3!} + \frac{2(2t)^4}{4!} + \frac{2(2t)^5}{5!} + \cdots$$

Then,

P' (t) =$2e^{2t} = 2 + 4t + 4t^2 + \frac{8t^4}{3} + \cdots$____ (1 Mark)

b)$\int_2^3 e^t \, dt = \left|\frac{e^t}{2}\right|$ = $\left[\frac{e^3}{2} - \frac{e^2}{2}\right]$____ (1 Mark)

=10.04 – 3.69 = 6.35 ____ (1 Mark)

53. Question

If the value of y is as follows

y	11	12	13	14	15

Find the value of

$$\sum Y$$

where Y= log y

a) 5.5567

b) 4.6527

c) 55.567

d) 46.567

Answer

a) 5.5567

54. Question

A tech company that is tracking the cumulative number of users for a newly launched mobile app over time. Instead of measuring in years, they might track the user growth on a monthly basis or by every 10,000 downloads milestone.

Let x be the cumulative milestone in the number of downloads (e.g., x=0 for the launch, x=1 for 10,000 downloads, x=2 for 20,000 downloads, and so on).

Let y be the cumulative number of users actively using the app.

Milestones (x)	0	1	2	3
Number of users(y)	1.05	2.10	3.85	8.30

Construct the normal equations to determine the best fit exponential equation to predict future user growth based on past trends.

Hint:

$$\sum Y = nA + B\sum x$$

$$\sum xY = A\sum x + B\sum x^2$$

Answer

x	*y*	**Y=log10(*y*)**	***x*²**	***x*·*Y***
0	1.05	0.0212	0	0
1	2.1	0.3222	1	0.3222
2	3.85	0.5855	4	1.1709
3	8.3	0.9191	9	2.7572
∑*x*=6	∑*y*=15.3	∑Y=1.8479	∑*x*2=14	∑x·Y=4.2504

∑Y=1.8479 _____ (1 Mark)

∑*x*·Y=4.2504_____ (1 Mark)

The Normal equations are

$1.8479 \;=\; 4A \;+6B$ _____ (1 Mark)
$4.250 \;=6\,A \;+14B$_____ (1 Mark)

55. Question

A research laboratory is studying the decay of a chemical pollutant in a lake over time. The concentration of the chemical (in milligrams per liter) decreases gradually as it breaks down. The table below shows the concentration of the chemical at different points in time after it was first introduced into the water.

Time (x) in years	0	5	8	12	20
Concentration (Y)mg/L	3.0	1.5	1.0	0.55	0.18

a)Construct the normal equations to determine the best fit exponential equation for the concentration of the chemical changes over the years.
Hint:

$$\sum Y \;=\; nA \;+B\sum x$$

$$\sum xY \;=\; A\sum x \;+B\sum x^2$$

b) As the laboratory wants to model the changes in the concentration of chemical over time (in years), fit an exponential model

$$y = a\, e^{bx}$$

for the given data to predict how long it will take for the chemical concentration to reach a safe level for aquatic life.
Hint:

$$y = a\, e^{bx}$$

takes the form

$$log\, y = log\, a + bx\, log\, e \Rightarrow Y = A + Bx$$

where,

$$Y = log\, y;\; A = log\, a\; and\; B = \; b\, log_{10}\, e$$

$$a = Antilog_{10}\, A\; and\; b = \frac{B}{log_{10}\, e}$$

Answer

a)

x	y	Y=log10(y)	x^2	x·Y
0	3	0.4771	0	0
5	1.5	0.1761	25	0.8805
8	1	0	64	0
12	0.55	-0.2596	144	-3.1156
20	0.18	-0.7447	400	-14.8945
∑x=45	∑y=6.23	∑Y= -0.3512	∑ x^2=633	∑x·Y=-17.1297

∑Y=-0.3512, ∑x·Y=-17.1297 ____ (1 Mark)

The Normal equations are

5A+ 45 B = -0.3512 ____ (1 Mark)
45A+633B=-17.1297____ (1 Mark)
b) A=0.4812, B=-0.0613____ (1 Mark)
a=3.0281 and b=-0.1411
$y = 3.0281\ e^{-0.1411x}$____ (1 Mark)

56. Question

The population of a state in the year 2000 was approximately 750,000. Assume that the population is increasing at a rate of 8% per year. Identify the exponential function that relates the total population as a function of t.

a) y= 750,000 $e^{0.08\,t}$
b) y= 750 $e^{0.08\,t}$
c) y= 750,000 $e^{-0.08\,t}$
d) y= 750 $e^{-0.08\,t}$

Answer

a) y= 750,000 $e^{0.08\,t}$

57. Question

The value of x in the exponential equation $(1/6)^{x-5} = 216$ is

a) x = 2
b) x = 5
c) x = 6
d) x = 4

Answer

a) x = 2

58. Question

Identify the coefficient of third term in the series e^{-2t}.

a) - 4/3
b) 8/3
c) - 1/3
d) 2/3

Answer

a) - 4/3

59. Question

If you sow a seed and observe the growth of the plant day by day. What will be the length of the plant on tenth day, if the growth of the plant is fitted as exponential model

$$y = 1.1\, e^{0.2x}$$

Answer

$$y = 1.1\, e^{0.2(10)}\_\_\_\_\text{ (1 Mark)}$$
$$y = 1.1\, e^{2}$$
$$y = 8.13 \text{ cm approximately}\_\_\_\_\text{ (1 Mark)}$$

60. Question

If the value of x is as follows

x	- 0.5	1.5	3.5	4.0	6.5

Find the value of

$$\sum x$$

a) 15.0
b) 14.5
c) 15.5
d) 3.0

Answer

a) 15.0

61. Question

Which of the following scenarios is best modeled by an exponential decay function?
a) The population of a city
b) The depreciation of a car's value
c) The growth of a tree
d) The spread of a rumor

Answer

b) The depreciation of a car's value

62. Question

A company in India specializes in producing organic cashew nuts. Over the years, demand for these high-quality cashews has been increasing in European markets. The table below shows how the quantity of cashew nuts (in metric tons) exported to Europe grows over time.

Years (x)	0	0.5	1	1.5	2	2.5
Quantity exported(y) in Metric Tons	0.10	.45	2.15	9.15	40.35	180.75

a) Construct the normal equations to determine the best fit exponential equation for the quantity of cashew nuts exported over the years.
Hint:

$$\sum Y = nA + B\sum x$$

$$\sum xY = A\sum x + B\sum x^2$$

b)As the company wants to model the export growth to forecast future demand and optimize production fit an exponential model

$$y = a\, e^{bx}$$

for the given data.
Hint:

$$y = a\, e^{bx}$$

takes the form

$$log\ y = log\ a + bx\ log\ e \Rightarrow Y = A + Bx$$

where,

$$Y = log\ y;\ A = log\ a\ and\ B = b\ log_{10}\ e$$

$$a = Antilog_{10}\ A\ and\ b = \frac{B}{log_{10}\ e}$$

Answer

a)

x	y	Y=log10(y)	x^2	x·Y
0	0.1	-1	0	0
0.5	0.45	-0.3468	0.25	-0.1734
1	2.15	0.3324	1	0.3324
1.5	9.15	0.9614	2.25	1.4421
2	40.35	1.6058	4	3.2117
2.5	180.75	2.2571	6.25	5.6427
∑x=7.5	∑y=232.95	∑Y=3.81	∑ x^2=13.75	∑x·Y=10.4556

∑Y=3.81____ (1 Mark)
∑x·Y=10.4556____ (1 Mark)
The Normal equations are

6*A*+7.5*B*=3.81 ____ (1 Mark)
7.5*A*+13.75*B*=10.4556____ (1 Mark)
b) A= - 0.9916, *B*=1.3013____ (1 Mark)
a=0.102 and b=2.9963

$y = 0.102\, e^{2.9963x}$____ (1 Mark)

63. Question

The value of the property in a particular block follows a pattern of exponential growth. In the year 2001, a company purchased a piece of property in this block. The value of the property in thousands of dollars, t years after 2001 is given by the exponential growth model

$$V = 375e^{0.065\,t}$$

Hint: $A = A_0\, e^{kt}$, $k > 0$

a) Find the price of the property in the year 2011.

b) When will the property be worth 750 (in thousands of dollars)

Answer

a) t=10

$V = 375e^{0.065\,t}$. ____ (1 Mark)

V= 718.3278 thousands of dollars____ (1 Mark)

b) V=750

$750 = 375e^{0.065\,t}$.____ (1 Mark)

$t = \frac{\ln 2}{0.065} = \frac{0.6931}{0.065} = 10.664$____ (1 Mark)

After 10.7 years the property will worth 750 dollars approximately. ____ (1 Mark)

64. Question

A solar panel manufacturing company in California has been increasing its production capacity due to the growing demand for renewable energy sources. Over the years, the company has tracked the production levels of solar panels. The following table shows the number of panels produced (in thousands) at different time intervals.

Years since production started (x)	0	2	4
Panels produced (y) in thousands	5.012	10	31.62

a)As the company wants to model the production levels of solar panels fit an exponential model

$$\boxed{y = a\, e^{bx}}$$

for the given data.

Also estimate the production after 6 years since the production of the company started

Hint:

The normal equations for the given data are,

$3A+6B=3.2$

$6A+20B=7.9998$

$$y = a\, e^{bx}$$

takes the form

$$log\ y = log\ a + bx\ log\ e \Rightarrow Y = A + Bx$$

where,

$$Y = log\ y;\ A = log\ a\ and\ B = \ b\ log_{10}\ e$$

$$a = Antilog_{10}\ A\ and\ b = \frac{B}{log_{10}\ e}$$

Answer

A=0.6667, B=0.2____ (1 Mark)

a= 4.6421, b =0.4605
$y = 4.6421e^{-0.4605x}$____ (1 Mark)

To estimate y for x=6

$y(6) = 4.6421e^{-0.4605(6)}$

∴y (6) = (4.6421) (15.8435)
∴y (6) = 73.5473
Therefore, the production after 6 years is 73.55 thousand approximately. ____ (1 Mark)

65. Question

The Lagrange function

$$F(x, y, z, \lambda)$$

for a constraint

$$\phi(x, y, z) = c$$

is defined as

a)

$$F(x, y, z, \lambda) = f(x, y, z) + \lambda(\phi(x, y, z) - c)$$

b)

$$F(x, y, z, \lambda) = \phi(x, y, z) + \lambda(f(x, y, z) - c)$$

c)

$$F(x, y, z, \lambda) = f(x, y, z) + \lambda(\phi(x, y, z) + c)$$

d)

$$F(x, y, z, \lambda) = f(x, y, z) + \phi(x, y, z)$$

Answer

a)

$$F(x, y, z, \lambda) = f(x, y, z) + \lambda(\phi(x, y, z) - c)$$

66. Question

The surface area S of the closed rectangular container is

a)

$$2xy + 2xz + yz$$

b)

$$xy + xz + yz$$

c)

$$2xy + 2xz + 2yz$$

d)

$$xyz$$

Answer

c)

$$2xy + 2xz + 2yz$$

67. Question

An engineer is studying a temperature field in space described by the function:

$$T = 400xyz^2.$$

The engineer aims to determine the hottest point or region of maximum temperature on the surface of a unit sphere, constrained by the equation:

$$x^2 + y^2 + z^2 = 1.$$

Frame the auxiliary function

$$F = T + \lambda\phi$$

and find the first order partial derivatives with respect to x, y, and z.

Answer

$$F(x, y, z, \lambda) = 400\,xyz^2 + \lambda(x^2 + y^2 + z^2 - 1) \text{------------------(1 mark)}$$

$$\frac{\partial f}{\partial x} + \lambda\frac{\partial \phi}{\partial x} = 400yz^2 + \lambda(2x) \text{------------------------------(1 mark)}$$

$$\frac{\partial f}{\partial y} + \lambda\frac{\partial \phi}{\partial y} = 400xz^2 + \lambda(2y) \text{--------------------------------(1 mark)}$$

$$\frac{\partial f}{\partial z} + \lambda\frac{\partial \phi}{\partial z} = 800xyz + \lambda(2z) \text{--------------------------------(1 mark)}$$

68. Question

Find the minimum value of the function

$$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$

if the minimum point is (6,0).

a) 34
b) 72
c) 108
d) 120

Answer

c) 108

69. Question

A company is designing a closed rectangular cardboard box with a fixed volume of 1000 cubic centimetre to package a new product. The box must have a closed top and bottom, and the company wants to minimize the amount of material (cardboard) used for the box. The goal is to determine the dimensions x, y, and z (length, width, and height) of the box that will minimize the surface area, while maintaining the given volume.

Identify the relationship among the length, width, and height of the rectangular cardboard box.

Hint:

$$F = 2xy + 2yz + 2zx + \lambda(xyz - 1000)$$

Answer

$$\frac{\partial F}{\partial x} = 0 \Rightarrow 2y + 2z + \lambda yz = 0$$
$$\frac{\partial F}{\partial y} = 0 \Rightarrow 2x + 2z + \lambda xz = 0$$ ---------------------------------------(2 marks)
$$\frac{\partial F}{\partial z} = 0 \Rightarrow 2x + 2y + \lambda xy = 0$$
$$(1)x - (2)y \Rightarrow 2z(x - y) = 0$$
$z \neq 0, x - y = 0, x = y$ --(1 mark)
$$(1)x - (3)z \Rightarrow 2xy - 2yz = 0$$
$$y(x - z) = 0$$
$$y \neq 0, x = z$$
$x = y = z$ ---(1 mark)

70. Question

A farmer is trying to maximize the growth rate

$$f(x, y)$$

of a particular crop, which depends on two environmental factors:

$$x,$$

the temperature (in degrees Celsius) and

$$y$$

the humidity (in percentage). The growth rate is modeled as:

$$z = f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$

where

$$z$$

represents how the environmental variables

$$x$$

and

$$y$$

affects crop growth. Find the stationary points of growth rate function of a crop.
Hint:

$$f_x$$

refers the instantaneous rate of change of crop growth with respect to temperature.

$$f_y$$

refers the instantaneous rate of change of crop growth with respect to humidity.

$$A = f_{xx}$$

refers the whether the growth rate z increases or decreases at an accelerating or decelerating rate as temperature changes.

$$B = f_{xy}$$

refers the sensitivity of growth to temperature changes with humidity.

$$C = f_{yy}$$

refers whether the growth rate z increases or decreases at an accelerating or decelerating rate as humidity changes.

Answer

$$f_x = 3x^2 + 3y^2 - 30x + 72\,;\, f_y = 6xy - 30y$$ -------(1)

$$f_x = 0 \Rightarrow f_x = 3x^2 + 3y^2 - 30x + 72 = 0 \Rightarrow x^2 + y^2 - 10x + 24 = 0$$ -------(*)

$$f_y = 0 \Rightarrow 6xy - 30y = 0 \Rightarrow y(x-5) = 0 \Rightarrow y = 0, x = 5$$ -------(1)

If $y = 0$, eqn (*) $\Rightarrow x = 6, x = 4$

If $x = 5$, eqn (*) $\Rightarrow y = 1, y = -1$

The stationary points are $(6,0), (4,0), (5,1), (5,-1)$. -------(2)

71. Question

An engineer is assigned the task of designing a rectangular water tank open at the top. The objective is to minimize the amount of material required to construct the tank, while ensuring that the tank has a fixed volume of 50 cubic meters. Let x, y, and z represent the length, width, and height of the tank, respectively.
Identify the relationship among x,y, and z. Also, calculate the dimensions of the tank (length, width, and height).
Hint:

$$F(x,y,z,\lambda) = xy + 2yz + 2zx + \lambda(xyz - 50),\ \frac{\partial F}{\partial x} = 0 \Rightarrow y + 2z + \lambda yz = 0,$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow x + 2z + \lambda xz = 0,\ \frac{\partial F}{\partial z} = 0 \Rightarrow 2x + 2y + \lambda xy = 0.$$

Answer

$(1)x - (2)y \Rightarrow 2z(x-y) = 0$

$z \neq 0, x - y = 0, x = y$ --(1 mark)

$(1)x - (3)z \Rightarrow xy - 2yz = 0$

$y(x - 2z) = 0$

$y \neq 0, x = 2z$

$x = y = 2z$ --(1 mark)

Put $y = x, z = \frac{x}{2}$ in $xyz = 50$

$\frac{x^3}{2} = 50 \Rightarrow x^3 = 100 \Rightarrow x = 4.6416\text{ cm} = y$ ----------------------(1 mark)

$\Rightarrow z = 2.3208\text{ cm}$ ---(1 mark)

72. Question

A saddle point occurs when the Hessian determinant AC-B^2 is

a) Positive

b) Negative

c) Zero

d) Undefined

Answer

b) Negative

73. Question

A flat circular plate is heated so that the temperature at any point

$$(x, y)$$

is

$$u(x, y) = x^2 + 2y^2 - x.$$

Find the coldest temperature on the plate.

Answer

$$u_x = 2x - 1;\ u_y = 4y;\ u_{xx} = 2;\ u_{xy} = 0;\ u_{yy} = 4$$ -------(1)

$$u_x = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = 1/2$$

$$u_y = 0 \Rightarrow 4y = 0 \Rightarrow y = 0$$

The stationary point is $(1/2, 0)$. -------(1)

	Stationary Point
	(1/2,0)
A	2>0
B	0
C	4
AC-B²	8>0
Conclusion	Minimum pt

-------(1)

The coldest point on the plate is (1/2,0) and the coldest temperature is -0.25. -------(1)

74. Question

For

$$F(x, y, z, \lambda) = x + 2y - 3z + \lambda(x^2 + y^2 + z^2 - 10),$$

the derivative

$$\frac{\partial F}{\partial x}$$

is

a)

$$1 - 2\lambda x$$

b)

$$1 + 2\lambda x$$

c)

$$1 - 2\lambda$$

d)

$$1 - \lambda$$

Answer

b)

$$1+2\lambda x$$

75. Question

The Hessian matrix $AC-B^2$ is evaluated at a critical point to determine

a)The gradient at that point

b)Whether the critical point is a local maximum, local minimum, or saddle point

c)The value of the function at that point

d)The continuity of the function

Answer

b)Whether the critical point is a local maximum, local minimum, or saddle point

76. Question

Find the maximum value of the function

$$f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$

if the maximum point is (4,0).

a) 34

b) 72

c) 108

d) 112

Answer

d) 112

77. Question

For

$$F(x,y,z,\lambda) = x^2 + 3y^2 + zx + \lambda(xy + z - 3),$$

the derivative

$$\frac{\partial F}{\partial y}$$

is

a)

$$6y+\lambda$$

b)

$$6y+\lambda x$$

c)

$$6y+\lambda(x+z)$$

d)

$$3y+\lambda y$$

Answer

b)

$$6y+\lambda x$$

78. Question

A company is designing a closed rectangular container to hold a certain volume of products while minimizing the amount of material used. The goal is to find the dimensions of the container that will provide the maximum volume with the least surface

area.
Let x, y, and z be the dimensions of the rectangular container (length, width, and height).
The volume of the container is constrained to be 5000 cubic centimeters:

$$V=xyz=5000\ cm^3$$

Frame the auxiliary function

$$F=f+\lambda\phi$$

to solve this constrained optimization problem.

Answer

$$F=2xy+2yz+2zx+\lambda(xyz-5000)$$ ------------------------(2 marks)

79. Question

A farmer is trying to maximize the growth rate

$$f(x,y)$$

of a particular crop, which depends on two environmental factors:

$$x,$$

the temperature (in degrees Celsius) and

$$y$$

the humidity (in percentage). The growth rate is modeled as:

$$z=f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$$

where

$$z$$

represents how the environmental variables

$$x$$

and

$$y$$

affects crop growth. Find the stationary points of growth rate function of a crop.
Hint:

$$f_x$$

refers the instantaneous rate of change of crop growth with respect to temperature.

$$f_y$$

refers the instantaneous rate of change of crop growth with respect to humidity.

$$A=f_{xx}$$

refers the whether the growth rate z increases or decreases at an accelerating or decelerating rate as temperature changes.

$$B=f_{xy}$$

refers the sensitivity of growth to temperature changes with humidity.

$$C=f_{yy}$$

refers whether the growth rate z increases or decreases at an accelerating or decelerating rate as humidity changes.

Answer

$f_x = 3x^2 + 3y^2 - 30x + 72$; $f_y = 6xy - 30y$ -------(1)

$f_x = 0 \Rightarrow f_x = 3x^2 + 3y^2 - 30x + 72 = 0 \Rightarrow x^2 + y^2 - 10x + 24 = 0$ -------(*)

$f_y = 0 \Rightarrow 6xy - 30y = 0 \Rightarrow y(x-5) = 0 \Rightarrow y = 0, x = 5$ -------(1)

If $y = 0$, eqn (*) $\Rightarrow x = 6, x = 4$

If $x = 5$, eqn (*) $\Rightarrow y = 1, y = -1$

The stationary points are $(6,0),(4,0),(5,1),(5,-1)$. -------(2)

80. Question

An engineer is assigned the task of optimizing a custom shipping container for a new line of delicate, ultra-thin televisions at our environmentally-conscious and cost-effective shipping company. The objective is to design a rectangular shipping container that maximizes the number of televisions it can hold while minimizing material usage, thus reducing costs and environmental impact. The number of televisions the container can hold is determined by the ratio of the volume of the container to the volume of a single television.

Consider the rectangular shipping container open at the top, is to have a volume of 256 cc. Let x, y, z be the dimensions of the rectangular container and S and V denote its surface area and volume are given by S=xy+2yz+2zx and V=xyz=256.

Determine the optimal dimensions x, y, and z that will maximize the number of televisions that can be safely transported. Also, calculate the amount of material (area in square centimetres) needed to construct the container.

Hint:

$$F(x,y,z) = (xy + 2xz + 2yz) + \lambda(xyz - 256)$$

Answer

$$\frac{\partial F}{\partial x} = 0 \Rightarrow y + 2z + \lambda yz = 0,$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow x + 2z + \lambda xz = 0$$

$$\frac{\partial F}{\partial z} = 0 \Rightarrow 2x + 2y + \lambda xy = 0.$$ -------------------------------------(2 marks)

$(1)x - (2)y \Rightarrow 2z(x-y) = 0$

$z \neq 0, x - y = 0, x = y$

$(1)x - (3)z \Rightarrow xy - 2yz = 0$

$y(x - 2z) = 0$

$y \neq 0, x = 2z$

$x = y = 2z$ --(1 mark)

Put $y = x, z = \frac{x}{2}$ in $xyz = 256$

$\frac{x^3}{2} = 256 \Rightarrow x^3 = 512 \Rightarrow x = 8$ cm = y ------------------------(1 mark)

$\Rightarrow$ z = 4 cm --(1 mark)

$S = xy + 2xz + 2yz$ =80 square centimetres------------------------(1 mark)