BUAN/OPRE 6398
Prescriptive Analytics

Time Series Forecasting
(Non-stationary Data)

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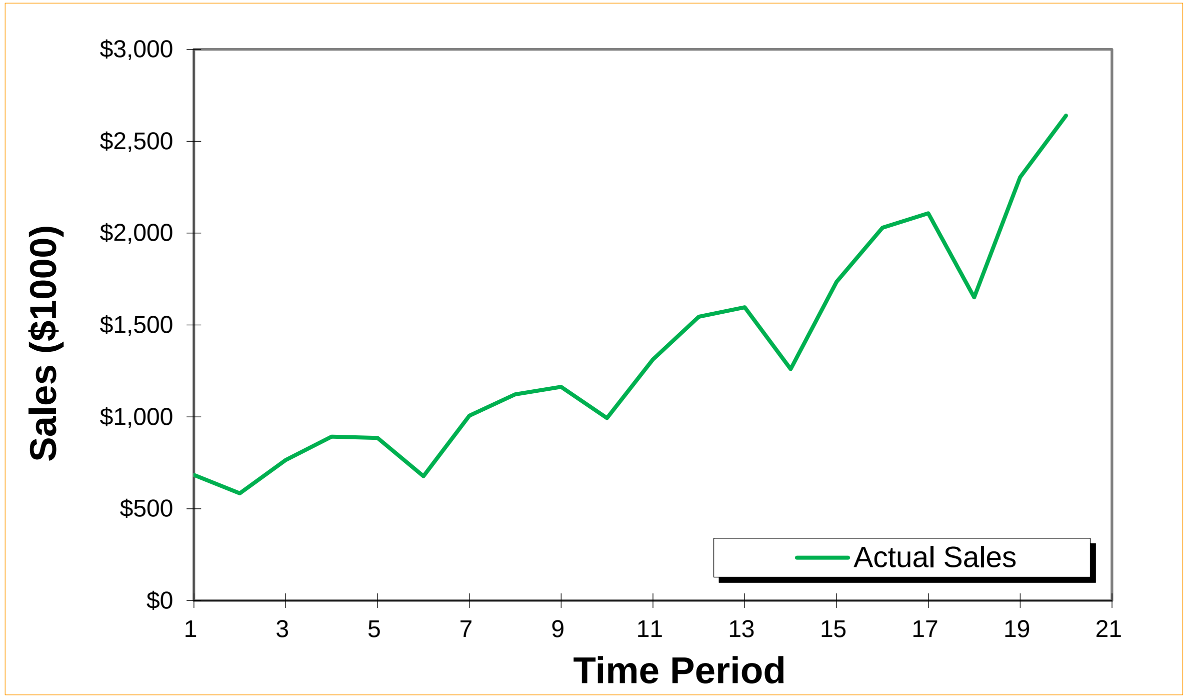
Lecture Outline

- Predicting Models for Nonstationary Data
  - Double Moving Average (FYI)
  - Double Exponential Smoothing (Holt’s Method)
  - Holt-Winter’s Method for Additive Seasonal Effects
  - Holt-Winter’s Method for Multiplicative Seasonal Effects

Nonstationary Data; An Example

- WaterCraft Inc. manufactures personal watercraft (jet skis).
- The company has enjoyed a fairly steady growth in sales of its products.
  - The managers of the company are preparing sales and manufacturing plans for the coming year.
  - Forecasts are needed of the level of sales that the company expects to achieve each quarter.

See file Fig11-19.xlsm



Double Moving Average Technique (FYI)

- Under the double MA technique,
  - The predicted value at period $t+n$ is:

$$\hat{Y}_{t+n} = E_t + nT_t$$

    - The expected level at period t: $E_t = 2M_t - D_t$
    - The expected trend at period t: $T_t = 2(M_t - D_t)/(k-1)$
- Where:
    - $M_t =$ Moving average over the current and past $k-1$ periods:

$$M_t = (Y_t + Y_{t-1} + \cdots + Y_{t-k+1})/k$$

    - $D_t =$ Double moving average over the current and past $k-1$ periods

$$D_t = (M_t + M_{t-1} + \cdots + M_{t-k+1})/k$$

Forecasting with the Double Moving Average Technique (FYI)

- Forecasts for time periods 21 to 24:
  - The current period is $t = 20$.

$$\hat{Y}_{t+n} = E_t + nT_t$$
$$\hat{Y}_{20+n} = E_{20} + nT_{20}$$

$$\hat{Y}_{21} = E_{20} + 1 * T_{20} = 2385.33 + 1 * 139.9 = 2525.23$$
$$\hat{Y}_{22} = E_{20} + 2 * T_{20} = 2385.33 + 2 * 139.9 = 2665.13$$
$$\hat{Y}_{23} = E_{20} + 3 * T_{20} = 2385.33 + 3 * 139.9 = 2805.03$$
$$\hat{Y}_{24} = E_{20} + 4 * T_{20} = 2385.33 + 4 * 139.9 = 2944.94$$

See file Fig11-20.xlsm

Double Exponential Smoothing Model (Holt's Method)

- The predicted value at period $t + n$ is:

$$\hat{Y}_{t+n} = E_t + nT_t$$

  - The expected level at period t: $E_t = \alpha Y_t + (1-\alpha)(E_{t-1} + T_{t-1})$; $0 \leq \alpha \leq 1$
  - The expected trend at period t: $T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$; $0 \leq \beta \leq 1$
- Forecasts for time periods 21 to 24:
  - The current period is $t = 20$.

$$\hat{Y}_{20+n} = E_{20} + nT_{20}$$

$$\hat{Y}_{21} = E_{20} + 1 * T_{20} = 2336.8 + 1 * 152.1 = 2488.9$$
$$\hat{Y}_{22} = E_{20} + 2 * T_{20} = 2336.8 + 2 * 152.1 = 2641.0$$
$$\hat{Y}_{23} = E_{20} + 3 * T_{20} = 2336.8 + 3 * 152.1 = 2793.1$$
$$\hat{Y}_{24} = E_{20} + 4 * T_{20} = 2336.8 + 4 * 152.1 = 2945.2$$

See file Fig11-24.xlsm

Holt-Winter's Method
Additive Seasonal Effects

- The predicted value at period $t + n$ is:

$$\hat{Y}_{t+n} = E_t + nT_t + S_{t+n-p}$$

  - The expected level at period t: $E_t = \alpha(Y_t - S_{t-p}) + (1-\alpha)(E_{t-1} + T_{t-1})$
  - The expected trend at period t: $T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$
  - The seasonal factor at period t: $S_t = \gamma(Y_t - E_t) + (1-\gamma)S_{t-p}$

$$0 \le \alpha, \beta, \gamma \le 1$$

- Forecasts for time periods 21 to 24:
  - The current period is $t = 20$.

$$\hat{Y}_{20+n} = E_{20} + nT_{20} + S_{20+n-4}$$

$$\hat{Y}_{21} = E_{20} + 1 * T_{20} + S_{17} = 2553.3 + 1 * 154.3 + 262.662 = 2670.3$$

$$\hat{Y}_{22} = E_{20} + 2 * T_{20} + S_{18} = 2553.3 + 2 * 154.3 + 312.593 = 2249.3$$

$$\hat{Y}_{23} = E_{20} + 3 * T_{20} + S_{19} = 2553.3 + 3 * 154.3 + 205.401 = 2921.6$$

$$\hat{Y}_{24} = E_{20} + 4 * T_{20} + S_{20} = 2553.3 + 4 * 154.3 + 386.116 = 3256.6$$

See file Fig11-27.xlsm

Holt-Winter’s Method
Multiplicative Seasonal Effects

- The predicted value at period $t+n$ is:

$$\hat{Y}_{t+n} = (E_t + nT_t)S_{t+n-p}$$

  - The expected level at period t: $E_t = \alpha(Y_t/S_{t-p}) + (1-\alpha)(E_{t-1} + T_{t-1})$
  - The expected trend at period t: $T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$
  - The seasonal factor at period t: $S_t = \gamma(Y_t/E_t) + (1-\gamma)S_{t-p}$

$$0 \le \alpha, \beta, \gamma \le 1$$

- Forecasts for time periods 21 to 24:
  - The current period is $t = 20$.

$$\hat{Y}_{20+n} = E_{20} + nT_{20} + S_{20+n-4}$$

$$\hat{Y}_{21} = (E_{20} + 1 * T_{20})S_{17} = (2217.6 + 1 * 137.3)1.152 = 2713.7$$

$$\hat{Y}_{22} = (E_{20} + 2 * T_{20})S_{18} = (2217.6 + 2 * 137.3)0.849 = 2114.9$$

$$\hat{Y}_{23} = (E_{20} + 3 * T_{20})S_{19} = (2217.6 + 3 * 137.3)1.103 = 2900.5$$

$$\hat{Y}_{24} = (E_{20} + 4 * T_{20})S_{20} = (2217.6 + 4 * 137.3)1.1190 = 3293.9$$

See file Fig11-30.xlsm

End of Lecture 10