BUAN/OPRE 6398
Prescriptive Analytics

Decision Analysis

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Lecture Outline

- Decision Rules
  - Non-probabilistic
    - Maximax
    - Maximin
    - Minimax Regret
  - Probabilistic
    - Expected Monetary Value (EMV)
    - Expected Opportunity Loss (EOL)
- Decision Trees
- Risk Analysis Techniques
  - Sensitivity Analysis
  - Tornado Charts
  - Strategy Tables
  - Strategy Charts

Introduction

- Models help managers gain insight and understanding but can’t make decisions.
- Decision-making often remains a challenging task due to the:
  1. Uncertainty regarding the future.
  2. Conflicting values or objectives.
- **Example:** You got an offer for a Data Analysis position from two companies: A and B
- Company A
  - In a new industry that could boom or bust.
  - Low starting salary but could increase rapidly.
  - Located near friends, family, and favorite sports team.
- Company B
  - An established firm with financial strength and commitment to employees.
  - Higher starting salary but slower advancement opportunity.
  - Distant location, offering few cultural or sporting activities.
- **Good Decisions vs. Good Outcomes**
  - A structured approach to decision-making can help us make good decisions, but it cannot guarantee good outcomes.
  - Good decisions sometimes result in bad outcomes.

Characteristics of Decision Problems

- **Alternatives/ Options:** Courses of action taken to make a decision. E.g.,
  - Working for Company A
  - Working for Company B
  - Rejecting both offers and keep searching
- **Criteria:** Factors that are important to the decision maker and are influenced by the alternatives.
  - Starting Salary
  - Career Potential
  - Location
- **States of Nature:** Future events that are not under the decision maker's control.
  - Company A grows
  - Company A bankrupts

An Example: Magnolia Inns

- Hartsfield International Airport in Atlanta, Georgia, has expanded many times to handle increasing air traffic.
- Commercial development around the airport prevents it from building more runways.
- Plans are being made to build another airport outside the city limits.

- Two possible locations for the new airport have been identified:
  - Location A
  - Location B
- A final decision will not be made for a year.
- The Magnolia Inns hotel chain intends to build a new facility near the new airport.

Payoff Matrix

Land purchased at:	Airport Location A	Airport Location B
A	$13	($12)
B	($8)	$11
A&B	$5	($1)
None	$0	$0

- For raw data, refer to the next slide.

Magnolia Inns, Raw Data

	Parcel of Land Near Location	
	A	B
Current purchase price	$18	$12
Present value of future cash flows if hotel and airport are built at the same location	$31	$23
Present value of future cash flows if hotel and airport are built at different location	$6	$4

(Note: All values are in millions of dollars.)

Magnolia Inn Problem's Characteristics

- **Decision Alternatives:**
  1. Buying the land at location A
  2. Buying the land at location B
  3. Buying at both locations
  4. Buying at none of the locations
- **Criteria:**
  – Net Present Value (payoff)
- **Possible States of Nature**
  1. The new airport will be built at location A.
  2. The new airport will be built at location B.

Decision Rules

- **Non-probabilistic Decision Rules:**
  - Those that do not assign any probability of occurrence to states of nature.
    - Maximax
    - Maximin
    - Minimax Regret
- **Probabilistic Decision Rules:**
  - Those that assign probabilities of occurrence to states of nature.
    - Expected Monetary Value (EMV)
    - Expected Opportunity Loss (EOL)
- Note: No decision rule is always the best, and each has its own weaknesses.

Decision Rules: Maximax and Maximin

- **Maximax:**
  - Identify the *maximum* payoff for each alternative.
  - Choose the alternative with the *largest* maximum payoff.

Land purchased at:	Airport Location A	B	MAX	
A	$13	($12)	**$13**	**<-- max**
B	($8)	$11	$11	
A&B	$5	($1)	$5	
None	$0	$0	$0	

- **Maximin**
  - Identify the *minimum* payoff for each alternative.
  - Choose the alternative with the *largest* minimum payoff.

Land purchased at:	Airport Location A	B	MIN	
A	$13	($12)	($12)	
B	($8)	$11	($8)	
A&B	$5	($1)	($1)	
None	$0	$0	**$0**	**<-- max**

Decision Rules:
Minimax Regret

- Identify the *maximum* regret (opportunity cost) for each alternative.
- Choose the alternative with the *smallest* maximum regret.
- The **regret value (opportunity cost)** of alternative i under the jth state od nature is:

$$g_{ij} = Max(r_j) - r_{ij}$$

  - r_{ij} = Payoff of alternative i under the jth state of nature
  - $Max(r_j)$ = The maximum payoff under the jth state of nature

Regret Matrix

Land purchased at:	Airport Location A	Airport Location B	MAX	
A	\$0	\$23	\$23	
B	\$21	\$0	\$21	
A&B	\$8	\$12	**\$12**	**<--min**
None	\$13	\$11	\$13	

Probabilistic Decision Rules

- Probabilistic decision rules can be used
  - if states of nature can be assigned probabilities representing their likelihood of occurrence.
- For decision problems that occur more than once, we can often estimate these probabilities from historical data.
- Many decision problems (such as the Magnolia Inns problem) represent one-time decisions where historical data for estimating probabilities do not exist.
  - In these cases, *subjective probabilities* are often assigned based on interviews with one or more domain experts.
- We will focus on techniques that can be used once appropriate probability estimates have been obtained.

Expected Monetary Value Rule

- This rule chooses the alternative with the largest expected monetary value (EMV):

$$EMV_i = \sum_j p_j r_{ij}$$

  - p_j = The probability of the jth state of nature
  - r_{ij} = Payoff for alternative i under the jth state of nature
- Suppose with a 40% chance the airport is built at location A and with a 60% chance it is built at location B.

EMV Matrix

Land purchased at:	Airport Location: A	B	EMV	
A	$13	($12)	($2.0)	
B	($8)	$11	**$3.4**	**<--max**
A&B	$5	($1)	$1.4	
None	$0	$0	$0.0	

Expected Opportunity Loss

- This rule selects the alternative with the smallest expected regret or opportunity loss (EOL):

$$EOL_i = \sum_j p_j g_{ij}$$

  - p_j = The probability of the jth state of nature
  - g_{ij} = Opportunity cost of alternative i under the jth state of nature

EOL Matrix

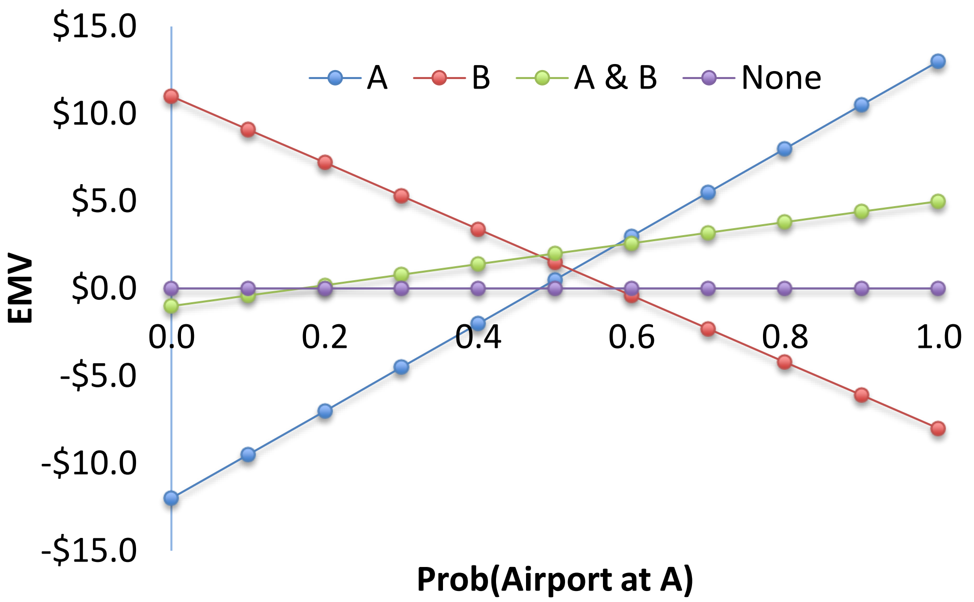
Land purchased at	Airport Location A	Airport Location B	EOL	
A	$0	$23	$13.8	
B	$21	$0	**$8.4**	**<--min**
A&B	$8	$12	$10.4	
None	$13	$11	$11.8	

- **NOTE:** The Maximum EMV and Minimum EOL rules always result in the selection of the same decision alternative.

Sensitivity Analysis

- Both EMV and EOL decision rules suggest purchasing a parcel of land at location B.
  - But this is under the assumption that $p_A = 40\%$. What if $p_A = 50\%$, $P_A = 55\%$, $P_A = 60\%$, etc?
- The sensitivity analysis helps to determine the best alternative under different values of P_A (and so of P_B).

P_A	EMV A	EMV B	EMV A&B	EMV None
	($2.0)	$3.4	$1.4	$0.0
0.0	(12.0)	**11.0**	(1.0)	0.0
0.1	(9.5)	**9.1**	(0.4)	0.0
0.2	(7.0)	**7.2**	0.2	0.0
0.3	(4.5)	**5.3**	0.8	0.0
0.4	(2.0)	**3.4**	1.4	0.0
0.5	0.5	1.5	**2.0**	0.0
0.6	3.0	(0.4)	2.6	0.0
0.7	**5.5**	(2.3)	3.2	0.0
0.8	**8.0**	(4.2)	3.8	0.0
0.9	**10.5**	(6.1)	4.4	0.0
1.0	**13.0**	(8.0)	5.0	0.0



- See file Fig14-1.xlsm (EMV Sensitivity Analysis)

EV of Perfect Information (PI) in Repeating Decision Problem

EV with PI = $\sum P_j max(r_j)$

EV of PI = EV with PI - maximum EMV

- Suppose the above decision problem was repeatable, and we could hire a consultant who could predict the future with 100% accuracy. I.e.,
  - 40% of the time, he would indicate that the airport will be built at A.
  - 60% of the time, he would indicate that the airport will be built at B.
- All else being equal, 40% of the time, Magnolia Inns buys the land at location A and 60% of the time at location B. Hence,

EV <u>with</u> PI = 0.4($13) + 0.6($11) = $11.8 (millions)

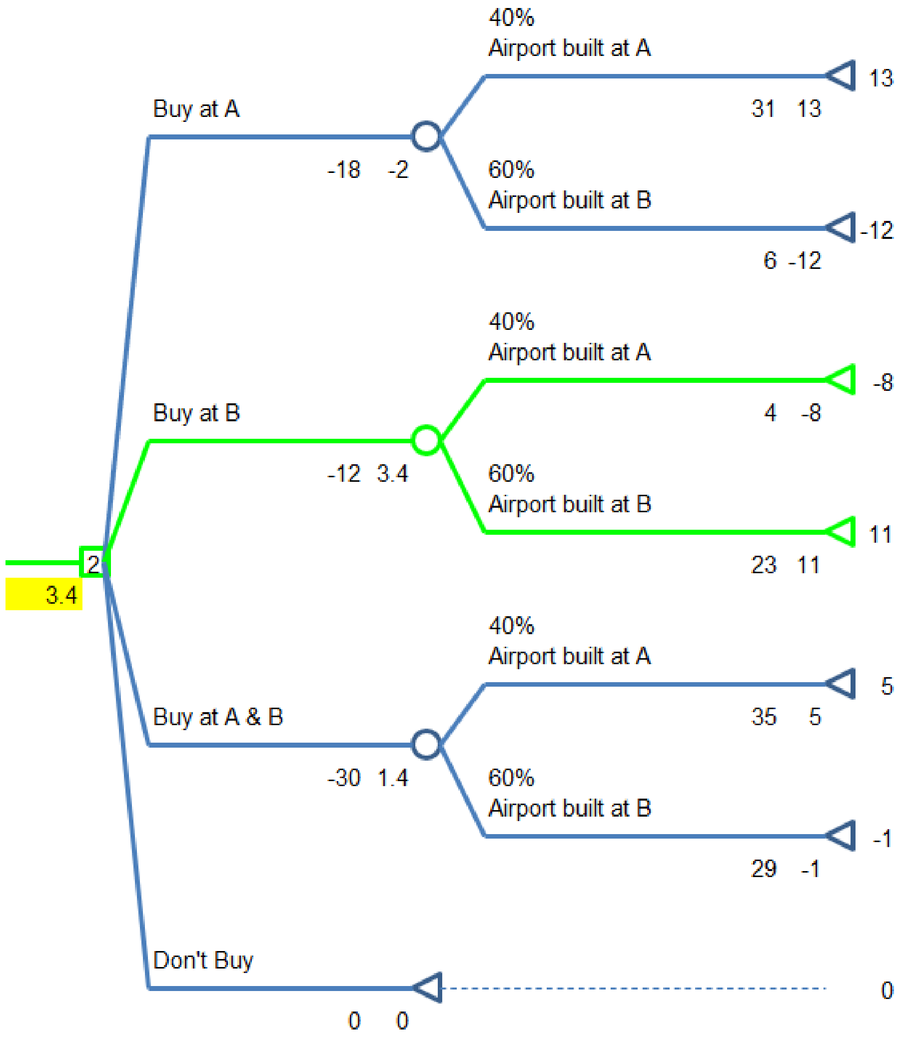
- Without perfect information, maximum EMV = $3.4 (millions). Thus,

EV <u>of</u> PI = $11.8 - $3.4 = $8.4 (millions)

- Note:

EV of PI = minimum EOL

Decision Tree for Magnolia Inns Hotel



- Based on EMV rule, buying land at location B (alternative 2) is the best alternative.

Max EMV = $3.4 (millions)

See file Fig14-19.xlsm

Multi-Stage Decision Example: COM-TECH

- The Occupational Safety and Health Administrator (OSHA) has decided to award an $85,000 research grant for using wireless communication technology to enhance safety in the coal industry.
- Steve Hinton, the owner of COM-TECH, is considering whether to apply for the grant.
- Steve would spend approximately $5,000 preparing the grant proposal and estimates a 50-50 chance of receiving the grant.
- If awarded, he would need to decide whether to use microwave, cellular, or infrared communication technology.

Technology	Equipment Cost
Microwave	$4,000
Cellular	$5,000
Infrared	$4,000

Multi-Stage Decision Example: COM-TECH (*con't*)

- Steve knows that he will also need to spend money on R&D, but he is not sure about the extract R&D costs.
- He has estimated the following best-case and worst-case R&D costs and associated probabilities.

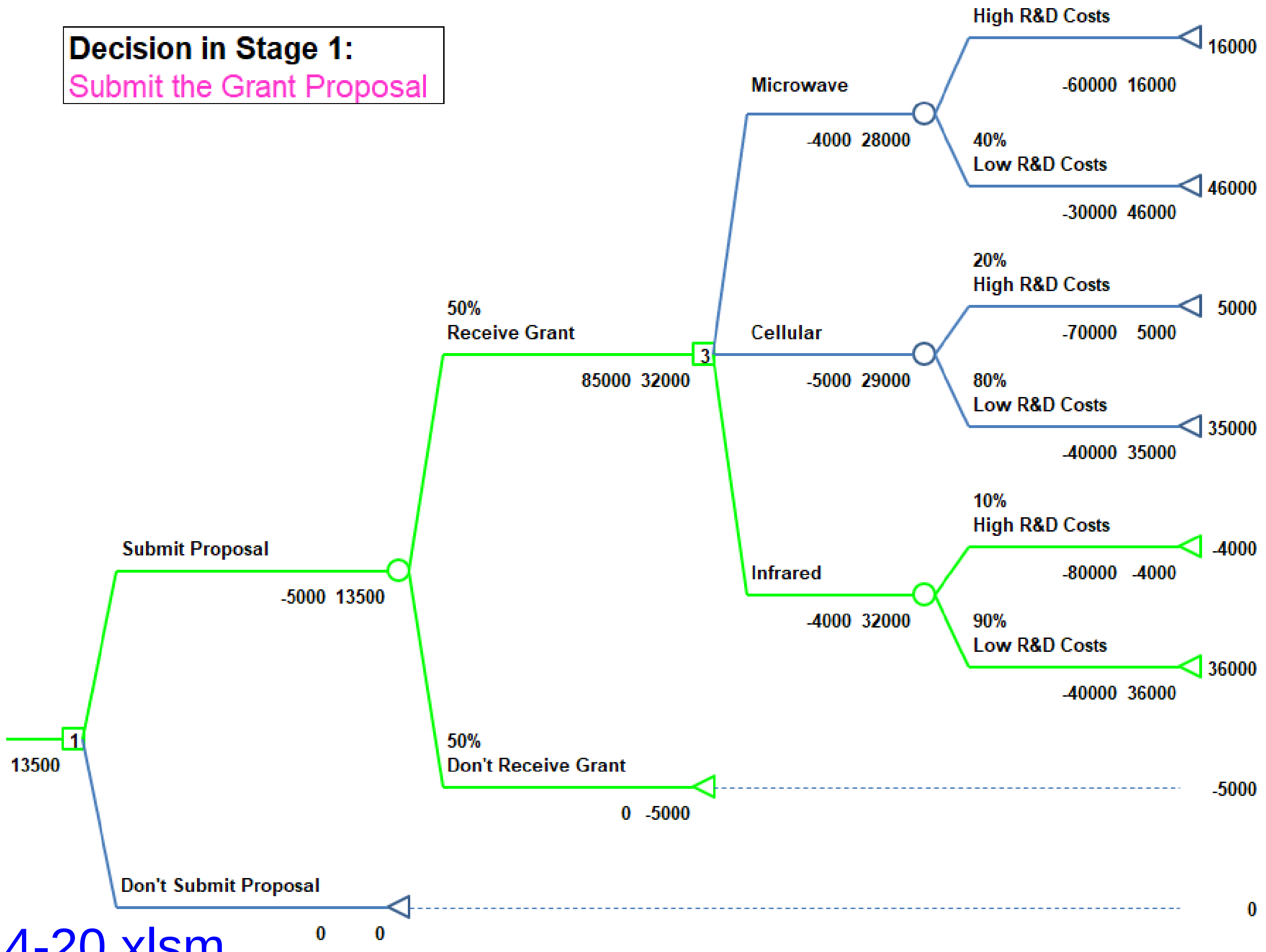
	Best Case		Worst Case	
	Cost	**Prob.**	**Cost**	**Prob.**
Microwave	$30,000	0.4	$60,000	0.6
Cellular	$40,000	0.8	$70,000	0.2
Infrared	$40,000	0.9	$80,000	0.1

- Steve needs to synthesize all the factors in this problem to decide whether to submit a grant proposal to OSHA.

COM-TECH Optimal Decisions

Decision in Stage 1:
Submit the Grant Proposal

Decision in Stage 2:
If grant received, choose the infrarad Technology



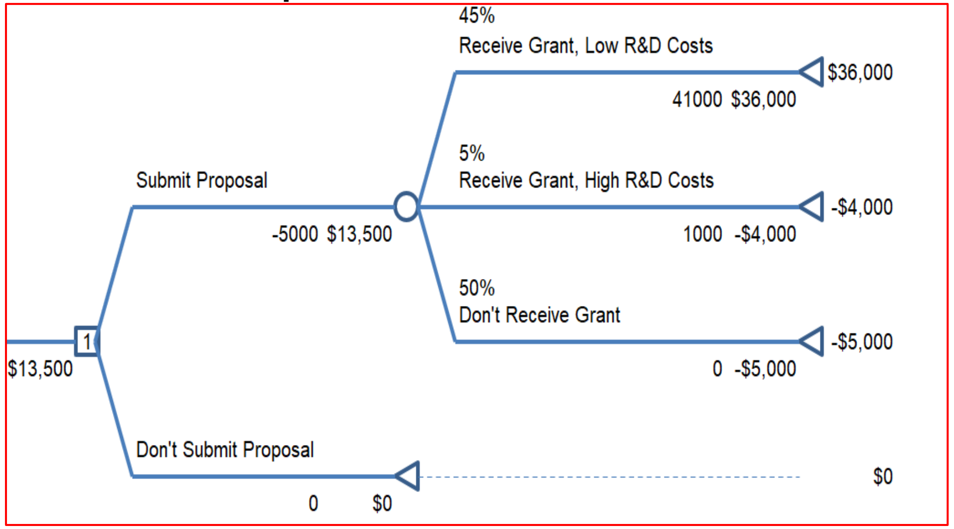
- See file Fig14-20.xlsm

Risk Profile

- The risk profile summarizes the makeup of EMV.
- The $13,500 EMV for COM-TECH was created as follows:

Event	Probability	Payoff
Receive grant and Low R&D costs (w/ infrared tech.)	(.5)(.9) = 0.45	$36,000
Receive grant and High R&D costs (w/ infrared tech.)	(.5)(.1) = 0.05	- $4,000
Don't receive grant	0.5	- $5,000
	EMV	$13,500

Risk profile decision tree



Sensitivity Analysis in a Decision Tree

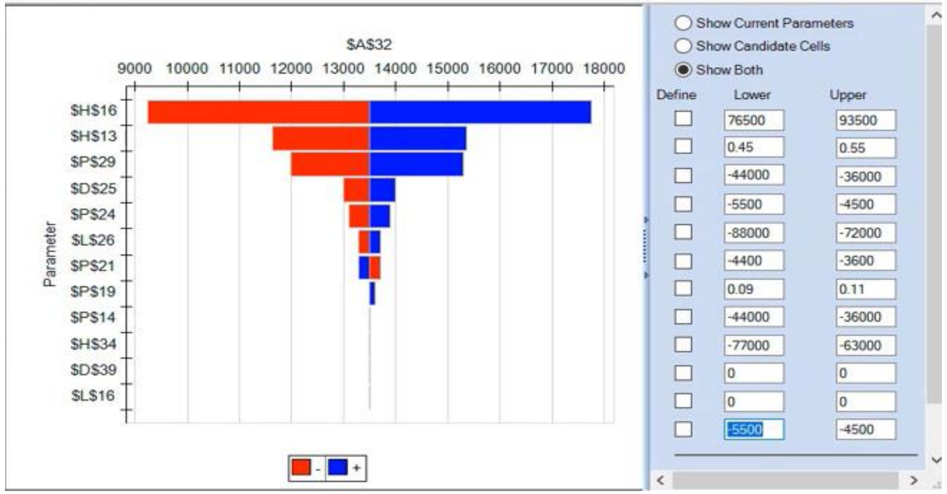
- How sensitive is the decision in the COM-TECH problem to changes in the probability estimates?
- We can use Solver to determine
  - The smallest "*probability of receiving the grant*" for which Steve should still be willing to submit the proposal.
- In Fig14-20.xlsm, formulate: H21 = 1 – H13. Then, …

  Min H13
  St: $0 \leq$ H13 ≤ 1
  B31 = 1

  - All else remaining constant, if the probability of receiving the grant is $\geq$ 13.51%, Steve should submit the proposal. Since in that case, the EMV of submitting the proposal is greater than 0.

Tornado Charts

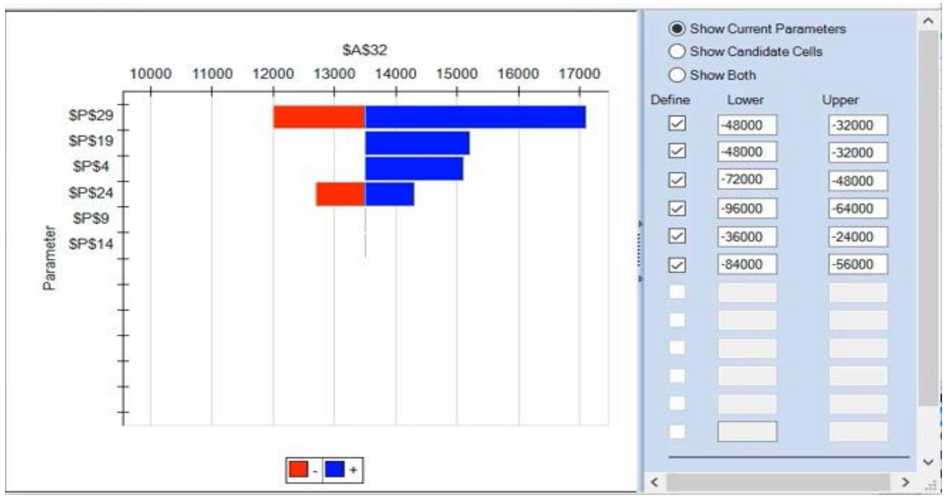
- How would EMV change if all inputs' value changes between, for example, $\pm 10\%$ of their original values, all else being constant?
- Which input has the largest effect on EMV?
- In Fig14-20.xlsm,
  - *Choose the EMV cell*
  - *ASP => Parameters => Identify*



- What if R&D costs change between, for example, $\pm 20\%$ of their initial values (V_0)?
- In Fig14-20.xlsm, formulate associate cells:

  PsiSenParam(-1.2V_0, -.8V_0, -V_0)

  - *Choose the EMV cell*
  - *ASP => Parameters => Identify*
  - *Show Current Parameters*



Strategy Tables

- How would the optimal decisions change if two inputs change simultaneously within some ranges? For example:

1. Probability of receiving the grant ranges between 0 and 1, incrementing by 0.10.
2. Probability of high R&D cost with the infrared technology ranges between 0 and 0.5, incrementing by 0.10.

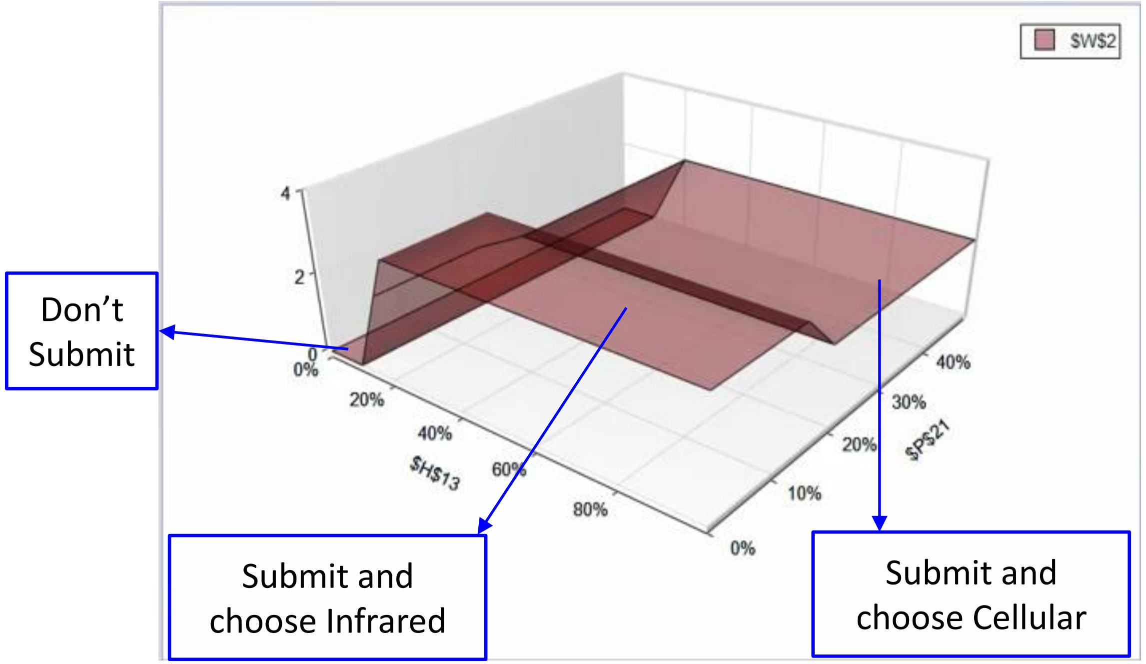
		Prob. Of High R&D Costs					
	Infrared	0.0	0.1	0.2	0.3	0.4	0.5
Prob. of Grant	0.0	Don't Submit	Don't Submit	Don't Submit	Don't Submit	Don't Submit	Don't Submit
	0.1	Don't Submit	Don't Submit	Don't Submit	Don't Submit	Don't Submit	Don't Submit
	0.2	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.3	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.4	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.5	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.6	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.7	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.8	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	0.9	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular
	1.0	Infrared	Infrared	Cellular	Cellular	Cellular	Cellular

- See file Fig14-25.xlsm

Strategy Charts

- A strategy Chart is the graphical representation of changes in optimal alternatives when two inputs change simultaneously.

- In Fig14-20.xlsm, formulate:
  - H13: =PsiSenParam(0,1)
  - P21: =PsiSenParam(0,.5)
  - W2: =IF(B31=2,0,J15)
- Then,
  - Select W2 => Analytic Solver
  - Charts => Sensitivity
  - Parameter Analysis
  - Vary Parameters Independently



End of Lecture 12