

## # DATA STRUCTURE

- \* Data Structure is a way of collecting & organizing data in such a way that we can perform operations on these data effectively.
- \* Anything that can store data can be called (as) datastructure, hence int, float, char etc. are the datastructure.
- \* They are known as primitive datastructure. (builtin)
- \* We have some complex datastructure, which are used to store large & connected data. These are known as abstract datastructure (User defined data structure). For ex- ~~LinkList~~, tree, graph, stack, Queue.
- \* All these datastructure allow us to perform different operations on data. We select these datastructure based on which type of operation is required. For example - Queue.

### DATA STRUCTURE.

#### PRIMITIVE

int char float double

↓ ADT

↓ USER DEFINED.

↓ Array List file

↓ Linear Non-linear  
queue stack tree graph

## # NEED OF DATA STRUCTURE .

- \* It gives different level of organization data . It tel
- \* It tells how data store & access in its elementary level .
- \* Provide operation on group of data , such as adding an item , deleting an item , updating an item , searching / an item etc.
- \* Provide a means to manage huge amount of data efficiently .
- \* Provide fast searching & sorting of data

## # OPERATIONS ON DATA STRUCTURE.

- \* Traversing
- \* Searching
- \* Inserting
- \* Deleting
- \* Sorting
- \* Merging

## # ELEMENTARY DATA ORGANIZATION

- \* Data - Data can be defined as representation of facts , concepts or instructions in a formalized manner suitable for communication , interpretation or processing by human or electric machine .
- b. Data is represented with the help of characters like alphabets , digit & special

character.

\* Data items - A set of character which are used together to represent specific data element is called data items or fields. for example- name of student in a class represented by dataitem, say Name

\* Record - Record is a collection of related dataitem.

\* File - file is a collection of logically related records. for example- payroll file might consist of payrecords for a company .

\* Entity - An entity is a person, place or thing, event or concepts about which information recorded .

\* Attributes - Attributes gives the characteristics or properties of entity .

\* Data value - A data value is a actual data or information contained in each attribute

# Primary & Secondary Key -

\* Primary Key - A field or a collection of fields in a record which identifies a record uniquely is called primary key . for example - ID NO.

\* Secondary Key - A field in a record which identifies the record but not uniquely is called Secondary Key.

## # ALGORITHM -

\* An algorithm is a finite set of instruction kept in a particular order to perform a particular task.

\* Algorithm is not a complete code or program. It is just a core logic (solution) of a problem which can be expressed as an high level formal description. It is also known as pseudo code.

\* An algorithm can be expressed in three ways

(i) In any natural language.

(ii) In a programming language or in

(iii) Or in form of flowchart.

OR PERFORMANCE

## \* EFFICIENCY OF ALGORITHM -

An algorithm is said to be efficient if fast if it takes less time to execute & consumed less memory space. The performance of an algorithm measured on the basis of two properties.

(i) Space Complexity

(ii) Time Complexity.

Programme under execution is known as process.

### \* SPACE COMPLEXITY -

It is amount of memory space required by an algorithm, during the execution of any algorithm.

Space complexity must be taken seriously for multiuser system & in situation where memory is limited.

An algorithm generally required space for following component.

#### (a) Instruction Space

It is a space required to store the executable version of programme. This space is fixed, but varies depending upon no. of lines of code in a program.

#### (b) Data Space.

It is the space required to store all the constants & variables (including temporary variable) values.

#### (c) Environment Space

It is the space required to store the environment information needed to resume the suspended process.

## \* TIME COMPLEXITY

The Time Complexity is a way to represent the amount of time required by the programme to run till its completion.

It is generally a good practice to try to keep the time required minimum, so that our algorithm complete its algorithm in minimum time possible.

## # ASYMPTOTIC NOTATION -

\* Asymptotic Notation are languages that allow us to analyze an algorithm's running time by identifying it's behaviour as the input size for an algorithm increases.

\* This is also known as algorithms growth rate.

\* Asymptotic Notation means a way to write down & talk about the fastest & the slowest possible running time of an algorithm.

## \* BIG O NOTATION

\* Big O is an <sup>asymptotic</sup> notation for worst case running time.

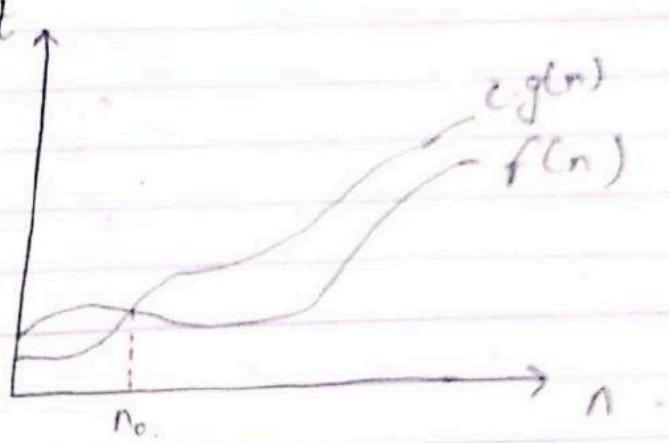
⑦ closet upper bound  $\rightarrow$  tight upper bound  
[www.aktutor.in](http://www.aktutor.in)

- This notation denotes the asymptotic tight upper bound of function  $f(n)$ .

Let  $f$  be a non-negative function then we define the three most uncommon asymptotic bound as follow-

$f(n)$  is  $O(g(n))$ . if there exist a two constant  $c \geq 0$  &  $n_0$  such that  $f(n) \leq c \cdot g(n)$  where  $c \geq 0$  &  $n_0 \leq n$

If  $0 \leq f(n) \leq c \cdot g(n)$ , we say that  $g(n)$  is tight upper bound of  $f(n)$ .



for example - let  $f(n) = 5n + 2$  &  $g(n) = n$

$$\text{for } c = 6$$

$$5n + 2 \leq 6 \cdot n$$

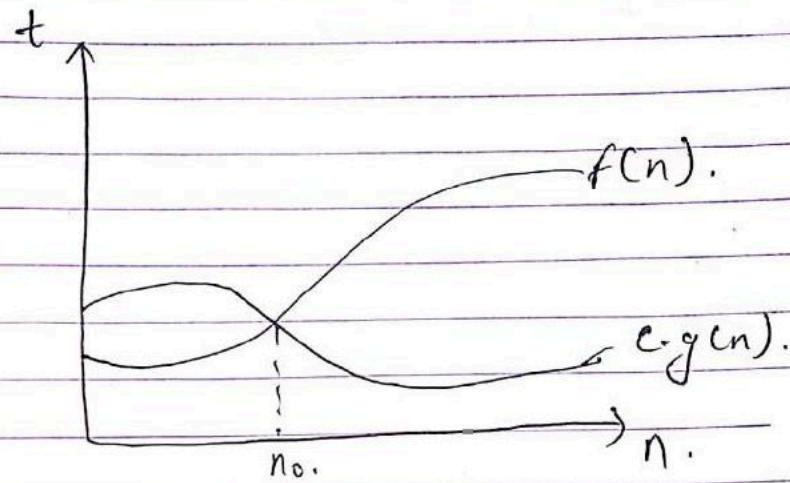
$$n = 2$$

## \* BIG SL NOTATION -

- It is an asymptotic notation for case running time.

- It provide asymptotically tight lower bound for given function  $f(n)$ .

•  $f(n)$  is said to be  $\Omega(g(n))$  if written as  $f(n) = \Omega(g(n))$ , if & only if (iff) there are +ve constant  $c$  &  $n_0$  such that  $0 \leq c \cdot g(n) \leq f(n)$ . for all  $n \geq n_0$ . if  $f(n) = \Omega(g(n))$  we say that  $g(n)$  is lower bound of  $f(n)$ .

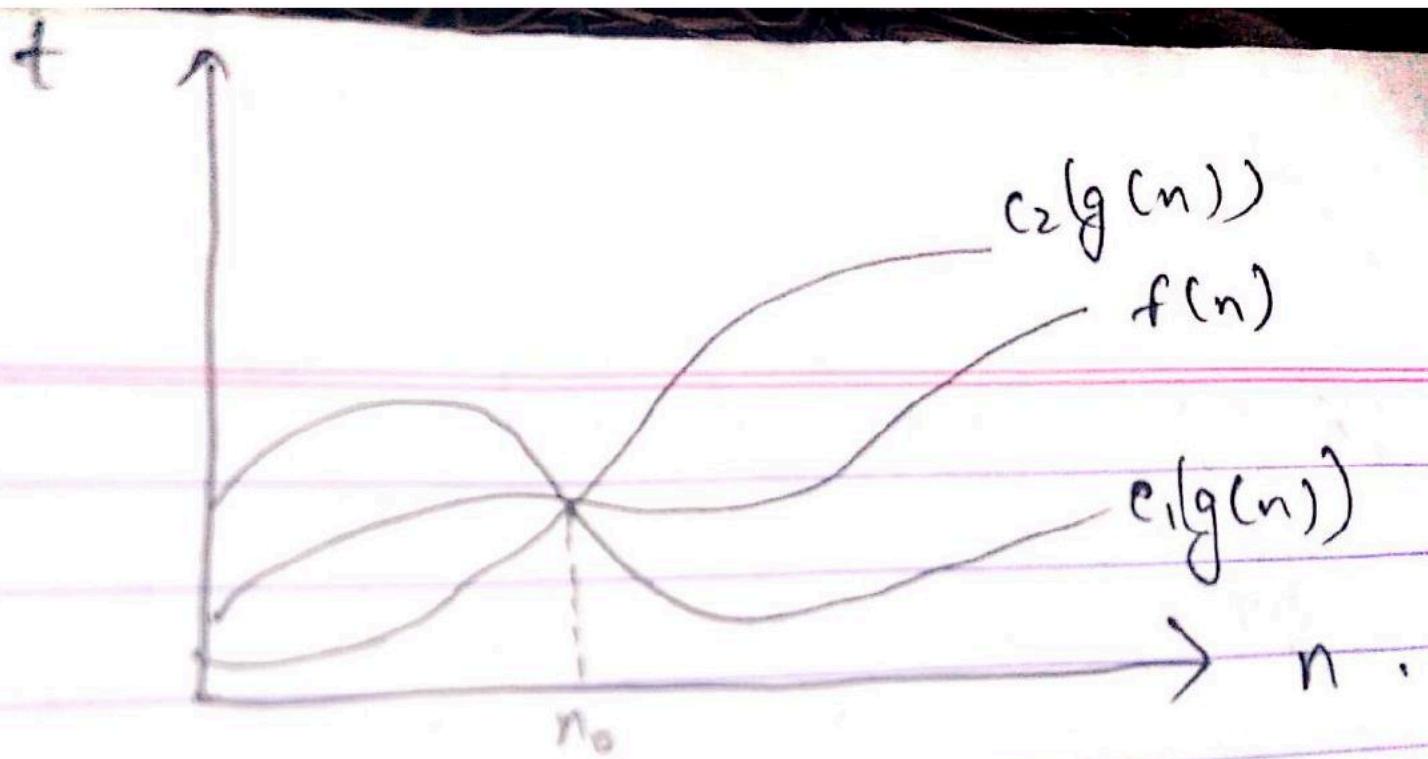


## \* BIG Θ NOTATION -

- It is an asymptotic notation for average case running time.

- It is an asymptotic notation to denote asymptotically tight upper bound & tight lower bound.

- We say  $f(n) = \Theta(g(n))$  written as  $f(n) = \Theta(g(n))$  iff there are +ve constant  $c_1, c_2$  &  $n_0$  such that  $0 \leq c_1(g(n)) \leq f(n) \leq c_2(g(n))$  &  $c_1 \neq c_2 > 0$  &  $n_0 \leq n$



Q    `for( i=1; i<=n; i++ ) {  
    for( j=1; j<=i; j++ ) {  
        for( K=1; K<=100; K++ )  
            printf( "ABC" );  
    }  
}`

$i = 1$	$1$	$2$	$3$	$\dots$	$K$	$\dots$
$j = 1$	$1$	$2$	$3$	$\dots$	$K$	$\dots$
$K = 100$	$100$	$100$	$100$	$\dots$	$100$	$\dots$
$P = 1 \times 100$	$2 \times 100$	$3 \times 100$	$\dots$	$K \times 100$	$\dots$	$\dots$

$$1 \times 100 + 2 \times 100 + 3 \times 100 + \dots + K \times 100.$$

$$100 (1+2+3+4+5+\dots+n) \\ 100 \left( \frac{n(n+1)}{2} \right) = O(n^2).$$

Q    `for( i=1; i<=n; i++ ) {  
    for( j=1; j<=i^2; j++ ) {  
        for( K=1; K<=n/2; K++ )  
            printf( "ABC" );  
    }  
}`

$i = 1$	$j = 2$	$i = 3$	$\dots$	$n$
$j = 1$	$j = 4$	$j = 9$	$\dots$	$n^2$
$K = n/2$	$K = n/2 \times 2$	$K = 9 \times n/2$	$\dots$	$n/2$
$(\times n/2)$	$\times n/2$	$\times 9 \times n/2$	$\dots$	$\dots$

PF  $1 \times \frac{n}{2} + 4 \times \frac{n}{2} + 9 \times \frac{n}{2} + \dots + n^2 \times \frac{n}{2}$ .

$$\frac{n}{2} + 4 \times \frac{n}{2} + 9 \times \frac{n}{2} + \dots + n^2 \times \frac{n}{2} = \frac{1}{2} (1^2 + 2^2 + 3^2 + \dots + n^2).$$

$$\frac{n}{2} \times \frac{n(n+1)(2n+1)}{6} = O(n^4).$$

Q for ( $i=1; i \leq n; i = i \times 2$ ),  
 $\text{printf}("ABC")$ ;

$$i=1 \quad 1 \leq n, 2 \leq n, 4 \leq n, 8 \leq n, 16 \leq n.$$

$$2^k = n \Rightarrow k = \log_2 n.$$

Q1. for ( $i = n/2; i \leq n; i++$ )  
 for ( $j = 1; j \leq n/2; j++$ ).  
 for ( $K = 1; K \leq n; K = K * 2$ ).  $O(n^4)$   
 $\text{printf}("ABC")$ ;

Q2. for ( $i = n/2; i \leq n; i++$ ).  
 for ( $j = 1; j \leq n; j = j * 2$ ).  
 for ( $K = 1; K \leq n; K = K * 2$ ).  $O(n^4)$   
 $\text{printf}("Shiv")$ ;

Q3  $A(C)$

while ( $n > 1$ )

$n = n/2$ .  $\rightarrow \frac{n}{2}$  times  $O(n)$   
 $\text{printf}("ABC")$ ;

## # ABSTRACT DATA TYPE - (ADT)

- \* Abstract Data Type is a process of providing only essentials & hiding the details is known as abstraction.
- \* Abstract Data Type is a type whose behaviour is defined by set of values & set of operations.
- \* The definition of ADT only mentions what operation are to be performed but not how these operations will be implemented.
- \* It involves what can be done in the data, not how has to be done.
- \* The user of data type only with the knowledge <sup>with</sup> ~~of~~ values that can take a operation that can be performed on them without any idea of how these types are implemented.

## # ARRAY

Array is a collection of homogenous data type.

Declaration -

Ex - `data type a [size];`

Actual Representation of memory -  
(Overview).

a	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

$a[1]$

2D - array declaration -

Ex - `data type a [size1][size2];`

`int a [5][4];`

a	0	1	2	3
0				
1				
2				
3				
4				

Overview Representation  
of memory.  
 $a[2][2]$ .

\* Row Major Order.

$$A[i, j] = B.S. + W[m[i]] + j].$$

$A[i, j]$  represents the address of  $i$ th row  $j$ th column.

B.S. stand for Base Address.

W stand for word size (size of data).  
M is a no. of column in an array.

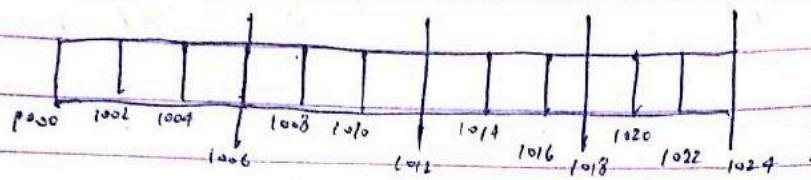
$$2000 + 4(6 \times 4 + 3)$$

$$2000 + 4(6 \times 4 + 3) = 2000 + 4(24 + 3) = 2000 + 108 = 2108$$

Ans

Q  $a[4][3]$ , whose B.A is 1000. Find B.A of  $a[3][2]$ .

$\Rightarrow$



$$a[3][2] = 1000 + 2(3 \times 3 + 2)$$

$$= \underline{1022}$$

Q If an array of size  $6 \times 5$  has an B.A. of 2000. What will be the B.A of  $a[3][4]$  R.M.O.

$$a[3][4] = 2000 + 4(5 \times 3 + 4)$$

$$a[3][4] = 2000 + 76$$

$$\Rightarrow \underline{2076}$$

C. M.O.

$$a[3][4] = 2000 + 4(6 \times 4 + 3)$$

$$= 2000 + 108$$

$$= 2108$$

\* COLUMN MAJOR ORDER.

When the indexing starts from zero.

$$A[i, j] = B \cdot A \cdot A M[j-1] + (i-1)$$

$$A[i, j] = B \cdot A + W(R(j) + i)$$

$$\begin{aligned}
 R &= B \cdot A + W(i - (R_i)C + j - (C_j)) \\
 &= BA + W(j - C_j)R + i - (R_i)C
 \end{aligned}
 \quad \left| \begin{array}{l} C = (C_{n-1}+1) \cdot (x+2) \\ R = C \\ 2 \quad 3 \quad 4 \\ \dots \quad \dots \quad \dots \\ 4 - C - 2 + 1 \end{array} \right. \quad \text{--- to } 4$$

## EFF SPARSE MATRIX -

- \* Matrix with maximum zero's element is called sparse matrix.
- \* A Matrix containing more no. zero's value than non-zero's / value is called sparse matrix.
- \* A matrix which is not sparse is called dense matrix.

## \* REPRESENTATION

A Sparse matrix can be represented by two method  $\textcircled{i}$  Triplet method if it is also known as array method -  $\textcircled{ii}$  Link List method.

## \* Representation of Sparse matrix using array -

- In this representation we consider only non-zero's value with their row & column index values.
- In this representation zeroth row stores total no. of rows, total no. of columns & total no. of non-zero's value in the sparse matrix.

for example consider a following sparse matrix

	0	1	2	3	4
0	0	0	0	1	0
1	0	2	0	0	0
2	0	0	2	3	0
3	0	0	0	0	1

$4 \times 5$

row	0	1	2	3
col.	3	1	2	3
value	1	2	2	3

$3 \times n$

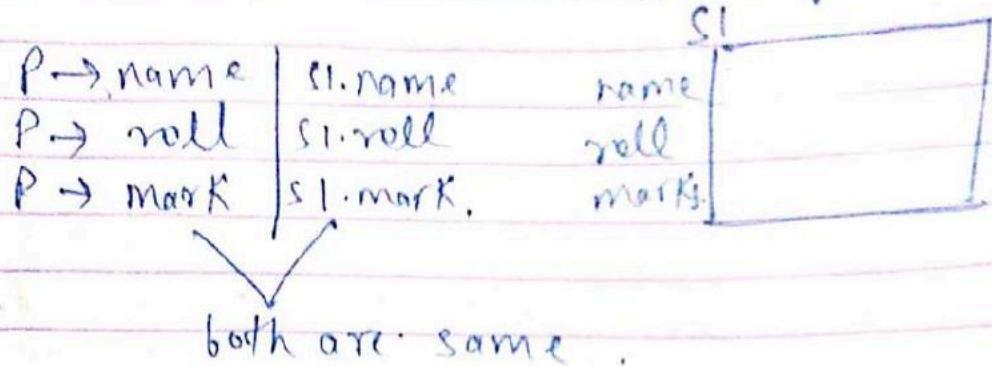
where  $n$  is non-zero values.

## \* / VIKR NISU -

\* Syntax for accessing a field by a pointer variable of structure type.

struct student s<sub>1</sub>, s<sub>2</sub>, \* p;

p = &s<sub>1</sub>; (storing address of structure variable to pointer var.)



## \* DYNAMIC MEMORY ALLOCATION

Allocation of memory during execution of program is known as dynamic memory allocation. This is done by three special function in C which are as follows:

- i) malloc()
- ii) calloc()
- iii) realloc()
- iv) free()

### (i) MALLOC

Malloc is used to create single unit of memory of given size during run time.

(7)

S = struct student \* malloc ((size of (student)));

Syntax for Malloc is -

pointer\_variable = (cast type) malloc (size in bytes);  
ex-      p = int \* malloc (size of (int));

(ii) CALLOC -

By using calloc function we create n element of corresponding datatype in contiguous memory allocation dynamically.

Syntax for Calloc() is →

pointer\_variable = (cast type) calloc (no of element,  
size of each element));  
ex-      p = (int\*) calloc (10, size of (int));

Q Create a student array using calloc →.  
struct student \*s;  
s = (struct student \*) calloc (10, size of (student));

s = (struct student \*) calloc (n, size of (struct student));  
s → name      |      s[0].name.  
(s+1) → name.      |      s[1].name.

Q WAP to create a student record having field name, rollno & marks. Print the detail of those student who had scored above 70% marks.

pointer\_variable = (cast type) realloc (ptr, new size in byte),  
free (s)

• If include <stdio.h>

struct marks

{  
    int m[5];  
};

struct student

{  
    char name[30];

    int rollno;

    int total;

    int per;

    struct marks sub;

};

void main()

{

    int i, j, n;

    struct student s[n], \*p;

    printf("Enter no. of Students ");

    scanf("%d", &n);

    struct student

    p = (struct student \*)calloc(n, sizeof(int));

    printf("Enter Student Details \n");

    for (i=0; i<n; i++)

{

        printf("Enter Student %d ", i+1);

        printf("Enter Student name ");

        scanf("%s", (p+i)→name);

        printf("Enter Student rollno ");

        scanf("%d", (p+i)→rollno);

        for (j=0; j<5; j++)

{

            printf("Enter subjects %d ", j+1);

16

8)  $\text{canf}(\text{obj}, (\text{p}+i) \rightarrow \text{sub.m}[j])$  ;

```
for(i=0; i<n; i++)
```

$$(\rho \mathbf{F}^i) \rightarrow \text{total} = 0;$$

```
for(j=0; j<5; j++)
```

$(p+i) \rightarrow \text{total} = (p+i) \rightarrow \text{total} + (p+i) \rightarrow \text{sub. m}[j];$

$(p+i) \rightarrow p.eff = (\text{float})(p+i) \rightarrow \text{total} / 5 ;$

~~for( p=0 ; i<n ; i++ )~~

if ((pt*i*) > per > 70 )  
{

```
printf("Student Name & Roll No.\n");  
scanf("%s %d", name, &roll);
```

```

printf("Student Name : %s", name);
printf("Roll No : %d", rollno);
for(j=0; j<5; j++)
{
    cout << " " << name[j];
}

```

```
printf("Subject mark of %d : %d\n", i, sub_marks[i]);
```

```

    printf("Subject mark of %d : %d\n",
    printf("Value of %d : (%d+i) → per

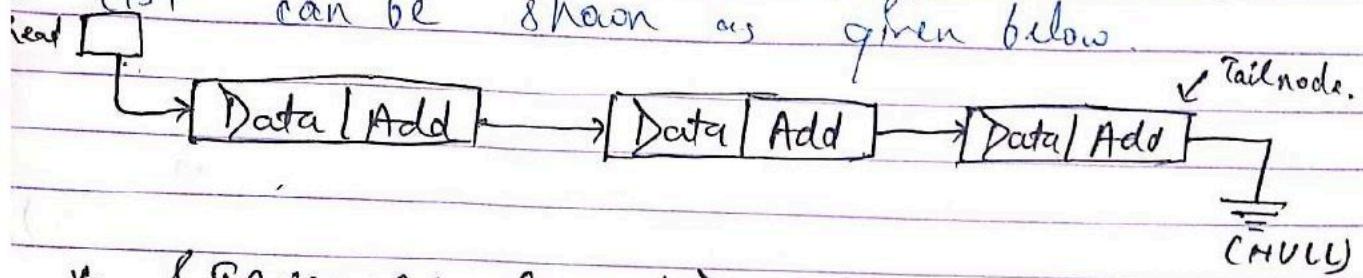
```

$\text{Profit} = (\text{Total Percentage is } \% \text{ of } , (p+i) \rightarrow \text{per})$

6

## # LINK LIST -

Link List is a collection of nodes where each node contains two part a data part & a address part where data part contains information of different field of record whereas address part is a self referential field which contains the address of next node. Link List has one special pointer variable known as head which contains the address of first node. The last node of link list which is pointing to nothing (null) is known as tail of link list. Link list can be shown as given below.



### \* STRUCTURE OF NODE IS

```

struct node
{
    datatype var1 ;
    datatype var2 ;
    :
    :
    datatype varn ;
} ;
```

} Address

Data

81

n → char null;

2

## \* SYNTAX FOR CREATING A NODE -

```

struct node
{
    int
    char a;
    struct node * link;
}; void main()
{
    struct node * p;
    p = (struct node *) malloc ( sizeof ( struct node ) );
}
for accessing a we use p → a .
for accessing next node p → next link .

```

## \* VARIOUS OPERATION PERFORMED ON LINKLIST

- i Searching
- ii Insert /
- iii delete
- iv Traverse

## \* TRAVERSE -

```

void Traverse ( struct node * p )
{
    while ( p != Null )
    {
        printf (" \n Value is %d ", p → a );
        p = p → link ;
    }
}

```

## \* INSERTION

- ① While inserting a node we create a new node dynamically to fill the value of each field given by user in data part.
- ② To search the position of node in existing list.
- ③ To link the node at desired position.

Insertion is done in three ways -

- i) Insertion at head. -

A new node is always inserted at head position.

- ii) Insertion at tail -

The new node is always inserted after the last node of existing list.

- iii) Insertion in between (At desired location).

The new node is inserted between two existing node in existing list

- i) At head.

```
void Insert_at_beginning(Struct node * phead)
```

```
Struct node * t;
```

```
t = (Struct node *) malloc(sizeof(Struct node));
```

```
printf("Enter any character ");
```

```
scanf("%c", t->data);
```

```
, t->link = p;
```

```
} p = t;
```

Inserion at desired location.

2. 9

### \* Insertion on Tail -

Void Insertionontail (struct node \* head)

```
{  
    struct node * n, * p;  
    n = (struct node *) malloc (sizeof (struct node));  
    printf ("Enter the character ");  
    scanf ("%c", &n->a)  
    n->link = NULL;  
    if (head == NULL)  
        head = n;  
    else  
    {  
        p = head;  
        while (p->link != NULL)  
            p = p->link;  
        p->link = n  
    }  
}
```

## \* SEARCHING -

```

void search( struct node *head, int n ) {
    int f = 0, c = 0;
    struct node *p;
    p = head;
    if ( p == NULL )
        printf( "\n Element Not found [list is empty] " );
    else
        {
            while ( p != NULL ) {
                if ( p->a == n )
                    printf( "\n Element found in position %d ", f + 1 );
                c++;
                p = p->link;
            }
            if ( c == 0 )
                printf( "\n Element Not found " );
        }
}

```

## # DELETION

- \* Deletion of Head Node -

```
void Delete (struct node *head)
```

```
struct node *p;
```

```
p = head;
```

```
if (p == NULL)
```

```
printf ("\\n Link List is empty ");
```

```
else
```

```
head = head->link;
```

```
free (p);
```

```
}
```

- \* Deletion of Tail Node -

```
void Delete (struct node *head)
```

```
{
```

```
struct node *p, *q;
```

```
p = head;
```

```
if (p == NULL)
```

```
printf ("\\n Link List is empty ");
```

```
else
```

```
q = p;
```

```
while (p->next != NULL)
```

```
q = p;
```

```
p = p->next
```

```
if (p == q)
```

```
{
```

```
head = NULL
```

```

    free(p);
}
else
{
    q->next = NULL;
    free(p);
}
}
}

```

\* Deletion in between.

```

void Delete (struct node * head),
{
    struct node * p, * q;
    int t;
    p = head;
    if (p == NULL)
        printf ("In Link list is empty");
    else
    {
        printf ("Enter the value");
        scanf ("%d", &t);
        q = p;
        while (p != NULL)
        {
            if (t == p->a)
                break;
            q = p;
            p = p->next;
        }
        if (q == p)
            printf ("Value not found");
        else
        {
            q->next = p->next;
            free(p);
        }
    }
}

```

```

} p = p->link;
if (p == NULL)
    printf("Value not found in list No
           node deleted");
else
{
    if (p == q)
    {
        head = NULL
        free(p);
    }
    else
    {
        q->next = p->next
        free(p);
    }
}

```

## # CIRCULAR LINK LIST

## ~~#~~ ① DOUBLY LINK LIST -

Insertion On Head -

Void Insert\_head ( )

```
struct node *p, *pre, *next;
p = (struct node*) malloc (sizeof(struct node));
printf ("Enter the value \n");
scanf ("%d", &p->a);
if (head == NULL)
{
```

```
    head = p;
    p->pre = NULL;
    p->next = NULL;
```

else

```
{
```

p->pre = NULL;
p->next = head;
head->pre = p;
head = p;

}

## ② Insertion On Tail

void Insert\_tail ( )

```
struct node *p, *q, *pre, *next;
p = (struct node*) malloc (sizeof(struct node));
printf ("Enter the value \n");
scanf ("%d", &p->a);
```

```

if (head == NULL)
{
    head = p;
    p->prev = NULL;
    p->next = NULL;
}
else
{
    q = head;
    while (q->next != NULL)
    {
        q = q->next;
    }
    q->next = p;
    p->next = NULL;
    p->prev = q;
}

```

DOUBLY

(iii)

Insertion in between on the basis of data.

```

void Insert_in-between()
{

```

```

    struct node *p, *q;
    p = (struct node *) malloc(sizeof(
        struct node));
    printf("Enter the value \n");
    scanf("%d", &p->a);
    if (head == NULL)
    {
        head = p;
        p->link = NULL;
    }
}
```

```

} p->pre = NULL;
else {
    q = head;
    if (q->link == NULL)
        if (q->a < p->a)
            q->link = p;
            p->pre = q;
            p->link = NULL;
        }
    else
        {
            p->pre = NULL;
            p->link = q;
            q->pre = p;
            head = p;
        }
}
}
else if (head == NULL)
{
    new->pre = NULL;
    new->next = q;
    q->pre = new;
    head = new;
}
else if (q->next == NULL && q->a < new->a)
}

```

P.T.O.

## Insertion in b/w in decen

```

void insert(void)
{
    p = head;
    node * pre, * ne, * new;
    new = (node *) malloc (size of (node));
    printf ("Enter the value ");
    scanf ("%d", & new->a);
    if (p == NULL)
    {
        head = new;
        new->next = NULL;
    }
    else
    {
        pre = head;
        ne = pre->next;
        if (ne == NULL)
        {
            if (pre->a > new->a)
                new->next = head;
            head = new;
        }
        else
        {
            while (ne != NULL && pre->a < new->a && ne->a < new->a)
            {
                pre = ne;
                ne = ne->next;
            }
            pre->next = new;
            new->next = ne;
        }
    }
}

```

$\text{new} \rightarrow \text{next} = \text{pre} \rightarrow \text{next}$   
}  $\text{pre} \rightarrow \text{next} = \text{new};$

In Doubly Link list continue,

$\text{new} \rightarrow \text{next} = \text{null};$   
 $\text{new} \rightarrow \text{pre} = q;$   
}  $q \rightarrow \text{next} \neq \text{new};$

else  
{

$\text{new} \rightarrow \text{pre} = q \rightarrow \text{pre}$   
 $\text{new} \rightarrow \text{next} = q;$   
 $q \rightarrow \text{pre} \rightarrow \text{next} = \text{new};$   
}  $q \rightarrow \text{pre} = \text{new};$

}

UNIT-2.

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STACK

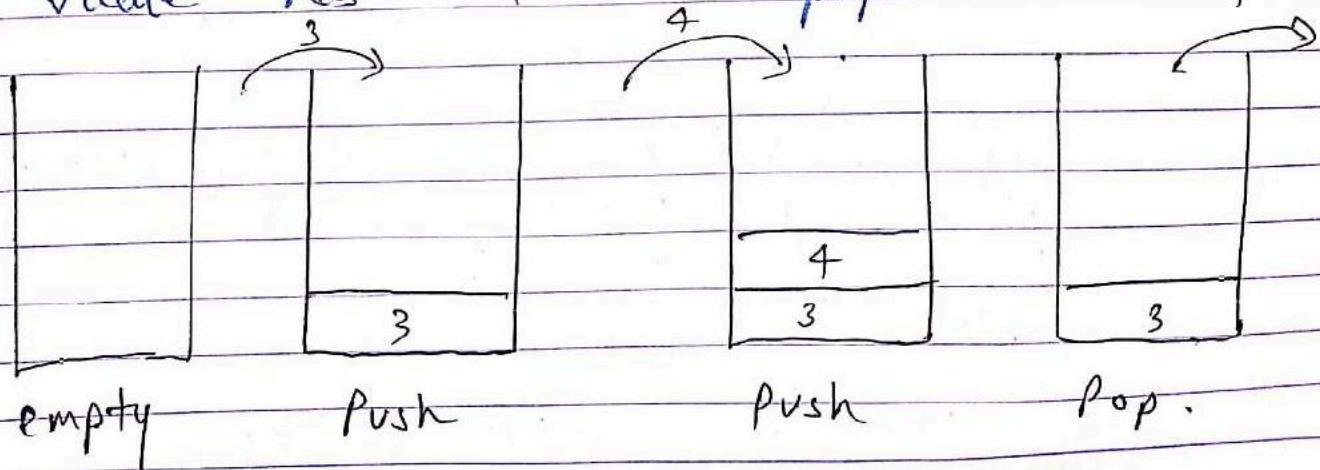
\* Stack is the data structure which is used to restore data in LIFO order. LIFO stand for Last In First Out order.

\* The element which is placed last is accessed first.

\* In stack terminology, insertion operation is called Push operation & removal operation is called Pop operation.

\* Both insertion & removal are allowed at only one end of the stack called TOP of the stack.

\* A stack data structure has a special variable called StackTop whose value denotes the position of stack where the value has to be inserted or from the value has to be pop.



\* Implementation of stack is done by using Array & Linklist.

## \* BASIC OPERATIONS OF STACK

- i Push()
- ii Pop()
- iii Peep()
- iv Isempty()
- v Isfull()

#include <stdio.h>

```

int isfull(void);
int isempty(void);
void push(void);
int peep(void);
int pop(void);
int a[50], top = 0;
void main(void)
{
    int i, n;
    char ch = 'y';
    while (toupper(ch) == 'Y')
    {
        printf("\n 1: PUSH \n 2: POP \n 3: PEEP\n");
        printf("\n Enter your choice ");
        scanf("%d", &i);
        switch (i)
        {
            Case 1: push(); break;
            Case 2: n = pop();
        }
    }
}

```

```

    deleted
printf ("In Value at top is %d ", n);
break;
Case 3 : n = peek();
printf ("In Value at top is %d ", n);
break;
default :
    printf ("In Wrong Choice ");
    printf ("Do you want to continue
Y/N ");
    scanf ("%c ", &ch);
}

int isempty ()
{
    if (top == 0)
        return 1;
    else
        return 0;
}

int isfull ()
{
    if (top == 50) // Size of stack array
        return 1;
    else
        return 0;
}

void push (void)
{
    int n;
    if (isfull ())

```

```

printf ("In stack is full");
else
{
    printf ("In enter the value ");
    scanf ("%d", &n);
    a[top] = n;
    top++;
}
printf ("Value inserted successfully");
int pop (void)
{
    int n;
    if (isempty ())
        printf ("Stack is empty ");
    else
        top--;
        n = a[top];
        return n;
}
printf ("Value deleted successfully");
int peep (void)
{
    int n;
    if (isempty ())
        printf ("Stack is empty ");
    else
        n = a[top - 1];
        return n;
}

```

IMP.

REVERSE PASS^

## # INFIX TO POSTFIX CONVERSION

$$\textcircled{Q} \quad (A+B/C * (D+E) - F)$$

Scan symbol

Stack

Postfix expression

Priority  
()   
+ or -   
\* or /   
+ or -

C	(	A
A	(	A
+	( +	A
B	( + *	AB
/	( + /	AB
C	( + *	ABC
*	( + * (	ABC / -
(	( + * (	ABC / -
D	( + * C	ABC / D
+	( + * C +	ABC / D
E	( + * ( +	ABC / DE
)	( + * E + )	ABC / DE +
-	( + * E - )	ABC / DE + * -
F	( + - )	ABC / DE + * - F -
)	( - )	ABC / DE + * - F - +

ABC / DE + \* - F - + is the postfix expression.

$$\textcircled{Q} \quad ((G * (F + E / D) - C * B) + A)$$

Scan

Stack

Postfix expression.

{

{ { } }

G

$$( ( G * ( F \uparrow E / D ) - C * B ) + A )$$

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*	((*)	G
C	((*)C	GF
F	((*)C	GF
\uparrow	((*)C\uparrow	GF
E	((*)C\uparrow	GF E
/	((*)C/A)	GF E \uparrow
D	((*)C/A)	GF E \uparrow D
)	((*)C(A))	GF E \uparrow D /
-	((*)C(A))	GF E \uparrow D / *
C	((C -	GF E \uparrow D / * C
*	((C - *)	GF E \uparrow D / * C
B	((C - *)	GF E \uparrow D / * C B
)	((C - *))	GF E \uparrow D / * C B * -
+	( +	GF E \uparrow D / * C B * -
A	( +	GF E \uparrow D / * C B * - A
)	( + ) <sub>empty</sub>	GF E \uparrow D / * C B * - A +

GF E \uparrow D / \* C B \* - A + is the postfix expression

# INFIX TO PREFIX CONVERSION -

Q  $((G * (F \uparrow E / D) - C * B) + A)$

$$(A + (B * C - (D / E \uparrow F) * G))$$

Reverse infix  $\rightarrow$  Postfix.  
Postfix  $\leftrightarrow$  Reverse.

$$( A + ( B * C - ( D / E \uparrow F ) * G ) ). \quad 20$$

Scan Symbol	Stack	Postfix Expression.
(	(	
A	(	A
+	( +	A
(	( + (	A
B	( + C	AB
*	( + C *	AB
C	( + C *	ABC
-	( + C * -	ABC *
(	( + ( - (	ABC *
D	( + ( - (	ABC * D
/	( + ( - ( /	ABC * D
E	( + ( - ( /	ABC * D E
↑	( + ( - ( / ↑	ABC * D E
F	( + ( - ( / ↑	ABC * D E F
)	( + ( - ( / ↑ )	ABC * D E F ↑ /
*	( + ( - * )	ABC * D E F ↑ /
G	( + ( - * )	ABC * D E F ↑ / G
)	( + ( - * )	ABC * D E F ↑ / G * -
	( + )	ABC * D E F ↑ / G * - +

Now Prefix is +-\*G/↑FED\*CB A

## # POSTFIX TO INFIX.

Q G F E ↑ D / \* C B \* - A +

Scan

Top of stack

Expression stack.

G

F

E

↑

G

F

E

E ↑ F

D

D

/

D / (E ↑ F)

\*

(D / (E ↑ F)) \* G

C

B

C

B

\*

B \* C

-

(B + C) - ((D / (E ↑ F)) \* G)

[ G ]

[ G ]

[ E ]

[ E ↑ F ]

[ D ]

[ D / (E ↑ F) ]

[ (D / (E ↑ F)) \* G ]

[ C ]

[ C ]

[ B \* C ]

[ (B + C) - ((D / (E ↑ F)) \* G) ]

A

A

$$\frac{A}{(B*C) - ((D/(E+F))*G)}$$

+

$$A + (B*C) - ((D/(E+F))*G)$$

$$\frac{A + (B*C) - ((D/(E+F))*G)}{}$$

# PREFIX TO INFIX.

Q

$$+ - * A B / C D E$$

Reverse  $\rightarrow$  E D C / B A \* - +

E

E

$$\frac{E}{}$$

D

D

$$\frac{D}{C}$$

C

C

$$\frac{C}{D}$$

/

C / D

$$\frac{C / D}{E}$$

B

B

$$\frac{B}{C / D}$$

A

A

$$\frac{A}{\frac{B}{\frac{C / D}{E}}}$$

\*

A \* B

$$\frac{A * B}{C / D}$$

$$- (A * B) - (C / D) \quad \boxed{\frac{(A * B) - (C / D)}{e}}$$

$$+ ((A * B) - (C / D)) + E \quad \boxed{((A * B) - (C / D)) + E}$$

Q) Convert <sup>Postfix</sup>  $AB - DC + F * /$  into infix.

A

A

 $\boxed{A}$ 

B

B

 $\boxed{\frac{B}{A}}$ 

-

 $(B - A)$  $\boxed{(B - A)}$ 

D

D

 $\boxed{\frac{D}{(B - A)}}$ 

E

E

 $\boxed{\frac{E}{\frac{D}{(B - A)}}}$ 

+

 $(E + D)$  $\boxed{\frac{(E + D)}{(B - A)}}$ 

F

F

 $\boxed{\frac{F}{\frac{(E + D)}{(B - A)}}}$ 

\*

 $F * (E + D)$  $\boxed{\frac{F * (E + D)}{(B - A)}}$

$$\frac{F \times (E+D)}{(B-A)}$$

$$\boxed{\frac{F \times (E+D)}{(B-A)}}$$

II Sessional start :-

## # RECURSION -

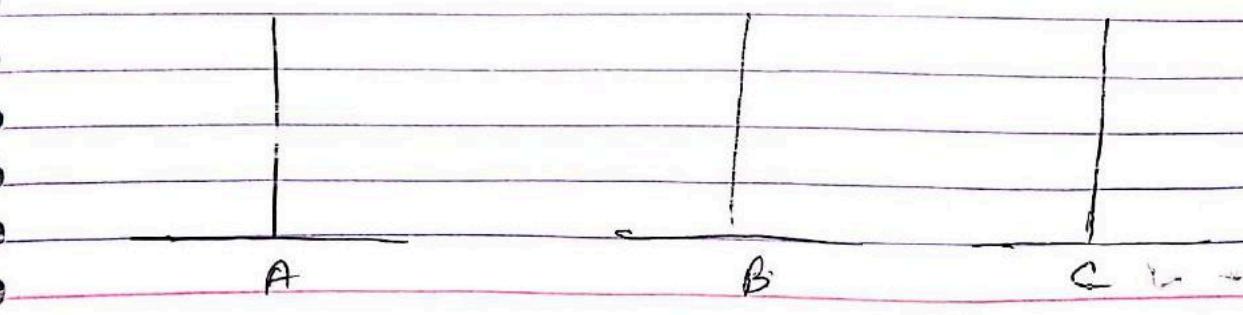
### \* TOWER OF HANOI

Tower of Hanoi is a mathematical puzzle where we have three poles or rod of  $n$  discs. The  $n$  disc is placed on one pole in decreasing order from bottom to top. The objective of puzzle is to move entire disc stack to another pole obeying the following rules-

Rule 1 - Only <sup>one</sup> disc can be move from one pole to another pole at a time.

Rule 2 - Each move consist of taking the upper disc from one of the stack & placing it on top of another stack it implies the disc can only be moved if it is upper most disc on the stack.

Rule 3 - No disc may be placed on the top of smaller disc



In General the pattern of movement of n disc will be -

Step 1 - Move n-1 disc from A  $\rightarrow$  B using C

Step 2 - Move a disc from A  $\rightarrow$  C

Step 3 - Move n-1 disc from B  $\rightarrow$  C using A.

By above movement we can see there is a recursive call in Step 1 & Step 3. The algorithm / function of Tower of Hanoi will be -

void TOH (int n, char A, char B, char C).

{ if (n > 0)

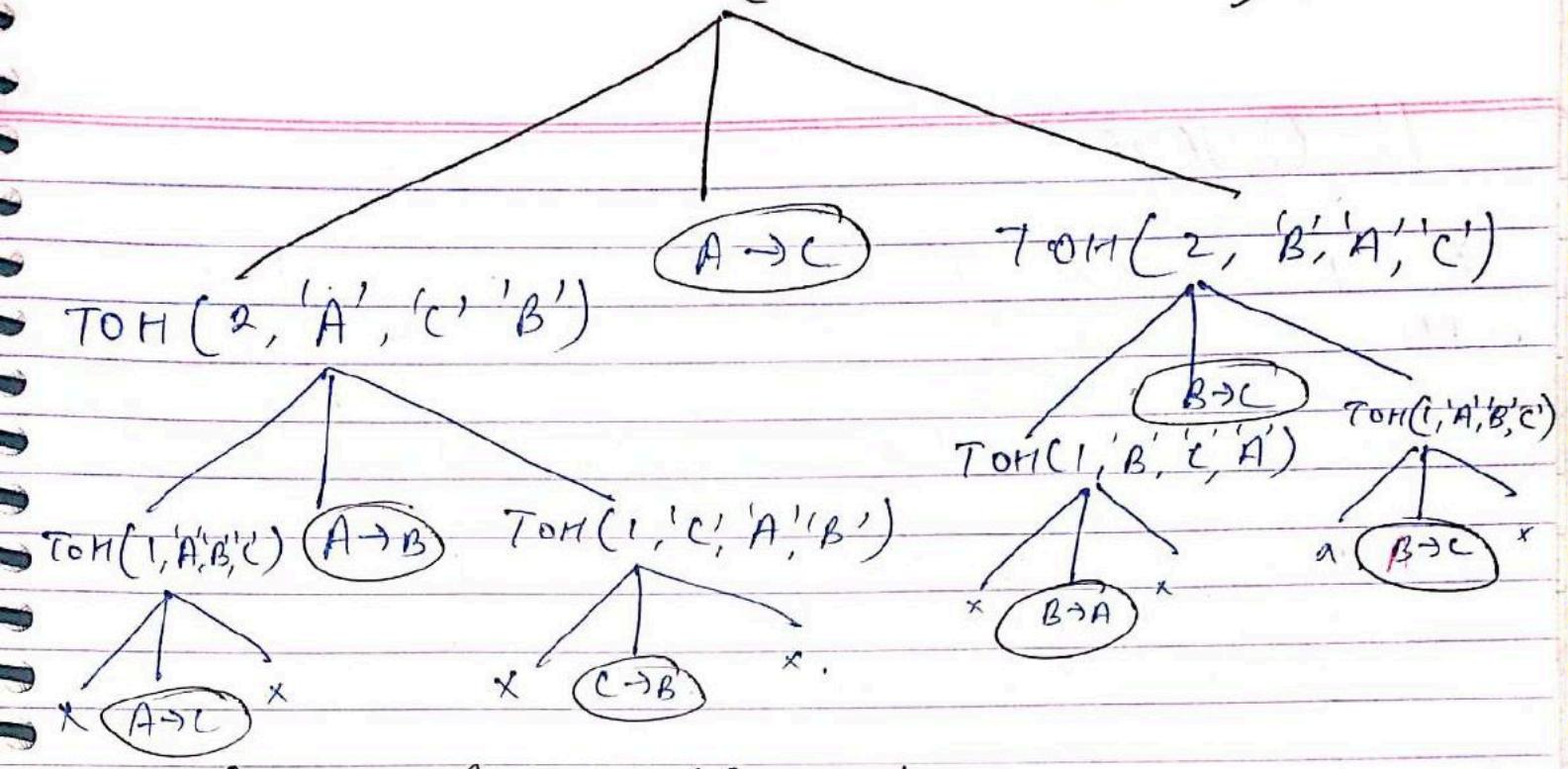
TOH (n-1, 'A', 'C', 'B')

printf ("Move Disc %c to %c ", A, C);

TOH (n-1, 'B', 'A', 'C');

}

}.



Order of execution step . -

$$(A \rightarrow C), (A \rightarrow B), (C \rightarrow B), (A \rightarrow C), (B \rightarrow A), (B \rightarrow C), \\ (\cancel{B} \rightarrow C)$$

Note - Total no. of movement of  $n$  disc will be  $2^n - 1$ .

In every step we have to show poles.

## # QUEUE -

- \* A queue is a linear data structure where the first element is inserted from one end called rear & deleted from other end called front.
- \* front points to the beginning of the queue whereas rear points to the end of the queue.
- \* Queue follows the FIFO (First in First Out)
- \* According to its FIFO structure element inserted first will also be removed first.
- \* In Queue, one end is used to insert data known as enqueue & the other end is used to delete data known as dequeue. The enqueue & dequeue are the two important function for any insertion & deletion operation performed on queue.

FUNCTION ( ).

```

⇒ int a[5], front = -1, rear = -1 ;
int isempty (void)
{
    if (front == -1 || front > rear).

```

```

    return 1 ;
else
    return 0 ;
}

```

```

int isfull(void)
{
    if ( Rear >= (size - 1) array )
        return 1 ;
    else
        return 0 ;
}

```

```

void enqueue(int x)
{
    if (isfull())
        printf("In Queue is full ");
    else if (Rear == -1 || front == -1)
        Rear = front = 0 ;
    else
        Rear++ ;
        a[Rear] = x ;
    }
}

```

```

int
void dequeue(void)
{
    int t ;
    if (isempty())
        printf("In Queue is empty ");
    else
    {
}

```

```

    t = a[front]
    f++;
    return t;
}

```

```

void display(void)
{
    int i;
    if (isempty(1))
        printf("Queue is Empty ");
    else
    {
        for (i = front; i <= rear; i++)
        {
            printf("%d ", a[i]);
        }
    }
}

```

## # CIRCULAR QUEUE .

### ⇒ FUNCTION ()

```

int isempty(void.)
{
    if (front == -1)
        return 1;
    else
        return 0;
}

```

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int isfull(void)

{

if ((front == 0 & & rear == size - 1) || (front == rear + 1))  
return 1;

else

return 0;

int. a[3] = q[2], {

}

a[3] = q[2] 3

void enqueue(int x)

{

if (isfull())

{

printf ("In Queue is full ");

Overflow

else

{

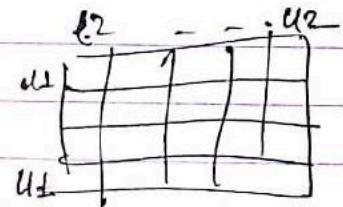
if (Rear == -1)

{

front = 0;

Rear = 0;

}.



else if (Rear == size - 1)

{

Rear = 0;

a[0:5]

(u1-l1+1)(u2-l2+1)

else

{

R = R + 1;

a[1:u1]

[u1-l1+1]

a[R] = x;

$$\begin{aligned} 5-1+1 &= 5 \\ 5-0+1 &= 6 \end{aligned}$$

}.

int cdequeue (void)

int x;

if ( isempty () )

} printf ( "\n", Underflow );  
return;

else

if ( x = a[front] );

if ( front == rear ) // single element.

{ front = -1;

Rear = -1;

} else if ( front == size - 1 )

{ front = 0;

else

{

front = front + 1;

}

return x;

} } .

void cdisplay (void)

int i;

if ( isempty () )

UNIT - 3

## # QUICK SORTING -

```
int partition(char *A, int p, int r)
```

```
    int x, i, j, t;
```

```
    x = A[r];
```

```
    i = p - 1;
```

```
    for (j = p; j <= r - 1; j++)
```

```
        if (A[j] <= x)
```

```
{
```

```
    i = i + 1;
```

```
    t = A[i];
```

```
A[i] = A[j];
```

```
A[j] = t;
```

```
}
```

```
t = A[i + 1];
```

```
A[i + 1] = A[r];
```

```
A[r] = t;
```

```
return i + 1;
```

```
}.
```

```
Quicksort (char *A, int p, int r).
```

```
int q;
```

```
if (p < r)
```

```
    q = partition(A, p, r);
```

```
    Quicksort (A, p, q - 1);
```

```
    Quicksort (A, q + 1, r);
```

```
} }
```

# MERGE SORT - ( $n \log_2 n$ )

Merge(A, p, q, r)

$$n_1 = q - p + 1;$$

$$n_2 = r - q + 1;$$

int  $L_1[n_1+1]$ ,  $L_2[n_2+1]$ ,  $L_1[n_1+1] = \infty$ ,  $L_2[n_2+1] = \infty$ ;

for ( $i = p$ ;  $i <= q$ ;  $i++$ )

$$L[i] = A[i];$$

for ( $j = 0$ ;  $j <= n_2$ );

$$R[j] = A[j].$$

$$i = 0;$$

$$j = 0;$$

for ( $K = p$ ;  $K <= r$ ;  $K++$ )

{

if ( $L[i] < R[j]$ )

$$A[K] = L[i];$$

$$i++;$$

}

else

$$A[K] = R[j];$$

$$j++;$$

} }.

Merge-Sort(A, p, r)

{

if ( $p < r$ )

{

$$q = (p+r)/2;$$

$$h = \log_2 n$$

$$\{2^{h+1} - 1\} = n \text{ (nodes)}$$

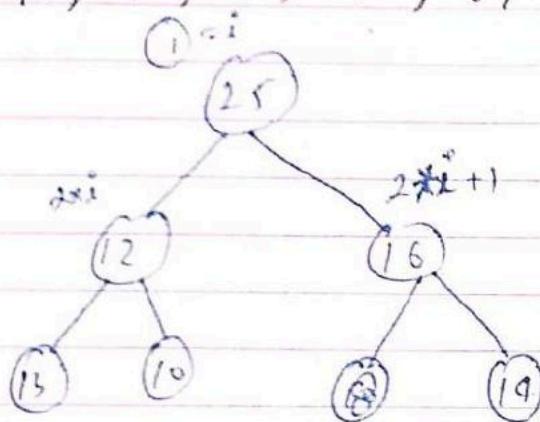
```

Merge-Sort(A, p, q);
Merge-Sort(A, q+1, r);
merge(A, p, q, r);
}

```

## # HEAP SORT

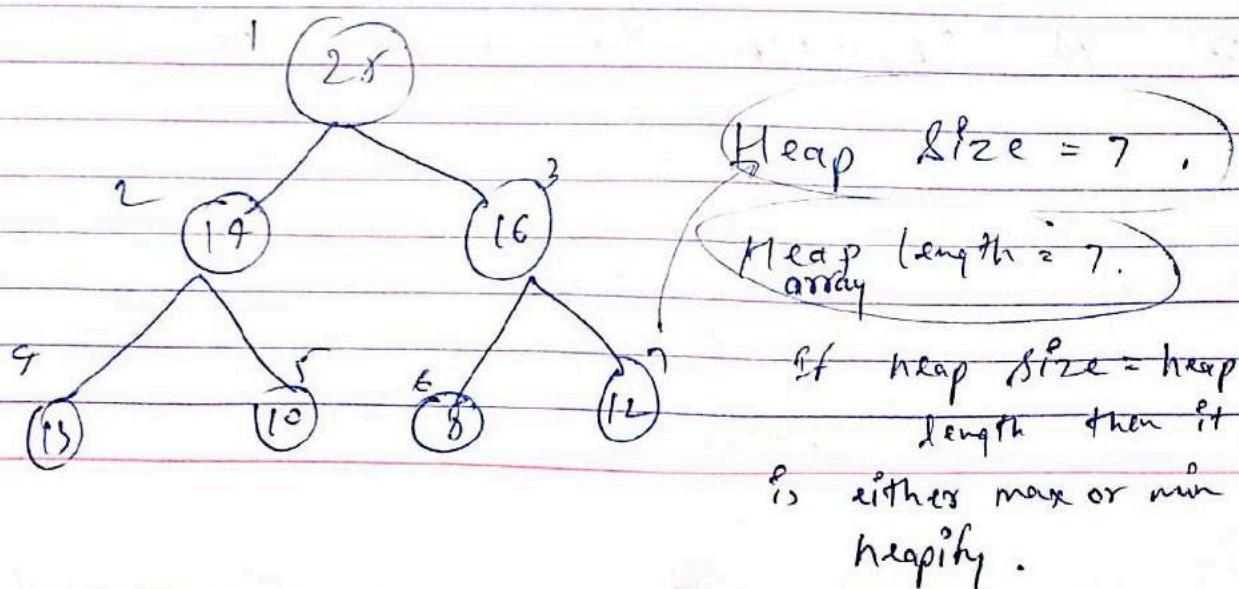
Construct almost binary tree of array  
 25, 12, 16, 13, 10, 8, 14



$$P(i^o) = \left[ \frac{i^o}{2} \right]$$

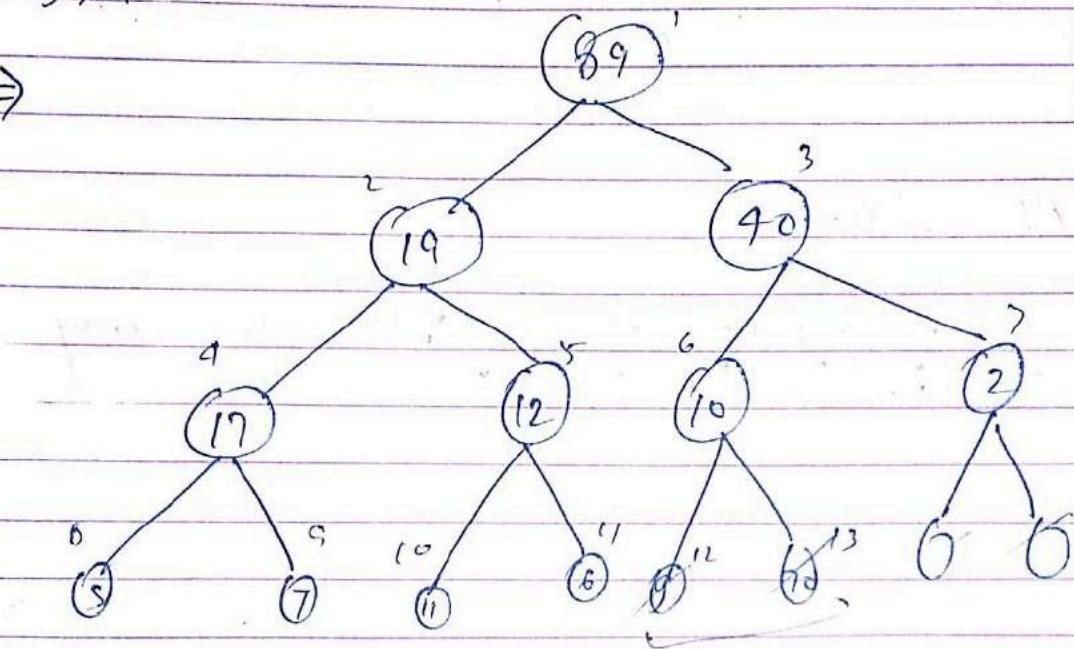
→ Heapsify      max Heapsify (for increasing order)  
 min Heapsify (for decreasing order)

Q      25, 14, 16, 13, 10, 8, 12. find Heap size



Q What is the heap size & heap length of given array - 89, 19, 40, 17, 12, 10, 2, 5, 7, 11, 6, 9, 70

⇒



Heap size = 13. + Heap length = 13.  
(Condition validates - 1)

Parent node from 1 to  $\left\lceil \frac{\text{size of array}}{2} \right\rceil$

Leaf node from  $\left\lceil \frac{\text{size of array}}{2} \right\rceil + 1$  to n.

## # ALGORITHM OF HEAP SORT -

{ Heap-Sort (A) }

① Build\_max\_heap (A)

for ( $i = A.length$  down to 2)

exchange  $a[1]$  with  $a[i]$

A.heapsize -- ;  
Max-heapify (A, i)

} build-Maxheap (A)

A.heapsize = A.length

for ( $i = 0$  to A.heapsize)

for ( $i \in A.length(n/2, down to 1)$ )

Max-heapify (A, i).

} max-heapify (A, i)

$$l = 2 * i$$

$$r = 2 * i + 1$$

if ( $l \leq A.heapsize \text{ and } A[l] \geq A[i]$ )

largest = l else largest = i

~~else if ( $r \leq A.heapsize \text{ and } A[r] > A[largest]$ )~~

largest = r.

if (largest  $\neq i$ )

exchange ( $A[i]$ ) with  $A[largest]$

Max-heapify (A, largest)

}

## # HASHING -

Q What is hashing?

Answer \* Hashing is the process of mapping large amount of data item to smaller table with the help of hashing function.

- \* Hashing is also known as message digest function.
- \* It is a technique to convert a range of key values into range of indexes of an array.
- \* It is used to facilitate next level searching method when compared with linear & binary search.
- \* Hashing allows to update & retrieve data entry in a constant time i.e.,  $O(1)$ , with the help of hashing function.
- \* Constant time  $O(1)$  means operation doesn't depend on size of data.
- \* Hashing is used with a database to enable to be retrieved more quickly.

Q What is hash function ?.

Answer A fixed process that convert a key & map it for a value of certain length which is called hash value or hash .

Hash value represents the original string of character , but it is normally smaller than the original value . The parameters of good hashing function are i.e., easy to compute , even distribution & minimize collision .

Q What is hash table ?.

Answer \* Hash table or hash map is a datastructure used to store Key value pairs .

\* It is a collection of item store to make it easy to find them later .

\* It uses a hash function to compute an index into an array of buckets or slot from which the desired value can be formed .

\* It is an array of list where each list is known as bucket ( a node of linked list )

\* It contains values based on the key .

- \* Hash table is used to implement the map interface & extends the dictionary class.
- \* Hash table is synchronized & contains only unique elements.

## # HASH FUNCTION TYPES - (Algorithm) H.W.

### i) DIVISION -

$$m = 1000 \quad (0-999) \quad (\text{No. of memory slots})$$

$$\text{Key} = 123456789$$

$$H(\text{Key}) = \text{Key \% } m$$

### ii) SQUARE METHOD -

$$m = 1000 \quad (0-999)$$

$$\text{Key} = 8492$$

$$H(\text{Key}) = (8492)^2 = 721(4064)$$

= 114 or 140,

### iii) Digit Extraction Method -

$$m = 10 \quad (0-9)$$

$$\text{Key} = 123456789$$

$$H(\text{Key}) = 5 \quad (4^{\text{th}} \text{ position})$$

$$m = 1000 \quad (0-999)$$

$$\text{Key} = 12345678910$$

$$H(\text{Key}) = 260(1,5,7)$$

### iv) Folding Method -

field  
Boundary

fold &  
shifting method.

### (a) Fold Boundary Method -

$$M = 1000 \quad (0-999)$$

Key = 123456789

$$H(\text{Key}) = 123 + 789 = 912 \text{ (index)}$$

Key = 789956789.

$$H(\text{Key}) = 789 + 789 = \underline{1578} \rightarrow 157 + 8 = 165 \text{ (index)}$$

### (b) Fold Shifting Method -

$$M = 1000 \quad (0-999)$$

Key = 123456789

$$H(\text{Key}) = (23 + 456 + 78 + 9) = \underline{1368} \rightarrow 136 + 8 = 144 \text{ (index)}$$

### (c) Binning Method -

$$M = 10 \quad (0-9)$$

Key Value Range (0-999)

0-99      Slot 0.

100-199      Slot 1

200-299      Slot 2

300-399      Slot 3

⋮ ⋮

900-999      Slot 9

## # COLLISION RESOLUTION TECHNIQUES -

Chaining (outside)

Open Addressing Method  
(Inside)

Linear Probing

Quadratic Probing

Double Hashing

## # COLLISION -

In hashing hash function used to compute hash value for a key. Hash value is then used as an index to store the key in the hash table. Hash function may return the same hash value for two or more key. When the hash value of a key map to an already occupied bucket of hash table it is called collision.

## # COLLISION RESOLUTION TECHNIQUE

Collision Resolution Technique is used for resolving or handling the collision. These techniques are classified as above given.

### \* Separate Chaining -

To handle the collision this technique creates a link list to the slot for which collision occurs. The new key is then inserted in the link list. These link list to the slot appears like a chain that's why this technique is known as

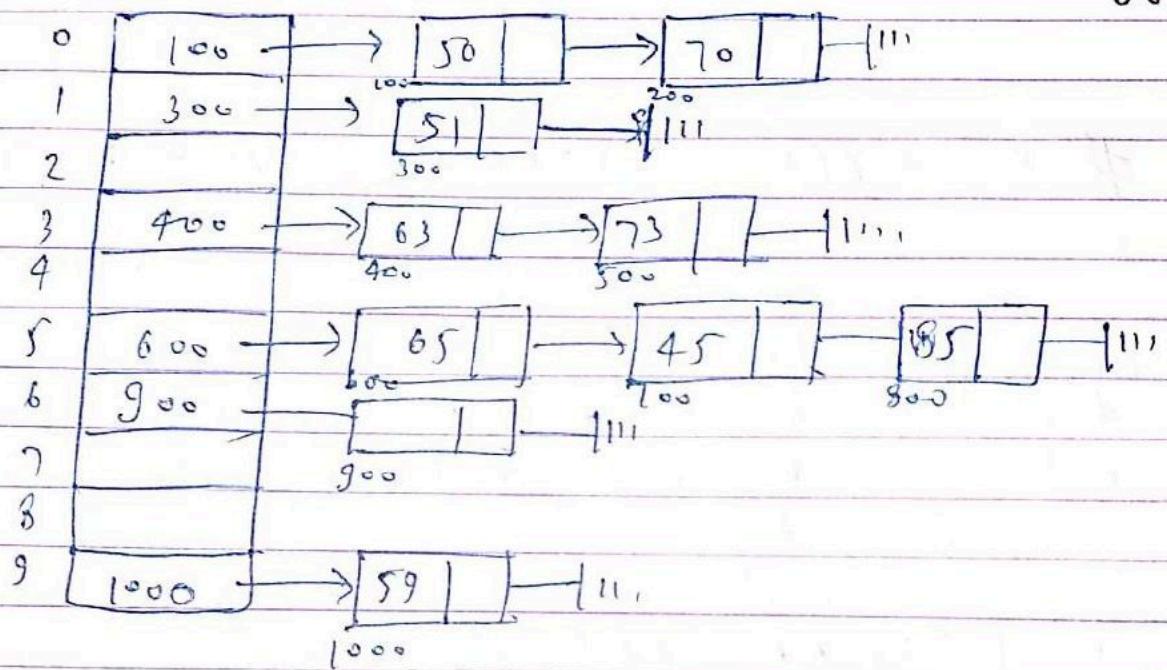
seperate chaining.

NOTE - The hash table contains array of pointers (addresses) which points to the first node of list of corresponding hash key index.

~~ex-~~

$$m = 10 \cdot (0 - 9)$$

elements = 50, 70, 51, 59, 65, 45, 85, 63, 73, 86



### \* TIME COMPLEXITY FOR SEARCHING -

(a) In worst case all the Key might mapped to the same bucket of hash table. It will

(b) In such case all Key present in single link list. So, time taken for searching in worst case is  $O(n)$

(c)

## • LOAD FACTOR ( $\alpha$ )

$$\alpha = \frac{\text{No. of element present in hash table}}{\text{Total size of hash table}}$$

Time complexity =  $O(\alpha)$

Ex- If elements keys (0-99) &  $m=10$ ,  
 $\therefore \alpha = \frac{100}{10} = 10$ .

## # OPEN ADDRESSING METHOD -

In open addressing unlike separate chaining all the keys are stored inside the hash table i.e., no key is stored outside the hash table memory.

Various type of open addressing method are -

### (i) LINEAR PROBING -

When a collision occurs we linearly probe for the next bucket. We keep probing until an empty bucket is found.

Ex-  $M=10$ , elements = 50, 60, 70, 71, 89, 92,

0	50	7	
1	60	8	
2	70	9	89
3	71	10	
4	92		
5			
6			

(ii)

## QUADRATIC PROBING .

In Quadratic Probing when collision occurs, we probe for  $i^2$ th bucket in  $i$ th operation iteration. We keep probing until an empty bucket is found.

Ex -

$M=10$ , elements - 50, 60, 70, 71, 92, 95, 89.

$$\begin{aligned} 50 \bmod 10 &= 0. \\ 60 \bmod 10 &= 0 \\ (60+1^2) \bmod 10 &= 1 \\ 70 \bmod 10 &= 0 \text{ coll.} \\ (70+1^2) \bmod 10 &= 1 \text{ coll.} \\ (70+2^2) \bmod 10 &= 4 \text{ coll.} \end{aligned}$$

0	50
1	60
2	71
3	92
4	70
5	95
6	
7	
8	
9	89.

(iii)

## DOUBLE HASHING -

In Double hashing we use another hash function say  $A_2(x)$  to look for  $i$ th  $h_1(x)$  bucket in  $i$ th iteration. It requires more computation time as two hash function needed to be computed.

UNIT - 4

Date - 10/10/19

GRAPH

- A Graph consist of two sets.
- A non-empty set  $V$  whose elements are called vertices.
  - A set of  $E$  of edges such that  $e \in E$  associated with ordered or unordered pair of element of  $V$ . The set  $E(G)$  is called edges set of  $G$ .

The Graph with vertices  $V$  & edges  $E$  is written as  $G(V, E)$  or  $G = (V, E)$

- \* If an edge  $e \in E$  is associated with an ordered or unordered pair  $(u, v)$  where  $u, v \in V$  Then,  $E$  is said to connect  $u \neq v$ ,  $u \neq v$  are called end point of edge  $e$ .
- \* An edge  $E$  is said to be incident with the vertices it join.

- \* The edge  $e$  that join vertices  $u \neq v$  it said to be incident on each of its end point  $u \neq v$ , i.e., connected by edge  $e$  in a graph is called adjacent vertex.

## # ISOLATED VERTEX -

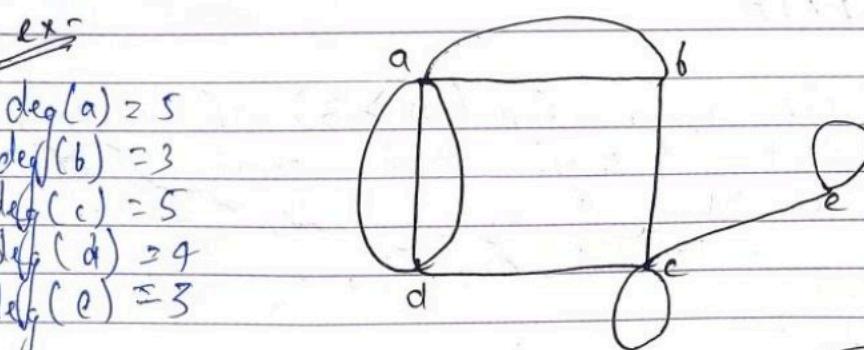
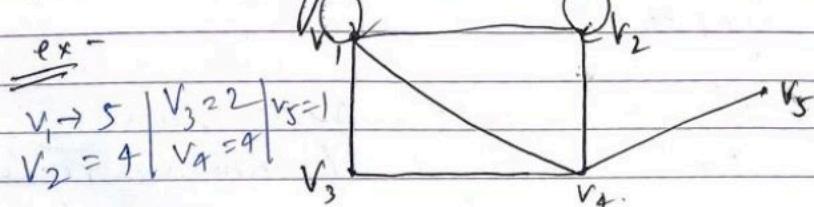
In a graph vertex that ~~have~~ is not adjacent to any vertex is called isolated vertex.

## # DEGREE OF VERTEX -

The degree of a vertex of an undirected graph is the number of edges incident with it.

NOTE -

The loop at vertex contributes twice to the degree of that vertex.



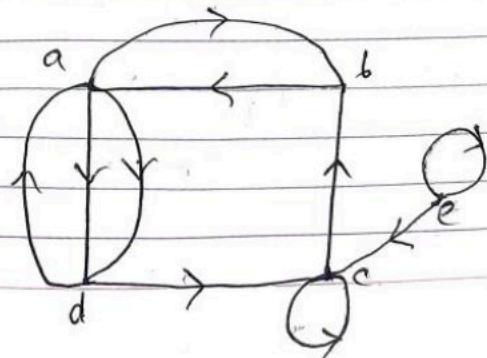
$$\text{Indeg}(a) = 2, \text{Outdeg}(a) = 3$$

$$\text{Indeg}(b) = 2, \text{Outdeg}(b) = 1$$

$$\text{Indeg}(c) = 3, \text{Outdeg}(c) = 2$$

$$\text{Indeg}(d) = 2, \text{Outdeg}(d) = 3$$

$$\text{Indeg}(e) = 1, \text{Outdeg}(e) = 2$$

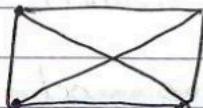
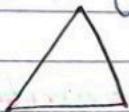


## # NULL GRAPH

Null graph is a graph which contain isolated graph vertex.

## # COMPLETE GRAPH

If all vertex are interconnected to all remaining vertex.

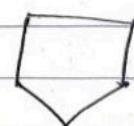
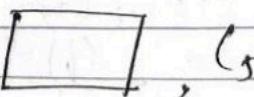
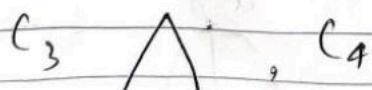
ex-

## # REGULAR GRAPH

If all vertex of graph have same degree then this graph is known as regular graph.

## # CYCLE GRAPH

The Cycle graph  $C_n$  ( $n \geq 3$ ) of length  $n$  is a connected graph which consist of  $n$  vertices say  $v_1, v_2, v_3, \dots, v_n$  &  $n$  edges  $E_n = \{ \{v_1, v_2\}, \{v_2, v_3\}, \dots, \{v_{n-1}, v_n\}, \{v_n, v_1\} \}$ .

ex-

## # WALK

A walk in a graph  $G$  is a finite alternative sequence of say  $v_0e_0, v_1e_1, v_2e_2, \dots, v_{n-1}e_{n-1}, v_n$  of vertices & edges of the graph such that edge  $e_i$  in the sequence joins vertices  $v_{i-1}, v_i$  where  $1 \leq i \leq n$ .

The end vertices  $v_0$  &  $v_n$  are called end terminals of the walk whereas  $v_1, v_2, \dots, v_{n-1}$  are called internal vertices of walk.

The number of edges in walk is called length of a walk.

## \* OPEN WALK

A walk is called open when its end terminals are distinct.

## \* CLOSED WALK

A walk is called closed when its end terminals are same.

NOTE - A walk may repeat both edge & vertices.

## \* TRAIL

A walk is called trail if all its edges are distinct but it can repeat the

Closed loop = Circuit

vertices.

A Trail is an open or closed depends on whether its end vertices are distinct or same.

### \* PATH -

A walk does not contain repeated vertex & edges is called Path.

### # SPANNING TREE -

A Spanning tree is a subset of graph  $G$  which has all the vertices covered with minimum possible no. of edges. Hence, a spanning tree does not have cycles & it cannot be disconnected.

Every connected & undirected graph  $G(V, E)$  has at least one spanning tree.

A disconnected graph does not have any spanning tree as it cannot be spanned to all its vertices.

### \* PROPERTIES OF SPANNING TREE

- (i) A graph  $G$  can have more than one spanning tree.
- (ii) A complete undirected graph can have  $(n^{n-2})$  where  $n$  is no. of vertices.
- (iii) All possible spanning tree of Graph  $G$  have same no. of vertices & edges.

we make graph V disconnected very implies  
 Spanning tree minimally connected subgraph.  
 ) Adding one edge to the spanning tree  
 will create a circuit or closed loop if  
 implies spanning tree is minimally acyclic  
 subgraph.

### MATHEMATICAL PROPERTIES OF S.T.

- i) From complete graph by removing maximum  $(e-n+1)$  edges we can construct a spanning tree where e is no. of edges & n is no. of vertices
- ii) A com spanning tree has  $(n-1)$  edges where n is no. of vertices;

### APPLICATIONS OF S.T.

- i) Civil network planning
- ii) Computer network routing protocol
- iii) Cluster analysis.

### # MINIMAL SPANNING TREE -

In weighted graph, a minimal spanning tree is a spanning tree that have minimum weight, that all other spanning

- (iv) Removing one edge from spanning tree will make graph disconnected. It implies spanning tree is minimally connected subgraph.
- (v) Adding one edge to the spanning tree will create a circuit or closed loop if implies spanning tree is minimally acyclic subgraph.

#### \* MATHEMATICAL PROPERTIES OF S.T.

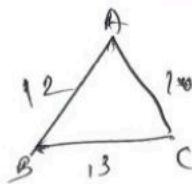
- (i) From complete graph by removing maximum  $(e-n+1)$  edges we can construct a spanning tree where  $e$  is no. of edges &  $n$  is no. of vertices.
- (ii) A com. spanning tree has  $(n-1)$  edges where  $n$  is no. of vertices.

#### \* APPLICATIONS OF S.T.

- (i) Civil network planning.
- (ii) Computer network routing protocol.
- (iii) Cluster analysis.

#### # MINIMAL SPANNING TREE -

In weighted graph, a minimal spanning tree is a spanning tree that have minimum weight, that all other spanning



tree of same graph. In real world situation, this weight can be measured as distance, congestion, traffic load or any arbitrary value denoted to edge.

## # ADJACENCY MATRIX -

If a connected graph  $G(V, E)$  can be represent into a 2-dimensional adjacency matrix of size  $n \times n$  where  $n$  is no. of vertices. By following rule.  $A[i^o][j^o] = 1$  if there exist an edge b/w from  $i^{th}$  vertex to  $j^{th}$  vertex.  $A[i^o][j^o] = 0$ . if there is no edge b/w  $i^{th}$  &  $j^{th}$  vertex.

$\leftarrow$

	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	0	1
C	1	1	0	1	1
D	1	0	1	1	0
E	0	1	1	0	0

$5 \times 5$

## # SPANNING TREE (ALGORITHM FOR CONSTRUCTING) USING BREADTH FIRST SEARCH (BFS).

Step 1 - Choose a vertex & designate it as the root.

Step 2 - Add all edges incident to this vertex, such that addition of edges does not produce cycle or circuit.

Step 3 - A new vertex added in previous

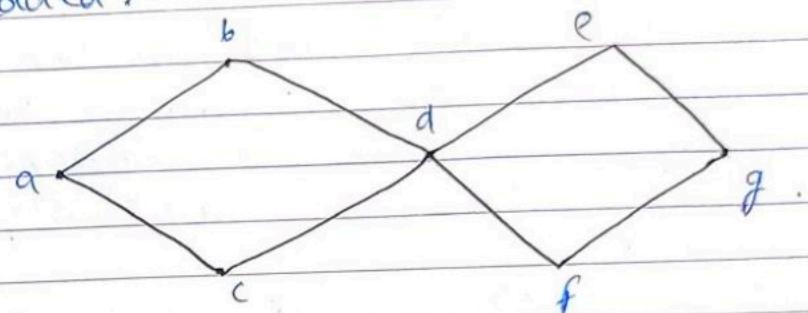
step become vertices at level 1 in the spanning tree, arbitrary order them

Step 4 - for each vertex of level 1 visit in order, add each edge incident to this vertex to the tree as long as it does not produce any cycle or circuit.

Step 5 - Arbitrary order the vertices at level 2.

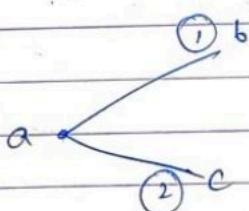
Step 6 - Continue the same procedure until all the vertices in the tree have been added.

Ex -



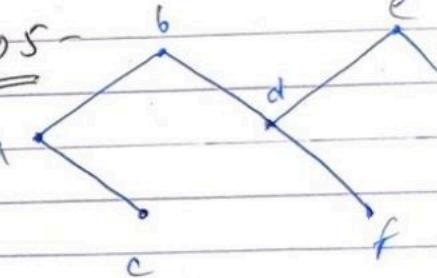
Step 1 -

a.

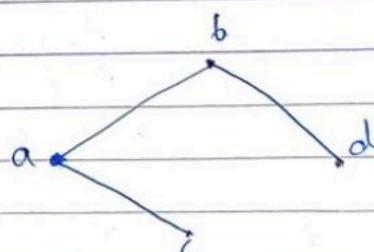


Step 2 -

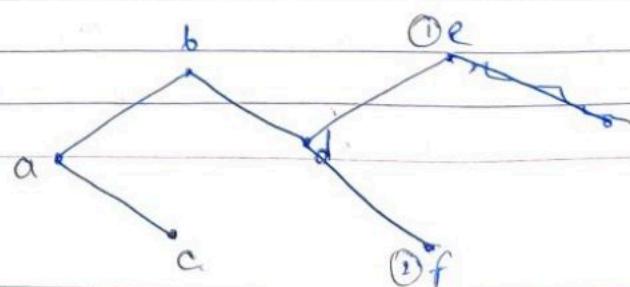
Step 5 -



Step 3 -



Step 4 -



## # DEPTH FIRST SEARCH ALGORITHM

Step 1 - Arbitrarily choose a vertex from a graph  $G(V)$  & designate it as root.

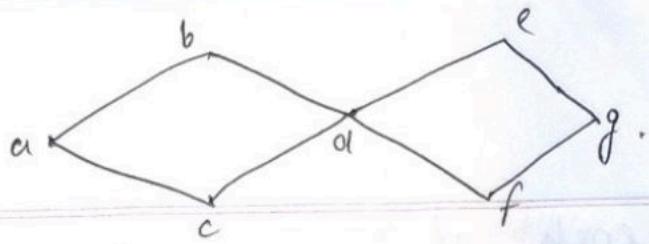
Step 2 - form a path starting at this vertex by successively adding edges as long as possible where each new edge is incident with the last vertex in the path without producing any cycle.

Step 3 - If the path goes through all the vertices of graph  $G$ . The tree consisting of this path is a spanning tree. Otherwise go to the next step.

Step 4 - Move back to the next to last vertex in the path & if possible, form a new path starting at this vertex passing through vertices that were not already visited.

Step 5 - If this cannot be done move back to another vertex in the path i.e., two vertex back in the path, repeat this procedure beginning at the last vertex visited.

Step 6 - Moving back up the path one vertex at a time, forming new path that are as long as possible until no more edges can be added.

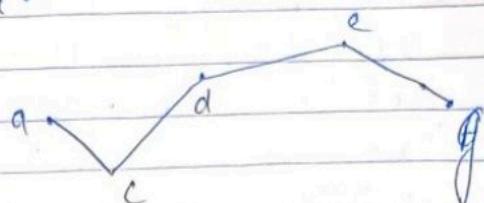


Step 1-

a.

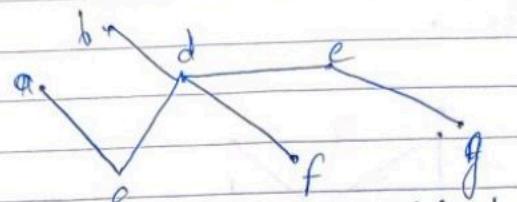
loop → within a vertex.

Step 2-



Circuit →

Step 3-



or minimal

# MST (MINIMUM SPANNING TREE).-  
(Based upon minimum weight).

# KRUSKAL's

# KRUSKAL's ALGORITHM -

This Algorithm provides an acyclic graph  $T$  of a connected weighted graph  $G$ , which is minimum spanning tree of  $G$ .

I/P

O/P - MST T

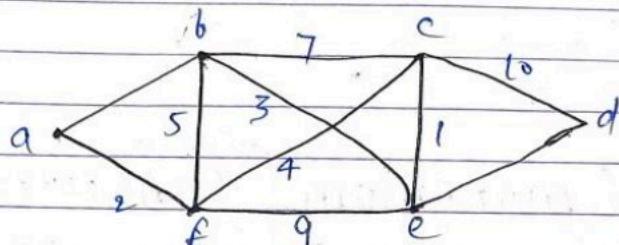
Step 1- List all the edges (which do not form a loop) of  $G$  in non-decreasing order of their weight.

Step 2- Select an edge of minimal weight (if more than one edge of minimum weight, arbitrary choose one of them).

Step 3- At each stage, select an edge of minimum weight from all the remaining edges of  $G$ . If it does not form a circuit with the previously selected edge in  $T$ .

Step 4- Repeat the step 3 until  $(n-1)$  edges are selected where  $n$  is no. of vertices.

Ex-



Step 1-

edges	ab	ce	af	be	cf	de	bf	bc	fe	ed
weight	1	1	2	3	4	4	5	7	9	10

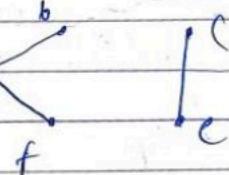
Step 2-

$$T = \{ab\}$$

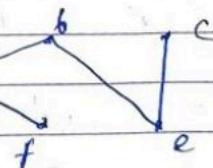
$$T = \{ab, ce\}.$$

Step 3-

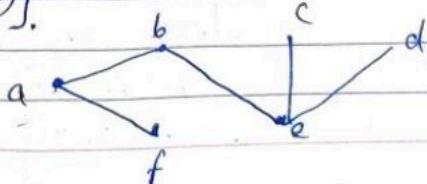
$$T = \{(a,b), (c,e), (a,f)\}$$



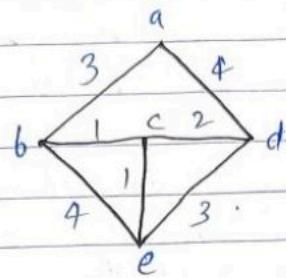
$$T = \{(a,b), (c,e), (a,f), (b,e)\}$$



$$T = \{(a,b), (c,e), (a,f), (b,e), (d,e)\}$$



Q Draw the MST of the given graph with help of Kruskal's algorithm.



Answer Step 1 -

edge	ab	ac	cd	de	ab	be	ad
weight	1	1	2	3	3	4	4

Step 2 -

$$T = \{ \text{bc}, \text{ce} \}$$

$b \longrightarrow c$

$$T = \{ \text{bc}, \text{ce}, \text{cd} \}$$

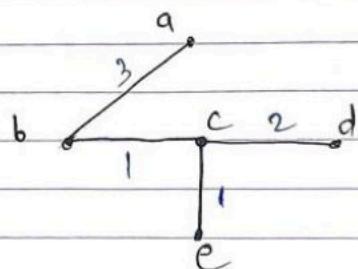
$b \longrightarrow \begin{cases} c \\ e \end{cases} \longrightarrow d$

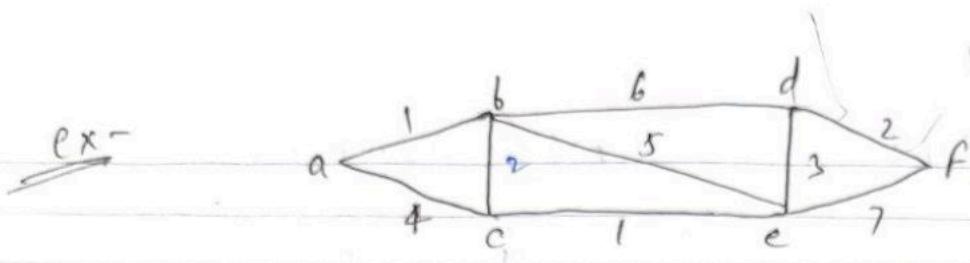
Step 3 -

$$T = \{ \text{bc}, \text{ce}, \text{cd}, \text{ab} \}$$

$b \longrightarrow \begin{cases} c \\ e \end{cases} \longrightarrow \begin{cases} d \\ a \end{cases}$

$$T = \{ \text{bc}, \text{ce}, \text{cd}, \text{ab} \}$$





Initial label is given by table.

Vertex	a	b	c	d	e	f
$L(V)$	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
T	a, b	c	d	e	f	

\* Iteration 1 -  $u = a$  has  $L(u) = 0$ .  $T$  becomes  $T - \{a\}$ . There are 2 edges incident with  $a$  i.e.,  $ab$  &  $ac$  where  $b, c \in T$ .

$$L(b) = \min(\text{old}(L(b)), L(a) + w(a, b))$$

$$= \min(\infty, 0+1) = 1$$

$$L(c) = \min(\text{old}(L(c)), L(a) + w(a, c))$$

$$= \min(\infty, 0+4) = 4$$

Since,  $L(b) < L(c)$  minimum label is of  $b$ . So,  $b$  vertex is chosen  $L(b) = 1$ .

Vertex	a	b	c	d	e	f
$L(V)$	0	1	4	$\infty$	$\infty$	$\infty$
T	b	c	d	e	f	

\* Iteration 2 -  $u = b$  has  $L(u) = 1$ .  $T$  becomes  $T - \{b\}$ . There are 3 edges incident with  $b$  i.e.,  $bc$ ,  $bd$ ,  $be$  where  $c, d, e \in T$ .

$$L(c) = \min(\text{old}(L(c)), L(b) + w(b, c))$$

$$= \min(4, 1+2) = 3.$$

$$L(d) = \min(\infty, 1+6) = 7$$

$$L(e) = \min(\infty, 1+5) = 6$$

Since,  $L(c) < L(e) < L(d)$ . minimum label is of c. So, c vertex is chosen  $L(c)=3$ .

Vertex	a	b	c	d	e	f
$L(V)$	0	1	3	7	6	$\infty$
T			c	d	e	f

\* Iteration 3-  $u=c$  has  $L(u)=3$ . T becomes  $T-\{c\}$ . There are only one edge incident with c i.e., ce where  $e \in T$ .

$$\begin{aligned} L(e) &= \min(L(d), L(c)+w(c,e)) \\ &= \min(6, 3+1) = 4 \end{aligned}$$

Since, only one vertex is there. minimum label is of e. So, e vertex is chosen  $L(e)=4$ .

Vertex	a	b	c	d	e	f
$L(V)$	0	1	3	7	4	$\infty$
T				d	e	f

\* Iteration-4  $u=e$  has  $L(e)=4$ . T becomes  $T-\{e\}$ . There are two edges incident with e i.e., ed & ef where  $d \neq f \in T$ .

$$\begin{aligned} L(d) &= \min(L(d), L(e)+w(e,d)) \\ &= \min(7, 4+3) = 7. \end{aligned}$$

$$L(f) = \min(\infty, 4+7) = 11.$$

Since,  $L(d) < L(f)$  minimum label is of d so, d vertex is chosen  $L(d)=7$ .

Vertex	a	b	c	d	e	f
L(V)	0	1	3	7	4	18
T				d	f	

\* Iteration - 5  $u = d$ . has  $L(d) = 7$ .  $T$  becomes  $T - \{d\}$ . There are only edge incident with  $d$  i.e.,  $df$  where  $f \in T$ .

$$\begin{aligned} L(f) &= \min(\text{old}(L(f)), L(d) + w(d, f)) \\ &= \min(18, 7 + 2) = 9. \end{aligned}$$

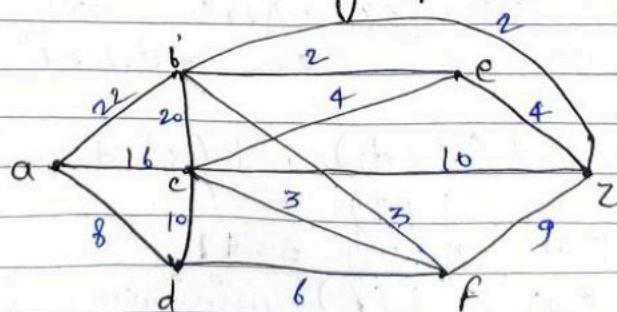
Since, only one vertex is there so, minimum label is of  $f$ ,  $f$  vertex is chosen  $L(f) = 9$ .

Vertex	a	b	c	d	e	f
L(V)	0	1	3	7	4	9
T						f,

Min. Path for moving a to f.

$$a \rightarrow b \rightarrow c \rightarrow e \rightarrow d \rightarrow f$$

# Q In below graph



Find the shortest distance b/w a & z.

Using Dijkstra's Rule -

STEP-3

For each vertex  $K$ , weight where  $K$  is an intermediate vertex from  $i$  to  $j$  vertex. Calculate a distance matrix  $\alpha$ .

for ( $K = 0$  to  $K \leq v - 1$ )

for ( $i = 0$  to  $i \leq v - 1$ )

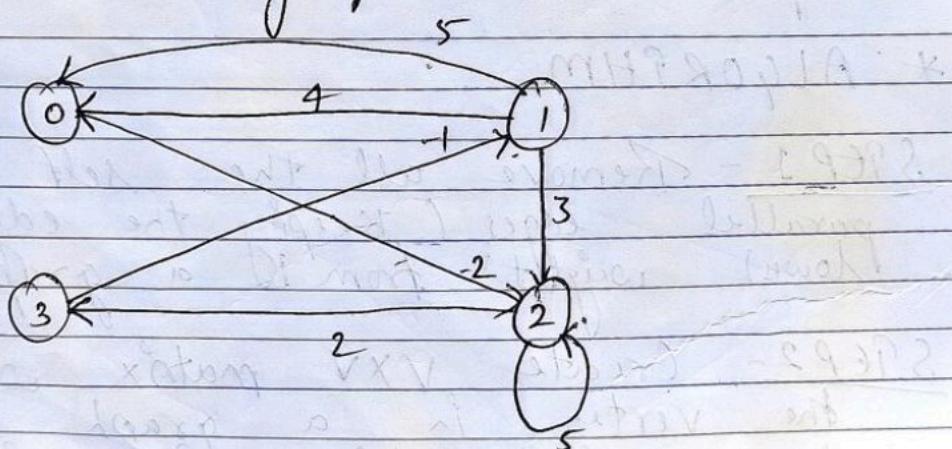
for ( $j = 0$  to  $v - 1$ )

if ( $d[i][j] > d[i][K] + d[K][j]$ )  $< d[i][j]$ )

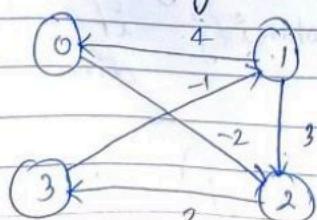
$d[i][j] = d[i][K] + d[K][j]$ .

Q

Consider a graph  $g$ .



Step 1 - Remove all self loop & parallel edges - the graph will be -



$$D = 0 \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & \infty & -2 & \infty \\ 1 & 4 & 0 & 3 & \infty \\ 2 & \infty & \infty & 0 & 2 \\ 3 & \infty & -1 & \infty & 0 \end{bmatrix}$$

Weight Matrix (Vertex X of a Graph Vertex Y)

$$P_0 = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & \infty & -2 & \infty \\ 1 & 4 & 0 & 2 \\ 2 & \infty & \infty & 0 & 2 \\ 3 & \infty & -1 & \infty & 0 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & \infty & -2 & \infty \\ 1 & 4 & 0 & 2 & \infty \\ 2 & \infty & \infty & 0 & 2 \\ 3 & 3 & -1 & 1 & 0 \end{bmatrix}$$

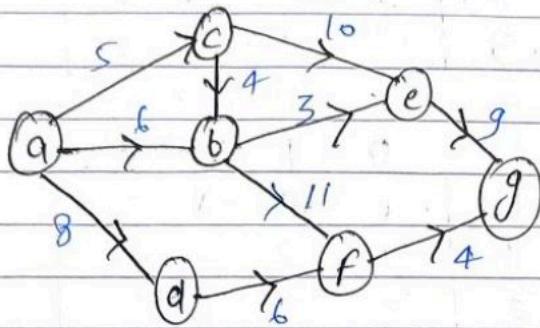
$$P_2 = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & \infty & -2 & 0 \\ 1 & 4 & 0 & 2 & 4 \\ 2 & \infty & \infty & 0 & 2 \\ 3 & 3 & -1 & 1 & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & -1 & -2 & 0 \\ 1 & 4 & 0 & 2 & 4 \\ 2 & 5 & -1 & 0 & 2 \\ 3 & 3 & -1 & 1 & 0 \end{bmatrix}$$

Above matrix is required matrix.

Sorting RMO CMO, Postfix,

Q Find the minimum distance b/w every pair of vertices of following graph.



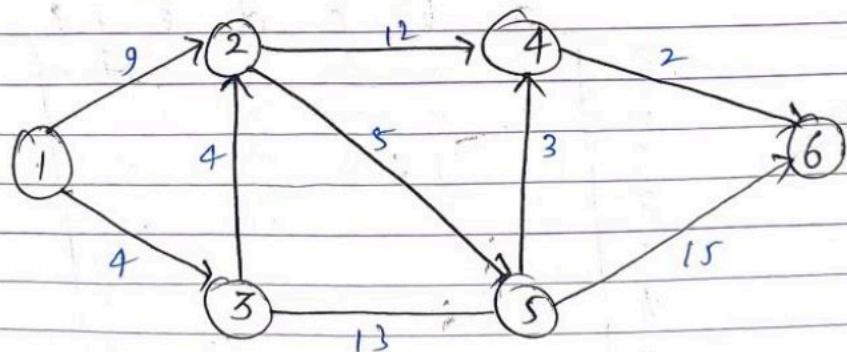
	a	b	c	d	e	f	g
a	0	6	5	8	$\infty$	$\infty$	$\infty$
b	$\infty$	0	$\infty$	2	3	11	$\infty$
c	$\infty$	4	0	$\infty$	10	$\infty$	$\infty$
d	$\infty$	$\infty$	$\infty$	0	$\infty$	6	$\infty$
e	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$	9
f	$\infty$	$\infty$	$\infty$	$\infty$	0	4	
g	$\infty$	$\infty$	$\infty$	$\infty$	8	8	0

3,4 or 5

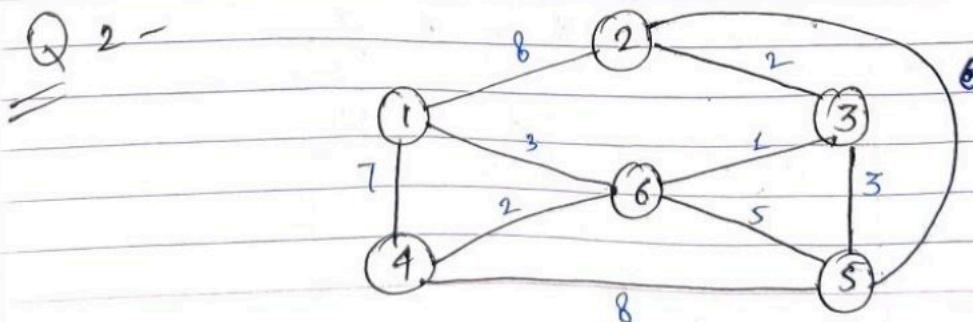
	a	b	c	d	e	f	g
a	0					14	
b		0				11	
c			0			$\infty$	
d				0		6	
e					0	$\infty$	
f		$\infty$	$\infty$	$\infty$	$\infty$	0	4
g						$\infty$	

	a	b	c	d	e	f	g
a	0						
b		0					
c			0				
d				0			
e					0		
f						0	
g							0

Q 1 -



Q 2 -



Solution - 2

$$D = \begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 8 & \infty & 7 & \infty & 3 \\ 2 & 8 & 0 & 2 & \infty & 6 & \infty \\ 3 & \infty & 2 & 0 & \infty & 3 & 1 \\ 4 & 7 & \infty & \infty & 0 & 8 & 2 \\ 5 & \infty & 6 & 3 & 8 & 0 & 5 \\ 6 & 3 & \infty & 1 & 2 & 5 & 0 \end{matrix}$$

$$D_1 = \begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 8 & \infty & 7 & \infty & 3 \\ 2 & 8 & 0 & 2 & 15 & 6 & 11 \\ 3 & \infty & 2 & 0 & \infty & 3 & 1 \\ 4 & 7 & 15 & \infty & 0 & 8 & 2 \\ 5 & \infty & 6 & 3 & 8 & 0 & 5 \\ 6 & 3 & 11 & 1 & 2 & 5 & 0 \end{matrix}$$

$$D_2 = \begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 8 & 10 & 7 & 14 & 3 \\ 2 & 8 & 0 & 2 & 15 & 6 & 11 \\ 3 & 10 & 2 & 0 & 17 & 3 & 1 \\ 4 & 7 & 15 & 17 & 0 & 8 & 2 \\ 5 & 14 & 6 & 3 & 8 & 0 & 5 \\ 6 & 3 & 11 & 1 & 2 & 5 & 0 \end{matrix}$$

$$D_3 = \begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 8 & 10 & 7 & 13 & 3 \\ 2 & 8 & 0 & 2 & 15 & 5 & 3 \\ 3 & 10 & 2 & 0 & 17 & 3 & 1 \\ 4 & 7 & 15 & 17 & 0 & 8 & 2 \\ 5 & 19 & 5 & 3 & 8 & 0 & 4 \\ 6 & 3 & 3 & 1 & 2 & 4 & 0 \end{matrix}$$

1	1	1	2	3	4	5	6
2	2	8	0	2	15	5	3
3	10	2	0	17	3	11	
4	7	15	17	0	8	2	
5	13	5	3	8	0	4	
6	3	3	1	2	4	0	

1	2	1	2	3	4	5	6
2	2	8	0	2	13	5	3
3	10	2	0	11	3	1	
4	7	13	11	0	8	2	
5	13	5	3	8	0	4	
6	3	3	1	2	4	0	

1	2	3	4	5	6
2	6	0	2	5	5
3	4	2	0	3	3
4	5	5	3	0	6
5	7	5	3	6	0
6	3	3	1	2	4

Date - 23/10/19

## UNIT - V

### TREE

Definition

Properties

Basic Terminology -

- (a) Root
- (b) Edge
- (c) Parent
- (d) Child
- (e) Sibling
- (f) Degree
- (g) Internal Nodes
- (h) Leaf Node
- (i) Level
- (j) Height
- (k) Depth
- (l) Subtree
- (m) Forest

### BINARY TREE -

Definition

Unlabelled Binary Tree      Labelled Binary Tree       $\nearrow$  formula depend upon height depth

Types of Binary Tree -

- (a) Rooted B.T.
- (b) Full / Strictly B.T.
- (c) Complete B.T.
- (d) Almost Complete B.T.
- (e) Skewed B.T.

Binary Search Tree .

Various formula or Properties of B.T.

A Graph in which no circuit is formed or self looping. Is K/a tree.  
A Node is collection of data & address.

## # BINARY TREE -

Binary Tree is the tree in which every node can have atmost two children. Each node can have 0,1 or 2 children but not more than two.

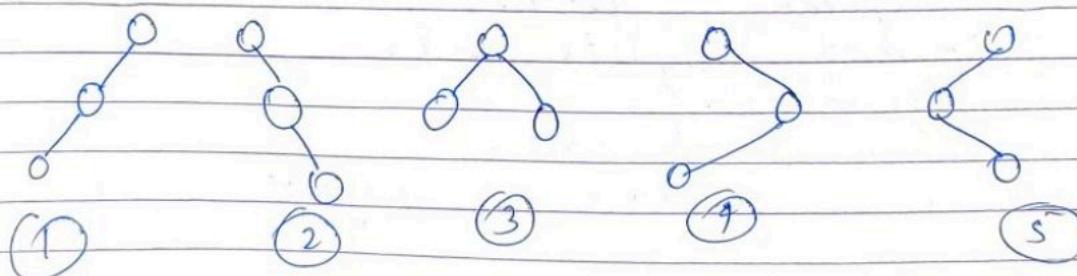
### \* Unlabelled Binary Tree -

A binary tree is unlabelled if its <sup>node</sup>s are not assigned any label.

No. of different binary tree possible with  $n$  unlabelled node  $\left( = \frac{2^n C_n}{n+1} \right)$

Ex- If we have three node then how many binary tree possible.

$$\frac{2^3 C_3}{3+1} = \frac{6!}{3!(6-3)!} / 4 = 5.$$



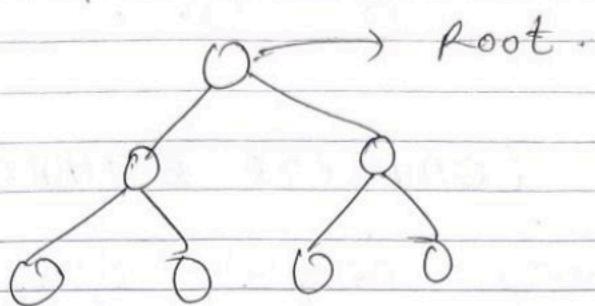
$$\frac{6 \times 5 \times 4 \times 3!}{4 \times 3! \times 3!}$$

No. of binary tree possible with  $n$  labelled nodes  $= \left( \frac{2^n C_n}{n+1} \right) * n!$

## \* TYPES OF BINARY TREE -

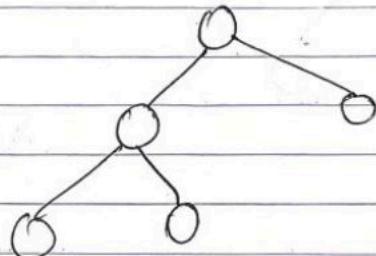
### (i) ROOTED BINARY TREE -

A rooted binary tree is one that has a root node & every node has atmost two children



### (ii) FULL OR STRICTLY BINARY TREE -

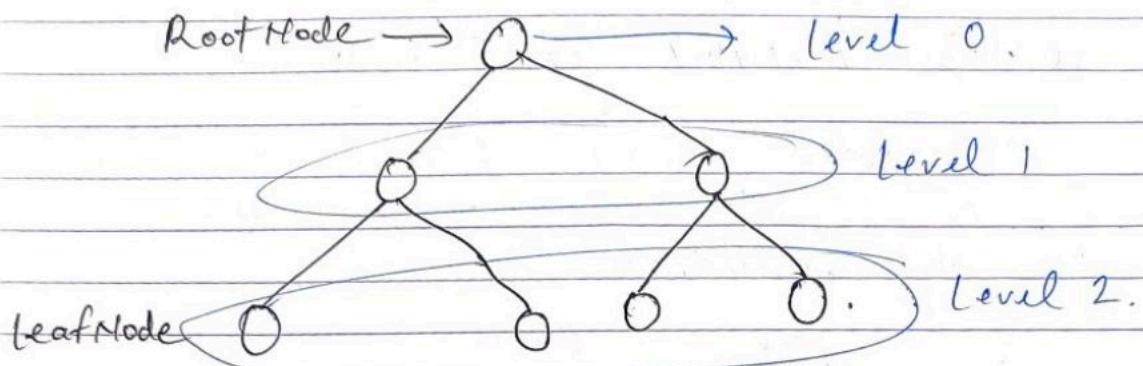
A full or strictly binary tree is a tree in which every node has nodes has 0 or 2 children



### (iii) COMPLETE BINARY TREE (PERFECT B.T.)

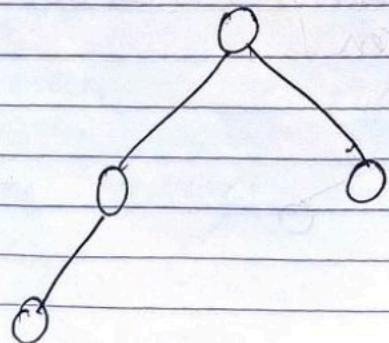
A complete or perfect binary tree is a tree in which every internal node has exactly two children & all nod leaf

nodes are at same level



#### (IV) ALMOST COMPLETE BINARY TREE

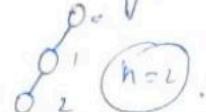
An almost complete binary tree is a tree in which all the levels are completely filled except possibly the last level & the last level must be strictly filled from left to right.



#### (V) SKewed BINARY TREE

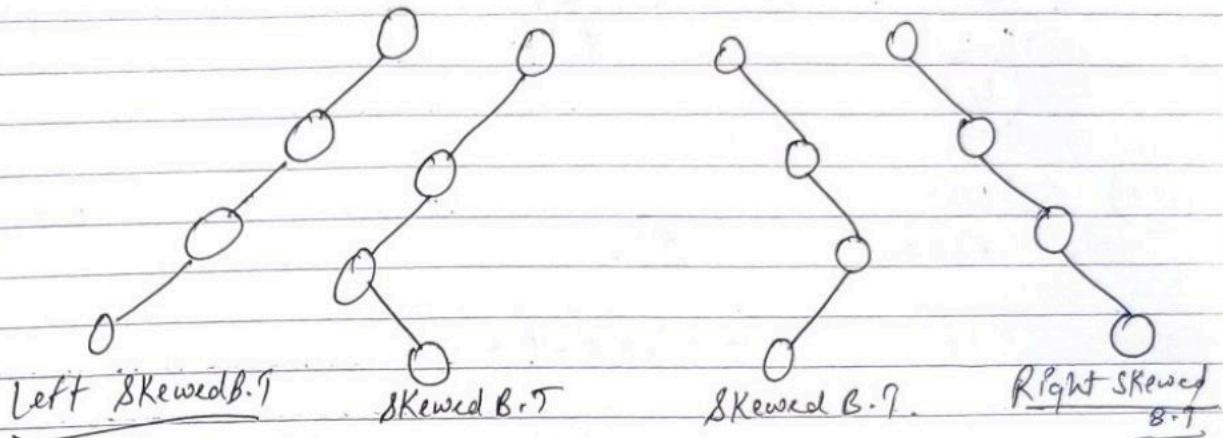
A skewed binary tree is the tree in which all the nodes except one node have one or only one child the

Height of the tree is a max. no. of edges b/w root node to last node.



Remaining nodes has no child.

A skewed binary tree is a tree of  $n$  node of  $n$  node such that it depth is  $n-1$

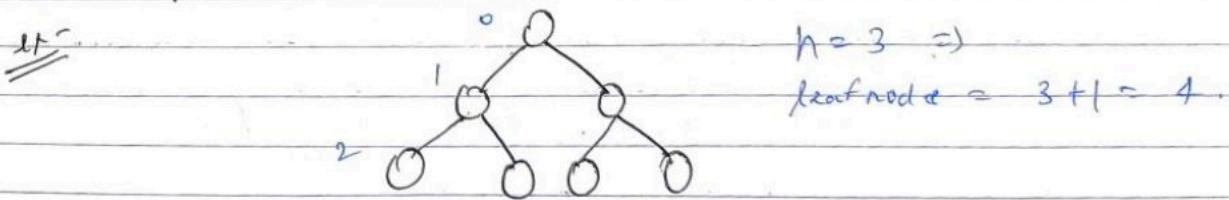


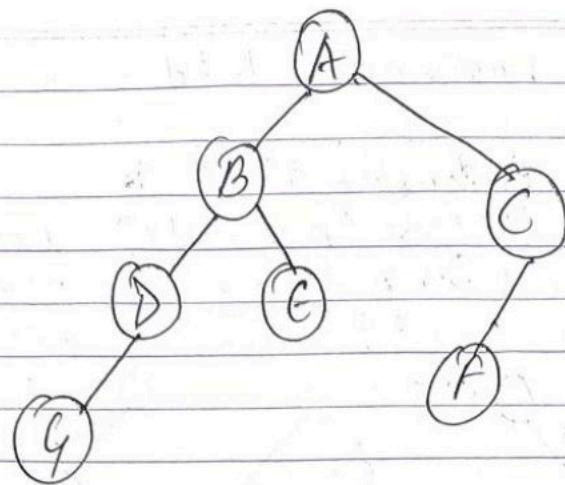
### \* PROPERTIES OF BINARY TREE -

i) Minimum no. of node in a binary tree of height  $h$  is equal to  $h+1$ .

ii) Maximum no. of node in a binary tree of height  $h$  is equal to  $2^{h+1} - 1$

iii) Total no. of leaf node in a binary tree is equal to total no. of degree two node + 1





- IV Maximum no. of node at level  $l$   
is equal to  $2^l$ .

## # TREE TRAVERSAL -

If is also known as tree search or tree display. Tree traversal is a form of graph traversal & refers to the process of visiting each node in a tree data structure exactly once.

Tree traversal techniques are depth first search which includes three techniques



i Preorder

ii Inorder

iii Postorder

Tree Traversal

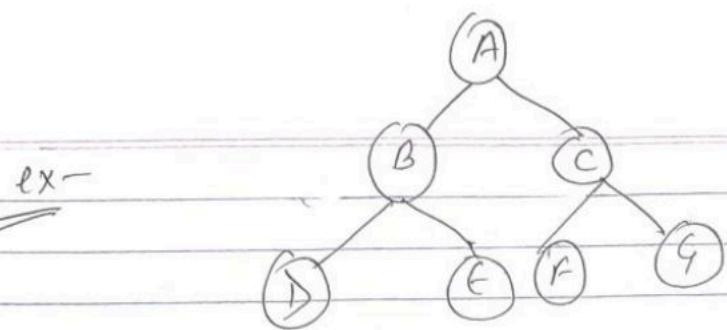
" "

" "

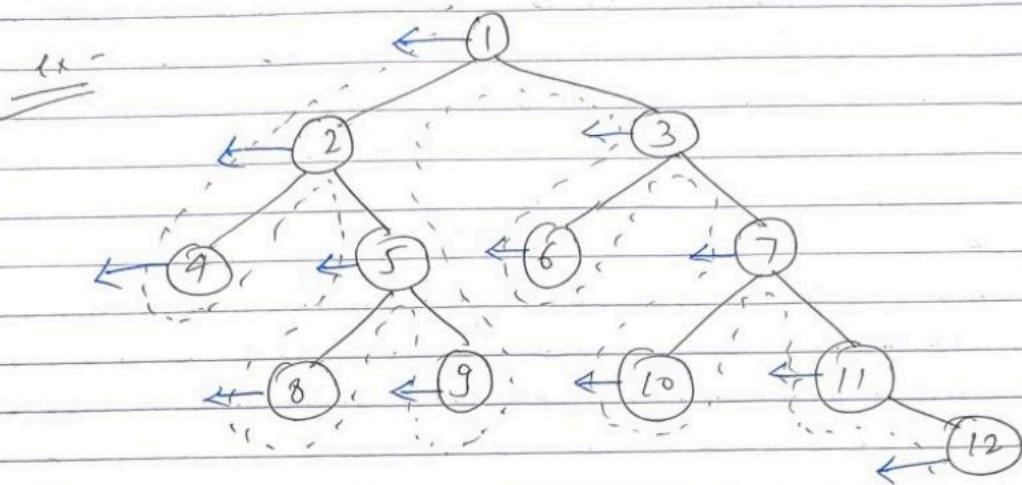
" "

## \* ALGORITHM OF TREE TRAVERSAL

- i Visit the root.
- ii Traverse left subtree i.e., called Preorder left subtree.
- iii Traverse right subtree i.e., called Preorder right subtree.



A B D E C F G .



1, 2, 4, 5, 8, 9, 3, 6, 7, 10, 11, 12  
Write a algo to traverse on tree in preorder.

void Preorder ( struct node \*root )

{  
    if ( .root != NULL )

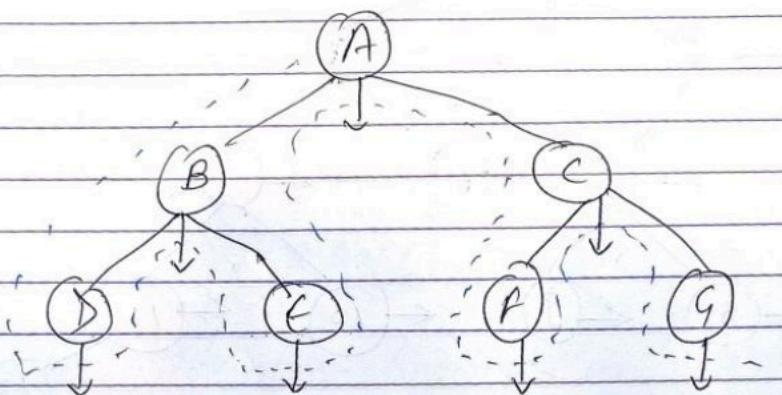
        printf ("% .d \t ", root->data );  
        preorder ( root-> left );  
        preorder ( root-> right );

}

## \* ALGORITHM FOR INORDER .

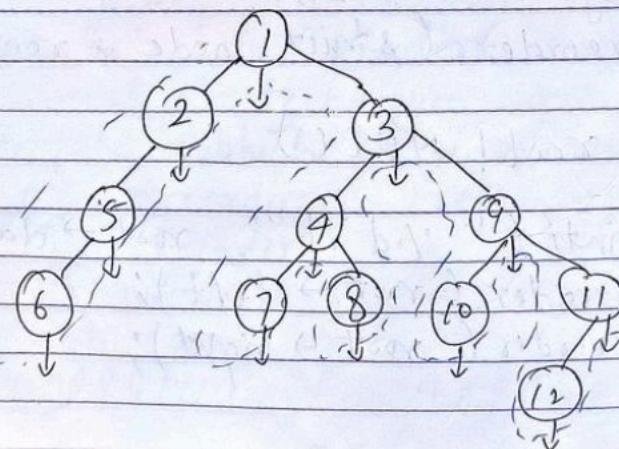
- ① TRAVERSE the left subtree i.e., call the inorder left subtree
- ② Visit the root.
- ③ Traverse the right subtree i.e., ^ inorder right subtree .

ex -



D, B, E, A, F, C, G .

ex -



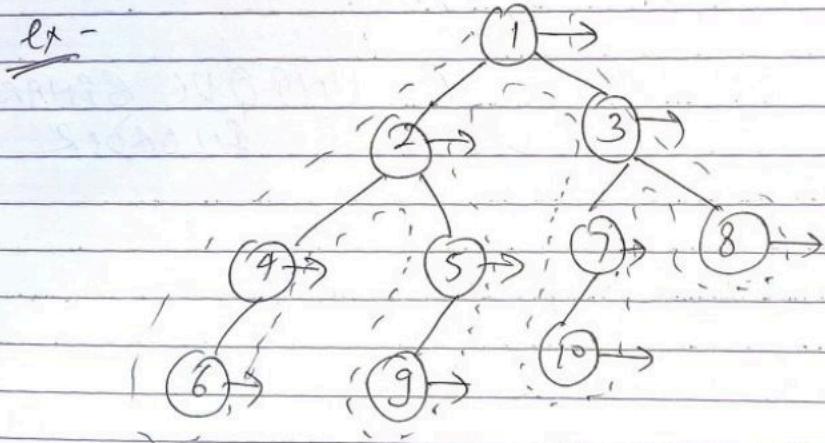
6, 5, 2, 1, 7, 4, 8, 3, 10, 9, 12, 11 .

Q WAP in C to implement algo of Inorder  
tree traversal.

```
void inorder( struct node * root )
{
    if (root != NULL)
    {
        inorder( root -> left );
        printf("%d ", root -> data );
        inorder( root -> right );
    }
}
```

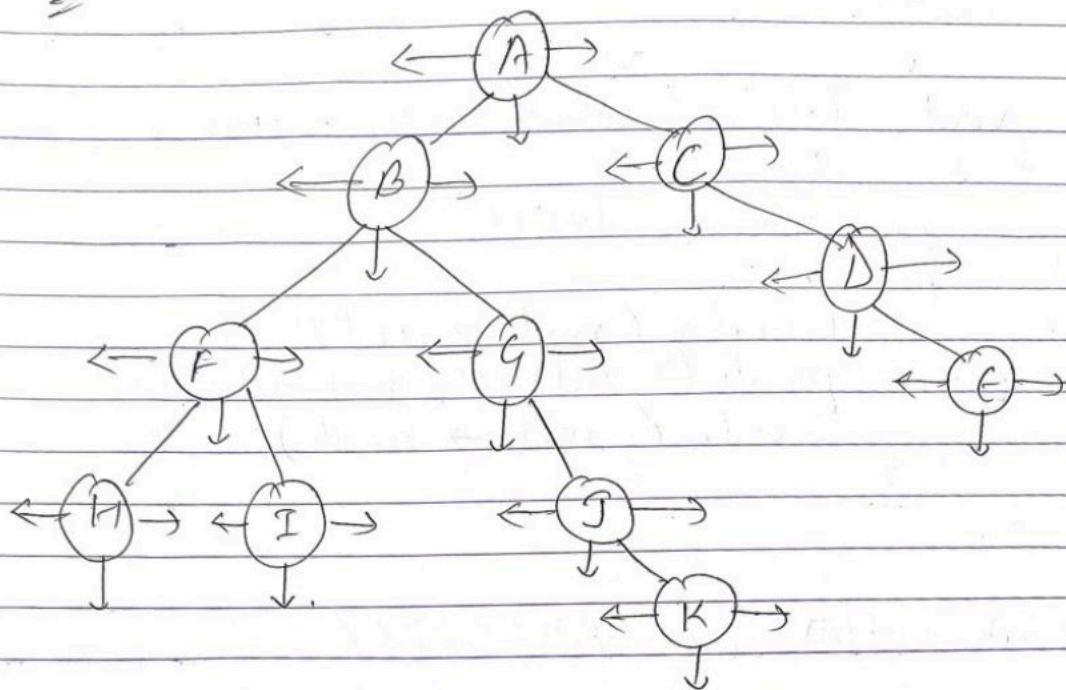
### ALGORITHM OF POSTORDER

- ① Traverse the right left subtree i.e., called postorder left subtree.
- ② Traverse the right subtree i.e., call postorder right subtree.
- ③ Visit the root.



6, 4, 9, 5, 2, 10, 7, 8, 3, 1 .

Q. Construct Preorder, Inorder & Postorder.



Pre A, B, F, H, I, G, J, K, C, D, E.

In H, F, I, B, G, J, K, A, C, D, E

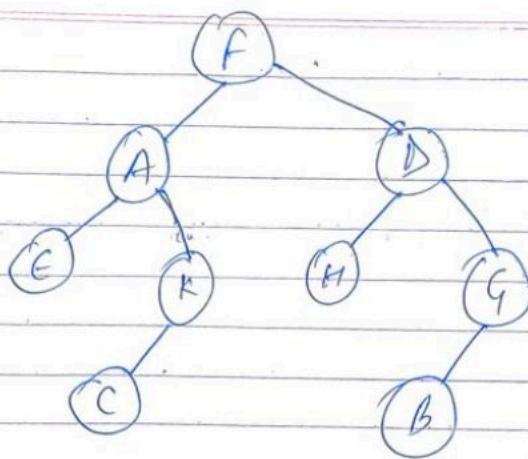
Post H, F, I, K, J, G, B, E, D, C, A.

Q # To construct a unique binary tree when Preorder & Inorder is given -

Inorder - E, A, C, K, F, H, D, B, G.

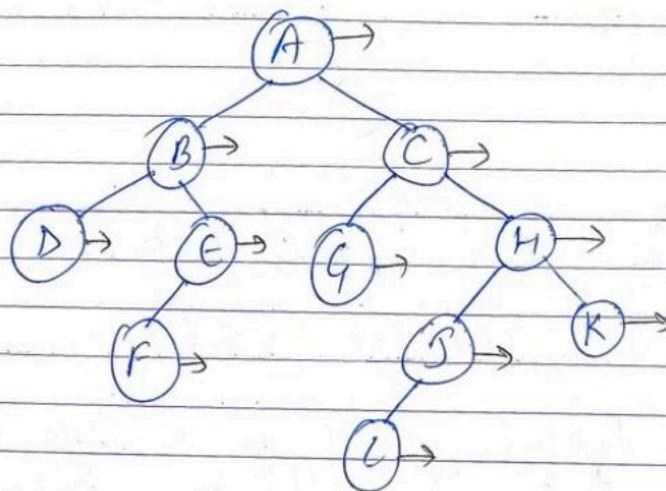
Preorder - F, A, E, K, C, D, H, G, B.

F  
ACK  
HDBG.



INORDER - D, B, F, E, A, G, C, L, I, H, K.

PREORDER - A, C, H, K, I, L, G, B, E, F, D.

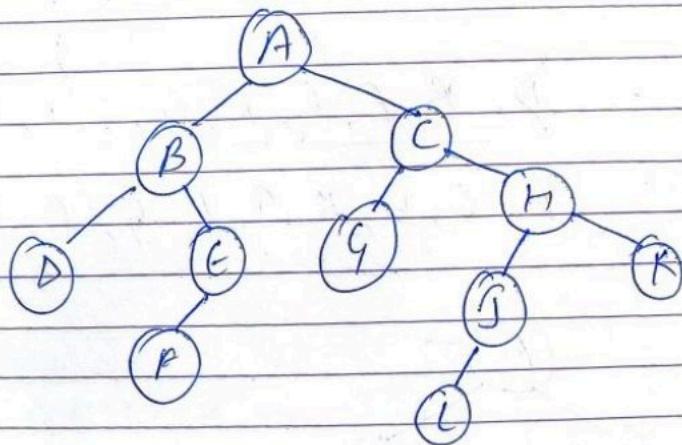


D, F, E, B, G, L, I, K, H, C, A.

# HOW TO CONSTRUCT A TREE WHEN  
POSTORDER & INORDER ARE GIVEN -

INORDER  $\{P, B, F, E, A, G, C, L, J, H, K\}$

POSTORDER D, F, E, B, G, L, J, K, H, C, A.  
FIND PREORDER -



Preorder - A, B, D, E, F, C, G, H, J, L, K.

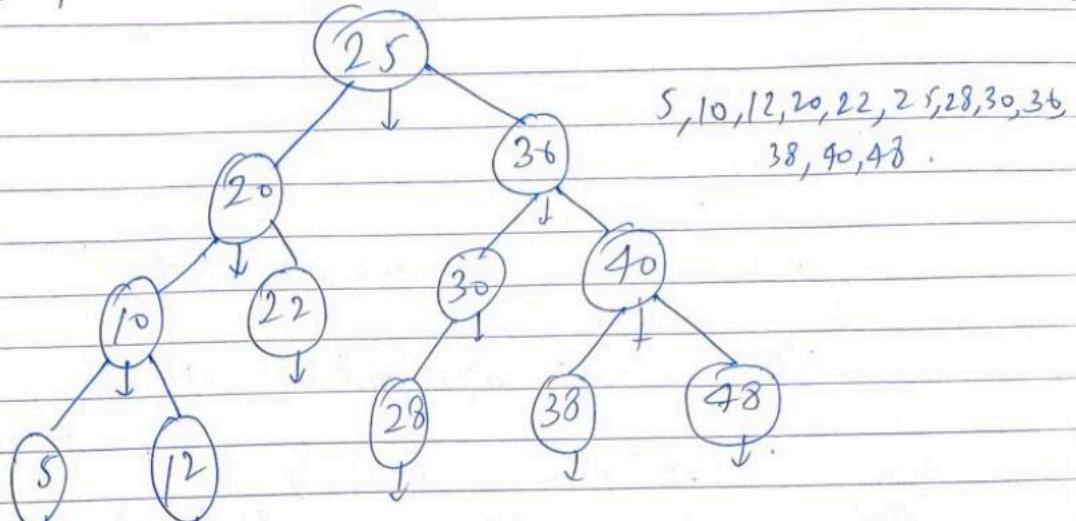
# BINARY SEARCH TREE - (B.S.T.)

Binary Search Tree is a special kind of binary tree in which every node are arranged in specific orders.

In binary search tree, each node contains -

- ① Only smaller value in its left sub tree.
- ② Only greater value in its right sub tree.

for example:- 25, 20, 10, 5, 12, 22, 36, 30, 28, 40, 38, 48.



\* No. of distinct B.S.T. possible with n distinct key =  $\frac{2^n C_n}{n+1}$

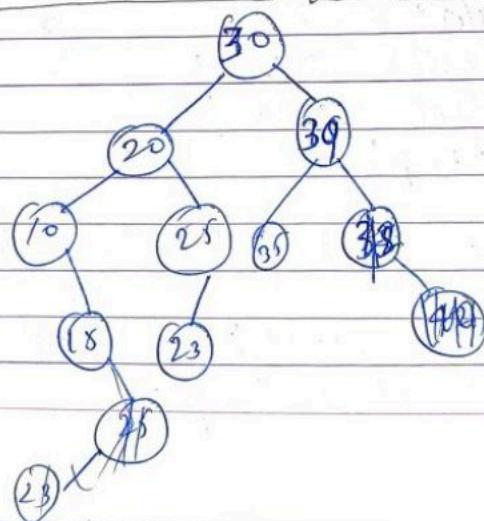
## # BINARY SEARCH TREE TRAVERSAL -

Q Preorder of binary search tree is

30, 20, 10, 15, 25, 23, 39, 35, 42

Find the Postorder of B.S.T.

10, 15, 20, 23, 25, 39, 35, 38, 39, 42



## # OPERATION OF B.S.T. -

### INSERTION, DELETION & UPDATION -

```

#include <stdio.h>
struct node;
{
    int data;
    struct node *left, *right;
};

struct node * insert(struct node *new, int key);
{
    if(new == NULL)
        return newnode(key);
    if(new->data < key)
        new->right = insert(new->right, key);
    if(new->data > key)
        new->left = insert(new->left, key);
    return new;
}

struct node * newnode(int key)
{
    struct node *temp;
    temp = (struct node *) malloc(sizeof(struct node));
    temp->data = key;
    temp->right = NULL;
    temp->left = NULL;
    return temp;
}

struct node * search(struct node *temp, int value)
{
    if(temp == NULL || temp->key == value)
        return temp;
}

```

Insert, particular element search, traversal (inorder-preorder, postorder)

```
if (temp->key > value)
    search (temp->left, value);
else
    search (temp->right, value);
```

## # HEIGHT OF A TREE

Longest Path from root to leaf node  
i.e., in that path the no. of edges.

## # AVL.

Balance factor of node = height of left subtree - height of right subtree.

## # TIME COMPLEXITY OF B.S.T. -

The time complexity of B.S.T.  
for insertion, searching & deletion operation  
is  $O(h)$  where  $h$  is the height of B.S.T.

### \* Worst Case Analysis of B.S.T. -

The worst case of B.S.T. is Skewed B.S.T. It happens when values are inserted either in increasing or decreasing order occurrence. In this (skewed B.S.T) case the height of B.S.T. becomes equivalent to  $n$  where  $n$  is no. of values inserted in B.S.T. Hence, for searching, insertion & deletion

order will be  $O(n)$ .

### \* BEST CASE ANALYSIS OF B.S.T. -

The Best Case of Analysis of B.S.T. will be a Balanced B.S.T. where the height of Balanced B.S.T. will be  $h = \log_2 n$ . Where  $n$  is the no. of nodes or value inserted. So, By above analysis we find if we balanced the tree by insertion & deletion operation we can reduce the time complexity for further searching, insertion & deletion operation. For balancing there are many algorithm that are -

- (i) AVL Tree
- (ii) Red-Black Tree.
- (iii) B-Tree.

### # AVL TREE -

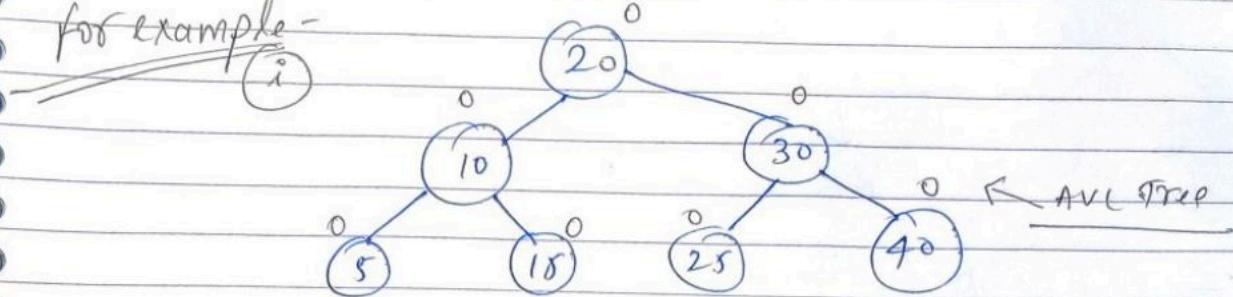
AVL Tree are self balancing B.S.T. where the difference of height of left subtree & right subtree of any node is not greater than 1.

#### \* Balanced Factor -

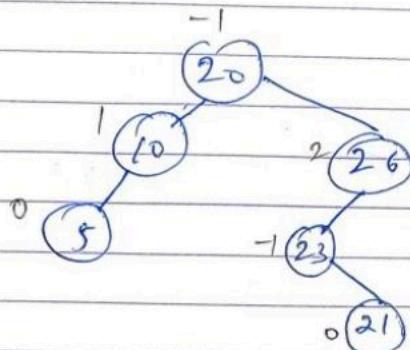
Balanced factor of any node in B.S.T. is equal to height of left subtree - height of right subtree.

So, AVL tree are those B.S.T. where Balance factor of each node will be either 0, -1 or 1.

for example -



(ii)



Not a AVL tree.

# HOW BALANCING IS DONE IN AVL TREE?

In AVL Tree after performing insertion & deletion, we check balanced factor of every node. If every node satisfy the balanced factor (i.e., 0, -1, 1) then the operation is concluded, if it is not so then we must make it balance. For balance for Balancing we use rotation operation which can be classified as .

## Rotation

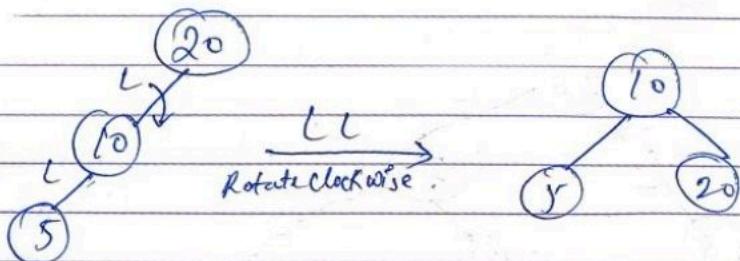
Single Rotation

LL      RR

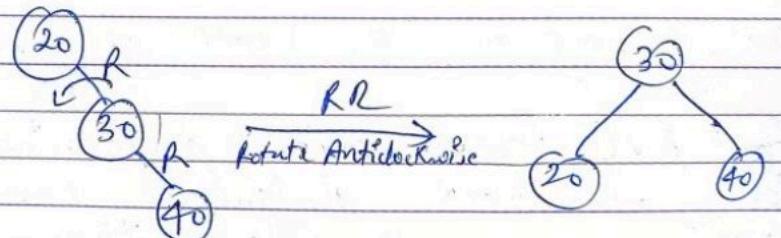
Double Rotation

LR      RL

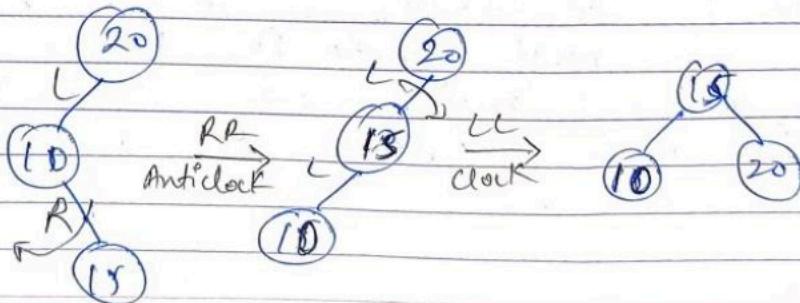
$\Rightarrow$  LL Rotation.



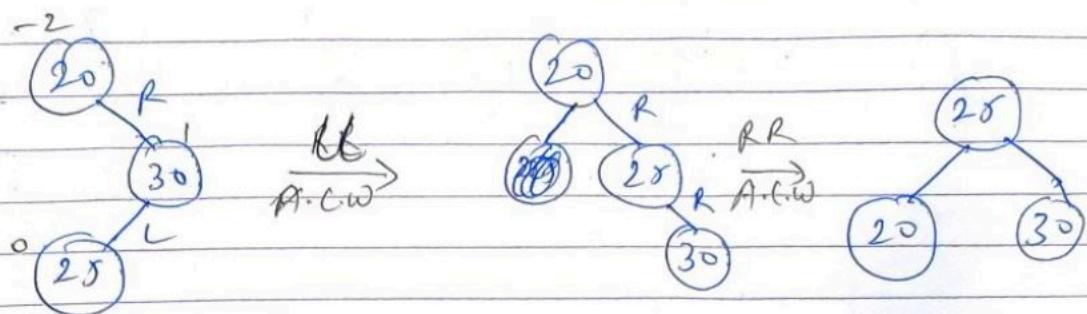
$\Rightarrow$  RR rotation



$\Rightarrow$  LR rotation

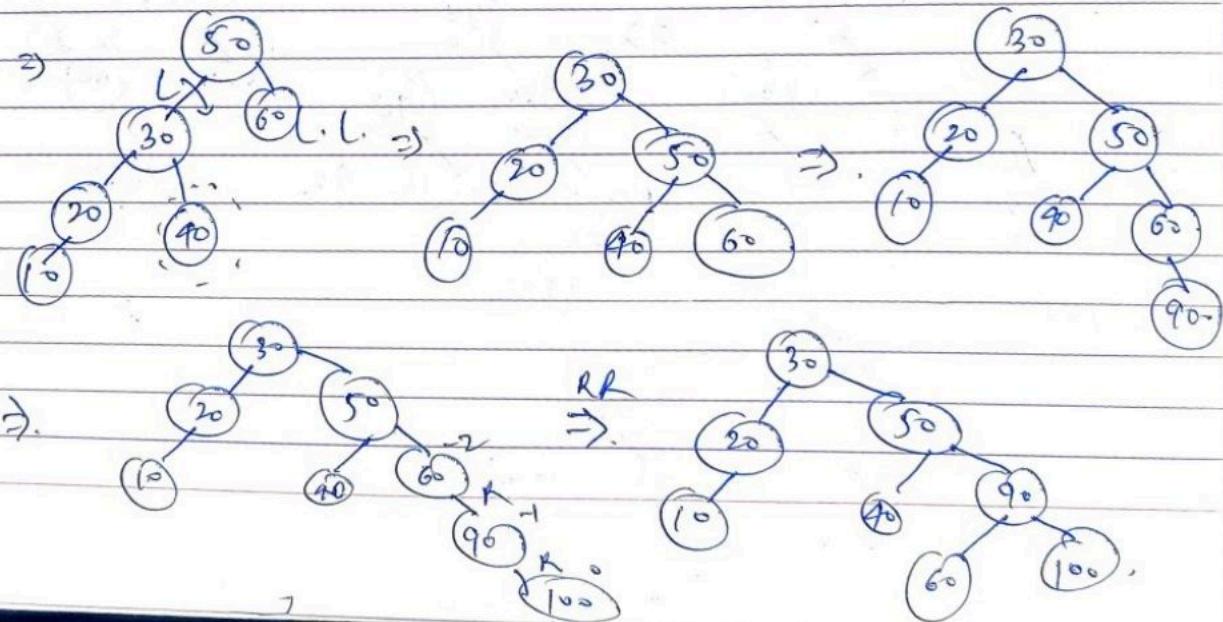
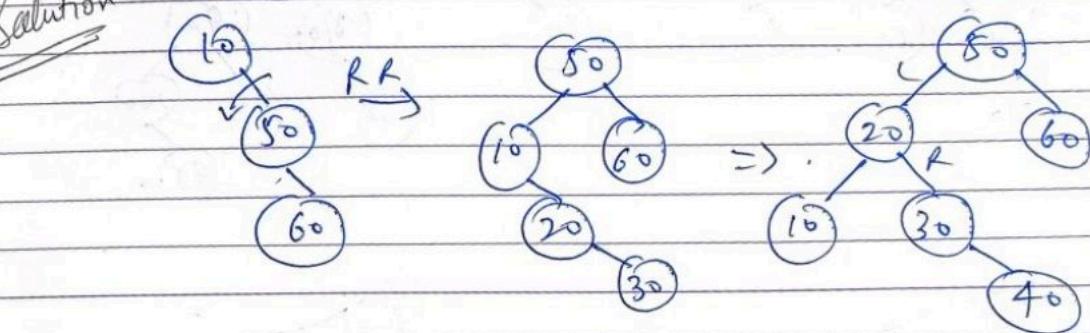


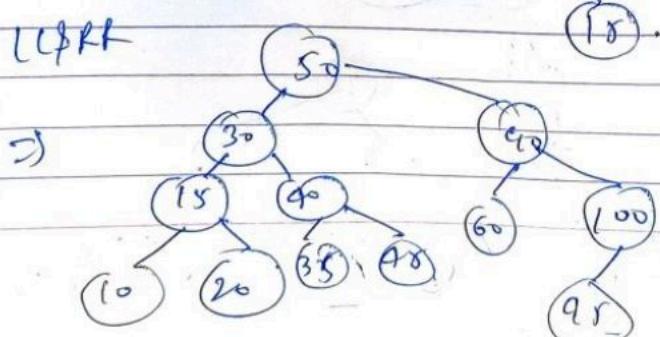
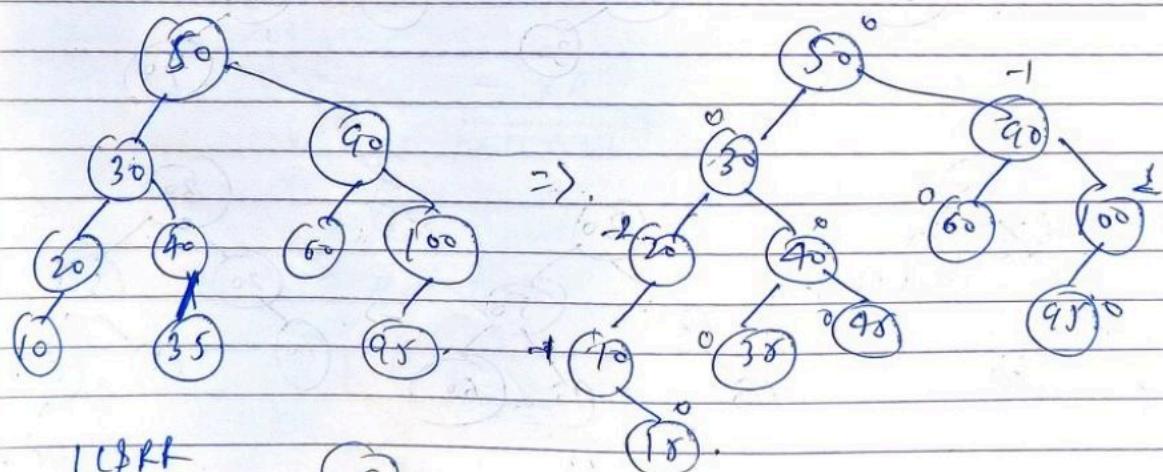
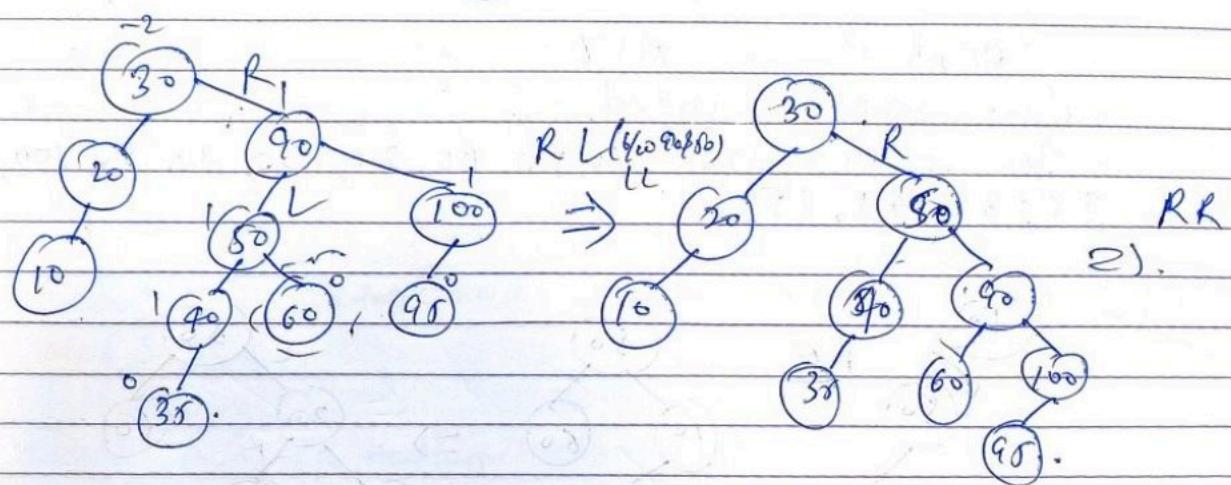
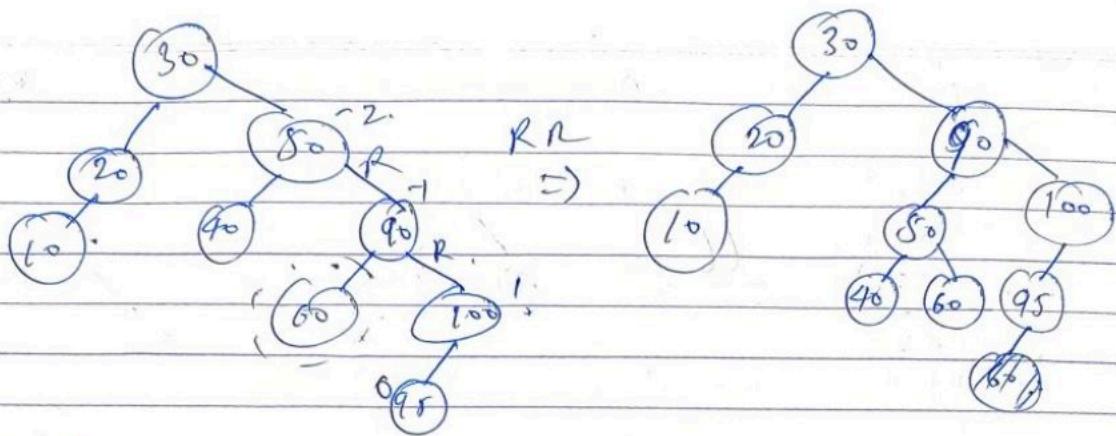
$\Rightarrow$  RL Rotation -



Ex- Construct a AVL Tree when the values are inserted in following sequence  
 $\&$  The values are 10, 50, 60, 20, 30, 40, 90, 100,  
 85, 35, 45, 15.

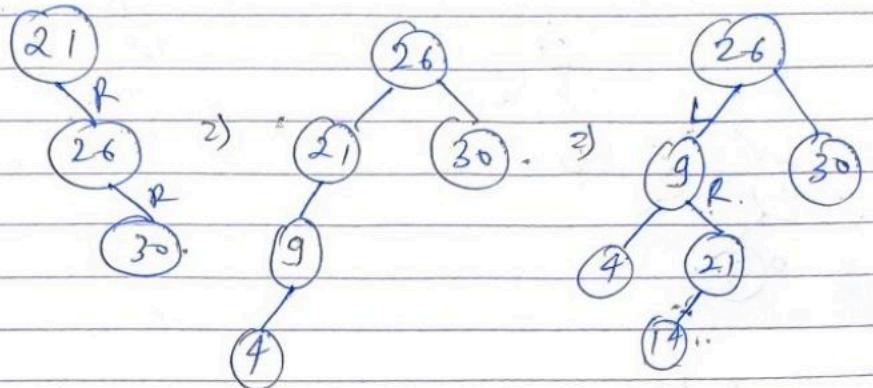
Solution-



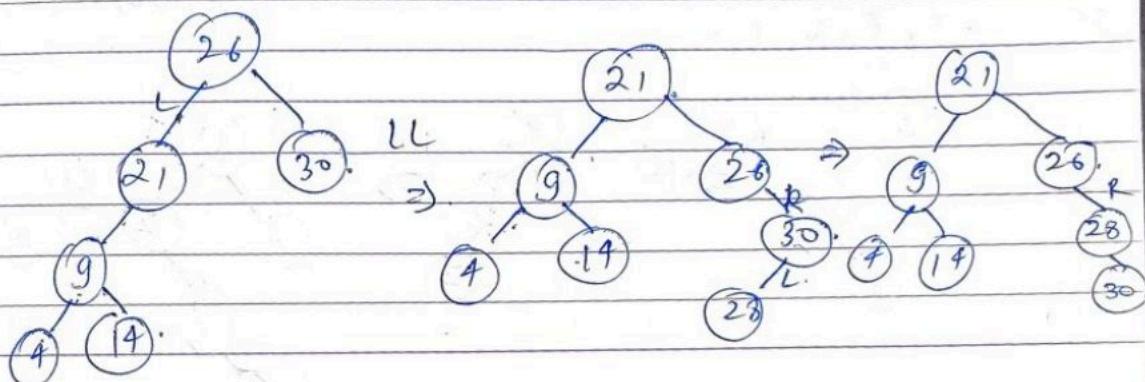


Q. AVL :- →

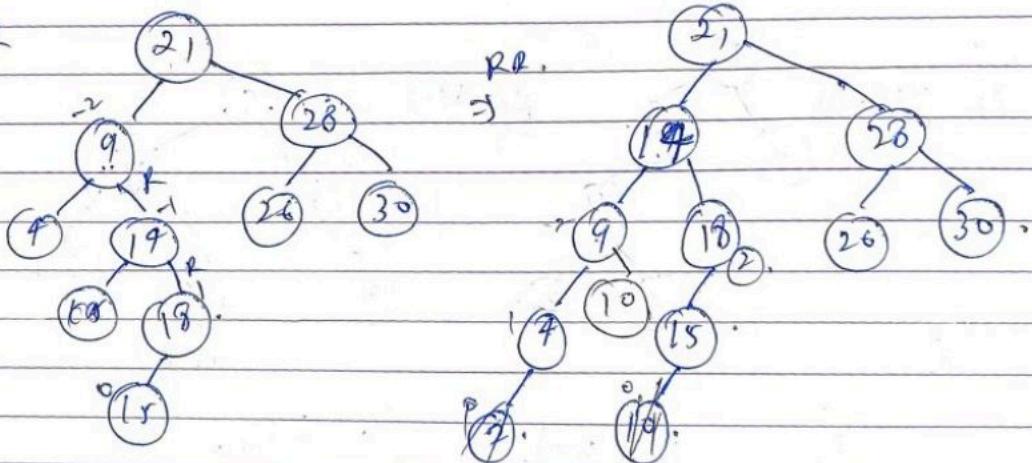
21, 26, 30, 9, 4, 14, 28, 18, 15, 10, 2, 3, 7

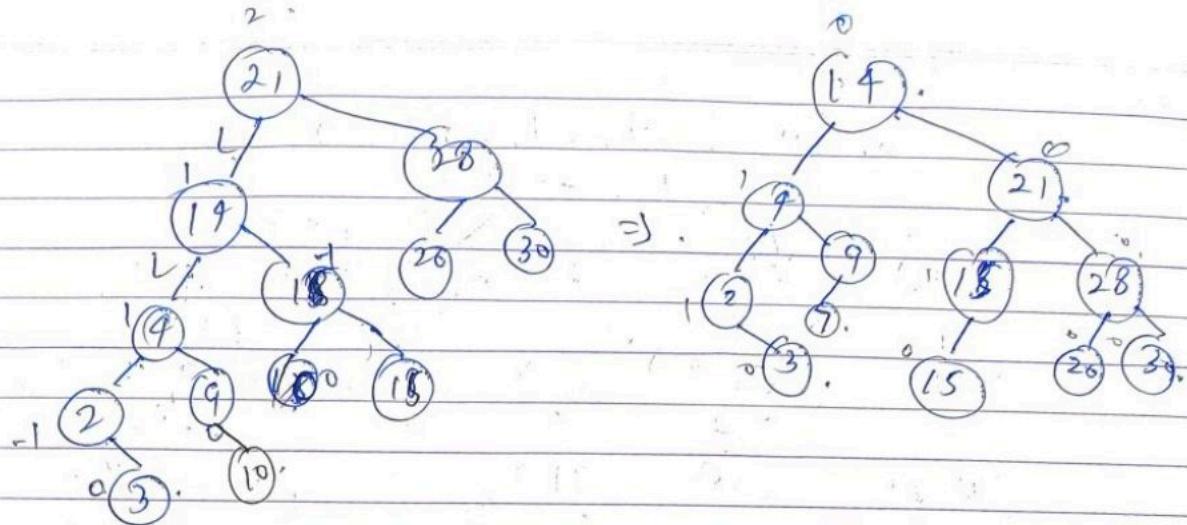


3.

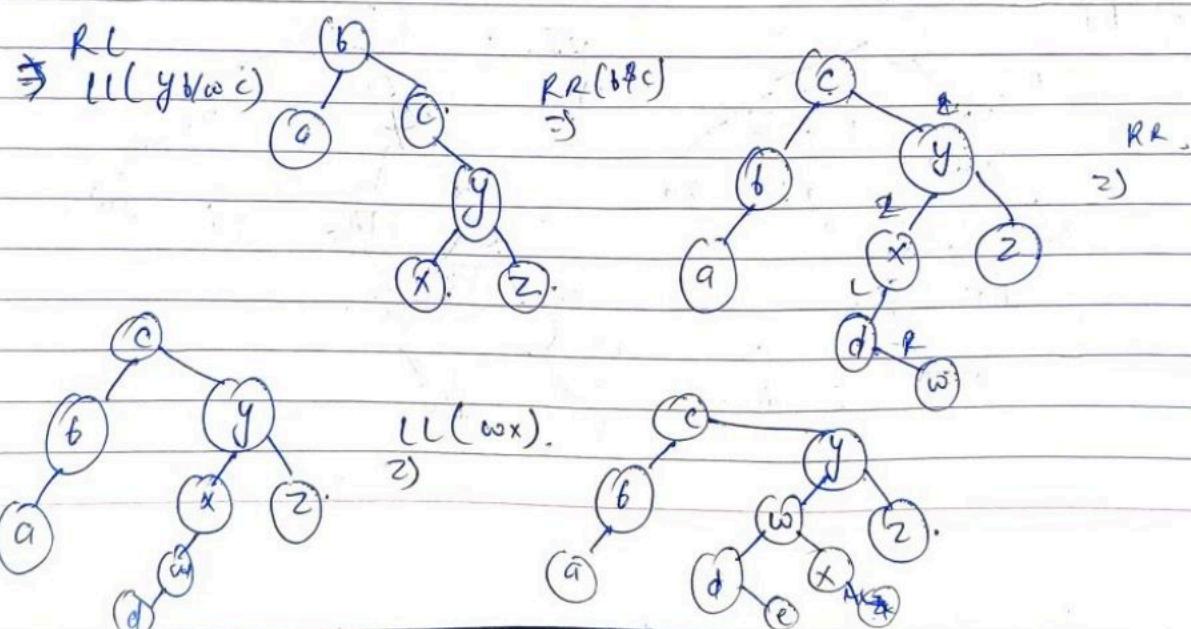
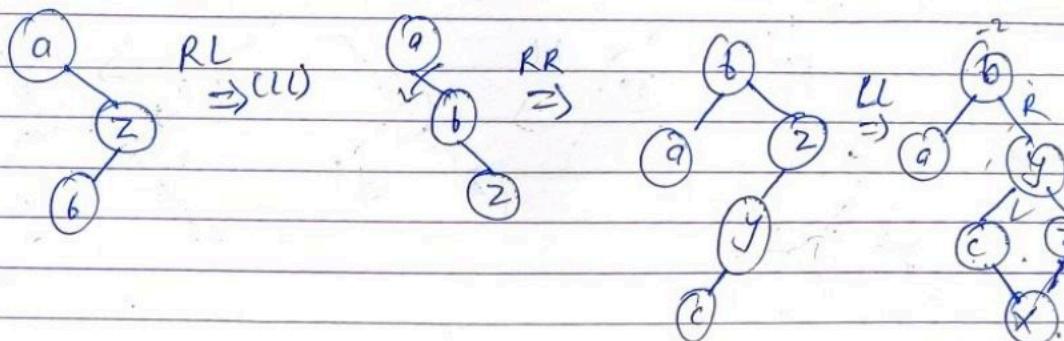


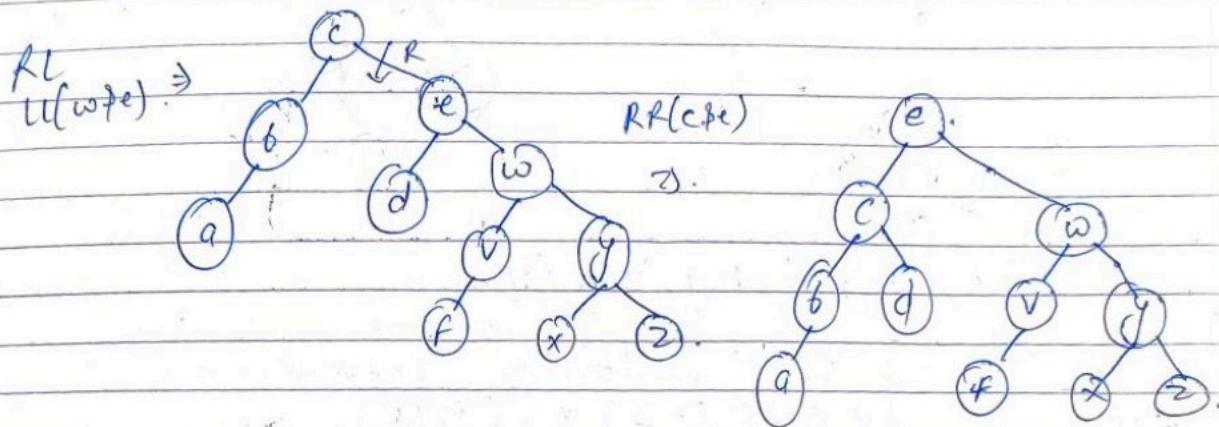
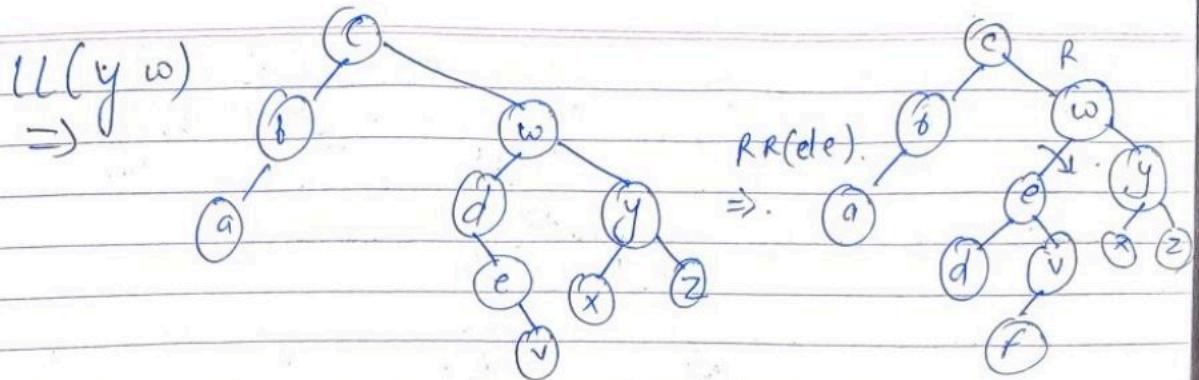
→ RR



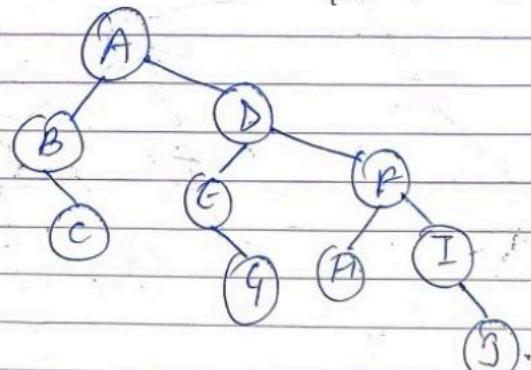


Q) Find the AVL Prece of a, z, b, y, c, x, d, w, e, v, f.





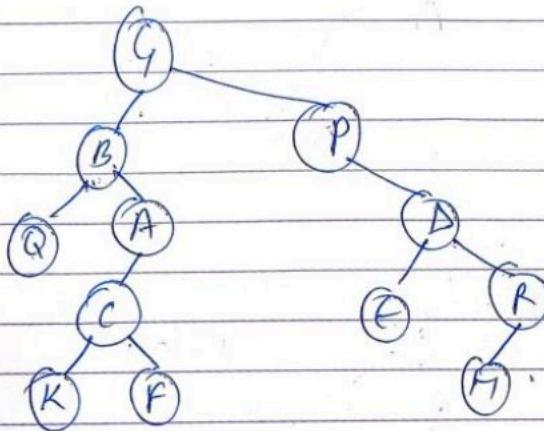
Q      Inorder  $\rightarrow \underline{B, C, A, E, G, D, H, F, I, S}$ ,  
 Preorder  $\rightarrow \underline{A, B, C, D, E, G, F, H, I, S}$ .



Post -  $C, B, G, E, H, S, I, F, D, A$ .

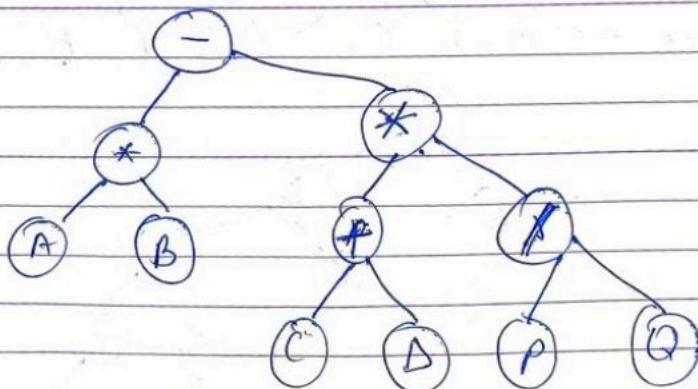
Q Inorder  $\Rightarrow Q, B, K, C, F, A, G, P, E, D, H, L.$   
Preorder  $\Rightarrow Q, B, Q, A, C, K, F, P, D, E, R, H.$   
Postorder  $\Rightarrow ?$

Solution:



Post - Q, R, F, C, A, B, E, H, L, D, P, G.

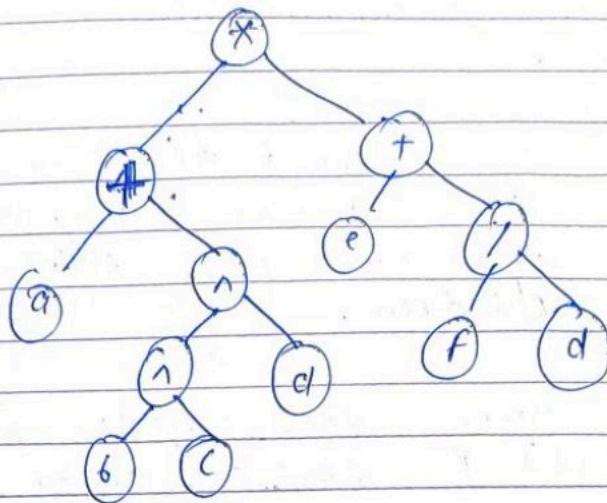
Q Draw the ^ following expression -  
 $A * B - C (C + D) * (P/Q)$ .



Preorder =  $- * A B$

Q

Convert the infix string  $(a + b \cap c \cap d) \times (e \cap f \cap d)$ . to reverse polish.



a, b, c, n, d, n, +, t, e, f, d, l, \*, .

Q

$6 + 2 \cap 3 + 9 / 3 - 4 * 5$ .

## Self Balance Tree

### B-Tree

- (i) B-Tree is a balanced 'm'-way tree where m is known as order of B-Tree.
- (ii) B-Tree is a generalization of binary search tree (B.S.T) in which a node can have more than one key elements & more than two children.
- (iii) In B-Tree each node maintains data (Keys value) in sorted order.
- (iv) All leaf node should be at same level
- (v) B-Tree of order m has following properties-

- (a) Every node has maximum 'm' children.
- (b) Minimum children : leaf  $\rightarrow 0$ .  
Root  $\rightarrow 2$ .

$$\text{Internal Node} \rightarrow \left\lceil \frac{m}{2} \right\rceil$$

$$\text{ex} - \left\lceil \frac{5}{2} \right\rceil = [2.5] = 3.$$

- (c) Every node has maximum  $(m-1)$  keys.
- (d) Minimum Keys : Root  $\rightarrow 1$ .  
all other node  $\rightarrow \left\lceil \frac{m}{2} \right\rceil - 1$ .

(V)

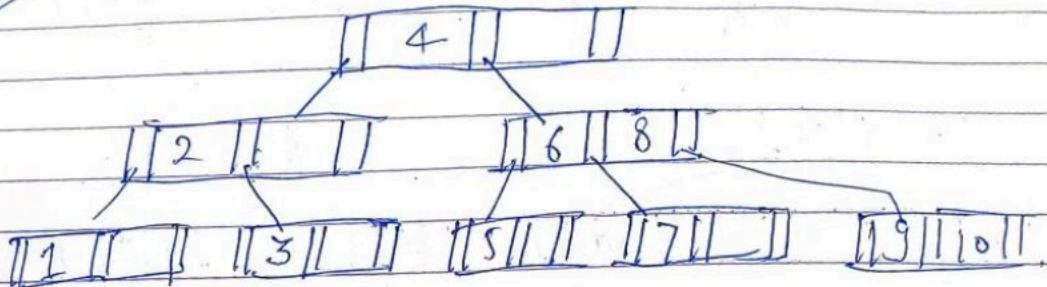
every node would be inserted on leaf.

NOTE - B-Tree will grow upward

Q Construct a B-Tree of order 3. by inserting values 1 to 10.

Solution-

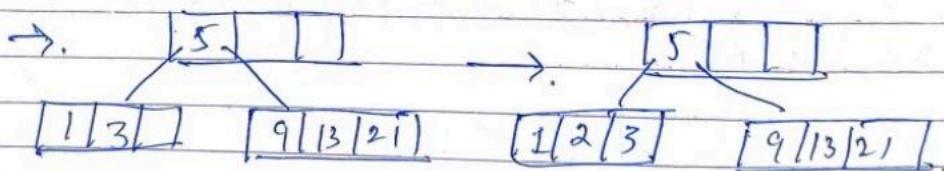
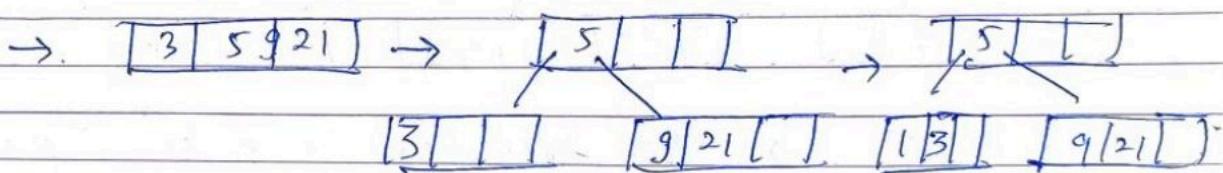
$$m=3, k=2$$

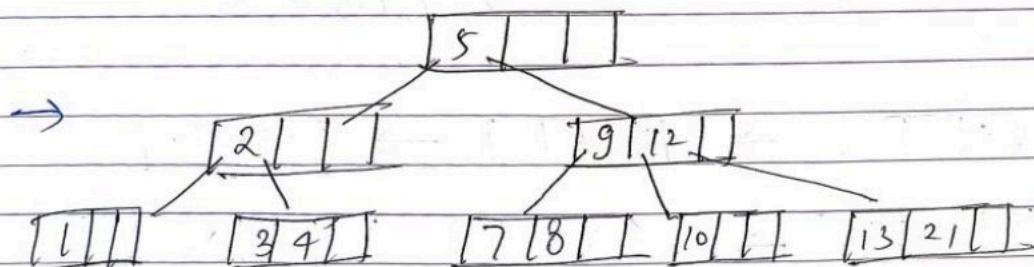
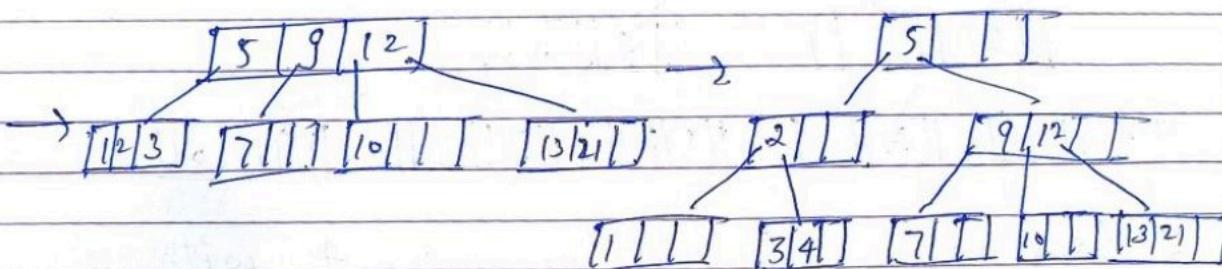
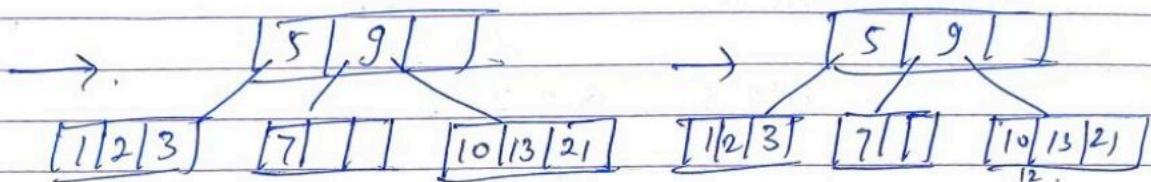
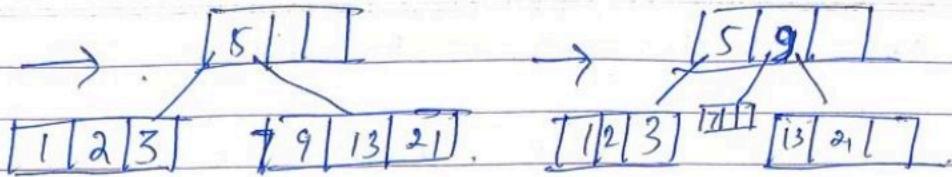


Q Construct a B-Tree of order 4 with following element - .

5, 3, 21, 9, 1, 13, 2, 7, 10, 4, 8.

Solution-





Q) Construct a B-tree of order 5 inserting the following sequence elements.

3, 14, 7, 1, 8, 5, 11, 17, 13, 6, 23, 12, 20, 26, 4, 16, 18, 24, 25, 19

$\Rightarrow$

[1, 3, 7, 14]

→

$\boxed{1 \ 3 \ 7 \ 14}$

8.

$\begin{array}{|c|c|c|} \hline 7 & & \\ \hline | & | & | \\ \hline 1 & 3 & | \\ \hline 8 & 14 & 1 \\ \hline \end{array}$

→ .

$\begin{array}{|c|c|c|} \hline 7 & & \\ \hline | & | & | \\ \hline 1 & 3 & 5 \\ \hline 8 & 14 & | \\ \hline \end{array}$

→ .

$\begin{array}{|c|c|c|} \hline 7 & & \\ \hline | & | & | \\ \hline 1 & 3 & 5 \\ \hline 8 & 11 & 14 \\ \hline \end{array}$

→ .

$\begin{array}{|c|c|c|} \hline 7 & & \\ \hline | & | & | \\ \hline 1 & 3 & 13 \\ \hline 8 & 11 & 14 \\ \hline \end{array}$

→

$\begin{array}{|c|c|c|} \hline 7 & 13 & \\ \hline | & | & | \\ \hline 1 & 3 & 5 \\ \hline 8 & 11 & 13 \\ \hline | & | & | \\ \hline 14 & 11 & 17 \\ \hline \end{array}$

→ .

$\begin{array}{|c|c|c|} \hline 7 & 13 & \\ \hline | & | & | \\ \hline 1 & 3 & 8 \\ \hline 8 & 11 & 11 \\ \hline | & | & | \\ \hline 14 & 12 & 1 \\ \hline \end{array}$

→ .

$\begin{array}{|c|c|c|} \hline 7 & 13 & \\ \hline | & | & | \\ \hline 1 & 3 & 5 \\ \hline 8 & 11 & 11 \\ \hline | & | & | \\ \hline 14 & 11 & 23 \\ \hline \end{array}$

→ .

$\begin{array}{|c|c|c|} \hline 7 & 13 & \\ \hline | & | & | \\ \hline 1 & 3 & 5 \\ \hline 8 & 11 & 12 \\ \hline | & | & | \\ \hline 14 & 11 & 23 \\ \hline \end{array}$

→ .

$\begin{array}{|c|c|c|} \hline 7 & 13 & \\ \hline | & | & | \\ \hline 1 & 3 & 6 \\ \hline 8 & 11 & 12 \\ \hline | & | & | \\ \hline 14 & 11 & 20 \\ \hline \end{array}$

→ .

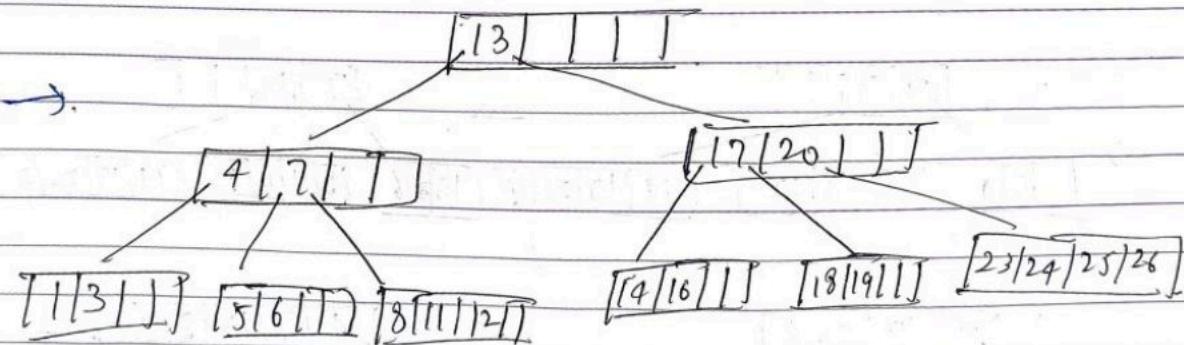
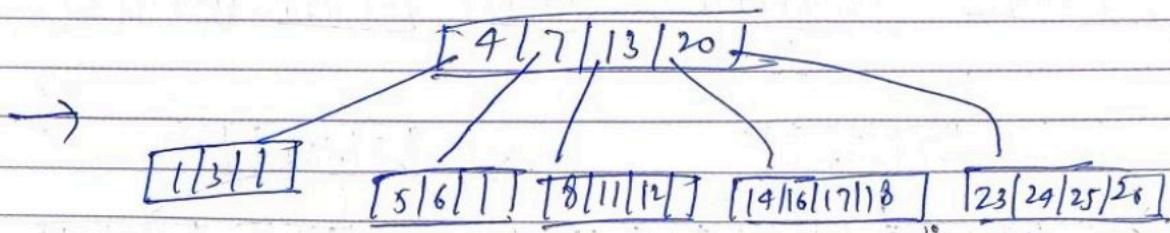
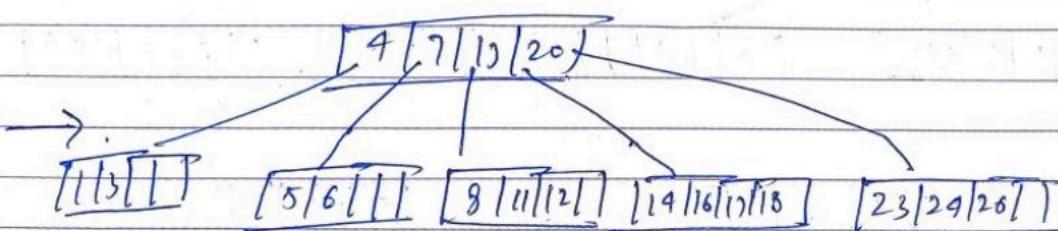
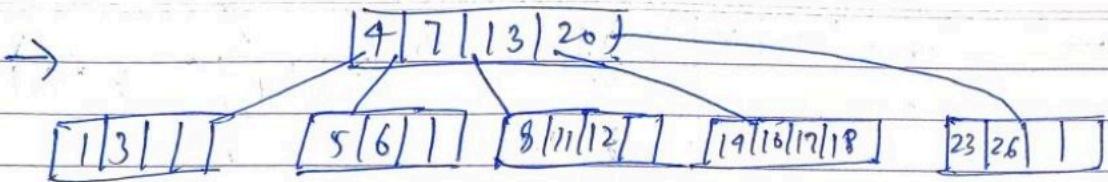
$\begin{array}{|c|c|c|} \hline 7 & 13 & 20 \\ \hline | & | & | \\ \hline 1 & 3 & 8 \\ \hline 8 & 11 & 12 \\ \hline | & | & | \\ \hline 14 & 11 & 7 \\ \hline | & | & | \\ \hline 23 & 26 & 1 \\ \hline \end{array}$

→ .

$\begin{array}{|c|c|c|} \hline 4 & 7 & 13 & 20 \\ \hline | & | & | & | \\ \hline 1 & 3 & 11 & \\ \hline 9 & 11 & | & | \\ \hline 8 & 11 & 12 & \\ \hline | & | & | & | \\ \hline 14 & 11 & 7 & \\ \hline | & | & | & | \\ \hline 23 & 26 & 1 & \\ \hline \end{array}$

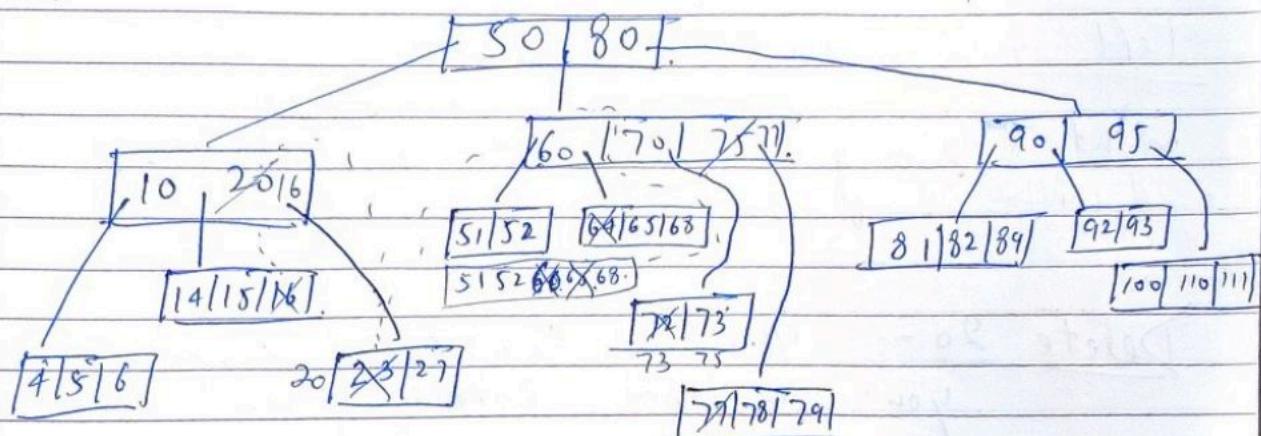
→ .

$\begin{array}{|c|c|c|} \hline 4 & 1 & 3 & 7 & 13 & 20 \\ \hline | & | & | & | & | & | \\ \hline 1 & 3 & 1 & \\ \hline 5 & 6 & 1 & \\ \hline 8 & 11 & 12 & \\ \hline | & | & | & | \\ \hline 14 & 16 & 11 & 7 \\ \hline | & | & | & | \\ \hline 23 & 26 & 1 & 1 \\ \hline \end{array}$



Q

## Deletion Operation on B-Tree.



B-Tree of order 5.

$$\text{min. Child} = \left\lceil \frac{m}{2} \right\rceil = 3.$$

$$\text{Max. Child} = m = 5.$$

$$\text{Min. Key} = \frac{m-1}{2} = 2.$$

$$\text{Max. Key} = m-1 = 4.$$

Left Borrow -

left sibling max. value will move to its parent & parent key will move to target node & then delete the target node.

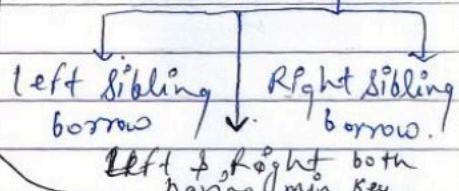
Right Borrow -

Right sibling min. value will move to its parent & parent key will move to target node in sorted order for

The value can be found on

- ① Root.
- ② Intermediate.
- ③ Leaf.

More than min. Keys. Exactly min. Key.



delete the target node.

Left & Right Borrow - Merges the target node with its right sibling or left sibling along with its parent.

Delete 2o - This is the case where you can not borrow parent node.

