GOVT. TOOL ROOM AND TRAINING CENTRE
KARNATAKA

REFERENCE NOTES

STRENGTH OF MATERIALS

SUBJECT CODE: DTDM IIIS 302

FOR
: DIPLOMA IN TOOL AND DIE MAKING
: DIPLOMA IN PRECISION MANUFACTURING

UNIT 1: SYSTEM OF FORCES
(Composition & resolution of forces)

UNIT – 1

- ✓ Introduction - Definition of SOM
- ✓ Physical quantities, Scalars and vectors
- ✓ Definition of a force and its unit
- ✓ Component forces and resultant force
- ✓ Composition of forces
- ✓ Resolution of force
- ✓ Law of parallelogram of forces
- ✓ Law of triangle of forces
- ✓ Law of polygon of forces
- ✓ Equilibrium of a body
- ✓ Lami's theorem (With proof)
- ✓ L) Problems (Analytical methods)

FORCE: - Force is an agent tends to produce or tends to destroy motion.
(Unit is N or kg)

- To pump a bicycle pump force is required.
- A horse applies force to pull a cart to set it in motion.
- To kick a ball.
- To require Force cut one piece in to the 2 pieces.

SCALAR AND VECTOR QUANTITY

SCALAR QUANTITY: - It is physical quantity, which has only magnitude but no direction.

Ex: - Force, Speed, Mass.

VECTOR QUANTITY: - It is physical quantity which has both magnitude and direction.

Ex: - Velocity, acceleration etc.

WHAT ARE THE CHARACTERISTICS OF FORCE:

1) It has magnitude of the force (10 N ,5 kg)
2) Line of action of the force (Inclined, Vertical) the direction of line along which the force acts.
3) Nature of the force (Push & Pull)

4) The point at which the force acts on the body. (centre of body & corner of body)

EFFECTS OF FORCES

1) It may change the motion of the body.
2) It may retard the motion of the body.
3) It may balance the forces already acting on the body (Equilibrium).
4) It may give rise to the internal stresses in the body, which acts.

RESULTANT FORCE (R)

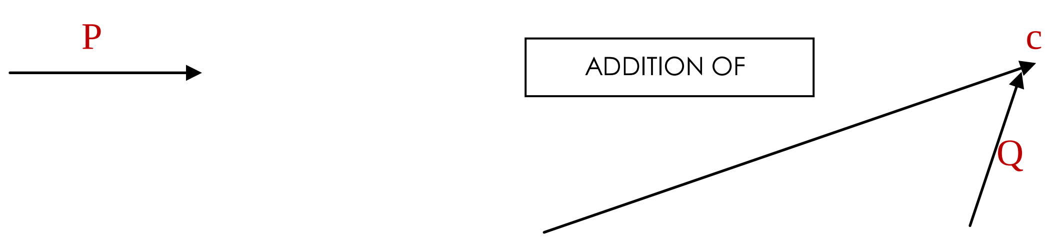
If number of force PQRS are acting simultaneously on a particle, it is possible to find out a single force which would replace them, that is which would produce same effect as produced by all the given force. This force is called “component force”.

COMPOSITION OF FORCE:

To find out a single resultant force this would produce same effect as produce by all the given forces. These have known as composition of forces.

ADDITION OF VECTORS:

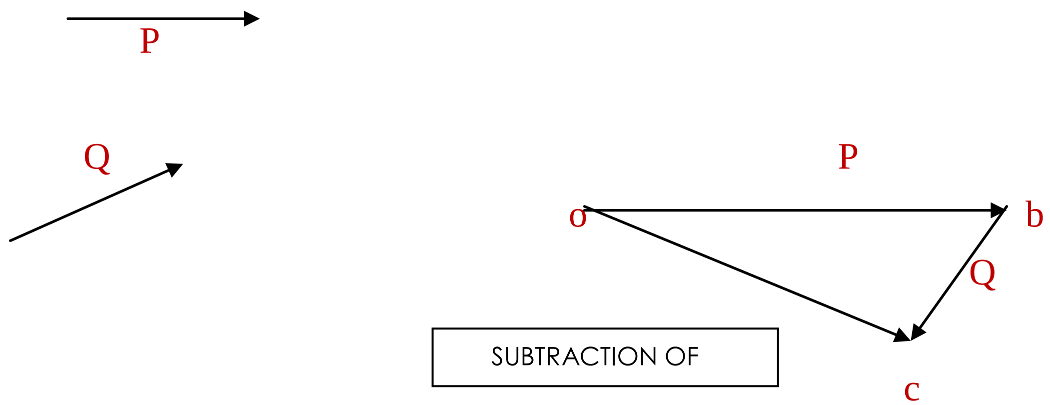
Let P & Q Two vectors, which are required to be added



Take a point 'o' & draw line ob parallel to & equal in magnitude to the vector 'P' , with convenient scale (1kg = 1cm). Similarly bc Then join oc with opposite direction will gives the required sum of the vectors P&Q.

SUBTRACTION OF VECTORS:

Let P & Q Two Vectors which are required to be subtracted..



Take a point 'o' & draw Ob parallel & equal in magnitude to the Vector 'P', with convenient scale. Through 'b' draws bc parallel & Equal magnitude to 'Q' to some scale. Then join oc with opposite direction will gives the required subtraction of the Vectors P & Q.

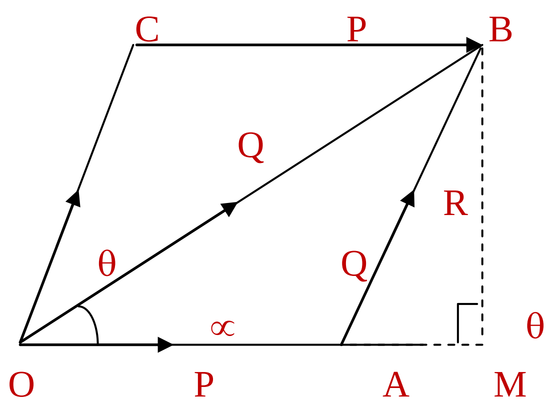
ANALYTICAL METHODS OF FINDING OUT THE RESULTANT FORCE (R)

There are 2 methods

- By the method of law of parallelogram of forces.

❖ By the method of resolution.

1) By the method of law of Parallelogram of forces :



"If two Vectors P & Q acting simultaneously on a particle be represented in magnitude & direction by the two adjacent sides of a parallelogram. Then the resultant may be represented in magnitude & direction by the diagonal of the parallelogram, which posses through their point of intersection".

PROOF: Consider the Δle BAM

$\text{Sin}\theta = BM/AB$

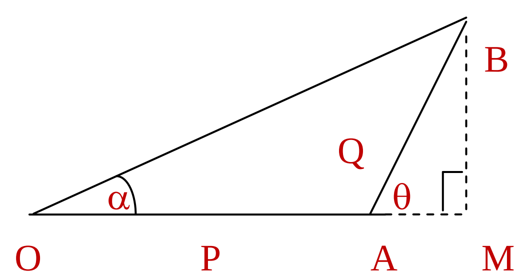
$BM = AB\ \text{Sin}\theta$

$BM = Q\ \text{Sin}\theta$ (ABIIOC)

$\text{Cos}\theta = AM/AB$

$AM = AB\ \text{Cos}\theta$

$AM = Q\ \text{Cos}\theta$



Consider the Δle OBM

$OB^2 = OM^2 + BM^2$

$OB = \sqrt{OM^2 + BM^2}$ (OA=P)

$OB = \sqrt{(OA+AM^2) + BM^2}$

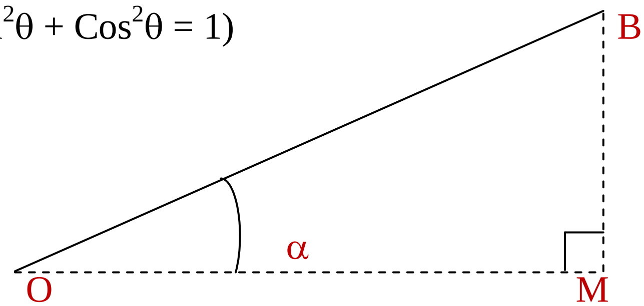
$OB = \sqrt{(P+Q\ Cos\theta)^2 + (Q\ Sin\theta)^2}$

$= \sqrt{P^2 + Q^2\ Cos^2\theta\ + 2PQ\ Cos\theta\ + Q^2\ Sin^2\theta}$

$= \sqrt{P^2+2PQCos\theta + Q^2\ (Cos^2\theta + Sin^2\theta)}$

$= \sqrt{P^2 + 2PQ\ Cos\theta + Q^2}$ $\quad (\quad sin^2\theta + Cos^2\theta = 1)$

$R = \sqrt{P^2+Q^2+2PQCos\theta}$



From the triangle OBM

$\tan\alpha = BM/OM$

$\tan\alpha = Q\ Sin\theta\ /\ P + Q\ Cos\theta$

$\alpha = Tan^{-1}\ Q\ Sin\theta\ /\ P + Q\ Cos\theta$

1) 2 Forces of 10 kg & 15 kg acts simultaneously at a point find the resultant force, if the angle between two forces is 30°.

$P = 10\ kg \qquad Q = 15\ kg \qquad \theta = 30°$

$R = \sqrt{P^2 + Q^2 + 2PQ\ Cos\ \theta}$

$= \sqrt{(10)^2 + (15)^2 + 2x10x15xCos30}$

$= \sqrt{100 + 225 + 300 + 0.86}$

$= \sqrt{100 + 225 + 258}$

$= \sqrt{583}$

$\therefore$ **R = 24.14 kg**

2) A push of 18 N & pull of 35N acts simultaneously at a point find the resultant of force, if the angle between 2 forces be 135°.

$P = -18N \qquad Q = 35N \qquad \theta = 135°$

$R = \sqrt{P^2+Q^2 + 2PQ\ Cos\theta}$

$= \sqrt{(-18)^2 + (35)^2 + 2x\text{-}18x35xcos135}$

$= \sqrt{324 + 1225 - 1260\ x\text{-}0.7}$

$= \sqrt{324 + 1225 + 891}$

$= \sqrt{2440}$

$\therefore$ **R = 49.39N**

3) Find the magnitude of 2 forces such that if they act at right angles there resultant is $\sqrt{10}$ N. But if they act at 60° there resultant is $\sqrt{13\ N}$.

1) At right angle (90°) $\qquad R = \sqrt{10\ N}$

2) At $\theta = 60°$ $\qquad R=\sqrt{13\ N}$

$\sqrt{10} = \sqrt{P^2 + Q^2 + 2PQCos90°}$

$\sqrt{10} = \sqrt{P+Q+2PQx0}$

$\sqrt{10} = \sqrt{P^2 + Q^2}$

$10 = P^2 + Q^2$

$\sqrt{13} = \sqrt{P+Q+2PQCos60*}$

$= \sqrt{P+Q+2PQ}$

$\sqrt{13} = \sqrt{P+Q+PQ}$

$13 = \underline{P+Q}+PQ$

$13 = 10+PQ$

$(P+Q)^2 = P^2 + Q^2 + 2PQ$

$= 10 + 2 \times 3$

$P + Q = \sqrt{16}$

$P + Q = 4 \quad \rightarrow (1)$

$(P - Q)^2 = P^2 + Q^2 - 2PQ$

$= 10 - 2x3$

$P - Q = \sqrt{4}$

$P - Q = 2 \quad \rightarrow (2)$

Add EQ (1) & (2)

$P + Q = 4$

$\underline{P - Q = 2}$

$2P = 6$

$P = 6/2$

$P = 3N$

$Q = 1N$

Equ No .1 $P + Q = 4$

$Q = 4-3=1N$

4) 2 forces whose magnitude are P & √2 P, act on a particle in direction at an angle of 135° to each other. Find the magnitude & direction of resultant.

$P = P \qquad Q= \sqrt{2}\,P \qquad \theta = 135°$

$R = \sqrt{P^2 + Q^2 + 2PQ\,Cos\theta}$

$= \sqrt{P + \sqrt{2}P + 2P + \sqrt{2}\,P \times -7}$

$= \sqrt{P + 2P - 2P}$

$= \sqrt{P}$

$\therefore$ **R = P**

$$\alpha = \tan^{-1} \frac{Q\,Sin\theta}{P + Q\,Cos\theta}$$

$= \tan^{-1} \underline{P\sqrt{2}\,sin135}$

$$= \frac{P + P\sqrt{2}\,\text{Cos}\,135}{P}$$

P-P = (P) = 0

∴ α = 90°

2) BY THE METHOD OF RESOLUTION

By resolution of a force is meant to split up the force into a number of its component without changing its effect on the body & force is resolved in to 2 mutually perpendicular directions.

PRINCIPLE OF RESOLUTION

"It states the algebraic sum of the resolved parts of forces in a given direction is equal to the resolved part of their resultant in the same direction ".

ANALYTICAL METHOD FOR THE RESULTANT FORCE

2 Methods for find R

a) Analytical

b) Graphical

STEPS:

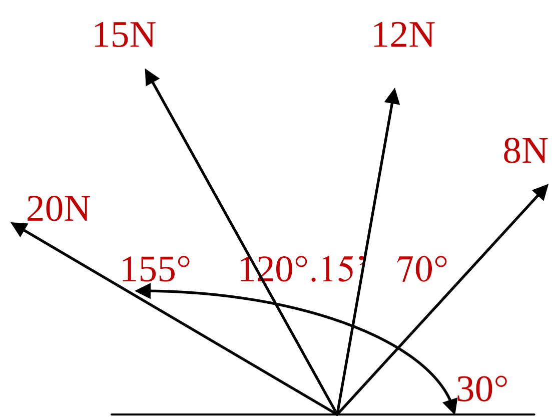
- Resolve all the forces horizontally & find ΣH.
- Resolve all the forces vertically & find ΣV.
- Then Resultant R= $\sqrt{\Sigma H^2 + \Sigma V^2}$
- tan α = $\Sigma V / \Sigma H$
- NOTE : - 1) When ΣH is positive the resultant makes an angle between 0° - 90° & 270° - 360° If ΣH is Negative the resultant makes an angle between 90° - 270°.
- ΣV is positive then R is makes an angle between 0°- 180° If it is negative 180° - 360°.

NOTE: Horizontal component of force= Cos θ

Vertical component of force =sinθ

PROBLEMS

1) Find the magnitude & direction of the resultant of the 4 concurrent forces of 8N, 12N, 15N, & 20N making angle of 30°, 70°, 120°.15’, & 155° respectively with a fixed line.



$\Sigma H = 8\cos 30° + 12\cos 70° + 15\cos 120°15' + 20\cos 155°$

$\therefore \Sigma H = -14.65N$

$\Sigma V = 8\sin 30° + 12\sin 70° + 15\sin 120°15' + 20\cos 155°$

$\therefore \Sigma V = 36.68N$

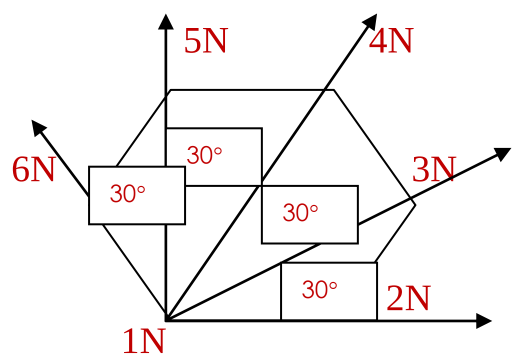
$R = \Sigma H + \Sigma V$
$= 214.62 + 1345.4$
$= 1560.04$
$\therefore R = 39.49N = 39.5N$
$\therefore \tan\alpha = \Sigma V / \Sigma H$
$= 36.68/-14.65$
$\tan\alpha = -2.50$
$\alpha = \tan(-2.5)$
$\alpha = -68°1'55''$

NOTE: - Since ΣV is '+'ve & ΣH is –ve therefore θ lies between 90° - 180°.

Actual $\theta = 180 - 68°12'$
$= 111°\,48'$

2) The force if 2,3,4,5 & 6N are acting on one of the angular point of a regular hexagon towards other 5 angular points taken in order find R & θ .

NOTE: - To finding included angle of any polygon = $\frac{2n-4}{n}$ X90



Where n = number of sides.

Included angle for hexagon

$= \frac{2n-4}{n} \times 90$

$= \frac{2\times6-4}{6} \times 90$

$= \frac{12-4}{6} \times 90$

= 120°

$\Sigma H = 2\cos0 + 3\cos30 + 4\cos60 + 5\cos90 + 6\cos120$
$= 2 + 2.59 + 2 + 0 + (-3)$
$\therefore \Sigma H = 3.59$
$= 3.60N$
$\Sigma V = 2\sin0 + 3\sin30 + 4\sin60 + 5\sin90 + 6\sin120$
$= 0 + 1.5 + 3.46 + 5 + 5.19$
$\therefore \Sigma V = 15.16N$
$R = \sqrt{\Sigma V^2 + \Sigma H^2}$

$= \sqrt{(15.16)^2 + (3.6)^2}$

$\therefore$ **R = 15.58N**

$\tan\theta = \Sigma V / \Sigma H$

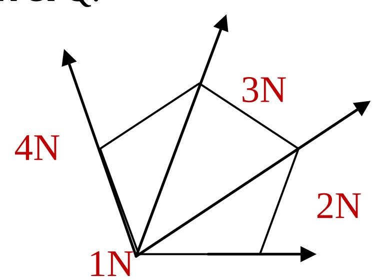
$= 15.16 / 3.6$

$= 4.211$

$\theta = \tan^{-1} 4.211$

$\therefore$ **θ = 76* 39'**

3) The forces of 1, 2, 3, 4N are acting on one of the angular points of a regular pentagon towards other 4 angular points taken in order Find R & Q.



Included angle for pentagon

$= \frac{2n-4}{n} X90$

$= \frac{10-4}{5} X90$

$= 108°$

$\Sigma H = 1 \cos 0° + 2 \cos 30° + 3 \cos 70° + 4 \cos 108°$

$= 1 + 1.73 + 1.02 + (-1.23)$

$\Sigma H = 2.52N$

$\Sigma V = 1 \sin 0° + 2 \sin 30° + 3 \sin 70° + 4 \sin 108°$

$= 0 + 1 + 2.81 + 3.80$

$\therefore \Sigma V = 7.61N$

$R = \sqrt{\Sigma V^2 + \Sigma H^2}$

$= \sqrt{(7.61)^2 + (2.52)^2}$

$= \sqrt{57.91 + 6.35}$

$= \sqrt{64.26}$

$\therefore$ **R = 8.01N**

$\tan\theta = \Sigma V / \Sigma H$

$= \dfrac{7.61}{2.52}$

$= 3.01N$

$\theta = \tan^{-1} 3.01$

$\therefore \theta = 71° \, 38'$

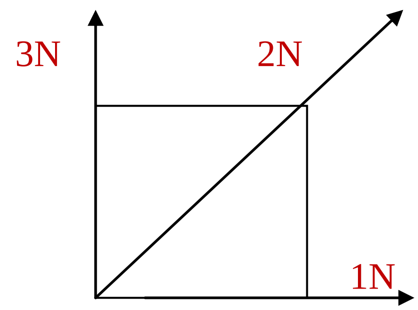
4) The force of 1, 2 & 3N are acting on one of the angular point of a regular square. Towards other 3 angular points taken in order. Find R &θ.

Included angle for square

$= \dfrac{2n-4}{n} \times 90$

$= \dfrac{2 \times 4 - 4}{4} \times 90°$

$= 90°$



$\Sigma H = 1 \cos 0° + 2 \cos 45° + 3 \cos 90°$

$= 1 + 1.41 + 0$

$\therefore \Sigma H = 2.41N$

$\Sigma V = 1 \sin 0° + 2 \sin 45° + 3 \sin 90°$

$= 0 + 1.41 + 3$

$\therefore \Sigma V = 4.41N$

$R = \sqrt{\Sigma V^2 + \Sigma H^2}$

$= \sqrt{(4.41)^2 + (2.41)^2}$

$= \sqrt{19.44 + 5.8}$

R = 5.02N

$\tan\theta = \Sigma V / \Sigma H$

$= \dfrac{4.41}{2.41}$

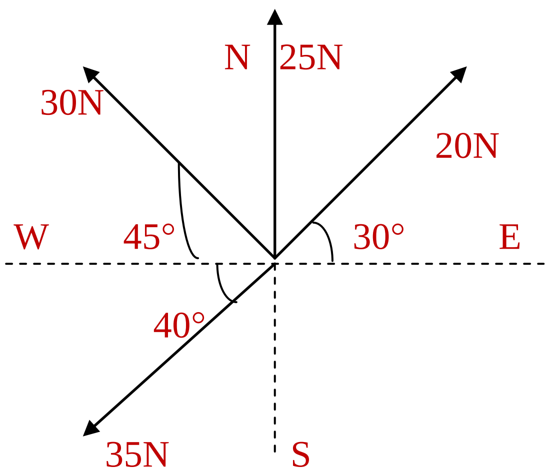
$= 1.82$

$\theta = \tan^{-1} 1.82$

$\therefore$ **θ = 61° 13'**

5) A particle is acted up on simultaneously by the following forces.
   a) 20N inclined 30° to north of east
   b) 25N towards the north
   c) 30N towards the north of west
   d) 35N inclined 40° to south of west

   Find R &θ.



$\Sigma H = 20 \cos 30° + 25 \cos 90° + 30 \cos 135° + 35 \cos 220°$

$\therefore$ **ΣH = -30.7N**

$\Sigma V = 20 \sin 30° + 25 \sin 90° + 30 \sin 135° + 35 \sin 220°$

$\therefore$ **ΣV = 33.7N**

$R = \sqrt{\Sigma V^2 + \Sigma H^2}$

$= \sqrt{(-30.7)^2 + (33.7)^2}$

$\therefore$ **R = 45.6N**

$\tan\theta = \Sigma V / \Sigma H$

$= \dfrac{33.7}{-30.7}$

$= -1.098$

$\theta = \tan^{-1}(-1.098)$

$\therefore \theta = 47° \; 42'$

Since ΣV is '+'ve & ΣH is '–'ve therefore θ lies between 90° to 180°

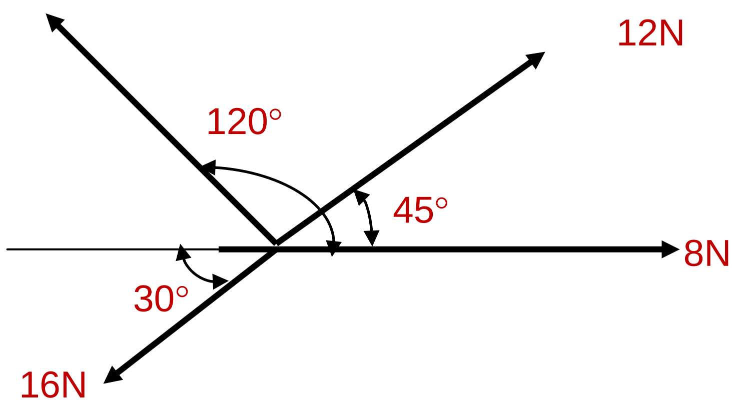
$\therefore$ Actual $\theta = 180 - 47° \; 42'$

$\therefore \theta = 132° \; 18'$

6) Following forces simultaneously act on a particle

a) 12N at 45° to the horizontal
b) 8N horizontally
c) 6N at 120° to the horizontal
d) 16N at 210° to the horizontal

Find R &θ.



$\Sigma H = 8 \cos 0° + 12 \cos 45° + 6 \cos 120° + 16 \cos 210°$

$= 8 + 8.48 + (-3) + (-13.85)$

$\therefore \Sigma H = -0.37N$

$\Sigma V = 8 \sin 0° + 12 \sin 45° + 6 \sin 120° + 16 \sin 210°$

$= 0 + 8.48 + 5.19 + (-8)$

$\therefore \Sigma V = 5.67N$

$R = \sqrt{\Sigma V^2 + \Sigma H^2}$

$= \sqrt{(-0.37)^2 + (5.67)^2}$

$= \sqrt{0.1369 + 32.14}$

$= \sqrt{32.28}$

$\therefore R = 5.68N$

$\text{Tan}\theta = \Sigma V / \Sigma H$

$= \dfrac{5.67}{-0.37}$

$= -15.32$

$\theta = \tan(-15.32)$

$\therefore \boldsymbol{\theta = 86° \, 16'}$

Since ΣV is '+'ve and ΣH is '–'ve therefore θ lies between 90° to 180°

$\therefore$ Actual $\theta = 180 - 86° \, 16'$

$\therefore \boldsymbol{\theta = 93°44'}$

7) The horizontal and vertical components of force acting at a point are 3N and 4N respectively. Find the magnitude and direction of the resultant

8) Two forces P= 40N & Q= 20N are acting simultaneously at a point. What is the resultant of these two forces if the angle between is 120^0

ASSIGNMENT

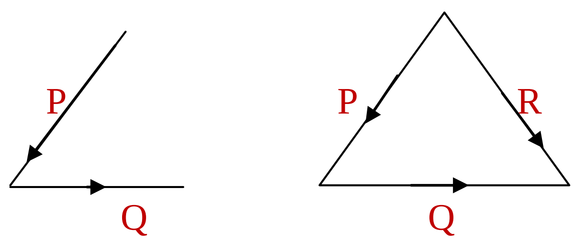
NOV.DEC. 2016

GRAPHICAL METHOD FOR RESULTANT FORCES

A) By triangle law of forces
B) By polygon law of forces
C) By the method of vector

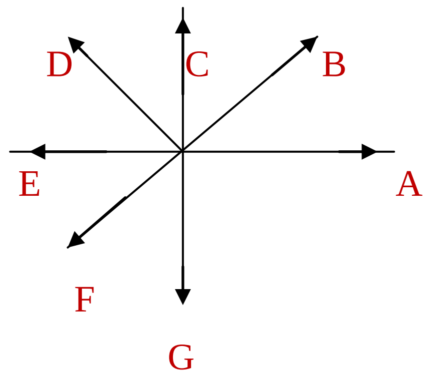
BY THE TRIANGLE LAW OF FORCES

"If 2 forces acting simultaneously on a particle be represented in magnitude and direction by the 2 sides of a triangle taken in order. Their resultant may represent in magnitude and direction by the 3rd side of the triangle taken in opposite order ".

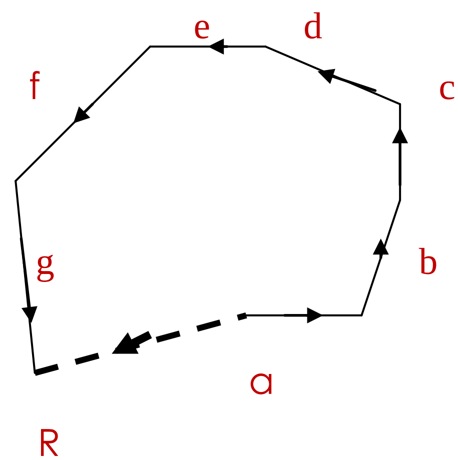


BY THE POLYGON LAW OF FORCES

"If a number of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon taken in order. Then resultant of all these forces may be represented in magnitude and direction by closing side of the polygon taken in opposite order.



Space diagram



Vector diagram

BY THE METHOD OF VECTOR FINDING OUT 'R'

STEPS:

1) First draw the space diagram for the given force.
2) Now select some suitable point then go on adding forces vectorially. (Based on the direction).
3) Then the closing side of the polygon with opposite direction wills gives the magnitude of the resultant force & its direction. (Based on the scale).

EQUILIBRIUM OF FORCES:

A set of forces whose resultant is 'zero' are called equilibrium of forces.

COLLINEAR FORCES

The forces whose lines of action lie on the same line are known as collinear forces

COPLANAR FORCES

The forces whose line of action lies on the same plain are known as coplanar forces.

CONCURRENT FORCES

The forces, which meet at one point, are known as concurrent forces.

COPLANAR - CONCURRENT FORCES

The forces whose line of action lies on the same plane and which meet at one point.

COPLANAR NON – CONCURRENT FORCES

The forces whose line of action lie on the same plane but not meet at one point.

NON – COPLANAR CONCURRENT FPRCES

The forces whose line of action lies not on the same planes and meets at one point.

NON COPLANAR – NON CONCURRENT FORCES

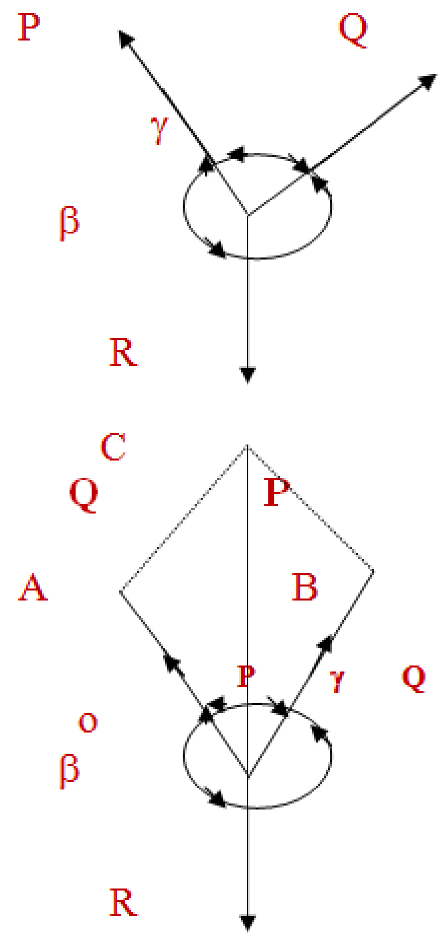
The forces whose line of action not lies on the same plane and not meet at one point is called Non coplanar – Non concurrent forces.

ANALYTICAL METHOD OF STUDY IN THE EQUILIBRIUM FORCES

Lami's Theorem (Only statement and proof in syllabus)

If three coplanar forces acting on a point be in equilibrium. Then each force is proportional to the sign of the angle between other two.

$$\frac{P}{\text{Sin}\,\alpha} = \frac{Q}{\text{Sin}\,\beta} = \frac{R}{\text{Sin}\gamma}$$



PROOF:

Consider three coplanar forces P, Q & R acting at a point O. Let the opposite angles to three forces be α, β & γ as shown in figure.

Now let us complete the parallelogram OACB with OA & OB & adjacent sides as shown in figure. We know that the resultant of two forces P & Q will be given by the diagonal OC both in magnitude and direction of the parallelogram OACB.

Since these forces are in equilibrium, therefore the resultant of the forces P & Q must be in line with 0D and equal to R, but in opposite direction.

From the geometry of the figure, we find

BC = P AND AC = Q

$\therefore \angle AOC = (180° - \beta)$

And $\angle ACO = \angle BOC = (180° - \alpha)$

$$\therefore \angle CAO = 180° - (\angle AOC + \angle ACO)$$
$$= 180° - [(180° - \beta) + (180° - \alpha)]$$
$$= 180° - 180° + \beta - 180° + \alpha$$
$$= \alpha + \beta - 180°$$

But $\alpha + \beta + \gamma = 360°$

Subtracting 180° from both sides of the above equation,

$(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$

Or $\angle CAO = 180° - \gamma$

We know that in triangle AOC,

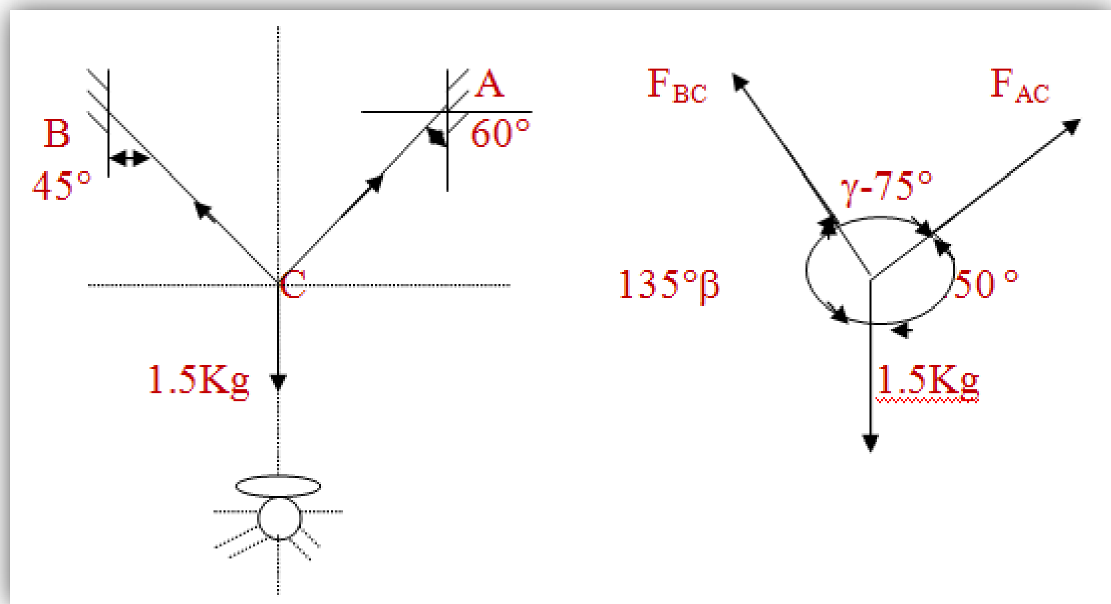
$$\frac{OA}{\sin\angle ACO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{}$$

$$\frac{OA}{\sin(180°-\alpha)} = \frac{AC}{\sin(180°-\beta)} = \frac{OC}{\sin(180°-\gamma)}$$

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} \text{ ------- } [\sin(180° - \theta) = \sin\theta]$$

Problems

1) An electric light fixture weighing 1.5 kg hangs from a point 'C' by 2 strings AC and BC. AC is inclined at 60° to the horizontal and BC is at 45* to the vertical using Lami's theorem determine the forces in strings AC & BC.



$$\frac{FAC}{\sin 126°52'} = \frac{FBC}{\sin 143°8'} = \frac{100}{\sin 90°}$$

$$FBC = \frac{100 \times \sin 143°8'}{\sin 90°}$$

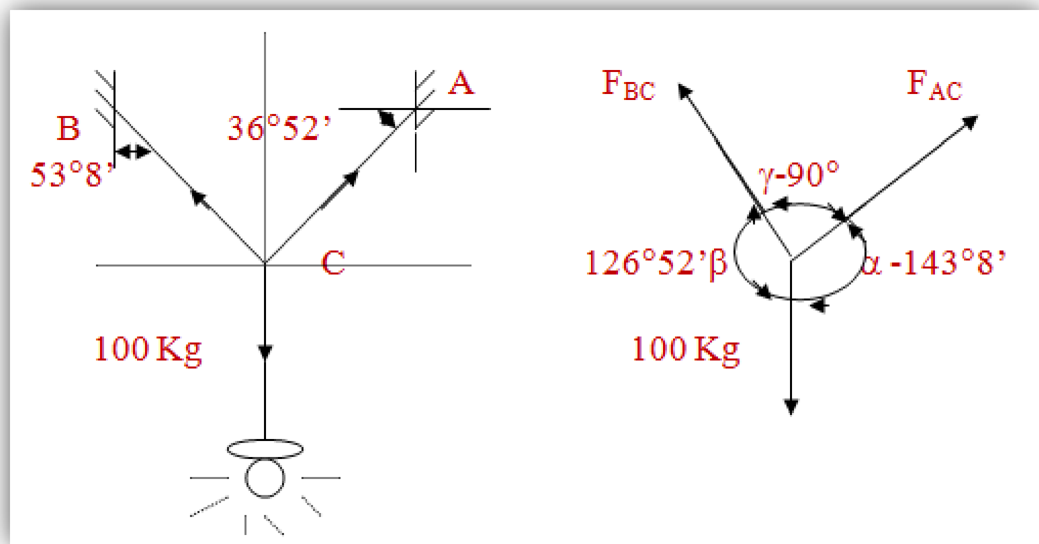
$\therefore$ FBC = **59.99** = **60 Kg**

$$\frac{FAC}{\sin 126°52'} = \frac{100}{\sin 90°}$$

$$FAC = \frac{100 \times \sin 126°52'}{\sin 90°}$$

FAC = 80/1 $\therefore$ **FAC = 80 kg**

2) Find the tension AC and BC.



$$\frac{F_{AC}}{\sin 126°52'} = \frac{F_{BC}}{\sin 143°8'} = \frac{100}{\sin 90°}$$

$$F_{BC} = \frac{100 \times \sin 143°8'}{\sin 90°}$$

$\therefore F_{BC}$ = **59.99** = **60 Kg**

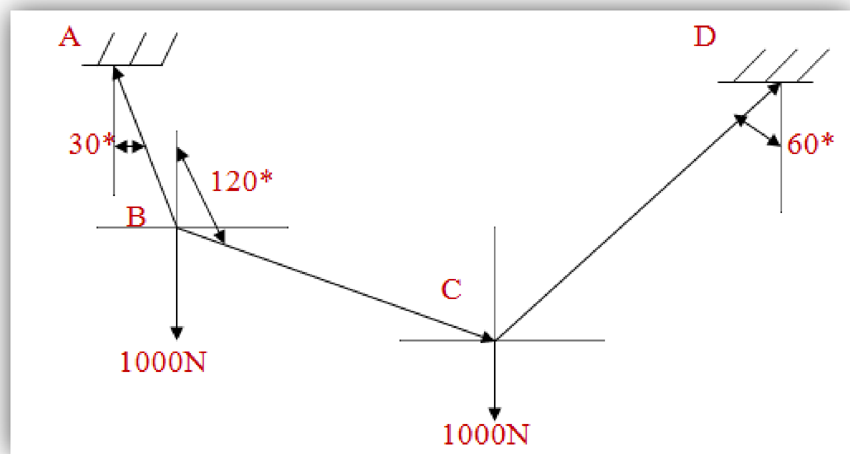
$$\frac{F_{AC}}{\sin 126°52'} = \frac{100}{\sin 90°}$$

$$F_{AC} = \frac{100 \times \sin 126°52'}{\sin 90°}$$

F_{AC} = 80/1 $\therefore$ **F_{AC} = 80 kg**

3) A string ABCD attached to 2 fixed points A and D, has 2 equal weights 1000N attached to it at B and C. The weights rest with the portion AB and CD inclined at an

angle of 30° and 60° respectively to the vertical. Find the tensions in the portion AB, BC and CD of the string. If inclination of the portion BC with the vertical is 120°.



Joint B

$$\frac{FAB}{\sin 60} = \frac{1000}{\sin 150} = \frac{FBC}{\sin 150}$$

$$FAB = \frac{\sin 60 \times 1000}{\sin 150}$$

$$= \frac{866.025}{0.5}$$

$\therefore$ **FAB = 1732.05 N**

$$FBC = \frac{\sin 150 \times 1000}{\sin 150}$$

$\therefore$ **FBC = 1000N**

Joint C

$$\frac{FCB}{\sin 120} = \frac{1000}{\sin 120} = \frac{FCD}{\sin 120}$$

$$FCB = \frac{\sin 120 \times 1000}{\sin 120}$$

$\therefore$ **FCB = 1000N**

$$FCD = \frac{1000 \times \sin 120}{\sin 120}$$

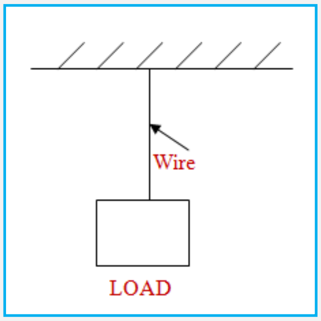
$\therefore$ **FCD = 1000N**

UNIT 2: SIMPLE STRESSESS AND STRAINS

UNIT – 2

- ✓ Stress and strain – Definition and units
- ✓ Tensile, compressive and shear stress and strains
- ✓ T. Stress – strain diagram : elasticity elastic range, proportional limit, yield stress, plastic range, ultimate stress
- ✓ Hook’s law, Young’s modulus,(E)
- ✓ Working stress and factor of satisfy
- ✓ Expression for the deformation of a body subjected to an axial force
- ✓ Simple problem

INTRODUCTION:



Whenever a load is attached to a thin hanging wire, it elongates and loads moves downwards. The amount, by which the wire elongates, depends upon the type of load and nature of wire material.

It experimentally found that as load moves downwards its sets up some resistance against the deformation.

- If the resistance(R) force is greater than applied force or load(F) no deformation its takes place.(R>F)
- If the resistance force is equal to the applied force or load deformation will be stop.(R=F)
- If the resistance force is less than the applied load in such a case deformation continuous until failure takes place.(R< F)

ELASTICITY:

The property of certain materials of returning back to their original position after removing the external force is known as Elasticity.

PERFECTLY ELASTIC BODY:

A body is set to be perfectly elastic if it returns back completely to its original shape and size after removal of external force.

Ex: Rubber band spring.

DEFNITIONS:

1. **Stress (p):** Whenever a system of external forces acting on a body, the body under goes deformation as the body will undergoes deformation it sets up resistance This resistance / unit area is known as stress.

Mathematically it can be expressed as Force / unit area.

$\therefore$ P = F/A Where P = Stress

A = Area F = Force or load.

UNIT OF STRESS:

M.K.S SYSTEM Kg / cm^2

SI system N / mm^2

2. **STRAIN (e):**

Whenever a system of external forces acting on a body, the body will under goes deformation. This deformation / unit length this known as strain.

e. = Δ l / l

Where Δ l = change in length

l = original length.

UNIT OF STRAIN: Constant or no unit.

NORMAL STRESS: The force ΔF can be resolved into components such that one Of them is along the outward drawn normal to the area ΔA and the other components lie in the plane of the area ΔA. Let ΔF_n be the normal component, then normal stress.

$$P = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} \quad \text{Da}$$

The normal stress may be tensile or compressive depending upon the forces acting on the material to be either of the pull or push type respectively. Tensile and compressive stresses together are called direct stresses.

$$p = F / A_o$$

NORMAL STRESS: It is the strain produced under the action of direct or normal stresses.

CONVENTIONAL OR ENGINEERING STRESS: It is defined as the ratio of loads F to the original area of cross-section A_o.

CONVENTIONAL OR ENGINEERING STRAIN: It is defined as the change in length per unit original length by definition.

$$\textbf{e or } \varepsilon = \frac{l - l_o}{lo} = \int_{lo}^{1} \frac{dl}{lo} = \frac{1}{lo}\int_{lo}^{1} dl$$

Where - l = changed or deformed length

lo =original length

dl = change in length

TRUE STRESS: It is defined as the ratio of loads F to the original area of cross section Ao. Thus

$$p = \frac{F}{A} \quad \frac{F}{A_o} \frac{A_o}{A} = p_0 \frac{A_o}{A}$$

For volume $Al = A_o l_o$ where $l = l_o$

$A = \frac{A_o}{1+e}$ where e = engineering strain

$p = p_0 (1+e)$

NATURAL STRAIN: It is defined as the change in length per unit instantaneous length. By definition, the natural strain

$$e = \int_{lo}^{1} \frac{dl}{l} = \int_{lo}^{1} l = \ln(1+e)$$

SUPERFICIAL STRAIN: It is defined as the change in area of cross section per unit original area is called superficial strain.

NORMAL STRAIN: It is the strain produced under the action of direct or normal stresses.

$$e_s = \frac{A - A_o}{A_o} = \frac{dA}{A_o}$$

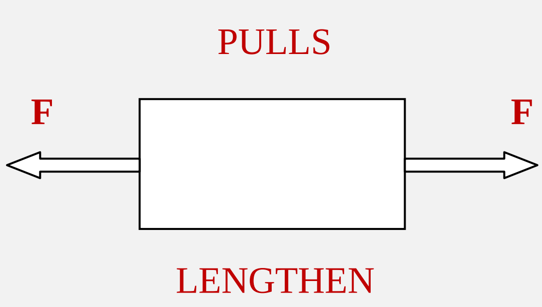
PROOF STRESS: It is the maximum stress, which can be applied to a material without allowing the material to fail.

FACTOR OF SAFETY: Because of uncertainties of loading conditions, we introduce a factor of safety, **defined as the ratio of the maximum stress to the allowable or working stress.** The maximum stress is generally taken as the yield stress. This is also called the “factor of ignorance ".

TYPES OF STRESSES AND STRAINS:

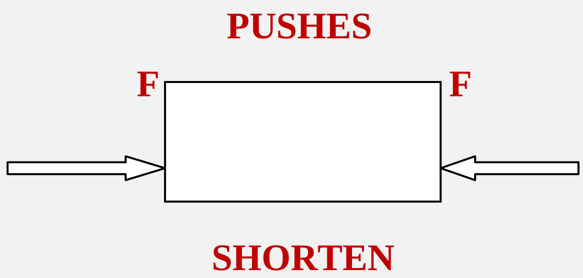
1. Tensile stress and strain.
2. Compressive stress and strain.
3. Shear stress and strain.

1. TENSILE STRESS AND STRAIN:



When a section is subjected to two equal and opposite pulls as the result of which the body tends to lengthen the stress induced is called **tensile stress** and corresponding strain is known as **tensile strain**.

2. COMPRESSIVE STRESS AND STRAIN:



When a section is subjected to 2 equal and opposite pushes as the result of which the body tends to shorten the stress induced called compressive stress and corresponding strain is known as compressive strain.

Note: - Tensile is ' +'ve

Compressive is ' -'ve

3. ELASTIC LIMIT:

Whenever some external system of force acts on a body. It undergoes some deformation. If the external forces causing deformation are removed the body spring back to its original position. It has been found that for a given section there is a limiting value of force up to end within which the deformation entirely disappears on removal of the force, **the value of intensity of stress corresponding to this limiting force is** called Elastic limits of the materials.

HOOKE'S LAW:

It states "When a material is loaded within an elastic limit the stress is proportional to strain"

Mathematically stress $\propto$ strain

$P \propto e$

$P = E\, e$

$\therefore \mathbf{E = P / e}$

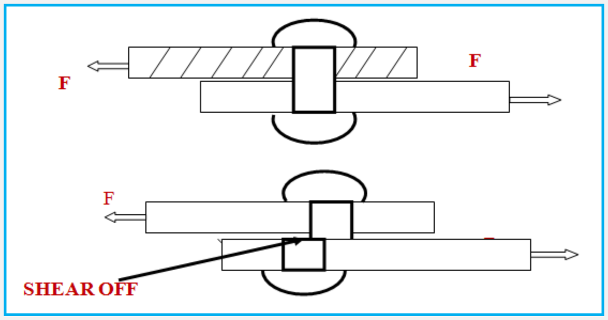
Where E = Young's modulus of elasticity.

Unit of E = kg / cm^2 or N / mm^2

Note: Other type of stresses.

1. Working stress
2. Longitudinal stress
3. Circumferential stress
4. Yield stress

4. SHEAR STRESS AND STRAIN:



When a section is subjected to 2 equal and opposite forces acting tangentially across its resisting section as a result of which the body tends to shear off a stress induced is called Shear stress. The corresponding strain is called Shear strain.

Young's modulus of elasticity (E)

We know that stress is proportional to strain that is $P \propto e$

$P = E\,e$ **$E = P/e$**

Where

E = Young's modulus constant

P = stress

e =strain

The unit of E is N/mm^2 OR kg/cm^2

Deformation of a body due to force acting on it.

Let us consider a body subjected to tensile stresses.

Let P = Stress induced on a body
F = Force
e. = Strain
A = Cross sectional area
l = Length of the body
Δl = change in length
E = young's modulus of elasticity

We know that,

$P = F/A$ $E = P/e$ $e = \Delta l / l$
$E = P/e$
$e = P/E$ $\Delta l / l = F / AE$
$\therefore$ **$\Delta l = Fl / AE$**

Unit of Δl is cm or mm.

Note: MKS system E is $10^6\ kg/cm^2$

SI system E is $10^5\ N/mm^2$

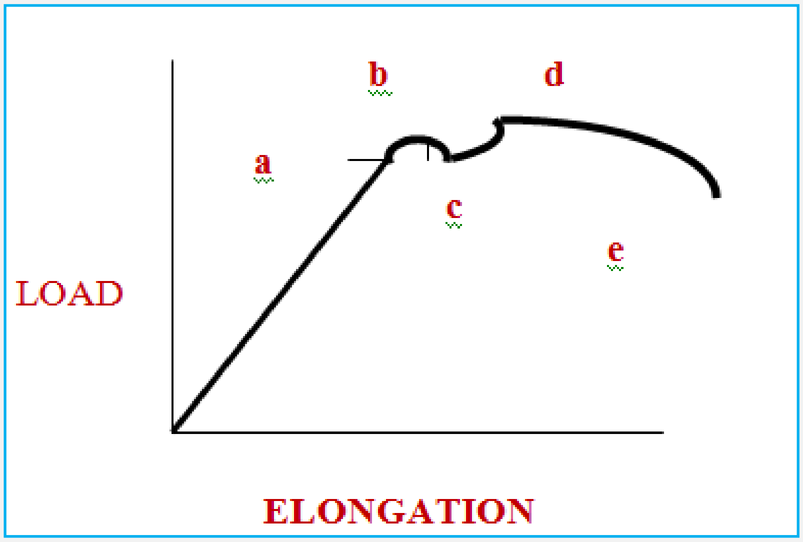
' E ' VALUES (YOUNGS MODULUS OR MODULUS OF ELASTICITY)

SL.NO	Material	'E' kg / $_{cm}$2 (MKS)'	E' N / $_{mm}$2 (SI system)
1	Steel	2.0×10^6 to 2.2×10^6	2.0×10^5 to 2.2×10^5
2	Wrought iron	1.9×10^6 to 2.6×10^6	1.9×10^5 to 2.6×10^5
3	Cast iron	1.0×10^6 to 1.6×10^6	1.0×10^5 to 1.6×10^5
4	Copper	0.9×10^6 to 1.1×10^6	0.9×10^5 to 1.1×10^5
5	Brass	0.8×10^6 to 0.9×10^6	0.8×10^5 to 0.9×10^5
6	Aluminum	0.6×10^6 to 0.8×10^6	0.6×10^5 to 0.8×10^5

Note: $\Delta l = Fl / AE$ OR $\Delta l = Pl / AE$ where P= Force.

MULTYPLAING FACTOR	PREFIX
10^{12}	TERA
10^{9}	GIGA
10^{6}	MEGA
10^{3}	KILO
10^{2}	HECTO
10	DECO
10^{-1}	DECI
10^{-2}	CENTI
10^{-3}	MILLI
10^{-6}	MICRO
10^{-9}	NANO
10^{-12}	PICO

STRESS AND STRAIN CURVE



For measuring the characteristic properties of a metal (say mild steel) it is subjected to increasing load and the extensions taking place are measured with an **extensometer** on plotting a graph b/w the loads and elongation's produced it is seen that in the beginning there is a straight line relationship. It continues up to ' **a** ' which is called the limit of proportional to extension.

This can also be expressed as “stress is proportional to strain ".

Point ' **b** ' denotes elastic limit. Below this point, the body regains its original position if the load is removed. Beyond this point the body does not recover its original position completely even if the load is removed.

A little after the elastic limit a considerable amount of stretching of the materials takes place even with a slight increase in load. The point **' c '** where it occurs is called the yield point.

At **' d '** the maximum or the ultimate load is reached. After this a waist or local construction is formed in the specimen and fracture occurs **(e)**.

PROBLEM

1. A rod 1m long and of 2cm cross section is subjected to a pull of 1 ton force. If - E = 2.0 X 10^6 kg / cm^2. Determine the elongation of the rod.

Given: l = lm = 100 cm

A =2X2 = 4 cm^2

F = 1t. = 1000 kg

E = 2.0X10^6 kg / cm^2

Δ l = ?

$$\Delta l = Fl / AE = \frac{1000X100}{4X2.0X10^6}$$

$$= \frac{1,00,000}{80,00,000} = 0.0125 \text{ cm.}$$

2. Determine the elongation of steel bar 1m long and 1.5 cm^2 cross sectional area when subjected to a tensile load of 1500 kg assumed E value suitably.

Given: I = 1m = 100 cm

A = 1.5 cm^2

F = 1500 kg

E = 2.0X10^6 kg / cm^2

$$\therefore \ \mathbf{\Delta l} = Fl / AE = \frac{1500X100}{1.5X2.0X10^6}$$

$$= \frac{150000}{3000000} = 0.05 \text{ CM or } 0.5 \text{ mm.}$$

3. A brass rod of 2-cm diameter and 1.5 m long is subjected to an axial pull of 4000 kg. Find the a) stress b) strain and c) Δl of the bar. Take $E = 1.0X10^6$ kg / cm^2

Given :

F = 4000kg

l = 150 cm

$E = 1.0X10^6$ kg / cm^2

Dia = 2 cm

A = 3.142 cm^2

$A = \Pi / 4 \text{ X } d^2$

$= \Pi / 4 \text{ X } 2^2 = \Pi / 4 \text{ X } 4$

A = 3.142 cm^2

a) P = F / A = 4000 / 3.142

$\therefore$ P = 1273 kg / cm^2

b) E = P / e, e = P / E = 1273 / 1.0X 10^6

= 1.273 X 10^{-3}

e = 0.001273

c) Δl = Fl / AE = 4000X150 / 3.142 X 1.0 X 10^6

= 600000 / 3142000

= 0.190 cm

4. A circular bar 1cm diameter and 0.6cm long was tested for young's modulus of elasticity. It was observed that under a pull of 1360 kg, the extension was 0.492 mm. Find E value in kg / cm$^{2.}$

ASSIGNMENT

Ans: - E= 21114.39 kg / cm^2

5. A steel bar of length 6m and of rectangular cross section 5cm x 2.5 cm supports load of 2 tonne. How much is the change in length of the bar?
Young's modulus of steel = $20X10^{10}$ N / m^2

ASSIGNMENT

NOV.DEC.2016

SI UNIT PROBLEMS

1. A wooden Tie in 75 mm wide 150-mm deep at 1.5 m long. It is subjected to an axial pull of 45000 N. The stretch of the member is found to be 0.6380 mm. Find P, e, and E.

Given: $A = l \times b$

$A = 75 \times 150 = 11250\ MM^2$

$F = 45000\ N$

$\Delta l = 0.6380\ mm$

$l = 1.5\ m = 150\ cm = 1500\ mm$

$P = F / A = \frac{45000}{11250} = 4\ N / MM^2$

$e = \Delta l / l = \frac{0.6380}{1500}$

$e = 0.000425$

$E = P / e = 4 / 0.000425$

$\mathbf{E = 9411.76\ N / mm^2}$

2. A wooden tie rod is 60-mm wide 120-mm deep and 1.5 m long. It is subjected to an axial pull of 30 KN. $\Delta l = 0.625$ mm. Find P, e, and E.

Given: $A = 60 \times 120 = 7200\ mm^2$

$F = 30\ KN = 30000\ N$

$\Delta l = 0.625\ mm$

$l = 1.5\ mm = 150\ cm = 1500\ mm$

$P = F / A = 30000 / 7200 = 4.16\ N / mm^2$

$e = \Delta l / l = 0.625 / 1500 = 0.000416$

$E = P / e = 4.166 / 0.000416$

$\mathbf{E = 10014.42\ N / mm^2}$

3. An elastic rod 2.5-mm in dia 200 mm in long extends by 0.25 mm under a tensile load of 40 KN. Find the stress, strain and E.

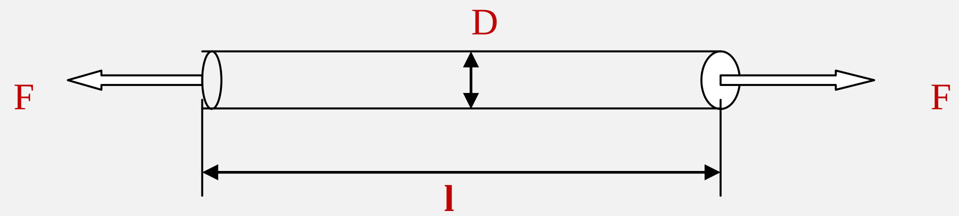
ASSIGNMENT
Ans. - E= 65176 N / mm²

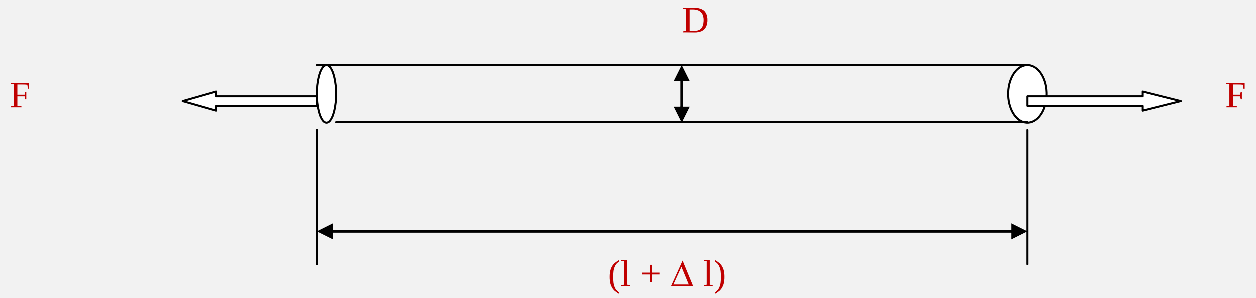
UNIT 3: ELASTIC CONSTANTS

UNIT – 3

- ✓ **Linear or primary strain**
- ✓ **Lateral OR secondary strain**
- ✓ **Poisson's ratio, shear stress and shear strain**
- ✓ **Volumetric strain, bulk modulus and modulus of rigidity**
- ✓ **Volumetric strain of a rectangular body subjected to axial force and 3 mutually perpendicular forces**
- ✓ **Relation between Young's modulus(E) and bulk modulus (with proof)**
- ✓ **Relation between Young's modulus, bulk modulus of rigidity (with proof)**
- ✓ **Simple problems**

Primary or Linear strain





Whenever some external force acts on a body it undergoes some deformation. Let us consider a circular bar subjected to tensile force.

Let-

L = Length of the bar

F= Force

d = diameter

Δ l = change in length

$(l \pm \Delta l) \rightarrow$ Linear strain

The deformation of the bar per unit length in the direction of the force is known as primary or linear strain.

Secondary or Lateral strain

Every direct stress is always accompanied by a strain in its own direction & an opposite kind of strain in every direction at right angles to it.

Such a strain known as secondary or lateral strain.

$(d \pm \Delta d) \rightarrow$ Lateral strain

Poisson's ratio (1/m)

It is the ratio of lateral strain to the linear strain.

$\therefore$ 1 / m = Lateral strain / Linear strain

Lateral strain = 1/m x e

Sl.No	Material	Poisson's ratio (l/m)
1	Steel	0.25 - 0.33
2	Cast iron	0.23 - 0.27
3	Copper	0.31 - 0.34
4	Brass	0.32 - 0.42
5	Aluminum	0.32 - 0.36
6	Rubber	0.45 - 0.50
7	Concrete	0.08 - 0.18

Problems

1) Steel bar 2cm long, 2cm wide & 1cm thick is subjected to a pull of 2 tons in the direction of its length. Find the 1) Change in length 2) Change in breadth. 3) Change in thickness.

Take E= $2.0X10^6$ kg / cm^2 and Poisson's ratio 0.3.

l = 200cm.

b = 2cm

t = 1cm

F= 2000kg.

E = $2.0X10^6$ kg / cm^2

1/m = 0.3

$\Delta l = Fl / AE$

$= 2000X200 / 2X2.0X10^6$

$\therefore \Delta l = 0.1cm$

e= $\Delta l / l = 0.1 / 200 = 0.0005$

Lateral strain = l/m x e

$\therefore$ Lateral strain = 0.3x0.0005 = 0.00015

Lateral strain = $\Delta b / b$

$\therefore \Delta b$ = Lateral strain x b

$\therefore \Delta b = 0.00015x2 = 0.0003cm$

Lateral strain = $\Delta t / t$

Δt = Lateral strain x t

$\therefore \Delta t = 0.00015x1 = 0.00015cm.$

Note: **1) Linear strain (e) = $\Delta l / l$**

2) Lateral strain (breath or width) = $\Delta b / b$

3) Lateral strain (thickness) = $\Delta t / t$.

2) A metal bar 5cmx5cm section is subjected to an axial compressive load 50 tons. The contraction of a 20cm gauge length is found to be 0.5mm and the increase in thickness 0,04mm. Find the value of E and Poisson's ratio.

$A = 5cm \times 5cm = 25cm^2$

$F = 50$ TON $= 50000$ kg

$l = 20cm$

$\Delta l = 0.5mm = 0.4cm$

$\Delta t = 0.04mm = 0.4cm$

$E = ?$

$1/m = ?$

$\mathbf{E = 0.8X10^6 \ kg / cm^2}$

Linear strain $= \Delta l / l = 0.05 / 20 = 0.0025$

Lateral strain $= \Delta t / t = 0.4 / 5 = 0.08$

$\therefore \mathbf{1 / m = 0.08 / 0.0025 = 0.32}$

$P = f/a = 50000 / 25 = 2000 \ kg / cm^2$

$e = P / E$ $\quad$ $e = P / e = 2000 / 0.0025$

Volumetric strain

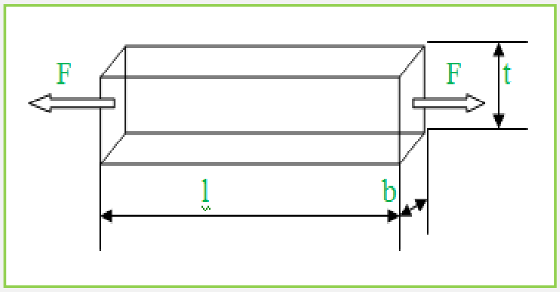
Volumetric strain is the ratio of change in volume to the original volume.

$Ev = \Delta v / v$

Where Δv = change in volume

V = original volume

VOLUMETRIC STRAIN OF A RECTANGULAR BODY SUBJECTED TO AXIAL FORCES.



Let us consider a rectangular bar subjected to axial tensile forces.

Let- l = length of bar

b = breadth

t = thickness

1/m = Poisson's ratio

Linear stress (P) F / A = F / bt

Linear strain (e) = P / E = F / btE

Lateral strain = 1 / m x linear strain

Lateral strain = 1/ m x F / btE = F / m btE

Change in thickness = Δ t = Lateral strain x t

= F / m btE x t

$\therefore \Delta$ l = F / m b E

Change in breadth = Δ b = Lateral strain X b

= F / mbtE X b

$\therefore$ Δ b = F / mtE

Change in length = $\therefore$ Δ l = Fl / btE

As the results of tensile force

Let the final length = l + Δ l

Final breadth = b - Δ b

Final thickness = t - Δ t

Final volume = (l+ Δ l) (b - Δ b) (t - Δ t)

= lbt [(1+ Δ l / l) (1- Δ b / b) (1 - Δ t / t)]

= lbt [1+ Δ l / l - Δb /b - Δ t / t - Δ l / l x Δb / b - Δ l / l x Δ t / t +
Δ b /b x Δ t / t + Δ l / l x Δb / b x Δ t / t]

$= lbt\ [1 + \Delta l / l - \Delta b / b - \Delta t / t\ ]$

We know that original volume = v = lbt

Change in volume = Final volume - original volume

$= lbt\ [1 + \Delta l / l - \Delta b / b - \Delta t / t)\ ] - lbt$

$= \Delta l / l - \Delta b / b - \Delta t / t$

$= lbt\ [fl / btE / l - F / mtE/b - F/mbE/t\ ]$

$= lbt\ [F / btE - F / mbtE - F / mbtE\ ]$

Let lbt = v

Change in volume = $v\ [F / btE - F / mbtE - F / mbtE]$

$\therefore$ Change in volume = $v \times F / btE\ [1 - 2 / m]$

original volume = lbt

Volumetric strain = $\Delta v / v = v\ F / btE\ [1 - 2 / m]$

$\Delta v / v = e\ (1 - 2 / m)$ Where $e = F / btE$

PROBLEM

1) A bar of 2cm length, 2cm breadth and 1.5cm thickness is subjected to a tensile load of 3000 kg. Find the final volume of the bar. If Poisson's ratio = 1/4 & E = 2.0X106 kg / cm$^{2.}$ (Unit of volume m^3 or cm^3) and find volumetric strain.

$l = 2cm = 200cm.$

$b = 2cm$

$t = 1.5cm$

$F = 3000kg$

$1/m = 1/4$

$E = 2.0X10^6$ kg / cm^2

$e = F/btE$

$= 3000/2X1.5X2.0X10^6 = 3000/6000000$

$\therefore e = 0.0005$

$\Delta v / v = e (1 - 2 / m)$

$\Delta v / lbt = 0.0005 (1 - 2/1/4)$

$\Delta v / 600 = 0.0005 (1 - 2x0.25)$

$\Delta v / 600 = 0.00025$

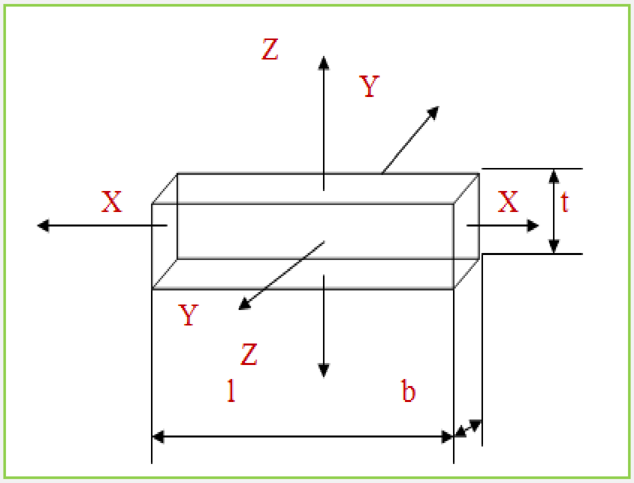
$\therefore \Delta v = 0.00025x600 = 0.15$ cm^3

Final volume = Original volume + change in volume

= 600 + 0.15

$\therefore$ Final volume = 600.15 cm^3

VOLUMETRIC STRAIN OF RECTANGULAR BODY SUBJECTED TO THREE MUTUALLYPERPENDICULAR FORCES.



Let us consider a rectangular body subjected to direct tensile stresses along 3 mutually perpendicular axis.

Let Px = stress in x- axis direction
Py = stress in y- axis direction
Pz = stress in z- axis direction
E = Young's modulus
1/m = Poisson's ratio
ex = Px / E

Strain in y- axis direction
ey = Py / E

Strain in z- axis direction
ez = Pz / E
The resulting strain in 3 directions
ex = Px / E – Py / Me – Pz / Me
ey = Py / E – Pz / Me – Px / Me
ez = Pz / E – Px / Me – Py / Me

Then volumetric strain = Δv / v = ex + ey + ez

PROBLEMS

1) A steel bar 25cm long, 5cmx5cm cross section is subjected to a pull of 30 tons in the direction of its length. Calculate the change in volume. If m = 4 and $E = 2.0X10^6$ kg / cm$^{2.}$

$\Delta v / v = ex + ey + ez$

$\Delta v / v = v (ex + ey + ez \quad \therefore v = lbt \quad \therefore v = 25x5x5x = 625\ cm^2$

$ex = Px / E – Py / Me – Pz / Me$

$Px / E = Fx / AE = 30000 / 25X2.0X10^6 = 30000/50000000 = 0.0006$

$Py / = Fy / mae = 0.25x0.0006 = 0.00015.$

$Pz / Me = Fz / Mae = 0.25X0.0006 = 0.00015$

$ex = Px / E – Py / Me – Pz / Me$

$= 0.0006 – 0.00015 – 0.00015$

$\therefore ex = 0.0006 – 0.0003 - = 0.0003$

$ey = Py / E – Pz / Me – Px / Me$

$= 0.0006 – 0.00015 –0.00015$

$ey = 0.0003$

$ez = Pz / E – Px / Me – Py / Me$

$= 0.0006 – 0.00015 – 0.00015$

$\therefore ez = 0.0003$

$\Delta v = v (ex + ey + ez)$

$= 625 (0.0003+0.0003+0.0003)$

$= 625 (0.0009)$

$\therefore \Delta v = 0.5625\ cm^3$

Note: Δv along x- axis = 625x.0003

$= 0.1875\ cm^3$

BULK MODULUS ('K' denotes bulk module)

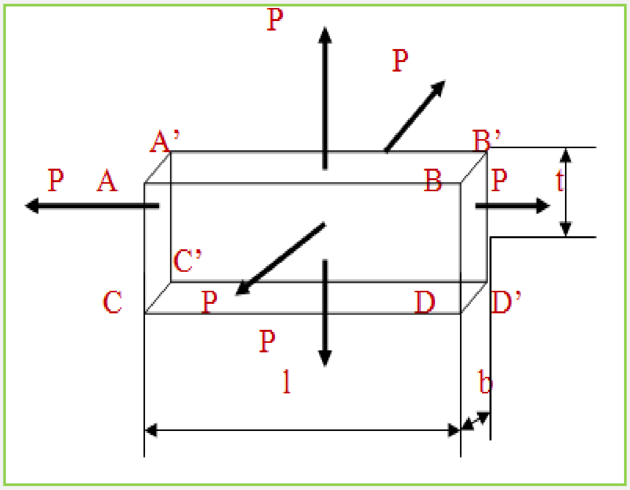
When a body is subjected to 3 mutually perpendicular stress of equal intensity the ratio of direct stress to the corresponding volumetric strain is known as bulk modules. Mathematically it can be expressed are -

$\therefore K = mE / 3(m-2)$ **or**

K = Direct stress / volumetric strain = $P / \Delta v / V$.

$\therefore K = P / \Delta v / V$.

RELATION BETWEEN THE BULK MODULE'S(K)ANDYOUNG'S MODULE'S(E)



Consider a cube ABCD, A' B' C' D'.
Let the cube is subjected to 3 mutually perpendicular tensile Stresses of equal intensity.

Let l = length of the cube P = direct stress.
V = volume = l^3
E = Young's module's
K = Bulk module's
Now consider the deformation of one side of cube (AB) under the action of the 3 mutually perpendicular stresses.

Strain due to stresses.

1) Tensile strain of P / E due to stresses on the faces BB', CC' & AA', DD'.
2) Compressive lateral strain of 1/m x P / E due to stresses on the faces AA', BB' & DD', CC'.
3) Compressive lateral strain 1/m x P / E due to stresses on the faces ABCD & A'B'C'D'.

The net tensile strain -

$\Delta l / l = P / E - 1/m \times P / E - 1/m \times P / E$

$\therefore \Delta l / l = P / E (1 - 2 / M)$

$V = l^3$ (Differentiate with respect to l)

$\Delta v / \Delta l = 3 l^3$

$\Delta v / \Delta = 3 l^3 x \Delta l / l^3$

$\Delta v / v = 3 l^3 Xp / e (1 - 2 / m)$

$\Delta v = 3 l^3 x P/ E (1 - 2/M)$

$$\therefore \mathbf{K} = \frac{\mathbf{mE}}{\mathbf{3(m-2)}}$$

RELATION B/W MODULE'S OF REGIDITY (C) AND (E)

$\therefore$ C = mE / 3(m + 1)

Sl. No	Material	Module's of rigidity (kg / cm^2)
1	Steel	$0.8x10^6 - 1\ 10^6$
2	Copper	$0.3x10^6 - 0.5\ x10^6$
3	Brass	$0.3x10^6 - 0.5\ x10^6$

PROBLEMS

1) For a given material young's module's is $1.0x10^6$ kg / cm^2 and module's of rigidity $0.4x10^6$kg / cm^2 Find the bulk module's and the lateral contraction of a round bar of 5cm diameter and 2.5cm long when stretched 0.25cm.

Solution:

$C = 0.4x10^6 kg / cm^2$

$E = 1.0x10^6 kg / cm^2$

K = ?

$C = mE / 2(m+1)$

$2(m+1) C = mE$

$2X0.4x10^6 (m+1) = 1.0x10^6 m$

$2X0.410^6 (m+1) - 1.0x10^6 m = 0$

$\therefore m = 4$

$K = mE / 3(m-2) = 4x1x10^6 / 3(4-2)$

$\therefore K = 0.67X10^6 kg / cm^3$

$e = \Delta l / l = 0.25 / 250 = 0.001$

$1/mxe$ = lateral strain

= 0.25x0.001

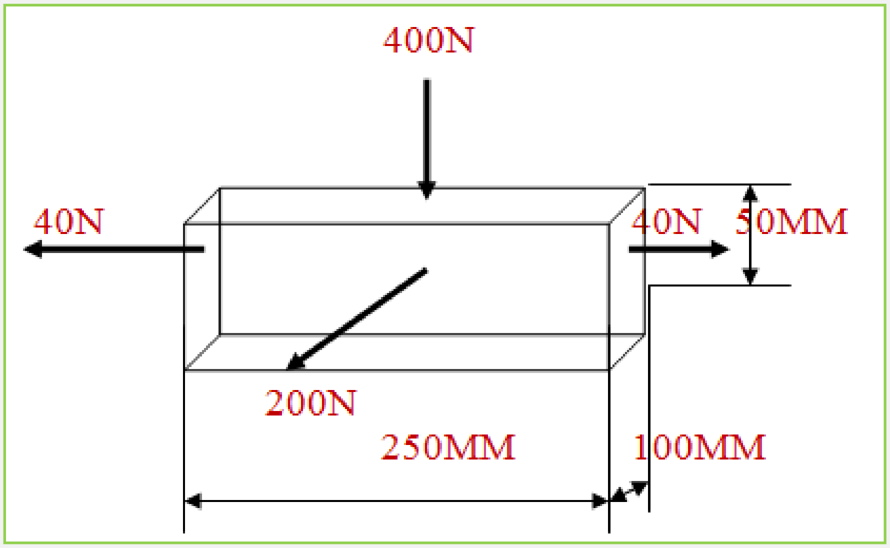
$\therefore$ lateral strain = 0.00025

lateral strain = $\Delta d / d$

Δd = lateral strain x d

$\therefore \Delta d = 0.00025x5 = 0.00125cm.$

2)



Take $E = 2x10^5$ N / mm^2, Poisson's ratio 1/m = 0.25. Find the change in volume that should be made in the 400 N loads in order that there should be no change in volume of the bar.

$\Delta v / v = ex + ey + ez$

$\Delta v = v\ (ex+ey+ez)$

$\therefore v = lbt$

$= 250x100x50$

$\therefore v = 1250000\ mm^3$

$\therefore ex = Px / E - Py / mE + Pz / Me.$

$Px = Fx / A = 40 / 100X50 = 0.008\ N / mm^2$

$Py = Fy / A = 200 / 250X50 = 0.016\ N / mm^2$

$Pz = Fz / A = 400 / 250\ X100 = 0.016\ N / mm^2$

$ex = Px / E - Py / mE + Pz / mE$

$\therefore ex = 0.008 / 2X10^5 - 0.016 / 2X10^5\ X\ 0.25 + 0.016 / 2X10^5\ X\ 0.25 = 0.00000004$

$ey = Py / E + Pz / mE - Px / mE$

$= 0.016 / 2x\ 10^5 + 0.016 / 2x\ 10^5 \times 0.25 - 0.008 / 2x\ 10^5 \times 0.25.$

$= 0.00000008 + 0.00000002 - 0.00000001$

$\therefore ey = 0.00000009.$

$\therefore ez = - Pz / E - Px / mE - Py / mE$

$= 0.016 / 2 \times 10^5 - 0.008 / 2x\ 10^5 \times 0.25 - 0.016 / 2x\ 10^5 \times 0.25$

$= - 0.00000008 - 0.00000001 - 0.00000002$

$\therefore ez = - 00000011$

$\Delta v / v = ex + ey + ez$

$\Delta v / v = v (ex + ey + ez)$

$= 1250000(0.00000004 + 0.00000009 - 0.00000011)$

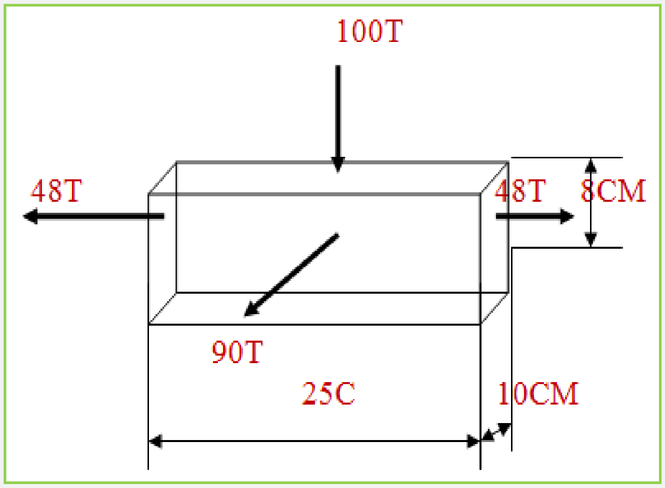
$= 1250000 (0.00000002)$

$\therefore \Delta v = 0.025\ mm^3$

$\therefore$ F = 500 N

Change in the 400N load should be 500-400 = 100N.

3)



Take $E = 2 \times 10^6\ kg / cm^3$, $1/M = 0.25$

Find the change in volume.

$\Delta v / v = ex + ey + ez$

$\Delta v / v = v (ex + ey + ez)$

$\therefore v = lbt$

$= 25 \times 10 \times 8$

$\therefore v = 2000\ cm^3$

$\therefore ex = Px / E - Py / mE + Pz / mE$

$Px = Fx / A = 48000 / 10 X = 600\ kg / cm^2$

$Py = Fy / A = 90000 / 25X8 = 450\ kg / cm^3$

$Pz = Fz / A = 100000 / 25X10 = 400\ kg / cm^3$

$ex = Px / E - Py / mE + Pz / mE$

$ex = 600 / 2 \times 10^6 - 450 / 2 \times 10^6 X 0.25 + 400 / 2 \times 10^6 X 0.25$

$= 0.0003 - 0.00005625 + 0.00005$

$\therefore ex = 0.00029375.$

$ey = Py / E + Pz / mE - Px / mE$

$= 450 / 2 \times 10^6 + 400 / 2 \times 10^6 X 0.25 - 600 / 2 \times 10^6 X 0.25$

$= 0.000225 + 0.00005 - 0.000075$

$\therefore ey = 0.0002.$

$ez = - Pz / E - Px / mE - Py / mE$

$= - 400 / 2 \times 10^6 - 600 / 2 \times 10^6 X 0.25 - 450 / 2 \times 10^6 X 0.25$

$= - 0.0002 - 0.000075 - 0.00005625$

$\therefore ez = - 0.00033125$

$\Delta v / v = ex + ey + ez$

$\Delta v = v (ex + ey + ez)$

$= 2000 (0.00029375 + 0.0002 - 0.00033125)$

$= 2000 (0.0001625)$

$\boldsymbol{\therefore \Delta v = 0.325\ cm^3}$

4) A bar of 30mm diameter is subjected to a pull of 6 tons. The measured extension on gauge length of 20cm is 0.9mm and the change in diameter is 0.0039mm. Calculate the Poisson's ratio and the values of the 3 module's (ECK).

Given:-ø of bar = 30mm = 3cm.

$A = \Pi / 4 (d)^2 = \Pi / 4 (3)^2 = 7.069\ cm^2$

F = 6 TONS = 6000kg.

l = 20cm

$\Delta l = 0.9mm = 0.09\ cm$

$\Delta d = 0.0039\ mm = 0.00039\ cm$

Young's module's of elasticity

$\therefore E = P / e$

$P = F / A = 6000 / 7.069 = 848.7 \quad kg / cm^2$

$e = \Delta l / l = 0.09 / 20 = 0.0045$

$\therefore E = P / e = 848.7 / 0.0045 = 188600\ kg / cm^2$

Poisson's ratio

1 / m = Lateral strain / Linear strain

$1 / m = \Delta d / d / \Delta l / l = 0.00039/3 / 0.09 / 20 = 0.00013 / 0.0045$

$\therefore 1 / m = 0.029$

$\therefore 1 / m = 1 / 0.029 = 34.48$

Bulk module's of elasticity.

$K = mE / 3(M-2)$

$= 34.48\ (188600) / 3\ (34.48 - 2)$

$\therefore K = 6502928 / 97.44 = 64106.15\ kg / cm^2$

Module's of rigidity.

$C = mE / 2(M+1) = 34.48X188600 / 2\ (34.48+1) = 6502928 / 70.96$

$= 91642.16\ kg / cm^2$

$= 0.09X10^6\ kg / cm^2$

Solutions: E= 188600 = 1.8 $X10^6\ kg / cm^2$ $\quad 1/M = 0.029 = 34.48 = m$

$K = 64106.15 = 0.06\ X10^6\ kg / cm^2 \qquad C = 91642.16 = 0.09X10^6\ kg / cm^2$

UNIT 4: STRAIN ENERGY AND IMPACT LOADING

UNIT – 4

- ✓ **Strain energy or resilience**
- ✓ **Proof resilience**
- ✓ **Proof stress**
- ✓ **Modulus of resilience**
- ✓ **General expression for the strain energy of a body**
- ✓ **Types of loading : gradual, sudden and impact**
- ✓ **Equation for the strain energy stored in a body under gradual and sudden of loading**
- ✓ **Simple problems**

STRAIN ENERGY AND IMPACT LOADING

Whenever a load attached to thin hanging wire it elongates and the load moves down wards by an amount equal to the extension of the wire.

As the load moves downwards it loses its potential energy. This energy is observed or stored in the stretched wire, which may be released by removing the load.

On removing the load the wire will spring back to its original position **"This energy which is observed in a body when strained within its elastic limit** "is known as strain energy.

Unit of strain energy: **N mm or kg cm.**

Resilience: The total strain energy stored in a body.

Proof Resilience: Maximum strain energy stored in a body.
When a body is stressed up to the elastic limit this stress is known as **proof stress.**

Module's of Resilience: The proof resilience per unit volume of a material is known as modules of resilience.

Types of Loading:

1) Gradually applied loading
2) Suddenly applied loading
3) With impact applied loading.

GRADUALLY APPLIED LOADING: Strain energy stored in a body when the load is applied gradually.

Ex: - When we lower a body with the help of a crane the body first touches the plat form on which it is to be placed.

On further releasing the chain the plat form goes on loading till it is fully loaded by the body. **It is most common practical way of loading.**

Gradually applied loading means the loading starts from '0' and increases gradually till the body is fully loaded.

Now consider a bar subjected to gradual load.

F= Force

E = Module's of elasticity of bar

A = Area of the bar

l = length of the bar

U = strain energy of the bar

Strain energy / work done

$= F \times \Delta l$	$F = 0+F / 2 = F / 2$
$= F / 2 \times \Delta l$	$e = \Delta l / l \quad \Delta l = el$
$= ½\ PAXel$	$P = f / a \quad F = PA$
$= ½\ Pal\ P / E$	$e = P / E$
$= ½\ P^2 Al / E$	$V = Al$
$= ½\ P^2\ V / E$	

$$\boxed{U = P^2 v / 2E}$$

SI Unit of Strain energy = N

SUDDENLY APPLIED LOADING: (only gradually and suddenly applied loads)

Strain energy stored in a body when a load is suddenly applied.
Ex: - When we lower a body with the help of a crane the body is first of all just above the plat form on which it is to the placed. If the chain breaks at once the whole body begins to act on the platform.
Let us consider a bar subjected to suddenly applied load.

F= Force
E = Module's of elasticity of bar
Δl = Elongation of the bar
P = stress in the bar
A = Area of the bar
l = length of the bar
U = strain energy of the bar

We know that work done

$= F \times S$

$U = F \times \Delta l$ $\quad\quad \Delta l = Fl / AE = Pl / E$

$P^2 V / 2E = F\Delta l$ $\quad\quad V = Al$

$P^2 Al / 2E = f = Pl / E$

$PA / 2 F$

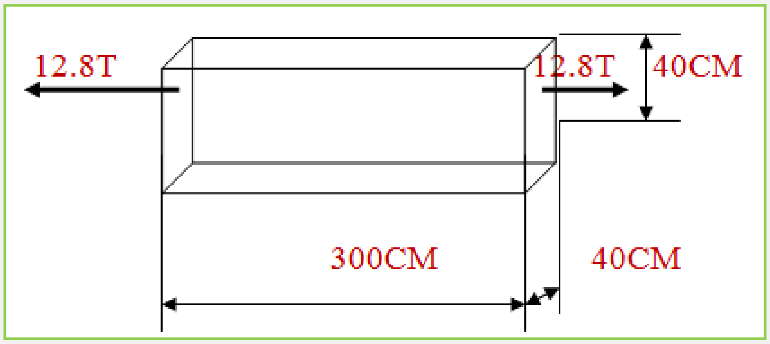
$\therefore P = 2F / A$

It means that the stress induced in this case is twice or double the stress induced when the same load is applied gradually.

1) **Gradually applied load**
   $U = P^2 V / 2E$ or $P^2 Al / 2E$
2) **Suddenly applied load**
   $P = 2 F / A$

Problems

1) A steel bar 40mm / 40mm in section 300cm long is subjected to an axial pull of 12.8 tons taking E = 2.0X 10^6 kg / cm^2 & Poisson's ratio as 0.3, calculate the alterations in length and sides of the bar. Calculate also the amount of energy stored in the bar during the extension.



$E = 2.0 X10^6 kg / cm^2$ 40mm / 40mm

F = 12.8 tons = 12800 kg = 4cm / 4cm

1/m = 0.3

l = 300cm.

$$\Delta l = Fl / AE = \frac{12800X300}{4X4X2.0X10^6}$$

$$= \frac{3840000}{32000000}$$

$\therefore \Delta l = 0.12cm$

$el = \Delta l / l = 0.12 / 300 = 0.0004$

eLat = 1/m X el

$\therefore$ el = 0.3X0.0004 = 0.00012

Δb = Lateral strain X b

$\therefore \Delta b = 0.00012X4 = 0.00048$

Δt = Lateral strain X t

$\therefore \Delta t = 0.00012x4 = 0.00048$

$U = P^2 Al / 2E$ $\quad$ $P = F/A = 12800 / 16 = 800$ kg / cm^2

$$= \frac{(800)^2 X16X300}{2X2X10^6} = \frac{3072000000}{4000000}$$

$\therefore U = 768$ kg / cm.

Note: 1) Derive the equation strain energy stored in a body ? or

Derive the equation strain energy stored in a body, when load is applied gradually.

Ans.: $U = P^2 v / 2E$

2) Prove that $U = P^2 v / 2E$ where P, A, l & E are there usual meaning.

3) An axial pull of 5 tons is suddenly applied to a steel rod 2m long and 10 cm^2 is cross section. Calculate the strain energy that can be observed.

Take $e = 2.0X10^6$ kg / cm^2

f = 5ton = 5000kg

l = 2m = 200cm

$E = 2.0X10^6$ kg / cm^2

A = 10 cm^2

$$U = P^2 AL/ 2E = \frac{(500)^2 X10X200}{2X2.0X10^6} \qquad P = F/A = 5000 / 10 = 500 \text{ kg / cm}^2$$

$$\therefore U = \frac{500000000}{4000000} = 125\text{kg cm}$$

Suddenly applied

$P = 2\ F/A$

$= 2X5000/10$

$\therefore P = 1000\ kg\ /\ cm^2$

$U = P^2\ Al\ /\ 2E$

$$= \frac{(1000)^2\ 10X200}{2X2.0X10^6} = \frac{2000000000}{40000000}$$

$\therefore$ **P = 500 kg cm**

4) A steel wire 2.5mm diameter is firmly held in a clamp from which it hangs vertically. An anvil the weight of which may be neglected is secured to the wire 1.8mm below the clamp. The wire is to be tested allowing a weight bored to slide over the wire to drop freely from 1mm above the anvil. Calculate the weight required to stress the wire to 100 kg / mm$^{2.}$ Assuming the wire to be elastic up to this stress. Take E = 2.1 X10^6 kg / cm^2

dia of the wire = 2.5 mm OR 0.25cm

$A= 0.0491\ cm^2$ $\qquad A= \Pi\ /\ 4\ (d)^2$

$l = 1.8m = 180cm$ $\qquad \therefore A = \Pi\ /\ 4\ (0.25)^2 = 0.0491\ cm^2$

$h = 1m = 100cm$

$P = 100\ kg\ /\ mm^2$

$\therefore P=10000\ kg\ /\ cm^2$

$$\therefore P = \sqrt{\frac{2EFh}{Al}} \qquad P^2 = 2EFh\ /\ Al$$

$$\therefore F = \frac{P^2\ Al}{2Eh} = \frac{(100000)^2 X0.0491X180}{2x2.0x10^6\ x100x2.1}$$

$$= \frac{883800000}{420000000}$$

$\therefore$ **F = 2.1kg.**

5) A bar of 1.2cm diameter gets stretched by 0.3cm under a study load of 800kg.What stress would be produced in the bar by a weight of 80kgf. Which falls through 8cm before commencing the stretching of the rod? Which is initially unstressed? Take E = 2.0×10^6 kg / cm^2

dia of bar = 1.2 cm

$A = 1.13\ cm^2$

$\Delta l = 0.3cm$

$F_1 = 800kg$

$F_2 = 80kg$

$h = 8cm$

$E = 2.0 \times 10^6\ kg / cm^2$

$A = \Pi / 4\ (d)^2$

$\therefore A = 1.13\ cm^2$

$\therefore U = F/A\left(1+\sqrt{1+Aeh / Fl}\right)$

$\therefore \Delta l = Fl / AE$

$0.3 = \dfrac{800\ l}{2260000}$

$678000 = 800\ l$

$\therefore l = 847.5cm$

$U = F/A\left(1+ \sqrt{1+Aeh / Fl}\right)$

$= 80 / 1.13\left(1+ \sqrt{1+\dfrac{1.13 \times 2 \times 10^6 \times 8}{80 \times 847.5}}\right)$

$= 70.79\left(1+\sqrt{1 +\dfrac{18080000}{67800}}\right)$

$= 70.79\left(1+\sqrt{1+266.6}\right)$

$= 70.79\left(1+\sqrt{267.6}\right)$

$= 70.79\ (1+16.36)$

$= 70.79\ (17.36)$

$\boldsymbol{\therefore U = 1220\ kg.\ cm}$

ASSIGNMENT

1. Calculate the strain energy that can be stored in steel bar 2m long and of $5cm^2$ cross section are subjected to a tensile stress 500 kg / $cm^{2.}$

   Take E = $2X10^6$ kg / $cm^{2.}$

   Ans.: $\therefore$ **U = 62.5 kg.cm**

2. A mild steel rod 4m long and 2.5cm diameter is subjected to a pull of 4.5 tons. Find the elongation of the rod, when the load is applied

   1) Gradually
   2) Suddenly.

   Take E = 2.0 $X10^6$ kg / cm^2

   Ans.: $\therefore$ **U = 1653.03 kg.cm**

3. An unknown weight falls 3cm on to a collar rigidly attached to the lower end of a vertical bar 4m long and 10 $cm^{2\ in}$ cross section. If the maximum instantaneous extension is found to be 0.366cm. Find the corresponding stress and the value of the unknown weight. Take E = 2X 10^6 kg / cm^2.

UNIT 5: CENTRE OF GRAVITY (G)

UNIT – 5

- ✓ Centre of gravity Centroid – definition and importance & method of finding centre of gravity
- ✓ Axis of reference and axis of symmetry
- ✓ CG of some regular bodies like triangle, square, cylinder, circle, semicircle & cut out hole section
- ✓ Simple Problems

CENTRE OF GRAVITY(G)

A point found out in a body, through which the resultant of all such parallel forces acts. This point through which the whole weight of the body acts, irrespective of the position of the body is known as Centre of gravity.

Centroid:

The plane geometrical figures like triangle, quadrilateral, circle, rectangle, square etc. will have only areas but no mass. The centre of the area of such figure is known as Centroid of the body.

Axis of reference:

The centre of gravity is always calculated with reference to some assumed axis known as axis of reference.

For calculating Y1 & Y_2 axis of reference is x^1, x-axis. For calculating x_1 & x_2 axis of reference is Y^1, Y-axis.

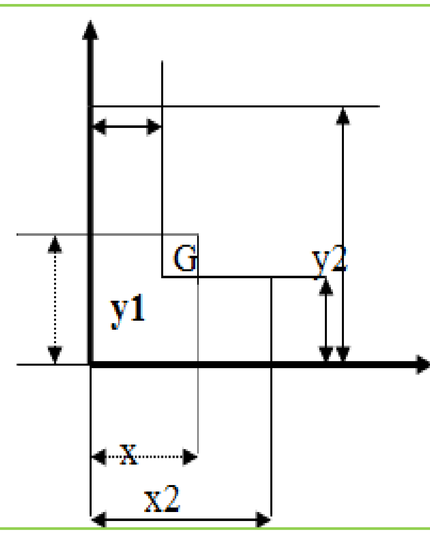
Y x1

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2 + \ldots a_n Y_n}{a_1 + a_2 + \ldots a_n}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + \ldots a_n X_n}{a_1 + a_2 + \ldots a_n}$$

y

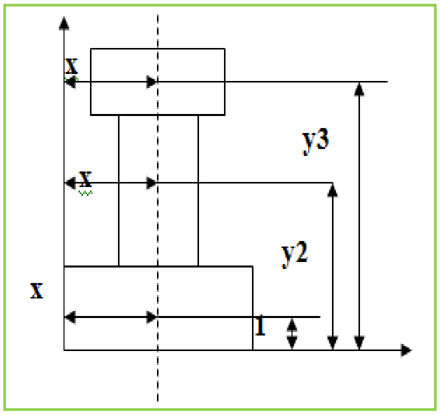
Axis of reference :



Axis of symmetry:

The centre of gravity of the body is symmetrical about x-x axis, but not in Y-Y axis. ∴Centre of gravity of the body will lie on the axis of symmetry.

Example I - Section:



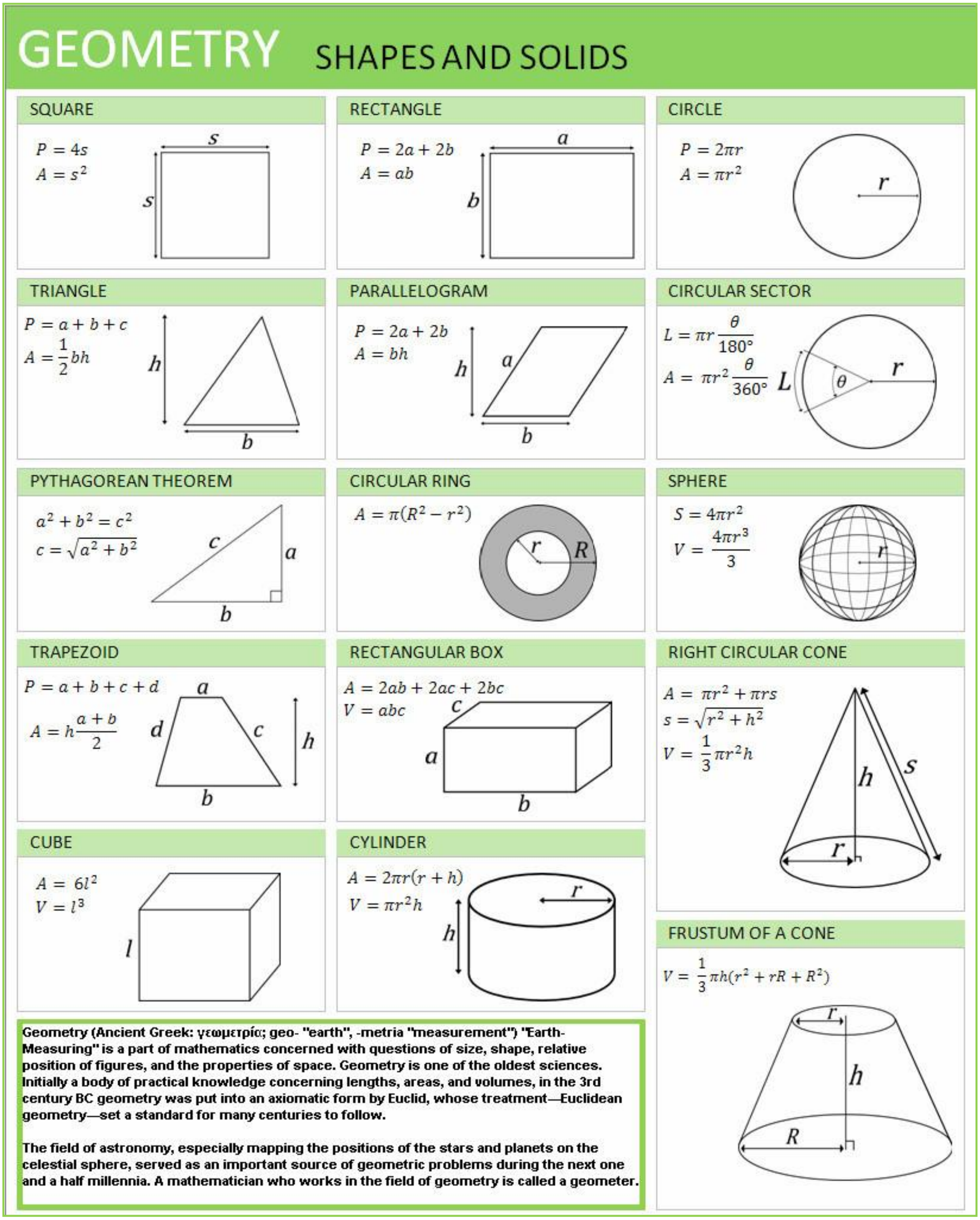
Axis of symmetry X-X AXIS

: It is symmetrical about x-x axis

Unsymmetrical about Y-Y axis.

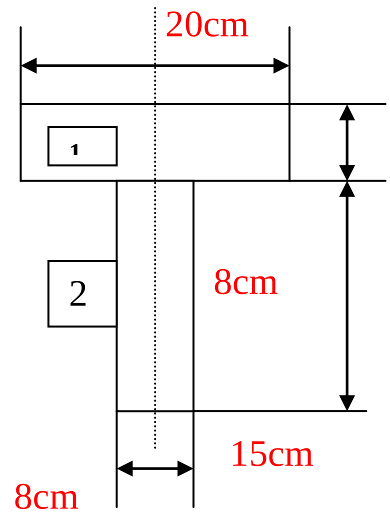
METHOD OF FINDING OUT CENTRE OF GRAVITY

- By method of geometrical consideration
- By method of moment
- By the method of graphics


GEOMETRY SHAPES AND SOLIDS
SQUARE
$P = 4s$
$A = s^2$
s
s
RECTANGLE
$P = 2a + 2b$
$A = ab$
a
b
CIRCLE
$P = 2\pi r$
$A = \pi r^2$
r
TRIANGLE
$P = a + b + c$
$A = \frac{1}{2}bh$
h
b
PARALLELOGRAM
$P = 2a + 2b$
$A = bh$
h
a
b
CIRCULAR SECTOR
$L = \pi r \frac{\theta}{180°}$
$A = \pi r^2 \frac{\theta}{360°}$
L
θ
r
PYTHAGOREAN THEOREM
$a^2 + b^2 = c^2$
$c = \sqrt{a^2 + b^2}$
c
a
b
CIRCULAR RING
$A = \pi(R^2 - r^2)$
r
R
SPHERE
$S = 4\pi r^2$
$V = \frac{4\pi r^3}{3}$
r
TRAPEZOID
$P = a + b + c + d$
$A = h\frac{a + b}{2}$
a
d
c
h
b
RECTANGULAR BOX
$A = 2ab + 2ac + 2bc$
$V = abc$
c
a
b
RIGHT CIRCULAR CONE
$A = \pi r^2 + \pi rs$
$s = \sqrt{r^2 + h^2}$
$V = \frac{1}{3}\pi r^2 h$
h
s
r
CUBE
$A = 6l^2$
$V = l^3$
l
CYLINDER
$A = 2\pi r(r + h)$
$V = \pi r^2 h$
r
h
FRUSTUM OF A CONE
$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$
r
h
R
Geometry (Ancient Greek: γεωμετρία; geo- "earth", -metria "measurement") "Earth-Measuring" is a part of mathematics concerned with questions of size, shape, relative position of figures, and the properties of space. Geometry is one of the oldest sciences. Initially a body of practical knowledge concerning lengths, areas, and volumes, in the 3rd century BC geometry was put into an axiomatic form by Euclid, whose treatment—Euclidean geometry—set a standard for many centuries to follow.
The field of astronomy, especially mapping the positions of the stars and planets on the celestial sphere, served as an important source of geometric problems during the next one and a half millennia. A mathematician who works in the field of geometry is called a geometer.

Problems

Find centre of gravity T-section.



1) Find centre of gravity T-section.

Rectangle (1) $a_1 = l \times b$

$= 20x8 = 160 \text{ cm}^2$

$X_1 = 20 / 2 = 10\text{cm}$

$Y_1 = 15+8 / 2 = 19 \text{ cm}$

Rectangle (2) $a_2 = l \times b$

$= 15X8 = 120 \text{ cm}^2$

$X_2 = 6+8 / 2 = 10\text{cm}$

$Y_2 = 15 / 2 = 7.5 \text{ cm}$

It is symmetrical about x-x axis.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

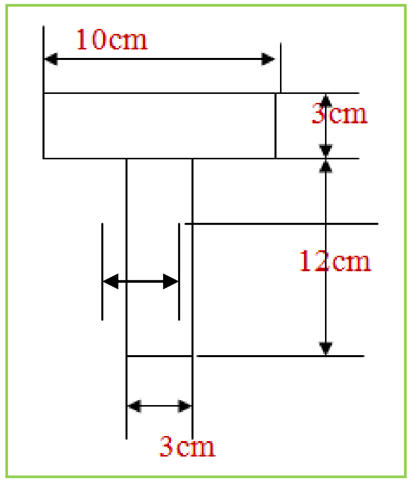
$= 2800 / 280 =$ **10cm**

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2}{a_1 + a_2}$$

$= 3940 / 280$

= 14.07 cm

2) Find centre of gravity. T- Section.



Rectangle (1) $a_1 = 10 \times 3$

$= 30\ cm^2$

$X_1 = 10 / 2 = 5cm$

$Y_1 = 12+3 / 2 = 13.5\ cm$

Rectangle (2) $a_2 = 3 \times 12$

$= 36\ cm^2$

$X_2 = 3.5+3 / 2 = 5cm$

$Y_2 = 12 / 2 = 6\ cm$

It is symmetrical about x-x axis.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

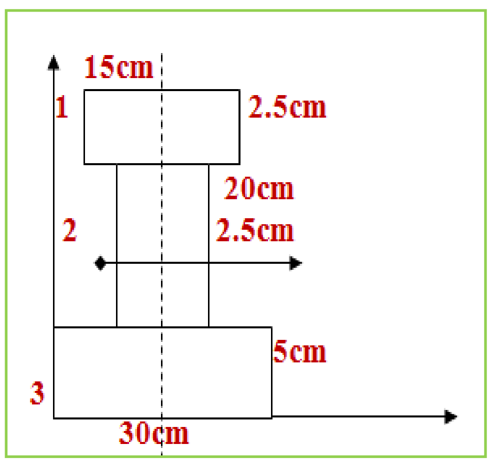
= 330 / 66 = 5 cm

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2}{a_1 + a_2}$$

= 621 / 66

= 9.40 cm

3) Find centre of gravity 1 - Section.



Rectangle (1) a1 = 15 x2.5

= 37.5 cm^2

X_1 = 7.5 + 15 / 2 = 15cm

Y_1 = 25+2.5 / 2 = 26.25 cm

Rectangle (2) a_2 = 2.5 x 20

= 50 cm^2

X_2 = 13.75+2.5 / 2 = 15cm

Y_2 = 5+20 / 2 = 15 cm

Rectangle (3) a_3 = 30 x 5

…. = 150 cm^2

X_3 = 30 / 2 = 15cm

Y_3 = 5 / 2 = 2.5 cm

It is symmetrical about x-x axis.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

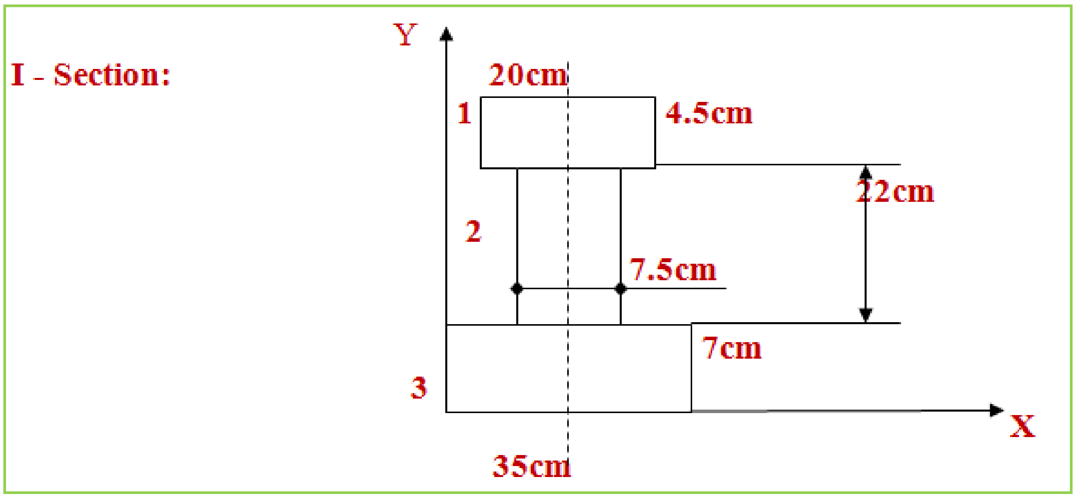
= 3562.5 / 237.5 = **15 cm**

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2 + a_3 Y_3}{a_1 + a_2 + a_3}$$

= 21.9.37 / 2237.5

= 8.88 cm

4) Find centre of gravity I - Section.



Rectangle (1) $a_1 = 20 \times 4.5$

$= 90\ cm^2$

$X_1 = 7.5 + 20 / 2 = 17.5$ cm

$Y_1 = 29+4.5 / 2 = 31.25$ cm

Rectangle (2) $a_2 = 7.5 \times 22$

$= 16.5\ cm^2$

$X_2 = 13.75+7.5 / 2 = 17.5$ cm

$Y_2 = 7+22 / 2 = 18$ cm

Rectangle (3) $a_3 = 35 \times 7$

= 245 cm2

$X_3 = 35 / 2 = 17.5$ cm

$Y_3 = 7 / 2 = 3.5$ cm

It is symmetrical about x-x axis.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

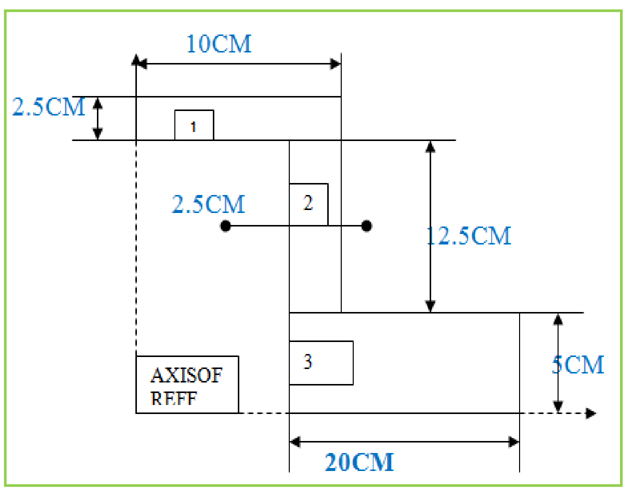
= 6151.25 / 351.5 = **17.5 cm**

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2 + a_3 Y_3}{a_1 + a_2 + a_3}$$

= 3967 / 351.5

= 11.28 cm

5) Find centre of gravity Z - Section.



Rectangle (1) $a_1 = 10 \times 2.5$

$= 25\ cm^2$

$X_1 = 10 / 2 = 5$ cm

$Y_1 = 17.5 + 2.5 / 2 = 18.75$ cm

Rectangle (2) $a_2 = 2.5 \times 12.5$

$= 31.25\ cm^2$

$X_2 = 7.5+2.5 / 2 = 8.75$ cm

$Y_2 = 5+ 12.5 / 2 = 11.25$ cm

Rectangle (3) $a_3 = 20 \times 5$

$= 100$ cm2

$X_3 = 7.5 +20 / 2 = 17.5$ cm

$Y_3 = 5 / 2 = 2.5$ cm

It is symmetrical about x-x axis.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

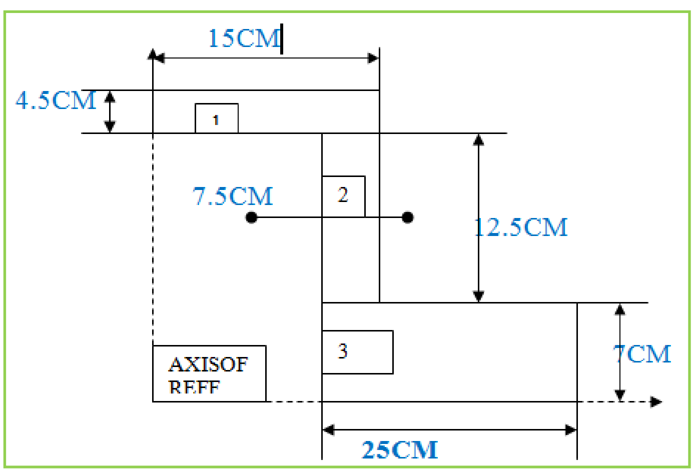
$= 2148.43 / 156.25 = 13.74$ **cm**

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2 + a_3 Y_3}{a_1 + a_2 + a_3}$$

$= 1070.31 / 156.25$

= 6.85 cm

6) Find centre of gravity Z – Section



Rectangle (1) $a_1 = 15 \times 4.5$

$= 67.5\ cm^2$

$X_1 = 15 / 2 = 7.5\ cm$

$Y_1 = 19.5 + 4.5 / 2 = 21.75\ cm$

Rectangle (2) $a_2 = 7.5 \times 12.5$

$= 93.75\ cm^2$

$X_2 = 7.5+7.5 / 2 = 11.25\ cm$

$Y_2 = 7+ 12.5 / 2 = 13.25\ cm$

Rectangle (3) $a_3 = 25 \times 7$

$= 175$ cm2

$X_3 = 7.5 +25 / 2 = 20\ cm$

$Y_3 = 7 / 2 = 3.5\ cm$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

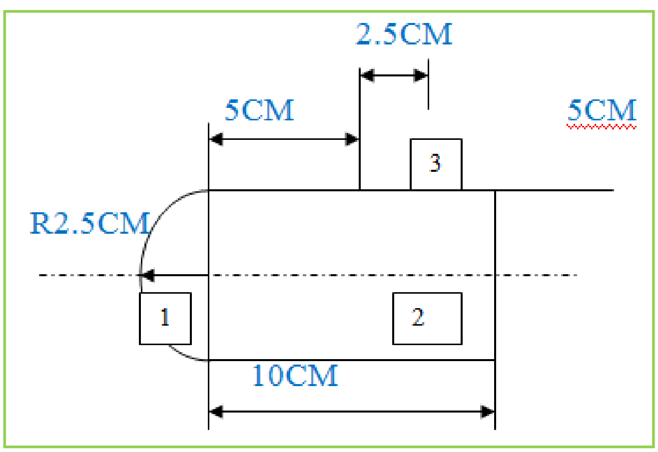
$= 5060.93 / 336.25 = 15.05\ cm$

$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2 + a_3 Y_3}{a_1 + a_2 + a_3}$$

$= 3322.8 / 336.25$

$= 9.88\ cm$

7) Find Centre of gravity.



Semi circle (1)

$$a_1 = \frac{3.142X\,(2.5)^2}{\mathbf{2}} = 9.81\ cm^2$$

$$X_1 = r - 4r / 3\ \Pi$$
$$= 1.44\ cm$$
$$Y_1 = 2.5\ cm$$

Rectangle (2) $a_2 = 10 \times 5$

$$= 50\ cm^2$$
$$X_2 = 2.5+10 / 2 = 7.5\ cm$$
$$Y_2 = 5 / 2 = 2.5\ cm$$

Triangle (3) $a_3 = ½ \times b \times h$

$$= 12.5\ cm2$$
$$X_3 = 7.5 +5 / 2 = 10\ cm$$
$$Y_3 = 5+ 5 / 3 = 6.66\ cm$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$
$$= 514.12 / 72.31 = 7.11\ \textbf{cm}$$

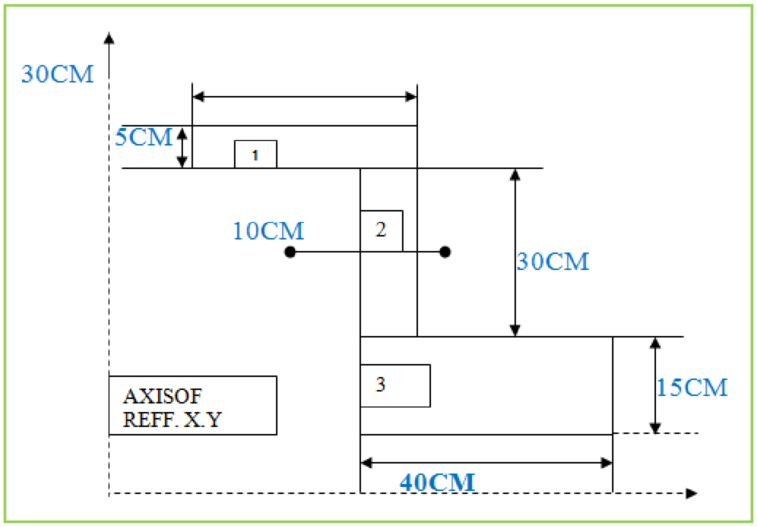
$$\bar{Y} = \frac{a_1 Y_1 + a_2 Y_2 + a_3 Y_3}{a_1 + a_2 + a_3}$$
$$= 3232.775 / 72.31$$
$$\mathbf{= 3.22\ cm}$$

For the below said fig. Find the following

8) Find centre of gravity. Take reference point is X= 10cm, Y = 10cm.

9) Find centre of gravity, Take reference point is X=10cm, Y= 25cm.

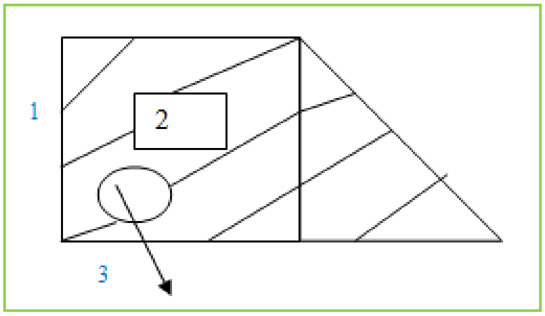
10) Find centre of gravity, take reference point is X = 25cm, Y= 10cm.



CENTRE OF GRAVITY OF SECTIONS WITH CUTOUT HOLES:

The centre of gravity of section with a cutout a hole is found out by considering in main section first as a complete one and then deducting the area of the cutout hole. That is by taking the area of cut out hole as negative.

Ex:

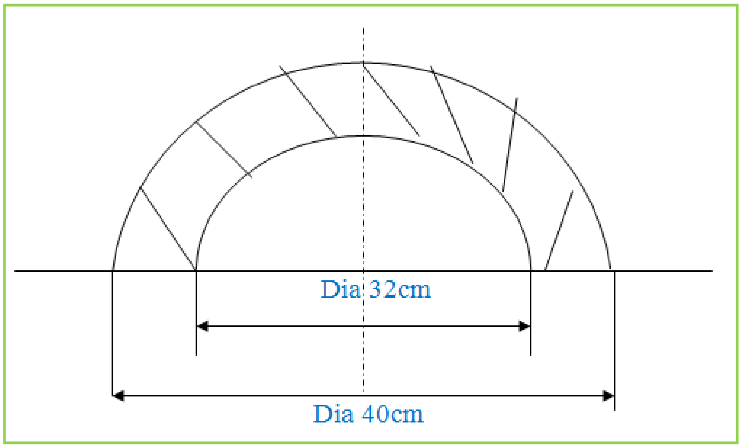


$$\bar{X} = \frac{a_1 x_1 - a_2 x_2 - a_3 x_3 + a_4 x_4}{a_1 - a_2 - a_3 + a_4}$$

$$\bar{Y} = \frac{a_1 Y_1 - a_2 Y_2 - a_3 Y_3 + a_4 Y_4}{a_1 - a_2 - a_3 + a_4}$$

Problems

1) Find centre of gravity of hatched section given below.



Semi circle (1)

$a_1 = \Pi r^2 / 2 = \Pi X(20)^2 / 2 = 628.4\ cm^2$

$X_1 = 20\ cm$

$Y_1 = 4r / 3\Pi = 80 / 9.426 = 8.4\ cm$

Semicircle (2) $a_2 = \Pi r^2 / 2 = \Pi X\ (16)^2 / 2 = 402.17\ cm2$

$X_2 = 20\ cm$

$Y_2 = 4r / 3\Pi = 64 / 9.426 = 6.7\ cm$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{628.4X20-402.17X20}{628.4 - 402.17} = \frac{4524.6}{226.23}$$

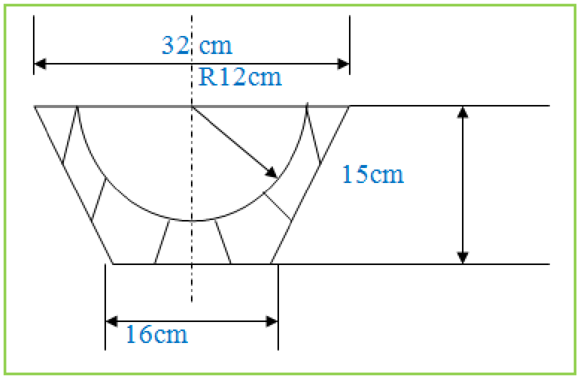
$$\therefore \bar{x} = 20\ cm.$$

$$\bar{Y} = \frac{a_1 Y_1 - a_2 Y_2}{a_1 - a_2}$$

$$= \frac{62.84X8.4-402.17X6.7}{628.4-402.17} = \frac{2569.39}{226.23}$$

$$\therefore \bar{Y} = \mathbf{11.3\ cm.}$$

2) Find centre of gravity of hatched section given below.



Trapezium (1)

$a_1 = h\,(a+b / 2) = 15\,(16+33 / 2)$

$x_1 = b / 2 = 32/2 = 16$ cm

$Y_1 = h/3\,(b+2a / b+a)$

$= 15/3\,\left(\dfrac{32+2(16)}{32+16}\right)$

$\therefore \bar{Y} = 6.67$ cm.

Semicircle (2)

$a_2 = \Pi r^2 / 2 = \Pi X\,(12)^2 / 2 = 226.2$ cm2

$X_2 = 16$ cm

$Y_2 = 4r / 3\Pi = 48 / 9.426 = 5.09$ cm

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

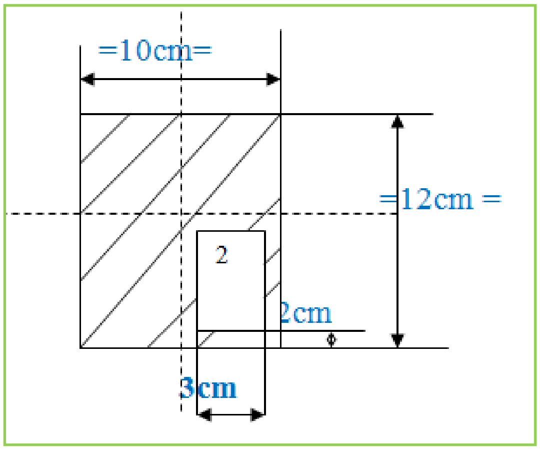
$$= \frac{360(16) - 226.2(16)}{360 - 226.2} = \frac{2140.8}{133.8} = 16 \text{ cm}$$

$$\bar{Y} = \frac{a_1 Y_1 - a_2 Y_2}{a_1 - a_2}$$

$$= \frac{360(6.67)-226.2(5.09)}{360-226.2} = \frac{1249.8}{133.8}$$

= 9.34 cm.

3) Find centre of gravity of hatched section given below.



Rectangle (1)

$A_1 = lxb$

$= 10X12 = 120\ cm^2$

$X_1 = l / 2 = 5\ cm$

$Y_1 = 12 / 2 = 6\ cm$

Rectangle (2)

$A_1 = lxb$

$= 3X4 = 12\ cm^2$

$X_2 = 6 + l / 2 = 7.5\ cm$

$Y_2 = 2 + 4 / 2 = 4\ cm$

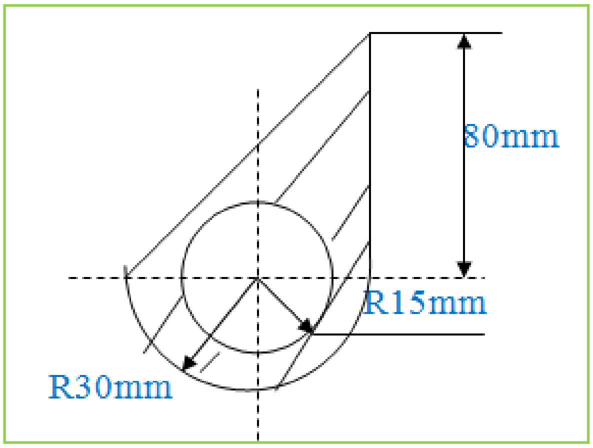
$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{120(5) - 12(7.5)}{120 - 12} = 672 / 108 = \bar{\mathbf{x}} = 4.72cm$$

Lily

$$\therefore \bar{\mathbf{Y}} = \mathbf{6.22\ cm}$$

4) Find centre of gravity of hatched section given below.



Triangle (1)

$A_1 = 1/2\ bh$

$= 1/2 X 60 X 80 = 2400\ mm^2$

$X_1 = 60 / 2 = 30$ cm

$Y_1 = 30 + 80 / 3 = 56.6$ mm

Semicircle (2)

$A_2 = \Pi r^2 = \frac{\Pi (30)^2}{2}$

$= 1413.9$ mm2

$X_2 = 30$ cm

$Y_2 = 30 - 4r / 3\Pi = 17.27$ mm.

Circle (3)

$A_3 = \Pi r^2 = \Pi (15)^2$

$= 706.9\ mm^2$

$X_3 = 30$ cm

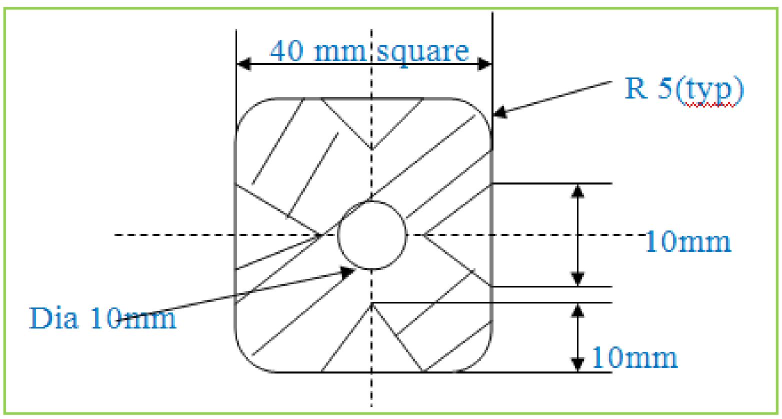
$Y_3 = 30$ cm

$$\bar{x} = \frac{a_1\, y_1 - a_2\, y_2 - a_3\, y_3}{a_1 - a_2 - a_3}$$

$$= \frac{2400(30) + 1413.9(30) - 706.9\,(30)}{2400+1413.9 - 706.9 - 706.9}$$

$$\therefore \bar{Y} = \frac{139051.05}{3107} = \mathbf{44.75\ cm}$$

5) Find centre of gravity of the shaded portion.



Square (1)

$A_1 = l \times b$

$= 40X40 - 21.45 = 1578.55\ mm^2$

$X_1 = b / 2 = 40 / 2 = 20$ cm

$Y_1 = l / 2 = 40 / 2 = 20$ cm

Triangle (2)

$A_2 = 1/2 \text{ X } b \text{ X } h$

$= 1/2X10X10 = 50 \text{ mm}^2$

$X_2 = h / 3 = 3.3 \text{ mm}$

$Y_2 = b / 2 = 15 + 10 / 2 = 20 \text{ mm}$

Triangle (3)

$A_3 = 1/2 \text{ X } 10 \text{ X } 10$

$= 50 \text{ mm}^2$

$X_3 = b / 2 = 3.3 \text{ mm}$

$Y_3 = h / 3 = 10 / 3 = 3.3 \text{ mm}$

Triangle (4) $A_4 = 1/2 \text{ X } 10 \text{ X } 10$

$= 50 \text{ mm}^2$

$X_4 = 30 + h - h / 3 = 30 + 6.6 = 36.6\text{mm.}$

$Y_4 = 15 + 10 / 2 = 20 \text{ mm}$

Triangle (5) $A_5 = 1/ 2 \text{ X } 10 \text{ X } 10$

$= 50 \text{ mm}^2$

$X_5 = 15 + 10 / 2 = 20 \text{ mm}$

$Y_5 = 30 + h - h / 3$

$= 30 + 6.6 = 36.6 \text{ mm}$

Triangle (6) $A_6 = \Pi r^2$

$= \Pi(5)^2 = 78.55 \text{ mm}^2$

$X_6 = 15 + 5 = 20 \text{ mm.}$

$Y_6 = 15 + 5 = 20 \text{ mm.}$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2 - a_3 x_3 - a_4 x_4 - a_5 x_5 - a_6 x_6}{a_1 - a_2 - a_3 - a_4 - a_5 - a_6}$$

$$= \frac{31571 - 165 - 1000 - 1830 - 1000 - 1571}{1578.5 \quad - 50 - 50 - 50 - 50 - 78.}$$

$$= \frac{31571 - 5566}{1300}$$

$$\bar{X} = 26005 / 1300 = \mathbf{20\ mm}$$

$$\bar{Y} = \frac{a_1 Y_1 - a_2 Y_2 - a_3 Y_3 - a_4 Y_4 - a_5 Y_5 - a_6 Y_6}{a_1 - a_2 - a_3 - a_4 - a_5 - a_6}$$

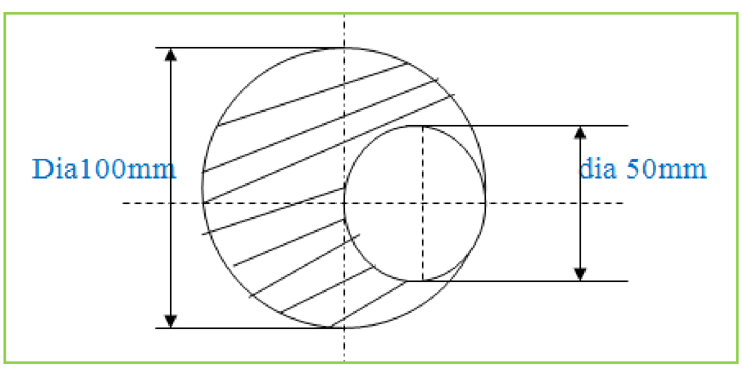
$$= \frac{31571 - 1000 - 165 - 1000 - 1830 - 1000 - 1571}{1300}$$

$$= \frac{31571 - 26005}{1300}$$

$$= 26005 / 1300$$

$$\therefore \bar{Y} = \mathbf{20\ mm.}$$

6) Find centre of gravity. of the shaded portion



Circle (1)

$A_1 = \Pi r^2$

$= \Pi(50)^2 = 7855\ mm^2$

$X_1 = 50$ mm.

$Y_1 = 50$ mm.

Circle (2)

$A_2 = \Pi r^2$

$= \Pi(25)^2 = 1963.75\ mm^2$

$X_2 = 50+25 = 75$ mm.

$Y_2 = 50$ mm.

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{7855(50) - 1963.75(75)}{7855 - 1963.75}$$

$$= \frac{392750 - 147281.25}{5891.25} = \frac{245468.75}{5891.25}$$

$$\therefore \bar{X} = \mathbf{41.7\ mm}$$

$$\bar{Y} = \frac{a_1 Y_1 - a_2 Y_2}{a_1 - a_2}$$

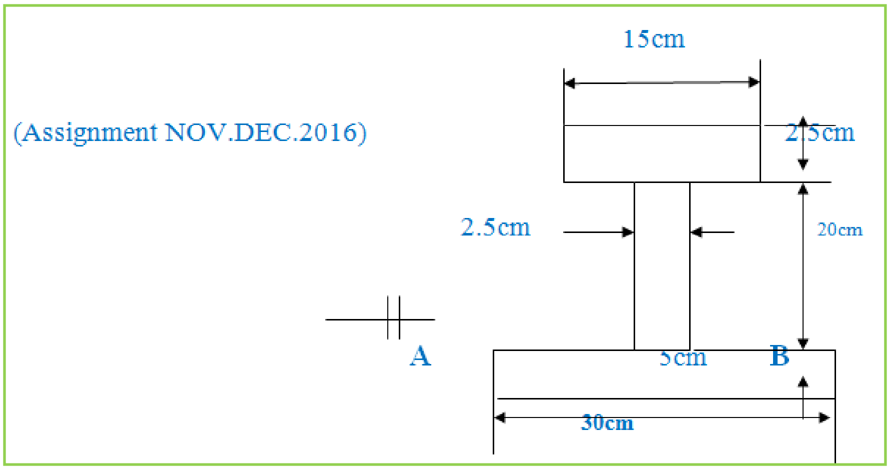
$$= \frac{7855(50) - 1963.75(50)}{5891.25} = \frac{392750 - 9818.75}{5891\ .25}$$

$$= \frac{31571 - 26005}{1300}$$

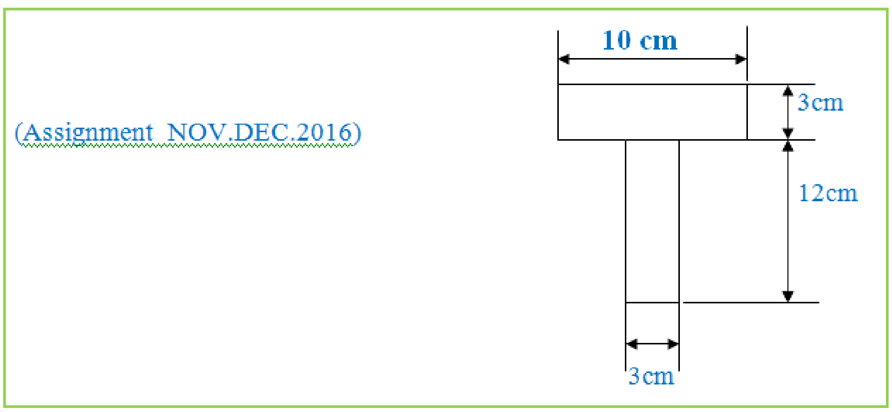
$$= 294562.5 / 5891.25$$

$$\therefore \bar{Y} = \mathbf{50\ mm.}$$

7) Find the centre of gravity of the I section from the base AB



8) Find the centre of gravity of the T section 10cmx15cmx3cm shown below.



UNIT 6: TORSION

UNIT – 6

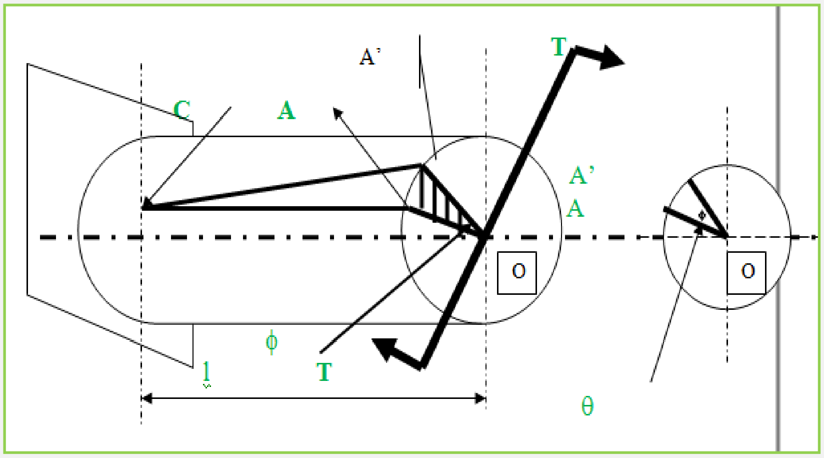
- ✓ **Torsion stress and strain**
- ✓ **Theory of pure torsion**
- ✓ **Angle of twist and shear strain – formulas**
- ✓ **Equation for the strength of a shaft**
- ✓ **Polar moment of inertia**
- ✓ **Power transmitted by a shaft**
- ✓ **Simple problems**

TORSION

A turning force is always applied to transmits energy by rotation this turning force is applied either rim of the pulley or keyed into the shaft.

The product of turning force and the distance between the point of application of force and the axis of the shaft is known as torque or turning moment or twisting moment.

NOTE: - Due to the torque every cross section of the shaft is subjected to shear stress.



Let us consider the shaft is subjected to torque. Due to the torque every cross section of the shaft is subjected to shear stress.

Let Angle ACA' $\propto$ in degrees
Angle AOA' = θ in radians
Fs = Shear stress
C = Modulus of rigidity
We know that

Shear strain $= \dfrac{\text{change in length}}{\text{Original length}}$

$$= \frac{AA'}{l}$$

$$\tan\phi = \phi$$

We also know that

The length of the arc AA' = Rθ

$$\phi = \frac{AA'}{l}$$

$$\phi = \frac{R\theta}{l}$$

Where R = Radius of shaft
θ in radians
l = length of shaft

If fs is the intensity of shear stress on the outer most layer and 'C' is the modulus of rigidity $= \dfrac{fs}{C}$

Now from equations $\dfrac{fs}{C} = \dfrac{R\theta}{l}$

$$T = \frac{fs}{R} = \frac{C\theta}{l}$$

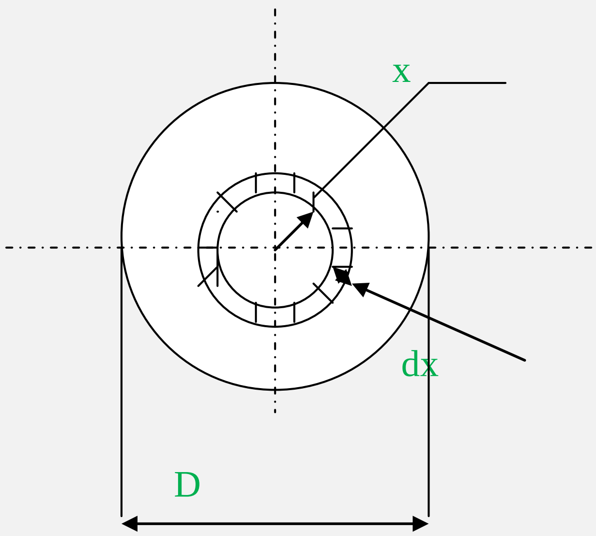
If q be the intensity of shear stress, any layer at a distance 'r' from the centre of the shaft, then

$$\frac{Q}{r} = \frac{fs}{R} = \frac{C\theta}{l}$$

Assumption for finding out shear stress in a circular shaft, subjected to torsion

1) The material of the shaft is uniform throughout.
2) The twist along the shaft is uniform.
3) Normal cross- section of the shafts, which was plane and circular before twist, remains plane and circular after twist.

STRENGTH OF A SOLID SHAFT:



Let D = diameter of shaft

R = Radius of shaft

dx = Elemental thickness

da = Elemental area

fs = Intensity of Shear stress

C= Modulus of rigidity

We know that the area of this ring

da = 2 Π x dx

and Shear stress at this section,

$$fx = fs \times \frac{X}{R}$$

Turning force = Stress x Area

Maximum torque it can transmit from one pulley to another.

Let us consider a solid shaft subjected to torque = T

$= fx \times da$

$$= fs \times \frac{X}{R} \times da$$

$$= fs \times \frac{X}{R} \times 2\Pi X\, dx$$

$$= \frac{2\Pi\, fs}{R} X^2\, dx$$

WE know that turning moment of this element,

dT = Turning force x Distance of the element from the axis of the shaft.

$$dT = \frac{2\Pi\, fs}{R} X^2\, dx\ X$$

$$dT = \frac{2\Pi\, fs}{R} X^3\, dx$$

$$T = \int_0^R \frac{2\Pi\, fs}{R} X^3\, dx$$

$$\frac{2\Pi\, fs}{R} \int_0^R X^3\, dx$$

$$= \frac{2\Pi\, fs}{R} [X^4/4]$$

$$= \frac{2\Pi\, fs}{R} R^4/4$$

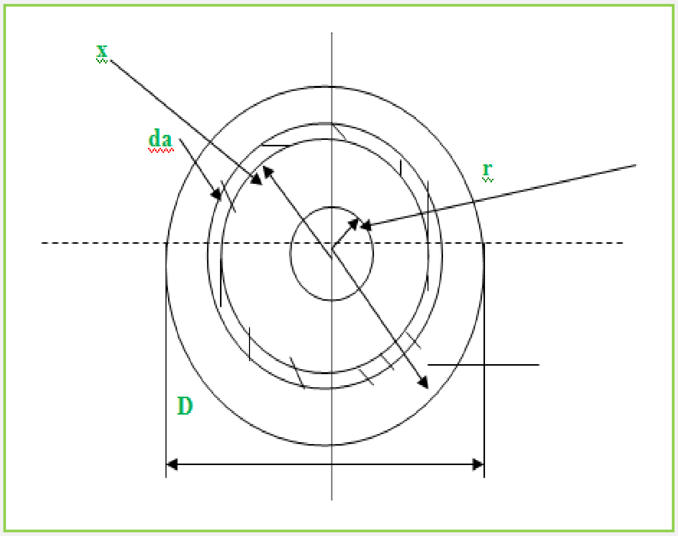
$$= \frac{\Pi}{2} fs\, R^3 \qquad (R = D/2,\ R^3 = D^3/8)$$

$$= \frac{\Pi}{2} fs \times D^3/8$$

$$= \frac{\Pi\, fs \times D^3}{16}$$

$$\therefore\ T = \frac{\Pi}{16} fs\, D^3$$

STRENGTH OF A HALLOW SHAFT



It means the maximum torque or power a hallow shaft can transmit from one pulley to another.

We know that the area of this ring,

da = 2 *X x dx

Shear stress at this section

fx = fs x X / R

Turning force = fX x da

= fs X / R x da

= fs X / R x 2*X dx

Turning Force $= \dfrac{2\Pi\ fx}{R} X^{2}\ dx$

dT = Turning force x X

$= \dfrac{2\Pi\ fx}{R} X^{2}\ dx$

$dT = \dfrac{2\Pi\ fx}{R} X^{3}\ dx$

R

$$T = \int_r^R \frac{2\Pi\, fs}{R} X^3\, dx$$

$$= \frac{2\Pi\, fs}{R} \int_r^R X^3\, dx$$

$$= \frac{2\Pi\, fs}{R} \left[X^4/4\right]_r^R$$

$$= \frac{2\Pi\, fs}{R} \frac{(R - r)}{4}$$

$$\therefore T = \frac{\Pi}{16}\, fs \frac{(D^4 - d^4)}{D}$$

POLAR MOMENT OF INERTIA(I):

The moment of inertia of a plane area with respect to an axis perpendicular to the plane of the figure is called polar moment of inertia.

$$\frac{Fs}{R} = \frac{C\theta}{l} \qquad (T = \Pi / 16\, fsD^3$$

$$= fs = \frac{16T}{\Pi D^3}$$

$$\frac{\frac{16T}{\Pi D^3}}{R} = \frac{C\theta}{I}$$

$$\frac{T}{\frac{\Pi D^4}{32}} X \frac{D}{2} = \frac{C\theta}{l} \qquad (J = \frac{\Pi}{32} D4$$

$$\frac{T}{J} = \frac{C\theta}{l}$$

Note: For solid shaft $J = \frac{\Pi D^4}{32}$

For hallow shaft $J = \frac{\Pi}{32} (D4 - d^{4)}$

POWER TRANSMITTED BY A SHAFT:

$$HP = \frac{2\,\Pi NT}{4500}\ \text{Kg. m}$$

$$P = \frac{2\,\Pi NT}{60000}\ \text{Nm}$$

Problems

1. A solid steel shaft has to transmit a torque of 10000 kg. m. If the shearing stress not to exceed 450 kg / cm^{2}. Find minimum diameter of the shaft.

T = 10000 kg. m = 1000000 kg. Cm

Fs = 450 kg / cm^2

$$T = 2\,\frac{\Pi}{16}\,fs\,(D^3)$$

$$D^3 = \frac{T\,x16}{\Pi\,fs}$$

$$= \frac{1000000X16}{\Pi\,X450}$$

$$= \frac{16000000}{1413.9}$$

$$D^3 = 11316.21$$

$$D = \sqrt[3]{11316.21}$$

$$\therefore \mathbf{D = 22.45\ cm}$$

2. A solid shaft is subjected to a torque of 1500kg.m. Find the necessary diameter of the shaft if the allowable shear stress is 600 kg / $cm^{2.}$ The allowable twist is 1^0 for every 20 diameters length of the shaft.

Take C = 1500 kg. m = 150000 kg.cm

$fs = 600\ kg/cm^2$

$\theta = 1^0 = 1 \times \Pi / 180$ radians

l = 20 dia.

$C = 0.8X10^6\ kg / cm^2$

$T = \Pi / 16\ fs\ D^3$

$$D^3 = \frac{16T}{\Pi fs} = \frac{150000X16}{\Pi X600}$$

$D^3 = 1273.2$

$D = \sqrt[3]{1273.2}$

D = 10.84 cm

WKT polar moment of inertia of a circular section.

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$J = \frac{\Pi}{32} D^4$$

$$\frac{150000}{\Pi XD^4} = \frac{0.8X10^6 (\Pi / 180)}{20 D}\ \ 32$$

Note: For solid shaft $J = \frac{\Pi}{32} D^4$

For hallow shaft $J = \frac{\Pi}{32} (D4 - d^4)$

POLAR MODULE'S:

Let T be the torsional moment of resistance of the section of a shaft of radius R and I_p the polar moment of inertia of the shaft section.

The intensity of shear stress 'q' at any point on the section distant 'r' from the axis of the shaft is given by

$$\frac{q}{r} = T / I_p.$$

The maximum shear stress fs occurs at the greatest radius R.

$\therefore\ fs = T / I_p.\ R$

or $T = fs.\ \ I_p / R$

or $T = fs.\ \ Z_p$

Where, $Z_p = I_p / R$

$$Z_p = \frac{\text{Polar moment of inertia of the shaft section}}{\text{Maximum radius}}$$

This ratio is called the polar modules.

Power transmitted by a shaft:

Let a shaft turning at N rpm transmit P kilowatts. Let the mean torque to which the shaft is subjected to be T Nm.

NOTE:

$\therefore$ Power transmitted = p = Mean torque x Angle turned per second.

$$= \frac{2\,\Pi\,N\,T}{60} \text{ WATTS}$$

$$P = \frac{2\Pi NT}{60000} \text{ kW ----- (SI Unit)}$$

$$HP = \frac{2\Pi NT}{4500} \text{ Kg.m----- (MKS Unit)}$$

Problems

1) A metal bar of 10mm diameter when subjected to a pull of 23.55 KN gave an elongation of 0.30 mm as a gauge length of 200 mm. In a torsion test on the same material, a maximum shear stress of 40.71 N / mm^2 was measured on a bar of 50 mm diameter, the angle of twist measured over a length of 300 mm being $0^0\ 21'$. Determine the passion's ratio of the material.

Solution: Tensile test data, d = 10 mm, P = 23.55 KN, δ = 0.30 mm, l = 200 mm

Strain $e = \frac{\delta l}{l} = \frac{0.30}{.200} = 0.0015$

Stress $f = \frac{23.55X103}{\frac{\Pi}{4}(10)^2} = 299.85\ N / mm^2$

Module's of elasticity, $E = f / e = \frac{299.85}{0.0015} = 1.999X10^5\ N / mm^2$

Torsion test data, d = 50 mm, l = 300 mm, $f_s = 40.71\ N / mm^{2,}$ $\theta = 0^0\ 21' = 7/20.\ \Pi / 180$ radians.

$f_s / R = C\theta / l$

$\therefore C = \frac{300.}{\frac{7}{20}\ \frac{\Pi}{180}}\ \frac{40.71}{25} = 0.79972\ X\ 10^5\ N / mm^2$

$E = 2C\ (1+1 / m)\ \therefore\ 1+\frac{1}{m} = \frac{E}{2C} = \frac{1.999X10^5}{2X0.79972X10^5} = 1.25,\ \therefore\ \frac{1}{m} = 0.25.$

∴ Poisson's ratio = 0.25.

2) In a tensile test, a test piece 25mm in diameter, 200 mm gauge length stretched 0.0975 mm under a pull of 50.000N. In a torsion test, the same rod twisted 0.025 radian over a length of 200 mm, when a torque of 400 Nm was applied. Evaluate the Poisson's ratio and the three elastic modules for the Material.

Solution:

d = 10 mm, Pull = 50.000 N, l = 200 mm

Tensile stress = $f = \dfrac{50.000}{\frac{\Pi (25)^2}{4}} = 101.86$ N / mm N / mm^2

Tensile stress = $e = \dfrac{0.0975}{200}$

$\therefore$ Elastic Module's = $E = f / e = \dfrac{101.86 \text{ X } 200}{0.0975} = 2.089 \text{ X } 10^5$ N / mm^2

$\dfrac{T}{I_p} = \dfrac{C\theta}{l}$

$\therefore C = \dfrac{T\,l}{I_P\ \theta} = \dfrac{400\text{X}1000\text{X}200}{\frac{\Pi\, 25^4}{32}\text{X } 0.025.} = 0.834 \text{ X } 105$ N / mm^2

We know, $E = 2C\,(1+1/m)$

$\therefore E = 2C\,(1+1/m)$

$\therefore 1+\dfrac{1}{m} = \dfrac{E}{2C} = \dfrac{2.089\text{X}}{2\text{X}0.0.834\text{X}10^5}$ E = 2C (1+1 / m)

$\therefore \dfrac{1}{m} = 0.25.$

$\therefore$ **Poisson's ratio = 0.252.**

We know that,

$E = 3K\,(1 - 2/m)$

$K = \dfrac{E}{3(1 - 2/m)}$

$= \dfrac{2.089 \text{ X } 10^5}{3(1-2\text{X}0.252)}$ N / mm^2 $= 1.404 \text{ X } 105$ N / mm^2

3) Find the diameter of the shaft required transmitting 60kw at 150r.p.m. if the maximum torque is likely to exceed the mean torque by 25% for a maximum permissible shear stress of 60N/mm^2. Find also the angle of twist for a length of 2.5 meters.

Take - $C = 8X10^4\ N/mm^2$

$P = \frac{2\Pi NT}{60000}$

$60 = \frac{2\Pi X150T}{60000}$

$T = \frac{60X60000}{2\Pi X150} = 3819.7Nm = 3819.7x1000Nm.$

The torque calculated above is the mean torque

$\therefore$ Max. Torque = T_{max} = 1.25XMEAN TORQUE

= 1.25X3819.7X1000= 4774.625X1000Nmm.

Polar module's $= \frac{I_p}{r_{max}} = \frac{\Pi d^4 x2}{32xd} = \frac{\Pi d^3}{16}$

$T_{max} = f$s X Polar methods.

$\therefore 4774.625X1000 = 60\ X \frac{\Pi d^3}{16}$

$\therefore \quad d^3 = \frac{4774.625\ X\ 1000\ X\ 16}{60\Pi}$

$\therefore \quad d = 74\ mm$

$T / I_p = C\theta / l$

$\therefore \theta = T / I_p = l / C = \frac{4774.625\ X\ 1000\ X\ 32}{\Pi\ X74^4}\ X\ \frac{2500}{8\ X\ 10^4}$

= 0.0507 radian = $2^0$54'

UNIT 7: THIN CYLINDERS

UNIT – 7

- ✓ **Introduction to thin cylinders, stresses in cylindrical shells**
- ✓ **Difference between thin and thick cylinders**
- ✓ **Stresses due to internal pressure : circumferential and longitudinal stresses**
- ✓ **Simple problems**
- ✓ **Design of thin cylinders with simple problems**

THIN CYLINDERS

PRESSURE VESSELS:

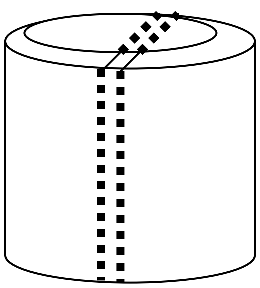
These are cylindrical and spherical forms containing fluids Such as barrels, tank, boilers, compressed air receivers, petrol tank are called Pressure vessels.

THIN & THICK CYLINDERS:

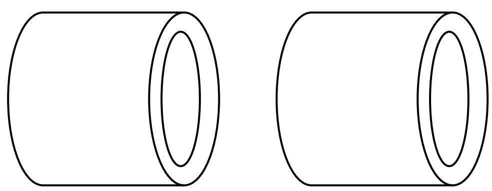
Thin cylinders: If the thickness of the wall of a shell is **less than 1/10th to 1/15th** of its diameter known as thin cylinder.

Thick cylinders: If the thickness of the wall of a shell is **greater than 1/10th to 1/15th** of its diameter known as thick cylinder.

STRESSES IN CYLINDRICAL SHELL DUE TO AN INTERNAL PRESSURE



(a) Split into 2 troughs.



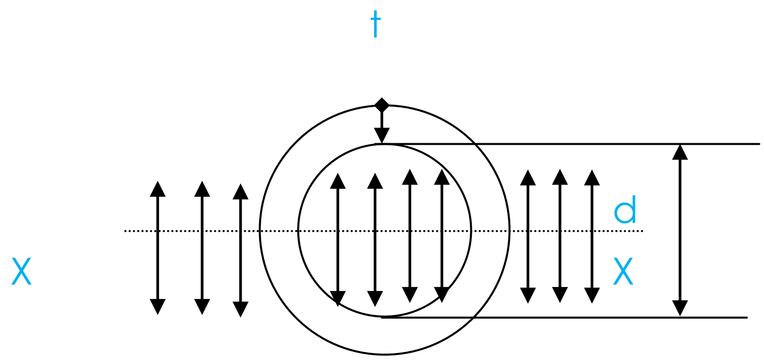
(b) Split into 2 cylinder

Whenever a cylinder shell is subjected to an internal pressure it is likely to fail by splitting up in any one of the 2 ways.

1) Split into 2 through (fig. a)
2) Split into 2 cylinders (fig. b)

In both the case the wall of the cylinder is subjected to an internal pressure that as a result of circumferential stress the cylinder has a tendency to split up.

CIRCUMFERENTIAL STRESS



Let us consider a body subjected to an internal pressure.

Let P = internal pressure
l = length of shell
d = dia of shell
t = thickness of shell
f_1 = circumferential stress

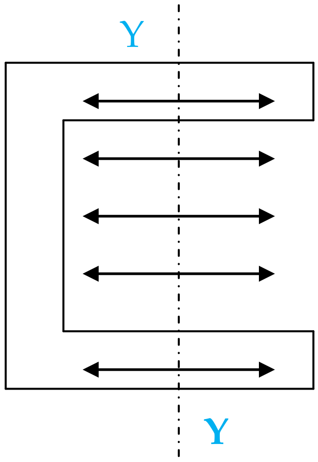
Total pressure along dia of the shell
= stress X Area
= Pdl

The circumferential stress = $\frac{\text{Total pressure}}{\text{Resisting section}}$

$f_1 = Pdl / 2tl$

$\therefore$ **$f_1 = Pd / 2t$**.

LONGITUDINAL STRESS:



Let us consider a same cylindrical shell subjected to an internal pressure that as a result of longitudinal stress the cylinder has a tendency to split up into 2 cylinders.

Let P = internal pressure

l = length of shell

d = dia of shell

t = thickness of shell

f_2 = longitudinal stress

$$f_2 = \frac{\text{total pressure}}{\text{resisting section}}$$

$$f_2 = \frac{P\Pi / 4\, d^2}{\Pi dt}$$

$\therefore$ **$f_2 = Pd / 4t$**.

Problems

1) A water main of 4m diameter and 4cm thickness is subjected to an internal pressure of 30 kg / cm^2 . Calculate the f_1 & f_2 (circumferential and longitudinal stresses)

$d = 4m = 400cm$

$f_1 = Pd / 2t = \frac{30X400}{2X4}$

$\therefore f_1 = 1500 \text{ kg / cm}^2$

$f_2 = Pd / 4t = \frac{30X400}{4X4}$

$\therefore$ **$f_2 = 750$ kg / cm^2**

2) A gas cylinder of internal dia **3m** & 6cm thickness. Find the allowable pressure of gas inside the cylinder. If tensile stress 2000 kg / cm^2.

$d = 3m = 300cm$

$f_1 = Pd / 2t$

$P = 2t\, f_1 / d$

$= 2X6X2000 / 300$

$\therefore$**P = 80 kg / cm^2**

3) A water main 160cm dia contains water at a pressure head of 200m. If the weight of the water is 1000 kg / cm^3. Find the thickness of metal required for the water main If $f_1 = 4000$ kg / cm^2

head = 200m = 20000cm

$\therefore f = Wh$

$= 1X20000 \text{ gm / cm}^2$

$f_1 = Pd / 2t$

$t = Pd / 2\, f_1 = \frac{20X160}{2X4000}$

$\therefore$**t = 0.4cm or 4mm**

4) A gas cylinder of internal diameter 40mm is 5 mm thick. If the tensile stress in the material is not to exceed 30 MPa, find the maximum pressure which can be allowed in the cylinder.

Given d= 40mm,t= 5mm $f_1 = 30$ MPa = 30 N/ mm^2

Let P= Maximum pressure which can be allowed in the cylinder

We know that circumferential stress (f1)

$$30 = Pd / 2t = \frac{P \times 40}{2 \times 5}$$

$$= 4P \quad \therefore P = 30/4 = 7.5 \text{ MPa}$$

Note:

a) Since the circumferential stress f_1 is double the longitudinal stress f_2 therefore in order to find the maximum pressure the given stress should be taken as circumferential stress.

b) 2. If however we take the given tensile stress of 30 N/mm^2 as the longitudinal stress then

$$30 = Pd / 4t = \frac{P \times 40}{4 \times 5}$$

$$= 2P \quad \therefore P = 30/2 = 15 \text{ MPa}$$

5) A cylindrical shell 2 m long and 1 m internal diameter is made up of 20mm thick plates. Find the circumferential and longitudinal stresses in the shell material, if it is subjected to an internal pressure of 5 MPa.

(ASSIGNMENT) Ans. 125 MPa : 62.5 MPa

6) A pipe of 100 mm diameter is carrying a fluid under a pressure of 4 MPa .what should be the minimum thickness of the pipe, if maximum circumferential stress in the pipe material is 12.5 MPa

(ASSIGNMENT) Ans. 16mm

Design of Thin Cylindrical shells

It means to calculate the thickness of a cylindrical shell for the given length, diameter and intensity of maximum internal pressure.

$f_1 = Pd/2t$ $\qquad\qquad$ $f_2 = Pd/4t$

Where t is the required thickness of the shell. If the thickness, so obtained is not a round figure, the next higher value is provided.

Note:
The thickness obtained from the longitudinal stress will be half of the thickness obtained from circumferential stress. Thus it should not be accepted.

1. A cylindrical shell of 500 mm diameter is required to withstand an internal pressure of 4 MPa. Find the minimum thickness of the shell, if maximum tensile strength in the plate material is 400MPa and efficiency of the joints is 65%.Take factor of safety as 5.

Given: d= 500 mm; P=4 MPa; Tensile strength= 400MPa=400N/mm^2 $\quad$ η= 65%

We know that allowable tensile stress (i.e. circumferential stress)

$$F_1 = \frac{\text{Tensile strength}}{\text{Factor of safety}}$$
$$= 400/5 = 80\text{N/mm}^2$$

And minimum thickness of the shell,

$$t = Pd/2f_1\eta = 4\text{x}500/2\text{x}80\text{x}0.65$$
$$= 19.2 \text{ say } 20\text{mm}$$

Change in Dimensions of a Thin Cylindrical Shell due to an Internal Pressure

Change in diameter of the shell (Δd) = $e_1 d = \frac{Pd^2}{2tE}(1 - 1/2m)$

Change in length of the shell (Δl) = $e_2 l = \frac{Pdl}{2tE}(1/2 - 1/m)$

1. A cylindrical thin drum 800mm in diameter and 4m long has a shell thickness of 10mm. If the drum is subjected to an internal pressure of 2.5MPa, determine its changes in diameter and length. Take E as 200GPa and Poisson's ratio as 0.25.

d=800mm; l=4 m=$4x10^3$mm; t=10mm; p=2.5 MPa=2.5N/mm^2; E=200 GPa= 200x10 ^{3}N/mm^2; 1/m, =0.25.

Change in diameter (Δd) = $e_1 d = \frac{Pd^2}{2tE}(1 - 1/2m)$

$$= \frac{2.5 \times 800^2}{2 \times 10 \times 200 \times 10^3}[1 - 0.25/2]$$

$$\mathbf{= 0.35mm}$$

Change in length (Δl) = $e_2 l = \frac{Pdl}{2tE}(1/2 - 1/m)$

$$= \frac{2.5 \times 800 \times 4 \times 10^3}{2 \times 10 \times 200 \times 10^3}[1/2 - 0.25]$$

$$\mathbf{= 0.5mm}$$

Change in volume of a Thin Cylindrical shell due to an Internal Pressure

Change in volume of a Thin Cylindrical shell due to an Internal Pressure is given by $\Delta V = V(e_2 + 2e_1)$.

1. A cylindrical vessel 2m long and 500mm in diameter with 10mm thick plates is subjected to an internal pressure of 3 MPa. Calculate the change in volume of the vessel. Take E= 200GPa and Poisson's ratio= 0.3 for the vessel material.

Given, $l = 2\ m = 2\times10^3$ mm; d= 500mm; t = 10 mm; P = 3 MPa; E = 200GPa; $200\times10^3\ N/mm^2$ and 1/m = 0.3

Circumferential strain

$$e_1 = \frac{Pd}{2tE}(1 - 1/2m) = \frac{3\times500}{2\times10\times200\times10^3} \times [1 - 0.3/2] = 0.32\times10^{-3} \quad \ldots\ldots.. 1$$

Longitudinal strain

$$e_2 = \frac{Pd}{2tE}(1/2 - 1/m) = \frac{3\times500\ xs}{2\times10\times200\times10^3}[1/2 - 0.3] = 0.075\times10^{-3} \quad \ldots\ldots.. 2$$

We also know that original volume of the vessel $V = \frac{\pi d^2}{4} x l$

$$= \frac{\pi 500^2}{4} x200$$

$$= \mathbf{392.7\times10^6 mm^3}$$

2. A steam boiler of 1.25 m in diameter is subjected to an internal pressure of 1.6 MPa. If the steam boiler is made up of 20mm thick plates, calculate the circumferential and longitudinal stresses. Take efficiency of the circumferential and longitudinal joints as 75% and 60% respectively.

ASSIGNMENT Ans. 67MPa; 42 MPa

3. A cylindrical shell 3 m long has 1 m internal diameter and 15mm metal thickness. Calculate the circumferential and longitudinal stresses, if the shell is subjected to an internal pressure of 1.5MPa. Also calculate the changes in dimensions of the shell. Take E = 200GPa; and Poisson's ratio = 0.3

   ASSIGNMENT Ans. 50MPa; 25 MPa; $\Delta l = 0.15mm$; $\Delta d = 0.21mm$

4. A cylindrical vessel 1.8 m long and 800mm in diameter is made up hick plates. Find the hoop and longitudinal stresses in the vessel, when it contains fluid under a pressure of 2.5 MPa. Also find the changes in length, diameter and volume of the vessel. Take E = 200GPa; and Poisson's ratio = 0.3

   ASSIGNMENT Ans.125MPa; 62.5 MPa; $\Delta l = 0.42mm$; $\Delta d = 0.23mm$

   $l\Delta v = 1074mm^3$

STRENGTH OF MATERIALS OBJECTIVE QUESTIONS.

FILLS IN THE BLANKS WITH APPROPRIATE WORDS:-

1	The system in which all the forces meet at one point are known as---------	concurrent
2	Velocity is an example of ------- quantity	Vector
3	Centre of gravity of a rectangle lines at the point of intersection of--------	Two diagonals
4	The ratio of lateral strain to linear strain is called-------	Poisson's ratio
5	-------- is a capacity to do work	Strain energy

6	Parallelogram law of force is applicable only to ----- forces	Two
7	Maximum power transmitted by a shaft is known as---	Strength of a shaft
8	Primary strain is also known as---------	Linear strain
9	Bulk modulus is the ratio of---------	Direct stress/volumetric strain
10	Lami's theorem is applicable only for -------	coplanar forces
11	The forces which are lying in the same line is ---------	Collinear forces
12	Formula for finding out circumferential stress--------	Pd/2t
13	$1MN/mm^2$------	$10^6 N/mm^2$
14	The stress induced in a body in case of a suddenly applied load----- the stress induced when a body is gradually loaded	Two times/Twice
15	Hoop stress is also known as-------stress	Circumferential
16	Strain energy per unit volume is known as--------	Modulus of resilience
17	Young's modulus is the ratio of ------	Stress/Strain
18	Modulus of rigidity is the ratio of -------	Shear stress/Shear strain
19	Wall thickness of the thin cylinder is--------	$1/10^{th}$-$1/15^{th}$ of its diameter
20	Factor of safety is the ratio of --------	Ultimate stress/Working stress
21	Multiplying factor of giga is -----	10^9
22	The unit of poisons ratio is ------	No units
23	Formula for strain energy is---------	$F^2/2E$ xAl
24	Formula used to find Torque is --------	Force x Radius
25	Centre of gravity of a semicircle is -------	$4r/3\pi$

MATCH THE FOLLOWING:-

Sl. no	A	B	ANSWER
1	Strain energy	a) $\pi/32\ xD^4$	Work done
2	Torque	b) Primary strain	Strength of a shaft
3	Split in to two cylinder	c) Back to its original position	Failure of cylinders
4	Centre of gravity	d) Triangle law of forces	Centroid
5	Lami's theorem	e) Work done	Coplanar force
6	Resolution of force	f) Strength of a shaft	Splitting up of force
7	General law of resultant force	g) Failure of cylinders	Triangle law of forces
8	Elasticity	h) Centroid	Back to its original position
9	Longitudinal strain	i) Coplanar force	Primary strain
10	Polar moment of inertia	j) Splitting up of force	$\pi/32\ xD^4$

MATCH THE FOLLOWING:-

Sl no	A	B	ANSWER
1	Lateral strain	a) μ	Secondary strain
2	Shear strain	b) moments method	Shear stress
3	Torque	c) unit less	EV
4	Power	d) stress	KW
5	Pressure	e) composition of force	N/mm2
6	Poisons ratio	f) KW	unit less
7	Center of gravity	g) EV	moments method
8	Resisting force	h) N/mm2	stress
9	Resultant force	i) Secondary strain	composition of force
10	Volume metric strength	j) Shear stress	μ

MATCH THE FOLLOWING:-

Sl no	A	B	ANSWER
1	Stress	a)π/16 x ι xD^3	c)Load/Area
2	Strain	b)2 πNT/60	d) Change in length/original length
3	Force	c)Load/Area	e)Mass X Acceleration
4	Power transmitted by shaft	d) Change in length/original length	b)2 πNT/60
5	Torque	e)Mass X Acceleration	a)π/16 x ι xD^3
6	Poisson's ratio	f) Centroid	i)1/m
7	Area	g)Force x distance	h) mm^2
8	Work done	h) mm^2	g)Force x distance
9	Volume	i)1/m	j)L x b x h
10	CG	j)L x b x h	f) Centroid

Multiple choice questions:

Sl no	A	ANSWERS
1	The product of force and distance is a) Energy b) power c) work done d) none	work done
2	The ratio of direct stress to volumetric strain is known as a)young modules b) bulk modules c) Poisons ratio d) Center of gravity	bulk modules
3	The unit of torque is A)N/m b) N-mm c) meter D) Newton	N-mm
4	The stress at which specimen breaks away is called a)ultimate stress b) yield stress c) breaking stress d) all of the above	breaking stress
5	The multiplying factor of the unit mega is a) 10^6 B) 10^9 C)10^3 D)10^{12}	10^6
6	A body is set to be in stable equilibrium a)if it does not return back to its original position b)if remains at rest in this position c) if it return back to its original position d) all of the above	if it return back to its original position
7	The process of replacing the given force system by its resultant is called a) Composition of force b) resolution force c) coplanar force d) resultant force	resultant force
8	Centre of gravity of semi circle from base is a)$4r/3\pi$ b) πr^2 c) $2\pi r$ d) $\pi d^2/16$	$4r/3\pi$
9	The maximum strain energy which can be stored in a body is known as a) Resilience b) resistance c) proof Resilience d) modules	proof Resilience

	Resilience	
10	Circumferential stress is also called as a)hoop stress b) longitudinal stress c) shear stress d) ultimate stress	hoop stress
11	The Poisson's ratio for steel varies from A) 0.20 to 0.25 b) 0.25 to 0.33 c) 0.35 to 0.40 d) 0.45 to 0.50	0.25 TO 0.33
12	When rectangular bar is subjected to a tensile stress then the volumetric strain is equal to A) ex(1-2/m) b) ex(1+2/m) c) ex(2-1/m) d)ex(2+1/m)	ex(1-2/m)
13	The CG of an equilateral triangle with each side(a) is ---- from any of the three sides a)ax√3/2 b) ax√2/3 c) a/2x√3 d)a/3x√2	a/2x√3
14	Strain energy stored in body when the load is gradually applied is equal to a)f^2/2xExV b) f^2/ExV c) f^2/2vxE d)) f^2/vxE	a)f^2/2xExV
15	When solid shaft is subjected to torsion the shear stress induced in the shaft at it center is a)zero b) minimum c) maximum d) average	zero
16	The term young's modulus is used for a) only for young person b) only for old person c) for young and person d) none of this	none of this
17	Shafts are used for--------- a)transmission of weight b) transmission of other object c) transmission of number of articles d) transmission of power	d) transmission of power
18	Thin cylinders are used for--------- a) Playing b) to make thick cylinders c) to transmit motion d) storing fluids and gasses.	d) Storing fluids and gasses.
19	Circumferential stresses are -------- the longitudinal stresses in thin cylinders a) Always b) smaller c) Twice d) with	c) Twice

20	Secondary strain is------- a) Change in area/original area b) change in weight/original weight c) change in breadth/original breadth d) change in velocity/original velocity.	c)change in breadth/original breadth
21	Strength of a material is------- a) Behavior of a material under external load b) lowering with the load c) increasing with the load d) Always constant	a)Behavior of a material under external load
22	Scalar quantities are--------- a)both magnitude and direction b) rotary motion and linear motion c) only magnitude no direction d) always in motion	c) only magnitude no direction
23	Vector quantities are--------- a)both magnitude and direction b) rotary motion and linear motion c) only magnitude no direction d) always in rest condition	a)both magnitude and direction
24	Resolution of a force is always done along------- a)X & Y axis b) X,Y, Z axis c) XX,YY,ZZ axis d) XY,YZ,XZ axis	a)X & Y axis
25	Centre of gravity of irregular body is find out by------- a)centrifugal force b) gravitational force c) Moments d) analytical method	c) Moments