GOVT.TOOL ROOM & TRAINING CENTRE
KARNATAKA

DIPLOMA IN TOOL & DIE MAKING / PRECISION MANUFACTURING AND MECHATRONICS COURSE

SUBJECT: ENGINEERING MATHEMATICS – I **SEMESTER: I**

UNIT 1: DETERMINANTS

I.FILL IN THE BLANKS **01 MARKS**

1. If $\begin{vmatrix} 2 & 4 \\ 3 & x \end{vmatrix} = 0$, the value of x is ____ (x = 6)
2. $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is ____ ($a_1b_2 - a_2b_1$)
3. If two rows or columns of a determinant are interchanged, the determinant is multiplied by _____ (-1)
4. The value of $\begin{vmatrix} 5 & -2 \\ 3 & 10 \end{vmatrix}$ is ____ (56)
5. The order of determinant $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is ____ (2)
6. $\begin{vmatrix} 8 & 4 \\ 3 & 0 \end{vmatrix}$ - $\begin{vmatrix} 2 & 5 \\ 0 & 2 \end{vmatrix}$ = ___ (-16)
7. In a determinant if two rows and columns are identical, the value of determinant is ____ (0)
8. If rows are changed into columns and columns into rows, the value of determinant is ____ (unaltered)
9. If a determinant has 2 rows and 2 columns, the order of determinant is ____ (2)
10. If A = $\begin{vmatrix} 5 & 8 \\ 3 & 2 \end{vmatrix}$ then 2A is ____$\left(\begin{vmatrix} 10 & 16 \\ 3 & 2 \end{vmatrix} \right)$
11. The value of $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$ is ____(-2)
12. Determinant of $\begin{pmatrix} 5 & 1 \\ 6 & 1 \end{pmatrix}$ is ___ (-1)
13. $\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix}$ is ___ (a^2)

14. $\begin{vmatrix} 1 & -5 \\ 6 & 3 \end{vmatrix}$ - $\begin{vmatrix} 5 & 8 \\ 0 & 4 \end{vmatrix}$ is ____ (13)

15. If A = $\begin{vmatrix} 8 & 1 \\ 0 & -4 \end{vmatrix}$ then -3A = _____ $\begin{vmatrix} -24 & -3 \\ 0 & -4 \end{vmatrix}$

16. $\begin{vmatrix} 4 & -2 \\ 6 & 0 \end{vmatrix}$ * $\begin{vmatrix} 1 & 9 \\ 0 & 4 \end{vmatrix}$ is ____ (48)

17. The value of $\begin{vmatrix} 2 & 0 & 0 \\ 2 & 1 & 3 \\ 7 & 9 & 0 \end{vmatrix}$ is ___ (-54)

18. The value of $\begin{vmatrix} 0 & -3 & 0 \\ 0 & 2 & 5 \\ 6 & 9 & 10 \end{vmatrix}$ is ____ (-90)

19. The order of determinant $\begin{vmatrix} 2 & 4 & 5 \\ 2 & 8 & 1 \\ 7 & 9 & 0 \end{vmatrix}$ is ___ (3)

20. If A = $\begin{vmatrix} 12 & 3 \\ -4 & 5 \end{vmatrix}$ and B = $\begin{vmatrix} 2 & 4 \\ 0 & 3 \end{vmatrix}$ then A * B is ___ (432)

II.MULTIPLE CHOICE QUESTIONS 01 MARKS

1. A^{-1} does not exists if IAI =
   a) 0
   b) 1
   c) 2
   d) -1
2. $\begin{vmatrix} 10 & 2 \\ 0 & 5 \end{vmatrix}$ * $\begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix}$ is
   a) 10
   b) 5
   c) 100
   d) 50
3. If $\begin{vmatrix} 2 & x \\ -1 & 5 \end{vmatrix}$ = 0 the value of x is
   a) 5
   b) 10

c) -10

d) 0

4. The value of $\begin{vmatrix} 3 & 4 & 6 \\ 6 & 8 & 12 \\ 1 & 0 & 1 \end{vmatrix}$ is

a) 0

b) 1

c) 10

d) 12

5. $A = \begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix}$ and $B = \begin{vmatrix} 12 & 5 \\ 7 & 2 \end{vmatrix}$ A – B is

a) 25

b) 10

c) 3

d) -4

6. If 6x+ 2y = 7 and 10x -14 y = 3 the value of x is

a) 0

b) 104

c) -104

d) 1

7. The value of is $\begin{vmatrix} 5 & -4 \\ 3 & 6 \end{vmatrix}$

a) 12

b) 18

c) 42

d) 9

8. If $A = \begin{pmatrix} 5 & 0 \\ -3 & -1 \end{pmatrix}$ then determinant of A is

a) 5

b) -15

c) -5

d) -3

9. If $\begin{vmatrix} x & 5 \\ 3 & 1 \end{vmatrix} = 0$ the value of x is

a) 5
b) 3
c) 1
d) 15

10. The value of is $\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$

a) abc
b) –abc
c) 0
d) None of these

11. If A = $\begin{vmatrix} tan\theta & sec\theta \\ 1 & 1 \end{vmatrix} =$

a) $\tan\theta - \sec\theta$
b) $\sec\theta - tan\theta$
c) $\tan\theta + \sec\theta$
d) $\sec\theta + tan\theta$

12. $\begin{vmatrix} 0 & -b & 0 \\ 0 & a & b \\ b & 9 & a \end{vmatrix}$ is

a) b^3
b) $-b^3$
c) $-ab^3$
d) ab^3

13. Determinant of $\begin{pmatrix} 3 & -4 \\ 1 & 3 \end{pmatrix}$ is

a) 10
b) 13
c) 15
d) 5

14. The value of $\begin{vmatrix} 1 & 0 & b \\ a & 0 & 0 \\ c & c & 0 \end{vmatrix}$

a) -abc

b) abc

c) 0

d) 1

15. If A = $\begin{vmatrix} 1 & sin\theta \\ sin\theta & 1 \end{vmatrix} =$

a) $\mathrm{Cos}^2\theta$

b) $\sin^2\theta$

c) $\mathrm{Cosec}^2\theta$

d) $\mathrm{Cot}^2\theta$

16. If A = $\begin{pmatrix} 2 & 11 \\ 1 & 12 \end{pmatrix}$ then determinant of A is

a) 24

b) 11

c) 35

d) 13

17. If A = $\begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix}$ and B= $\begin{vmatrix} 6 & 0 \\ 3 & 1 \end{vmatrix}$ then A + 2B is

a) 14

b) 12

c) 8

d) 6

18. The value of $\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ is

a) abc-1

b) abc

c) 1

d) 0

19. If $\begin{vmatrix} 6 & x \\ 4 & x \end{vmatrix} = 0$ the value of is

a) 1
b) 2
c) 0
d) None of these

20. If $\begin{vmatrix} 12 & 2x \\ 4 & x \end{vmatrix} = 4$ the value of is

a) 10
b) 8
c) 4
d) 1

III.ANSWER THE FOLLOWING QUESTION 02 MARKS

1. Solve for x if $\begin{vmatrix} 3 & 4 \\ 2 & x \end{vmatrix} = 0$

Ans: 3x-8 = 0

3x = 8

x = 8/3

2. Simplify $\begin{vmatrix} 5 & -2 \\ -3 & 6 \end{vmatrix}$

Ans: 5x6-(-3x2) = 30 - 6 = 24

3. Evaluate $\begin{vmatrix} a & 0 & 0 \\ -a & b & 0 \\ x & y & c \end{vmatrix}$

Ans: a(bxc-yx0) - 0 + 0 = abc

4. Find the value of $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

Ans: = cos θ * cos θ – (sin θ * -sin θ)

= $\text{Cos}^2\,\theta - (-\sin^2\theta)$

= $\text{Cos}^2\,\theta + \sin^2\theta$

5. If $\begin{vmatrix} 1-x & 1 \\ 6 & 2 \end{vmatrix} = 0$ find the value of x

Ans: (1-x)*2 – 6*1 = 0

2-2x – 6 = 0

-2x – 4 = 0

-2x = 4

x = 4/-2 = -2

6. Simplify $\begin{vmatrix} 3 & 2 \\ 5 & 6 \end{vmatrix}$ - $\begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix}$

   Ans: = (3*6 – 5*2) – (4*5 – 6*3)

   = (18 – 10) – (20 – 18)

   = 8 – 2 = 6

7. Find the value of x if $\begin{vmatrix} 2 & 3x \\ 5 & 10 \end{vmatrix} = 5$

   Ans: 2*10 – 5*3x = 5

   20 – 15x = 5

   -15x = 5 – 20

   X = $\frac{-15}{-15}$

   X = 1

8. Simplify $\begin{vmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{vmatrix}$

   Ans: = 1 *1 – (tan θ * -tan θ)

   = 1 + $\tan^2\theta$

   = $\sec^2\theta$

9. Evaluate $\begin{vmatrix} 2 & -6 \\ -5 & 4 \end{vmatrix}$

   Ans: = 2* 4 – (-5* -6)

   = 8 – 30

   = 22

10. Evaluate $\begin{vmatrix} 1 & 0 & 0 \\ 9 & -5 & 0 \\ 3 & 4 & -2 \end{vmatrix}$

    Ans: = 1(-5 * -2 – 4*0) -0 +0

    = 1(10 – 0)

    = 10

11. Simplify $\begin{vmatrix} 1 & -cot\theta \\ cot\theta & 1 \end{vmatrix}$

Ans: $= 1 - (-\,cot^2\theta)$

$= 1 + cot^2\theta$

$= cosec^2\theta$

12. Find x if $\begin{vmatrix} 3 & 2-x \\ 1 & 10 \end{vmatrix} = 0$

Ans: 30 - (2 - x) = 0

28 + x = 0

X= -28

13. Find y if $\begin{vmatrix} 1+y & x \\ 0 & 4 \end{vmatrix} = 0$

Ans: 4(1+y) - 0 = 0

4+ 4y = 0

4y = -4

Y = -1

14. Evaluate $\begin{vmatrix} 9 & 4 \\ -3 & 11 \end{vmatrix}$

Ans: = 99 - (-12) = 99 + 12 = 111

15. Evaluate $\begin{vmatrix} 1 & 4 & 3 \\ 2 & 0 & 0 \\ 5 & 2 & 1 \end{vmatrix}$

Ans: = 1(0 - 0) - 4 (2 - 0) +3 (4 - 0)

= 0 - 8 + 12 = 4

16. Evaluate $\begin{vmatrix} sin\theta & -cos\theta & 1 \\ cos\theta & sin\theta & 1 \\ 0 & 0 & 1 \end{vmatrix}$

Ans: $= sin\theta\ (sin\theta - 0) + cos\theta(cos\theta - 0) + 1(0)$

$= sin^2\theta + cos^2\theta = 1$

17. Simplify $\begin{vmatrix} x & y & z \\ 0 & 0 & -1 \\ y & x & 1 \end{vmatrix}$

Ans: = x (0 + x) - y (0 + y)+z (0)

= x^2 - y^2

18. Evaluate $\begin{vmatrix} 2 & 1 & 5 \\ 1 & 0 & 3 \\ 1 & 2 & 10 \end{vmatrix}$

Ans: = 2(0 - 6) - 1(10 - 3) + 5(2 - 0)

= -12 - 7 +10 = -9

19. If A= $\begin{vmatrix} 2 & 7+3x \\ 1 & 5x \end{vmatrix}$ = 0 find x

Ans: 10x - (7+3x) = 0

7x - 7 = 0

7x = 7

X = 1

20. If A = $\begin{vmatrix} 3 & 15 \\ 1 & 10 \end{vmatrix}$ and B = $\begin{vmatrix} 2 & 7 \\ 1 & 20 \end{vmatrix}$ find A * B

Ans: A = 30 - 15 = 15

B = 40 - 7 = 33

A * B = 15 * 33 = 495

IV.ANSWER THE FOLLOWING QUESTIONS 03 MARKS

1. Evaluate $\begin{vmatrix} 7 & 8 & 4 \\ 3 & 2 & 6 \\ 2 & 4 & 5 \end{vmatrix}$

Ans : = 7(2*5 – 4*6) – 8(3*5 – 2*6) + 4(3*4 – 2*2)

= 7(10-24) – 8(15 – 12) + 4(12 – 4)

= 7(-14) – 8(3) + 4(8)

= -98-24 +32

= -90

2. Find the unknown value if $\begin{vmatrix} 1 & x & 2 \\ 3 & -1 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 2$

Ans: 1(-2-1)- x(6-0)+2(3-0) = 2

1(-3) – x(6) +2(3) = 2

-3-6x+6 = 2

-6x+3 = 2

-6x = 2-3 = -1

X = -1/-6 = 1/6

3. Without expanding show that $\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix} = 0$

Ans : LHS $\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$

$= \begin{vmatrix} 1 & x & x+y+z \\ 1 & y & x+y+z \\ 1 & z & x+y+z \end{vmatrix} (c_3 = c_2 + c_3)$

$= (x+y+z) \begin{vmatrix} 1 & x & 1 \\ 1 & y & 1 \\ 1 & z & 1 \end{vmatrix}$

$= (x+y+z) . 0 \quad (c_1 \equiv c_3)$

= 0 = RHS

4. Solve for x and y, if 7x+5y = 16 and 2x-3y =9

Ans:

$\Delta = \begin{vmatrix} 7 & 5 \\ 2 & -3 \end{vmatrix}$ = 7*-3 – 2*5 = -21 – 10 = -31

$\Delta_x = \begin{vmatrix} 16 & 5 \\ 9 & -3 \end{vmatrix}$ = 16*-3 – 9*5 = -48 – 45 = -93

$\Delta_y = \begin{vmatrix} 7 & 16 \\ 2 & 9 \end{vmatrix}$ = 7*9 – 2*16 = 63 – 32 = 31

$X = \frac{\Delta x}{\Delta} = \frac{-93}{-31} = 3 \quad y = \frac{\Delta y}{\Delta} = \frac{31}{-31} = -1$

5. Solve for R_2 using determinant method, if $6R_1+2R_2=7$ and $10R_1-14R_2=3$

Ans: $\Delta = \begin{vmatrix} 6 & 2 \\ 10 & -14 \end{vmatrix} = 6*-14 - 10*2 = -84 - 20 = -104$

$\Delta_2 = \begin{vmatrix} 6 & 7 \\ 10 & 3 \end{vmatrix} = 6*3 - 10*7 = 18 - 70 = -52$

$R_2 = \frac{\Delta 2}{\Delta} = \frac{-52}{-104} = \frac{1}{2}$

6. Solve for x by Cramer's rule $x+4y=70$ and $-3x+2y=0$

Ans: $\Delta = \begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix} = 1*2 - (-3*4) = 2+12 = 14$

$\Delta_x = \begin{vmatrix} 70 & 4 \\ 0 & 2 \end{vmatrix} = 70*2 - 0*4 = 140 - 0 = 140$

$X = \frac{\Delta x}{\Delta} = \frac{140}{-14} = 10$

7. Solve y by Cramer's rule $5x+2y = 4$ and $3x+2y = 8$

Ans: $\Delta = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 5*2 - (3*2) = 10 - 6 = 4$

$\Delta_y = \begin{vmatrix} 5 & 4 \\ 3 & 8 \end{vmatrix} = 5*8 - 3*4 = 40 - 12 = 28$

$Y = \frac{\Delta y}{\Delta} = \frac{28}{4} = 7$

8. Solve x by determinants method $x+3y = -1$ and $5x + 4y = -7$

Ans: $\Delta = \begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix} = 4 - 15 = -11$

$\Delta_y = \begin{vmatrix} -1 & 3 \\ -7 & 4 \end{vmatrix} = -4 + 21 = 17$

$X = \frac{\Delta x}{\Delta} = \frac{17}{-11}$

9. Without expanding prove that $\begin{vmatrix} 10 & 12 & 17 \\ 6 & 7 & 8 \\ 4 & 5 & 9 \end{vmatrix} = 0$

Ans $= \begin{vmatrix} 4 & 5 & 9 \\ 6 & 7 & 8 \\ 4 & 5 & 9 \end{vmatrix}$ $(R_1 = R_1 - R_2)$

$= 0 \quad (R_1 \equiv R_2)$

10. Evaluate $\begin{vmatrix} 3 & 2 & -1 \\ 2 & 4 & 6 \\ 10 & 5 & 3 \end{vmatrix}$

Ans:

$= 3 (12 – 30) – 2 (6 – 60) – 1(10 - 40)$

$= 3 (-18) – 2 (- 54) – 1(-30)$

$= -54 + 108 + 30$

$= 84$

11. Evaluate $\begin{vmatrix} 91 & 92 & 93 \\ 94 & 95 & 96 \\ 97 & 98 & 99 \end{vmatrix}$

Ans:

$$= \begin{vmatrix} 91 & 92 & 93 \\ 94 & 95 & 96 \\ 97 & 98 & 99 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ 97 & 98 & 99 \end{vmatrix} \quad (R_1 = R_1 - R_2 \; and \; R_2 = R_2 - R_3)$$

$$= 0 \qquad (R_1 \equiv R_2)$$

12. Find the value of $\begin{vmatrix} 2\sqrt{5} & \sqrt{12} \\ \sqrt{3} & \sqrt{5} \end{vmatrix}$

$= 2\sqrt{5} \times \sqrt{5} - \sqrt{3} \times \sqrt{12}$

$= 2x\,5 - 6 = 10 - 6 = 4$

13. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+b \end{vmatrix} = ab$

Ans: LHS $= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+b \end{vmatrix}$

$$= \begin{vmatrix} 0 & 0 & 1 \\ -a & a & 1 \\ 0 & -b & 1+b \end{vmatrix} \quad (C_1 = C_1 - C_2 \; and \; C_2 = C_2 - C_3)$$

$= 0 – 0 + 1(ab – 0)$

$= ab = RHS$

14. Evaluate $\begin{vmatrix} 0 & b & -c \\ -a & 0 & a \\ c & -a & 0 \end{vmatrix}$

Ans: = 0 - b (0 - ac) - c(a^2 - 0)

= abc - a^2c

15. Find the value of m if $\begin{vmatrix} 2+m & 16 \\ 2 & 4 \end{vmatrix} = 0$

Ans: (2 + m) 4 - 32 = 0

8 + 4m -32 = 0

4m = 24

m = 6

16. Find the value of x if $\begin{vmatrix} 4x & 8 \\ x & -3 \end{vmatrix} = 10$

$$-12x - 8x = 10$$

$$-20x = 10$$

$$x = \frac{10}{-20} = \frac{1}{-2}$$

V.ANSWER THE FOLLOWNG QUESTION **05 MARKS**

1. Simplify $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

Ans: Subtracting the second column from first and third column from second

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c^2 \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix} \quad (c_1 = c_1 - c_2) \,\&(c_2 = c_2 - c_3)$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a+b)(a-b) & (b+c)(b-c) & c^2 \end{vmatrix}$$

Taking out (a-b) from first column and (b-c) from second column

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ (a+b) & (b+c) & c^2 \end{vmatrix}$$

= (a-b) (b-c) [0 – 0 + 1(b+c – (a+b))]

= (a-b) (b-c) [b+c – a – b)

= (a-b) (b-c) (c-a)

2. Find the value of x if $\begin{vmatrix} x & 2 & x+3 \\ 3 & 5 & 8 \\ x+1 & 7-x & 12 \end{vmatrix} = 0$

Ans:

x[60 - (7-x)8] - 2[36 –(x+1)8] + (x+3) [3(7-x) – (x+1) 5] = 0

x(60 – 56 + 8x) – 2 (36 – 8x – 8) + (x+3) (21 – 3x – 5x – 5) = 0

x (4 +8x) – 2 (28 – 8x) + (x+3) (16 – 8x) = 0

$4x + 8x^2 - 56 + 16x + 16x - 8x^2 + 48 - 24x = 0$

12x – 8 = 0

X = 8/12 = 2/3

3. Solve for x and y using determinants method if 4x-y =1 and x+4y = 13

Ans: $\Delta = \begin{vmatrix} 4 & -1 \\ 1 & 4 \end{vmatrix} = 16 - (-1) = 17$

$\Delta_x = \begin{vmatrix} 1 & -1 \\ 13 & 4 \end{vmatrix} = 4 - (-13) = 17$

$\Delta_y = \begin{vmatrix} 4 & 1 \\ 1 & 13 \end{vmatrix} = 52 - 1 = 51$

$X = \frac{\Delta x}{\Delta} = \frac{17}{17} = 1 \quad y = \frac{\Delta y}{\Delta} = \frac{51}{17} = 3$

4. Solve for z using Cramer's rule if

2x+3y-z = 4

x+2y-5z = -2

3x-y+3z = 5

Ans :

$$\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 2 & -5 \\ 3 & -1 & 3 \end{vmatrix}$$

= 2 (6 – 5) – 3 (3 + 15) -1 (-1 – 6)

= 2 – 54 + 7 = -45

$$\Delta_z = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & -2 \\ 3 & -1 & 5 \end{vmatrix}$$

= 2 (10 – 2) -3 (5 + 6) + 4 (-1 -6)

= 16 -33 – 28 = -45

$$Z = \frac{\Delta z}{\Delta} = \frac{-45}{-45} = 1$$

5. Solve the equation by Cramer's rule

   2y-z = 0

   x+3y = -4

   3x+4y = 4

   Ans:

$$\Delta = \begin{vmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

$$= 0 - 2(0 - 0) - 1(4 - 9)$$

$$= 0 - 0 + 5 = 5$$

$$\Delta_x = \begin{vmatrix} 0 & 2 & -1 \\ -4 & 3 & 0 \\ 4 & 4 & 0 \end{vmatrix}$$

$$= 0 - 2(0 - 0) - 1(-16 - 12)$$

$$= 0 - 0 + 28 = 28$$

$$\Delta_y = \begin{vmatrix} 0 & 0 & -1 \\ 1 & -4 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

$$= 0 - 0 - 1(4 + 12) = -16$$

$$\Delta_z = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 4 \end{vmatrix}$$

$$= 0 - 2(4 + 12) + 0$$

$$= -32$$

$$X = \frac{\Delta x}{\Delta} = \frac{28}{5} \quad y = \frac{\Delta y}{\Delta} = \frac{-16}{5} \quad z = \frac{\Delta z}{\Delta} = \frac{-32}{5}$$

6. Solve for x and y by determinants method

   2x+y+z = 1

   x+y+2z = 0

   5x+3y+3z = 2

   Ans:

$$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 5 & 3 & 3 \end{vmatrix}$$

$$= 2(3 - 6) - 1(3 - 10) + 1(3 - 5)$$

$$= -6 + 7 - 2 = -1$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & 3 \end{vmatrix}$$

$= 1 (3 – 6) -1(0 – 4) +1 (0 – 2)$

$= -3 +4 – 2 = -1$

$\Delta_y = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 5 & 2 & 3 \end{vmatrix}$

$= 2 (0 – 4) – 1(3 – 10) + 1(2 – 0)$

$= -8 + 7 + 2 = 1$

$X = \frac{\Delta x}{\Delta} = \frac{-1}{-1} = 1 \quad y = \frac{\Delta y}{\Delta} = \frac{1}{-1} = -1$

7. Solve for x by Cramer's rule
   $-x +3y -2z = 5$
   $4x –y -3z = -8$
   $2x +2y -5z = 7$
   Ans:

$\Delta = \begin{vmatrix} -1 & 3 & -2 \\ 4 & -1 & -3 \\ 2 & 2 & -5 \end{vmatrix}$

$= -1 (5 +6) – 3 (-20 + 6) – 2(8 + 2)$

$= -1 (11) -3 (-14) -2 (10)$

$= -11 + 42 – 20 = 11$

$\Delta_x = \begin{vmatrix} 5 & 3 & -2 \\ -8 & -1 & -3 \\ 7 & 2 & -5 \end{vmatrix}$

$= 5 (5 +6) – 3 (40 + 21) – 2(-16 + 7)$

$= 5 (11) -3 (61) – 2 (-9)$

$= 55 – 183 + 18$

$= -110$

$X = \frac{\Delta x}{\Delta} = \frac{-110}{11} = -10$

8. Prove that $\begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix} = 0$

   Ans: LHS $= \begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$

   Taking out 4 from first column we get,

$$= \ 4 \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

$$= 4 \begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & c+a \\ 1 & a+b+c & a+b \end{vmatrix} \quad (c_2 = c_2 + c_3)$$

$$= 4\,(a+b+c) \begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$$

$$= 4\,(a+b+c)\,.0 \qquad (c_1 \equiv c_2)$$

$$= \ 0 \ = \text{RHS}$$

9. Find the value of m and n if m+4n = 70 and 2m-3n = 0

Ans: $\Delta = \begin{vmatrix} 1 & 4 \\ 2 & -3 \end{vmatrix} = -3 - 8 = -11$

$$\Delta_m = \begin{vmatrix} 70 & 4 \\ 0 & -3 \end{vmatrix} = -210 - 0 = -210$$

$$\Delta_n = \begin{vmatrix} 1 & 70 \\ 2 & 0 \end{vmatrix} = 0 - 140 = -140$$

$m = \frac{\Delta_m}{\Delta} = \frac{-210}{-11} = \frac{210}{11}$ and $n = \frac{\Delta_n}{\Delta} = \frac{-140}{-11} = \frac{140}{11}$

10. Solve for x and y if 3x-4y-2 = 0 and 4x-y-7 =0

Ans: $\Delta = \begin{vmatrix} 3 & -4 \\ 4 & -1 \end{vmatrix} = -3 + 16 = 13$

$$\Delta_x = \begin{vmatrix} 2 & -4 \\ 7 & -1 \end{vmatrix} = -2 + 28 = 26$$

$$\Delta_y = \begin{vmatrix} 3 & 2 \\ 4 & 7 \end{vmatrix} = 21 - 8 = 13$$

$x = \frac{\Delta_x}{\Delta} = \frac{26}{13} = 2$ and $y = \frac{\Delta_y}{\Delta} = \frac{13}{13} = 1$

11. Solve for P and D if P +2D = 16 and 4P -7D = 4

Ans: $\Delta = \begin{vmatrix} 1 & 2 \\ 4 & -7 \end{vmatrix} = -7 - 8 = -15$

$\Delta_P = \begin{vmatrix} 16 & 2 \\ 4 & -7 \end{vmatrix} = -112 - 8 = -120$

$\Delta_D = \begin{vmatrix} 1 & 16 \\ 4 & 4 \end{vmatrix} = 4 - 64 = -60$

$P = \frac{\Delta_P}{\Delta} = \frac{-120}{-15} = 8$ and $Q = \frac{\Delta_D}{\Delta} = \frac{-60}{-15} = 4$

12. Prove that $\begin{vmatrix} -b & c & a \\ c & -a & b \\ a & b & -c \end{vmatrix} = a^3 + b^3 + c^3 + abc$

$Ans: LHS = -b(ac - b^2) - c(-c^2 - ab) + a(cb + a^2)$

$= -abc + b^3 + c^3 + abc + abc + a^3$

$= a^3 + b^3 + c^3 + abc =$ RHS

VI.ANSWER THE FOLLOWING QUESTIONS 08 MARKS

1. Solve by Cramer's rule
   3x-y+z = 19
   x+y+z = 21
   2x-2y+3z =22

   Ans :

$$\Delta = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & -2 & 3 \end{vmatrix}$$

= 3 (3+2) +1 (3-2) + 1(-2 – 2)

= 15 +1 – 4 = 12

$$\Delta_x = \begin{vmatrix} 19 & -1 & 1 \\ 21 & 1 & 1 \\ 22 & -2 & 3 \end{vmatrix}$$

= 19(3+2) +1 (63-22) + 1(-42 – 22)

= 95 + 41 – 64 = 72

$$\Delta_y = \begin{vmatrix} 3 & 19 & 1 \\ 1 & 21 & 1 \\ 2 & 22 & 3 \end{vmatrix}$$

$= 3\,(63 - 22) - 19\,(3-2) + 1(22 - 42)$

$= 123 - 19 - 20 = 84$

$$\Delta_z = \begin{vmatrix} 3 & -1 & 19 \\ 1 & 1 & 21 \\ 2 & -2 & 22 \end{vmatrix}$$

$= 3\,(22+42) + 1\,(22 - 42) + 19\,(-2 -2)$

$= 192 - 20 - 76 = 96$

$X = \frac{\Delta x}{\Delta} = \frac{72}{12} = 6 \quad y = \frac{\Delta y}{\Delta} = \frac{84}{12} = 7 \quad z = \frac{\Delta z}{\Delta} = \frac{96}{12} = 8$

2. Solve by Cramer's rule
   $2x+3y+z = 2$
   $x-2y-3z = 1$
   $-3x-3y+z = 0$

   Ans:

$$\Delta = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & -3 \\ -3 & -3 & 1 \end{vmatrix}$$

$= 2\,(-2 - 9) - 3\,(1 - 9) + 1(-3 - 6)$

$= -22 + 24 - 9 = -7$

$$\Delta_x = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & -3 \\ 0 & -3 & 1 \end{vmatrix}$$

$= 2\,(-2 - 9) - 3(1 - 0) + 1\,(-3 - 0)$

$= -22 - 3 - 3 = -28$

$$\Delta_y = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 1 & -3 \\ -3 & 0 & 1 \end{vmatrix}$$

$= 2\,(1- 0) - 2(1 - 9) + 1(0 + 3)$

$= 2 + 16 + 3 = 21$

$$\Delta_z = \begin{vmatrix} 2 & 3 & 2 \\ 1 & -2 & 1 \\ -3 & -3 & 0 \end{vmatrix}$$

$= 2\,(0 + 3) - 3(0 + 3) + 2(-3 - 6)$

$= 6 - 9 - 18 = -21$

$X = \frac{\Delta x}{\Delta} = \frac{-28}{-7} = 4 \quad y = \frac{\Delta y}{\Delta} = \frac{21}{-7} = -3 \quad z = \frac{\Delta z}{\Delta} = \frac{-21}{-7} = 3$

3. Solve by determinants method
   x+y+z = 1
   5x-2y+3z = -8
   6x+y-2z = 23

   Ans:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 5 & -2 & 3 \\ 6 & 1 & -2 \end{vmatrix}$$

$= 1\,(4 – 3) – 1(\,-10 – 18\,) + 1(\,5 + 12\,)$

$= 1 + 28 + 17 = 46$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ -8 & -2 & 3 \\ 23 & 1 & -2 \end{vmatrix}$$

$= 1\,(\,4 – 3) -1\,(\,16 – 69\,) + 1(\,-8 + 46\,)$

$= 1 + 53 + 38 = 92$

$$\Delta_y = \begin{vmatrix} 1 & 1 & 1 \\ 5 & -8 & 3 \\ 6 & 23 & -2 \end{vmatrix}$$

$= 1\,(16 – 69\,) -1\,(\,-10 -18\,) + 1(\,115 + 48)$

$= -53 + 28 + 163 = 138$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 1 \\ 5 & -2 & -8 \\ 6 & 1 & 23 \end{vmatrix}$$

$= 1\,(-46 + 8\,)\ -1(\,115 + 48\,) + 1(\,5 + 12)$

$= -38 – 163 + 17 = -184$

$X = \frac{\Delta x}{\Delta} = \frac{92}{46} = 2 \quad y = \frac{\Delta y}{\Delta} = \frac{138}{46} = 3 \quad z = \frac{\Delta z}{\Delta} = \frac{-184}{46} = -4$

4. Evaluate $\begin{vmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \\ 4 & 2 & 3 \end{vmatrix} \times \begin{vmatrix} 1 & 3 & 4 \\ 4 & 2 & 1 \\ 3 & 4 & 2 \end{vmatrix}$

Ans :

$= [\,2\,(\,9 – 4\,) – 1\,(\,9 – 8\,) + 4\,(\,6 – 12\,)\,] \times [\,1\,(\,4 – 4\,) – 3\,(\,8 – 3\,) + 4\,(\,16 – 6\,)\,]$

$= [\,2\,(\,5\,) – 1\,(\,1\,) + 4\,(\,-6\,)\,] \times [\,1\,(\,0\,) – 3\,(\,5\,) + 4\,(\,10\,)\,]$

$= [\,10 – 1 - 24\,]\,[\,0 – 15 + 40\,]$

$= -15 \times 25$

$= -375$

5. Evaluate $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

Ans: $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

Taking out a from first row, b from second row and c from third row we get,

$= abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

Taking out a from first column, b from second column and c from third column we get,

$= a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= a^2 b^2 c^2 [-1 (1 - 1) - 1(-1 - 1) + 1(1 + 1)]$

$= a^2 b^2 c^2 [-1 (0) - 1(-2) + 1(2)]$

$= a^2 b^2 c^2 [0 + 2 + 2]$

$= 4 a^2 b^2 c^2$

6. Solve for x, y and z by determinant method
$2x + 3y - z = 1$
$4x + y - 3z = 11$
$3x - 2y + 5z = 21$

Ans :

$\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 1 & -3 \\ 3 & -2 & 5 \end{vmatrix}$

$= 2 (5 - 6) - 3(20 + 9) -1 (-8 - 3)$

$= 2 (-1) - 3 (29) -1 (-11)$

$= -2 - 87 + 11 = -78$

$\Delta_x = \begin{vmatrix} 1 & 3 & -1 \\ 11 & 1 & -3 \\ 21 & -2 & 5 \end{vmatrix}$

$= 1 (5 - 6) - 3 (55 + 63) -1 (-22 -21)$

$= 1 (-1) - 3 (118) + 43$

$= -1 -354 + 43 = - 312$

$$\Delta_y = \begin{vmatrix} 2 & 1 & -1 \\ 4 & 11 & -3 \\ 3 & 21 & 5 \end{vmatrix}$$

$= 2(55+63) - 1(20+9) - 1(84-33)$

$= 2(118) - 1(29) - 1(51)$

$= 236 - 29 - 51 = 156$

$$\Delta_z = \begin{vmatrix} 2 & 3 & 1 \\ 4 & 1 & 11 \\ 3 & -2 & 21 \end{vmatrix}$$

$= 2(21+22) - 3(84-33) + 1(-8-3)$

$= 2(43) - 3(51) + 1(-11)$

$= 86 - 153 - 11 = -78$

$X = \frac{\Delta x}{\Delta} = \frac{-312}{-78} = 4 \quad y = \frac{\Delta y}{\Delta} = \frac{156}{-78} = -2 \quad z = \frac{\Delta z}{\Delta} = \frac{-78}{-78} = 1$

7. Solve for x, y and z if 3x-y+z = 0 , x+y-2z = 3 and 3x+z = 4

Ans: $$\Delta = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 1 & -2 \\ 3 & 0 & 1 \end{vmatrix}$$

$= 3(1+0) + 1(1+6) + 1(0-3)$

$= 3 + 7 - 3 = 7$

$$\Delta_x = \begin{vmatrix} 0 & -1 & 1 \\ 3 & 1 & -2 \\ 4 & 0 & 1 \end{vmatrix}$$

$= 0 + 1(3+8) + 1(0-4)$

$= 11 - 4 = 7$

$$\Delta_y = \begin{vmatrix} 3 & 0 & 1 \\ 1 & 3 & -2 \\ 3 & 4 & 1 \end{vmatrix}$$

$= 3(3+8) - 0 + 1(4-9)$

$= 33 - 0 - 5 = 28$

$$\Delta_z = \begin{vmatrix} 3 & -1 & 0 \\ 1 & 1 & 3 \\ 3 & 0 & 4 \end{vmatrix}$$

$= 3(4-0) + 1(4-9) + 0$

$= 12 - 5 = 7$

$x = \frac{\Delta_x}{\Delta} = \frac{7}{7} = 1$ and $y = \frac{\Delta_y}{\Delta} = \frac{28}{7} = 4 z = \frac{\Delta_z}{z} = \frac{7}{7} = 1$

8. Show that $\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = (a+2b)(a-b)^2$

$$LHS = \begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix}$$

$$= \begin{vmatrix} a+2b & b & b \\ a+2b & a & b \\ a+2b & b & a \end{vmatrix} \quad (c_1 = c_1 + c_2 + c_3)$$

$$= (a+2b)\begin{vmatrix} 1 & b & b \\ 1 & a & b \\ 1 & b & a \end{vmatrix}$$

$$= (a+2b)\begin{vmatrix} 1 & b & b \\ 0 & a-b & 0 \\ 0 & b-a & a-b \end{vmatrix} (R_2 = R_2 - R_1 \; and \; R_3 = R_3 - R_2)$$

$$= (a+2b)\,[1\,(a-b)^2 - 0]$$

$$= (a+2b)\ (a-b)^2 = \text{RHS}$$

9. Prove that $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = (x+a+b)(x-a)((x-b)$

$$LHS = \begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$$

$$= \begin{vmatrix} x+a+b & a & b \\ x+a+b & x & b \\ x+a+b & b & x \end{vmatrix} \quad (c_1 = c_1 + c_2 + c_3)$$

$$= (x+a+b)\begin{vmatrix} 1 & a & b \\ 1 & x & b \\ 1 & b & x \end{vmatrix}$$

$$= (x+a+b)\begin{vmatrix} 1 & a & b \\ 0 & x-a & 0 \\ 0 & b-x & x-b \end{vmatrix} (R_2 = R_2 - R_1 \; and \; R_3 = R_3 - R_2)$$

$= (x + a + b)\ [1\ (x - a)(x - b) - 0]$

$= (x + a + b)(x - a)(x - b) = \text{RHS}$

10. if $\begin{vmatrix} 2 & m-1 & -3 \\ 1 & -2 & 4 \\ 3 & -1 & 5 \end{vmatrix} = 3m - 1$ find the value of m

Ans: 2(-10 + 4) - (m - 1)(5 - 12) - 3(-1 + 6) = 3m - 1

2 (-6) - (m - 1) (-7) - 3(5) = 3m - 1

-12 + 7m - 7 - 15 = 3m - 1

7m - 3m = -1 +34

4m = 33

$m = \frac{33}{4}$

UNIT 2: MATRICES

I.FILL IN THE BLANKS 01 MARKS

1. A Matrix in which all the elements are zero is called Null Matrix
2. A scalar matrix in which all the diagonal elements are one is called Identity Matrix
3. A Matrix with a single column of elements is called Column Matrix
4. A Matrix with a single Row of elements is called Row Matrix
5. The rectangular array of elements enclosed in a bracket is Known as Matrices
6. A square matrix in which rows are change into columns and columns into rows then matrix is called as Symmetric matrix
7. $A = \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$ the order of the matrix A is 2
8. $If\ A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}\ then\ 3A\ is\ \begin{bmatrix} -3 & 6 \\ 6 & 3 \end{bmatrix}$
9. Transpose of a row matrix is column matrix
10. Transpose of a column matrix is Row matrix
11. The Transpose of cofactor matrix is a Adjoint Matrix
12. The Transpose of a rectangular matrix is Rectangular matrix

13. [a b c] is a **Row** matrix

14. The order of the matrix [2 5 7] is **1X3**

15. If a matrix has equal number of columns and rows then it is said to be a **square** matrix.

16. If determinant of a matrix is equal to zero, then it is said to be **Singular** matrix

17. Vertically arranged elements in a matrix are called **column** Matrix

18. In a unit matrix, diagonal elements are equal to **one**

19. If $A = \begin{bmatrix} 0 & 5 \\ 2 & 3 \end{bmatrix}$ then A^T is $\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$

20. Addition of matrices is defined if order of the matrices is **same**

II.MULTIPLE CHOICE QUESTIONS 01 MARKS

1. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$ then 2A is

a) $\begin{bmatrix} 4 & 6 \\ 2 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 3 \\ 10 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix}$

2. A matrix having one column but any number of rows is called as

a) *unit matrix* b) *Row matrix*

c) *column matrix* d) *Null matrix*

3. A matrix having one Row but any number of columns is called as

a) *unit matrix* b) *Row matrix*

c) *column matrix* d) *Null matrix*

4. ***The value of*** $\begin{vmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{2} & \mathbf{2} \\ \mathbf{3} & \mathbf{3} & \mathbf{3} \end{vmatrix}$

a) 0 b) 1

c) 2 d) 6

5. **A Diagonal matrix whose all the diagonal elements are same is called as:**

a) *Unit matrix* b) *Scalar matrix*

c) *Row matrix* d) *column matrix*

6. $\boldsymbol{A\ matrix\ A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}\ is\ a}$

a) *Unit matrix* b) *Scalar matrix*

c) *Row matrix* d) *column matrix*

7. **If the order of matrix A is m×p. And the order of B is p×n. Then the order of matrix AB is ?**

a) $n \times p$ ***b) $m \times n$***

c) $n \times p$ d) $n \times m$

8. **A square matrix in which all elements except at least one element in diagonal are zeros is said to be a**

a) Identical matrix b) null/zero matrix

c) Column matrix d) ***Diagonal matrix***

9. **Matrices obtained by changing rows and columns is called**

a) Rectangular Matrix **b) Transpose**

c) Symmetric d) None of Above

10. **If |A| = 0, then A is**

a) Zero matrix **b) singular matrix**

c) Non-singular matrix d)0

11. **Two matrices A and B are added if**

a) Both are rectangular

b) Both have same order

c) No of columns of A is equal to columns of B

d) No of rows of A is equal to no of columns of B

12. **A square or a rectangular array of numbers written within square brackets in a definite order in rows and columns is known as**

a) Formula

b) Determinant

c) *Matrix*

d) equation

13. **Two matrices A & B are multiplied to get BA If**

a) Both are rectangular

b) Both have same order

c) No columns A is equal to columns B

d) **Both are square matrices**

14. **[0 0 0] is a**

a) Scalar matrix

b) Diagonal matrix

c) Identity matrix

d) Null matrix

15. If $\boldsymbol{A \neq 0, then\ A\ is\ a}$

a) Zero matrix

b) singular matrix

c) **Non singular matrix**

d) Diagonal matrix

16. **A Matrix having m rows and n columns with m=n said to be a**

a) Rectangular matrix

b) **Square matrix**

c) Identity matrix

d) scalar matrix

17. **If a matrix has m rows and n columns then order is**

a) m+n

b) n x n

c) m x m

d) m x n

18. If determinant of a matrix A is Zero than ________

a) A is a Singular matrix b) A is a non-Singular matrix

c) Can't say d) none of the above

19. **We can add two matrices having real numbers A and B if their**

a) Order is same b) Rows are same

c) Columns are same d) Elements are same

20. **Generally the matrices are denoted by**

a) Capital letters b) Numbers

c) Small letters d) Operational signs

21. **Any element from the set of real numbers is also called a**

a) Scalar b) Factor

c) Transpose d) coefficient

22. **Horizontally arranged elements in a matrix is called**

a) *Unit matrix* b) *Scalar matrix*

c) ***Row matrix*** d) *column matrix*

III.ANSWER THE FOLLOWING QUESTION 02 MARKS

1. Write the transpose of matrix $A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

   Solution: $A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

   $\boldsymbol{A^T} = \begin{bmatrix} \mathbf{5} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{bmatrix}$

2. Define unit matrix

   Solution: A Scalar matrix in which all the diagonal elements are one is called a unit matrix.

3. In which matrix all the elements are equal to zero?

   Solution: Null Matrix

4. Write a formula to calculate Inverse of matrix.

   Solution: $\boldsymbol{A^{-1} = \frac{Adj\ A}{|A|}}$

5. In which matrix, only one columns of elements but any number of rows.

   Solution: Column Matrix

6. In which matrix, only one row of elements but any number of columns.

   Solution: Row Matrix

7. If $A = \begin{bmatrix} 4 & 2 \\ -3 & 4 \end{bmatrix}$ then find Adj A

   Solution: $\boldsymbol{Adj\ A = \begin{bmatrix} 4 & -2 \\ 3 & 4 \end{bmatrix}}$

8. $A = \begin{bmatrix} 3 & -1 \\ 5 & 4 \end{bmatrix}$ $\mathrm{B} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ Find $A - B$

   Solution: $\boldsymbol{A - B = \begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}}$

9. Define Null matrix.

   Solution: A matrix in which all the elements are equal to zero is called as null or zero matrix.

10. If $A = \begin{bmatrix} -1 & 4 \\ 2 & 4 \end{bmatrix}$ then find 2A.

    Solution: $A = \begin{bmatrix} -1 & 4 \\ 2 & 4 \end{bmatrix}$

    $$2A = 2\begin{bmatrix} -1 & 4 \\ 2 & 4 \end{bmatrix}$$

$$\boldsymbol{2A} = \begin{bmatrix} \mathbf{-2} & \mathbf{8} \\ \mathbf{4} & \mathbf{16} \end{bmatrix}$$

11. Which matrix obtained if transpose of Column matrix.

    Solution: Row Matrix

12. Mention the order of the matrix if $A = \begin{bmatrix} 5 & 5 & 3 \\ 0 & 1 & 5 \end{bmatrix}$

    Solution: order of the matrix is 2 X 3

13. A= $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ mention the type of matrix of A.

    Solution: Null Matrix or zero matrix

14. $A = \begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix}$ $\mathrm{B} = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$ Find $B - A$

    Solution: $B - A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix}$

    $$\boldsymbol{B - A} = \begin{bmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}$$

15. If $A = \begin{bmatrix} 1 & -1 \\ 3 & 4 \end{bmatrix}$ then find Adj A

    Solution: $\boldsymbol{Adj\ A} = \begin{bmatrix} \mathbf{4} & \mathbf{1} \\ \mathbf{-3} & \mathbf{1} \end{bmatrix}$

16. Define square matrix.

    Solution: A matrix in which the number of rows is equal to number of column is called square matrix.

17. A= $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Mention the type of matrix A.

    Solution: unit or Identity matrix

18. Define symmetric matrix.

Solution: A square matrix is said to be symmetric if it remains to same when rows are changed into column and column in to rows.

19. $A = \begin{bmatrix} 6 & -1 \\ -2 & 4 \end{bmatrix}$ Find $-3A$

Solution: $-3A = -3\begin{bmatrix} 6 & -1 \\ -2 & 4 \end{bmatrix}$

$$\boldsymbol{-3A} = \begin{bmatrix} \mathbf{-18} & \mathbf{3} \\ \mathbf{6} & \mathbf{-12} \end{bmatrix}$$

20. $A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ $\mathrm{B} = \begin{bmatrix} 3 & -3 \\ 1 & 4 \end{bmatrix}$ Find $A - B$

Solution:

$$A - B = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ 1 & 4 \end{bmatrix}$$

$$\boldsymbol{A - B} = \begin{bmatrix} \mathbf{-1} & \mathbf{3} \\ \mathbf{-4} & \mathbf{-3} \end{bmatrix}$$

IV.ANSWER THE FOLLOWING QUESTIONS 03 MARKS

1. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 5 & 7 \\ 6 & 2 & 1 \end{bmatrix}$ Find 3A+B

Solution: $\boldsymbol{3A + B} = 3\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 7 \\ 6 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 3 & -6 & 9 \\ 0 & 6 & 15 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 7 \\ 6 & 2 & 1 \end{bmatrix}$$

$$\boldsymbol{3A + B} = \begin{bmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{16} \\ \mathbf{6} & \mathbf{8} & \mathbf{16} \end{bmatrix}$$

2. Find the value of 2A+4B If A= $\begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$ and B= $\begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix}$

Solution: $2A + 4B = 2\begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix} + 4\begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 8 \\ 6 & 10 \end{bmatrix} + \begin{bmatrix} 12 & 20 \\ 24 & 28 \end{bmatrix}$$

$$\boldsymbol{2A + 4B} = \begin{bmatrix} \mathbf{14} & \mathbf{28} \\ \mathbf{30} & \mathbf{38} \end{bmatrix}$$

3. Find the value of 3A-2B If A= $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} -1 & 5 \\ 2 & 3 \end{bmatrix}$

Solution: $3A - 2B = 3\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} - 2\begin{bmatrix} -1 & 5 \\ 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 6 \\ 9 & 3 \end{bmatrix} - \begin{bmatrix} -2 & 10 \\ 4 & 6 \end{bmatrix}$$

$$\boldsymbol{3A - 2B = \begin{bmatrix} 8 & -4 \\ 5 & -3 \end{bmatrix}}$$

4. If A= $\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 5 \\ -2 & 3 \end{bmatrix}$ Find AB

Solution: $\boldsymbol{A.B = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -2 & 3 \end{bmatrix}}$

$$\boldsymbol{= \begin{bmatrix} 6-4 & 15+6 \\ -2-8 & -5+12 \end{bmatrix}}$$

$$\boldsymbol{A.B = \begin{bmatrix} 2 & 21 \\ -10 & 7 \end{bmatrix}}$$

5. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ Find the matrix $A + A^1$

Solution: $A + A^1 = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$$\boldsymbol{A + A^1 = \begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}}$$

6. If A= $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 5 & 2 \\ -1 & 3 \end{bmatrix}$ Find $(A+B)^T$

Solution: $A + B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -1 & 3 \end{bmatrix}$

$$A + B = \begin{bmatrix} 8 & 1 \\ 1 & 4 \end{bmatrix}$$

$$\boldsymbol{(A + B)^T = \begin{bmatrix} 8 & 1 \\ 1 & 4 \end{bmatrix}}$$

7. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ Find A^2

Solution: $A^2 = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 4+3 & 2+0 \\ 6+0 & 3+0 \end{bmatrix}$$

$$\boldsymbol{A^2} = \begin{bmatrix} \mathbf{7} & \mathbf{2} \\ \mathbf{6} & \mathbf{3} \end{bmatrix}$$

8. If $\begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}+\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 4 & 5 \end{bmatrix}$ Find the values of $x\ and\ y$.

Solution:

$$\begin{bmatrix} 3 & 3 \\ x+2 & y+1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 4 & 5 \end{bmatrix}$$

$x + 2 = 4$	$y + 1 = 5$
$x = 4 - 2$	$y = 5 - 1$
$\boldsymbol{x = 2}$	$\boldsymbol{y = 4}$

9. Find the value of 3A-2B If A= $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} -1 & 5 \\ 2 & 3 \end{bmatrix}$

Solution: $3A - 2B = 3\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} - 2\begin{bmatrix} -1 & 5 \\ 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 6 \\ 9 & 3 \end{bmatrix} - \begin{bmatrix} -2 & 10 \\ 4 & 6 \end{bmatrix}$$

$$\boldsymbol{3A - 2B} = \begin{bmatrix} \mathbf{8} & \mathbf{-4} \\ \mathbf{5} & \mathbf{-3} \end{bmatrix}$$

10. Find the value of 2A-B If A= $\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 3 & 0 \\ 3 & 4 \end{bmatrix}$

Solution: $2A - B = 2\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 3 & 4 \end{bmatrix}$$

$$\boldsymbol{2A - B} = \begin{bmatrix} \mathbf{3} & \mathbf{8} \\ \mathbf{1} & \mathbf{-2} \end{bmatrix}$$

11. If A= $\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 5 \\ -2 & 3 \end{bmatrix}$ Find BA

Solution: $BA = \begin{bmatrix} 2 & 5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 6-5 & 4+20 \\ -6-3 & -4+12 \end{bmatrix}$$

$$\boldsymbol{BA} = \begin{bmatrix} \mathbf{1} & \mathbf{24} \\ \mathbf{-9} & \mathbf{8} \end{bmatrix}$$

12. $If\ \begin{bmatrix} x & 3 \\ 2 & y \end{bmatrix}+\begin{bmatrix} 2 & 6 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 4 & 6 \end{bmatrix}$ Find the values of $x\ and\ y$.

Solution: $\begin{bmatrix} x & 3 \\ 2 & y \end{bmatrix}+\begin{bmatrix} 2 & 6 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 4 & 6 \end{bmatrix}$

$$\begin{bmatrix} x+2 & 9 \\ 4 & y+1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 4 & 6 \end{bmatrix}$$

$x + 2 = 5$ $\qquad\qquad$ $y + 1 = 6$

$\boldsymbol{x = 3}$ $\qquad\qquad$ $\boldsymbol{y = 5}$

13. If $A = \begin{bmatrix} 3 & 4 \\ -2 & 3 \end{bmatrix}$ Find the matrix $2A + A^1$

Solution: $2A + A^1 = 2\begin{bmatrix} 3 & 4 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 8 \\ -4 & 6 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix}$$

$$\boldsymbol{2A + A^1} = \begin{bmatrix} \mathbf{9} & \mathbf{6} \\ \mathbf{0} & \mathbf{9} \end{bmatrix}$$

14. If $A = \begin{bmatrix} 0 & -2 \\ 3 & 3 \end{bmatrix}$ Find the matrix $A - A^1$

Solution: $A - A^1 = \begin{bmatrix} 0 & -2 \\ 3 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -2 & 3 \end{bmatrix}$

$$\boldsymbol{A - A^1} = \begin{bmatrix} \mathbf{0} & \mathbf{-5} \\ \mathbf{5} & \mathbf{0} \end{bmatrix}$$

15. Find the Adjoint of A= $\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$

Solution: $\boldsymbol{Adj\ A} = \begin{bmatrix} \mathbf{-1} & \mathbf{-4} \\ \mathbf{-1} & \mathbf{2} \end{bmatrix}$

16. If A= $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $FindA^2$

Solution: $A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix}$$

$$\boldsymbol{A^2} = \begin{bmatrix} \mathbf{2} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{2} \end{bmatrix}$$

17. If A= $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ $Find A^2$

Solution: $A^2 = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 4+4 & 8+12 \\ 2+3 & 4+9 \end{bmatrix}$$

$$\boldsymbol{A^2} = \begin{bmatrix} \mathbf{8} & \mathbf{20} \\ \mathbf{5} & \mathbf{13} \end{bmatrix}$$

18. If A= $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ $Find A^2$

Solution: $A^2 = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 4+3 & 2+0 \\ 6+0 & 3+0 \end{bmatrix}$$

$$\boldsymbol{A^2} = \begin{bmatrix} \mathbf{7} & \mathbf{2} \\ \mathbf{6} & \mathbf{3} \end{bmatrix}$$

19. If A= $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 5 & 2 \\ -2 & 3 \end{bmatrix}$ $Find\ (A+B)^T$

Solution: $(A+B) = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -2 & 3 \end{bmatrix}$

$$(A+B) = \begin{bmatrix} 8 & 1 \\ 0 & 4 \end{bmatrix}$$

$$(\boldsymbol{A}+\boldsymbol{B})^{\boldsymbol{T}} = \begin{bmatrix} \mathbf{8} & \mathbf{0} \\ \mathbf{1} & \mathbf{4} \end{bmatrix}$$

20. Find the Adjoint of A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Solution: $\boldsymbol{Adj\ A} = \begin{bmatrix} \boldsymbol{d} & \boldsymbol{-b} \\ \boldsymbol{-c} & \boldsymbol{a} \end{bmatrix}$

V.ANSWER THE FOLLOWNG QUESTION 05 MARKS

1. If $A=\begin{bmatrix}6 & 3 & 2\\1 & 1 & 6\\5 & 6 & 3\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 2\\6 & 10 & 5\\4 & 4 & 0\end{bmatrix}$ Find AB

Solution:

$$AB=\begin{bmatrix}6 & 3 & 2\\1 & 1 & 6\\5 & 6 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\6 & 10 & 5\\4 & 4 & 0\end{bmatrix}$$

$$AB=\begin{bmatrix}6+18+8 & -6+30+8 & 12+15+0\\1+6+24 & -1+10+24 & 2+5+0\\5+36+12 & -5+60+12 & 10+30+0\end{bmatrix}$$

$$\boldsymbol{AB}=\begin{bmatrix}\mathbf{32} & \mathbf{32} & \mathbf{27}\\\mathbf{31} & \mathbf{33} & \mathbf{7}\\\mathbf{53} & \mathbf{67} & \mathbf{40}\end{bmatrix}$$

2. If $A=\begin{bmatrix}2 & 3 & 5\\1 & 6 & 5\\4 & 3 & 5\end{bmatrix}$ and $B=\begin{bmatrix}0 & 2 & 3\\5 & 3 & 3\\1 & 1 & 4\end{bmatrix}$ Find 5A-4B

Solution:

$$5A-4B=5\begin{bmatrix}2 & 3 & 5\\1 & 6 & 5\\4 & 3 & 5\end{bmatrix}-4\begin{bmatrix}0 & 2 & 3\\5 & 3 & 3\\1 & 1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}10 & 15 & 25\\5 & 30 & 25\\20 & 15 & 25\end{bmatrix}-\begin{bmatrix}0 & 8 & 12\\20 & 12 & 12\\4 & 4 & 16\end{bmatrix}$$

$$\mathbf{5}\boldsymbol{A}-\mathbf{4}\boldsymbol{B}=\begin{bmatrix}\mathbf{10} & \mathbf{7} & \mathbf{13}\\\mathbf{-15} & \mathbf{18} & \mathbf{13}\\\mathbf{16} & \mathbf{11} & \mathbf{9}\end{bmatrix}$$

3. If $A=\begin{bmatrix}1 & 2 & 4\\1 & 6 & 1\\4 & 3 & 5\end{bmatrix}$ and $B=\begin{bmatrix}0 & 2 & 3\\5 & 3 & 3\\1 & 1 & 4\end{bmatrix}$ Find AB

Solution:

$$AB=\begin{bmatrix}1 & 2 & 4\\1 & 6 & 1\\4 & 3 & 5\end{bmatrix}\begin{bmatrix}0 & 2 & 3\\5 & 3 & 3\\1 & 1 & 4\end{bmatrix}$$

$$= \begin{bmatrix} 00+10+4 & 2+6+4 & 3+6+16 \\ 00+30+1 & 2+18+1 & 3+18+4 \\ 00+15+5 & 8+9+5 & 12+9+20 \end{bmatrix}$$

$$\boldsymbol{AB} = \begin{bmatrix} \mathbf{14} & \mathbf{12} & \mathbf{25} \\ \mathbf{31} & \mathbf{21} & \mathbf{25} \\ \mathbf{20} & \mathbf{22} & \mathbf{41} \end{bmatrix}$$

4. Solve for x and y by matrix method $3x + 4y = 7 \quad 7x - y = 6$

Solution:

$$\boldsymbol{3x + 4y = 7}$$

$$\boldsymbol{7x - y = 6}$$

Let $A = \begin{bmatrix} 3 & 4 \\ 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$$Adj\ A = \begin{bmatrix} -1 & -4 \\ -7 & 3 \end{bmatrix}$$

$|A| = [-3 - 28]$

$|A| = -31$

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$\boldsymbol{A^{-1}} = \frac{\begin{bmatrix} \mathbf{-1} & \mathbf{-4} \\ \mathbf{-7} & \mathbf{3} \end{bmatrix}}{\mathbf{-31}}$$

$\boldsymbol{X = A^{-1}\, B}$

$$= \frac{1}{-31}\begin{bmatrix} -1 & -4 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$= \frac{1}{-31}\begin{bmatrix} -7 - 24 \\ -49 + 18 \end{bmatrix}$$

$$= \frac{1}{-31}\begin{bmatrix} -31 \\ -31 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\therefore \boldsymbol{x = 1 \ \ and\ y = 1}$

5. Find the Inverse of A= $\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

$Adj\ A = \begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix}$

$|A| = [20 - 6]$

$|A| = 14$

$A^{-1} = \frac{Adj\ A}{|A|}$

$\boldsymbol{A^{-1}} = \frac{\begin{bmatrix} \mathbf{4} & \mathbf{-3} \\ \mathbf{-2} & \mathbf{5} \end{bmatrix}}{\mathbf{14}}$

6. A= $\begin{bmatrix} 1 & 3 \\ -6 & 7 \end{bmatrix}$ B= $\begin{bmatrix} 7 & -8 \\ 6 & 2 \end{bmatrix}$ Show that AB≠ BA

Solution:

$AB = \begin{bmatrix} 1 & 3 \\ -6 & 7 \end{bmatrix} \begin{bmatrix} 7 & -8 \\ 6 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 7+18 & -8+6 \\ -42+42 & 48+14 \end{bmatrix}$

$\boldsymbol{AB} = \begin{bmatrix} \mathbf{25} & \mathbf{-2} \\ \mathbf{0} & \mathbf{64} \end{bmatrix}$

$BA = \begin{bmatrix} 7 & -8 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -6 & 7 \end{bmatrix}$

$BA = \begin{bmatrix} 7+48 & 21-56 \\ 6-12 & 18+14 \end{bmatrix}$

$\boldsymbol{BA} = \begin{bmatrix} \mathbf{55} & \mathbf{-35} \\ \mathbf{-6} & \mathbf{32} \end{bmatrix}$

∴ **AB ≠ BA**

7. If $\begin{bmatrix} 2x+3 & 0 \\ 0 & 1 \end{bmatrix}$ is a unit matrix , then find the value of 'x'

Solution:

$$\begin{bmatrix} 2x+3 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\begin{bmatrix} 2x+3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore 2x + 3 = 1$

$2x = 3 - 1$

$x = \frac{2}{2}$

$\therefore \boldsymbol{x = 1}$

8. If the matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ 5 & 1 & 0 \\ 1 & 0 & x \end{bmatrix}$ $and\ |A| = 0$ then find the value of $'x'$.

Solution:

$if\ |A| = 0$ then

$$\begin{vmatrix} 2 & -1 & 3 \\ 5 & 1 & 0 \\ 1 & 0 & x \end{vmatrix} = 0$$

$\rightarrow \quad 2[x - 0] + 1[5x - 0] + 3[0 - 1] = 0$

$\rightarrow \quad 2x + 5x - 3 = 0$

$7x = 3$

$x = \frac{3}{7}$

$\boldsymbol{x = \frac{3}{7}}$

9. $If\ A = \begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix}$ $Find\ A^2 - 5A + 7I, Where\ I\ is\ a\ unit\ matrix\ of\ order\ 2.$

Solution:

$let\ A^2 - 5A + 7I$

$$A^2 = \begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 25+6 & -10-4 \\ -15-6 & 6+4 \end{bmatrix}$$

$$\boldsymbol{A^2 = \begin{bmatrix} 31 & -14 \\ -21 & 10 \end{bmatrix}}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 31 & -14 \\ -21 & 10 \end{bmatrix} - 5\begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 31 & -14 \\ -21 & 10 \end{bmatrix} - \begin{bmatrix} 25 & -10 \\ -15 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 31 - 25 + 7 & -14 + 10 + 0 \\ -21 + 15 + 0 & 10 - 10 + 7 \end{bmatrix}$$

$$\boldsymbol{A^2 - 5A + 7I = \begin{bmatrix} 13 & -4 \\ -6 & 7 \end{bmatrix}}$$

10. $If\ A = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}\ Find\ A\ X\ AdjA$

Solution:

$$Adj\ A = \begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix}$$

$$then\ AX\ Adj\ A = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 + 10 & -15 + 15 \\ -8 + 8 & 10 + 12 \end{bmatrix}$$

$$\boldsymbol{AX\ Adj\ A = \begin{bmatrix} 22 & 0 \\ 0 & 22 \end{bmatrix}}$$

11. $f\ A = \begin{bmatrix} 4 & 3 \\ 0 & 2 \end{bmatrix}\ Find\ \ AdjA\ XA$

Solution: $\boldsymbol{Adj\ A = \begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}}$

$$AdjA\ XA = \begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 + 0 & 6 - 6 \\ 0 + 0 & 0 + 8 \end{bmatrix}$$

$$\boldsymbol{AdjA\ XA = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}}$$

12. If $\begin{bmatrix} 3x - 2 & 0 \\ 0 & 2 \end{bmatrix}$ is a unit matrix , then find the value of 'x'

Solution: let $\begin{bmatrix} 3x - 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (since it is a unit matrix)

$$3x - 2 = 1$$

$$3x = 3$$

$$\boldsymbol{x = 1}$$

13. Find the Inverse of A= $\begin{bmatrix} 1 & 3 \\ 1 & -3 \end{bmatrix}$

Solution: let

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$\boldsymbol{Adj\ A} = \begin{bmatrix} \mathbf{-3} & \mathbf{-3} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}$$

$$\text{A}= \begin{bmatrix} 1 & 3 \\ 1 & -3 \end{bmatrix}$$

$$|A| = [-3 - 3]$$

$$|\boldsymbol{A}| = \mathbf{-6}$$

$$\boldsymbol{A^{-1}} = \frac{\begin{bmatrix} \mathbf{-3} & \mathbf{-3} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}}{\mathbf{-6}}$$

14. If the matrix $A = \begin{bmatrix} x & 0 & 3 \\ 2 & 1 & 0 \\ 1 & 2 & 5 \end{bmatrix}$ $and\ |A| = 0$ then find the value of $'x'$.

Solution: let $\begin{bmatrix} x & 0 & 3 \\ 2 & 1 & 0 \\ 1 & 2 & 5 \end{bmatrix} = 0$

$$\boldsymbol{x}[\mathbf{5 - 0}] - \mathbf{0}[\mathbf{10 - 0}] + \mathbf{3}[\mathbf{4 - 1}] = \mathbf{0}$$

$$\mathbf{5}\boldsymbol{x} - \mathbf{0} + \mathbf{9} = \mathbf{0}$$

$$\mathbf{5}\boldsymbol{x} = \mathbf{-9}$$

$$\boldsymbol{x} = \frac{\mathbf{-9}}{\mathbf{5}}$$

15. If A= $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ and B= $\begin{bmatrix} 1 & 0 & 3 \\ 5 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix}$ Find BA

Solution: $BA = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2+0+3 & 1+0+9 & 0+0+15 \\ 10+3+0 & 5+0+0 & 0+3+0 \\ 0+1+4 & 0+0+12 & 0+1+20 \end{bmatrix}$$

$$\boldsymbol{BA} = \begin{bmatrix} \mathbf{5} & \mathbf{10} & \mathbf{15} \\ \mathbf{13} & \mathbf{5} & \mathbf{3} \\ \mathbf{5} & \mathbf{12} & \mathbf{21} \end{bmatrix}$$

16. $If\ A = \begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix}$ $Find\ 2A - 5A + 7I, Where\ I\ is\ a\ unit\ matrix\ of\ order\ 2.$

Solution:

$$2A - 5A + 7I = 2\begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix} - 5\begin{bmatrix} 5 & -2 \\ -3 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -4 \\ -6 & 4 \end{bmatrix} - \begin{bmatrix} 25 & -10 \\ -15 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10 - 25 + 7 & -4 + 10 + 0 \\ -6 + 15 + 0 & 4 - 10 + 7 \end{bmatrix}$$

$$\boldsymbol{2A - 5A + 7I} = \begin{bmatrix} \mathbf{-8} & \mathbf{6} \\ \mathbf{9} & \mathbf{1} \end{bmatrix}$$

17. $\text{A}= \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ $\text{B}= \begin{bmatrix} 5 & 2 \\ -1 & 3 \end{bmatrix}$ $Prove$ that $(\text{A} + \text{B})^{\text{T}} = \text{A}^{\text{T}} + \text{B}^{\text{T}}$

Solution:

$$(\boldsymbol{A} + \boldsymbol{B}) = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -1 & 3 \end{bmatrix}$$

$$(\boldsymbol{A} + \boldsymbol{B}) = \begin{bmatrix} \mathbf{8} & \mathbf{1} \\ \mathbf{1} & \mathbf{4} \end{bmatrix}$$

$$(\mathbf{A} + \mathbf{B})^{\mathbf{T}} = \begin{bmatrix} \mathbf{8} & \mathbf{1} \\ \mathbf{1} & \mathbf{4} \end{bmatrix}$$

$$\boldsymbol{A^T} = \begin{bmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix} \quad \textbf{and} \quad \boldsymbol{B^T} = \begin{bmatrix} \mathbf{5} & \mathbf{-1} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}$$

$$\text{A}^{\text{T}} + \text{B}^{\text{T}} = \begin{bmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix} + \begin{bmatrix} \mathbf{5} & \mathbf{-1} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}$$

$$\mathbf{A}^{\mathbf{T}} + \mathbf{B}^{\mathbf{T}} = \begin{bmatrix} \mathbf{8} & \mathbf{1} \\ \mathbf{1} & \mathbf{4} \end{bmatrix}$$

$$\therefore\ (\mathbf{A} + \mathbf{B})^{\mathbf{T}} = \mathbf{A}^{\mathbf{T}} + \mathbf{B}^{\mathbf{T}}$$

18. $If\ A = \begin{bmatrix} 3 & -9 \\ -4 & 7 \end{bmatrix}$ $Find\ AA^1$

Solution:

$$\boldsymbol{AA^1} = \begin{bmatrix} 3 & -9 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -9 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 9 + 81 & -12 - 63 \\ -12 - 63 & 16 + 49 \end{bmatrix}$$

$$= \begin{bmatrix} 90 & -75 \\ -75 & 65 \end{bmatrix}$$

$$\boldsymbol{AA^1} = \begin{bmatrix} \mathbf{90} & \mathbf{-75} \\ \mathbf{-75} & \mathbf{65} \end{bmatrix}$$

19. $Find\ x\ if\ [5 \quad -1 \quad 2] \begin{bmatrix} 4 \\ x \\ 1 \end{bmatrix} = 15$

Solution: $[5 \quad -1 \quad 2] \begin{bmatrix} 4 \\ x \\ 1 \end{bmatrix} = 15$

$$[20 - x + 2] = 15$$

$$[22 - x] = 15$$

$$-x = 15 - 22$$

$$-x = -7$$

$$\boldsymbol{x = 7}$$

20. $Find\ the\ value\ of\ x\ if\ the\ matrix \begin{bmatrix} 2 & 3 & 4 \\ -4 & x & -8 \\ 5 & 6 & 7 \end{bmatrix} is\ singular$

Solution:

$$\begin{bmatrix} 2 & 3 & 4 \\ -4 & x & -8 \\ 5 & 6 & 7 \end{bmatrix} = 0$$

$$2[7x + 48] - 3[-28 + 40] + 4[-24 - 5x] = 0$$

$$14x + 96 - 3[12] - 96 - 5x = 0$$

$$9x + 96 - 36 - 96 = 0$$

$$9x = 36$$

$$x = \frac{36}{9}$$

$$\boldsymbol{x = 4}$$

VI.ANSWER THE FOLLOWING QUESTIONS 08 MARKS

1. If $A = \begin{bmatrix} 6 & 3 & 4 \\ 2 & 1 & 5 \\ 1 & 0 & 9 \end{bmatrix}$ Find $A^2 + 3A + 2I$

Solution:

$$A^2 = \begin{bmatrix} 6 & 3 & 4 \\ 2 & 1 & 5 \\ 1 & 0 & 9 \end{bmatrix} \begin{bmatrix} 6 & 3 & 4 \\ 2 & 1 & 5 \\ 1 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 36+6+4 & 18+3+0 & 24+15+36 \\ 12+2+5 & 6+1+0 & 8+5+45 \\ 6+0+9 & 3+0+0 & 4+0+81 \end{bmatrix}$$

$$\boldsymbol{A^2} = \begin{bmatrix} \mathbf{46} & \mathbf{21} & \mathbf{75} \\ \mathbf{19} & \mathbf{7} & \mathbf{58} \\ \mathbf{15} & \mathbf{3} & \mathbf{85} \end{bmatrix}$$

$$A^2 + 3A + 2I = \begin{bmatrix} 46 & 21 & 75 \\ 19 & 7 & 58 \\ 15 & 3 & 85 \end{bmatrix} + 3 \begin{bmatrix} 6 & 3 & 4 \\ 2 & 1 & 5 \\ 1 & 0 & 9 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 46 & 21 & 75 \\ 19 & 7 & 58 \\ 15 & 3 & 85 \end{bmatrix} + \begin{bmatrix} 18 & 9 & 12 \\ 6 & 3 & 15 \\ 3 & 0 & 27 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 46+18+2 & 21+9+0 & 75+12+0 \\ 19+6+0 & 7+3+2 & 58+15+0 \\ 15+3+0 & 3+0+0 & 85+27+2 \end{bmatrix}$$

$$\boldsymbol{A^2 + 3A + 2I} = \begin{bmatrix} \mathbf{66} & \mathbf{30} & \mathbf{87} \\ \mathbf{25} & \mathbf{12} & \mathbf{73} \\ \mathbf{18} & \mathbf{3} & \mathbf{116} \end{bmatrix}$$

2. Find the Adjoint of $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 6 & 5 \\ 4 & 3 & 2 \end{bmatrix}$

Solution:

$$let\ A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 6 & 5 \\ 4 & 3 & 2 \end{bmatrix}$$

$cofact\ of\ 2 = +[12 - 15] = +[-3] = -3$

$cofact\ of\ 3 = -[2 - 20] = -[-18] = 18$

$cofact\ of\ 5 = +[3 - 24] = +[-21] = -21$

$cofact\ of\ 1 = -[6 - 15] = -[-9] = 9$

$cofact\ of\ 6 = +[4 - 20] = +[-18] = -18$

$cofact\ of\ 5 = -[6 - 12] = -[-6] = 6$

$cofact\ of\ 4 = +[15 - 30] = +[-15] = -15$

$cofact\ of\ 3 = -[10 - 5] = -[5] = -5$

$cofact\ of\ 2 = +[12 - 3] = +[9] = 9$

$$cofact\ of\ A = \begin{bmatrix} -3 & 18 & -21 \\ 9 & -18 & 6 \\ -15 & -5 & 9 \end{bmatrix}$$

$$\boldsymbol{Adj\ A} = \begin{bmatrix} \mathbf{-3} & \mathbf{9} & \mathbf{-15} \\ \mathbf{18} & \mathbf{-18} & \mathbf{-5} \\ \mathbf{-21} & \mathbf{6} & \mathbf{9} \end{bmatrix}$$

3. Find the Adjoint of $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 3 & 2 \\ -1 & 2 & 1 \end{bmatrix}$

Solution:

$$let\ A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 3 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

$cofact\ of\ 1 = +[3 - 4] = +[-1] = -1$

$cofact\ of - 2 = -[4 - 2] = -[2] = -2$

$cofact\ of\ 3 = +[8 - 3] = +[5] = 5$

$cofact\ of\ 4 = -[-2-6] = -[-8] = 8$

$cofact\ of\ 3 = +[1+3] = +[4] = 4$

$cofact\ of\ 2 = -[2-2] = -[0] = 0$

$cofact\ of\ -1 = +[-4-9] = +[-13] = -13$

$cofact\ of\ 2 = -[2-12] = -[-10] = 10$

$cofact\ of\ 1 = +[3+8] = +[11] = 11$

$$cofact\ of\ A = \begin{bmatrix} -1 & -2 & 5 \\ 8 & 4 & 0 \\ -13 & 10 & 11 \end{bmatrix}$$

$$\boldsymbol{Adj\ A} = \begin{bmatrix} \mathbf{-1} & \mathbf{8} & \mathbf{-13} \\ \mathbf{-2} & \mathbf{4} & \mathbf{10} \\ \mathbf{5} & \mathbf{0} & \mathbf{11} \end{bmatrix}$$

4. Find the Inverse of $A = \begin{bmatrix} 1 & -3 & -1 \\ 2 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$

Solution:

$$let\ A = \begin{bmatrix} 1 & -3 & -1 \\ 2 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$$

$|A| = +1[8-15] - (-)3[4-20] - 1[6-16]$

$= +1[-7] + 3[-16] - 1[-10]$

$= -7 - 48 + 10$

$\boldsymbol{|A| = -45}$

$$let\ A = \begin{bmatrix} 1 & -3 & -1 \\ 2 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$$

$cofact\ of\ 1 = +[8-15] = +[-7] = -7$

$cofact\ of\ -3 = -[4-20] = -[-16] = 16$

$cofact\ of - 1 = +[6 - 16] = +[-10] = -10$

$cofact\ of\ 2 = -[-6 + 3] = -[-3] = 3$

$cofact\ of\ 4 = +[2 + 4] = +[6] = 6$

$cofact\ of\ 5 = -[3 + 12] = -[15] = -15$

$cofact\ of\ \ 4 = +[-15 + 4] = +[-11] = -11$

$cofact\ of\ 3 = -[5 + 2] = -[7] = -7$

$cofact\ of\ 2 = +[4 + 6] = +[10] = 10$

$$cofact\ of\ A = \begin{bmatrix} -7 & 16 & -10 \\ 3 & 6 & -15 \\ -11 & -7 & 10 \end{bmatrix}$$

$$\boldsymbol{Adj\ A} = \begin{bmatrix} \mathbf{-7} & \mathbf{3} & \mathbf{-11} \\ \mathbf{16} & \mathbf{6} & \mathbf{-7} \\ \mathbf{-10} & \mathbf{-15} & \mathbf{10} \end{bmatrix}$$

$$\boldsymbol{A^{-1}} = \frac{\boldsymbol{Adj\ A}}{\boldsymbol{|A|}}$$

$$A^{-1} = \frac{\begin{bmatrix} \mathbf{-7} & \mathbf{3} & \mathbf{-11} \\ \mathbf{16} & \mathbf{6} & \mathbf{-7} \\ \mathbf{-10} & \mathbf{-15} & \mathbf{10} \end{bmatrix}}{-45}$$

$$\boldsymbol{A^{-1}} = \frac{\mathbf{-1}}{\mathbf{45}} \begin{bmatrix} \mathbf{-7} & \mathbf{3} & \mathbf{-11} \\ \mathbf{16} & \mathbf{6} & \mathbf{-7} \\ \mathbf{-10} & \mathbf{-15} & \mathbf{10} \end{bmatrix}$$

5. Find the Inverse of $A = \begin{bmatrix} 2 & 4 & -1 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Solution:

$$let\ A = \begin{bmatrix} 2 & 4 & -1 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

$$|A| = +2[-1+1] - 4[0-2] - 1[0-2]$$

$$= 2[0] - 4[-2] - 1[-2]$$

$$= 0 + 8 + 2$$

$$|\boldsymbol{A}| = \mathbf{10}$$

$$let\ A = \begin{bmatrix} 2 & 4 & -1 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

$$cofact\ of\ 2 = +[-1+1] = +[0] = 0$$

$$cofact\ of\ 4 = -[0-2] = -[-2] = 2$$

$$cofact\ of - 1 = +[0-2] = +[-2] = -2$$

$$cofact\ of\ 0 = -[-4+1] = -[-3] = 3$$

$$cofact\ of\ 1 = +[-2+2] = +[0] = 0$$

$$cofact\ of\ 1 = -[2-8] = -[-6] = 6$$

$$cofact\ of\ 2 = +[4+1] = +[5] = 5$$

$$cofact\ of\ 1 = -[2-0] = -[2] = -2$$

$$cofact\ of - 1 = +[2-0] = +[2] = 2$$

$$cofact\ of\ A = \begin{bmatrix} 0 & 2 & -2 \\ 3 & 0 & 6 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\boldsymbol{Adj\ A} = \begin{bmatrix} \mathbf{0} & \mathbf{3} & \mathbf{5} \\ \mathbf{2} & \mathbf{0} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{6} & \mathbf{2} \end{bmatrix}$$

$$\boldsymbol{A^{-1}} = \frac{\boldsymbol{Adj\ A}}{|\boldsymbol{A}|}$$

$$\boldsymbol{A^{-1}} = \frac{\begin{bmatrix} 0 & 3 & 5 \\ 2 & 0 & -2 \\ -2 & 6 & 2 \end{bmatrix}}{10}$$

$$\boldsymbol{A^{-1}} = \frac{\mathbf{1}}{\mathbf{10}} \begin{bmatrix} \mathbf{0} & \mathbf{3} & \mathbf{5} \\ \mathbf{2} & \mathbf{0} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{6} & \mathbf{2} \end{bmatrix}$$

6. $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 & 1 \\ 4 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ Prove that $(AB)^T \neq B^T A^T$

Solution:

$$let\ AB = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 4 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4+1 & 4+1+2 & 2+2+1 \\ 0+8+2 & 2+2+4 & 1+4+2 \\ 0+4+4 & 2+1+8 & 1+2+4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & 7 & 5 \\ 10 & 8 & 7 \\ 8 & 11 & 7 \end{bmatrix}$$

$$(\boldsymbol{AB})^{\boldsymbol{T}} = \begin{bmatrix} \mathbf{5} & \mathbf{10} & \mathbf{8} \\ \mathbf{7} & \mathbf{8} & \mathbf{11} \\ \mathbf{5} & \mathbf{7} & \mathbf{7} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 4 \end{bmatrix} \qquad\qquad B = \begin{bmatrix} 0 & 2 & 1 \\ 4 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix} \qquad B^T = \begin{bmatrix} 0 & 4 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+2+1 & 8+1+2 & 2+2+1 \\ 0+4+1 & 4+2+2 & 1+4+1 \\ 0+4+4 & 4+2+8 & 1+4+4 \end{bmatrix}$$

$$\boldsymbol{B^T A^T} = \begin{bmatrix} \mathbf{3} & \mathbf{11} & \mathbf{5} \\ \mathbf{5} & \mathbf{8} & \mathbf{6} \\ \mathbf{8} & \mathbf{14} & \mathbf{9} \end{bmatrix}$$

$\therefore \ (\boldsymbol{AB})^{\boldsymbol{T}} \neq \boldsymbol{B^T A^T}$

7. Find the Adjoint of $A = \begin{bmatrix} 2 & 3 & -1 \\ -1 & 0 & 5 \\ 4 & 1 & -2 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & 3 & -1 \\ -1 & 0 & 5 \\ 4 & 1 & -2 \end{bmatrix}$

$cofact\ of\ 2 = +[0-5] = +[-5] = -5$

$cofact\ of\ 3 = -[2-20] = -[-18] = 18$

$cofact\ of -1 = +[-1-0] = +[-1] = -1$

$cofact\ of -1 = -[-6+1] = -[-5] = 5$

$cofact\ of\ 0 = +[-4+4] = +[0] = 0$

$cofact\ of\ 5 = -[2-12] = -[-8] = 8$

$cofact\ of\ 4 = +[15-0] = +[15] = 15$

$cofact\ of\ 1 = -[10-1] = -[9] = -9$

$cofact\ of -2 = +[0+3] = +[3] = 3$

$$cofact\ of\ A = \begin{bmatrix} -5 & 18 & -1 \\ 5 & 0 & 8 \\ 15 & -9 & 3 \end{bmatrix}$$

$$\boldsymbol{Adj\ A} = \begin{bmatrix} \mathbf{-5} & \mathbf{5} & \mathbf{15} \\ \mathbf{18} & \mathbf{0} & \mathbf{-9} \\ \mathbf{-1} & \mathbf{8} & \mathbf{3} \end{bmatrix}$$

8. Find the cofactor matrix of $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 5 & 1 & 0 \end{bmatrix}$

Solution: $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 5 & 1 & 0 \end{bmatrix}$

$cofact\ of\ 1 = +[0-0] = 0$

$cofact\ of\ 2 = -[0-0] = 0$

$cofact\ of\ 3 = +[-1-10] = +[-11] = -11$

$cofact\ of\ -1 = -[0-3] = -[-3] = 3$

$cofact\ of\ 2 = +[0-15] = +[-15] = -15$

$cofact\ of\ 0 = -[1-10] = -[-9] = 9$

$cofact\ of\ 5 = +[0-6] = +[-6] = -6$

$cofact\ of\ 1 = -[0+3] = -[3] = -3$

$cofact\ of\ 0 = +[2+2] = +[4] = 4$

$$\boldsymbol{cofact\ of\ A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-11} \\ \mathbf{3} & \mathbf{-15} & \mathbf{9} \\ \mathbf{-6} & \mathbf{-3} & \mathbf{4} \end{bmatrix}$$

9. Find the Inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

Solution:

$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

$|A| = 1[1-6] - 2[3-4] + 3[9-2]$

$= 1[-5] - 2[-1] + 3[7]$

$= -5 + 2 + 21$

$|\boldsymbol{A}| = \mathbf{18}$

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

$cofact\ of\ 1 = +[1-6] = +[-5] = -5$

$cofact\ of\ 2 = -[3-4] = -[-1] = 1$

$cofact\ of\ 3 = +[9-2] = +[7] = 7$

$cofact\ of\ 3 = -[2-9] = -[-7] = 7$

$cofact\ of\ 1 = +[1-6] = +[-5] = -5$

$cofact\ of\ 2 = -[3-4] = -[-1] = 1$

$cofact\ of\ 2 = +[4-3] = +[1] = 1$

$cofact\ of\ 3 = -[2-9] = -[-7] = 7$

$cofact\ of\ 1 = +[1-6] = +[-5] = -5$

$$cofact\ of\ A = \begin{bmatrix} -5 & 1 & 7 \\ 7 & -5 & 1 \\ 1 & 7 & -5 \end{bmatrix}$$

$$Adj\ A = \begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}$$

$$\boldsymbol{A^{-1} = \frac{Adj\ A}{|A|}}$$

$$= \frac{\begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}}{18}$$

$$\boldsymbol{A^{-1} = \frac{1}{18}\begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}}$$

10. Find the Adjoint of $A = \begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -4 \\ 2 & 3 & 1 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & -4 \\ 2 & 3 & 1 \end{bmatrix}$$

$$cofact\ of - 1 = +[1 + 12] = +[13] = 13$$

$$cofact\ of\ 2 = -[0 + 8] = -8$$

$$cofact\ of - 2 = +[0 - 2] = +[-2] = -2$$

$$cofact\ of\ 0 = -[2 + 6] = -[8] = -8$$

$$cofact\ of\ 1 = +[-1 + 4] = +[3] = 3$$

$$cofact\ of - 4 = -[-3 - 4] = -[-7] = 7$$

$$cofact\ of\ 2 = +[-8 + 2] = +[-6] = -6$$

$$cofact\ of\ 3 = -[4 + 0] = -4$$

$$cofact\ of\ 1 = +[-1 + 0] = -1$$

$$cofact\ of\ A = \begin{bmatrix} 13 & -8 & -2 \\ -8 & 3 & 7 \\ -6 & -4 & -1 \end{bmatrix}$$

$$\boldsymbol{Adj\ of\ A} = \begin{bmatrix} \mathbf{13} & \mathbf{-8} & \mathbf{-6} \\ \mathbf{-8} & \mathbf{3} & \mathbf{-4} \\ \mathbf{-2} & \mathbf{7} & \mathbf{-1} \end{bmatrix}$$

UNIT 3: BINOMIAL THEOREM

I.FILL IN THE BLANKS **01 MARKS**

1. The value of $^{15}C_{13}$ is ________

   Ans: 105

2. The value of $^{n}C_0 + {}^{n}C_n$ is ______

   Ans: 2

3. ______is the Third term in the expansion of $(1\text{-}3/x)^{10}$

   Ans:$405/x^2$

4. The middle term in the expansion of $(2\text{-}3/x)^{10}$ is ____

   Ans: 6^{th} term $^{10}C^5$ $(6^5/x^5)$

5. The constant term in $(x^2+1/x^3)^{10}$ the value of r is _______

   Ans: $r = 4$

6. ______is the formula to find midterm in the expansion

   Ans: (Power/2 +1)

7. ______is the Formula to find r^{th} term in the expansion

   Ans: $T_{r+1} = {}^{n}C_r\, x^{n-r}\, a^r$

8. The fourth term of $(3x - 5y)^6$ is______

   Ans: $-67,500x^3 y^3$

9. The coefficient of x 5 is the expansion of (x + 1 x) ^{9} is_______

   Ans: 36

10. The second term of $(2x - y)^4$ is_____

    Ans: $-32x^3 y$

II.MULTIPLE CHOICE QUESTIONS 01 MARKS

1. The fifth term in the expansion of $(4x/5 -- 5/2x)^8$ is
   a) 1110 b) 1120 c)1100 d) 1010

2. Middle term in $(x/3–3/x)^{10}$ is
   a) 152 b) -152 c)-252 d) 252

3. The term Independent of x is $(x^2 +1/x)^{12}$
   a) 395 b) 295 c) 405 d) 495

4. The coefficient of x^5 in $(x-1/x)^{11}$ is
   a) 165 b) -165 c) 175 d) 35

5. The coefficient of x^{-3} in $(4x/5 – 5/2x)^7$ is
   a) 125 b) 625/3 c) 353/3 d) None

6. $(r +1)^{th}$ term of $(x+a)^n$ is
   a) nCr X^r a^{n-r} b) nCr $X^n a^{n-r}$ c)nCr $X^{n-r} a^r$ d)nNr x^{-n} a^{n+1}

7. Number of terin in $(1 -- x)^n$
   a)n b) n+1 c)n-1 d) n2

8. Number of terms in $(1 - x)^{n-1}$ is
   a) n +2 b) n-1 c) n+1 d)n

9. The first approximation of $(1-x^2)^{1/4}$ according to bionominal theorem is
   a) $1+(1/4)x^2$ b) $1-(1/4)x^2$ c) 1-(1/4)x d) 1+(1/4)x

10. The value of $(1.01)^{1/5}$ according to first approximation is
    a) 1.05 b) 1.005 c) 1.02 d) 1.002

11. Co-efficient of x^5 in the expansion of $(1+x^2)^5$ $(1+x)^4$ is

    a) 40 b) 50 c) 30 d) 60

12. The term independent of x in the expansion of $(x+1/x)^{2n}$ is

    (a) $\dfrac{1.3.5.--(2n-1).2^n}{n!}$ (b) $\dfrac{1.3.5.--(2n-1).2^n}{n!\,n!}$

(c) $\dfrac{1.3.5.--(2n-1)}{n!}$ (d) $\dfrac{1.3.5.--(2n-1)}{n!\,n!}$

13. If 6th term in the expansion of $\left[\dfrac{1}{x^{8/3}} + x^2 \log_{10} x\right]^8$ is 5600, then x is equal to

a) 5 b) 4 c) 8 d) 10

14. If coefficient of $x^2 y^3 z^4$ in $(x + y + z)^n$ is A, then coefficient of x^4y^4z is

(a) 2A (b) $\dfrac{nA}{2}$ c) $\dfrac{A}{2}$ d) None of these

15. The coefficient of x^6 in $\{(1 + x)^6 + (1 + x)^7 + ...+ (1 + x)^{15}\}$ is

a) $^{16}C_9$ b) $^{16}C_5 - {}^6C_5$ c) $^{16}C_6 - 1$ d) None of these

16. If $(1 + x)^{10} = a_0 + a_1x + a_2x^2 + ...+ a_{10}x^{10}$ then $(a_0 - a_2 + a_4 - a_6 + a_8 - a_{10})^2 + (a_1 - a_3 + a_5 - a_7 + a_9)^2$ is equal to

a) 3^{10} b) 2^{10} c) 2^9 d) None of these

17. The term independent of x in the expansion of $\left(1 + 2x + \dfrac{2}{x}\right)^3$ is...........

(a) $3_{c_0} + 3_{c_1}$ b) $3_{c_0} + 2.3_{c_1}$ c) 2.3_{c_1} d) 5

18. The sum $\dfrac{1}{2}\ {}^{10}C_0 - {}^{10}C_1 + 2.\,{}^{10}C_2 - 2^2.\,{}^{10}C_3 + ...+ 2^9.\,{}^{10}C_{10}$ is equal to

a) $\dfrac{1}{2}$ b) 0 c) $\dfrac{1}{2}.3^{10}$ d) None of these

19. If the second, third and fourth terms in the expansion of $(a+b)^n$ are 135, 30 and 10/3 respectively, then

(a) a = 3 b) b = 1/3 c) n = 5 d) All the above

20. The expansion $\left[x+\left(x^3-1\right)^{\frac{1}{2}}\right]^5+\left[x-\left(x^3-1\right)^{\frac{1}{2}}\right]^5$ is a polynomial of degree ……

a) 8 b) 9 c) 7 d) 10

III.ANSWER THE FOLLOWING QUESTION **02 MARKS**

1. What is Binomial theorem?

   Ans: This the theorem which is used to expand two algebraical expressions

2. Write midpoint formula when power is odd.

   Ans: (Power+1 /2) and (Power+1 /2 +1)

3. Write midpoint formula when power is even.

   Ans: (Power/2 +1)

4. What is formula to find r^{th} term?

   Ans: $T_{r+1} = {}^nC_r \, x^{n-r} \, a^r$

5. Find 12! And 10!

   Ans: 479.0016 X 10^{06} and 3.6288X10^{06}

6. Formula for ${}^nC_{r.}$

   Ans: (n! / r! n-r!)

7. Find ${}^{12}C_5$

   Ans:972

8. Find ${}^8C_4$

   Ans:70

9. What is ${}^nC_{n?}$

   Ans: 1

10. What is $(a+b)^2$

    Ans: a^2+b^2+2ab

11. Find c_4^7

Ans: $= \frac{7!}{4!(7-4)!}$

$=\frac{7!}{4!3!}$

$=\frac{7x6x5x4x3x2x1}{4x3x2x1x3x2x1}$

$=35$

12. Find $c_5^8$where n=8 and r=5

Ans: $= \frac{8!}{5!(8-5)!}$

$=\frac{8!}{5!3!}$

$=\frac{8x7x6x5x4x3x2x1}{5x4x3x2x1x3x2x1}$

$=42$

13. Find $c_2^4$where n=4 and r=2

Ans: $= \frac{4!}{2!(4-2)!}$

$=\frac{4!}{2!2!}$

$=\frac{4x3x2x1}{2x1x2x1}$

$=06$

14. what is ^nC_r

Ans: $\boldsymbol{c_r^n}=\frac{\boldsymbol{n}!}{\boldsymbol{r}!(\boldsymbol{n}-\boldsymbol{r})!}$

15. Find $^8C_6 + {}^{12}C_5$

Ans:820

IV.ANSWER THE FOLLOWING QUESTIONS 03 MARKS

1. Using the binomial theorem, expand $(x + 2)^6$.

Solution :

$(a + b)^n = a^n + ({}^nC_1)a^{n-1}b + ({}^nC_2)a^{n-2}b^2 + \dots + ({}^nC_{n-1})ab^{n-1} + b^n$

$=x^6+12x^5+60x^4+160x^3+240x^2+192x+64$

2. Using the binomial theorem, expand $(2x + 3)^4$

Solution :

Using the binomial theorem: $(a +b)^4 = a^4 +4a^3 b +6a^2 b^2 +4ab^3 +b^4$.

Let a = 2x and b = 3.

Then $(2x +3)^4 = (2x)^4 +4(2x)^3 \times 3+6(2x)^2 \times 3^2 +4(2x)\times 3^3 +3^4$

$=$ **$16x^4 +96x^3 +216x^2 +216x +81$**.

3. Find the co-efficient of z4 in the expansion of $(5 + z)^8$

Sol: As expansion is of the form $(a + x)^n$, so r^{th} term

$= a^{n-r+1} x^{r-1} [\{n (n - 1) (n - 2) \dots (n - r + 2)\} \div (r - 1)!]$.

z4 will come in 5^{th} term.

Hence we have to find the 5^{th} term of the expansion.

Here r = 5 and n = 8.

So 5^{th} term of $(5 + z)^8 =5^{8-5+1}. z^{5-1}$.

$= 5^4 . z^4 . 70 = 625x70x4 = 43750 z^4$

Hence coefficient is z^4 is 43750.

4. Find the co-efficient of p5 in the expansion of $(p + 2)^6$.

Sol: As expansion is of the form $(x + a)^n$, so r^{th} term

$= x^{n-r+1} a^{r-1} [\{n(n-1) (n - 2) \dots (n - r + 2)\} \div (r - 1)!]$.

So x^5 will come when r = 2 and n = 6.

Hence we have to find the 2nd term of the expansion.

So r = 2 and n = 6.

So 2^{nd} term of $(p + 2)^6 = p^{6-2+1} . 2^{6-1}$.

$= p^5 . 2^5 . 6 = 192\ p^5$

Hence coefficient of p^5 is 192.

5. Find the third term in the expansion of $(3 + y)^6$

Sol: As expansion is of the form $(a + x)^n$, so r^{th} term

$= a^{n-r+1} x^{r-1} [\{n(n-1) (n - 2) ... (n - r + 2)\} \div (r - 1)!]$.

Here r = 3 and n = 6.

So 3^{rd} term of $(3 + y)^6 = 3^{6-3+1} . y^{3-1} . [(6x5)/2]$

$=3^4 . y^2 . 15 = 1215\ y^2$

6. Find $(a + b)^3$

$(a +b)^3 = (a +b)^2 (a +b)$

$= (a^2 +2ab +b^2)(a +b)$

$= (a^3 +2a^2 b + ab^2)+(a^2 b +2ab_2 +b^3)$

$= a^3 +3a^2 b +3ab^2 +b^3$

7. Expand $(2x + 3)^4$

Using the binomial theorem: $(a +b)^4 = a^4 +4a^3 b +6a^2 b^2 +4ab^3 +b^4$.

Let a = 2x and b = 3.

Then $(2x +3)^4 = (2x)^4 +4(2x)^3 \times 3+6(2x)^2 \times 3^2 +4(2x)\times 3^3 +3^4$

= $\mathbf{16x^4 +96x^3 +216x^2 +216x +81}$.

8. Find the midterm in the expansion $(2x+5)^6$

Solution :

Midpoint when power is even (Power/2 +1)

=(6/2 +1)

=4 (T_4 is the midterm)

$= T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

$T_{3+1} = 20\ (2x)^3\ (125)$

9. Find the midterm in the expansion $(3y+4)^7$

when power is odd

(Power+1 /2) and (Power+1 /2 +1)

So T_4 and T_5 are the midterms

$35,81y^4,64$ & $35,27y^3,256$

10. Find T_5 in the expansion $(3+4x)^8$

Solution :

$T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

$T_{4+1} = {}^8C_4\ 3^4\ (4x)^4$

$1.4515*10^{06}\ x^4$ **(70, 81,256x^4)**

11. Find the 7^{th} term of $\left(2+\frac{x^2}{2}\right)^{11}$

Using the formula $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

$$T_{6+1} = \binom{11}{6} 2^{11-6} \left(\frac{x^2}{2}\right)^6$$

$$= 462\left(2^5\right)\left(\frac{1}{2}\right)^6 \left(x^2\right)^6$$

$$= 231x^{12}$$

12. Expand the following $(1 - x + x\ 2\ )\ ^4$

Solution Put $1 - x = y$. Then

$(1 - x + x^2)^4 = (y + x^2)^4$

$= {}^4C_0\, y^4\, (x^2)^0 + {}^4C_1\, y^3\, (x^2)^1 + {}^4C_2\, y^2\, (x^2)^2 + {}^4C_3\, y\, (x^2)^3 + {}^4C_4\, (x^2)^4$
$= y^4 + 4y^3 x^2 + 6y^2 x^4 + 4y\, x^6 + x^8$
$= (1 - x)^4 + 4x^2 (1 - x)^3 + 6x^4 (1 - x)^2 + 4x^6 (1 - x) + x^8$
$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$

13. Find the co efficient of x^4 in the expansion $(2x^2 - \frac{3}{x})^8$

Solution: Here**,**

X=$2x^2$,a=$-\frac{3}{x}$ and n=8

T_{r+1}=$C_r^n x^{n-r} a^r$

T_{r+1}=$C_r^8 (2x^2)^{8-r} (-\frac{3}{x})^r$

T_{r+1}=$C_r^8 2^{8-r} (x^2)^{8-r}$ $(-\frac{3}{x})^r$

T_{r+1}=$C_r^8 2^{8-r} (x^{16-2r})$ $(-\frac{3}{x})^r$ ………………………………(1)

Take (x^{16-2r})=x^4

Compare the power 16-2r=4

16-4=2r

12=2r

6=r

Put r=6 in equation(1)

T_{6+1}=$C_6^8 2^{8-6} (x^{16-2x6})$ $(-\frac{3}{x})^6$

T_{6+1}=$C_6^8 2^2 (x^4)$ $(-\frac{3}{x})^6$

Co efficient of x^4=$C_6^8 2^2$ $(-\frac{3}{x})^6$

14. Find the third term in in expansion $(x - 2y)^5$
Solution: We know that ,

T_{r+1}=$C_r^n x^{n-r} a^r$

X=x ,a =-2y and n=5

Find r?

$T_{r+1}=T_3$

That means,

r+1=3 and r=3-1=2

$T_{2+1}=C_2^5 x^{5-2}(-2y)^2$

$T_3=10x^3(-2)^2y^2$

$T_3=10x^3 4y^2$

$T_3=40x^3y^2$

15. Find the middle term in the expansion of $(ax+\frac{b}{x})^{12}$

Solution:

Number of terms in the expansion =12+1=13

Therefore middle term is 7th term that is , $T[(\frac{n}{2}+1)$ gives middle term

i.e., $\frac{12}{2}+1=7]$

$T_{r+1}= T_7 \Rightarrow r = 6$

Now , $x \rightarrow ax$, $a \rightarrow b/x$, $n \rightarrow 12$.

Using, $T_{r+1} = {}^nC_r\ X^{n-r}.\ a^r$

We get ,

$T_{6+1} = {}^{12}C_6\ (ax)^{12-6}(\frac{b}{x})^6$

$= {}^{12}C_6\ (ax)^{12-6}\ (\frac{b}{x})^6$

$= {}^{12}C_6\ a^6.x^6.\left(\frac{b}{x}\right) 6$

$T_7 = {}^{12}C_6\ a^6.b^6$ or ${}^{12}C_6\ (ab)^6$

16. Find the constant or independent of x in $(x^5+4)^6$

Solution:

Where x=x^5 , a=4 and n=6 We know that ,

T_{r+1}=$C_r^n x^{n-r} a^r$

T_{r+1}=$C_r^6 (x^5)^{6-r} 4^r$

T_{r+1}=$C_r^6 x^{5x6-5r} 4^r$

T_{r+1}=$C_r^6 x^{30-5r} 4^r$……………………1

For constant or independent take x^0

Compare x^{30-5r}=x^0

30-5r=0

30=5r

6=r

Put the r=6 in equation (1)

T_{6+1}=$C_6^6 x^{30-5x6} 4^6$

T_{6+1}=$C_6^6 x^{30-30} 4^6$

T_{6+1}=$C_6^6 x^0 4^6$

Co efficient of x^0=$C_6^6 4^6$

=1x4x4x4x4

=4096

17. Find the third term in in expansion $(x-2y)^5$

Solution:

We know that

T_{r+1}=$C_r^n x^{n-r} a^r$

X=x ,a =-2y and n=5

Find r?

$T_{r+1}=T_3$

That means,

r+1=3 and r=3-1=2

$T_{2+1}=C_2^5 x^{5-2}(-2y)^2$

$T_3=10x^3(-2)^2y^2$

$T_3=10x^3 4y^2$

$T_3=40x^3y^2$

18. Expand $(a+b)^5$

$$(a+b)^5 = {}^5C_0a^5 + {}^5C_1a^4b + {}^5C_2a^3b^2 + {}^5C_3a^2b^3 + {}^5C_4ab^4 + {}^5C_5b^5$$

$$= 1a^5 + \frac{5}{1}a^4b + \frac{5\cdot4}{1\cdot2}a^3b^2 + \frac{5\cdot4\cdot3}{1\cdot2\cdot3}a^2b^3 + \frac{5\cdot4\cdot3\cdot2}{1\cdot2\cdot3\cdot4}ab^4 + \frac{5\cdot4\cdot3\cdot2\cdot1}{1\cdot2\cdot3\cdot4\cdot5}b^5$$

$$= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

19. Expand $(x-1)^6$.

$$(x-1)^6 = x^6 - \underline{6}x^5\cdot 1 + \underline{15}x^4\cdot 1^2 - \underline{20}x^3\cdot 1^3 + \underline{15}x^2\cdot 1^4 - \underline{6}x\cdot 1^5 + 1^6$$

$$= x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1.$$

20. Expand $(x+2)^3$.

$$(x+2)^3 = x^3 + 3x^2\cdot 2 + 3x\cdot 2^2 + 2^3 = x^3 + 6x^2 + 12x + 8$$

V.ANSWER THE FOLLOWNG QUESTION **05 MARKS**

1. Evaluate $(\sqrt{2}+1)^5+(\sqrt{2}-1)^5$

   Sol:

   We have

   $(x + y)^5 + (x - y)^5 = 2[5C_0\ x^5 + 5C_2\ x^3 y^2 + 5C_4\ xy^4]$

   $= 2(x^5 + 10\ x^3 y^2 + 5xy^4)$

   Now $(\sqrt{2} + 1)^5 + (\sqrt{2} - 1)^5 = 2[(\sqrt{2})^5 + 10(\sqrt{2})^3(1)^2 + 5(\sqrt{2})(1)^4]$

   $=58\sqrt{2}$

2. **Expand $\left(3x + \frac{1}{x}\right)^5$ by using binomial theorem**

Solution:

Binomial theorem,

$(x + a)^n = x^n + c_1^n x^{n-1} a + c_2^n x^{n-2} a^2 + c_3^n x^{n-3} a^3 \ldots\ldots c_{n-1}^n x^1 a^n + a^n$

Where x= 3x and a=-$\frac{1}{x}$ and n=5

$$\left(3x\ \frac{1}{x}\right)^5 = (3x)^5 + c_1^5 (3x)^{(5-1)} \frac{1}{x} + c_2^5 (3x)^{(5-2)} \left(-\frac{1}{x}\right)^2 + c_3^5 (3x)^{(5-3)} \left(-\frac{3}{x}\right)^3 + c_4^5 (3x)^{(5-4)} \left(-\frac{1}{x}\right)^4 + \left(-\frac{1}{x}\right)^5$$

$$\left(3x - \frac{1}{x}\right)^5 = 3^5 x^5 + \frac{5x4x3x2x1}{4x3x2x1} (3x)^{(4)} - \frac{1}{x} + \frac{5x4x3x2x1}{2x1x3x2x1} (3x)^{(3)} \frac{-1^2}{x^2} + \frac{5x4x3x2x1}{3x2x1x2x1} 3x^{(2)} \frac{-1^3}{x^3} + \frac{5x4x3x2x1}{4x3x2x1x2} 3x^{(1)} \frac{-1^4}{x^4} + \frac{-1^5}{x5}$$

$\left(3x-\frac{1}{x}\right)^5$=243$x^5-405x^3$+270x - $\frac{90}{x}$+$\frac{15}{x^3}$ $-\frac{1}{x^5}$

3. Simplify:
$(2+\sqrt{3})^4$+$(2-\sqrt{3})^4$ using binomial theorem

Solution:

$(x+a)^4$=x^4+$C_1^4x^3$a+$C_2^4x^2a^2$+$C_3^4x^1a^3$+a^4

Where x=2 a=$\sqrt{3}$

$(2+\sqrt{3})^4$=2^4+$C_1^4 2^3\sqrt{3}$+$C_2^4 2^2\sqrt{3}^2$+$C_3^4 2^1\sqrt{3}^3$+$\sqrt{3}^4$

$(2+\sqrt{3})^4$=2^4+$C_1^4 8\sqrt{3}$+$C_2^4 4\sqrt{3}^2$+$C_3^4 2\sqrt{3}^3$+$\sqrt{3}^4$

Similarly ,

$(2-\sqrt{3})^4$=2^4-$C_1^4 2^3\sqrt{3}$+$C_2^4 2^2\sqrt{3}^2$-$C_3^4 2^1\sqrt{3}^3$+$\sqrt{3}^4$

$(2+\sqrt{3})^4$=2^4-$C_1^4 8\sqrt{3}$+$C_2^4 4\sqrt{3}^2$-$C_3^4 2\sqrt{3}^3$+$\sqrt{3}^4$

$(2+\sqrt{3})^4$+$(2-\sqrt{3})^4$=??

$(2-\sqrt{3})^4$=2^4-$C_1^4 2^3\sqrt{3}$+$C_2^4 2^2\sqrt{3}^2$-$C_3^4 2^1\sqrt{3}^3$+$\sqrt{3}^4$

+

$(2+\sqrt{3})^4$=2^4-$C_1^4 8\sqrt{3}$+$C_2^4 4\sqrt{3}^2$-$C_3^4 2\sqrt{3}^3$+$\sqrt{3}^4$

$(2+\sqrt{3})^4$+$(2-\sqrt{3})^4 = 2x2^4$+2x$C_2^4 2^2\sqrt{3}^2$+2x$\sqrt{3}^4$

=32+12x12+2x9

=32+144+18

=194

4. Simplify :
$(\sqrt{5}+1)^5$-$(\sqrt{5}-1)^5$ using binomial solution

Solution:

$(x+a)^5$=x^5+$C_1^5 x^4$a+$C_2^5 x^3 a^2$+$C_3^5 x^2 a^3$+$C_4^5 x^1 a^4$+a^4

Where,

x=$\sqrt{5}$ and a= 1 n=5

$(\sqrt{5}+1)^5$=$\sqrt{5}^5$+$\sqrt{5}^4 C_1^5 11$+$C_2^5\sqrt{5}^3 1^2$+$C_3^5\sqrt{5}^2 1^3$+$C_4^5\sqrt{5}^1 1^4$+1^4

= $\sqrt{5}^5$+C_1^5+$C_2^5\sqrt{5}^3$+$C_3^5\sqrt{5}^2$+$C_4^5\sqrt{5}^1$+1

Similarly ,

$(\sqrt{5}-1)^5$=$\sqrt{5}^5$-$\sqrt{5}^4 C_1^5 1$+$C_2^5 \sqrt{5}^3 1^2$-$C_3^5 \sqrt{5}^2 1^3$+$C_4^5 \sqrt{5}^1 1^4$-1^4

= $\sqrt{5}^5$-C_1^5+$C_2^5\sqrt{5}^3$-$C_3^5\sqrt{5}^2$+$C_4^5\sqrt{5}^1$-1

$(\sqrt{5}+1)^5$-$(\sqrt{5}-1)^5$ =($\sqrt{5}^5$+$\sqrt{5}^4 C_1^5$+$C_2^5\sqrt{5}^3$+$C_3^5\sqrt{5}^2$+$C_4^5\sqrt{5}^1$+1)-($\sqrt{5}^5$-

$\sqrt{5}^4 C_1^5$+$C_2^5\sqrt{5}^3$-$C_3^5\sqrt{5}^2$+$C_4^5\sqrt{5}^1$-1)

=2$\sqrt{5}^4 C_1^5$+2$C_3^5\sqrt{5}^2$+2

=2x25x5+2x10x5+2

=50+100+10

=160

5. Find the middle term in the expression $(2x-\frac{x^2}{3})^{11}$
Solution:

No of term =11+1= 12 it is even so middle term is there

First Middle term equal to $\frac{n+1}{2}$=$\frac{11+1}{2}$=6 or T_6

T_{r+1}=T_6=T_{5+1}

r= 5 , n=11and a=$-\frac{x^2}{3}$ and x=2x

T_{r+1}=$C_r^n x^{n-r} a^r$

$T_6 = T_{5+1}$=$C_5^{11}(2x)^{11-5}(-\frac{x^2}{3})^5$

$T_6 = T_{5+1}$=$C_5^{11}(2x)^6(-\frac{x^2}{3})^5$

$T_6 = T_{5+1}$=$C_5^{11} 2^6 x^6 \frac{-x^{2^5}}{3^5}$

=$T_6 = T_{5+1}$=$C_5^{11} 2^6 x^6 \frac{-x^{2^5}}{3^5}$

$T_6 = T_{5+1}$=$C_5^{11} 2^6 x^6 \frac{-x^{10}}{3^5}$

$T_6 = T_{5+1}$=$C_5^{11} 2^6 x^{10} \frac{1}{3^5}$

Second Middle term equal to $\frac{n+3}{2}$=$\frac{11+3}{2}$=7 or T_7

T_{r+1}=T_7=T_{6+1}

Here ,

r= 6 , n=11and a=$-\frac{x^2}{3}$ and x=2x

T_{r+1}=$C_r^n x^{n-r} a^r$

$T_7 = T_{6+1}$=$C_6^{11}(2x)^{11-6}(-\frac{x^2}{3})^6$

$T_7 = T_{6+1}$=$C_6^{11}(2x)^5(-\frac{x^2}{3})^6$

$T_7 = T_{56+1}$=$C_6^{11}2^5x^5\,\frac{-x^{2^6}}{3^6}$

Type equation ere.

$T_6 = T_{6+1}$=$C_6^{11}2^5x^5\,\frac{-x^{2^6}}{3^6}$

$T_7 = T_{6+1}$=$C_6^{11}2^5x^5\,\frac{-x^{12}}{3^6}$

6. Find the co efficient of x^6 in the expansion $(2x^3 - \frac{a}{x^3})^{10}$

Solution:

Here,

X=2x^3,a=$-\frac{a}{x^3}$ and n=10

T_{r+1}=$C_r^n x^{n-r} a^r$

T_{r+1}=$C_r^{10}(2x^3)^{10-r}(-\frac{a}{x^3})^r$

T_{r+1}=$C_r^{10}2^{10-r}(x^3)^{10-r}$ $(-\frac{a}{x^3})^r$

T_{r+1}=$C_r^{10}2^{10-r}(x^{30-3r})$ $\frac{-a^r}{x^{3r}}$

T_{r+1}=$C_r^{10}2^{10-r}(x^{30-3r-3r})$ $-a^r$

T_{r+1}=$C_r^{10}2^{10-r}(x^{30-6r})$ $-a^r$ ………………………..1

Take (x^{30-6r})=x^6

Compare the powers 30-6r=6

30-6=6r

24=6r

4=r

Put r=4 in equation (1)

T_{4+1}=$C_4^{10}2^{10-4}(x^{30-6x4})$ $(-a\ )^4$

T_{4+1}=$C_4^{10}2^6(x^{30-24})(-a)^4$

T_{4+1}=$C_4^{10}2^6(x^4)$ $(-a)^4$

Co efficient of x^6=$C_4^{10}2^6a^4$

$T_7 = T_{6+1}$=$-C_6^{11}2^5x^{17}\frac{1}{3^6}$

7. **Find the middle term in the expansion of (ax+$\frac{b}{x}$)^{12}**

Solution:

Number of terms in the expansion =12+1=13

Therefore middle term is 7th term that is , T[($\frac{n}{2}$+1) gives middle term

i.e., $\frac{12}{2} + 1 = 7$]

T_{r+1}= T_7⇒ r = 6

Now , x⟶ax, a⟶b/x, n⟶12.

Using, $T_{r+1} = {}^nC_r\, X^{n-r}.\, a^r$

We get ,

$T_{6+1} = {}^{12}C_6\, (ax)^{12-6}(\frac{b}{x})^6$

$= {}^{12}C_6\, (ax)^{12-6}\, (\frac{b}{x})^6$

$= {}^{12}C_6\ a^6.x^6.\left(\frac{b}{x}\right)6$

$T_7 = {}^{12}C_6\ a^6.b^6$ or ${}^{12}C_6\ (ab)^6$

8. **Find the middle term in the expansion of** $(x\sqrt{y} + \frac{2}{y\sqrt{x}})^9$

Solution:

No of terms = 9+1=10

Therefore , middle terms are T_5 and T_6

i.e., $\frac{9+1}{2}$ and $\frac{9+3}{2}$ are middle terms

5^{th} and 6^{th} are middle terms.

$x \rightarrow x\sqrt{y}$

$a \rightarrow \frac{2}{y\sqrt{x}}$

$n \rightarrow 9$

To find T_5: put r =4 in $(r+1)^{th}$ term

$T_{r+1} = {}^nC_r\ X^{n-r}\ a^r$

$T_{4+1} = {}^9C_4\ (x\sqrt{y})^{9-}\ (\frac{2}{y\sqrt{x}})^4$

$= {}^9C_4\ (x\sqrt{y})^5\ .(\frac{2}{y\sqrt{x}})^4$

$= {}^9C_4.x^5.(\sqrt{y})^5 * \frac{2^4}{y^4..(\sqrt{x})4}$

$= {}^9C_4.2^4.\frac{x^5\ .y^2\sqrt{y}}{y^4\ .x^2}$

$= {}^9C_4.16.x^3/y^{3/2}$

To find $T_{6,=?}$

put r = 5 in $(r+1)^{th}$ term.

$$T_{5+1} = {}^9C_5\,(x\sqrt{y})^{9-5}\,.\,(\frac{2}{y\sqrt{x}})^4$$

$= {}^9C_5\,(x\sqrt{y})^4.2^5/(\,y\sqrt{x})^5$

$= {}^9C_5.X^4.Y^2.\;\; 2^5/Y^5.X^2\sqrt{X}$

$T_6 = {}^9C_5.\,2^5.\;\; X^{3/2}/Y^3$

$= {}^9C_5.32.\,X^{3/2}/Y^3$

9. Find the coefficient of x^{-3} in $(\sqrt{x}+\frac{5}{x})^{18}$

Solution :

$x\rightarrow \sqrt{x}$, $a\rightarrow 5/x$, $n \rightarrow 18$

$T_{r+1} = {}^nC_r.\,X^{n-r}.a^r$

$T_{r+1} = {}^{18}C_r.(\sqrt{x}\,)^{18-r}\,.(\frac{5}{x})^r$

$= {}^{18}C_r.x^{9-r/2}.5^r.x^{-r}$

$= {}^{18}C_r.\ 5^r.x^{9-r/2-r}$

$T_{r+1} = {}^{18}C_r.\ 5^r.x^{9-3r/2}$

Set $x^{9-3r/2} = x^{-3}$ to get term containing x^{-3}

i.e., $9-\frac{3r}{2} = -3$

$9+3=\frac{3r}{2}$

i.e.,

$12 = \frac{3r}{2}$,

i.e.,

$r = 8$

Put $r = 8$ in (1)

$T_{8+1} = {}^{18}C_8.\ 5^8\, x^{-3}$

Coefficient of x^{-3} is ${}^{18}C_8.\ 5^8$

Expand $\left(2x+\frac{3}{x}\right)^4$ by using binomial theorem

SOLUTION:

Binomial theorem ,

$(x+a)^n$=x^n+$c_1^n x^{n-1}$a+$c_2^n x^{n-2}a^2$+$c_3^n x^{n-3}a^3$……$c_{n-1}^n x^1 a^n$+a^n

Where,

x= 2x and a= $\frac{3}{x}$ and n=4

$\left(2x+\frac{3}{x}\right)^4$=$(2x)^4$+$c_1^4 2x^{(4-1)}\frac{3}{x}$+$c_2^4 2x^{(4-2)}\left(\frac{3}{x}\right)^2$+$c_3^4 2x^{(4-3)}\left(\frac{3}{x}\right)^3$+$\left(\frac{3}{x}\right)^3$

$\left(2x+\frac{3}{x}\right)^4$=$2^4x^4$+$\frac{4x3x2x1}{3x2x1}(2x)^{(3)}\frac{3}{x}$+$\frac{4x3x2x1}{2x1x2x1}(2x)^{(2)}\frac{3}{x^2}^2$+$\frac{4x3x2x1}{3x2x1x1}2x^{(1)}\frac{3}{x^3}^3$+$\frac{3}{x^4}^4$

$\left(2x+\frac{3}{x}\right)^4$=$2^4x^4$+$\frac{4}{1}2^3x^3\frac{3}{x}$+$\frac{4x3}{2x1}2^2x^2\frac{9}{x^2}$+$\frac{4}{1}2^1x^1\frac{27}{x^3}$+$\frac{81}{x^4}$

$\left(2x+\frac{3}{x}\right)^4$=$16x^4$+$96x^2$+216 +$\frac{216}{x^2}$+$\frac{81}{x^4}$

10. Find the value of the greatest term in the expansion of $\sqrt{3}(1+(1/\sqrt{3}))^{20}$.

Ans: **Solution:**

Let T_{r+1} be the greatest term, then $T_r < T_{r+1} > T_{r+2}$

Consider : $T_{r+1} > T_r$

$=> {}^{20}C_r \; (1/\sqrt{3})^r > 20C_{r-1}(1/\sqrt{3})^{r-1}$

$=> ((20)!/(20-r)!r!) \, (1/(\sqrt{3})^r) > ((20)!/(21-r)!(r-1)!) \, (1/(\sqrt{3})^{r-1})$

$=> r < 21/(\sqrt{3}+1)$

$=> r < 7.686$ .... (i)

Similarly, considering $T_{r+1} > T_{r+2}$

$=> r > 6.69$ .... (ii)

From (i) and (ii), we get

$r = 7$

Hence greatest term $= T_8 = 25840/9$

11. Find the coefficient of x^{50} in the expansion of $(1+x)^{1000} + 2x(1+x)^{999} + 3x^2 (1+x)^{998} + ... + 1001x^{1000}$.

Solution:

Let $S = (1 + x)^{1000} + 2x(1+x)^{999} + ... + 1000x^{999} (1+x) + 1001 x^{1000}$

This is an Arithmetic Geometric Series with $r = x/(1+x)$ and $d = 1$.

Now $(x/(1+x)) S = x(1 + x)^{999} + 2x^2 (1 + x)^{998} + ... + 1000x^{1000} + 1000x^{1001}/(1+x)$

Subtracting we get,

$(1 - (x/(x+1))) S = (1+x)^{1000} + x(1+x)^{999} + ... + x^{1000} - 1001x^{1000}/(1+x)$

or $S = (1+x)^{1001} + x(1+x)^{1000} + x^2(1+x)^{999} + ... + x^{1000} (1+x) - 1001x^{1001}$

This is G.P. and sum is

$S = (1+x)^{1002} - x^{1002} - 1002x^{1001}$

So the coeff. of x^{50} is $= {}^{1002}C_{50}$

12. If the coefficient of $(2r + 4)^{th}$ and $(r - 2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal then find the value of r.

Solution:

The general term of $(1 + x)^n$ is $T_{r+1} = C_r x^r$

Hence coefficient of $(2r + 4)^{th}$ term will be

$T_{2r+4} = T_{2r+3+1} = {}^{18}C_{2r+3}$

and coefficient or $(r - 2)^{th}$ term will be

$T_{r-2} = T_{r-3+1} = {}^{18}C_{r-3}$.

$=> {}^{18}C_{2r+3} = {}^{18}C_{r-3}$.

$=> (2r + 3) + (r-3) = 18$ ($\because {}^nC_r = {}^nC_K => r = k$ or $r + k = n$)

$\therefore \quad r = 6$

13. Find the coefficient of the independent term of x in expansion of $(3x - (2/x^2))^{15}$.

Solution:

The general term of $(3x - (2/x^2))^{15}$ is written, as $T_{r+1} = {}^{15}C_r (3x)^{15-r} (-2/x^2)^r$. It is independent of x if,

$15 - r - 2r = 0 => r = 5$

$\therefore$ $T_6 = {}^{15}C_5(3)^{10}(-2)^5 = - {}^{16}C_5\, 3^{10}\, 2^5$.

14. **Expand $(x^2 + 3)^6$**

Solution:

$(x^2 + 3)^6 = {}_6C_0\,(x^2)^6(3)^0 + {}_6C_1\,(x^2)^5(3)^1 + {}_6C_2\,(x^2)^4(3)^2 + {}_6C_3\,(x^2)^3(3)^3$

$+ {}_6C_4\,(x^2)^2(3)^4 + {}_6C_5\,(x^2)^1(3)^5 + {}_6C_6\,(x^2)^0(3)^6$

Then simplifying gives

$(1)(x^{12})(1) + (6)(x^{10})(3) + (15)(x^8)(9) + (20)(x^6)(27)$

$+ (15)(x^4)(81) + (6)(x^2)(243) + (1)(1)(729)$

$= \mathbf{x^{12} + 18x^{10} + 135x^8 + 540x^6 + 1215x^4 + 1458x^2 + 729}$

15. **Expand $(2x - 5y)^7$**

Solution:

$(2x - 5y)^7 = {}_7C_0\,(2x)^7(-5y)^0 + {}_7C_1\,(2x)^6(-5y)^1 + {}_7C_2\,(2x)^5(-5y)^2$

$+ {}_7C_3\,(2x)^4(-5y)^3 + {}_7C_4\,(2x)^3(-5y)^4 + {}_7C_5\,(2x)^2(-5y)^5$

$+ {}_7C_6\,(2x)^1(-5y)^6 + {}_7C_7\,(2x)^0(-5y)^7$

$(1)(128x^7)(1) + (7)(64x^6)(-5y) + (21)(32x^5)(25y^2) + (35)(16x^4)(-125y^3)$

$+ (35)(8x^3)(625y^4) + (21)(4x^2)(-3125y^5) + (7)(2x)(15625y^6)$

$+ (1)(1)(-78125y^7)$

$= \mathbf{128x^7 - 2240x^6y + 16800x^5y^2 - 70000x^4y^3 + 175000x^3y^4 - 262500x^2y^5}$

$\mathbf{+ 218750xy^6 - 78125y^7}$

16. **What is the fourth term in the expansion of $(3x - 2)^{10}$?**

Solution:

$(3x - 2)^{10} = {}_{10}C_0\,(3x)^{10-0}(-2)^0 + {}_{10}C_1\,(3x)^{10-1}(-2)^1 + {}_{10}C_2\,(3x)^{10-2}(-2)^2$

$+ {}_{10}C_3\,(3x)^{10-3}(-2)^3 + {}_{10}C_4\,(3x)^{10-4}(-2)^4 + {}_{10}C_5\,(3x)^{10-5}(-2)^5$

$+ {}_{10}C_6\,(3x)^{10-6}(-2)^6 + {}_{10}C_7\,(3x)^{10-7}(-2)^7 + {}_{10}C_8\,(3x)^{10-8}(-2)^8$

$+ {}_{10}C_9\ (3x)^{10-9}(-2)^9 + {}_{10}C_{10}\ (3x)^{10-10}(-2)^{10}$

17. **Find the tenth term in the expansion of $(x + 3)^{12}$.**
Solution:

To find the tenth term Binomial Theorem, using the number 10 – 1 = 9 as my counter:

$${}_{12}C_9\ (x)^{12-9}(3)^9 = (220)x^3(19683) = \mathbf{4330260x^3}$$

19. Write out the expansion of $(x + y)^7$.
Solution: $(x + y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$.

20. Write out the expansion of $(2x + 3y)^4$.
Solution:

$$(2x + 3y)^4 = (2x)^4 + 4(2x)^3(3y) + 6(2x)^2(3y)^2 + 4(2x)(3y)^3 + (3y)^4$$

$$= 16x^4 + 4(8x^3)(3y) + 6(4x^2)(9y^2) + 4(2x)(27y^3) + 81y^4$$

$$= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4.$$

UNIT 4: LOGARITHM

I.FILL IN THE BLANKS **01 MARKS**

1. The logarithm of one to the any base is equal to _____(zero)
2. the value of $\log_3 3$ is equal to _____(one)
3. The value of log 1is = _____(zero)
4. The value of $\log_2 2^2$ = _______(2)
5. The logarithm of zero to the any base is equal to = _______(-∞)
6. The logarithm of any number to the same base is equal to _____(1)
7. Log m + Log n = ______(log mn)
8. Log m + Log n + Log p = __________(log mnp)
9. The logarithm of one to the any base is equal to the _______(zero)
10. The value of $\log_a a^5$ = _______(5)

11. $\log_{10} 5$ - $\log_{10} 3 =$ ________($\log_{10} \frac{5}{3}$)

12. The value of $5^{\log_5 3} =$ ______(3)

13. The value of $\log_2 8 + \log_2 6 =$ ______($\log_2 14$)

14. The value of $\log_3 7 + \log_3 12 + \log_3 8 =$ ________(3 or $\log_3 27$)

15. The value of $\log_{10} \sqrt{10} =$ ______($\frac{1}{2}$)

II.MULTIPLE CHOICE QUESTIONS **01 MARKS**

1. The logarithm of any number to the same base is equal to _____

   A) 1

   B) 0

   C) SAME NUMBER

   D) NONE OF THE ABOVE

2. Log m + Log n = ______

   A) Log mn

   B) Log (m+n)

   C) Log m/n

   D) Log n/m

3. $\log_b a$ is can be written as

   A) $\log_a b$

   B) log a/log b

   C) log b/log a

   D) none of the above

4. Log m - Log n = ______

   A) Log mn

   B) Log (m+n)

   C) Log m/n

   D) Log n/m

5. Log m + Log n + Log p = ______

   A) Log mnp

   B) Log (m+n+p)

   C) Log (mn+p)

   D) Log (m+np)

6. $\log_a m^K$ can be written as

   A) $\log_a m$

   B) $\frac{1}{k}\log_a m$

   C) $\boldsymbol{k \log_a m}$

   D) none of the above

7. $\log_{a^k} m$ is can be written as

   A) $\log_a m$

   B) $\boldsymbol{\frac{1}{k}\log_a m}$

   C) $k \log_a m$

   D) NONE OF THE ABOVE

8. The value of $\log_{10} 80 + \log_{10} 20 =$ _____

   A) $\log_{10} 100$

   B) 2

   C) $2\log_{10} 10$

   D) ALL OF THE ABOVE

9. The value of $\log_{10} \sqrt[3]{10} =$ _______

   A) 3

   B) $\boldsymbol{\frac{1}{3}}$

   C) $\frac{1}{3}\log_{10} 3$

   D) $\log_{10} 10$

10. The value of $7^{\log_7 2} =$ _______

   A) 2

   B) 7

C) $\log_7 2$

D) $\log_2 7$

11. The logarithm of zero to the any base is equal to = ___ _____

A) $+\infty$

B) $-\infty$

C) $\pm\infty$

D) none of the above

12. The value of $\log_4 20 + \log_4 30 + \log_4 14 =$ _________

A) $\log_4 64$

B) $3\log_4 4$

C) 3

D) all of the above

13. The value of $\log_{3^2} 3^5 =$ _______

A) $\log_3 3$

B) $5\log_3 3$

C) $\frac{5}{2}\log_3 3$

D) all of the above

14. The value of $\log_6 36 - \log_6 9 =$ _________

A) $\log_6 27$

B) $\log_6 9$

C) $\log_6 4$

D) all of the above

15. The value of $\log_{100} 1$

A) 1

B) 100

C) $\log_1 100$

D) 0

III.ANSWER THE FOLLOWING QUESTION 02 MARKS

1. DEFINE LOGARITHM

   ANS : IF '$a^X = m$' IS SAID TO BE LOGARITHM OF 'm' TO THE BASE 'a' IS EQUAL TO 'X'

   $\log_a m = X$

   $a^X = m$

FIND THE VALUE OF X

2. $\log_4 64 = X$ $\log_a m = X$

   SOL : $4^X = 64$ $a^X = m$

   $4^X = 4^3$

   BOTH SIDE BASES ARE EQUAL, HENCE CANCEL THE BASES

   $\therefore$ x=3

3. $\log_x 625 = 2$ $\log_a m = X$

   SOL : $x^2 = 625$ $a^X = m$

   $x^2 = 25^2$

   BOTH SIDE POWERS ARE EQUAL, HENCE CANCEL THE POWERS

   $\therefore$ x=25

4. $\log_7 x = 2$ $\log_a m = X$

   SOL : 7^2 =x $a^X = m$

   $\therefore$x=49

5. MENTION ANY TWO RULES OF THE LOGARITHM

   ANS : PRODUCT RULE = $\log_a m + \log_a n = \log_a m\,n$

   QUOTIENT RULE = $\log_a m$ - $\log_a n = \log_a m\,/n$

6. MENTION POWER RULE AND CHNGE OF BASE RULE OF THE LOGARITHM

   ANS : POWER RULE : $k \log_a m = \log_a m^k$ AND $\log_{a^k} n = \frac{1}{k}\log_a n$

   CHNGE OF BASE RULE :$\log_a b = \frac{\log b}{\log a}$

IV.ANSWER THE FOLLOWING QUESTIONS 03 MARKS

1. $\log_{\sqrt{2}} 16$ =x $\log_a m = X$

   SOL :$\sqrt{2}^{x} = 16$ $a^X = m$

   $= 2^{(1/2)X} = 2^4$

BOTH SIDE BASES ARE EQUAL, HENCE CANCEL THE BASES

$= \frac{x}{2} = 4$

$\therefore x = 8$

2. $\log_{0.1} 100 = x$ $\quad\quad$ $\log_a m = X$

SOL : $0.1^x = 100$ $\quad\quad$ $a^X = m$

$= \frac{1}{10}^{(x)} = 10^2$

$= 10^{-1(x)} = 10^2$

BOTH SIDE BASES ARE EQUAL, HENCE CANCEL THE BASES

$-x = 2$

$\therefore x = -2$

3. $\log_{10}(x - 9) = 2$ $\quad\quad$ $\log_a m = X$

SOL : $10^2 = x - 9$ $\quad\quad$ $a^X = m$

$100 + 9 = x$

$\therefore x = 109$

4. PROVE THAT $\log_3 5 X \log_7 3 X \log_5 7 = 1$

SOL :APPLY THE 'LAW OF CHANGE OF BASE' $\log_a b = \frac{\log b}{\log a}$

LHS $= \log_3 5 X \log_7 3 X \log_5 7$

$= \frac{\log 5}{\log 3} X \quad \frac{\log 3}{\log 7} X \frac{\log 7}{\log 5}$

CANCELS THE ALL LIKE TERMS

$= 1$

5. SHOW THAT $\log \frac{x}{y} + \log \frac{y}{z} + \log \frac{z}{x} = 0$

SOL :APPLY THE 'QUOTIENT RULE' TO THE ABOVE PROBLEM

$\log_a m - \log_a n = \log_a m/n$

LHS $= \log \frac{x}{y} + \log \frac{y}{z} + \log \frac{z}{x}$

$= \log x - \log y + \log y - \log z + \log z - \log x$

$= 0$

6. IF $\log_m x + \log_m y + \log_m z = 0$ SHOW THAT $xyz = 1$

SOL :APPLY THE EXTENSION OF PRODUCT RULE $\log_a m + \log_a n + \log_a p = \log_a m\, n\, p$

$= \log_m x + \log_m y + \log_m z = 0$

$\log_m x\,yz = 0$

$xyz = m^0 \therefore xyz = 1$

V.ANSWER THE FOLLOWNG QUESTION **05 MARKS**

1. PROVE THAT $2\log\frac{16}{15} + \log\frac{25}{24} - \log\frac{32}{27} = 0$

   SOL : APPLY THE POWER RULE AND PRODUCT RULE

   i.e $\log_a m^k = k\log_a m \;\; and \;\; \log_a m + \log_a n = \log_a mn$

   LHS $= 2\log\frac{16}{15} + \log\frac{25}{24} - \log\frac{32}{27}$

   $= \log\left(\frac{16}{15}\right)^2 + \log\frac{25}{24} - \log\frac{32}{27}$

   $= \log\frac{256}{225} + \log\frac{25}{24} - \log\frac{32}{27}$

   $= \log\left(\frac{256}{225} \times \frac{25}{24}\right) - \log\frac{32}{27}$

   $= \log\frac{32}{27} - \log\frac{32}{27}$

   $= 0$

2. SHOW THAT $\log_2 2 - \log_4 2 + \log_8 2 - \log_{16} 2 = \frac{7}{12}$

   SOL : APPLY THE POWER RULE TO THE BASES i.e $\log_{a^k} n = \frac{1}{k}\log_a n$

   LHS $= \log_2 2 - \log_4 2 + \log_8 2 - \log_{16} 2$

   $= 1 - \log_{2^2} 2 + \log_{2^3} 2 - \log_{2^4} 2$

   $= 1 - \frac{1}{2}\log_2 2 + 1/3\log_2 2 - \frac{1}{4}\log_2 2$

   $= 1 - \frac{1}{2} + 1/3 - \frac{1}{4}$

   $= \frac{24-12+8-6}{24}$

   $= \frac{14}{24} = \frac{7}{12}$

3. PROVE THAT $\log_3 3 + \log_9 3 + \log_{27} 3 - \log_{81} 3 = 7/12$

   SOL : APPLY THE POWER RULE i.e $\log_{a^k} n = \frac{1}{k}\log_a n$ AND PROPERTY OF LOG i.e $\log_c c = 1$

   LHS $= \log_3 3 + \log_9 3 + \log_{27} 3 - \log_{81} 3$

   $= 1 - \log_{3^2} 3 + \log_{3^3} 3 - \log_{3^4} 3$

   $= 1 - \frac{1}{2}\log_3 3 + 1/3\log_3 3 - \frac{1}{4}\log_3 3$

   $= 1 - \frac{1}{2} + 1/3 - \frac{1}{4}$

   $= \frac{24-12+8-6}{24}$

   $= \frac{14}{24} = \frac{7}{12}$

4. PROVE THAT $\log\frac{8}{49} + \log\frac{28}{3} + \log\frac{21}{16} = \log 2$

SOL :APPLY THE EXTENSION OF PRODUCT RULE $\log_a m + \log_a n + \log_a p = \log_a m\,n\,p$

LHS $= \log\frac{8}{49} + \log\frac{28}{3} + \log\frac{21}{16}$

$= \log\left(\frac{8}{49} \times \frac{28}{3} \times \frac{21}{16}\right)$

$= \log 2$

5. SHOW THAT $\log\frac{75}{16}$ - 2 $\log\frac{5}{6} + \log\frac{8}{27} = \log 2$

SOL :APPLY THE POWER RULE AND QUOTIENT RULES OF THE LOG TO ABOVE PROBLEM

i.e$\log_a m^k = k\log_a m$ $\;and\;$ $\log_a m$ - $\log_a n = \log_a m/n$

LHS=$\log\frac{75}{16}$ - 2 $\log\frac{5}{6} + \log\frac{8}{27}$

$= \log\frac{75}{16} - \log\left(\frac{5}{6}\right)^2 + \log\frac{8}{27}$

$= \log\frac{75}{16} - \log\frac{25}{36} + \log\frac{8}{27}$

$= \log\frac{\frac{75}{16}}{\frac{25}{36}} + \log\frac{8}{27}$

$= \log\frac{75}{16} \times \frac{25}{36} + \log\frac{8}{27}$

$= \log\frac{27}{4} + \log\frac{8}{27}$

$= \log\left(\frac{27}{4} \times \frac{8}{27}\right)$

$= \log 2$

6. SOLVE FOR X , $\log_9 x + \log_3 x = 6$

SOL : APPLY THE POWER RULE TO THE BASES i.e$\log_{a^k} n = \frac{1}{k}\log_a n$ $\qquad$ $\log_a m = X$

$= \log_9 x + \log_3 x = 6$ $\qquad$ $a^X = m$

$\log_{3^2} x + \log_3 x = 6$

$\frac{1}{2}\log_3 x + \log_3 x = 6$

$\log_3 x \left(1 + \frac{1}{2}\right) = 6$

$\log_3 x \left(\frac{3}{2}\right) = 6$

$\log_3 x = 6\left(\frac{2}{3}\right)$

$\log_3 x = 4$

$x = 3^4$

$x = 81$

VI.ANSWER THE FOLLOWING QUESTIONS 08 MARKS

1. IF $\frac{1}{\log_3 x}+\frac{1}{\log_9 x}+\frac{1}{\log_{27} x}=6$ SHOW THAT $x = 3$

SOL : APPLY THE POWER RULE AND CHANGE OF BASE RULE OF THE LOGARITHM

i.e$k \log_a m = \log_a m^k$ and $\log_a b = \frac{\log b}{\log a}$

$= \frac{1}{\log_3 x}+\frac{1}{\log_9 x}+\frac{1}{\log_{27} x}=6$

$= \frac{1}{\frac{\log x}{\log 3}}+\frac{1}{\frac{\log x}{\log 9}}+\frac{1}{\frac{\log x}{\log 27}}=6$

$= \frac{\log 3}{\log x}+\frac{\log 9}{\log x}+\frac{\log 27}{\log x}=6$

$= \log_x 3+\log_x 9+\log_x 27=6$

$= \log_x 3+\log_x 3^2+\log_x 3^3=6$

$= \log_x 3+2\log_x 3+3\log_x 3=6$

$= (1+2+3)\log_x 3=6$

$= \log_x 3=\frac{6}{6}$

$= \log_x 3=1$

$x^1=3$

$x= 3$

2. IF $X=\log_a bc$, $Y=\log_b ca$, $Z=\log_c ba$ FIND $\frac{1}{1+X}+\frac{1}{1+Y}+\frac{1}{1+Z}$

SOL : APPLY THE PROPERTY , EXTENSION OF PRODUCT RULE AND CHANGE OF BASE RULE OF THE LOGARITHM

i.e$\log_a a=\log_b b=\log_c c=1$ and $\log_a m + \log_a n+\log_a p=\log_a mnp$,

$\log_a b=\frac{\log b}{\log a}$

$= \frac{1}{1+X}+\frac{1}{1+Y}+\frac{1}{1+Z}$

$= \frac{1}{\log_a a+\log_a bc}+\frac{1}{\log_b b+\log_b ca}+\frac{1}{\log_c c+\log_c ba}$

$= \frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}$

$= \frac{1}{\frac{\log abc}{\log a}}+\frac{1}{\frac{\log abc}{\log b}}+\frac{1}{\frac{\log abc}{\log c}}$

$= \frac{\log a}{\log abc}+\frac{\log b}{\log abc}+\frac{\log c}{\log abc}$

$= \log_{abc} a+\log_{abc} b+\log_{abc} c$

$= \log_{abc} abc$

$= 1$

3. Derive Product rule and Quotient rule of the Logarithm

ANS : Product rule : $\log_a m + \log_a n=\log_a mn$

Let $\log_a mn = x \rightarrow a^x = mn$ ------>(1)

$\log_a m = y \rightarrow a^y = m$------>(2)

$\log_a n = z \rightarrow a^z = n$ ------>(3)

From equation (1) becomes $a^x = mn$

$a^x = a^y a^z$

$a^x = a^{y+z}$

Both sides Bases are equal hence cancel each other

$x = y + z$

$\log_a mn = \log_a m + \log_a n$

Quotient rule : $\log_a m$ -$\log_a n = \log_a m/n$

Let $\log_a m/n = x \rightarrow a^x = m/n$ ------>(1)

$\log_a m = y \rightarrow a^y = m$------>(2)

$\log_a n = z \rightarrow a^z = n$ ------>(3)

From equation (1) becomes $a^x = m/n$

$a^x = a^y / a^z$

$a^x = a^{y-z}$

Both sides Bases are equal hence cancel each other

$x = y - z$

$\log_a m/n = \log_a m$ - $\log_a n$

UNIT 5: UNITS & MEASUREMENT OF ANGLES

I.FILL IN THE BLANKS 01 MARKS

1. To convert radian into degree, multiply by _______

Ans: $\frac{180}{\Pi}$

2. $\frac{5\,\Pi}{6}$ radians are equal to _______

Ans: 150^o

3. 180^o =_________

Ans: Π^c or Π rad

4. 1^o = _______

Ans: $\left(\frac{\Pi^c}{180}\right)$

5. $\frac{7\,\Pi}{}$ radians are equal to _______

3

Ans: 420^{o}

6. $150^{o} =$ ________

Ans: $\frac{5\Pi^{c}}{6}$

7. $\frac{2\Pi^{c}}{9} =$ ______

Ans: 40^{o}

8. $60^{o} =$ ________

Ans: $\left(\frac{\Pi^{c}}{3}\right)$

9. $\frac{3\Pi}{5}$ radians are equal to ______

Ans: 108^{o}

10. $\frac{2\Pi^{c}}{3} =$ ______

Ans: 120^{o}

11. $30^{o} =$ ________

Ans: $\left(\frac{\Pi^{c}}{6}\right)$

12. In sexagesimal system a right angle is divided in to 90 equal parts called _______

Ans: Degrees

13. In circular system angle is measured in a unit called ________

Ans: Radian

14. $\frac{\Pi}{15}$ radians are equal to ______

Ans: 12^{o}

15. $125^{o} =$ ______

Ans: $\frac{25\Pi^c}{36}$

II.MULTIPLE CHOICE QUESTIONS **01 MARKS**

1. Θ degree = _______ radians
   a) π/180 . Θ
   b) 180/ π . Θ
   c) Θ
   d) Θ/2

   Ans: a) π/180 . Θ

2. The length of an arc of a circle is given by _____
   a) $l = r^2 \Theta$
   b) $l = r \Theta$
   c) $l^2 = r \Theta$
   d) $l = r/ \Theta$

   Ans: b) $l = r \Theta$

3. Area of sector of circle of radius 'r' and angle 'Θ' then A is given by
   a) $r^2 \Theta$
   b) $\frac{1}{2} r^2 \Theta^2$
   c) $\frac{1}{2} r^2 \Theta$
   d) $2 r^2 \Theta$

   Ans: c) $\frac{1}{2} r^2 \Theta$

4. Π/4 radians are equal to _______
   a) 30^o
   b) 45^o
   c) 60^o
   d) 90^o

   Ans: b) 45^o

5. 90^o = ________
    a) $(\pi/2)^c$
    b) $(\pi/3)^c$
    c) $2\pi^c$
    d) π^c

    Ans: a) $(\pi/2)^c$

6. 2Π radians are equal to _______
    a) 360^o
    b) 270^o
    c) 180^o
    d) 90^o

    Ans: a) 360^o

7. In centesimal system a right angle is divided in to 100 equal parts called __________
    a) Minutes
    b) Seconds
    c) Grades
    d) Degrees

    Ans: c) Grades

8. To convert degrees in to radians multiple by _________
    a) $180/\pi$
    b) $\pi/180$
    c) π^c
    d) **2**π^c

    Ans: b) $\pi/180$

9. $10\ \pi^c$ = _________
    a) 180^o
    b) 1800^o

c) 1080^o

d) 1008^o

Ans: b) 1800^o

10. 105^o = ________

a) $\frac{5\pi^c}{6}$

b) $\frac{7\pi^c}{12}$

c) $\frac{7\pi^c}{5}$

d) $\frac{3\pi^c}{2}$

Ans: b) $\frac{7\pi^c}{12}$

11. $\frac{5\pi}{9}$ radians are equal to ______

a) 100^o

b) 120^o

c) 180^o

d) 50^o

Ans: a) 100^o

12. $\frac{7\pi}{6}$ radians are equal to ______

a) 225^o

b) 120^o

c) 210^o

d) 200^o

Ans: c) 210^o

13. 225^o = ________

a) $\frac{5\pi^c}{4}$

b) $\frac{5\pi^c}{6}$

c) $7\pi^c$

6

d) $\frac{5\pi^c}{2}$

Ans: a) $\frac{5\pi^c}{4}$

14. 300^o = ________

a) $\frac{5\pi^c}{6}$

b) $\frac{5\pi^c}{4}$

c) $\frac{5\pi^c}{8}$

d) $\frac{5\pi^c}{3}$

Ans: d) $\frac{5\pi^c}{3}$

15. $\frac{\pi}{12}$ radians are equal to ______

a) 12^o

b) 15^o

c) 20^o

d) 10^o

Ans: b) 15^o

III.ANSWER THE FOLLOWING QUESTION **02 MARKS**

1. Express 1 radian into degree?

Ans: 1 radian $=\frac{180^0}{\Pi}$

$=57^o, 11^1, 42^{11}$

2. Define Radian?

Ans: Radian is a plane angle subtended at the center by an arc of a circle such that radius of circle is equal to length of the arc.

i.e if arc AB = radius = OA then $\underline{|AOB}$ **=**1radian or 1^c

3. Express $\frac{7\pi}{5}$ radians into degree?

Ans: $\frac{7\pi}{5}$ radian = $\frac{7\pi}{5} x \frac{180^0}{\pi}$

$= 252^0$

4. Convert $\frac{3\pi}{2}$radians into degree?

Ans: $\frac{3\pi}{2}$ radian = $\frac{3\pi}{2} x \frac{180^0}{\pi}$

$= 270^0$

5. Express 120^0 into radians?

Ans: $120^0 = 120 \text{ x } \frac{\pi}{180}$

$= \frac{2\pi}{3}$radian

$= 2.094$ radian

6. Express 75^o into radians?

Ans: $75^o = 75 \text{ x } \frac{\pi}{180}$

$= 0.4167\ \pi^c$

$= 1.309$ radian

7. Express 2.35 radian in degrees?

Ans: 2.35 radian = $2.35 \text{ x } \frac{180^0}{\pi}$

$= 134.62^o$

8. Express 135^o into radians?

Ans: $135^o = 135 \text{ x } \frac{\pi}{180}$

$=3\ \pi/4$ radian

9. Write the 3 systems to measure a given angle in units and measurements of angles.

Ans: 1. Sexagesimal system

2. Centesimal system

3. Circular system

10. Express 450^o into radian?

Ans: $450^o = 450 \text{ x} \frac{\pi}{180}$

$= \underline{5\ \pi}$ radian

2

11. Express 780^o into radian?

Ans: $780^o = 780 \times \frac{\pi}{180}$

$= 13\frac{\pi}{3}$ radian

12. Convert $\frac{25\pi}{4}$ radians into degree?

Ans: $\frac{25\pi}{4}$ radian $= \frac{25\pi}{4} \times \frac{180}{\pi}$

$= 1125^0$

13. Convert $\frac{15\pi}{12}$ radians into degree?

Ans: $\frac{15\pi}{12}$ radian $= \frac{15\pi}{12} \times \frac{180^0}{\pi}$

$= 225^0$

IV.ANSWER THE FOLLOWING QUESTIONS **03 MARKS**

1. Express 2 radians into degree, minutes, and seconds.

Soln:

$$2 \text{ radian} = 2 \times \frac{180^0}{\Pi} = \frac{360^0}{\pi}$$

$= 114.58^o$

$= 114^o, 34.8^1$

$= 114^o, 34^1, 48^{11}$

2. Express $5^o, 20', 50''$ in radian measure

Soln: $5^o, 20', 50'' = 5^o + \frac{20^o}{60} + \frac{50^0}{60 \times 60}$

$= 5 + \frac{1}{3} + \frac{1}{72}$

$= \frac{360 + 24 + 1}{72}$

$= (385/72)^o = 5.347^o$

$5^o, 20', 50'' = 5.347 \times \frac{\pi}{180} = 0.0297\pi^c$

$= 0.093$ radian

3. An arc length of 6cm subtends an angle of 110^0 at center of the circle. Find the radius of the circle

Soln:

Given, l=6cm

$\Theta = 110^0$

r=?

$$\Theta = 110^0 \text{x} \frac{\pi}{180}$$

$= 1.920$ rad

We know that, $l = r\Theta$

$$\therefore r = \frac{l}{\Theta} = \frac{6}{110^0}$$

$$= \frac{6}{1.920}$$

$\therefore r = 3.125$cm

4. Express $123^0, 24', 24''$ into radian measures

Soln: $$123^o, 24', 24'' = 123^o + \frac{24^0}{60} + \frac{24^0}{60 \text{ x } 60}$$

$$= 123^0 + (0.4)^0 + (0.0067)^0$$

$$= 123.4067^0$$

$$\therefore 123^o, 24', 24'' = 123.4067 \text{ x } \frac{\pi}{180}$$

$$= 0.6855 \text{ x } \pi^c$$

$$= 2.154 \text{ radian}$$

5. Find the length of an arc of circle of radius 3 cm subtending an angle of 60^0.

Soln: Given $r = 3$ cm

$\Theta = 60^0$

$l = ?$

$$\Theta = 60 \text{ x } \frac{\pi}{180^0} = 1.047 \text{ rad}$$

$\therefore\ l = r\ \Theta$

$l = 3 \times 1.0473$

$l = 3.1419 = 3.142$ cm

6. Express $15^0, 16', 40''$ into radian measures.

Soln: $15^o,16',40'' = 15^o + \frac{16^0}{60} + \frac{40^0}{60 \times 60}$

$= 15^0 + (0.27)^0 + (0.011)^0$

$= 15.281^0$

$\therefore\ 15^o, 16', 40'' = 15.281 \times \frac{\pi}{180}$

$= 0.0849 \times \pi^c$

$= 0.267$ radian

7. Express $32^0, 48', 15''$ into radian measures.

Soln: $32^o,48',15'' = 32^o + \frac{48^0}{60} + \frac{15^0}{60 \times 60}$

$= 32^0 + (0.8)^0 + (0.0042)^0$

$= 32.8042^0$

$\therefore 32^o, 48', 15'' = 32.8042 \times \frac{\pi}{180}$

$= 0.1822 \times \pi^c$

$= 0.572$ radian

8. Express $50^0, 37', 30''$ into radian measures.

Soln: $50^o,37',30'' = 50^o + \frac{37^0}{60} + \frac{30^0}{60 \times 60}$

$= 50^0 + (0.62)^0 + (0.0083)^0$

$= 50.6283^0$

$\therefore 50^o, 37', 30'' = 50.6283 \times \frac{\pi}{180}$

$= 0.281 \times \pi^c$

$= 0.883$ radian

9. Express 1.235 radians into degree, minutes and seconds.

Soln: 1.235 radian $= 1.235 \times \frac{180}{\pi}$

$= 70.75^o$

$= 70^o, 45'$

10. Express 0.75 radians into degree, minutes and seconds.

Soln: 0.75 radian $= 0.75 \times \frac{180}{\pi}$

$= 42.97^o$

$= 42^o, 58', 12''$

11. Convert 0.4 radian into degrees, minutes and second.

Soln: 0.4 radian $= 0.4 \times \frac{180}{\Pi}$

$= 22.92^o$

$= 22^o, 55', 12''$

12. Convert 0.056 radian into degrees, minutes and second.

Soln: 0.056 radian $= 0.056 \times \frac{180}{\Pi}$

$= 3.208^o$

$= 3^o, 12', 28''$

13. Express $175^o, 45'$ into radians.

Soln: $175^o, 45' = 175^o + \frac{45}{60}$

$= 175^o + (0.75)$

$= 175.75^o$

$\therefore 175^o, 45' = 175.75 \times \frac{\pi}{180}$

$= 0.976\pi^c$

$= 3.066$ radian

V.ANSWER THE FOLLOWNG QUESTION 05 MARKS

1. In a circle central angle of a sector is 175^0 and the sector has area of $7.5\ m^2$. Find the radius of the circle.

Soln: Given, $\Theta = 175^0$

$A = 7.5\ m^2$

$r = ?$

$\Theta = 175^0 = 175 \times \frac{\pi}{180} = 3.055$ rad

$\therefore$ Area of sector $A = \frac{1}{2} r^2 \Theta$

$r^2 = \frac{2A}{\Theta}$

$= \frac{2 \times 7.5}{3.055}$

$r^2 = 4.91$

$r = \sqrt{4.91}$

$r = 2.21$ m

2. Find the area of the sector of a circle of radius 20 cm with central angle 36^0.

Soln: Given ,$r = 20$ cm

$\Theta = 36^0$

$A = ?$

$\Theta = 36^0 = 36 \times \frac{\pi}{180} = 0.63$ rad

$A = \frac{1}{2} r^2 \Theta = \frac{1}{2} (20)^2 \times 0.63$

$= \underline{252}$

$= 126\ cm^2$

3. Calculate the area of a sector whose radius is 5 m and subtends an angle of 45^0 at the center.

Soln: Given, $r = 5$ m

$\Theta = 45^0$

$A = ?$

$\Theta = 45 \times \underline{\pi} = 0.786$ rad

180^0

$A = \frac{1}{2} r^2\Theta$

$= \frac{1}{2}(5)^2 (0.786)$

$= \frac{1 \times 25 \times 0.786}{2}$

$= \frac{19.65}{2} = 9.83 \text{ m}^2$

4. In a circle of radius 25 cm find the length of the arc subtended by a central angle of 60^o

Soln: Given, r = 25 cm

$\Theta = 60^o$

l =?

$\Theta = 60 \times \frac{\pi}{180} = 1.05$ rad

$\therefore l = r\Theta$

$= 25 \times 1.05$

$l = 26.25$ cm

5. A satellite is orbiting in a circular path of radius 5000 km round the Centre of the earth what is the distance travelled by the satellite when it sweeps through an angle of 60^o.

Soln: Given, r = 5000 km

$\Theta = 60^o$

l =?

$\Theta = 60 \times \frac{\pi}{180} = 1.047$ rad

$\therefore l = r\Theta$

$= 5000 \times 1.047$

$l = 5235$kms

6. A wire of length 10 cm is bent into an arc of a circle of radius 5 cm. Find the angle subtended at the center of the arc.

Soln: Given, r = 5 cm

$\Theta = ?$

$l = 10$ cm

$l = r\Theta$

$\therefore \Theta = \frac{l}{r} = \frac{10}{5}$

$\therefore \Theta = 2$ rad

$\therefore \Theta = 2 \times \frac{180}{\pi}$

$\Theta = 114.57^o$

7. A wire of length 15 cm is bent into an arc of a circle of radius 5 cm. Find the central angle subtended by the wire in degrees.

Soln: Given, r = 10 cm

$\Theta = ?$

$l = 15$ cm

$l = r\Theta$

$\therefore \Theta = \frac{l}{r} = \frac{15}{10}$

$\therefore \Theta = 1.5$ rad

$\therefore \Theta = 1.5 \times \frac{180}{\pi}$

$\Theta = 85.93^o$

8. A wire of length 14 cm is bent into an arc of radius 7 cm. Find the angle subtended in degrees at the center.

Soln: Given, r = 7 cm

$\Theta = ?$

$l = 14$ cm

$l = r\Theta$

$\therefore \quad \Theta = \frac{l}{r} = \frac{14}{7}$

$\therefore \Theta = 2$ rad

$\therefore \Theta = 2 \times \frac{180}{\pi}$

$\Theta = 114.58^o$

9. Find the radius of circle when an arc of length 12 cm subtends 50^o at the center.

Soln: Given, $l = 12$ cm

$\Theta = 50^o$

$r = ?$

$\Theta = 50^o = 50 \times \frac{\pi}{180} = 0.873$ rad

$\therefore l = r\Theta$

$r = \frac{l}{\Theta} = \frac{12}{0.873}$

$\therefore r = 13.74$ cm

10. Find the central angle of a circle of radius 25 cm if its arc length is 26.25 cm.

Soln: Given, $r = 25$ cm

$l = 26.25$ cm

$\Theta = ?$

$\therefore l = r\Theta$

$\Theta = \frac{l}{r} = \frac{26.25}{25}$

$\Theta = 1.05$ rad

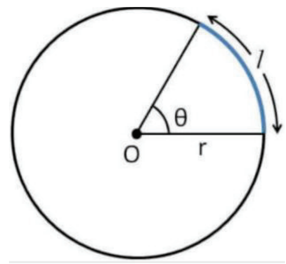
$\therefore \Theta = 1.05 \times \frac{180}{\Pi}$

$\Theta = 60.15^o$

VI.ANSWER THE FOLLOWING QUESTIONS 08 MARKS

1. Derive length of an Arc of a circle $l = r\,\Theta$

   Ans:



Statement: If an arc 'l' subtends an angle

of Θ radians at the center of circle of radius

'r' then $\Theta = \frac{l}{r} = \frac{Arc}{radius}$

$\therefore l = r\,\Theta$

Proof:

Let 'O' be the center of the circle of radius 'r'.
Let OA = r be the radius
Let A, B, C are three points on the circumference of a circle. Join OA, OB and OC.

let OA = r $\therefore$ Arc AB = r

The angle Θ^c subtended by an arc 'l' at the center of the circle.
Let $\angle AOB = 1^c$ and $\angle AOC = \Theta^c$ and Arc AC = l

Since the arcs are proportional to the subtended angles.

From geometry.

$$\frac{Arc\ AC}{Arc\ AB} = \frac{\angle AOC}{\angle AOB}$$

$$\frac{l}{r} = \frac{\Theta^c}{1^c} = \frac{\Theta \times 1^c}{1^c}$$

i.e $\frac{l}{r} = \Theta$

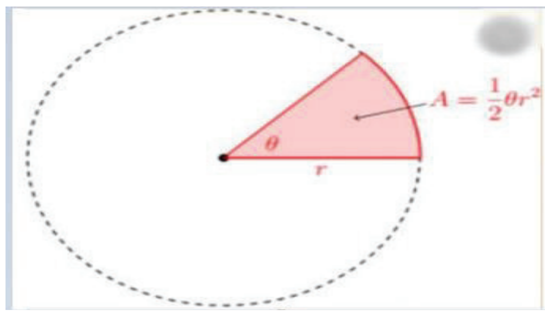
$\therefore \quad l = r\,\Theta$

2. Derive Area of sector $A = \frac{1}{2} r^2 \Theta$

Ans:

Statement: The area of sector of a circle $A = \frac{1}{2} r^2 \Theta$

Where r = radius, Θ = sectorial angle



Proof: Let A,B and C are the points on Circumference of a circle of radius ‘r’ and ‘O’ is the center of the circle.

Let $\angle AOB = \Theta^c$, Draw $OC \perp OA$

From geometry.

$$\frac{\text{Area of Sector AOB}}{\text{Area of Sector AOC}} = \frac{\angle AOB}{\angle AOC}$$

$$\therefore \text{ Area of sector AOB} = \frac{\angle AOB}{\angle AOC} \times \text{Area of sector AOC}$$

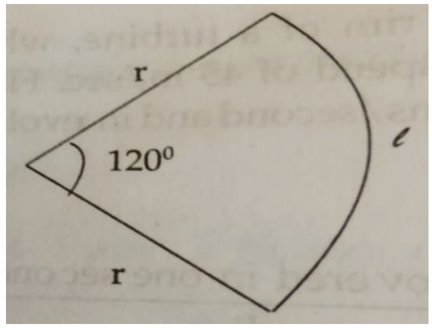
$$= \frac{\Theta^c}{(\pi/2)^c} \times \frac{1}{4} \times \text{Area of circle}$$

$$= \frac{2\Theta}{\Pi} \times \frac{1}{4} \times \pi r^2$$

$$= \text{Area of sector AOB} = \frac{1}{2} r^2 \Theta$$

$$\therefore \quad A = \frac{1}{2} r^2 \Theta$$

3. A circular sector has perimeter 250 cm and central angle 120^o. Find its area.
Ans:



Given perimeter, $l + 2r = 250$ cm

$\Theta = 120^o$

$A = ?$

We have, arc $l = r\,\Theta$

Putting the value of 'l' in $l + 2r$

$\therefore l + 2r = 250$

$r.\Theta + 2r = 250$

$r(\Theta + 2) = 250$

$\therefore r = \dfrac{250}{\Theta + 2}$

$= \dfrac{250}{2.095 + 2}$

$\Theta = 120^o$

$= 120 \times \dfrac{\pi}{180}$

$\Theta = 2.095$ rad

$\therefore r = \dfrac{250}{4.095}$

$r = 61.05$ cm

$\therefore A = \dfrac{1}{2} r^2\,\Theta$

$A = \dfrac{1}{2} (61.05)^2 \times 2.095$

$A = \dfrac{1}{2} \times 7808.27$

$\therefore A = 3904.13\ cm^2$

4. If an arc of a circle of radius 4 cm subtends an angle 15^0 at the Centre. Find the arc length and area of sector formed?

Soln: Given $r = 4$ cm

$\Theta = 15^0$

$l = ?$ and $A = ?$

$l = r\Theta$ $\qquad\qquad$ $\Theta = 15^0 \times \frac{\pi}{180}$

$= 4 \times 15^0$

$= 4 \times 0.262$ $\qquad\qquad$ $= 0.262$ rad

$l = 1.047$ cm

and area of sector $A = \frac{1}{2} r^2 \Theta$

$A = \frac{1}{2} (4)^2 \times 0.262$

$= \frac{1}{2} \times 16 \times 0.262$

$= 8 \times 0.262$

$= 2.096\ cm^2$

5. Find the arc length and area of sector given that arc radius is 7 cm and angle at the center is 15^o.

Soln: Given, $r = 7$ cm

$\Theta = 15^o$

$l = ?, A = ?$

$\Theta = 15^o = 15 \times \frac{\pi}{180} = 0.262$ rad

$\therefore l = r\Theta$

$= 7 \times 0.262$

$l = 1.834$ cm

And Area of sector $A = \frac{1}{2} r^2\Theta$

$= \frac{1}{2}(7)^2 \times 0.262$

$= \frac{1}{2} \times 12.838$

$A = 6.42\ cm^2$

UNIT 6: TRIGONOMETRIC RATIOS

I.FILL IN THE BLANKS 01 MARKS

1. $sin^2\theta + cos^2\theta = \underline{1}$
2. 5cos0°+sin90° = <u>6</u>
3. tan90° is <u>undefined</u>
4. $(1+tan^2A)(1+sinA)(1-sinA) = \underline{1}$

II.MULTIPLE CHOICE QUESTIONS 01 MARKS

1. If $3\cot\theta = 2$, then the value of $\tan\theta$

   a) 2/3 b) 3/2

   c) ½ d) 0

   Ans: b)

2. If $\tan\theta = \frac{a}{b}$ then the value of $\frac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta}$ is

   a) $\frac{a^2-b^2}{a^2+b^2}$ b) $\frac{a^2+b^2}{a^2-b^2}$

   c) $\frac{a}{a^2+b^2}$ d) $\frac{b}{a^2+b^2}$

   Ans: b)

3. The ratios of sides of a right triangle w.r.t its acute angle are known as

   a) trigonometric identities b) compound angle

   c) trigonometric ratios of the angles d) none of these

   Ans: c) trigonometric ratios of the angles

4. Which of the following is the value of sin 90°?

   a) ½ b) 0

   c) -1 d) 1

   Ans: d)

5. If sinA – cosA = 0, then the value of $\sin^4A+\cos^4A$ is

a) 2 b) 1

c) $\frac{1}{\sqrt{2}}$ d) ½

Ans: d)

III.ANSWER THE FOLLOWING QUESTION **02 MARKS**

1. What is the tangent ratio?

   Ans: It is a tool we use with the right triangles.

2. If $\Delta ABC, C = 90°$, then what is the value of sin (A+B) ?

   Ans: The value of sin (A+B) is 1

3. What is the minimum value of sinA, $0 \leq A \leq 90°$?

   Ans: 0

4. Define acute angle.

   Ans: Acute angle is the small angle which is less than 90°.

5. What is the function of cotangent?

Ans: $\cot\boldsymbol{\theta} = \frac{adjacent}{opposite}$

IV.ANSWER THE FOLLOWING QUESTIONS **03 MARKS**

1. Prove that $\sec^4\boldsymbol{\theta} - 1 = 2\tan^2\boldsymbol{\theta} + \tan^4\boldsymbol{\theta}$

Ans: L.H.S $= \sec^4\boldsymbol{\theta} - 1$

$$= (\sec^2\boldsymbol{\theta} - 1)(\sec^2\boldsymbol{\theta} + 1)$$

$$= \tan^2\boldsymbol{\theta}\ (1+\tan^2\boldsymbol{\theta}+1)$$

$$= \tan^2\boldsymbol{\theta}\ (2+\tan^2\boldsymbol{\theta})$$

$$= 2\tan^2\boldsymbol{\theta} + \tan^4\boldsymbol{\theta} = \text{R.H.S}$$

2. Find the value of $5\sin 30° + 3\tan 45°$

Ans: $5\sin 30° + 3\tan 45°$

$= 5\ (1/2) + 3\ (1)$

$= \frac{5+6}{2}$

$= \frac{11}{2}$

3. Find the value of $\sin^2 30° - \cos^2 30°$

Ans: $\sin^2 30° - \cos^2 30°$

$= (1/2)^2 - (\sqrt{3}/2)^2$

$= 1/4 - 3/4$

$= -2/4$

$= -1/2$

4. Find the value of $\cos 0° + \sin 30°$

Ans: $\cos 0° + \sin 30°$

$= 1 + (1/2)$

$= 3/2$

5. How Sine function is calculated?

Ans: In trigonometry, a right triangle the sine of an angle is the length of the opposite side divided by length of the side.

6. List any three trigonometric functions.

Ans: Trigonometric functions are (any 3)

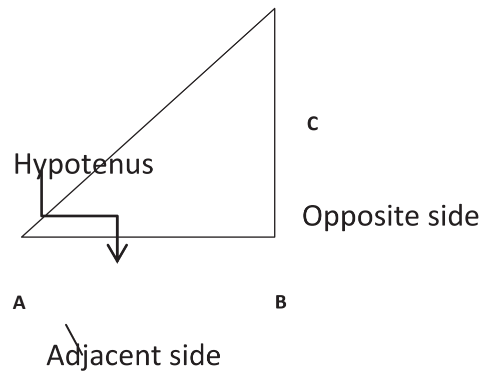
- Sine
- Cosine
- Tangent
- Cosecant
- Secant

- Cotangent

V.ANSWER THE FOLLOWNG QUESTION 05 MARKS

1. Prove that $\boldsymbol{sin^2\theta}+\boldsymbol{cos^2\theta}=1$

Proof:



In a right angle triangle $AB^2+BC^2= AC^2$

Dividedboth side by AC^2

$$= \frac{AB^2+BC^2}{AC^2}$$

$$= \frac{AC^2}{AC^2}$$

$$= 1$$

$$= \frac{AB^2}{AC^2}+\frac{BC^2}{AC^2} = 1$$

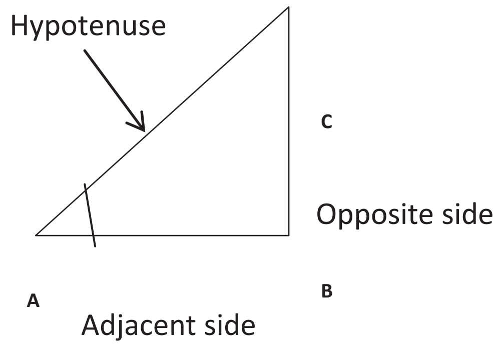
$$= \left(\frac{\mathrm{AB}}{AC}\right)^2+\left(\frac{\mathrm{BC}}{\mathrm{AC}}\right)^2 = 1$$

But in triangle ABC $\frac{AB}{AC} = cos\theta$ and $\frac{BC}{AC} = sin\theta$

$$sin^2\theta + cos^2\,\theta = 1$$

2. Show that $\boldsymbol{sin^2\theta}$-$\tan^2\boldsymbol{\theta}$=1

Proof:



In a right angle triangle $AB^2+BC^2= AC^2$

Divided both side by AB^2

$$= \frac{AB^2 + BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

$$= 1$$

$$= \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

$$= \left(\frac{\mathrm{AB}}{AB}\right)^2 + \left(\frac{\mathrm{BC}}{\mathrm{AB}}\right)^2 = \left(\frac{\mathrm{AC}}{\mathrm{AB}}\right)^2$$

$$= 1 + \left(\frac{\mathrm{BC}}{\mathrm{AB}}\right)^2 = \left(\frac{\mathrm{AC}}{\mathrm{AB}}\right)^2$$

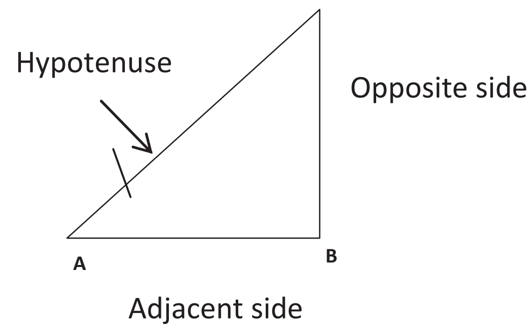
But in triangle ABC $\frac{BC}{AB} = tan\theta$

$$\frac{AC}{AB} = sec\theta$$

$$1 + tan^2\,\theta = sec^2\theta$$

$$sec^2\,\theta - tan^2\,\theta = 1$$

3. Show that $\boldsymbol{cosec^2\theta\text{-}cot^2\theta}=1$



In a right angle triangle,

$AB^2+BC^2= AC^2$

Divide both side by BC^2

$\frac{AB^2+BC^2}{AB^2}=\frac{AC^2}{BC^2}=1$

$$= \frac{AB^2}{BC^2}+\frac{BC^2}{BC^2}=\frac{AC^2}{BC^2}$$

$$\left(\frac{\mathrm{AB}}{BC}\right)^2+\left(\frac{\mathrm{BC}}{\mathrm{BC}}\right)^2=\left(\frac{\mathrm{AC}}{\mathrm{BC}}\right)^2$$

$$\left(\frac{\mathrm{BC}}{\mathrm{AB}}\right)^2+1=\left(\frac{\mathrm{AC}}{\mathrm{BC}}\right)^2$$

But in triangle ABC,

$$\frac{AB}{BC}=cot\theta$$

$$\frac{AC}{BC}=cosec\theta$$

$$cot^2\theta+1=cosec^2\theta$$

$$cosec^2\,\theta-cot^2\,\theta=1$$

4. **Show that** $\frac{sinA}{1+cosA} - \frac{1+cosA}{sinA} = 2cosecA$

L.H.S $= \frac{sinA}{1+cosA} - \frac{1+cosA}{sinA}$

Take LCM

$= \frac{(sinA)(sinA)+(1-cosA)(1+cosA)}{(1+cosA)(sinA)}$

$= \frac{sin^2A+(1+cosA)^2}{(1+cosA)(sinA)}$

$= \frac{sin^2A+1+cos^2A+2cosA}{(1+cosA)(sinA)}$

$= \frac{1+1+2cosA}{(1+cosA)(sinA)}$

$= \frac{2+2cosA}{(1+cosA)(sinA)}$

$= \frac{2(1+cosA)}{(1+cosA)(sinA)}$

$= \frac{2}{(sinA)}$

$=$ 2cosecA

$=$ R.H.S

5. **If x= rcosA and y= rsinA show that** $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$

Let x = rcosA

$\frac{x}{r} = $ cosA

$\frac{x^2}{r^2} = cos^2A$ -----------------(1)

Y = rsinA

$\frac{y}{r} = $ sinA

$\frac{Y^2}{r^2} = \sin^2 A$ ------------------(2)

Equations (1)+(2) $= \frac{x^2}{r^2} + \frac{y^2}{r^2}$

$= \cos^2 A + \sin^2 A$

$= 1$

6. **Prove that $\sqrt{\frac{1-sin\theta}{1+sin\theta}}$ =secθ+tanθ**

L.H.S $= \sqrt{\frac{1-\sin\theta}{1+\text{Sin}\theta} + \frac{1-\sin\theta}{1-\text{Sin}\theta}}$

$= \sqrt{\frac{(1-\sin\theta)^2}{1+\sin^2\theta}}$

$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$

$=\frac{1-\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= sec\theta - tan\theta$

7. **Prove that $\frac{1}{1-sinA} + \frac{1}{1+sinA} = 2\sec^2 A$**

L.H.S $= \frac{1}{1-sinA} + \frac{1}{1+sinA}$

$= \frac{(1+sinA)+(1-sinA)}{(1-sinA)(1+sinA)}$

$= \frac{2}{1-sin^2 A}$

$= \frac{2}{cos^2 A}$

$= 2\sec^2 A =$ R.H.S

8. Find the value of $\sin^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{3\pi}{4}\right) + \sin^2\left(\frac{5\pi}{4}\right) + \sin^2\left(\frac{7\pi}{4}\right)$.

VI.ANSWER THE FOLLOWING QUESTIONS 08 MARKS

1. **Prove that** $\frac{1+sin\theta}{1-sin\theta}-\frac{1-sin\theta}{1+sin\theta}=4\tan\theta\sec\theta$

L.H.S $= \frac{1+sin\theta}{1-sin\theta}-\frac{1-sin\theta}{1+sin\theta}$

Take LCM

$=\frac{(1+sin\theta)(1+sin\theta)+(1-sin\theta)(1-sin\theta)}{(1-sin\theta)(1-sin\theta)}$

$=\frac{(1+sin\theta)+(1-sin\theta)}{(1-sin\theta)(1-sin\theta)}$

$=\frac{(1+2sin^2\theta)+sin2\theta-(1-2sin\theta+2sin^2\theta)}{(1-sin\theta)(1-sin\theta)}$

$=\frac{1+2sin\theta+2sin^2\theta-1+2sin\theta-sin^2\theta}{1-sin2\theta}$

$=\frac{1+2sin\theta+2sin^2\theta-1+2sin\theta-sin^2\theta}{cos2\theta}$

$=\frac{4sin\theta}{cos\theta}x\frac{1}{cos\theta}=4\tan\theta sec\theta$

L.H.S=R.H.S

2. **Prove that** $\frac{1+cos\theta}{1-cos\theta}-\frac{1-cos\theta}{1+cos\theta}=4\text{cosec}\theta cot\theta$

L.H.S $= \frac{1+cos\theta}{1-cos\theta}-\frac{1-cos\theta}{1+cos\theta}$

Take LCM

$=\frac{(1+cos\theta)(1+cos\theta)+(1-cos\theta)(1-cos\theta)}{(1-cos\theta)(1+cos\theta)}$

$=\frac{(1+cos\theta)^2+(1-cos\theta)^2}{(1-cos\theta)(1+cos\theta)}$

$$\frac{1 + 2\cos\theta + \cos^2\theta - (1 - 2\cos\theta + \cos^2\theta)}{(1 - \cos\theta)(1 + \cos\theta)}$$

$$\frac{1 + 2\cos\theta + \cos^2\theta - 1 + 2\cos\theta - \cos^2\theta}{(1 - \cos^2\theta)}$$

$$\frac{1 + 2\cos\theta + \cos^2\theta - 1 + 2\cos\theta - \cos^2\theta}{\sin^2\theta}$$

$\frac{4cos\theta}{sin\theta}x\frac{1}{sin\theta}$=4cosec$\theta cot\theta$

L.H.S = R.H.S

UNIT 7: ALLIED ANGLES

I.FILL IN THE BLANKS 01 MARKS

1. Sin(180°– ϴ) = _______ (sinθ)
2. Cos(270–ϴ) = ________ (-sinθ)
3. Sin(180°+ ϴ) = _______ (-sinθ)
4. Cos(90–ϴ) = ________ (sinθ)
5. tan(180 + ϴ) = ________ (tanθ)
6. Cos(90 + ϴ) = ________ (-sinθ)
7. Sin(90° + ϴ) = _______ (cosθ)
8. In _____ quadrant, both tan and cot are positive and other ratios are negative.(third)
9. Cot(360 + ϴ) = ________ (cotθ)
10. In _____ quadrant, all trignometric ratios are positive.(first)
11. Sin(– ϴ) = _______ (-sinθ)
12. Sec(270 + ϴ) = ________(cosecθ)

II.MULTIPLE CHOICE QUESTIONS 01 MARKS

1. Sine and cosec are positive in ______ quadrant.

a. First b.Second c. Third d.Fourth

2. Cos and sec are positive in _____ quadrant.

a. First b.Second c. Third d.Fourth

3. ______ are called complemetry angles.

a. ϴ and (90 - ϴ) b. ϴ and (90 + ϴ) c. ϴ and (180 - ϴ) d. ϴ and (180 + ϴ)

4. ______ are called supplementary angles.

a. ϴ and (90 - ϴ) b. ϴ and (90 + ϴ) c. ϴ and (180 - ϴ) d. ϴ and (180 + ϴ)

5. tan (- ϴ) = ________

a. tan ϴ b. cot ϴ c. -tan ϴ d. -cot ϴ

6. Cosec (-ϴ) = ______

a. cosec ϴ b. sec ϴ c. - cosec ϴ d. -sec ϴ

7. Cosec (360 – ϴ) = _______

a. cosec ϴ b. sec ϴ c. - cosec ϴ d. -sec ϴ

8. Sec (360 + ϴ) = _______

a. cosec ϴ b. sec ϴ c. - cosec ϴ d. -sec ϴ

9. Sec (270 + ϴ) = ______

a. tan ϴ b. -tan ϴ c. cosec ϴ d. -cosec ϴ

10. The value of cos (-60°) = _______

a. 0 b. 1 c. $\frac{1}{2}$ d.- $\frac{1}{2}$

III.ANSWER THE FOLLOWING QUESTION 02 MARKS

1. Define allied angles.

The allied angles of ϴ are the angles which are in the form of $\pm$ ϴ, 90 $\pm$ ϴ, 180 $\pm$ ϴ, 270 $\pm$ ϴ,

A) 360 $\pm$ ϴ,

2. When do you say an angle is positive and when it is negative?

A) an angle is said to be formed when a ray rotates from initial position to final position positive. if it rotates in anti clockwise direction and it is said to be negative if it rotates in clockwise direction.

3. Mention different allied angles.

A) 90 $\pm$ ϴ, 180 $\pm$ ϴ, 270 $\pm$ ϴ, 360 $\pm$ ϴ,

4. Mention complementary angles and supplementary angles.

A) ϴ and (90 - ϴ) are called complementary angles, ϴ and (180 - ϴ) are called supplementary angles.

5. Find the value of sin120°.

A) $\sin(120°) = \sin(180° - 60°) = \sin(60°) = \frac{\sqrt{3}}{2}$

6. Find the value of tan (-315°).

A) tan (-315)= -tan(315) = -tan(360-45) = -(-tan(45)) = tan(45) = 1

7. Find the value of cosec (-225°).

A) cosec -225= -cosec(180+45)=-(-cosec45) = -(-√2) = √2

8. Find the value of sin (360°).

A) sin (360°) = sin (360°+0°) = sin (0°) = 0

9. Name the quadrants in which **tan** is positive and negative.

A) tan is positive in I and III quadrant and tan is negative in II and IV quadrant

10. Find the value of cosec (-60°).

A) cosec (-60°) = -cosec60 = $-\frac{2}{\sqrt{3}}$

11. Find the value of sec (-120°).

A) sec (-120°) = sec(120) = sec (180°- 60°) = - sec (60°)= - 2

12. Find the value of cos (120°).

A) cos(180°- 60°) = - cos(60°)= $-\frac{1}{2}$

IV.ANSWER THE FOLLOWING QUESTIONS 03 MARKS

1. Find the value of cos 480°.

A) cos 480°

=cos (360° + 120°)

= Cos120°

= cos (180° - 60°) = - cos 60°

= $-\frac{1}{2}$

2. Find the value of cosec (-660°).

A) cosec (-660°) = -cosec (660°) = -cosec (720°- 60°)

= -(-cosec60°)

= $\frac{2}{\sqrt{3}}$

3. Find the value of sin (-870°).

A) sin (-870°) = -sin (180°x5- 30°)

= -sin(30°)

= $-\frac{1}{2}$

4. Find the value of cot (990°).

A) cot (990°) = cot (180°x5+0) = cot (0°)

= undefined

5. Find the value of sin (1140°).

A) sin (1140°) = sin (180°x6+60°) = sin(60°)

= $\frac{\sqrt{3}}{2}$

6. Find the value of cosec (-1110°).

A) cosec (-1110°) = -cosec(180°x6+30°) = -cosec(30°)

= -2

7. Write all the trignometric ratios of angle (-Θ).

A) sin(-Θ) = - sin Θ cosec(-Θ) = - cosec Θ

cos(-Θ) = cosΘ sec(-Θ) = sec Θ

tan(-Θ) = - tan Θ cot (-Θ) = - cotΘ

8. Write all the trignometric ratios of angle (270 - Θ).

A) sin(270 - Θ) = -cosΘ cosec(270 - Θ) = -sec Θ

cos(270 - Θ) = - sin Θ sec(270 - Θ) = -cosec Θ

tan(270 - Θ) = cotΘ cot (270 - Θ) = tan Θ

9. Write all the trignometric ratios of angle (360 - Θ).

A) sin(360 - Θ) = -sin Θ cosec(360 - Θ) = -cosec Θ

cos(360 - Θ) = cosΘ sec(360 - Θ) = sec Θ tan(360 - Θ) = -tan Θ cot (360 - Θ) = -cotΘ

10. Write all the trignometric ratios of angle (90 - Θ).

A) sin(90 - Θ) = cosΘ cosec(90 - Θ) = sec Θ

cos(90 - Θ) = sin Θ sec(90 - Θ) = cosec Θ

tan(90 - Θ) = cotΘ cot (90 - Θ) = tan Θ

11. Write the trignometric ratios of angle (180 + Θ).

A) sin(180+ Θ) = -sin Θ cosec(180+ Θ) = -cosec Θ

cos(180+ Θ) =cosΘ sec(180+ Θ) = sec Θ

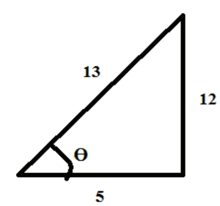
tan(180+ Θ) = -tan Θ cot (180+ Θ) = -cotΘ

12. If sinΘ = $\frac{12}{13}$ and 0° <Θ< $\frac{\pi}{2}$, find all remaining trigonometric ratios.

A) sin(Θ) = $\frac{12}{13}$ cosec(Θ) = $\frac{13}{12}$

cos(Θ) = $\frac{5}{13}$ sec(Θ) = $\frac{13}{5}$

tan(Θ) = $\frac{12}{5}$ cot (Θ) = $\frac{5}{12}$



V.ANSWER THE FOLLOWNG QUESTION 05 MARKS

1. Prove that sin150° + cos300° - tan135° + sec660° = 4

A) sin(180-30) + cos(360-60) - tan(360-45) + sec(360-60) [sec(660) = sec(300)]

= sin 30 + cos 60 - (-tan(45)) + sec 60

= $\frac{1}{2}$ + $\frac{1}{2}$+ 1 + 2

= 1+1+2

= 4

2. Prove that sec(180° − A).cosec(90° + A) - cot(90° + A).tan(180° + A) = -1

A) -secA.(-secA) – (-tanA) tan(A)

= -sec^2A-tan^2A

= -1

3. Show that tan225°x cot 405° + tan765° x cot675° = 0

A) tan(180+45) x cot (360+45) + tan(720+45) x cot(720-45)

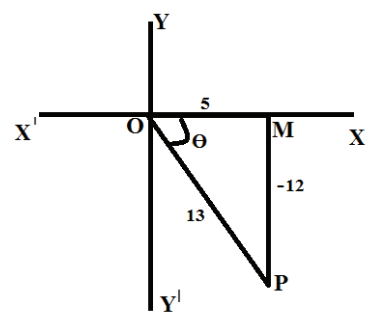
=tan(45) x cot (45) + tan(45) x (-cot(45))

=1+(-1)

=0

4. If secϴ = $\frac{13}{5}$ and ϴ lies in IVth quadrant. Find the value of 4cotϴ - cosecϴ

A) ϴ lies in IVth quadrant



OP =13, OM=5 hence by phytagores theorem MP= - 12

cotϴ = $\frac{5}{-12}$ and cosecϴ = $\frac{13}{-12}$

now 4cotϴ - cosecϴ

$4(\frac{5}{-12})$- $\frac{13}{-12}$

$= \frac{-20+13}{12}$

$= \frac{-7}{12}$

5. Find the value of cos570°. sin150°. sin330°. cos390°

A) cos570°. sin150°. sin330°. cos390°

cos(360+150) sin(180-30) sin(360-30)cos(360+30)

cos(150) sin(30) (- sin(30)) cos(30)

cos(180-30) sin(30)(- sin(30)) cos(30)

-cos(30) sin(30) (- sin(30)) cos(30)

$(-\frac{\sqrt{3}}{2})$x $\frac{1}{2}$ x(- $\frac{1}{2}$) x$\frac{\sqrt{3}}{2}$

$= \frac{3}{16}$

6. If $\frac{\pi}{2}$ <A< π , cosA = $\frac{-5}{13}$ find the value of $\frac{\text{sinA}-\text{cosA}}{sinA+CosA}$

A) If cosA = $\frac{-5}{13}$ then sinA = $\frac{12}{13}$

Now $\frac{\text{sinA}-\text{cosA}}{sinA+CosA} = \frac{\frac{12}{13}-\frac{-5}{13}}{\frac{12}{13}+\frac{-5}{13}} = \frac{\frac{17}{13}}{\frac{7}{13}} = \frac{17}{7}$

7. Find the value of $\tan^2\frac{\pi}{6} + \tan^2\frac{5\pi}{6}$

A) $\tan^2\frac{\pi}{6} + \tan^2 \pi - \frac{\pi}{6}$

$= \tan^2\frac{\pi}{6} + \tan^2 - \frac{\pi}{6}$

$= \tan^2\frac{\pi}{6}$ - $\tan^2\frac{\pi}{6}$

$= 0$

8. If cosΘ = $\frac{5}{13}$ and 270° <Θ<360°,find the value of $\frac{5\text{cosec}\Theta}{\tan\Theta + sec\Theta}$

A) Θ lies in IVth quadrant

MP is negative, MP= -12, OM=5, OP=13

If cosΘ = $\frac{5}{13}$ then cosecΘ = $\frac{13}{-12}$, tanΘ = $\frac{-12}{5}$, secΘ = $\frac{13}{5}$

Now $\frac{5\text{cosec}\Theta}{\tan\Theta + sec\Theta}$

$= \frac{5(\frac{13}{-12})}{\frac{-12}{5}+\frac{13}{5}}$

$= \frac{-\frac{65}{12}}{\frac{-12+13}{5}}$

$= -\frac{65}{12}$ x $\frac{5}{1}$

$= -\frac{325}{12}$

VI.ANSWER THE FOLLOWING QUESTIONS 08 MARKS

1. If secΘ = $\frac{13}{5}$ and 270° <Θ<360°, find the value of $\frac{3\sin\Theta - 2cos\Theta}{9\cos\Theta + 4sin\Theta}$

A) Θ lies in IVth quadrant

MP is negative, MP= -12, OM=5, OP=13

Sin$\Theta = \frac{-12}{13}$, cos$\Theta = \frac{5}{13}$

Now $\frac{3\sin\Theta - 2cos\Theta}{9\cos\Theta + 4sin\Theta}$

$$= \frac{3\left(\frac{-12}{13}\right)-2\left(\frac{5}{13}\right)}{9\left(\frac{5}{13}\right)+4\left(\frac{-12}{13}\right)}$$

$$= \frac{\left(\frac{-36}{13}\right)-\left(\frac{10}{13}\right)}{\left(\frac{45}{13}\right)+\left(\frac{-48}{13}\right)}$$

$$= \frac{\left(\frac{-46}{13}\right)}{\left(\frac{-3}{13}\right)} = \frac{46}{3}$$

2. If tan$\Theta = \frac{24}{7}$and 180° <Θ<270°,find the value of $\frac{3\sin\Theta + 4cos\Theta}{4\sin\Theta - 3cos\Theta}$

A) MP is negative, MP = -24, OM=-7, OP=25

Sin$\Theta = \frac{-24}{25}$, cos$\Theta = \frac{-7}{25}$

Now $\frac{3\sin\Theta + 4cos\Theta}{4\sin\Theta - 3cos\Theta}$

$$= \frac{3\left(\frac{-24}{25}\right)+4\left(\frac{-7}{25}\right)}{4\left(\frac{-24}{25}\right)-3\left(\frac{-7}{25}\right)}$$

$$= \frac{\left(\frac{-72}{25}\right)+\left(\frac{-28}{25}\right)}{\left(\frac{-96}{25}\right)-\left(\frac{-21}{25}\right)}$$

$$= \frac{(-100)}{(-75)}$$

$$= \frac{4}{3}$$

3. If cos$\Theta = \frac{-5}{13}$ and 90° <Θ<180°,find the value of $\frac{13\sin\Theta + 5sec\Theta}{5\tan\Theta - 13scos\Theta}$

A) OM=-5, OP=13, MP = 12

Sin$\Theta = \frac{12}{13}$, sec $\Theta = \frac{13}{-5}$, tan $\Theta = \frac{12}{-5}$, cos $\Theta = \frac{-5}{13}$

Now $\frac{13\sin\Theta + 5sec\Theta}{5\tan\Theta - 13scos\Theta}$

$= \frac{13\left(\frac{12}{13}\right)+5\left(\frac{13}{-5}\right)}{5\left(\frac{12}{-5}\right)-13\left(\frac{-5}{13}\right)}$

$= \frac{(12+13)}{(-12+5)}$

$= \frac{-1}{-7}$

$= \frac{1}{7}$

4. Find the value of $\sin^2(\frac{\pi}{4}) + \sin^2(\frac{3\pi}{4}) + \sin^2(\frac{5\pi}{4}) + \sin^2(\frac{7\pi}{4})$.

A) $\sin^2(\frac{\pi}{4}) + \sin^2(\frac{3\pi}{4}) + \sin^2(\frac{5\pi}{4}) + \sin^2(\frac{7\pi}{4})$

$= \sin^2(45) + \sin^2(135) + \sin^2(225) + \sin^2(315)$.

$= (\frac{1}{\sqrt{2}})^2 + [\sin(180\text{-}45)]^2 + [\sin(180+45)]^2 + [\sin(360\text{-}45)]^2$

$= \frac{1}{2} + \sin^2(45) + \sin^2(45) + \sin^2(45)$

$= \frac{1}{2} + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{4}{2}$

$= 2$

5. If $\tan\Theta = \frac{4}{5}$ and $180° < \Theta < 270°$, find the value of $\frac{5\sin\Theta + 7cos\Theta}{6\cos\Theta - 3sin\Theta}$.

A) OM=-5, MP=-4, OP = $\sqrt{41}$

$\text{Sin}\Theta = \frac{-4}{\sqrt{41}}$, $\cos\Theta = \frac{-5}{\sqrt{41}}$

Now $\frac{5\sin\Theta + 7cos\Theta}{6\cos\Theta - 3sin\Theta}$

$= \frac{5(\frac{-4}{\sqrt{41}}) + 7(\frac{-5}{\sqrt{41}})}{6(\frac{-5}{\sqrt{41}}) - 3(\frac{-4}{\sqrt{41}})}$

$= \frac{(\frac{-20}{\sqrt{41}}) + (\frac{-35}{\sqrt{41}})}{(\frac{-30}{\sqrt{41}}) - (\frac{-12}{\sqrt{41}})}$

$= \frac{\left(\frac{-45}{\sqrt{41}}\right)}{\left(\frac{-18}{\sqrt{41}}\right)}$

$= \frac{15}{6}$

6. If cose$\Theta = \frac{-13}{12}$and π <Θ< $\frac{3\pi}{2}$, find the value of $\frac{4\sin\Theta - 2cos\Theta}{\sin\Theta + 2cos\Theta}$.

A) OM=-5, MP=-12, OP = 13

$\text{Sin}\Theta = \frac{-12}{13}$, $\cos\Theta = \frac{5}{13}$

Now $\frac{4\sin\Theta - 2cos\Theta}{\sin\Theta + 2cos\Theta}$

$= \frac{4\left(\frac{-12}{13}\right) - 2(\frac{5}{13})}{\left(\frac{-12}{13}\right) + 2(\frac{5}{13})}$

$= \frac{\left(\frac{-48}{13}\right) - (\frac{10}{13})}{\left(\frac{-12}{13}\right) + (\frac{10}{13})}$

$= \frac{\left(\frac{-58}{13}\right)}{\left(\frac{-2}{13}\right)}$

$= \frac{58}{2}$

$= 29$

7. If cos$\Theta = \frac{2}{3}$ and $\frac{3\pi}{2}$ <Θ<2π,find the value of $\frac{5\sin\Theta - 6cos\Theta}{6\sin\Theta - 5cos\Theta}$

A) OM=2, MP=-$\sqrt{5}$, OP = 3

$\text{Sin}\Theta = \frac{-\sqrt{5}}{3}$,

Now $\frac{5\sin\Theta - 6cos\Theta}{6\sin\Theta - 5cos\Theta}$

$$= \frac{5\left(\frac{-\sqrt{5}}{3}\right)-6(\frac{2}{3})}{6\left(\frac{-\sqrt{5}}{3}\right)-5(\frac{2}{3})}$$

$$= \frac{\left(\frac{-5\sqrt{5}}{3}\right)-(\frac{12}{3})}{\left(\frac{-6\sqrt{5}}{3}\right)-(\frac{10}{3})}$$

$$= \frac{\left(\frac{-5\sqrt{5}-12}{3}\right)}{\left(\frac{-6\sqrt{5}-10}{3}\right)}$$

$$= \frac{-5\sqrt{5}-12}{6\sqrt{5}-10}$$

8. If $\sin\Theta = \frac{4}{5}$ and $\frac{\pi}{2} < \Theta < \pi$, find the value of $\frac{3\sin\Theta - \cos\Theta}{4\operatorname{cosec}\Theta + 3\tan\Theta}$.

A) OM=-3, MP=4, OP = 5

$\sin\Theta = \frac{4}{5}$, $\cos\Theta = \frac{-3}{5}$, $\operatorname{cosec}\Theta = \frac{5}{4}$, $\tan\Theta = \frac{4}{-3}$

Now $\frac{3\sin\Theta - \cos\Theta}{4\operatorname{cosec}\Theta + 3\tan\Theta}$

$$= \frac{3\left(\frac{4}{5}\right)-(\frac{-3}{5})}{4\left(\frac{5}{4}\right)+3(\frac{4}{-3})}$$

$$= \frac{\left(\frac{12}{5}\right)+(\frac{3}{5})}{5-4}$$

$$= \frac{15}{5}$$

$$= 3$$

9) If $\tan\Theta = \frac{12}{15}$ and $180° < \Theta < 270°$, find the value of $\frac{3\sin\Theta + 4\cos\Theta}{3\sin\Theta - 4\cos\Theta}$

A) OM=-15, MP=-12, OP = $\sqrt{369}$

$\sin\Theta = \frac{-12}{\sqrt{369}}$, $\cos\Theta = \frac{-15}{\sqrt{369}}$

Now $\frac{3\sin\Theta + 4\cos\Theta}{3\sin\Theta - 4\cos\Theta}$

$$= \frac{3\left(\frac{-12}{\sqrt{369}}\right)+4(\frac{-15}{\sqrt{369}})}{3\left(\frac{-12}{\sqrt{369}}\right)-4(\frac{-15}{\sqrt{369}})}$$

$= \frac{\left(\frac{-36}{\sqrt{369}}\right)+\left(\frac{-60}{\sqrt{369}}\right)}{\left(\frac{-36}{\sqrt{369}}\right)-\left(\frac{-60}{\sqrt{369}}\right)}$

$= \frac{-96}{24}$

= -4

10. Simplify $\frac{\sin(\pi - A)}{\cos(\frac{\pi}{2} - A)} + \frac{\tan(\pi + A)}{\cos(\frac{3\pi}{2} + A)} + \frac{\sin(-A)}{\cos(\frac{\pi}{2} + A)}$

A) $\frac{\sin(\pi - A)}{\cos(\frac{\pi}{2} - A)} + \frac{\tan(\pi + A)}{\cos(\frac{3\pi}{2} + A)} + \frac{\sin(-A)}{\cos(\frac{\pi}{2} + A)}$

$= \frac{\sin(180 - A)}{\cos(90 - A)} + \frac{\tan(180 + A)}{\cos(270 + A)} + \frac{\sin(-A)}{\cos(90 + A)}$

$= \frac{\sin A}{\sin A} + \frac{\tan A}{\sin A} + \frac{-\sin A}{-\sin A}$

$= 1 + \frac{\sin A}{\cos A \sin A} + 1$

$= 2 + \frac{1}{\cos A}$

=2+secA

UNIT 8: VECTORS ALGEBRA

I.FILL IN THE BLANKS **01 MARKS**

1) Vector quantity has both _______ and direction.(magnitude)
2) Unit vector, $\hat{a} =$ _________ $\left(\frac{\vec{a}}{|\vec{a}|}\right)$
3) The magnitude of vector $\vec{a} = xi + yj + zk$ is __________ $\left(\sqrt{x^2 + y^2 + z^2}\right)$
4) The projection of $\vec{b}$ on $\vec{a}$ is ________ $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|}\right)$
5) If two vectors are perpendicular, then $\vec{a}.\vec{b} =$ ___________(zero)
6) The position vector of $\vec{A}$ and $\vec{B}$ is given by $\overrightarrow{AB} =$ ___________ $(\overrightarrow{OB} - \overrightarrow{OA})$
7) The sine of angle between two vectors is given by $\sin\theta =$ ____________ $\left(\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\right)$
8) The cosine of angle between two vectors is given by $\cos\theta =$ ___________ $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right)$
9) Vectors lying in the same plane are called ___________ $(coplanar\ vectors)$
10) The projection of $\vec{a}$ on $\vec{b}$ is ___________ $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right)$
11) If two vectors have same magnitude and direction are called _________ $(equal\ vectors)$

12) If two vectors have same magnitude but have opposite directions are called __________ (*unlike vectors*)

13) The magnitude of vector $\overrightarrow{AB} = 2i + j + 2k$ is _________ ($\sqrt{9}$ *or* 3)

14) If $\vec{a} = (2,4,-3)$ then in terms of (i,j,k) is ____________ $(2i + 4j - 3k)$

15) The magnitude of 5i-12j is ___________ (13)

16) If $\vec{a} = i + 2j - 3k$ then $|\vec{a}|$ is ___________ $(\sqrt{14})$

17) If $\vec{a} = (2,4)$ then in terms of (i,j,k) is ____________ (2i+4j)

18) If $\vec{a} = (4,2,-4)$ then $2\vec{a} =$ __________ {(8, 4,-8) or (8i+4j-8k)}

19) If $\vec{a} = (-3,2,1)$ then $2\vec{a} =$ __________ {(-9, 6, 3) or (-9i+6j+3k)}

20) If $\vec{a} = (-2,3)$ & $\vec{b} = (-1,5)$ then $\vec{a} + \vec{b} =$ _________ {(-3i+8j) or (-3, 8)}

II.MULTIPLE CHOICE QUESTIONS 01 MARKS

1) A vector whose magnitude is one is called

a) Equal vector b) Collinear vector

c) Unit vector d) None

Ans: c

2) If $\vec{a} = 3i - 2j + k$ then $|\vec{a}|$ is

a) $\sqrt{14}$ b) $\sqrt{10}$

b) $\sqrt{6}$ d) 0

Ans: a

3) If $\vec{a} = i + j + k, \vec{b} = 2i - 3j + 6k$ then $\vec{a} + \vec{b}$ is

a) 3i+2j+7k b) i-2j+6k

c) 3i-2j+7k d) None

Ans: c

4) If $\vec{a} = i + 2j - 3k, \vec{b} = 3i - 5j + 2k$ find $\vec{a}.\vec{b}$

a) -10 b) 20

c)-13 d) -1

Ans: c

5) A vector whose magnitude is zero is called

a) zero vector b) Co-initial vector

c) Coplanar vector d) None

Ans: a

6) If $\vec{a} = 7i - 4j + 5k$ and $\vec{b} = 3i + 4j - 7k$ then $\vec{a} - \vec{b}$ is

a)8i+12j-4k b) i-12j+4k

c) 12i-4j+8k d) 4i-8j+12k

Ans: d

7) If $\vec{a} = 2i + 3j,\ \vec{b} = -i + 5j$ then $\vec{a} + \vec{b}$ value is

a) -i+8j b) i+8j

c) 8i-j d) -i-8j

Ans: b

8) Projection of $\vec{a}\ on\ \vec{b}$ is

a) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ b) $\frac{|\vec{a}.\vec{b}|}{|\vec{b}|}$

c) $\frac{\vec{a}.\vec{b}}{\vec{b}}$ d) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Ans: d

9) If $\vec{a} = (-2,3)$ & $\vec{b} = (-1,5)$ then $\vec{a} + \vec{b}$

a) (-1,11) b) (11,-1)

c) (11,-7) d) (-7,11)

Ans: b

10) If $\vec{a} = (5,4,1)$ then inverse of $\vec{a}$ is

a) (-5,-4,-1) b) (5,-4,1)

c) (-5,-4,1) d) none

Ans: a

11) If $\vec{a} = (2,1,-1)$ & $\vec{b} = (1,-2,1)$ then $\vec{a} + \vec{b}$

a) 3i-j+k b) 3i+j-k

c) 3i-j d) 3i+j+k

Ans: c

12) If $\vec{a} = 2i - j + k$ & $\vec{b} = 3i + j - k$ then $\vec{a}.\vec{b}$

a) 5 b) -4

c) 16 d) 4

Ans: d

13) If $\vec{a} = (5,-2,3)$& $\vec{b} = (2,3,-1)$ then $\vec{a}.\vec{b}$

a) 1 b) 0

c) 2 d) -1

Ans: a

14) Vectors $\lambda i + 2j + 2k$ and $i - j + k$ are perpendicular to each other then value of λ is

a) 0 b) -1

c) 2 d) none

Ans: a

15) If $\vec{a} + \vec{b} = (3,2,8)$ and $\vec{c} = (1,1,1)$ then$(\vec{a} + \vec{b}).\vec{c}$ value is

a) -13 b) 13

c) 11 d) 12

Ans: b

16) If $\vec{a} = i + j + 2k$ & $\vec{b} = 2i - j + k$ then $\vec{a} - \vec{b}$ is

a) 2i+j+k b) i-2j-k

c) –i+2j+k d) i-2j+k

Ans: c

17) If $\vec{a} \times \vec{b} = -i + 2j - k$ then $|\vec{a} \times \vec{b}|$ is

a) $\sqrt{5}$ b) 6

c)$\sqrt{4}$ d)$\sqrt{6}$

Ans: d

18) If $\vec{a} = (5,-2,3)$ & $\vec{b} = (2,3,-1)$ then $\vec{a} - \vec{b}$ is

a) (3,-5,4) b) (-3,-5,4)

c) (5,-3,4) d) (-3,5,-4)

Ans: a

19) If $\vec{a} = \cos\theta\, i - \sin\theta\, j$ then $|\vec{a}|$ is

a) 1 b) 0

c) $\sqrt{2}$ d) none

Ans: a

20) If $2\vec{a} + 3\vec{b} = i + 21j$ then inverse of $2\vec{a} + 3\vec{b}$ is

a) –i+21j b) 21i+j

c) –i-21j d) i-21j

Ans: c

III.ANSWER THE FOLLOWING QUESTION 02 MARKS

1. If $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ find $|\vec{a}|$?

$|\vec{a}| = \sqrt{(3)2 + (-2)2 + (1)2}$

$= \sqrt{9+4+1} = \sqrt{14}$

2. Find the magnitude $\vec{A} = 5\hat{\imath}$- $12\hat{\jmath}$?

$|\vec{A}| = \sqrt{X2 + Y2}$

$= \sqrt{(5)2 + (-12)2} = \sqrt{25 + 224} = \sqrt{249}$.

3. Find $\vec{a} \,.\, \vec{b}$ Where $\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$ - $\hat{k}$ and $\vec{b}$ =$3\hat{\imath} - 2\hat{\jmath} + \hat{k}$?

$\vec{a} \,.\, \vec{b} = (2\hat{\imath} + 3\hat{\jmath}$ - $\hat{k})$* $(3\hat{\imath} - 2\hat{\jmath} + \hat{k})$

= 6-6-1

= -1

4. Define vectors with example?

=> Physical quantities which passes both magnitude and direction is known as vectors.

Eg: force, velocity, acceleration etc;

5. Explain any two types of vectors?

=> <u>Unit vectors</u>: A vector whose magnitude is unity is called unit vector.

If $|\vec{AB}|$=1, then AB is a unit vector.

Zero vectors: A vector whose magnitude is zero is called a zero vector (or) null vector.

It is written as $\mathbf{0}$ vector.

6. Define work done

Work is said to be done and also defined as product of foce and displacement.

W= F.S

7. Find the magnitude $\vec{A} = 4\hat{\imath} + 3\hat{\jmath}$ -2$\hat{k}$?

$= \sqrt{(4)2 + (3)2 + (4)2}$

$= \sqrt{16 + 9 + 4}$

$=\sqrt{29}$

8. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, find?

$|\vec{a}|= \sqrt{(2)2 + (3)2 + (4)2}$

$= \sqrt{4 + 9 + 16}$ $= \sqrt{29}$

IV.ANSWER THE FOLLOWING QUESTIONS 03 MARKS

1. If $\vec{a}$=2$\hat{\imath}$-$\hat{\jmath}$+$\hat{k}$ and $\vec{b}$=$\hat{\imath}$+2$\hat{\jmath}$+$\hat{k}$, find $\vec{b}$ and $\vec{a}$?

Ans: $\vec{a} . \vec{b}$ = (2$\hat{\imath}$-$\hat{\jmath}$+$\hat{k}$)*($\hat{\imath}$+2$\hat{\jmath}$+$\hat{k}$)

= (2-2+1) = 1

So,$|\vec{a}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$

Projection of $\vec{b}$ on $\vec{a}$ is, $\frac{\vec{a}.\vec{b}}{|\vec{a}|} = \frac{1}{\sqrt{6}}$.

2. If $\vec{a}$=3$\hat{\imath}$+$\hat{\jmath}$+2$\hat{k}$ and $\vec{b}$= $\hat{\imath}$-2$\hat{\jmath}$+3$\hat{k}$, find the magnitude of $\vec{a}$ +$\vec{b}$?

Ans : $\vec{a} + \vec{b} = (3\hat{\imath}+\hat{\jmath}+2\hat{k}) + (\hat{\imath}-2\hat{\jmath}+3\hat{k})$

$= (3\hat{\imath}+\hat{\jmath}+2\hat{k} - \hat{\imath}-2\hat{\jmath}+3\hat{k})$

$= (2i - j + 5k)$

So, $\left|\overrightarrow{a + b}\right| = \sqrt{2^2 + (-1^2) + 5^2} = \sqrt{4+1+25} = \sqrt{30}$

3. If $\vec{A}$=(7,-4,5) and $\vec{B}$=(3,4,-7) ,find magnitude $\vec{A}$- $\vec{B}$?

Ans : $\vec{A}$- $\vec{B}$ = (7i-4j+5k)-(3i+4j-7k)

= (7i-4j+5k - 3i-4j+7k)

= (4i-8j+12k)

$\left|\vec{A} - \vec{B}\right| = \sqrt{4^2 + (-8^2) + 12^2} = \sqrt{16+64+144} = \sqrt{224}$

4. Find the midpoint of AB where $\vec{A}=7\hat{\imath}-8\hat{\jmath}+9\hat{k}$ and $\vec{B}=5\hat{\imath}+6\hat{\jmath}+7\hat{k}$?

Ans : AB = $\frac{\vec{a}+\vec{b}}{2} = \frac{(7\hat{\imath}-8\hat{\jmath}+9\hat{k})+(5\hat{\imath}+6\hat{\jmath}+7\hat{k})}{2}$

$= \frac{12i-2j+16k}{2}$

= 6i-1j+8k

5. Find the midpoint of AB where $\vec{A}=2\hat{\imath}+3\hat{\jmath}+5k$ and $\vec{B}=6\hat{\imath}+7\hat{\jmath}+8\hat{k}$?

Ans : AB = $\frac{(2\hat{\imath}+3\hat{\jmath}+5k)+(6\hat{\imath}+7\hat{\jmath}+8\hat{k})}{2}$

$= \frac{8i + 10j + 13k}{2}$

6. If $\vec{a}=2\hat{\imath}-\hat{\jmath}+\hat{k}$&$\vec{b}=\hat{\imath}+2\hat{\jmath}+\hat{k}$, find projection of $\vec{b}$ on $\vec{a}$?

Ans : $\vec{a} \cdot \vec{b} = (2\hat{\imath}-\hat{\jmath}+\hat{k})*(\hat{\imath}+2\hat{\jmath}+\hat{k})$

= (2-2+1)

= 1

So, $|\vec{a}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$

Projection of $\vec{b}$ on $\vec{a}$ is, $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{1}{\sqrt{6}}$.

7. Find unit vector of $\vec{a} = 5i - j + 2k$

Solution: Unit vector = $\frac{\vec{a}}{|\vec{a}|}$

$$= \frac{5i - j + 2k}{\sqrt{(5)^2 + (-1)^2 + (2)^2}}$$

$$= \frac{5i - j + 2k}{\sqrt{30}}$$

8. If $\vec{a} = (2,3,4)$ and $\vec{b} = (-3,2,1)$ find $|\vec{a} - \vec{b}|$

Solution: $\vec{a} - \vec{b} = (2i + 3j + 4k) - (-3i + 2j + k)$

$$\vec{a} - \vec{b} = 5i + j + 3k$$

$$|\vec{a} - \vec{b}| = \sqrt{(5)^2 + (1)^2 + (3)^2}$$

$$|\vec{a} - \vec{b}| = \sqrt{35}$$

9. If $\vec{A} = (3, -5)$ and $\vec{B} = (-5,6)$ find the position vectors of A and B and also find $|\overrightarrow{AB}|$

Solution: Position vectors, $\overrightarrow{OA} = 3i - 4j$, $\overrightarrow{OB} = -5i + 6j$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\overrightarrow{AB} = (-5i + 6j) - (3i - 5j)$$

$$\overrightarrow{AB} = -8i + 10j$$

$$|\overrightarrow{AB}| = \sqrt{(-8)^2 + (10)^2}$$

$$|\overrightarrow{AB}| = \sqrt{164}$$

10. If $\vec{a} = i + j + k$, $\vec{b} = 2i + 3j + 4k$, find the magnitude of $2\vec{a} + 3\vec{b}$

Solution: $2\vec{a} + 3\vec{b} = 2i + 2j + 2k + 6i + 9j + 12k$

$$2\vec{a} + 3\vec{b} = 8i + 11j + 14k$$

$$|\overrightarrow{2a} + 3\vec{b}| = \sqrt{8^2 + 11^2 + 14^2}$$

$$= \sqrt{64 + 121 + 196}$$

$= \sqrt{381}$

11. Find $\vec{a}.\vec{b}$ where $\vec{a} = 2i + 3j - k$ and $\vec{b} = 3i - 2j + k$

Solution: $\vec{a}.\vec{b} = (2i + 3j - k)\ (3i - 2j + k)$

$=2\times3+3\times(-2)+(-1)\times1$

$= 6 - 6 - 1$

$\vec{a}.\vec{b} = -1$

12. When $\vec{a} = 2i + 3j - 4k, \vec{b} = 5i + k$ find $3\vec{a} + 4\vec{b}$

Solution: $3\vec{a} = 6i + 9j - 12k$

$4\vec{b} = 20i + 4k$

$3\vec{a} + 4\vec{b} = (6i + 9j - 12k) + (20i + 4k)$

$3\vec{a} + 4\vec{b} = 26i + 9j - 8k$

13. If $\vec{a} = (2,1,-1)$ and $\vec{b} = (1,-2,1)$ find $|\vec{a} + \vec{b}|$

Solution: $\vec{a} = 2i + j - k$, $\quad \vec{b} = i - 2j + k$

$\vec{a} + \vec{b} = (2i + j - k) + (i - 2j + k)$

$= 3i - j$

$|\vec{a} + \vec{b}| = \sqrt{3^2 + 1^2}$

$|\vec{a} + \vec{b}| = \sqrt{10}$

14. If $\vec{a} = i + j + k$ and $\vec{b} = 2i + 3j + 4k$ find $|\overrightarrow{2a} + 3\vec{b}|$

Solution: $2\vec{a} = 2i + 2j + 2k$

$4\vec{b} = 6i + 9j + 12k$

$2\vec{a} + 3\vec{b} = (2i + 2j + 2k) + (6i + 9j + 12k)$

$= 8i + 11j + 14k$

$\left|2\vec{a} + 3\vec{b}\right| = \sqrt{8^2 + 11^2 + 14^2}$

$= \sqrt{64 + 121 + 196}$

$\left|2\vec{a} + 3\vec{b}\right| = \sqrt{381}$

15. If $\vec{a} = i + 2j + 3k$ and $\vec{b} = 4i - j - 5k$ find $\vec{a} + \vec{b}$ and $\left|\vec{a} + \vec{b}\right|$

Solution: $\vec{a} = i + 2j + 3k$, $\vec{b} = 4i - j - 5k$

$\vec{a} + \vec{b} = (i + 2j + 3k) + (4i - j - 5k)$

$\vec{a} + \vec{b} = 5i + j - 2k$

$\left|\vec{a} + \vec{b}\right| = \sqrt{5^2 + 1^2 + 2^2}$

$\left|\vec{a} + \vec{b}\right| = \sqrt{30}$

16. If $\vec{a} = 2i - j + k$ and $\vec{b} = 3i + j - k$ find $\vec{a}.\vec{b}$

Solution: $\vec{a}.\vec{b} = (2i - j + k).(3i + j - k\ )$

$= 2 \times 3 - 1 \times 1 - 1 \times 1$

$\vec{a}.\vec{b} = 6 - 1 - 1 = 4$

17. Find the $\overrightarrow{AB}$ and its magnitude if $\vec{a} = (3,5,-6)\ and\ \vec{b} = (-4,2,0)$

Solution: $\overrightarrow{OA} = 3i + 5j - 6k$

$\overrightarrow{OB} = -4i + 2j$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = (-4i + 2j) - (3i + 5j - 6k)$

$\overrightarrow{AB} = -7i - 3j + 6k$

Magnitude of $\overrightarrow{AB} = \left|\overrightarrow{AB}\right| = \sqrt{-7^2 + -3^2 + 6^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{94}$

18. If $\vec{A} = (3, -4,2)$ *and* $\vec{B} = (-6,8,4)$ find position vector of $\vec{A}$ & $\vec{B}$ and also find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$

Solution: $\overrightarrow{OA} = 3i - 4j + 2k$

$\overrightarrow{OB} = -6i + 8j + 4k$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = (-6i + 8j + 4k) - (3i - 4j + 2k)$

$\overrightarrow{AB} = -9i + 12j + 2k$

Magnitude of $\overrightarrow{AB} = \left|\overrightarrow{AB}\right| = \sqrt{-9^2 + 12^2 + 2^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{229}$

19. If $\vec{a} = (7, -4,5)$ and $\vec{b} = (3,4, -7)$ find $\left|\vec{a} - \vec{b}\right|$

Solution: $\vec{a} - \vec{b} = (7i - 3j + 5k) - (3i + 4j - 7k)$

$\vec{a} - \vec{b} = 4i - 8j + 12k$

$\left|\vec{a} - \vec{b}\right| = \sqrt{(4)^2 + (-8)^2 + (12)^2}$

$\left|\vec{a} - \vec{b}\right| = \sqrt{324}$

$\left|\vec{a} - \vec{b}\right| = 18$

20. If $\vec{a} = i + 2j - 3k$, $\vec{b} = 3i - 5j + 2k$, find the magnitude of $3\vec{a} - 2\vec{b}$

Solution: $3\vec{a} = 3i + 6j - 9k$

$2\vec{b} = 6i - 10j + 4k$

$3\vec{a} - 2\vec{b} = (3i + 6j - 9k) - (6i - 10j + 4k)$

$= 3i + 16j - 13k$

$\left|3\vec{a} - 2\vec{b}\right| = \sqrt{3^2 + 16^2 + -13^2}$

$= \sqrt{9 + 256 + 169}$

$\left|3\vec{a} - 2\vec{b}\right| = \sqrt{434}$

21. If $\vec{a} = 4i + j + k$ and $\vec{b} = 3i + 4j + 5k$ find $\vec{a}.\vec{b}$

Solution: $\vec{a}.\vec{b} = (4i + j + k).(3i + 4j + 5k\ )$

$= 4 \times 3 + 1 \times 4 + 1 \times 5$

$\vec{a}.\vec{b} = 12 + 4 + 5 = 21$

V.ANSWER THE FOLLOWNG QUESTION 05 MARKS

1. Find AB its magnitude and unit vector along AB vector, if A= (3, 5,-6) and B = (-4, 2, 0)?

=> OA= $(3\hat{\imath}+5\hat{\jmath}-6\hat{k})$ and OB= $(-4\hat{\imath}+2\hat{\jmath}+o\hat{k})$

So, AB= OB-OA

$= (-4\hat{\imath}+2\hat{\jmath}+o\hat{k}) - (3\hat{\imath}+5\hat{\jmath}-6\hat{k})$

$= (-4\hat{\imath}+2\hat{\jmath}+o\hat{k} - 3\hat{\imath}-5\hat{\jmath}+6\hat{k})$

$= (-7\hat{\imath}-3\hat{\jmath}+6\hat{k})$

AB = $\sqrt{(-7)2 + (-3)2 + (6)2} = \sqrt{49 + 9 + 36} = \sqrt{94}$

Then unit vector, a^ $= \frac{\vec{a}}{|\vec{a}|} = \frac{-7\hat{\imath}-3\hat{\jmath}+6\hat{k}}{\sqrt{94}}$.

2. If $\vec{a}$ =2$\hat{\imath}$+3$\hat{\jmath}$, $\overrightarrow{b}$= -$\hat{\imath}$+5$\hat{\jmath}$, that are vectors ab, find $\vec{a} + \overrightarrow{b}$ and $2\vec{a} + 3\overrightarrow{b}$?what are the inverse of $2\vec{a} + 3\overrightarrow{b}$?

i) $\vec{a} + \overrightarrow{b}$= 2$\hat{\imath}$+3$\hat{\jmath}$-$\hat{\imath}$+5$\hat{\jmath}$ = $\hat{\imath}$+8$\hat{\jmath}$

ii) $2\vec{a} + 3\overrightarrow{b}$ = 2(2$\hat{\imath}$+3$\hat{\jmath}$) +3(-$\hat{\imath}$+5$\hat{\jmath}$)

= (4$\hat{\imath}$+6$\hat{\jmath}$)-(-$\hat{\imath}$+5$\hat{\jmath}$)

= (4$\hat{\imath}$+6$\hat{\jmath}$ - 3$\hat{\imath}$+15$\hat{\jmath}$)

= (1$\hat{\imath}$+21$\hat{\jmath}$)

3. ST the vectors 2$\hat{\imath}$+5$\hat{\jmath}$+6$\hat{k}$ and 7$\hat{\imath}$+2$\hat{\jmath}$+4$\hat{k}$ are orthogonal vectors?

=> $\vec{a} * \overrightarrow{b}$ =0

= (2$\hat{\imath}$+5$\hat{\jmath}$+6$\hat{k}$) $*$ (7$\hat{\imath}$+2$\hat{\jmath}$+4$\hat{k}$) =0

= 14+10-24

= 24-24

=0

4. Find the cosine of angle b\w the vector 6$\hat{\imath}$+2$\hat{\jmath}$-$\hat{k}$ and 3$\hat{\imath}$-4$\hat{\jmath}$+2$\hat{k}$?

=>cos Θ = $\vec{a}\ \overrightarrow{b}$ / $|\vec{a}|\ |\vec{b}|$

= (6$\hat{\imath}$+2$\hat{\jmath}$-$\hat{k}$)(3$\hat{\imath}$-4$\hat{\jmath}$+2$\hat{k}$) / $\sqrt{(6)2 + (2)2 + (-1)2}\ \sqrt{(3)2 + (-4)2 + (2)2}$

= 18-8-2 / $\sqrt{36 + 4 + 1}\ \sqrt{9 + 16 + 4}$

= 8/ $\sqrt{41}\ \sqrt{29}$

5. Find $\vec{a} * \overrightarrow{b}$ where $\vec{a}$= $2\hat{\imath}$+$3\hat{\jmath}$-$\hat{k}$, $\overrightarrow{b}$ =$3\hat{\imath}$-$2\hat{\jmath}$+$4\hat{k}$?

$$=> \vec{a} * \overrightarrow{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 3 & -2 & 4 \end{vmatrix}$$

$$i \begin{vmatrix} 3 & -1 \\ -2 & 4 \end{vmatrix} -j \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} +k \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$$

= i (12-2) –j (8+3) k (-4-9) = 10i -11j+ 13k.

6. Find $\vec{a} * \overrightarrow{b}$ where $\vec{a}$= $3\hat{\imath}$-$2\hat{\jmath}$+$4\hat{k}$, $\overrightarrow{b}$ = $2\hat{\imath}$+$3\hat{\jmath}$-$\hat{k}$?

$$=> \vec{a} * \overrightarrow{b} = \begin{vmatrix} i & j & k \\ 3 & -2 & 4 \\ 2 & 3 & 0 \end{vmatrix}$$

$$i \begin{vmatrix} -2 & 4 \\ 3 & 0 \end{vmatrix} -j \begin{vmatrix} 3 & 4 \\ 2 & 0 \end{vmatrix} +k \begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix}$$

= i (0-12) –j (0-8) +k (9+4) = -12i – (-8j) +13k.

7. Explain dot product and cross product.

Ans: Dot product: let $\vec{a}$ & $\overrightarrow{b}$ be two non zero vectors. The scalar product is dot product of $\vec{a}$ & $\overrightarrow{b}$ is written as $\vec{a}$. $\overrightarrow{b}$ and is equal to product of magnitude of $\vec{a}$ & $\overrightarrow{b}$ are the cosine of the angle Θ b\w them.

Cross product: It is denoted by $\vec{a} * \overrightarrow{b}$ are the vectors $|\vec{a}| \left|\overrightarrow{b}\right|$ sinΘ n^, is the angle b\w the vectors $\vec{a}$ & $\overrightarrow{b}$, s.t $\vec{a}$. $\overrightarrow{b}$, ^n form a right handed system. ie; $\vec{a} * \overrightarrow{b} = |\vec{a}|\left|\overrightarrow{b}\right|$ sinΘ n^.

8. Explain magnitude of vector with one example?

Ans: it is the length of the vector. The magnitude of the vector is denoted as$|\vec{a}|$.

Formulas for the magnitude of vectors in two and three dimensions in terms of their coordinates are derived.

Ie; $|\vec{a}|= \sqrt{a2 + b2}$,for two dimensional vector

$|\vec{a}|=\sqrt{a2+b2+c2}$, for three dimensional vectors.

Ex: Find the magnitude $\vec{A} = 4\hat{\imath} + 3\hat{\jmath}$ -2$\hat{k}$?

$= \sqrt{(4)2 + (3)2 + (4)2}$

$= \sqrt{16+9+4}$

$=\sqrt{29}$

9. If $\vec{a} = i + 2j - 3k$, $\vec{b} = 3i - 5j + 2k$ find the magnitude of $3\vec{a} - 2\vec{b}$

Solution: $3\vec{a} - 2\vec{b} = 3(i + 2j - 3K) - 2(3i - 5j + 2k)$

$3\vec{a} - 2\vec{b} = (3i + 6j - 9K) - (6i - 10j + 4k)$

$3\vec{a} - 2\vec{b} = 3i + 16j - 13k$

$\left|3\vec{a} - 2\vec{b}\right| = \sqrt{(-3)^2 + (16)^2 + (-13)^2}$

$= \sqrt{9 + 256 + 169}$

$= \sqrt{434}$

10. Find the value of, if the vectors $3i + 2j - 5k$ and $6i + \lambda j + 2k$ are orthogonal vectors

Solutions: Let$\vec{a} = 3i + 2j - 5k$, $\vec{b} = 6i + \lambda j + 2k$

For orthogonal vectors, $\vec{a}.\vec{b} = 0$

$(3i + 2j - 5k).(6i + \lambda j + 2k) = 0$

$18+2\lambda+10 = 0$

$2\lambda = -28$

$\lambda = -14$

11. If $\vec{a} = i + j + 2k$ and $\vec{b} = 2i - j + k$ then show that $(\vec{a} + \vec{b})$ is perpendicular to $(\vec{a} - \vec{b})$

Solution: $\vec{a} + \vec{b} = (i + j + 2k) + (2i - j + k)$

$$\vec{a}+\vec{b}=3i+0+3k$$

$$\vec{a}-\vec{b}=\ -i+2j+k$$

Consider $(\vec{a}+\vec{b})\ (\vec{a}-\vec{b})=(3i+3k).(-i+2j+k)$

$$=-3+3=0$$

Hence $(\vec{a}+\vec{b})$ perpendicular $(\vec{a}-\vec{b})$

12. find the cosine of the angle between the vectors 6i+2j-k and 3i-4j+2k

Solution: $\vec{a}.\vec{b}=(6i+2j-k).(3i-4j+k)$

$$=18-8-1$$

$$\vec{a}.\vec{b}=9$$

$|\vec{a}|=\sqrt{6^2+2^2+(-1)^2}=\sqrt{41}$

$|\vec{b}|=\sqrt{3^2+(-4)^2+(1)^2}=\sqrt{26}$

$$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta=\frac{9}{\sqrt{41}.\sqrt{26}}$$

13. Find the moment of Force $\vec{F}=4i+2j-k$ acting at a point $A(1,2,-3)$ about the point $B(2.-3,4)$

Solution: $\vec{F}=4i+2j-k$

$$\overrightarrow{BA}=(1-2)i+(2+3)j+(-3-4)k$$

$$\overrightarrow{BA}=-i+5j-7k$$

Moment of $\vec{F}$about $\vec{B}=\overrightarrow{BA}\times\vec{F}$

$$=\begin{vmatrix} i & j & k \\ -1 & 5 & -7 \\ 4 & 2 & -1 \end{vmatrix}$$

Moment of $\vec{F}$about $\vec{B}=9i-29j-22k$

14. Find $\left|\vec{a} \times \vec{b}\right|$ If $\vec{a} = i - 7j + 7k$, $\vec{b} = 3i - 2j + 2k$

Solution: $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$

$\vec{a} \times \vec{b} = i(-14 + 14) - j(2 - 21) + k(-2 + 21)$

$= i \times 0 + 19j + 19k$

$\vec{a} \times \vec{b} = 19j + 19k$

$\left|\vec{a} \times \vec{b}\right| = \sqrt{19^2 + 19^2} = 19\sqrt{2}$

15. A force $\vec{F} = 2i + j - 2k$ acting on a particle at $(3,2,2)$ displaces it to the point $(1,3,-1)$ find the work done

Solution: Let $\overrightarrow{OA} = 3i + 2j - 2k$, $\overrightarrow{OB} = i + 3j - k$

$S = OB - OA$

$=(i + 3j - k) - (3i + 2j - 2k)$

S$= -2i + j - 3k$

Work done $= \vec{F}.\vec{S}$

$= (2i + j - 2k)(-2i + j - 3k)$

$= -4 + 1 + 6$

$= 3\ units$

16. Find the work done by force $\vec{F} = 2i + 3j + k$ in moving a body from $i + 2j - 4k\ to\ 2i + 5j + 3k$

Solution: $\overrightarrow{OA} = i + 2j - 4k$

$\overrightarrow{OB} = 2i + 5j + 3k$

$\overrightarrow{AB} = OB - OA$

$\overrightarrow{AB}$ =$(2i + 5j + 3k) - (i + 2j - 4k)$

$\overrightarrow{AB} = i + 3j + 7k$

$Work\ done = \vec{F}.\vec{B}$

$= (2i + 3j + k).(i + 3j + 7k)$

$Work\ done = 2 + 9 + 7$

$Work\ done = 18\ units$

17. Find the projection of $\vec{b}\ on\ \vec{a}$ if $\vec{a} = 2i - j + 4k$ & $\vec{b} = 6i + 3j - 7k$

Solution: $Projection\ \vec{b}\ on\ \vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|}$

$= \frac{(2i-j+4k\ ).(6i+3j-7k)}{\sqrt{(2)^2+(-1)^2+4^2}}$

$= \frac{12-3-28}{\sqrt{21}}$

$Projection\ \vec{b}\ on\ \vec{a} = \frac{-19}{\sqrt{21}}$

18. Find the projection $\vec{a}\ on\ \vec{b}$ if $\vec{a} = i + 2j - 3k$ & $\vec{b} = 3i - 5j + 4k$

Solution: $Projection\ \vec{a}\ on\ \vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \frac{(i+2j-3k\ ).(3i-5j+4k)}{\sqrt{(3)^2+(-5)^2+4^2}}$

$= \frac{3-10-12}{\sqrt{50}}$

$Projection\ \vec{b}\ on\ \vec{a} = \frac{-19}{\sqrt{50}}$

19. Show that the vectors $3i - 2j + k, i - 3j + 5k$ & $2i + j - 4k$ from a right angled triangle

Solution: Let $\vec{a} = 3i - 2j + k, =\ i - 3j + 5k, \vec{c} = 2i + j - 4k$

Therefore $\vec{a}$, $\vec{b}$, $\vec{c}$ form the sides of triangle

Now, $\vec{a} = \sqrt{9+4+1} = \sqrt{14}$

$\vec{b} = \sqrt{1+9+25} = \sqrt{35}$

$\vec{c} = \sqrt{4+1+16} = \sqrt{21}$

$|\vec{b}|^2 = |\vec{a}|^2 + |\vec{c}|^2$

$35 = 14 + 21$

$35 = 35$

$\therefore$ The vectors form right angled triangle.

20. Find the cosine of angle between the vectors where $\vec{a} = 2i + 3j - k$ & $\vec{b} = 3i - 2j + 4k$

Solution: Let θ be the angle between them

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(2i+3j-k).(3i-2j+4k)}{\sqrt{4+9+1}\sqrt{9+4+16}}$$

$$\cos\theta = \frac{-4}{\sqrt{14}\sqrt{29}}$$

$$\cos\theta = \frac{-4}{\sqrt{406}}$$

21. If vectors $\lambda i + 5j - 6k$ and $7i + 2j + 4k$ are orthogonal find λ

Solution: Orthogonal vectors

$$\vec{a}.\vec{b} = 0$$

$(\lambda i + 5j - 6k).(7i + 2j + 4k) = 0$

$7\lambda + 10 - 24 = 0$

$7\lambda = 14$

$\therefore \lambda = 2$

VI.ANSWER THE FOLLOWING QUESTIONS 08 MARKS

1. Find the projection of $\overrightarrow{b}$ on $\vec{a}$, if $\vec{a} = 5\hat{\imath}+2\hat{\jmath}-4\hat{k}$ and $\overrightarrow{b}=2\hat{\imath}-5\hat{\jmath}+6\hat{k}$?

   Ans : $\vec{a}\,.\,\overrightarrow{b} = (5\hat{\imath}+2\hat{\jmath}-4\hat{k})*(2\hat{\imath} - 5\hat{\jmath} + 6\hat{k})$

   $= (10-10-24)$

   $= -24$

   So, $|\vec{a}| = \sqrt{5^2 + 2^2 + (-4)^2} = \sqrt{25 + 4 + 16} = \sqrt{45}$

   Projection of $\overrightarrow{b}$ on $\vec{a}$ is, $\frac{\vec{a}\,.\overrightarrow{b}}{|\vec{a}|} = \frac{-24}{\sqrt{45}}$.

2. If the vectors of a triangle have position vector i) $4\hat{\imath}+5\hat{\jmath}+6\hat{k}$ ii) $5\hat{\imath}+6\hat{\jmath}+4\hat{k}$ iii) $6\hat{\imath}+4\hat{\jmath}+5\hat{k}$, prove that triangle is a equilateral triangle?

=> OA= $4\hat{\imath}+5\hat{\jmath}+6\hat{k}$, OB=$5\hat{\imath}+6\hat{\jmath}+4\hat{k}$, OC= $6\hat{\imath}+4\hat{\jmath}+5\hat{k}$

*) AB =OB-OA

$= (5\hat{\imath}+6\hat{\jmath}+4\hat{k}) - (4\hat{\imath}+5\hat{\jmath}+6\hat{k})$

$= (5\hat{\imath}+6\hat{\jmath}+4\hat{k} - 4\hat{\imath}-5\hat{\jmath}-6\hat{k})$

$= 1\hat{\imath}+1\hat{\jmath}-2\hat{k}$

*) BC= OC-OB

$= (6\hat{\imath}+4\hat{\jmath}+5\hat{k}) - (5\hat{\imath}+6\hat{\jmath}+4\hat{k})$

$= (6\hat{\imath}+4\hat{\jmath}+5\hat{k} - 5\hat{\imath}-6\hat{\jmath}-4\hat{k})$

$= 1\hat{\imath}\text{-}2\hat{\jmath}+1\hat{k}$

*) CA=OA-OC

$= (4\hat{\imath}+5\hat{\jmath}+6\hat{k}) - (6\hat{\imath}+4\hat{\jmath}+5\hat{k})$

$= (4\hat{\imath}+5\hat{\jmath}+6\hat{k} - 6\hat{\imath}\text{-}4\hat{\jmath}\text{-}5\hat{k})$

$=\text{-}2\hat{\imath}+1\hat{\jmath}+1\hat{k}$

Ie: $AB = \sqrt{(1)2 + (1)2 + (2)2}=\sqrt{6}$

BC= $\sqrt{(1)2 + (-2)2 + (1)2}=\sqrt{6}$

CA= $\sqrt{(-2)2 + (1)2 + (1)2}=\sqrt{6}$

Ie; AB =BC=CA.

3. PT the points where position vector $4\hat{\imath}+1\hat{\jmath}+3\hat{k}$, $\hat{\imath}+3\hat{\jmath}+2\hat{k}$, $2\hat{\imath}+0\hat{\jmath}+7\hat{k}$,are vectors of a right angle triangle?

=>

OA=$4\hat{\imath}+1\hat{\jmath}+3\hat{k}$, OB=$\hat{\imath}+3\hat{\jmath}+2\hat{k}$, OC= $2\hat{\imath}+0\hat{\jmath}+7\hat{k}$

*) AB =OB-OA

$= (\hat{\imath}+3\hat{\jmath}+2\hat{k})$ - $(4\hat{\imath}+1\hat{\jmath}+3\hat{k})$

$= (\hat{\imath}+3\hat{\jmath}+2\hat{k}\text{-} 4\hat{\imath}\text{-}1\hat{\jmath}\text{-}3\hat{k})$

$= \text{-}3\hat{\imath}\text{-}2\hat{\jmath}\text{-}1\hat{k}$

*) BC= OC-OB

$= (2\hat{\imath}+0\hat{\jmath}+7\hat{k})\text{-}(\hat{\imath}+3\hat{\jmath}+2\hat{k})$

$= (2\hat{\imath}+0\hat{\jmath}+7\hat{k}\text{-} \hat{\imath}\text{-}3\hat{\jmath}\text{-}2\hat{k})$

$= \hat{\imath}$-3$\hat{\jmath}$-5$\hat{k}$

*) CA=OA-OC

$= (4\hat{\imath}+1\hat{\jmath}+3\hat{k}) - (2\hat{\imath}+0\hat{\jmath}+7\hat{k})$

$= (4\hat{\imath}+1\hat{\jmath}+3\hat{k} - 2\hat{\imath}-0\hat{\jmath}-7\hat{k})$

=2$\hat{\imath}$-1$\hat{\jmath}$+-4$\hat{k}$

Ie: $AB = \sqrt{(3)2 + (2)2 + (-1)2} = \sqrt{9+4+1} = \sqrt{14}$

BC= $\sqrt{(1)2 + (3)2 + (5)2} = \sqrt{1+9+25} = \sqrt{35}$

CA= $\sqrt{(2)2 + (1)2 + (4)2} = \sqrt{4+1+16} = \sqrt{21}$

Ie; $AB^2+CA^2 = BC^2 = (\sqrt{14})^2 + (\sqrt{21})^2 = (\sqrt{35})$

=> 14+21=35

=> 35 =35

Hence it is right angle triangle.

4. Examine whether the following force are in equilibrium? A) 3$\hat{\imath}$-5$\hat{\jmath}$+7$\hat{k}$, -2$\hat{\imath}$+$\hat{\jmath}$+$\hat{k}$, -4$\hat{\jmath}$-5$\hat{k}$?

=> F =$F^1+F^2+F^3$

= (3$\hat{\imath}$-5$\hat{\jmath}$+7$\hat{k}$) + (-2$\hat{\imath}$+$\hat{\jmath}$+$\hat{k}$) + (-4$\hat{\jmath}$-5$\hat{k}$)

= (3$\hat{\imath}$-5$\hat{\jmath}$+7$\hat{k}$-2$\hat{\imath}$+$\hat{\jmath}$-$\hat{k}$-4$\hat{\jmath}$-5$\hat{k}$)

= 1$\hat{\imath}$-8$\hat{\jmath}$+1$\hat{k}$.

5. Find the perpendicular to both vectors $\vec{a}$= 3$\hat{\imath}$+2$\hat{\jmath}$-4$\hat{k}$, $\vec{b}$ = 2$\hat{\imath}$-$\hat{\jmath}$+5$\hat{k}$?

$$=> \vec{a} * \overrightarrow{b} = = \begin{vmatrix} i & j & k \\ 3 & 2 & -4 \\ 2 & -1 & 5 \end{vmatrix}$$

$$i \begin{vmatrix} 2 & -4 \\ -1 & 5 \end{vmatrix} - j \begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix} + k \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}$$

= i (10-4) –j (15+8)+ k (-3-4) = 6i -23j-7k

Now, $|\vec{a} * \overrightarrow{b}| = \sqrt{(6)2 + (-23)2 + (-7)2} = \sqrt{36 + 529 + 49} = \sqrt{614}$.

6. Explain all the types of vectors?

=> The types of vectors are given below;

*) null vector

*) unit vector

*) equal vector

*) negative vector

*) co-initial vector

*) collinear vector

*) co-planer vector

*) proper vector

*) position vector.

7. If $\vec{a} = 3\hat{\imath}+4\hat{\jmath}+5\hat{k}$, $\overrightarrow{b} = 4\hat{\imath}+5\hat{\jmath}+6\hat{k}$, find $\vec{a}*\overrightarrow{b}$?

$$\vec{a} * \overrightarrow{b} = \begin{vmatrix} i & j & k \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$$

$$\vec{a} * \overrightarrow{b} = \begin{matrix} i & j & k \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{matrix}$$

$$\mathbf{i}\begin{vmatrix}4 & 5\\5 & 6\end{vmatrix} - j\begin{vmatrix}3 & 5\\4 & 6\end{vmatrix} + k\begin{vmatrix}3 & 4\\4 & 5\end{vmatrix}$$

= i (24-25) –j (18-20) + k (15-16) = -1i +2j-1k

Now, $|\vec{a} * \vec{b}| = \sqrt{(-1)2 + (2)2 + (-1)2} = \sqrt{1+4+1} = \sqrt{6}$.

8. Find the projection of $\vec{a}$ on $\vec{b}$ and also projection of $\vec{b}$ on $\vec{a}$

Where $\vec{a} = 2i + 3j - k$ and $\vec{b} = 3i - 2j + 4k$

Solution: Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{(2i+3j-k)(3i-2j+4k)}{\sqrt{3^2+(-2)^2+4^2}}$$

$$= -\frac{4}{\sqrt{29}}$$

Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|}$

$$= \frac{(2i+3j-k)(3i-2j+4k)}{\sqrt{2^2+3^2+(-1)^2}}$$

$$= \frac{(2i+3j-k)(3i-2j+4k)}{\sqrt{4+9+1}}$$

$$= -\frac{4}{\sqrt{14}}$$

9. Find $|\vec{a} \times \vec{b}|$ and $|\vec{b} \times \vec{a}|$ where $\vec{a} = 2i + 3j - k$, $\vec{b} = 3i - 2j + 4k$

Solution: $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 3 & -4 & 4 \end{vmatrix}$

$$= i(12 - 2) - j(8 + 3) + k(-4 - 9)$$

$$\vec{a} \times \vec{b} = 10i - 11j - 13k$$

$$|\vec{a} \times \vec{b}| = \sqrt{10^2 + (-11)^2 + (-13)^2}$$

$$|\vec{a} \times \vec{b}| = \sqrt{100 + 121 + 169}$$

$$|\vec{a} \times \vec{b}| = \sqrt{390}$$

$$\vec{b} \times \vec{a} = \begin{vmatrix} i & j & k \\ 3 & -2 & 4 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= i(2-12) - j(-3-8) + k(9+4)$$

$$\vec{b} \times \vec{a} \ = -10i + 11j + 13k$$

$$\left|\vec{b} \times \vec{a}\right| = \sqrt{10^2 + (-11)^2 + (-13)^2}$$

$$\left|\vec{b} \times \vec{a}\right| = \sqrt{100 + 121 + 169}$$

$$\left|\vec{b} \times \vec{a}\right| = \sqrt{390}$$

10. Find the work done by the force $(4i - 3j + 2k)$ in moving a particle from the point $A(5,4,1)$ to $B(4,1,6)$

Solution: $\vec{F} = 4i - 3j + 2k$

$$\overrightarrow{OA} = 5i + 4j + k$$

$$\overrightarrow{OB} = 4i + j + 6k$$

$$\vec{r} = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\vec{r} = \overrightarrow{AB} = (4-5)i + (1-4)j + (6-1)k$$

$$\vec{r} = -i - 3j + 5k$$

Work done, $W = \vec{F}.\vec{r}$

$$W = (4i - 3j + 2k)(-i - 3j + 5k)$$

$$W = 15$$

11. If $\vec{a} = 2i - 3j + k, \vec{b} = i + 4j - 2k$ find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ also find the sine of angle between the vectors.

Solution: $|\vec{a}| = \sqrt{4+9+1} = \sqrt{14}$

$$\left|\vec{b}\right| = \sqrt{1 + 16 + 4} = \sqrt{21}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 1 & 4 & -2 \end{vmatrix}$$

$\vec{a} \times \vec{b} = i(6-4) - j(-4-1) + k(8+3)$

$\vec{a} \times \vec{b} = 2i + 5j + 11k$

$\left|\vec{a} \times \vec{b}\right| = \sqrt{4+25+121} = \sqrt{150}$

Unit vector $\perp^{r}$ to $\vec{a}$ & $\vec{b} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$$= \frac{1}{\sqrt{150}}(2i + 5j + 11k)$$

$$\sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{|a||b|}$$

$$\sin\theta = \frac{\sqrt{150}}{\sqrt{14}\sqrt{21}} = \frac{\sqrt{25\times 6}}{\sqrt{49\times 6}}$$

$\sin\theta = \frac{5}{7}$

12. Find the sine of the angle between the vectors $i + j - k$ and $2i + 3j + 4k$

Solution: $|\vec{a}| = \sqrt{16+4+9} = \sqrt{29}$

$\left|\vec{b}\right| = \sqrt{4+9+16} = \sqrt{29}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 4 & -2 & -3 \\ 2 & -3 & 4 \end{vmatrix}$$

$\vec{a} \times \vec{b} = 7i - 6j + k$

$\left|\vec{a} \times \vec{b}\right| = \sqrt{49+36+1} = \sqrt{86}$

$$\sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{|a||b|}$$

$\sin\theta = \frac{\sqrt{86}}{\sqrt{3}\sqrt{29}}$

13. Find the cosine of the angle between $4i - 2j - 3k$ & $2i - 3j + 4k$

Solution: Consider, $\vec{a}.\vec{b} = (4i - 2j - 3k).(2i - 3j + 4k)$

$$= 8 + 6 - 12$$

$$\vec{a}.\vec{b} = 2$$

$$|\vec{a}| = \sqrt{4^2 + (-2)^2 + (-3)^2}$$

$$|\vec{a}| = \sqrt{29}$$

$$|\vec{b}| = \sqrt{2^2 + (-3)^2 + (4)^2}$$

$$|\vec{b}| = \sqrt{29}$$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta = \frac{2}{\sqrt{29\sqrt{29}}}$$

$$\cos\theta = \frac{2}{29}$$

14. Find the projection of $\vec{b}$ on $\vec{a}$ and $\vec{a}$ on $\vec{b}$ if $\vec{a} = 5i + 2j - 4k$ & $\vec{b} = 2i - 5j + 6k$

Solution: $\vec{a} = 5i + 2j - 4k$, $\vec{b} = 2i - 5j + 6k$

$$|\vec{a}| = \sqrt{5^2 + 2^2 + (-4)^2} = \sqrt{45} = 3\sqrt{5}$$

$$|\vec{b}| = \sqrt{2^2 + (-5)^2 + (6)^2} = \sqrt{65}$$

$$\vec{a}.\vec{b} = (5i + 2j - 4k).(2i - 5j + 6k)$$

$$= 10 - 10 - 24$$

$$\vec{a}.\vec{b} = -24$$

Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|}$

$$= \frac{-24}{3\sqrt{5}}$$

Projection of $\vec{b}$ on $\vec{a} = \frac{-8}{\sqrt{5}}$

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{-24}{\sqrt{65}}$$

Projection of $\vec{a}$ on $\vec{b} = \frac{-24}{\sqrt{65}}$

15. Find the unit vector perpendicular to $\vec{a} = 2i - j + k$ & $\vec{b} = -i - 3j - 4k$

Solution: $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ -1 & -3 & -4 \end{vmatrix}$

$\vec{a} \times \vec{b} = i(4+3) - j(8+1) + k(-6-1)$

$\vec{a} \times \vec{b} = 7i + 7j - 7k$

$|\vec{a} \times \vec{b}| = \sqrt{49 + 49 + 49}$

$= 7\sqrt{3}$

Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$$= \frac{7(i + j - k)}{7\sqrt{3}} = \frac{i + j - k}{\sqrt{3}}$$

16. Find the unit vector in the direction perpendicular to the vectors $2i - 5j + k$ and $5i + j + 7k$

Solution: $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -5 & 1 \\ 5 & 1 & 7 \end{vmatrix}$

$\vec{a} \times \vec{b} = i(-35 - 1) - j(14 - 5) + k(2 + 25)$

$\vec{a} \times \vec{b} = -36i - 9j + 27k$

$|\vec{a} \times \vec{b}| = \sqrt{(-36)^2 + (-9)^2 + 27^2}$

$= \sqrt{1296 + 81 + 729}$

$|\vec{a} \times \vec{b}| = \sqrt{2106}$

Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$$= \frac{-36i - 9j + 27k}{\sqrt{2106}}$$

17. Find the sine of the angle between the vectors $\vec{a} = 3i + 2j - 4k$ and $\vec{b} = 2i - j + 5k$

Solution: $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & 2 & -4 \\ 2 & -1 & 5 \end{vmatrix}$

$$= i(10 - 4) - j(15 + 8) + k(-3 - 4)$$

$$= 6i - 23j - 7k$$

$|\vec{a} \times \vec{b}| = \sqrt{(6)^2 + (-23)^2 + (-7)^2} = \sqrt{614}$

$|\vec{a}| = \sqrt{9 + 4 + 16} = \sqrt{29}$

$|\vec{b}| = \sqrt{4 + 1 + 25} = \sqrt{30}$

$|\vec{a}||\vec{b}| = \sqrt{870}$

If θ is angle between $\vec{a}$ and $\vec{b}$, then

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$\sin\theta = \frac{\sqrt{614}}{\sqrt{870}}$$

18. Find the sine of the angle between the vectors $\vec{a} = i + 2j + 3k$ and $\vec{b} = 3i - 2j + k$

Solution: $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

$|\vec{a}| = \sqrt{1 + 4 + 9} = \sqrt{14}$

$|\vec{b}| = \sqrt{9 + 4 + 1} = \sqrt{14}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$

$$= i(2 + 6) - j(1 - 9) + k(-2 - 6)$$

$$\vec{a} \times \vec{b} = 8i + 8j - 8k$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{64 + 64 + 64} = \sqrt{192}$$

$$\sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{|\vec{a}|\left|\vec{b}\right|} = \frac{\sqrt{192}}{\sqrt{14}\sqrt{14}}$$

$$\sin\theta = \frac{\sqrt{192}}{14}$$

19. A force $\vec{F} = 2i + j + k$ acting at a point (-3, 2, 1). Find the magnitude of moment of force F about the point (2, 1, 2)

Solution: $\vec{F} = 2i + j + k$

$$\overrightarrow{AB} = (2i + j + 2k) - (-3i + 2j + k)$$

$$\overrightarrow{BA} = 5i - j + k$$

Moment of force $= \overrightarrow{BA} \times \vec{F}$

$$= \begin{vmatrix} i & j & k \\ 5 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= i(-1 - 1) - j(5 - 2) + k(5 + 2)$$

$$= -2i - 3j + 7k$$

Magnitude $= \sqrt{(-2)^2 + (-3)^2 + (7)^2}$

$$= \sqrt{4 + 9 + 49}$$

Magnitude$= \sqrt{62}$