AE667-Assignment2

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Contents

1 Status of program

Program is working fine.

2 Starting Data

2.1 Drone Design Parameters

- Weight of 4 motor and blades : 2.5kg
- Weight of Propeller: 1kg
- Weight of Battery: 3.5kg
- structural weight: 3kg
- Radius of motor: 0.3m
- Number of blades: 3
- Thrust: 220N

- a: 5.75
- $C_l = a\alpha$
- $C_d = 0.0113 + 1.25\alpha^2$

2.2 Assumptions

- We will be using Blade Element Momentum theory for power and thrust calculations here so all assumptions in momentum theory are taken here too
- Apply Momentum Theory on a strip of the disk and Apply BE theory on the strips of the blades and combining them
- The atmospheric condtions are normal without large disturbance
- No Swirl
- compressibility effects and stalling are ignored
-

3 Approach, Theory and Implementation

3.1 Version 1: Without the Prandtl Tip Loss model

1. We started program with defining parameters such as density, Forward velocity, no. of blades and rpm. Then we defined blade dimension Radius, Root cut out and cord length.
2. Defined Taper, Twist and solidity functions such that we can vary them linearly by giving slope and initial value. We can set them constant too by setting slope to zero.
3. Defined lamda function for calculating value of λ for given any r value.
   where $\lambda = \sqrt{(\frac{\sigma a}{16} - \frac{\lambda_c}{2})^2 + \frac{\sigma a \theta r}{8R}} - (\frac{\sigma a}{16} - \frac{\lambda_c}{2})$
   $\lambda = \frac{V+v}{\Omega R}$
   $\lambda_c = \frac{V}{\Omega R}$
   σ is solidity
   a is lift curve slope
   θ is twist value
   For σ and θ calculation we use the function defined in point 1 and 2. our function take r as input and lambda (r) return λ value at that r

4. defining function for calculating lambda

```
#Will return the value of lambda for given r
def lamda(r):
    lambda_c = V/(omega*R)                          #lambda_c value
    temp1 = a*solidity(r)/16 - lambda_c/2
    temp2 = (solidity(r)*a*linear_twist(r)*r)/(8*R)
    temp3 = solidity(r)*a/16 - lambda_c/2
    ans = (temp1**2 + temp2)**0.5 - temp3
    return ans
```

4. Defined functions for calculating the Lift and Drag coefficients named as Cl and Cd.
   $C_l = a\alpha_{effective}$
   $C_d = C_{d(min)} + \epsilon\alpha^2_{effective}$
   where $\alpha_{effective} = \theta - atan(\frac{\lambda R}{r})$
   For Calculating C_l and C_d we need r value and functions for calculation of λ and θ which are defined above

5. Next we defined function for calculation of Thrust and Torque calculation on element of blade named as dT and dQ respectively.
   $dT = b * 0.5 * \rho * (U_T^2 + U_P^2) * c * (C_l Cos(\phi) - C_d Sin(\phi))$
   $dQ = b * 0.5 * \rho * r * (U_T^2 + U_P^2) * c * (C_d Cos(\phi) + C_l Sin(\phi))$
   Where $U_T = \Omega r$, $U_P = \lambda R \Omega$ and $\phi = atan(\frac{\lambda R}{r})$
   For other calculations such as C_l,C_d,λ,c we have used functions defined above named as Cl,Cd,lamda,linear-taper respectively for given value of r **Code Snippet**

```
#Function for calculating Thrust on element of blade at r
def dT(r):
    U_T = omega*r
    U_P = lamda(r)*R*omega
    theta_r = linear_twist(r)
    C_r = linear_taper(r)
    phi = math.atan(lamda(r)*R/r)
    ans = 0.5*rho*b*(U_T**2 + U_P**2)*C_r*(Cl(r)*math.cos(phi) - Cd(r)*math.sin(phi))
    return ans

#Function for calculating Torque on element of blade at r
def dQ(r):
    U_T = omega*r
    U_P = lamda(r)*R*omega
    theta_r = linear_twist(r)
    C_r = linear_taper(r)
    phi = math.atan(lamda(r)*R/r)
    ans = 0.5*rho*r*b*(U_T**2 + U_P**2)*C_r*(Cl(r)*math.cos(phi) + Cd(r)*math.sin(phi))
    return ans
```

6. For calculating Total Thrust and Total Torque we have used the quad function from scipy.integrate for integrating dT and dQ respectively.

We integrated dT and dQ for variable r from Rrc to R using quad fucntion
after calculating total torque and thrust we have calculated C_T and C_Q using the fromula as:
$C_T = \frac{T}{\rho \pi \Omega^2 R^4}$
$C_Q = \frac{Q}{\rho \pi R^5 \Omega^2}$
Now for Total power P we have $P = \Omega Q$
and $C_P = C_Q$

7. Now for calculating C_T and C_Q for any given value of twist(theta0) we just have defined parameters named: rho, V, blades, rpm, R,Rrc, chord length=C0(for constant taper(C1=0)), C0 and C1(for linear taper), a, Cdmin, eps and then run the point number 1 to 7 under Part a: Without the Prandtl Tip loss Model.

   For linear twist we have to specify the value of theta1 also which is slope of theta vs r with all these parameters.

3.2 Version 2: With the Prandtl Tip Loss model

1. Point number 1 and 2 will remain same as defined for above part(Without Tip Loss model)

2. We defined two functions named f and Factor for calcualting the value for f and F for any given value of r and lamda(which is value of λ in previous iteration.
   Expression of f and F are :
   $F = (\frac{2}{\pi}) * cos^{-1}(e^{-f})$
   $f = \frac{b}{2}\frac{(1-\frac{r}{R})}{\lambda}$
   for calculating f and F we need to give value of r and λ(as real number which is λ value in previous iteration)

3. This part is the main difference between Tip loss and without Tip loss model. In this part we defined the function for calculating the value of λ for given r value.
   Here expression for the λ is:
   $\lambda = \sqrt{(\frac{\sigma a}{16F} - \frac{\lambda_c}{2})^2 + \frac{\sigma a \theta r}{8RF}} - (\frac{\sigma a}{16F} - \frac{\lambda_c}{2})$
   $\lambda = \frac{V+v}{\Omega R}$
   $\lambda_c = \frac{V}{\Omega R}$
   σ is solidity
   a is lift curve slope
   θ is twist value
   For calculating we have used iterative approach. We have defined the function find_lamda_r(r) which is give the value of λ at given r for Tip loss model.

Approach: We have defined some starting λ value lambda0 as 10(arbitrary value) and F value 1. Using this F value we have calculated new λ named as temp_lamda. Now if difference between lambda0 and temp_lambda is more than 10^{-8} we will calculate the F value using temp_lambda and set lambda0 as temp_lamda. Now we will iterate this till the value of lambda0 and temp_lambda will have same eight decimal digits. if we encounter the zero value of temp_lambda we will break loop with output given as zero.

Code Snippet

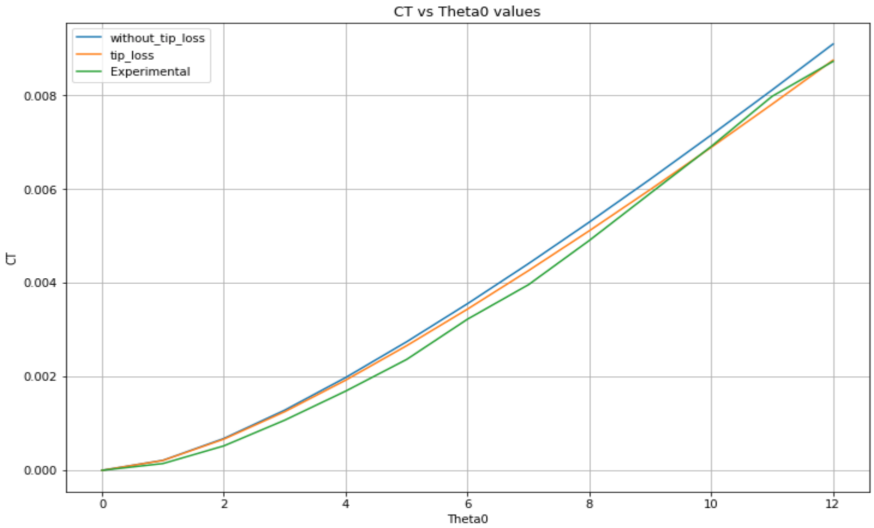
```
#Function for calculating value of lambda after taking tip loss into account
def find_lamda_r(r):
    lamda_0 = 10
    delta = 100
    F=1
    while delta > 10**-8 :
        lambda_c = V/(omega*R)
        temp1 = a*solidity(r)/(16*F) - lambda_c/2
        temp2 = (solidity(r)*a*linear_twist(r)*r)/(8*R*F)
        temp3 = solidity(r)*a/(16*F) - lambda_c/2
        temp_lamda = (temp1**2 + temp2)**0.5 - temp3
        if temp_lamda == 0:
            break
        delta = abs(lamda_0 - temp_lamda)
        F = Factor(r,temp_lamda)
        #print(F)
        lamda_0 = temp_lamda
    return temp_lamda
```

4. Now we have defiend the function for calculating the value for C_l, C_d,dT and dQ at any given r value named as Cl_tip_loss(r), Cd_tip_loss(r), dT_tip_loss(r) and dQ_tip_loss(r). These functions are remains same as we defined in above part except the value of λ which will be replaced by λ calculated using tip loss model as described in point 3.

5. Now for calculating Total Thrust and Total Torque we have used the quad function from scipy.integrate for integrating dT and dQ respectively. for dT and dQ we use the function defined in the point 4 which are dT_tip_loss(r) and dQ_tip_loss(r) respectively. C_T and C_Q expression also remains same as we defined for without Tip Loss model but here expression of λ is changed which indirectly appear in C_T and C_Q expression due its effect on total thrust and torque calculation.

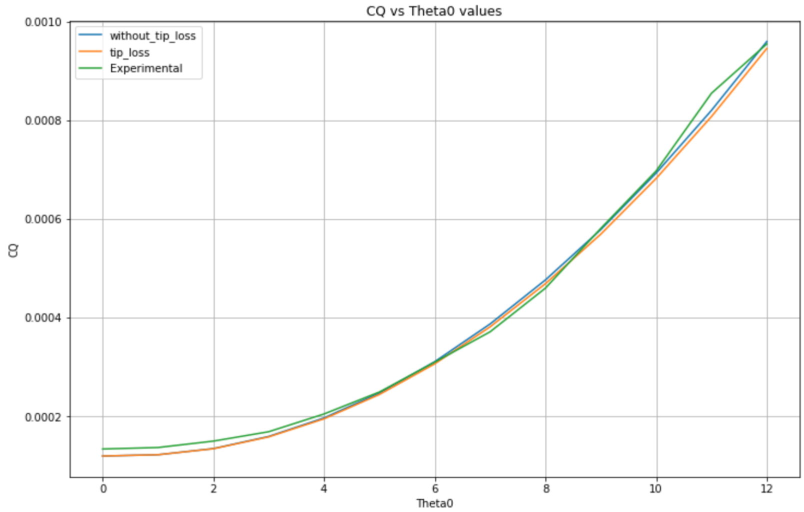
4 Results and discussion

1. C_T **vs** θ_0 **Plots**



- **Observations**
  - As we can see the value of C_T taken from reference paper after modification is nearly the same with out Tip loss as well as without Tip loss model
  - After taking the Tip loss into account we know that the total Thrust will reduce so same can be seen here as the without Tip loss model is bit above the Tip loss model
  - There is no case of stalling is seen because we haven't consider it in the our program.
  - As we increase the Twist value the Thrust will increase due to this the CT curve as we can see have positive slope
  - C_T estimated from our program is little high from experimental value because we ignores the swirl losses and vibration of blade
  - At zero Twist the value of C_T from paper as well from both of our model is zero due to nearly symmetric airfoil
  - Blade element momentum theory turns out to be a better approximation for Reynolds Number near the blade tip of about 105 – 106 but tip loss correction improves the program value and results are more close to the actual experimental values as compared to CT values without tip loss correction

2. C_Q **vs** θ_0 **Plots**



- **Observations**
  - As we can see the value of C_Q taken from reference paper after modification is nearly the same with out Tip loss as well as without Tip loss model
  - CQ value for zero geometric angle of attack is positive because of the interaction of air with rotor blade surface which lead to skin friction drag which amounts to torque
  - As geometric angle of increases the pressure drag will increase so the C_Q values
  - When geometric angle of attack increases, blades will create a wake region for the blade after it. due to this the decrease in drag encountered by rotor blades
  - Some error is due to above mentioned phenomena in values of C_Q as we haven't considered the wake phenomena in our program

3. **Sectional Thrust Generation**

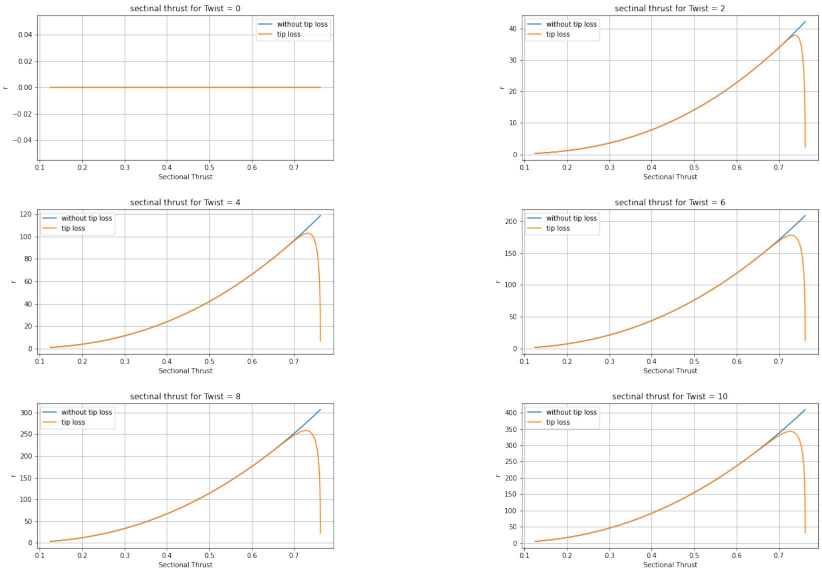


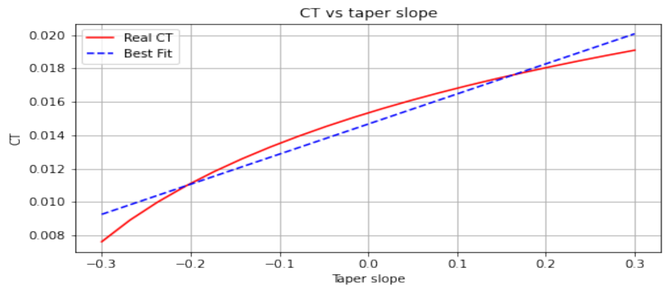
Figure 1: Sectional Thrust vs r

- **Observation**
  - For sub-figure 1 at zero twist there is no Thrust generation so there is no effect of Tip loss correction
  - As we know the value of Prandtl Tip loss factor value is 1 till the r/R value is around 0.8-0.85 so the value of sectional Thrust is almost same for Tip loss and without Tip loss model after that the value of Tip loss factor decreases so the Thrust generation will also reduces as we approach the end of blades
  - As angle of attack increases, sectional thrust per unit span and total thrust generated also increases & effect of tip loss model increases same we can see in the above graphs
  - Total Thrust generation will also less for Tip loss model as compared to without Tip loss model.
  - Maximum value of sectional Thrust is at near the end od blade.

4. **Variation Performance of rotor with Twist, Taper, and Solidity**
   Expression for the above quantities:
   Twist = θ_0 + Twist Slope*(r-Rrc)
   Taper = C_0 + Taper Slope*(r-Rrc)
   Solidity which is $\sigma = \frac{b*c}{\pi R}$; here only quantity which we can vary is c and we know c is linearly varying so will be solidity. We don't need to check separately for solidity after checking for taper.
   For visualizing the effect of Taper and Twist we have to consider the variation of four quantities which are θ_0, Twist Slope, C_0, Taper slope. We will set C_0 constant to 0.08 such that there is no overlap at root position. SO Now we have see effect of changing one variable keeping other 2 constant.

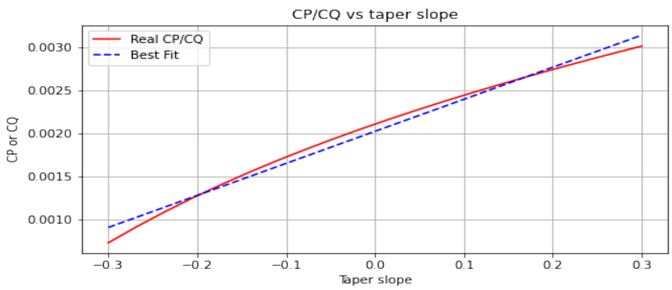
   - **Taper**
     Taper = C_0 + Taper Slope*(r-Rrc)
     As stated earlier we set the C_0 to 0.08. Now we have see how C_T,C_Q and Power varies with changing slope.
     For analysing the performance we set other variable to some constant values(not optimized but reasonable) and ovary only Taper slope
     Other variables:
     R = 0.3, Rrc=0.05, RPM=3000, Number of blades = 3, $\theta_0 = 12$ and Twist slope = $\theta_1 = -3$.
     Plots

     (a) C_T **vs Taper slope**
     Observation: We can see from the plot that as Taper slope increases the value of CT increases but for choosing the optimum taper slope we will consider those value of taper which have more change of rate of C_T. In the below plot. we can see that rate of change of slope is getting saturated after taper value of 0.1. So we have to choose the value of taper slope between the -0.1 to 0.1 because in this range the plot is above best fit.



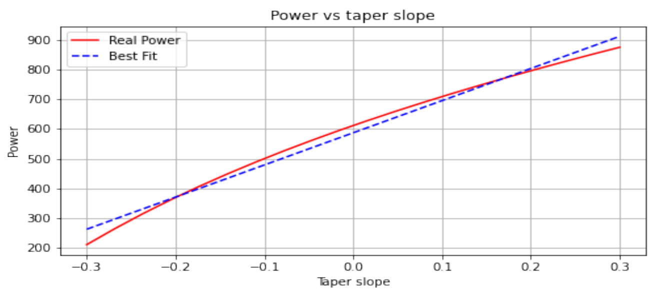
(b) C_P **or** C_Q **vs Taper slope**

Observation: From the plot below we can see the C_P value is increases with increase in taper value. As we seen from previous plot that Thrust also increases with increase in taper slope. So we will choose value of taper slope such that it is between -0.1 to 0.1, produce 55N thrust and have minimum value of C_P(Min Power). so the value of taper slope for our rotor is around -0.05(also depends on the what we choose the value of twist)



(c) **Power vs Taper slope**

Similar to above only difference of constant multiplication



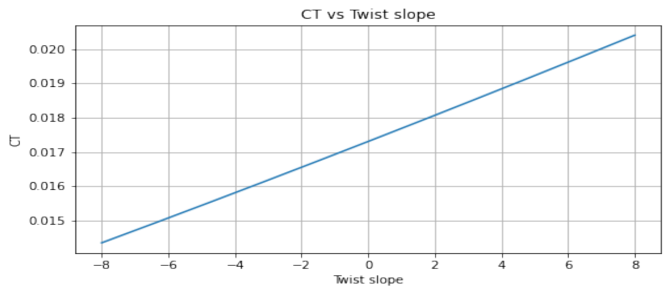
- **Twist**

  Same as taper case we will set C_0 to 0.08. Now we have to see the effect of θ_0 and Twist slope on performance. So first we keep θ_0 constant and vary Twist slope and later we will do the opposite. Now other parameters we will used as deifned for Taper. We will also choose C_1(Taper slope)=0.055(some reasonable value)
  Now we will see the effect of Twist slope effect on C_T,C_P and Power.

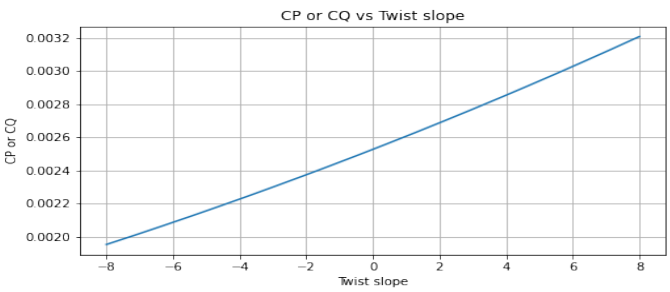
  Changing Twist Slope Plots
  $\theta_0 = 12$
  Observations: From the plots (a),(b),(c),(d) that increase in the value of Twist slope will leads to increase in all quantities(C_T,C_P,C_T/C_P and power). So for choosing the value of twist slope we will choose that minimum value which gives the lowest Power requirement as well as produce 55N(12kg Payload)

  As the twist increases, lift at sections increase which increase thrust and CT.

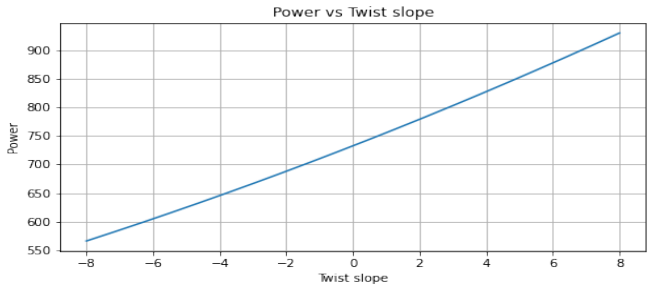
  (a) C_T **vs Twist slope**



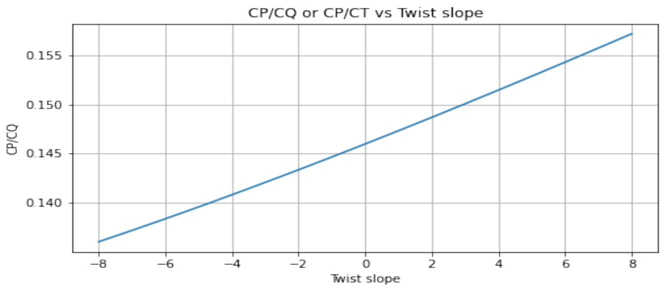
  (b) C_P **or** C_Q **vs Twist slope**



(c) **Power vs Twist slope**



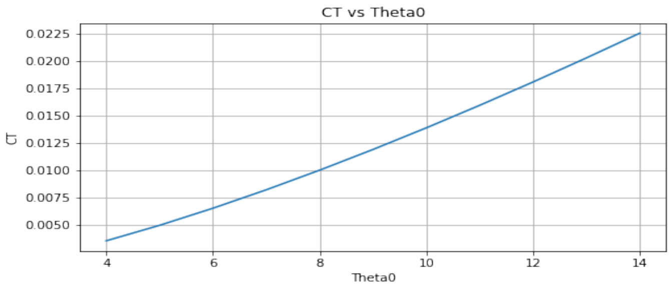
(d) **CP/CT vs Twist slope**



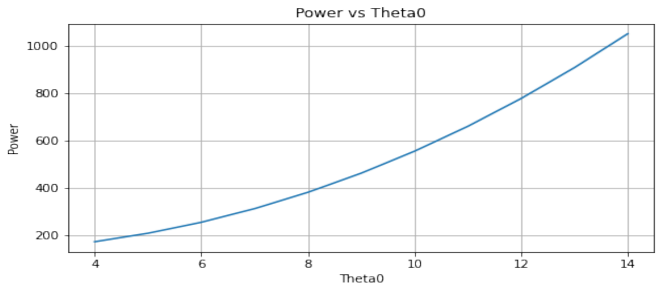
Changing θ_0 Value Plots

Observations: From below plots (a) and (b) We can see as we increase the value of $\theta 0$, value of C_T as well power increase. so we want to choose low value of θ_0 but if we choose low value of θ_0 the thrust value is also low at low Twist slope value. So we can see there is trade of between the θ_0 and Twist slope, we can only choose the low value for one variable. So I have chosen the Twist value low because that is giving the minimum power.

(a) C_T **vs** θ_0 **slope**



(b) **Power vs** θ_0 **slope**



5. **Design parameters converged for the UAV rotor**

Parameters	Value
Density	1.225
R	0.3
Rrc	0.05
RPM	3000
Number of blades	3
a	5.75
Cd_{min}	0.0113
eps	1.25
C0(Taper at root)	0.08
C1(Taper slope)	-0.05
θ_0	14 degree
θ_1(Twist slope)	-4
Total Thrust	220N

How to converge to above mentioned Parameters

- We have taken radius value from previous assignment
- We have choosen RPM such that tip mach number would be less then 0.3
- Rrc and C0 such that there is no overlap at root
- $C_{d(min)}$,a and epsilon i have choosen from the paper given for reference
- Now we have to choose the Taper slope($C1$), Twist Slope(θ_1) and θ_0 values.
- From last assignment we know that we will need total thrust fo 220N so 55N for one rotor.
- Now we have to choose the ramining parameters such that there is minimum power requirement for producing 55N Thurst.
- From the plots mentioned in point 5 above we can see that value of taper slope should be between -0.1 to 0.1.
- We know that if we increase the Twist slope or θ_0 keeping other constant the Thrust,CT and Power requirement all will increase. so we will try different combination such that our Power requirement will be minimum.
- For minimizing power we will some taper slope between -0.1 to 0.1 and change the theta0 and Twist slope so that we can generate 55N thrust. so there are not much combination. like first we fix taper slope and then some θ_0 then there are only 1-2 value of

Twist slope s.t. we can generate 55N Thrust. we will repeat this same changing θ_0 and then again taper slope.

- I have tried different combination and below is the table consist of those parameters which generate 55N. We will choose those parameters which are feasible as well as require minimum power.

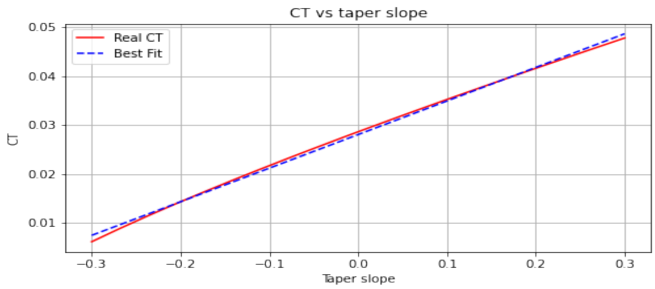
C_0	Taper slope	θ_0	Twist slope	Power	Thrust
0.08	0.05	10	13	786	55
0.08	0.05	12	2	773	55
0.08	0.05	13	-4	756	55
0.08	0.05	14	-10	747	55
0.08	0.08	10	11.5	790	55
0.08	0.08	12	0.5	770	55
0.08	0.08	13	-5	760	55
0.08	0.08	14	-11	752	55
0.08	0.1	10	11	793	55
0.08	0.1	12	-0.3	772	55
0.08	0.1	13	-6	762	55
0.08	0.1	14	-11.6	755	55
0.08	-0.05	12	7.4	758	55
0.08	-0.05	13	1.8	751	55
0.08	-0.05	14	-4	742	55
0.08	-0.1	12	11.5	760	55
0.08	-0.1	13	5.7	750	55
0.08	-0.1	14	-0.1	745	55

- We can Power minimizes at the Values of:
  $C_0 = 0.08$, $C_1 = -0.05$, $\theta_0 = 14$ degree and Twist slope=-4

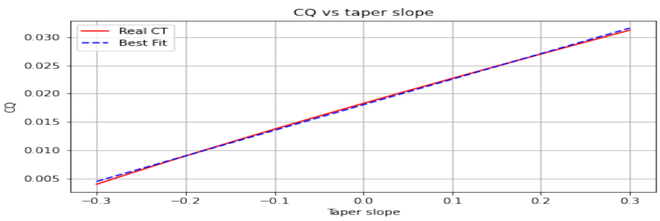
6. **Variation in performance of propeller by varying twist, taper, and solidity** Expression for the above quantities:
Twist = θ_0 + Twist Slope*(r-Rrc)
Taper = C_0 + Taper Slope*(r-Rrc)
Solidity which is $\sigma = \frac{b*c}{\pi R}$; here only quantity which we can vary is c and we know c is linearly varying so will be solidity. We don't need to check separately for solidity after checking for taper.
For visualizing the effect of Taper and Twist we have to consider the variation of four quantities which are θ_0, Twist Slope, C_0, Taper slope. We will set C_0 constant to 0.08 such that there is no overlap at root position. Twist slope also we have set to zero for propeller blades. SO Now we have see effect of changing one variable keeping other one constant.
We have to optimize only θ_0 value and Taper slope.

- **Taper Variation**
  Taper = C_0 + Taper Slope*(r-Rrc)
  As stated earlier we set the C_0 to 0.08. Now we have see how C_T,C_Q and Power varies with changing slope.
  For analysing the performance we set other variable to some constant values(not optimized but reasonable) and ovary only Taper slope
  Other variables:
  R = 0.3, Rrc=0.05, RPM=1500, Number of blades = 2, $\theta_0 = 45$ and Twist slope = $\theta_1 = 0$.
  Plots

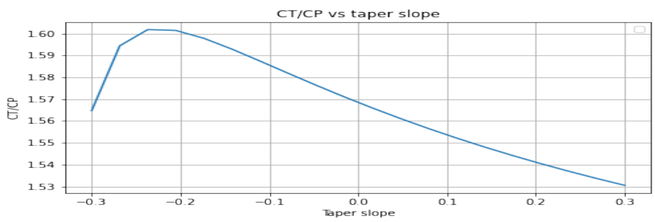
  (a) CT vs Taper slope
  Observation: From the plot below we can see that negative taper slope has more rate of change of C_T. So Taper value is chosen between -0.1 to 0
  Also we can observe that CT increases with increase in taper slope as at twist = 45 the effective angle of attack is positive near the half end of the blade which mainly contribute to positive thrust and CT value.
  So we will chose that value of Taper slope which produce required thrust and have lowest negative value.



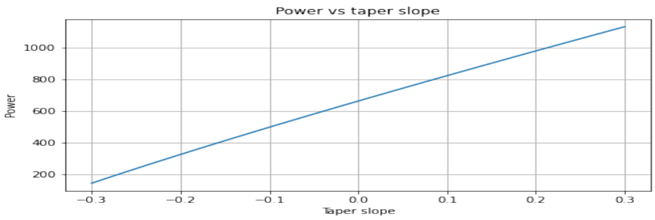
(b) CP vs Taper slope
Observation: The value of power is increases linearly with increase in Taper slope that is expected so no such information in chosing the Taper slope.



(c) CT/CP vs Taper slope
Observation: This plot and C_T plots are most important plots for choosing Taper slope. As we can see that for positive Taper slope CT/CP ratio is very low so positive value are discarded by both the plots. From plot it seems like Taper slope around -0.2 is very good but at this slope the Thrust requirement is not satisfied. So we want to choose the minimum value of Taper slope which satisfies our Thrust requirement as well as minimises the Power.

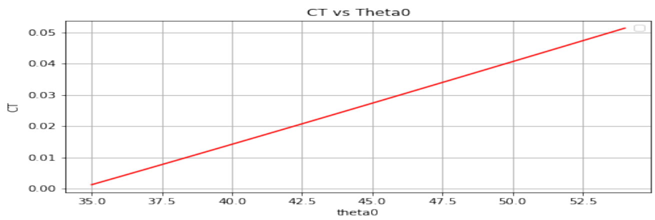


(d) Power vs Taper slope

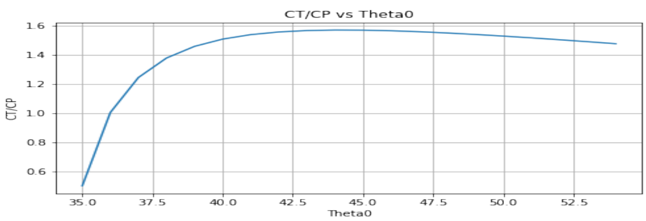


- **Twist variation Plots**

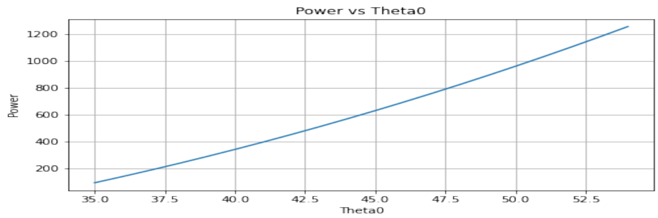
  (a) CT vs θ_0
  Observation: Plot is as expected the value of Thrust produce is increases with increase in θ_0. The curve is approximate linear so there is not much information about choosing the value of θ_0.



  (b) CT/CP vs θ_0
  Observation: This is most important plot for choosing the value of θ_0. We can see thet CT/CP value is increasing upto certain θ_0 then saturates. so we will θ_0 value for which out Thrust requirement meets and CT/CP curve is start saturating. From curve we can see that θ_0 value is betwenn 42-46 degree.



(c) Power vs θ_0



7. **Design parameters converged on for the UAV propeller**

Parameters	Value
Density	1.225
R	0.3
Rrc	0.05
RPM	1500
Number of blades	2
a	5.75
Cd_{min}	0.0113
eps	1.25
C0(Taper at root)	0.08
C1(Taper slope)	-0.039
θ_0	45 degree
θ_1(Twist slope)	0
Total Thrust	40N

How to converge to above mentioned Parameter

- We have taken radius value from previous assignment
- We have choosen RPM such that tip mach number would be less then 0.15
- Rrc and C0 such that there is no overlap at root
- $C_{d(min)}$,a and epsilon i have choosen from the paper given for reference
- Now we have to decide parameters for twist and taper
- This time it is quite easy then rotor design due to nature of curve of CT/CP vs θ_0 as well as Taper slope. From plot CT/CP vs taper slope(in above point 6) we can see that as we go towards the positive taper slope the CT/CP decreasing sharply. so now we have to choose some negative value of taper slope such that

the Thrust generation is around 20N for each motor(total 40N). So I have taken -0.039 as taper slope.

- for choosing θ_0; from plot CT/CP we can see that after increasing θ_0 to some value the CT/CP value saturates. so we have to chosse the minimum value of θ_0 such that for taper slope -0.039 the Thrust generation is around 20N.
- So we have choosen value of θ_0 as 45 degree for which Thrust is 20N and the Power requirement is 600W.

Comparison

Parameters	Rotor	Propeller
Density	1.225	1.225
R	0.3	0.3
Rrc	0.05	0.05
RPM	3000	1500
Number of blades	3	2
a	5.75	5.75
Cd_{min}	0.0113	0.0113
eps	1.25	1.25
C0(Taper at root)	0.08	0.08
C1(Taper slope)	-0.05	-0.039
θ_0	14 degree	45 degree
θ_1(Twist slope)	-4	0
Total Thrust	220N	40N

- RPM is taken 1500 with considering Tip Mach number smaller than 0.16 to operate in a regime in which compressibility effects are negligible and no shock is formed at blade tip at that RPM for propeller but for Rotor it is taken 3000 so that tip mach number smaller less then 0.3
- Twist for rotor blade decreases with increase in r whereas it is constant for propeller blade
- Value of taper slope is different because of requirement of Thrust as well as different variation of CP/CT
- Number of blades is 2 for propeller and 3 for rotor
- Due to difference in Twist variation the effective angle for the rotor blade has always positive value so rotor will generate positive sectional thrust per unit span at every point on the blade except at R Rrc where sectional thrust per unit span is 0
- While Propeller blade as both negative and positive effective angle of attack so there is negative sectional thrust per unit span

near the root cut off and positive sectional thrust per unit span near the end of the blade

- R, Rrc and C0 are same for both

References

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[4] Blade Element Momentum Theory Slide from Course AE667: Prof. Dhwanil Shukla