proof of the cross-ratio theorem

The projective cross-ratio theorem
Cross-ratio is invariant under the projective group P(2).

proof
Suppose that A = [**a**], B = [**b**], C = [**c**] and D = [**d**] are distinct, collinear p-points, and that **t** is a projective transformation.

As **t** is projective, A' = **t**(A), B' = **t**(B), C' = **t**(C) and D' = **t**(D) are distinct and collinear.

Now, **t** has the form **t**([**x**]) = [M**x**], where M is a non-singular 3x3 matrix, so we have A' = [M**a**], B' = [M**b**], C' = [M**c**] and D' = [M**d**].

As in the porism, **c** = α**a** + β**b**, and **d** = γ**a** + δ**b**.
The map **x** -> M**x** is *linear*, so that M**c** = αM**a** + βM**b**, and M**d** = γM**a** + δM**b**
Hence the ratio for (A',B',C',D') is βγ/αδ, *i.e.* is equal to that for (A,B,C,D).

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