11.9.3.2

EE23BTECH11040-MANOJ KUMAR AMBATIPUDI*

QUESTION:

Find the 12th term of a G.P. whose 8th term is 192 and common ratio is 2.

SOLUTION:

The general term of a G.P. is $a_0 r^n$ where a_0 is the first term, r is the common difference and n is the number indicating $(n+1)^{th}$ term of the sequence.

$$\implies a_n = a_0 r^n \tag{1}$$

Given, $a_7 = 192$, $r = 2$. On substituting, we get

$$\implies a_0 2^7 = 192 \tag{2}$$

$$\implies 128 a_0 = 192 \tag{3}$$

$$\implies \boxed{a_0 = \frac{3}{2} = 1.5} \tag{4}$$

Therefore, on substituting back, we get

$$a_n = 1.5 \times 2^n \tag{5}$$

$$\therefore a_{11} = 1.5 \times 2^{11} = 3072 \tag{6}$$

Fig. 1. Plot of the general term taken from Python

General term can also be written as

$$\boxed{x(n) = 1.5 \times 2^n u(n)} \tag{7}$$

Now on Z-Transforming, the expression which we get is

$$X(z) = \sum_{-\infty}^{\infty} 1.5 \times 2^n z^{-n} u(n) \tag{8}$$

$$\implies X(z) = \sum_{-\infty}^{\infty} 1.5 \times \left(\frac{2}{z}\right)^n u(n) \tag{9}$$

For the above series to converge, modulus of common ratio should be less than 1.

$$\implies r = \left|\frac{2}{z}\right| < 1 \tag{10}$$

$$\implies |z| > 2 \tag{11}$$

Therefore for all values given above, the above sequence shall converge.
The expression of $x(n)$ is

$$u(n) = \begin{cases} 1, & \forall \ n > 0 \\ 0, & \forall \ n < 0 \end{cases}$$

On simplifying $X(z)$, we get

$$\boxed{X(z) = \frac{3}{z-2} u(z)} \ \forall \ |z| > 2 \tag{12}$$

The expression of u(z) is

$$u(z) = \begin{cases} 0, & \forall \ z < 0, z \in Z \\ 1, & \forall \ z > 0, z \in Z \end{cases}$$

Now the expression simplifies to
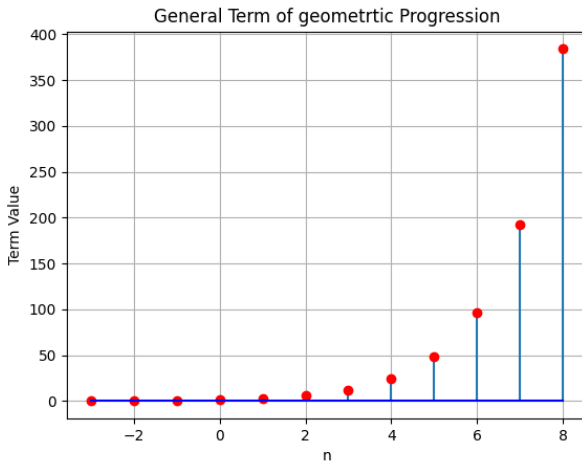
$$\boxed{X(z) = \frac{3}{z-2}} \ \forall \ z > 2 \tag{13}$$

Variable	Description	value
a_0	First Term in G.P.	1.5
n	Describing the order of term	None
a_7	8th term	192
r	common ratio	2
a_{11}	12th term	3072
x(n)	General term of sequence	None
u(n),u(z)	Unit Step Functions	Given Before
X(z)	Z-Transform Equation	None
z	frequency	None

TABLE 1
VARIABLES USED