12.10.6

EE23BTECH11040-MANOJ KUMAR AMBATIPUDI*

Question:
A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.
(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.
(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?
Solution:
Let the wave equations of the 2 waves coming to interfere be

$$y_1 = A_1 sin(2\pi f_1 t) + A_2 sin(s\pi f_2 t) \tag{1}$$
$$y_2 = A_1 sin(2\pi f_1 t + \phi_1) + A_2 sin(2\pi f_2 t + \phi_2) \tag{2}$$

Where A_1 and A_2 are amplitudes of waves and f_c is the frequency of the waves. Using principle of superposition, we get

$$y = y_1 + y_2 \tag{3}$$

where y is the resultant wave and y_1,y_2 are initial waves.

$$y = A_1(sin(2\pi f_1 t) + sin(2\pi f_1 t + \phi_1))+ \tag{4}$$
$$A_2(sin(2\pi f_2 t) + sin(2\pi f_2 t + \phi_2)) \tag{5}$$

using

$$sin(c) + sin(d) = 2sin(\frac{c+d}{2})cos(\frac{c-d}{2}) \tag{6}$$

we get

$$y = 2A_1 sin(2\pi f_1 t + \frac{\phi_1}{2})cos(\frac{\phi_1}{2})+ \tag{7}$$
$$2A_2 sin(2\pi f_2 t + \frac{\phi_2}{2})cos(\frac{\phi_2}{2}) \tag{8}$$

Now for constructive interference to happen,

$$cos(\frac{\phi_1}{2}) = +/-1 \text{ and } cos(\frac{\phi_2}{2}) = +/-1$$

$$\implies \phi_1 = 2n_1\pi \text{ and } \phi_2 = 2n_2\pi \tag{9}$$

Equation (9) is the condition for constructive Interference.
Now we will derive the condition for constructive interference in a YDSE setup where the light waves due to both the sources coincide.
The path difference suffered by 2 waves due to corresponding phase difference is given by,

$$\Delta x_1 = \frac{\lambda_1}{2\pi} 2n_1\pi = n_1\lambda \tag{10}$$
$$\Delta x_2 = \frac{\lambda_2}{2\pi} 2n_2\pi = n_2\lambda \tag{11}$$

In a YDSE setup, the 2 sources are seperated by a distance "d", the distance between screen and mid point of sources is "D" and θ_1 is the angle made by point of interest with horizontal line. The path difference between the 2 waves interfereing at the point of interest is given by

$$\Delta x_1 = dsin(\theta_1) \tag{12}$$
$$\Delta x_2 = dsin(\theta_2) \tag{13}$$

Now, from (12) and (13), we can write,

$$dsin(\theta_1) = n\lambda_1 \tag{14}$$
$$dsin(\theta_2) = n\lambda_2 \tag{15}$$

Now, for small values of θ_1 and θ_2, we can approximate

$$sin(\theta_1) = \frac{y_1}{D} \tag{16}$$
$$sin(\theta_2) = \frac{y_2}{D} \tag{17}$$

upon substituting in (14),(15) and rearranging, we get

$$y_1 = n_1 \frac{D\lambda_1}{d} \tag{18}$$
$$y_2 = n_2 \frac{D\lambda_2}{d} \tag{19}$$

Now for interference to happen at same points,

$$y_1 = y_2 \tag{20}$$
$$\implies n_1\lambda_1 = n_2\lambda_2 \tag{21}$$

Now,we shall use the above equations to solve the questions.
(a) Finding the distance of the third bright fringe on the screen from the central maximum for wavelength

$$\lambda_1 = 650\,\text{nm} \tag{22}$$

The path difference for constructive interference is given by:

$$\Delta x_1 = n_1 \lambda_1 \tag{23}$$

where $n_1 = 3$ for the third bright fringe.
Substitute the values:

$$\Delta x_1 = 3 \times 650 = 1950\,\text{nm} \tag{24}$$

$$y_1 = \Delta x_1 \cdot \frac{D}{d} = 1950 \times \frac{D}{d} \tag{25}$$

Where "D" is the distance of screen from sources, "d" is the distance between sources and "y_1" is the distance from Central Maxima.
(b) The least distance from the central maximum where the bright fringes due to both wavelengths coincide: Given

$$\lambda_1 = 650\text{nm}, \lambda_1 = 520\text{nm}$$

$$n_1 \lambda_1 = n_2 \lambda_2 \tag{26}$$

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{650}{520} \tag{27}$$

$$\frac{n_2}{n_1} = \frac{5}{4} \tag{28}$$

Now we can assume $n_2 = 5$k and $n_1 = 4$k where k is some positive integer.
For smallest value of y_1 we shall take k=1.
Path difference $\Delta x_1 = 4 \times 650 = 2600$nm

Now $y_1 = \Delta x_1 \frac{D}{d} = 2600 \frac{D}{d} nm$
Answer for (b):
Hence, at a least distance of $2600 \times \frac{D}{d}$nm the bright fringes due to both wavelengths coincide.

Variable	Description	Value
y_1	Wave Equation of First Wave	none
y_2	Wave Equation of Second Wave	none
y	Resultant Wave	none
f_c	Frequency of waves	none
A_1	Amplitude of First Wave	none
A_2	Amplitude of Second Wave	none
ϕ_1,ϕ_2	Phase Difference(before Interference)	none
I_{net}	Intensity after interference	none
n_1, n_2	Non Negative Integer Values	0,1,2....
Δx_1,Δx_2	Path Differences	In soln.
λ_1,λ_2	Wavelengths	650,520
y_1, y_2	Distance between central maxima and point	In soln.
d	distance between the slits	None
D	Distance between Slits and Screen	None
θ_1, θ_2	Angular Distance From Central Maxima	None

TABLE 0
VARIABLES AND THEIR VALUES