NCERT 12/10/6

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Question:
A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.
(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.
(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?
Solution: Consider single source interference. Let

Variable	Description	Value
y_1	Wave Equation of First Wave	none
y_2	Wave Equation of Second Wave	none
Y	Resultant Wave	none
f_c	Frequency of wave	none
A	Amplitude of Wave	none
λ_1,λ_2	Wavelengths	650,520
d	distance between the 2 slits	None
D	Distance from slits and Screen	None

TABLE 1
VARIABLES AND THEIR VALUES

the wave equations of the 2 waves coming from λ source be y_1,y_2. Assume an impulse emitted at source, travelling through the path labeled **Wave 1** be t_1 and for travelling through path **Wave 2** be t_2 respectively. The corresponding wave equations are

$$y_1 = Asin(2\pi f_c t_1) \tag{1}$$
$$y_2 = Asin(2\pi f_c t_2) \tag{2}$$

t_2 can be written in terms of t_1 as

$$t_2 = t_1 + \Delta t \tag{3}$$

Where the term Δt arises because of paths chosen. the values of t_1, t_2 are

$$t_1 = \frac{\sqrt{\left(\frac{d}{2} - b_1\right)^2 + (a_1)^2} + \sqrt{\left(\frac{d}{2} - y\right)^2 + (D)^2}}{c} \tag{4}$$
$$t_2 = \frac{\sqrt{\left(\frac{d}{2} + y\right)^2 + (D)^2} + \sqrt{\left(\frac{d}{2} + b_1\right)^2 + (a_1)^2}}{c} \tag{5}$$

Assuming the approximations

$$b_1 << a_1 \tag{6}$$
$$b_1 \sim d \tag{7}$$
$$d << D \tag{8}$$
$$(1+x)^n \simeq 1 + nx \tag{9}$$

We get

$$\Delta t = \frac{\frac{db_1}{a_1} + \frac{dy}{D}}{c} \tag{10}$$

On substituting t_2 in (2) and applying superposition theorem, we get

$$Y = y_1 + y_2 \tag{11}$$
$$Y = 2A\sin\left(2\pi f_c\left(t_1 + \frac{\Delta t}{2}\right)\right)\cos\left(2\pi f_c\left(\frac{\Delta t}{2}\right)\right) \tag{12}$$

For constructive interference

$$f\Delta t = n \tag{13}$$

substituting in (10), we get

$$\frac{db_1}{a_1} + \frac{dy}{D} = n\lambda \tag{14}$$

The equation (14) is the condition for constructive interference. For 2 sources, we do it as follows. From Fig. 2,

$$\frac{db_1}{a_1} + \frac{dy}{D} = n_1\lambda_1 \tag{15}$$
$$\frac{db_2}{a_2} + \frac{dy}{D} = n_2\lambda_2 \tag{16}$$

Now, for lights from both sources to interfere at a point constructively,

$$n_1\lambda_1 = n_2\lambda_2 \tag{17}$$

(a) From Table 1 and (14), taking $b_1 = 0$

$$n_1 = 3 \tag{18}$$
$$y = \frac{n_1\lambda_1 D}{d} \tag{19}$$
$$y = 1950\frac{D}{d} \tag{20}$$

(b) From Table 1, (14), (17) and taking $b_1 = 0$, $b_2 = 0$

$$n_1\lambda_1 = n_2\lambda_2 \tag{21}$$

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{650}{520} \tag{22}$$

$$\frac{n_2}{n_1} = \frac{5}{4} \tag{23}$$

Now we can assume n_2 = 5k and n_1 = 4k where k is some positive integer. For smallest value of y we shall take $k = 1$.

$$y = \Delta x_1 \frac{D}{d} = 2600\frac{D}{d}nm \tag{24}$$

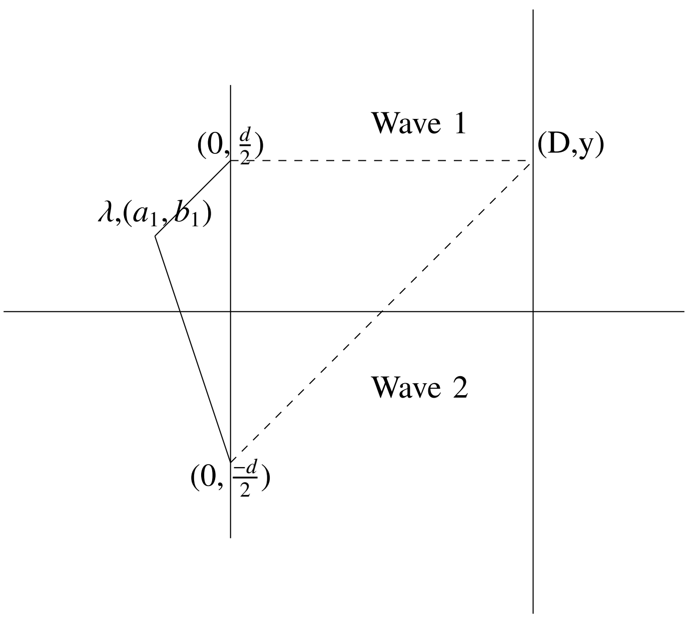

Fig. 1. Single Source YDSE

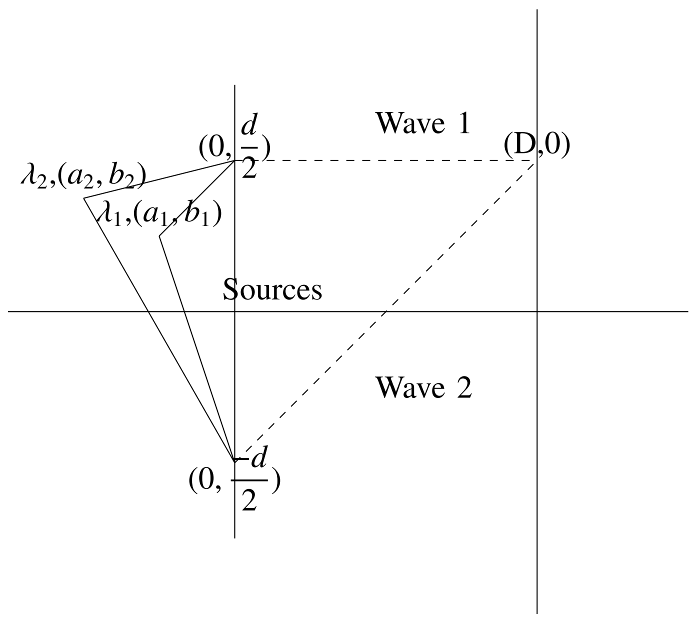

Fig. 2. 2 sources interference