NCERT: 11.9.3.2

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QUESTION:

Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and common ratio is 2.

SOLUTION:

$$\implies x(n) = x(0)\, r^n, \quad \forall \ n \geq 0 \tag{1}$$

Hence, general term can also be written as

$$x(n) = x(0)\, r^n u(n) \tag{2}$$

where

$$u(n) = \begin{cases} 1, & \forall \ n > 0 \\ 0, & \forall \ n < 0 \end{cases}$$

Now on Z-transforming, we get

$$X(z) = \sum_{-\infty}^{\infty} x(n)\, z^{-n} u(n) \tag{3}$$

On referring to **??**, we get

$$X(z) = \sum_{-\infty}^{\infty} x(0)\, r^n z^{-n} u(n) \tag{4}$$

The above series is convergent iff,

$$\left|\frac{r}{z}\right| < 1 \tag{5}$$

On expanding and solving, we get

$$X(z) = x(0)\, \frac{r}{z - r} \tag{6}$$

Referring to Table **??**

$$\implies r = 2 \tag{7}$$

$$\implies x(7) = 192 \tag{8}$$

$$\implies x(0)\, 2^7 = 192 \tag{9}$$

$$\implies 128 x(0) = 192 \tag{10}$$

$$\implies \boxed{x(0) = \frac{3}{2} = 1.5} \tag{11}$$

The general term is written as

$$x(n) = 1.5 \times 2^n \tag{12}$$

On simplifying $X(z)$, we get

$$\boxed{X(z) = \frac{3}{z - 2}} \quad \forall \ z > 2 \tag{13}$$

On referring to **??**, we get

$$x(11) = 1.5 \times 2^{11} = 3072 \tag{14}$$

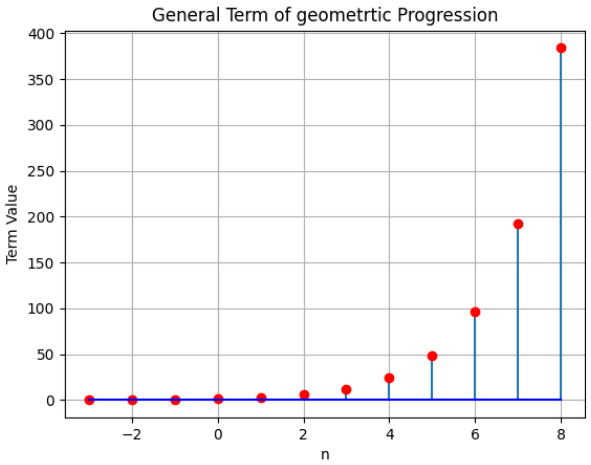

Fig. 1. Plot of the general term taken from Python3

Variable	Description	value
$x(7)$	8^{th} term	192
r	common ratio	2
$x(0)$	first term	1.5
$x(11)$	12^{th} term	3072
$x(n)$	General term of sequence	None
$X(z)$	Z-Transform Equation	None

TABLE 1
VARIABLES USED