NCERT: 11.9.3.2

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QUESTION:

Find the 12th term of a G.P. whose 8th term is 192 and common ratio is 2.

SOLUTION:

General term can also be written as

$$x(n) = x(0) r^n u(n) \quad (1)$$

Now on Z-transforming, we get

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n} u(n) \quad (2)$$

Referring to (**??**) we get

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad (3)$$

Referring to Table 1

$$r = 2 \quad (4)$$
$$\implies x(7) = 192 \quad (5)$$
$$\implies x(0) 2^7 = 192 \quad (6)$$
$$\implies 128 x(0) = 192 \quad (7)$$
$$\implies x(0) = \frac{3}{2} = 1.5 \quad (8)$$

The general term is written as

$$x(n) = 1.5 \times 2^n \quad (9)$$

On referring to (3) and Table 1, we get

$$\boxed{X(z) = \frac{3}{z - 2}} \quad \forall \ z > 2 \quad (10)$$

On referring to 1 for 12th term, we get
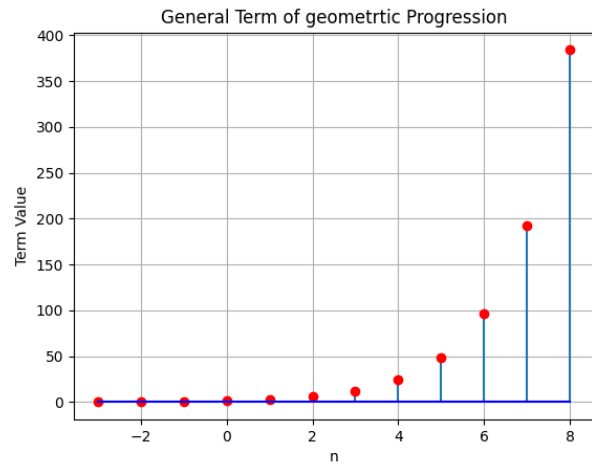
$$x(11) = 1.5 \times 2^{11} = 3072 \quad (11)$$

Fig. 1. Plot of the general term taken from Python3

Variable	Description	value
r	common ratio	2
$x(7)$	eighth term	192
$x(n)$	General term of sequence	None
$X(z)$	Z-Transform Equation	None

TABLE 1
VARIABLES USED AND THEIR DESCRIPTIONS