12.10.6

EE23BTECH11040-MANOJ KUMAR AMBATIPUDI*

Question:
A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.
(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.
(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?
Solution:
Let the wave equations of the 2 waves coming from λ_1 source be y_1,y_2. Let the wave equations of the 2 waves coming from λ_2 source be y_3,y_4,

$$y_1 = A_1 \sin(2\pi f_1 t) \tag{1}$$
$$y_2 = A_1 \sin(2\pi f_1 t + \phi_1 + \phi_2) \tag{2}$$
$$y_1 = A_2 \sin(2\pi f_2 t) \tag{3}$$
$$y_2 = A_2 \sin(2\pi f_2 t + \phi_3 + \phi_4) \tag{4}$$

Where $\phi_1, \phi_2, \phi_3, \phi_4$ are phase differences arisen because of position of source.

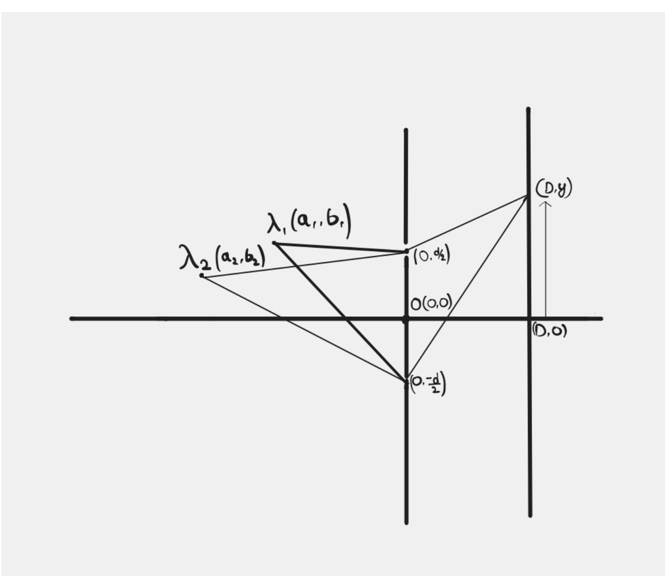

Fig. 1. YDSE SETUP

Using principle of superposition, we get

$$y = y_1 + y_2 \tag{5}$$
$$Y = y_3 + y_4 \tag{6}$$
$$\implies y = A_1(\sin(2\pi f_1 t) + \sin(2\pi f_1 t + \phi_1 + \phi_2)) \tag{7}$$
$$\implies Y = A_2(\sin(2\pi f_2 t) + \sin(2\pi f_2 t + \phi_3 + \phi_4)) \tag{8}$$

using

$$\sin(c) + \sin(d) = 2\sin(\frac{c+d}{2})cos(\frac{c-d}{2}) \tag{9}$$

we get

$$y = 2A_1 \sin(2\pi f_1 t + \frac{\phi_1 + \phi_2}{2})cos(\frac{\phi_1 + \phi_2}{2}) \tag{10}$$
$$Y = 2A_2 \sin(2\pi f_2 t + \frac{\phi_3 + \phi_4}{2})cos(\frac{\phi_3 + \phi_4}{2}) \tag{11}$$

Now for constructive interference to happen,

$$cos(\frac{\phi_1 + \phi_2}{2}) = \pm 1 \text{ and}$$
$$cos(\frac{\phi_3 + \phi_4}{2}) = \pm 1$$

$$\implies \phi_1 + \phi_2 = 2n_1\pi \text{ and } \phi_3 + \phi_4 = 2n_2\pi \tag{12}$$

path difference in terms of phase difference is

$$\Delta x = \Delta\phi \frac{\lambda}{2\pi} \tag{13}$$

Consider wave from λ_1 source. The phase difference ϕ_1, ϕ_2 corresponding path differences are calculated as

$$\Delta x_1 = \sqrt{\left(\frac{d}{2} + b_1\right)^2 + (a_1)^2} - \sqrt{\left(\frac{d}{2} - b_1\right)^2 + (a_1)^2} \tag{14}$$

$$\Delta x_2 = \sqrt{\left(\frac{d}{2} + y\right)^2 + (D)^2} - \sqrt{\left(\frac{d}{2} - y\right)^2 + (D)^2} \tag{15}$$

Assuming the approximations

$$b_1 << a_1 \tag{16}$$
$$b_1 \sim d \tag{17}$$
$$d << D \tag{18}$$
$$(1 + x)^n \simeq 1 + nx \tag{19}$$

We get

$$\Delta x_1 = \frac{db_1}{a_1} \tag{20}$$
$$\Delta x_2 = \frac{dy}{D} \tag{21}$$
$$\tag{22}$$

similarly, path difference of λ_2 source is

$$\Delta x_3 = \frac{db_2}{a_2} \tag{23}$$

The sign of path difference is based on the positioning of sources.($Table$ **??**)
Depending on the condition, we use appropriate sign convention and calculate path difference as

$$\Delta x = \Delta x_1 + \Delta x_2 \tag{24}$$

for λ_1 source and

$$\Delta x = \Delta x_3 + \Delta x_2 \tag{25}$$

for λ_2 source.

(a) Finding the distance of the third bright fringe on the screen from the central maximum for wavelength

$$\lambda_1 = 650\,\text{nm} \tag{26}$$

The path difference for constructive interference is given by:

$$\Delta x_1 = n_1\lambda_1 \tag{27}$$

where $n_1 = 3$ for the third bright fringe.
Substitute the values:

$$\Delta x_1 = 3 \times 650 = 1950\,\text{nm} \tag{28}$$
$$y_1 = \Delta x_1 \cdot \frac{D}{d} = 1950 \times \frac{D}{d} \tag{29}$$

(b) The least distance from the central maximum where the bright fringes due to both wavelengths coincide: Given

$$\lambda_1 = 650\text{nm}, \lambda_1 = 520\text{nm}$$

$$n_1\lambda_1 = n_2\lambda_2 \tag{30}$$
$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{650}{520} \tag{31}$$
$$\frac{n_2}{n_1} = \frac{5}{4} \tag{32}$$

Now we can assume $n_2 = 5$k and $n_1 = 4$k where k is some positive integer.
For smallest value of y_1 we shall take k=1.
Path difference $\Delta x_1 = 4 \times 650 = 2600$nm

Now $y_1 = \Delta x_1 \frac{D}{d} = 2600\frac{D}{d} nm$
Answer for (b):

Variable	Description	Value
y_1	Wave Equation of First Wave	none
y_2	Wave Equation of Second Wave	none
y	Resultant Wave	none
f_c	Frequency of waves	none
A_1	Amplitude of First Wave	none
A_2	Amplitude of Second Wave	none
ϕ_1,ϕ_2	Phase Difference(before Interference)	none
I_{net}	Intensity after interference	none
n_1, n_2	Non Negative Integer Values	0,1,2....
Δx_1,Δx_2	Path Differences	In soln.
λ_1,λ_2	Wavelengths	650,520
y_1, y_2	Distance between central maxima and point	In soln.
d	distance between the slits	None
D	Distance between Slits and Screen	None
θ_1, θ_2	Angular Distance From Central Maxima	None

TABLE 1
VARIABLES AND THEIR VALUES

Δx measured on $x < 0, y < 0$	$-ve$
Δx measured on $x < 0, y > 0$	$+ve$

TABLE 2
SIGN CONVENTION FOR PATH DIFFERENCE