GATE 22 EE/46

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QUESTION: Let a causal LTI system be governed by the following differential equation,

$$y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t) \tag{1}$$

where $x(t)$ and $y(t)$ are the input and output respectively. It's impulse response is (GATE EE-2022)

Solution:

From (1), corresponding Laplace transform,

$$Y(s) + \frac{1}{4}(sY(s) - y(0)) = 2X(s) \tag{2}$$

Since it is causal LTI system,

$$y(0) = 0 \tag{3}$$

$$Y(s) + \frac{1}{4}sY(s) = 2X(s) \tag{4}$$

$$Y(s) = X(s)\frac{8}{4+s} \tag{5}$$

$$H(s) = \frac{8}{4+s} \quad ROC : Re(s) > -4 \tag{6}$$

Using Inverse Laplace formula,

$$h(t) = \frac{1}{2\pi j}\lim_{T\to\infty}\int_{c-jT}^{c+jT} e^{st}H(s)\,ds \tag{7}$$

Where c is some number in the ROC of $H(s)$. Substituting $c = 0$,

$$h(t) = \frac{1}{2\pi j}\lim_{T\to\infty}\int_{-jT}^{+jT} e^{st}\frac{8}{4+s}ds \tag{8}$$

Using Cauchy's Residue Theorem,

$$h(t) = \lim_{s\to-4}(s+4)\frac{8e^{st}}{s+4} \tag{9}$$

$$= \lim_{s\to-4} 8e^{st} \tag{10}$$

$$= 8e^{-4t} \tag{11}$$

Since the system is causal,

$$h(t) = 8e^{-4t}u(t) \tag{12}$$

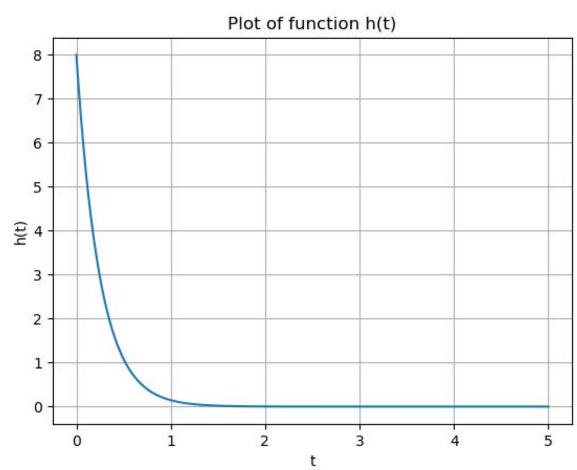

Fig. 1. Plot of $h(n)$, taken from python3