12.10.6

EE23BTECH11040-MANOJ KUMAR AMBATIPUDI*

Question:
A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.
(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.
(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Solution:
Let the wave equations of the 2 waves coming to interfere be

$$y_1 = A_1 sin(2\pi f_c t) \tag{1}$$
$$y_2 = A_2 sin(2\pi f_c t + \phi) \tag{2}$$

Where A_1 and A_2 are amplitudes of waves and f_c is the frequency of the waves. Using principle of superposition, we get

$$y = y_1 + y_2 \tag{3}$$

where y is the resultant wave and y_1,y_2 are initial waves. Expanding $sin(2\pi f_c t + \phi)$, we get

$$y = (A_1 + A_2 cos(\phi))sin(2\pi f_c t) + (A_2 sin(\phi))sin(2\pi f_c t) \tag{4}$$

Assuming

$$A_1 + A_2 cos(\phi) = Rcos(\theta) \tag{5}$$
$$A_2 sin(\phi) = Rsin(\theta) \tag{6}$$

here R is the resultant and θ is the phase difference we get,

$$y = Rsin(2\pi f_c t + \theta) \tag{7}$$

and

$$R^2 = A_1^2 + A_2^2 + 2A_1 A_2 cos(\theta) \tag{8}$$

For any Electromagnetic Wave,
Intensity(I) α (Amplitude)^2

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2} cos(\theta) \tag{9}$$

From (9), it is clear that maximum intensity occurs when

$$\theta = 2n\pi \tag{10}$$

For any wave, phase difference θ is given by

$$\theta = \frac{2\pi\Delta x}{\lambda} \tag{11}$$

Now, path difference Δx is given as

$$\Delta x = \frac{\lambda}{2\pi}\theta \tag{12}$$
$$\implies \Delta x = \frac{\lambda}{2\pi}2n\pi \tag{13}$$
$$\implies \Delta x = n\lambda \tag{14}$$

Equation (14) is the condition for constructive Interference.

Now we will derive the condition for constructive interference in a YDSE setup.

In a YDSE setup, the 2 sources are seperated by a distance "d", the distance between screen and mid point of sources is "D" and θ is the angle made by point of interest with horizontal line. The path difference between the 2 waves interfereing at the point of interest is given by

$$\Delta x = dsin(\theta) \tag{15}$$

Now, from (14) and (15), we can write,

$$dsin(\theta) = n\lambda \tag{16}$$

Now, for small values of θ, we can approximate

$$sin(\theta) = \frac{y}{D} \tag{17}$$

upon substituting in (16) and rearranging, we get

$$y = n\frac{D\lambda}{d} \tag{18}$$

Now,we shall use the above equation to solve the questions.
(a) Finding the distance of the third bright fringe

on the screen from the central maximum for wavelength

$$\lambda_1 = 650\,\text{nm} \tag{19}$$

The path difference for constructive interference is given by:

$$\Delta x_1 = m\lambda_1 \tag{20}$$

where m = 3 for the third bright fringe.

Substitute the values:

$$\Delta x_1 = 3 \times 650 = 1950\,\text{nm} \tag{21}$$

$$y = \Delta x_1 \cdot \frac{D}{d} = 1950 \times \frac{D}{d} \tag{22}$$

Where ”D” is the distance of screen from sources, ”d” is the distance between sources and ”y” is the distance from Central Maxima.

(b) The least distance from the central maximum where the bright fringes due to both wavelengths coincide:

Given

$$\lambda_1 = 650\text{nm}, \lambda_1 = 520\text{nm}$$

Let Δx be the common path difference for both wavelengths:

$$\Delta x = m\lambda_1 = n\lambda_2 \tag{23}$$

$$\frac{n}{m} = \frac{\lambda_1}{\lambda_2} = \frac{650}{520} \tag{24}$$

$$\frac{n}{m} = \frac{5}{4} \tag{25}$$

Now we can assume n = 5 and m = 4.
Path difference $\Delta x = 4 \times 650 = 2600$nm

Now $y = \Delta x \cdot \frac{D}{d} = 2600 \times \frac{D}{d}$

where ”y” is the distance from central maxima, ”D” is the distance of screen from sources and ”d” is the distance between sources.

Answer for (b):

Hence, at a least distance of $2600 \times \frac{D}{d}$ the bright fringes due to both wavelengths coincide.