11.9.3.2

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QUESTION:

Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and common ratio is 2.

SOLUTION:

The general term of a G.P. is ar^{n-1} where a is the first term, r is the common difference and n is the number indicating n^{th} term of the sequence.

$$\implies a_n = ar^{n-1} \tag{1}$$

Given, $a_8 = 192$, $r = 2$. On substituting, we get

$$\implies a2^{8-1} = 192 \tag{2}$$

$$\implies a2^7 = 192 \tag{3}$$

$$\implies 128a = 192 \tag{4}$$

$$\implies \boxed{a = \frac{3}{2} = 1.5} \tag{5}$$

Therefore, on substituting back, we get

$$a_n = 1.5 \times 2^{n-1} \tag{6}$$

$$\therefore a_{12} = 1.5 \times 2^{11} = 3072 \tag{7}$$

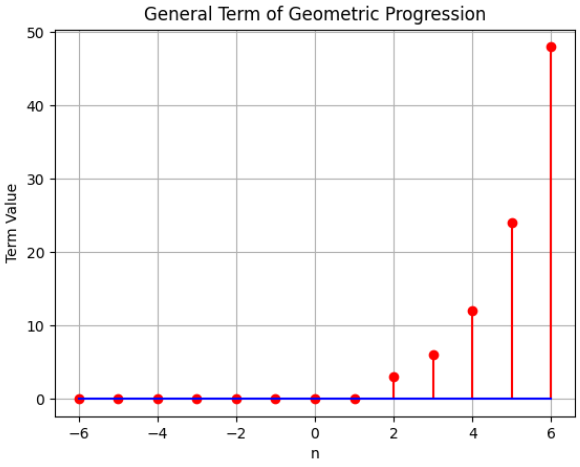

Fig. 0. Plot of the general term taken from Python

General term can also be written as

$$\boxed{x(n) = 3 \times 2^n} \tag{8}$$

Now on Z-Transforming, the expression which we get is

$$X(z) = \sum_{-\infty}^{\infty} 3 \times 2^n z^{-n} u(n) \tag{9}$$

$$\implies X(z) = \sum_{-\infty}^{\infty} 3 \times \left(\frac{2}{z}\right)^n u(n) \tag{10}$$

For the above series to converge, modulus of common ratio should be less than 1.

$$\implies r = \left|\frac{2}{z}\right| < 1 \tag{11}$$

$$|z| > 2 \tag{12}$$

Therefore for all values given above, the above sequence shall converge. On simplifying $X(z)$, we get

$$\boxed{X(z) = \frac{6}{z-2}} \quad \forall \ |Z| > 2 \tag{13}$$