

11.9.3.2

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QUESTION:

Find the 12th term of a G.P. whose 8th term is 192 and common ratio is 2.

SOLUTION:

The general term of a G.P. is $x(0)r^n$ where $x(0)$ is the first term, r is the common difference and n is the number indicating $(n+1)^{th}$ term of the sequence.

$$\Rightarrow x(n) = x(0)r^n \quad (1)$$

Given, $x(7) = 192$, $r = 2$. On substituting, we get

$$\Rightarrow x(0)2^7 = 192 \quad (2)$$

$$\Rightarrow 128x(0) = 192 \quad (3)$$

$$\Rightarrow \boxed{x(0) = \frac{3}{2} = 1.5} \quad (4)$$

Therefore, on substituting back, we get

$$x(n) = 1.5 \times 2^n \quad (5)$$

$$\therefore x(11) = 1.5 \times 2^{11} = 3072 \quad (6)$$

General term can also be written as

$$\boxed{x(n) = 1.5 \times 2^n u(n)} \quad (7)$$

Now on Z-Transforming, the expression which we get is

$$X(z) = \sum_{-\infty}^{\infty} 1.5 \times 2^n z^{-n} u(n) \quad (8)$$

$$\Rightarrow X(z) = \sum_{-\infty}^{\infty} 1.5 \times \left(\frac{2}{z}\right)^n u(n) \quad (9)$$

For the above series to converge, modulus of common ratio should be less than 1.

$$\Rightarrow r = \left|\frac{2}{z}\right| < 1 \quad (10)$$

$$\Rightarrow |z| > 2 \quad (11)$$

Therefore for all values given above, the above sequence shall converge.

The expression of $x(n)$ is

$$u(n) = \begin{cases} 1, & \forall n > 0 \\ 0, & \forall n < 0 \end{cases}$$

On simplifying $X(z)$, we get

$$\boxed{X(z) = \frac{3}{z-2} U(z)} \quad \forall |z| > 2 \quad (12)$$

The expression of $U(z)$ is

$$U(z) = \begin{cases} 0, & \forall z < 0, z \in \mathbb{Z} \\ 1, & \forall z > 0, z \in \mathbb{Z} \end{cases}$$

Now the expression simplifies to

$$\boxed{X(z) = \frac{3}{z-2}} \quad \forall z > 2 \quad (13)$$

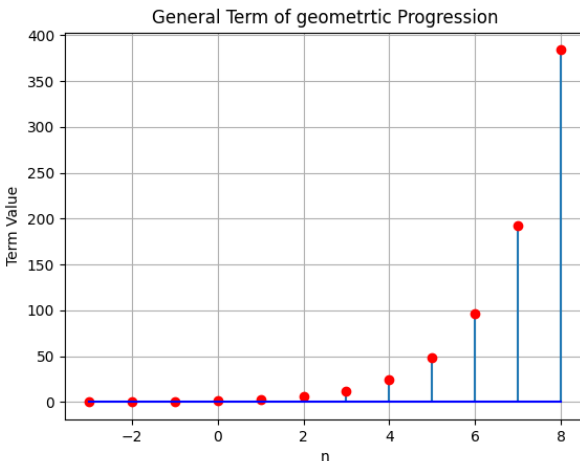


Fig. 1. Plot of the general term taken from Python

Variable	Description	value
$x(0)$	First Term in G.P.	1.5
n	Describing the order of term	None
$x(7)$	8^{th} term	192
r	common ratio	2
$x(11)$	12^{th} term	3072
$x(n)$	General term of sequence	None
$X(z)$	Z-Transform Equation	None

TABLE 1
VARIABLES USED