

NCERT 11.9.1.13Q

EE22BTECH11015 - DHANUSH V NAYAK*

Question: Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

$$a_1 = a_2 = 2, \quad a_n = a_{n-1} - 1, \quad n > 2$$

Solution:

| Parameter | Description | Value |
|-----------|-----------------------|---|
| $x(0)$ | First term | 2 |
| $x(1)$ | Second term | 2 |
| ROC | Region of convergence | $\{z : \sum_{n=-\infty}^{\infty} x(n)z^{-n} < \infty\}$ |

TABLE 1
PARAMETER TABLE

$$x(n+2) - x(n+1) = -1, n \geq 0 \quad (1)$$

Substituting $n = 0, n = 1, n = 2$ in equation (1) we get:

$$x(2) = x(1) - 1 = 1 \quad (2)$$

$$x(3) = x(2) - 1 = 0 \quad (3)$$

$$x(4) = x(3) - 1 = -1 \quad (4)$$

The corresponding series is:

$$2 + 2 + 1 + 0 + (-1) + \dots$$

One sided Z-transform is defined as :

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \quad (5)$$

Time shifting property for one-sided Z-transform:

$$x(n+k) \xrightarrow{Z} z^k \left(X(z) - x(0) - \frac{x(1)}{z} - \dots - \frac{x(k-1)}{z^{k-1}} \right) \quad (6)$$

$$\Rightarrow x(n+2) \xrightarrow{Z} z^2 \left(X(z) - 2 - \frac{2}{z} \right) \quad (7)$$

$$\Rightarrow x(n+1) \xrightarrow{Z} z(X(z) - 2), \quad (8)$$

Applying one-sided Z-transform on equation(1) and using results of equation(7) and (8)

$$z^2 \left(X(z) - 2 - \frac{2}{z} \right) - z(X(z) - 2) = -U(z) \quad (9)$$

$$X(z) = \frac{2 - 2z^{-1} - z^{-2}}{(1 - z^{-1})^2}, |z| > 1 \quad (10)$$

Using partial fractions

$$X(z) = \frac{2z^{-1}}{(1 - z^{-1})} - \frac{z^{-2}}{(1 - z^{-1})^2} + 2 \quad (11)$$

$$(12)$$

Substituting results of equation (??) to (??) in equation (11):

$$x(n) = 2u(n) + (1 - n)u(n - 1) \quad (13)$$

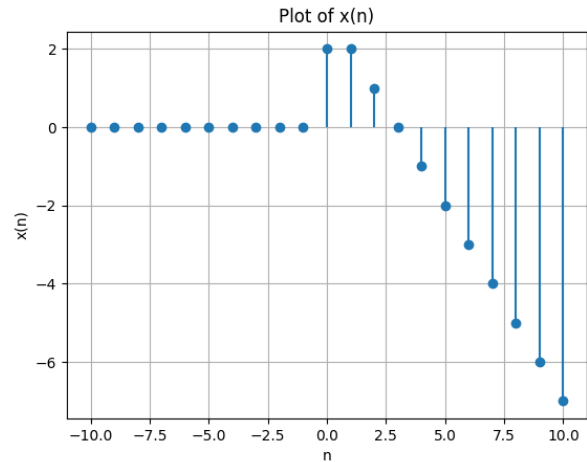


Fig. 1. Stem Plot of $x(n)$