NCERT 11.9.1.13Q

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Question: Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

$a_1 = a_2 = 2, \quad a_n = a_{n-1} - 1, \quad n > 2$

Solution:

Parameter	Description	Value
$x(0)$	First term	2
$x(1)$	Second term	2
ROC	Region of convergence	$\left\{ z : \left| \sum_{n=-\infty}^{\infty} x(n) z^{-n} \right| < \infty \right\}$

TABLE 1

PARAMETER TABLE

$$x(n) - x(n-1) = 2u(n) - 2u(n-1) - u(n-2) \tag{1}$$

Time shifting property for one-sided Z-transform:

$$x(n-k) \overset{\mathcal{Z}}{\longleftrightarrow} z^{-k}\left(X(z) + zx(-1) + \ldots + z^k x(-k)\right) \tag{2}$$

$$\implies x(n-1) \overset{\mathcal{Z}}{\longleftrightarrow} z^{-1}(X(z)) \tag{3}$$

$$\implies u(n-1) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{1-z^{-1}}, |z| > 1 \tag{4}$$

$$\implies u(n-2) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-2}}{1-z^{-1}}, |z| > 1 \tag{5}$$

Applying one-sided Z-transform on equation(1) and using results of equation(3) to (5)

$$X(z) - z^{-1}X(z) = \frac{2}{(1-z^{-1})} - \frac{z^{-2}}{(1-z^{-1})} - \frac{2z^{-1}}{(1-z^{-1})} \tag{6}$$

$$X(z) = \frac{2 - 2z^{-1} - z^{-2}}{(1-z^{-1})^2}, |z| > 1 \tag{7}$$

Using partial fractions

$$X(z) = \frac{2z^{-1}}{(1-z^{-1})} - \frac{z^{-2}}{(1-z^{-1})^2} + 2 \tag{8}$$

Taking inverse Z-transform by results of equation (**??**) and (4) in equation (5):

$$x(n) = 2u(n) + (1-n)u(n-1) \tag{9}$$
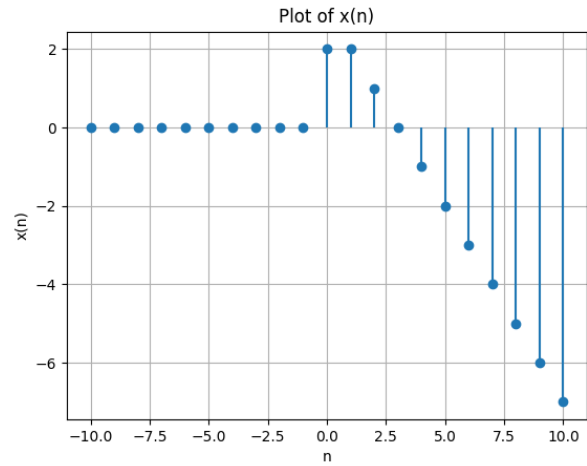

Fig. 1. Stem Plot of $x(n)$

Substituting $n = 0, 1, 2, 3, 4$ in equation(9) :

$$x(0) = 2 \tag{10}$$

$$x(1) = 2 \tag{11}$$

$$x(2) = x(1) - 1 = 1 \tag{12}$$

$$x(3) = x(2) - 1 = 0 \tag{13}$$

$$x(4) = x(3) - 1 = -1 \tag{14}$$