NCERT 11.9 13Q

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Question: Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:
$a_1 = a_2 = 2, \quad a_n = a_{n-1} - 1, \quad n > 2$
Solution:

$$x_{n+1} = x_n - 1, n > 1 \tag{1}$$

Substituting $n = 2, n = 3, n = 4$ in equation (1) we get:

$$x(2) = x(1) - 1 = 1 \tag{2}$$
$$x(3) = x(2) - 1 = 0 \tag{3}$$
$$x(4) = x(3) - 1 = -1 \tag{4}$$

The corresponding series is: $2 + 2 + 1 + 0 + (-1) +$

Parameter	Description	Value
$x(0)$	First term	2
$x(n)$	General term	$2u(n) + (1-n)u(n-2)$
ROC	Region of convergence	$\left\{z : \left\vert\sum_{n=-\infty}^{\infty} x(n)z^{-n}\right\vert < \infty\right\}$

TABLE 1
PARAMETER TABLE

By time shifting property:

$$x(n-k) \overset{\mathcal{Z}}{\longleftrightarrow} z^{-k}X(z) \tag{5}$$
$$\implies u(n-1) \overset{\mathcal{Z}}{\longleftrightarrow} z^{-1}U(z), |z| > 1 \tag{6}$$
$$\implies u(n-2) \overset{\mathcal{Z}}{\longleftrightarrow} z^{-2}U(z), |z| > 1 \tag{7}$$

By Differentiation Property:

$$nx(n) \overset{\mathcal{Z}}{\longleftrightarrow} -zX'(z) \tag{8}$$
$$\implies nu(n-2) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{2z^{-2} - z^{-3}}{(1-z^{-1})^2}, |z| > 1 \tag{9}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{10}$$
$$= \sum_{n=-\infty}^{\infty} (2u(n) + (1-n)u(n-2))z^{-n} \tag{11}$$

Using results of (7) and (9):

$$X(z) = \frac{2 - 2z^{-1} - z^{-2}}{(1-z^{-1})^2}, |z| > 1 \tag{12}$$

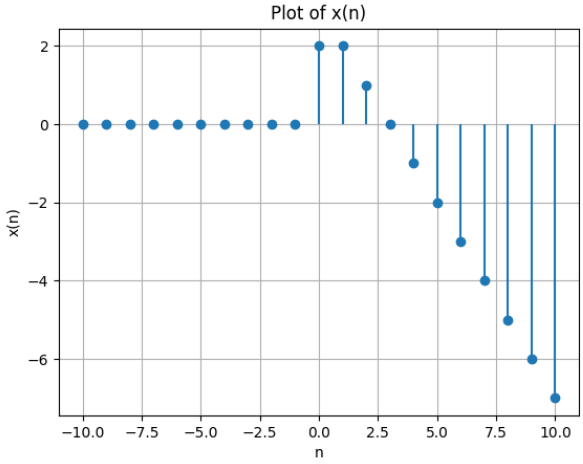

Fig. 1. Stem Plot of $x(n)$