GATE-EE-Q14

EE23BTECH11015 - DHANUSH V NAYAK*

Question: Consider a unity-gain negative feedback system consisting of the plant $G(s)$ and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

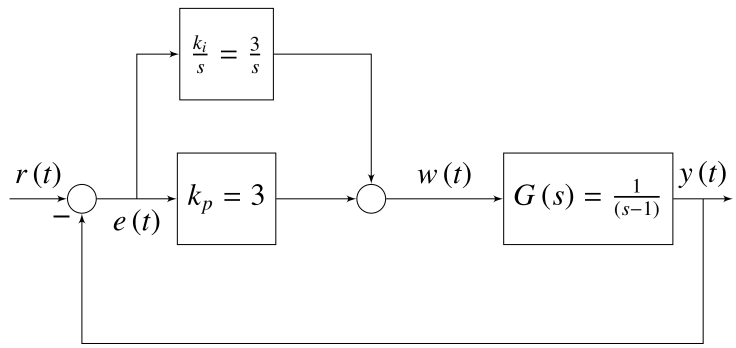

Fig. 1. Block Diagram of System

Solution:

Parameter	**Description**	**Value**
K_p	Proportional Gain	3
K_i	Integral Gain	1
$r(t)$	Reference Input	$u(t)$
$w(t)$	Controller Output	?
$y(t)$	Plant Output	?
$e(t)$	Error Input	$r(t) - y(t)$

TABLE 1
PARAMETER TABLE

From Fig. 1

$$e(t) = r(t) - y(t) \tag{1}$$

In frequency domain:

$$E(s) = U(s) - Y(s) \tag{2}$$
$$= \frac{1}{s} - Y(s) \tag{3}$$

$$w(t) = K_p e(t) + K_i \int_0^t e(t)\,dt \tag{4}$$
$$= 3u(t) + tu(t) \tag{5}$$

In frequency domain:

$$W(s) = 3E(s) + \frac{1}{s}E(s) \tag{6}$$

Substituting equation(3) in equation(6):

$$W(s) = \left(\frac{1}{s} - Y(s)\right)\left(3 + \frac{1}{s}\right) \tag{7}$$
$$= \frac{(s-1)(3s+1)}{s(s+1)^2}, Re(s) > -1 \tag{8}$$

$$Y(s) = G(s)W(s) \tag{9}$$
$$Y(s) = \frac{3s+1}{s(s+1)^2}, Re(s) > -1 \tag{10}$$

Final Value Theorem:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \tag{11}$$

Applying Final Value Theorem on Y(s):

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{12}$$
$$= 1 \tag{13}$$

Applying Final Value Theorem on W(s):

$$\lim_{t\to\infty} w(t) = \lim_{s\to 0} sW(s) \tag{14}$$
$$= -1 \tag{15}$$

Taking partial fraction of equation(10) :

$$Y(s) = \frac{1}{s} + \frac{2}{(s+1)^2} - \frac{1}{s+1} \tag{16}$$

$$\tag{17}$$

$$tx(t) \xleftrightarrow{\mathcal{L}} -\frac{dF(s)}{ds} \tag{18}$$

$$e^{-at}x(t) \xleftrightarrow{\mathcal{L}} F(s+a) \tag{19}$$

By using equation (18) and (19):

$$e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+1} \tag{20}$$

$$te^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+1)^2} \tag{21}$$

$$\therefore y(t) = u(t) + 2te^{-t}u(t) - e^{-t}u(t) \tag{22}$$

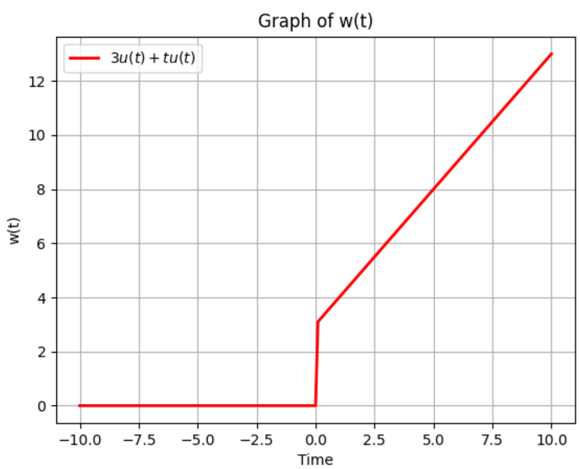

Fig. 2. Plot of $w(t)$

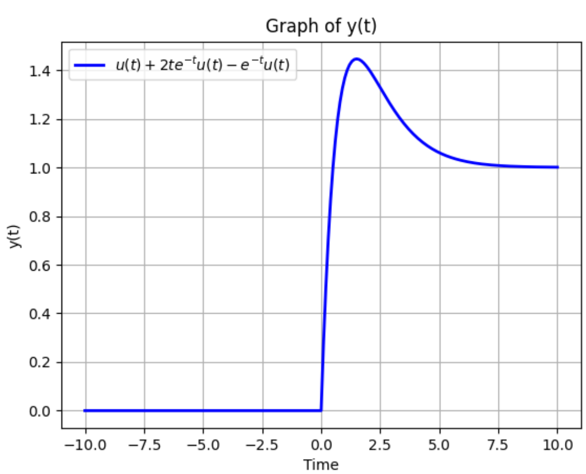

Fig. 3. Plot of $y(t)$