

GATE-EC-Q46

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Question: The outputs of four systems (S_1, S_2, S_3, S_4) corresponding to the input signal $\sin(t)$, for all time t , are shown in the figure. Based on the given information, which of the four systems is/are definitely NOT LTI(linear and time-invariant)?

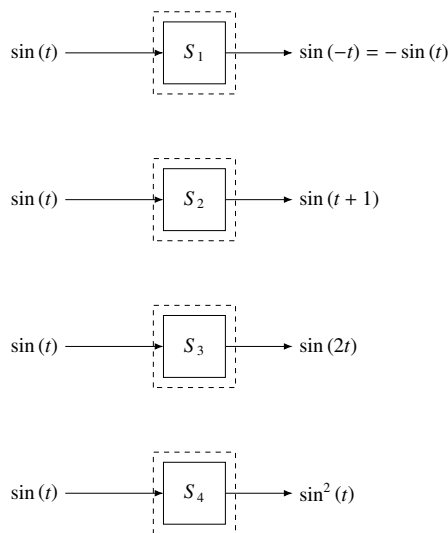


Fig. 1. Block Diagram of Systems

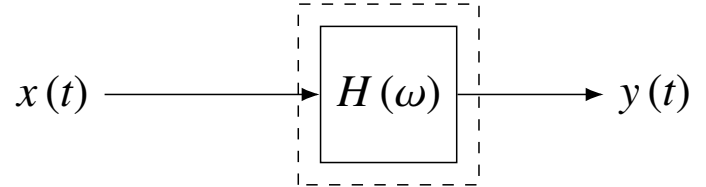


Fig. 2. Block Diagram of LTI System

For an LTI system :

$$y(t) = h(t) * x(t) \quad (1)$$

$$Y(\omega) = H(\omega) X(\omega) \quad (2)$$

$H(\omega)$ is a complex exponential :

$$H(j\omega) = |H(j\omega)| e^{j\Phi(\omega)} \quad (3)$$

$x(t) = \sin(t)$, and $\omega_o = 1 \text{ rad/sec}$

$$X(\omega) = j\pi (\delta(\omega + \omega_0) - \delta(\omega - \omega_0)) \quad (4)$$

Now,

$$Y(\omega) = (\delta(\omega + \omega_0) - \delta(\omega - \omega_0)) \pi |H(\omega)| e^{j\Phi(\omega)} \quad (5)$$

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Solution:

$$x(t) \delta(t - t_o) = x(t_o) \delta(t - t_o) \quad (6)$$

Using property (6) in (5) :

$$Y(\omega) = j\pi |H(-\omega_0)| e^{j\Phi(-\omega_0)} \delta(\omega + \omega_0) - j\pi |H(\omega_0)| e^{j\Phi(\omega_0)} \delta(\omega - \omega_0) \quad (7)$$

Given $h(t)$ a real-time domain signal, $H(\omega)$ is conjugate symmetric.

$$H(\omega) = H(-\omega) \quad (8)$$

$$\Phi(-\omega_0) = -\Phi(\omega_0) \quad (9)$$

Therefore,

$$Y(\omega) = j\pi |H(\omega_0)| \left(e^{-j\Phi(\omega_0)} \delta(\omega + \omega_0) - e^{j\Phi(\omega_0)} \delta(\omega - \omega_0) \right) \quad (10)$$

Parameter	Description
(S_1, S_2, S_3, S_4)	Systems Given
$\sin(t)$	Input
$H(\omega)$	Transfer Function
$X(\omega)$	Fourier-Transform of input
$Y(\omega)$	Fourier-Transform of output
$\Phi(\omega)$	Phase of Transfer Function

TABLE I
PARAMETER TABLE

Taking Inverse Fourier Transform,

$$\delta(\omega - \omega_0) \xleftrightarrow{\mathcal{F}} \frac{1}{2} e^{j\omega_0 t} \quad (11)$$

$$\delta(\omega + \omega_0) \xleftrightarrow{\mathcal{F}} \frac{1}{2} e^{-j\omega_0 t} \quad (12)$$

$$\Rightarrow y(t) = j\pi |H(\omega_0)| \frac{1}{2} \left(e^{-j(\omega_0 t + \Phi(\omega_0))} - e^{j(\omega_0 t + \Phi(\omega_0))} \right) \quad (13)$$

$$\Rightarrow y(t) = |H(\omega_0)| \sin(\omega_0 t + \Phi(\omega_0)) \quad (14)$$

$\omega_0 = 1$ rad/sec :

$$y(t) = |H(1)| \sin(t + \Phi(1)) \quad (15)$$

From (15) we can see output cant have scaled frequency nor a squared output. But can have a shifted output or amplitude-scaled output.

So, S_3 and S_4 cannot be LTI system.