NCERT 11.9.1.13Q

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Question: Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

$a_1 = a_2 = 2, \quad a_n = a_{n-1} - 1, \quad n > 2$

Solution:

TABLE 1

Parameter	Description	Value
$x(0)$	First term	2
$x(1)$	Second term	2
ROC	Region of convergence	$\left\{ z : \left\lvert \sum_{n=-\infty}^{\infty} x(n) z^{-n} \right\rvert < \infty \right\}$
$x_h(n)$	Homogenous solution	—
$x_t(n)$	Transient solution	—

PARAMETER TABLE

$$x(n+1) - x(n) = -u(n-1), n \geq 0 \qquad (1)$$

Homogeneous solution :-

$$x(n+1) - x(n) = 0 \qquad (2)$$

$$x_h(n) = cu(n) \qquad (3)$$

From table 1

$$x(0) = 2 \qquad (4)$$

$$\therefore c = 2 \qquad (5)$$

$$x_h(n) = 2u(n) \qquad (6)$$

Transient solution :

$$x(n+1) - x(n) = -u(n-1) \qquad (7)$$

One sided Z-transform is defined as :

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \qquad (8)$$

Time shifting property :

$$x(n-k) \overset{Z}{\longleftrightarrow} z^{-k} X(z) \qquad (9)$$

$$\implies x(n+1) \overset{Z}{\longleftrightarrow} z(X(z)) \qquad (10)$$

$$\implies u(n-1) \overset{Z}{\longleftrightarrow} \frac{z^{-1}}{1 - z^{-1}} \qquad (11)$$

Applying one-sided Z-transform on equation(7) and using results of equation(10) and (11)

$$z(X(z)) - X(z) = -\frac{z^{-1}}{1 - z^{-1}} \qquad (12)$$

$$X(z) = \frac{z^{-2}}{(1 - z^{-1})^2}, |z| > 1 \qquad (13)$$

Substituting result of equation (**??**) in equation (13):

$$x_t(n) = (1 - n) u(n - 1) \qquad (14)$$
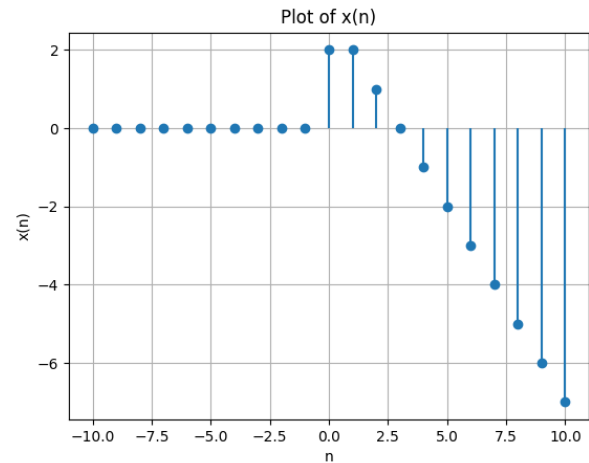
Now,

$$x(n) = x_h(n) + x_t(n) \qquad (15)$$

$$= 2u(n) + (1 - n) u(n - 1) \qquad (16)$$

Fig. 1. Stem Plot of $x(n)$