

NCERT 12.7 12Q

EE23BTECH11015 - DHANUSH V NAYAK*

Question: An LC circuit contains a $50\mu H$ inductor and a $50\mu F$ capacitor with an initial charge of $10mC$. The resistance of the circuit is negligible. Let the instant the circuit is closed by $t = 0$.

a) What is the total energy stored initially? Is it conserved during LC oscillations?

b) What is the natural frequency of the circuit?

c) At what time is the energy stored (i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?

d) At what times is the total energy shared equally between the inductor and the capacitor?

e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?
(NCERT 12.7 12Q)

Solution:

Parameter	Description	Value
L	Inductance	$50\mu H$
C	Capacitance	$50\mu F$
$E(0)$	Initial Energy of Capacitor	?
$q(0)$	Initial Charge on Capacitor	$10mC$
f	Natural Frequency	$\frac{1}{2\pi\sqrt{LC}}$

TABLE 1
PARAMETER TABLE

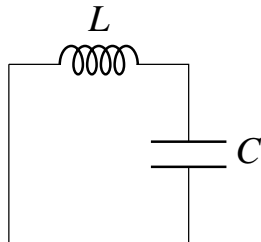


Fig. 1. LC Circuit

(a) Initial energy stored :

$$E(0) = \frac{q(0)^2}{2C} \quad (1)$$

$$= 1J \quad (2)$$

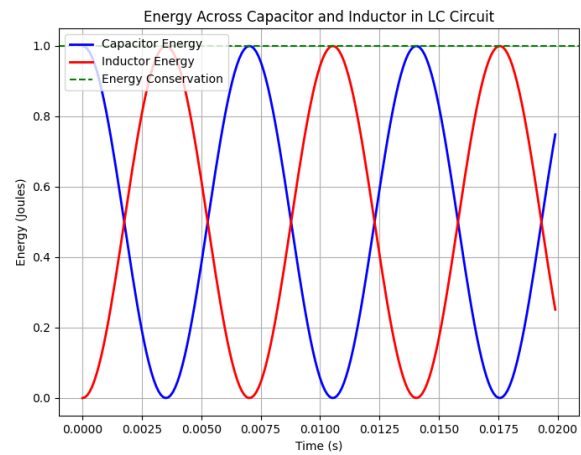


Fig. 2. Energy is Conserved During Oscillations total energy being limited to the initial energy

(b) Natural frequency:

From Table 1:

$$f = \frac{1}{2\pi\sqrt{LC}} \quad (3)$$

$$= 3183.10Hz \quad (4)$$

The Laplace domain circuit :

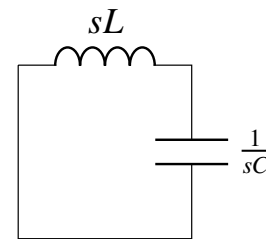


Fig. 3. LC Circuit in lapalace domain

The impedance of circuit :

$$Z = sL + \frac{1}{sC} \quad (5)$$

s can be expressed in resonant frequency as :

$$s = 2\pi f \quad (6)$$

$$Z = \left(2\pi f L - \frac{1}{2\pi f C} \right) j \quad (7)$$

$$|Z| = 2\pi f L - \frac{1}{2\pi f C} \quad (8)$$

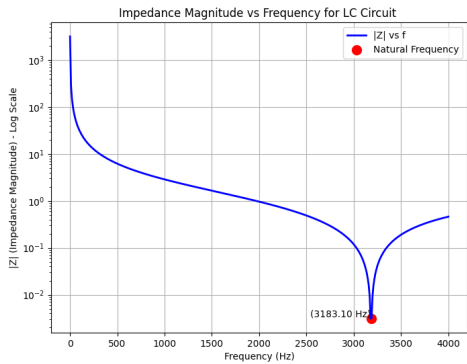


Fig. 4. The frequency at which impedance is minimum is natural frequency at $f = 3183.10\text{Hz}$

(c) At any given time the charge on the capacitor is given by:

$$q(t) = q(0) \cos 2\pi f t, T = \frac{1}{f} \quad (9)$$

$$i(t) = \frac{dq}{dt} \quad (10)$$

$$= q(0) 2\pi f \sin(2\pi f t + \pi) \quad (11)$$

The energy stored is completely electrical when $i(t) = 0$. At times $t = 0, \frac{T}{2}, T, \frac{3T}{2}, \dots$

The energy stored is completely magnetic when $q(t) = 0$ At times $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \dots$

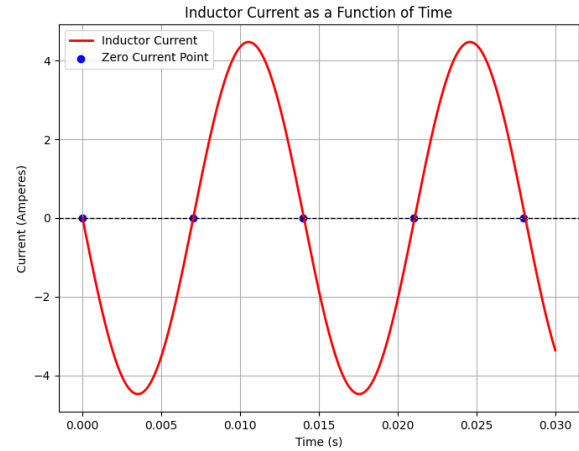


Fig. 5. Energy is completely electrical at $t = 0, \frac{T}{2}, T, \frac{3T}{2}, \dots$

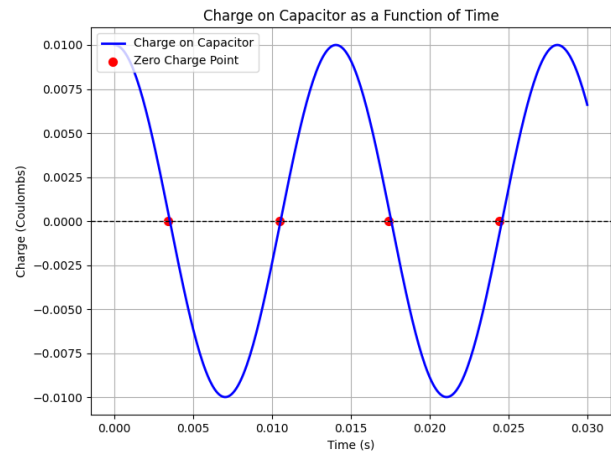


Fig. 6. Energy is completely magnetic at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \dots$

(d) When energy is equally shared then the capacitor has half of the maximum energy.

$$\frac{(q(t))^2}{2C} = \frac{1}{2} \quad (12)$$

$$q(0)^2 (\cos 2\pi f t)^2 = \frac{1}{2} \quad (13)$$

$$\cos 2\pi f t = \frac{1}{\sqrt{2}} \quad (14)$$

$$t = \frac{(2n+1)T}{8} \quad (15)$$

Hence, the total energy is equally shared between the inductor and capacitor at the time, $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \dots$

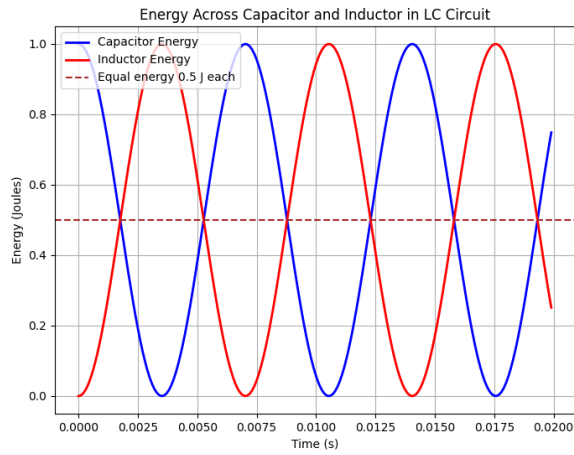


Fig. 7. At the intersection points with 0.5 J horizontal line both capacitor and inductor have equal energy.

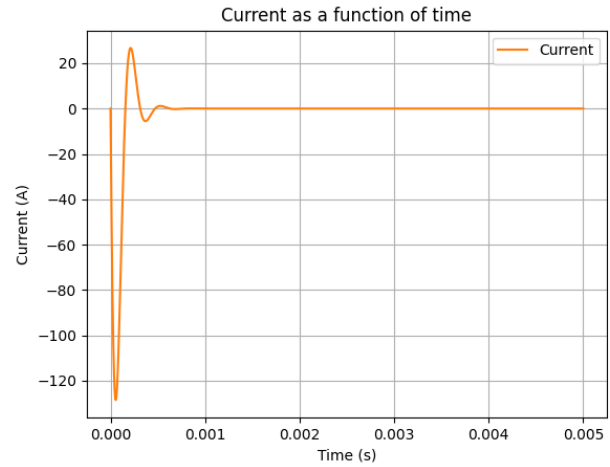
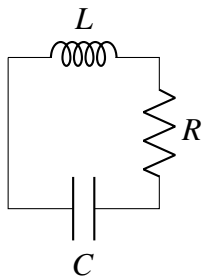


Fig. 9. Graph of current in the circuit. Becomes zero after all the energy is dissipated

(e) Once the resistor is added to the LC circuit, it starts dissipating energy in the form of heat.



Eventually, all the initial energy is dissipated in the form of heat.

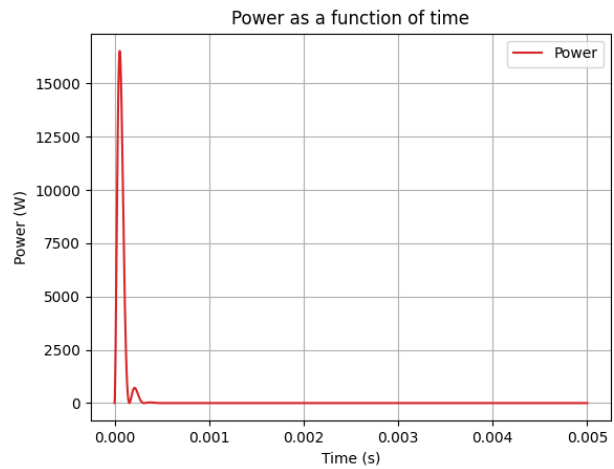


Fig. 10. Power Dissipated across resistor , $R = 1\Omega$

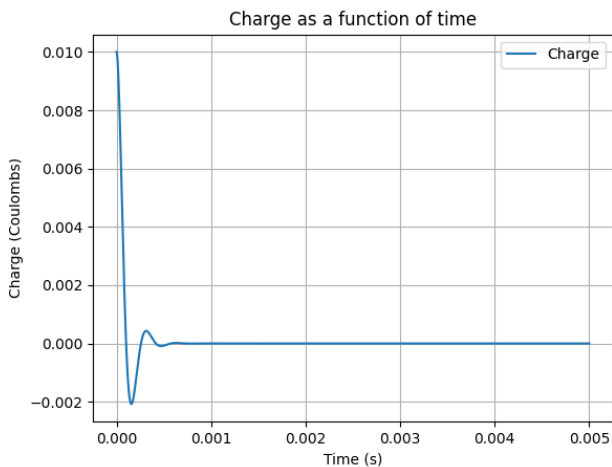


Fig. 8. Charge Across Capacitor decays as time proceeds