

GATE-EE-Q14

EE23BTECH11015 - DHANUSH V NAYAK*

Question: Consider a unity-gain negative feedback system consisting of the plant $G(s)$ (given below) and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

Solution:

Parameter	Description	Value
K_p	Proportional Gain	3
K_i	Integral Gain	1
$r(t)$	Reference Input	$u(t)$
$w(t)$	Controller Output	?
$y(t)$	Plant Output	?
$F(s)$	Feedback Gain	1
$e(t)$	Error Input	$r(t) - y(t)$

TABLE I
PARAMETER TABLE

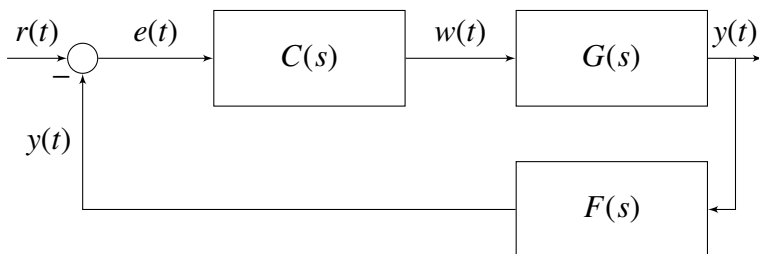


Fig. 1. Block Diagram of System

$$e(t) = r(t) - y(t) \quad (1)$$

$$E(s) = U(s) - Y(s) \quad (2)$$

$$= \frac{1}{s} - Y(s) \quad (3)$$

Now from Fig. 1,

$$w(t) = K_p e(t) + K_i \int_0^t e(t) dt \quad (4)$$

$$W(s) = 3E(s) + \frac{1}{s}E(s) \quad (5)$$

$$Y(s) = G(s)W(s) \quad (6)$$

From equation (3) and (5):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, \text{Re}(s) > -1 \quad (7)$$

By Final Value Theorem:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s) \quad (8)$$

From (8):

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) \quad (9)$$

$$= 1 \quad (10)$$

Substituting equation(3) in equation(5):

$$W(s) = \left(\frac{1}{s} - Y(s) \right) \left(3 + \frac{1}{s} \right) \quad (11)$$

$$= \frac{(s-1)(3s+1)}{s(s+1)^2}, \text{Re}(s) > -1 \quad (12)$$

Applying Final Value Theorem on $W(s)$:

$$\lim_{t \rightarrow \infty} w(t) = \lim_{s \rightarrow 0} sW(s) \quad (13)$$

$$= -1 \quad (14)$$