

NCERT 11.9.1.13Q

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Question: Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

$$a_1 = a_2 = 2, \quad a_n = a_{n-1} - 1, \quad n > 2$$

Solution:

Parameter	Description	Value
$x(0)$	First term	2
$x(1)$	Second term	2
ROC	Region of convergence	$\{z : \sum_{n=-\infty}^{\infty} x(n)z^{-n} < \infty\}$

TABLE I
PARAMETER TABLE

$$x(n+1) - x(n) = -u(n-1) \quad (1)$$

Time shifting property for one-sided Z-transform:

$$x(n+k) \xleftrightarrow{Z} z^k \left(X(z) - x(0) - \frac{x(1)}{z} - \dots - \frac{x(k-1)}{z^{k-1}} \right) \quad (2)$$

$$\Rightarrow x(n+1) \xleftrightarrow{Z} z(X(z) - 2) \quad (3)$$

$$\Rightarrow u(n-1) \xleftrightarrow{Z} \frac{z^{-1}}{1-z^{-1}}, |z| > 1 \quad (4)$$

Applying one-sided Z-transform on equation(1) and using results of equation(3) and (4)

$$z(X(z) - 2) - X(z) = -\frac{z^{-1}}{1-z^{-1}} \quad (5)$$

$$X(z) = \frac{2 - 2z^{-1} - z^{-2}}{(1-z^{-1})^2}, |z| > 1 \quad (6)$$

Using partial fractions

$$X(z) = \frac{2z^{-1}}{(1-z^{-1})} - \frac{z^{-2}}{(1-z^{-1})^2} + 2 \quad (7)$$

Taking inverse Z-transform by results of equation (??) and (4) in equation (7):

$$x(n) = 2u(n) + (1-n)u(n-1) \quad (8)$$

Substituting $n = 0, 1, 2, 3, 4$ in equation(8) :

$$x(0) = 2 \quad (9)$$

$$x(1) = 2 \quad (10)$$

$$x(2) = x(1) - 1 = 1 \quad (11)$$

$$x(3) = x(2) - 1 = 0 \quad (12)$$

$$x(4) = x(3) - 1 = -1 \quad (13)$$

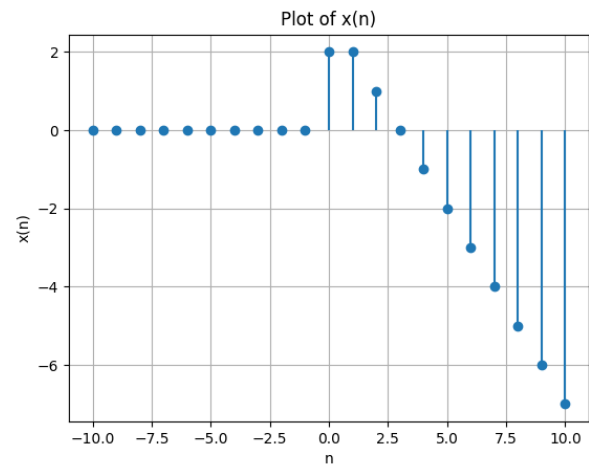


Fig. 1. Stem Plot of $x(n)$