GATE-EE-Q14

EE23BTECH11015 - DHANUSH V NAYAK*

Question: Consider a unity-gain negative feedback system consisting of the plant $G(s)$ and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

Solution:

Parameter	**Description**	**Value**
K_p	Proportional Gain	3
K_i	Integral Gain	1
$r(t)$	Reference Input	$u(t)$
$w(t)$	Controller Output	?
$y(t)$	Plant Output	?
$F(s)$	Feedback Gain	1
$C(s)$	Controller Gain	$3 + \frac{1}{s}$
$e(t)$	Error Input	$r(t) - y(t)$

TABLE 1
Parameter Table

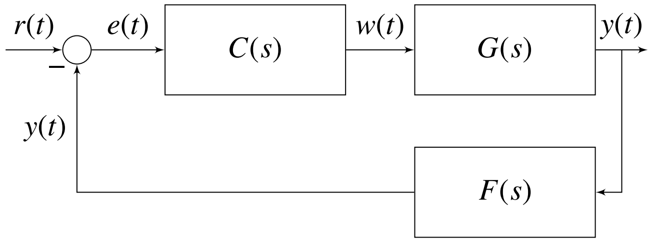

Fig. 1. Block Diagram of System

$$e(t) = r(t) - y(t) \tag{1}$$

In frequency domain:

$$E(s) = U(s) - Y(s) \tag{2}$$

$$= \frac{1}{s} - Y(s) \tag{3}$$

Now from Fig. 1,

$$w(t) = K_p e(t) + K_i \int_0^t e(t)\, dt \tag{4}$$

In frequency domain:

$$W(s) = 3E(s) + \frac{1}{s}E(s) \tag{5}$$

$$Y(s) = G(s)\, W(s) \tag{6}$$

From equation (3) and (5):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, Re(s) > -1 \tag{7}$$

By Final Value Theorem:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \tag{8}$$

From (8):

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{9}$$

$$= 1 \tag{10}$$

Substituting equation(3) in equation(5):

$$W(s) = \left(\frac{1}{s} - Y(s)\right)\left(3 + \frac{1}{s}\right) \tag{11}$$

$$= \frac{(s-1)(3s+1)}{s(s+1)^2}, Re(s) > -1 \tag{12}$$

Applying Final Value Theorem on W(s):

$$\lim_{t\to\infty} w(t) = \lim_{s\to 0} sW(s) \tag{13}$$

$$= -1 \tag{14}$$