

# NCERT 11.9.1.13Q

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**Question:** Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

$$a_1 = a_2 = 2, \quad a_n = a_{n-1} - 1, \quad n > 2$$

**Solution:**

| Parameter | Description           | Value   |
|-----------|-----------------------|---|
| $x(0)$    | First term            | 2   |
| $x(1)$    | Second term           | 2   |
| ROC       | Region of convergence | $\{z :  \sum_{n=-\infty}^{\infty} x(n)z^{-n}  < \infty\}$ |

TABLE 1  
PARAMETER TABLE

Applying one-sided Z-transform on equation(1) and using results of equation(7) and (8)

$$z(X(z) - 2) - X(z) = -\frac{z^{-1}}{1 - z^{-1}} \quad (9)$$

$$X(z) = \frac{2 - 2z^{-1} - z^{-2}}{(1 - z^{-1})^2}, |z| > 1 \quad (10)$$

Using partial fractions

$$X(z) = \frac{2z^{-1}}{(1 - z^{-1})} - \frac{z^{-2}}{(1 - z^{-1})^2} + 2 \quad (11)$$

$$(12)$$

Substituting results of equation (??) to (??) in equation (11):

$$x(n) = 2u(n) + (1 - n)u(n - 1) \quad (13)$$

$$x(n + 1) - x(n) = -u(n - 1), n \geq 0 \quad (1)$$

Substituting  $n = 1, n = 2, n = 3$  in equation (1) we get:

$$x(2) = x(1) - 1 = 1 \quad (2)$$

$$x(3) = x(2) - 1 = 0 \quad (3)$$

$$x(4) = x(3) - 1 = -1 \quad (4)$$

The corresponding series is:

$$2 + 2 + 1 + 0 + (-1) + \dots$$

One sided Z-transform is defined as :

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \quad (5)$$

Time shifting property for one-sided Z-transform:

$$x(n + k) \xrightarrow{Z} z^k \left( X(z) - x(0) - \frac{x(1)}{z} - \dots - \frac{x(k-1)}{z^{k-1}} \right) \quad (6)$$

$$\Rightarrow x(n + 2) \xrightarrow{Z} z^2 \left( X(z) - 2 - \frac{2}{z} \right) \quad (7)$$

$$\Rightarrow u(n - 1) \xrightarrow{Z} \frac{z^{-1}}{1 - z^{-1}}, |z| > 1 \quad (8)$$

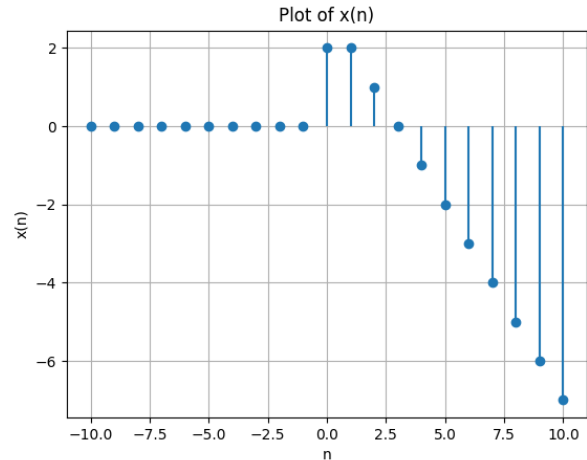


Fig. 1. Stem Plot of  $x(n)$