ASSIGNMENT - 3

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⑥ Big Omega Notation Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

Sol:

$g(n) \geq c\, n^3$

$g(n) = n^3 + 2n^2 + 4n$

for finding constants c and n_0

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

divide both sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

Here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$1 + \frac{2}{n} + \frac{4}{n^2}$

example: $c = \frac{1}{2}$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq \frac{1}{2}$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq \frac{1}{2}$ ($n \geq 1$ ($n_0 = 1$)

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeed $\Omega(n^3)$

7\. Big theta Notation: determine whether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or Not.

Sol:

$c_1 n^2 \leq h(n) \leq c_2 n^2$

&n upper bond $h(n)$ is $O(n^2)$

&n lower bond. $h(n)$ is $\Omega(n^2)$

upper bond $(O(n^2))$:

$h(n)(= 4n^2 + 3n$

$h(n) \leq c_2 n^2$

$4n^2 + 3n \leq c_2 n^2$

$4n^2 + 3n \leq 5n^2$

let $c_2 = 5$

Divide both sides by n^2

$$4 + 3/n \le 5$$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ ($c_2 = 5$, $n_0 = 1$)

lower bond:-

$h(n) = 4n^2 + 3n$

$h(n) \ge c_1 n^2$

$4n^2 + 3n \ge c_1 n^2$

lets $c_1 = 4 \Rightarrow 4n^2 + 3n \ge 4n^2$

divide both sides n^2

$$4 + \frac{3}{n} \ge 4$$

$h(n) = 4n^2 + 3n$ $(c_1 = 4, n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $\Theta(n^2)$

(8) let $f(n) = n^3 - 2n^2 + n$ and $g(n) - n^2$ show whether $f(n) = \Omega(g(n))$ is true or false and justify your answer.

$f(n) \ge g(n)$

Substituting $f(n)$ and $g(n)$ into this inequality we get

$$n^3 - 2n^2 + n > c(-n^2)$$

find c and no holds $n \ge n_0$

$$n^3 - 2n^2 + n \ge -cn^2$$

$$n^3 - c(-2)n^2 \ge 0$$

$(c = 1)$

$$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \ge 0$$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n))$ is true

(9)

Determine whether $h(n) = n\log n + n$ is $\Theta(n\log n)$ prove a rigorous proof for your conclusion.

Sol: $c_1 n \log n \leq h(n) \leq c_2 n \cdot \log n$

Upper bond:

$h(n) \leq c_2 n \log n$.

$h(n) = n\log n + n$

$n\log n + n \leq c_2 n \log n$

divide on both sides by $n\log n$

$1 + \frac{n}{n\log n} \leq c_2$

$1 + \frac{1}{\log n} \leq c_2$

$1 + \frac{1}{\log n} \leq 2 \quad (c_2 = 2)$

Then $h(n)$ is $O(n\log n)$ $(c_2 = 2, n_0 = 2)$

lower bond:-

$h(n) \geq c_1 \cdot n\log n$

$h(n) = n\log n + n$

$n\log n + n \geq c_1 n \log n$

divide both sides

$1 + \frac{1}{\log n} \geq c_1$

$1 + \frac{1}{\log n} \geq c_1$

$1 + \frac{1}{\log n} \geq 1$

$\frac{1}{\log n} \geq n$ for all $n \geq 1$

$h(n)$ is $\Omega(n \cdot \log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$.

(10) Solve the following recurrance relation and find the order of growth for solutions $T(n) = 4T(n/2) + n^2$, $T(1) = 1$

Sol:

$T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$T(n) = aT(n/2) + f(n)$

$a = 4$, $b = 2$, $f(n) = n^2$

applying master theorem $\left(T(n) = \Theta(n^{\log_b a})\right)$

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b a - 1})$

$f(n) = \Theta(n^{\log_b a})$, Then, $T(n) = \Theta(n^{\log_b a} \cdot \log n)$

Calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = O(n^2)$

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$. (case 2)

$f(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \cdot \log n) = \Theta(n^2 \log n)$

Order of growth

$T(n) = 4T(n/2) + n^2$. with $T(1) = 1$ is $\Theta(n^2 \log n)$.