DATA 605 - Homework 13

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```r
library(ggplot2)
library(psych)
library(dplyr)
library(knitr)
library(tidyr)
```

Ex.1 - intergration by substitution

solve $\int 4e^{-7x}dx$

Solution:

we chose $u = -7x$ which means $du = -7dx$ and $dx = -\frac{1}{7}du$

$\int 4e^{-7x}dx = \int -\frac{4}{7}e^u du = -\frac{4}{7}e^u + c = -\frac{4}{7}e^{-7x} + c$

Ex.2 - differential equation

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function $ N(t) $ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

Solution:

$\frac{dN}{dt} = -\frac{3150}{t^4} - 220$

$dN = (-\frac{3150}{t^4} - 220)dt$

$\int dN = \int(-\frac{3150}{t^4} - 220)dt$

$N(t) = \frac{-3150}{-3t^3} - 220t + c$

$N(t) = \frac{1050}{t^3} - 220t + c$

$N(1) = \frac{1050}{1^3} - 220 * 1 + c = 6530 => c = 6530 - 1050 + 220 = 5700$

$N(t) = \frac{1050}{t^3} - 220t + 5700$

Ex.3 - intergral approximation

Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$.

We can simply use geometry $(a * b)$ for the size of each box: $1 * 1 + 3 * 1 + 5 * 1 + 7 * 1 = 16$

or we can integrate $f(x) = 2x - 9$ from 4.5 to 8.5:

$\int_{4.5}^{8.5} 2x - 9dx = [x^2 - 9x]_{4.5}^{8.5} = (8.5^2 - 9 * 8.5) - (4.5^2 - 9 * 4.5) = 16$

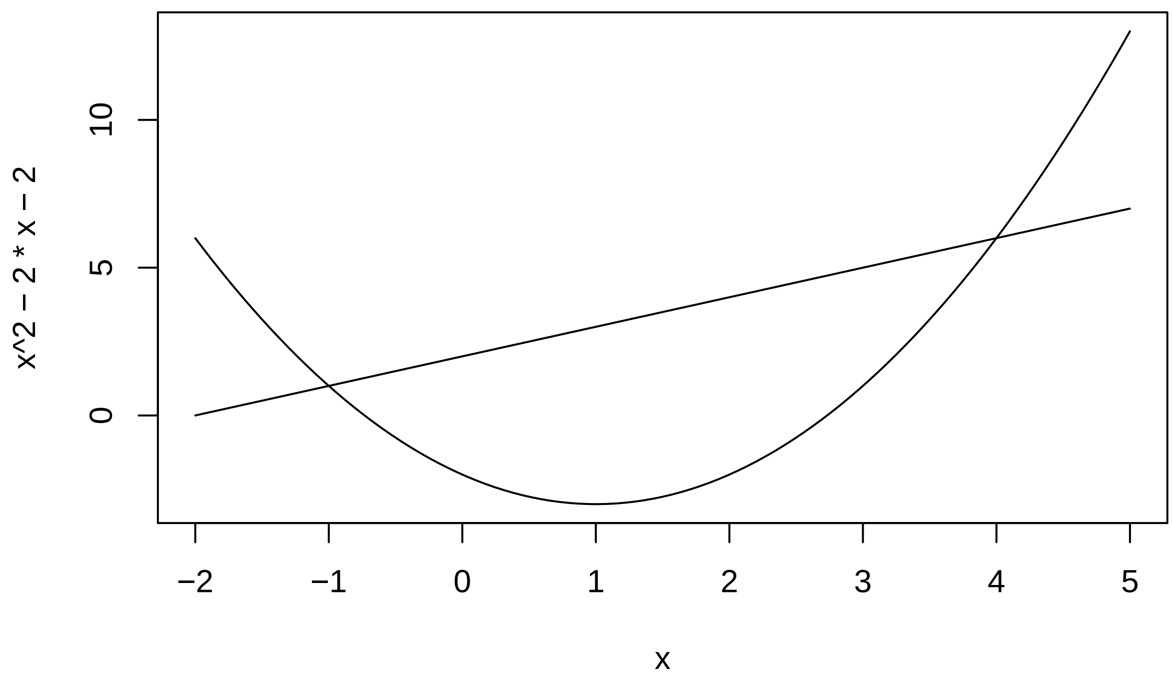
this only works perfectly since f(x) is linear

Ex.4 - area between 2 graphs

Find the area of the region bounded by the graphs of the given equations. $y = x^2 - 2x - 2$, $y = x + 2$

Solution:

```
curve(x^2 -2*x-2, -2, 5)
curve(x+2, add = TRUE, -2,5)
```



From the graph we can see that the intersects are -1 and 4 which we can double check by inserting these values into the equations:

$x = -1$: $y = x^2 - 2x - 2 = 1 + 2 - 2 = 1$, $y = x + 2 = -1 + 2 = 1$

$x = 4$: $y = x^2 - 2x - 2 = 16 - 8 - 2 = 6$, $y = x + 2 = 4 + 2 = 6$

Area:

$Area = \int_{-1}^{4} x + 2dx - \int_{-1}^{4} x^2 - 2 * x - 2dx$

$= \int_{-1}^{4} x + 2 - (x^2 - 2 * x - 2)dx$

$= \int_{-1}^{4} x + 2 - x^2 + 2 * x + 2dx$

$= \int_{-1}^{4} -x^2 + 3 * x + 4dx$

$= [-\frac{1}{3}x^3 + \frac{3}{2} * x^2 + 4x]_{-1}^{4}$

$= 18\frac{2}{3} - -2\frac{1}{6} = 20\frac{5}{6}$

Ex.5 - optimizing storage

A beauty supply store expects to sell 110 flat irons during the next year. It costs USD3.75 to store one flat iron for one year. There is a fixed cost of USD8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

Solution:

we assume that the sales happen regularly so that on average storage capacity is half of the order size.

x = number of orders:

cost $= 8.25 * x + 3.75 * 110/(2x) = 8.25 * x + 206.25 * x^{-1}$

cost' $= 8.25 - 206.25 * x^{-2}$

setting the derivative to zero will give us all minima and maxima of the cost function:

cost' $= 8.25 - 206.25 * x^{-2} = 0$

$x^{-2} = \frac{8.25}{206.25}$

$x = \pm\sqrt{\frac{206.25}{8.25}} = \pm 5$

Since we cannot have negative amount of orders the only possible solution is 5 which means that every order should be 22 flat irons

Ex.6 - integration by parts

Use integration by parts to solve the integral below.

$\int ln(9x)x^6dx$

Solution:

We chose:

$u = ln(9x)$ and $dv = x^6dx$

additionally:

$du = d(ln(9x))/dx = d(ln(9) + ln(x))/dx = d(ln(x))/dx = 1/xdx$

$v = \frac{1}{7}x^7$

$\int ln(9x)x^6dx = ln(9x) * \frac{1}{7}x^7 - \int \frac{1}{7}x^7\frac{1}{x}dx$

$=ln(9x) * \frac{1}{7}x^7 - \frac{x^7}{49} + c$

Ex.7 - prob density

Determine whether f (x) is a probability density function on the interval [1, e6]. If not, determine the value of the definite integral.

$f(x) = \frac{1}{6x}$

Solution:

We need to check wether f(x) > 0 and the integral over the interval = 1.

f(x) > 0 for x>0 so only the integral = 1 is left to show:

$\int_1^{e^6} \frac{1}{6x} = [\frac{1}{6} ln(x)]_1^{e^6} = \frac{1}{6} ln(e^6) - \frac{1}{6} ln(1) = 1 - 0 = 1$

So it does represent a prob density.

Github (both PDF and RMarkdown):

https://github.com/chilleundso/Data605_CompMath/tree/master/Homework13