DATA 605 - Homework 7

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```
library(ggplot2)
library(psych)
library(dplyr)
library(knitr)
library(tidyr)
```

1) Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y .

We can think of this as a combinatorical problem:

In order for $Y_n = y$ we know that at least one $X_i = y$ and all others are great than or equal to y.

Once we know how many ways we can achieve this we can divide by the total number of ways n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k can be chosen (which is k^n).

We can start with y=1: This includes all combinations of X_i except those where not a single one has value 1. We can therefore take all possible combinations (again, this is k^n) and subtract the amount of combinations when none are 1 ($(k-1)^n$):

$k^n - (k-1)^n$

We can now itteratively go higher with y:

For y=2 we go similarly to the above we start with all possibilitites and subtract all combinations where all $X_i > 2$ (which has $(k-2)^n$ combinations) and obviously we subtract all combinations where Y=1 (which is a disjointed group and we know from above is $k^n - (k-1)^n$):

$k^n - (k-2)^n - ((k^n - (k-1)^n)$

$= -(k-2)^n + (k-1)^n = (k-1)^n - (k-2)^n$

For Y= 3 (all - all $X_i > 3$ - $Y = 2$ - $Y = 1$):

$k^n - [(k-3)^n] - [(k-1)^n - (k-2)^n] - [k^n - (k-1)^n]$

We see the k^n and $(k-1)^n$ cancel out leaving us with:

$(k-2)^n - (k-3)^n$

The pattern shows that more generally there are the following amount of ways to achieve Y=y:

$(k-y+1)^n - (k-y)^n$

And therefore the probability distribtion is:

$\frac{(k-y+1)^n-(k-y)^n}{k^n}$

2) Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years.)

Probability to not fail 8 years in a row:

```
p=1/10

(1-p)^8
```

```
## [1] 0.4304672
```

Using R: pgeom is cumulative i.e. prob that it fails on or before n+1 trial (we want the complement) :

```
1-pgeom(8-1,0.1)
```

```
## [1] 0.4304672
```

Expected value:

```
E=1/p
E
```

```
## [1] 10
```

standard variation:

```
Sd=sqrt((1-p)/p^2)
Sd
```

```
## [1] 9.486833
```

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

Probability to not fail 8 years in a row:

```
l=1/10

exp(-8*.1)
```

```
## [1] 0.449329
```

Using R: pexp is cumulative i.e. prob that it fails on or before n trial (we want the complement) :

```r
1-pexp(8,l)
```

```
## [1] 0.449329
```

Expected value:

```r
E=1/l
E
```

```
## [1] 10
```

standard variation:

```r
Sd=sqrt(1/l^2)
Sd
```

```
## [1] 10
```

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

Probability to not fail 8 years in a row:

```r
n = 8
k = 0
p = 0.1
q=1-p

choose(n,k) * p^(k) * q^(n-k)
```

```
## [1] 0.4304672
```

Using R: dbinom with 8 years with zero breaks:

```r
dbinom(k, n, p)
```

```
## [1] 0.4304672
```

Expected value:

```r
E=n*p
E
```

```
## [1] 0.8
```

standard variation:

```
Sd=sqrt(n*p*q)
Sd
```

```
## [1] 0.8485281
```

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson Probability to not fail 8 years in a row:

```
l <- 8/10
k <- 0

exp(-l)*l^0/factorial(0)
```

```
## [1] 0.449329
```

Using R: ppois:

```
ppois(k,l)
```

```
## [1] 0.449329
```

Expected value:

```
E=l
E
```

```
## [1] 0.8
```

standard variation:

```
Sd=sqrt(l)
Sd
```

```
## [1] 0.8944272
```

Github (both PDF and RMarkdown):

https://github.com/chilleundso/Data605_CompMath/tree/master/Homework7