DATA 605 - Homework 9

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```
library(ggplot2)
library(psych)
library(dplyr)
library(knitr)
library(tidyr)
```

Ex.11 (page 363)

The price of one share of stock in the Pilsdorff Beer Company (see Exer- cise 8.2.12) is given by Y_n on the nth day of the year. Finn observes that the differences $X_n = Y_{n+1} - Y_n$ appear to be independent random variables with a common distribution having mean $\mu = 0$ and variance $\sigma^2 = \frac{1}{4}$. If $Y_1 = 100$, estimate the probability that Y_{365} is:

(a) 100.
(b) 110.
(c) 120.

Solution:

```
m = 0
var = .25
s = sqrt(var)
n = 364
```

(a) P(Y_{365} 100):

Since $\mu = 0$ we expect this random walk to equally end above or below the starting value therefore 50/50 chance:

```
x1 <- (100 - 100)/sqrt(n)
pnorm(x1, m, s, lower.tail = FALSE)
```

```
## [1] 0.5
```

(b) P(Y_{365} 110):

```
x2 <- (110 - 100)/sqrt(n)
pnorm(x2, m, s, lower.tail = FALSE)
```

```
## [1] 0.1472537
```

(c) P(Y_{365} 120):

```
x3 <- (120 - 100)/sqrt(n)
pnorm(x3, m, s, lower.tail = FALSE)
```

```
## [1] 0.01801584
```

From 10 we know the mean of the mean of the minimum of n exp densities is the mean of one density divided by n. Therefore, since we have 100 lightbubs each of which has an average life expecancy of 1000hours the minimum (first broken one) will be 1000/100 = 10 hours

Problem 2

Calculate the expected value and variance of the binomial distribution using the moment generating function.

Solution:

binomial distribution: $\binom{n}{k}p^x(1-p)^{n-x}$

moment generating function:

g(t) = $\sum_{j=0}^{n}\binom{n}{j}p^j q^{n-j}e^{tj}$

g(t) = $\sum_{j=0}^{n}\binom{n}{j}(pe^t)^j q^{n-j}$

g(t) = $(pe^t+q)^n$ = $(pe^t+(1-p)^n$

derivatives through wolfram alpha:

g'(t) = $npe^t(pe^t-p+1)^{n-1}$

g"(t) = $npe^t(pe^t-p+1)^{n-2}(npe^t-p+1)$

Exp = g'(0) = $npe^0(pe^0-p+1)^{n-1}$ = $np(p-p+1)^{n-1}$ = $np(1)^{n-1}$ = np

g"(0) = $npe^0(pe^0-p+1)^{n-2}(npe^0-p+1)$ = $np(p-p+1)^{n-2}(np-p+1)$ = $np(1)^{n-2}(np-p+1)$ = $np(np-p+1)$

Var(X) = $g''(0)-g'(0)^2$ = $np(np-p+1)-(np)^2$ = \$(np)^2 - np^2 + np - (np)^2 \$ = $np-np^2$ = $np(1-p)$

Problem 3

Calculate the expected value and variance of the exponential distribution using the moment generating function.

Solution:

exponential distribution: \$ e^{- x} \$

moment generating function:

g(t) = $\int_{j=0}^{\infty}\lambda e^{-\lambda j}e^{tj}dj$

g(t) = $\lambda\int_{j=0}^{\infty}e^{(t-\lambda)j}dj$

g(t) = $\lambda[\frac{e^{(t-\lambda)j}}{(t-\lambda)}]_{j=0}^{\infty}$

g(t) = $\lambda[\frac{e^{(t-\lambda)j}}{(t-\lambda)}]_{j=0}^{\infty}$

for $t < \lambda$

g(t) = $\lambda(0 - \frac{1}{t-\lambda}) = \frac{\lambda}{\lambda-t}$

g'(t) = $\frac{\lambda}{(\lambda-t)^2}$

g"(t) = $\frac{2\lambda}{(\lambda-t)^3}$

Exp = $g'(0) = \frac{\lambda}{(\lambda)^2} = \frac{1}{\lambda}$

g"(0) = $\frac{2\lambda}{(\lambda)^3} = \frac{2}{\lambda^2}$

Var = g''(0) - g'(0)^2 = $\frac{2}{\lambda^2} - (\frac{1}{\lambda})^2 = \frac{1}{\lambda^2}$

Github (both PDF and RMarkdown):

https://github.com/chilleundso/Data605_CompMath/tree/master/Homework9