DATA 605 - Homework 3

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```r
library(ggplot2)
library(psych)
library(dplyr)
library(knitr)
library(tidyr)
```

Problem Set 1

```r
matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3),nrow=4,ncol=4,byrow = T)
```

1)

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

We will do following operations:

R2 = R2 + R1

R4 = R4 - 5 * R1

```r
matrix(c(1,2,3,4,0,2,4,7,0,1,-2,1,0,-6,-17,-23),nrow=4,ncol=4,byrow = T)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]    0    2    4    7
## [3,]    0    1   -2    1
## [4,]    0   -6  -17  -23
```

We will do following operations:

R3 = R3 - .5*R2

R4 = R4 + 3* R2

```r
matrix(c(1,2,3,4,0,2,4,7,0,0,0,-2.5,0,0,-5,-2),nrow=4,ncol=4,byrow = T)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3  4.0
## [2,]    0    2    4  7.0
## [3,]    0    0    0 -2.5
## [4,]    0    0   -5 -2.0
```

We swap the 3rd and 4th row

```r
matrix(c(1,2,3,4,0,2,4,7,0,0,-5,-2,0,0,0,-2.5),nrow=4,ncol=4,byrow = T)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3  4.0
## [2,]    0    2    4  7.0
## [3,]    0    0   -5 -2.0
## [4,]    0    0    0 -2.5
```

Subtract R2 from R1:

```r
matrix(c(1,0,-1,-3,0,2,4,7,0,0,-5,-2,0,0,0,-2.5),nrow=4,ncol=4,byrow = T)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0   -1 -3.0
## [2,]    0    2    4  7.0
## [3,]    0    0   -5 -2.0
## [4,]    0    0    0 -2.5
```

Then multiply R2 by 1/2 , R3 by -1/5 and R4 by -5/2:

```r
matrix(c(1,0,-1,-3,0,1,2,7/2,0,0,1,2/5,0,0,0,1),nrow=4,ncol=4,byrow = T)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0   -1 -3.0
## [2,]    0    1    2  3.5
## [3,]    0    0    1  0.4
## [4,]    0    0    0  1.0
```

Finally, we do and see that it results in the 4x4 identity matrix i.e. full rank / rank 4:

R1 = R1 + 3 * R4

R2 = R2 -3.5 * R4

R3 = R3 -2/5 * R4

R1 = R1 + R3

R2 = R2 - 2 * R3

```r
matrix(c(1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1),nrow=4,ncol=4,byrow = T)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

2) The maximum rank is n (since m > n) and there cannot be more then n linear independent rows in a matrix with n variables. The minimum is 1 if all rows are a linear combination of each other.

2) We can see that R2 = 3 * R1 and R3 = 2 * R1. We could therefore do:

R2 = R2 - 3 * R1

R3 = R3 - 2 * R1

and end up with

```
matrix(c(1,2,1,0,0,0,0,0,0),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    0    0    0
## [3,]    0    0    0
```

Which means that the rank of the matrix is 1.

Problem Set 2

Since A is an upper triangular matrix A - x*Id will also be an upper traingular matrix and therefore the determinant will be purely the product of the main diagonal:

(1-x) * (4-x) * (6-x) = 0

therefore eigenvalues are 1, 4 and 6.

Eigenvector for lambda1 = 1:

```
matrix(c(0,2,3,0,3,5,0,0,5),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]    0    2    3
## [2,]    0    3    5
## [3,]    0    0    5
```

R2 = R2 - 3/2R1

```
matrix(c(0,2,3,0,0,5-9/2,0,0,5),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]    0    2  3.0
## [2,]    0    0  0.5
## [3,]    0    0  5.0
```

we can tranform this to

```
matrix(c(0,1,3/2,0,0,1,0,0,0),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]    0    1  1.5
## [2,]    0    0  1.0
## [3,]    0    0  0.0
```

and finally to:

```r
matrix(c(0,1,0,0,0,1,0,0,0),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]    0    1    0
## [2,]    0    0    1
## [3,]    0    0    0
```

therefore for the eigenvactor v1 of lambda1 x2=0, x3=0 and x1=1

```r
matrix(c(1,0,0),nrow=3,ncol=1,byrow = T)
```

```
##      [,1]
## [1,]    1
## [2,]    0
## [3,]    0
```

for lambda2 = 4 we start with the below matrix:

```r
matrix(c(-3,2,3,0,0,5,0,0,2),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]   -3    2    3
## [2,]    0    0    5
## [3,]    0    0    2
```

We see that we can tranform it to:

```r
matrix(c(1,-2/3,0,0,0,1,0,0,0),nrow=3,ncol=3,byrow = T)
```

```
##      [,1]       [,2] [,3]
## [1,]    1 -0.6666667    0
## [2,]    0  0.0000000    1
## [3,]    0  0.0000000    0
```

we see x3 = 0 and x1 = 2/3 * x2 setting x2 to 1 we get eigenvector v2 for lambda2=4:

```r
matrix(c(2/3,1,0),nrow=3,ncol=1,byrow = T)
```

```
##           [,1]
## [1,] 0.6666667
## [2,] 1.0000000
## [3,] 0.0000000
```

for lambda3 = 6 we start with the below matrix:

```r
matrix(c(-5,2,3,0,-2,5,0,0,0),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]   -5    2    3
## [2,]    0   -2    5
## [3,]    0    0    0
```

We can normalize using the first non zero element:

```r
matrix(c(1,-2/5,-3/5,0,1,-5/2,0,0,0),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]    1 -0.4 -0.6
## [2,]    0  1.0 -2.5
## [3,]    0  0.0  0.0
```

We add 2/5 of row 2 to row 1

```r
matrix(c(1,-2/5+(1*(2/5)),-3/5+((-5/2)*(2/5)),0,1,-5/2,0,0,0),nrow=3,ncol=3,byrow = T)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0 -1.6
## [2,]    0    1 -2.5
## [3,]    0    0  0.0
```

we see x3 = 1 and x1 = 1.6 and x2 = 2.5 setting x2 to 1 we get eigenvector v2 for lambda3=6:

```r
matrix(c(8/5,5/2,1),nrow=3,ncol=1,byrow = T)
```

```
##      [,1]
## [1,]  1.6
## [2,]  2.5
## [3,]  1.0
```

Github (both PDF and RMarkdown):

https://github.com/chilleundso/Data605_CompMath/tree/master/Homework3