DATA 605 - Homework 14

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```r
library(ggplot2)
library(psych)
library(dplyr)
library(knitr)
library(tidyr)
```

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion.

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$

f(x) = 1/(1−x)

$f(x) = \frac{1}{(1-x)} = (1-x)^{-1}$

$f'(x) = \frac{1}{(1-x)^2}$

$f''(x) = \frac{2}{(1-x)^3}$

$f'''(x) = \frac{6}{(1-x)^4}$

$f^{(4)}(x) = \frac{24}{(1-x)^5}$

$f^{(n)}(x) = \frac{n!}{(1-x)^{n+1}}$

for x = 0 this simplifies to $f^{(n)}(0) = n!$

for $a = 0$ we can see that the n! offset each other in each summand

$f(x) = 1 + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$

$f(x) = 1 + x^1 + x^2 + x^3 + x^4 + ...$

f(x) = exp(x)

$f(x) = f^{(n)}(x) = e^x$ for all n and therefore $f^{(n)}(0) = e^0 = 1$

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$ for a = 0:

$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

f(x) = ln(1+x)

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x}$

$f^2(x) = -\frac{-1}{(1+x)^2}$

$f^3(x) = \frac{2!}{(1+x)^3}$

$f^4(x) = -\frac{3!}{(1+x)^4}$

$f^n(x) = (-1)^{(n+1)}\frac{(n-1)!}{(1+x)^n}$

we plug this into the below:

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$

we can see that n! and (n-1)! simplify to $\frac{1}{n}$ therefire for a=0

$\sum_{n=0}^{\infty}(-1)^{(n+1)}\frac{x^n}{n!}$

Github (both PDF and RMarkdown):

https://github.com/chilleundso/Data605_CompMath/tree/master/Homework14