DATA 605 - Homework 15

Manolis Manoli

```r
library(ggplot2)
library(psych)
library(dplyr)
library(knitr)
library(tidyr)
```

1. Regression Line

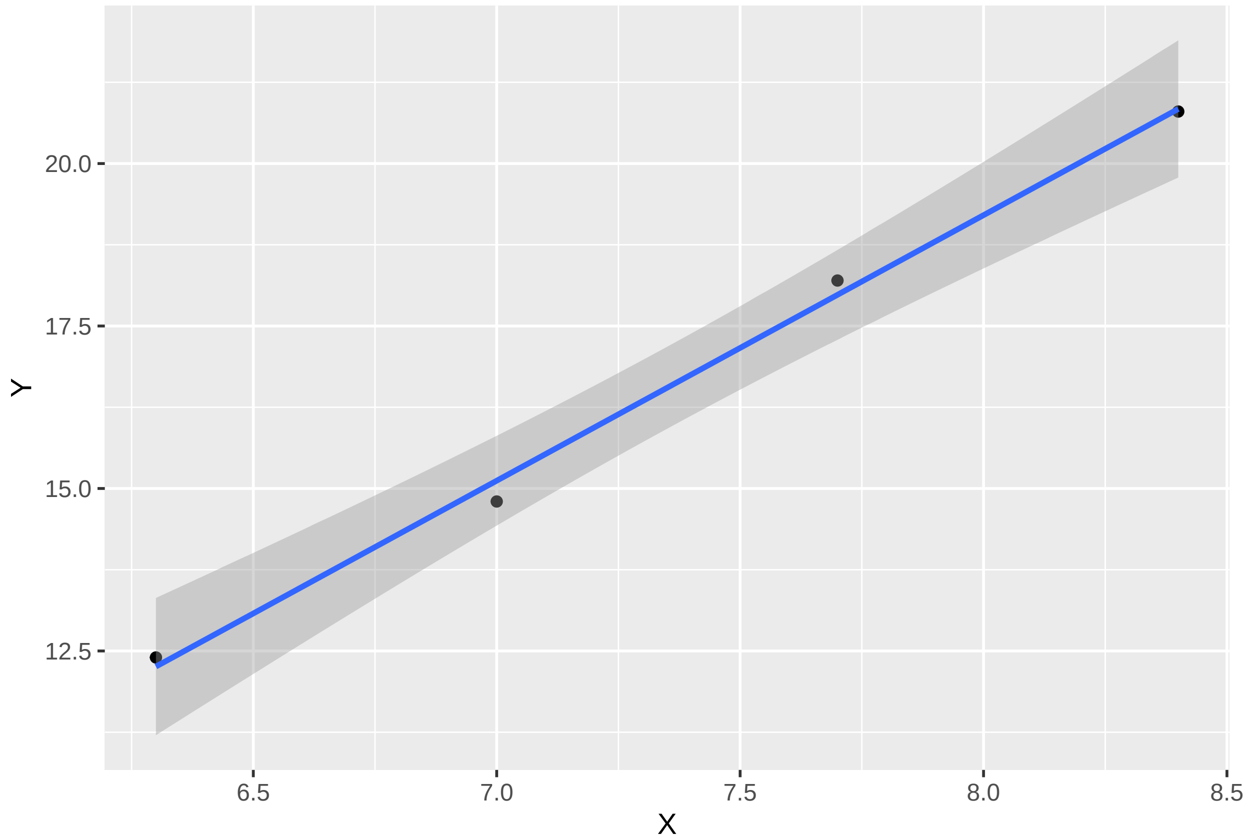
Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary.(5.6, 8.8), (6.3, 12.4), (7, 14.8), (7.7, 18.2), (8.4, 20.8)

```r
regdata <- matrix(c( 6.3, 12.4 ,  7, 14.8 ,  7.7, 18.2 ,  8.4, 20.8 ),ncol=2,byrow=TRUE)
colnames(regdata) <- c("X","Y")
regdf = as.data.frame(regdata)
```

```r
regr = lm(regdf$Y ~ regdf$X)
summary(regr)
```

```
##
## Call:
## lm(formula = regdf$Y ~ regdf$X)
##
## Residuals:
##     1     2     3     4
##  0.14 -0.32  0.22 -0.04
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13.4800     1.3848  -9.734   0.0104 *
## regdf$X       4.0857     0.1874  21.807   0.0021 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2933 on 2 degrees of freedom
## Multiple R-squared:  0.9958, Adjusted R-squared:  0.9937
## F-statistic: 475.6 on 1 and 2 DF,  p-value: 0.002096
```

```r
ggplot(regdf, aes(x = X, y = Y)) +
geom_point() +
geom_smooth(method='lm', formula= y ~ x )
```




```
inter = summary(regr)$coefficients[1,1]
slope = summary(regr)$coefficients[2,1]
```

The fitted line has the formula: y = -13.48 + 4.086 * x

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion.

2. Curve Discussion

Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma. $f(x,y) = 24x - 6xy^2 - 8y^3$

$f(x,y) = 24x - 6xy^2 - 8y^3$

$f_x = 24 - 6y^2$

$f_y = -12xy - 24y^2$

$f_{xx} = 0$

$f_{xy} = -12y$

$f_{yx} = -12y$

$f_{yy} = -12x - 48y$

find where the first derivatives are zero:

$f_x = 24 - 6y^2 = 0 <=> 6y^2 = 24 <=> y^2 = 4 <=> y = \pm 2$

we can plug the two solutions into f_y.

$y = 2$:

$f_y = -24x - 96 = 0 <=> 24x = -96 <=> x = -4$

First point is (-4,2)

$f(-4, 2) = -64$

(-4,2,-64)

$y = -2$:

$f_y = 24x - 96 = 0 <=> 24x = 96 <=> x = 4$

Second point is (4,-2)

$f(4, -2) = 64$

(4,-2,64)

we need to calculate $D = f_{xx}f_{yy} - f_{xy}^2$

Since $f_{xx} = 0$ $D = -f_{xy}^2 = -(-12y)^2$ which is always negative (for non zero y) therefore a sattlepoint:

Sattle-points : (-4,2,-64) and (4,-2,64)

3. Grocery Store

A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Step 1

Find the revenue function R(x, y).

$R(x, y) = x(81 - 21x + 17y) + y(40 + 11x - 23y) = 81x + 40y - 21x^2 - 23y^2 + 28xy$

Step 2

What is the revenue if she sells the "house" brand for USD 2.30 and the "name" brand for USD 4.10?

```
x = 2.3

y = 4.1

81 * x + 40*y  - 21* x^2  - 23*y^2 + 28 *y*x
```

```
## [1] 116.62
```

4. Cost minimum

A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

Since we know $x + y = 96$ we can transfor the formula with 2 variables into a formula with 1 substituting y with 96 - x:

$C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}(96 - x)^2 + 7x + 25(96 - x) + 700$

$= \frac{1}{6}x^2 + \frac{1}{6}(96 - x)^2 + 7x + 25(96 - x) + 700$

$C'(x, y) = \frac{1}{3}x - \frac{1}{3}(96 - x) - 18 = \frac{2}{3}x - 50$

Setting this to zero results in x=75

$C''(x, y) = \frac{2}{3}$ therefore a minimum

5. Integration

Evaluate the double integral on the given region.

$\int\int_R e^{(8x+3y)} dA$; $R : 2 \leq x \leq 4$ and $2 \leq y \leq 4$

Write your answer in exact form without decimals.

$\int\int_R e^{(8x+3y)} dA = \int\int_R e^{8x} * e^{3y} dA$

Since the respective integrals are finite we can take them apart:

$\int_2^4 e^{8x} dx \int_2^4 e^{3y} dy$

$\frac{1}{8}[e^{8x}]_2^4 * \frac{1}{3}[e^{3y}]_2^4$

$\frac{1}{24} * (e^{32} - e^{16}) * (e^{12} - e^6)$

$\frac{1}{24} * (e^{44} - e^{38} - e^{28} + e^{22})$

Github (both PDF and RMarkdown):

https://github.com/chilleundso/Data605_CompMath/tree/master/Homework15