DATA 605 - Homework 8

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```r
library(ggplot2)
library(psych)
library(dplyr)
library(knitr)
library(tidyr)
```

Ex.11 (page 303)

10) Let X 1 , X 2 , ..., X n be n independent random variables each of which has an exponential density with mean µ. Let M be the minimum value of the X j . Show that the density for M is exponential with mean µ/n. Hint: Use cumulative distribution functions.
11) A company buys 100 lightbulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out? (See Exercise 10.)

Solution:

From 10 we know the mean of the mean of the minimum of n exp densities is the mean of one density divided by n. Therefore, since we have 100 lightbubs each of which has an average life expecancy of 1000hours the minimum (first broken one) will be 1000/100 = 10 hours

Ex.14 (page 303)

Assume that X 1 and X 2 are independent random variables, each having an exponential density with parameter λ. Show that Z = X 1 − X 2 has density f Z (z) = (1/2)$\lambda e^{-\lambda|z|}$.

Solution:

exponential distribution has density function: $\lambda e^{-\lambda x}$

Z = X + (-Y):

$f_Z(z) = \int_{-\infty}^{\infty} f_{X_1}(x) f_{-X_2}(z-x)dx$

since $f_{-X_2}(z-x) = f_{X_2}(x-z)$

$f_Z(z) = \int_{-\infty}^{\infty} f_{X_1}(x) f_{X_2}(x-z)dx$

First case: z<0

we use the exponential distribtion function:

$= \int_{-\infty}^{\infty} \lambda e^{-\lambda x} \lambda e^{-\lambda(x-z)} dx$

since $f_{X_1}(x) = 0$ for $x < 0$ we can integrate from 0 to ∞

Using exponential calculation rules:

$= \int_0^\infty \lambda^2 e^{\lambda z - \lambda 2x} dx$

$= [\frac{\lambda^2}{-2\lambda} e^{\lambda z - \lambda 2x}]_0^\infty$

for x going to infinity the expression will go to zero. therefore, we are only left with the the negative term inserting zero into x:

$= -\frac{\lambda^2}{-2\lambda} e^{\lambda z - \lambda 2 * 0} = \frac{\lambda}{2} e^{\lambda z}$

Second case: z>0

we use the exponential distribtion function:

$= \int_{-\infty}^\infty \lambda e^{-\lambda x} \lambda e^{-\lambda(x-z)} dx$

since $f_{X_2}(x) = 0$ for $x < 0$ and in this case we have $f_{X_2}(x - z) = 0$ we can integrate from z to ∞

Using exponential calculation rules:

$= \int_z^\infty \lambda^2 e^{\lambda z - \lambda 2x} dx$

$= [\frac{\lambda^2}{-2\lambda} e^{\lambda z - \lambda 2x}]_z^\infty$

for x going to infinity the expression will go to zero. therefore, we are only left with the the negative term inserting z into x:

$= -\frac{\lambda^2}{-2\lambda} e^{\lambda z - \lambda 2 * z} = \frac{\lambda}{2} e^{-\lambda z}$

in summary:

for z>0: $f_Z(z) = \frac{\lambda}{2} e^{-\lambda z}$

for z<0: $f_Z(z) = \frac{\lambda}{2} e^{\lambda z}$

Therefore:

$f_Z(z) = \frac{\lambda}{2} e^{-\lambda |z|}$

Ex.1 (page 320)

Let X be a continuous random variable with mean μ = 10 and variance $\sigma^2 = 100/3$. Using Chebyshev's Inequality, find an upper bound for the following probabilities.

(a) $P(|X - 10| \geq 2)$.
(b) $P(|X - 10| \geq 5)$.
(c) $P(|X - 10| \geq 9)$.
(d) $P(|X - 10| \geq 20)$.

In general $P(|X - \mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2}$

Solution:

We know $\sigma^2 = 100/3$. Therefore we can find the upper bound by dividing σ^2 by the various ϵ^2 in each part:

(a) upperbound is: $\frac{\sigma^2}{\epsilon^2} = \frac{100}{3} * \frac{1}{2^2} = \frac{100}{3} * \frac{1}{4} = \frac{25}{3}$

(b) upperbound is: $\frac{\sigma^2}{\epsilon^2} = \frac{100}{3} * \frac{1}{5^2} = \frac{100}{3} * \frac{1}{25} = \frac{100}{75} = \frac{4}{3}$

(c) upperbound is: $\frac{\sigma^2}{\epsilon^2} = \frac{100}{3} * \frac{1}{9^2} = \frac{100}{3} * \frac{1}{81} = \frac{100}{243}$

(d) upperbound is: $\frac{\sigma^2}{\epsilon^2} = \frac{100}{3} * \frac{1}{20^2} = \frac{100}{3} * \frac{1}{400} = \frac{1}{12}$

Github (both PDF and RMarkdown):

https://github.com/chilleundso/Data605_CompMath/tree/master/Homework8