DATA 605 - Homework 5

Manolis Manoli

```
library(ggplot2)
library(psych)
library(dplyr)
library(knitr)
library(tidyr)
```

Problem Set 1

1) Choose independently two numbers B and C at random from the interval [0, 1] with uniform density.

Prove that B and C are proper probability distributions.

Since this is a continuous distribution we cannot define p(x) for x in [0, 1]. the prob density function for U(a,b) f(x) is:

f(x) = 1/(b-a) for x in [a,b]

f(x) = 0 for x NOT in [a,b]

in our case 1/(b-a) = 1

Integral of f(x) = 1 between 0 and 1:

F(x)=x

F(1)-F(0)=1-0=1

2) Note that the point (B,C) is then chosen at random in the unit square.

Find the probability that

a) B + C < 1/2

since they are independent B+C is equivalent to a U[0,2] distribution. Clearly the probability of a value less than 1/2 in a U[0,2] is 1/4

b) BC < 1/2

F_XY(z) = P[XY<z]

= int[x=0 to 1] P[Y<z/x] f_X(x)dx

= int[x=0 to z] dx + int[x=z to 1] z/x dx

= z - z log(z)

therefore the density is:

f(z) = -log(z)

f(1/2) = -log(1/2) ~= 0.69

c) abs(B-C) <1/2

if we think about it graphically, where we use a 2dim graph an plot B along the x-axis and C along the y-axis. This results in a 2 dim 1x1 box of possile combinations. If we can find the area where all combination of B and C for which abs(B-C) <1/2 we have the solution.

The line x=y will result in B-C = 0 which clearly satisfies the condition. No from any point along this main diagonal we can increase or decrease either B or C by up to 1/2. this creates 2 parralel lines along our main diagonal x=y: x=y-1/2 and x=y+1/2.

within these two graphs is our area of possible combinations that satisfies the condition.

The opposite of this area are the two triangles:

(0,1/2),(1/2,1),(0,1)

(1/2,0),(1,1/2),(1,0)

both of these have size 1/8 and a combined size of 1/4. Therefore the size of our satisfying area and the probability is 3/4.

d) max(B,C) <1/2

This is equivalent to B<1/2 and C<1/2. Therefore the probability is 1/2 * 1/2 = 1/4

e) min(B,C) <1/2

This is equivalent to B<1/2 or C<1/2. Given, P(A) + P(B) - P(A and B) and with the above (d) we know that this

p(A or B) = p(A)+p(B) -p(A and B) =

1/2 + 1/2 - 1/4 = 3/4

Github (both PDF and RMarkdown):

https://github.com/chilleundso/Data605_CompMath/tree/master/Homework5