Surgical: Institutional ranking

This example considers mortality rates in 12 hospitals performing cardiac surgery in babies. The data are shown below.

Hospital	No of ops	No of deaths
A	47	0
B	148	18
C	119	8
D	810	46
E	211	8
F	196	13
G	148	9
H	215	31
I	207	14
J	97	8
K	256	29
L	360	24

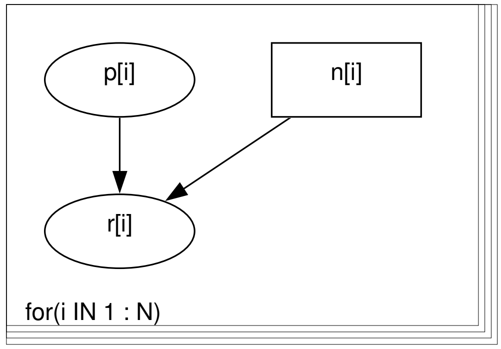
The number of deaths r_i for hospital *i* are modelled as a binary response variable with `true' failure probability p_i:

$$r_i \sim \text{Binomial}(p_i, n_i)$$

We first assume that the true failure probabilities are *independent* (*i.e.*fixed effects) for each hospital. This is equivalent to assuming a standard non-informative prior distribution for the p_i's, namely:

$$p_i \sim \text{Beta}(1.0, 1.0)$$

Graphical model for fixed effects surgical example:



BUGS *language for fixed effects surgical model:*

```
model
{
   for( i in 1 : N ) {
      p[i] ~ dbeta(1.0, 1.0)
    r[i] ~ dbin(p[i], n[i])
   }
}
```

Data (click to open)

Inits (click to open)

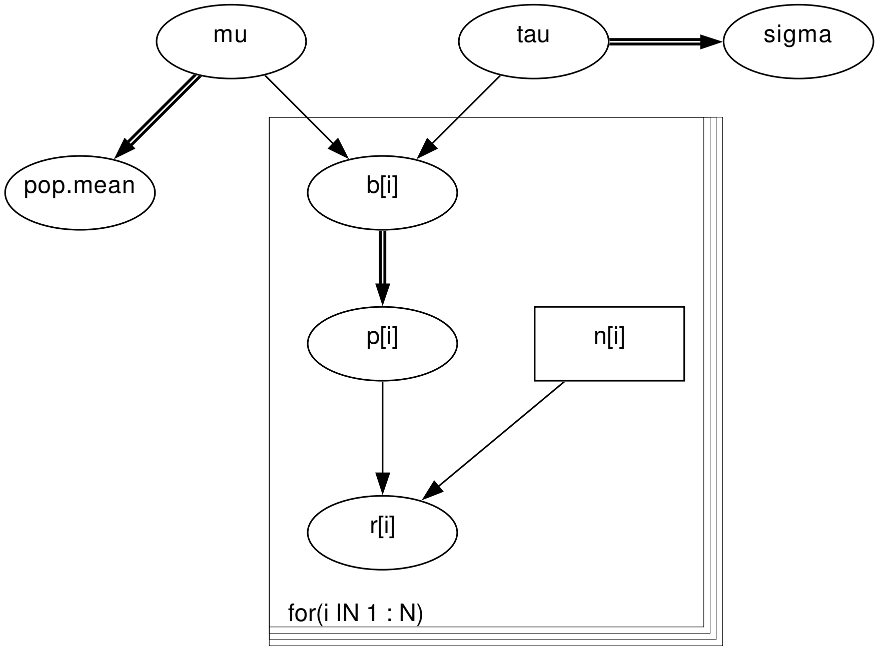
A more realistic model for the surgical data is to assume that the failure rates across hospitals are *similar* in some way. This is equivalent to specifying a *random effects* model for the true failure probabilities p_i as follows:

$$\mathrm{logit}(p_i) = b_i$$

$$b_i \sim \mathrm{Normal}(\mu, \tau)$$

Standard non-informative priors are then specified for the population mean (logit) probability of failure, μ, and precision, τ.

Graphical model for random effects surgical example:



BUGS *language for random effects surgical model:*

```
model
{
   for( i in 1 : N ) {
      b[i] ~ dnorm(mu,tau)
      r[i] ~ dbin(p[i],n[i])
      logit(p[i]) <- b[i]
      }
   pop.mean <- exp(mu) / (1 + exp(mu))
   mu ~ dnorm(0.0,1.0E-6)
   sigma <- 1 / sqrt(tau)
   tau ~ dgamma(0.001,0.001)
}
```

Data (click to open)

Inits (click to open)

Results

A burn in of 1000 updates followed by a further 10000 updates gave the following estimates of surgical mortality in each hospital for the fixed effect analysis

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
p[1]	0.02009	0.01946	2.085E-4	6.091E-4	0.01441	0.07178	1001	10000
p[2]	0.1266	0.0271	2.67E-4	0.07853	0.125	0.1845	1001	10000
p[3]	0.07436	0.02371	2.349E-4	0.03492	0.07181	0.1265	1001	10000
p[4]	0.05789	0.00824	8.136E-5	0.04264	0.05762	0.07487	1001	10000
p[5]	0.04237	0.01388	1.096E-4	0.01972	0.04086	0.07362	1001	10000
p[6]	0.07081	0.01811	1.935E-4	0.0397	0.06931	0.1098	1001	10000
p[7]	0.06686	0.02025	1.872E-4	0.03259	0.06493	0.111	1001	10000
p[8]	0.1473	0.02393	2.681E-4	0.1039	0.146	0.1983	1001	10000
p[9]	0.07216	0.0179	1.59E-4	0.04093	0.07071	0.1104	1001	10000
p[10]	0.09078	0.0288	3.122E-4	0.04274	0.08817	0.1531	1001	10000
p[11]	0.1165	0.02009	2.074E-4	0.08	0.1155	0.1589	1001	10000
p[12]	0.06906	0.01345	1.261E-4	0.04518	0.06816	0.0977	1001	10000

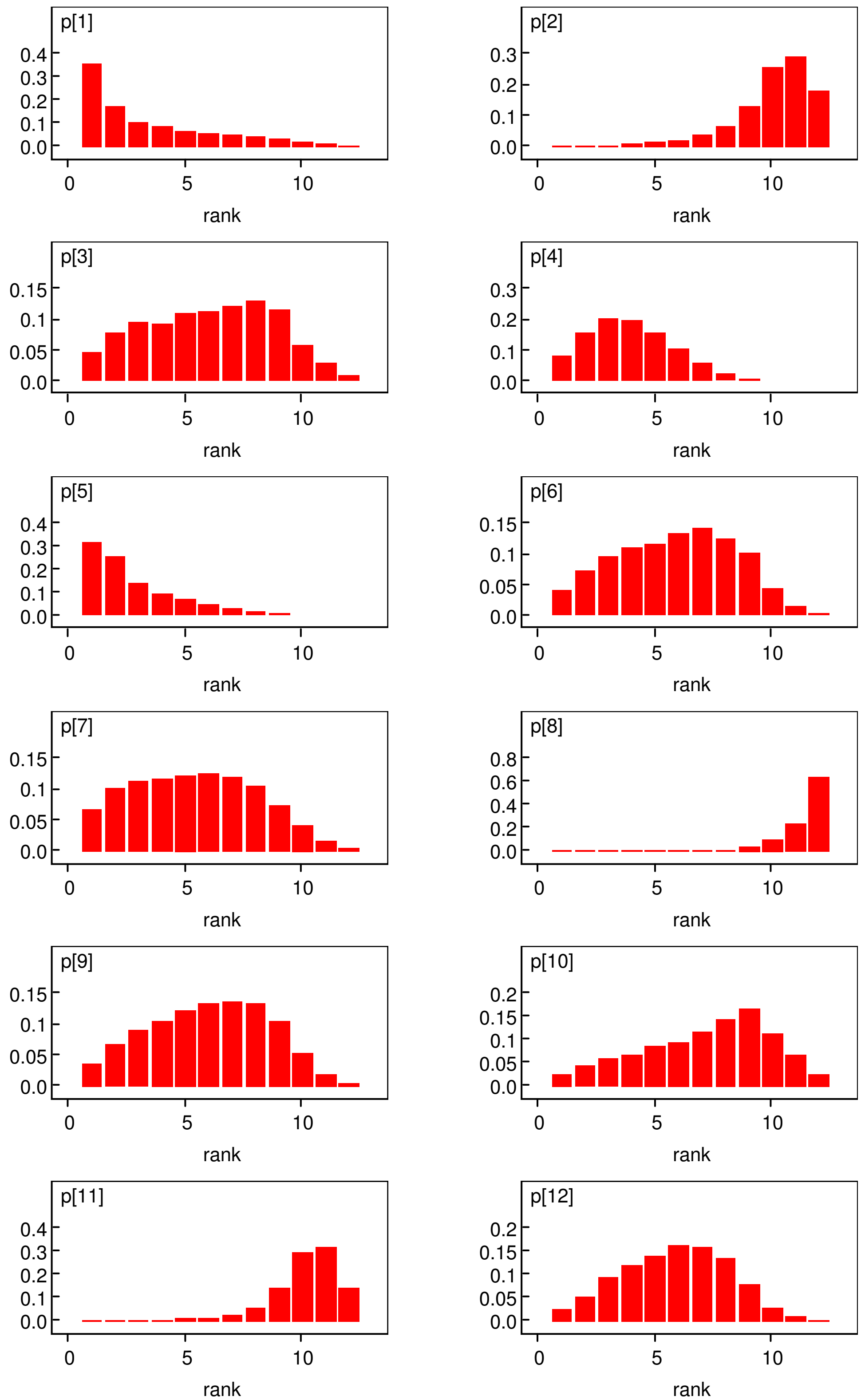
and for the random effects analysis

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
mu	-2.558	0.1554	0.002585	-2.884	-2.551	-2.266	1001	10000
p[1]	0.05302	0.01948	3.565E-4	0.01802	0.05221	0.09348	1001	10000
p[2]	0.1029	0.02196	2.976E-4	0.06712	0.1006	0.152	1001	10000
p[3]	0.07044	0.01727	1.978E-4	0.0397	0.06916	0.1079	1001	10000
p[4]	0.0593	0.007985	1.212E-4	0.04458	0.05897	0.07591	1001	10000
p[5]	0.05187	0.01329	2.269E-4	0.02791	0.05102	0.07961	1001	10000
p[6]	0.06903	0.01448	1.564E-4	0.04284	0.06854	0.1004	1001	10000
p[7]	0.06682	0.01602	1.773E-4	0.03835	0.06595	0.1009	1001	10000
p[8]	0.1226	0.02244	4.014E-4	0.08196	0.1217	0.1698	1001	10000
p[9]	0.0698	0.01432	1.508E-4	0.04432	0.06901	0.1004	1001	10000
p[10]	0.07851	0.01955	2.03E-4	0.04506	0.07662	0.1217	1001	10000
p[11]	0.1021	0.01761	2.283E-4	0.07158	0.1009	0.1398	1001	10000
p[12]	0.06858	0.01168	1.301E-4	0.04745	0.06805	0.09349	1001	10000
pop.mean	0.07259	0.01028	1.696E-4	0.05293	0.07235	0.09401	1001	10000
sigma	0.4028	0.16	0.003672	0.1577	0.3793	0.7872	1001	10000

A particular strength of the Markov chain Monte Carlo (Gibbs sampling) approach implemented in *BUGS* is the ability to make inferences on arbitrary functions of unknown model parameters. For example, we may compute the *rank* probabilty of failure for each hospital at each iteration. This yields a sample from the posterior distribution of the ranks.

The figures below show the posterior ranks for the estimated surgical mortality rate in each hospital for the random effect models. These are obtained by setting the rank monitor for variable p (select the "Rank" option from the "Statistics" menu) after the burn-in phase, and then selecting the "histogram" option from this menu after a further 10000 updates. These distributions illustrate the considerable uncertainty associated with 'league tables': there are only 2 hospitals (H and K) whose intervals exclude the median rank and none whose intervals fall completely within the lower or upper quartiles.

Plots of distribution of ranks of true failure probability for random effects model:


p[1]
p[2]
p[3]
p[4]
p[5]
p[6]
p[7]
p[8]
p[9]
p[10]
p[11]
p[12]
rank