Surgical

On calcul ~~on~~ la proba de chaque paramètre à estimer sachant tout le monde.

$$\pi(b_i | N, r_i) \propto \pi(b_i | N, \sigma) \, \pi(r_i | b_i)$$

or $\pi(b_i | N, \sigma^2) \sim \mathcal{N}(N, t)$

et $\pi(r_i | b_i) \sim \text{Binomial}(p_i, n_i)$

or $\text{logit}(p_i) = b_i \Rightarrow p_i = \frac{e^{b_i}}{1+e^{b_i}}$

$$\Rightarrow \pi(b_i | N, t) \sim \text{Binom}\left(\frac{e^{b_i}}{1+e^{b_i}}, n_i\right)$$

Donc $\pi(b_i | N, t, r_i) \propto e^{-\frac{1}{2}\left(\frac{b_i - N}{t}\right)^2} \left(\frac{e^{b_i}}{1+e^{b_i}}\right)^{b_i} \left(1 - \frac{e^{b_i}}{1+e^{b_i}}\right)^{n_i - b_i}$

pas de forme explicite Donc On utilise Metropolis à l'intérieur de Gibbs!

$$\pi(N | b_i, r_i) \propto \pi(N) \prod_{i=1}^{n} \pi(b_i | N, \sigma^2)$$

or $\pi(N) \sim \mathcal{N}(0, \sigma_N^2)$ et

$\pi(b_i | N, \sigma^2) \sim \mathcal{N}(N, t)$

$$\pi(N | b_i, r_i) \propto e^{-\frac{1}{2}\left(\frac{N - 0}{\sigma_N}\right)^2} \prod_{i=1}^{n_i} e^{-\frac{1}{2}\left(\frac{b_i - N}{t}\right)^2}$$

$$\propto e^{-\frac{1}{2}\left(\frac{N}{\sigma_N}\right)^2} e^{-\frac{1}{2}\sum_{i=1}^{n_i}\left(\frac{b_i - N}{t}\right)^2}$$

$$\propto e^{-\frac{1}{2}\left[\left(\frac{N}{\sigma_N}\right)^2 + \sum_{i=1}^{n_i}\left(\frac{b_i - N}{t}\right)^2\right]}.$$

$$-\frac{1}{2}\left\{ \frac{\mu^2}{(\sigma_N^2)} + \frac{\sum_{i=1}^{n_i} b_i^2 - 2N\sum_{i=1}^{n_i} b_i + n_i N^2}{t^2} \right\}$$

$$\propto -\frac{1}{2}\left[\frac{t^2\mu^2 + \sigma_N^2 \sum_{i}^{n_i} b_i^2 - 2\sigma_N^2 N \sum b_i + \sigma_N^2 n_i N^2}{\sigma_N^2 t^2} \right]$$

$$\propto -\frac{1}{2\sigma_N^2 t^2}\left[N^2 \sigma_N^2 n_i - 2\sigma_N^2 N \sum b_i + t^2 \mu^2 + \sigma_N^2 \sum_{i=1}^{n_i} b_i^2 \right\}$$

on $\boxed{\sigma^2 = \frac{1}{t} = t^{-1} = t^2}$ - pour la suite même notat°

$$\propto -\frac{1}{2}\left\{ t^2 N^2 + \sigma_N^2 \sum b_i^2 - 2\sigma_N^2 N \sum b_i + \sigma_N^2 n_i N^2 \right\}$$

$$\propto -\frac{1}{2}\left\{ N^2 (t^2 + \sigma_N^2 n_i) - 2\sigma_N^2 N \sum b_i + \sigma_N^2 \sum b_i^2 \right\}$$

$$\propto -\frac{1}{2}\left\{ t^2 + \sigma_N^2 n_i \left(N - \frac{\sigma_N^2 \sum b_i}{t^2 + \sigma_N^2 n_i} \right)^2 \right\}$$

$$\propto e^{-\frac{1}{2}\left[\frac{(t^2 + \sigma_N^2 n_i)}{\sigma_N^2 t^2}\right]\left[N - \frac{\sigma_N^2 \sum b_i}{t^2 + \sigma_N^2 n_i} \right]^2}$$

$$\boxed{\pi(N \mid \ldots) \propto \mathcal{N}\left(\frac{\sigma_N^2 \sum b_i}{t^2 + \sigma_N^2 n_i} ; \frac{\sigma_N^2 t^2}{t^2 + \sigma_N^2 n_i} \right)}$$

$\sigma^2 = t^{-1} =$ $\Rightarrow t^{-1} = \sigma^2$

$$\pi(t \mid N, r_i, b_i) \propto \pi(t) \prod_{i=1}^{n_i} \pi(b_i \mid N, t)$$

on $\pi(t) \sim \gamma(a, b)$

$\pi(b_i \mid N, t) \sim \mathcal{N}(N, t)$

$$\Rightarrow \pi(t \mid N, r_i, b_i) \propto t^{a-1} b^a e^{-bt} \left(\frac{1}{\sqrt{2\pi t^{-1}}} \right)^{n_i/2} e^{-\frac{1}{2t^{-1}} \sum_{i=1}^{n_i} (b_i - N)^2}$$

$$\propto t^{a-1}\, t^{n_i/2}\, e^{-bt - \frac{1}{2t^{-1}} \sum_{i=1}^{n_i} (b_i - N)^2}$$

$$\boxed{\pi(t \mid \ldots) \propto \gamma\left(a + \frac{n_i}{2} ; b + 2^{-1} \sum_{i=1}^{n_i} (b_i - N)^2 \right)}$$