Multi-Layer Perceptrons (Lab2:**part 3**)

Train a Multi-Layer perceptron on "*a1a*" dataset, using the cross-entropy loss with ℓ -2 regularization.

```
!wget -t inf https://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/binary/a1a
from sklearn.datasets import load_svmlight_file
X, y = load_svmlight_file("a1a")
from sklearn.neural_network import MLPClassifier
from sklearn import preprocessing
from sklearn.metrics import log_loss
import numpy as np
from sklearn.model_selection import train_test_split
import matplotlib.pyplot as plt
from sklearn.model_selection import learning_curve
```

```
alpha = np.logspace(-5, 1,num=6) #alphas are expected to be in general <=1 (not necessary though)
sizes=[ 0.4, 0.3, 0.2, 0.1]

#initializations
training_error=np.zeros((4, 6))
validation_error=np.zeros((4, 6))
```

```
for i,size in enumerate(sizes):  #iterate over 4 different dataset splits
  X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=size, random_state=42)

  for j,a in enumerate(alpha):
```

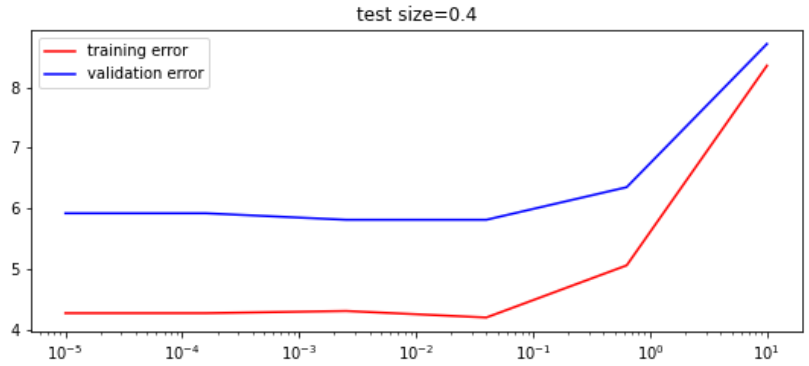
```
        mlp = MLPClassifier( solver='adam',alpha=a, activation = 'logistic',
                            hidden_layer_sizes=(16,8),
                            max_iter=500, random_state=1)
        mlp.fit(X_train,y_train)

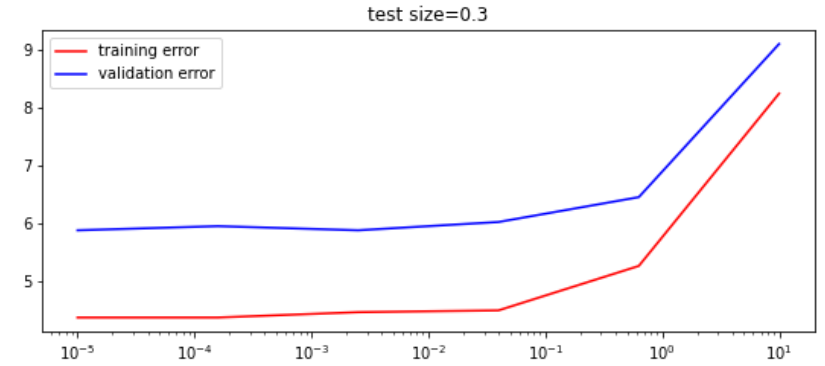
        training_error[i][j]=log_loss(y_train, mlp.predict(X_train))
        validation_error[i][j]= log_loss(y_test, mlp.predict(X_test))
```

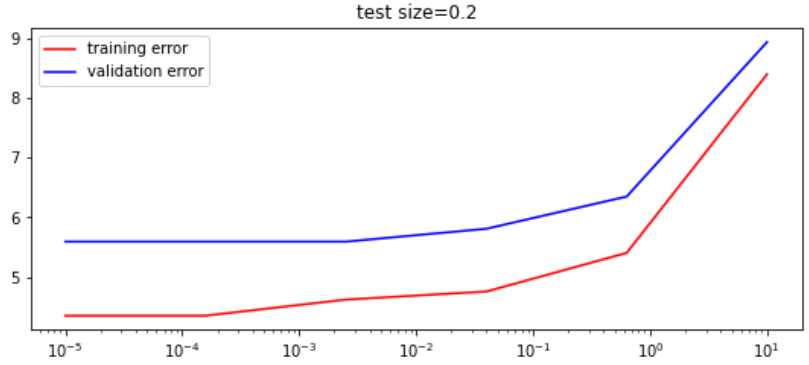
Analysing training / validation errors and their relation , as a function of α parameter, for different test sizes.

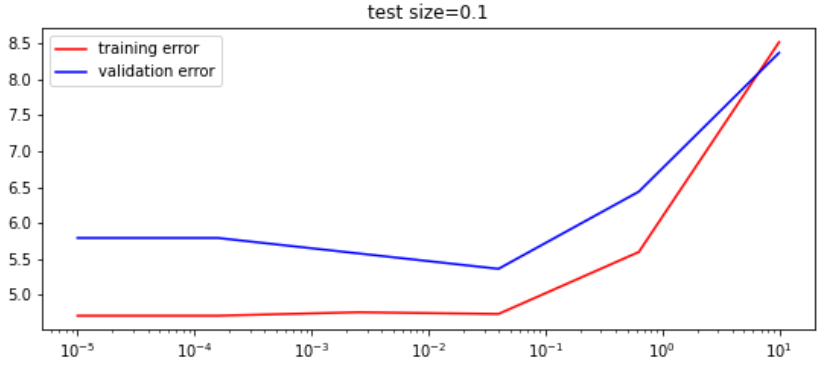
```
#plots
plt.rcParams["figure.figsize"] = [20, 8]
fig, axs = plt.subplots(2, 2)
for i,size in enumerate(sizes):
  if i<2:
    axs[0, i].semilogx(alpha, training_error[i,:], '-r')
    axs[0, i].semilogx(alpha, validation_error[i,:], '-b')
    axs[0, i].set_title('test size={}'.format(size))
    axs[0, i].legend(('training error','validation error'),loc="upper left")

  else:
    axs[1, i-2].semilogx(alpha, training_error[i,:], '-r')
    axs[1, i-2].semilogx(alpha, validation_error[i,:], '-b')
    axs[1, i-2].set_title('test size={}'.format(size))
    axs[1, i-2].legend(('training error','validation error'),loc="upper left")
```









Observations

In all 4 cases, the results regarding the α (or "λ") $\ell 2$-regularization factor are coherent with theory:

- For extremely **low values** of α our classification algorithm shows **HIGH VARIANCE**. That is to say *training* and *validation error* are far apart from each other and as the α is not "*penalizing*" enough, the MLP seems to overfit. Clearly the optimization at this point is focusing more on minimizing the training error.
- All the way to the other extreme, **high values** of α (eg.10) are somehow "*over-regularizing*", in a way that the losses converge to really high levels. This situation of **HIGH BIAS** mainly shows that we're penalizing too much and in fact the MLP doesn't get the chance to actually learn and perform well even on the training set to begin with.
- For all cases, a "*sweet-spot*" for α appears to be somewhere in between 10^{-2} and 10^{-1}

- We are expecting for the variance to be mitigated as we would raise the number of training data points (decrease the test size). It is visible that for test size smaller that 30%, the two curves begin to come closer as we're strengthening the regularization. Of course by changing the dataset split proportions, or by simply adding new data, after one point makes no difference, as get to the *high bias* region.

Furthermore:

As we can see from the learning curves extracted with the cross-validation method below, a stronger α factor causes a small drop in accuracy in exchange for better convergence.

Accuracy learning curves as we add more data for $\alpha = 0.001$ and $\alpha = 0.1$

```
mlp_no_l2 = MLPClassifier( solver='adam', activation = 'logistic',
                                hidden_layer_sizes=(16,8),
                                max_iter=500, random_state=1)
train_sizes, train_scores, test_scores, fit_times, _ = learning_curve(mlp, X, y, cv=10 ,return_times=True) #cross validation of 30 fo

mlp_l2= MLPClassifier( solver='adam', alpha=0.1, activation = 'logistic',
                                hidden_layer_sizes=(16,8),
                                max_iter=500, random_state=1)
train_sizes_l2, train_scores_l2, test_scores_l2, fit_times_l2, _ = learning_curve(mlp_l2, X, y, cv=10 ,return_times=True)

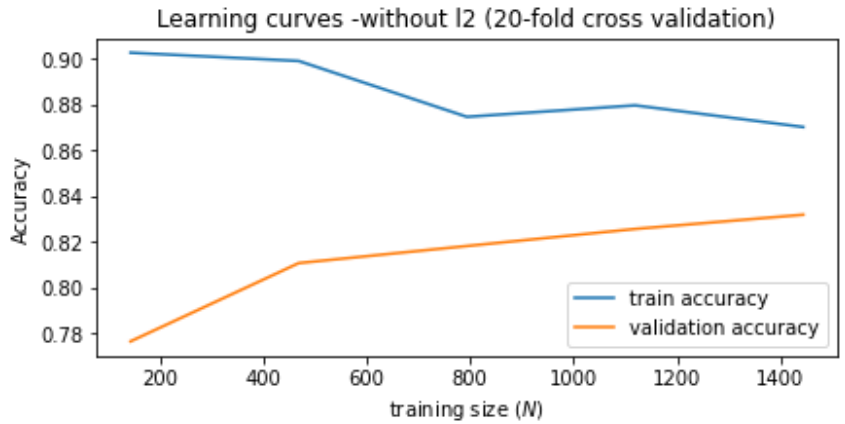


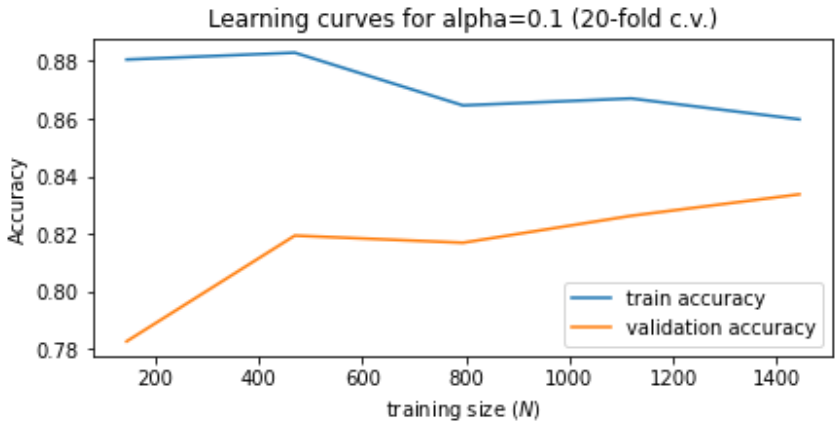
plt.figure(figsize=(15,3))
plt.subplot(1, 2, 1)
plt.plot(train_sizes,np.mean(train_scores,axis=1))
plt.plot(train_sizes,np.mean(test_scores,axis=1))
plt.xlabel("training size ($N$)")
plt.ylabel("Accuracy")
plt.title('Learning curves -without l2 (20-fold cross validation)')
plt.legend(('train accuracy', 'validation accuracy'))

plt.subplot(1, 2, 2)
plt.plot(train_sizes_l2,np.mean(train_scores_l2,axis=1))
```

```
plt.plot(train_sizes_l2,np.mean(test_scores_l2,axis=1))
plt.xlabel("training size ($N$)")
plt.ylabel("Accuracy")
plt.title('Learning curves for alpha=0.1 (20-fold c.v.)')
plt.legend(('train accuracy', 'validation accuracy'))
```

```
<matplotlib.legend.Legend at 0x7f8d17665390>
```






```
alpha = np.logspace(-5, 1,num=6) #alphas are expected to be in general <=1 (not necessary though)


#initializations
training_error=np.zeros((4, 6))
validation_error=np.zeros((4, 6))


layer=[16]
for i in range(4):
  X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.1, random_state=42) #fixed test size of 10%

  for j,a in enumerate(alpha):

        mlp = MLPClassifier( solver='adam',alpha=a, activation = 'logistic',
```

```
                                hidden_layer_sizes=(layer[i]),
                                max_iter=500, random_state=1)
        mlp.fit(X_train,y_train)

        training_error[i][j]=log_loss(y_train, mlp.predict(X_train))
        validation_error[i][j]= log_loss(y_test, mlp.predict(X_test))
    layer.append(int(layer[i]/2)) #adding layer of half size for next iteration
    print(layer[i])
```

Observing error behaviour as a function of α parameter, for different numbers of layers.

```
plt.rcParams["figure.figsize"] = [20, 8]
fig, axs = plt.subplots(2, 2)
for i,size in enumerate(sizes):
  if i<2:
    axs[0, i].semilogx(alpha, training_error[i,:], '-r')
    axs[0, i].semilogx(alpha, validation_error[i,:], '-b')
    axs[0, i].set_title('{} layer(s) with {} neurons '.format(len(layer[:i+1]), layer[:i+1]))
    axs[0, i].legend(('training error','validation error'),loc="upper left")

  else:
    axs[1, i-2].semilogx(alpha, training_error[i,:], '-r')
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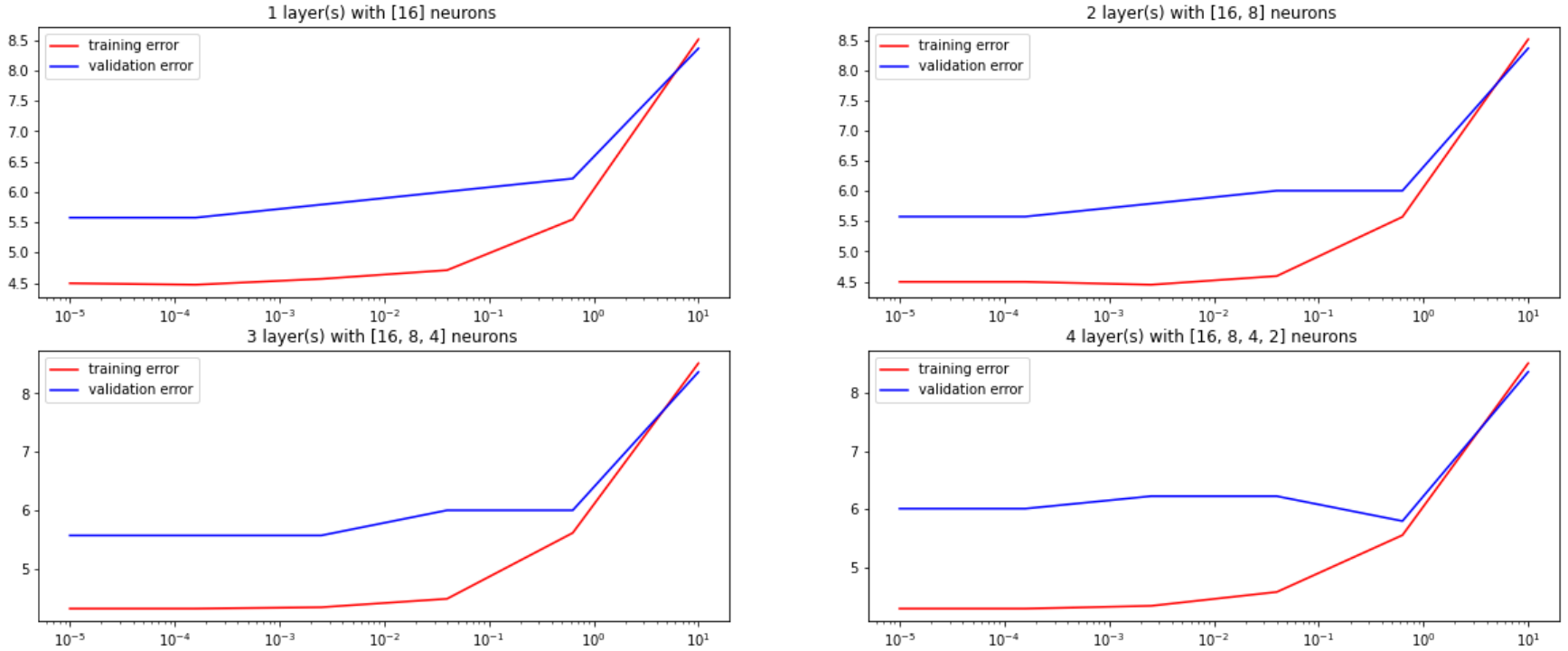

By aggregating **more layers** to our MLP, we're increasing the complexity of the model. Since we're facing a high-variance problem that's not helpful at all. We also see that the **penalizing effect becomes stronger** : the **error starts climbing earlier** with respect to α factor as we increase the **depth** of the network.

QUESTION: How do you construct a network with multiple layers? Is the order of function calls important when constructing the network?

Using the convenient structure of ***sequential*** model, we can pile up **hierarchicaly** the layers in the correct order, according to the desired *depth* and goals.

QUESTION: What happens when you call the module "Linear" with "bias = False"?

The module *linear* introduces a layer as a series of linear transformations ($w^T x + b$). Setting "bias=False" it means that we choose $b = 0$ or better say $w_0 = 0$ (not learning a bias).

QUESTION: How can we add non-linear activation function to the intermediate layers?

In the context of "*Sequential*" structure , we can always use a non linear activation function σ, by adding a line of (e.g.)

```
nn.ReLU()
```

if we desire a Rectified linear unit function, **right after each corresponding "linear" fully-connected layer** It will apply this nonlinearity to all units within the previous layer.

QUESTION: Why do we normalize the input images?

Normilizing the pixels of images according to the idea of a Gaussian curve centered at zero, allows the algorithm to converge faster. Usualy for RGB we devide by 256 so every pixel has a range of [0,1] .

QUESTION: How do you construct a network with multiple layers? Is the order of function calls important when constructing the network?

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QUESTION: In dropout regularization, why do we scale the values by a factor of 1/p?

When we apply Dropout regularization during training in favour of the robustness of our model, we want to pay back for the imbalance we create to the models' weights towards an accurate evaluation (in testing phase we don't apply dropouts).

QUESTION: When training a neural network, we often have to construct the network from multiple components. Which are the components of the network in the case of the shallow model with a single layer? Which are the different functions for constructing these components, adding the dataset, and training the network?

Mathematicaly: For a single layer we're focusing on all the different neurons within that layer. In practice we're working with the actual weights that correspond to each specific feature. The main construction we care about is the W matrix,which for a single layer can be simplified by a single vector of size $d+1$ (if we consider d the feature dimensions) and contains all the representantive weight parameters, absorbing also the bias as a w_0 component. The layer in general can be a typical **dense connection layer,** or a shared-parameters layer with **sparse connections** (as the **convolution layer**).

Pytorch (not far away from Tensorflow/Keras idea):

- for fully connected layers we call

```
nn.Linear(in_features, out_features, bias=True)
```

- for a convolution layer

```
nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0,  bias=True, padding_mode='zeros')
```

where *kernel_size* is the size of the filter, *stride* is the # of pixels to be slided, and *padding* the number of pixels to surround our image-grid.

- for "pooling" (shrinking the size of feuture map)

```
nn.MaxPool2d(kernel_size, ...)
nn.AvgPool2d(kernel_size, ...)
```

(MaxPool2d--> center pixel corresponds to the maximum of all pixels in a pooling-frame whereas AvgPool2d-->center pixel corresponds to the average of all pixels in a pooling-frame)

- for output we use activation of **softmax** (for multiclass probabilities) of simply **sigmoid** function (for binary classification)

```
nn.Softmax
nn.Sigmoid
```

QUESTION: What is the purpose of the module "Flatten"?

```
nn.Flatten(start_dim=1, end_dim=- 1)
```

This layer repares the data to become the input of a fully-connected network (if eg. we have a pixel-volume of 3D "flatten" layer **shrinks that to a 1D tensor**).