Branches of math

- Calculus
  - the study of change
  - to do with relationships between quantities
  - Two major branches
    1. Differential calculus - the study of rates of change and slopes of curves
    2. Integral calculus - the study of areas under and between curves
  - The two branches are related using the *fundemental theory of calculus*
  - A calculus refers to any system of calculation guided by the symbolic manipulation of expressions e.g. lambda calculus
- Algebra
  - the study of operations and how they can be applied to solve equations
  - from an arabic word meaning the *reunion of broken parts*
  - the study of symbols and the rules for manipulating symbols
  - a unifying thread of almost all of mathematics
  - 2 forms: *Elementary* and *Advanced*
    * Elementary algebra differs from arithmetic in the use of abstractions, such as using letters to stand for numbers that are either unknown or allowed to take on many values
  - Can also be a specialized kind of *mathematical object* e.g. *linear algebra*
- Geometry
  - the study of shape
- Arithmetic
  - The study of numbers, especially the properties of the traditional operations between them — addition, subtraction, multiplication and division.
  - A part of *number theory* - until the 19th century they were pretty much the same thing.
  - The oldest and most elementary branch of mathematics - origins around 2000 BCE ish.
  - Any *set* of objects on which *all four* of the basic operations can be performed is called a **field** i.e. if you can add, subtract, multiply, divide these objects and they follow the normal rules then they are a *field*.
  - each *field* has some special elements:
    * identity element

            - the element which when operated on with any other element returns the other element
        - inverse element
            - the element which when operated on with a particular element returns the identity element
    - 4 basic operations:
        - addition
            - commutative and associative
            - identity element is 0
            - inverse element of A is -A
            - can be performed using geometry using sticks!
        - subtraction
            - the inverse of addition
            - not commutative, not associative
            - because it is less convenient when things are not commutative and associative you can think of $a - b$ as $a + (-b)$ instead.
        - multiplication
            - the second basic operation of arithmetic
            - is commutative and associative
            - distributive over addition and subtraction - ???
            - identity element is 1
            - inverse element of an element is its reciprocal i.e. $x*(1/x) = 1$
        - division
            - the inverse of multiplication
            - anything divided by 0 is not defined
            - not commutative, not associative
            - because it is less convenient when things are not commutative and associative you can think of a/b as $a * (1/b)$ instead.
    - note that addition and multiplication are the real *basic* operations - subtraction and division are just their inverses.
    - Also are other more advanced operations:
        - percentages
        - square roots
        - exponentiation
        - logarithms

Aside: without the concept of 0 our positional number scheme would not be possible!

Top level divisions of modern mathematics

1. Algebra

2. Geometry
3. Analysis
4. Number theory
    - arithmetic is in this division

Numbers WIP

The heirarchy of groups of numbers:

- Real numbers $\mathbb{R}$
    - includes
        - transcendental numbers
        - irrational numbers
        - fractions
- Complex numbers
- Imaginary numbers
- Natural numbers
- Rational numbers
- Irrational numbers
- Transcendental numbers

Functions

- functions can be written in english or more succicently using mathemtatical symbols
- functions are the same iff they produce the same output for the same input
- it does not matter how they "look" i.e. they don't have to have the same "rule" or the same "way of turning the input into the output"

Important functions

- $f(x) = x$ the identity function
- $f(x) = c$ the constant function
- $f(x) = ax + b$
- $f(x) = x^n$
- $f(x) = 2x^3 + 5x^2 - 2x + 1$
    - a cubic *polynomial*
- $f(x) = \sqrt{x}$

- $f(x) = |x|$
- $f(x) = sin(x)$ (transcendental)
- $f(x) = cos(x)$ (transcendental)
- $f(x) = tan(x)$ (transcendental)

Functions can be *composed* into a sort of conveyor belt that pipes the output of one function into the input of another e.g. $f(g(x))$ or $f \circ g$.

Polynomials

- a polynomial is an expression consisting of variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents.
  - http://en.wikipedia.org/wiki/Polynomial

tancendental functions

> A transcendental function is an analytic function that does not satisfy a polynomial equation, in contrast to an algebraic function. (The polynomials are sometimes required to have rational coefficients.) In other words, a transcendental function "transcends" algebra in that it cannot be expressed in terms of a finite sequence of the algebraic operations of addition, multiplication, and root extraction. Examples of transcendental functions include the exponential function, the logarithm, and the trigonometric functions.

Defn: domain of a function

- The set of input values can I put into a function and get a *valid* output
- Note: it is a *set* of values
- Example: for $f(x) = x^2$ the domain is $\{x \in \mathbb{R}\}$ or *x is contained in the real numbers*
- Examples:
  - for $f(x) = \sqrt{x}$ the domain is $\{x \in [-1, \infty)\}$
  - for $f(x) = 1/\sqrt{x}$ the domain is $\{x \in \mathbb{R} | x \neq 0\}$

Defn: range of a function

- The *set* of all possible values the output of a function can take on given all possible inputs

Square root

The square root function takes a number an spits out a new number. When you multiply that new number by itself you get back the original number.

There are two possible outputs of this function for any given number - the positive and negative roots. This means that the function could do two different things and still be "correct" and we can't say for sure which should happen.

The problem is not tow inputs producing the same out put, it is two outputs for the same input

This is an ambiguity - it maps one input value to two possible output values. In code we could probably have this thing just return an array of the possible answers but mathematics is not comfortable with that becasue ???

Math realy wants a function that takes *one* number and returns *one* number so we *define* the square root function to be

> The *non negative* number which squares to x

This means that by convention we always pick the positive root. We can also have the "complete and accurate square root function" that returns two values but this is not the common usage.

A somewhat surprising consequence of defining the square root function in this way is that

$\sqrt{x^2} \neq x$

but we **can** say that

$\sqrt{x^2} = |x|$

The domain of $\surd$ is $\{x \in [0, \infty)\}$

- the domain is described here using an *interval*
- The [means to include the 0
- the) means to exclude the ∞ (because infinity is not a number)

Aside: Infinity (∞) is not a number

```
# wolfram alpha command
plot sqrt(x), x=-10 to 10
```