

{ HASHING } :

↳ searching / insertion / deletion $\rightarrow O(1)$

eg $\rightarrow 682, 761, 494, 567, 869$

Hashing : finding a unique index for all elements to store them. ↳ hash value

682, 761, 494, 567, 869
 $=2$ $=1$ $=4$ $=7$ $=9$ ← hash values

$$h(K) = K \bmod 10$$

↳ hash funcⁿ

Types of Hash functions :

1. Division method : $h(K) = K \bmod m$
 suppose $m=11$
 $1276 \bmod 11 = 0$

2. Mid-Square method :

$$h(K) = K^2 \text{ & extract middle digits}$$

$$\begin{array}{l} \text{eg } \rightarrow K = 60 \\ \quad \quad \quad K^2 = 3600 \\ \quad \quad \quad \quad \downarrow \\ \quad \quad \quad 60 = \text{hash value} \end{array}$$

$$\begin{array}{l} \text{eg. } k=13 \\ \quad \quad \quad k^2 = 169 \\ \quad \quad \quad \quad \quad 6 = \text{hash value} \end{array}$$

10 buckets	Hash Table
0	
1	761
2	682
3	
4	494
5	
6	
7	567
8	
9	869

3. Digit folding method : fold key into equal parts K_1, K_2, K_3, K_4, K_5

$$\text{eg. } K = 12345$$

$$\begin{array}{r} 12 \\ + 34 \\ + 5 \\ \hline 51 = \text{hash value} \end{array}$$

$$\begin{array}{r} K_1, K_2 \\ + K_3, K_4 \\ + K_5 \\ \hline \text{Hash value} \end{array}$$

4. Multiplication method : $\xrightarrow{\text{size of hashtable/no. of buckets}}$

$$h(K) = \lfloor M(KA \bmod 1) \rfloor$$

$\text{eg} \rightarrow K = 12345, A = 0.01, M = 10$

$$\begin{aligned}
 h(K) &= \text{floor} (10 (12345 \times 0.01 \% 1)) \\
 &= \text{floor} (10 (123.45 \% 1)) \\
 &= \text{floor} (10 \times 0.45) \\
 &= \text{floor} (4.5) = 4 \Rightarrow \underline{\text{hash value}}
 \end{aligned}$$

Collisions: when two elements have same hash value.

$\text{eg} \rightarrow h(K) = K \% 10.$

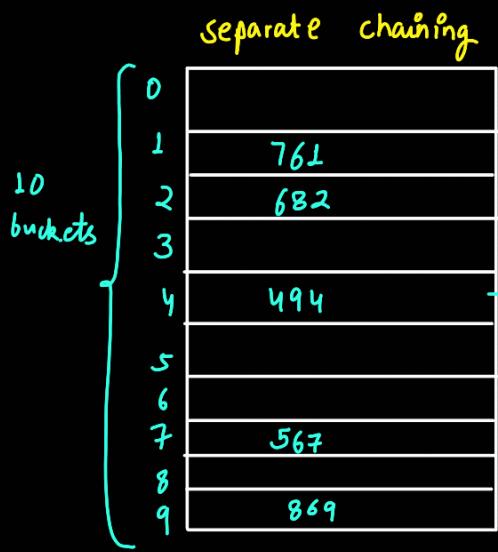
hash val. = $\begin{matrix} 234, 564 \\ 4 \quad 4 \end{matrix}$

Handling collisions: → 1. Open hashing (closed addressing)
→ 2. Closed hashing

Open hashing (closed addressing):

↪ separate chaining

$$\begin{array}{ll}
 \text{eg} \rightarrow & 682, 761, 494, 567, 869, 634, 794 \\
 \text{hash values:} & 2 \quad 1 \quad 4 \quad 7 \quad 9 \quad 4 \quad 4 \\
 h(K) &= K \% 10
 \end{array}$$



* Searching: $O(l)$ ↴ length of list.

(using linked list)

Closed Hashing (open addressing / linear Probing): ↴ kitne chances lage

value nach-table mein
insert karne ke liye.

Eg → 682, 761, 494, 567, 869, 634, 794.
 ↓ ↳ 3 probes
 2 probes

Hash var. = 2 1 4 7 9 4 4
 ↳ no. of buckets

linear probing

store at $(h(k) + i) \% M \quad \forall 0 \leq i \leq M$

$$\begin{aligned} \text{eg} \rightarrow h(794) &= (4+0) \% 10 = 4 \quad \text{filled} \\ &= (4+1) \% 10 = 5 \\ &= (4+2) \% 10 = 6 \Rightarrow \text{empty} \end{aligned}$$

add 794
to 6th bucket

Searching: O(probes)

Quadratic Probing: store at $(h(k) + i^2) \% M \quad \forall 0 \leq i \leq M$

↳ to prevent formation
of clusters

Eg → 794

$$h(k) = 794 \% 10 = 4$$

$$\begin{aligned} \text{at } (4+0^2) \% 10 &= 4 \times \quad \text{occupied} \\ (4+1^2) \% 10 &= 5 \times \\ (4+2^2) \% 10 &= 8 \checkmark \quad \text{vacant} \end{aligned}$$

Quadratic probing

0	
1	761
2	682
3	
4	494
5	634
6	
7	567
8	794
9	869

Double Hashing:

682, 761, 494, 567, 869, 634

0	
1	761
2	682
3	
4	494

$$h_1(k) = k \% 10$$

$$h_2(k) = k \% 5$$

store at $(h_1(k) + i^2 h_2(k)) \% M \quad 0 \leq i \leq M$

	5
	6
	7
	567
	634
	869

$$h(634) = \left[634 / 10 + 0 (634 / 5) \right] / 10$$

$$= 4 / 10 = 4 (\text{X occupied})$$

$$i=1 \Rightarrow = [4 + 1(4)] / 10 = \underline{\underline{8}} \checkmark (\text{vacant})$$

Load factor \Rightarrow $n \rightarrow \text{no. of elements}$
 $m \rightarrow \text{no. of buckets}$

$$\boxed{\text{load factor} = n/m} \Rightarrow \text{signifies average entries in one bucket.}$$

↳ Should be ideally low

Rehashing: when load factor exceeds a certain limit, we again rehash the given data.

L.F. > limit

generally $\boxed{\text{Load factor limit} = 0.75}$

* in rehashing, we increase the size of hash table and redistribute the elements again.

↓ very costly (time ↑↑)

→ closed addressing

Implementing Hash Table (Separate Chaining): (V.S code)

dynamic container HASHMAPS :

★ STL container which stores key-value pairs.

key	value
Name	Roll no.
Arijit	1234
Shubh	9928
Kartik	9854

★ The elements are stored in ascending/descending w.r.t. keys.

$$\begin{aligned} A &\rightarrow 12 \\ B &\rightarrow 98 \\ C &\rightarrow 05 \end{aligned}$$

→ by default

★ Maps cannot have duplicate values.

$A \rightarrow []$

$A \rightarrow []$

\times

* Implemented through BST (binary search trees).

* declaration → `#include <map>`

`map < key-type, value-type > map-name;`

e.g. → `map < string, int > phone-directory; // ascending order by default.`

e.g. → `map < datatype 1, datatype 2, greater < datatype 1 > > map-name;`

* Initialization:

`map < key-type, value-type > map-name = { {key1, val1}, {key2, val2}, {key3, val3} }`

e.g. → `map < string, int > directory = { {"Naman", 23456}, {"Chatur", 12987} }`

* Insertion → I. `directory.insert(make-pair ("ABC", 386));`
 $O(\log N)$ II. `directory[key] = value; // preferred.`

* Printing elements: for each loop: key, value pair
`for (auto element : map) {
 key = element.first;
 value = element.second;
}`

* updation → `directory["Chatur"] = 9927;`

More Member Funcⁿ:

iterators are part of STL library
used to traverse STL containers
like lists, stacks, queues etc.

$O(\log N)$

`erase()`

`mp.erase(iterator);`

`mp.erase(key);`

`mp.erase(start_itr, end_itr);`

`swap()`

`mp1, mp2`

\downarrow
Same type.

* `mp1.swap(mp2);`

`clear()`

`mp.clear();`

↓
 delete range from
 start to end (excluding
 end)
 ↓
 $O(n)$
 range

→ swap(mp1, mp2);

empty()

$\text{mp}.\text{empty}();$
 → returns true if empty else
 false

size()

$\text{mp}.\text{size}();$

max-size()

$\text{mp}.\text{max_size}();$

↓
 max. storage allocated
 to the map.

find() → $O(\log N)$

$\text{mp}.\text{find}(\text{key});$
 ⇒ returns itr. to the element if present,
 else returns $\text{mp}.\text{end}();$
 ↓
 itr after the last
 element in map

count()

$\text{mp}.\text{count}(\text{key});$
 ⇒ no. of occurrences of key.
 returns 1 if element is
 present else 0.

upper-bound();

returns an itr to next greater
 element.

$\{ \{ 10, "Name 1" \} \}$
 $\{ 20, "Name 2" \} \leftarrow \text{mp}.\text{lower_bound}(20)$
 $\{ 30, "Name 3" \} \leftarrow \text{mp}.\text{upper_bound}(20);$

lower-bound();

returns itr to element if present else
 itr to next greater element.

$\text{mp}.\text{lower_bound}(20);$

$\text{mp}.\text{begin}(), \text{mp}.\text{end}(), \text{mp}.\text{rbegin}(), \text{mp}.\text{rend}()$

$\left[\begin{matrix} K_1 & V_1 \\ K_2 & V_2 \\ K_3 & V_3 \end{matrix} \right]$ ← $\text{mp}.\text{rend}();$
 ← $\text{mp}.\text{begin}();$

$[k_y \quad v_y] \leftarrow mp.\text{rbegin}();$
 $\leftarrow mp.\text{end}();$

→ efficient → implemented using hashing.
Unordered Map: Same as map but elements are stored in unordered manner.

insertion/deletion/search - $O(1)$

also find(), insert(), erase - $O(1)$ avg. time complexity.
↳ $O(n)$ worst case

→ using BST(implementation)

MULTIMAP: same as ordered map, the only diff. is that you can
include <map> also store duplicate keys in a multimap.

insert/delete/search : $O(\log N)$

* note: in multmaps, we can only insert new elements using insert func',
we can't use $mp[\text{key}] = \text{value}$. \times

Unordered multimap → duplicate keys allowed and stored in unordered manner.
↳ member functions $\Rightarrow O(1)$

★ equal_range() func' in Multimap :

* Questions → in VS code.

- ⇒ isomorphic
- ⇒ longest subarray with zero sum.
- ⇒ target sum indices.

Ques → Given an array arr[] of length N, find the length of the longest subarray with a sum equal to 0.

input = { 15, -2, 2, -8, 1, 7, 10, 23 }

Output = 5

⇒ using prefix sum and Hashmaps.

