Approximate Algorithm example

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The approximate algorithm example here is take from the below book.

1. Vazirani, V. V., 2005, Algorytmy aprosksymacyjne (in Polish). WNT, Warsaw, Poland. (English original: Vazirani, V. V., 2003, Approximation algorithms. Springer)

Vertex covering problem
For a given graph $G = (V, E)$, where V is the set of vertices and E the set of edges, find the smallest subset $V' \subseteq V$ of vertices from V such that each edge from E has an ending in V'. The vertex covering problem is NP–hard.

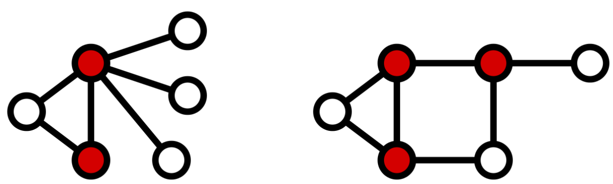

Figure 1: Minimum vertex covering examples.
By Miym—Own work, CC BY-SA 3.0, `https://en.wikipedia.org/wiki/Vertex_cover`

Maximal matching problem
For a given graph $G = (V, E)$ find a *matching*, i.e. set M of edges in E such that they do not have any common vertices, that is maximal (i.e. no edge can be added to it).

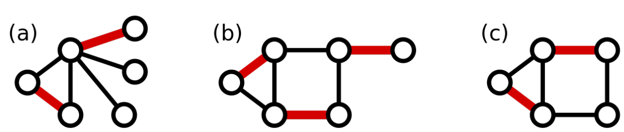

Figure 2: Maximal matching examples.
By Miym—Own work, CC BY-SA 3.0, `https://en.wikipedia.org/wiki/Matching_(graph_theory)`

For any given graph a maximal matching can be found in polynomial time using a greedy approach.

1. Set $M := \emptyset$
2. Choose any edge, e.
3. Set $M := M \cup \{e\}$.
4. Remove the ends of the edge from the graph (i.e. remove e's vertices from V and edges with e's vertices from E).
5. Repeat steps 2–4 until the graph is empty (i.e. $V = \emptyset$ and $E = \emptyset$).

Remark Notice that any maximal matching induces a vertex covering set (not necessarily the smallest). If it were not the case, i.e. that there would be edges that do not have endings in the set of vertices induced by the matching, then these edges could be added to the matching.

Approximate algorithm for the vertex covering problem
Finding a maximal matching is a 2–approximate algorithm for the vertex covering problem.
Proof If OPT is the power of V', i.e. $OPT = |V'|$, then $|M| \leq OPT$ as any vertex cover must contain at least one ending of the matching (since its induced vertices cover each edge). But then $2|M| \leq 2OPT$ is the size of the covering from the approximate algorithm. We hence obtain $S_{\text{approx}} \leq 2S_{\text{opt}}$.