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# Machine learning – Block 3

Måns Magnusson  
Department of Statistics, Uppsala University

Autumn 2022

- Practicalities
- Introduction
- Feed-Forward Neural Networks
  - Feed-Forward Neural Networks
  - Hyper-parameters
- Regularization
- Optimization of Neural Networks
  - Local minima
  - Plateaus and Saddle Points
  - Parameter initialization
- Neural Networks in Practice



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# Evaluation assignment 2

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# This week's lecture

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- **Practicalities**

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- Feed-Forward Neural Networks
- Regularization of Neural Networks
- Neural Network Optimization



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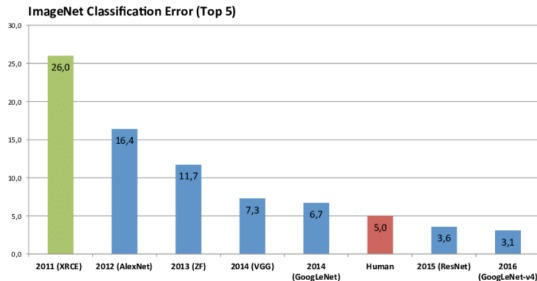
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## Section 2

### Introduction



Figure: ImageNet performance (Roessler, 2019)



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Word error rate on Switchboard trained against the Hub5'00 dataset

Year	Model	Percentage error
2011	CDNN	16.1
2012	DNN+IRM	18.5
2014	DNN+LSTM	12.8
2014	DNN+MER	12.5
2015	DNN	16.0
2015	Deep Speech + FSH	12.6
2015	Deep Speech	20.0
2015	FSH 2015	8.0
2016	FSH 2016	6.9
2017	RNNLM	6.9
2017	CNN+LSTM	6.2
2017	CNN+LSTM	6.1
2017	Microsoft 2016	5.8

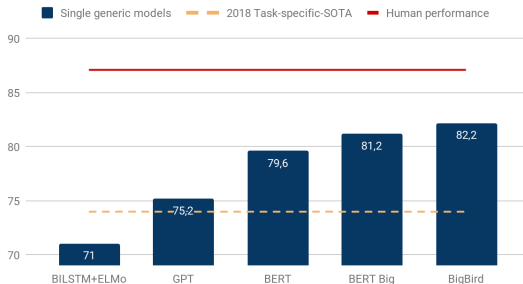
**Figure:** Speech recognition performance (source: <https://eff.org/ai/metrics>)



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# The Hype: Natural Language Processing

GLUE scores evolution over 2018-2019



**Figure:** General Language Understanding (source: <https://www.programmersought.com/article/4251948498/>)

Work is very much ongoing:

<https://gluebenchmark.com/leaderboard>



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# The Hype

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- Although - Neural Networks is not a silver bullet





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- Although - Neural Networks is not a silver bullet
- Remember the **Bayes error**



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- Although - Neural Networks is not a silver bullet
- Remember the **Bayes error**
- Some times a linear regression (or Random Forest) is enough



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## Section 3

# Feed-Forward Neural Networks



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# The Feed-Forward Network

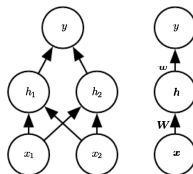


Figure 6.2: An example of a feedforward network, drawn in two different styles. Specifically, this is the feedforward network we use to solve the XOR example. It has a single hidden layer containing two units. (Left) In this style, we draw every unit as a node in the graph. This style is very explicit and unambiguous but for networks larger than this example it can consume too much space. (Right) In this style, we draw a node in the graph for each entire vector representing a layer's activations. This style is much more compact.

**Figure:** A simple feed-forward network (Goodfellow et al, 2017)

**Important concepts:**

Layers, neurons, input, output, weights, bias, architecture



# Different Architectures for Different Purposes

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- Different networks for different purposes
  - **Convolutional Neural Networks:** Computer Vision



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- Different networks for different purposes
    - **Convolutional Neural Networks**: Computer Vision
    - **Recurrent Neural Networks**: Speech Audio (?)



# Different Architectures for Different Purposes

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- Different networks for different purposes
  - **Convolutional Neural Networks**: Computer Vision
  - **Recurrent Neural Networks**: Speech Audio (?)
  - **Transformers/Attention**: Textual data
- The Neural Network Zoo: <https://www.asimovinstitute.org/neural-network-zoo/>



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# Areas of Use: All fields

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- Supervised learning
- Unsupervised learning
- Reinforcement learning





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# Why and when neural nets?

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- Learning feature representations
- Needs a lot of data to learn complex representations
- Good for sensor data (high-dimensional)



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# Why and when neural nets?

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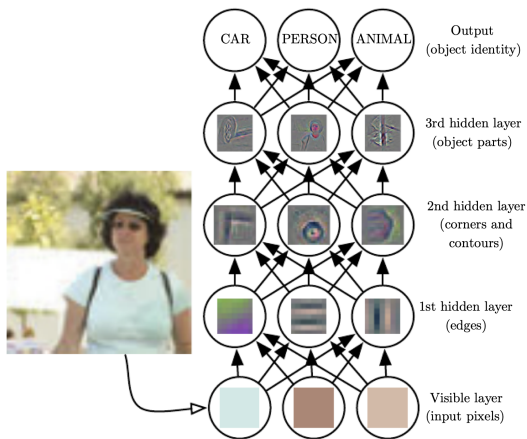
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- Learning feature representations
- Needs a lot of data to learn complex representations
- Good for sensor data (high-dimensional)
- When should we not use neural networks?



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# Learning Representations



**Figure:** Learning representations can be crucial (Goodfellow et al, 2017, Fig. 1.2)



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# The Feed-Forward Network

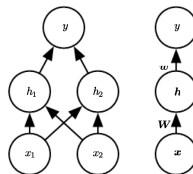


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**Figure:** A simple feed-forward network (Goodfellow et al, 2017, Fig. 6.2)

In mathematical notation:

$$y_i = \mathbf{w}^T g(\mathbf{W}^T \mathbf{x}_i + \mathbf{b}_1) + \mathbf{b}_2$$



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$$\mathbf{W} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \mathbf{b}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{b}_2 = (0)$$

$$g(z) = \text{ReLU}(z) = \max(0, z), x_i = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$y_i = \begin{pmatrix} 1 \\ -2 \end{pmatrix}^T g\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) + (0)$$

$$y_i = \begin{pmatrix} 1 \\ -2 \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (0) = 1$$



# The Feed-Forward Network

A feed-forward network for one observation ( $x_i$ ).

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$$\underbrace{\mathbf{h}_1}_{1 \times k_1} = g_1 \left( \underbrace{\mathbf{x}^T}_{1 \times p} \underbrace{\mathbf{W}_1}_{p \times k_1} + \underbrace{\mathbf{b}_1}_{1 \times k_1} \right)$$

$\vdots$

$$\underbrace{\mathbf{h}_l}_{1 \times k_l} = g_l \left( \underbrace{\mathbf{h}_{l-1}^T}_{1 \times k_{l-1}} \underbrace{\mathbf{W}_l}_{k_{l-1} \times k_l} + \underbrace{\mathbf{b}_l}_{1 \times k_l} \right)$$

$\vdots$

$$\underbrace{\hat{\mathbf{y}}}_{1 \times m} = g_L \left( \underbrace{\mathbf{h}_{L-1}^T}_{1 \times k_{L-1}} \underbrace{\mathbf{W}_L}_{k_{L-1} \times m} + \underbrace{\mathbf{b}_L}_{1 \times m} \right)$$

$$\hat{y} = f_L(f_{L-1}(\dots f_1(x)\dots))$$

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# Activation functions ( $g_l$ )

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- Sometimes use notation  $\sigma$  as in  $\sigma(Wh + b)$

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# Activation functions ( $g_l$ )

- Sometimes use notation  $\sigma$  as in  $\sigma(Wh + b)$
- Historically  $g(z)$  has been the sigmoid or or hyperbolic tangent (tanh)

$$g_{\text{sigmoid}}(z) = \frac{e^z}{e^z + 1} = \frac{1}{1 + e^{-z}}$$

$$g_{\text{tanh}}(z) = \frac{\sinh z}{\cosh z} = \frac{e^{2z} - 1}{e^{2z} + 1}$$

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- Now, usually variants of Rectified linear unit (ReLU)

$$g_{\text{ReLU}}(z) = \max(0, z)$$

- Easier to estimate with SGD
  - Easier for deep models
- Last activation is the output function  $g_L$ , usually a softmax (if classification)

$$f(z_i) = \frac{e^{z_i}}{\sum_{j=1}^J e^{z_j}}$$



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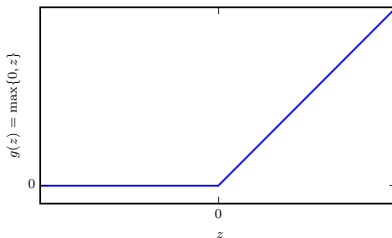


Figure: Rectified Linear Unit (Goodfellow et al, 2017, Fig. 6.3)



# Universal Approximation Theorem

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"A feed-forward neural network with a linear output layer and at least one hidden layer with any 'squashing' activation function can approximate any Borel measurable function from one finite-dimensional space to another with any desired non-zero amount of error, provided that the network is given enough hidden units." (Goodfellow et al. 2017, p. 198)

- Also holds for ReLU
- No guarantee we can learn the network
- No guarantee that it will generalize
- No indication of how large the network need to be

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# Hyper-parameters in feed-forward networks

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- The number of layers
- The number of neurons
- Activation functions
- The type of layers (CNN, MaxPooling, Multi-head attention)



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# How to choose parameters

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- Trial and error on validation sets
  - Art rather than science
  - Specialized approaches (Bayesian Optimization)



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- Trial and error on validation sets
  - Art rather than science
  - Specialized approaches (Bayesian Optimization)
  - Grid search (combinatorial explosion)
    - Really bad with many parameters with less effects
    - If we have 5 irrelevant parameters we try 3 values for: 125 training per relevant run
    - Instead use...



# How to choose parameters

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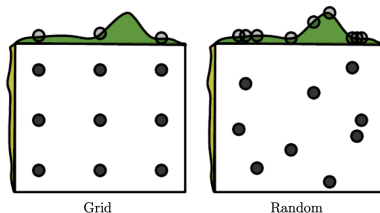
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- Random search



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## Grid search vs. Random Search



**Figure:** Grid search and random search (Goodfellow et al, 2017, Fig. 11.2)





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## Section 4

# Regularization



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# Regularization of Neural Networks

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- Reduce training error but improve test/validation error



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# Regularization of Neural Networks

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- Reduce training error but improve test/validation error
  - Neural Networks are extremely flexible / high model capacity



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# Regularization of Neural Networks

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- Reduce training error but improve test/validation error
  - Neural Networks are extremely flexible / high model capacity
  - Regularization is crucial for good generalizability of NN



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# Regularization of Neural Networks

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- Let

$$\tilde{J}(W, b) = J(W, b) + \alpha\Omega(W),$$

where  $J(W, b)$  is the cost function and  $\alpha\Omega(W)$  is the penalty for the weight matrices.

- $\alpha$  is the strength of the penalty.



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- Let

$$\Omega_1(W) = \sum_i \sum_j |w|_{i,j},$$

and

$$\Omega_2(W) = \sum_i \sum_j w_{i,j}^2,$$

be the  $L_1$  and  $L_2$  regularization respectively.

- We can then get the cost function

$$\tilde{J}(W, b) = J(W, b) + \sum_l \alpha_l \Omega_2(W_l),$$



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## Weight decay / Norm penalty

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- Lets define the cost function as

$$\begin{aligned}\tilde{J}(w) &= J(w) + \alpha\Omega_2(w) \\ &= J(w) + \alpha w^T w\end{aligned}$$

- Then the gradient update becomes

$$\nabla_w \tilde{J}(w) = \nabla_w J(w) + 2\alpha w$$

- To update our weights with gradient descent

$$\begin{aligned}w &\leftarrow w - \epsilon(\nabla_w J(w) + 2\alpha w) \\ w &\leftarrow (1 - 2\alpha\epsilon)w - \epsilon\nabla_w J(w)\end{aligned}$$





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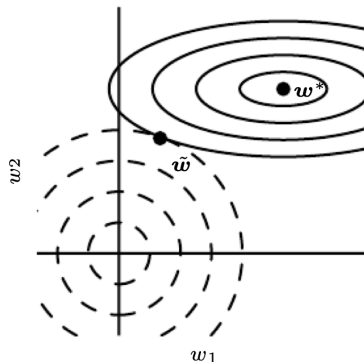


Figure:  $L_2$  regularization (Goodfellow et al, 2017, Fig. 7.1)



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# Early Stopping

- Stop optimization early based on validation error
- Rerun to that number of epochs (hyperparameter)
- Can be shown to be equivalent (under strict assumptions) to  $L_2$  regularization

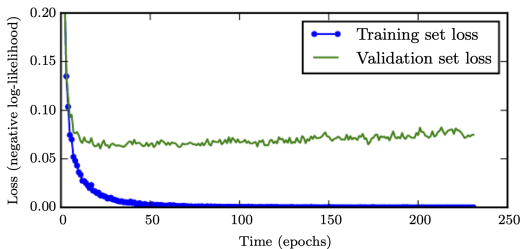
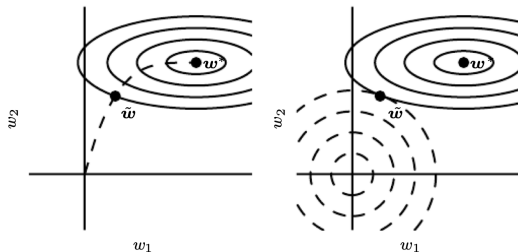


Figure: Early Stopping (Goodfellow et al, 2017, Fig. 7.3)



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# Early Stopping



**Figure:** Early Stopping (Goodfellow et al, 2017, Fig. 7.4)



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- In each iteration:
  - Sample an indicator  $I_i$  for each node  $i$
  - Set the value  $h_i$  to 0 with probability  $p$
- The dropout probability is typically 0.8 for input nodes and 0.5 for hidden nodes
- Forces the network to
  - not rely on individual nodes
  - spread out the weights over more nodes
- Can be seen as an ensemble method

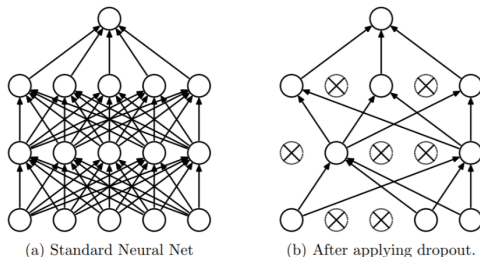


Figure 1: Dropout Neural Net Model. **Left:** A standard neural net with 2 hidden layers. **Right:** An example of a thinned net produced by applying dropout to the network on the left. Crossed units have been dropped.

Figure: Dropout (Srivastava et al, 2014)

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# Other regularization techniques

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- In CNN: Dataset augmentation
- Get more data...



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## Section 5

# Optimization of Neural Networks



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# Neural Network Learning

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- Usually, a lot of data and many parameters ( $\theta = (W, b)$ )





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- Usually, a lot of data and many parameters ( $\theta = (W, b)$ )
- We usually minimize our training cost function

$$J(\theta) = \sum_i^N L(\text{NN}(x_i), y_i) + \Omega(\theta),$$

where  $L$  is the observation level loss,  $\text{NN}()$  is our neural network and  $\Omega$  is the regularization term.



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- **Learning Target:** Find  $\hat{\theta}$  that minimize the **generalization** error



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- Stochastic Gradient Descent methods

$$\theta_t = \theta_{t-1} - \eta_t \hat{\nabla} J(\theta_{t-1})$$

- In practice, better optimizers are used:
  - Adam
  - RMSprop



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  - Adam
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- To compute gradients ( $\hat{\nabla} J(\theta_{t-1})$ ):
  - **Backpropagation** algorithm (chain-rule for derivatives)



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# Problems: Local Minima

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- Neural Networks cost function  $J(\theta)$  are (usually) **not a convex** function
  - We can have local minima
  - **When will this be a problem?**



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- A problem if local optima has a **high cost**



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# Problems: Plateaus and Saddle Points

- Another problem is **saddle points**

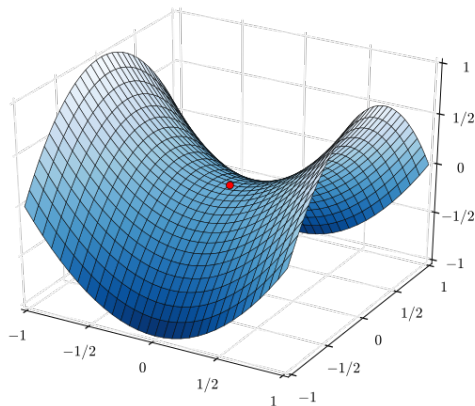


Figure: Saddle point for  $z = x^2 - y^2$

- What is the gradient in a saddle point?



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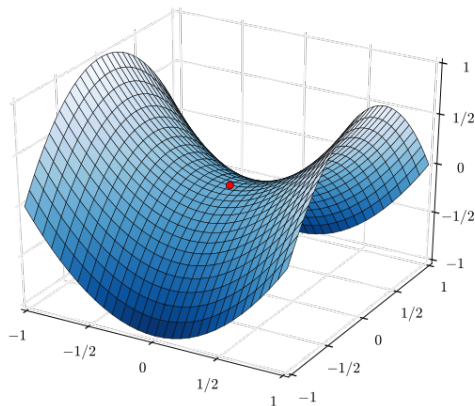


Figure: Saddle point for  $z = x^2 - y^2$

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- Difficult problem
  - Many local minima (weight space symmetry)
  - Plateaus and saddle points
    - Gradient is small - but not a minimum or maximum
    - Saddle points increases with the number of dimensions (?)
    - Large areas with small change in cost function



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# Initial values

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- We need to have **starting values** for gradient descent

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# Initial values

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# Initial values

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  - Numerical problems



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  - Otherwise the same units will be updated in the same way (deterministically)



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- Initialization can be seen as a hyperparameter
- Bad initial values might
  - Bad convergence (local optimum)
  - Numerical problems
- We want to **break symmetry** between layers
  - Otherwise the same units will be updated in the same way (deterministically)
- Good practice
  - Initialize values randomly close to zero (uniform or normal)



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## Section 6

# Neural Networks in Practice



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# TensorFlow

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- Framework for large-scale machine learning and Neural Networks
- Developed by Google
- Can be used both from R and Python
- Used in both research and production
- What Tensorflow does:
  - Computing gradients (autodiff) for Neural Networks
  - Enable use of graphical processing units (GPU) and Tensor processing Units (TPU)
  - Enable training using common optimizers (such as Adam, RMSprop)
- Tensorflow Probability is a probabilistic programming framework using TF





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# (Py)Torch

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- Similar to TensorFlow
- Developed by Meta AI
- Can be used both from R and Python
- Used in both research and production
- **pyro** is a probabilistic programming framework using torch





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# Keras

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- Syntax for 'building' Neural Networks
- Available both in R and Python
- TensorFlow or Torch as backend

