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Machine learning – Block 7

Måns Magnusson
Department of Statistics, Uppsala University

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
 - The probabilistic decoder
 - The encoder
 - Training a variational autoencoder
- Probabilistic Topic Models
 - Latent Dirichlet Allocation
 - Estimating the LDA model

Autumn 2025



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This week's lectures

- **Practicalities**
- **Introduction**
- **Autoencoders**
- **The Variational Autoencoder**
 - The probabilistic decoder
 - The encoder
 - Training a variational autoencoder
- **Probabilistic Topic Models**
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- **Variational autoencoders**
- **Probabilistic Topic Models**
- **(Diffusion models)**
- **Large Language Models**



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Section 2

Introduction



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Why variational autoencoders and topic models?

- Popular approaches in industry and academia
- Probabilistic methods for unsupervised learning





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Why variational autoencoders and topic models?

- Popular approaches in industry and academia
- Probabilistic methods for unsupervised learning
- Aim of this lecture:
 - Describe the models
 - How to estimate these models
 - Explain what they are used for





Use Cases

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- Variational autoencoders: Unsupervised modeling of **images**
- Topic models: Unsupervised modeling of **documents**



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- Variational autoencoders: Unsupervised modeling of **images**
- Topic models: Unsupervised modeling of **documents**
- Used for:
 - Identify "**closeness**" in high-dimensional data



Use Cases

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- Variational autoencoders: Unsupervised modeling of **images**
- Topic models: Unsupervised modeling of **documents**
- Used for:
 - Identify "**closeness**" in high-dimensional data
 - **Visualize/analyze** data



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Use Cases

- Variational autoencoders: Unsupervised modeling of **images**
- Topic models: Unsupervised modeling of **documents**
- Used for:
 - Identify "**closeness**" in high-dimensional data
 - **Visualize/analyze** data
 - **Compression**





Use Cases

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- Variational autoencoders: Unsupervised modeling of **images**
- Topic models: Unsupervised modeling of **documents**
- Used for:
 - Identify "**closeness**" in high-dimensional data
 - **Visualize/analyze** data
 - **Compression**
 - **Feature construction**



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- Variational autoencoders: Unsupervised modeling of **images**
- Topic models: Unsupervised modeling of **documents**
- Used for:
 - Identify "**closeness**" in high-dimensional data
 - **Visualize/analyze** data
 - **Compression**
 - **Feature construction**
 - Analyze underlying **patterns**



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Use Cases: Examples

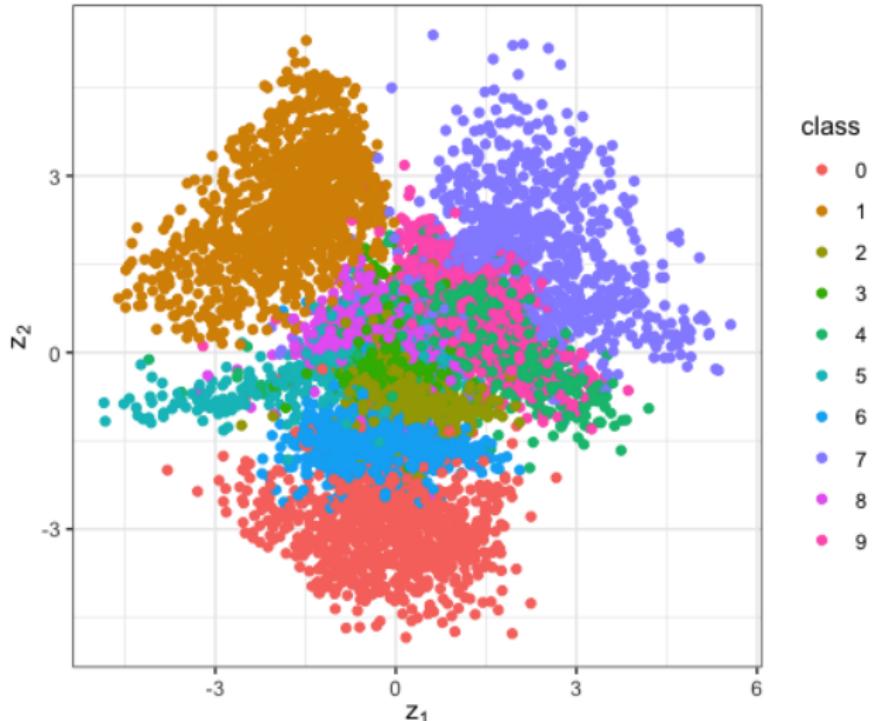


Figure: The latent state of MNIST using an Variational Autoencoder



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Section 3

Autoencoders



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Autoencoder

- An autoencoder is a neural network (e.g. feed-forward) that takes an input x and predict (the same) x (r , reconstruction).



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Autoencoder

- An autoencoder is a neural network (e.g. feed-forward) that takes an input x and predict (the same) x (r , reconstruction).
- Three parts:
 - **encoder** $f(x)$ (or $e(x)$)
 - **code** h
 - **decoder** $g(h)$ (or $d(h)$)

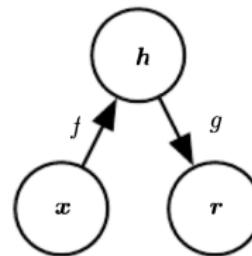


Figure: A Neural Autoencoder (Goodfellow et al, 2018)



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Autoencoder loss (Reconstruction error)

For a deterministic autoencoder:

$$h = e_\phi(x), \quad \hat{x} = d_\theta(h)$$

A common reconstruction loss is:

$$L(\theta, \phi) = \|x - \hat{x}\|_2^2 = \|x - d_\theta(e_\phi(x))\|_2^2$$

Minimizing squared error is equivalent (up to constants) to maximizing the log-likelihood of a Gaussian decoder with fixed variance:

$$x \mid h \sim \mathcal{N}(d_\theta(h), \sigma^2 I).$$



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The Undercomplete Autoencoder

- More interesting: an **undercomplete** autoencoder:
Dimension of code is **lower** than that of x

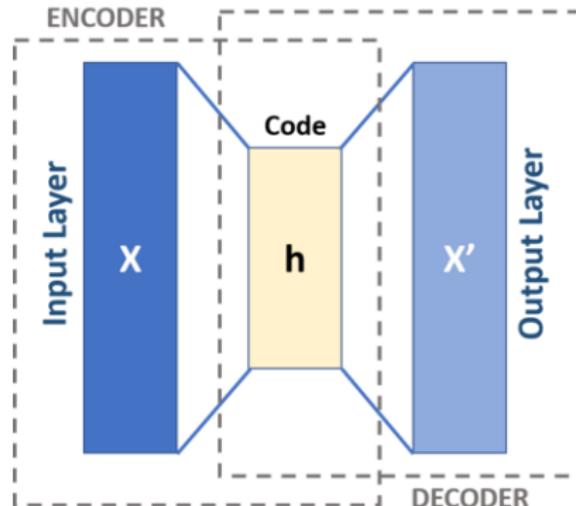


Figure: A Neural Autoencoder (Wikipedia)



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PCA and autoencoders

- A linear autoencoder: $e_\phi(x) = W_\phi x$, and $d_\theta(h) = W_\theta h$
- We want to minimize the loss (ignoring b /the mean):

$$L(\theta, \phi) = \sum_{i=1}^N \|x_i - W_\theta W_\phi x_i\|_2^2$$



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- Remember PCA loss:

$$L(P) = \sum_{i=1}^N (x_i - P_q P_q^T x_i)^2,$$

where P is an orthogonal matrix of rank q .



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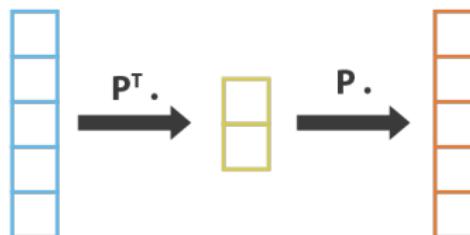
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- Remember **PCA loss**:

$$L(P) = \sum_{i=1}^N (x_i - P_q P_q^T x_i)^2,$$

where P is an orthogonal matrix of rank q .

- Hence: PCA can be seen as a linear autoencoder (under squared loss and orthonormal constraints)





Deep Autoencoders

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- Deep Autoencoder: An autoencoder with **multilayer neural networks** as encoder and decoder
 - can be seen as a **non-linear PCA**
 - learn **nonlinear representations**



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Deep Autoencoders

- Deep Autoencoder: An autoencoder with **multilayer neural networks** as encoder and decoder
 - can be seen as a **non-linear PCA**
 - learn **nonlinear representations**
- Problem: Deep autoencoders need to be **regularized** to not **overfit** the latent state





Probabilistic PCA as a decoder

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-
- Problem: Autoencoders (as PCA) are not probabilistic models:
 - cannot generate data.
 - no notion of uncertainty





Probabilistic PCA as a decoder

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- Problem: Autoencoders (as PCA) are not probabilistic models:
 - cannot **generate** data.
 - no notion of **uncertainty**
- We would like something like probabilistic PCA for deep autoencoders



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- Remember the pPCA model (with z as latent variable):

$$x_i \sim N(\mathbf{b} + \mathbf{W} z_i, \sigma^2 I)$$



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probabilistic PCA as an decoder

- Remember the pPCA model (with z as latent variable):

$$x_i \sim N(\mathbf{b} + \mathbf{W} z_i, \sigma^2 I)$$

- Now, swap the simple parameters with a neural network

$$x_i \sim N(\text{NeuralNetwork}_\theta(z_i), \sigma I)$$



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- This is an example of a Deep Latent Variable model (DLVM)



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- This is an example of a Deep Latent Variable model (DLVM)
- One example of a DLVM is the Variational Autoencoder



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Section 4

The Variational Autoencoder



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The Variational Autoencoder (VAE)

- The variational autoencoder (VAE) is a **deep probabilistic autoencoder**
- Used for unsupervised learning of **images**





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The Variational Autoencoder (VAE)

- The variational autoencoder (VAE) is a **deep probabilistic autoencoder**
- Used for unsupervised learning of **images**
- Consists of:
 1. The (probabilistic) encoder $q(z|\phi, x)$: **inference model**





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 2. Sample z from encoded x : the **latent state**





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- Encoding the **latent state as a distribution** forces the space to be "reasonable" /reduces overfitting



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- Encoding the **latent state as a distribution** forces the space to be "reasonable" /reduces overfitting
- VAEs get their name from **variational inference** (used in training)



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The Variational Autoencoder

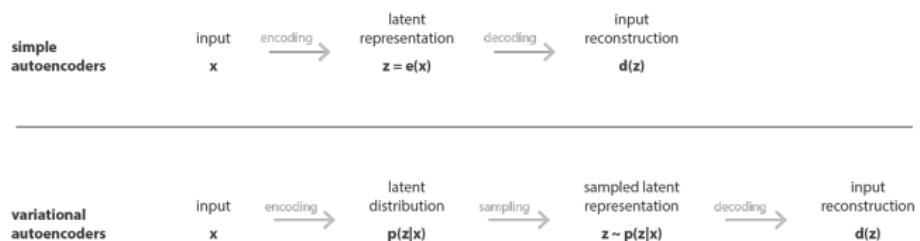


Figure: Autoencoder vs. the Variational Autoencoder (Rocca, 2019)



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The Variational Autoencoder

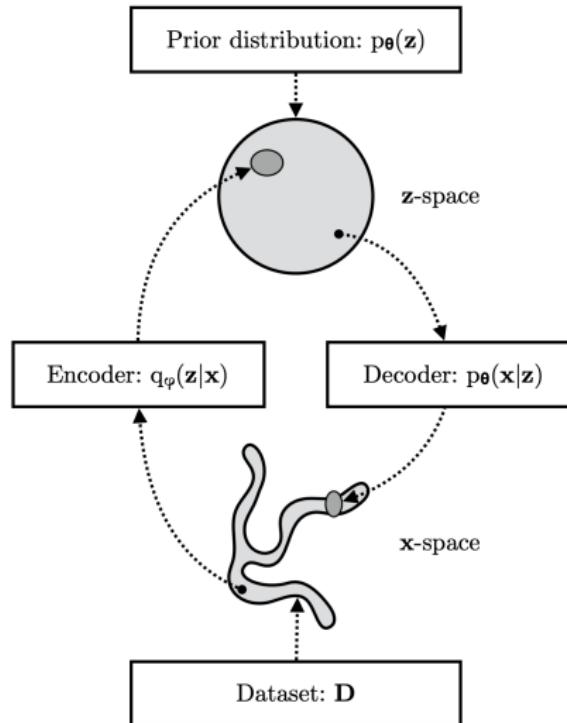


Figure: The Variational Autoencoder (Kingma and Welling, 2018, Fig. 2.1)



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The Prior over the Latent Space

In a variational autoencoder, the latent variable is assumed to follow a simple prior:

$$z \sim p(z) = \mathcal{N}(0, I).$$

This prior defines where encoded data points are allowed to live in latent space.

- Prevents arbitrary or irregular latent encodings
- Encourages continuity and smooth interpolation
- Enables easy generation by sampling from $p(z)$



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The probabilistic decoder

- The probabilistic decoder $p(x|\theta, z)$ (**observation model**)
- Usually a Normal distribution:

$$x_i \sim N(\text{NeuralNetwork}(z, \theta), \sigma_\theta^2 I)$$

- x_i for observation i depends non-linearly on the latent state z_i



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- A probabilistic linear decoder: **pPCA**

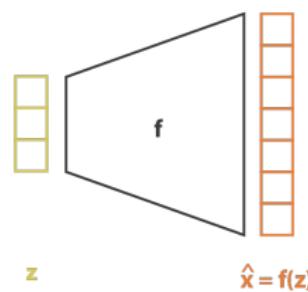


Figure: The Decoder (Rocca, 2019)



The probabilistic encoder

- The probabilistic encoder $q(z|x, \phi)$ (**inference model**)

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The probabilistic encoder

- The probabilistic encoder $q(z|x, \phi)$ (**inference model**)
- We assume that $q_\phi(z|x)$ follows a specific distribution.
Commonly:

$$z \sim N(\mu, \Sigma_\phi)$$

where $\Sigma_\phi = \text{diag}(\sigma_{1,\phi}^2, \dots, \sigma_{d,\phi}^2)$ (diagonal covariance) in practice.



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- A neural network learns the parameters μ and Σ_ϕ

$$\mu(x) = \text{NeuralNetwork}(x, \phi_\mu),$$

and

$$\log \sigma_{d,\phi}^2(x) = \text{NeuralNetwork}(x, \phi_\Sigma).$$



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$$\mu(x) = \text{NeuralNetwork}(x, \phi_\mu),$$

and

$$\log \sigma_{d,\phi}^2(x) = \text{NeuralNetwork}(x, \phi_\Sigma).$$

- **Result:** z_i for observation i depends non-linearly on x_i

Note: the decoder variance is typically fixed (e.g. $\sigma_\theta^2 I$), while the encoder learns a diagonal covariance.



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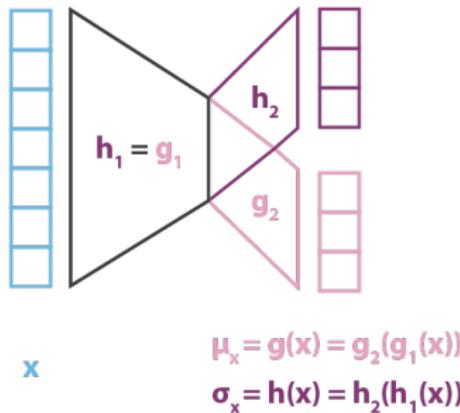


Figure: The Encoder (Rocca, 2019)



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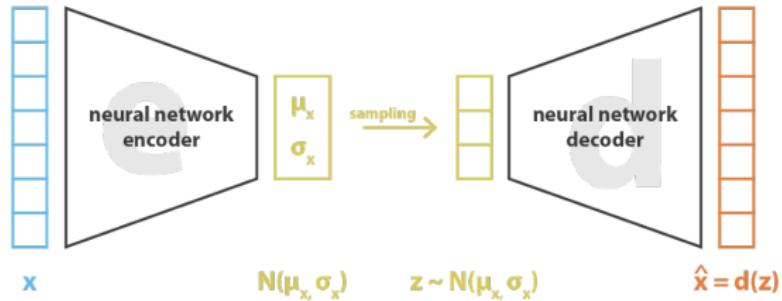


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The Variational Autoencoder

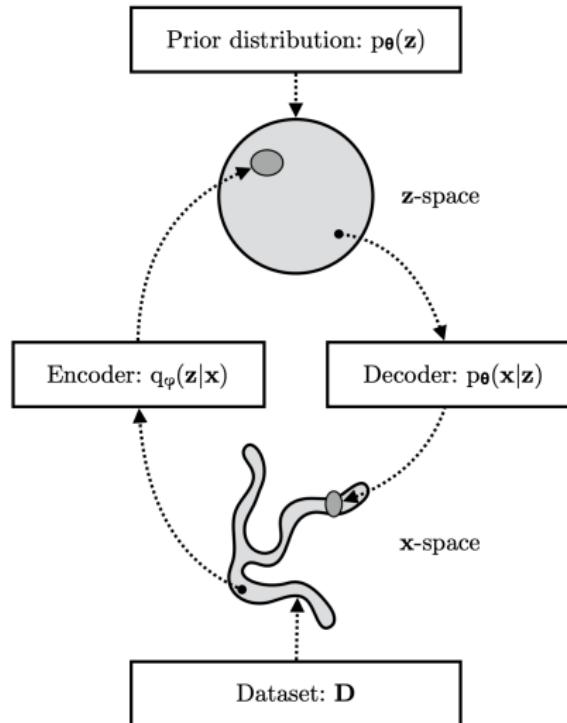


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Training a VAE

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- **Goal:** estimating ϕ, θ (and z_i)



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Training a VAE

- **Goal:** estimating ϕ, θ (and z_i)
- The encoder and decoder are (usually) complicated (no close form solution)
- Estimate ϕ and θ



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Training a VAE

- **Goal:** estimating ϕ, θ (and z_i)
- The encoder and decoder are (usually) complicated (no close form solution)
- Estimate ϕ and θ
- Target:
- Maximize $\log p(x)$ (i.e. Explain the data as well as possible)



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The Kullback–Leibler (KL) Divergence

What is it?

A measure of how well one probability distribution approximates another.

Definition

For two distributions p and q :

$$D_{\text{KL}}(p \parallel q) = \mathbb{E}_p \left[\log \frac{p(X)}{q(X)} \right]$$

Key properties

- $D_{\text{KL}}(p \parallel q) \geq 0$
- $D_{\text{KL}}(p \parallel q) = 0$ iff $p = q$
- Not symmetric: $D_{\text{KL}}(p \parallel q) \neq D_{\text{KL}}(q \parallel p)$

Interpretation

- Measures information lost when using q to approximate p
- Direction matters: *which distribution is the truth*



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From the marginal likelihood to the ELBO

Goal: Compute or maximize the marginal log-likelihood

$$\log p_\theta(x)$$

Step 1: Introduce the latent variable

$$\log p_\theta(x) = \log \int p_\theta(x, z) dz$$

Step 2: Introduce a variational distribution

$$\log p_\theta(x) = \log \int q_\phi(z | x) \frac{p_\theta(x, z)}{q_\phi(z | x)} dz = \log \mathbb{E}_{q_\phi(z|x)} \left[\frac{p_\theta(x, z)}{q_\phi(z | x)} \right]$$

(This equality always holds as long as $q_\phi(z | x) > 0$.)

Step 3: Apply Jensen's inequality

(For a concave function f , $f(\mathbb{E}[X]) \geq \mathbb{E}[f(X)]$)

$$\log p_\theta(x) \geq \mathbb{E}_{q_\phi(z|x)} \left[\log \frac{p_\theta(x, z)}{q_\phi(z | x)} \right]$$



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The Evidence Lower Bound (ELBO)

Definition: The Evidence Lower Bound (ELBO) is

$$\mathcal{L}_{\theta, \phi}(x) = \mathbb{E}_{q_{\phi}(z|x)} \left[\log \frac{p_{\theta}(x, z)}{q_{\phi}(z|x)} \right]$$

Key relationship:

$$\mathcal{L}_{\theta, \phi}(x) = \log p_{\theta}(x) - D_{KL}(q_{\phi}(z|x) \| p_{\theta}(z|x))$$

(KL divergence is always non-negative.)

Interpretation:

- Maximizing the ELBO *increases* $\log p_{\theta}(x)$
- Minimizes the gap between $q_{\phi}(z|x)$ and $p_{\theta}(z|x)$
- The KL term regularizes the latent space by pushing $q_{\phi}(z|x)$ toward the prior $\mathcal{N}(0, I)$.

Important:

$$\mathcal{L}_{\theta, \phi}(x) \leq \log p_{\theta}(x), \quad \text{with equality iff } q_{\phi}(z|x) = p_{\theta}(z|x)$$



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Optimizing the ELBO

Objective

We want to maximize the ELBO with respect to encoder and decoder parameters:

$$\max_{\theta, \phi} \mathcal{L}_{\theta, \phi}(x) = \mathbb{E}_{q_\phi(z|x)} [\log p_\theta(x, z) - \log q_\phi(z | x)]$$





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Problem

- The expectation is over $q_\phi(z | x)$
- We need gradients with respect to ϕ and θ
- Direct differentiation through sampling is not possible



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Problem

- The expectation is over $q_\phi(z | x)$
- We need gradients with respect to ϕ and θ
- Direct differentiation through sampling is not possible

Solution

- Approximate the expectation using Monte Carlo
- Rewrite the sampling step so gradients can flow (reparameterization trick)



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The Reparameterization Trick

Assumption

The variational posterior is Gaussian:

$$q_\phi(z \mid x) = \mathcal{N}(\mu_\phi(x), \sigma_\phi^2(x)I)$$

Key idea

Instead of sampling $z \sim q_\phi(z \mid x)$, write:

$$z = g(\phi, x, \epsilon) = \mu_\phi(x) + \sigma_\phi(x) \odot \epsilon, \quad \epsilon \sim \mathcal{N}(0, I)$$

Why this helps

- Randomness is now isolated in ϵ
- $z = g(\phi, x, \epsilon)$ is a differentiable function of ϕ
- Enables backpropagation through the sampling step



Monte Carlo Optimization of the ELBO

ELBO rewritten

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$$\mathcal{L}_{\theta, \phi}(x) = \mathbb{E}_{p(\epsilon)} [\log p_{\theta}(x, g(\phi, x, \epsilon)) - \log q_{\phi}(g(\phi, x, \epsilon) \mid x)]$$

Monte Carlo approximation

Using one sample $\epsilon^{(1)} \sim \mathcal{N}(0, 1)$:

$$\mathcal{L}_{\theta, \phi}(x) \approx \log p_{\theta}(x, g(\phi, x, \epsilon^{(1)})) - \log q_{\phi}(g(\phi, x, \epsilon^{(1)}) \mid x)$$

Optimization

- Use stochastic gradient ascent (or descent on $-\text{ELBO}$)
- Gradients flow through $\mu_{\phi}(x)$ and $\sigma_{\phi}(x)$ and we can compute $\nabla \mathcal{L}_{\theta, \phi}(x)$
- One or few samples ϵ per datapoint is usually sufficient



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The Autoencoding Variational Bayes Algorithm

Algorithm 1: Stochastic optimization of the ELBO. Since noise originates from both the minibatch sampling and sampling of $p(\epsilon)$, this is a doubly stochastic optimization procedure. We also refer to this procedure as the *Auto-Encoding Variational Bayes* (AEVB) algorithm.

Data:

\mathcal{D} : Dataset

$q_\phi(\mathbf{z}|\mathbf{x})$: Inference model

$p_\theta(\mathbf{x}, \mathbf{z})$: Generative model

Result:

θ, ϕ : Learned parameters

$(\theta, \phi) \leftarrow$ Initialize parameters

while SGD not converged **do**

$\mathcal{M} \sim \mathcal{D}$ (Random minibatch of data)

$\epsilon \sim p(\epsilon)$ (Random noise for every datapoint in \mathcal{M})

 Compute $\tilde{\mathcal{L}}_{\theta, \phi}(\mathcal{M}, \epsilon)$ and its gradients $\nabla_{\theta, \phi} \tilde{\mathcal{L}}_{\theta, \phi}(\mathcal{M}, \epsilon)$

 Update θ and ϕ using SGD optimizer

end

Figure: The Autoencoding Variational Bayes Algorithm (Kingma and Welling, 2018, Algo. 1)



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The Autoencoding Variational Bayes Algorithm

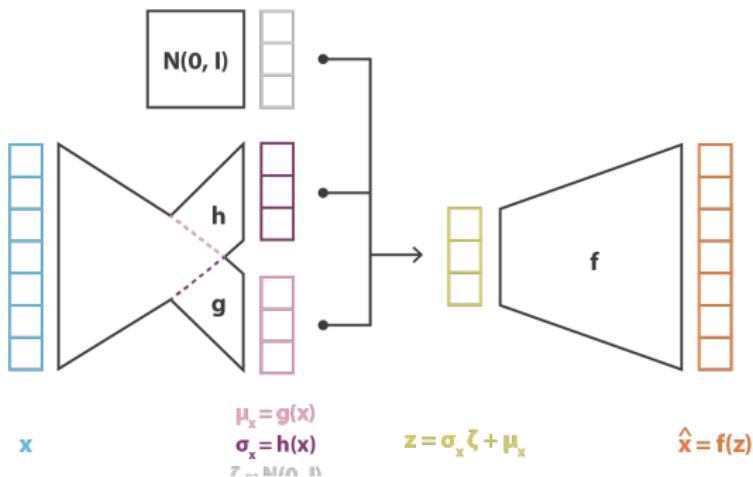


Figure: The Autoencoding Variational Bayes Algorithm (Rocca, 2019)



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Summary

- Benefits of VAE:

- Get a more **interpretable** latent state
- We can estimate **uncertainty** (but it's usually bad)
- **Regularize** the latent state
- We can inject knowledge in our latent state



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Summary

- Benefits of VAE:
 - Get a more **interpretable** latent state
 - We can estimate **uncertainty** (but it's usually bad)
 - **Regularize** the latent state
 - We can inject knowledge in our latent state
- Still much ongoing research



Figure: Examples of images generated with a deep hierarchical Variational Autoencoder (Vahdat and Kautz, 2020)



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VAE vs Diffusion Models

Variational Autoencoders (VAEs)

- Latent variable model: $z \sim p(z)$
- Trained by maximizing the ELBO
- Fast sampling (one forward pass)
- Compact, interpretable latent space
- Often produce blurrier samples

Diffusion Models

- No explicit low-dimensional latent space
- Learn to reverse a noise process
- Trained via denoising objectives
- Very high-quality samples
- Slow sampling (many steps)

Takeaway: VAEs prioritize efficient inference and representation learning, while diffusion models prioritize sample quality.



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Section 5

Probabilistic Topic Models



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- Unsupervised method for **textual data**



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- Unsupervised method for **textual data**
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- The most common model: **Latent Dirichlet Allocation**
- A **mixed membership** model (a mixture of multinomial mixtures model)



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- Topic model builds on the **distributional hypothesis**



Probabilistic Topic Models

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- Use cases:
 - **Explore** document collections



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 - **Analyzing large corpora** using statistical methods
- Example: All ears media monitoring of speech data



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The Dirichlet Distribution

- Probability distribution over the simplex with K categories:

$$f(\mathbf{x}|\boldsymbol{\alpha}) = \frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^K x_i^{\alpha_i - 1}, \text{ where } B(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^K \Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^K \alpha_i\right)},$$

and $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_K)$



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and $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_K)$

- The probability distribution has the support on the simplex, that is

$$\sum_{i=1}^K x_i = 1 \text{ and } x_i \geq 0 \text{ for all } i \in [1, K]$$



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- The probability distribution has the support on the simplex, that is

$$\sum_{i=1}^K x_i = 1 \text{ and } x_i \geq 0 \text{ for all } i \in [1, K]$$

- The parameters $\boldsymbol{\alpha}$ can be seen as **pseudo-counts** and

$$\mathbb{E}[x_i] = \frac{\alpha_i}{\sum_{j=1}^K \alpha_j}$$



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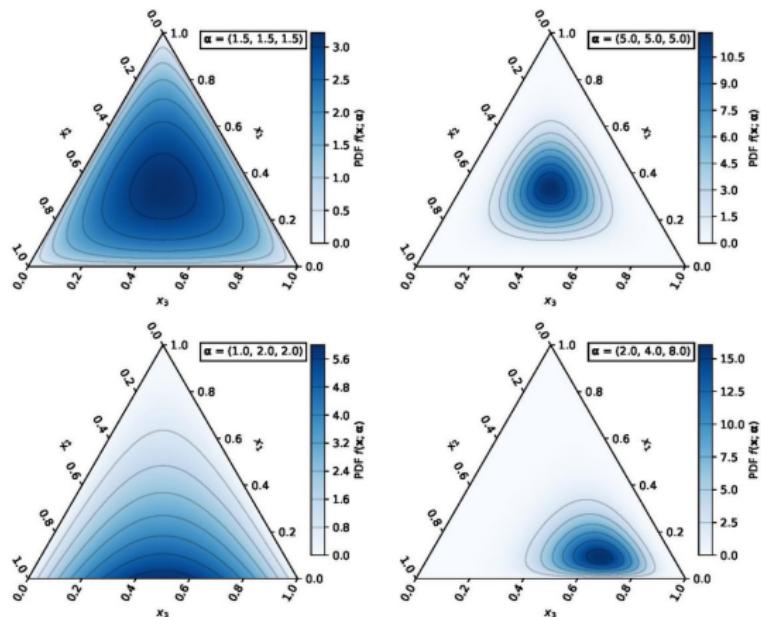


Figure: The Dirichlet Distribution (Wikipedia)



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The distributional hypothesis

- Harris (1954) and Firth (1957):
“Word is characterized by the company it keeps”





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- Semantics (broadly defined) is captured by **context**





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The distributional hypothesis

- Harris (1954) and Firth (1957):
“Word is characterized by the company it keeps”
- Semantics (broadly defined) is captured by **context**
- Rough definition: **word windows** of different sizes
- Different window sizes, different **semantic** content:
 - Word embeddings (context: word windows)
 - Topic models (context: documents)

Example

1. “A friend in need is a friend indeed.”
2. “She is my friend indeed.”



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Latent Dirichlet Allocation

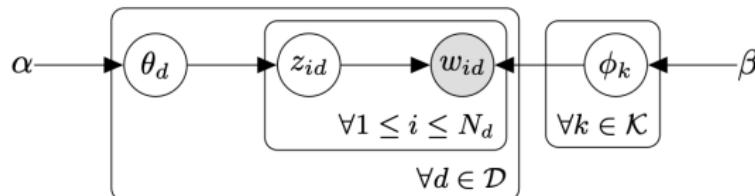


Figure: The Latent Dirichlet Allocation Model

where ϕ_k is the k th row in Φ (of dimension $K \times V$) and θ_d is the d th row in Θ (of dimension $D \times K$).



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Generative model for LDA

Relies on the **bag-of-word** assumption

1. For each component k to K :
 - 1.1 $\phi_k \sim \text{Dirichlet}(\beta)$
2. For each document d :
 - 2.1 $\theta_d \sim \text{Dirichlet}(\alpha)$
 - 2.2 For each token i :
 - 2.2.1 $z_{id} \sim \text{Categorical}(\theta_d)$
 - 2.2.2 $w_{id} \sim \text{Categorical}(\phi_{z_{id}})$



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Example of parameters z , Θ and Φ

w_1	boat	shore	bank		
z_1	1	1	1		
w_2	Zlatan	boat	shore	money	bank
z_2	2	1	1	3	3
w_3	money	bank	soccer	money	
z_3	3	3	2	3	



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w_3	money	bank	soccer	money			
z_3	3	3	2	3			

$\Phi =$		boat	shore	soccer	Zlatan	bank	money
	Topic 1	0.35	0.35	0.05	0.05	0.15	0.05
	Topic 2	0.025	0.025	0.45	0.45	0.025	0.025
	Topic 3	0.025	0.025	0.025	0.025	0.45	0.45



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$\Phi =$		boat	shore	soccer	Zlatan	bank	money
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	Topic 3	0.025	0.025	0.025	0.025	0.45	0.45

$\Theta =$		Topic 1	Topic 2	Topic 3
	doc 1	0.96	0.02	0.02
	doc 2	0.3	0.2	0.5
	doc 3	0.05	0.35	0.6



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Closing arguments were heard yesterday in the Federal bankruptcy fraud trial of Stephen J. Sabbeth, whose legal problems have raised doubts about his ability to continue as leader of the Nassau County Democratic Party.

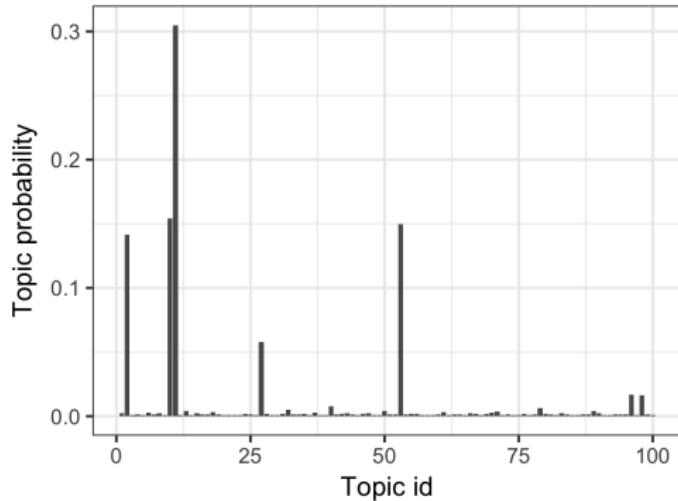
Mr. Sabbeth is charged with trying to conceal \$750,000 from his bank creditors by hiding the money in a secret account in his wife's maiden name, rather than use it to pay creditors when his lumber business went into bankruptcy 10 years ago.

– The New York Times 25th of February 1999



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The estimated topic proportion ($\hat{\theta}_d$)





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Topic top words

Topic	Top words (by ϕ_{kv})
2	party election voters campaign democratic
10	bank banks loans loan insurance savings
11	trial prison jury prosecutors convicted guilty
53	investigation inquiry documents investigators

Table: The words with highest probability ($p(w|k)$) for topic 2, 10, 11 and 53.



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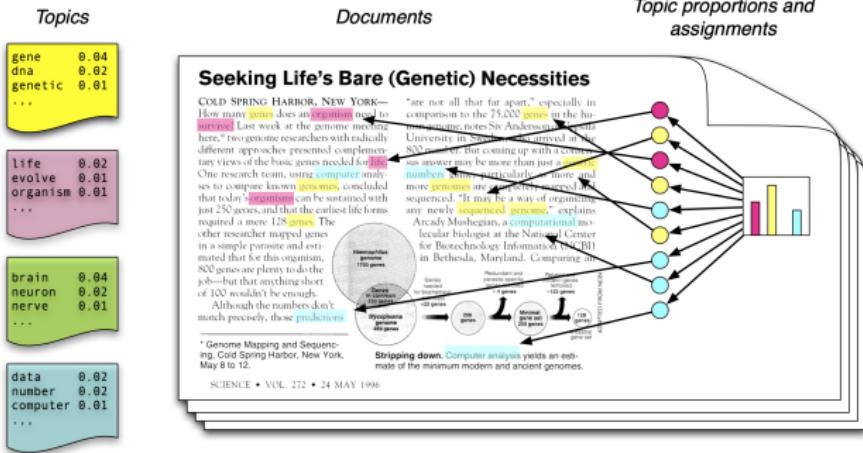


Figure: The Latent Dirichlet Allocation Model (Blei 2012, Fig. 1)



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-
- Common inference approaches
 1. Variational inference
 2. Markov Chain Monte Carlo (MCMC)





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- Common inference approaches
 1. Variational inference
 2. Markov Chain Monte Carlo (MCMC)
- The Gibbs sampler is usually preferred
- Similar to (Stochastic) EM





Gibbs sampler for LDA

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The basic Gibbs sampler:

1. We want to estimate z, Φ, Θ :



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The basic Gibbs sampler:

1. We want to estimate z, Φ, Θ :
2. Sample topic indicators (latent variable)

$$p(z = k | \Phi, \Theta) \propto \phi_{v,k} \theta_{k,d}$$





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3. Sample model parameters

$$\theta_d | \mathbf{z} \sim Dir(\mathbf{n}^{(d)} + \alpha)$$

$$\phi_k | \mathbf{z} \sim Dir(\mathbf{n}^{(v)} + \beta)$$

where $\mathbf{n}^{(d)}$ is the number of tokens by topic in document d and $\mathbf{n}^{(v)}$ is the number of tokens by topic for word type v .



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Integrating out (collapsing) Θ and Φ

$$p(\mathbf{z}|\mathbf{w}) = \int \int p(\mathbf{z}, \Theta, \Phi | \mathbf{w}) \cdot p(\mathbf{z}, \Theta, \Phi) d\Phi d\Theta$$

will result in the following Gibbs sampler:

$$p(z_i = k | w_i, \mathbf{z}_{\neg i}) \propto \underbrace{\frac{n_k^{(v)} + \beta}{n_k^{(v)} + V\beta}}_{type-topic (\Phi)} \cdot \underbrace{\frac{(n_k^{(d)} + \alpha)}{topic-doc (\Theta)}}_{}$$

where $n^{(v)}$ and $n^{(d)}$ are count matrices of size $D \times K$ and $K \times V$.



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Example of $n^{(v)}$ and $n^{(d)}$

w_1	boat	shore	bank		
z_1	1	1	1		
w_2	Zlatan	boat	shore	money	bank
z_2	2	1	1	3	3
w_3	money	bank	soccer	money	
z_3	3	3	2	3	



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$$n^{(v)} = \begin{matrix} & \text{boat} & \text{shore} & \text{soccer} & \text{Zlatan} & \text{bank} & \text{money} \\ \text{boat} & 2 & 2 & 0 & 0 & 1 & 0 \\ \text{shore} & 0 & 0 & 1 & 1 & 0 & 0 \\ \text{soccer} & 0 & 0 & 0 & 0 & 2 & 2 \end{matrix}$$



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$$n^{(d)} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 2 & 3 \end{bmatrix}$$



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- Setting K , α and β



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 - Reducing the vocabulary: stopwords, rare words, stemming





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 - "Junk" topics





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- Setting K , α and β
- Reducing the vocabulary: stopwords, rare words, stemming
- "Junk" topics
- We can analyze the topic indicators z directly





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Research Example: Swedish Immigration Discourse

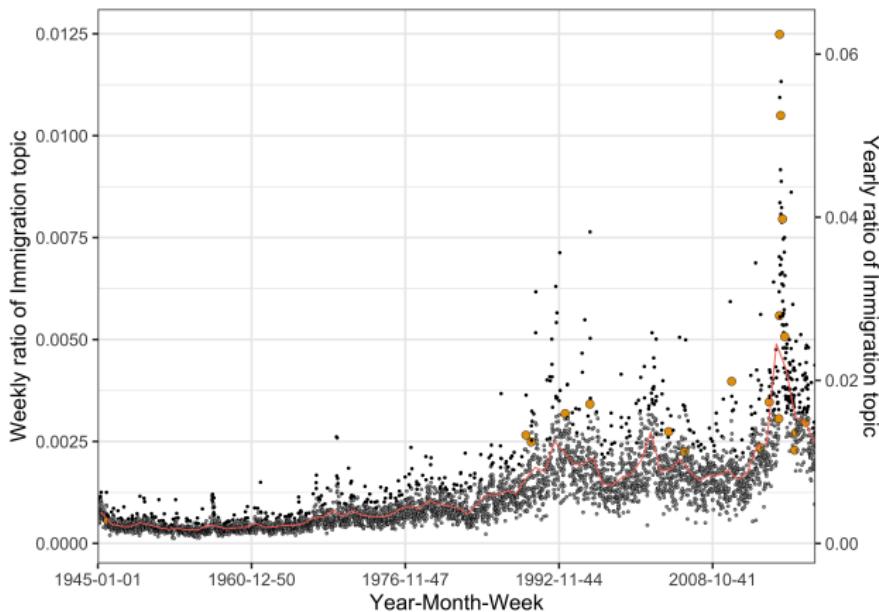


Figure: The Immigration topic in Swedish Newspapers (Hurtado Bodell et al, 2024)



Summary: Topic Models

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 - The **Latent Dirichlet Allocation** is a popular model





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- Topic models are **unsupervised** models for textual data
- The **Latent Dirichlet Allocation** is a popular model
- A **mixed membership model** (a mixture of multinomial mixtures model)



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- Topic models are **unsupervised** models for textual data
- The **Latent Dirichlet Allocation** is a popular model
- A **mixed membership model** (a mixture of multinomial mixtures model)
- Use Gibbs samplers for estimation