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- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

Machine learning – Block 8

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This week's lectures

- Introduction to Reinforcement learning

Introduction to Reinforcement Learning

- Another type of Machine Learning:
  - Supervised Learning
  - Unsupervised Learning
  - Reinforcement learning

Introduction to Reinforcement Learning

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  - Supervised Learning
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  - Reinforcement learning
- Computational approach of learning from interaction

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- Closest to human and animal learning: trial, error, and planning.

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- Computational approach of learning from interaction
- Closest to human and animal learning: trial, error, and planning.
- The learner is *not* told which actions to take
- Connections to:
  - Game Theory
  - Control Theory
  - Multi-agent systems
  - Swarm intelligence
  - Information theory
  - Statistics

Introduction to Reinforcement Learning

- Goal: maximize return over a sequence of actions

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- Three characteristics:
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  2. No instructions
  3. Reward signals over a long period of time

Recent Achievements

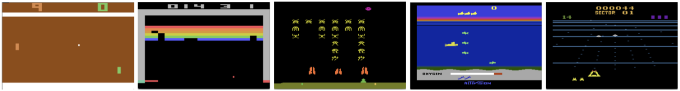

Figure: Mnih et al (2013) "Playing Atari with Deep Reinforcement Learning"

Recent Achievements

Figure: Lee Sedol vs. Alpha Go in 2016

Recent Achievements, also

- Industry automation: RL is used to reduce the energy cost of datacenter cooling

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- Industry automation: RL is used to reduce the energy cost of datacenter cooling
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- A/B testing and personalized recommendations

The different parts in RL

1. The Agent: The learning agent.

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3. Actions: Made by the agent and affects the environment.
4. Reward: The evaluation of an action. A scalar value. "Pleasure and pain" of actions.
5. Return: The aggregated reward over a long period.

The different parts in RL

1. Agents:
   1.1 Have a goal (maximize return)
   1.2 Sense aspect of their environment
   1.3 Choose actions
   1.4 Possibility to improve performance over time

The different parts in RL

1. Agents:
   1.1 Have a goal (maximize return)
   1.2 Sense aspect of their environment
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   1.4 Possibility to improve performance over time
2. Usually an uncertainty about the environment
3. Represent uncertainty of environment: Probability

Sub-elements of agents

1. Policy: How the agent choose actions. Determines behaviour.
2. Model: The agent's model of the environment. Used for planning

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- Outside the agent: Reward signal: The instant value of an action
- Problem: Balance the trade-off between long-term and short-term rewards

Supervised, Unsupervised and Reinforcement Learning

1. Static vs. Dynamic

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4. Need for exploration
5. Evaluates actions - not only instruct actions

Exploration vs Exploitation

- Goal: Maximize the return (the total reward), i.e.

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Exploration vs Exploitation

- Goal: Maximize the return (the total reward), i.e.
- Exploit the best actions
- Explore to know the best actions

Evolution vs Learning

- Set a policy without learning: Evolutionary Methods
- Good when the agent cannot sense the environment

Evolution vs Learning

- Set a policy without learning: Evolutionary Methods
- Good when the agent cannot sense the environment
- Example: Bacteria don't learn, they evolve

Setting the goal for the Agent

- Setting the goal: defining the reward signal (reward function)
- Example: If you want the agent to do something quick, give -1 per action.

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Setting the goal for the Agent

- Setting the goal: defining the reward signal (reward function)
- Example: If you want the agent to do something quick, give -1 per action.
- We should give rewards for correct behaviour
- Do not use reward to guide how to reach the goal
- Be careful what you wish for...

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Section 2

Bandits

The k-armed bandit problem

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where $\mathbb{E}(R_t|A_t = a) = q^\star(a)$.

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- This is a tabular method/problem:
  We can represent the actions in a table.
- Tabular methods works in small problems
  e.g. A/B testing and dynamic web pages.

Exploration vs. Exploitation

- Two types of actions:
  1. Exploitation: Choose the action with highest expected reward (short term)
  2. Exploration: Choose action to improve $Q_t(a)$, but reduces the reward (long term)
- The conflict between exploration and exploitation

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ϵ-greedy

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- For any $\epsilon > 0$, $Q_t(a) \to q^\star(a)$

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ϵ-greedy

- We estimate $q^\star(a)$ using $Q_t(a)$ as

$$Q_T(a) = \frac{1}{N(a)} \sum_{t}^{T-1} R_{t, A_t = a} \,,$$

where $N(a)$ is the total number of times action a has been taken.

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  - Large $\mathcal{A}$
  - Non-stationarity

Bandit example

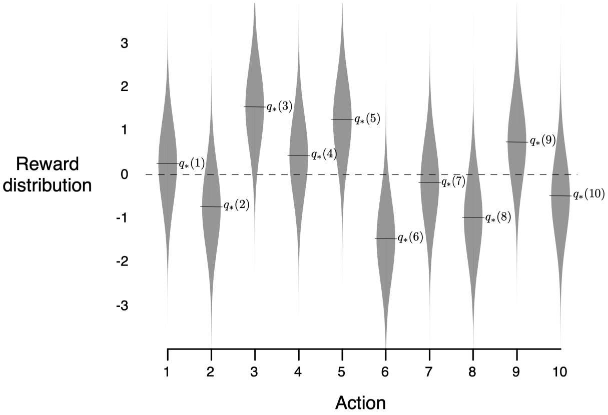

Figure: The 10-armed bandit environment (Sutton and Barto, 2017, Fig. 2.1)

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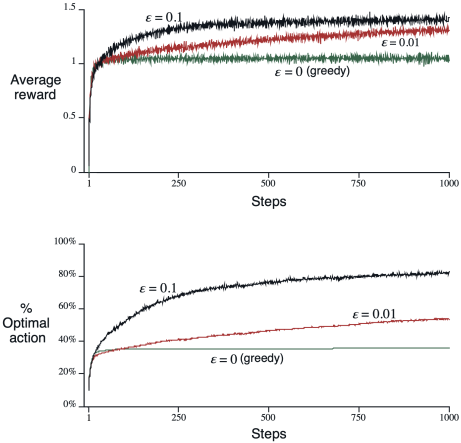

Figure: The ϵ-greedy algorithm result in the 10-armed bandit (Sutton and Barto, 2017, Fig. 2.2)

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Bandit example: Optimistic initialization

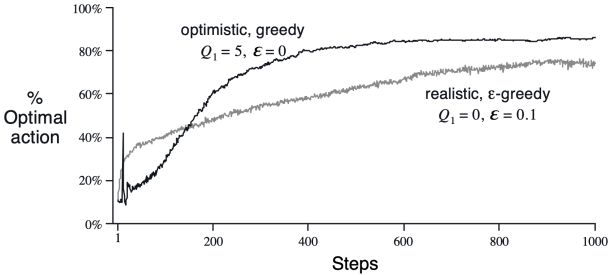

Figure: The ϵ-greedy algorithm and optimistic initialization (Sutton and Barto, 2017, Fig. 2.3)

Efficient computation and non-stationarity

- Compute $Q_t(a)$ on the fly:

$$Q_T(a) = Q_{T-1} + \frac{1}{N_t(a)}(R_{t,A_t=a} - Q_{T-1}(a))$$

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- Handling non-stationarity:

$$Q_T(a) = Q_{T-1} + \alpha(t)(R_{t,A_t=a} - Q_{T-1}(a))$$

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Efficient computation and non-stationarity

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- Examples:
  - $\alpha(t) = 1$: $Q_T(a) = R_{t,A_t=a}$

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  - $\alpha(t) = \frac{1}{N_t(a)}$: Average reward

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- $Q_T(a) \rightarrow q^\star(a)$, if:

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  - $\alpha(t) = \frac{1}{N_t(a)}$: Average reward
- $Q_T(a) \rightarrow q^\star(a)$, if:
  1. $\sum_t^\infty \alpha_t = \infty$
  2. $\sum_t^\infty \alpha_t^2 < \infty$
- Where have we seen these criterias before?

The ϵ-greedy algorithm

A simple bandit algorithm

Initialize, for $a = 1$ to k:

$Q(a) \leftarrow 0$

$N(a) \leftarrow 0$

Repeat forever:

$$A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1-\varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$$

$R \leftarrow bandit(A)$

$N(A) \leftarrow N(A) + 1$

$Q(A) \leftarrow Q(A) + \frac{1}{N(A)}\left[R - Q(A)\right]$

Figure: The ϵ-greedy algorithm

The Upper-Confidence-Bound method

- Explore based on our uncertainty of $Q_t(a)$

The Upper-Confidence-Bound method

- Explore based on our uncertainty of $Q_t(a)$
- The Upper-Confidence-Bound (UCB) method

$$A_t = \arg\max_a \left(Q_t + c\sqrt{\frac{\log t}{N_t(a)}} \right)$$

An analogy:

$$A_t = \arg\max_a \left(Q_t + c\sqrt{\frac{\hat{\sigma}^2(a)}{N_t(a)}} \right)$$

The UCB algorithm

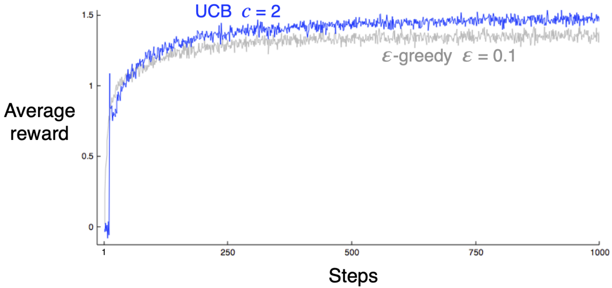

Figure: The UCB algorithm

The Bayesian Bandit: Thompson sampling

- A Bayesian Bandit
  1. Setup a likelihood for R, $p(R|a, \theta)$
  2. Setup a prior for θ, $p(\theta)$
  3. Compute posterior for θ, $p(\theta|R, a)$
  4. Choose action A_t proportional to

$$\int I[\mathbb{E}(R|\theta, a^\star) = \arg\max_{a'} \mathbb{E}(R|\theta, a')]p(\theta|R, a)d\theta$$

  where I is the indicator function.
- Repeat step 3-4

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  where I is the indicator function.
- Repeat step 3-4
- Monte Carlo approximation of step 4:
  1. Draw one sample from the posterior $\tilde{\theta}$

$$\tilde{\theta} \sim p(\theta|R, a)$$

  2. Conditional on $\tilde{\theta}$, choose action A_t proportional to

$$\arg\max_{a'} \mathbb{E}(R|\tilde{\theta}, a')] \,,$$

  i.e. sample A_t based on the probability of being the best action.

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Section 3

Markov Decision Processes

The Markov Decision process

- Bandits does not have a *state*.

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- An action might change the environment.
- An action might be different in different states

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The Markov Decision process

- Bandits does not have a *state*.
- An action might change the environment.
- An action might be different in different states
- Example: In chess, we want to make a move based on the current position of all pieces
- To capture this we use a Markov Decision process
- One of the most important concepts in Reinforcement Learning

The Markov Decision process

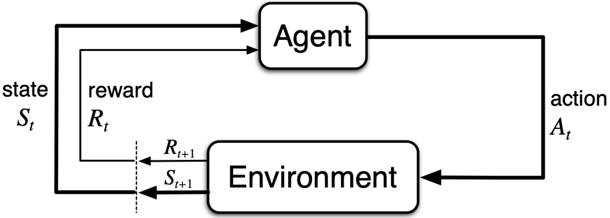

Figure: The (finite) Markov Decision Process (Sutton and Barto, 2017, Fig 3.1)

- States $S_t \in \mathcal{S}$: Basis for action
- Actions $A_t \in \mathcal{A}$
- Rewards $R_t \in \mathbb{R}$

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The Markov Decision process

- Boundry between Agent and Environment:
  - The total control of the action

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  - Reward is external to agent: Pain and pleasure
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The Markov Decision process

- Boundry between Agent and Environment:
  - The total control of the action
  - Reward is external to agent: Pain and pleasure
  - The agent should not be able to change the reward function
- The policy ($\pi(A_t|S_t = s)$):
  - We make an action given the current state S_t
- The goal: (Again) maximize return $G_t = R_{t+1} + ... + R_T$

Return and discount

- Two type of interactions
  - Episodic: $T < \infty$, has terminal state
  - Continuing: $T = \infty$
- Discounting:

$$G_t = R_{t+1} + \gamma R_{t+3} + \gamma^2 R_{t+3} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

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- Discount rate γ:
  - $0 \leq \gamma \leq 1$

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- Discount rate γ:
  - $0 \leq \gamma \leq 1$
  - $\gamma = 1$: No discount
  - $\gamma = 0$: Full discount: Only next reward counts
  - $\gamma < 1$ and R_t is bounded: $G_t < \infty$

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  - $\gamma < 1$ and R_t is bounded: $G_t < \infty$
- For episodic problem we assume $R_{T+i} = 0$ for all $i \in \mathbb{N}^+$

The Markov Decision Process

- The Markov Decision process (MDP):

$$P(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a) \quad (1)$$

- Eq. (1) fully specify a MDP

The Markov Decision Process

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- Eq. (1) fully specify a MDP
- Markov property:

$$P(S_{t+1} = s', R_{t+1} = r | S_1 = s, A_1 = a, ..., S_t = s, A_t = a) = P(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a)$$

- The MDP is a good approximation or model:
  All models are wrong, but some are useful.

The Markov Decision Process Marginals

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- The Markov Decision process (MDP):

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- From Eq. (1) we can get marginals of interest:
  - State-action rewards:

$$r(s, a) = \mathbb{E}(R_{t+1} | S_t = s, A_t = a)$$

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$$r(s, a) = \mathbb{E}(R_{t+1} | S_t = s, A_t = a)$$

  - State-transition probability:

$$p(s' | s, a) = P(S_{t+1} = s' | S_t = s, A_t = a)$$

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The Value function

- The value function $v_\pi(s)$:
  the long-term value of s given a policy $\pi(a|s)$, i.e.

$$v_\pi(s) = \mathbb{E}_\pi(G_t|S_t = s) = \mathbb{E}_\pi\left(\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}|S_t = s\right)$$

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- Informally: How "good" is a state for the agent with the policy π.

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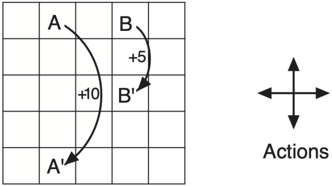
- This is the Bellman equation for $v_\pi(s)$:
  The relationship between the values of the state and its successor states.
- Bellman equation is the basis for computing $v_\pi(s)$ (not part of this course)

The value function



3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

Figure: The gridworld equiprobable policy value function

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The Optimal Policy

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- The optimal value function: $v_{\pi_\star}(s) = v_\star(s)$
- The optimal policy $\pi_\star$ is greedy wrt $v_{\pi_\star}(s)$:
  The best long term strategy

The Optimal Policy

- Computing optimal value function might be impossible: we need to estimate/re-estimate/approximate it
- Example: Chess, we cannot compute the optimal long-term moves, we need to approximate/estimate (based on computational budget)

The Optimal Policy

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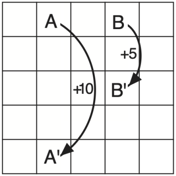
- Example: Chess, we cannot compute the optimal long-term moves, we need to approximate/estimate (based on computational budget)
- We might also estimate $v_\star(s)$ better for commonly encountered states

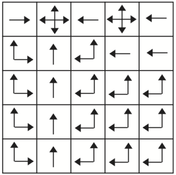
The value function



Gridworld

22.0	24.4	22.0	19.4	17.5
19.8	22.0	19.8	17.8	16.0
17.8	19.8	17.8	16.0	14.4
16.0	17.8	16.0	14.4	13.0
14.4	16.0	14.4	13.0	11.7

v_*



π_*

Figure: The gridworld optimal value function and policy