UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Machine learning – Block 7

Måns Magnusson
Department of Statistics, Uppsala University

Autumn 2024

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

This week's lectures

- Variational autoencoders
- Probabilistic Topic Models
- (Diffusion models)
- Large Language Models

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Section 2

Introduction

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Why variational autoencoders and topic models?

- Popular approaches in industry and academia
- Probabilistic methods for unsupervised learning

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Why variational autoencoders and topic models?

- Popular approaches in industry and academia
- Probabilistic methods for unsupervised learning
- Aim of this lecture:
  - Describe the models
  - How to estimate these models
  - Explain what they are used for

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Use Cases

- Variational autoencoders: Unsupervised modeling of images
- Topic models: Unsupervised modeling of documents

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Use Cases

- Variational autoencoders: Unsupervised modeling of images
- Topic models: Unsupervised modeling of documents

- Used for:
  - Identify "closeness" in high-dimensional data

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Use Cases

- Variational autoencoders: Unsupervised modeling of images
- Topic models: Unsupervised modeling of documents

- Used for:
  - Identify "closeness" in high-dimensional data
  - Visualize/analyze data

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Use Cases

- Variational autoencoders: Unsupervised modeling of images
- Topic models: Unsupervised modeling of documents

- Used for:
  - Identify "closeness" in high-dimensional data
  - Visualize/analyze data
  - Compression

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Use Cases

- Variational autoencoders: Unsupervised modeling of images
- Topic models: Unsupervised modeling of documents

- Used for:
  - Identify "closeness" in high-dimensional data
  - Visualize/analyze data
  - Compression
  - Feature construction

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Use Cases

- Variational autoencoders: Unsupervised modeling of images
- Topic models: Unsupervised modeling of documents

- Used for:
  - Identify "closeness" in high-dimensional data
  - Visualize/analyze data
  - Compression
  - Feature construction
  - Analyze underlying patterns

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Use Cases: Examples

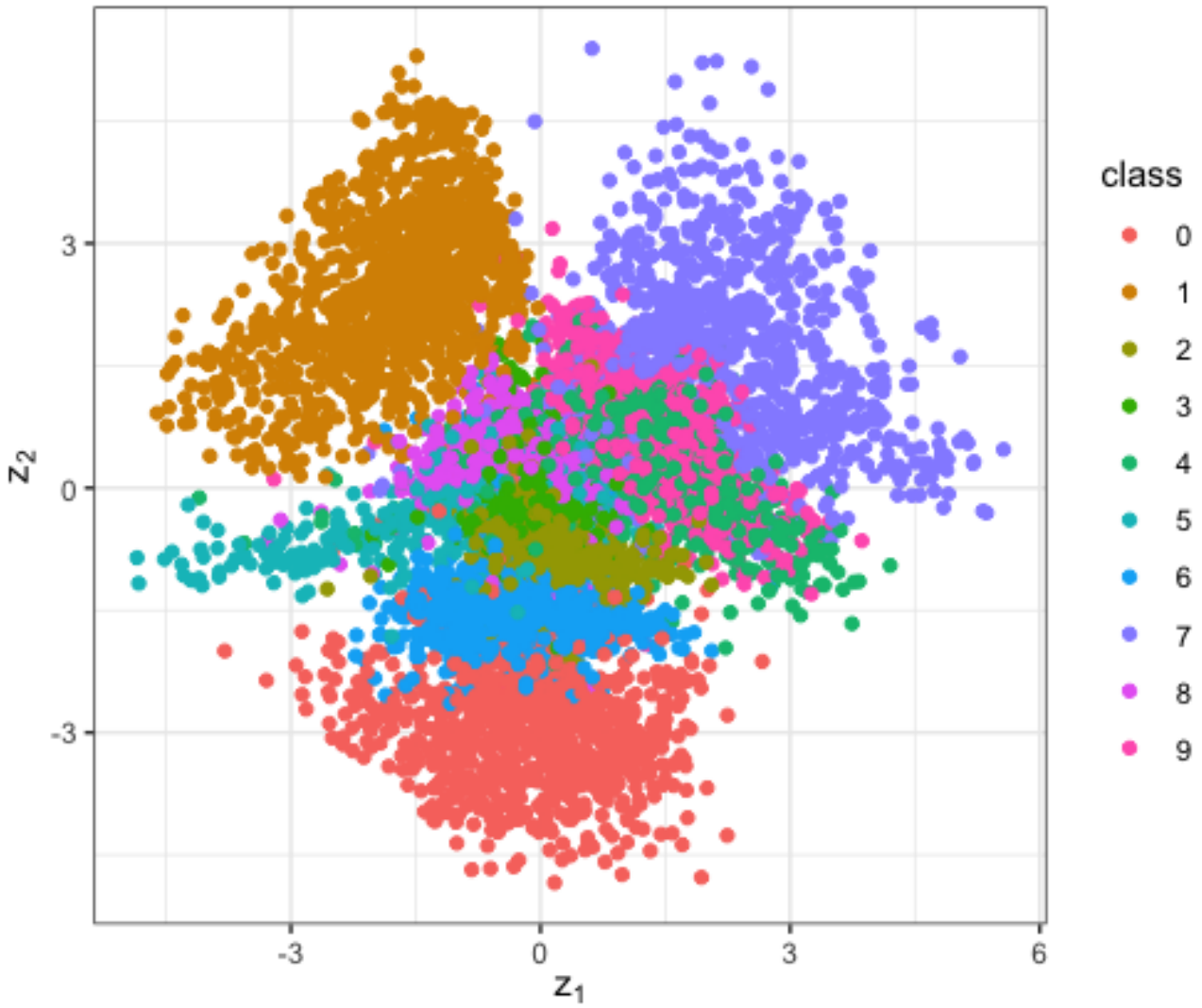


Figure: The latent state of MNIST using an Variational Autoencoder

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Section 3

Autoencoders

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Autoencoder

- An autoencoder is a neural network (e.g. feed-forward) that take an input x and predict (the same) x (r, reconstruction).

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Autoencoder

- An autoencoder is a neural network (e.g. feed-forward) that take an input x and predict (the same) x (r, reconstruction).
- Three parts:
  - encoder $f(x)$ (or $e(x)$)
  - code h
  - decoder $g(h)$ (or $d(z)$)

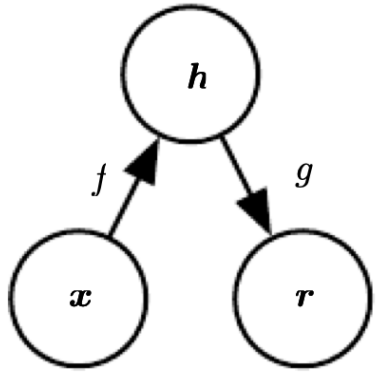


Figure: A Neural Autoencoder (Goodfellow et al, 2018)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Autoencoder

- An autoencoder is a neural network (e.g. feed-forward) that take an input x and predict (the same) x (r, reconstruction).
- Three parts:
  - encoder $f(x)$ (or $e(x)$)
  - code h
  - decoder $g(h)$ (or $d(z)$)

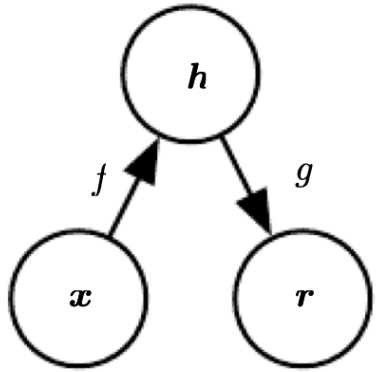


Figure: A Neural Autoencoder (Goodfellow et al, 2018)

- Loss function (reconstruction error):

$$L(\phi, \theta) = (x - d_\phi(e_\theta(x)))^2$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Undercomplete Autoencoder

- More interesting: an undercomplete autoencoder: Dimension of code is lower than that of x

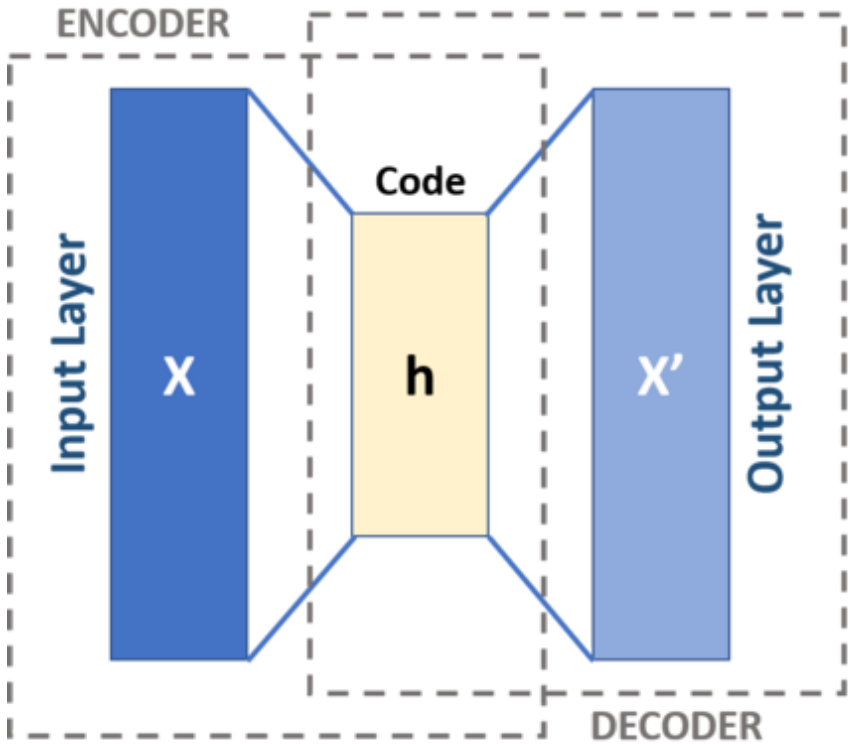


Figure: A Neural Autoencoder (Wikipedia)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

PCA and autoencoders

- A linear autoencoder: $e_\theta(x) = W_\phi$, and $d_\theta(x) = W_\phi$
- We want to minimize the loss (ignoring b/the mean):

$$L(\phi, \theta) = \sum_{i=1}^{N} (x_i - W_\theta W_\phi x_i)^2$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

PCA and autoencoders

- A linear autoencoder: $e_\theta(x) = W_\phi$, and $d_\theta(x) = W_\phi$
- We want to minimize the loss (ignoring b/the mean):

$$L(\phi, \theta) = \sum_{i=1}^{N} (x_i - W_\theta W_\phi x_i)^2$$

- Remember PCA loss:

$$L(P) = \sum_{i=1}^{N} (x_i - P_q P_q^T x_i)^2 \,,$$

where P is an orthogonal matrix of rank q.

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

PCA and autoencoders

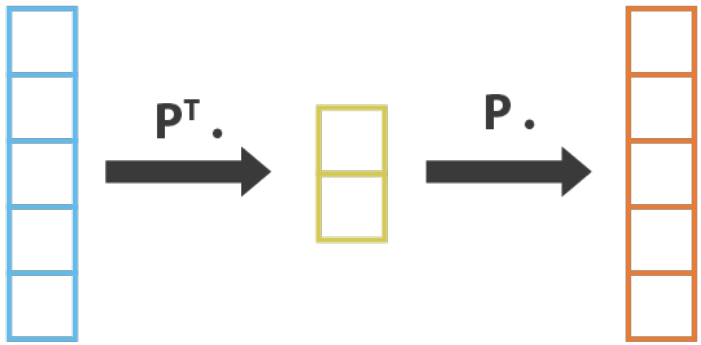
- A linear autoencoder: $e_\theta(x) = W_\phi$, and $d_\theta(x) = W_\phi$
- We want to minimize the loss (ignoring b/the mean):

$$L(\phi, \theta) = \sum_{i=1}^{N} (x_i - W_\theta W_\phi x_i)^2$$

- Remember PCA loss:

$$L(P) = \sum_{i=1}^{N} (x_i - P_q P_q^T x_i)^2 \,,$$

  where P is an orthogonal matrix of rank q.
- Hence: PCA can be seen as an autoencoder



UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Deep Autoencoders

- Deep Autoencoder: An autoencoder with multilayer neural networks as encoder and decoder
  - can be seen as a non-linear PCA
  - learn nonlinear representations

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Deep Autoencoders

- Deep Autoencoder: An autoencoder with multilayer neural networks as encoder and decoder
  - can be seen as a non-linear PCA
  - learn nonlinear representations
- Problem: Deep autoencoders needs to be regularized to not overfit the latent state

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

probabilistic PCA as an decoder

- Problem: Autoencoders (as PCA) are not probabilistic models:
  - cannot generate data.
  - no notion of uncertainty

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

probabilistic PCA as an decoder

- Problem: Autoencoders (as PCA) are not probabilistic models:
  - cannot generate data.
  - no notion of uncertainty
- We would like something like probabilistic PCA for deep autoencoders

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

probabilistic PCA as an decoder

- Remember the pPCA model (with z as latent variable):

$$x_i \sim N(b + W z_i^T, \sigma I)$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

probabilistic PCA as an decoder

- Remember the pPCA model (with z as latent variable):

$$x_i \sim N(b + W z_i^T, \sigma I)$$

- Now, swap the simple parameters with a neural network

$$x_i \sim N(\text{NeuralNetwork}_\phi(z_i), \sigma I)$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

probabilistic PCA as an decoder

- Remember the pPCA model (with z as latent variable):

$$x_i \sim N(b + W z_i^T, \sigma I)$$

- Now, swap the simple parameters with a neural network

$$x_i \sim N(\text{NeuralNetwork}_\phi(z_i), \sigma I)$$

- This is an example of a Deep Latent Variable model (DLVM)
  (a probabilistic decoder)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

probabilistic PCA as an decoder

- Remember the pPCA model (with z as latent variable):

$$x_i \sim N(b + W z_i^T, \sigma I)$$

- Now, swap the simple parameters with a neural network

$$x_i \sim N(\text{NeuralNetwork}_\phi(z_i), \sigma I)$$

- This is an example of a Deep Latent Variable model (DLVM)
  (a probabilistic decoder)
- One example of a DLVM is the Variational Autoencoder

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Section 4

The Variational Autoencoder

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder (VAE)

- The variational autoencoder (VAE) is a deep probabilistic autoencoder
- Used for unsupervised learning of images

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder (VAE)

- The variational autoencoder (VAE) is a deep probabilistic autoencoder
- Used for unsupervised learning of images
- Consists of:
  1. The (probabilistic) encoder $q(z|\phi, x)$: inference model

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder (VAE)

- The variational autoencoder (VAE) is a deep probabilistic autoencoder
- Used for unsupervised learning of images
- Consists of:
  1. The (probabilistic) encoder $q(z|\phi, x)$: inference model
  2. Sample z from encoded x: the latent state

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder (VAE)

- The variational autoencoder (VAE) is a deep probabilistic autoencoder
- Used for unsupervised learning of images
- Consists of:
  1. The (probabilistic) encoder $q(z|\phi, x)$: inference model
  2. Sample z from encoded x: the latent state
  3. The (probabilistic) decoder $p(x|\theta, z)$: observation model

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder (VAE)

- The variational autoencoder (VAE) is a deep probabilistic autoencoder
- Used for unsupervised learning of images
- Consists of:
  1. The (probabilistic) encoder $q(z|\phi, x)$: inference model
  2. Sample z from encoded x: the latent state
  3. The (probabilistic) decoder $p(x|\theta, z)$: observation model
- Encoding the latent state as a distribution forces the space to be "reasonable"/reduces overfitting

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder (VAE)

- The variational autoencoder (VAE) is a deep probabilistic autoencoder
- Used for unsupervised learning of images
- Consists of:
  1. The (probabilistic) encoder $q(z|\phi, x)$: inference model
  2. Sample z from encoded x: the latent state
  3. The (probabilistic) decoder $p(x|\theta, z)$: observation model
- Encoding the latent state as a distribution forces the space to be "reasonable"/reduces overfitting
- VAEs get their name from variational inference (used in training)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder

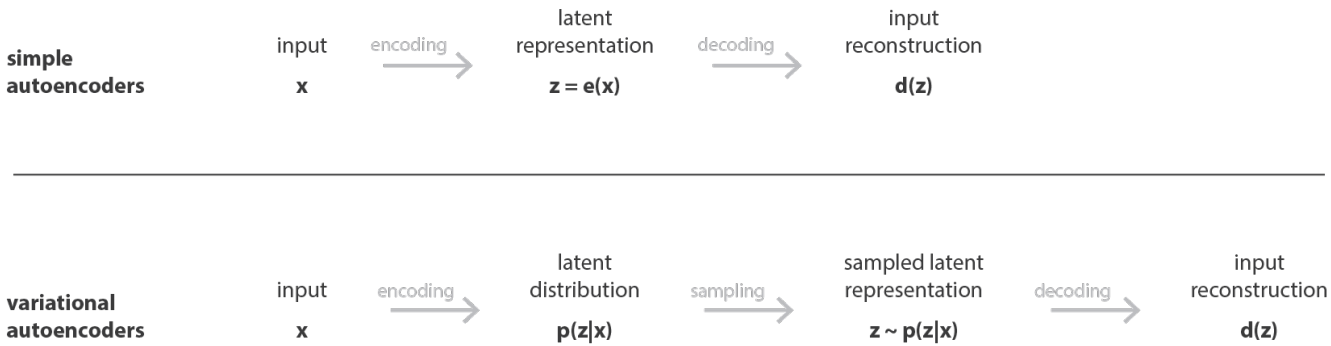


Figure: Autoencoder vs. the Variational Autoencoder (Rocca, 2019)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder

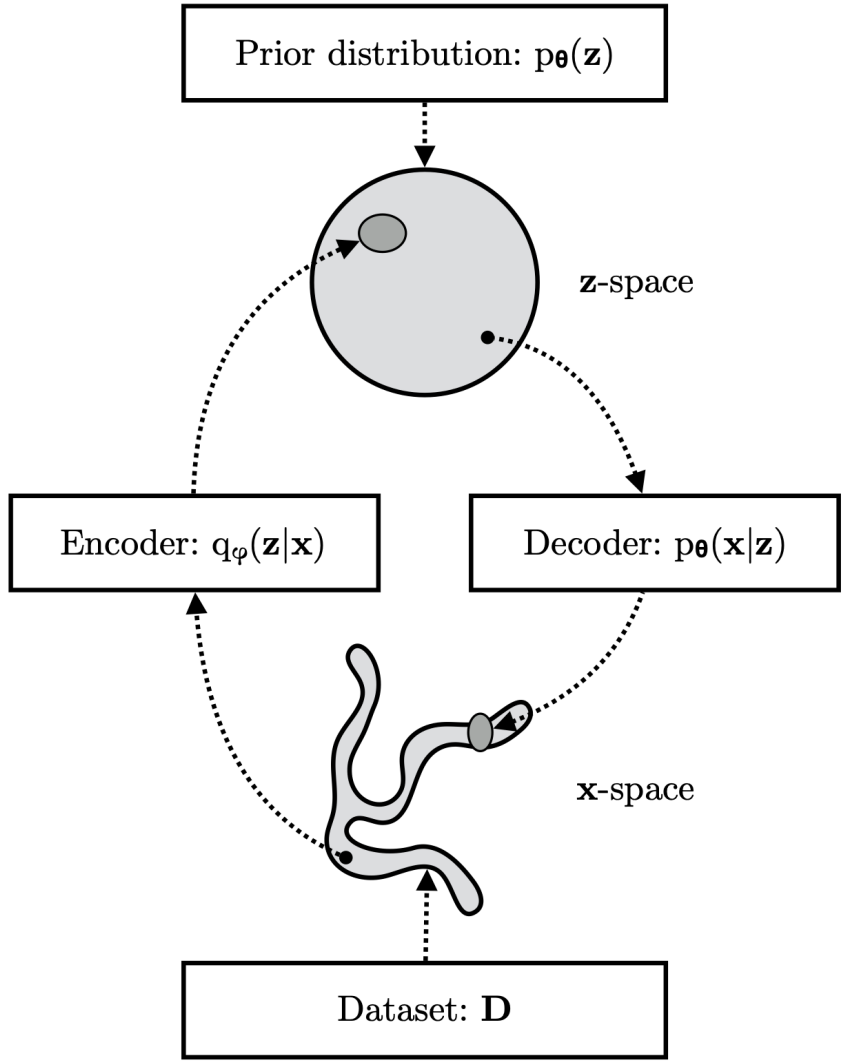


Figure: The Variational Autoencoder (Kingma and Welling, 2018, Fig. 2.1)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The probabilistic decoder

- The probabilistic decoder $p(x|\theta, z)$ (observation model)
- Usually a Normal distribution:

$$x_i \sim N(\text{NeuralNetwork}(z, \theta), cI)$$

- x_i for observation i depends non-linearly on the latent state z_i

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The probabilistic decoder

- The probabilistic decoder $p(x|\theta, z)$ (observation model)
- Usually a Normal distribution:

$$x_i \sim N(\text{NeuralNetwork}(z, \theta), cI)$$

- x_i for observation i depends non-linearly on the latent state z_i
- A probabilistic linear decoder: pPCA

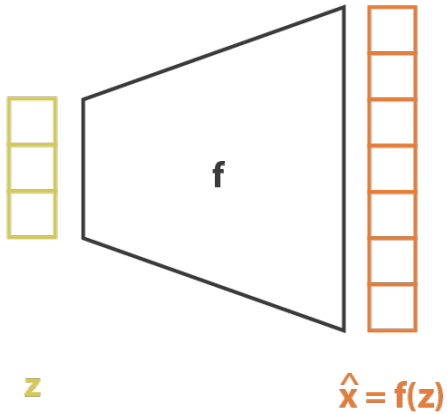


Figure: The Decoder (Rocca, 2019)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The probabilistic encoder

- The probabilistic encoder $q(z|x, \phi)$ (inference model)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The probabilistic encoder

- The probabilistic encoder $q(z|x, \phi)$ (inference model)
- We assume that $q_\phi(z|x)$ follows a specific distribution. Commonly:

$$z \sim N(\mu, \Sigma)$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The probabilistic encoder

- The probabilistic encoder $q(z|x, \phi)$ (inference model)
- We assume that $q_\phi(z|x)$ follows a specific distribution. Commonly:

$$z \sim N(\mu, \Sigma)$$

- A neural network learns the parameters μ and Σ

$$\mu = \text{NeuralNetwork}(x, \phi_\mu)\,,$$

and

$$\Sigma = \text{NeuralNetwork}(x, \phi_\Sigma)\,.$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The probabilistic encoder

- The probabilistic encoder $q(z|x, \phi)$ (inference model)
- We assume that $q_\phi(z|x)$ follows a specific distribution. Commonly:

$$z \sim N(\mu, \Sigma)$$

- A neural network learns the parameters μ and Σ

$$\mu = \text{NeuralNetwork}(x, \phi_\mu)\,,$$

and

$$\Sigma = \text{NeuralNetwork}(x, \phi_\Sigma)\,.$$

- Common assumption: Σ is a diagonal matrix.

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The probabilistic encoder

- The probabilistic encoder $q(z|x, \phi)$ (inference model)
- We assume that $q_\phi(z|x)$ follows a specific distribution. Commonly:

$$z \sim N(\mu, \Sigma)$$

- A neural network learns the parameters μ and Σ

$$\mu = \text{NeuralNetwork}(x, \phi_\mu)\,,$$

and

$$\Sigma = \text{NeuralNetwork}(x, \phi_\Sigma)\,.$$

- Common assumption: Σ is a diagonal matrix.
- Result: z_i for observation i depends non-linearly on x_i

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The probabilistic encoder

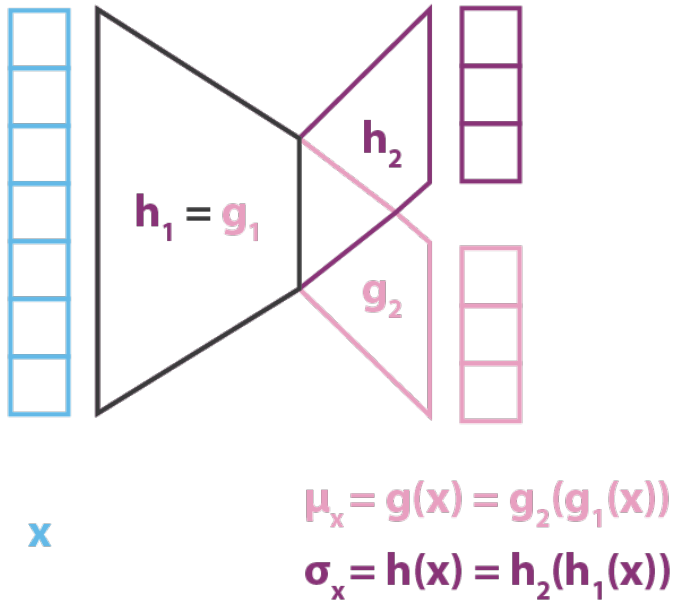


Figure: The Encoder (Rocca, 2019)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder

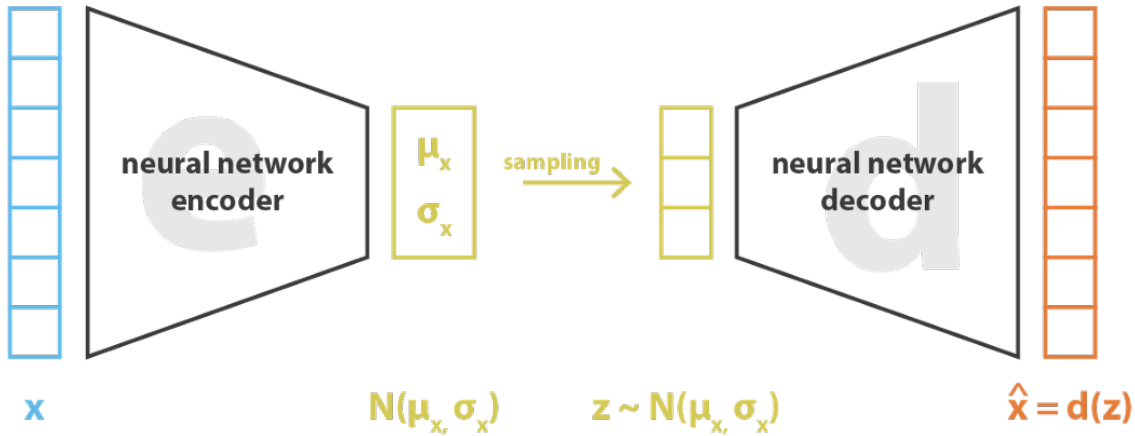


Figure: The Variational Autoencoder (Rocca, 2019)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Variational Autoencoder

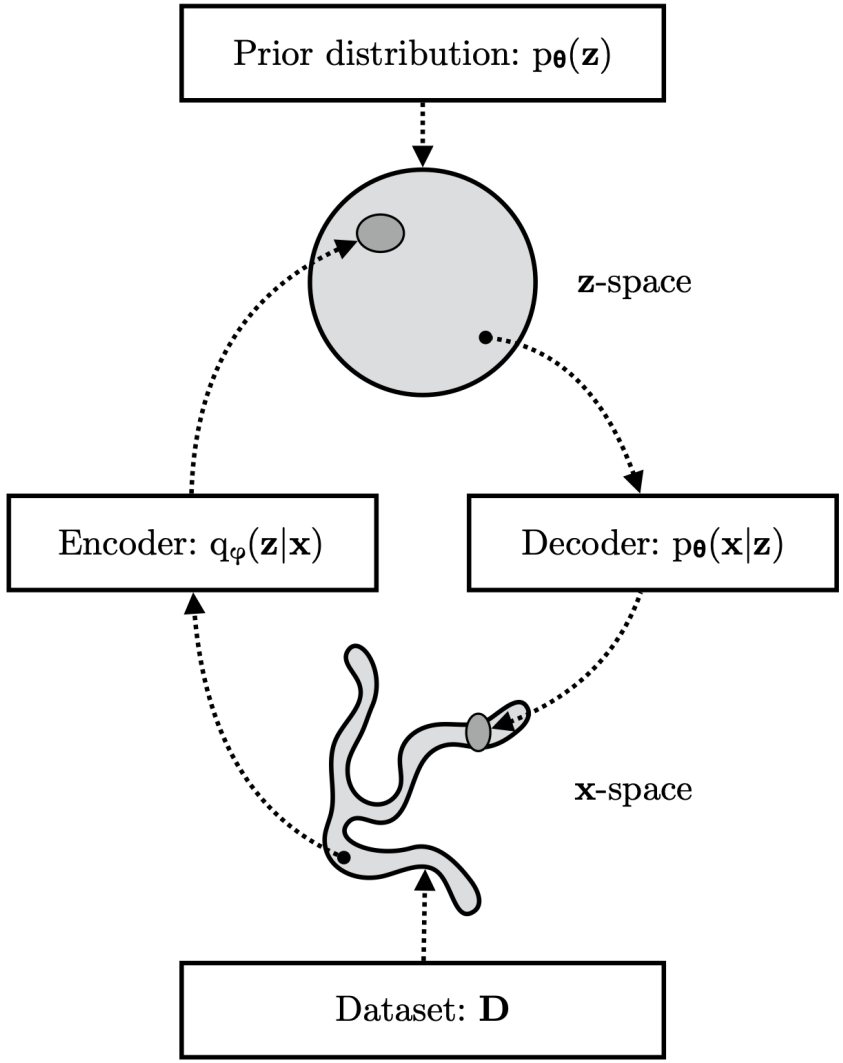


Figure: The Variational Autoencoder (Kingma and Welling, 2018, Fig. 2.1)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Training a VAE

- Goal: estimating ϕ, θ (and z_i)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Training a VAE

- Goal: estimating ϕ, θ (and z_i)
- The encoder and decoder are (usually) complicated (no close form solution)
- Estimate ϕ and θ

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Training a VAE

- Goal: estimating ϕ, θ (and z_i)
- The encoder and decoder are (usually) complicated (no close form solution)
- Estimate ϕ and θ
- Target:
  - Maximize $\log p(x)$
    (Explain the data as well as possible)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Training a VAE

- Goal: estimating ϕ, θ (and z_i)
- The encoder and decoder are (usually) complicated (no close form solution)
- Estimate ϕ and θ
- Target:
  - Maximize $\log p(x)$
    (Explain the data as well as possible)
- Optimization target:
  Maximize the variational lower bound or evidence lower bound (ELBO)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The marginal log-likelihood

$$
\begin{aligned}
\log p_\theta(x) &= \int q_\phi(z|x) \log p_\theta(x) dz \\
&= \mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x)] \\
&= \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{p_\theta(x,z)}{p_\theta(z|x)}\right)\right], \text{using } p(z|x) = \frac{p(x,z)}{p(x)} \\
&= \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{p_\theta(x,z)}{q_\phi(z|x)}\frac{q_\phi(z|x)}{p_\theta(z|x)}\right)\right] \\
&= \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{p_\theta(x,z)}{q_\phi(z|x)}\right)\right] + \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{q_\phi(z|x)}{p_\theta(z|x)}\right)\right] \\
&= \underbrace{\mathcal{L}_{\theta,\phi}(x)}_{\text{ELBO}} + D_{KL}(q_\phi(z|x)||p_\theta(z|x))
\end{aligned}
$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The marginal log-likelihood

$$
\begin{aligned}
\log p_\theta(x) &= \int q_\phi(z|x) \log p_\theta(x) dz \\
&= \mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x)] \\
&= \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{p_\theta(x,z)}{p_\theta(z|x)}\right)\right], \text{using } p(z|x) = \frac{p(x,z)}{p(x)} \\
&= \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{p_\theta(x,z)}{q_\phi(z|x)}\frac{q_\phi(z|x)}{p_\theta(z|x)}\right)\right] \\
&= \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{p_\theta(x,z)}{q_\phi(z|x)}\right)\right] + \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{q_\phi(z|x)}{p_\theta(z|x)}\right)\right] \\
&= \underbrace{\mathcal{L}_{\theta,\phi}(x)}_{\text{ELBO}} + D_{KL}(q_\phi(z|x)||p_\theta(z|x))
\end{aligned}
$$

$$
\underbrace{\mathcal{L}_{\theta,\phi}(x)}_{\text{ELBO}} = \log p_\theta(x) - D_{KL}(q_\phi(z|x)||p_\theta(z|x))
$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Kullback-Leibler divergence

- The Kulback-Leibler divergence: a way of measuring the "distance" between probability distributions

$$D_{KL}(q_\phi(z|x)||p_\theta(z|x)) = \mathbb{E}_{q_\phi(z|x)}\left[\log\left(\frac{q_\phi(z|x)}{p_\theta(z|x)}\right)\right]$$

$$D_{KL}(q_\phi(z|x)||p_\theta(z|x)) \geq 0$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Training target

- Optimization target: Maximize the ELBO

$$\mathcal{L}_{\theta,\phi}(x) = \log p_\theta(x) - D_{KL}(q_\phi(z|x)||p_\theta(z|x))$$

- ELBO is a lower bound for the marginal log-likelihood

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Training target

- Optimization target: Maximize the ELBO

$$\mathcal{L}_{\theta,\phi}(x) = \log p_\theta(x) - D_{KL}(q_\phi(z|x)||p_\theta(z|x))$$

- ELBO is a lower bound for the marginal log-likelihood
- Maximizing the ELBO will do two things:
  - Maximize the marginal log-likelihood $\log p_\theta(x)$: Better generative model/decoder
  - Minimize the KL-divergence between $q_\phi(z|x)$ and $p_\theta(z|x)$: Better approximation of the latent space/encoder

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Optimizing the ELBO

- Stochastic Gradient Ascent to maximize:

$$\mathcal{L}_{\theta,\phi}(x) = \sum_{i}^{N} \mathcal{L}_{\theta,\phi}(x_i)$$

$$= \sum_{i}^{N} \mathbb{E}_{q_\phi(z_i|x_i)} \left[\log\left(p_\theta(x_i, z_i)\right) - \log(q_\phi(z_i|x_i))\right]$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Optimizing the ELBO

- Stochastic Gradient Ascent to maximize:

$$\mathcal{L}_{\theta,\phi}(x) = \sum_{i}^{N} \mathcal{L}_{\theta,\phi}(x_i)$$

$$= \sum_{i}^{N} \mathbb{E}_{q_\phi(z_i|x_i)} \left[\log\left(p_\theta(x_i, z_i)\right) - \log(q_\phi(z_i|x_i))\right]$$

- Two problems:
  1. How do we compute the expectation?
     Solution: Monte Carlo Approximation

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Optimizing the ELBO

- Stochastic Gradient Ascent to maximize:

$$\mathcal{L}_{\theta,\phi}(x) = \sum_{i}^{N} \mathcal{L}_{\theta,\phi}(x_i)$$

$$= \sum_{i}^{N} \mathbb{E}_{q_\phi(z_i|x_i)} \left[\log\left(p_\theta(x_i, z_i)\right) - \log(q_\phi(z_i|x_i))\right]$$

- Two problems:
  1. How do we compute the expectation?
     Solution: Monte Carlo Approximation
  2. How compute the gradient wrt ϕ?
     Solution: Change of variables: $z = g(\epsilon, \phi, x)$
     This is called the reparametrization trick

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Optimizing the ELBO

- Using the reparametrization trick and Monte Carlo approximation, we get:

$$\begin{aligned}\mathcal{L}_{\theta,\phi}(x) =& \mathbb{E}_{q_\phi(z|x)}\left[\log\left(p_\theta(x,z)\right) - \log(q_\phi(z|x))\right] \\ =& \mathbb{E}_{p(\epsilon)}\left[\log\left(p_\theta(x,g(\epsilon,\phi,x))\right) - \log(q_\phi(g(\epsilon,\phi,x)|x))\right] \\ \approx& \log\left(p_\theta(x,g(\epsilon,\phi,x))\right) - \log(q_\phi(g(\epsilon,\phi,x)|x))\end{aligned}$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Optimizing the ELBO

- Using the reparametrization trick and Monte Carlo approximation, we get:

$$\begin{aligned}\mathcal{L}_{\theta,\phi}(x) =& \mathbb{E}_{q_\phi(z|x)}\left[\log\left(p_\theta(x,z)\right) - \log(q_\phi(z|x))\right] \\ =& \mathbb{E}_{p(\epsilon)}\left[\log\left(p_\theta(x, g(\epsilon,\phi,x))\right) - \log(q_\phi(g(\epsilon,\phi,x)|x))\right] \\ \approx& \log\left(p_\theta(x, g(\epsilon,\phi,x))\right) - \log(q_\phi(g(\epsilon,\phi,x)|x))\end{aligned}$$

- A common approach: do the MC approximation with only one sample per datapoint x_i.

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Optimizing the ELBO

- Using the reparametrization trick and Monte Carlo approximation, we get:

$$\begin{aligned}\mathcal{L}_{\theta,\phi}(x) =& \mathbb{E}_{q_\phi(z|x)}\left[\log\left(p_\theta(x,z)\right) - \log(q_\phi(z|x))\right] \\ =& \mathbb{E}_{p(\epsilon)}\left[\log\left(p_\theta(x,g(\epsilon,\phi,x))\right) - \log(q_\phi(g(\epsilon,\phi,x)|x))\right] \\ \approx& \log\left(p_\theta(x,g(\epsilon,\phi,x))\right) - \log(q_\phi(g(\epsilon,\phi,x)|x))\end{aligned}$$

- A common approach: do the MC approximation with only one sample per datapoint x_i.
- We approximate both $\mathcal{L}_{\theta,\phi}(x)$ and $\nabla\mathcal{L}_{\theta,\phi}(x)$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Optimizing the ELBO

- Using the reparametrization trick and Monte Carlo approximation, we get:

$$\begin{aligned}\mathcal{L}_{\theta,\phi}(x) =& \mathbb{E}_{q_\phi(z|x)}\left[\log\left(p_\theta(x,z)\right) - \log(q_\phi(z|x))\right] \\ =& \mathbb{E}_{p(\epsilon)}\left[\log\left(p_\theta(x, g(\epsilon,\phi,x))\right) - \log(q_\phi(g(\epsilon,\phi,x)|x))\right] \\ \approx& \log\left(p_\theta(x, g(\epsilon,\phi,x))\right) - \log(q_\phi(g(\epsilon,\phi,x)|x))\end{aligned}$$

- A common approach: do the MC approximation with only one sample per datapoint x_i.
- We approximate both $\mathcal{L}_{\theta,\phi}(x)$ and $\nabla\mathcal{L}_{\theta,\phi}(x)$
- Sometimes called a doubly stochastic algorithm.

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Autoencoding Variational Bayes Algorithm

Algorithm 1: Stochastic optimization of the ELBO. Since noise originates from both the minibatch sampling and sampling of $p(\boldsymbol{\epsilon})$, this is a doubly stochastic optimization procedure. We also refer to this procedure as the *Auto-Encoding Variational Bayes* (AEVB) algorithm.

Data:

$\mathcal{D}$: Dataset

$q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{x})$: Inference model

$p_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{z})$: Generative model

Result:

$\boldsymbol{\theta}, \boldsymbol{\phi}$: Learned parameters

$(\boldsymbol{\theta}, \boldsymbol{\phi}) \leftarrow$ Initialize parameters

while *SGD not converged* **do**

  $\mathcal{M} \sim \mathcal{D}$ (Random minibatch of data)

  $\boldsymbol{\epsilon} \sim p(\boldsymbol{\epsilon})$ (Random noise for every datapoint in $\mathcal{M}$)

  Compute $\tilde{\mathcal{L}}_{\boldsymbol{\theta},\boldsymbol{\phi}}(\mathcal{M}, \boldsymbol{\epsilon})$ and its gradients $\nabla_{\boldsymbol{\theta},\boldsymbol{\phi}} \tilde{\mathcal{L}}_{\boldsymbol{\theta},\boldsymbol{\phi}}(\mathcal{M}, \boldsymbol{\epsilon})$

  Update $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ using SGD optimizer

end

Figure: The Autoencoding Variational Bayes Algorithm (Kingma and Welling, 2018, Algo. 1)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Autoencoding Variational Bayes Algorithm

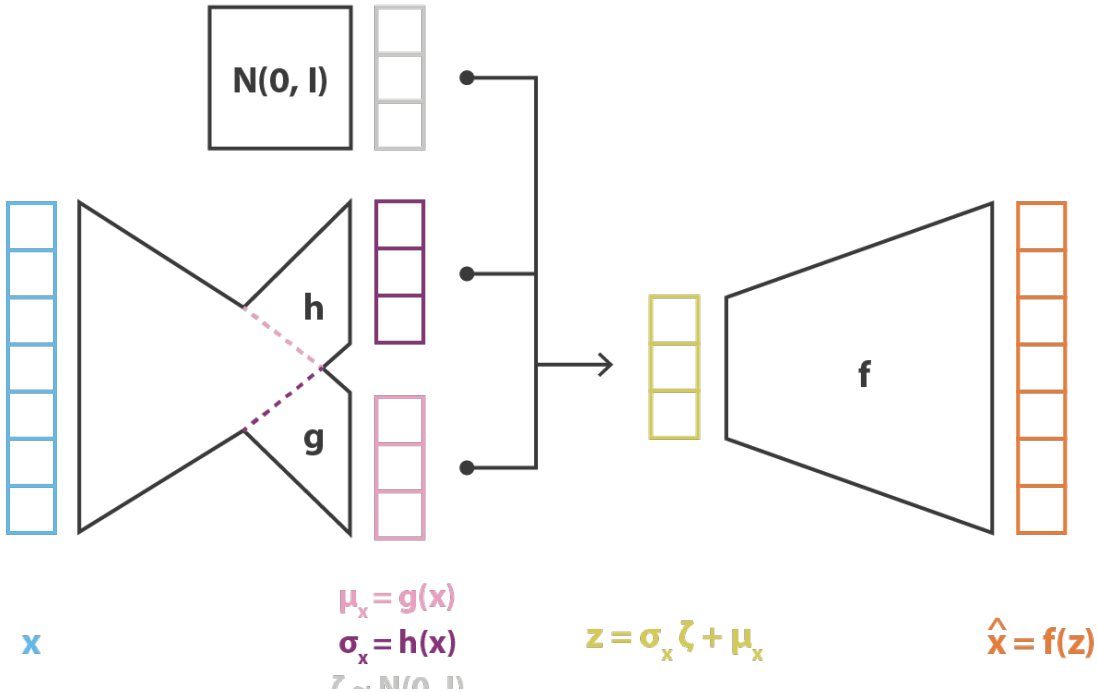


Figure: The Autoencoding Variational Bayes Algorithm (Rocca, 2019)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Summary

- Benefits of VAE:
  - Get a more interpretable latent state
  - We can estimate uncertainty
  - Regularize the latent state
  - We can inject knowledge in our latent state

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Summary

- Benefits of VAE:
  - Get a more interpretable latent state
  - We can estimate uncertainty
  - Regularize the latent state
  - We can inject knowledge in our latent state
- Still much ongoing research
- Diffusion models is getting increased attention as a rival method

Figure: Examples of images generated with a deep hierarchical Variational Autoencoder (Vahdat and Kautz, 2020)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Section 5

Probabilistic Topic Models

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Probabilistic Topic Models

- Unsupervised method for textual data

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Probabilistic Topic Models

- Unsupervised method for textual data
- Popular in industry and academia to analyze large corpora

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Probabilistic Topic Models

- Unsupervised method for textual data
- Popular in industry and academia to analyze large corpora
- The most common model: Latent Dirichlet Allocation
- A mixed membership model (a mixture of multinomial mixtures model)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Probabilistic Topic Models

- Unsupervised method for textual data
- Popular in industry and academia to analyze large corpora
- The most common model: Latent Dirichlet Allocation
- A mixed membership model (a mixture of multinomial mixtures model)
- Topic model builds on the the distributional hypothesis

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Probabilistic Topic Models

- Unsupervised method for textual data
- Popular in industry and academia to analyze large corpora
- The most common model: Latent Dirichlet Allocation
- A mixed membership model (a mixture of multinomial mixtures model)
- Topic model builds on the the distributional hypothesis
- Use cases:
  - Visualize document collections

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Probabilistic Topic Models

- Unsupervised method for textual data
- Popular in industry and academia to analyze large corpora
- The most common model: Latent Dirichlet Allocation
- A mixed membership model (a mixture of multinomial mixtures model)
- Topic model builds on the the distributional hypothesis
- Use cases:
  - Visualize document collections
  - Analyzing large corpora using statistical methods

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Probabilistic Topic Models

- Unsupervised method for textual data
- Popular in industry and academia to analyze large corpora
- The most common model: Latent Dirichlet Allocation
- A mixed membership model (a mixture of multinomial mixtures model)
- Topic model builds on the the distributional hypothesis
- Use cases:
  - Visualize document collections
  - Analyzing large corpora using statistical methods
- Example: All ears media monitoring of speech data

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Dirichlet Distribution

- Probability distribution over the simplex with K categories:

$$f(\mathbf{x}|\boldsymbol{\alpha}) = \frac{1}{\mathrm{B}(\boldsymbol{\alpha})} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

where

$$\mathrm{B}(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^{K} \Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^{K} \alpha_i\right)},$$

and where

$$\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_K)$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Dirichlet Distribution

- Probability distribution over the simplex with K categories:

$$f(\mathbf{x}|\boldsymbol{\alpha}) = \frac{1}{\mathrm{B}(\boldsymbol{\alpha})} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

where

$$\mathrm{B}(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^{K} \Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^{K} \alpha_i\right)},$$

and where

$$\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_K)$$

- The probability distribution has the support on the simplex, that is

$$\sum_{i=1}^{K} x_i = 1 \text{ and } x_i \geq 0 \text{ for all } i \in [1, K]$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Dirichlet Distribution

- Probability distribution over the simplex with K categories:

$$f(\mathbf{x}|\boldsymbol{\alpha}) = \frac{1}{\mathrm{B}(\boldsymbol{\alpha})} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

where

$$\mathrm{B}(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^{K} \Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^{K} \alpha_i\right)},$$

and where

$$\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_K)$$

- The probability distribution has the support on the simplex, that is

$$\sum_{i=1}^{K} x_i = 1 \text{ and } x_i \geq 0 \text{ for all } i \in [1, K]$$

- The parameters $\boldsymbol{\alpha}$ can be seen as pseudo-counts

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Dirichlet Distribution

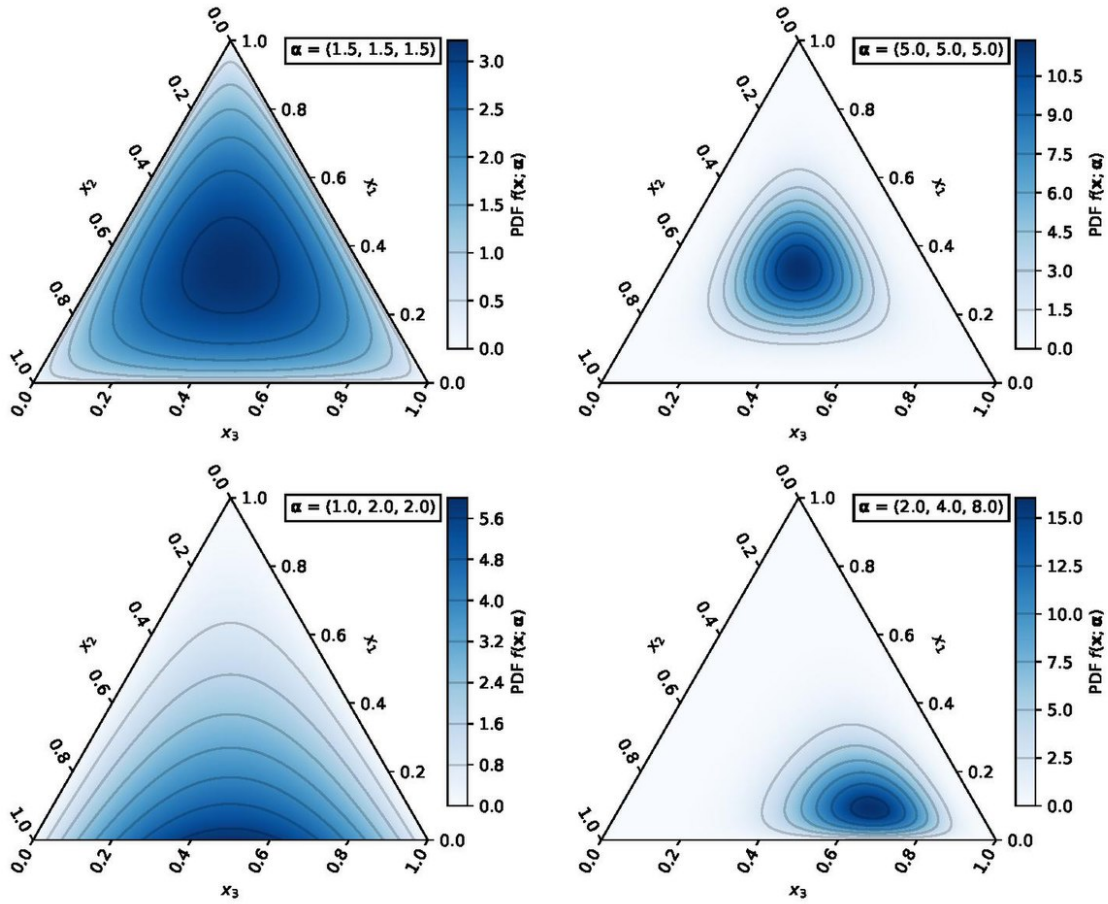


Figure: The Dirichlet Distribution (Wikipedia)

Interactive plot here.

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The distributional hypothesis

- Harris (1954) and Firths (1957):
  "Word is characterized by the company it keeps"

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The distributional hypothesis

- Harris (1954) and Firths (1957):
  "Word is characterized by the company it keeps"
- Semantics (broadly defined) is captured by context

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The distributional hypothesis

- Harris (1954) and Firths (1957):
  "Word is characterized by the company it keeps"
- Semantics (broadly defined) is captured by context
- Rough definition: word windows of different sizes

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The distributional hypothesis

- Harris (1954) and Firths (1957):
  "Word is characterized by the company it keeps"
- Semantics (broadly defined) is captured by context
- Rough definition: word windows of different sizes
- Different window sizes, different semantic content:
  - Word embeddings (context: word windows)
  - Topic models (context: documents)

Example

1. "A friend in need is a friend indeed."
2. "She is my friend indeed."

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Latent Dirichlet Allocation

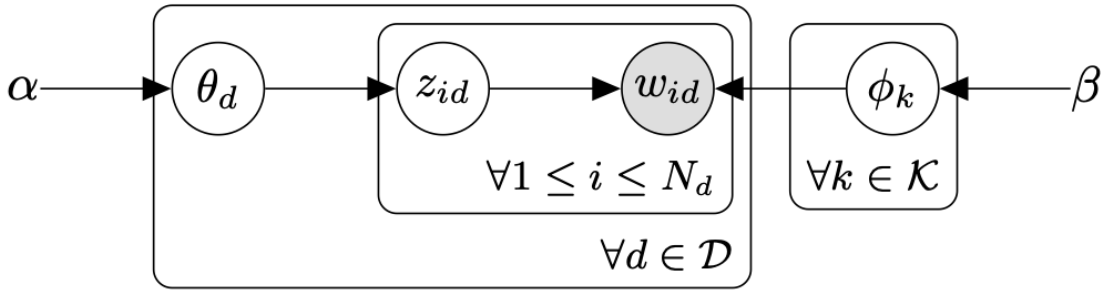


Figure: The Latent Dirichlet Allocation Model

where ϕ_k is the kth row in Φ (of dimension $K \times V$) and θ_d is the dth row in Θ (of dimension $D \times K$).

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Generative model for LDA

Relies on the bag-of-word assumption

1. For each component k to K:
   - 1.1 $\phi_k \sim \text{Dirichlet}(\beta)$
2. For each document d:
   - 2.1 $\theta_d \sim \text{Dirichlet}(\alpha)$
   - 2.2 For each token i:
     - 2.2.1 $z_{id} \sim \text{Categorical}(\theta_d)$
     - 2.2.2 $w_{id} \sim \text{Categorical}(\phi_{z_{id}})$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Example of parameters $\mathbf{z}$, Θ and Φ

$\mathbf{w}_1$	boat	shore	bank		
$\mathbf{z}_1$	1	1	1		
$\mathbf{w}_2$	Zlatan	boat	shore	money	bank
$\mathbf{z}_2$	2	1	1	3	3
$\mathbf{w}_3$	money	bank	soccer	money	
$\mathbf{z}_3$	3	3	2	3	

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Example of parameters $\mathbf{z}$, Θ and Φ

$\mathbf{w}_1$	boat	shore	bank			
$\mathbf{z}_1$	1	1	1			
$\mathbf{w}_2$	Zlatan	boat	shore	money	bank	
$\mathbf{z}_2$	2	1	1	3	3	
$\mathbf{w}_3$	money	bank	soccer	money		
$\mathbf{z}_3$	3	3	2	3		

$\Phi =$

	boat	shore	soccer	Zlatan	bank	money
Topic 1	0.35	0.35	0.05	0.05	0.15	0.05
Topic 2	0.025	0.025	0.45	0.45	0.025	0.025
Topic 3	0.025	0.025	0.025	0.025	0.45	0.45

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Example of parameters $\mathbf{z}$, Θ and Φ

$\mathbf{w}_1$	boat	shore	bank			
$\mathbf{z}_1$	1	1	1			
$\mathbf{w}_2$	Zlatan	boat	shore	money	bank	
$\mathbf{z}_2$	2	1	1	3	3	
$\mathbf{w}_3$	money	bank	soccer	money		
$\mathbf{z}_3$	3	3	2	3		

$\Phi =$

	boat	shore	soccer	Zlatan	bank	money
Topic 1	0.35	0.35	0.05	0.05	0.15	0.05
Topic 2	0.025	0.025	0.45	0.45	0.025	0.025
Topic 3	0.025	0.025	0.025	0.025	0.45	0.45

$\Theta =$

	Topic 1	Topic 2	Topic 3
doc 1	0.96	0.02	0.02
doc 2	0.3	0.2	0.5
doc 3	0.05	0.35	0.6

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Closing arguments were heard yesterday in the Federal bankruptcy fraud trial of Stephen J. Sabbeth, whose legal problems have raised doubts about his ability to continue as leader of the Nassau County Democratic Party.

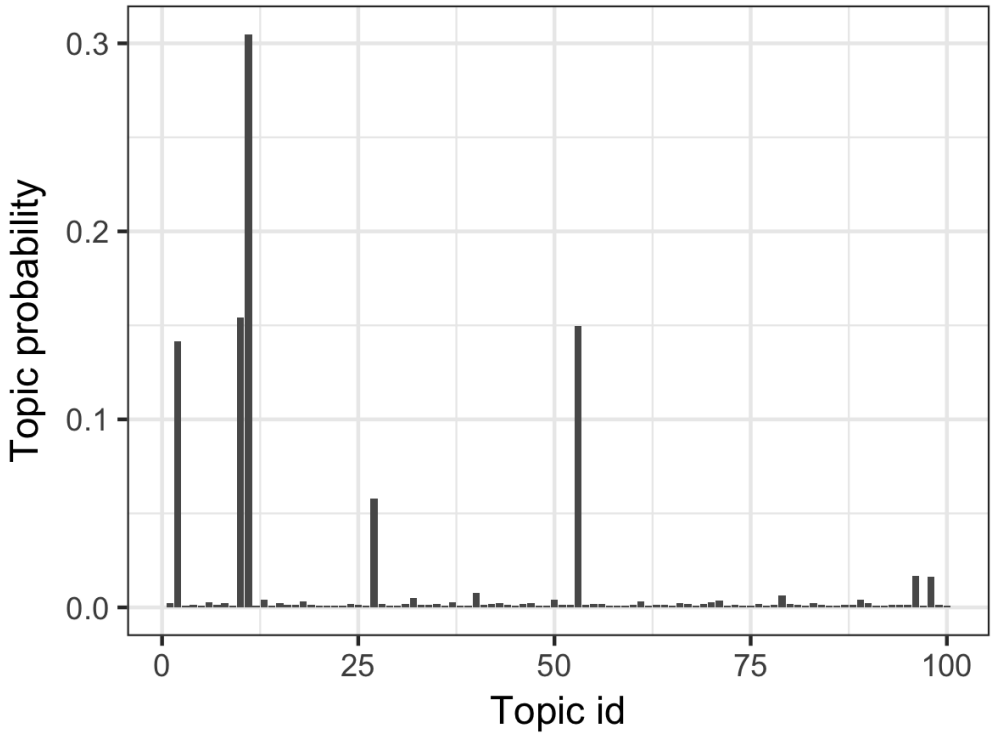
Mr. Sabbeth is charged with trying to conceal $750,000 from his bank creditors by hiding the money in a secret account in his wife's maiden name, rather than use it to pay creditors when his lumber business went into bankruptcy 10 years ago.

– The New York Times 25th of Febuary 1999

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The estimated topic proportion ($\hat{\theta}_d$)



UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Topic top words

Topic	Top words (by ϕ_{kv})
2	party election voters campaign democratic
10	bank banks loans loan insurance savings
11	trial prison jury prosecutors convicted guilty
53	investigation inquiry documents investigators

Table: The words with highest probability ($p(w|k)$) for topic 2, 10, 11 and 53.

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

The Latent Dirichlet Allocation Model

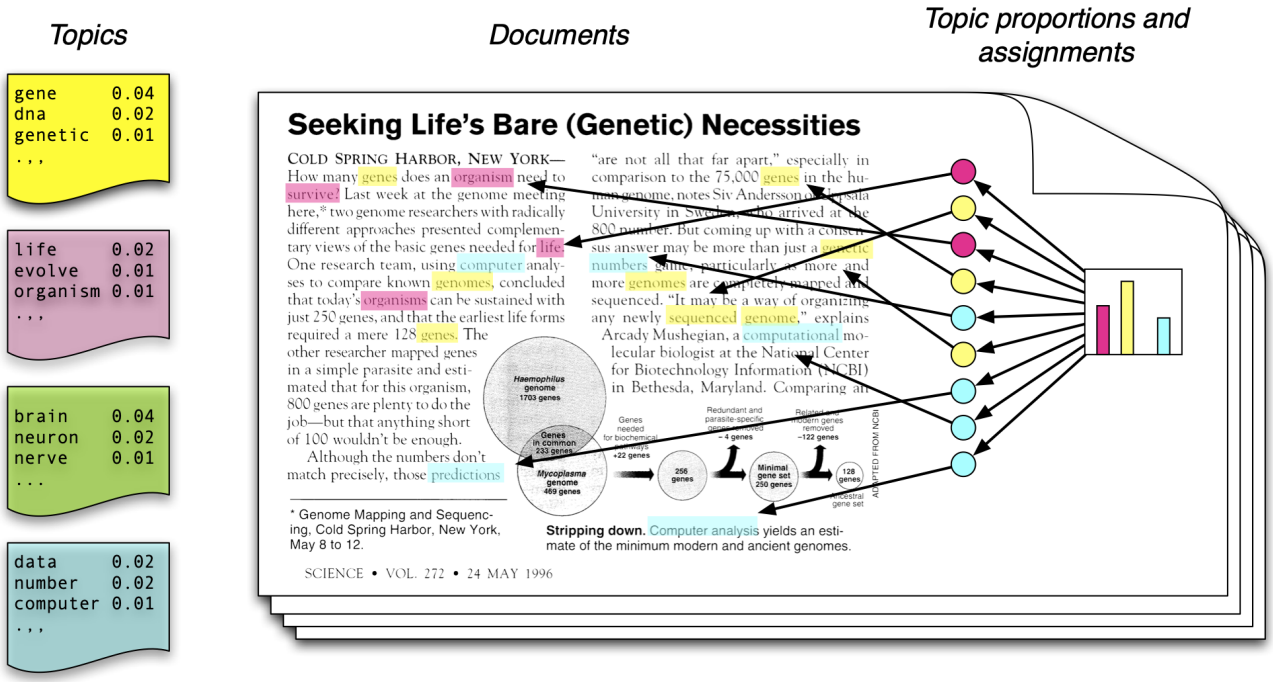


Figure: The Latent Dirichlet Allocation Model (Blei 2012, Fig. 1)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Inference

- Common inference approaches
  1. Variational inference
  2. Markov Chain Monte Carlo (MCMC)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Inference

- Common inference approaches
  1. Variational inference
  2. Markov Chain Monte Carlo (MCMC)
- The Gibbs sampler is usually prefered

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Inference

- Common inference approaches
  1. Variational inference
  2. Markov Chain Monte Carlo (MCMC)
- The Gibbs sampler is usually prefered
- Similar to (Stochastic) EM

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Gibbs sampler for LDA

The basic Gibbs sampler:

1. We want to estimate z, Φ, Θ:

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Gibbs sampler for LDA

The basic Gibbs sampler:

1. We want to estimate z, Φ, Θ:
2. Sample topic indicators (latent variable)

$$p(z = k|\Phi, \Theta) \propto \phi_{v,k}\theta_{k,d}$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Gibbs sampler for LDA

The basic Gibbs sampler:

1. We want to estimate z, Φ, Θ:
2. Sample topic indicators (latent variable)

$$p(z = k|\Phi, \Theta) \propto \phi_{v,k}\theta_{k,d}$$

3. Sample model parameters

$$\theta_d|\mathbf{z} \sim Dir(\mathbf{n}^{(d)} + \alpha)$$

$$\phi_k|\mathbf{z} \sim Dir(\mathbf{n}^{(v)} + \beta)$$

where $\mathbf{n}^{(d)}$ is the number of tokens by topic in document d and $\mathbf{n}^{(v)}$ is the number of tokens by topic for word type v.

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Gibbs sampler for LDA

Integrating out (collapsing) Θ and Φ

$$p(\mathbf{z}|\mathbf{w}) = \int\int p(\mathbf{z}, \Theta, \Phi|\mathbf{w}) \cdot p(\mathbf{z}, \Theta, \Phi) d\Phi d\Theta$$

will result in the following Gibbs sampler:

$$p(z_i = k|w_i, \mathbf{z}_{\neg i}) \propto \underbrace{\frac{n_k^{(v)} + \beta}{n_k^{(v)} + V\beta}}_{type-topic\ (\Phi)} \cdot \underbrace{(n_k^{(d)} + \alpha)}_{topic-doc\ (\Theta)}$$

where $n^{(v)}$ and $n^{(d)}$ are count matrices of size $D \times K$ and $K \times V$.

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Example of $n^{(v)}$ and $n^{(d)}$

$\mathbf{w}_1$	boat	shore	bank		
$\mathbf{z}_1$	1	1	1		
$\mathbf{w}_2$	Zlatan	boat	shore	money	bank
$\mathbf{z}_2$	2	1	1	3	3
$\mathbf{w}_3$	money	bank	soccer	money	
$\mathbf{z}_3$	3	3	2	3	

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Example of $n^{(v)}$ and $n^{(d)}$

$\mathbf{w}_1$	boat	shore	bank		
$\mathbf{z}_1$	1	1	1		
$\mathbf{w}_2$	Zlatan	boat	shore	money	bank
$\mathbf{z}_2$	2	1	1	3	3
$\mathbf{w}_3$	money	bank	soccer	money	
$\mathbf{z}_3$	3	3	2	3	

$$n^{(v)} = \begin{array}{cccccc} \text{boat} & \text{shore} & \text{soccer} & \text{Zlatan} & \text{bank} & \text{money} \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 \end{array}$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Example of $n^{(v)}$ and $n^{(d)}$

$\mathbf{w}_1$	boat	shore	bank		
$\mathbf{z}_1$	1	1	1		
$\mathbf{w}_2$	Zlatan	boat	shore	money	bank
$\mathbf{z}_2$	2	1	1	3	3
$\mathbf{w}_3$	money	bank	soccer	money	
$\mathbf{z}_3$	3	3	2	3	

$$n^{(v)} = \begin{array}{cccccc} \text{boat} & \text{shore} & \text{soccer} & \text{Zlatan} & \text{bank} & \text{money} \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 \end{array}$$

$$n^{(d)} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 2 & 3 \end{bmatrix}$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Topic Models as non-negative matrix factorization

$$\begin{bmatrix} n_{dv} \\ (D \times V) \end{bmatrix} \approx \begin{bmatrix} \Theta \\ (D \times K) \end{bmatrix} \times \begin{bmatrix} \Phi \\ (K \times V \end{bmatrix}$$

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Practicalities

- Setting K, α and β

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Practicalities

- Setting K, α and β
- Reducing the vocabulary: stopwords, rare words, stemming

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Practicalities

- Setting K, α and β
- Reducing the vocabulary: stopwords, rare words, stemming
- "Junk" topics

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Practicalities

- Setting K, α and β
- Reducing the vocabulary: stopwords, rare words, stemming
- "Junk" topics
- We can analyze the topic indicators z directly

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Research Example: Swedish Immigration Discourse

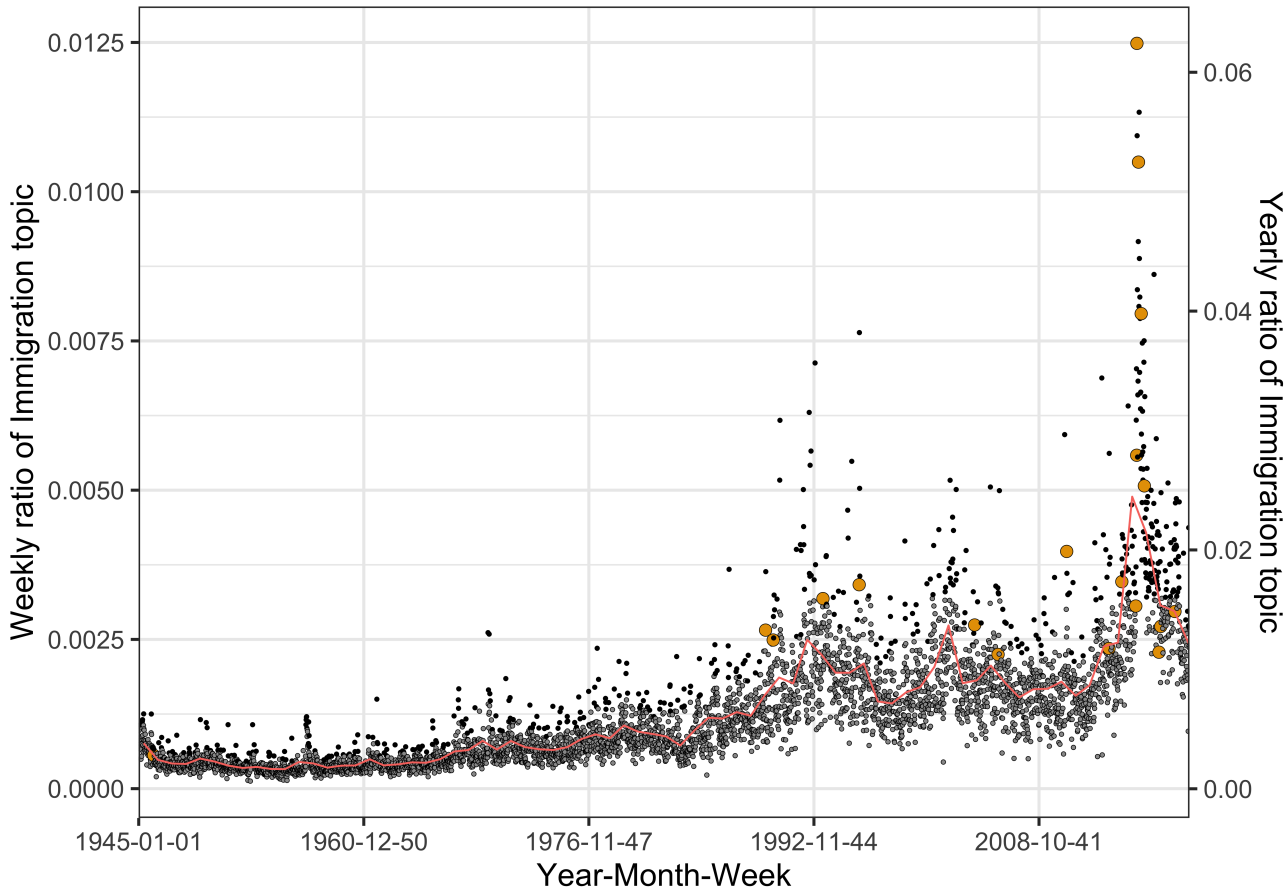


Figure: The Immigration topic in Swedish Newspapers (Hurtado Bodell et al, 2024)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Summary: Topic Models

- Topic models are unsupervised models for textual data

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Summary: Topic Models

- Topic models are unsupervised models for textual data
- The Latent Dirichlet Allocation is a popular model

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Summary: Topic Models

- Topic models are unsupervised models for textual data
- The Latent Dirichlet Allocation is a popular model
- A mixed membership model (a mixture of multinomial mixtures model)

UPPSALA UNIVERSITET

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
  - The probabilistic decoder
  - The encoder
  - Traing a variational autoencoder
- Probabilistic Topic Models
  - Latent Dirichlet Allocation
  - Estimating the LDA model

Summary: Topic Models

- Topic models are unsupervised models for textual data
- The Latent Dirichlet Allocation is a popular model
- A mixed membership model (a mixture of multinomial mixtures model)
- Use Gibbs samplers for estimation