UPPSALA
UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

Machine learning – Block 8

Måns Magnusson
Department of Statistics, Uppsala University

Autumn 2023

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

This week's lectures

- Introduction to Reinforcement learning

Introduction to Reinforcement Learning

- Another type of Machine Learning:
  - Supervised Learning
  - Unsupervised Learning
  - Reinforcement learning

Introduction to Reinforcement Learning

- Another type of Machine Learning:
  - Supervised Learning
  - Unsupervised Learning
  - Reinforcement learning
- Computational approach of learning from interaction

Introduction to Reinforcement Learning

- Another type of Machine Learning:
  - Supervised Learning
  - Unsupervised Learning
  - Reinforcement learning
- Computational approach of learning from interaction
- Closest to human and animal learning: trial, error, and planning.

Introduction to Reinforcement Learning

- Another type of Machine Learning:
  - Supervised Learning
  - Unsupervised Learning
  - Reinforcement learning
- Computational approach of learning from interaction
- Closest to human and animal learning: trial, error, and planning.
- The learner is *not* told which actions to take

Introduction to Reinforcement Learning

- Another type of Machine Learning:
  - Supervised Learning
  - Unsupervised Learning
  - Reinforcement learning
- Computational approach of learning from interaction
- Closest to human and animal learning: trial, error, and planning.
- The learner is *not* told which actions to take
- Connections to:
  - Game Theory
  - Control Theory
  - Multi-agent systems
  - Swarm intelligence
  - Information theory
  - Statistics

Introduction to Reinforcement Learning

- Goal: maximize return over a sequence of actions

Introduction to Reinforcement Learning

- Goal: maximize return over a sequence of actions
- Three characteristics:
  1. Closed-loop: early actions affects later actions

Introduction to Reinforcement Learning

- Goal: maximize return over a sequence of actions
- Three characteristics:
    1. Closed-loop: early actions affects later actions
    2. No instructions

Introduction to Reinforcement Learning

- Goal: maximize return over a sequence of actions
- Three characteristics:
    1. Closed-loop: early actions affects later actions
    2. No instructions
    3. Reward signals over a long period of time

Recent Achievements

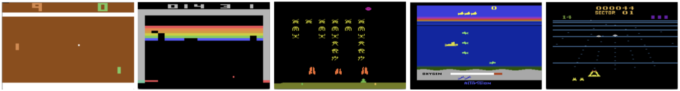


Figure: Mnih et al (2013) "Playing Atari with Deep Reinforcement Learning"

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

UPPSALA UNIVERSITET

Recent Achievements

Figure: Lee Sedol vs. Alpha Go in 2016

Recent Achievements

Figure: ChatGPT in 2022

Recent Achievements, also

- Industry automation: RL is used to reduce the energy cost of datacenter cooling

Recent Achievements, also

- Industry automation: RL is used to reduce the energy cost of datacenter cooling
- Automated trading

Recent Achievements, also

- Industry automation: RL is used to reduce the energy cost of datacenter cooling
- Automated trading
- Elevator scheduling

Recent Achievements, also

- Industry automation: RL is used to reduce the energy cost of datacenter cooling
- Automated trading
- Elevator scheduling
- A/B testing and personalized recommendations

The different parts in RL

1. The Agent: The learning agent.

The different parts in RL

1. The Agent: The learning agent.
2. The Environment: Where the agent performs actions.

The different parts in RL

1. The Agent: The learning agent.
2. The Environment: Where the agent performs actions.
3. Actions: Made by the agent and affects the environment.

The different parts in RL

1. The Agent: The learning agent.
2. The Environment: Where the agent performs actions.
3. Actions: Made by the agent and affects the environment.
4. Reward: The evaluation of an action. A scalar value. Pleasure and pain.

The different parts in RL

1. The Agent: The learning agent.
2. The Environment: Where the agent performs actions.
3. Actions: Made by the agent and affects the environment.
4. Reward: The evaluation of an action. A scalar value. Pleasure and pain.
5. Return: The aggregated reward over a long period.

The different parts in RL

1. Agents:
   1.1 Have a goal (maximize return)
   1.2 Sense aspect of their environment
   1.3 Choose actions
   1.4 Possibility to improve performance over time

The different parts in RL

1. Agents:
   1.1 Have a goal (maximize return)
   1.2 Sense aspect of their environment
   1.3 Choose actions
   1.4 Possibility to improve performance over time
2. Usually an uncertainty about the environment
3. Represent uncertainty of environment: Probability

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

Sub-elements of agents

1. Policy: How the agent choose actions. Determines behaviour.
2. Model: The agent's model of the environment. Used for planning

Sub-elements of agents

1. Policy: How the agent choose actions. Determines behaviour.
2. Model: The agent's model of the environment. Used for planning
3. Value function: The long-term value (the expected long-term return following a policy)

Sub-elements of agents

1. Policy: How the agent choose actions. Determines behaviour.
2. Model: The agent's model of the environment. Used for planning
3. Value function: The long-term value (the expected long-term return following a policy)

- Outside agent: Reward signal: The instant value of an action

Sub-elements of agents

1. Policy: How the agent choose actions. Determines behaviour.
2. Model: The agent's model of the environment. Used for planning
3. Value function: The long-term value (the expected long-term return following a policy)

- Outside agent: Reward signal: The instant value of an action
- Problem: Balance the trade-off between long-term and short-term rewards

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

Supervised, Unsupervised and Reinforcement Learning

1. Static vs. Dynamic

UPPSALA
UNIVERSITET

Supervised, Unsupervised and Reinforcement Learning

1. Static vs. Dynamic
2. No Gold Standard

Supervised, Unsupervised and Reinforcement Learning

1. Static vs. Dynamic
2. No Gold Standard
3. Multiple-Decision Process: Return vs. reward

Supervised, Unsupervised and Reinforcement Learning

1. Static vs. Dynamic
2. No Gold Standard
3. Multiple-Decision Process: Return vs. reward
4. Need for exploration

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

Supervised, Unsupervised and Reinforcement Learning

1. Static vs. Dynamic
2. No Gold Standard
3. Multiple-Decision Process: Return vs. reward
4. Need for exploration
5. Evaluates actions - not only instruct actions

Exploration vs Exploitation

- Goal: Maximize the return (the total reward), i.e.

Exploration vs Exploitation

- Goal: Maximize the return (the total reward), i.e.
- Exploit the best actions

Exploration vs Exploitation

- Goal: Maximize the return (the total reward), i.e.
- Exploit the best actions
- Explore to know the best actions

Evolution vs Learning

- Set a policy without learning: Evolutionary Methods
- Good when agent cannot sense the environment

Evolution vs Learning

- Set a policy without learning: Evolutionary Methods
- Good when agent cannot sense the environment
- Example: Bacteria don't learn, they evolve

Setting the goal for the Agent

- Setting the goal: defining the reward signal (reward function)
- Example: If you want the agent to do something quick, give -1 per action.

Setting the goal for the Agent

- Setting the goal: defining the reward signal (reward function)
- Example: If you want the agent to do something quick, give -1 per action.
- We should give rewards for correct behaviour
- Do not use reward to guide how to reach the goal
- Be careful what you wish for...

Section 2

Bandits

The k-armed bandit problem

- Goal: Maximize the total or average reward after N actions

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

The k-armed bandit problem

- Goal: Maximize the total or average reward after N actions
- The actions: Choose between k arms, i.e. $A_t \in \{1, ..., k\}$

The k-armed bandit problem

- Goal: Maximize the total or average reward after N actions
- The actions: Choose between k arms, i.e. $A_t \in \{1, ..., k\}$
- The reward signal:

$$R_t \sim p(R_t|a)\,,$$

where $\mathbb{E}(R_t|A_t = a) = q^{\star}(a)$.

The k-armed bandit problem

- Goal: Maximize the total or average reward after N actions
- The actions: Choose between k arms, i.e. $A_t \in \{1, ..., k\}$
- The reward signal:

$$R_t \sim p(R_t|a),$$

  where $\mathbb{E}(R_t|A_t = a) = q^\star(a)$.
- $q^\star(a)$ is unknown.

The k-armed bandit problem

- Goal: Maximize the total or average reward after N actions
- The actions: Choose between k arms, i.e. $A_t \in \{1, ..., k\}$
- The reward signal:

$$R_t \sim p(R_t|a)\,,$$

  where $\mathbb{E}(R_t|A_t = a) = q^\star(a)$.
- $q^\star(a)$ is unknown.
- The estimated (expected) value if action a at step t: $Q_t(a)$.

The k-armed bandit problem

- Goal: Maximize the total or average reward after N actions
- The actions: Choose between k arms, i.e. $A_t \in \{1, ..., k\}$
- The reward signal:

$$R_t \sim p(R_t|a)\,,$$

where $\mathbb{E}(R_t|A_t = a) = q^\star(a)$.

- $q^\star(a)$ is unknown.
- The estimated (expected) value if action a at step t: $Q_t(a)$.

- This is a tabular method/problem:
  We can represent the actions in a table.
- Tabular methods works in small problems
  e.g. A/B testing and dynamic web pages.

Exploration vs. Exploitation

- Two types of actions:
  1. Exploitation: Choose the action with highest expected reward (short term)
  2. Exploration: Choose action to improve $Q_t(a)$, but reduces the reward (long term)
- The conflict between exploration and exploitation

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

ϵ-greedy

- ϵ-greedy: $P(\text{exploration}) = \epsilon$

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

ϵ-greedy

- ϵ-greedy: $P(\text{exploration}) = \epsilon$
    - Exploitation:

$$A_t = \arg\max_a Q_t(a)$$

ϵ-greedy

- ϵ-greedy: $P(\text{exploration}) = \epsilon$
  - Exploitation:
    $$A_t = \arg\max_a Q_t(a)$$
  - Exploration:
    $$A_t \sim U(1, ...k)$$

ϵ-greedy

- ϵ-greedy: $P(\text{exploration}) = \epsilon$
  - Exploitation:
    $$A_t = \arg\max_a Q_t(a)$$
  - Exploration:
    $$A_t \sim U(1, ...k)$$
- $Q_1(a) = 0$ (or used to encourage initial exploration)

UPPSALA UNIVERSITET

ϵ-greedy

- ϵ-greedy: $P(\text{exploration}) = \epsilon$
  - Exploitation:
    $$A_t = \arg\max_a Q_t(a)$$
  - Exploration:
    $$A_t \sim U(1, ...k)$$
- $Q_1(a) = 0$ (or used to encourage initial exploration)
- For any $\epsilon > 0$, $Q_t(a) \to q^\star(a)$

ϵ-greedy

- We estimate $q^\star(a)$ using $Q_t(a)$ as

$$Q_T(a) = \frac{1}{N(a)} \sum_{t}^{T-1} R_{t,A_t=a} \,,$$

where $N(a)$ is the total number of times action a has been taken.

ϵ-greedy

- We estimate $q^\star(a)$ using $Q_t(a)$ as

$$Q_T(a) = \frac{1}{N(a)} \sum_{t}^{T-1} R_{t,A_t=a} \, ,$$

  where $N(a)$ is the total number of times action a has been taken.
- When should we explore?
  - Large $V(R_t)$

ϵ-greedy

- We estimate $q^{\star}(a)$ using $Q_t(a)$ as

$$Q_T(a) = \frac{1}{N(a)} \sum_{t}^{T-1} R_{t,A_t=a} \,,$$

  where $N(a)$ is the total number of times action a has been taken.
- When should we explore?
  - Large $V(R_t)$
  - Large $\mathcal{A}$

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

ϵ-greedy

- We estimate $q^\star(a)$ using $Q_t(a)$ as

$$Q_T(a) = \frac{1}{N(a)} \sum_{t}^{T-1} R_{t,A_t=a} \,,$$

  where $N(a)$ is the total number of times action a has been taken.

- When should we explore?
  - Large $V(R_t)$
  - Large $\mathcal{A}$
  - Non-stationarity

Bandit example

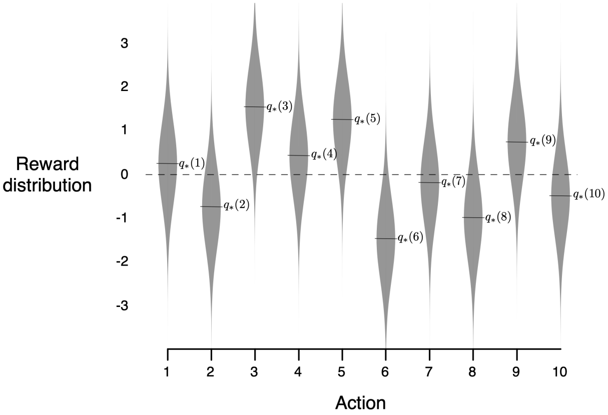


Figure: The 10-armed bandit environment (Sutton and Barto, 2017, Fig. 2.1)

Bandit example

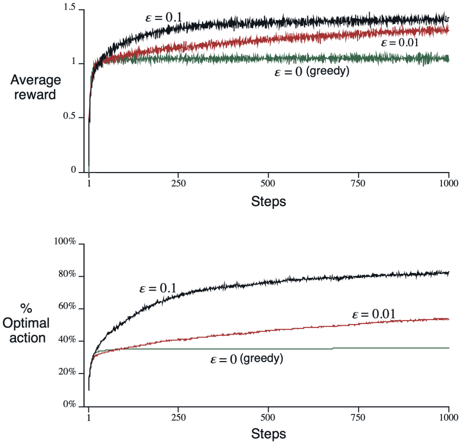


Figure: The ϵ-greedy algorithm result in the 10-armed bandit (Sutton and Barto, 2017, Fig. 2.2)

Bandit example: Optimistic initialization

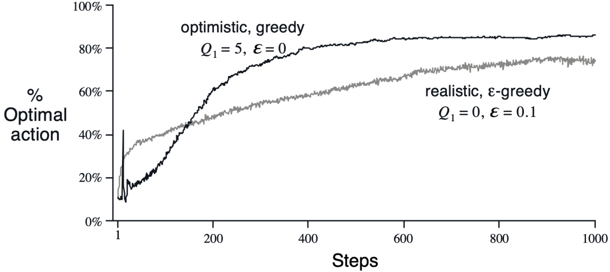


Figure: The ϵ-greedy algorithm and optimistic initialization (Sutton and Barto, 2017, Fig. 2.3)

Efficient computation and non-stationarity

- Compute $Q_t(a)$ on the fly:

$$Q_T(a) = Q_{T-1} + \frac{1}{N_t(a)}(R_{t,A_t=a} - Q_{T-1}(a))$$

Efficient computation and non-stationarity

- Compute $Q_t(a)$ on the fly:

$$Q_T(a) = Q_{T-1} + \frac{1}{N_t(a)}(R_{t,A_t=a} - Q_{T-1}(a))$$

- Handling non-stationarity:

$$Q_T(a) = Q_{T-1} + \alpha(t)(R_{t,A_t=a} - Q_{T-1}(a))$$

Efficient computation and non-stationarity

$$Q_T(a) = Q_{T-1} + \alpha(t)(R_{t,A_t=a} - Q_{T-1}(a))$$

- Examples:
  - $\alpha(t) = 1$: $Q_T(a) = R_{t,A_t=a}$

Efficient computation and non-stationarity

$$Q_T(a) = Q_{T-1} + \alpha(t)(R_{t,A_t=a} - Q_{T-1}(a))$$

- Examples:
  - $\alpha(t) = 1$: $Q_T(a) = R_{t,A_t=a}$
  - $\alpha(t) = 0$: $Q_T(a) = Q_1(a)$

Efficient computation and non-stationarity

$$Q_T(a) = Q_{T-1} + \alpha(t)(R_{t,A_t=a} - Q_{T-1}(a))$$

- Examples:
  - $\alpha(t) = 1$: $Q_T(a) = R_{t,A_t=a}$
  - $\alpha(t) = 0$: $Q_T(a) = Q_1(a)$
  - $\alpha(t) = \frac{1}{N_t(a)}$: Average reward

Efficient computation and non-stationarity

$$Q_T(a) = Q_{T-1} + \alpha(t)(R_{t,A_t=a} - Q_{T-1}(a))$$

- Examples:
  - $\alpha(t) = 1$: $Q_T(a) = R_{t,A_t=a}$
  - $\alpha(t) = 0$: $Q_T(a) = Q_1(a)$
  - $\alpha(t) = \frac{1}{N_t(a)}$: Average reward
- $Q_T(a) \rightarrow q^{\star}(a)$, if:

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

Efficient computation and non-stationarity

$$Q_T(a) = Q_{T-1} + \alpha(t)(R_{t,A_t=a} - Q_{T-1}(a))$$

- Examples:
  - $\alpha(t) = 1$: $Q_T(a) = R_{t,A_t=a}$
  - $\alpha(t) = 0$: $Q_T(a) = Q_1(a)$
  - $\alpha(t) = \frac{1}{N_t(a)}$: Average reward
- $Q_T(a) \to q^\star(a)$, if:
  1. $\sum_t^\infty \alpha_t = \infty$
  2. $\sum_t^\infty \alpha_t^2 < \infty$
- Where have we seen these criterias before?

The ϵ-greedy algorithm

A simple bandit algorithm

Initialize, for $a = 1$ to k:

$Q(a) \leftarrow 0$
$N(a) \leftarrow 0$

Repeat forever:

$$A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1-\varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$$

$R \leftarrow bandit(A)$
$N(A) \leftarrow N(A) + 1$
$Q(A) \leftarrow Q(A) + \frac{1}{N(A)}\left[R - Q(A)\right]$

Figure: The ϵ-greedy algorithm

The Upper-Confidence-Bound method

- Explore based on our uncertainty of $Q_t(a)$

The Upper-Confidence-Bound method

- Explore based on our uncertainty of $Q_t(a)$
- The Upper-Confidence-Bound (UCB) method

$$A_t = \arg\max_a \left(Q_t + c\sqrt{\frac{\log t}{N_t(a)}} \right)$$

An analogy:

$$A_t = \arg\max_a \left(Q_t + c\sqrt{\frac{\hat{\sigma}^2(a)}{N_t(a)}} \right)$$

The UCB algorithm

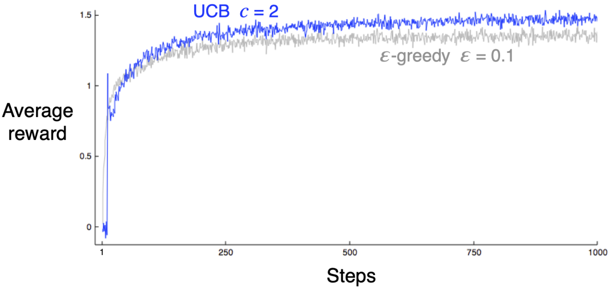


Figure: The UCB algorithm

The Bayesian Bandit: Thompson sampling

- A Bayesian Bandit
  1. Setup a likelihood for R, $p(R|a, \theta)$
  2. Setup a prior for θ, $p(\theta)$
  3. Compute posterior for θ, $p(\theta|R, a)$
  4. Choose action A_t proportional to

  $$\int I[\mathbb{E}(R|\theta, a^\star) = \arg\max_{a'} \mathbb{E}(R|\theta, a')]p(\theta|R, a)d\theta$$

  where I is the indicator function.
- Repeat step 3-4

The Bayesian Bandit: Thompson sampling

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

- A Bayesian Bandit
  1. Setup a likelihood for R, $p(R|a,\theta)$
  2. Setup a prior for θ, $p(\theta)$
  3. Compute posterior for θ, $p(\theta|R,a)$
  4. Choose action A_t proportional to

$$\int I[\mathbb{E}(R|\theta, a^\star) = \arg\max_{a'} \mathbb{E}(R|\theta, a')]p(\theta|R,a)d\theta$$

     where I is the indicator function.
- Repeat step 3-4
- Monte Carlo approximation of step 4:
  1. Draw one sample from the posterior $\tilde{\theta}$

$$\tilde{\theta} \sim p(\theta|R,a)$$

  2. Conditional on $\tilde{\theta}$, choose action A_t

$$A_t = \arg\max_{a'} \mathbb{E}(R|\tilde{\theta}, a')]$$

Section 3

Markov Decision Processes

The Markov Decision process

- Bandits does not have a *state*.

UPPSALA UNIVERSITET

The Markov Decision process

- Bandits does not have a *state*.
- An action might change the environment.
- An action might be different in different states

The Markov Decision process

- Bandits does not have a *state*.
- An action might change the environment.
- An action might be different in different states
- Example: In chess, we want to make a move based on the current position of all pieces

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

The Markov Decision process

- Bandits does not have a *state*.
- An action might change the environment.
- An action might be different in different states
- Example: In chess, we want to make a move based on the current position of all pieces
- To capture this we use a Markov Decision process
- One of the most important concepts in Reinforcement Learning

The Markov Decision process

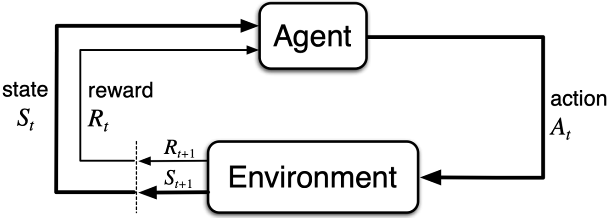


Figure: The (finite) Markov Decision Process (Sutton and Barto, 2017, Fig 3.1)

- States $S_t \in \mathcal{S}$: Basis for action
- Actions $A_t \in \mathcal{A}$
- Rewards $R_t \in \mathbb{R}$

The Markov Decision process

UPPSALA UNIVERSITET

- Boundry between Agent and Environment:
  - The total control of the action

The Markov Decision process

- Boundry between Agent and Environment:
  - The total control of the action
  - Reward is external to agent: Pain and pleasure
  - The agent should not be able to change the reward function

The Markov Decision process

- Boundry between Agent and Environment:
  - The total control of the action
  - Reward is external to agent: Pain and pleasure
  - The agent should not be able to change the reward function
- The policy ($\pi(A_t|S_t = s)$):
  - We make an action given the current state S_t
- The goal: (Again) maximize return $G_t = R_{t+1} + ... + R_T$

Return and discount

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

- Two type of interactions
  - Episodic: $T < \infty$, has terminal state
  - Continuing: $T = \infty$
- Discounting:

$$G_t = R_{t+1} + \gamma R_{t+3} + \gamma^2 R_{t+3} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

UPPSALA UNIVERSITET

Return and discount

- Two type of interactions
  - Episodic: $T < \infty$, has terminal state
  - Continuing: $T = \infty$
- Discounting:

$$G_t = R_{t+1} + \gamma R_{t+3} + \gamma^2 R_{t+3} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- Discount rate γ:
  - $0 \leq \gamma \leq 1$

Return and discount

- Two type of interactions
  - Episodic: $T < \infty$, has terminal state
  - Continuing: $T = \infty$
- Discounting:

$$G_t = R_{t+1} + \gamma R_{t+3} + \gamma^2 R_{t+3} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- Discount rate γ:
  - $0 \leq \gamma \leq 1$
  - $\gamma = 1$: No discount
  - $\gamma = 0$: Full discount: Only next reward counts
  - $\gamma < 1$ and R_t is bounded: $G_t < \infty$

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

Return and discount

- Two type of interactions
  - Episodic: $T < \infty$, has terminal state
  - Continuing: $T = \infty$
- Discounting:

$$G_t = R_{t+1} + \gamma R_{t+3} + \gamma^2 R_{t+3} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- Discount rate γ:
  - $0 \leq \gamma \leq 1$
  - $\gamma = 1$: No discount
  - $\gamma = 0$: Full discount: Only next reward counts
  - $\gamma < 1$ and R_t is bounded: $G_t < \infty$
- For episodic problem we assume $R_{T+i} = 0$ for all $i \in \mathbb{N}^+$

UPPSALA UNIVERSITET

The Markov Decision Process

- The Markov Decision process (MDP):

$$P(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a) \quad (1)$$

- Eq. (1) fully specify a MDP

UPPSALA UNIVERSITET

The Markov Decision Process

- The Markov Decision process (MDP):

$$P(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a) \quad (1)$$

- Eq. (1) fully specify a MDP
- Markov property:

$$P(S_{t+1} = s', R_{t+1} = r | S_1 = s, A_1 = a, ..., S_t = s, A_t = a) = P(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a)$$

- The MDP is a good approximation or model:
  All models are wrong, but some are useful.

UPPSALA UNIVERSITET

The Markov Decision Process Marginals

- The Markov Decision process (MDP):

$$P(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a) \quad (1)$$

- From Eq. (1) we can get marginals of interest:
  - State-action rewards:

$$r(s, a) = \mathbb{E}(R_{t+1} | S_t = s, A_t = a)$$

The Markov Decision Process Marginals

- The Markov Decision process (MDP):

$$P(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a) \; (1)$$

- From Eq. (1) we can get marginals of interest:
  - State-action rewards:

$$r(s, a) = \mathbb{E}(R_{t+1} | S_t = s, A_t = a)$$

  - State-transition probability:

$$p(s' | s, a) = P(S_{t+1} = s' | S_t = s, A_t = a)$$

The Value function

- The value function $v_\pi(s)$:
  the long-term value of s given a policy $\pi(a|s)$, i.e.

$$v_\pi(s) = \mathbb{E}_\pi(G_t|S_t = s) = \mathbb{E}_\pi\left(\sum_{k=0}^{\infty}\gamma^k R_{t+k+1}|S_t = s\right)$$

The Value function

- The value function $v_\pi(s)$:
  the long-term value of s given a policy $\pi(a|s)$, i.e.

$$v_\pi(s) = \mathbb{E}_\pi(G_t|S_t = s) = \mathbb{E}_\pi\left(\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}|S_t = s\right)$$

- Informally: How "good" is a state for the agent with the policy π.

The Value function

- Estimating $v_\pi(s)$ is one of the most important problem in RL

The Value function

- Estimating $v_\pi(s)$ is one of the most important problem in RL
- Value functions are recursive:

$$\begin{aligned} v_\pi(s) &= \mathbb{E}_\pi(G_t | S_t = s) \\ &= \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)(r + \gamma v_\pi(s')) \end{aligned}$$

- This is the Bellman equation for $v_\pi(s)$:
  The relationship between the values of the state and its successor states.

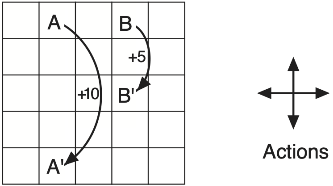
The Value function

- Estimating $v_\pi(s)$ is one of the most important problem in RL
- Value functions are recursive:

$$\begin{aligned} v_\pi(s) &= \mathbb{E}_\pi(G_t | S_t = s) \\ &= \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)(r + \gamma v_\pi(s')) \end{aligned}$$

- This is the Bellman equation for $v_\pi(s)$:
  The relationship between the values of the state and its successor states.
- Bellman equation is the basis for computing $v_\pi(s)$ (not part of this course)

The value function



3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

Figure: The gridworld equiprobable policy value function

The Optimal Policy

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

- A policy π is better than π' if $v_\pi(s) \geq v_{\pi'}(s)$ for all $s \in \mathcal{S}$.

The Optimal Policy

- A policy π is better than π' if $v_\pi(s) \geq v_{\pi'}(s)$ for all $s \in \mathcal{S}$.
- A policy that is better or equal to all other policies is the optimal policy $\pi_\star$.

The Optimal Policy

- A policy π is better than π' if $v_\pi(s) \geq v_{\pi'}(s)$ for all $s \in \mathcal{S}$.
- A policy that is better or equal to all other policies is the optimal policy $\pi_\star$.
- The optimal value function: $v_{\pi_\star}(s) = v_\star(s)$

The Optimal Policy

- A policy π is better than π' if $v_\pi(s) \geq v_{\pi'}(s)$ for all $s \in \mathcal{S}$.
- A policy that is better or equal to all other policies is the optimal policy $\pi_\star$.
- The optimal value function: $v_{\pi_\star}(s) = v_\star(s)$
- The optimal policy $\pi_\star$ is greedy wrt $v_{\pi_\star}(s)$:
  The best long term strategy

UPPSALA UNIVERSITET

- Introduction to Reinforcement Learning
- Bandits
- Markov Decision Processes

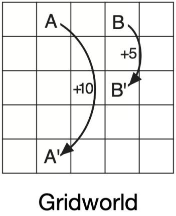
The Optimal Policy

- Computing optimal value function might be impossible: we need to estimate/re-estimate/approximate it
- Example: Chess, we cannot compute the optimal long-term moves, we need to approximate/estimate (based on computational budget)

The Optimal Policy

- Computing optimal value function might be impossible: we need to estimate/re-estimate/approximate it
- Example: Chess, we cannot compute the optimal long-term moves, we need to approximate/estimate (based on computational budget)
- We might also estimate $v_\star(s)$ better for commonly encountered states

The value function



22.0	24.4	22.0	19.4	17.5
19.8	22.0	19.8	17.8	16.0
17.8	19.8	17.8	16.0	14.4
16.0	17.8	16.0	14.4	13.0
14.4	16.0	14.4	13.0	11.7

v_*

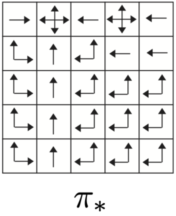


Figure: The gridworld optimal value function and policy