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Machine learning – Block 7

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Department of Statistics, Uppsala University

Autumn 2022

- Practicalities
- Introduction
- Autoencoders
- The Variational Autoencoder
 - The probabilistic decoder
 - The encoder
 - Training a variational autoencoder
- Probabilistic Topic Models
 - Latent Dirichlet Allocation
 - Estimating the LDA model



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Assignment 6: Evaluation

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● ?



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This week's lectures

- **Practicalities**

- Introduction

- Autoencoders

- The Variational
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- Probabilistic Topic
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- Latent Dirichlet Allocation
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- Variational autoencoders
- Probabilistic Topic Models



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Section 2

Introduction



Why variational autoencoders and topic models?

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- Popular approaches in **industry and academia**
- **Probabilistic** methods for unsupervised learning



Why variational autoencoders and topic models?

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- Popular approaches in industry and academia
- Probabilistic methods for unsupervised learning
- Aim of this lecture:
 - Describe the models
 - How to estimate these models
 - Explain what they are used for



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Use Cases

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- Variational autoencoders: Unsupervised modeling of **images**
- Topic models: Unsupervised modeling of **documents**



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- Variational autoencoders: Unsupervised modeling of images
- Topic models: Unsupervised modeling of documents
- Used for:
 - Identify "closeness" in high-dimensional data



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- Used for:
 - Identify "**closeness**" in high-dimensional data
 - **Visualize/analyze** data



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 - Used for:
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 - **Visualize/analyze** data
 - **Compression**



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 - Feature construction



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 - Used for:
 - Identify "**closeness**" in high-dimensional data
 - **Visualize/analyze** data
 - **Compression**
 - **Feature construction**
 - Analyze underlying **patterns**



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Use Cases: Examples

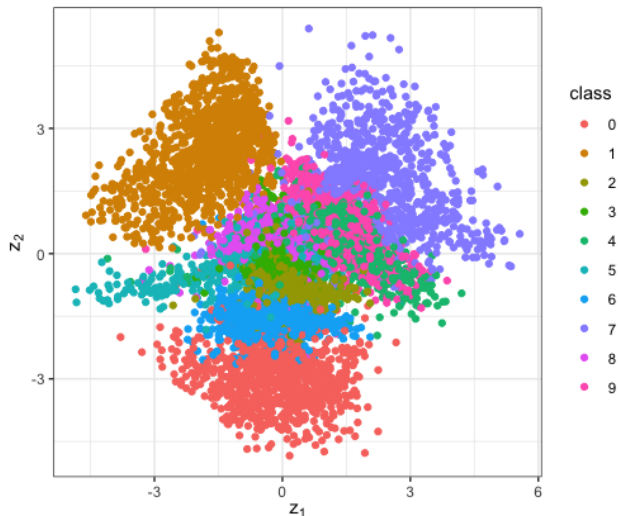


Figure: The latent state of MNIST using an Variational Autoencoder



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Section 3

Autoencoders



Autoencoder

- An autoencoder is a neural network (e.g. feed-forward) that take an input x and predict (the same) x (r).

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Autoencoder

- An autoencoder is a neural network (e.g. feed-forward) that take an input x and predict (the same) x (r).
- Three parts:
 - encoder $f(x)$ (or $e(x)$)
 - code h
 - decoder $g(h)$ (or $d(z)$)

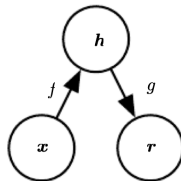


Figure: A Neural Autoencoder (Goodfellow et al, 2018)



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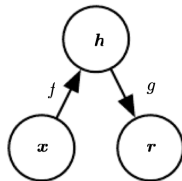


Figure: A Neural Autoencoder (Goodfellow et al, 2018)

- Loss function (reconstruction error):

$$L(\phi, \theta) = (x - d_{\phi}(e_{\theta}(x)))^2$$



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The Undercomplete Autoencoder

- More interesting: an **undercomplete** autoencoder:
Dimension of code is **lower** than that of x

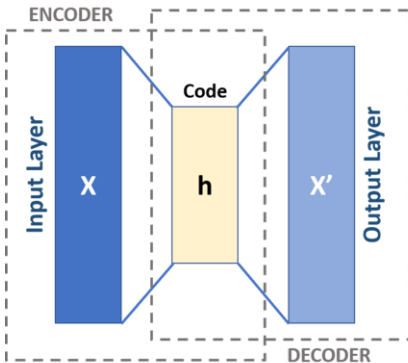


Figure: A Neural Autoencoder (Wikipedia)



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PCA and autoencoders

- A linear autoencoder: $e_\theta(x) = W_\phi$, and $d_\theta(x) = W_\phi$
- We want to minimize the loss (ignoring b /the mean):

$$L(\phi, \theta) = \sum_{i=1}^N (x_i - W_\theta W_\phi x_i)^2$$



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- Remember **PCA loss**:

$$L(P) = \sum_{i=1}^N (x_i - P_q P_q^T x_i)^2,$$

where P is an orthogonal matrix of rank q .



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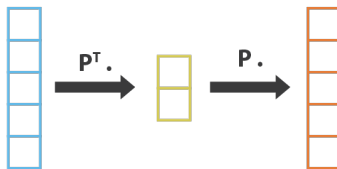
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- Remember **PCA loss**:

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- **Hence:** PCA can be seen as an autoencoder





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- Deep Autoencoder: An autoencoder with multilayer neural networks as encoder and decoder
 - can be seen as a non-linear PCA
 - learn nonlinear representations



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- **Deep** Autoencoder: An autoencoder with **multilayer neural networks** as encoder and decoder
 - can be seen as a non-linear PCA
 - learn nonlinear representations
- Problem: Deep autoencoders need to be **regularized** to not **overfit** the latent state



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- Problem: Autoencoders (as PCA) are not probabilistic models:
 - cannot generate data.
 - no notion of uncertainty



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- Problem: Autoencoders (as PCA) are not probabilistic models:
 - cannot generate data.
 - no notion of uncertainty
- We would like something like probabilistic PCA for (deep) autoencoders



- Remember the pPCA model (with z as latent variable):

$$x_i \sim N(\mathbf{b} + \mathbf{W}z_i^T, \sigma I)$$

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probabilistic PCA as an decoder

- Remember the pPCA model (with z as latent variable):

$$x_i \sim N(\mathbf{b} + \mathbf{W}z_i^T, \sigma I)$$

- Now, swap the simple parameters with a neural network

$$x_i \sim N(\text{NeuralNetwork}_{\phi}(z_i), \sigma I)$$



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- This is an example of a Deep Latent Variable model (a probabilistic decoder)



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- Another example is the Variational Autoencoder



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Section 4

The Variational Autoencoder



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The Variational Autoencoder (VAE)

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- The variational autoencoder (VAE) is a **deep probabilistic autoencoder**
- Used for unsupervised learning of **images**



The Variational Autoencoder (VAE)

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- The variational autoencoder (VAE) is a **deep probabilistic autoencoder**
- Used for unsupervised learning of **images**
- Consists of:
 1. The (probabilistic) encoder $q(z|\phi, x)$: **inference model**



The Variational Autoencoder (VAE)

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 2. Sample z from encoded x : the **latent state**



The Variational Autoencoder (VAE)

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The Variational Autoencoder (VAE)

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- Encoding the **latent state as a distribution** forces the space to be "reasonable"/reduces overfitting



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- Encoding the **latent state as a distribution** forces the space to be "reasonable"/reduces overfitting
- VAEs get their name from **variational inference** (used in training)



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The Variational Autoencoder

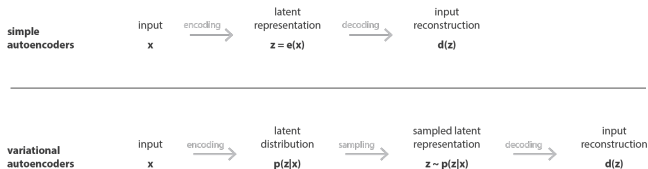


Figure: Autoencoder vs. the Variational Autoencoder (Rocca, 2019)



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The Variational Autoencoder

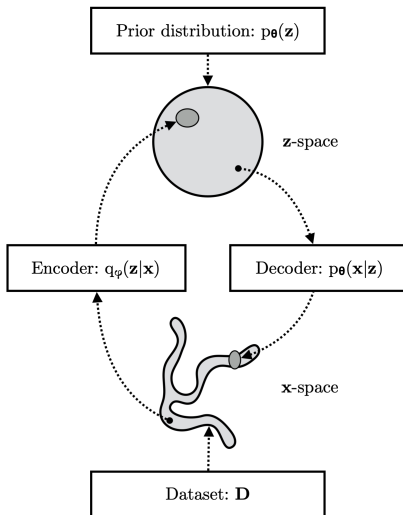


Figure: The Variational Autoencoder (Kingma and Welling, 2018, Fig. 2.1)



The probabilistic decoder

- The probabilistic decoder $p(x|\theta, z)$ (**observation model**)
- Usually a Normal distribution:

$$x_i \sim N(\text{NeuralNetwork}(z, \theta), cI)$$

- x_i for observation i depends non-linearly on the latent state z_i

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The probabilistic decoder

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- Usually a Normal distribution:

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- x_i for observation i depends non-linearly on the latent state z_i
- A probabilistic linear decoder: **pPCA**

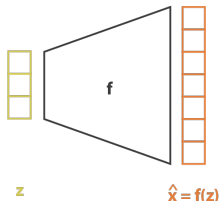


Figure: The Decoder (Rocca, 2019)



The probabilistic encoder

- The probabilistic encoder $q(z|x, \phi)$ (inference model)

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- The probabilistic encoder $q(z|x, \phi)$ (**inference model**)
- We assume that $q_\phi(z|x)$ follows a specific distribution.
Commonly:

$$z \sim N(\mu, \Sigma)$$

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- A neural network learns the parameters μ and Σ

$$\mu = \text{NeuralNetwork}(x, \phi_\mu),$$

and

$$\Sigma = \text{NeuralNetwork}(x, \phi_\Sigma).$$



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- Common assumption: Σ is a diagonal matrix.
- **Result:** z_i for observation i depends non-linearly on x_i



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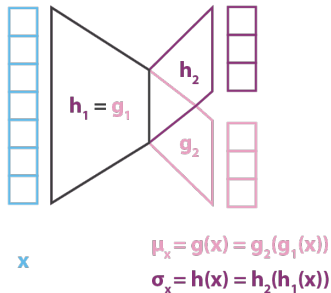


Figure: The Encoder (Rocca, 2019)



The Variational Autoencoder

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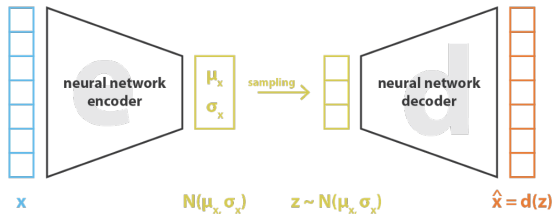


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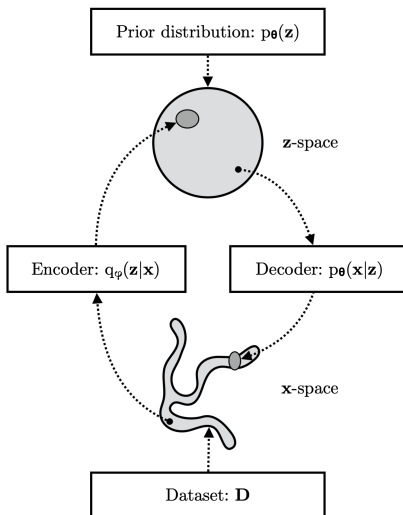


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Training a VAE

- **Goal:** estimating ϕ , θ (and z_i)

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- **Goal:** estimating ϕ , θ (and z_i)
- The encoder and decoder are (usually) complicated (no close form solution)
- Estimate ϕ and θ using gradient ascent



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- **Goal:** estimating ϕ , θ (and z_i)
- The encoder and decoder are (usually) complicated (no close form solution)
- Estimate ϕ and θ using gradient ascent
- Target:
 - Maximize $\log p(x)$
(Explain the data as well as possible)



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- **Goal:** estimating ϕ , θ (and z_i)
- The encoder and decoder are (usually) complicated (no close form solution)
- Estimate ϕ and θ using gradient ascent
- **Target:**
 - Maximize $\log p(x)$
(Explain the data as well as possible)
- **Optimization target:**
Maximize the variational lower bound or **evidence lower bound (ELBO)**



$$\begin{aligned}\log p_{\theta}(x) &= \int q_{\phi}(z|x) \log p_{\theta}(x) dz \\ &= \mathbb{E}_{q_{\phi}(z|x)} [\log p_{\theta}(x)] \\ &= \mathbb{E}_{q_{\phi}(z|x)} \left[\log \left(\frac{p_{\theta}(x, z)}{p_{\theta}(z|x)} \right) \right], \text{ using } p(z|x) = \frac{p(x, z)}{p(x)} \\ &= \mathbb{E}_{q_{\phi}(z|x)} \left[\log \left(\frac{p_{\theta}(x, z)}{q_{\phi}(z|x)} \frac{q_{\phi}(z|x)}{p_{\theta}(z|x)} \right) \right] \\ &= \mathbb{E}_{q_{\phi}(z|x)} \left[\log \left(\frac{p_{\theta}(x, z)}{q_{\phi}(z|x)} \right) \right] + \mathbb{E}_{q_{\phi}(z|x)} \left[\log \left(\frac{q_{\phi}(z|x)}{p_{\theta}(z|x)} \right) \right] \\ &= \underbrace{\mathcal{L}_{\theta, \phi}(x)}_{\text{ELBO}} + D_{KL}(q_{\phi}(z|x) || p_{\theta}(z|x))\end{aligned}$$

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$$\underbrace{\mathcal{L}_{\theta, \phi}(x)}_{\text{ELBO}} = \log p_{\theta}(x) - D_{KL}(q_{\phi}(z|x) || p_{\theta}(z|x))$$

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The Kullback-Leibler divergence

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- **The Kullback-Leibler divergence:** a way of measuring the "distance" between probability distributions

$$D_{KL}(q_{\phi}(z|x)||p_{\theta}(z|x)) = \mathbb{E}_{q_{\phi}(z|x)} \left[\log \left(\frac{q_{\phi}(z|x)}{p_{\theta}(z|x)} \right) \right]$$

$$D_{KL}(q_{\phi}(z|x)||p_{\theta}(z|x)) \geq 0$$



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- **Optimization target:** Maximize the ELBO

$$\mathcal{L}_{\theta, \phi}(x) = \log p_{\theta}(x) - D_{KL}(q_{\phi}(z|x) || p_{\theta}(z|x))$$

- ELBO is a lower bound for the marginal log-likelihood (similar to the EM algorithm)



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- **Optimization target:** Maximize the ELBO

$$\mathcal{L}_{\theta, \phi}(x) = \log p_{\theta}(x) - D_{KL}(q_{\phi}(z|x) || p_{\theta}(z|x))$$

- ELBO is a lower bound for the marginal log-likelihood (similar to the EM algorithm)
- Maximizing the ELBO will do two things:
 - Maximize the marginal log-likelihood $\log p_{\theta}(x)$:
Better generative model/decoder
 - Minimize the KL-divergence between $q_{\phi}(z|x)$ and $p_{\theta}(z|x)$:
Better approximation of the latent space/encoder



- Stochastic Gradient *Ascent* to maximize:

$$\begin{aligned}\mathcal{L}_{\theta,\phi}(x) &= \sum_i^N \mathcal{L}_{\theta,\phi}(x_i) \\ &= \sum_i^N \mathbb{E}_{q_\phi(z_i|x_i)} [\log(p_\theta(x_i, z_i)) - \log(q_\phi(z_i|x_i))]\end{aligned}$$

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- Two problems:
 1. How do we compute the expectation?
Solution: **Monte Carlo Approximation**

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- Two problems:
 1. How do we compute the expectation?
Solution: **Monte Carlo Approximation**
 2. How compute the gradient wrt ϕ ?
Solution: **Change of variables**: $z = g(\epsilon, \phi, x)$
This is called **the reparametrization trick**

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- Using the reparametrization trick and Monte Carlo approximation, we get:

$$\begin{aligned}\mathcal{L}_{\theta, \phi}(x) &= \mathbb{E}_{q_{\phi}(z|x)} [\log(p_{\theta}(x, z)) - \log(q_{\phi}(z|x))] \\ &= \mathbb{E}_{p(\epsilon)} [\log(p_{\theta}(x, g(\epsilon, \phi, x))) - \log(q_{\phi}(g(\epsilon, \phi, x)|x))] \\ &\approx \log(p_{\theta}(x, g(\epsilon, \phi, x))) - \log(q_{\phi}(g(\epsilon, \phi, x)|x))\end{aligned}$$

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- A common approach: do the MC approximation with only one sample per datapoint x_i .



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Optimizing the ELBO

- Using the reparametrization trick and Monte Carlo approximation, we get:

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- A common approach: do the MC approximation with only **one sample per datapoint x_i** .
- We approximate both $\mathcal{L}_{\theta, \phi}(x)$ and $\nabla \mathcal{L}_{\theta, \phi}(x)$



- Using the reparametrization trick and Monte Carlo approximation, we get:

$$\begin{aligned}\mathcal{L}_{\theta,\phi}(x) &= \mathbb{E}_{q_{\phi}(z|x)} [\log(p_{\theta}(x, z)) - \log(q_{\phi}(z|x))] \\ &= \mathbb{E}_{p(\epsilon)} [\log(p_{\theta}(x, g(\epsilon, \phi, x))) - \log(q_{\phi}(g(\epsilon, \phi, x)|x))] \\ &\approx \log(p_{\theta}(x, g(\epsilon, \phi, x))) - \log(q_{\phi}(g(\epsilon, \phi, x)|x))\end{aligned}$$

- A common approach: do the MC approximation with only **one sample per datapoint** x_i .
- We approximate both $\mathcal{L}_{\theta,\phi}(x)$ and $\nabla \mathcal{L}_{\theta,\phi}(x)$
- Sometimes called a **doubly stochastic** algorithm.

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The Autoencoding Variational Bayes Algorithm

Algorithm 1: Stochastic optimization of the ELBO. Since noise originates from both the minibatch sampling and sampling of $p(\epsilon)$, this is a doubly stochastic optimization procedure. We also refer to this procedure as the *Auto-Encoding Variational Bayes* (AEVB) algorithm.

Data:

\mathcal{D} : Dataset

$q_\phi(\mathbf{z}|\mathbf{x})$: Inference model

$p_\theta(\mathbf{x}, \mathbf{z})$: Generative model

Result:

θ, ϕ : Learned parameters

$(\theta, \phi) \leftarrow$ Initialize parameters

while *SGD not converged* **do**

$\mathcal{M} \sim \mathcal{D}$ (Random minibatch of data)

$\epsilon \sim p(\epsilon)$ (Random noise for every datapoint in \mathcal{M})

 Compute $\tilde{\mathcal{L}}_{\theta, \phi}(\mathcal{M}, \epsilon)$ and its gradients $\nabla_{\theta, \phi} \tilde{\mathcal{L}}_{\theta, \phi}(\mathcal{M}, \epsilon)$

 Update θ and ϕ using SGD optimizer

end

Figure: The Autoencoding Variational Bayes Algorithm (Kingma and Welling, 2018, Algo. 1)



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The Autoencoding Variational Bayes Algorithm

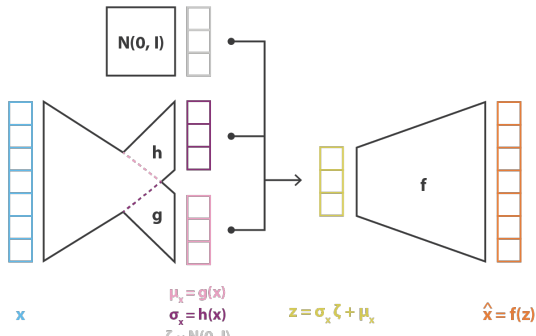


Figure: The Autoencoding Variational Bayes Algorithm (Rocca, 2019)



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Summary

- Benefits of VAE:
 - Get a more interpretable latent state
 - We can estimate uncertainty
 - Regularize the latent state
 - We can inject knowledge in our latent state



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Summary

- Benefits of VAE:
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 - The blurry image problem



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 - Get a more **interpretable** latent state
 - We can estimate **uncertainty**
 - **Regularize** the latent state
 - We can inject knowledge in our latent state
- Problems:
 - The blurry image problem
- Still much ongoing research:



Figure: Examples of images generated with a deep hierarchical Variational Autoencoder (Vahdat and Kautz, 2020)



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Probabilistic Topic Models



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Section 5

Probabilistic Topic Models



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Probabilistic Topic Models

- Unsupervised method for **textual data**

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- Unsupervised method for **textual data**
 - Popular in industry and academia to **analyze large corpora**



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 - The most common model: **Latent Dirichlet Allocation**
 - A **mixed membership** model (a mixture of multinomial mixtures model)



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- Example: **All ears** media monitoring of speech data



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The Dirichlet Distribution

- Probability distribution over the simplex with K categories:

$$f(\mathbf{x}|\boldsymbol{\alpha}) = \frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^K x_i^{\alpha_i-1}$$

where

$$B(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^K \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^K \alpha_i)},$$

and where

$$\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_K)$$



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$$\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_K)$$

- The probability distribution has the support on the simplex, that is

$$\sum_{i=1}^K x_i = 1 \text{ and } x_i \geq 0 \text{ for all } i \in [1, K]$$



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- The parameters $\boldsymbol{\alpha}$ can be seen as **pseudo-counts**



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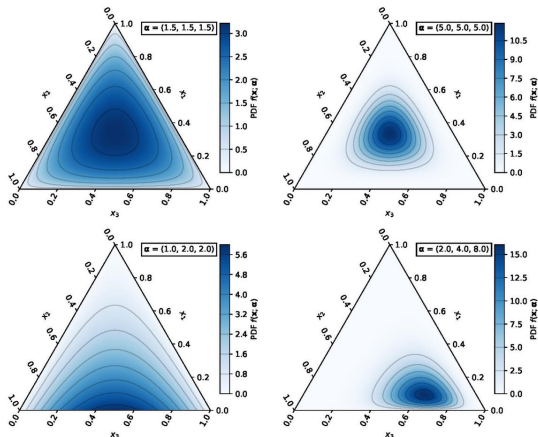


Figure: The Dirichlet Distribution (Wikipedia)



The distributional hypothesis

- Harris (1954) and Firths (1957):
“Word is characterized by the company it keeps”

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- Semantics (broadly defined) is captured by **context**



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- Rough definition: **word windows** of different sizes



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The distributional hypothesis

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“Word is characterized by the company it keeps”
- Semantics (broadly defined) is captured by **context**
- Rough definition: **word windows** of different sizes
- Different window sizes, different **semantic** content:
 - Word embeddings (context: word windows)
 - Topic models (context: documents)

Example

1. “A friend in need is a friend indeed.”
2. “She is my friend indeed.”



Latent Dirichlet Allocation

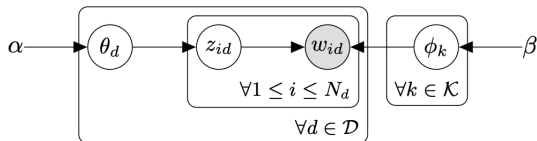


Figure: The Latent Dirichlet Allocation Model

where ϕ_k is the k th row in Φ (of dimension $K \times V$) and θ_d is the d th row in Θ (of dimension $D \times K$).



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Relies on the **bag-of-word** assumption

1. For each component k to K :

1.1 $\phi_k \sim \text{Dirichlet}(\beta)$

2. For each document d :

2.1 $\theta_d \sim \text{Dirichlet}(\alpha)$

2.2 For each token i :

2.2.1 $z_{id} \sim \text{Categorical}(\theta_d)$

2.2.2 $w_{id} \sim \text{Categorical}(\phi_{z_{id}})$



Example of parameters \mathbf{z} , Θ and Φ

\mathbf{w}_1	boat	shore	bank		
\mathbf{z}_1	1	1	1		
\mathbf{w}_2	Zlatan	boat	shore	money	bank
\mathbf{z}_2	2	1	1	3	3
\mathbf{w}_3	money	bank	soccer	money	
\mathbf{z}_3	3	3	2	3	

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\mathbf{w}_3	money	bank	soccer	money	
\mathbf{z}_3	3	3	2	3	

		boat	shore	soccer	Zlatan	bank	money
$\Phi =$	Topic 1	0.35	0.35	0.05	0.05	0.15	0.05
	Topic 2	0.025	0.025	0.45	0.45	0.025	0.025
	Topic 3	0.025	0.025	0.025	0.025	0.45	0.45



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Example of parameters \mathbf{z} , Θ and Φ

\mathbf{w}_1	boat	shore	bank		
\mathbf{z}_1	1	1	1		
\mathbf{w}_2	Zlatan	boat	shore	money	bank
\mathbf{z}_2	2	1	1	3	3
\mathbf{w}_3	money	bank	soccer	money	
\mathbf{z}_3	3	3	2	3	

		boat	shore	soccer	Zlatan	bank	money
$\Phi =$	Topic 1	0.35	0.35	0.05	0.05	0.15	0.05
	Topic 2	0.025	0.025	0.45	0.45	0.025	0.025
	Topic 3	0.025	0.025	0.025	0.025	0.45	0.45

		Topic 1	Topic 2	Topic 3
$\Theta =$	doc 1	0.96	0.02	0.02
	doc 2	0.3	0.2	0.5
	doc 3	0.05	0.35	0.6



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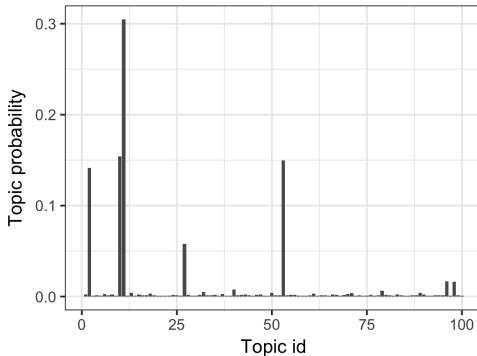
Closing arguments were heard yesterday in the Federal bankruptcy fraud trial of Stephen J. Sabbeth, whose legal problems have raised doubts about his ability to continue as leader of the Nassau County Democratic Party.

Mr. Sabbeth is charged with trying to conceal \$750,000 from his bank creditors by hiding the money in a secret account in his wife's maiden name, rather than use it to pay creditors when his lumber business went into bankruptcy 10 years ago.

— The New York Times 25th of February 1999



The estimated topic proportion ($\hat{\theta}_d$)



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Topic	Top words (by ϕ_{kv})
2	party election voters campaign democratic
10	bank banks loans loan insurance savings
11	trial prison jury prosecutors convicted guilty
53	investigation inquiry documents investigators

Table: The words with highest probability ($p(w|k)$) for topic 2, 10, 11 and 53.



The Latent Dirichlet Allocation Model

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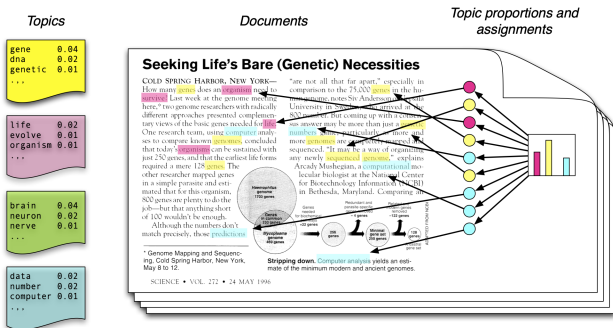


Figure: The Latent Dirichlet Allocation Model (Blei 2012, Fig. 1)



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Inference

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- Common inference approaches
 1. Variational inference
 2. Markov Chain Monte Carlo (MCMC)



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- Common inference approaches
 1. Variational inference
 2. Markov Chain Monte Carlo (MCMC)
- The Gibbs sampler is usually preferred
- Similar to (Stochastic) EM



The basic Gibbs sampler:

1. We want to estimate z, Φ, Θ :

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The basic Gibbs sampler:

1. We want to estimate z, Φ, Θ :
2. Sample topic indicators (latent variable)

$$p(z = k | \Phi, \Theta) \propto \phi_{v,k} \theta_{k,d}$$

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Gibbs sampler for LDA

The basic Gibbs sampler:

1. We want to estimate z, Φ, Θ :
2. Sample topic indicators (latent variable)

$$p(z = k | \Phi, \Theta) \propto \phi_{v,k} \theta_{k,d}$$

3. Sample model parameters

$$\theta_d | \mathbf{z} \sim \text{Dir}(\mathbf{n}^{(d)} + \alpha)$$

$$\phi_k | \mathbf{z} \sim \text{Dir}(\mathbf{n}^{(v)} + \beta)$$

where $\mathbf{n}^{(d)}$ is the number of tokens by topic in document d and $\mathbf{n}^{(v)}$ is the number of tokens by topic for word type v .



Integrating out (collapsing) Θ and Φ

$$p(\mathbf{z}|\mathbf{w}) = \int \int p(\mathbf{z}, \Theta, \Phi | \mathbf{w}) \cdot p(\mathbf{z}, \Theta, \Phi) d\Phi d\Theta$$

will result in the following Gibbs sampler:

$$p(z_i = k | w_i, \mathbf{z}_{-i}) \propto \underbrace{\frac{n_k^{(v)} + \beta}{n_k^{(v)} + V\beta}}_{\text{type-topic } (\Phi)} \cdot \underbrace{(n_k^{(d)} + \alpha)}_{\text{topic-doc } (\Theta)}$$

where $n^{(v)}$ and $n^{(d)}$ are count matrices of size $D \times K$ and $K \times V$.

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Example of $n^{(v)}$ and $n^{(d)}$

\mathbf{w}_1	boat	shore	bank		
\mathbf{z}_1	1	1	1		
\mathbf{w}_2	Zlatan	boat	shore	money	bank
\mathbf{z}_2	2	1	1	3	3
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\mathbf{w}_3	money	bank	soccer	money	
\mathbf{z}_3	3	3	2	3	

	boat	shore	soccer	Zlatan	bank	money
$n^{(v)} =$	2	2	0	0	1	0
	0	0	1	1	0	0
	0	0	0	0	2	2

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\mathbf{w}_3	money	bank	soccer	money	
\mathbf{z}_3	3	3	2	3	

$$n^{(v)} = \begin{array}{ccccc} & \text{boat} & \text{shore} & \text{soccer} & \text{Zlatan} & \text{bank} & \text{money} \\ \begin{array}{c} 2 \\ 0 \\ 0 \end{array} & \begin{array}{c} 2 \\ 0 \\ 0 \end{array} & \begin{array}{c} 2 \\ 0 \\ 0 \end{array} & \begin{array}{c} 0 \\ 1 \\ 0 \end{array} & \begin{array}{c} 0 \\ 1 \\ 0 \end{array} & \begin{array}{c} 1 \\ 0 \\ 2 \end{array} & \begin{array}{c} 0 \\ 0 \\ 2 \end{array} \end{array}$$

$$n^{(d)} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 2 & 3 \end{bmatrix}$$



Topic Models as non-negative matrix factorization

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$$\begin{bmatrix} n_{dv} \\ (D \times V) \end{bmatrix} \approx \begin{bmatrix} \Theta \\ (D \times K) \end{bmatrix} \times \begin{bmatrix} \Phi \\ (K \times V) \end{bmatrix}$$



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- Setting K , α and β



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- Setting K , α and β
- Reducing the vocabulary: stopwords, rare words, stemming



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- Setting K , α and β
 - Reducing the vocabulary: stopwords, rare words, stemming
 - "Junk" topics



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- Setting K , α and β
- Reducing the vocabulary: stopwords, rare words, stemming
- "Junk" topics
- We can analyze the topic indicators z directly



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Research Example: Swedish Immigration Discourse

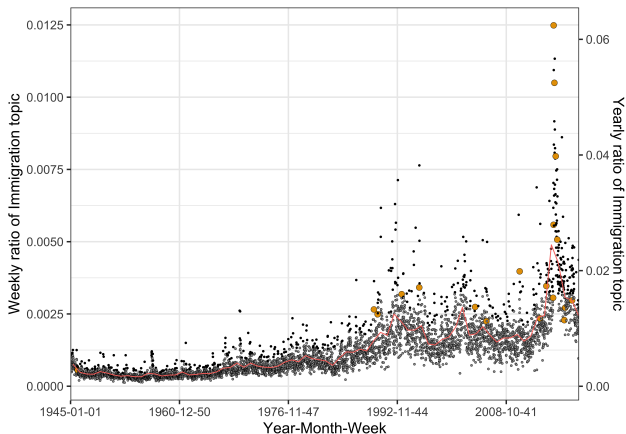


Figure: The Immigration topic in Swedish Newspapers (Hurtado Bodell et al, not in print)



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- Topic models are unsupervised methods for textual data



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- Topic models are unsupervised methods for textual data
- The Latent Dirichlet Allocation is a popular model



Summary: Topic Models

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- Topic models are unsupervised methods for textual data
- The Latent Dirichlet Allocation is a popular model
- A mixed membership model (a mixture of multinomial mixtures model)



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- Topic models are unsupervised methods for textual data
- The Latent Dirichlet Allocation is a popular model
- A mixed membership model (a mixture of multinomial mixtures model)
- Usually use Gibbs samplers for estimation