Lösningsförslag till Tentamen 100608 i 732G26 Surveymetodik med uppsats

1. a) $\hat{t} = N \cdot \bar{y} = 55560 \cdot (5540/200) = 55560 \cdot 27.70 = 1539012$ timmar

   99% konfidensintervall (z = 2.576):

$$N \cdot \bar{y} \pm 2.576 \cdot N \cdot \sqrt{\left(1 - \frac{n}{N}\right) \cdot \frac{s^2}{n}} \Rightarrow 55560 \cdot \left(27.70 \pm 2.576 \cdot \sqrt{\left(1 - \frac{200}{55560}\right) \cdot \frac{15^2}{200}}\right)$$

$$\Rightarrow 1539012 \pm 151531 = (1387481, 1690543)$$

   b) Bredden för intervall för $t \le 280000$ ➔ Bredden för intervall för $\mu \le 5.04$.

$$n_0 \ge \frac{4 \cdot 2.576^2 \cdot 15^2}{5.04^2} \approx 236$$

$$\Rightarrow n = \frac{236}{1 + \frac{236}{55560}} \approx 235$$

   c) Sätt de 10 individernas värden till 0 och räkna om medelvärde och standardavvikelse.
   Beteckna övertäckningens antal, medelvärde och standardavvikelse med $n_ö$, $\bar{x}_ö$ resp. $s_ö$.

$$\bar{u} = \frac{5540 - 10 \cdot 28.5}{200} = 26.275$$

$$s_u = \sqrt{\frac{(n-1) \cdot s^2 + n \cdot \bar{x}^2 - \left((n_ö - 1) \cdot s_ö^2 + n_ö \cdot \bar{x}_ö^2\right) - n \cdot \bar{u}^2}{n-1}} =$$

$$= \sqrt{\frac{199 \cdot 15^2 + 200 \cdot 27.70^2 - \left(9 \cdot 10.9^2 + 10 \cdot 28.5^2\right) - 200 \cdot 26.275^2}{199}} \approx 16$$

   Ny punktskattning: $55560 \cdot 26.275 \approx 1459839$

   99% K.I. $55560 \cdot \left(26.275 \pm 2.576 \cdot \sqrt{\left(1 - \frac{200}{55560}\right) \cdot \frac{16^2}{200}}\right) \approx 1459839 \pm 161633 =$

$$= (1298206, 1621472)$$

2. a) Stratifierat urval. Punktskattning av medeltal:

$$\bar{y}_{str} = \frac{N_1}{N} \cdot \bar{y}_1 + \frac{N_2}{N} \cdot \bar{y}_2 = 0.929 \cdot \frac{3350}{150} + 0.071 \cdot \frac{1800}{50} \approx 23.304$$

   Punktskattning av total

$$\hat{t}_{str} = 55560 \cdot 23.304 \approx 1294770$$

   99% konfidensintervall för medeltal

$$\bar{y}_{str} \pm 2.576 \cdot \sqrt{\left(\frac{N_1}{N}\right)^2 \cdot \left(1 - \frac{n_1}{N_1}\right) \cdot \frac{s_1^2}{n_1} + \left(\frac{N_2}{N}\right)^2 \cdot \left(1 - \frac{n_2}{N_2}\right) \cdot \frac{s_2^2}{n_2}}$$

$$N_1 = 0.969 \cdot 55560 \approx 53838 \Rightarrow N_2 = 1722 \Rightarrow$$

$$23.304 \pm 2.576 \cdot \sqrt{(0.929)^2 \cdot \left(1 - \frac{150}{51615}\right) \cdot \frac{10.3^2}{150} + (0.071)^2 \cdot \left(1 - \frac{50}{3945}\right) \cdot \frac{9.5^2}{50}}$$

$\Rightarrow 23.304 \pm 3.466$

⇨ 99% konfidensintervall för total

$$55560 \cdot (23.304 \pm 3.466) \approx 1294770 \pm 192571 = (1102199, 1487341)$$

b) Optimal allokering blir här Neyman-allokering eftersom man kan anta lika stora kostnader.

$$n_h = n \cdot \frac{N_h \cdot S_h}{N_1 \cdot S_1 + N_2 \cdot S_2} = n \cdot \frac{\frac{N_h}{N} \cdot S_h}{\frac{N_1}{N} \cdot S_1 + \frac{N_2}{N} \cdot S_2}$$

S_h approximeras med s_h från den första undersökningen.

$n = 200 \Rightarrow$

$$n_1 = 200 \cdot \frac{0.969 \cdot 10.3}{0.969 \cdot 10.3 + 0.071 \cdot 9.5} \approx 187$$

$\Rightarrow n_2 = 200 - 187 = 13$

3. Tvåstegs klusterurval

a) Väntevärdesriktig skattning:

$$\hat{p} = \hat{\bar{y}}_{unb} = \frac{\hat{t}_{unb}}{M_0} = \frac{N}{M_0 \cdot n} \cdot \sum_{i \in S} M_i \cdot \bar{y}_i$$

$$10\% \text{ urval} \Rightarrow \text{Avrunda till närmaste heltal} \Rightarrow \frac{55}{5954 \cdot 3} \cdot \left(92 \cdot \frac{2}{9} + 83 \cdot \frac{1}{8} + 149 \cdot \frac{3}{15}\right) \approx 0.187$$

$$\hat{t}_{unb} = \frac{55}{3} \cdot \left(92 \cdot \frac{2}{9} + 83 \cdot \frac{1}{8} + 149 \cdot \frac{3}{15}\right) \approx 1111.36$$

$$\hat{V}(\hat{p}) = \frac{1}{M_0^2} \cdot \hat{V}(\hat{t}_{unb}) = \frac{1}{M_0^2} \cdot \left[N^2 \cdot \left(1 - \frac{n}{N}\right) \cdot \frac{s_t^2}{n} + \frac{N}{n} \cdot \sum_{i \in S} \left(1 - \frac{m_i}{M_i}\right) \cdot M_i^2 \cdot \frac{s_i^2}{m_i} \right]$$

Räkna med $m_i / M_i \approx 0.1$. $s_i^2 = \dfrac{\hat{p}_i \cdot (1 - \hat{p}_i)}{m_i - 1}$

$$\Rightarrow \frac{1}{5954^2} \cdot \left[55^2 \cdot \left(1 - \frac{3}{55}\right) \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \left(\left(92 \cdot \frac{2}{9} - \frac{1111.36}{55}\right)^2 + \left(83 \cdot \frac{1}{8} - \frac{1111.36}{55}\right)^2 + \left(149 \cdot \frac{3}{15} - \frac{1111.36}{55}\right)^2 \right) + \right.$$

$$\left. + \frac{55}{3} \cdot \left((1 - 0.1) \cdot 92^2 \cdot \frac{(2/9) \cdot (7/9)}{8} + (1 - 0.1) \cdot 83^2 \cdot \frac{(1/8) \cdot (7/8)}{7} + (1 - 0.1) \cdot 149^2 \cdot \frac{(3/15) \cdot (12/15)}{14} \right) \right] \approx$$

0.0027

➔ 95% konfidensintervall

$$0.187 \pm 1.96 \cdot \sqrt{0.0027} \approx 0.187 \pm 0.101 \approx (0.086, 0.288)$$

b) Upprepa beräkningarna, men med nya m_i och med justerad ändlighetskorrektion i steg 2, dvs. 1 – 0.05 ist.f. 1 – 0.1

$$\hat{p} = \frac{55}{5954 \cdot 3} \cdot \left(92 \cdot \frac{2}{5} + 83 \cdot \frac{1}{4} + 149 \cdot \frac{3}{7} \right) \approx 0.3738 \quad \left(\frac{55}{5954 \cdot 3} \cdot \left(92 \cdot \frac{2}{4} + 83 \cdot \frac{1}{4} + 149 \cdot \frac{3}{7} \right) \approx 0.4022 \right)$$

$$\hat{t}_{unb} = \frac{55}{3} \cdot \left(92 \cdot \frac{2}{5} + 83 \cdot \frac{1}{4} + 149 \cdot \frac{3}{7} \right) \approx 2225.80$$

$$\hat{V}(\hat{p}) = \frac{1}{5954^2} \cdot \left[55^2 \cdot \left(1 - \frac{3}{55}\right) \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \left(\left(92 \cdot \frac{2}{5} - \frac{2225.80}{55} \right)^2 + \left(83 \cdot \frac{1}{4} - \frac{2225.80}{55} \right)^2 + \left(149 \cdot \frac{3}{7} - \frac{2225.80}{55} \right)^2 \right) + \right.$$

$$\left. + \frac{55}{3} \cdot (1 - 0.05) \left(\cdot 92^2 \cdot \frac{(2/5) \cdot (3/5)}{4} + 83^2 \cdot \frac{(1/4) \cdot (3/4)}{3} + 149^2 \cdot \frac{(3/7) \cdot (4/7)}{6} \right) \right] \approx 0.0137$$

➔ 95% konfidensintervall

$$0.3738 \pm 1.96 \cdot \sqrt{0.0137} \approx 0.374 \pm 0.229 \approx (0.145, 0.603)$$

4. a) Kvotskattning:

$$\hat{\bar{y}}_r = \hat{B} \cdot \bar{x}_U = \frac{3655.89}{249721} \cdot \frac{135730000}{381960} \approx 5.202 = 5202 \text{ kronor}$$

➔ 95% konfidensintervall:

$$\hat{V}(\hat{\bar{y}}_r) = \left(1 - \frac{n}{N}\right) \cdot \left(\frac{\bar{x}_U}{\bar{x}} \right)^2 \cdot \frac{1}{n} \cdot \frac{1}{n-1} \cdot \sum_{i \in S} (y_i - \hat{B} x_i)^2 =$$

$$= \left(1 - \frac{n}{N}\right) \cdot \left(\frac{\bar{x}_U}{\bar{x}} \right)^2 \cdot \frac{1}{n} \cdot \frac{1}{n-1} \cdot \left(\sum_{i \in S} y_i^2 + \hat{B}^2 \cdot \sum_{i \in S} x_i^2 - 2 \cdot \hat{B} \cdot \sum_{i \in S} y_i \cdot x_i \right) =$$

$$= \left(1 - \frac{700}{381960}\right) \cdot \left(\frac{135730000/381960}{249721/700} \right)^2 \cdot \frac{1}{700} \cdot \frac{1}{699} \cdot \left(23881.3 + \left(\frac{3655.89}{249721} \right)^2 \cdot 92032277 - \right.$$

$$\left. - 2 \cdot \frac{3655.89}{249721} \cdot 1400947 \right) \approx 0.005236$$

⇨ $5.202 \pm 1.96 \cdot \sqrt{0.005236} \approx 5.202 \pm 0.142 = (5060, 5344)$ kronor

b) Regressionsskattning:

$$\hat{\bar{y}}_{reg} = \bar{y} + \hat{B}_1 \cdot (\bar{x}_U - \bar{x})$$

$$\hat{B}_1 = \frac{\sum_{i\in S}(x_i-\bar{x})\cdot(y_i-\bar{y})}{\sum_{i\in S}(x_i-\bar{x})^2} = \frac{\sum_{i\in S} x_i\cdot y_i - \frac{\left(\sum_{i\in S} x_i\right)\cdot\left(\sum_{i\in S} y_i\right)}{n}}{\sum_{i\in S} x_i^2 - \frac{\left(\sum_{i\in S} x_i\right)^2}{n}} =$$

$$= \frac{1400947 - 248721\cdot 3655.89/700}{92032277 - 248721^2/700} \approx 0.0279$$

$$\Rightarrow \hat{\bar{y}}_{reg} = \frac{3655.89}{700} - 0.0279\cdot\left(\frac{135730000}{381960} - \frac{248721}{700}\right) \approx 5.222 = 5222 \text{ kronor}$$

95% konfidensintervall:

$$SE\left(\hat{\bar{y}}_{reg}\right) = \sqrt{\left(1-\frac{n}{N}\right)\cdot\frac{1}{n}\cdot s_y^2\cdot\left(1-r^2\right)}$$

$$s_y^2 = \frac{\sum_{i\in S} y_i^2 - \frac{\left(\sum_{i\in S} y_i\right)^2}{n}}{n-1} = \frac{23881.3 - 3655.89^2/700}{699} \approx 6.8493$$

$$r^2 = \frac{\sum_{i\in S} x_i\cdot y_i - \frac{\left(\sum_{i\in S} x_i\right)\cdot\left(\sum_{i\in S} y_i\right)}{n}}{\sqrt{\left(\sum_{i\in S} x_i^2 - \left(\sum_{i\in S} x_i\right)^2/n\right)\cdot\left(\sum_{i\in S} y_i^2 - \left(\sum_{i\in S} y_i\right)^2/n\right)}} =$$

$$= \frac{1400947 - 248721\cdot 3655.89/700}{\sqrt{\left(92032277 - 248721^2/700\right)\cdot\left(23881.3 - 3655.89^2/700\right)}} \approx 0.7704$$

$\Rightarrow$

$$5.222 \pm 1.96\cdot\sqrt{\left(1-\frac{700}{381960}\right)\cdot\frac{1}{700}\cdot 6.8493\cdot\left(1-0.7704^2\right)} \approx 5.222 \pm 0.123 =$$

(5099,5345) kronor