Message Passing Simplicial Network on SRG

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Bootcamp_2

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Contents

Introduction

Problem and Approach

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Simplex, WL

- Informally, a simplex is a topological structure on graphs.
  - A zero-simplex is a vertex;
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- A simplex can have an orientation but not for SRG.

Simplicial WL Test

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Simplicial WL Algorithm

1. Given a simplicial complex $\mathbb{K}$, all the simplices $\sigma \in \mathbb{K}$ are initialised with the same colour.
2. Given the colour c_σ^t of simplex σ at iteration t, compute c_σ^{t+1} by perfectly hashing the multi-set of colours belonging to the adjacent simplices of σ.
3. The algorithm stops once a stable colouring is reached. If the colour histogram is not the same, the there's no isomorphism.

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- The model does message aggregation on the different boundary adjacency conditions of simplices in $\mathbb{K}$.
- Just as in k-WL tests, the update operation takes into account the different types of incoming messages and the previous colour of the simplex and then hash the multi-set of colours.

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Data and Implementation

- Strongly regular graphs of at most 35 vertices were studied.
- Implementation incomplete...

References

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