Message Passing Simplicial Network on SRG

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Problem and Approach

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Consider local higher-order interactions - simplicial complexes.

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Simplex, WL

- Informally, a simplex is a topological structure on graphs.
  - A zero-simplex is a vertex;
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- Formally, a simplicial complex is a collection of nonempty subsets of a vertex set V that contain all the singleton subsets of V and is closed under the operation of taking subsets.
- A simplex can have an orientation but not for SRG.

Simplicial WL Test

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This test is built from the WL test to derive a message-passing procedure that can retain the expressive power of the test[1].

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Simplicial WL Algorithm

1. Given a simplicial complex $\mathbb{K}$, all the simplices $\sigma \in \mathbb{K}$ are initialised with the same colour.
2. Given the colour c_σ^t of simplex σ at iteration t, compute c_σ^{t+1} by perfectly hashing the multi-set of colours belonging to the adjacent simplices of σ.
3. The algorithm stops once a stable colouring is reached.If the colour histogram is not the same, the there's no isomorphism.

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- The model does message aggregation on the different boundary adjacency conditions of simplices in $\mathbb{K}$.
- ust as in k-WL tests, the update operation takes into account the different types of incoming messages and the previous colour of the simplex and then hash the multi-set of colours.

Data and Implementation

- Strongly regular graphs of at most 35 vertices were studied.
- The code is incomplete, for not many failure rates were registered.

References

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