xample

Sketch root locus for

$$s(s+3)(s^2+2s+2) + K = 0.$$

$S \quad S$

$\quad\;\; S^2$

$$G(s) = \frac{K}{s(s+3)(s^2+2s+2)}$$

Sol: Step 1: Find the Poles and zeros.

Poles $= 0, -3, -1+j, -1-j$

zeros = none

→ Number of poles = 4

Number of zeros = 0

Step 2 : Find Centroid:

$$-\sigma_m = \frac{\sum \text{poles} - \sum \text{zeros}}{\text{Poles} - \text{zeros}}$$

$$= \frac{\sum(0-3-1-1) - \sum(0)}{4-0}$$

$$= \frac{-5}{4} = -1.25$$

Step 3 : Find angles of asymptotes $= \frac{(2q+1)180^\circ}{\text{no. of poles} - \text{no. of zeroes}}$

q = number of poles − zeroes − 1

$$\boxed{q = 0, 1, 2, 3}$$

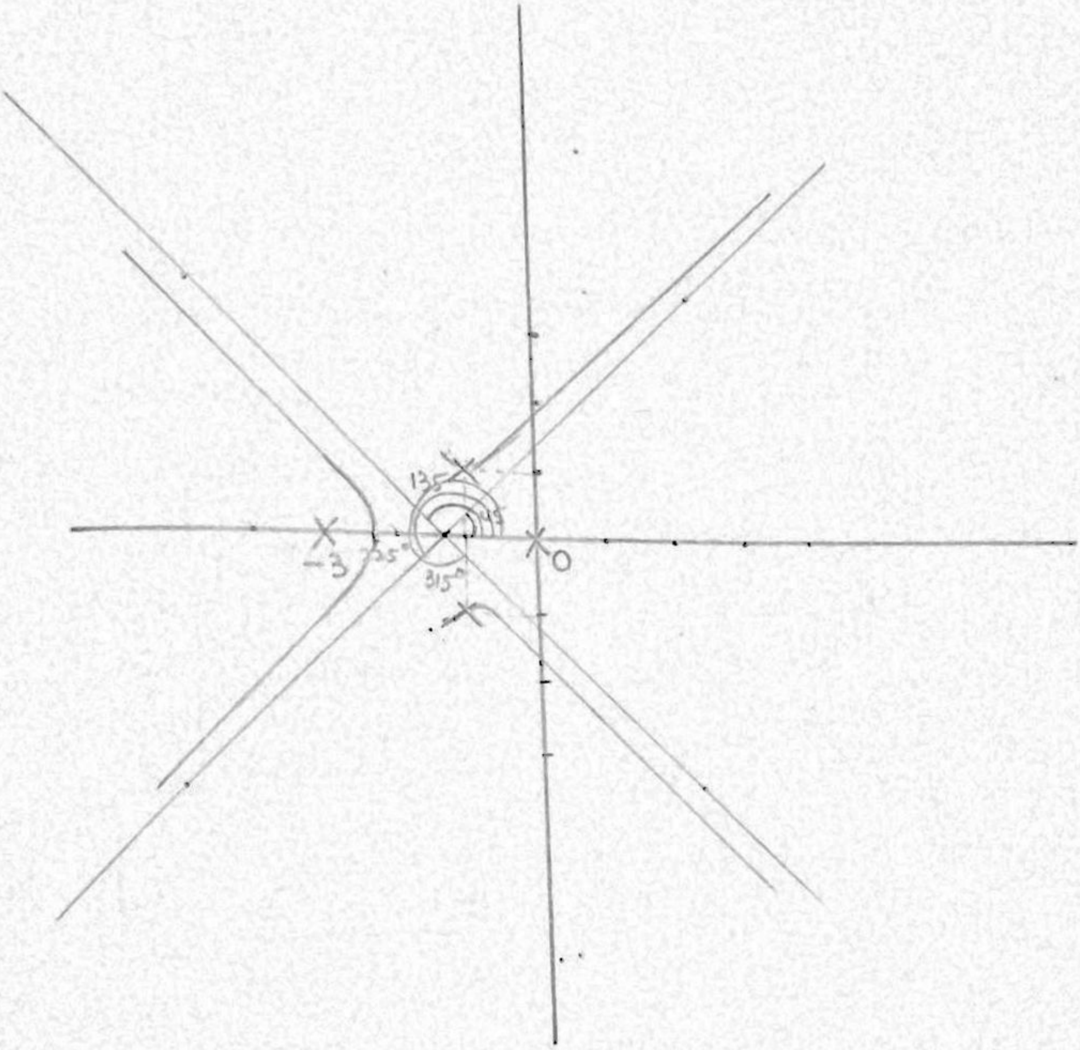
$q = 0$ then $\frac{(2(0)+1)180}{4} = 45^\circ$.

$q = 1$ then $\frac{(2(1)+1)180}{4} = 135^\circ$

$q = 2$ then $\frac{(2(2))+1 \times 180}{4} = 225^\circ$

$q=3$ then $\frac{(2(3)+1)\times 180}{4} = 315^\circ$

Step 4: Root locus always start from pole and end at zero.



Step 5: Find breakaway point.

characteristic eqn $= 1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+3)(s^2+2s+2)} = 0$$

$$1 + \frac{K}{s^4 + 5s^3 + 8s^2 + 6s} = 0$$

$$s^4 + 5s^3 + 8s^2 + 6s + K = 0$$

$$K = -[s^4 + 5s^3 + 8s^2 + 6s]$$

$$\frac{dK}{ds} = -[4s^3 + 15s^2 + 16s + 6]$$

or $4s^3 + 15s^2 + 16s + 6 = 0$

$s = -2.28$; $-0.73 + 0.34j$; $-0.73 - 0.34j$

breakaway point is approximated at -2.28.

Step 6 : To find out imaginary axis crossover through Routh Hurwitz table.

characteristic eqn => $s^4 + 5s^3 + 8s^2 + 6s + K = 0$.

$$\begin{array}{c|ccc}
s^4 & 1 & 8 & K \\
s^3 & 5 & 6 & 0 \\
s^2 & 34/5 & K & 0 \\
s^1 & \frac{-25K+204}{34} & 0 & 0 \\
s^0 & K & 0 & 0
\end{array}$$

$1 \times 6 -$

$8 \times 5 - 6$

$$\frac{-25K+204}{34} = 0.$$

$$25K = 204$$

$$K = \frac{204}{25}.$$

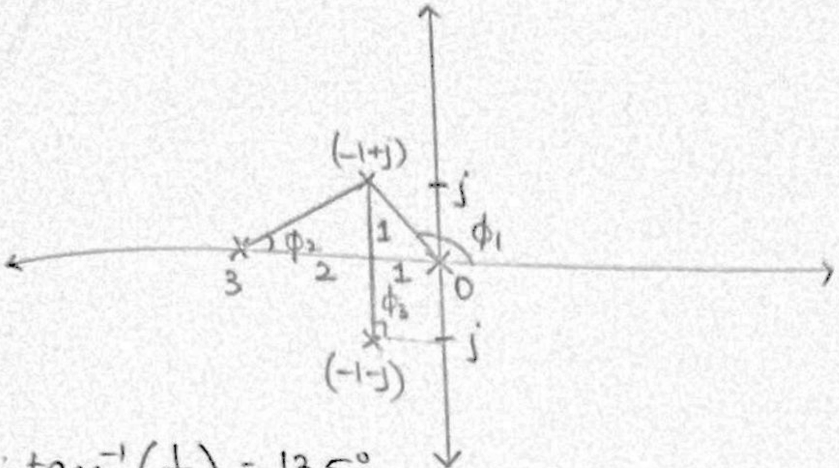
Now auxillary eqn

$$\frac{34}{5}s^2 + \frac{204}{25} = 0.$$

$$s^2 = -\frac{204}{25} \times \frac{5}{34} = -1.2$$

$$s = \pm 1.09j$$

step 7: Angle of departure



$\phi_1 = 180 - \tan^{-1}\left(\frac{1}{1}\right) = 135^\circ$

$\phi_3 = 90^\circ$

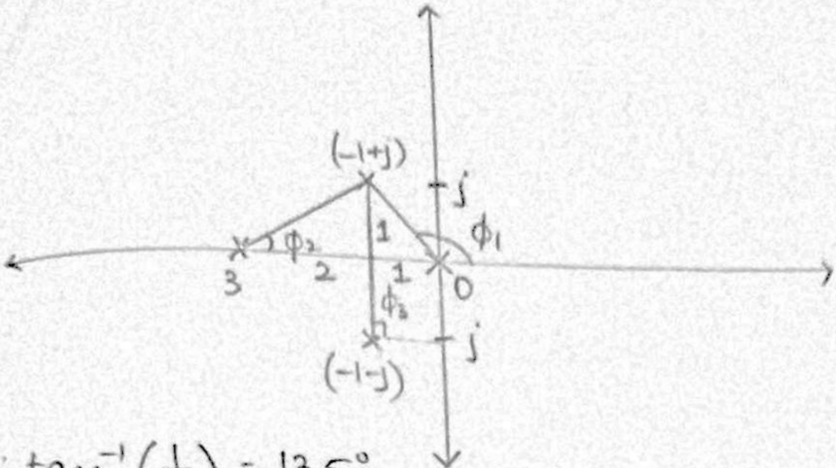
$\phi_2 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56^\circ$

angle of departure $= 180^\circ - (\phi_1 + \phi_2 + \phi_3)$

$= 180^\circ - (90 + 135 + 26.56)$

$= -71.56^\circ$.

step 7: Angle of departure



$\phi_1 = 180 - \tan^{-1}\left(\frac{1}{1}\right) = 135°$

$\phi_3 = 90°$

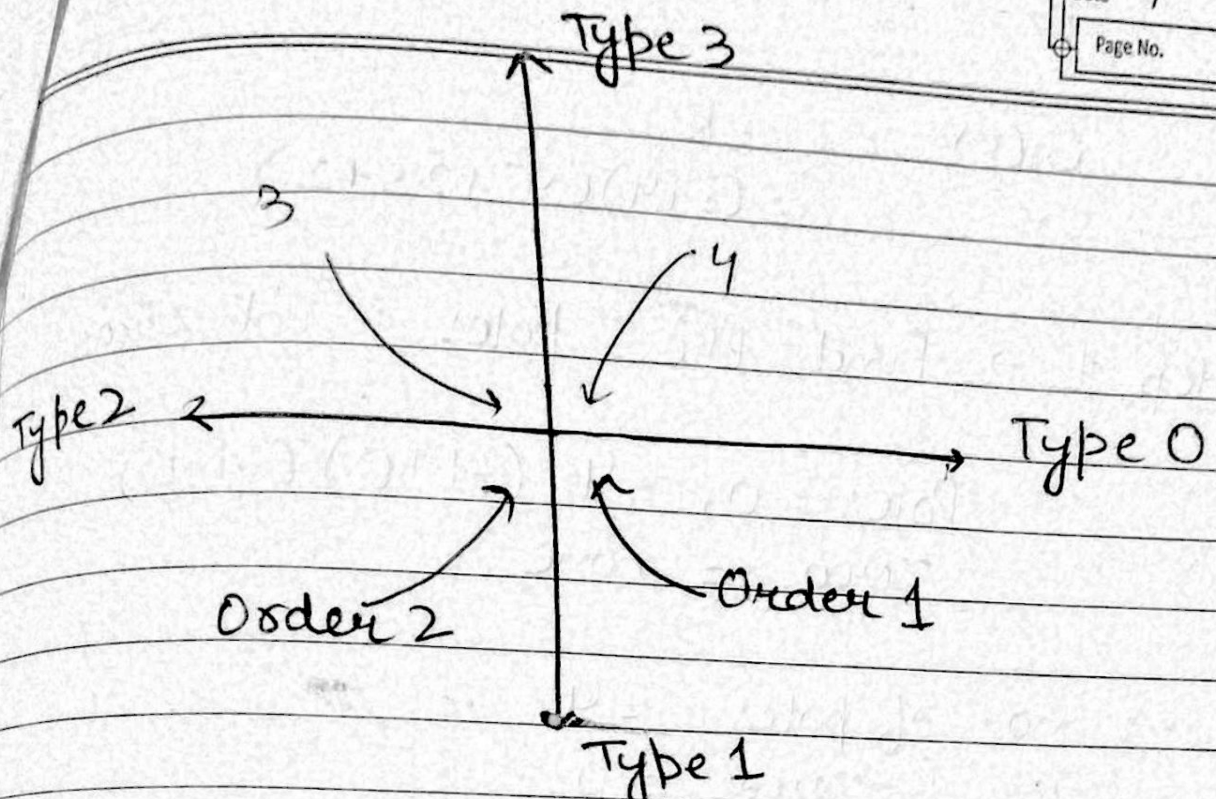
$\phi_2 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56°$

angle of departure $= 180° - (\phi_1 + \phi_2 + \phi_3)$

$= 180° - (90 + 135 + 26.56)$

$= -71.56°$.

Date / /
Page No.



Type = starting point
order = end point

$$G(s) = \frac{K}{s(s+4)(s^2+2s+2)}$$

Step 1 → Find the poles and zeros

Poles = 0, −4, (−1+i) (−1−i)
zero = none

→ no. of poles = 4
" " zeros = 0.

Step 2 → Find centroid.

$$\sigma_m = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{\text{no. of poles} - \text{no. of zeros}}$$

$$= \frac{\Sigma(0-4-1-1) - \Sigma 0}{4-0}$$

$$= \frac{-6}{4} = -1.5$$

Step 3
angle of asymptodes $= \frac{(2q+1)180°}{\text{no. of poles} - \text{no. of zeros}}$

q = (poles − zeros − 1)
q = 0, 1, 2, 3

$$= \frac{(2q+1)180}{4}$$

$q=0$ ⓘ $= 45° = \frac{180}{4}$

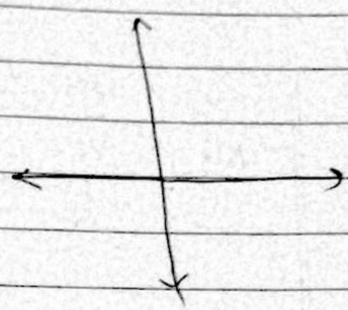
$q=1$ ② $= \frac{3\times180^\circ}{4} = 135^\circ$

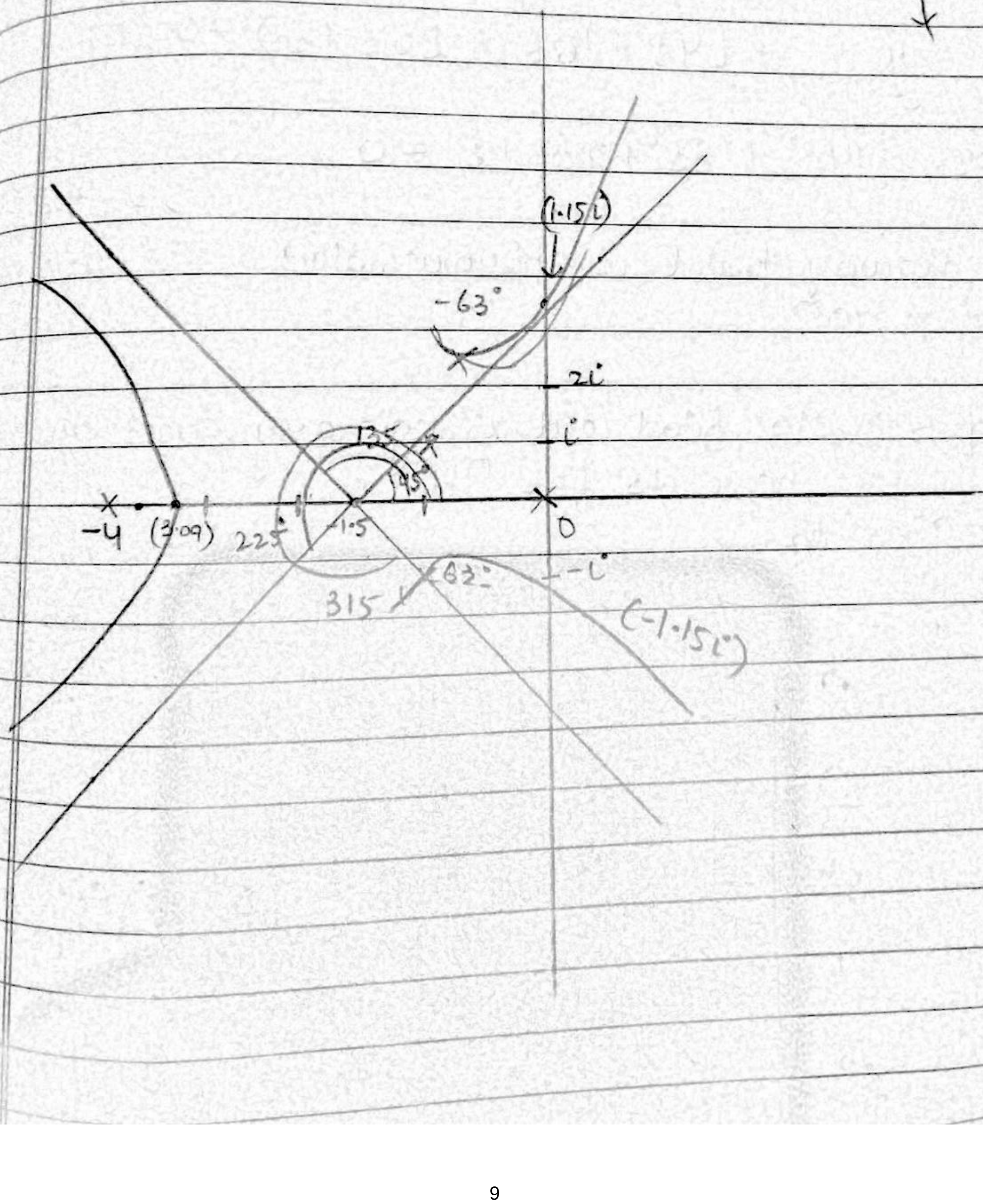
$q=2$ ③ $= \frac{5\times180^\circ}{4} = 225^\circ$

$q=3$ ④ $= 7\times\frac{180}{4} = 315^\circ$

step 4.

root locus start at pole and ends at zero (x⟶•)





Step 5

breakaway point —

charac eqn $= 1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+4)(s^2+2s+2)} = 0$$

or $s^4 + 2s^3 + 2s^2 + 4s^3 + 8s^2 + 8s + K = 0$

Now

$$K = -[s^4 + 6s^3 + 10s^2 + 8s]$$

$$\frac{dk}{ds} = -[4s^3 + 18s^2 + 20s + 8) = 0$$

or $4s^3 + 18s^2 + 20s + 8 = 0$

$s = -3.09$

$= -0.70 + 0.38j$

$= \quad " - "$

breakaway point is approximated at -3.09

Step 6 → To find out imaginary axis crossover we have to for RH

Char eqn

$$s^4 + 6s^3 + 10s^2 + 8s + K = 0$$

$$\frac{8 - 60}{6}$$

s^4	1	10	K
s^3	6	8	0
s^2	$\frac{26}{3}$	K	0
s^1	$\frac{\frac{208}{3} - 6K}{26/3}$	0	0
s^0	K	0	0.

$(s^2+3s)(s^2+2s+2)$

$s^4 + 2s^3 + 2s^2 + 3s^3$

$+ 6s^2 + 6s$

$s^4 + 5s^3 + 8s^2 + 6s$

$$\Rightarrow \frac{200}{3} - 6K = 0$$

$$\boxed{K = \frac{200}{18}}$$

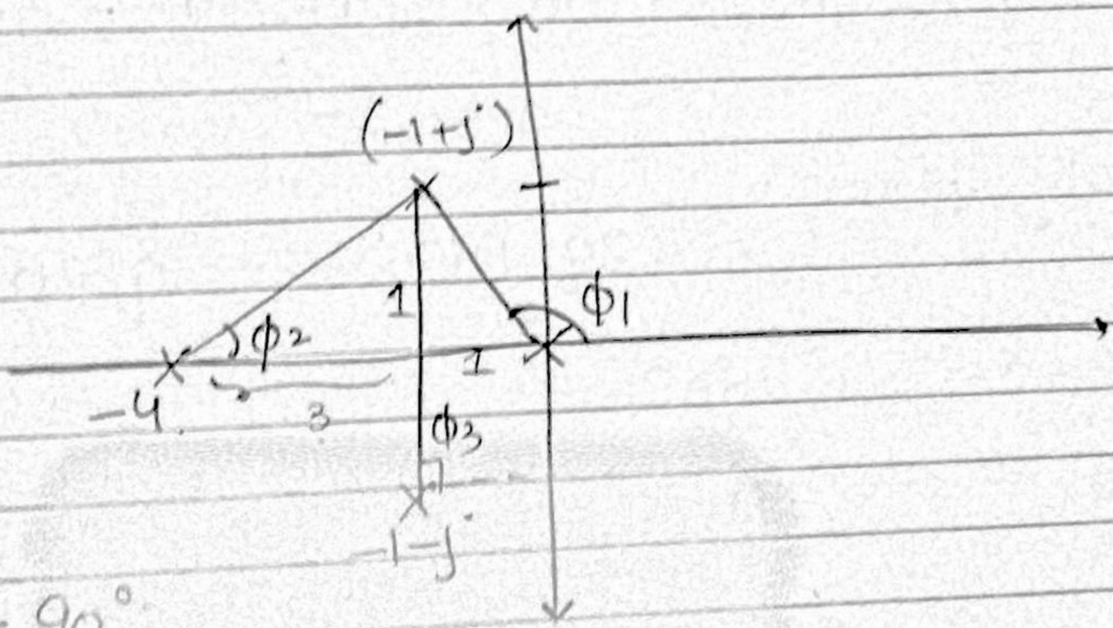
Now auxillary eqn.

$$\frac{26}{3}s^2 + K = 0$$

$$s^2 = \frac{-3K}{26} = -\frac{3}{26} \times \frac{200}{18} = -\frac{4}{3}$$

$$s = \pm j\frac{2}{\sqrt{3}} = \pm 1.15i$$

Step 7 angle of departure



$$\phi_3 = 90^\circ$$

$$\phi_1 = 180^\circ - \tan\left(\frac{1}{1}\right) = 135^\circ$$

$$\phi_2 = \tan^{-1}\left(\frac{1}{3}\right) = 18.4$$

Date / /
Page No.

angle of departure

$$180° - (\phi_1 + \phi_2 + \phi_3)$$

$$180 - (90° + 135° + 18.4)$$

$$= -63°$$

Step 8

Q2 $G(s) = \frac{K}{s(s+2)(s+4)}$

Step 1 poles $= 0, -2, -4 = (3)$
zeros $=$ none

step 2 find centroid $= \frac{\Sigma(0-2-4) - \Sigma(0)}{3}$

$$= -2$$

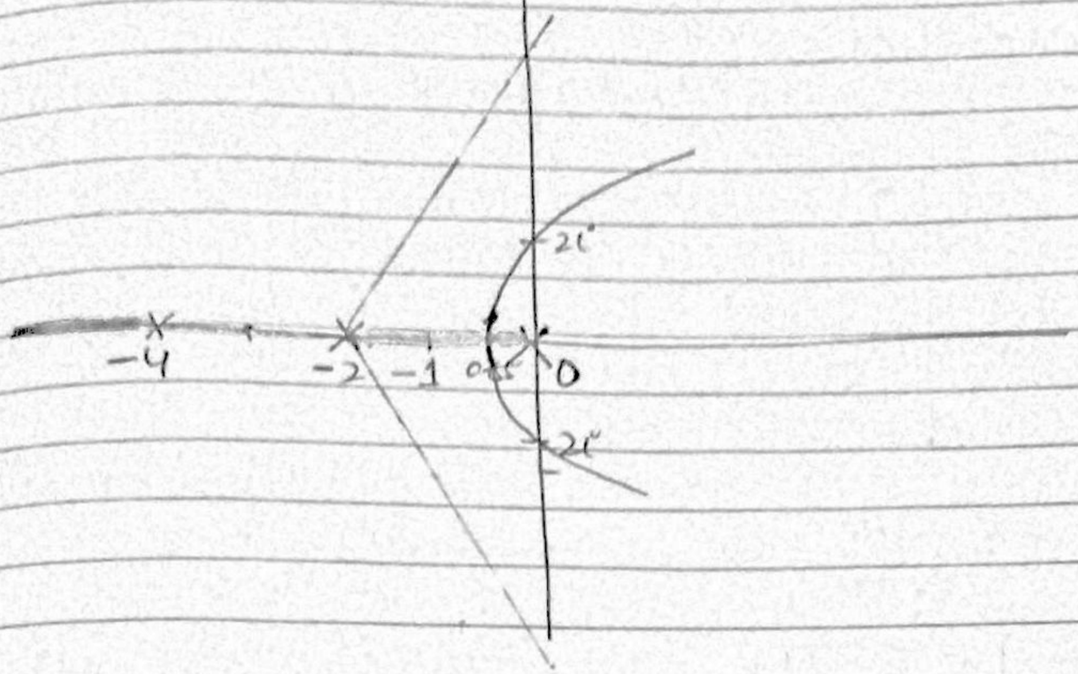
asymptotes,

$$\frac{(2q+1)180°}{3} \qquad q = 0, 1, 2$$

$q=0$ $60°$

$q=1$ $180°$

$q=2$ $300°$



step 5 breakaway point

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$s^3 + 6s^2 + 8s + K = 0 \quad \text{— (1)}$$

$$K = -[s^3 + 6s^2 + 8s]$$

$$\frac{ds}{dK} = -[3s^2 + 12s + 8] = 0$$

$$3s^2 + 12s + 8 = 0$$

$$s = -0.84 ; -3.15$$

Step 6
find out imaginary axis crossover.

char eqn $s^3 + 6s^2 + 8s + K = 0$

s^3	1 8
s^2	6 K
s	$\frac{24-K}{6}$ 0
s^0	K.

$\therefore \frac{24-K}{6} = 0.$

$\boxed{K = 24}$

$6s^2 + 24 = 0.$

$s^2 = -4$

$s = \pm 2i$

capacitor

$S = +20 \quad \frac{1}{S} = -20$

$S^2 = 40$

$S^3 = 60 \quad \frac{1}{S^2} = -40$

Date / / Page No.

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{10}{j\omega(1+0.4j\omega)(1+0.1j\omega)}$$

magnitude plot

Corner frequencies $\quad \omega_l = 0.1$

$\omega_{c1} = \frac{1}{0.4} = 2.5\ rad/s$

$\omega_{c2} = \frac{1}{0.1} = 10\ rad/s$

$\omega_h = 100$

Term	Corner frequency	slope	change in slope
$\frac{10}{j\omega}$	—	-20	—
$\frac{1}{1+0.4j\omega}$	2.5	-20	$-20-20=-40$
$\frac{1}{1+0.1j\omega}$	10	-20	-60

magnitude

when $\omega = \omega_l \quad A = 20\log\left|\frac{10}{j\omega}\right| \Rightarrow 20\log\left|\frac{10}{0.1}\right| = 40db$

$\omega = \omega_{c1} \quad A = 20\log\left|\frac{10}{j\omega}\right| \Rightarrow 20\log\left|\frac{10}{2.5}\right| = 12\ dB$

Date / /
Page No.

$\omega = \omega_{c2}$ $A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}}\right] + A_{\omega c1}$

$A = -40 \times \log \left| \frac{10}{2.5} \right| + 12 = -12\,dB$

$\omega_h = A = -60 \times \log \left| \frac{100}{10} \right| - 12 = -72\,dB$

0.1 1