Designing via Root Locus
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Motivation for Design

The root locus method is a powerful tool for analyzing and designing control systems. It allows us to understand the behavior of a system as its poles move across the complex plane with varying parameter values. By examining the root locus plot, we can make informed decisions to improve the system's performance, stability, and transient response.

Design Philosophy

When designing using the root locus method, we aim to achieve the desired system characteristics by carefully selecting the controller's parameters. The design philosophy involves the following key steps:

1. Determine the desired closed-loop system characteristics, such as settling time, overshoot, and stability margins.
2. Sketch the root locus plot for the open-loop transfer function.
3. Analyze the root locus plot to understand the system's behavior as the gain varies.
4. Adjust the system's poles by adding appropriate compensators.
5. Iteratively modify the compensators and re-analyze the root locus plot until the desired system response is achieved.

Types of Compensation Techniques

There are various compensation techniques used in root locus design to shape the desired system response. Some common types include:

1. **Proportional (P) Control:** Adding a gain term to the system transfer function to modify the system's response.
2. **Proportional-Integral (PI) Control:** Combining a gain term and an integrator to improve steady-state performance.
3. **Proportional-Derivative (PD) Control:** Combining a gain term and a derivative element to enhance transient response and stability.
4. **Lead Compensator:** Introducing a zero and a pole to shift the root locus towards desired locations, improving system response.
5. **Lag Compensator:** Introducing a pole and a zero to adjust the root locus and increase stability margins.
6. **Lead-Lag Compensator:** Combining both lead and lag compensators to achieve desired performance and stability.

Physical Realization of Compensation

Implementation of the active controllers and the passive compensators.

- Active-Circuit Realization
- Passive-Circuit Realization

Active-Circuit Realization

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} \tag{1}$$

- The transfer function of an inverting operational amplifier is given in eqn(1).
- By judicious choice Z_1 and Z_2, the circuit shown in figure 1 can be used as a building block to implement the compensators and controllers, such as PID controllers.

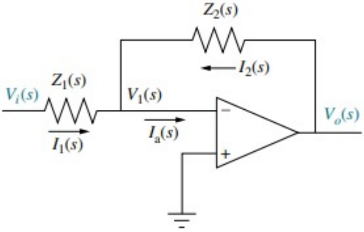

Figure: Operational amplifier configured for transfer function realization

Physical Realization of Compensation

- A lag-lead compensator can be formed by cascading the lag compensator with the lead compensator as shown in figure 2

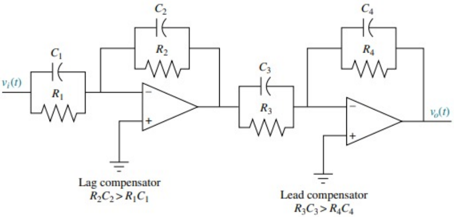

Figure: Lag-lead compensator implemented with operational amplifiers

Passive Circuit Realization

Lag, lead, and lag-lead compensators can also be implemented with passive networks

The lag-lead transfer function can be put in the following form:

$$G_c(s) = \frac{(s + \frac{1}{T_1})(s + \frac{1}{T_2})}{(s + \frac{1}{\alpha T_1})(s + \frac{\alpha}{T_2})} \quad (2)$$

- where $\alpha < 1$. Thus, the terms with T_1 form the lead compensator,and the terms with T_2 form the lag compensator.
- A possible realization using the passive networks uses an operational amplifier to provide isolation as shown in figure 3

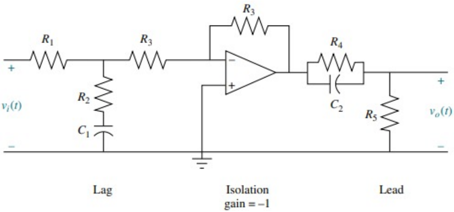

Figure: Lag-lead compensator implemented with cascaded lag and lead networks with isolation

Conclusion

Designing via root locus provides a systematic approach to control system design. By understanding the motivation, design philosophy, and types of compensation techniques, we can effectively shape the root locus and achieve the desired closed-loop system response.