QUANT
Concept Videos
Classification of Numbers

Recurring Decimals to Fractions

L C M   H C F

$\log_a x = \dfrac{\log_b x}{\log_b a}$

-5 -4 -3 -2 -1 0 1 2 3 4 5

2	18 24 36
2	9 12 18
3	9 6 9
3	3 2 3
	1 2 1

$\ln(x) = y$

$e^y = x$

$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

$\dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\log_6 36 = 2$

$x = 1.33333$

$10x = 13.333$

$9x = 1.2$

$9x = \dfrac{4}{3}$

Numbers
- Real Numbers
  - Rational
    - Fractions
    - Integers
      - Negative
      - Whole numbers
        - 0
        - Positive Integers
          - 1
          - Prime
          - Composite
  - Irrational
- Complex Numbers

2	4	7
3	8	5
6	1	9

Divisibility Rules Chart

2	if the last digit even (0, 2, 4, 6 or 8)
3	if the sum of the digits is divisible by 3
4	if the last two digits form a number divisible by 4
5	if the last digit is 0 or 5
6	if the number is divisible by both 2 and 3
9	if the sum of the digits is divisible by 9
10	if the last digit is 0

For any two positive integers A and B,
HCF (a,b) × LCM (a,b) = a × b

117 → 39, 3; 39 → 13, 3

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Classification of Numbers

Recurring Decimals to Fractions

1. $0.\overline{3} = \frac{3}{9} = \frac{1}{3}$

2. $0.\overline{63} = \frac{63}{99} = \frac{7}{11}$

Classification of Numbers

Recurring Decimals to Fractions

3. $0.1\overline{36} = \dfrac{1.\overline{36}}{10}$

$$= \frac{1 + \frac{36}{99}}{10}$$

$$= \frac{15}{11 \times 10}$$

$$= \frac{3}{22}$$

Classification of Numbers

Recurring Decimals to Fractions

4. $0.2\overline{6} = \dfrac{2 + 0.\overline{6}}{10}$

$$= \frac{2 + \frac{6}{9}}{10}$$

$$= \frac{8}{30}$$

$$= \frac{4}{15}$$

Classification of Numbers

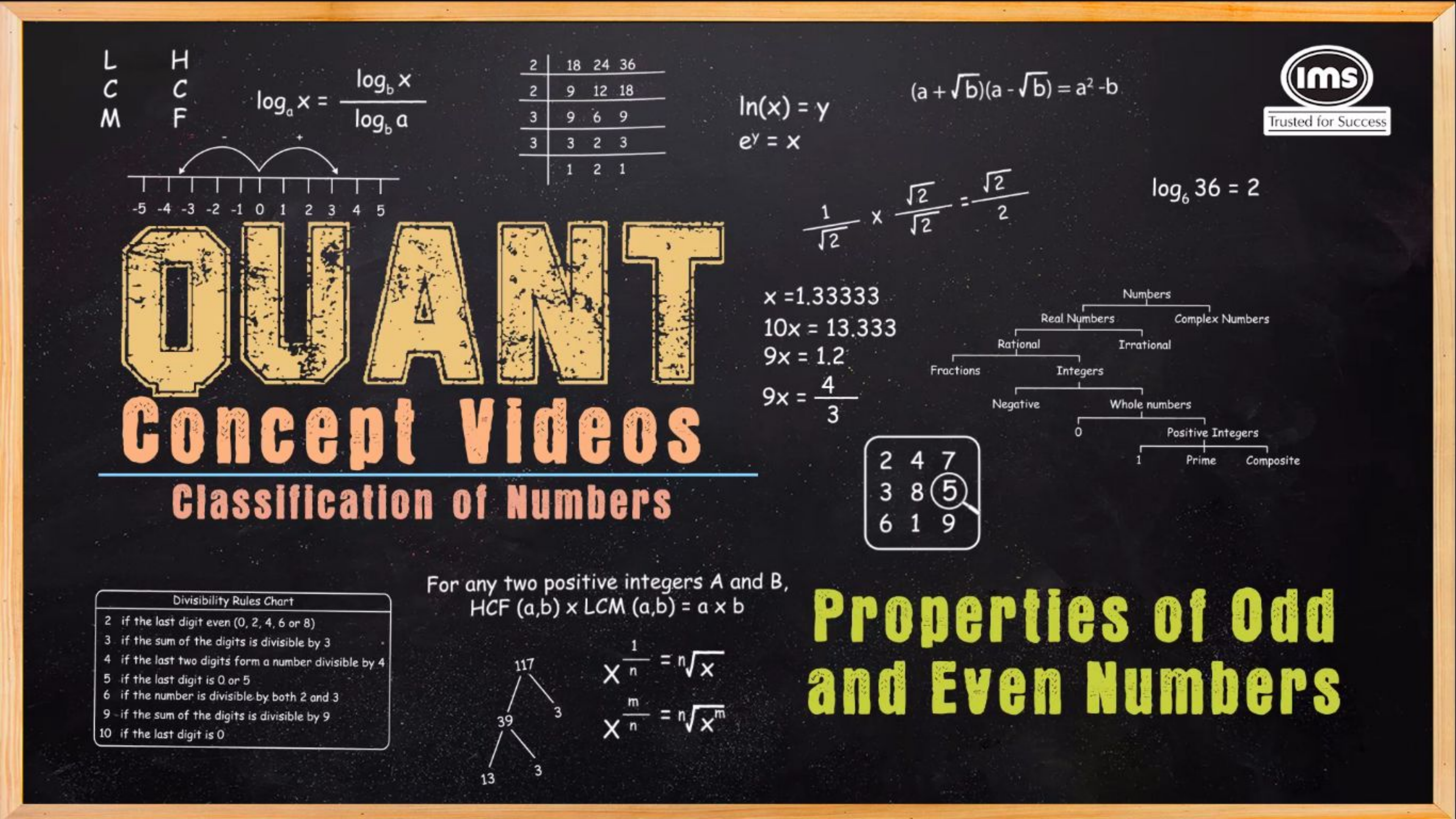
Recurring Decimals to Fractions

5. $5.2\overline{7} = \dfrac{52 + 0.\overline{7}}{10}$

$$= \frac{52 + \frac{7}{9}}{10}$$

$$= \frac{475}{90}$$

$$= \frac{95}{18}$$



QUANT
Concept Videos
Classification of Numbers
Properties of Odd and Even Numbers
Trusted for Success
L C M
H C F
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Numbers

Properties of Odd and Even Numbers

p	q	$p + q$	$p - q$	pq	p^q
Odd	Odd	Even	Even	Odd	Odd
Odd	Even	Odd	Odd	Even	Odd
Even	Odd	Odd	Odd	Even	Even
Even	Even	Even	Even	Even	Even

Numbers

Properties of Odd and Even Numbers

Sum and Difference:

Odd + Even = Odd;

Even + Even = Even; Odd + Odd = Even

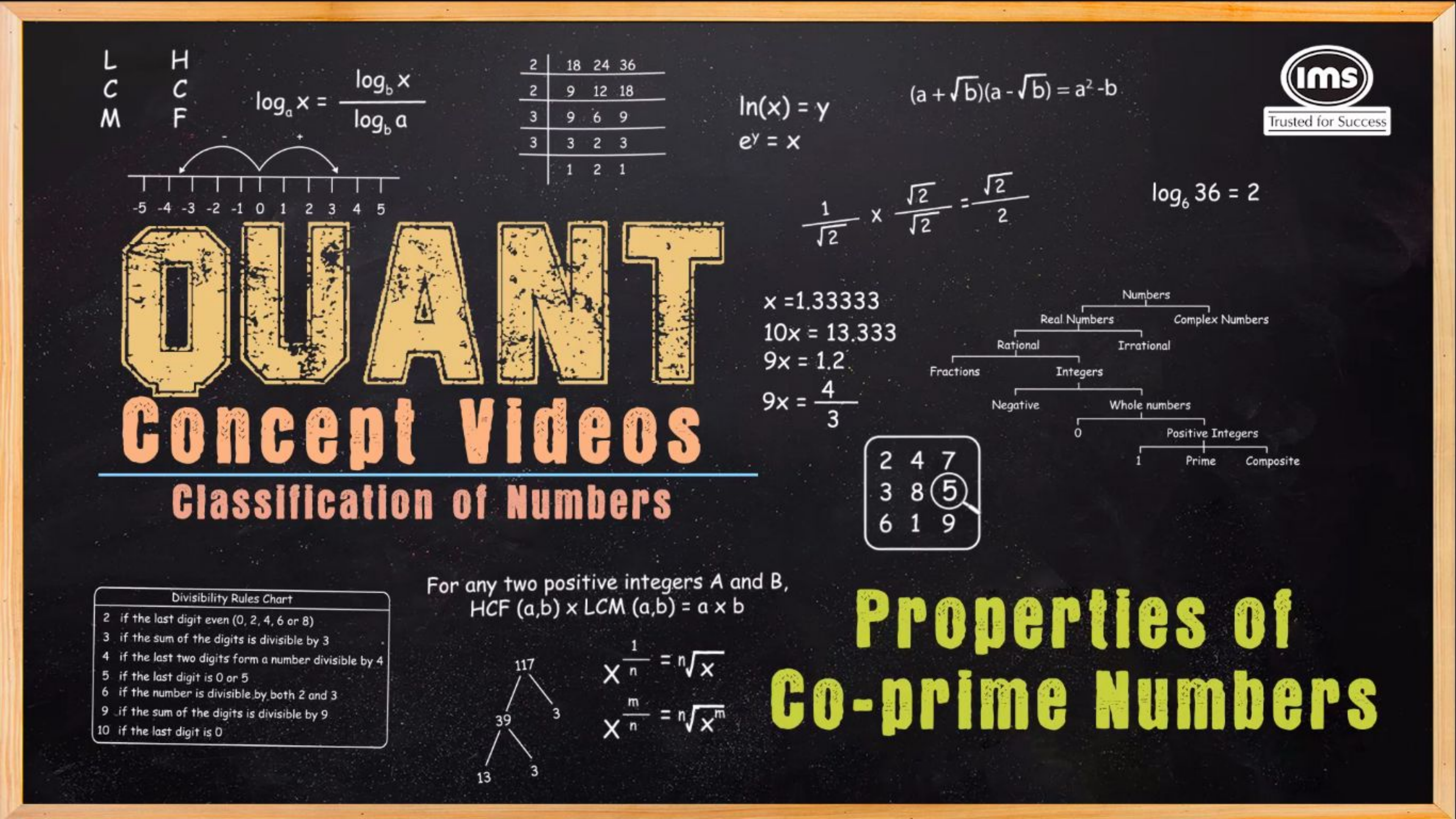
Product:

Odd × Odd = Odd;

Even × Odd = Even; Even × Even = Even

Exponent (p^q):

Always depends on the value of the base.


QUANT
Concept Videos
Classification of Numbers
Properties of Co-prime Numbers
IMS
Trusted for Success
LCM HCF
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Classification of Numbers

Relatively Prime or Co-Prime Numbers

Two or more numbers having 1 as their only common factor are termed as relatively prime or co-prime numbers.

Following are some important points to note:

- Two prime numbers are always relatively prime
- Two consecutive numbers are always relatively prime
- HCF of two relatively prime numbers is 1
- LCM of two relatively prime numbers is the product of the two numbers

Classification of Numbers

Relatively Prime or Co-Prime Numbers

Example 1

Number of ways in which a natural number can be written as a product of two co-prime numbers

$$
\begin{aligned}
90 &= 1 \times 90 \\
&= 2 \times 45 \\
&= 3 \times 30 \ ✘ \\
&= 5 \times 18 \\
&= 6 \times 15 \ ✘ \\
&= 9 \times 10
\end{aligned}
$$

Classification of Numbers

Relatively Prime or Co-Prime Numbers

Example 1

For any number N, if N has ‘m’ prime factors, then the number of ways in which N can be written as a product of two co-primes is equal to 2^{m-1}.

$90 = 2^1 \times 3^2 \times 5^1$

$m = 3$

Number of ways $= 2^{3-1} = 2^2$

$= 4$

Classification of Numbers

Relatively Prime or Co-Prime Numbers

Example 2

Find the smallest natural number which can be written as a product of two co-primes in 8 ways.

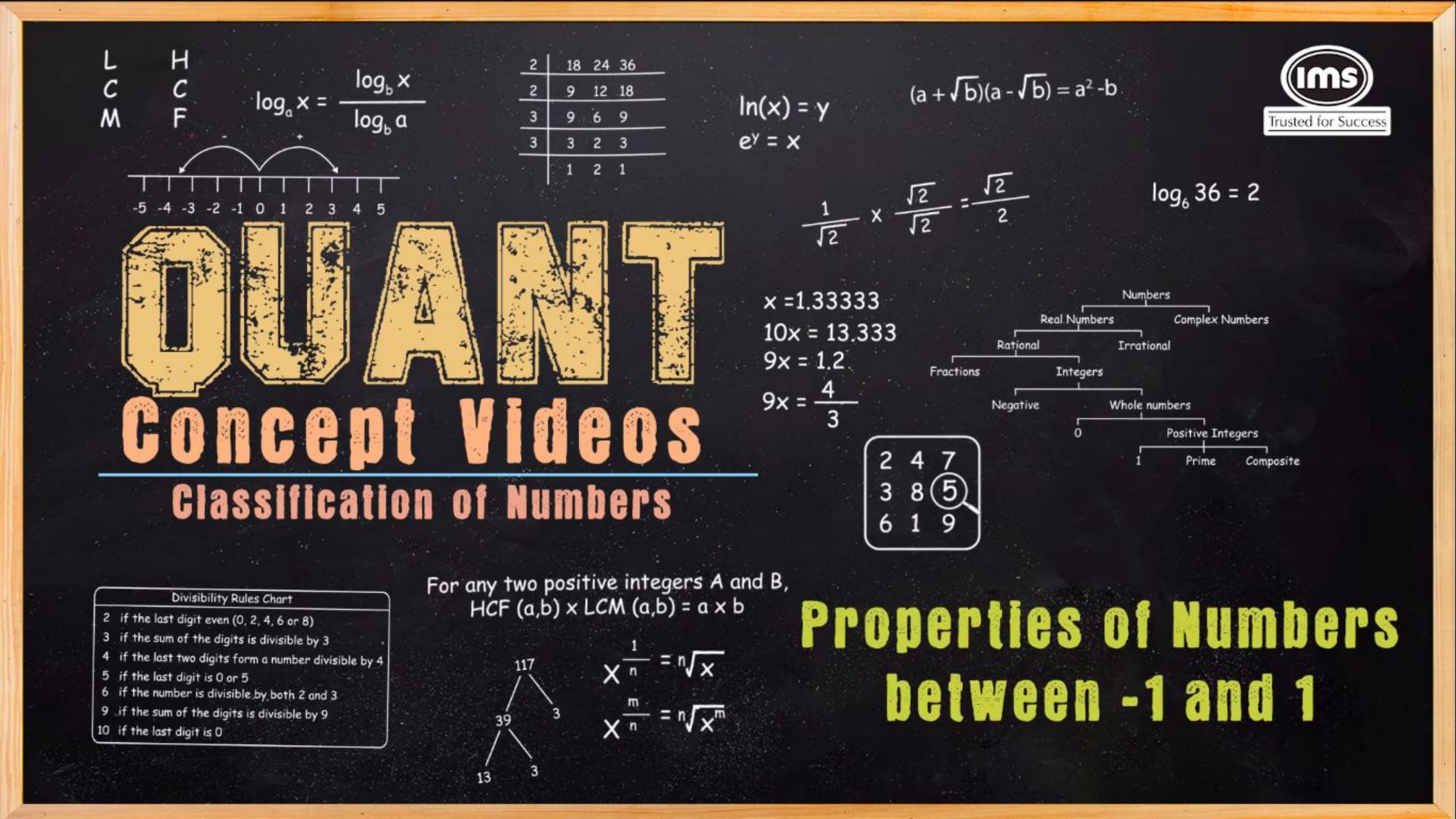
$2^{m-1} = 8$

$\quad\;\; = 2^3$

$\therefore m = 4$

Hence, the smallest natural number is:

$\Rightarrow 2 \times 3 \times 5 \times 7 = 210$



QUANT
Concept Videos
Classification of Numbers
Properties of Numbers between -1 and 1
Trusted for Success
L C M
H C F
log_a x = log_b x / log_b a
ln(x) = y
e^y = x
(a + √b)(a - √b) = a² - b
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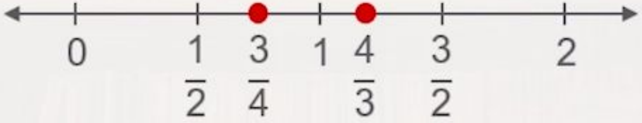
Classification of Numbers

Properties of numbers between – 1 and 1

1. The reciprocal of a number between 0 and 1 is greater than the number itself.

   Example 2:



$$x = \frac{3}{4}$$

$$\frac{1}{x} = \frac{4}{3} > \frac{3}{4}$$

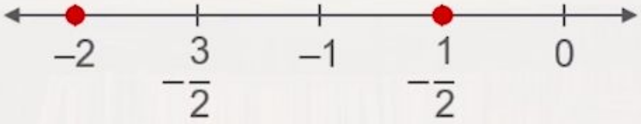
$$\frac{1}{x} > x \qquad [0 < x < 1]$$

Classification of Numbers

Properties of numbers between – 1 and 1

2. The reciprocal of a number between –1 and 0 is less than the number itself.

Example 1:



$$x = -\frac{1}{2}$$

$$\frac{1}{x} = -2 < -\frac{1}{2}$$

Classification of Numbers

Properties of numbers between – 1 and 1

3. The square of a number between 0 and 1 is less than the number itself.

Example 1:

$x = \frac{1}{2}$

$x^2 = \frac{1}{4} < \frac{1}{2}$

$x^2 < x \qquad [0 < x < 1]$

Classification of Numbers

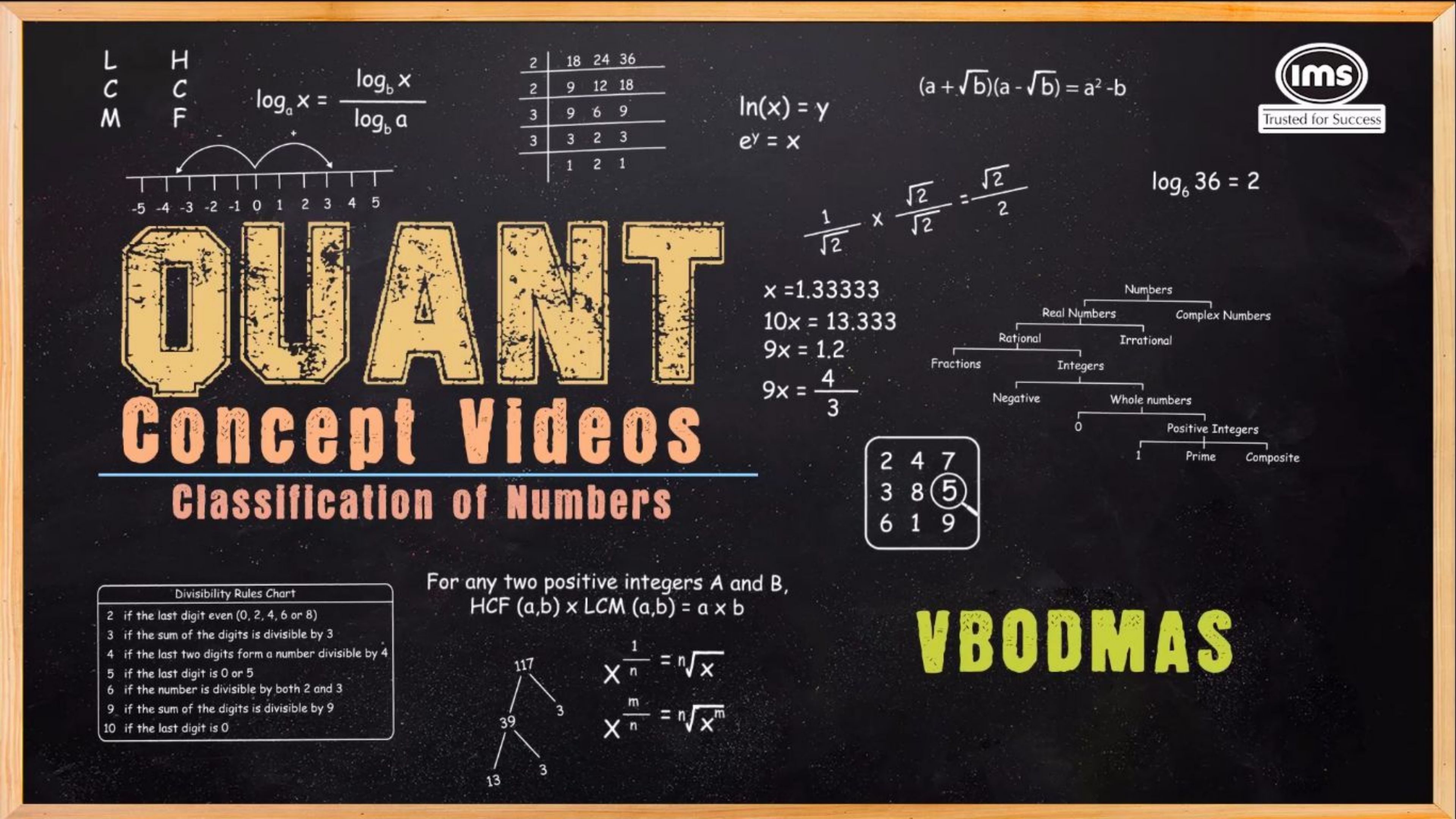
Properties of numbers between – 1 and 1

4. The cube of a number between –1 and 0 is always greater than the number itself.

Example:

$x = -\frac{1}{2}$

$x^3 = -\frac{1}{8}$



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Classification of Numbers
IMS
Trusted for Success
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117
39
3
13
3
$x^{\frac{1}{n}} = \sqrt[n]{x}$
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VBODMAS

Classification of Numbers

VBODMAS

VBODMAS is the rule applicable when a series of mathematical operations is required to be performed. It is an acronym for:

V - Vinculum (Bar)

B - Bracket ( ), { }, [ ]

O - Of

D - Division

M - Multiplication

A - Addition

S - Subtraction

Classification of Numbers

VBODMAS

Example 1

$$\left[\frac{1}{4} \text{ of } \frac{4}{3}\left\{(2 \times 3) + \overline{4 \times 5}\right\} + \frac{1}{3}\right]$$

$$= \left[\frac{1}{3}\{6 + 20\} + \frac{1}{3}\right]$$

$$= \frac{1}{3} \times 26 + \frac{1}{3}$$

$$= \frac{26}{3} + \frac{1}{3}$$

$$= \frac{27}{3}$$

$$= 9$$

Classification of Numbers

VBODMAS

Example 2

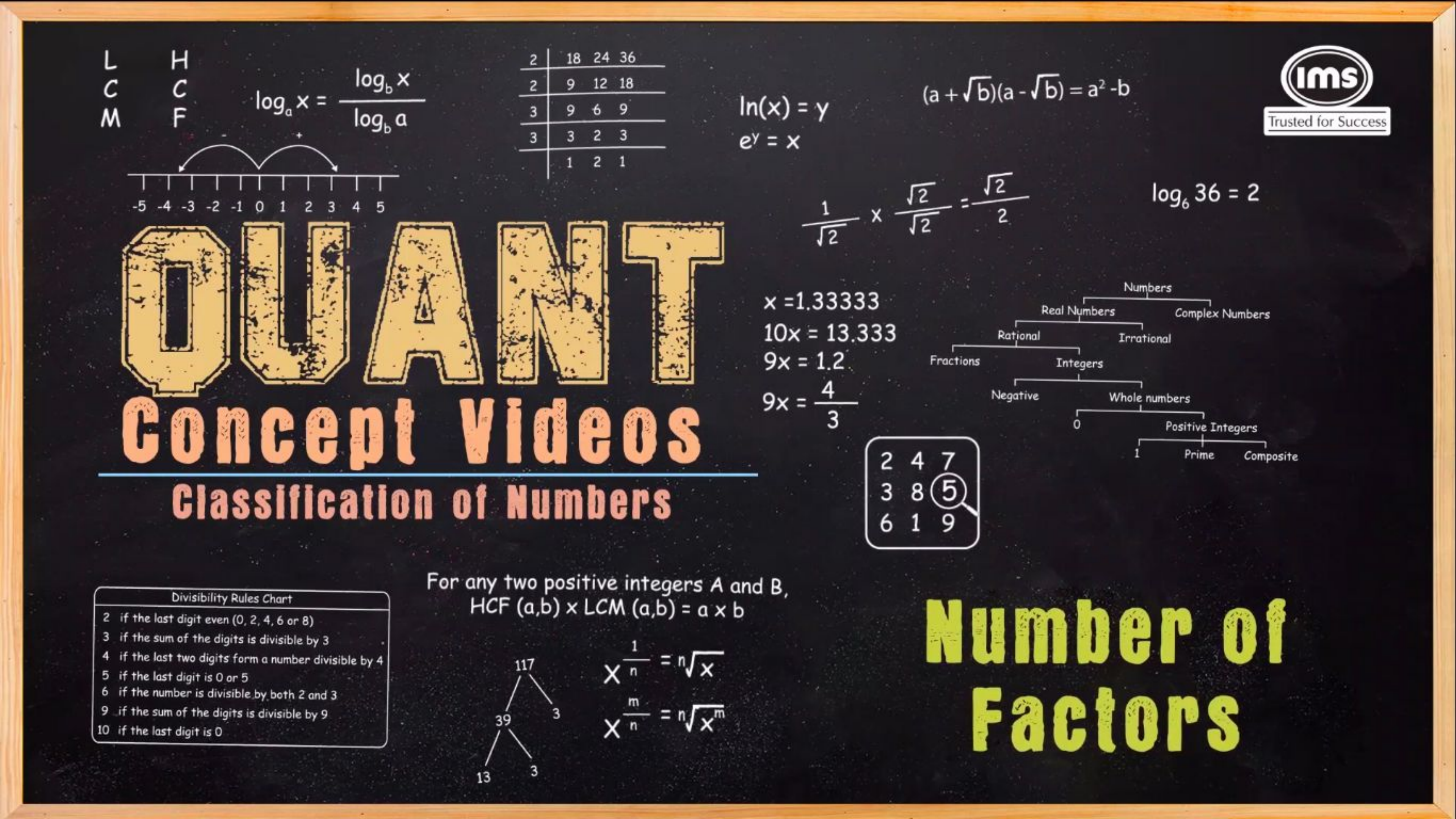
$$\frac{(1 + 3) \times 2 \div 4}{2 \times 2 - 2 \div 2 + 2}$$

$$= \frac{4 \times \frac{2}{4}}{2 \times 2 - 1 + 2}$$

$$= \frac{2}{4 - 1 + 2}$$

$$= \frac{2}{3+2}$$

$$= \frac{2}{5}$$


QUANT
Concept Videos
Classification of Numbers
Number of Factors
IMS
Trusted for Success
L C M
H C F
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1/√2 × √2/√2 = √2/2
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HCF (a,b) × LCM (a,b) = a × b
x^(1/n) = ⁿ√x
x^(m/n) = ⁿ√(x^m)

Number of Factors

Number of Factors of a Number

What are Factors?

All numbers which can divide a given number completely are factors of that number.

Number of Factors

Number of Factors of a Number

Number of Factors of 24

1×24

2×12

3×8

4×6

So, 24 has 8 factors.

Number of Factors of 60

1×60

2×30

3×20

4×15

5×12

6×10

So, 60 has 12 factors.

Number of Factors

Number of Factors of a Number

Number of Factors of a number ‘N’

If $N = x^a \times y^b \times z^c$ ……… where x, y, z are prime numbers

then the number of possible factors is $(a + 1)(b + 1)(c + 1)$ …….

Number of Factors

Number of Factors of a Number

Find the factors of 600.

$600 = 2^3 \times 3^1 \times 5^2$

So, according to formula for number of factors, the factors of 600

$= (3 + 1) \times (1 + 1) \times (2 + 1)$

$= 4 \times 2 \times 3$

$= 24$

Number of Factors

Number of Factors of a Perfect Square

Find the factors of 900.

$900 = 2^2 \times 3^2 \times 5^2$

Therefore, the number of factors of 900 will be equal to

$(2 + 1) \times (2 + 1) \times (2 + 1)$

$= 3 \times 3 \times 3$

$= 27$

From the above example you will notice that since 900 is a perfect square, the powers of a, b, c, ….. will always be an even number and hence the Number of factors of a perfect square will always be an odd number.

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Classification of Numbers

Advanced Concepts in Factors

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L C M H C F

$\log_a x = \frac{\log_b x}{\log_b a}$

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2	18	24	36
2	9	12	18
3	9	6	9
3	3	2	3
	1	2	1

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117 → 39, 3; 39 → 13, 3

$x^{\frac{1}{n}} = \sqrt[n]{x}$

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Classification of Numbers

Number of Odd Factors

Number of odd factors of 900

$900 = 2^2 \times 3^2 \times 5^2$

To calculate the number of odd factors, consider $3^2 \times 5^2$

Number of odd factors $= (2 + 1)(2 + 1)$

$= 9$

Classification of Numbers

Number of Even Factors

Number of even factors of 900

= Total number of factors – Number of odd factors

= 27 – 9

= 18

Classification of Numbers

Sum of Factors

$60 = 2^2 \times 3^1 \times 5^1$

Sum of factors $= (1 + 2 + 4)(1 + 3)(1 + 5)$

$= 7 \times 4 \times 6$

$= 168$

$100 = 2^2 \times 5^2$

Sum of factors $= (1 + 2 + 4)(1 + 5 + 25)$

$= 7 \times 31$

$= 217$

Classification of Numbers

Example

$8008 = 2^3 \times 7^1 \times 11^1 \times 13^1$

Total factors $= (3 + 1)(1 + 1)(1 + 1)(1 + 1)$

$= 32$

Odd factors $= (1 + 1)(1 + 1)(1 + 1)$

$= 8$

Even factors $= 32 - 8$

$= 24$

Sum of factors $= (1 + 2 + 4 + 8)(1 + 7)(1 + 11)(1 + 13)$

$= 20160$

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Concept Videos

Classification of Numbers

Highest Power in Factorial I

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L H
C C
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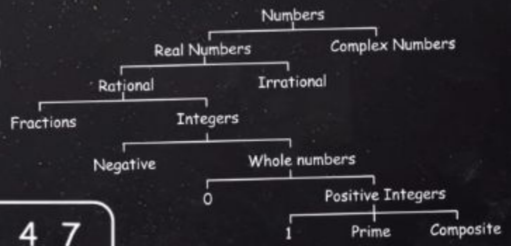

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$$117 \to 39, 3;\quad 39 \to 13, 3$$

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Highest Power in Factorial

How many powers of 5 does 25! have

If 25! is completely divisible by 5^n, then find the largest value of 'n'.

$25! = 1 \times 2 \times 3 \times 4 \times 5 \times \ldots..10 \times \ldots..15 \times \ldots..20 \times \ldots..25$

5×5

$1 + 1 + 1 + 1 + 2 = 6$

Highest Power in Factorial

How many powers of 7 does 200! have

If 200! is completely divisible by 7^n, then find the largest value of 'n'.

$200! = 200 \times 199 \times \ldots\ldots \times 1$

$\frac{200}{7} = 28$

$\frac{200}{49} = 4$

$\overline{\quad 32 \quad}$

$200! = 200 \times 199 \times \ldots\ldots \times 1$

$\frac{200}{7} = 28$

$\frac{28}{7} = 4$

$\overline{\quad 32 \quad}$

Highest Power in Factorial

How many powers of 5 does 300! have

If 300! is completely divisible by 5^n, then find the largest value of 'n'.

$$300! = 300 \times 299 \times \ldots.. \times 1$$

$$\frac{300}{5} = 60$$

$$\frac{300}{25} = 12$$

$$\frac{300}{125} = 2$$

$$\overline{\underline{\ \ 74\ \ }}$$

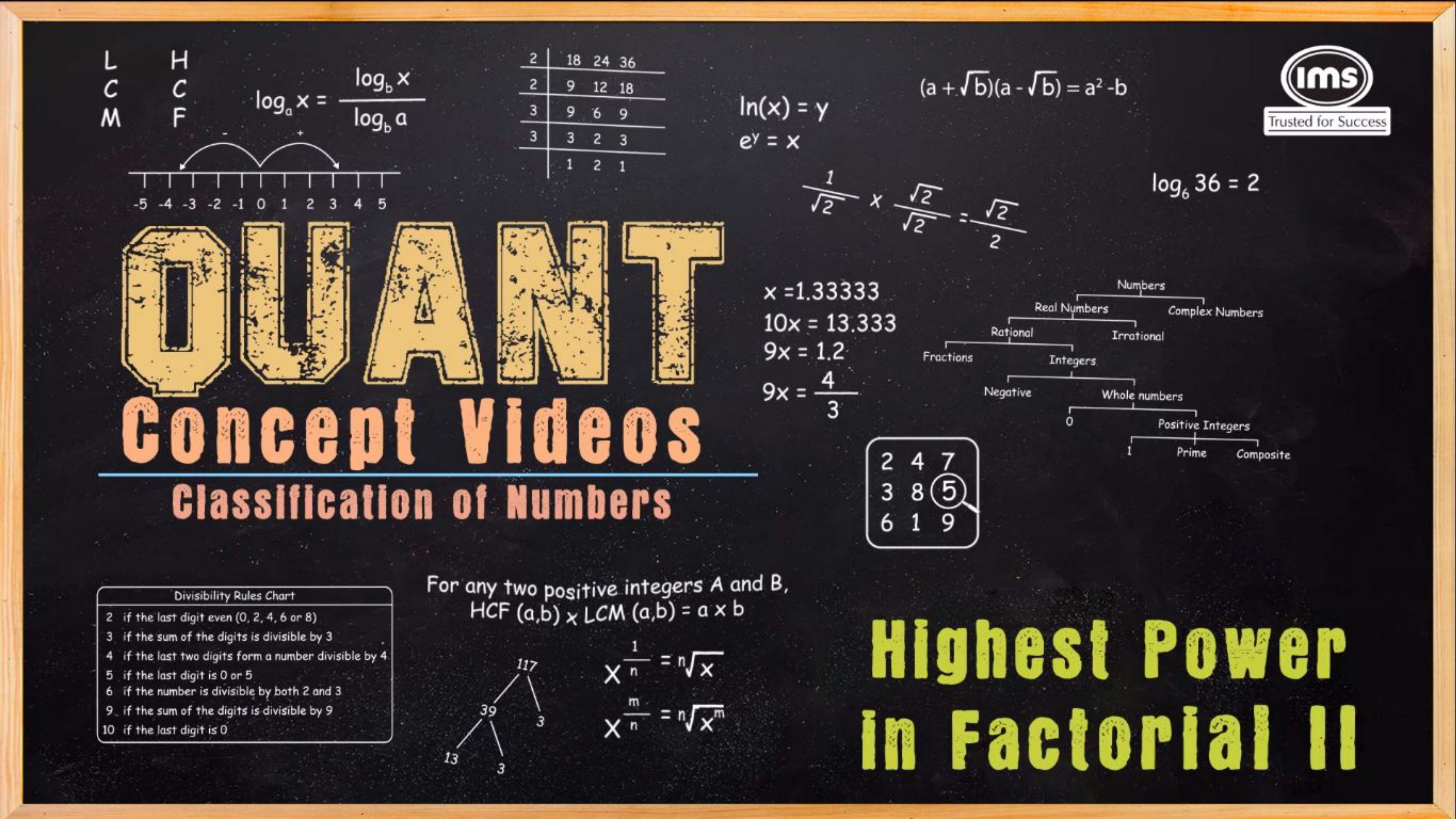
$$300! = 300 \times 299 \times \ldots.. \times 1$$

$$\frac{300}{5} = 60$$

$$\frac{60}{5} = 12$$

$$\frac{12}{5} = 2$$

$$\overline{\underline{\ \ 74\ \ }}$$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Highest Power in Factorial

How many powers of 6 does 100! have

'6' is not a PRIME number

2×3

$100! = 100 \times 99 \times \ldots\ldots \times 1$

$\frac{100}{2} = 50$

$\frac{50}{2} = 25$

$\frac{25}{2} = 12$

$\frac{12}{2} = 6$

$\frac{6}{2} = 3$

$\frac{3}{2} = 1$

$\overline{\underline{\quad 97 \quad}}$

$100! = 100 \times 99 \times \ldots\ldots \times 1$

$\frac{100}{3} = 33$

$\frac{33}{3} = 11$

$\frac{11}{3} = 3$

$\frac{3}{3} = 1$

$\overline{\underline{\quad 48 \quad}}$

Highest Power in Factorial

How many powers of 6 does 100! have

So, now there are 97 powers of 2 and 48 powers of 3.

Hence, maximum powers of 6 will be 48.

We can conclude that if a number is made up of only 2 prime numbers then we need to check for only the larger of the 2 numbers.

Highest Power in Factorial

How many powers of 9 does 200! have

If 200! is completely divisible by 9^n, then find the largest value of 'n'

9 is not a PRIME number, but $9 = 3^2$

$200! = 200 \times 199 \times \ldots\ldots \times 1$

$\frac{200}{3} = 66$ $\quad$ $\frac{22}{3} = 7$

$\frac{66}{3} = 22$ $\quad$ $\frac{7}{3} = 2$

97

200! has maximum 97 powers of 3

Hence, the maximum powers of 9 will be 48.

Highest Power in Factorial

How many zeroes does 250! end in?

We need to find powers of 10 and 10 is not a PRIME number

$250! = 250 \times 249 \times \ldots.. \times 1$

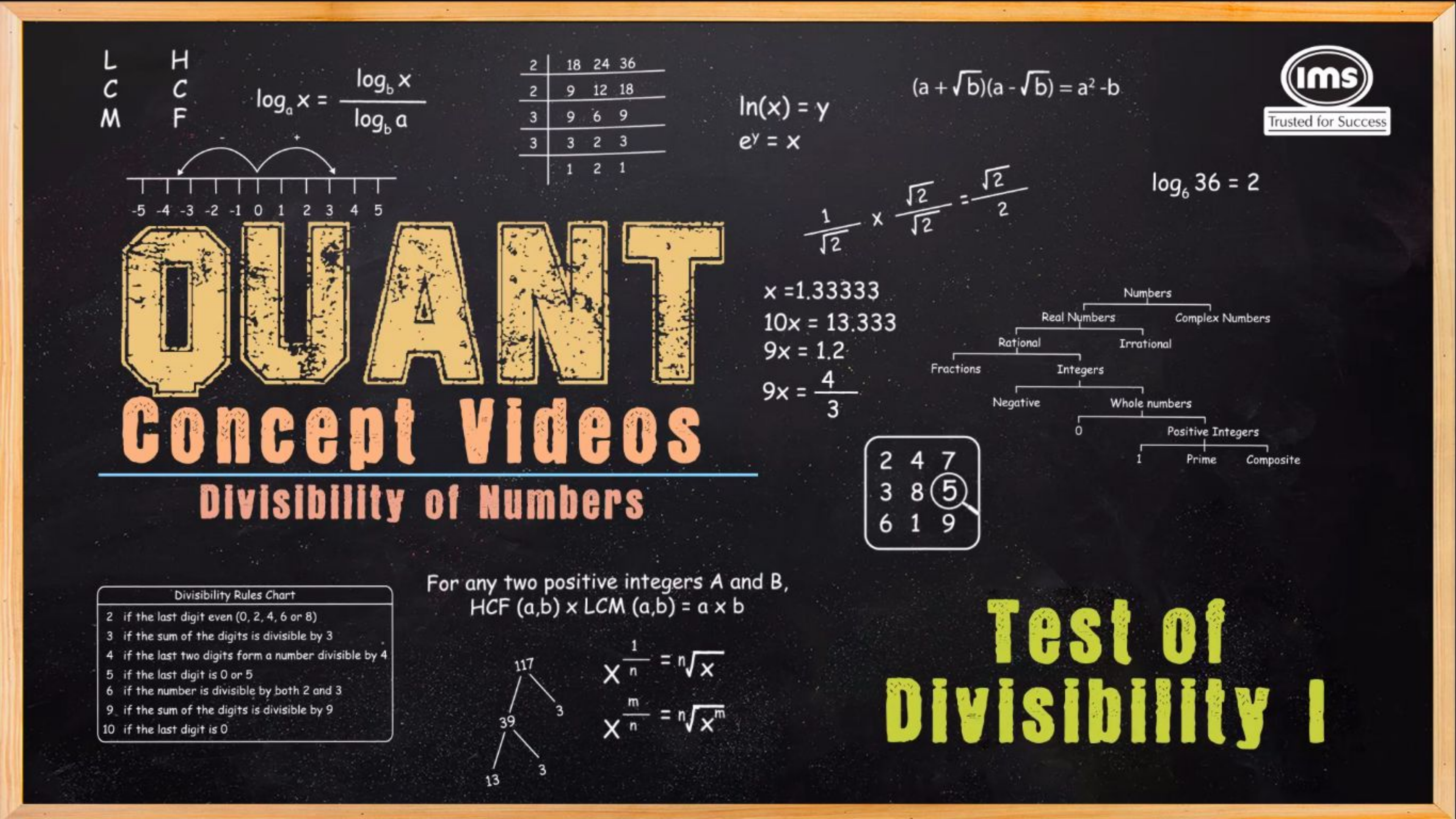
$\frac{250}{5} = 50$

$\frac{50}{5} = 10$

$\frac{10}{5} = 2$

$$\begin{array}{r} \\ \hline 62 \\ \hline \end{array}$$

250! Will end in 62 zeroes.



L C M
H C F
$\log_a x = \frac{\log_b x}{\log_b a}$
-5 -4 -3 -2 -1 0 1 2 3 4 5
2 | 18 24 36
2 | 9 12 18
3 | 9 6 9
3 | 3 2 3
| 1 2 1
$\ln(x) = y$
$e^y = x$
$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$
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$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
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Concept Videos
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Test of
Divisibility I

Divisibility of Numbers

Tests of Divisibility

Tests for Powers of 2

Test for 2 $\rightarrow$ Last digit has to be divisible by 2

e.g. All even numbers will be divisible by 2

Test for 4 $\rightarrow$ Last 2 digits have to be divisible by 4

e.g. 345678, 5566768

Divisibility of Numbers

Tests of Divisibility

Tests for Powers of 2

Test for 8 → Last 3 digits have to be divisible by 8

e.g. 7634568, 12345678

Divisibility of Numbers

Tests of Divisibility

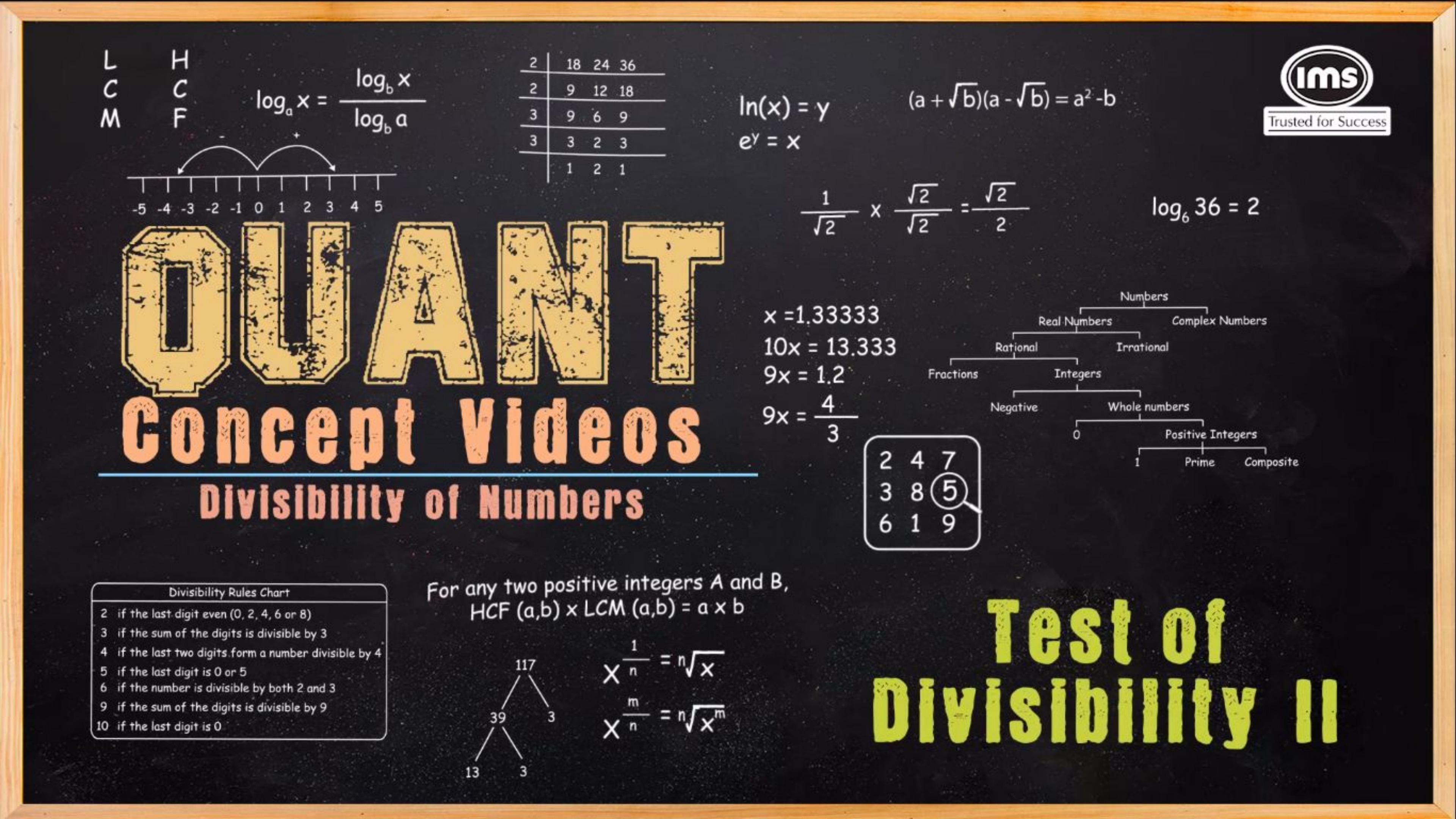
Tests for Powers of 3

Test for 3 → Sum of the digits has to be divisible by 3

e.g. 75689023, 123456

Test for 9 → Sum of the digits has to be divisible by 9

e.g. 345678, 5566768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Concept Videos
Divisibility of Numbers
Test of Divisibility II
IMS
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LCM HCF
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Divisibility of Numbers

Tests of Divisibility

Tests for Powers of 5

Test for 5 → Last digit has to be 0 or 5

Test for 25 → Last 2 digits have to be divisible by 25

i.e. last 2 digits can be 25, 50, 75, 00

Divisibility of Numbers

Tests of Divisibility

Tests for 11

Test for 11 → Add the alternate digits, and find two sums. If the difference of these sums is 0 or divisible by 11, then the number is divisible by 11.

e.g. 35678045348

$3+6+8+4+3+8$

$= 32$

$5+7+0+5+4$

$= 21$

$\therefore 32 - 21 = 11$

Divisibility of Numbers

Tests of Divisibility

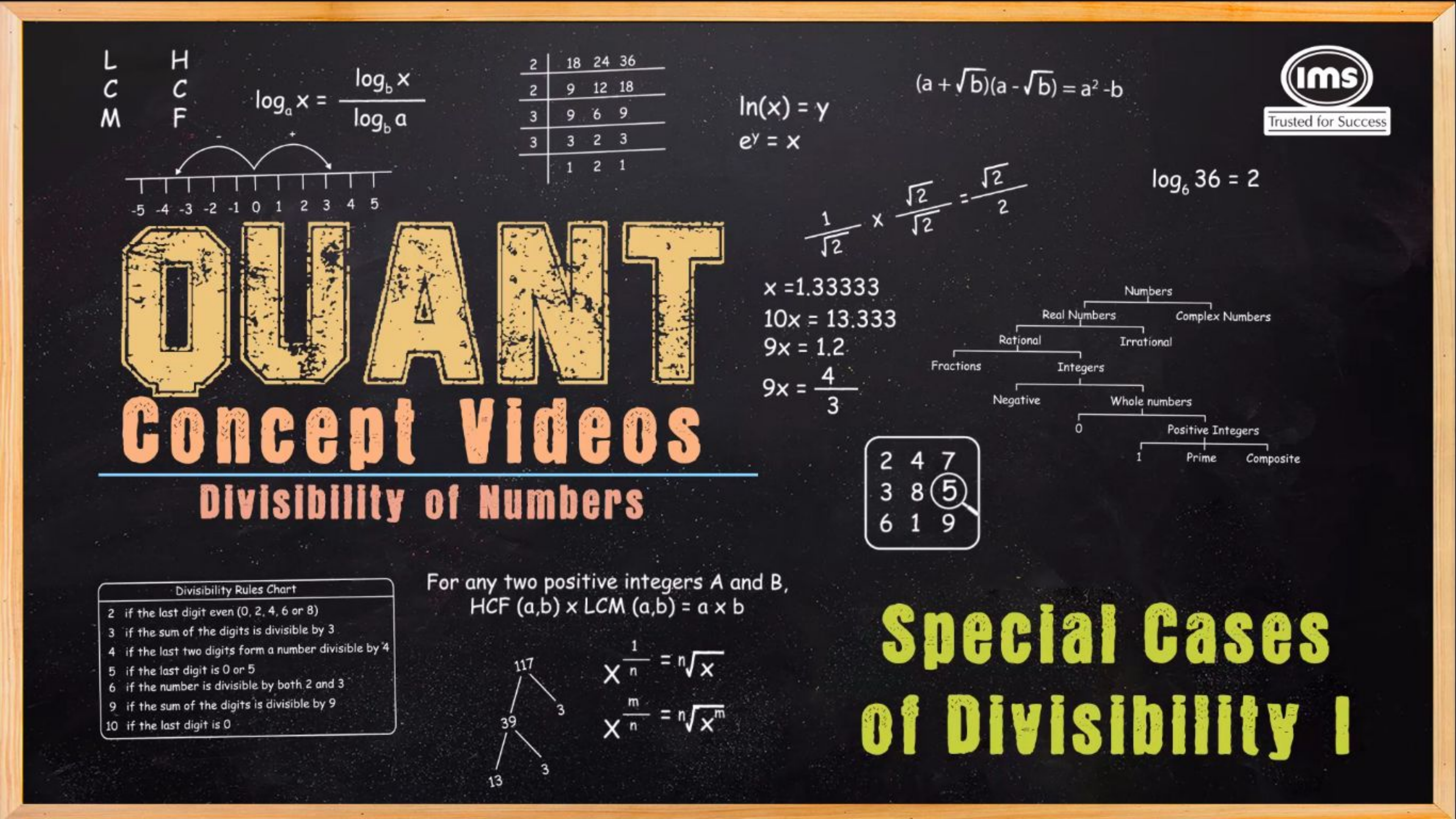
Tests for Composite Numbers

Test for 6 → The number has to be divisible by both 2 and 3.

e.g. 78936, 1234789

Test for 12 → The number has to be divisible by both 3 and 4.

e.g. 78936, 1234789


QUANT
Concept Videos
Divisibility of Numbers
Special Cases of Divisibility I
Trusted for Success
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Divisibility of Numbers

Special Cases of Divisibility

A four digit number formed with 2 digits repeating (xyxy)

e.g. 7373

$$7373 = 7300 + 73 = 73(100 + 1) = 73(101)$$

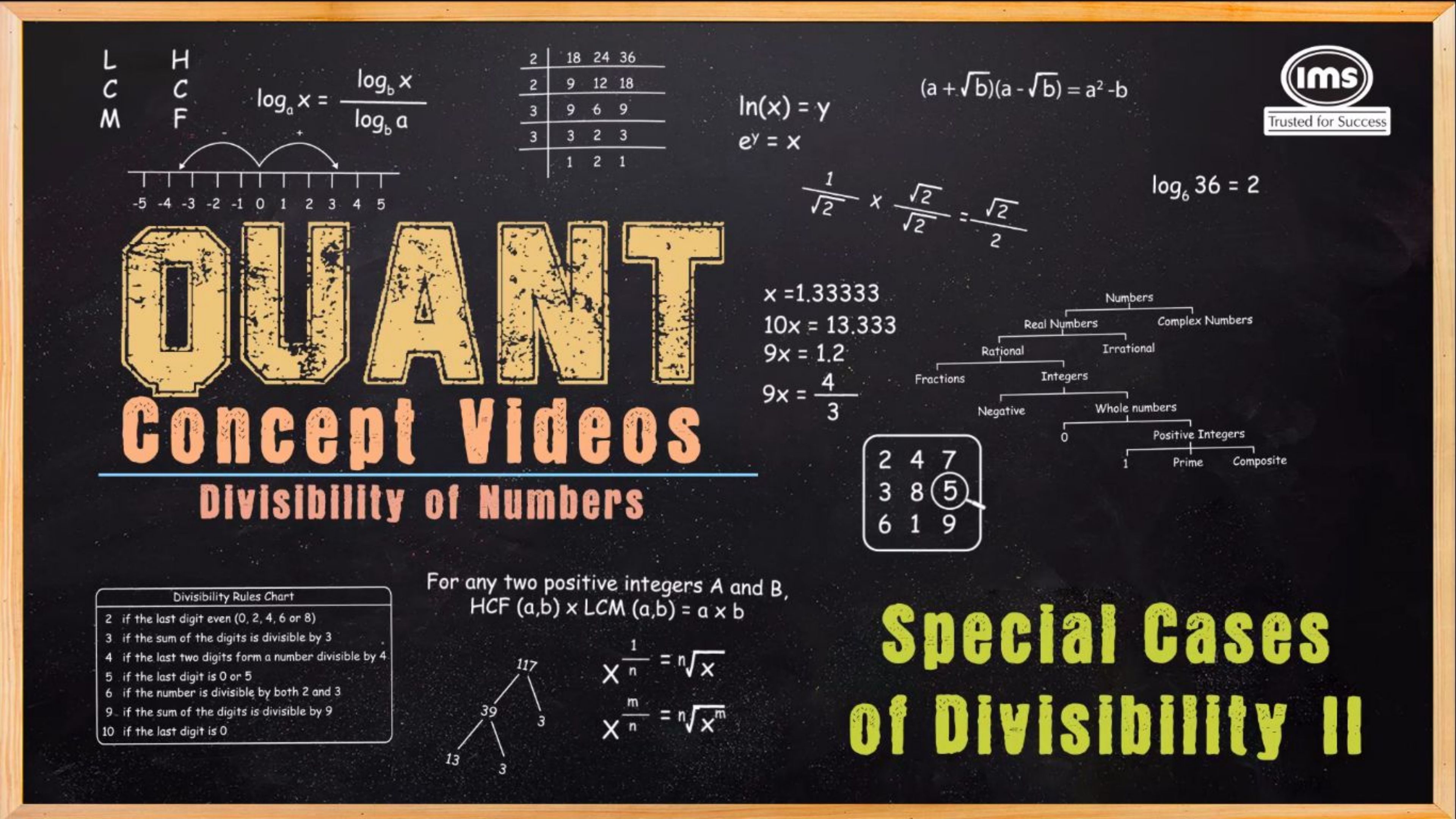
Any number in the form of xyxy will always be divisible by 101

A six digit number formed with 3 digits repeating (xyzxyz)

e.g. $731731 = 731000 + 731 = 731(1000 + 1) = 731(1001)$

$$1001 = 7 \times 11 \times 13$$

Any number in the form of xyzxyz will always be divisible by 1001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Concept Videos
Divisibility of Numbers
Special Cases of Divisibility II
IMS
Trusted for Success
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Divisibility of Numbers

Special Cases of Divisibility

Two ‘2’ digit numbers formed with their digits interchanged (xy and yx)

Tens	Units
x	y

Original Number = $10x + y$

Interchanged number = $10y + x$

Sum of the 2 numbers = $11(x + y)$

Therefore, sum will always be divisible by 11 and sum of the 2 digits.

Difference of the 2 numbers = $9(x – y)$

Therefore, difference will always be divisible by 9 and difference of the 2 digits.

Divisibility of Numbers

Special Cases of Divisibility

e.g. If the digits are 7 and 5, the two numbers will be 75 and 57.
Their sum will be 132, which is divisible by both 11 and 12.
Their difference will be 18, which is divisible by both 9 and 2.

Divisibility of Numbers

Special Cases of Divisibility

Two ‘3’ digit numbers formed with their digits interchanged (xyz and zyx)

Hundreds	Tens	Units
x	y	z

Original number = $100x + 10y + 1z$

Reversed number = $100z + 10y + 1x$

Difference of the 2 numbers = $99(x - z)$

Therefore, difference will always be divisible by 99 (hence by 9 and 11) and difference of the extreme digits.

e.g. If the digits are 7, 4 and 2 the two numbers will be 742, 247.

Their difference will be 495, which is divisible by both 99 and 5.

L C M

H C F

$$\log_a x = \frac{\log_b x}{\log_b a}$$

2	18	24	36
2	9	12	18
3	9	6	9
3	3	2	3
	1	2	1

$\ln(x) = y$

$e^y = x$

$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

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$\log_6 36 = 2$

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

QUANT

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HCF and LCM

$x = 1.33333$

$10x = 13.333$

$9x = 1.2$

$9x = \frac{4}{3}$

Numbers
- Real Numbers
  - Rational
    - Fractions
    - Integers
      - Negative
      - Whole numbers
        - 0
        - Positive Integers
          - 1
          - Prime
          - Composite
  - Irrational
- Complex Numbers

2	4	7
3	8	5
6	1	9

Divisibility Rules Chart

2	if the last digit even (0, 2, 4, 6 or 8)
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For any two positive integers A and B,
HCF (a,b) × LCM (a,b) = a × b

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Highest Common Factor

HCF and LCM

Highest Common Factor (HCF)

HCF is the largest number which divides all the given numbers.

Example

Find HCF of 24 and 60.

Factors of 24 = {1, 2, 3, 4, 6, 8, 12, 24 }

Factors of 60 = {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 }

Common Factors = {1, 2, 3, 4, 6, 12}

Highest Common Factor = 12

HCF and LCM

Methods of Finding HCF

Factorization Method

Find HCF of 24 and 60.

$$
\begin{array}{l|l}
2 & 24,\ 60 \\ \hline
2 & 12,\ 30 \\ \hline
3 & 6,\ 15 \\ \hline
 & 2,\ \ 5 \rightarrow \text{Co-prime Numbers}
\end{array}
$$

HCF = $2 \times 2 \times 3 = 12$

HCF and LCM

Methods of Finding HCF

Division Method

$$\begin{array}{l} \quad\;\; 2 \\ 24\overline{)\,60} \\ \;\, -48 \quad 2 \\ \qquad 12\overline{)\,24} \\ \qquad\;\; -24 \\ \qquad\quad\;\; 00 \end{array}$$

$\therefore$ HCF = 12

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HCF and LCM

Least Common Multiple

L C M — H C F

$\log_a x = \dfrac{\log_b x}{\log_b a}$

2	18	24	36
2	9	12	18
3	9	6	9
3	3	2	3
	1	2	1

$\ln(x) = y$

$e^y = x$

$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

-5 -4 -3 -2 -1 0 1 2 3 4 5

$\dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\log_6 36 = 2$

$x = 1.33333$

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$9x = \dfrac{4}{3}$

Numbers: Real Numbers, Complex Numbers; Real Numbers: Rational, Irrational; Rational: Fractions, Integers; Integers: Negative, Whole numbers; Whole numbers: 0, Positive Integers; Positive Integers: 1, Prime, Composite

2	4	7
3	8	5
6	1	9

Divisibility Rules Chart

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For any two positive integers A and B,
HCF (a,b) × LCM (a,b) = a × b

117 → 39, 3; 39 → 13, 3

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

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HCF and LCM

Least Common Multiple (LCM)

LCM is the smallest number which is divisible by all the given numbers.

Example

Find LCM of 24 and 60.

Multiples of 60 = {60, 120, …………….}

Multiples of 24 = {24, 48, 72, 96, 120}

Least Common Multiple = 120

HCF and LCM

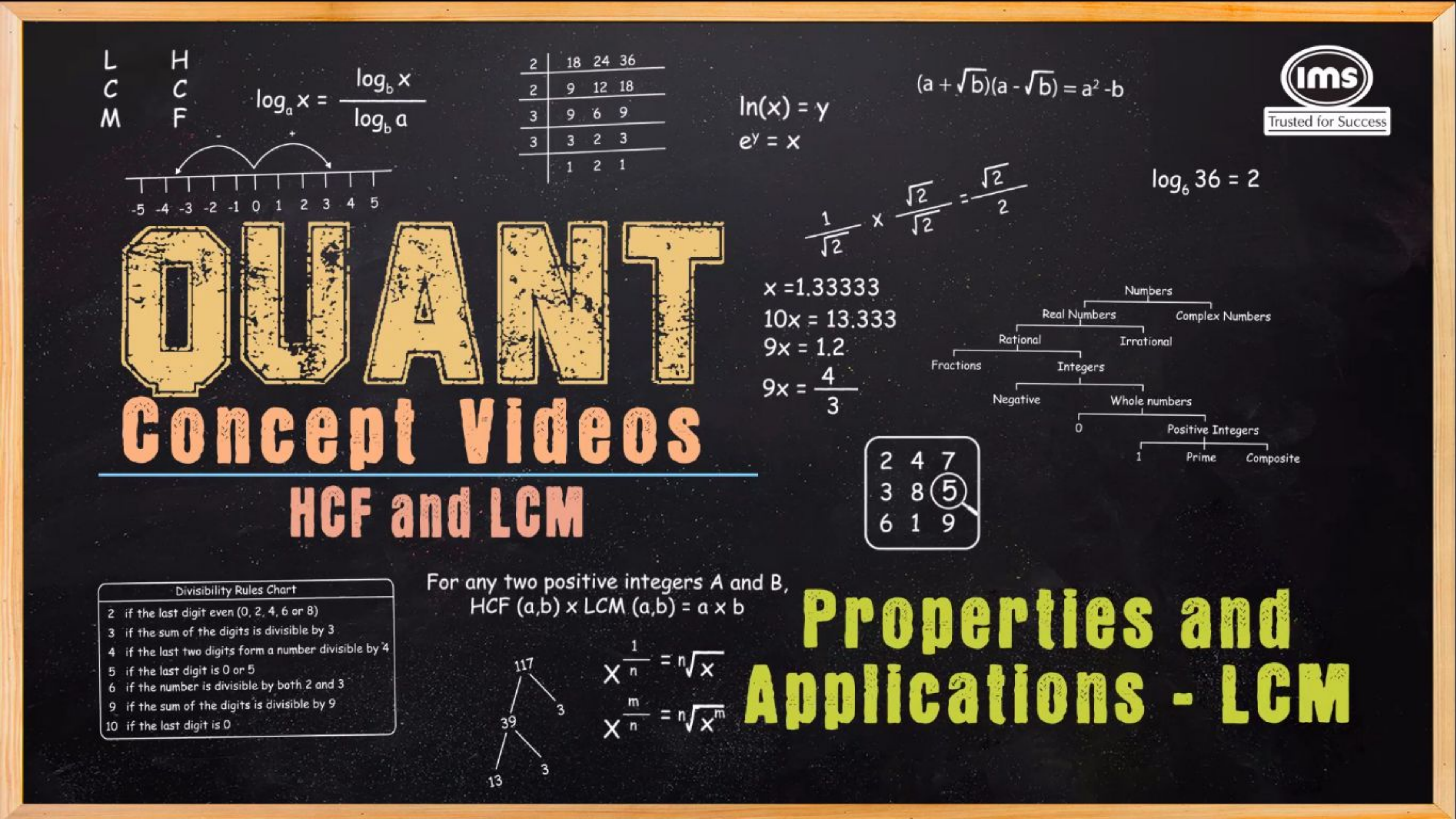
Method of Finding LCM

Factorization Method

Find LCM of 24 and 60.

2	24, 60
2	12, 30
3	6, 15
2	2, 5
	1, 5

LCM = $(2 \times 2 \times 3) \times 2 \times 5 = 120$



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HCF and LCM
Properties and Applications - LCM
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L C M
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HCF and LCM

Properties of LCM

1. LCM of a set of numbers $\underline{\ \geq\ }$ the largest number.
2. LCM of a set of numbers is a multiple of all the given numbers as well as the HCF.

HCF and LCM

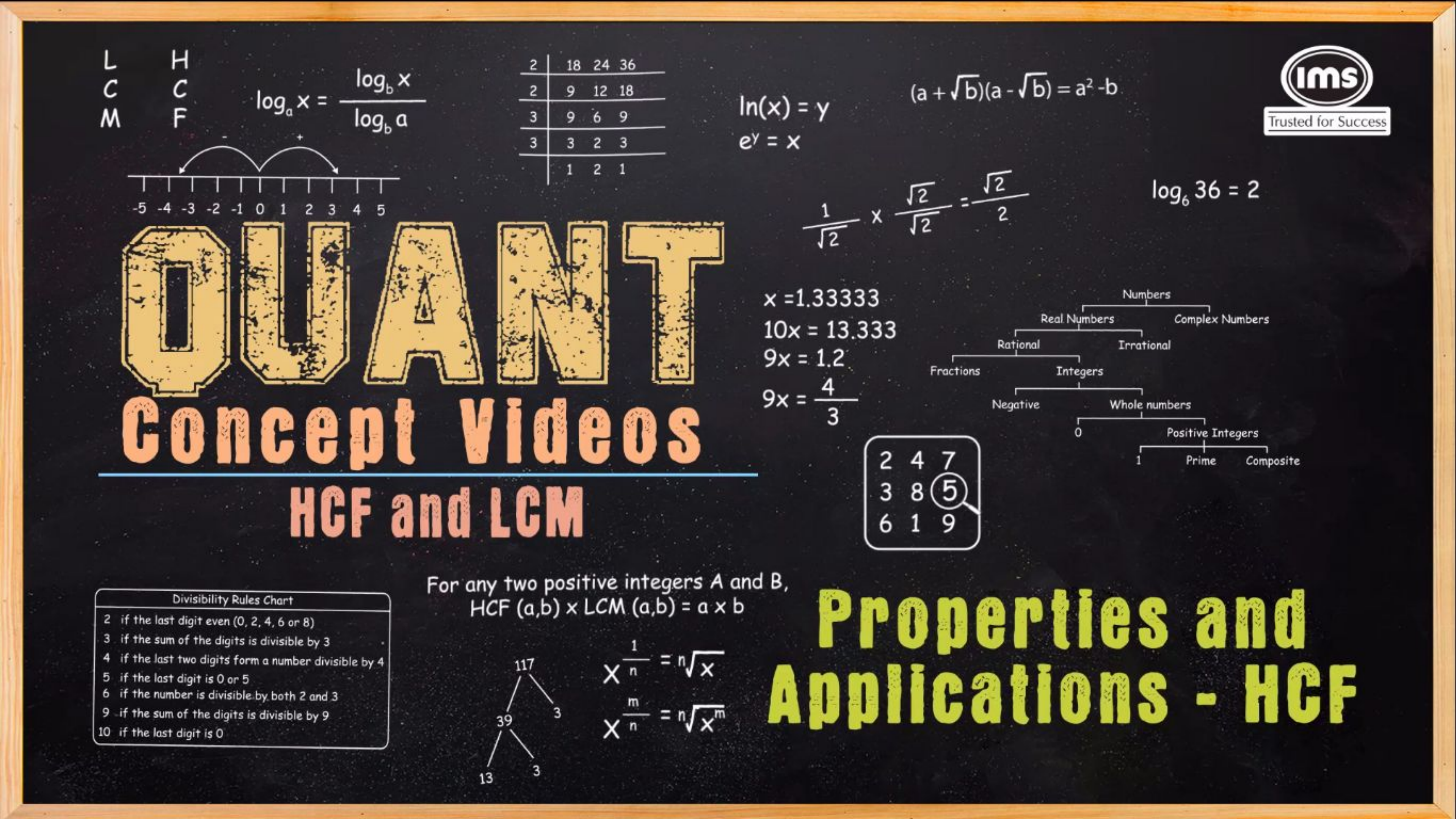
Applications

Michael, Nicholas and Oahan work in an office. They initially meet in the cafeteria at 9:00 am for a break. M, N and O take a break after every 12 minutes, 24 minutes and 60 minutes respectively to go to the cafeteria. Find the time at which they will meet the second time together.

$\therefore$ Time they will take to meet in the cafeteria together again
= 120 minutes i.e. 2 hours.

+ 2 hours =

11:00 am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HCF and LCM
Properties and Applications - HCF
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HCF and LCM

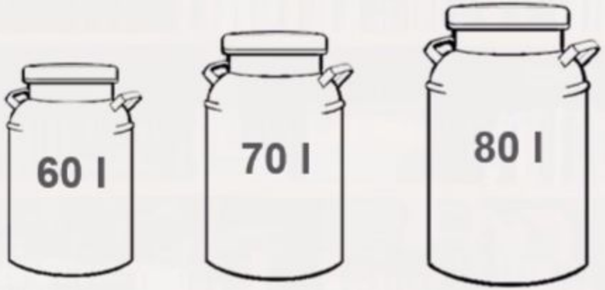
Properties of HCF

1. HCF of a set of numbers $\underline{\ \leq\ }$ the smallest number.
2. HCF is a factor of all the given numbers as well as the LCM.

HCF and LCM

Applications

Three cans of the capacity of 60 l, 70 l and 80 l are completely filled with milk. Find the least number of equal capacity bottles needed to fill them with a fixed quantity. Also, find the size of the bottle.



HCF of 60, 70, 80, = 10

$\therefore$ Capacity of each bottle = 10 l

$\therefore$ Number of bottles $= \frac{60}{10} + \frac{70}{10} + \frac{80}{10} = 21$

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HCF and LCM

HCF and LCM of Fractions

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L C M   H C F

$$\log_a x = \frac{\log_b x}{\log_b a}$$

2	18	24	36
2	9	12	18
3	9	6	9
3	3	2	3
	1	2	1

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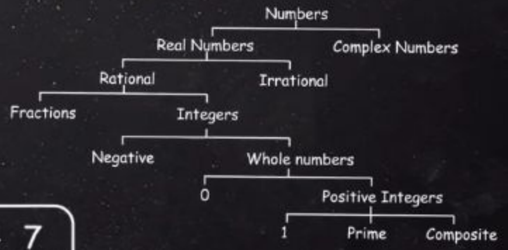

2	4	7
3	8	5
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For any two positive integers A and B,
HCF (a,b) × LCM (a,b) = a × b

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

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HCF and LCM

Fractions

$$\text{HCF of Fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

$$\text{LCM of Fractions} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

Note: Always reduce the fraction to the lowest form.

HCF and LCM

Fractions

Find the HCF and LCM of $\frac{7}{4}$, $\frac{12}{15}$ and $\frac{21}{9}$

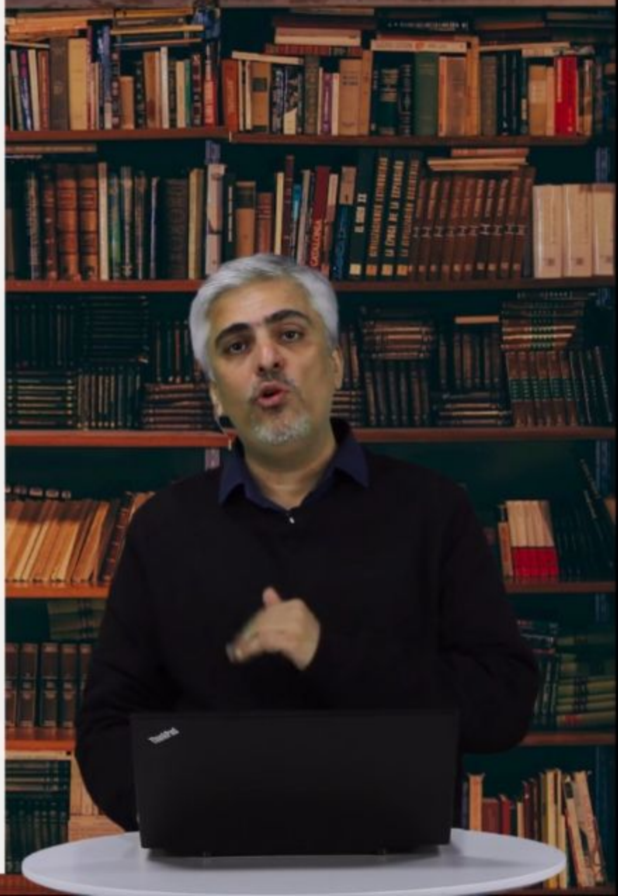
$$\text{HCF of Fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

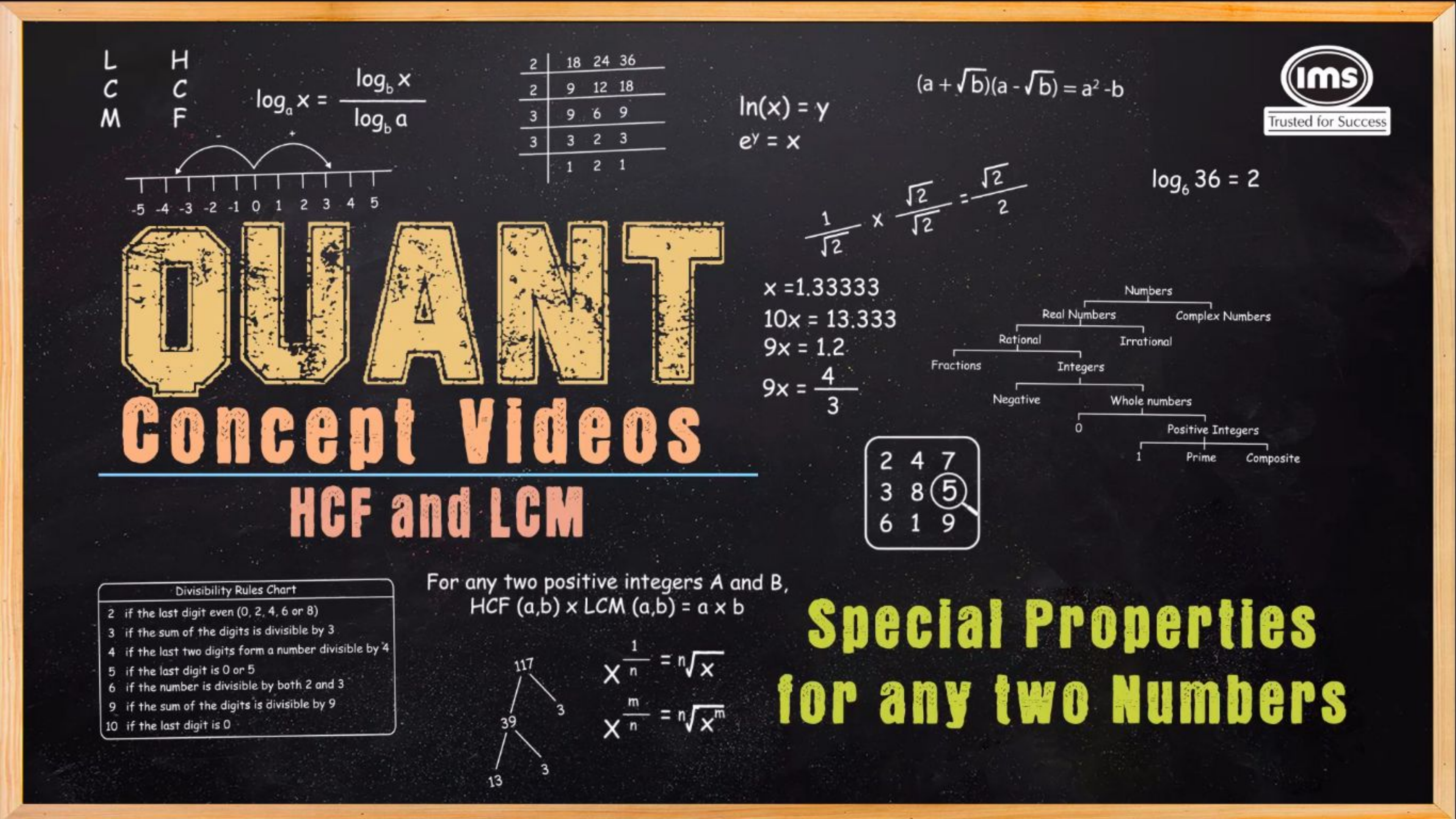
$\therefore$ The fractions are: $\frac{7}{4}, \frac{4}{5}, \frac{7}{3}$

$$= \frac{\text{HCF }(7, 4, 7)}{\text{LCM }(4, 5, 3)} = \frac{1}{60}$$

$$\text{LCM of Fractions} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

$$= \frac{\text{LCM }(7, 4, 7)}{\text{HCF }(4, 5, 3)} = \frac{28}{1}$$

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HCF and LCM
Special Properties for any two Numbers
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$X^{\frac{m}{n}} = \sqrt[n]{x^m}$

HCF and LCM

Special properties for any two Numbers

For two numbers ‘a’ and ‘b’, if ‘b’ is a multiple of ‘a’, then

HCF = a

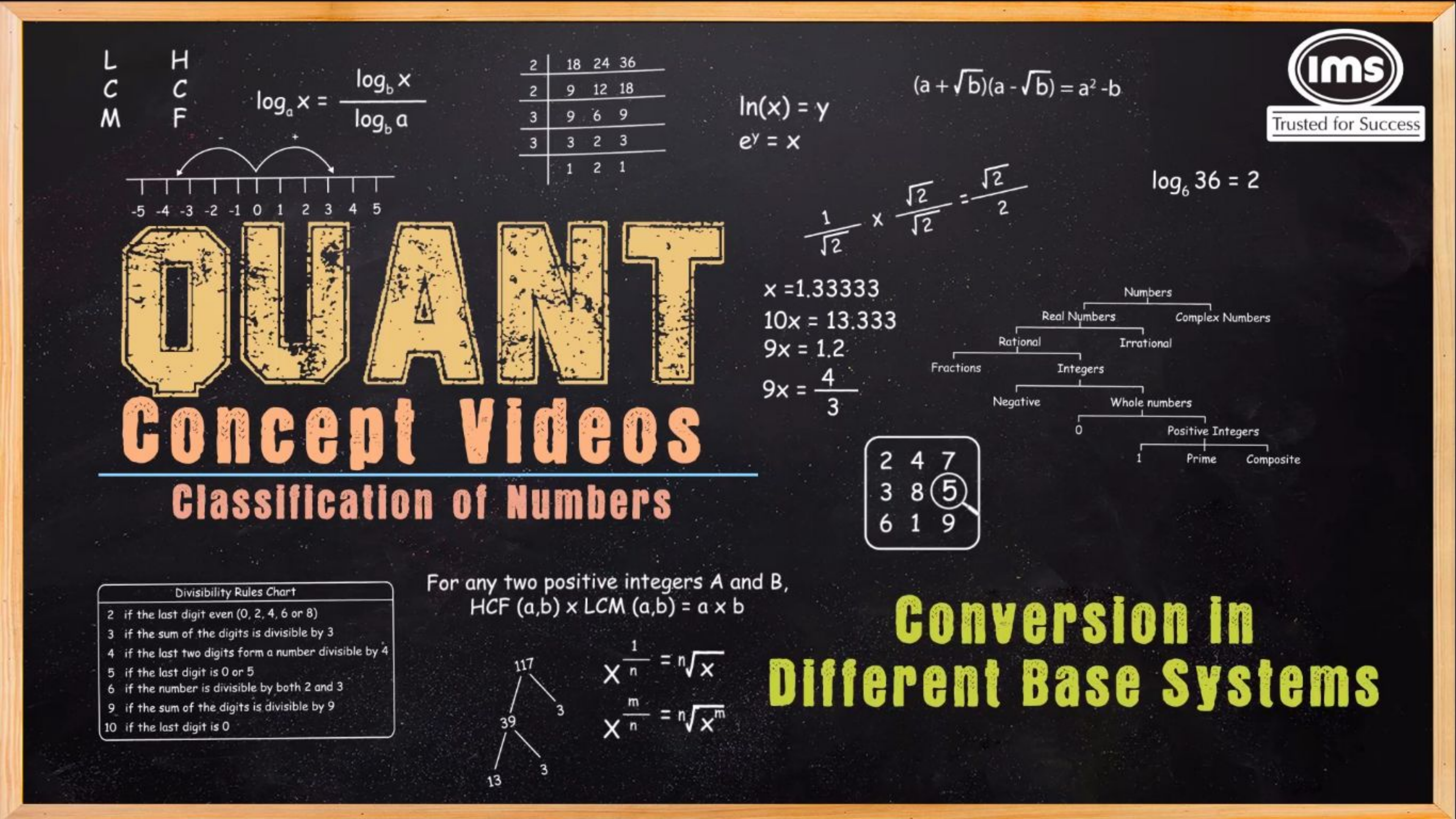
LCM = b

Product of two numbers = HCF × LCM

For any two co-prime numbers,

HCF = 1

LCM = Product of two numbers



IMS
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Conversion in
Different Base Systems

Classification of Numbers

Base System

Base 10 $\Rightarrow$ digits from 0 – 9

Base 7 $\Rightarrow$ digits from 0 – 6

Base 2 $\Rightarrow$ digits 0 and 1

Base 16 $\Rightarrow$ digits 0 – 9 and A - F

Classification of Numbers

Base System

Conversions from Decimal to any other Base

$(93)_{10} \Rightarrow (1011101)_2$

2	93	1
2	46	0
2	23	1
2	11	1
2	5	1
2	2	0
	1	1

$= (1011101)_2$

Classification of Numbers

Base System

$(726)_{10} \Rightarrow (2055)_7$

7	726	5
7	103	5
7	14	0
	2	2

$= (2055)_7$

Classification of Numbers

Base System

Base 16 Digits 0 – 9 and A – F

$(1805)_{10} \Rightarrow (70\ D)_{16}$

$$\begin{array}{r|r l}
16 & 1805 & \quad 13 \uparrow \\ \hline
16 & 112 & \quad 0 \\ \hline
 & 7 & \quad 7
\end{array}$$

$= (70\ D)_{16}$

Classification of Numbers

Base System

For any other Base to Decimal

$(1011101)_2 \Rightarrow (93)_{10}$

$= 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4 + 0 \times 2^5 + 1 \times 2^6$

$= 1 + 0 + 4 + 8 + 16 + 0 + 64$

$= (93)_{10}$

$(2055)_7 \Rightarrow (\quad)_{10}$

$= 5 \times 7^0 + 5 \times 7^1 + 0 \times 7^2 + 2 \times 7^3$

$= 5 + 35 + 0 + 686$

$= (726)_{10}$

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Classification of Numbers

Arithmetic Operations in Base System

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H C F

$\log_a x = \frac{\log_b x}{\log_b a}$

-5 -4 -3 -2 -1 0 1 2 3 4 5

2	18	24	36
2	9	12	18
3	9	6	9
3	3	2	3
	1	2	1

$\ln(x) = y$

$e^y = x$

$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

$\log_6 36 = 2$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$x = 1.33333$

$10x = 13.333$

$9x = 1.2$

$9x = \frac{4}{3}$

Numbers
- Real Numbers
  - Rational
    - Fractions
    - Integers
      - Negative
      - Whole numbers
        - 0
        - Positive Integers
          - 1
          - Prime
          - Composite
  - Irrational
- Complex Numbers

2	4	7
3	8	5
6	1	9

Divisibility Rules Chart	
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117 → 39, 3; 39 → 13, 3

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Classification of Numbers

Arithmetic Operations of Bases

Addition of numbers in base 10

$$
\begin{array}{r}
1\ 1\ 1\ \\
(7\,9\,3\,4)_{10} \\
+\ (8\,4\,6\,8)_{10} \\
\hline
(1\,6\,4\,0\,2)_{10}
\end{array}
$$

$$
\begin{array}{r}
1\ 1\ 1\ \\
(7\,9\,3\,4)_{10} \\
+\ (8\,4\,6\,8)_{10} \\
\hline
(1\,6\,4\,0\,2)_{10}
\end{array}
$$

Subtraction of numbers in base 10

$$
\begin{array}{r}
13\ 9\ \\
5\ \cancel{3}\ \cancel{10}\ 12 \\
(1\,\cancel{6}\,\cancel{4}\,\cancel{0}\,\cancel{2})_{10} \\
-\ \ (7\,9\,3\,4)_{10} \\
\hline
(8\,4\,6\,8)_{10}
\end{array}
$$

Classification of Numbers

Arithmetic Operations of Bases

Addition of numbers in base 8

$$
\begin{array}{r}
{\scriptstyle 1\ \ 1\ \ 1\ \ 1\ \ \ } \\
(7\ 3\ 6\ 5)_8 \\
+\ (4\ 4\ 7\ 7)_8 \\
\hline
(1\ 4\ 0\ 6\ 4)_8
\end{array}
$$

Subtraction of numbers in base 8

$$
\begin{array}{r}
{\scriptstyle 11\ \ 7\ \ 13\ \ \ \ \ } \\
{\scriptstyle 0\ \ \cancel{3}\ \ \cancel{8}\ \ \cancel{5}\ \ 12} \\
(\cancel{1}\ \cancel{4}\ \cancel{0}\ \cancel{6}\ \cancel{4})_8 \\
-\ \ (7\ 3\ 6\ 5)_8 \\
\hline
(4\ 4\ 7\ 7)_8
\end{array}
$$

Classification of Numbers

Trusted for Success

Arithmetic Operations of Bases

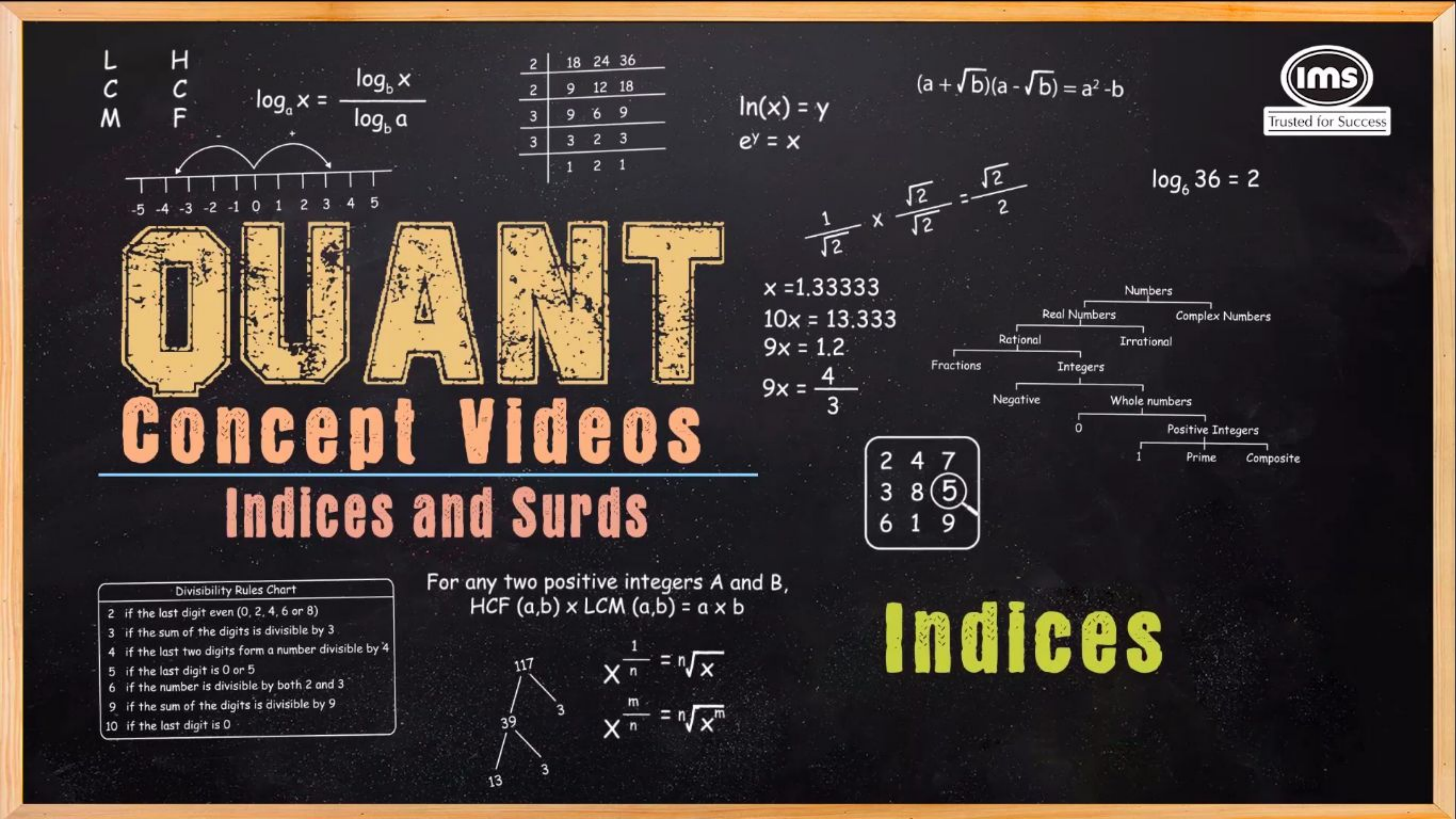
Addition of numbers in base 16

$$\begin{array}{r} (7\,9\,A)_{16} \\ +\ (A\,B\,8)_{16} \\ \hline (1\,2\,5\,2)_{16} \end{array} \;\Rightarrow\; \begin{array}{rrrr} {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & \\ & 7 & 9 & (10) \\ + & (10) & (11) & 8 \\ \hline 1 & 2 & 5 & 2 \end{array}$$

A	10
B	11
C	12
D	13
E	14
F	15

Subtraction of numbers in base 16

$$\begin{array}{r} (1\,2\,5\,2)_{16} \\ -\ \ (7\,9\,A)_{16} \\ \hline (A\,B\,8)_{16} \end{array} \;\Rightarrow\; \begin{array}{rrrr} & {\scriptstyle 17} & {\scriptstyle 20} & \\ {\scriptstyle 0} & \cancel{\scriptstyle 1} & \cancel{\scriptstyle 4} & {\scriptstyle 18} \\ \cancel{1} & \cancel{2} & \cancel{5} & \cancel{2} \\ - & 7 & 9 & (10) \\ \hline & A & B & 8 \end{array}$$



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Indices and Surds
Indices
L C M
H C F
$\log_a x = \frac{\log_b x}{\log_b a}$
$\ln(x) = y$
$e^y = x$
$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$
$\log_6 36 = 2$
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HCF (a,b) × LCM (a,b) = a × b
$X^{\frac{1}{n}} = \sqrt[n]{x}$
$X^{\frac{m}{n}} = \sqrt[n]{x^m}$

Indices and Surds

Introduction to Indices

$a^5 = a \times a \times a \times a \times a$

$a^m = a \times a \times a \times \ldots\ldots$ m times

'a' is the base

'm' is the index

Indices and Surds

Law of Indices

1. $a^m \times a^n = a^{m+n}$

2. $a^m \times a^n \times a^p = a^{m+n+p}$

3. $\frac{a^m}{a^n} = a^{m-n}$

4. $(a^m)^n = a^{mn}$

5. $\sqrt[n]{a} = a^{\frac{1}{n}}$

Indices and Surds

Law of Indices

6. $\sqrt[n]{a^p} = a^{\frac{p}{n}}$

7. $a^{-n} = \dfrac{1}{a^n}$

8. $a^0 = 1$ [except when $a = 0$]

9. $(a \times b)^m = a^m \times b^m$

10. $\left(\dfrac{a}{b}\right)^m =$

Indices and Surds

Properties of Indices

1. If $a^m = a^n$, then $m = n$, except when $a = 1, -1, 0$

2. If $a^m = b^m$, then $a = b$ (if m is odd) and $a = \pm b$ (if m is even)

Indices and Surds

Example

If $5^a = 26$ and $125^b = 676$

Then,

(1) $3a = 2b$ (2) $a = 2b$ (3) $2a = 3b$ (4) $b = 2a$

$125^b = 676$

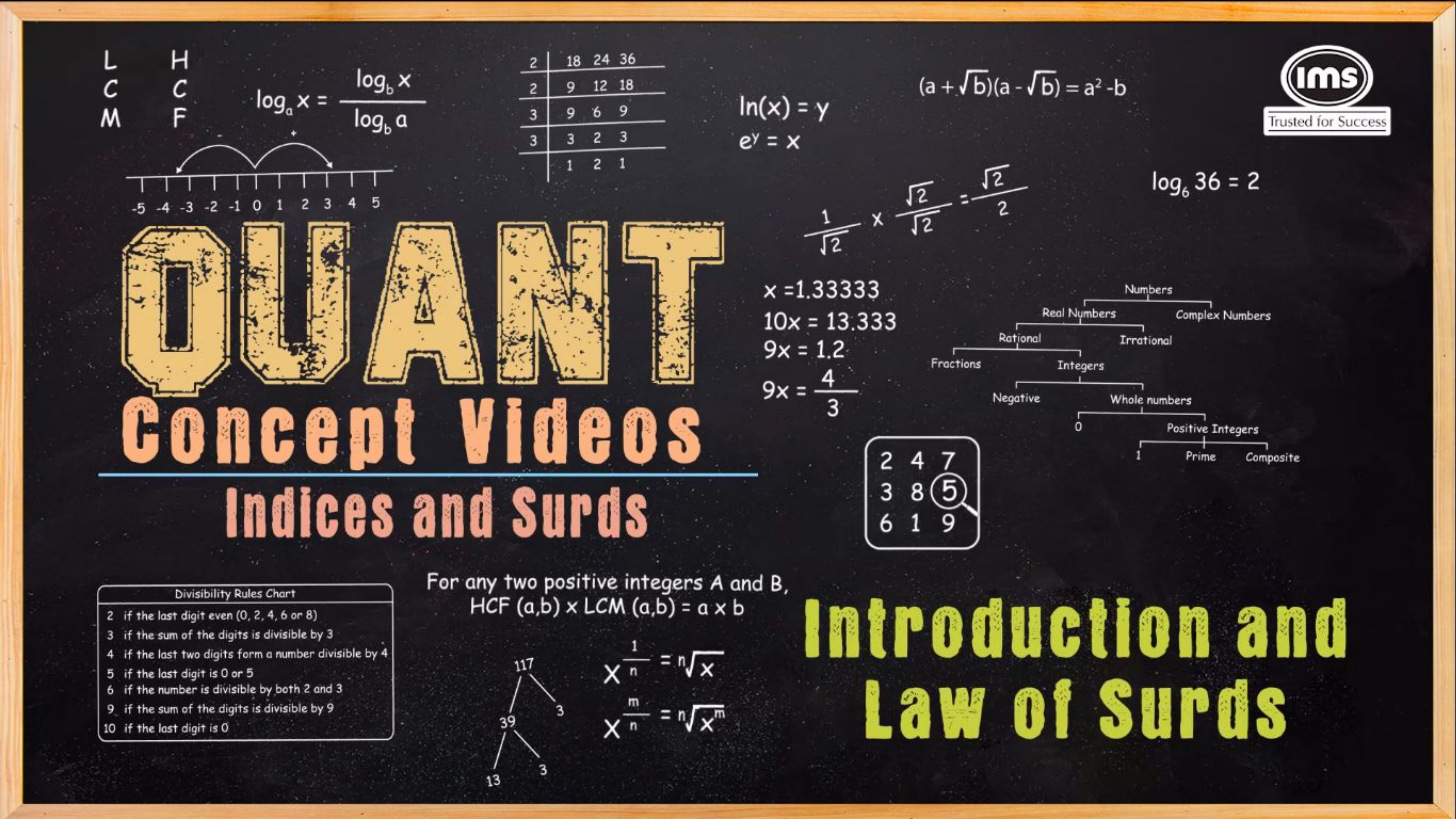
$125^b = (26)^2$

$125^b = (5^a)^2$

$(5^3)^b = (5^a)^2$

$5^{3b} = 5^{2a}$

$3b = 2a$



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Indices and Surds
Introduction and Law of Surds
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LCM HCF
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Indices and Surds

Introduction to Surds

Square Root $\Rightarrow \sqrt{5}$

Cube Root $\Rightarrow \sqrt[3]{7}$

Indices and Surds

Law of Surds

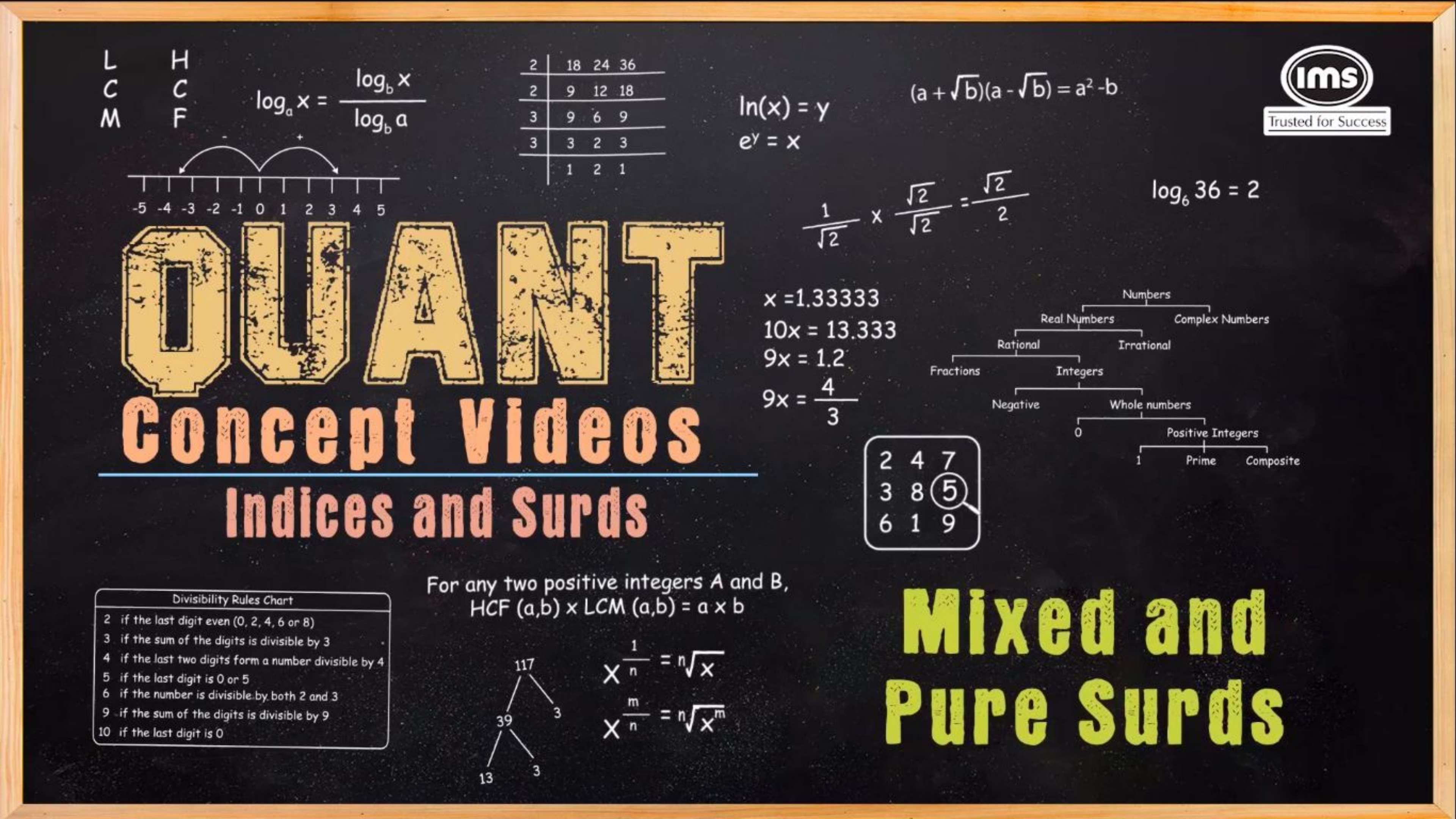
1. $\left(\sqrt[n]{a}\right)^n = a$

2. $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

5. $\left(\sqrt[n]{a}\right)^m = \sqrt[\frac{n}{m}]{a}$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Indices and Surds
Mixed and Pure Surds
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L C M
H C F
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Indices and Surds

Pure and Mixed Surds

Pure Surds

$\sqrt{45}$, $\sqrt{7}$, $\sqrt[3]{23}$

Mixed Surds

$2\sqrt{7}$, $5\sqrt[3]{7}$, $7\sqrt[4]{5}$

Indices and Surds

Pure and Mixed Surds

Conversion (Mixed $\Rightarrow$ Pure)

$$2\sqrt{7} = \sqrt{7 \times 4} = \sqrt{28}$$

$$5\sqrt[3]{7} = \sqrt[3]{7 \times 125} = \sqrt[3]{875}$$

Pure $\Rightarrow$ Mixed

$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

$$\sqrt[3]{54} = \sqrt[3]{27 \times 2} = 3\sqrt[3]{2}$$

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Indices and Surds

Rationalisation and Square Roots

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L H
C C
M F

$\log_a x = \dfrac{\log_b x}{\log_b a}$

-5 -4 -3 -2 -1 0 1 2 3 4 5

2	18 24 36
2	9 12 18
3	9 6 9
3	3 2 3
	1 2 1

$\ln(x) = y$

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117 → 39, 3
39 → 13, 3

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Indices and Surds

Rationalization

$$\left[\frac{12}{3\sqrt{2}+2\sqrt{3}}\right] \times \left[\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\right]$$

$$\left[\frac{12\left(3\sqrt{2}-2\sqrt{3}\right)}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}\right] = \left[\frac{12\left(3\sqrt{2}-2\sqrt{3}\right)}{18-12}\right]$$

$$= \left[\frac{12\left(3\sqrt{2}-2\sqrt{3}\right)}{6}\right]$$

$$= \left[\frac{2\left(3\sqrt{2}-2\sqrt{3}\right)}{1}\right]$$

Indices and Surds

Square Root of Surds

Positive square root of the surds of type $a + 2\sqrt{b}$:

$$\sqrt{7 + 2\sqrt{12}}$$

$$= \sqrt{4 + 3 + 2\sqrt{4 \times 3}}$$

$$= \sqrt{2^2 + (\sqrt{3})^2 + 2(2)(\sqrt{3})}$$

since, $a^2 + b^2 + 2ab = (a + b)^2$

$$= \sqrt{(2 + \sqrt{3})^2}$$

$$= (2 + \sqrt{3})$$

Indices and Surds

Square Root of Surds

Positive square root of the surds of type $a - 2\sqrt{b}$:

$$\sqrt{24 - \sqrt{252}}$$

$$= \sqrt{24 - 2\sqrt{63}}$$

$$= \sqrt{(\sqrt{21})^2 + (\sqrt{3})^2 - 2\sqrt{21}\,.\,\sqrt{3}}$$

since, $a^2 + b^2 - 2ab = (a - b)^2$

$$= \sqrt{(\sqrt{21} - \sqrt{3})^2}$$

$$= \sqrt{21} - \sqrt{3}$$

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Classification of Numbers

Square Roots

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H C F

$$\log_a x = \frac{\log_b x}{\log_b a}$$

-5 -4 -3 -2 -1 0 1 2 3 4 5

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	1	2	1

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117
39 3
13 3

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Classification of Numbers

Square Root using Long Division method

Example 1

Find the square root of 13010449

	3 6 0 7
3	13 | 01 | 04 | 49
+ 3	– 9
66	401
+ 6	– 396
720	504
+ 0	– 0
7207	50449
+ 7	– 50449
7214	0

Classification of Numbers

Square Root using Long Division method

Example 2

Find the square root of 5.23

	2. 2 8 6
2	5. | 2 3 | 0 0 | 0 0
+ 2	– 4
4 2	1 2 3
+ 2	– 8 4
4 4 8	3 9 0 0
+ 8	– 3 5 8 4
4 5 6 6	3 1 6 0 0
+ 6	– 2 7 3 9 6
4 5 7 2	4 2 0 4

$\therefore \sqrt{5.23} = 2.286$

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Classification of Numbers

Trusted for Success

L C M   H C F

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$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

Classification of Numbers

Imaginary / complex

$$\sqrt{-1} = i$$

$$i = \sqrt{-1}$$

$$i^2 = (\sqrt{-1})^2 = \boxed{-1}$$

$$i^3 = i^2 \times i = -i$$

$$i^4 = (i^2)^2 = (-1)^2 = 1$$

$$\sqrt{-36} = \sqrt{36} \times \sqrt{-1} = \pm 6i$$

$$\sqrt{-54} = \sqrt{54} \times \sqrt{-1} = \pm 3\sqrt{6}\, i$$

$$\boxed{\begin{matrix} 1 \\ i \\ -1 \\ -i \end{matrix}}$$

$$i^{57} = i^{56} \times i = (i^2)^{28} \times i = (-1)^{28} \times i = i$$

$$i^{74} = (i^2)^{37} = (-1)^{37} = \boxed{-1}$$

Classification of Numbers

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Numbers
(Digits: 10) + Numerals
(0, 1, 2, 3, 4, 5, 6, 7, 8, 9) → Base system 10

- **Real Numbers** — Numbers on number line
  - **Rational (Q)** — $\boxed{Q = \frac{p}{q};\ q \neq 0}$ (p, q are integers)
    - **Integers (I)** / Z — (–3, –2, –1, 0, 1, 2, 3)
      - **Whole Numbers** — (0, 1, 2,……)
        - **Natural (Positive)** — (1, 2, 3,……)
        - **0 (Neutral)**
      - **Negative Numbers** — (–3, –2, –1)
    - **Fractions (f)** — $\left(\frac{p}{q};\ q \neq 0, 1\right)$ → **Decimals (d)**
      - **Proper** — $\frac{p}{q};\ p < q$
      - **Improper** — $\frac{p}{q};\ p > q$
      - **Mixed** — Integer + Fraction; $I\frac{r}{s} = \frac{Is + r}{s}$
  - **Irrational** — Cannot be expressed in $\frac{p}{q}$ form; $\sqrt{2}$, π,……
- **Imaginary Numbers** — a + ib (a, b are constants); $i = \sqrt{-1}$

Decimals (d)
- **Terminating** — Denominator with powers of only 2 & 5 (0.2, 0.4,……)
- **Non-terminating** — $(0.3\bar{3}, 0.16\bar{6}, \ldots\ldots)$
  - **Recurring** — (rational $0.3333\ldots = 0.\bar{3}$)
  - **Non-recurring** — (irrational 0.315219…)

$\sqrt{-36} \neq$ Real no.

$2.5 = \frac{5}{2}$

$0.\bar{3} = 1/3$

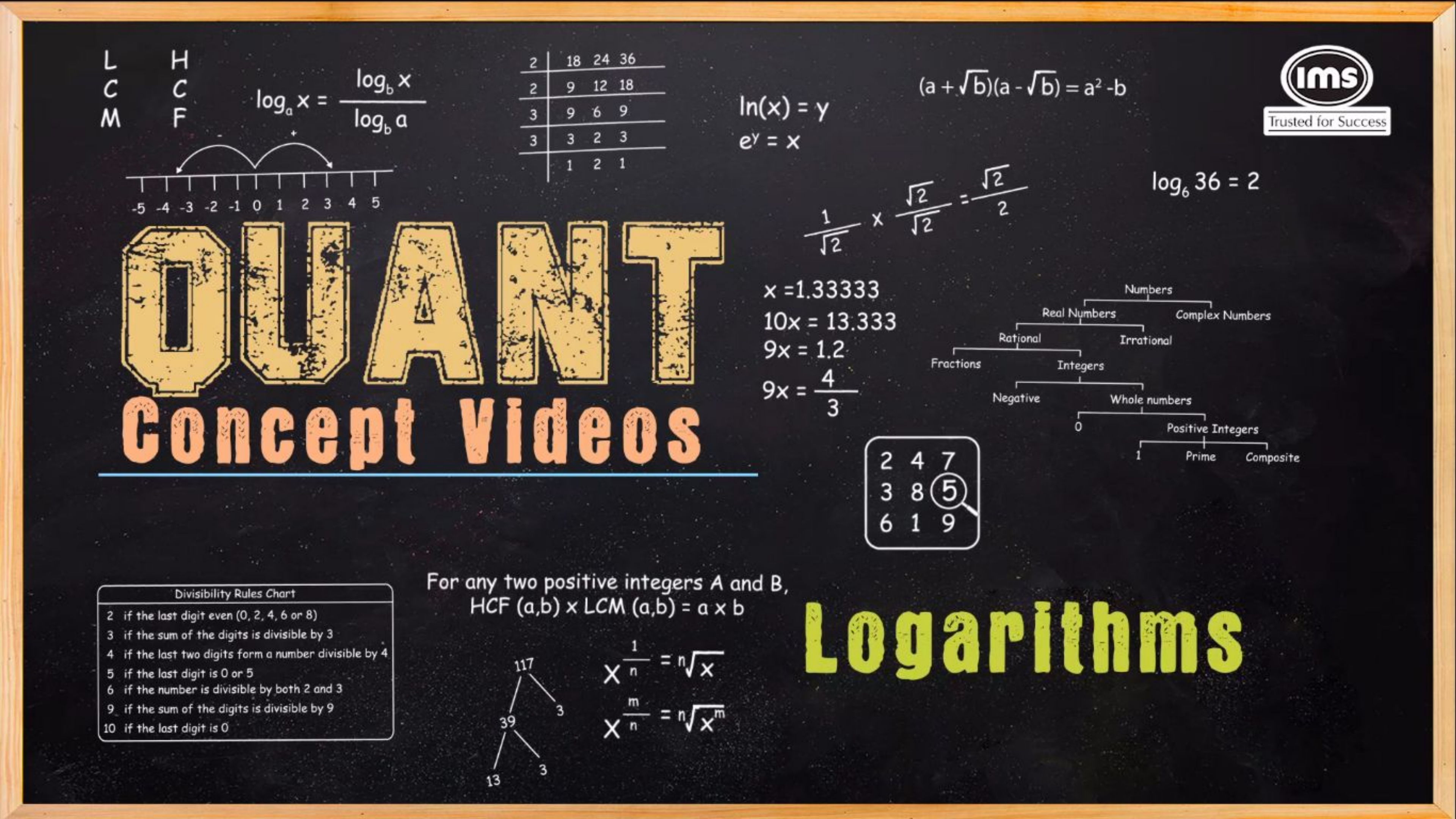
$\frac{22}{7} \approx \pi$

$i = \sqrt{-1}$

$\frac{\cancel{2}}{\cancel{6}_3}$

$= \boxed{2\frac{1}{3}} = 7/3$

$\frac{7}{3} \Rightarrow$ Improper



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1/√2 × √2/√2 = √2/2
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Logarithms

Logarithms

Introduction

Indices to Logarithm

If $a^x = b$,

Then, $\log_a b = x$

$2^5 = 32$, $\log_2 32 = 5$

Logarithm to Indices

If $\log_a b = x$

Then, $a^x = b$

$\log_3 243 = 5$ $3^5 = 243$

Logarithms

Laws of Logarithms

1. $\log 1 = 0$

2. $\log_a a = 1$

3. $\log_a(m \times n) = \log_a m + \log_a n$

4. $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

5. $\log_a m^r = r \log_a m$

6. $\log_a m = \frac{\log_b m}{\log_b a}$

Logarithms

Laws of Logarithms

7. $\log_a b \times \log_b a = \dfrac{\log b}{\log a} \times \dfrac{\log a}{\log b} = 1$

8. $\text{Log}_{a^\beta} b^\alpha = \dfrac{\log b^\alpha}{\log a^\beta} = \dfrac{\alpha \log b}{\beta \log a} = \dfrac{\alpha}{\beta} \times \log_a b$

9. $a^{\log_a N} = N$

Let $a^{\log_a N} = x$

$$\log x = \log a^{\log_a N}$$

$$\log x = \log_a N. \log a$$

$$\log x = \frac{\log N}{\log a} \times \log a$$

$$\log x = \log N$$

$$\therefore x = N$$

Logarithms

Examples

1. $\log(5x - 10) - \log(12 - x) = 1 + \log 7$. Evaluate x.

$$\log\left(\frac{5x-10}{12-x}\right) = \log(10 \times 7)$$

$$\frac{5x-10}{12-x} = 70$$

$$x - 2 = 168 - 14x$$

$$15x = 170$$

$$x = \frac{34}{3}$$

Logarithms

Examples

2. If log 2 = x   log 3 = y

Find log 144 in terms of ‘x’ & ‘y’

$$144 = 2^4 \times 3^2$$

$$\begin{aligned} \log(144) &= \log(2^4 \times 3^2) \\ &= 4\log 2 + 2\log 3 \\ &= 4x + 2y \end{aligned}$$

Logarithms

Examples

3. $\log_5 \log_3 (x - 2) = 1$ find 'x'

$$\log_5 a = 1$$

$$a = 5$$

$$\log_3 (x - 2) = 5$$

$$x - 2 = 3^5$$

$$x - 2 = 243$$

$$x = 245$$

QUANT
Concept Videos
Classification of Numbers

Last Digit or Units Digit I

LCM HCF

$\log_a x = \frac{\log_b x}{\log_b a}$

-5 -4 -3 -2 -1 0 1 2 3 4 5

2	18	24	36
2	9	12	18
3	9	6	9
3	3	2	3
	1	2	1

$\ln(x) = y$

$e^y = x$

$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\log_6 36 = 2$

$x = 1.33333$

$10x = 13.333$

$9x = 1.2$

$9x = \frac{4}{3}$

Numbers
- Real Numbers
  - Rational
    - Fractions
    - Integers
      - Negative
      - Whole numbers
        - 0
        - Positive Integers
          - 1
          - Prime
          - Composite
  - Irrational
- Complex Numbers

2	4	7
3	8	5
6	1	9

Divisibility Rules Chart

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5	if the last digit is 0 or 5
6	if the number is divisible by both 2 and 3
9	if the sum of the digits is divisible by 9
10	if the last digit is 0

For any two positive integers A and B,
HCF (a,b) × LCM (a,b) = a × b

117 → 39, 3; 39 → 13, 3

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Last Digit or Units Digit

Basic

$32 \times 73 \times 48$

$2 \times 3 \times 8$

8

Last Digit or Units Digit

Cyclicity of 2

Find the last digit of $(2)^{73}$.

$2^1 = 2$	$2^5 = 3\boxed{2}$	2
$2^2 = 4$	$2^6 = 6\boxed{4}$	4
$2^3 = 8$	$2^7 = 12\boxed{8}$	8
$2^4 = 1\boxed{6}$	$2^8 = 25\boxed{6}$	6

Last Digit or Units Digit

Example

Find the last digit of $(2)^{7583451}$.

$\frac{7583451}{4}$ $\boxed{R = ?}$

$\frac{51}{4}$ $\boxed{R = ?}$

R = 3

$(2)^{7583451}$

$= (2)^3$

$= 8$

2

4

8

6

Last Digit or Units Digit

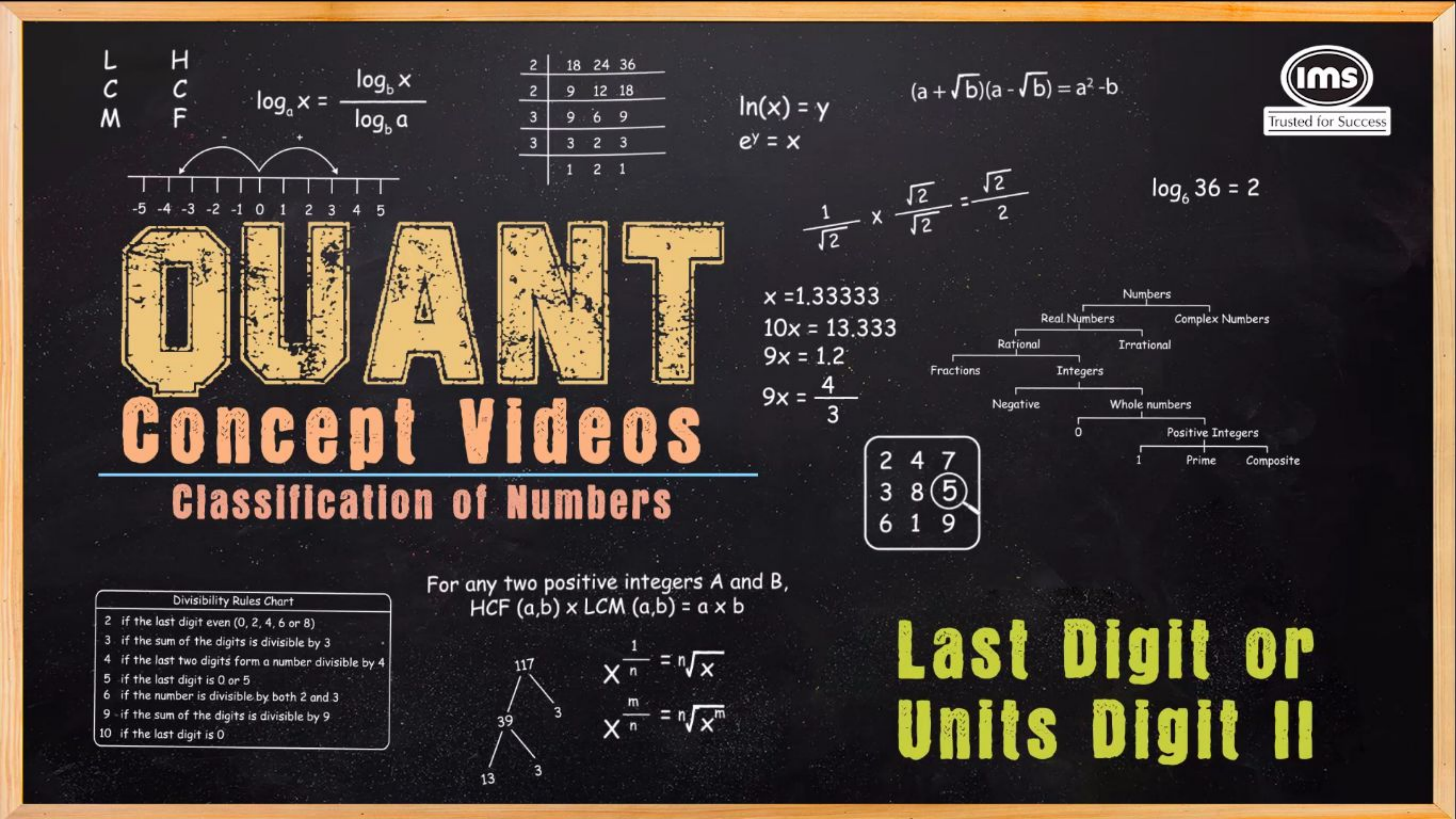
Cyclicity of 3

Find the last digit of $(2753)^{2984}$.

$(3)^{2984}$

		3
$3^1 = 3$	$3^5 = 3$	9
$3^2 = 9$	$3^6 = 9$	7
$3^3 = 7$	$3^7 = 7$	1
$3^4 = 1$	$3^8 = 1$	

$\therefore (3)^{84} \equiv 1$



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Classification of Numbers
Last Digit or Units Digit II
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L C M
H C F
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ln(x) = y
e^y = x
(a + √b)(a - √b) = a² - b
log_6 36 = 2
1/√2 × √2/√2 = √2/2
x = 1.33333
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Last Digit or Units Digit

Cyclicity of Other Digits

0, 1, 5, 6 ends in 0, 1, 5, 6

Cyclicity of Other 9 and 4

$9^1 = 9$	$4^1 = 4$
$9^2 = 1$	$4^2 = 6$
$9^3 = 9$	$4^3 = 4$
$9^4 = 1$	$4^4 = 6$
$9^{\text{Odd number}} \equiv 9$	$4^{\text{Odd number}} \equiv 4$
$9^{\text{Even number}} \equiv 1$	$4^{\text{Even number}} \equiv 6$

Last Digit or Units Digit

Cyclicity of 7 and 8

$7^1 = 7$	$8^1 = 8$
$7^2 = 9$	$8^2 = 4$
$7^3 = 3$	$8^3 = 2$
$7^4 = 1$	$8^4 = 6$

Last Digit or Units Digit

Example

$$\left((763)^{2197}\right)^{3471}$$

$$(3)^{2197 \times 3471}$$

$$\frac{2197 \times 3471}{4}$$

Rem = ?

$$\frac{1 \times 3}{4} \equiv 3$$

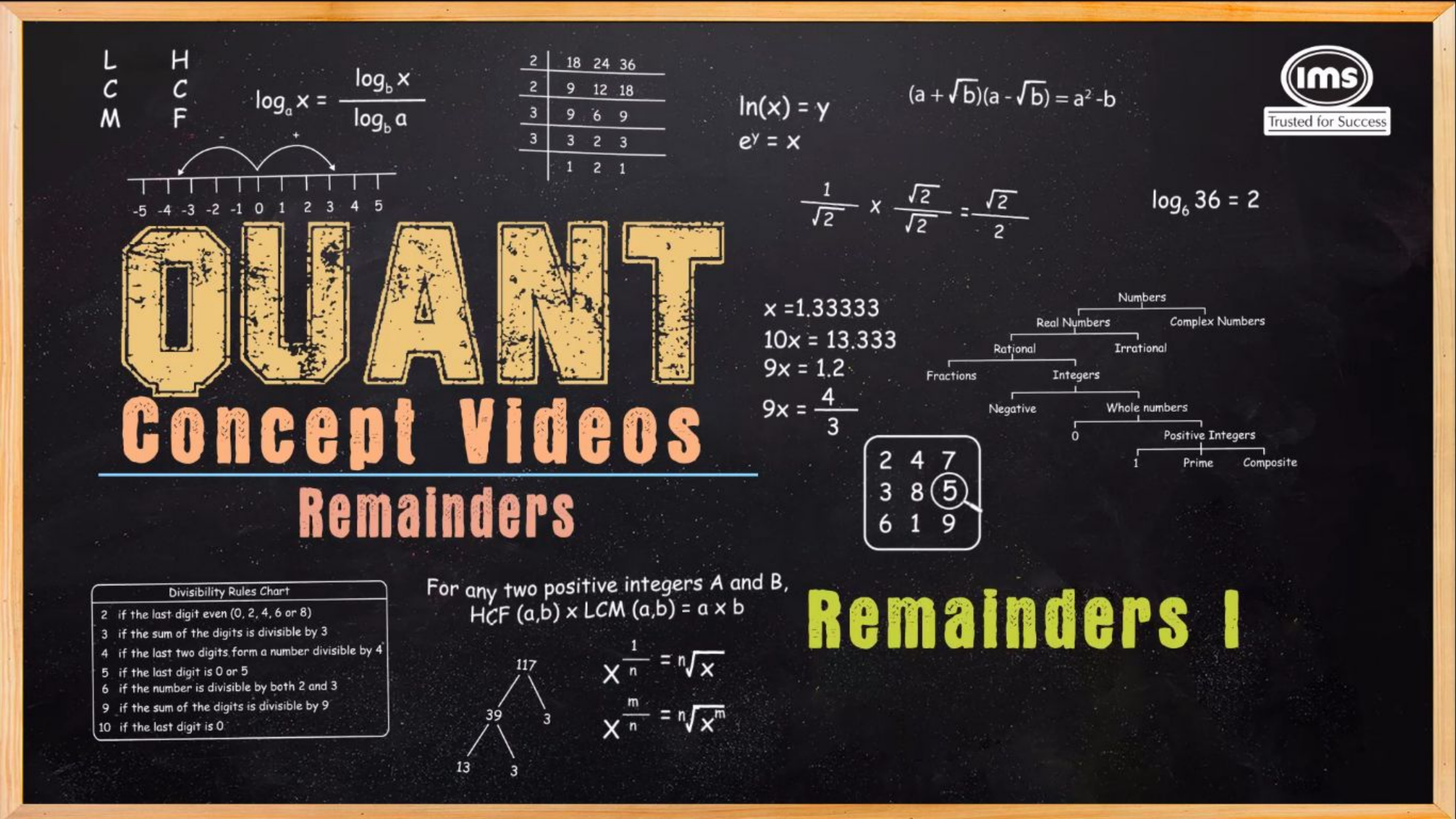
$$3^3 \equiv 7$$

3

9

7

1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Remainders I
IMS
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$x^{\frac{1}{n}} = \sqrt[n]{x}$
$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Remainders

Find the Remainder

$$\frac{3^{435}}{242}$$

$$= \frac{(3^5)^{87}}{242}$$

$$= \frac{(243)^{87}}{242}$$

$$= \frac{(242 + 1)^{87}}{242}$$

$$= \frac{(1)^{87}}{242} \qquad = 1$$

Remainders

Find the Remainder

$$\frac{3^{66}}{242}$$

$$= \frac{3^{65} \times 3^{1}}{242}$$

$$= \frac{3^{65}}{242} \times \frac{3^{1}}{242}$$

$$= 1 \times 3$$

$$= 3$$

Remainders

Find the Remainder

$$\frac{3^{75}}{244}$$

$$= \frac{(3^5)^{15}}{244}$$

$$= \frac{(243)^{15}}{244}$$

$$= \frac{(244-1)^{15}}{244}$$

$$= \frac{(-1)^{15}}{244} \quad = -1 \quad = 243$$

Remainders

Find the Remainder

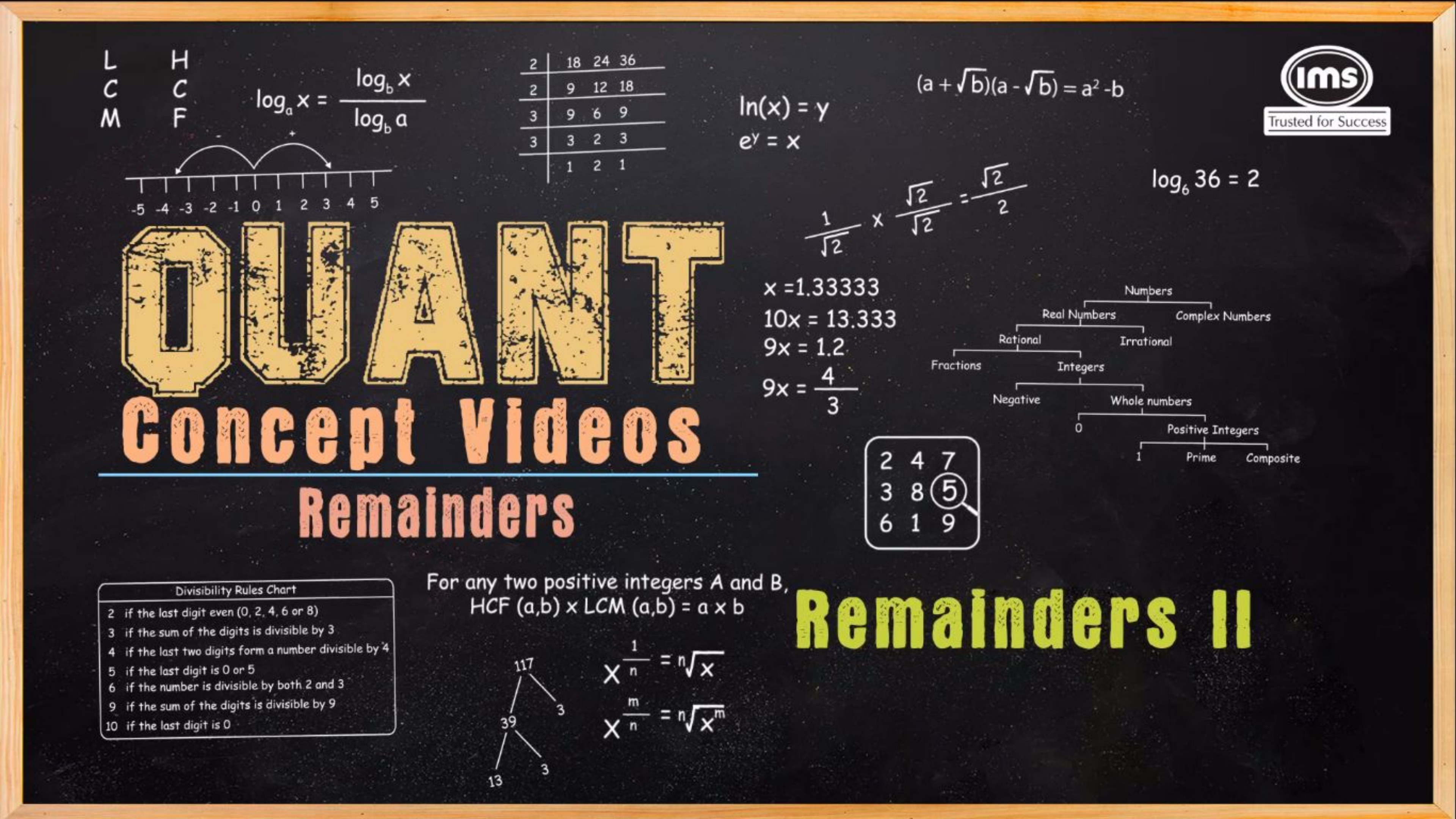
$$\frac{3^{78}}{244} = \frac{3^{75} \times 3^3}{244}$$

$$\frac{3^{75}}{244} \times \frac{3^3}{244} = -1 \times 27$$

$$= -27$$

$$= 244 - 27$$

$$= 217$$



LCM
HCF
$\log_a x = \frac{\log_b x}{\log_b a}$
-5 -4 -3 -2 -1 0 1 2 3 4 5
2 | 18 24 36
2 | 9 12 18
3 | 9 6 9
3 | 3 2 3
1 2 1
$\ln(x) = y$
$e^y = x$
$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$
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117
39
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$x^{\frac{1}{n}} = \sqrt[n]{x}$
$x^{\frac{m}{n}} = \sqrt[n]{x^m}$
Remainders II

Remainders

Method of Cyclicity

$$\frac{3^{76}}{11}$$

$$\frac{3^1}{11} = 3$$

$$\frac{3^2}{11} = 9$$

$$\frac{3^3}{11} = 5$$

$$\frac{3^4}{11} = \frac{5 \times 3}{11} = 4$$

$$\frac{3^5}{11} = \frac{4 \times 3}{11} = 1$$

$$\frac{3^6}{11} = \frac{1 \times 3}{11} = 3$$

$$\therefore \frac{3^{75}}{11} \Rightarrow \text{Remainder} = 1$$

$$\therefore \frac{3^{76}}{11} \Rightarrow \text{Remainder} = 3$$

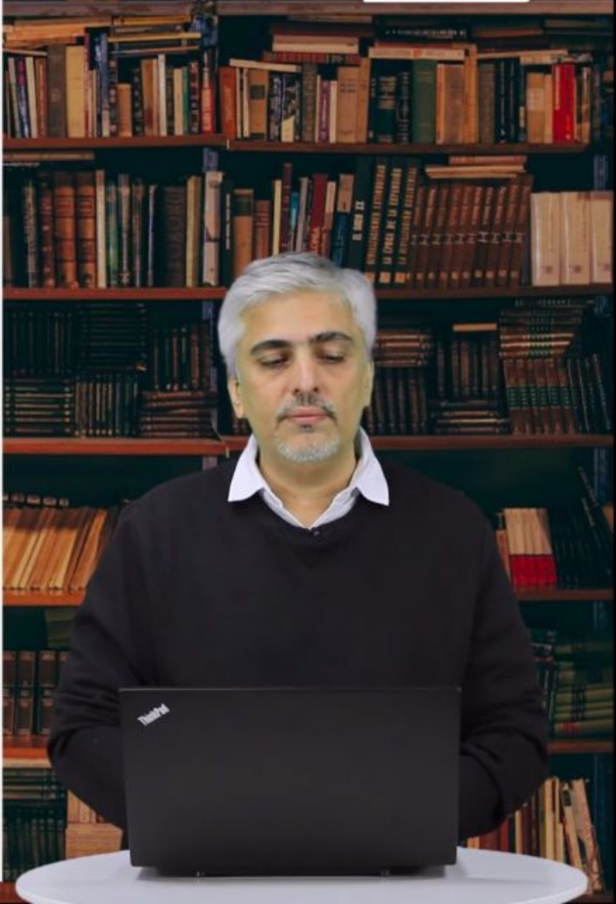
3

9

5

4

1



Remainders

Method of Cyclicity

$$\frac{(2225)^{78}}{11}$$

$$\frac{2225}{11} \Rightarrow \text{Remainder} = 3$$

$$= \frac{(3)^{78}}{11}$$

$\therefore$ Remainder = 5

3

9

5

4

1

Remainders

Method of Cyclicity

$$\frac{(17838)^{593}}{7}$$

$$\frac{17838}{7} \Rightarrow \text{Remainder} = 2$$

$$\therefore \frac{(2)^{593}}{7}$$

$$\frac{2^1}{7} = 2$$

$$\frac{2^2}{7} = 4$$

$$\frac{2^3}{7} = 1$$

$$\frac{2^4}{7} = 2$$

$$\text{Now, } \frac{593}{3} \Rightarrow \text{Remainder} = 2$$

$$\therefore \text{Final Remainder} = 4$$

2

4

1

QUANT

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Remainders

Chinese Remainder Theorem

L C M H C F

$$\log_a x = \frac{\log_b x}{\log_b a}$$

-5 -4 -3 -2 -1 0 1 2 3 4 5

2	18	24	36
2	9	12	18
3	9	6	9
3	3	2	3
	1	2	1

$\ln(x) = y$

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$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

Remainders

Chinese Remainder Theorem

Example 1

Find the remainder when 2^{26} is divided by 77

$77 = 7 \times 11$

Calculating the remainders for $\left(\frac{2^{26}}{7}\right)$ and $\left(\frac{2^{26}}{11}\right)$ separately

a) $\text{Rem}\left(\frac{2^{26}}{7}\right) = \text{Rem}\left(\frac{(2^3)^8 \cdot 2^2}{7}\right) = 4$

$\therefore\ 2^{26} = 7a + 4$

Remainders

Chinese Remainder Theorem

Example 1

Find the remainder when 2^{26} is divided by 77

a) $\text{Rem}\left(\frac{2^{26}}{11}\right) = \text{Rem}\left(\frac{(2^5)^5 .\ 2^1}{11}\right)$

$$= \text{Rem}\left(\frac{-1 .\ 2}{11}\right) = 9$$

$\therefore\ 2^{26} = 11b + 9$

Remainders

Chinese Remainder Theorem

Example 1

Find the remainder when 2^{26} is divided by 77

$2^{26} = 7a + 4$

$2^{26} = 11b + 9$

9, 20, 31, 42, 53.

$\therefore 7a + 4 = 11b + 9 = 53$

$\Rightarrow \text{Rem}\left(\frac{2^{26}}{77}\right) = 53$

Remainders

Chinese Remainder Theorem

Example 2

Find the remainder when 2^{51} is divided by 143

$143 = 11 \times 13$

Calculating the remainders for $\left(\frac{2^{51}}{11}\right)$ and $\left(\frac{2^{51}}{13}\right)$ separately

a) $\text{Rem}\left(\frac{2^{51}}{11}\right) = \text{Rem}\left(\frac{(2^5)^{10} . 2^1}{11}\right)$

$= \text{Rem}\left(\frac{(-1)^{10} . 2^1}{11}\right) = 2$

Remainders

Chinese Remainder Theorem

Example 2

Find the remainder when 2^{51} is divided by 143

b) $\text{Rem}\left(\dfrac{2^{51}}{13}\right)$

$$= \text{Rem}\left(\frac{(2^6)^8 . 2^3}{13}\right) = \text{Rem}\left(\frac{(-1)^8 . 8}{13}\right) = 8$$

$\therefore 2^{51} = 13b + 8$

Remainders

Chinese Remainder Theorem

Example 1

Find the remainder when 2^{51} is divided by 143

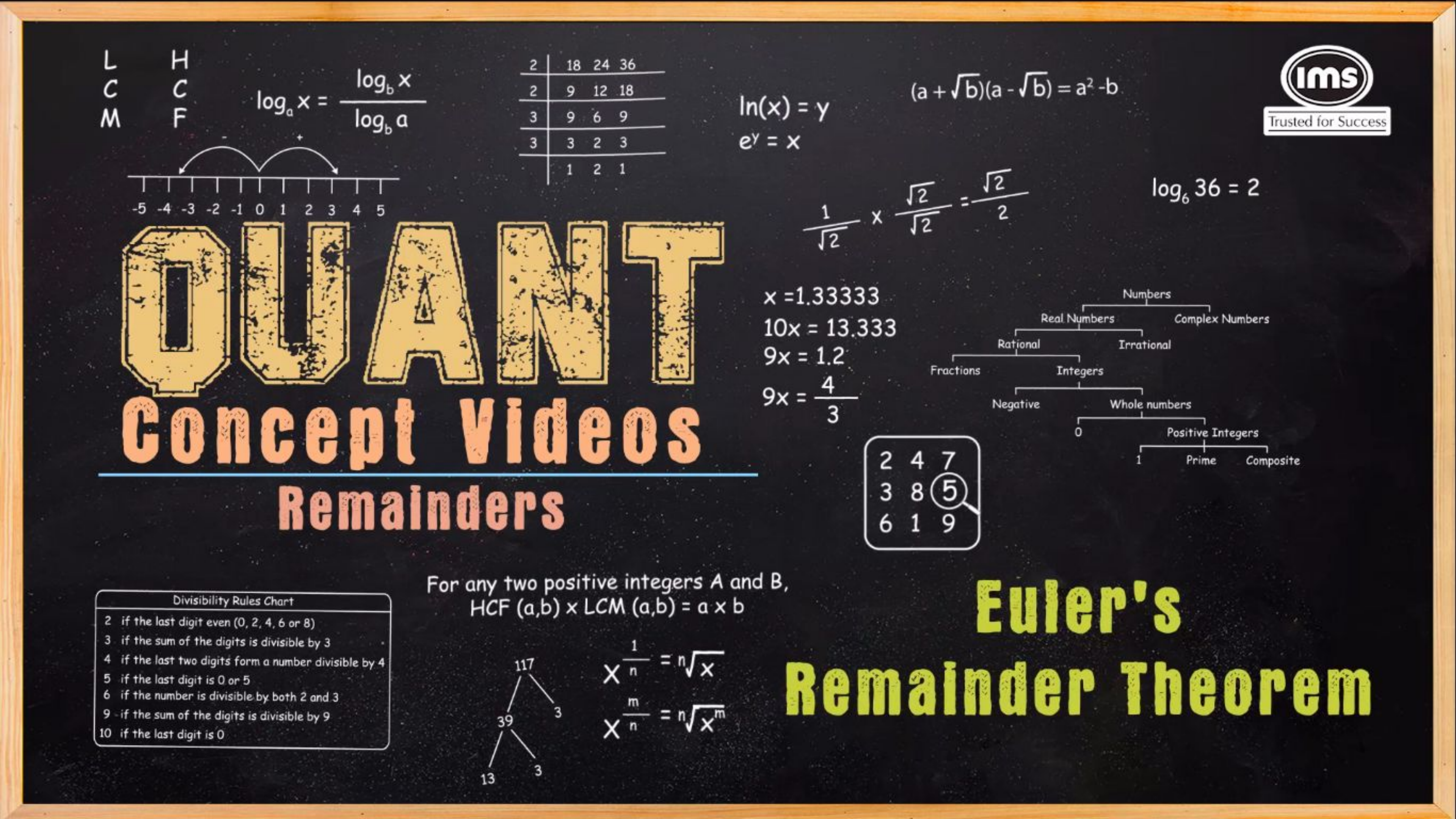
$2^{51} = 11a + 2$

$2^{51} = 13b + 8$

8, 21, 34, 47, 60, 73, 86, 99, 112,

$\therefore 11a + 2 = 13b + 8 = 112$

$\Rightarrow \text{Rem}\left(\frac{2^{51}}{143}\right) = 112$



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Remainders
Euler's Remainder Theorem
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e^y = x
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1/√2 × √2/√2 = √2/2
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9 if the sum of the digits is divisible by 9
10 if the last digit is 0
117
39
3
13
3

Remainders

Euler's Function

Consider a natural number 'n' which can be written as

$n = a^p \times b^q \times c^r$

where a, b, c are the prime factors of n

The number of positive integers less than n and co-prime to n are given by

$$\Phi(n) = n\left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{b}\right)\left(1 - \frac{1}{c}\right)$$

where $\Phi(n)$ is the Euler's function

Remainders

Euler’s Function

Example

Calculate the Euler’s function of 63

$n = 63$

$63 = 3^2 \times 7$

$$\Phi(63) = 63\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{7}\right)$$

$$= 63\left(\frac{2}{3}\right)\left(\frac{6}{7}\right)$$

$$= 36$$

Remainders

Euler’s Remainder Theorem

According to Euler’s Remainder Theorem,

Remainder of $\left(\frac{m^{\Phi(n)}}{n}\right) = 1$

where m and n are relatively prime numbers

Remainders

Euler's Remainder Theorem

Example 1

Find the remainder when 3^{50} is divided by 35

$m = 3, n = 35$

$$\Phi(35) = 35\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right) = 24$$

$$\therefore \text{Rem}\left(\frac{3^{24}}{35}\right) = 1$$

$$\text{Rem}\left(\frac{3^{50}}{35}\right) = \text{Rem}\left(\frac{3^{24} \cdot 3^{24} \cdot 3^{2}}{35}\right) = 1.1.9 = 9$$

Remainders

Euler's Remainder Theorem

Example 2

Find the remainder when 31^{2162} is divided by 1001

$m = 31,\ n = 1001$

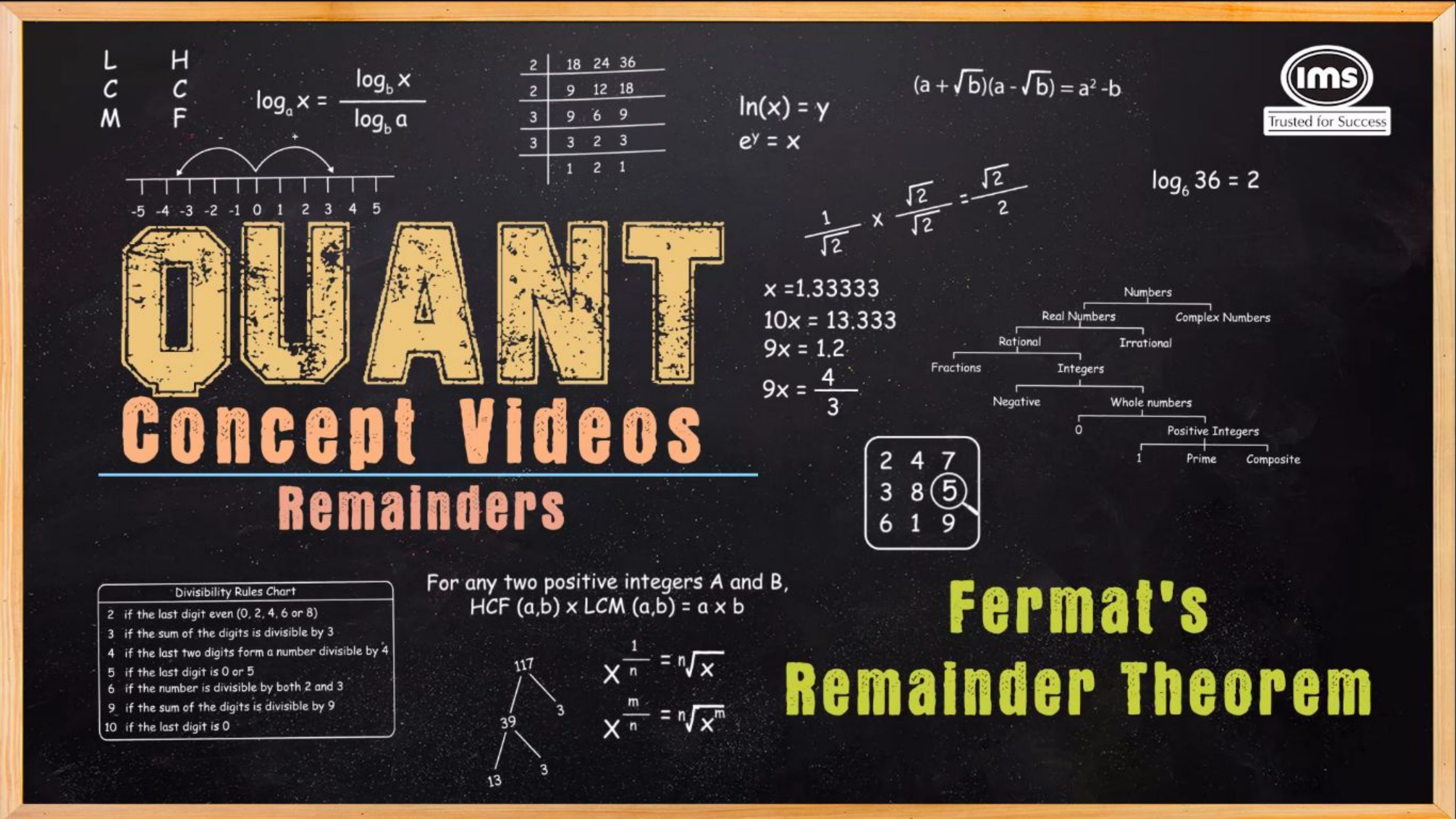
$$\Phi(1001) = 1001\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{11}\right)\left(1 - \frac{1}{13}\right) = 720$$

$$\therefore \text{Rem}\left(\frac{31^{720}}{1001}\right) = 1$$

$$\text{Rem}\left(\frac{31^{2162}}{1001}\right) = \text{Rem}\left(\frac{31^{720}.\ 31^{720}.\ 31^{720}.\ 31^{2}}{1001}\right)$$

$$= 1.1.1.961$$

$$= 961$$



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$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Remainders

Fermat's Little Theorem

According to Fermat's Little Theorem,

Remainder of $\left(\frac{m^{n-1}}{n}\right) = 1$

where n is a prime number and m and n are relatively prime

Remainders

Fermat's Little Theorem

Example 1

Find the remainder when 7^{98} is divided by 13

$m = 7,\ n = 13$

According to Fermat's little theorem,

$$\text{Rem}\left(\frac{7^{13-1}}{13}\right) = \text{Rem}\left(\frac{7^{12}}{13}\right) = 1$$

$$\therefore \text{Rem}\left(\frac{7^{98}}{13}\right) = \text{Rem}\left(\frac{(7^{12})^{8} \cdot 7^{2}}{13}\right)$$

$$= 1 \times 10$$

$$= 10$$

Remainders

Fermat's Little Theorem

Example 2

Find the remainder when 37^{1784} is divided by 163

$m = 37$, $n = 163$

According to Fermat's little theorem,

$$\text{Rem}\left(\frac{37^{163-1}}{163}\right) = \text{Rem}\left(\frac{37^{162}}{163}\right) = 1$$

$$\therefore \text{Rem}\left(\frac{37^{1784}}{163}\right) = \text{Rem}\left(\frac{(37^{162})^{11} . 37^{2}}{163}\right)$$

$$= 1 \times 65$$

$$= 65$$