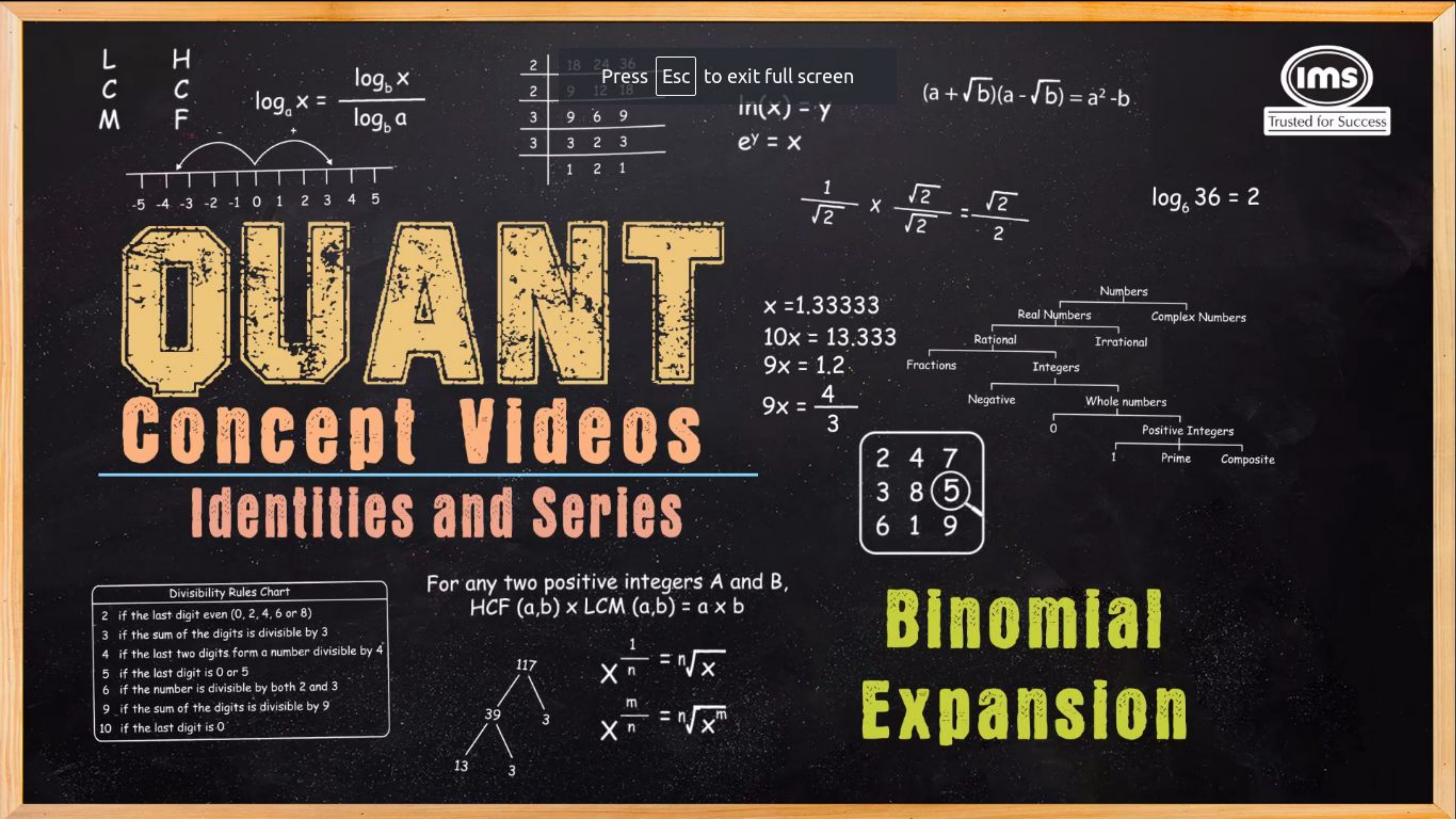

QUANT
Concept Videos
Identities and Series
Binomial Expansion
IMS
Trusted for Success
Press Esc to exit full screen
L C M
H C F
log_a x = log_b x / log_b a
ln(x) = y
e^y = x
(a + √b)(a - √b) = a² - b
log_6 36 = 2
x = 1.33333
10x = 13.333
9x = 1.2
9x = 4/3
Numbers
Real Numbers
Complex Numbers
Rational
Irrational
Fractions
Integers
Negative
Whole numbers
0
Positive Integers
1
Prime
Composite
Divisibility Rules Chart
2 if the last digit even (0, 2, 4, 6 or 8)
3 if the sum of the digits is divisible by 3
4 if the last two digits form a number divisible by 4
5 if the last digit is 0 or 5
6 if the number is divisible by both 2 and 3
9 if the sum of the digits is divisible by 9
10 if the last digit is 0
For any two positive integers A and B,
HCF (a,b) × LCM (a,b) = a × b

Identities

Binomial Expansion

$(a + b)^n = \ {}^nC_0 a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 \ldots\ldots\ldots\ {}^nC_n\, b^n$

Pascal's Triangle

$$
\begin{array}{ccccccccccc}
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

$(a + b)^5 = \ a^5 + 5\, a^4 b + 10\, a^3 b^2 + 10\, a^2 b^3 + 5\, a b^4 + b^5$

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Concept Videos

Functions

Introduction to Functions

Inequality Symbols

$<$ Less than, $>$ Greater than, $\leq$ Less than or equal to, $\geq$ Greater than or equal to, $\neq$ Not equal to

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

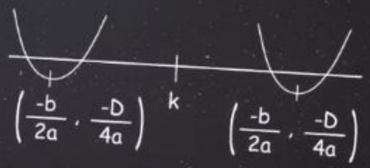
$x > y \quad (x, y \longrightarrow \text{positive})$

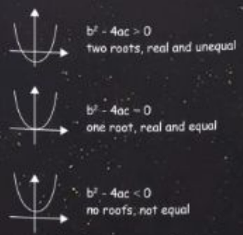
$$\frac{1}{x} < \frac{1}{y}$$

$ax^2 + bx + c = 0$

sum of roots $= \frac{-b}{a}$

Product of roots $= \frac{c}{a}$





$T_n \longrightarrow$ n^{th} Term

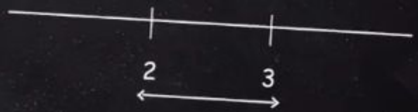
$r \longrightarrow$ Common ratio

$n \longrightarrow$ Number of Terms

Sum $\longrightarrow$ Sum of all Geometric progression

$x^2 - 5x + 6 < 0$

$(x - 3)(x - 2) < 0$



$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$t_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}[a + l]$

1, 2, 4, 8, 16, 32, ...

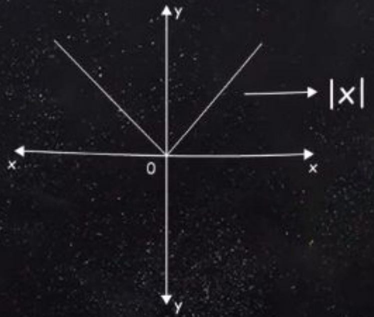
$r = 2$

$t_n = (2)^{n-1}$

$s_n = (2^n - 1)$

$t_n = (r)^{n-1}$

$s_n = \frac{a(r^n - 1)}{(r - 1)}$



Functions

Introduction

Definition

A function from a set A to a set B is a relation which associates every element of the set A to a unique element of set B and is denoted by $f: A \to B$.

If f is a function from A to B and $(x, y) \in f$, then we write it as

$y = f(x)$.

Functions

Introduction

Domain

The set of all first components of the ordered pairs in a function F is called the domain of the function F.

Range

The set of all second components of the ordered pairs in a function F is called the range of the function F.

Co-domain

If F is a function from A to B, then set B is called the co-domain of the Function F.

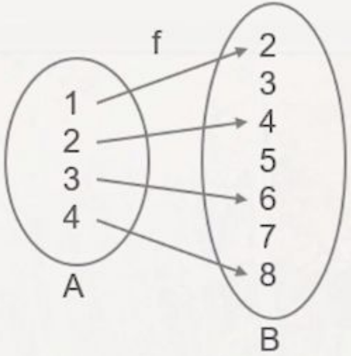
Note that the range set is a subset of co-domain. This subset may be proper or improper.

Functions

Example

Let A = {1, 2, 3, 4}, B = {2, 3, 4, 5, 6, 7, 8}

$f(x) = 2x$



This relation is a function from set A to set B.

Here, $f(1) = 2$, $f(2) = 4$, $f(3) = 6$, $f(4) = 8$

The set of all values of function f is {2, 4, 6, 8}.

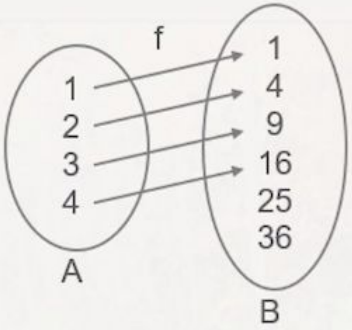
This set is called range of the function f.

Functions

Example

In the figure for $f: A \rightarrow B$

$f(x) = x^2$



$A = \{1, 2, 3, 4\}$ is domain

$B = \{1, 4, 9, 16, 25, 36\}$ is co-domain

Set $\{1, 4, 9, 16\}$ is the range of the function f.

Inequality Symbols

$<$ Less than, $>$ Greater than, $\leq$ Less than or equal to, $\geq$ Greater than or equal to, $\neq$ Not equal to

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x > y \quad (x, y \longrightarrow \text{positive})$

$$\frac{1}{x} < \frac{1}{y}$$

$ax^2 + bx + c = 0$

$\text{sum of roots} = \frac{-b}{a}$

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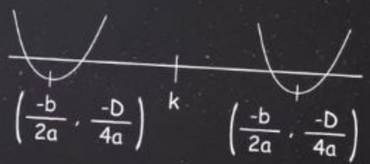

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Functions

$b^2 - 4ac > 0$ two roots, real and unequal

$b^2 - 4ac = 0$ one root, real and equal

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2 3

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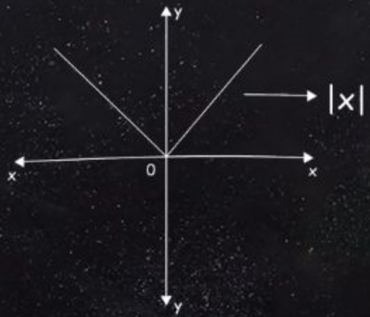


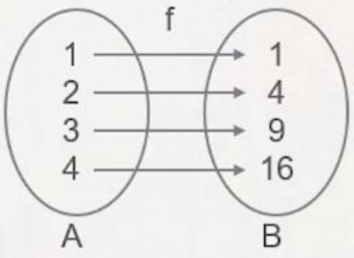
Types of Functions

Functions

Types of Functions

One-one Function

A function f: A $\rightarrow$ B is said to be one-one function, if different elements in A have different images in B.



Consider the function f: A $\rightarrow$ B such that each element of its range set is the value of the function at only one element of the domain set.

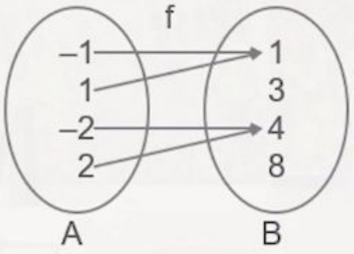
In this case, f: A $\rightarrow$ B is one-one function.

Functions

Types of Functions

Many-one Function

If the function f: $A \rightarrow B$ is such that two or more elements in a set A have the same image in set B i.e. there is at least one element in B which has more than one pre image in A then the function f is called many-one function.



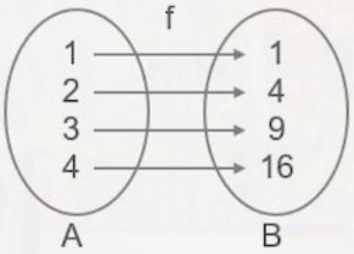
In this case, f: $A \rightarrow B$ is many-one function.

Functions

Types of Functions

Onto Function

If the function f: $A \to B$ is such that each element in B is the image of some element in A, then f is said to be a onto function. In this case range of function f is same as its co-domain B.



Hence, f: $A \to B$ is an onto function.

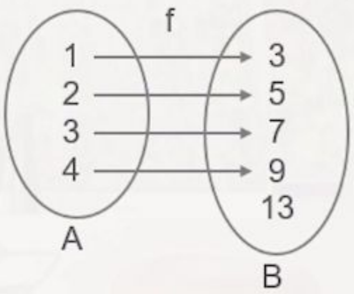
Range = Co-domain

Functions

Types of Functions

Into Function

If the function f: A → B is such that there exists at least one element in B which is not the image of any element in A, then f is said to be an into function. In this case, the range of a function f is a proper subset of its co-domain.



Hence, f: A → B is an into function.

Range ≠ Co-domain

Functions

Example 1

Find the domain and range of the following functions:

(i) $f(x) = x^2$ (ii) $f(x) = \dfrac{5 - x}{x - 3}$

Solution:

(i) $f(x) = x^2$

$\therefore$ Domain = set of all real numbers

Range = $\{x/x \in R \text{ and } x \geq 0\}$

Functions

Example 1

Find the domain and range of the following functions:

(i) $f(x) = x^2$ (ii) $f(x) = \dfrac{5 - x}{x - 3}$

Solution:

(ii) $f(x) = \dfrac{5 - x}{x - 3}$

f(x) is not defined,

when $x - 3 = 0$

i.e., when $x = 3$

$\therefore$ Domain of $f = R - \{3\}$

Let $y = f(x) = \dfrac{5 - x}{x - 3}$

$\therefore\ xy - 3y = 5 - x$

$\therefore\ x(y + 1) = 5 + 3y$

$\therefore\ x = \dfrac{5 + 3y}{y + 1}$

which is not defined,

i.e., when $y = -1$

$\therefore$ Range of $f = R - \{-1\}$

Functions

Example 2

If $f(x) = f(3x - 1)$, for $f(x) = x^2 - 4x + 11$, find x.

Solution:

$f(x) = x^2 - 4x + 11$

Also, $f(x) = f(3x - 1)$

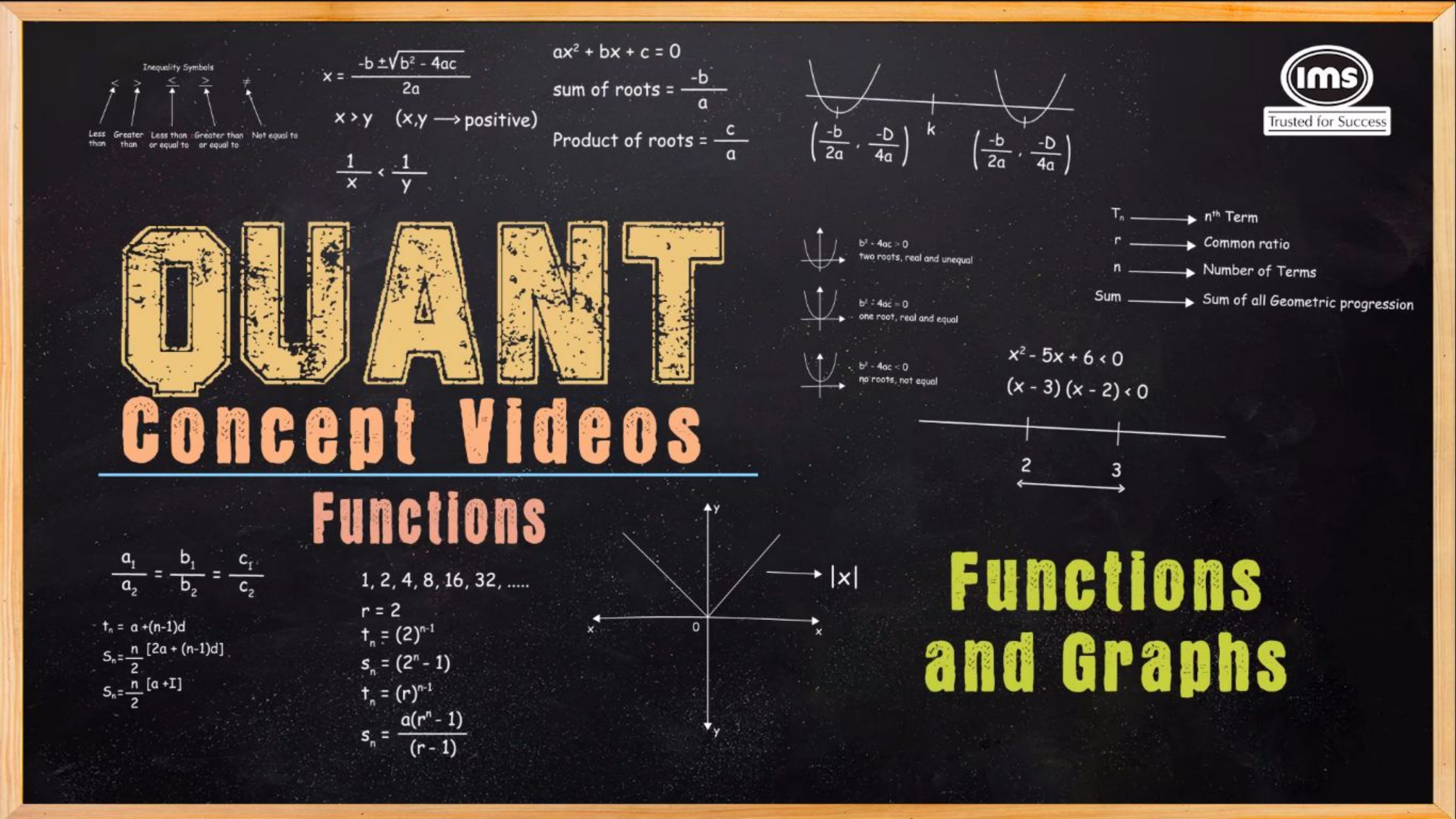
$\therefore\ x^2 - 4x + 11 = (3x - 1)^2 - 4(3x - 1) + 11$

$\therefore\ x^2 - 4x = 9x^2 - 6x + 1 - 12x + 4$

$\therefore\ 8x^2 - 14x + 5 = 0$

$\therefore (2x - 1)(4x - 5) = 0$

$\therefore\ x = \frac{1}{2}$ or $x = \frac{5}{4}$



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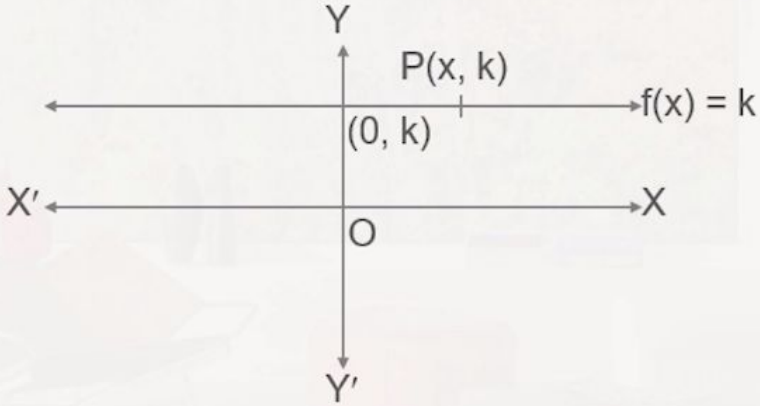
$s_n = \frac{a(r^n - 1)}{(r - 1)}$
y x 0 x y
$\longrightarrow |x|$
Functions and Graphs

Functions

Particular Types of Functions & their Graphs

Constant Function

A function f defined by f(x) = k, for all $x \in R$, where k is a constant, is called a constant function. The graph of a constant is a line parallel to the X-axis, intersecting Y-axis at (0, k).



For example, f(x) = 5 is a constant function.

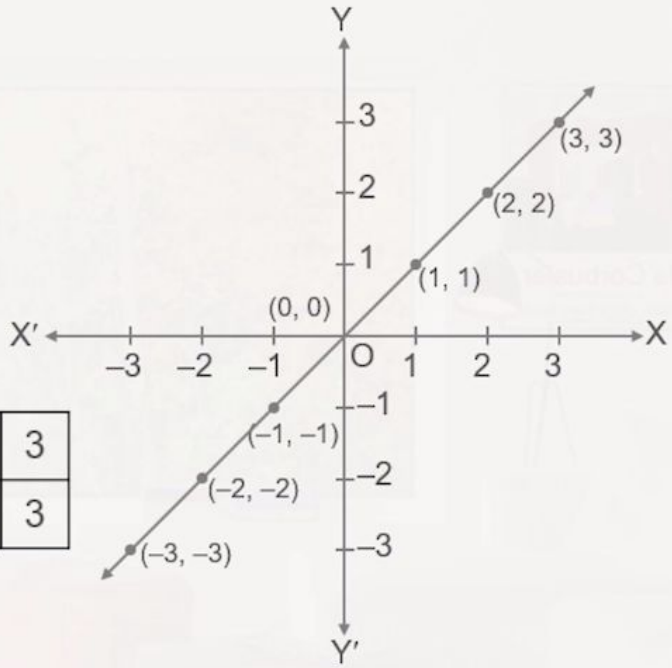
Functions

Particular Types of Functions & their Graphs

Identity Function

The function f(x) = x, is called an identity function. The graph of the identity function is the line which bisects the first and the third quadrants.

x	–3	–2	–1	0	1	2	3
f(x)	–3	–2	–1	0	1	2	3



Functions

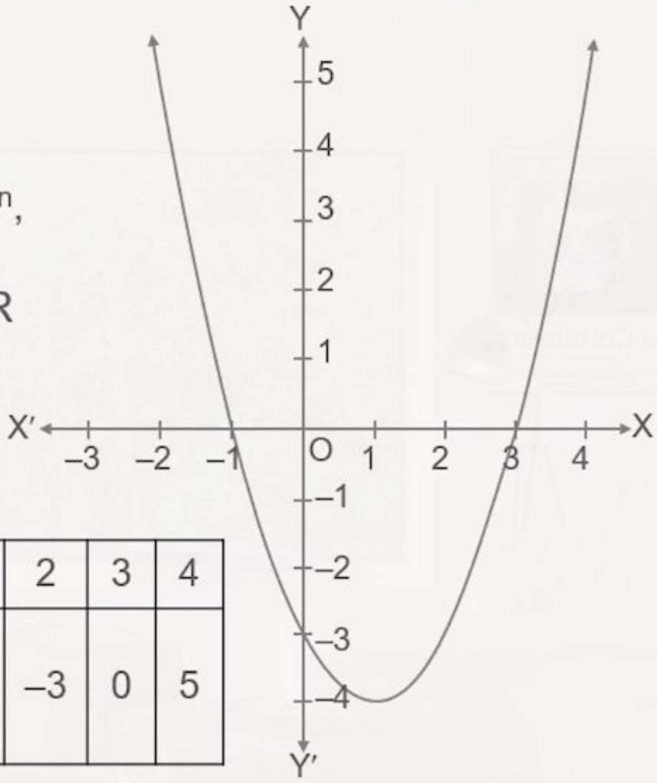
Particular Types of Functions & their Graphs

Polynomial Function

A function of the form
$f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$,
where n is a non-negative integer and $a_0, a_1, a_2, \ldots, a_n \in R$ is called a polynomial function.

e.g., $f(x) = x^2 - 2x - 3$ for $x \in R$

x	–2	–1	0	1	2	3	4
$f(x) = x^2 - 2x - 3$	5	0	–3	–4	–3	0	5



Functions

Particular Types of Functions & their Graphs

Modulus Function

Let f: R $\rightarrow$ R, the function f(x) = |x| such that

$$|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$$

is called modulus or absolute value function.

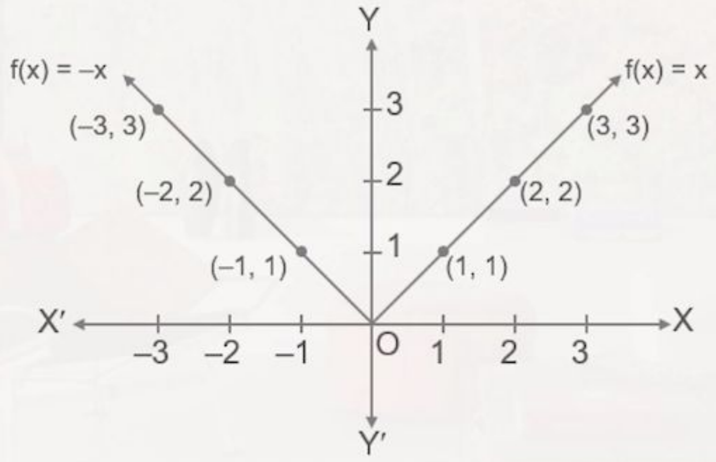
Functions

Particular Types of Functions & their Graphs

Modulus Function

Consider table of same values $f(x) = |x|$

x	–3	–2	–1	0	1	2	3
f(x)	–3	–2	–1	0	1	2	3



Functions

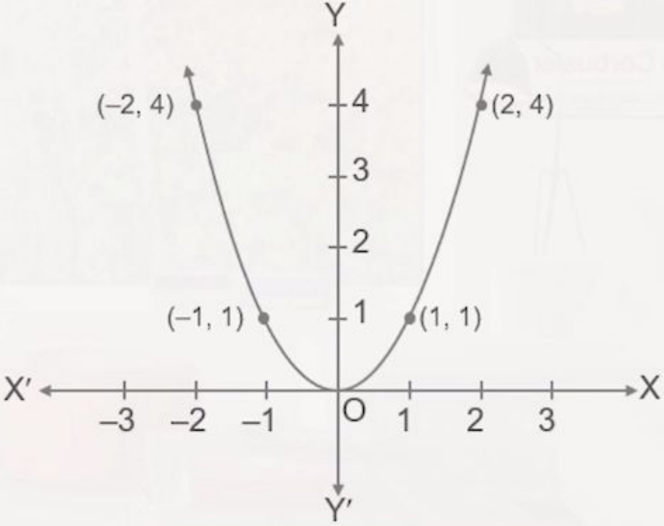
Particular Types of Functions & their Graphs

Even Function

A function f is said to be an even function, if $f(-x) = f(x)$ for all $x \in R$ Let $f: R \to R : f(x) = x^2$ for all $x \in R$ Domain of $f = R$, range of $f = \{x/x \in R, x \geq 0\}$

We have

x	–2	–1	0	1	2
$f(x) = x^2$	4	1	0	1	4



Functions

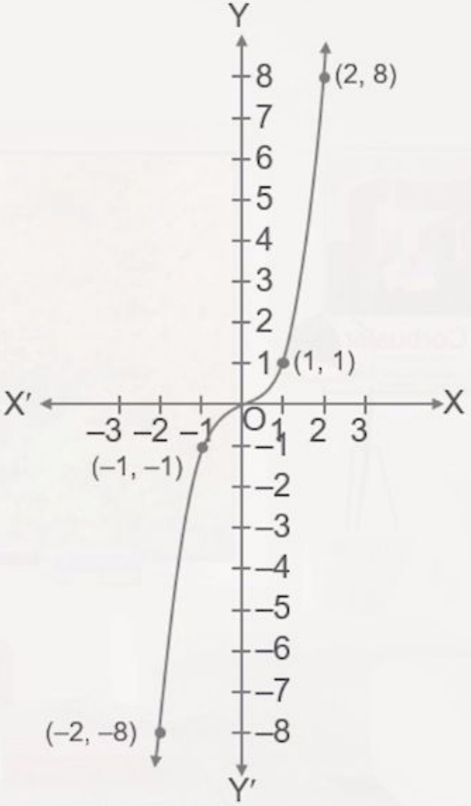
Particular Types of Functions & their Graphs

Odd Function

A function f is called an odd function, if $f(-x) = -f(x)$ for all $x \in R$ Let $f: R \to R : f(x) = x^3$ for all $x \in R$ Then, domain of $f = R$ and range of $f = R$.

We have

x	–2	–1	0	1	2
$f(x) = x^3$	–8	–1	0	1	8

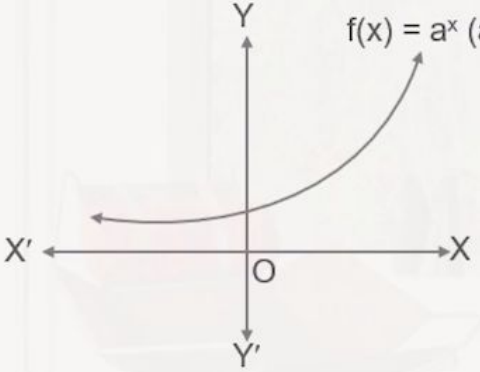


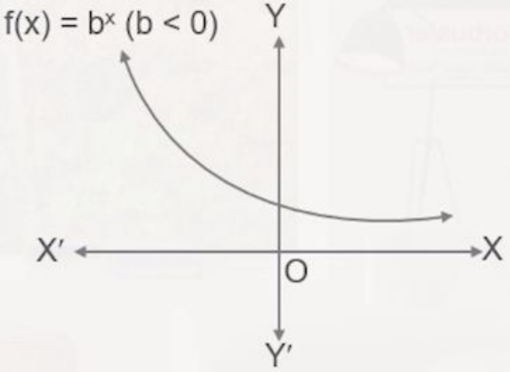
Functions

Particular Types of Functions & their Graphs

Exponential Function

Let f: $R \rightarrow R^+$. The function f is defined by $f(x) = a^x$, where $a > 0$, $a \neq 1$ is called an exponential function.





Functions

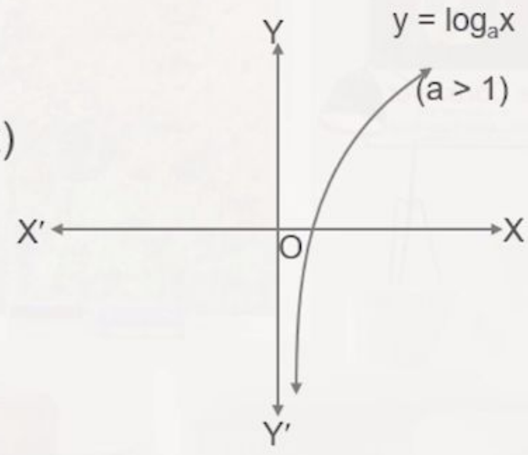
Particular Types of Functions & their Graphs

Logarithmic Function

Let ‘a’ be a positive real number with $a \neq 1$, if $a^y = x$, $x \in R$ then y is called the logarithm of x with base ‘a’ and we write it as $y = \log_a x$.

i.e. A function $f: R^+ \to R$ defined by $f(x) = \log_a x$ is called logarithmic function.

$\therefore$ $f = \{(x, \log_a x)/x \in R,\}$ $a > 0$, $a \neq 1\}$



Functions

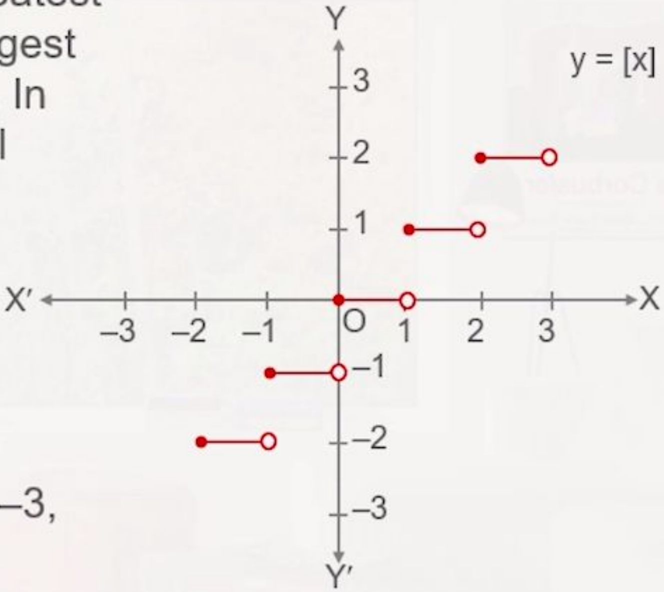
Greatest Integer Function

The Greatest Integer Function is denoted by $y = [x]$.

For all real numbers, x, the greatest integer function returns the largest integer less than or equal to x. In essence, it rounds down a real number to the nearest integer.

For example:
$[1] = 1$, $[1.5] = 1$, $[3.7] = 3$,
$[4.3] = 4$

Beware!
$[-2] = -2$, $[-1.6] = -2$, $[-2.1] = -3$,
$[-5.5] = -6$

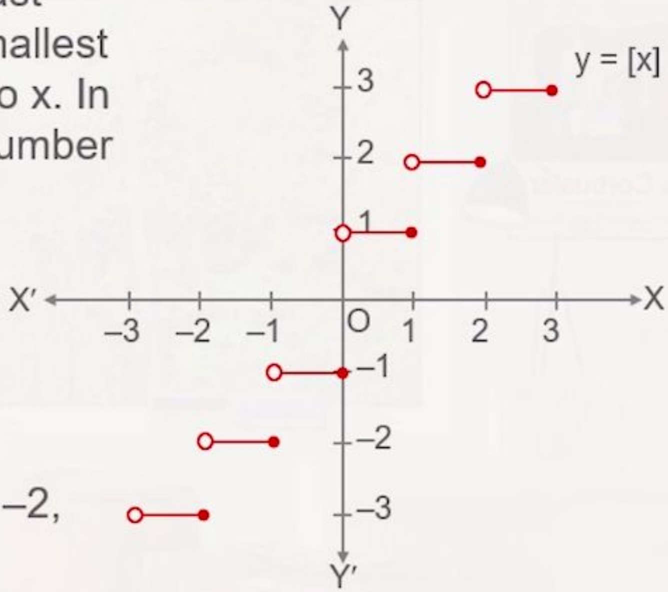


Functions

Least Integer Function

The least Integer Function is denoted by y = (x).

For all real numbers, x, the least integer function returns the smallest integer greater than or equal to x. In essence, it rounds up a real number to the nearest integer.



For example:
[1] = 1, [1.5] = 2, [3.7] = 4,
[4.3] = 5

Beware!
[–2] = –2, [–1.6] = –1, [–2.1] = –2,
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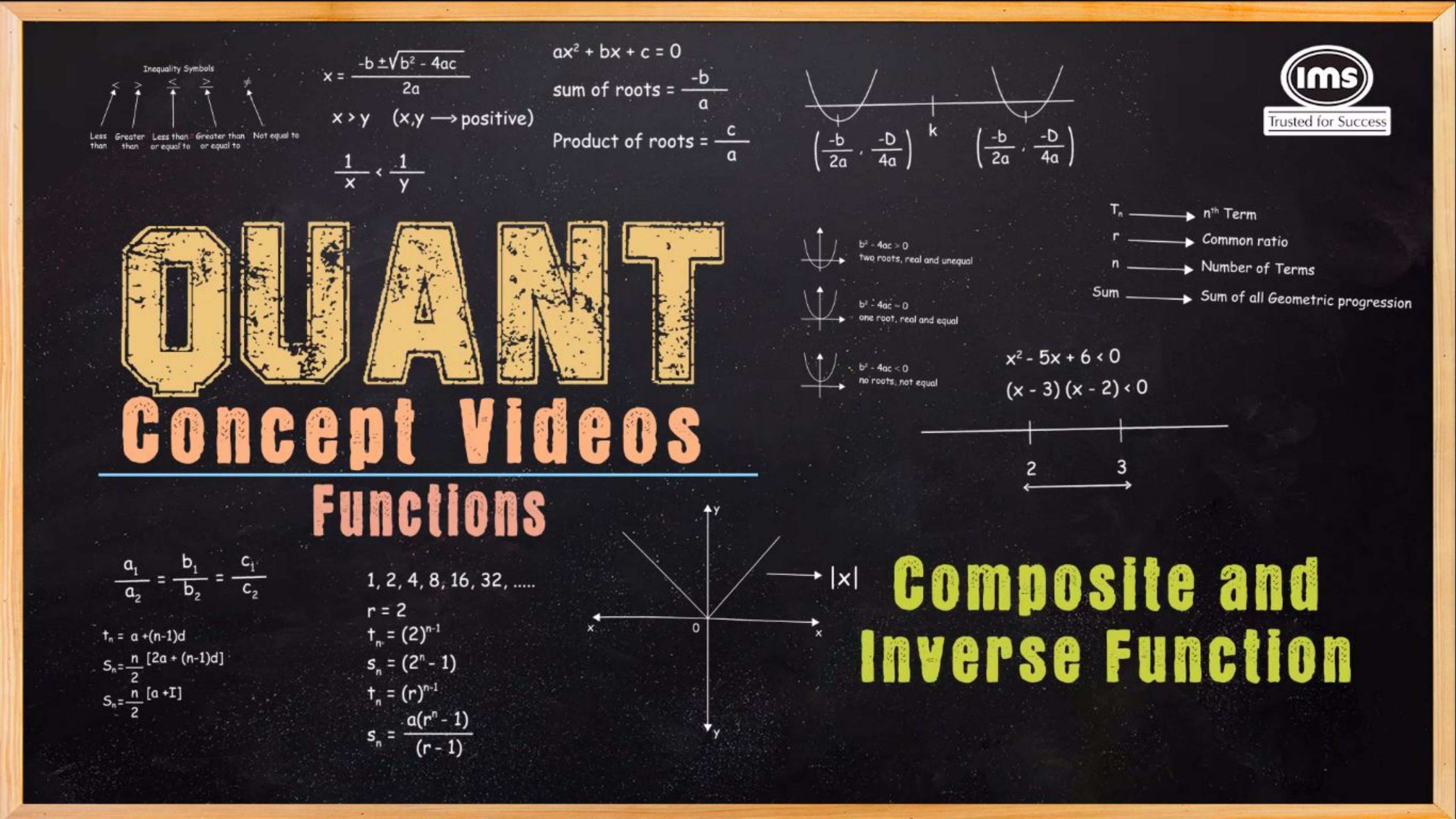

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Functions
Composite and Inverse Function
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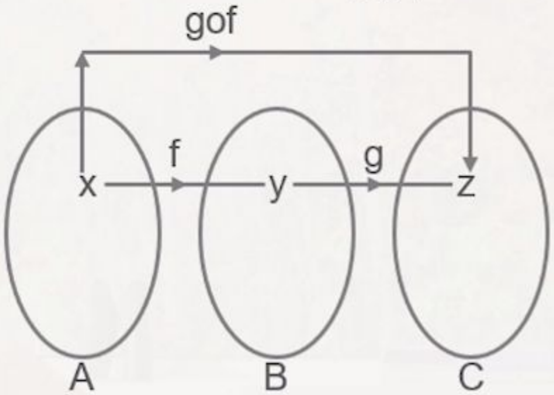
$t_n = (2)^{n-1}$
$s_n = (2^n - 1)$
$t_n = (r)^{n-1}$
$s_n = \frac{a(r^n - 1)}{(r - 1)}$
y x 0 x y
$\longrightarrow |x|$

Functions

Composite Function

If f: A $\rightarrow$ B and g: B $\rightarrow$ C are two functions then the composite function of f and g is the function gof: A $\rightarrow$ C given by (gof)(x) = g[f(x)], for all x $\in$ A Let z = g(y) then z = g(y) = g[f(x)] $\in$ C



This shows that every element x of the set A is related to one and only one element z = g[f(x)] of C. This gives rise to a function from the set A to the set C. This function is called the composite of f and g. Note that (fog)(x) $\neq$ (gof)(x).

Functions

Example

Find: (i) gof (ii) fog, where $f(x) = x - 2$, $g(x) = x^2 + 3x + 1$

$f(x) = x - 2$ and $g(x) = x^2 + 3x + 1$

(i) $(gof)(x) = g[f(x)] = g(x - 2)$

$$= (x - 2)^2 + 3(x - 2) + 1$$

$$= x^2 - 4x + 4 + 3x - 6 + 1$$

$$= x^2 - x - 1$$

(ii) $(fog)x = f[g(x)] = f(x^2 + 3x + 1)$

$$= x^2 + 3x + 1 - 2$$

$$= x^2 + 3x - 1$$

Functions

Inverse Function

If a function f: A $\rightarrow$ B is one-one and onto function defined by y = f(x), then the function g: B $\rightarrow$ A defined by g(y) = x is called the inverse of f and is denoted by f^{-1}.

Thus f^{-1}: B $\rightarrow$ A is defined by $x = f^{-1}(y)$

We also write if $y = f(x)$ then $x = f^{-1}(y)$

Note that if the function is not one-one nor onto, then its inverse does not exist.

Functions

Example 1

Find the inverse of the following function:

(i) $f(x) = 3x - 4$ (ii) $f(x) = \dfrac{3x + 2}{4x - 1}$

Solution:

(i) $y = 3x - 4$

$\therefore \quad 3x = y + 4$

$\therefore \quad x = \dfrac{y + 4}{3}$

$\therefore \quad f^{-1}(x) = \dfrac{x + 4}{3}$

(ii) $y = \dfrac{3x + 2}{4x - 1}$

$\therefore \quad 4xy - y = 3x + 2$

$\therefore \quad 4xy - 3x = y + 2$

$\therefore \quad x(4y - 3) = y + 2$

$y = \dfrac{y + 2}{4y - 3}$

$\therefore \quad f^{-1}(x) = \dfrac{x + 2}{4x - 3}$

Functions

Example 2

If $f(x) = \dfrac{x+3}{4x-5}, x \neq \dfrac{5}{4}$; $g(x) = \dfrac{3+5x}{4x-1}, x \neq \dfrac{1}{4}$; show that $(fog)(x) = x$.

Solution:

$$f(x) = \frac{x+3}{4x-5},\ g(x) = \frac{3+5x}{4x-1}$$

$$\therefore\ (fog)x = f[g(x)]$$

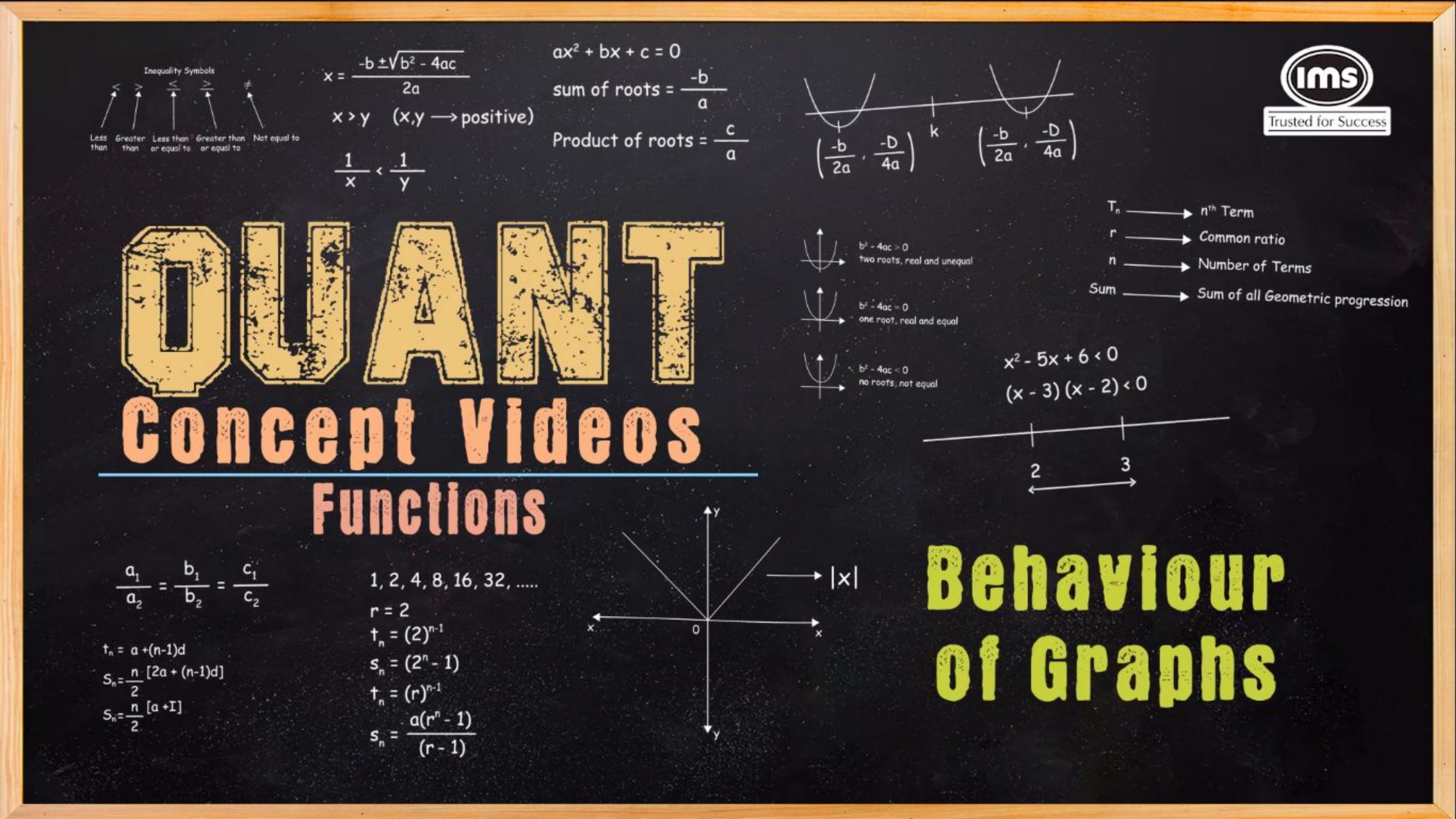
$$= f\left(\frac{3+5x}{4x-1}\right)$$

$$= \frac{\dfrac{3+5x}{4x-1} + 3}{4\left(\dfrac{3+5x}{4x-1}\right) - 5}$$

$$= \frac{\dfrac{3+5x+12x-3}{4x-1}}{\dfrac{12+20x-20x+5}{4x-1}}$$

$$= \frac{17x}{17}$$

$$= x$$



QUANT
Concept Videos
Functions
Behaviour of Graphs
IMS
Trusted for Success
Inequality Symbols
Less than
Greater than
Less than or equal to
Greater than or equal to
Not equal to
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x > y \quad (x, y \longrightarrow \text{positive})$
$\frac{1}{x} < \frac{1}{y}$
$ax^2 + bx + c = 0$
sum of roots $= \frac{-b}{a}$
Product of roots $= \frac{c}{a}$
$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$
k
$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$
$b^2 - 4ac > 0$ two roots, real and unequal
$b^2 - 4ac = 0$ one root, real and equal
$b^2 - 4ac < 0$ no roots, not equal
$T_n \longrightarrow n^{th}$ Term
$r \longrightarrow$ Common ratio
$n \longrightarrow$ Number of Terms
Sum $\longrightarrow$ Sum of all Geometric progression
$x^2 - 5x + 6 < 0$
$(x - 3)(x - 2) < 0$
2
3
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$t_n = a + (n-1)d$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_n = \frac{n}{2}[a + l]$
1, 2, 4, 8, 16, 32, ...
$r = 2$
$t_n = (2)^{n-1}$
$s_n = (2^n - 1)$
$t_n = (r)^{n-1}$
$s_n = \frac{a(r^n - 1)}{(r - 1)}$
y
x
0
x
y
$|x|$

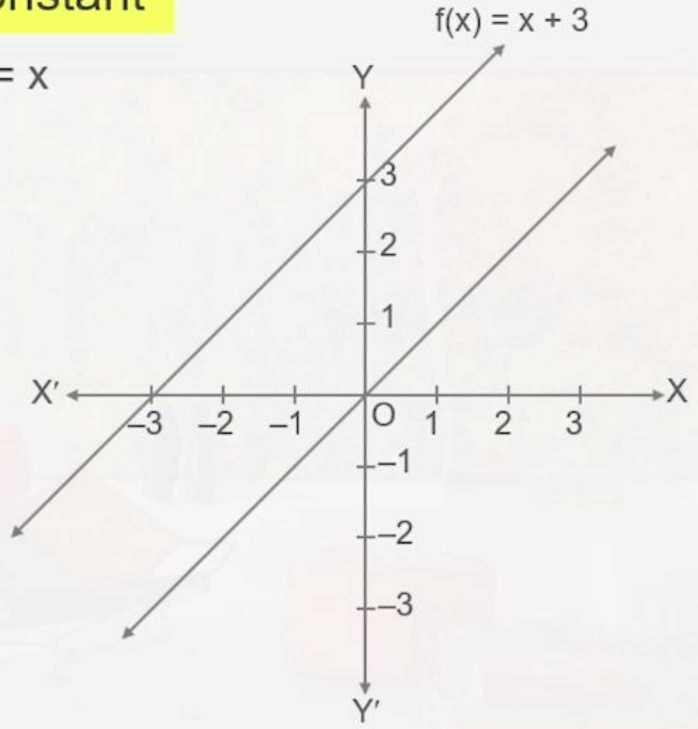
Functions

Behaviour of Graphs

Adding a Constant

Graph of: $f(x) = x$

$f(x) = x + 3$



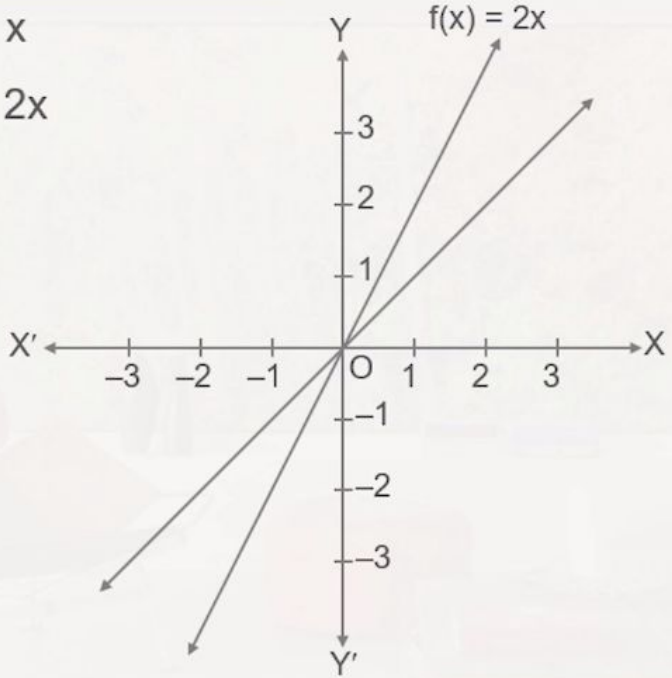
Functions

Behaviour of Graphs

Multiplying by a Constant

Graph of: $f(x) = x$

$f(x) = 2x$



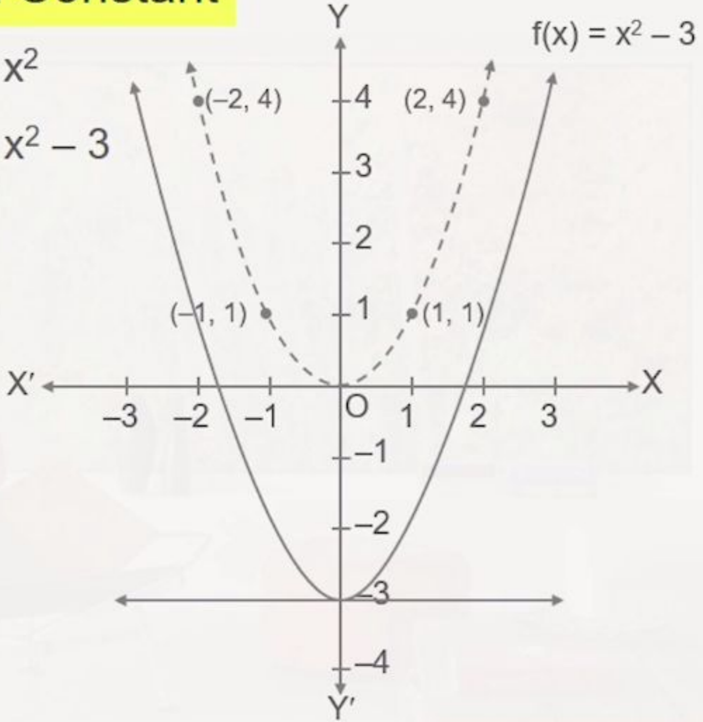
Functions

Behaviour of Graphs

Subtracting a Constant

Graph of: $f(x) = x^2$

$= x^2 - 3$

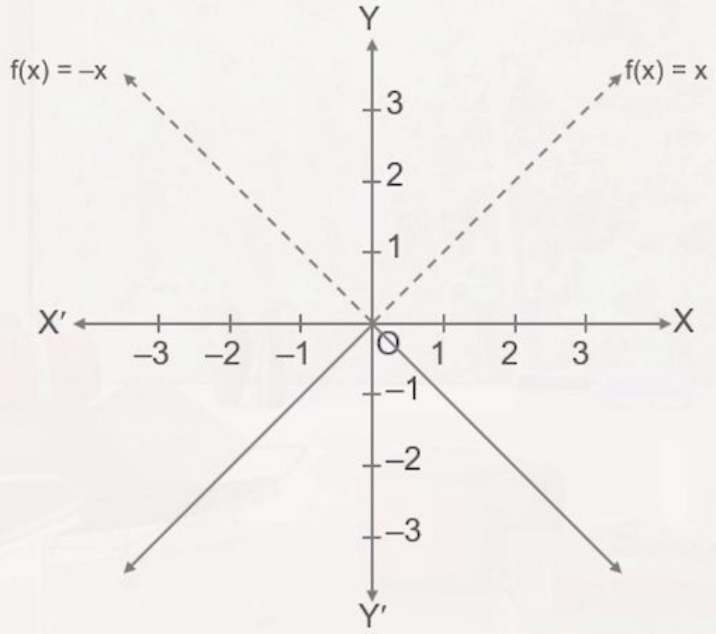


Functions

Graphs of different functions

Graph of: $f(x) = |x|$

$-|x|$

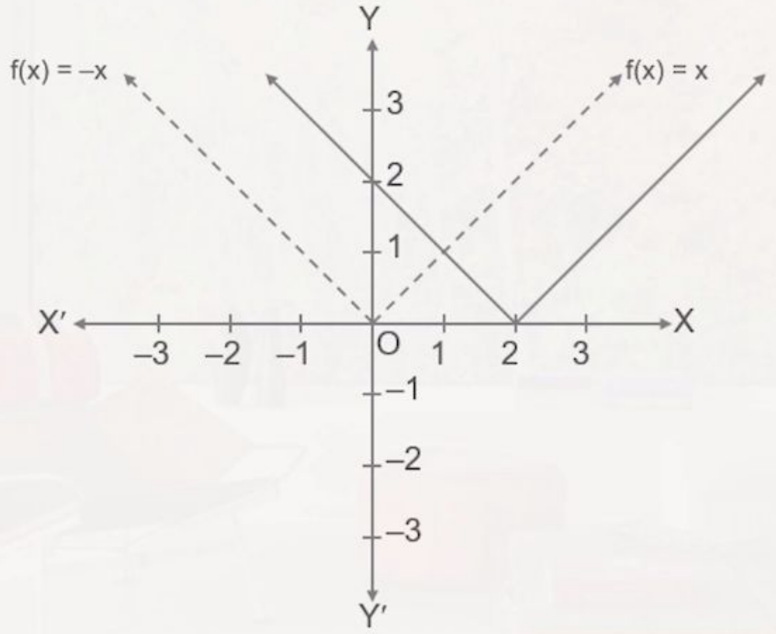


Functions

Graphs of different functions

Graph of: $f(x) = |x|$

$= |x - 2|$



Functions

Graphs of different functions

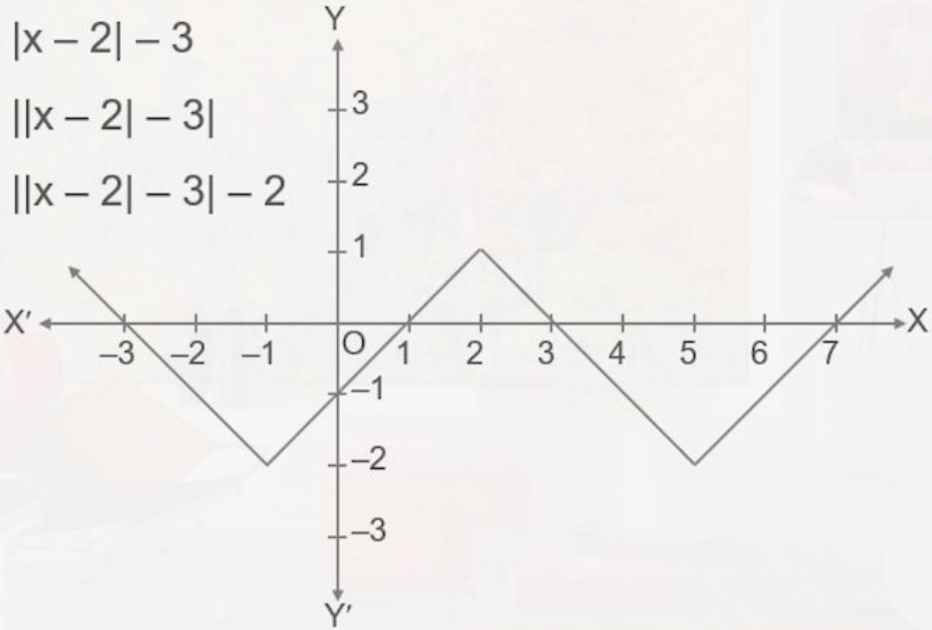
Graph of: $f(x) = ||x - 2| - 3| - 2$

Step 1 $f(x) = |x - 2|$

Step 2 $f(x) = |x - 2| - 3$

Step 3 $f(x) = ||x - 2| - 3|$

Step 4 $f(x) = ||x - 2| - 3| - 2$

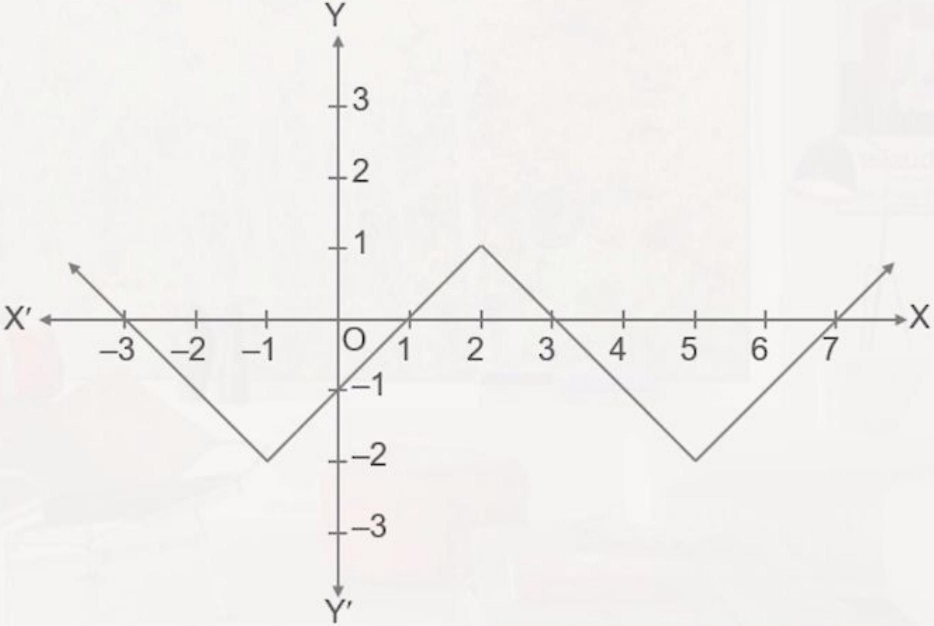


Functions

Number of Solutions

$f(x) = ||x - 2| - 3| - 2 = 0$

$x = -3, 1, 3, 7$



Functions

Number of Solutions

$f(x) = ||x - 2| - 3| - 2 = 0$

$||x - 2| - 3| = 2$

$|x - 2| - 3 = 2 \text{ or } -2$

$|x - 2| = 5 \text{ or } 1$

$x - 2 = 5, -5, 1 \text{ or } -1$

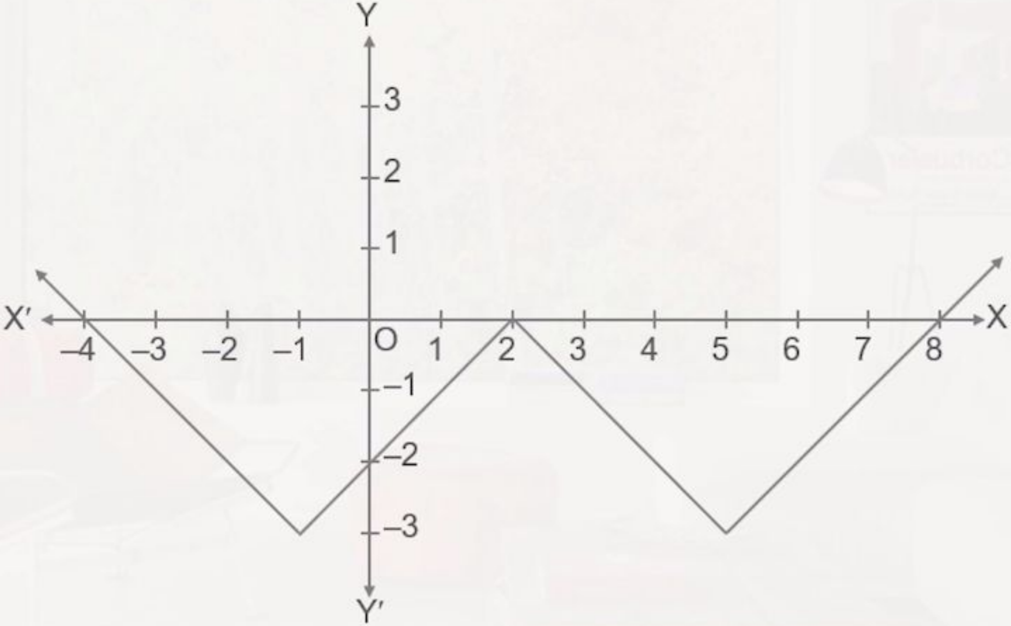
$x = 7, -3, 3, 1$

Functions

Number of Solutions

$f(x) = ||x - 2| - 3| - 3 = 0$

$x = 8, -4$ or 2



Functions

Number of Solutions

$f(x) = ||x - 2| - 3| - 3 = 0$

$||x - 2| - 3| - 3 = 0$

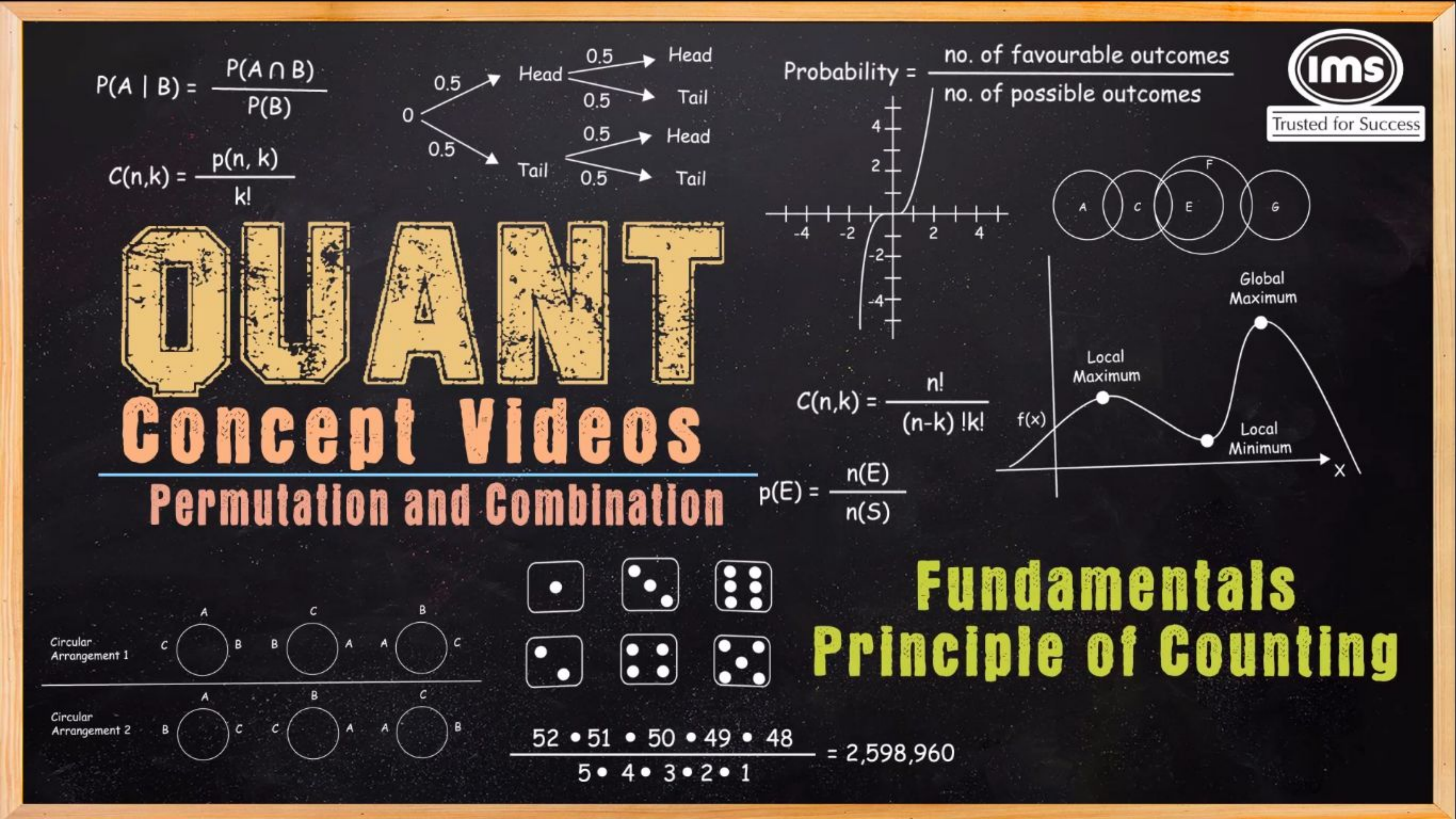
$||x - 2| - 3| = 3$

$|x - 2| - 3 = 3$ or -3

$|x - 2| = 6$ or 0

$x - 2 = 6, -6$ or 0

$x = 8, -4$ or 2



P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
0
0.5
Head
0.5
Head
0.5
Tail
0.5
Tail
0.5
Head
0.5
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
IMS
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QUANT
Concept Videos
Permutation and Combination
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Global Maximum
Local Maximum
Local Minimum
f(x)
X
Circular Arrangement 1
Circular Arrangement 2
Fundamentals
Principle of Counting
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Permutation and Combination

Fundamentals Principle of Counting

In how many ways can you select 1 Shirt and 1 Trouser from 2 Shirts and 3 Trousers ?

	T1			
S1		S1 T1	S1 T2	S1 T3
	T2			
S2		S2 T1	S2 T2	S2 T3
	T3			

∴ Total number of way is $2 \times 3 = 6$

Permutation and Combination

Fundamentals Principle of Counting

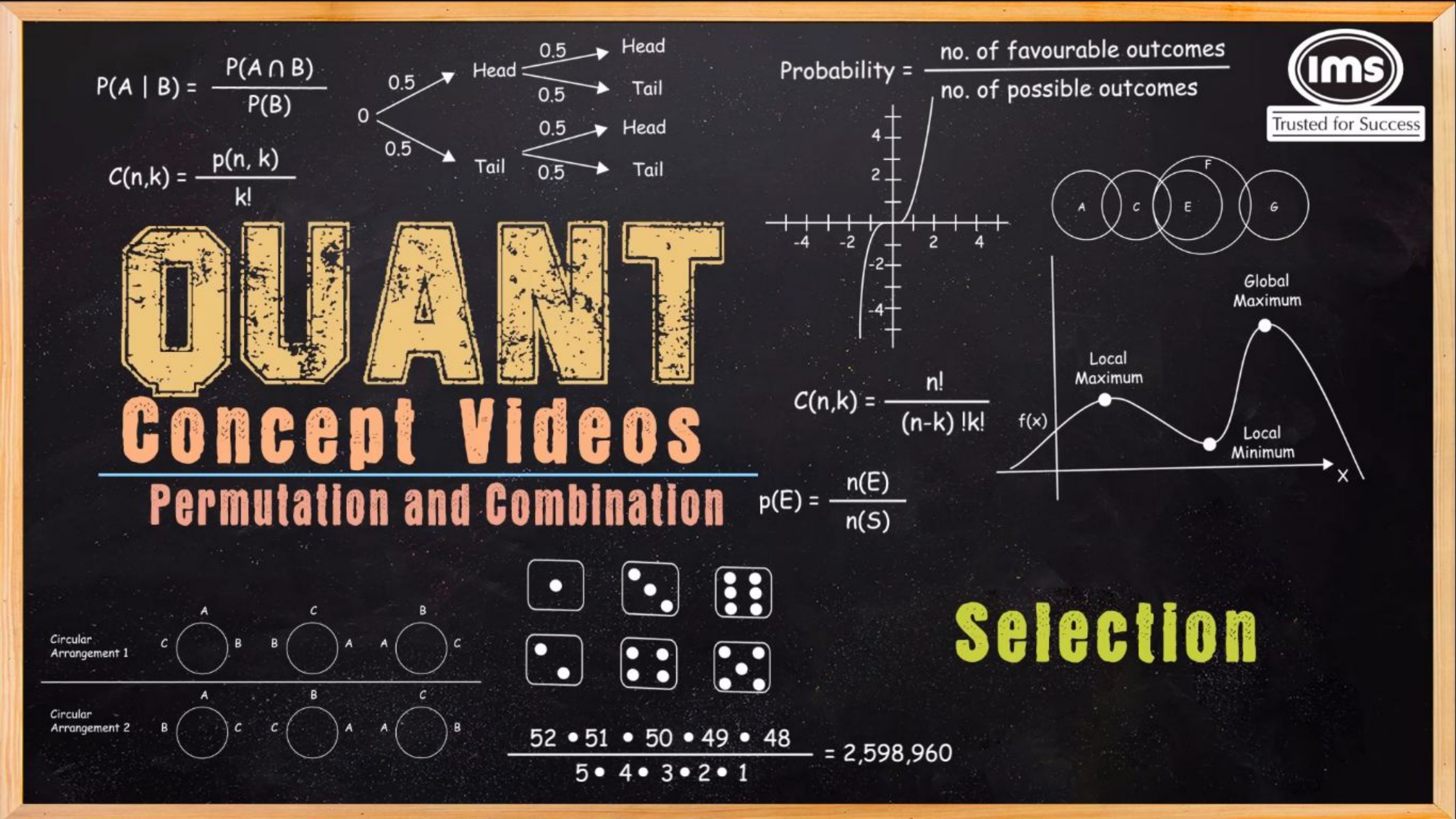
2 boys and 3 girls in a class. How to appoint 1 class monitor?

1 Boy OR 1 Girl

2 + 3 = 5

AND → ‘Multiplication’

OR → ‘Addition’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Concept Videos
Permutation and Combination
Selection
IMS
Trusted for Success
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
0
0.5
0.5
Head
Tail
0.5
0.5
0.5
0.5
Head
Tail
Head
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
A
C
E
F
G
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Global Maximum
Local Maximum
Local Minimum
f(x)
X
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Permutation and Combination

Selection

Quiz Team of 2 out of 5

AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

Permutation and Combination

Selection

${}^{n}C_{r} \rightarrow$ Selecting ‘r’ objects out of ‘n’

$${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

100 people and we have to select 2 = ${}^{100}C_{2}$ ways

$$\therefore\ {}^{100}C_{2} = \frac{100!}{98!\,2!} = \frac{100 \times 99 \times 98!}{98! \times 2!} = 4950$$

Trusted for Success

Permutation and Combination

Selection

$$^nC_r = {}^nC_{n-r}$$

$$\frac{n!}{(n-r)!\,r!} = \frac{n!}{r!(n-r)!}$$

A cricket team of 11 has to be selected out of 15

$^{15}C_{11}$ OR $^{15}C_4$

$= 1365$

Permutation and Combination

Selection

A team of 6 members is to be formed out of 5 Teachers, 5 Lawyers and 9 Accountants.

1. In how many ways can the team be formed?

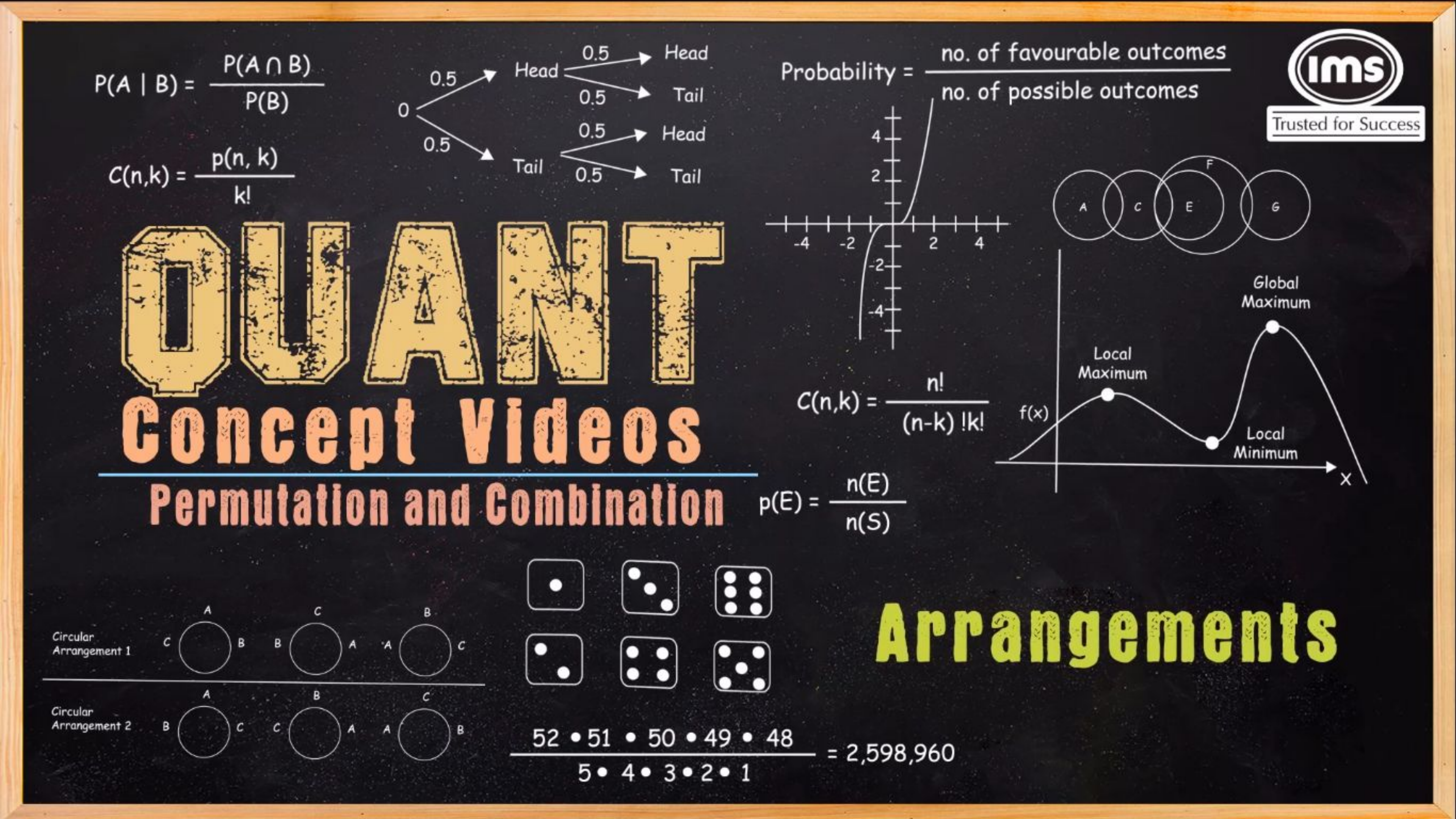
   $^{19}C_6$

2. In how many ways can the team be formed if the team should contain 2 Teachers, 3 Accountants and 1 Lawyer?

   $^{5}C_2 \times {}^{9}C_3 \times {}^{5}C_1$

3. In how many ways can the team be formed if it should contain exactly 3 Teachers?

   $^{5}C_3 \times {}^{14}C_3$



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Concept Videos
Permutation and Combination
Arrangements
IMS
Trusted for Success
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
0
0.5
Head
0.5
Head
0.5
Tail
0.5
Tail
0.5
Head
0.5
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
A
C
E
F
G
Global Maximum
Local Maximum
Local Minimum
f(x)
X
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Permutation and Combination

Arrangements

5 boys

5 empty chairs

4 boys remaining

1boy on a chair & 4 empty chairs

$5 \times 4 \times 3 \times 2 \times 1 = 5!$

In general ‘n’ people in ‘n’ places can be arranged in n! ways.

Permutation and Combination

Arrangements

5 boys

3 chairs

$5 \times 4 \times 3 = 60$ ways

$$^{n}P_{r} = \frac{n!}{(n-r)!} = {}^{n}C_{r} \times r!$$

$$^{5}P_{3} = \frac{5!}{2!} = 60 \text{ ways}$$

Permutation and Combination

Arrangements

Six boys and three girls have to stand in a row for a photograph.

1. In how many ways can this be done, if there is no restriction?

   (B1, B2, B3, B4, B5, B6, G1, G2, G3)

   $= 9!$

2. In how many ways can this be done, if all boys are together and all girls are together?

   (B1, B2, B3, B4, B5, B6) (G1, G2, G3)

   $= 6! \times 3! \times 2!$

Permutation and Combination

Arrangements

3. In how many ways can this be done, if all boys are together?

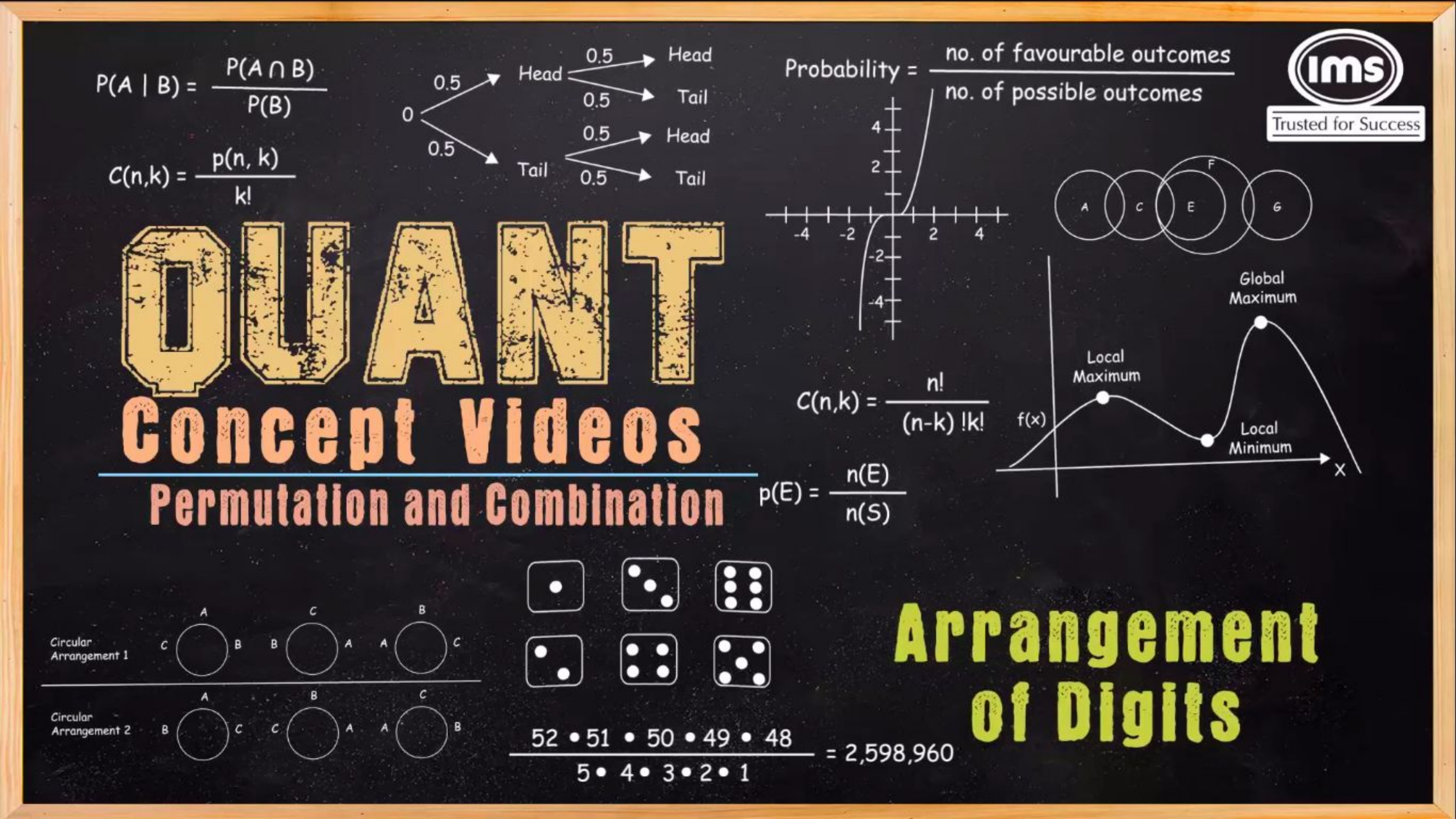
   (B1, B2, B3, B4, B5, B6) (G1) (G2) (G3)

   $= 6! \times 4!$

4. In how many ways can this be done, if no 2 girls are together?

   _ B1 _ B2 _ B3 _ B4 _ B5 _ B6 _

   $= 6! \times {}^{7}P_{3}$


QUANT
Concept Videos
Permutation and Combination
Arrangement of Digits
IMS
Trusted for Success
$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
$C(n,k) = \frac{p(n, k)}{k!}$
0
0.5
Head
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
$C(n,k) = \frac{n!}{(n-k)!k!}$
$p(E) = \frac{n(E)}{n(S)}$
Global Maximum
Local Maximum
Local Minimum
f(x)
X
Circular Arrangement 1
Circular Arrangement 2
$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960$

Permutation and Combination

Arrangement of Digits

If nothing is mentioned, repetition is allowed.

How many five digit numbers can be formed using the digits 0, 1, 3, 5, 6, 8, 9 under the following conditions:

1. They are odd

$$\underline{6} \times \underline{7} \times \underline{7} \times \underline{7} \times \underline{4} = 8232$$

2. They are divisible by 50

$$\underline{6} \times \underline{7} \times \underline{7} \times \underline{2} \times \underline{1} = 588$$

Permutation and Combination

Arrangement of Digits

How many five digit numbers can be formed using the digits 0, 1, 3, 5, 6, 8, 9 under the following conditions:

3. They are even and repetition of digits is not allowed.

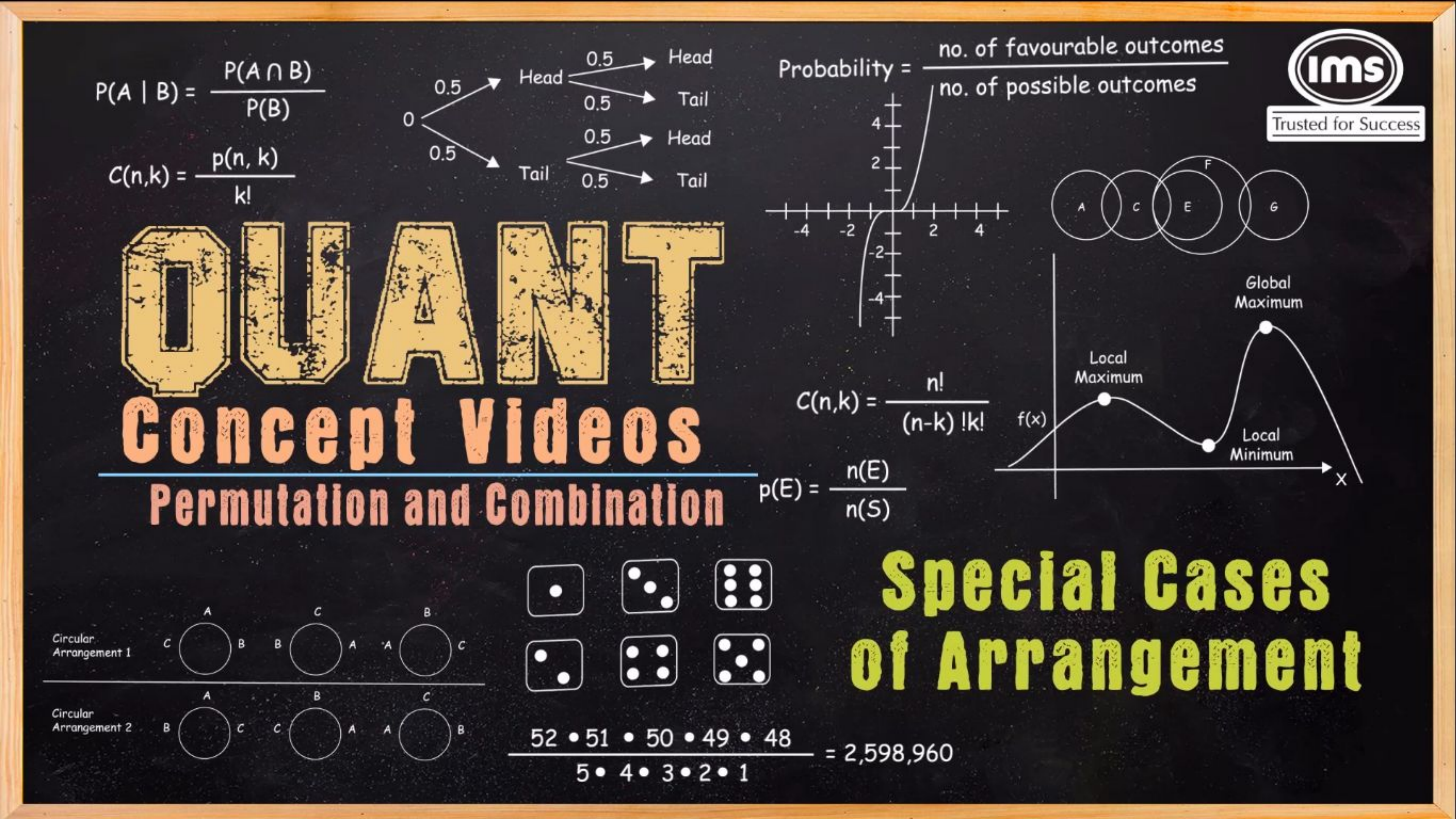
Case 1: Ending in 0: $\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{1} = 360$

Case 2: Ending in 6, 8: $\underline{5} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} = 600$

Hence, in all 960 numbers

4. They are greater than 50000 and repetition is not allowed?

$\underline{4} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} = 1440$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Special Cases
of Arrangement
IMS
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P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
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0.5
Head
Tail
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C
E
F
G
Global Maximum
Local Maximum
Local Minimum
f(x)
X
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Permutation and Combination

Special Cases of Arrangement

Identical Elements

In how many different ways can you arrange the letters of the word MISSISSIPPI?

M, IIII, SSSS, PP

$$\frac{11!}{4!\,4!\,2!}$$

Permutation and Combination

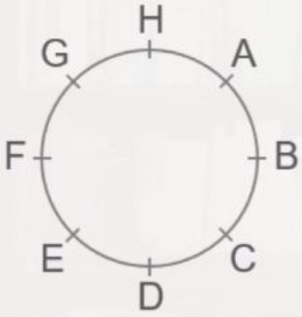
Special Cases of Arrangement

Circular Arrangement

8 boys A, B, C, D, E, F, G and H have to be seated on 8 chairs kept in a circle.

1. Number of ways in which they can be seated.

   7!



Permutation and Combination

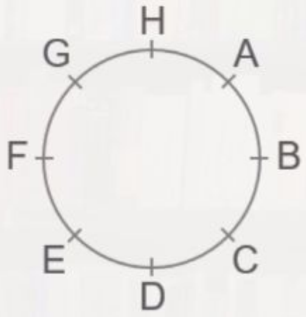
Special Cases of Arrangement

Circular Arrangement

8 boys A, B, C, D, E, F, G and H have to be seated on 8 chairs kept in a circle.

2. Number of ways in which they can be seated such that C and D always sit together.

   $2! \times 6!$



Permutation and Combination

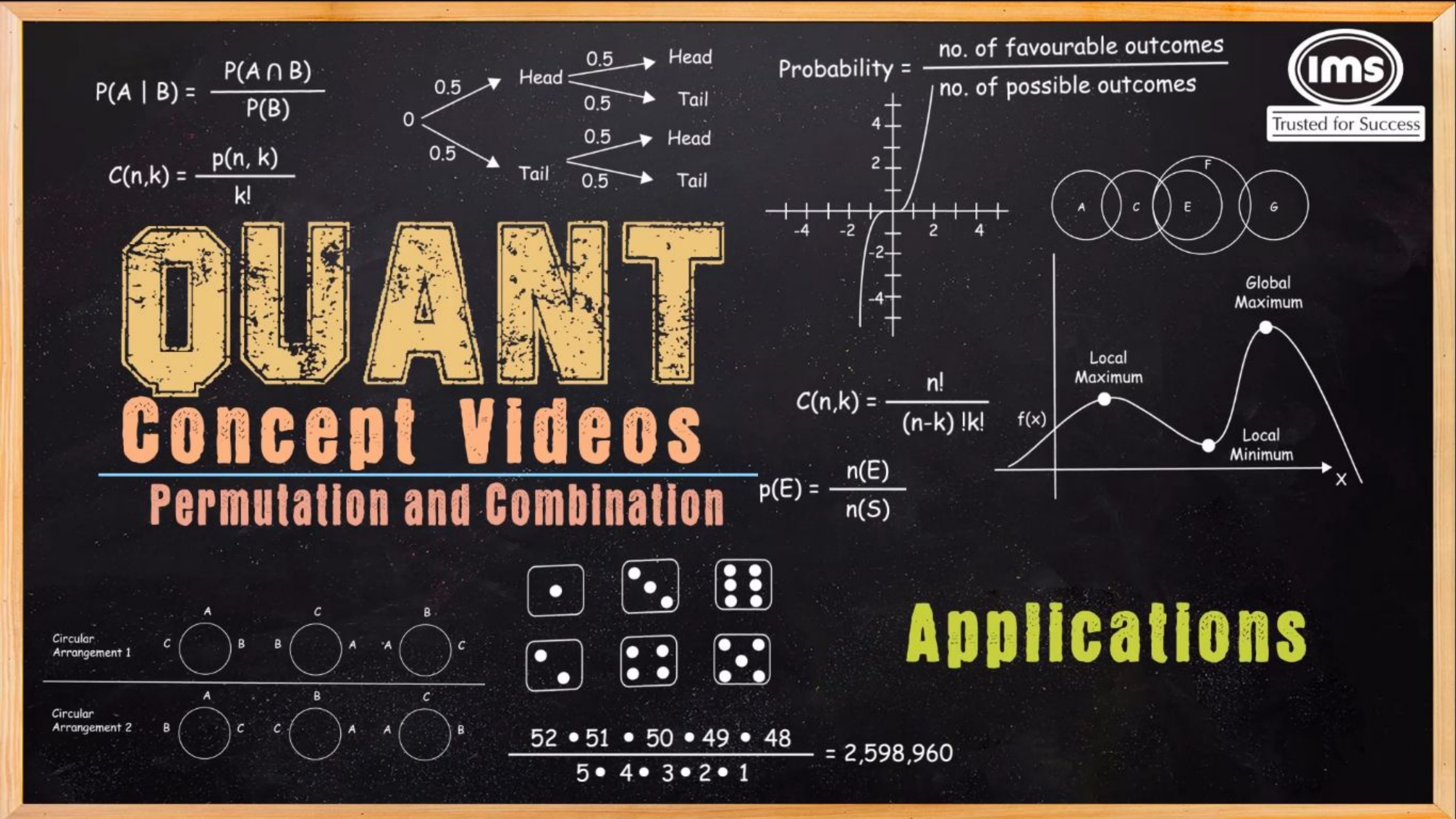
Special Cases of Arrangement

Circular Arrangement

8 boys A, B, C, D, E, F, G and H have to be seated on 8 chairs kept in a circle.

3. Number of ways in which they can be seated such that C and D never sit together.

   $7! - 6! \times 2!$



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Applications
IMS
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C(n,k) = p(n, k) / k!
0
0.5
Head
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Global Maximum
Local Maximum
Local Minimum
f(x)
x
Circular Arrangement 1
Circular Arrangement 2
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Permutation and Combination

Applications

1. In a room with 10 people, if every person shakes the hand of every other person, how many handshakes will happen?

   $^{10}C_2$

2. If instead of handshake its a gesture of Namaste?

   $^{10}C_2 \times 2$

3. How many matches will be played in the league stages of IPL 2040 if each of the 12 teams plays with every other team twice?

   $^{12}C_2$ x 2

Permutation and Combination

Applications

4. If there are 11 non-collinear points, how many lines can be formed?

   $^{11}C_2$

5. If there are 5 collinear and 6 non collinear then?

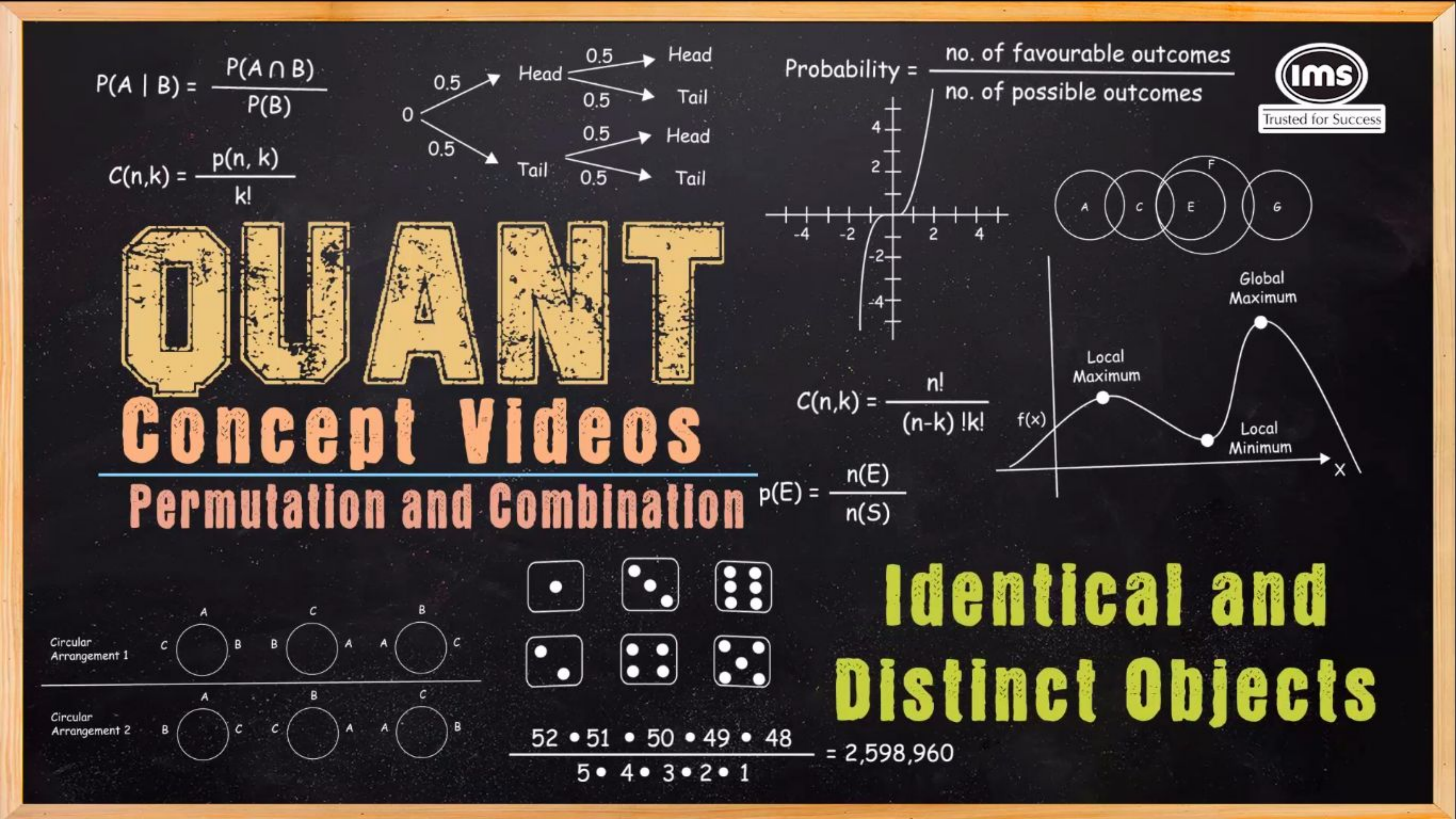
   $^{11}C_2 - {}^{5}C_2 + 1$

6. If there are 7 points in one line and 6 on another, how many triangles can be formed?

   $^{7}C_2 \times {}^{6}C_1 + {}^{6}C_2 \times {}^{7}C_1$

7. How many diagonals does a hexagon have?

   $^{6}C_2 - 6$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Permutation and Combination
Identical and
Distinct Objects
Trusted for Success
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
Head
Tail
0.5
Probability = no. of favourable outcomes / no. of possible outcomes
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Global Maximum
Local Maximum
Local Minimum
f(x)
x
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Permutations and Combinations

Distinct Objects

A basket contains 10 fruits.

1. Number of ways of selecting 3 fruits out of the basket: $^{10}C_3$
2. Number of ways of selecting 8 fruits out of the basket: $^{10}C_8$
3. Number of ways of not selecting any fruit from the basket:
   $^{10}C_0 = 1$
4. Number of ways of selecting fruits from this basket:
   $^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 \ldots\ldots\ldots\ldots {}^{10}C_{10} = 2^{10}$
5. Number of ways of selecting at least 1 fruit from this basket:
   $2^{10} - 1$
6. Number of ways of selecting at least 2 fruits from this basket:
   $2^{10} - ({}^{10}C_0 + {}^{10}C_1)$

Trusted for Success

Permutations and Combinations

Distinct Objects

A basket contains 5 mangoes, 8 oranges and 10 apples.

1. Number of ways of selecting fruits from this basket: 2^{23}
2. Number of ways of selecting at least 1 fruit from this basket:
   $2^{23} - 1$
3. Number of ways of selecting at least 1 fruit of each type from this basket:
   $(2^5 - 1)\,(2^8 - 1)\,(2^{10} - 1)$

Permutations and Combinations

Identical Objects

A basket contains 10 identical fruits.

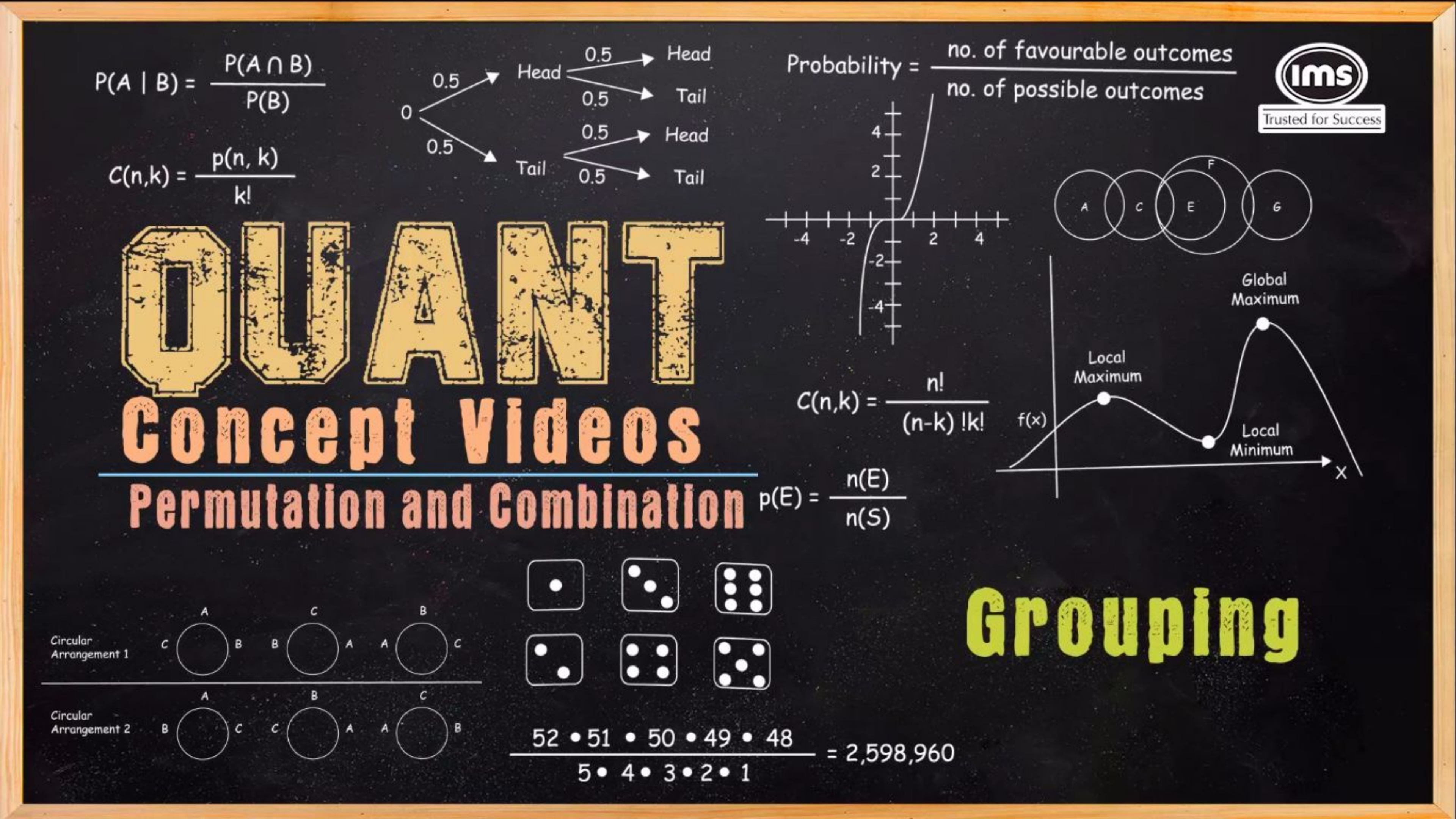
1. Number of ways of selecting 3 fruits out of the basket:
   1 way
2. Number of ways of selecting 8 fruits out of the basket:
   1 way
3. Number of ways of not selecting any fruit from the basket:
   1 way
4. Number of ways of selecting fruits from this basket:
   11 ways
5. Number of ways of selecting at least 1 fruit from this basket:
   10 ways
6. Number of ways of selecting at least 2 fruits from this basket:
   9 ways

Permutations and Combinations

Identical Objects

A basket contains 5 identical mangoes, 8 identical oranges and 10 identical apples.

1. Number of ways of selecting fruits from this basket:
   $6 \times 9 \times 11$
2. Number of ways of selecting at least 1 fruit from this basket:
   $(6 \times 9 \times 11) - 1$
3. Number of ways of selecting at least 1 fruit of each type from this basket:
   $5 \times 8 \times 10$



QUANT
Concept Videos
Permutation and Combination
Grouping
IMS
Trusted for Success
$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
$C(n,k) = \frac{p(n, k)}{k!}$
Probability = no. of favourable outcomes / no. of possible outcomes
$C(n,k) = \frac{n!}{(n-k)!k!}$
$p(E) = \frac{n(E)}{n(S)}$
$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960$
Head
Tail
0.5
Global Maximum
Local Maximum
Local Minimum
f(x)
X
Circular Arrangement 1
Circular Arrangement 2

Permutations and Combinations

Dividing Objects into Groups

9 students have to be divided into 3 groups of 2, 3 and 4

$^9C_2 \times {}^7C_3 \times {}^4C_4$

9 students have to be divided into 3 groups of 2, 2 and 5

$$\frac{^9C_2 \times {}^7C_2 \times {}^5C_5}{2!}$$

9 students have to be divided into 3 groups of 3 each

$$\frac{^9C_3 \times {}^6C_3 \times {}^3C_3}{3!}$$

9 students have to be divided into 3 groups of 3 each, such that one of the groups will go to the science exhibition, second for the education fair and the third for the sports mania.

$$\frac{^9C_3 \times {}^6C_3 \times {}^3C_3 \times 3!}{3!}$$

Permutations and Combinations

Distribution of Objects

In how many ways can 5 identical Chocolates be distributed between 2 girls?

0	5
1	4
2	3
3	2
4	1
5	0

= 6 ways

CCCCCP

$$= \frac{6!}{5!\ 1!}$$

= 6 ways

Permutations and Combinations

Distribution of Objects

'n' identical objects to be distributed among 'r' people in

${}^{n+r-1}C_{r-1}$ ways.

Find the number of non-negative integral solution to the equation: $x + y + z = 12$

$= {}^{12+3-1}C_2 = {}^{14}C_2 = 91$ ways

CCCCCCCCCCCCPP

$$= \frac{14!}{12!\ 2!}$$

$= 91$ ways

Permutations and Combinations

Distribution of Objects

In how many ways can 5 identical Chocolates be distributed between 2 girls such each of them has to get at least 1 chocolate?

1	4
2	3
3	2
4	1

CCCCPC

CCCPCC

CCPCCC

CPCCCC

Permutations and Combinations

Distribution of Objects

In how many ways can 5 identical Chocolates be distributed between 2 girls such each of them has to get at least 1 chocolate?

1	4
2	3
3	2
4	1

CCCCPC

CCCPCC

CCPCCC

CPCCCC

Permutations and Combinations

Distribution of Objects

‘n’ identical objects to be distributed among ‘r’ people in $^{n-1}C_{r-1}$ ways (Such that each person gets at least one object).

Find the number of positive integral solution to the equation:

$x + y + z = 12$

$= {}^{12-1}C_{3-1}$

$= {}^{11}C_2$ ways

$= 55$ ways

Permutations and Combinations

Distribution of Objects

Find the number of positive integral solution to the equation:

$x + y + z = 12$

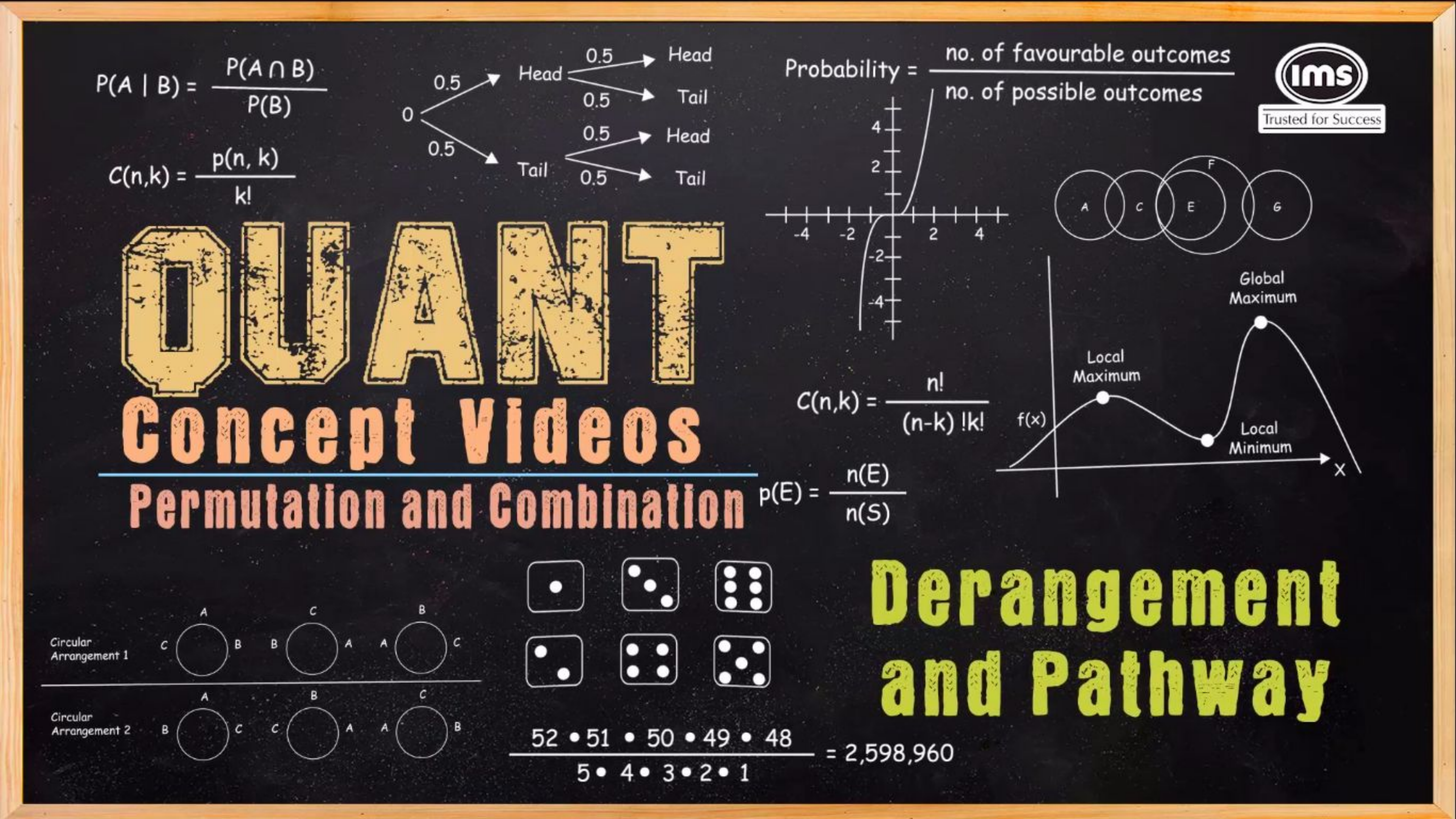
Initially let us give 1 to each of x, y, and z

Now we have 9 to be distributed between 3

$= {}^{9+3-1}C_{3-1}$

${}^{11}C_2$ ways

$= 55$ ways


QUANT
Concept Videos
Permutation and Combination
Derangement
and Pathway
IMS
Trusted for Success
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
Probability = no. of favourable outcomes / no. of possible outcomes
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Head
Tail
0.5
Global Maximum
Local Maximum
Local Minimum
f(x)
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Permutations and Combinations

Trusted for Success

Derangement

Permutations of elements of a set such that none of the elements appear in their original position is known as derangement

Consider n objects such that no object is at its desired place (or is deranged).

Number of derangements of n objects =

$$n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots (-1)^n \frac{1}{n!}\right]$$

Permutations and Combinations

Derangement

Example

5 men carrying identical bags meet for dinner at a restaurant. At the end of the meal, each of them picks up a bag. In how many ways could none of them pick his own bag?

n = 5

$$\text{Number of ways} = 5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right]$$

$$= 120\left[1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right]$$

$$= 120 \times \frac{44}{120}$$

= 44 ways

Permutations and Combinations

Pathways and Routes

Consider a grid such that movement can happen only horizontally or vertically (unless specified)

Number of ways of going from one point to another tracing the minimum possible distance is given by

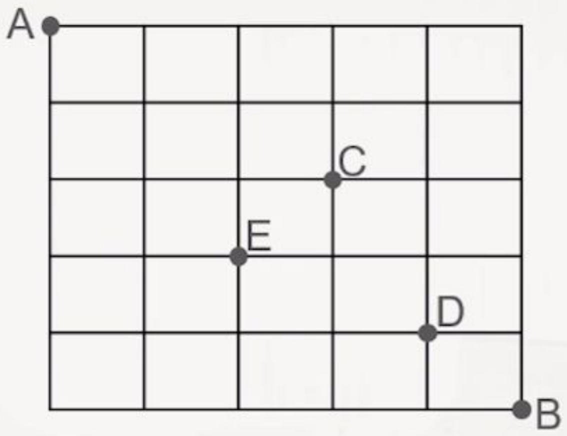
$^{m+n}C_m$ or $^{m+n}C_n$ ways

m = number of vertical moves
n = number of horizontal moves

A – C: $^5C_2$ or $^5C_3$ ways

E – C: $^2C_1$ ways

and so on….



Permutations and Combinations

Pathways and Routes

Example

1. If a person wants to visit A, E, C, D and B, in that order, in how many shortest routes can he do that?

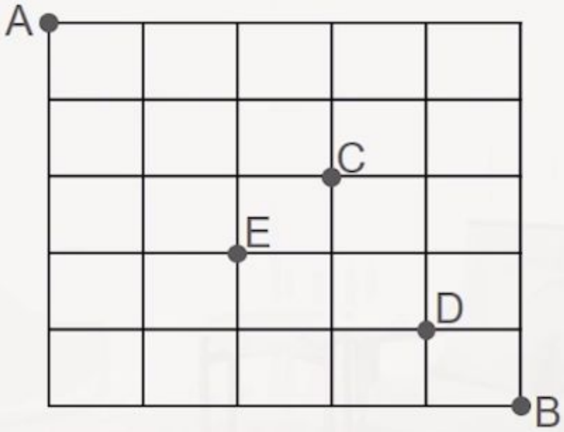
Route to be traced is
A – E – C – D – B

Number of shortest routes

$= {}^{5}C_{3} \times {}^{2}C_{1} \times {}^{3}C_{2} \times {}^{2}C_{1}$

$= 10 \times 2 \times 3 \times 2$

$= 120$



Trusted for Success

Permutations and Combinations

Pathways and Routes

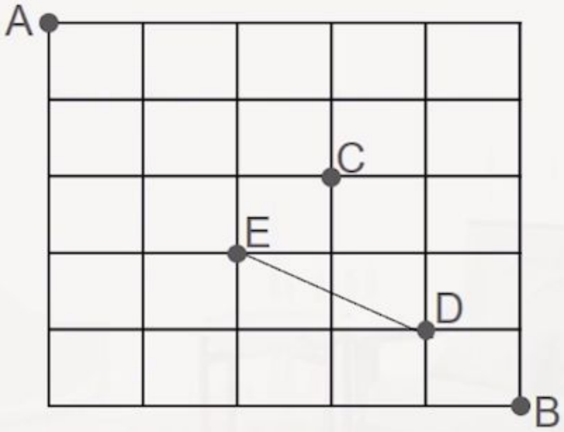
Example

2. If there is an underground subway from E to D, what is the shortest path from A to B via ED.

Route to be traced is
A – ED – B

Number of shortest routes

$= {}^{5}C_{3} \times 1 \times {}^{2}C_{1}$

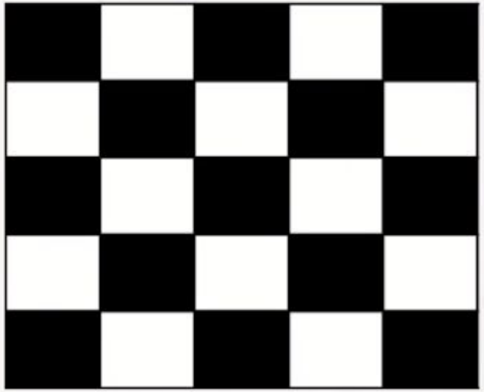


Permutations and Combinations

Number of Squares & Rectangles

Example

Consider a 5 × 5 chessboard. Calculate the number of squares and rectangles in the same.



Size	No. of Squares
1 × 1	25
2 × 2	16
3 × 3	9
4 × 4	4
5 × 5	1

Total number of squares in the grid

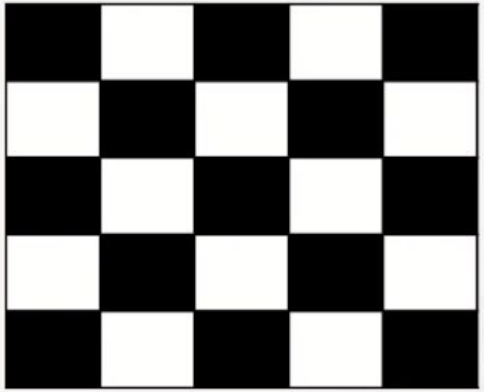
= 55

Permutations and Combinations

Number of Squares & Rectangles

Example

Consider a 5 × 5 chessboard. Calculate the number of squares and rectangles in the same.

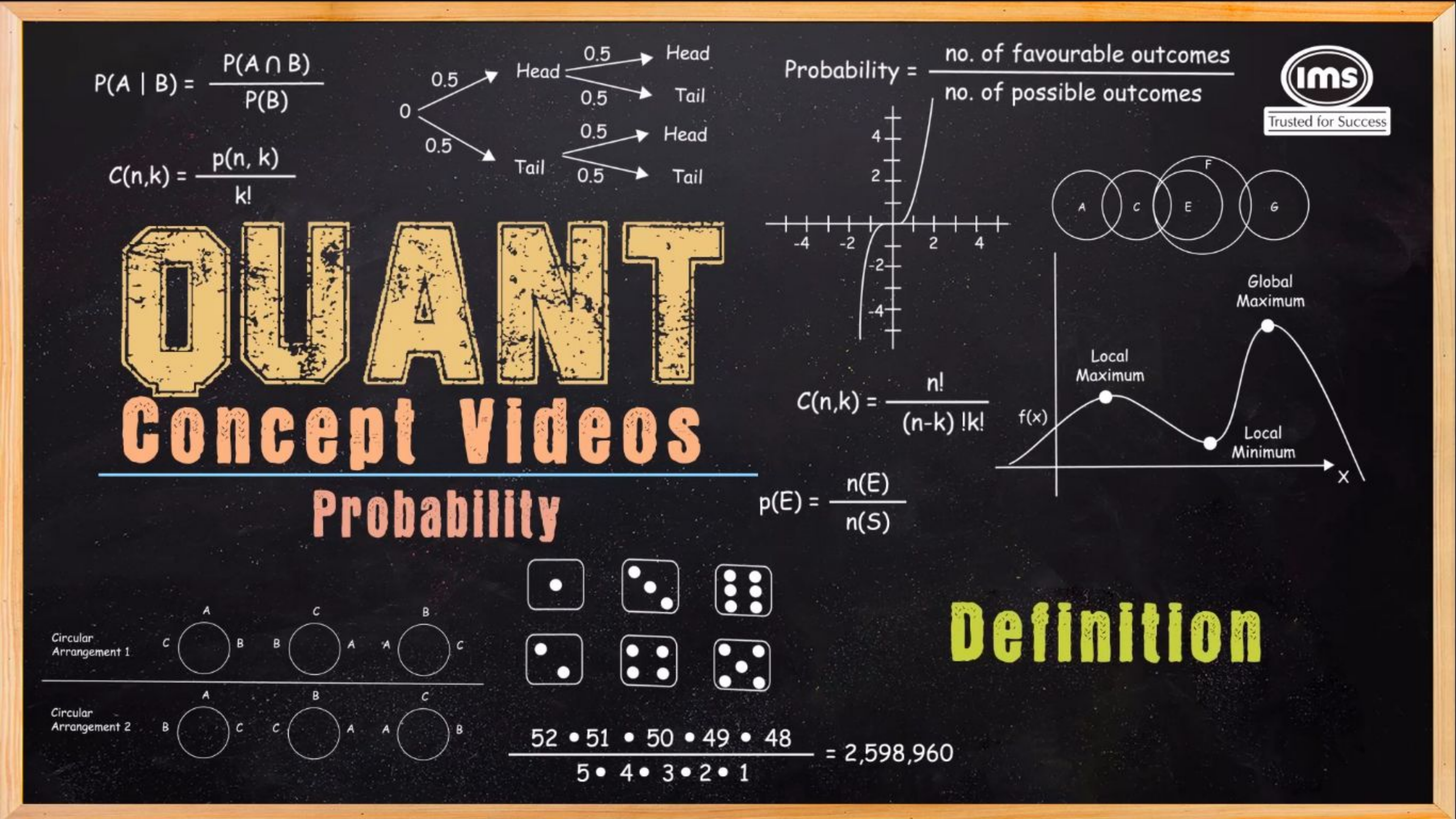


Total number of rectangles in the grid

$= {}^{5+1}C_2 \times {}^{5+1}C_2$

$= {}^{6}C_2 \times {}^{6}C_2$

$= 225$



QUANT
Concept Videos
Probability
Definition
IMS
Trusted for Success
$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
$C(n,k) = \frac{p(n, k)}{k!}$
0
0.5
Head
0.5
Head
0.5
Tail
0.5
Tail
0.5
Head
0.5
Tail
Probability = $\frac{\text{no. of favourable outcomes}}{\text{no. of possible outcomes}}$
A
C
E
F
G
$C(n,k) = \frac{n!}{(n-k)!k!}$
$p(E) = \frac{n(E)}{n(S)}$
Global Maximum
Local Maximum
Local Minimum
f(x)
X
Circular Arrangement 1
Circular Arrangement 2
$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960$

Probability

Definition

Probability is a number that reflects the chance or likelihood that a particular event will occur. Probabilities can be expressed as proportions that range from 0 to 1, and it can also be expressed as percentages ranging from 0% to 100%.

Experiment: A measurement process that produces quantifiable results.

Sample Space (S): Set of all possible outcomes from an experiment.

Event: An event is any subset of a sample set.

$$\text{Probability of Event A occurring} = \frac{n(A)}{n(S)}$$

Probability

Definition

Mutually Exclusive Events

Two events that have no outcome in common are called mutually exclusive events.

Probability

Definition

Independent Events

Events A and B are independent events if the probability of Event B occurring is the same whether or not Event A occurs.

Example

A fair coin is tossed two times.

What is the Probability that a head comes up on the second toss?

Ans. $\frac{1}{2}$

Probability

Definition

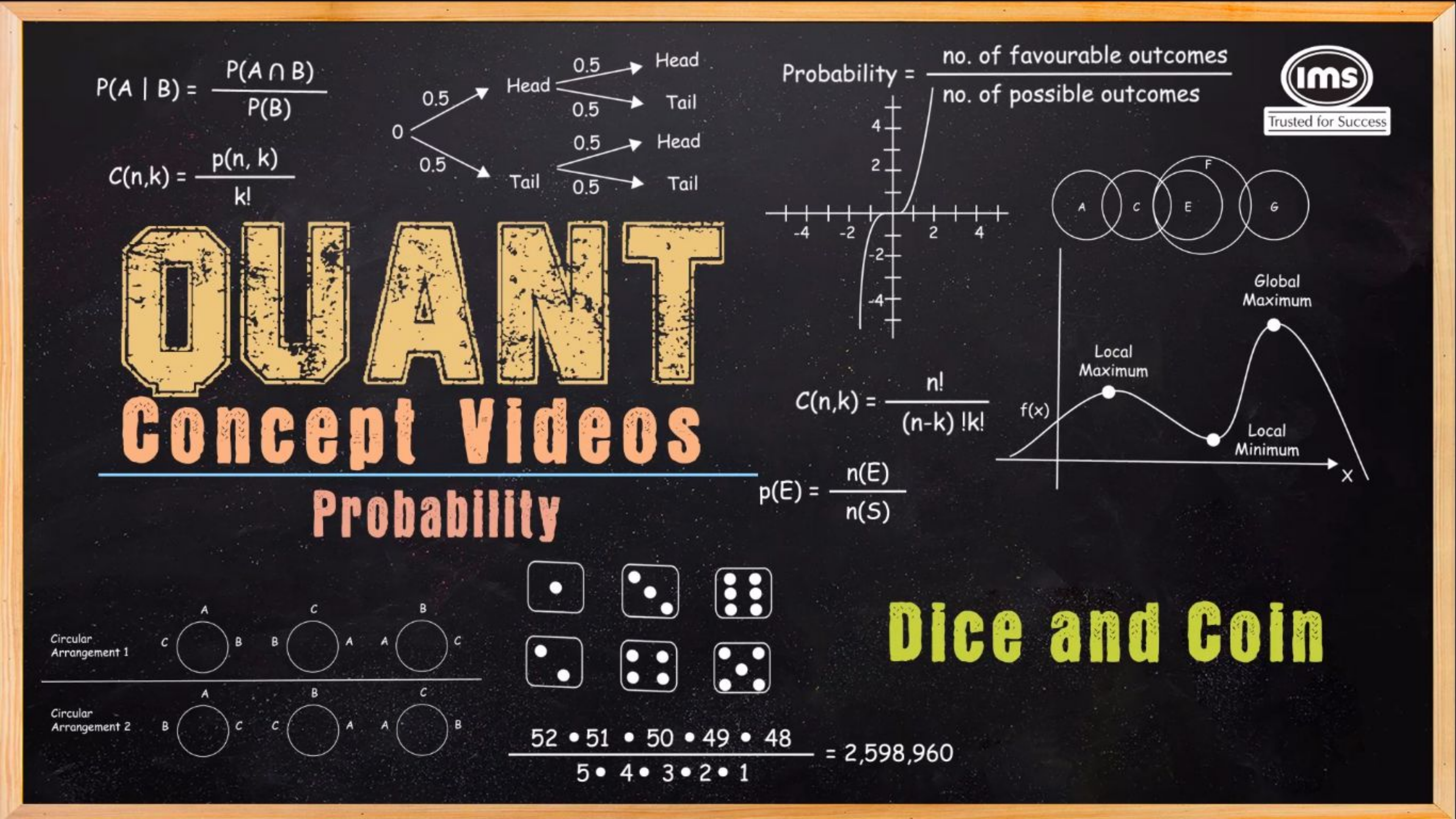
Independent Events

When two events are independent, the probability of both occurring is the product of the probabilities of the individual events.

If events A and B are independent, then the probability of both A and B occurring is:

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$


QUANT
Concept Videos
Probability
Dice and Coin
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
0
0.5
Head
0.5
Head
0.5
Tail
0.5
Tail
0.5
Head
0.5
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
IMS
Trusted for Success
A
C
E
F
G
Global Maximum
Local Maximum
Local Minimum
f(x)
x
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Probability

Dice is rolled

Single Die

Sample Space = {1, 2, 3, 4, 5, 6}

$\therefore$ n(S) = 6

Probability

P(1)	P(2)	P(3)	P(4)	P(5)	P(6)
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

P(Even Number) = $\frac{3}{6}$

P(Prime Number) = $\frac{3}{6}$

Probability

Dice is rolled

Two Dice

Sample Space = {(1, 1), (1, 2), (1, 3), ...

........., (6, 4), (6, 5), (6, 6)}

$\therefore$ n(S) = 36

Probability

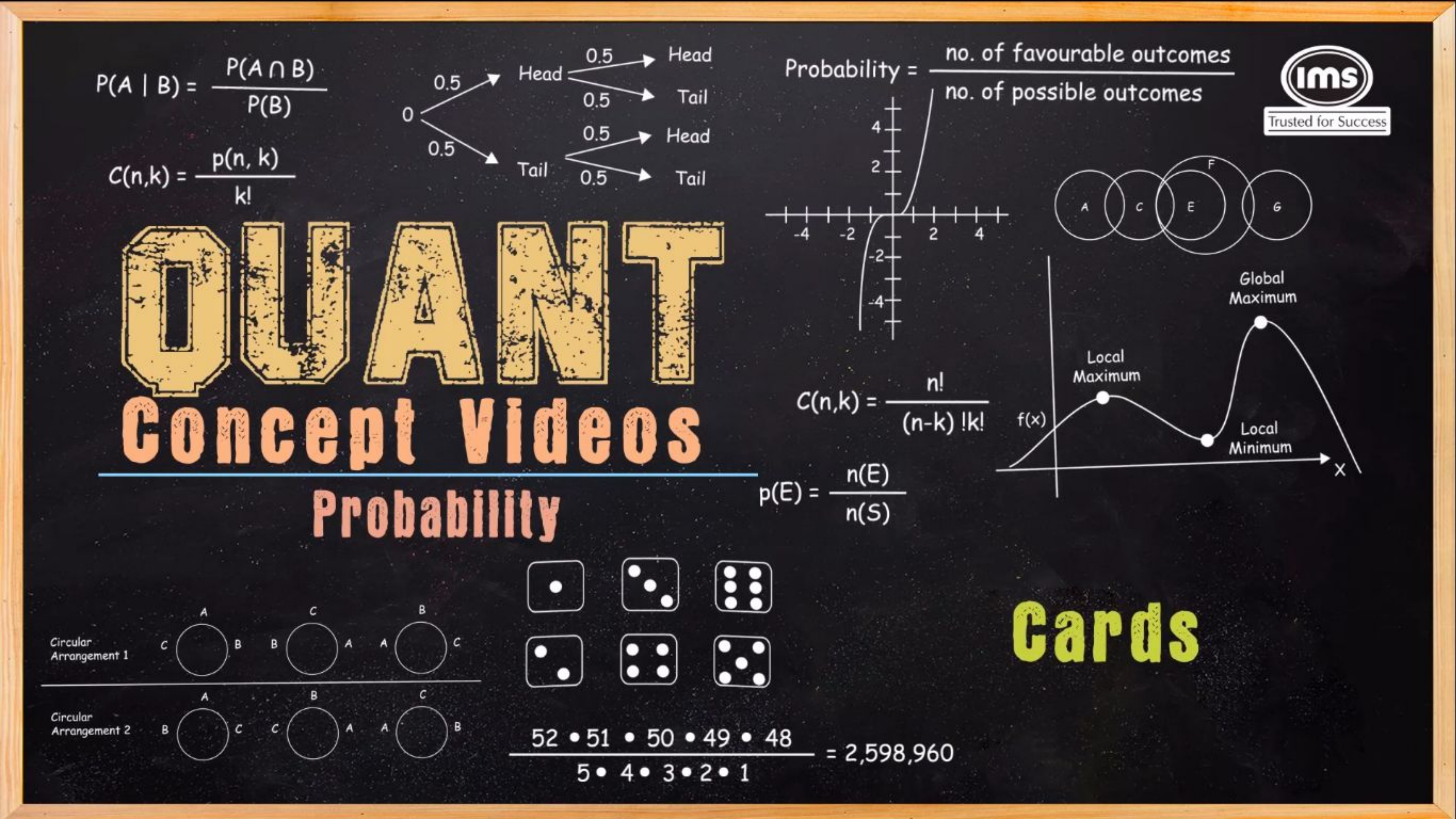
P(2)	P(3)	P(4)	P(5)	P(6)	P(7)	P(8)	P(9)	P(10)	P(11)	P(12)
$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

P(Sum less than equal to 10) = $1 - \left(\frac{3}{36}\right) = \frac{33}{36}$

Probability

Coin is tossed

Single Coin	2 Coins	3 Coins
Sample Space {H, T} n(S) = 2	Sample Space {H, H H, T T, H T, T} n(S) = 4	Sample Space {H, H, H H, H, T H, T, H H, T, T T, H, H T, H, T T, T, H T, T, T} n(S) = 8



$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
$C(n,k) = \frac{p(n, k)}{k!}$
0
0.5
Head
0.5
Head
0.5
Tail
0.5
Tail
0.5
Head
0.5
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
IMS
Trusted for Success
A
C
E
F
G
Global Maximum
Local Maximum
Local Minimum
f(x)
X
QUANT
Concept Videos
Probability
$C(n,k) = \frac{n!}{(n-k)!k!}$
$p(E) = \frac{n(E)}{n(S)}$
Circular Arrangement 1
Circular Arrangement 2
Cards
$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960$

Probability

Example

From a normal pack of cards, two cards are drawn. Find the probability that one of them is king and the other is queen if:

1. The two cards are drawn successively with replacement:

   $P = \frac{4}{52} \times \frac{4}{52} \times 2$ OR $P = \frac{8}{52} \times \frac{4}{52}$

2. The two cards are drawn successively without replacement:

   Method 1: $\frac{8}{52} \times \frac{4}{51} = \frac{8}{663}$

   Method 2: $\frac{{}^{4}C_{1} \times {}^{4}C_{1}}{{}^{52}C_{2}} = \frac{4 \times 4}{\frac{52 \times 51}{2}} = \frac{8}{663}$

Probability

Example

From a normal pack of cards, three cards are drawn one after the other without replacement. Find the probability that

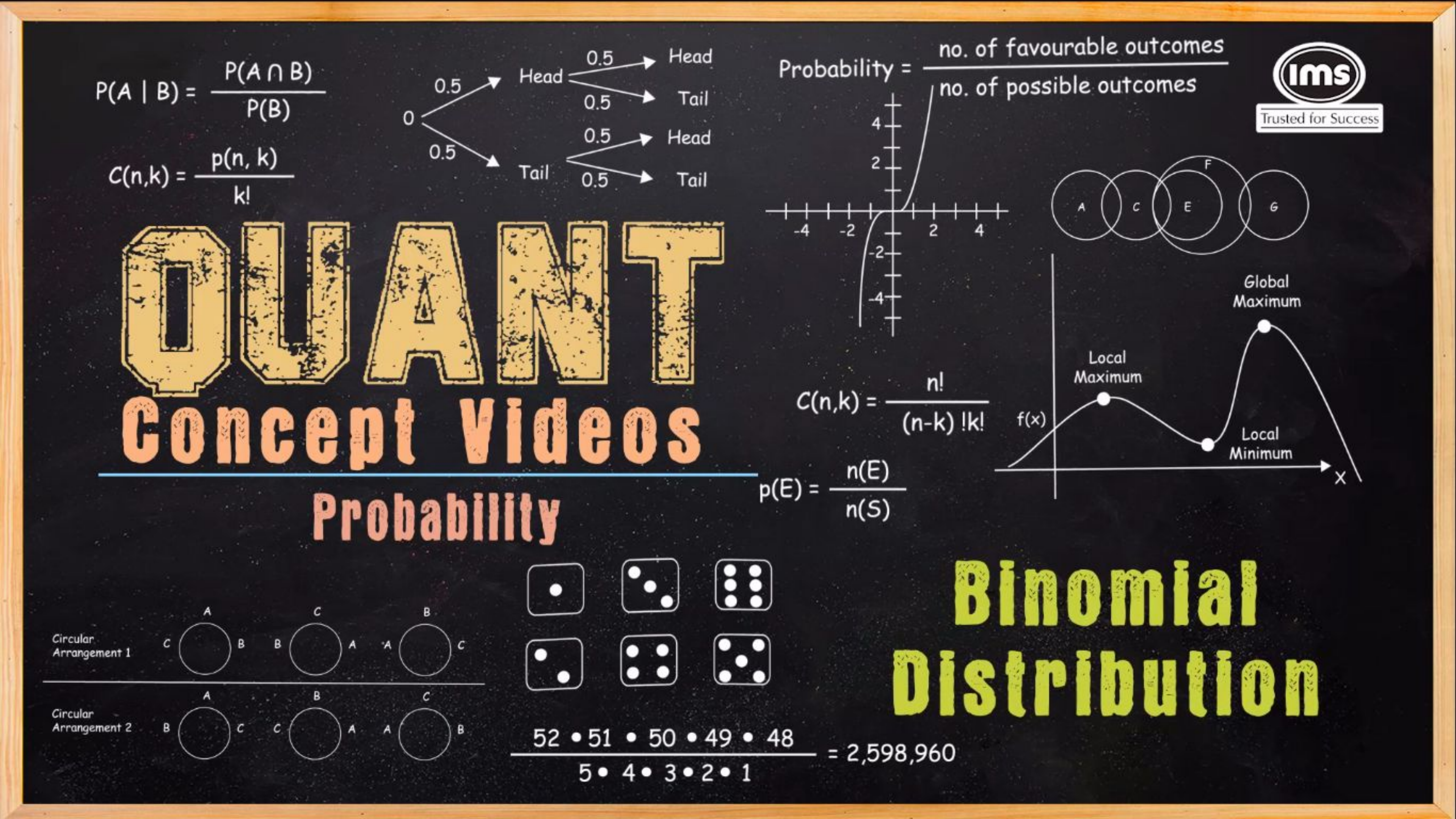
1. All 3 cards drawn are of the same suit

$$P = \frac{52}{52} \times \frac{12}{51} \times \frac{11}{50} \quad \text{OR} \quad P = \frac{{}^{4}C_1 \times {}^{13}C_3}{{}^{52}C_3}$$

2. All 3 cards drawn are of different suits

Method 1: $\frac{52}{52} \times \frac{39}{51} \times \frac{26}{50}$

Method 2: $\frac{{}^{4}C_3 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_3}$


QUANT
Concept Videos
Probability
Binomial Distribution
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
0
0.5
Head
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
IMS
Trusted for Success
A
C
E
F
G
Global Maximum
Local Maximum
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f(x)
X
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Probability

Binomial Distribution

If in an experiment involving ‘n’ events, ‘p’ is the probability of success and ‘q’ is the probability of failure, then the probability that there will be ‘x’ successes will be given by:

$P(x) = {}^{n}C_{x}\, p^{x}\, q^{(n-x)}$

Probability

Binomial Distribution

If an unbiased die is rolled 6 times,

What is the probability that the number ‘5’ appears four times?

$n = 6,\ x = 4,\ p = \frac{1}{6}$ and $q = \frac{5}{6}$

$P(4) = {}^{6}C_{4} \left(\frac{1}{6}\right)^{4} \left(\frac{5}{6}\right)^{2}$

What is the probability that the number 5 appears at least once?

P(At least Once) = 1 – P(None)

$n = 6,\ x = 0 \quad p = \frac{1}{6}$ and $q = \frac{5}{6}$

$= 1 - \left({}^{6}C_{0} \left(\frac{1}{6}\right)^{0} \left(\frac{5}{6}\right)^{6}\right)$

Probability

Odds in Favour and Against

Odds are 5 : 3 in favour of India winning the world cup:

Probability that India wins the world cup = $\frac{5}{8}$

Probability that India does not win the world cup = $\frac{3}{8}$

Odds are 2 : 9 against India winning the world cup:

Probability that India wins the world cup = $\frac{9}{11}$

Probability that India does not win the world cup = $\frac{2}{11}$

Probability

Example

Three archers A, B and C are shooting a target. Odds are 3 : 2 in favor of A hitting the target, 2 : 1 in favor of B hitting the target, and 3 : 1 against C hitting the target.

$P(A) = \frac{3}{5}$ $P(B) = \frac{2}{3}$ $P(C) = \frac{1}{4}$

$P(A') = \frac{2}{5}$ $P(B') = \frac{1}{3}$ $P(C') = \frac{3}{4}$

1. What is the probability that all three of them would hit the target?

   $P(A) \times P(B) \times P(C) = \frac{3}{5} \times \frac{2}{3} \times \frac{1}{4}$

Probability

Example

2. What is the probability that the target will not be hit?

$$P(A') \times P(B') \times P(C') = \frac{2}{5} \times \frac{1}{3} \times \frac{3}{4} = \frac{1}{10}$$

3. What is the probability that the target will be hit?

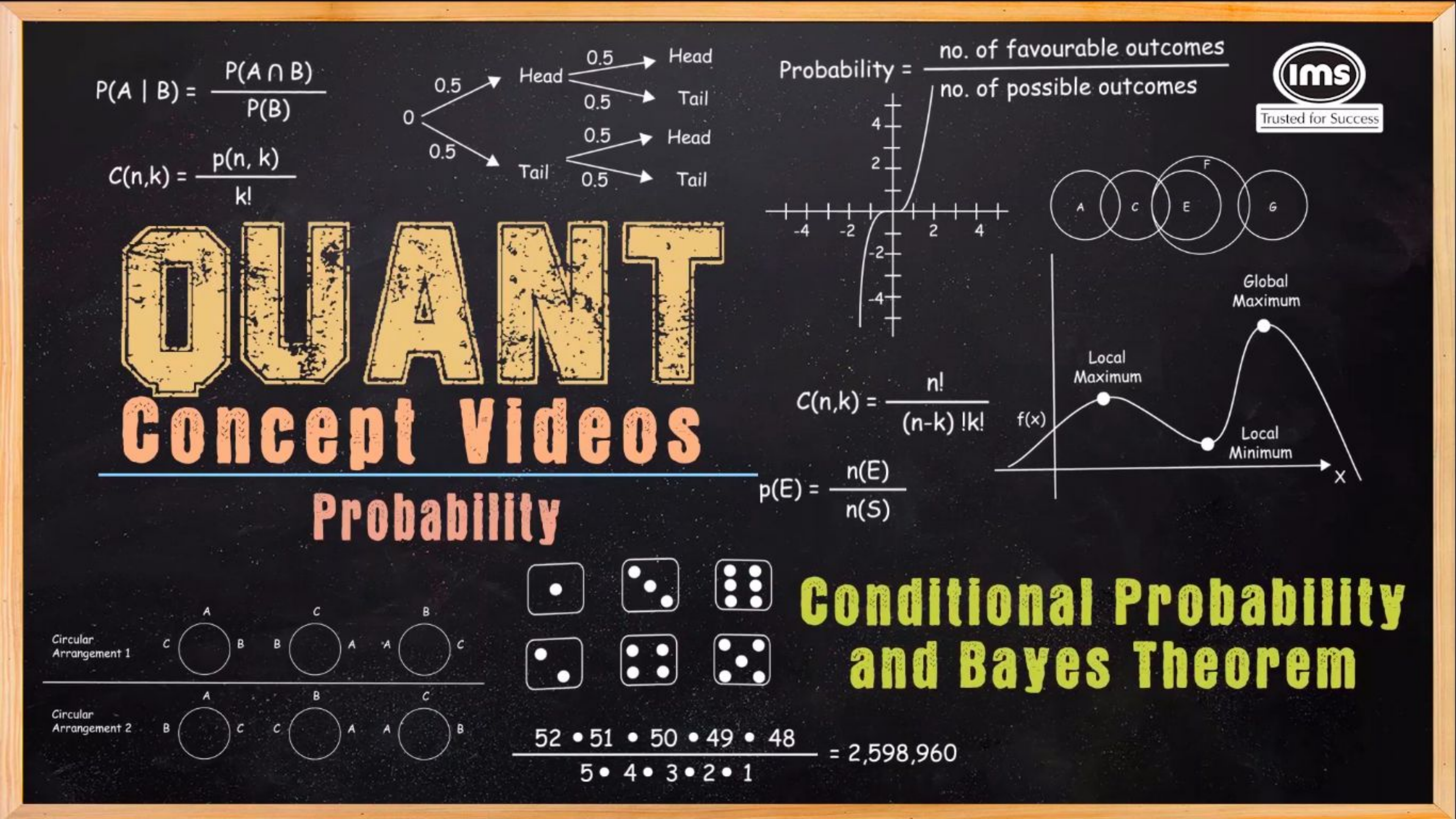
$$= 1 - P(A') \times P(B') \times P(C')$$

$$= 1 - \left(\frac{2}{5} \times \frac{1}{3} \times \frac{3}{4}\right) = \frac{9}{10}$$

4. What is the probability that exactly 2 of them will hit the target?

$$P(A) \times P(B) \times P(C') + P(A) \times P(B') \times P(C) + P(A') \times P(B) \times P(C)$$

$$= \frac{3}{5} \times \frac{2}{3} \times \frac{3}{4} + \frac{3}{5} \times \frac{1}{3} \times \frac{1}{4} + \frac{2}{5} \times \frac{2}{3} \times \frac{1}{4} = \frac{5}{12}$$



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Concept Videos
Probability
Conditional Probability and Bayes Theorem
IMS
Trusted for Success
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
Probability = no. of favourable outcomes / no. of possible outcomes
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Head
Tail
0.5
Global Maximum
Local Maximum
Local Minimum
f(x)
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Probability

Conditional Probability

There are 2 urns. The first urn contains 5 red balls and 6 black balls and the second one contains 6 red and 5 black balls. One ball is randomly selected and transferred from the first urn to the second. Then, a ball is drawn from the second urn.

1. What is the probability that the ball drawn from the second urn is red in colour?

   $P(R_1) \times P(R_2/R_1) + P(B_1) \times P(R_2/B_1)$

   $$\frac{5}{11} \times \frac{7}{12} + \frac{6}{11} \times \frac{6}{12} = \frac{71}{132}$$

Probability

Conditional Probability

2. What is the probability that the ball drawn from the second urn is NOT of the same colour as the one which is transferred?

$$P(R_1) \times P(B_2/R_1) + P(B_1) \times P(R_2/B_1)$$

$$\frac{5}{11} \times \frac{5}{12} + \frac{6}{11} \times \frac{6}{12} = \frac{61}{132}$$

Probability

Bayes' theorem

Example

In a workshop, 80% screws manufactured are blue screws and 20% are red screws. Out of the blue screws manufactured, 10% screws are faulty while out of the red screws manufactured 5% screws are faulty. Calculate:

1. The probability that a randomly selected screw is faulty
2. The probability that a randomly selected faulty screw is blue

Probability

Bayes' theorem

Solution

P(Blue) = 0.8

P(Faulty / Blue)

= 10% of 0.8 = 0.08

P(Red) = 0.2

P(Faulty / Red) = 5% of 0.2 = 0.01

	Blue	Red	Total
Faulty	0.08	0.01	0.09
Non - Faulty	0.72	0.19	0.91
Total	0.80	0.20	1.00

1. The probability that a randomly selected screw is faulty

   = 0.09

Probability

Bayes' theorem

Solution

P(Blue) = 0.8

P(Faulty / Blue)

= 10% of 0.8 = 0.08

P(Red) = 0.2

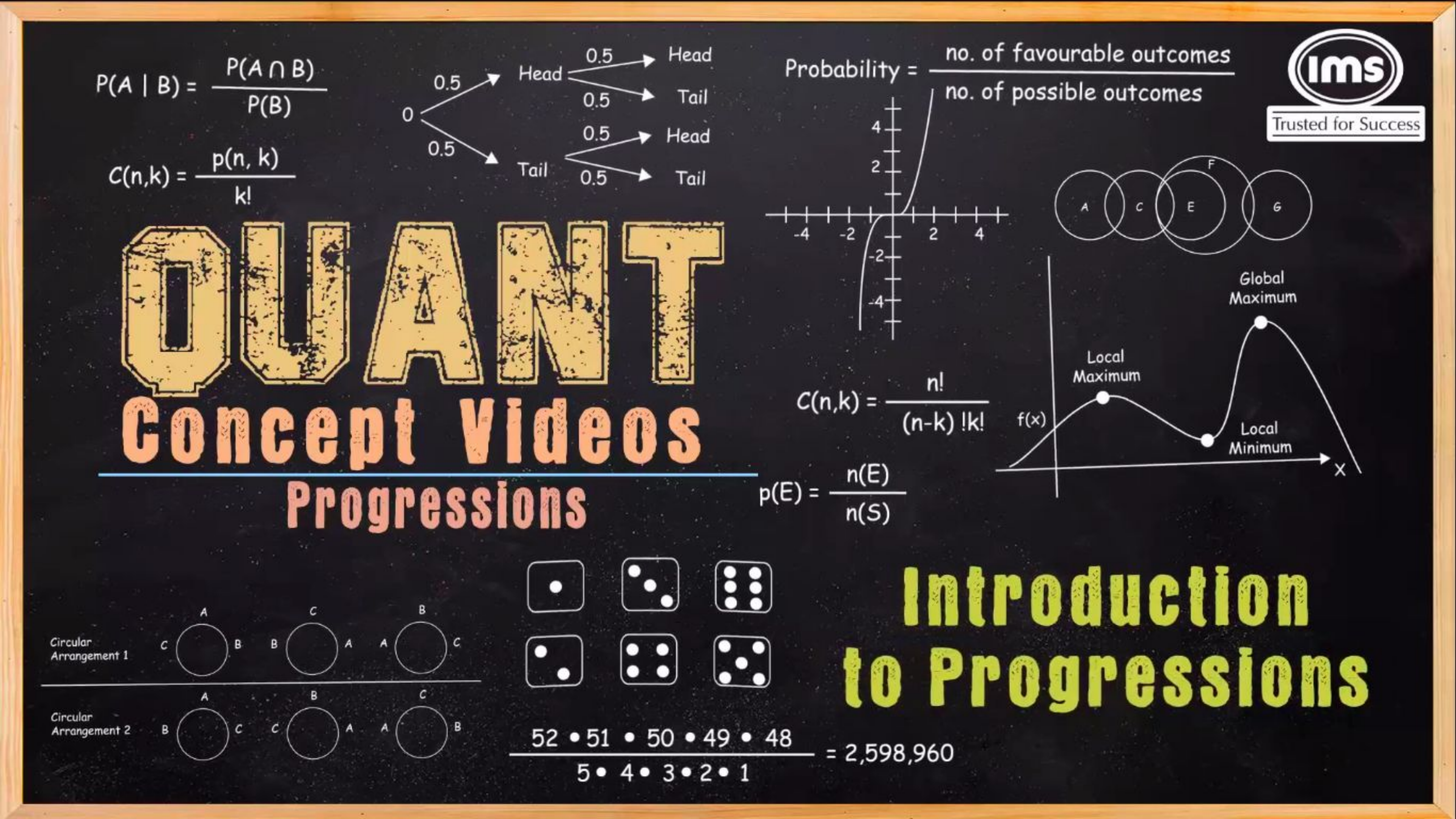
P(Faulty / Red) = 5% of 0.2 = 0.01

	Blue	Red	Total
Faulty	0.08	0.01	0.09
Non - Faulty	0.72	0.19	0.91
Total	0.80	0.20	1.00

2. The probability that a randomly selected faulty screw is blue

$$P(A/B) = \frac{P(A) \times P(B/A)}{P(A) \times P(B/A) + P(A') \times P(B/A')}$$

$$P(\text{Blue / Faulty}) = \frac{0.1 \times 0.8}{0.1 \times 0.8 + 0.05 \times 0.2} = \frac{8}{9}$$



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Concept Videos
Progressions
Introduction to Progressions
IMS
Trusted for Success
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
Probability = no. of favourable outcomes / no. of possible outcomes
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Head
Tail
Global Maximum
Local Maximum
Local Minimum
f(x)
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Progressions

Introduction

A **Progression** is a series that advances in a logical and predictable pattern.

Three Types of Progression

Arithmetic Progression (AP)

Geometric Progression (GP)

Harmonic Progression (HP)

Progressions

Types of Progressions

Arithmetic Progression, sequence of numbers such that the difference of any two successive terms of the sequence is constant.

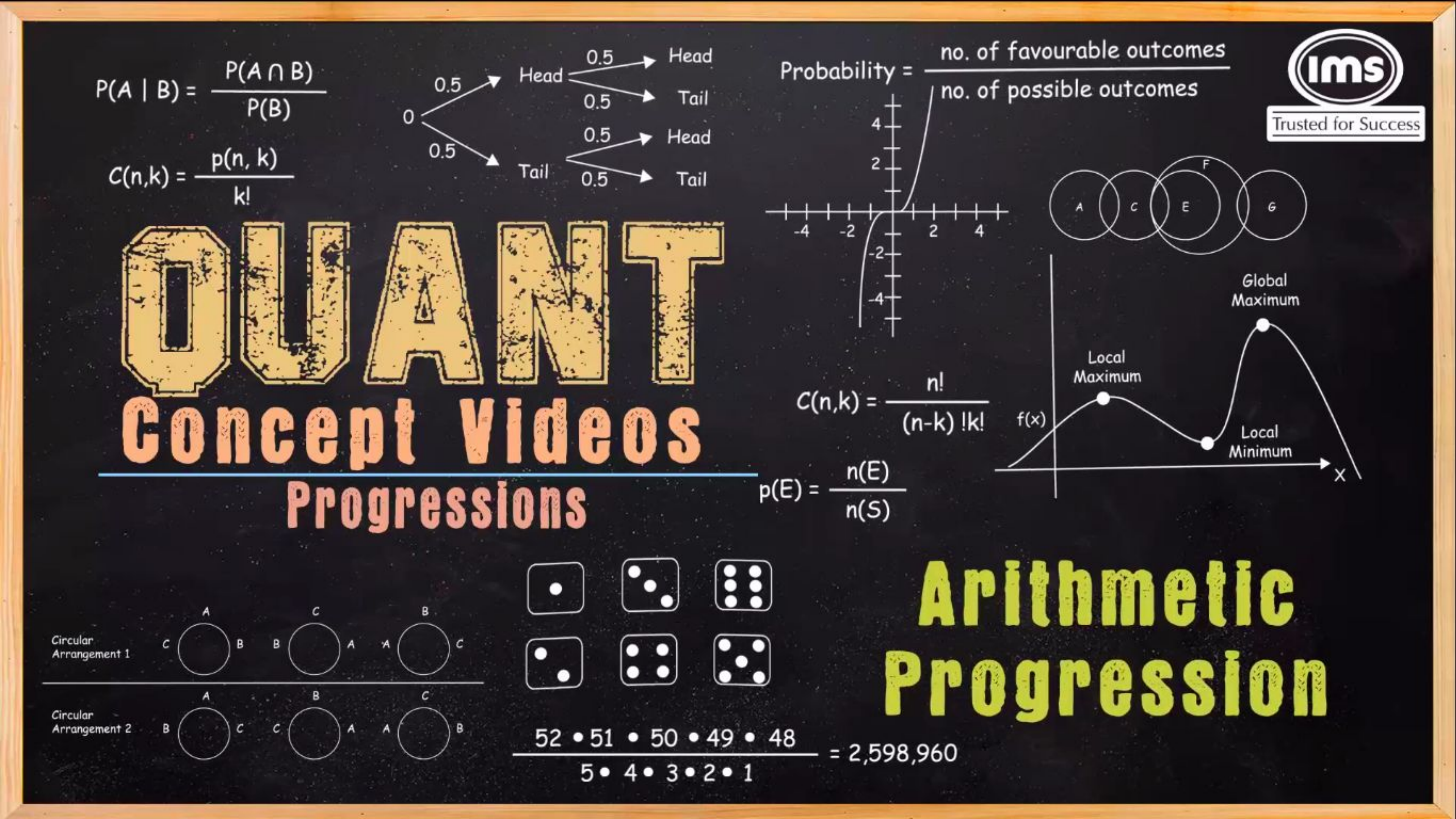
2, 5, 8, 11, …..

Geometric Progression, sequence of numbers such that the ratio of any two successive terms of the sequence is constant.

2, 6, 18, 54, …..

Harmonic Progression, sequence of numbers such that their reciprocals form an arithmetic progression.

$\frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \ldots..$


QUANT
Concept Videos
Progressions
Arithmetic Progression
IMS
Trusted for Success
P(A | B) = P(A ∩ B) / P(B)
C(n,k) = p(n, k) / k!
Head
Tail
Probability = no. of favourable outcomes / no. of possible outcomes
C(n,k) = n! / (n-k) !k!
p(E) = n(E) / n(S)
Global Maximum
Local Maximum
Local Minimum
f(x)
Circular Arrangement 1
Circular Arrangement 2
52 • 51 • 50 • 49 • 48 / 5 • 4 • 3 • 2 • 1 = 2,598,960

Progressions

Arithmetic Progression

The difference of any two successive terms of the sequence is constant.

First Term $(T_1) = a$

Common difference = d

$a, a + d, a + 2d, a + 3d, \ldots\ldots\ldots$

$\therefore \; T_n = a + (n - 1)d$

$\therefore \; T_{100} = a + 99d$

Progressions

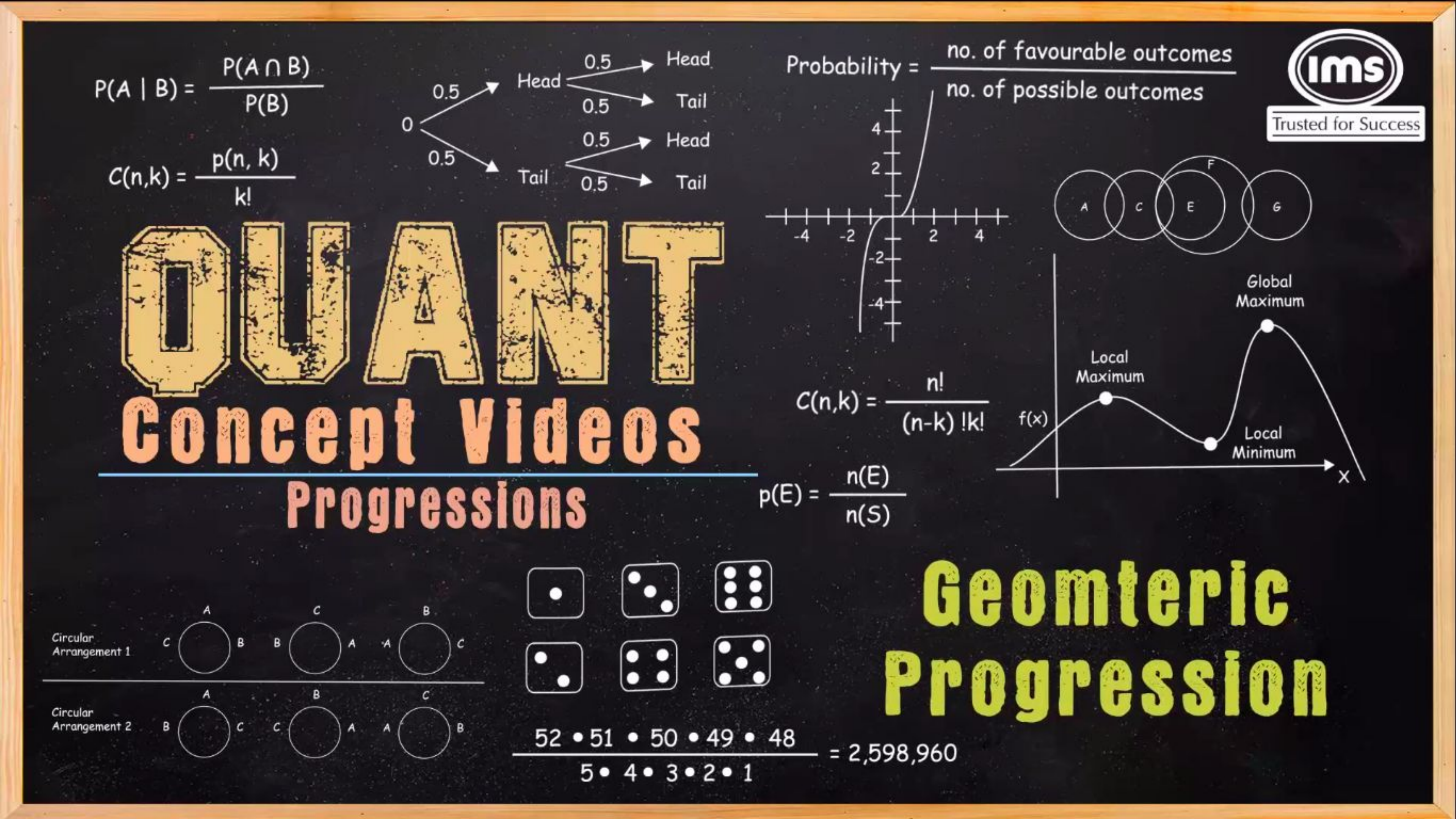
Arithmetic Progression

1, 4, 7, 10, 13, 16, 19, 22

$\therefore$ Sum of this series = 23 × 4

$$\therefore S_n = \frac{n}{2}[T_1 + T_n]$$

$$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$$



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Progression
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Progressions

Geometric Progression

The ratio of any two successive terms of the sequence is constant.

First Term (T_1) = a

Common Ratio = r

a, ar, ar^2, ar^3, ………

$\therefore T_n = ar^{n-1}$

$\therefore T_{20} = ar^{19}$

Progressions

Geometric Progression

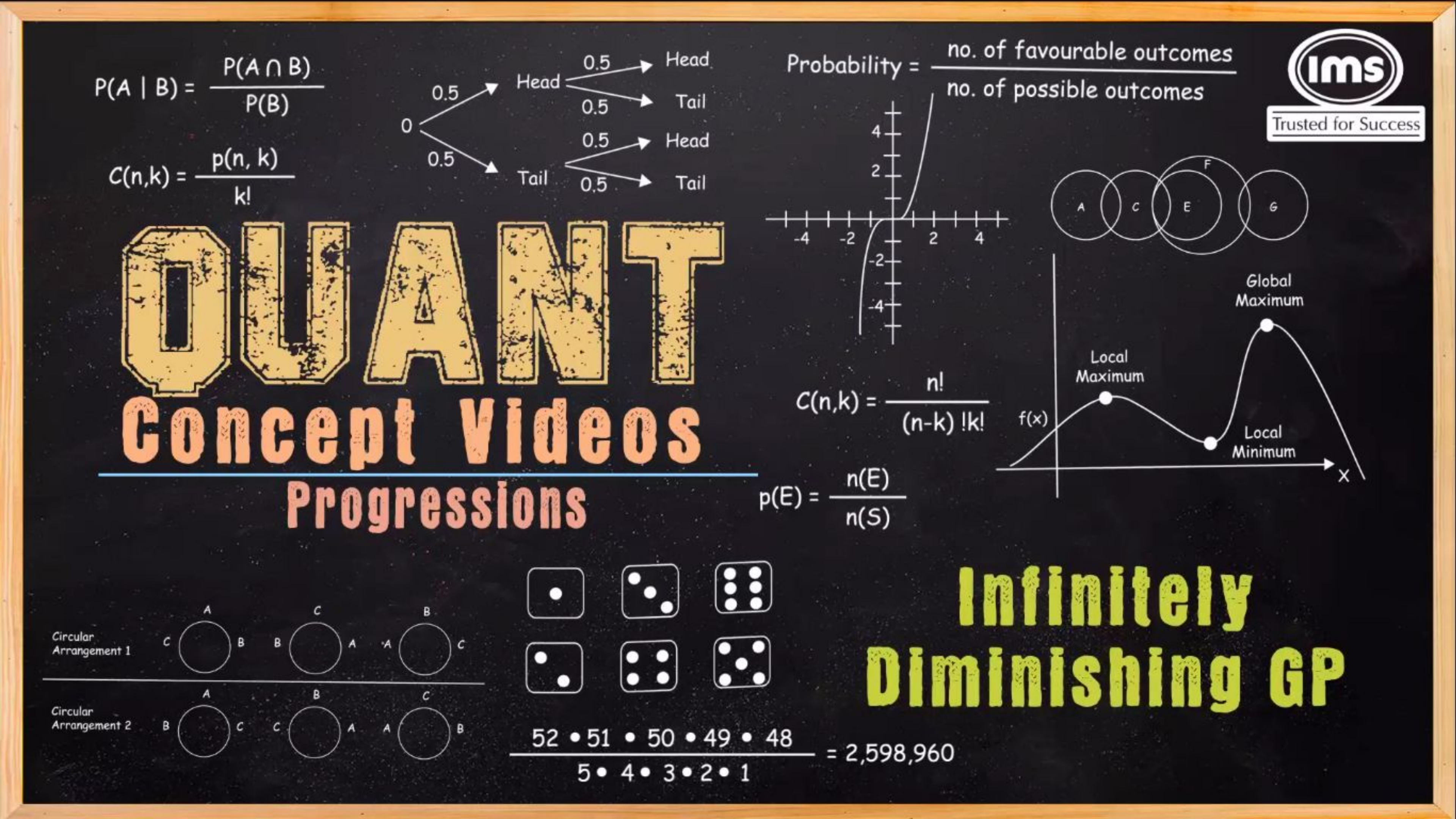
1 , 3, 9, 27, 81, 243, 729, 2187………

$$\boxed{S_n = a\left[\frac{r^n - 1}{r - 1}\right]}$$

$$\therefore \quad S_8 = 1\left[\frac{3^8 - 1}{3 - 1}\right]$$

$$= \frac{6561 - 1}{2}$$

$$= 3280$$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Progressions
Infinitely
Diminishing GP
IMS
Trusted for Success
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Progressions

Infinitely Diminishing Geometric Progression

$$486,\ 243,\ \frac{243}{2},\ \frac{243}{4},\ \frac{243}{8},\ \ldots\ldots\ldots$$

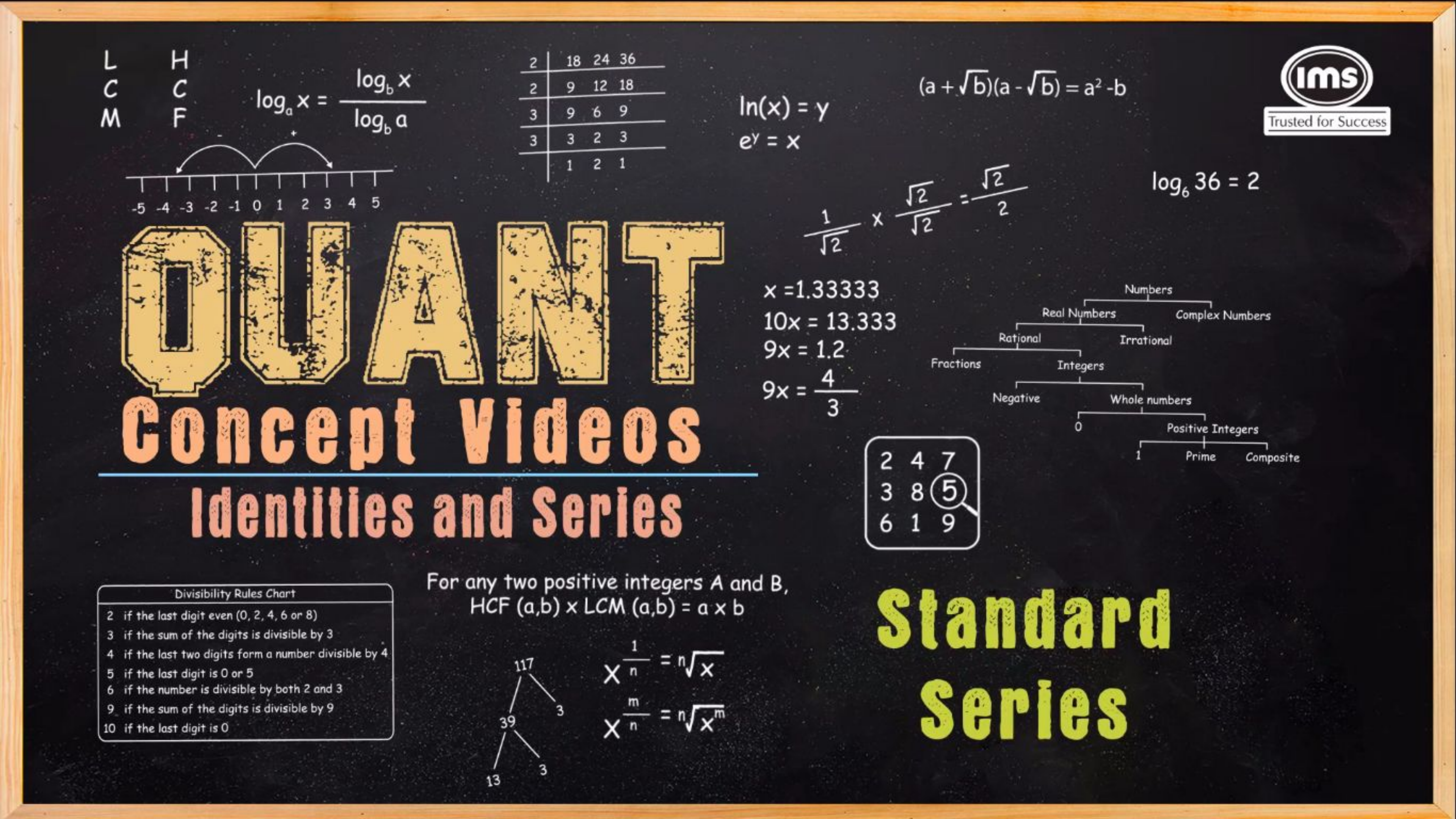
$$\boxed{S_\infty = \frac{a}{1 - r}}$$

$$\therefore\ S_\infty = \frac{486}{1 - \frac{1}{2}}$$

$$= \frac{486}{\frac{1}{2}}$$

$$= 486 \times \frac{2}{1}$$

$$= 972$$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Concept Videos
Identities and Series
Standard Series
IMS
Trusted for Success
Divisibility Rules Chart
2 if the last digit even (0, 2, 4, 6 or 8)
3 if the sum of the digits is divisible by 3
4 if the last two digits form a number divisible by 4
5 if the last digit is 0 or 5
6 if the number is divisible by both 2 and 3
9 if the sum of the digits is divisible by 9
10 if the last digit is 0
For any two positive integers A and B,
HCF (a,b) x LCM (a,b) = a x b
Numbers
Real Numbers
Complex Numbers
Rational
Irrational
Fractions
Integers
Negative
Whole numbers
0
Positive Integers
1
Prime
Composite

Standard Series

1. Sum of first ‘n’ natural numbers = $1 + 2 + 3 + 4 \ldots\ldots n$ terms

$$= \frac{n(n+1)}{2}$$

2. Sum of first ‘n’ odd numbers = $1 + 3 + 5 + 7 \ldots\ldots n$ terms

$$= n^2$$

3. Sum of first ‘n’ even numbers = $2 + 4 + 6 + 8 \ldots\ldots n$ terms

$$= n(n+1)$$

Standard Series

4. Sum of the squares of first 'n' natural numbers

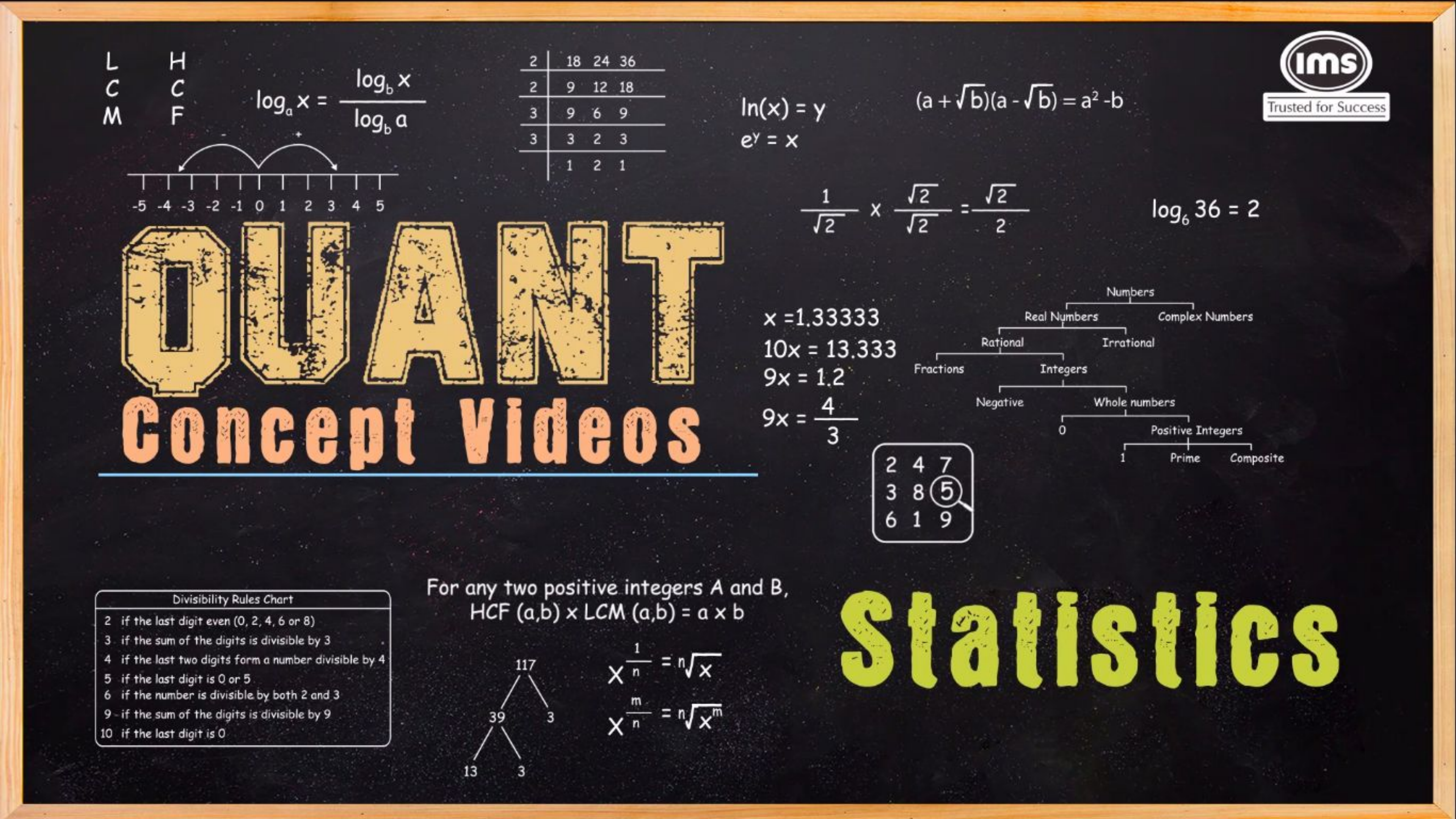
$$= 1^2 + 2^2 + 3^2 + \ldots\ldots n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

5. Sum of the cubes of first 'n' natural numbers

$$= 1^3 + 2^3 + 3^3 + \ldots\ldots n^3$$

$$= \left[\frac{n(n+1)}{2}\right]^2$$



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Concept Videos
Statistics
Trusted for Success
L C M
H C F
$\log_a x = \frac{\log_b x}{\log_b a}$
-5 -4 -3 -2 -1 0 1 2 3 4 5
$\ln(x) = y$
$e^y = x$
$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$
$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
$\log_6 36 = 2$
x =1.33333
10x = 13.333
9x = 1.2
$9x = \frac{4}{3}$
Numbers
Real Numbers
Complex Numbers
Rational
Irrational
Fractions
Integers
Negative
Whole numbers
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Positive Integers
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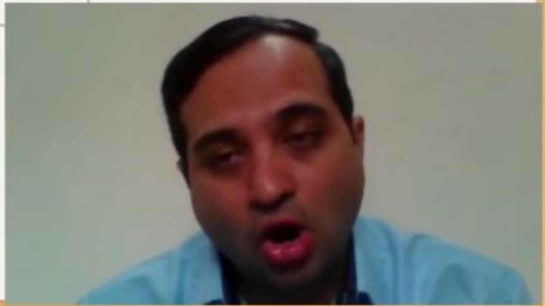
5 if the last digit is 0 or 5
6 if the number is divisible by both 2 and 3
9 if the sum of the digits is divisible by 9
10 if the last digit is 0
For any two positive integers A and B,
HCF (a,b) x LCM (a,b) = a x b
117
39
3
13
3
$x^{\frac{1}{n}} = \sqrt[n]{x}$
$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Simple Average / Arithmetic Mean

$$\frac{\text{Sum of all Values}}{\text{Number of Values}}$$

Arithmetic Mean of two numbers lies between the two numbers.



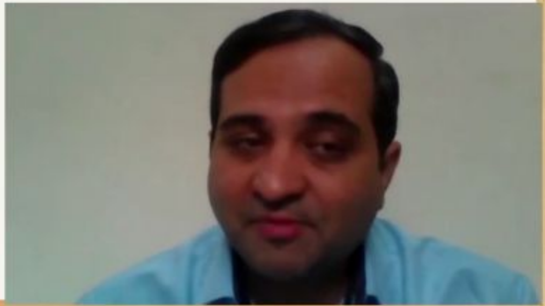
Example 1

The arithmetic mean of 5, 9, 11, 24, 31 is :

1) 11 2) 31 3) 24 4) 16

$$\frac{5 + 9 + 11 + 24 + 31}{5} = \frac{80}{5} = 16$$

Option (4)



Example 2

The mean for the set of observations given below is 6.2. Find x.

x_i	7	8	5	9	3
f_i	1	2	X	2	2

$$\text{Mean } = \bar{x} = \frac{\Sigma f_i \, xi}{\Sigma f_i}$$

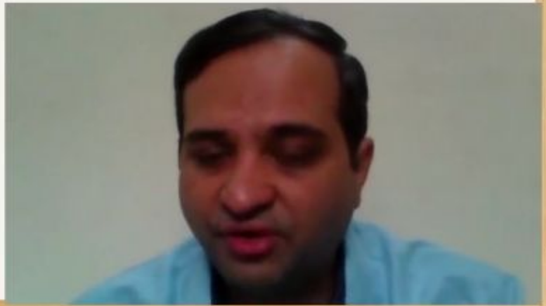
$$6.2 = \frac{7 + 16 + 5x + 18 + 6}{7 + x}$$

$$\therefore 6.2\,(7 + x) = 47 + 5x$$

$$\therefore 43.4 + 6.2x = 47 + 5x$$

$$\therefore 1.2x = 3.6$$

$$\therefore x = 3$$



Mean

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Geometric Mean

$$\sqrt{a \times b}$$

$$\sqrt[3]{a \times b \times c}$$

$$\sqrt[4]{a \times b \times c \times d}$$

Harmonic Mean

$$\frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a + b}$$

$$\frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}} \checkmark$$

$$\frac{4}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$$

Relation between Means

AM GM HM

$$\frac{\cancel{a+b}}{\cancel{2}} \qquad \sqrt{ab} \qquad \frac{\cancel{2}ab}{\cancel{a+b}}$$

$AM \times HM = (GM)^2$ For 2 numbers

$AM \geq GM \geq HM$ For more than 2 numbers

Median/Mode/Range

Steps to find median.

1. Arrange the numbers in ascending or descending order.
2. If there are odd number of numbers Median is the middle number and if there are even number of numbers Median is the average of the middle two numbers.

Mode

Mode is the number that occurs most frequently in a given set of numbers.

Range

Range is defined as greatest measurement minus the least measurement.

Example 3

The median for the set of observations 5, 13, 17, 8, 9, 11, 21, 17 is ?

1) 4.5 2) 8.5 3) 12 4) 17

5 8 9 11 13

12

Option (3)

Example 4

For the data given below compute the mean and median

13, 15, 18, 19, 14, 20, 23, 17, 21

$$\text{Mean} = \bar{x} = \frac{\text{sum of observation}}{\text{number of observation}}$$

$$= \frac{13 + 15 + 18 + 19 + 14 + 20 + 23 + 17 + 21}{9} = \frac{160}{9}$$

For median, arranging the data in ascending order
13, 14, 15, 17, 18, 19, 20, 21, 23

Here, $n = 9$ (odd)

$$\therefore \text{Median} = \left(\frac{9+1}{2}\right)^{th} \text{term} = 5^{th} \text{ term} = 18$$

Example 6

The mean and the mode of a data are 30 and 21 respectively. The median of the data is?

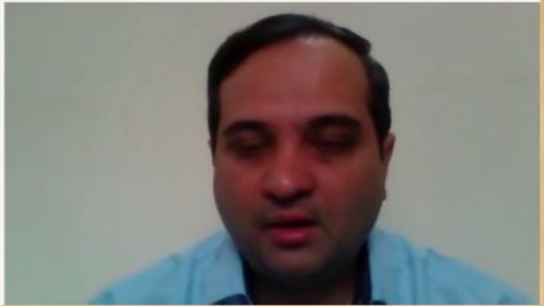
1) 25.5   2) 26   3) 27   4) 25

Mode = 3 (Median) – 2 (Mean)

21 = 3 (Median) – 2 (30)

Median = 27

Option (3)



Example 7

Find the range for the data:

14, 13, 12, 19, 21, 17, 13, 9, 16, 17, 14, 18

1) 12 2) 18 3) 21 4) 14

Range = 21 – 9

12

Option (1)

Example 8

Find the mean deviation from the median for the following data:

33, 27, 31, 42, 35, 28

1) 3 2) 0 3) 3.67 4) 4

27 28 31 33

Median 32

Mean deviation $\dfrac{1+5+1+10+3+4}{6} = \dfrac{24}{6}$

4

Option (4)

Example 9

Find the mean deviation from the mean for the following data:

32, 27, 31, 42, 35, 31

1) 3 2) 0 3) 3.67 4) 4.2

Mean $\dfrac{32 + 27 + 31 + 42 + 35 + 31}{6} = 33$

Mean deviation $\dfrac{1 + 6 + 2 + 9 + 2 + 2}{6} = \dfrac{22}{6}$

3.67

Option (3)

Example 10

Find the standard deviation of 5, 6, 8 and 9

1. Calculate the arithmetic mean of the set of values. 7

2. Find the square of the differences between each value and the mean.

   4 1 1 4

3. Add the squared differences and divide it by the number of values to get the average of the squared differences. This value is also called as **Variance**.

   $$\frac{4 + 1 + 1 + 4}{4} = 2.5$$ Variance

4. Find the non negative square root of the value obtained in step 3 above.

   Standard Deviation = $\sqrt{2.5}$

Example 11

Find the standard deviation for the data.

12, 8, 9, 11, 10

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) 2 4) 3

Mean 10

Variance $\dfrac{4 + 4 + 1 + 1 + 0}{5} = 2$

$\therefore$ Standard Deviation $\sqrt{2}$

Option (1)

Example 12

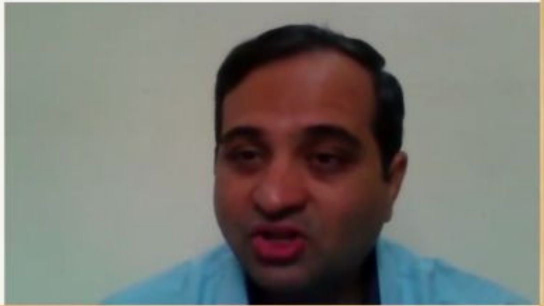
Find the variance for the data 3, 5, 6, 7, 9, 18

1) 8.33 2) 48 3) 23.33 4) 24.3

Mean 8

Variance $\frac{25 + 9 + 4 + 1 + 1 + 100}{6} = \frac{140}{6}$

23.33



Example 13

1, 2, 2, 3		4, 5, 5, 6		3, 6, 6, 9	
Mean	2	Mean	5	Mean	6
Median	2	Median	5	Median	6
Mode	2	Mode	5	Mode	6
Range	2	Range	2	Range	6
SD	$\sqrt{\frac{1}{2}}$	SD	$\sqrt{\frac{1}{2}}$	SD	$3\sqrt{\frac{1}{2}}$

Example 14

The arithmetic mean of the series $3x_1 + 1, 3x_2 + 1, 3x_3 + 1, \ldots\ldots, 3x_n + 1$ is a.

Then the arithmetic mean of $\frac{2x_1}{3}, \frac{2x_2}{3}, \frac{2x_3}{3}, \ldots\ldots\ldots, \frac{2x_n}{3}$ is

1) $\frac{2}{3}a$ 2) $\frac{3}{2}a$ 3) $\frac{2a-1}{3}$ 4) $\frac{2a-2}{9}$

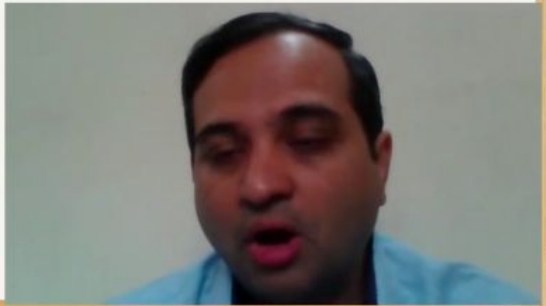
$$AM\,(3x_1 + 1, 3x_2 + 1, 3x_3 + 1, \ldots\ldots, 3x_n + 1) = a$$

$$AM\,(3x_1, 3x_2, 3x_3, \ldots\ldots, 3x_n) = a - 1$$

$$AM\,(x_1, x_2, x_3, \ldots\ldots, x_n) = \frac{a-1}{3}$$

$$AM\,(2x_1, 2x_2, 2x_3, \ldots\ldots, 2x_n) = \frac{2a-2}{3}$$

$$AM\left(\frac{2x_1}{3}, \frac{2x_2}{3}, \frac{2x_3}{3}, \ldots\ldots\ldots, \frac{2x_n}{3}\right) = \frac{2a-2}{9}$$



Example 15

The variance of a, b, c, d is 25 then the variance of 3a, 3b, 3c, 3d is ?

1) 75 2) 225 3) 15 4) 625

Variance (a, b, c and d) 25

Standard Deviation (a, b, c and d) 5

Standard Deviation (3a, 3b, 3c and 3d) 15

Variance (3a, 3b, 3c and 3d) 225

Example 16

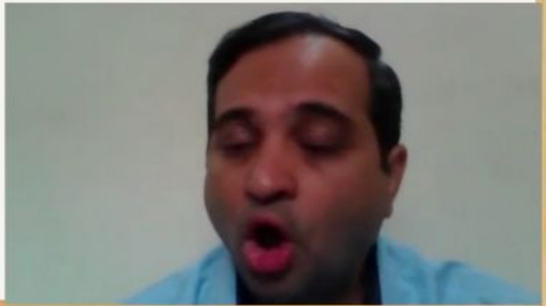
In a certain set of observations, the coeffcient of variation is 0.8 and the mean is 20, then find S.D.

$$C.V. = \frac{\sigma}{x} \times 100$$

CV = Coefficient of variation

σ = Standard Deviation

x = Mean



Variance

Variance is the average of the squared deviations from mean.

If $x_1, x_2, x_3, \ldots. x_n$ are the observations and their mean is $\bar{x}$, then

$$\text{Variance (V)} = \frac{\sum_{i=1}^{n} (xi - \bar{x})^2}{n}$$

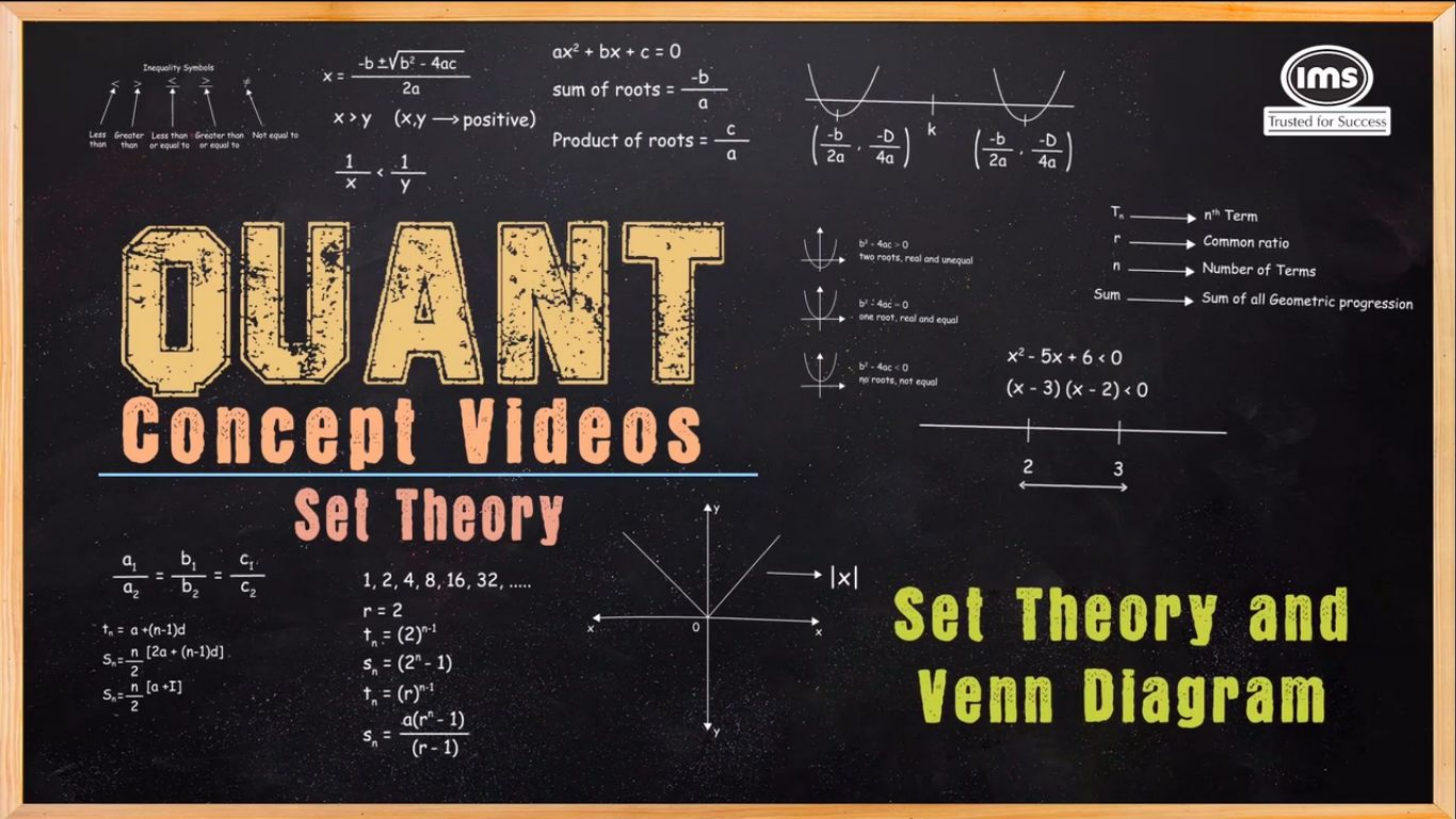
$$\text{Variance (V)} = \frac{\sum_{i=1}^{n} x_i^2}{n} - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2$$

Standard Deviation

The positive square root of the variance is called Standard Deviation (σ). is the average of the squared deviations from mean.

If $x_1, x_2, x_3, \ldots. x_n$ are the observations and their mean is $\bar{x}$, then

$$\sigma = +\sqrt{\text{Variance}} = +\sqrt{\frac{\sum_{i=1}^{n}(xi-\bar{x})^2}{n}} = +\sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n} - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2}$$



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Set Theory
Set Theory and Venn Diagram
IMS
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Inequality Symbols
Less than
Greater than
Less than or equal to
Greater than or equal to
Not equal to
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x > y$ ($x, y \longrightarrow$ positive)
$\frac{1}{x} < \frac{1}{y}$
$ax^2 + bx + c = 0$
sum of roots = $\frac{-b}{a}$
Product of roots = $\frac{c}{a}$
$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$ k $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$
$b^2 - 4ac > 0$ two roots, real and unequal
$b^2 - 4ac = 0$ one root, real and equal
$b^2 - 4ac < 0$ no roots, not equal
$T_n \longrightarrow$ n^{th} Term
$r \longrightarrow$ Common ratio
$n \longrightarrow$ Number of Terms
Sum $\longrightarrow$ Sum of all Geometric progression
$x^2 - 5x + 6 < 0$
$(x - 3)(x - 2) < 0$
2 3
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$t_n = a + (n-1)d$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_n = \frac{n}{2}[a + l]$
1, 2, 4, 8, 16, 32, ...
$r = 2$
$t_n = (2)^{n-1}$
$s_n = (2^n - 1)$
$t_n = (r)^{n-1}$
$s_n = \frac{a(r^n - 1)}{(r - 1)}$
$|x|$

Set Theory

Trusted for Success

A set is a well-defined collection of objects.

- Elements of a set are unordered, synonymous terms.
- Sets are usually denoted by capital letters.
- Elements of a set are represented by small letters.

$A = \{a, b, c, d -$.

Set Theory

Standard Sets

N : the set of all natural numbers

$N = \{1, 2, 3, 4, \text{------}\}$

Z : the set of all integers

$Z = \{\text{----}, -3, -2, -1, 0, \underline{1, 2, 3, \text{----}}\}$

Z+ : the set of positive integers

Q : the set of all rational numbers

Q+ : the set of positive rational numbers

R : the set of all real numbers

R+ : the set of positive real numbers

Set Theory

Cardinality/Order of a set

No. of elements in a set.

Eg : $A = \{1,2,3\}$

$\therefore |A| = 3$

Set Theory

Empty Set

A set which doesn't contain any element is called the empty set or null set, denoted by symbol φ or { }. ϕ $\{ \}$

e.g. : let R = {x : 1< x < 2, x is a natural number} $\{ \}$

$N = \{1, 2, 3, \ldots\ldots\}$

Finite & Infinite Sets

A set which is empty or consist of a definite numbers of elements is called finite otherwise, the set is called infinite.

e.g. : let K be the set of the days of the week. Then K is finite $K = \{M, T, W, \ldots S\}$

let R be the set of points on a line. Then R is infinite R

Set Theory

Equal Sets

Given two sets K & R are said to be equal if they have exactly the same elements

e.g. : let K = {1,2,3,4} & R = {1,2,3,4}

then K = R

$K = \{1, 2, 3\}$ $R = \{a, b, c\}$

$\boxed{R \subseteq K}$

$A = \{1, 2\}$

$B = \{1, 2\}$

$A = \{2\}$ ✓

Subsets

A set R is said to be subset (⊆) of a set K if every element of R is also an element K. Denoted as R ⊆ K

PROPER SUBSET : If A ⊆ B but not equal to B then A is called as a proper subset of B. Denoted as A ⊂ B

Set Theory

Trusted for Success

Power Set

The set of all subsets of a given set is called power set of that set.

The collection of all subsets of a set K is called the power set of denoted by P(K).

In P(K) every element is a set. If K= {1,2,3}

P(K) = {φ, {1}, {2}, {3}, {1,2}, {1,3},{2,3},{1,2,3}}

$\boxed{|P(K)| = 2^n}$ where n is the no of elements in set K.

No of Proper subsets of K = |P(K)| - 1 = 7

A = { }

B = {1}

Set Theory

Trusted for Success

Universal Set

Real

Universal set is set which contains all object, including itself.

e.g. : the set of real number would be the universal set of all other sets of number.

NOTE : excluding negative root (imaginary) (complex)

Set Theory

Operation on Sets

$A = \{1, 2, 3\}$

Union of Sets : the union of two sets A and B is the set C which consist of all those elements which are either in A or B or in both.

Some Properties Of The Operation Of Union

1) A U B = B U A (commutative law)

2) (A U B) U C = A U (B U C) (associative law)

3) A U φ = A (law of identity element)

4) A U A = A (idempotent law)

5) U U A = U (law of U)

Set Theory

Operation on Sets

Intersection of Sets : The Intersection of two sets A and B is the set C which consist of only the common elements which are in both A and B.

Some Properties Of The Operation Of Intersection

1) $A \cap B = B \cap A$ (commutative law)

2) $(A \cap B) \cap C = A \cap (B \cap C)$ (associative law)

3) $\Phi \cap A = \Phi$, $U \cap A = A$ (law of Φ and U)

4) $A \cap A = A$ (idempotent law)

5) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (distributive law)

Set Theory

Complement of Sets

Let $U = \{1, 2, 3\}$ ✓

$U = \{1, 2, 3\}$

Now the set of all those element of U which doesn't belong to A will be called as A complement (A').

$A = \{1, 2\}$ ✓ $\therefore A' = \{3\}$ ✓

Properties of Complements of Sets

1) Complement laws :
   a) $A \cup A' = U$
   b) $A \cap A' = \Phi$

$U = \{a, b, c\}$

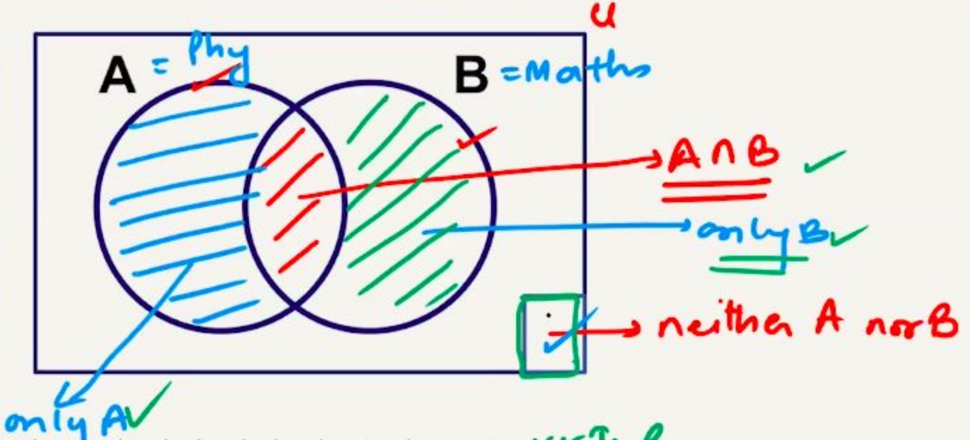
$\Phi = \{\ \}$

$(\Phi)' = \{a, b, c\}$

2) De Morgan's laws :
   a) $(A \cup B)' = A' \cap B'$
   b) $(A \cap B)' = A' \cup B'$
3) Laws of double complementation : $(A')' = A$
4) Laws of empty set and universal set : $\Phi' = U$ & $U' = \Phi$

Set Theory

For Two Variables (venn diag)



$|A \cup B| = |A| + |B| - |A \cap B|$ → vv Imp.

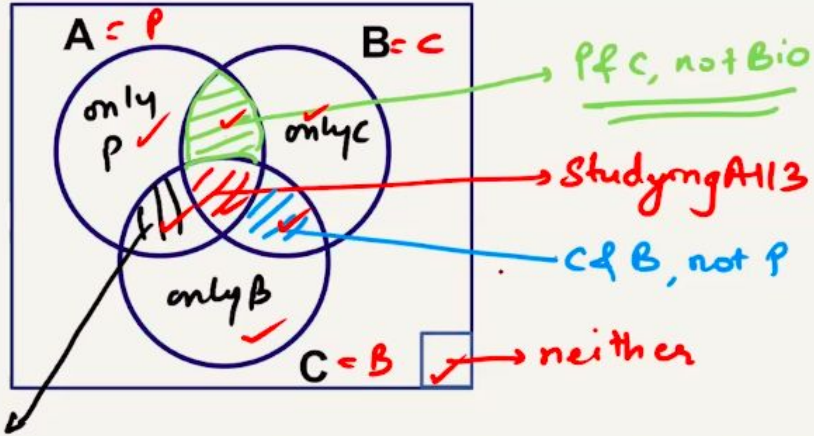
[In order to get the universe, we will also need to add those who does not belong to any category]

$A \cup B = U - \{A \cup B\}'$

Set Theory

For Three Variables

20 study P & C



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

only A + only B + only C + ~~P~~ (A, B, C') . . .

Set Theory

IMS
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Example

In a class of 50 students, 30 students play cricket, 29 play hockey and

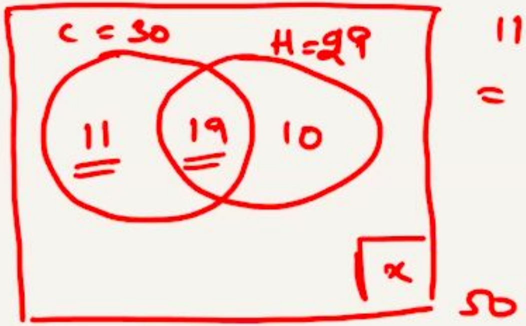
19 students play both the games. $(A \cup B)$

(a)How many students play at least one of the two games. $= 40$ ✓

(b)How many students play exactly one of the two games. $11 + 10 = 21$ ✓

(c)How many students play neither of the two games. $x = 50 - 40$

$= \boxed{10}$ ✓



$11 + 19 + 10$

$= 40 \ (A \cup B)$